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A DICTIONARY

OF

APPLIED PHYSICS

EDITED BY

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K.C.B., D.Sc., F.R.S.

IN FIVE VOLUMES

VOL. I

MECHANICS—ENGINEERING—HEAT

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PREFACE

DURING the past twenty-five years the applications of Physics to Industry have grown enormously. The National Physical Laboratory was opened in 1900, while Universities and Technical Colleges have multiplied, and recent years have seen the growth of the Department of Scientific and Industrial Research, with its Research Associations in many fields, its studentships, and its skilled Research workers.

Meanwhile, the results of the labours of the past are, for the most part, scattered in the *Proceedings* of learned Societies or stored in the brains of the active workers to whose efforts they are due.

To find out what are the latest methods of Calorimetry, what exactly is known about the laws of Friction, how far has the theory of the Steam Engine advanced, what are the principles on which methods of accurate gauging or of the determination of the many factors which come into the lay-out of a big electrical plant, the design of a Dynamo, or the methods of Pyrometry are based, means a long search in Libraries and, not infrequently, a futile journey to some place where it is hoped the wished-for information may be found.

The Science of Aeronautics, the Design of Optical Instruments, the Methods of Metallurgy, the Construction of Clocks, Telescopes or Microscopes, the Laws of Music and Acoustics are all based on Physics.

The manufacturer who is concerned with these and, indeed, with countless other subjects must know, not perhaps all that has been done—that would be too heavy a task—but where he may find the latest and most accurate information on the subject with which he is mainly concerned. This it has been the object of the *Dictionary of Applied Physics* to give. Applied Physics is a wide subject and the task has been a heavy one.

The Dictionary will appear in five volumes of 800-1000 pages each, and, as will be seen from the names of some of the principal contributors, the Editor has been fortunate in securing the help of those most competent to write on each subject. His thanks are due, in the first place, to these colleagues, without whose cordial help the Dictionary could not have been produced. He is also indebted to a number of Scientific Societies whose Councils have allowed use of illustrations from their *Proceedings* to be freely made. Among these should be mentioned in particular the Royal Society, the Institution of Mechanical Engineers, and the Institution of Electrical Engineers. The same help has been readily afforded by a number of Publishers.

It is clear that, with so large a range of subjects, any individual worker will, probably, be concerned mainly with one branch, and, with this in view, the volumes have been arranged, as far as possible, in subjects. To obtain information as to the latest advances of Applied Electricity it will not be necessary to purchase the sections of the Dictionary dealing with Aeronautics or Meteorology. The arrangement in each volume is alphabetical, but, at the same time, it has been thought best to deal with each main subject—for example, the Thermodynamics of the Steam Engine—in a continuous article; references are given, each in its own alphabetical position, to the headings of the various sections of an article and to the more important subjects which it includes.

R. T. G.

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Calorimetry, Method of Mixtures.
Calorimetry, Methods based on Change of State.
Calorimetry, Quantum Theory.
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Pyrometry, Total Radiation.
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— A —

Absolute Scale of Temperature (Kelvin). If a perfectly reversible heat-engine takes in a quantity of heat Q_1 at temperature T_1 and rejects Q_2 at temperature T_2, then $Q_1/T_1 = Q_2/T_2$ provided the temperatures T_1 and T_2 are measured on the absolute thermodynamic scale. Hence the ratio of two temperatures on that scale is equal to the ratio of the heat taken in to the heat rejected by any perfectly reversible engine working between those temperatures. See "Thermodynamics," §§ (17), (22); "Engines, Thermodynamics of Internal Combustion," § (7).

Absolute Zero, Definitions of, on "Gas" and "Work" Scales. See "Thermodynamics," § (4).

Absorption Dynamometers. See "Dynamometers," § (2).

Absorption of Radiation as affecting the Readings of Radiation Pyrometers. See "Pyrometry, Total Radiation," § (19).

Abutment Pumps:
  Fixed. See "Air-pumps," § (25).
  Movable. See *ibid.* § (14).

Acceleration Images. See "Kinematics of Machinery," § (4) (iv.).

Accumulators, Hydraulic. See "Hydraulics," § (55).

Adiabatic Change. A change in the volume and pressure of a body carried out reversibly in such a way that no heat is allowed to pass to or from the body. See also "Thermodynamics," §§ (15), (38).

Adiabatic Equation for a Perfect Gas. See "Engines, Thermodynamics of Internal Combustion," § (20); "Thermodynamics," § (15).

Adiabatic Expansion of a Fluid. See "Thermodynamics," § (38).

Adiabatic and Isothermal Changes. See "Engines, Thermodynamics of Internal Combustion," § (3); "Thermodynamics," §§ (15), (16).

Aerodynamic Pumps. See "Air-pumps," § (26).

Aerodynamic Tacheometer: For measuring number of revolutions per unit time by means of air pressure differences. See "Meters," § (6), Vol. III.

Aero-engine, The Rolls-Royce "Eagle." See "Petrol Engine, The Water-cooled," § (6) (i.).

Aerostatic Pumps, Theory of. See "Air-pumps," § (8).

Air, Constituents of, separated by Fractional Distillation. See "Gases, Liquefaction of," § (2).

Air, Index of Refraction of, used as secondary standard of temperature in the range above 500° C. See "Temperature, Realisation of Absolute Scale of," § (91) (ii.).

Air, Specific Heat of:
  At high temperature. See "Gases, Specific Heat of, at High Temperatures."
  At 59° C. and various pressures, tabulated values obtained by Holborn and Jacob. See "Calorimetry, Electrical Methods of," § (16), Table X.
  Variation with pressure over the range ·1 to 1200 atmospheres, determined by Holborn and Jacob. See *ibid.* § (16).

Air (free from CO_2), Specific Heats of: tabulated values obtained by Scheel and Heuse. See "Calorimetry, Electrical Methods of," § (15), Table IX.

Air and other Gases, Specific Heat of, determined at room and low temperatures by the continuous flow electrical method, by Scheel and Heuse. See "Calorimetry, Electrical Methods of," § (15).

Air and Steam Meters, Calibration of. See "Meters for Measurement of Steam," § (5), Vol. III.

Air-compression Refrigerating Machines. See "Refrigeration," § (4).

Air-lift Pump. See "Hydraulics," II. § (45).

Air Meters. See "Coal-gas and Air Meters," Vol. III.

AIR-PUMPS

INTRODUCTION

§ (1) COMPRESSORS, EVACUATORS, BLOWERS.—An air- or gas-pump is a device whereby gas is transferred from a low-pressure vessel (L.P.V.) to a high-pressure vessel (H.P.V.). The term "vessel" includes the free atmosphere, and the term "gas" includes vapours. It is assumed, unless the contrary is stated, that the L.P.V. and H.P.V. are at the same temperature.

If the L.P.V. and H.P.V. are separated by a gas-tight partition, and if the gas is not a saturated vapour, the pump will diminish the pressure in the L.P.V. and increase it in the H.P.V.; it will act at the same time as a compressor and as an evacuator. In practice one of the two vessels is almost always maintained at atmospheric pressure, and variations of pressure in the other vessel alone are important. If this condition is fulfilled, a compressor may be defined as a pump of which the L.P.V. is at atmospheric pressure, an evacuator as one of which the H.P.V. is at atmospheric pressure.

The L.P.V. and H.P.V. are seldom completely separated, except in laboratory evacuators; there is a continual stream of gas from one to the other. If the energy required to produce this stream is comparable with the whole work done on the gas, the pump may be termed a "blower," or, if it is of one constructional type, a "fan." The distinction between pumps and blowers, though formally indefinite, is perfectly clear in practice. Blowers are usually, but not always, compressors, producing pressures greater than atmospheric. In blowers there can be no single and definite p_H or p_L, capable of general scientific definition; but there is usually some pair of places along the stream of gas passing through the blower at which it is obviously convenient to measure p_H and p_L. These places may be regarded for our purpose as constituting the H.P.V. and L.P.V.

§ (2) NOTATION.—Suffixes L and H denote quantities referring to the L.P.V. or H.P.V. Many of the formulae given will still be true if the suffixes L and H are interchanged; this feature is indicated by writing before them ("L or H").

p, p_L, p_H = pressure.
p_0 = initial pressure (the same for L.P.V. and H.P.V.).
$p_L{}^0$, $p_H{}^0$ = final pressures.
$\kappa = p_H{}^0/p_L{}^0$ = range.
Π = atmospheric pressure.
P = vapour pressure.
V_L, V_H = volumes of L.P.V. and H.P.V.
u_L, u_H = maximum and minimum volumes of "cylinder."
T_0, T_L, T_H = absolute temperature of atmosphere, L.P.V. and H.P.V.
m = mass of gas.
v = velocity.
S = volumetric speed.
W = work.
w = power.
ρ = density.
η = viscosity.
ϵ = friction coefficient.
$E = E_{\text{mech.}}$ = mechanical efficiency.
$E_{\text{vol.}}$ = volumetric efficiency.

§ (3) WORKING CHARACTERISTICS.—Pumps may be distinguished either according to their working characteristics or according to the principles on which the action depends. Of the working characteristics the following are the most important of those applicable to pumps of all types:

Range of Pressure.—If any pump be worked continuously between closed vessels, there will ultimately be established in them constant pressures, $p_H{}^0$, $p_L{}^0$. By the range of pressure is meant either (*a*) the ratio $p_H{}^0/p_L{}^0$, or (*b*) the difference $p_H{}^0 - p_L{}^0$. (*a*) is generally the more important quantity and will here be termed the "range," denoted by κ; for it is often approximately independent of the absolute values $p_H{}^0$, $p_L{}^0$. But it is never completely independent; for all pumps have a minimum below which they will not reduce p_L whatever is the value, above this limit, of p_H, and all have a maximum p_H, though it may be determined only by mechanical strength.

The range of a pump of any given type may be increased by working two or more similar pumps in series to form a "composite" pump, the L.P.V. of one being the H.P.V. of the next. In all important cases, the range of the composite pump is approximately or exactly the product of the ranges of the components. But a composite pump can also be built up of components of different types; no general statement can be made about the relation between the range of such a composite pump and those of its components.

§ (4) SPEED OF PUMPING.—The speed is the rate at which gas is transferred from the L.P.V. to the H.P.V. The amount of gas is usually estimated by its volume at the pressure of the L.P.V., whether the pump is a compressor or an evacuator. The speed so estimated is called the "volumetric speed," S, and is expressed in volume per unit time.

Measurements of S are usually made by observations of the change of p_H in a compressor or of p_L in an evacuator, the H.P.V. or L.P.V. being completely closed. If δp during the measurement is small compared with p_H or p_L, the gas may be regarded as perfect. Consequently for an evacuator

$$S = \frac{1}{\rho}\frac{dm}{dt} = \frac{V_L}{p_L}\frac{dp_L}{dt} = V_L\frac{d}{dt}(\log_e p_L). \quad . \quad (1)$$

In a compressor, p_L is constant and equal to Π. Therefore

$$S = \frac{V_H}{\Pi}\frac{dp_H}{dt}. \quad . \quad . \quad . \quad (2)$$

In blowers the volume involved in S is usually estimated at p_H. It is conveniently measured by some type of flow-meter placed in the outlet or inlet pipe. If the pressure at the point where the meter is placed differs considerably from p_H, a correction must be applied to the readings of the meter.

S is usually a function of p_H and p_L, as well as of the nature of the pump; but there are important exceptions. The range κ or the maximum difference of pressure $p_H{}^0 - p_L{}^0$ is given by a pair $(p_H{}^0, p_L{}^0)$ such that $S = 0$.

§ (5) The Efficiency.—Several kinds of efficiencies are recognised as applicable to pumps and blowers; of these the mechanical efficiency, or the ratio of the useful work done to the total work expended, is alone applicable to all types. Both terms of the ratio need further definition to rid them of ambiguity. The work expended is usually taken to mean either (*a*) the work expended on the gas in giving to it energy, compressive, kinetic, or thermal, or (*b*) the work supplied to the mechanism of which the pump consists, including that lost in friction of solid or liquid parts. The efficiency reckoned with (*a*) is often termed the "gas" efficiency; that reckoned with (*b*) the "over-all" efficiency.

In pumps, where the L.P.V. and H.P.V.'s are separate, the useful work is always taken to be that required to transfer the gas that has actually passed from the former to the latter. This work will be least if the transference is effected reversibly; if the L.P.V. and H.P.V. are at the same temperature, the reversible transference must be isothermal, and any change of temperature during the process involves the expenditure of more work. If the transference is reversible and isothermal, the work required to transfer a mass of gas between the atmosphere at constant pressure Π and a closed vessel, the pressure in which is changed by the transference from Π to p_1, is given by

$$W = \int_{\Pi}^{p_1} (\Pi - p)dV, \quad . \quad . \quad . \quad (3)$$

where $V = f(p)$ is the isothermal characteristic of the mass of the gas occupying the closed vessel at the pressure p_1. If the gas is perfect, (3) applied to an evacuator or compressor becomes

$$(\text{L or H}) \quad W = p_H V_H \int_{\Pi}^{p_H} (\Pi - p)d\left(\frac{1}{p}\right) \quad . \quad (4)$$

$$= p_H V_H \left(\log_e \frac{p_H}{\Pi} - 1 + \frac{\Pi}{p_H}\right). \quad (5)$$

If $\Pi - p_H = \delta p$ is small, (5) becomes

$$(\text{L or H}) \quad W = \tfrac{1}{2}\frac{(\delta p)^2}{\Pi}V_H. \quad . \quad (6)$$

If p_L is small, as in a high evacuator, it becomes

$$W = (\Pi - p_L)V_L. \quad . \quad . \quad (7)$$

In some text-books, the work done by the atmosphere is left out of account; the term in Π is omitted from (3), and the second and third terms in the bracket from (5). But since work is always done by or on the atmosphere in compressing or evacuating, the efficiencies reckoned without these terms would seem to have no practical significance.

In blowers the useful effect is usually estimated by the volume of gas produced at a given pressure. The work required to force a mass of gas from the atmosphere into a vessel in which the pressure is maintained constant and equal to p_H (by increasing the volume of the vessel as the gas enters) is given by

$$W = (p_H - \Pi)V, \quad . \quad . \quad . \quad (8)$$

where V is the volume of the gas at pressure p_H. Consequently, if S is the volumetric speed, and w the work done per second,

$$w = (p_H - \Pi)S. \quad . \quad . \quad . \quad (9)$$

The stream of gas issuing from a blower possesses kinetic energy. If the work expended in giving to it this energy is to be included as useful work, there must be added within the bracket in (9) the term $p_v = \frac{1}{2}\rho v^2$. It is often impossible practically to convert this kinetic energy into energy of any other form without stopping the flow which is the main purpose of the blower; accordingly the total efficiency, as it is called, reckoned from the relation

$$w = (p_H + p_v - \Pi)S \quad . \quad . \quad (9a)$$

is often misleading. But it may be noted that ideally it is always possible to reduce p_v to zero without changing S, and thus to "convert velocity into pressure"; for if the cross section of the stream is made infinitely large, an infinitely small v will give a finite S.

In addition to the mechanical efficiency, there is recognised for many pumps a quantity known as the volumetric efficiency. But since this quantity cannot be defined generally for all types of pump, it will be discussed in connection with those to which it applies.

§ (6).—The remaining working characteristics common to all pumps are less capable of precise measurement; but they are none the less important. They include simplicity and convenience, first cost and cost of maintenance, adaptation to available sources of power, and so on. When several types of pump are permissible, it is usually these characteristics rather than any measurable efficiency which determine the choice. They will be noticed in connection with particular types.

§ (7) Principles of Action.—For the detailed consideration of the various types of pump, it is more convenient to adopt a classification based upon the principles underlying the action. Here pumps fall into three great classes:

A. Aerostatic.
B. Aerodynamic.
C. Molecular or High-vacuum.

In an *aerostatic* pump the transference of gas is effected by forces that are at any instant in statical equilibrium. For any particular pump the range of pressure is independent of the speed of working within wide limits; the pump can be worked infinitely slowly without loss of range or of efficiency. (This statement is not strictly true when the viscosity of a lubricating liquid is used to secure gas-tightness; such pumps are dynamic, but not aerodynamic.) In all practical examples the statical forces are those due to compression, but those due to change of temperature might conceivably be used.

In an *aerodynamic* pump the forces on the gas are dynamical, and vary with the motion of the parts of the pump; they cease when the speed of working becomes infinitely small, so that the range and speed of the pump vanish together. These dynamical forces arise from the inertia or viscosity of the gas.

The distinction between the two classes can be expressed less formally, but perhaps more clearly, by saying that in the first class, but not in the second, the action is "positive" in the engineering sense; or that, while it is impossible to blow through a pump of the first class, it is possible to blow through one of the second.

In both these classes the forces are such as are associated with a continuous medium. In the third class the action is due to "forces" appreciable on the molecular but not on the molar scale. The class would properly be termed "*molecular*," but since that term has been appropriated to a particular member of it, the less scientific expression "*high-vacuum*" pumps will be used.

In addition to these three classes of pump, there are some methods of evacuation which scarcely satisfy the definition of pumping, but will be conveniently noticed briefly at the end of this article.

A. Aerostatic Pumps

§ (8) Range and Speed.—The working part is always a vessel (U) of variable volume, u. U is

(1) connected to the L.P.V. when its volume is a maximum u_L;
(2) disconnected from the L.P.V. and the volume decreased to the minimum u_H;
(3) connected to the H.P.V.;
(4) disconnected from the H.P.V. and the volume increased to u_L.

This ideal cycle is never attained in practice but forms the basis of any calculations. Even if the ideal cycle were attained, the general formulae giving the relation between p_H, p_L, p_0 after a number of cycles n would be extremely complicated. But if it is assumed that the gas is perfect, and that the transference is isothermal, the relation between $(p_H)_n$ and $(p_H)_{n+1}$, the values of p_H after the nth and $(n+1)$th cycles, is

$$\text{(L or H)}\quad (p_H)_{n+1} = \frac{(p_H)_n V_H + p_L u_L}{V_H + u_H}. \qquad (10)$$

In a compressor or evacuator we have from (10)

$$\text{(L or H)}\quad \frac{(p_H)_n}{\Pi} = \frac{u_L}{u_H} + \left(1 - \frac{u_L}{u_H}\right)\left(1 + \frac{u_H}{V_H}\right)^{-n}, \quad (11)$$

(11) is true whatever the ratio of u to the V's; if it is small and if N, the number of cycles per unit of time, is large, (11) becomes

$$\text{(L or H)}\quad \frac{p_H}{\Pi} = \frac{u_L}{u_H} - \left(\frac{u_L}{u_H} - 1\right)\exp.\left(-\frac{Nu_H}{V_H + u_H}t\right). \qquad (12)$$

From (12) and (1) we obtain for the volumetric speed of an evacuator

$$S = \frac{NV_L}{V_L + u_L}\left(u_L - \frac{\Pi}{p_L}u_H\right), \qquad (13)$$

and for that of a compressor

$$S = \frac{NV_H}{V_H + u_H}\left(u_L - \frac{p_H u_H}{\Pi}\right). \qquad (14)$$

The range of pressure is given by $S = 0$. Consequently

$$\text{(L or H)}\quad \frac{p_H{}^0}{\Pi} = \frac{u_L}{u_H}. \qquad (15)$$

The range, measured by the ratio of the pressures, is independent of the initial pressure and of the volumes of the L.P.V. and H.P.V.

These relations become very simple when u_H, the volume of the "dead space," is zero. Then (11)-(14) become

$$\frac{p_H}{\Pi} = 1 + \frac{nu_L}{V_H}\ \text{(compressor)}, \qquad (16)$$

$$\frac{p_{\mathrm{L}}}{\Pi}=\left(\frac{\mathrm{V}_{\mathrm{L}}}{\mathrm{V}_{\mathrm{L}}+u_{\mathrm{L}}}\right)^{n} \text{ (evacuator)}, \quad . \quad (17)$$

$$\mathrm{S}=\mathrm{N}u_{\mathrm{L}} \text{ (compressor)}, \quad . \quad . \quad (18)$$

$$\mathrm{S}=\mathrm{N}u_{\mathrm{L}}\frac{\mathrm{V}_{\mathrm{L}}}{\mathrm{V}_{\mathrm{L}}+u_{\mathrm{L}}} \text{ (evacuator)}. \quad . \quad (19)$$

Thus the volumetric speed of such a pump would be independent of the pressure against which it worked; its range would be infinite.

§ (9) VOLUMETRIC EFFICIENCY.—In no actual pump is the ideal cycle performed; the yield is always less than that given by (10)-(14), and *a fortiori* less than that given by (16)-(19) on the assumption of no dead space. The deficiency is due to incomplete "connection" and "disconnection" of the L.P.V. and H.P.V. with U and with each other, *i.e.* to leakage and to a failure to establish pressure equilibrium.

The comparison of an actual with an ideal pump is made in terms of the "volumetric efficiency" ($\mathrm{E}_{\mathrm{vol.}}$), which may be roughly defined as the ratio of the number of cycles in which an ideal pump would produce a given effect to the number of cycles in which the actual pump produces the same effect. The ideal pump is assumed to have the same u_{L} (*e.g.* cylinder volume) as the actual pump; it is also usually assumed to have no dead space u_{H}. This last assumption is not necessary, for the effect of the dead space can be readily calculated if the pump is otherwise ideal; but in the pumps for which the conception of volumetric efficiency is most important, u_{H} is always made as small as possible, and its magnitude is important in judging the excellence of the design.

The "effect" by which $\mathrm{E}_{\mathrm{vol.}}$ is estimated must be defined. It is usually either (1) the attainment of a given p_{L}/Π or p_{H}/Π, or (2) the transference of a given quantity of gas with a fixed p_{L} or p_{H}. In either case, $\mathrm{E}_{\mathrm{vol.}}$ is a function of p_{L} or p_{H}, and the value of this pressure must be stated. For a given p_{L} or p_{H}, $\mathrm{E}_{\mathrm{vol.}}$ will not in general be the same for (1) and for (2).

If (1) is adopted, and if n is the number of cycles in which the actual pump establishes the assigned p_{H}/Π or p_{L}/Π, then from (16) and (17)

$$\mathrm{E}_{\mathrm{vol.}}=\frac{(p_{\mathrm{H}}-\Pi)\mathrm{V}_{\mathrm{H}}}{nu_{\mathrm{L}}\Pi} \text{ (compressor)} \quad . \quad . \quad (20)$$

$$=\frac{\log(p_{\mathrm{L}}/\Pi)}{n\log[\mathrm{V}_{\mathrm{L}}/(u_{\mathrm{L}}+\mathrm{V}_{\mathrm{L}})]} \text{ (evacuator)}. \quad (21)$$

If (2) is adopted, and if N is the number of cycles per unit time required for a given volumetric speed S, we have from (18) or (19)

$$\mathrm{E}_{\mathrm{vol.}}=\frac{\mathrm{S}}{\mathrm{N}u_{\mathrm{L}}} \text{ (compressor)}, \quad . \quad (22)$$

$$\mathrm{E}_{\mathrm{vol.}}=\frac{\mathrm{S}(\mathrm{V}_{\mathrm{L}}+u_{\mathrm{L}})}{\mathrm{N}u_{\mathrm{L}}\mathrm{V}_{\mathrm{L}}} \text{ (evacuator)}. \quad . \quad (23)$$

If the H.P.V. or L.P.V. is not at atmospheric temperature, (20)-(23) must be corrected by the substitution of $\mathrm{V}_{\mathrm{H}}/\mathrm{T}_{\mathrm{H}}$, $\mathrm{V}_{\mathrm{L}}/\mathrm{T}_{\mathrm{L}}$, $u_{\mathrm{L}}/\mathrm{T}_{0}$ for V_{H}, V_{L}, u_{L}.

§ (10) OTHER CHARACTERISTICS.—The advantage which aerostatic pumps possess over other types lies in the great range which can be obtained with them. They are, therefore, well suited for the production of very high or very low pressures in a single operation; but extreme pressures can be obtained with other types combined into composite pumps. They are in general less well suited for the transference of large volumes of gas under moderate differences of pressure, although some types (*Ac*, *d*) are used for this purpose.

Their disadvantage is that they cannot exhaust vapours satisfactorily, especially when designed for a large range, for the vapours condense in U when its volume is reduced and do not pass readily into the H.P.V.; when the volume is increased again, they evaporate once more, return to the L.P.V., and keep p_{H} permanently at or above the vapour pressure of the substance. Permanent gas mixed with the vapour is removed very slowly after its partial pressure in the L.P.V. has fallen to that of the vapour.

The various types of aerostatic pump are distinguished by the construction of U and the means adopted for connecting and disconnecting it with the L.P.V. and H.P.V. The following sub-classes include conveniently all the important types:

A*a*. Solid Piston Pumps.
A*b*. Liquid Piston Pumps.
A*c*. Flexible Container Pumps.
A*d*. Rotary Aerostatic Pumps.

A*a*. *Solid Piston Pumps*

§ (11) THE VON GUERICKE PUMP.—This is the oldest type of gas-pump, and its invention is generally attributed to Otto von Guericke (1672); it was probably developed from the similar water-pump. It has still a wider sphere of use than any other type, being used for the attainment of pressures from 1000 atmos. to 10^{-6} mm., and for volumetric speeds from many cubic feet to a few cubic millimetres per minute. It is equally familiar in heavy engineering, in delicate laboratory work, and, as the tyre pump, in everyday life. Broadly, the advantages of the type are a great range of pressure and great mechanical strength; the disadvantages, cumbrousness and mechanical inefficiency. It is unrivalled for high-pressure compressors, and for small portable laboratory evacuators; for all other purposes it can be replaced by other types. However, it is still widely used even for blowers, the purpose for which its disadvantages as compared with other types are most marked. Its survival is probably due partly to its long history and to its resemblance to the reciprocating steam-engine, of which the constructional problems have been studied so completely.

The principle of the pump is familiar to all. The vessel U is a cylinder in which moves a piston. Connection is made to the L.P.V. and H.P.V. either (*a*) through ports in the cylinder wall opened and covered by the piston, or (*b*) through valves moved "positively" by the piston or the mechanism that actuates it, or (*c*) through valves opened and closed by the excess gas pressure. (*c*) is the oldest arrangement and the simplest to construct, but it represents a departure from the ideal cycle and necessarily reduces the range below the ratio u_L/u_H; for the connection between U and the L.P.V. or H.P.V. ceases before the pressures have become equal. It is still standard practice in high-pressure compressors; in evacuators for moderate vacua (*a*) is often used; in those intended for the lowest possible pressures, one at least of the valves must be of type (*b*).

§ (12).—Three kinds of piston pump may be considered rather more fully. The *high-pressure compressor*, shown diagrammatically in *Fig.* 1, is always composite. It would not be impossible to obtain a range of pressure of 200 and a final pressure of 200 atmos. by a simple pump, but there are several reasons why the multi-stage pump is preferable. Thus, it is possible to cool the gas between the stages by the "intercoolers" C. By spacing the cranks evenly round the crank-shaft a more even torque and a balanced motion can be obtained. The construction of each pump can be adapted to the pressures between which it has to work; the thickness of the metal can be increased, as shown in the figure, as the pressure increases; special piston packings and forced lubrication can be used in the H.P. cylinders. Some makers prefer water to oil as a lubricant at high pressures, and at the highest pressures the substitution seems necessary because oil would burn explosively.

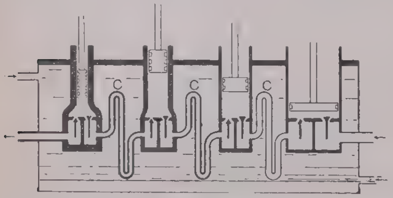

FIG. 1.

The volumetric efficiency of such a pump should be not less than 80 per cent; the gas efficiency also about 80 per cent; and the over-all efficiency about 60 per cent. The work done on the gas may be measured, for the determination of the gas efficiency, by indicator diagrams taken from the cylinders. These efficiencies are determined largely by the completeness of the cooling, which is one of the most important features of these pumps; they are also determined by leakage and by throttling at the valves.

§ (13).—*Fig.* 2 shows a large-scale two-stage evacuator, such as is used for the condensers of steam-engines and for working pneumatic tubes. The slide-valves are similar to those of a steam-engine, but to secure smooth working connection is made between the L.P.V. and the H.P.V. when the piston is in the extreme position. The pistons are connected in tandem. If the pump is to be worked by a reciprocating steam-engine it would be possible to use the same piston-rod for the driving piston, and thus to avoid rotary motions and bearings and to reduce moving parts to a minimum; but this arrangement is seldom adopted; pump and motor are usually separate.

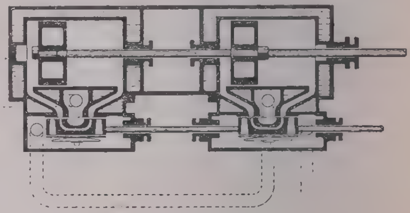

FIG. 2.

The volumetric efficiency should be about 85 per cent when $p_L=20$ cm. of mercury or more; about 80 per cent at $p^L=5$ cm.; for lower values of p_L, $E_{vol.}$ will fall rapidly, and p_L^0 will not be less than 1 cm. The over-all efficiency should be not less than 50 per cent at the higher pressures.

§ (14).—*Fig.* 3 shows part of a *laboratory evacuator* in very general use. The pump is composite with two stages; the high-pressure member presents no special features and is not shown; it is connected to H. In the low-pressure member shown, the piston is covered with oil which is ejected at the end of the stroke through the valve V, carrying the air with it. At the same time the crank J, worked by the piston guide K, forces oil into the space O by means of the oil-pump R; from O the oil flows on to the top of the piston as it descends. It is claimed that the pump will attain 10^{-6} mm., if the gas and oil are free from vapour. A drying tube with P_2O_5 is necessary in the connection L to the L.P.V., and another, which can be filled with $CaCl_2$, is desirable in the outlet of the H.P. cylinder to keep the oil dry. The pump is very efficient and convenient down to pressures of 0·01 mm., but to obtain the highest vacua of which the pump is capable needs great care in its treatment.

A piston pump for extremely low pressures has also been developed by Gaede[1] (1); in principle it does not differ greatly from that shown. In order to free the oil from water, and thus to dispense with a drying agent, the oil is forced through a special woven material which effects a separation of the two liquids. Gaede claims that his pump without any drying agent will attain a pressure of 10^{-5} mm.

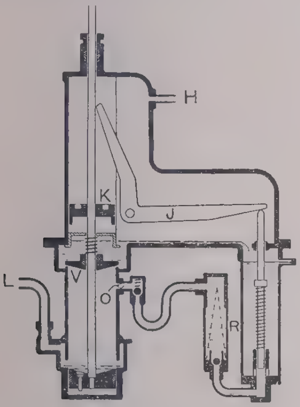

Fig. 3.

Ab. *Liquid Piston Pumps*

§ (15) Torricelli's Pump (2), (3).—It is impossible to make a perfect fit or a gas-tight joint between solid bodies movable relatively to each other; and therefore all truly solid piston pumps have some leakage and some dead space. Leakage can be wholly prevented and dead space very nearly abolished by using a liquid in place of a solid for the moving portion of the vessel U. The ratio $(p_{\mathrm{H}}{}^0/p_{\mathrm{L}}{}^0)$ can be increased by the substitution, and higher vacua (or higher compressions—though this result is not so important practically) obtained in a single operation. In fact a liquid is actually used in this manner, in pumps which are usually regarded as of the solid piston type. In the pump described last, the oil covering the "piston" and passing through the valves is really the piston, and the same purpose is served, in part at least, by the lubricating liquid of other pumps of section A*a*. However, the typical liquid piston pumps were developed historically from Torricelli's, and not Guericke's, method of evacuation; and the distinction between solid and liquid pistons, though slight from the standpoint of scientific principle, is perfectly clear in all practical examples.

In the Torricellian pump the vessel to be evacuated is completely filled with a liquid of density ρ; the open end is placed beneath the free surface of a liquid, which is usually the same as that filling the vessel. If, in this position, any part of the vessel is at a height above the free liquid surface greater than h_0, where $h_0\rho g = (\Pi - \mathrm{P})$, the surface of the liquid will sink to the height h_0, and the upper part of the vessel will contain only the vapour of the liquid at the pressure P corresponding to the prevailing temperature. As a liquid for such a pump, mercury is especially suitable, both on account of its high density and small "barometric height," h_0, and on account of its low vapour pressure.

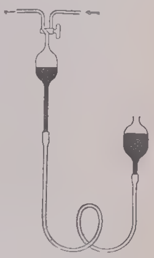

Fig. 4.

This method of evacuation has the obvious disadvantage that the whole L.P.V. has to be filled with liquid and inverted. A very obvious modification of it was described in principle by Swedenborg (1722) and put into a practical form by Geissler (1855). The action is clear from *Fig.* 4; the mercury is alternately raised and lowered and the two-way cock alternately connected to the air and to the vessel to be evacuated. The pump is a true liquid piston pump, differing from Guericke's pump only in the nature of the piston and the valves.

§ (16) The Töpler Pump.—A greatly improved form of the pump originally due to Töpler (4), but realised practically by Hagen and Neesen (5), is shown in *Fig.* 5; no stop-cocks are required. It was used in many classical researches on low vacua at the end of the nineteenth century, being at that time rivalled only by the Sprengel pump (see below) as a means of attaining low pressures.

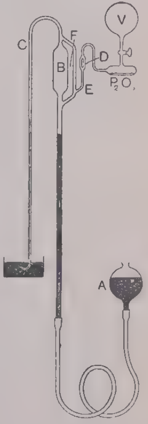

Fig. 5.

By raising the reservoir A, mercury is driven up into the cylinder B, thereby driving out the gas from the cylinder through the capillary tube C, from which it may be collected in the mercury trough. Mercury is prevented from flowing over into the vessel to be exhausted (V) by the small glass valve D.

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[1] Figures in brackets refer to references at the end of the article.

On lowering the reservoir, the mercury flows back from the cylinder, and when the mercury reaches the lower part of the cylinder gas will enter from the vessel V through the side-tube E, ready for expulsion at the next stroke.

The *lowering* of the reservoir must be done very slowly so long as the pressure in V is more than 2-3 mm., otherwise air entering the cylinder through the side-tube E will carry mercury violently up the tube F, and this may easily shatter the glass-work of the cylinder.

When the vacuum becomes high, the *raising* of the reservoir must be done very slowly, otherwise the "hammer" of the mercury at the top of its travel in the cylinder will break the cylinder head.

The reservoir A should have about twice the capacity of B. The volume of the cylinder B is fixed by the work required from the pump, but in ordinary laboratory models is from 500 to 600 c.c. The capillary tube C is about 800 mm. long and 1 mm. bore. The tube F should have a bore 12 mm., and the tube E about 4 mm. This ensures that gas entering from E to F will tend to travel in bubbles upward through the mercury in F, instead of carrying the mercury solidly up with it.

The P_2O_5 tube is, of course, inserted to absorb water-vapour, which presents the same obstacle to this as to all aerostatic pumps.

The lowest pressure which it is possible to obtain with this pump is determined as in (15), by the ratio u_L/u_H. u_L is the volume of B, while u_H is the volume of the smallest bubble which the mercury will carry down the capillary C. This volume is approximately that enclosed between the convex surfaces of two mercury meniscuses which just touch; it decreases with the bore of the capillary. But there is some gas adhering to the glass in addition to actual bubbles left behind by the mercury, which makes it useless to decrease that bore beyond a certain limit. The lowest pressure recorded as attained by a Töpler pump is 0·000025 mm.—in addition, of course, to the vapour pressure of the mercury.

The working of a Töpler by hand is extremely tedious, for several hours may be required to reach the limit of pressure. Numberless devices for rendering its action automatic have been proposed; electrical contacts worked by the mercury, or else the weight of the mercury, are used to control the operation. But no description of them is necessary to-day, for the problem of the automatic Töpler seems to have been solved finally by Gaede (6), who proposed to move the solid vessel rather than the liquid piston.

§ (17) GAEDE ROTARY MERCURY PUMP.—The general action of the pump can be seen from *Figs.* 6, 7, which are vertical sections parallel and at right angles to the front of the pumps. The outer casing A contains mercury to about two-thirds of its height, and is connected through the pipe R with a rough vacuum pump (*e.g.* an ordinary piston pump) capable of maintaining a pressure of about 10 mm. An ingenious device shown in *Fig.* 8 is used for cutting off the rough pump after the preliminary exhaustion. Inside this casing rotates a drum B, made of porcelain, to the side of which is attached a smaller porcelain drum C. The two drums communicate through the port D.

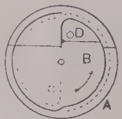

FIG. 6.

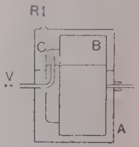

FIG. 7.

It will be seen from this diagram that if B is rotated in the direction of the arrow, the portion of B above the mercury will communicate with the pipe V, connected to the vessel to be exhausted, so long as the port D is not immersed. As soon as D is immersed, the communication with V is closed, and when the tail end E of the drum rises above the mercury, the gas will be passed on to the rough vacuum.

In the pump as actually manufactured two or three drums are spaced symmetrically on the same axle; a second drum is indicated by the dotted line in *Fig.* 6. The speed of the pump is thereby increased, but the construction made much more complicated.

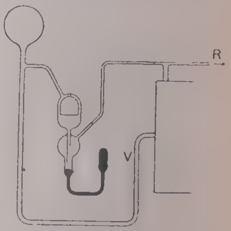

FIG. 8.

The range of the Gaede pump is probably somewhat less than that of the Töpler, for the conditions for the expulsion of small bubbles from U are less favourable. But since $p_H{}^0$ is less, owing to the use of the auxiliary pump and since its speed is much greater, the least pressure practically attainable is at least as low; a pressure of 0·00005 mm. is well within its power. The volumetric speed is not constant, as it would be according to (19); it is about 110 cm.^{3}/sec. at 0·01 mm., and falls off continuously at the lowest pressures.

§ (18) THE SPRENGEL PUMP.—A liquid piston pump, using mercury but working on a somewhat different principle from those just described, was invented by Sprengel in 1865 (7). It has over the Geissler (and later Töpler) pump the greater advantage that its action is more nearly continuous and automatic. The gas is carried out of the L.P.V. by the fall of mercury down a capillary tube, as in the Töpler pump; but it is not forced into that tube; it enters the tube under the pressure in the L.P.V. and is there trapped between successive drops of mercury falling into the top of the tube from a reservoir. In another and more convenient arrangement (*Fig.* 9) the drops are formed in the capillary by the entrance of gas from the L.P.V. through a side tube. But the principle is the same; the liquid column in the capillary will break up into drops, trapping the gas between them, if the gain in surface tension energy due to the formation of a liquid-gas surface is greater than the loss in hydrostatic energy due to the accompanying displacement of the liquid. The precise calculation of the conditions for drop formation is complicated; but it is clear that the bore of the capillary must be below the limit at which drops could be formed in the tube without completely occupying its cross-section.

The volumetric speed of a "single-fall" Sprengel is extremely small, not more than a few cubic millimetres per second. It can be increased by connecting in parallel several capillary tubes all fed from the same reservoir, and thus making a "multiple-fall" pump. The lowest pressure attainable is fixed by the same considerations as in the Töpler pump, but it may be increased by small quantities of air carried into the vacuum by the stream of mercury from the reservoir exposed to the atmosphere. Many devices have been suggested for avoiding this defect. (See (3).)

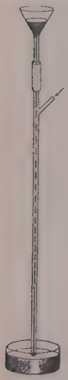

FIG. 9.

The Sprengel pump can be made completely automatic, if it is arranged that the mercury which has fallen down the capillary is restored periodically to the reservoir. Such an automatic form was at one time in universal use for the exhaustion of electric lamps (8). It is shown in *Fig.* 10, and the method of operation is obvious from that figure.

The outlet is connected to an auxiliary pump maintaining a pressure of a few cm. The capillary fall tubes A, into which mercury flows from D through the jets C, are therefore relatively short (about 20 cm.). The bent tube B enables the end of the exhaustion to be seen by the disappearance of gas-bubbles from the mercury.

The reservoir E is normally connected to H; an occasional admission of air, in order to restore the mercury to D, is controlled by a simple timing device set once and for all. It can also be controlled by the weight of the mercury in the reservoir.

The common laboratory mercury still is in effect a Sprengel pump whereby gases introduced by the mercury are removed.

For high-vacuum work, mercury pumps of these types, with the possible exception of the Gaede pump, are obsolete, and replaced by those of Class C. They may, however, be used as auxiliary pumps in series with those of that class when it is desired to collect gases pumped out from a vessel. For this purpose the Töpler pump is most suitable.

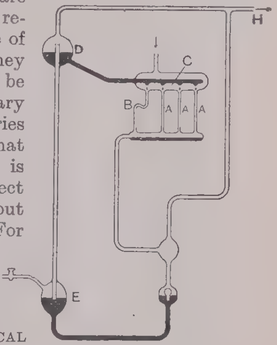

FIG. 10.

§ (19) CHEMICAL AND OTHER PUMPS.—The foregoing pumps are designed to attain low pressures. But liquid piston pumps are also of service for pumping chemically active gases, which would attack any of the metals or other materials suitable for the construction of solid piston pumps. Thus for the compression of chlorine, pumps are used of which the cylinder and valves are made of lead-covered steel, while the piston consists of oil or sulphuric acid. The liquid piston is set in motion either by compressed air or by a solid piston working in one limb of a U-tube, the other limb of which is the chlorine pump.

Laboratory pumps essentially similar to the Töpler, but using oil or glycerine or sulphuric acid in place of mercury, have also been used for some purposes. The lower densities of these liquids enable the pumps to be made of glass with a volume much greater than that set by the mechanical strength of the glass if mercury were used; or the gas to be pumped may be one which attacks mercury.

Again, air compressors for high pressures have been constructed in which water is used as the piston in order that the cooling of the gas may be more efficient. In some of these metal chains hanging into the water from the top of the piston have been used to facilitate the transference of heat between the gas and the liquid.

Ac. *Flexible Containers*

§ (20) BELLOWS.—In these pumps the vessel U has flexible walls and its volume is varied by changing its shape. The advantages of the type are high mechanical efficiency due to the absence of friction, simplicity of construction, and consequent cheapness and reliability. On the other hand, they have a small range of pressure, partly because the ratio u_H/u_L is comparatively great, partly because the flexibility of the walls makes it impossible to use them at pressures differing greatly from atmospheric. They are usually made for hand-working or very low power mechanical drives.

The earliest examples of the type are the lungs of air-breathing animals; the later improvements of the original model for this purpose are insignificant. The type is almost equally familiar in bellows of all kinds, for blowing up a fire, for vacuum cleaning, for piano-players and squeaking toys; in the fountain-pen filler, now partially replaced by a piston pump; in the bulb for scent and other sprays. But it is also used for less commonplace purposes.

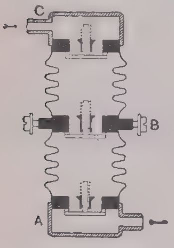

FIG. 11.

The action of the blacksmith's bellows is shown diagrammatically in *Fig.* 11. The end plates A and C are fixed, while B is given a reciprocating motion. Some bellows of this kind for smiths' hearths are several feet in diameter, with the flexible sides of leather. The air is usually pumped into a reservoir bellows in which a pressure of about 6 in. of water is maintained by a weight.

Fig. 11 also represents, on a different scale, a useful laboratory pump. The bellows are here made from the inner tube of a motor tyre, the corrugations being obtained by large metal rings inside the tube and small rings outside placed alternately. The cylindrical discs A, B, C are of aluminium, and B is driven by a crank from a small motor.

§ (21).—The most elaborate pump of this class is the organ bellows, shown on *Fig.* 12; the bellows compressor, called the "feeder" bellows, is at F. The upper board A of the feeder is fixed and the lower board B is hinged to A by a leather joint. The wedge-shaped volume between A and B is enclosed by the wooden ribs R; the ribs are hinged to each other and to A and B with leather and cloth. The board B falls by its own weight and the air enters through the flat valve V_1. Air is compressed in the feeder by a hand lever, and the air is driven through the valves V_2 into the reservoir bellows D, from which the air passes to the organ. The pressure on D depends on the weight W on the top board. To ensure a uniform pressure in D whether it is expanded or contracted a double set of ribs is used, the upper set R_3 folding outwards and lower set R_2 folding inwards; the frame H between the two sets of ribs is connected by a mechanism shown so that both halves of the reservoir bellows expand equally.

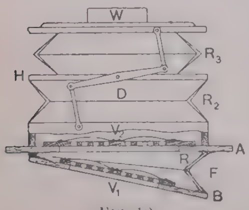

FIG. 12.

§ (22).—At the opposite extreme of the type is the squeezed rubber tube pump (*Fig.* 13) (10).

This pump consists of a rubber tube A_1A_2 (*Fig.* 5) wrapped inside a hollow cylinder B and squeezed by two or more rollers C_1 and C_2 so that the way through the tube is stopped at the squeezed portion. The rollers C_1 and C_2 roll round the inside of the cylinder driven by the shaft D, and gas (or liquid) is transferred from A_1 to A_2 as the shaft revolves.

The squeezed portions act as pistons, and these "pistons" are formed at A_1 and travel along the tube to A_2 where they disappear.

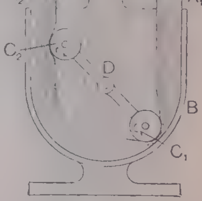

FIG. 13.

The action of the pump is somewhat similar to the rotary pump of *Fig.* 16, with the important difference that the "piston" of the rotary pump requires to be carried across from A_2 to A_1 and is thus liable to cause leakage of air or liquid. The tube pump has no dead-space and is only limited in range by the strength and tightness of the rubber tube to resist the pressure difference; but its speed is small and mechanically it is inefficient. The tube pump is particularly suitable for transferring gases or liquids without contamination, as the plain tube can be easily cleaned and no other portion of the pump can come into contact with the fluid being pumped.

An even simpler pump of the same type can clearly be made out of a plain piece of rubber tubing pressed with the fingers.

Ad. Rotary Pumps

§ (23) BLOWERS.—In this class the variation of the volume of U is effected by the rotary motion of solid bodies constituting part of its walls. The pumps are usually driven by power, and are intended for continuous action; they compete therefore with solid piston pumps rather than with other types of Class A. Over solid piston pumps they have the advantages in mechanical efficiency; in mechanical simplicity and consequently in cost, both initially and for upkeep; in compactness; in steadiness of air current when used as blowers. They have disadvantage in a smaller range of pressure, in greater leakage, and usually in noisiness. But pumps of this type compete also with those of Class B (especially B*g*). For blowers type B*g* generally has the advantage in mechanical efficiency and simplicity. For compressors there is little to choose between B*g* and A*d*, for though the latter has the greater range both types would be replaced by A*a* if a greater range were required. For evacuators B*g* is useless, while A*d* provides the standard machines of modern practice for all pressures between 10 mm. and ·01 mm.

The chief examples of the class can be divided into two groups, one (1) developed from the Root Blower, the other (2) from the Beale Blower. The development has been so gradual that it is difficult to associate any of the pumps, or even the two archetypes, with the name of any inventor. The groups are usually distinguished by the nature of the "abutment," that is the line or surface dividing the H.P.V. from the L.P.V.; group (1) is then characterised by a movable abutment, (2) by a fixed abutment; for though in (2) the bodies forming the abutment move, the line or surface which is the abutment is at rest relatively to the housing.

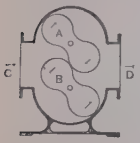

FIG. 14.

§ (24) MOVABLE ABUTMENT.—These pumps are chiefly used for moving large quantities of gas against a small pressure difference (say up to 5-10 feet water pressure).

An early example of this type was exhibited at the Paris Exhibition by Elihu Root in 1867 (*Fig.* 14). It consists of two two-toothed wheels, A, B, which are made to revolve at the same rate in opposite directions by means of gears outside the box or pump body; the space included between the wheels and the housing is U, and its volume varies with the position of the wheels. If they rotate as shown in the figure air would be sucked in at C and delivered through D.

To prevent leakage the wings are machined as accurately as possible, and are often covered with wood or other packing material. The volumetric efficiency against small pressures (say 10 inches of water) may be as high as 96 per cent, while at higher pressures (6 feet of water) it will drop to 80 per cent. The over-all mechanical efficiency is about 75 per cent.

Fig. 15 shows another type where the impeller vanes V are fastened at one end to a disc which causes them to rotate around the fixed core. The vanes after the delivery stroke come into the openings in the rotating body A (called the idler), which is caused to rotate at the same speed as the vanes by gears on the outside. On rotating further the vanes come into the suction chamber, whence they start again on the compression stroke.

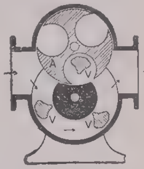

FIG. 15.

The pump is more complicated than the Root, but several advantages are claimed. Thus surfaces can be used to separate the two chambers where lines only are possible in the Root blower; air is compressed by one rotating part only; there is no contact between parts moving with different velocity, and thus there is less friction; the mechanical construction is simple and cheap; the pulsations of gas are reduced.

In another type the rotating parts are spiral vanes, which give a more even delivery of gas and make less noise; the mechanism is not easily shown in a diagram. Many other devices have been adopted, some varying widely in detail from those mentioned, but all based on the same principle; descriptions of them are to be found in makers' catalogues.

§ (25) FIXED ABUTMENT.—These pumps, of which the Beale blower is an early example, are used extensively as compressors, as blowers, and as evacuators. They are used in gasworks for pumping the gas to the holders, and in the factory or laboratory for attaining pressures down to ·001 mm. The general principle employed is shown in *Fig.* 16.

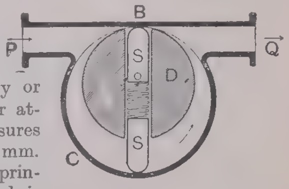

FIG. 16.

The cylinder D rotates about an axis O, so that it touches the containing cylinder C at the fixed abutment B. A slot in D carries the plates or "scrapers," the outer ends of which are held against the containing cylinder C. The plates divide the space between C and D into two parts; as D rotates the volumes of these two parts vary in a manner readily seen from the figure, in which P is the suction and Q the compression inlet.

The friction of the scrapers on the cylinder involves considerable loss and wear, and many alternative arrangements have been devised to avoid it. In one, contact between the scrapers and the cylinder is preserved by a circular guide with its centre coincident with that of C. In another there is a single solid scraper which slides freely in the slot in D; the section of the cylinder D is not circular, but such that the two ends of the scraper are in contact with the walls whatever the position of D. In another the scrapers are pushed out by springs, but they bear on an idly rotating cylinder C′ fitting closely within C and pierced with holes; the clearance between C and C′ is made so small that the leakage between the two is inappreciable; the friction is thus reduced to that of C′ on its bearings. In any pump of this type, the number of scrapers may be increased. *Fig.* 17 shows a type with three or more scrapers, hinged at the central axis M of the box B, and sliding in cylindrical stuffing boxes C fastened to the rotating drum D; this drum touches the casing in the fixed abutment E.

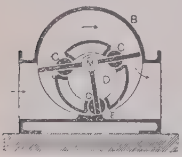

FIG. 17.

In pumps of this type designed as evacuators oil is always introduced into the cylinder to prevent leakage and to fill up the dead space. The scrapers are usually arranged as in *Fig.* 16. In order to prevent hammering by the oil when the vacuum is high, a valve is fitted which limits the quantity of oil flowing from the compression side; it also helps to separate the air from the oil. Such evacuators are often run in tandem, or one of them is used in series with an auxiliary pump of some other kind. If p_H is 4 to 10 mm., p_L may be reduced to ·0001 mm.

Scraper vacuum pumps are largely used as auxiliaries to high-vacuum pumps (C); they are also the chief type employed in the preliminary evacuation of electric incandescent lamps, which are subsequently "cleaned-up" by the discharge (D).

B. AERODYNAMIC PUMPS

§ (26).—In aerodynamical pumps, the pressures are functions of S and of the velocity of the gas in different parts of the apparatus. The fundamental connection between the pressure and velocity of any fluid is given by the familiar hydrodynamical equation

$$\delta(p+\tfrac{1}{2}\rho v^2)=0, \quad . \quad . \quad . \quad (24)$$

or

$$p_1-p_2=\tfrac{1}{2}\rho(v_2{}^2-v_1{}^2). \quad . \quad . \quad . \quad (25)$$

It is deduced on the assumptions (1) that the energy required to change the pressure of a volume V of the gas from p_1 to p_2 is (p_1-p_2)V, (2) that the energy of any such change of pressure which occurs is equal to the change in the kinetic energy of the gas. (1) implies that the fluid is incompressible,[1] or that the change in pressure is infinitely small; (2) that there is no loss or gain of energy to or from other sources, *e.g.* friction of the moving gas. By "the pressure" must be understood the force per unit area on a surface at rest relative to the gas; in a frictionless fluid it is equal at any point to the "static" pressure on a surface parallel to the flow at that point, but moving relatively to the gas; the "dynamic" pressure, or that on a surface perpendicular to the flow and at rest relative to the pump, is $p+\frac{1}{2}\rho v^2$, and, when (24) is true, is constant along the whole stream.

Be. *Injectors and Ejectors—Gaseous Stream*

(General Reference (11))

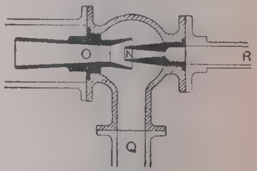

FIG. 18.

§ (27).—These are wholly analogous to liquid jet pumps (see "Hydraulics"). A gas or vapour (called "the fluid" to distinguish it from the gas to be pumped) is forced through a tube N from a reservoir R, at pressure p_R, into a larger tube O communicating with the atmosphere (see *Fig.* 18). If the flow satisfied assumption (2) above, the stream would not be brought to rest in the atmosphere unless p_R were equal to Π, and if $p_R=\Pi$ there would be no stream. But owing to viscosity and friction, p_R may be greater than Π, so that a high velocity v_N is obtained in N, and yet v may be zero when the atmosphere is reached. In these conditions, the difference $\Pi-p$, where p is the pressure at N, is not so great as $\frac{1}{2}\rho v_N{}^2$ given by (24); but it is still finite and of the same sign; p is less than Π.

If gas in the space surrounding N is given access to the stream through the gap between N and O, it will flow into the stream of fluid and be carried away by the stream, so long as its pressure is greater than p. The fluid streaming from N to O will suck gas through the pipe Q and will act as an evacuator or "ejector." In an "injector" or compressor, Q communicates with the atmosphere, and the space with which O communicates, and in which the stream comes to rest, is at a pressure greater than Π, but, of course, still much less than p_R. p is equal to $p_L{}^0$ in an ejector and to $p_H{}^0$ in a compressor; but if there is a continual stream of gas through the pump, p_L or p_H will not be equal to p, (1) because of the drop due to flow of the connecting tubes Q and O, (2) because the mixing of the gas with the fluid affects greatly the velocity and pressure of the latter.

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[1] Confusion is sometimes introduced by a failure to observe that the chief part of the pressure of a gas is inseparable from its compressibility; it is not due, like that of a liquid, to its weight. The pressure of an incompressible gas is a meaningless conception. The application of the theory to gases is justified only because, for small changes of pressure at constant temperature, $pd\mathrm{V}=-\mathrm{V}dp$.

In calculating the performance of a pump, allowance has to be made for departures from (25) owing to friction and viscosity. The allowance is usually made by introducing on the right hand of (25) an empirical factor ζ, less than 1, and writing

$$p_1 - p_2 = \tfrac{1}{2}\zeta\rho(v_2{}^2 - v_1{}^2), \quad . \quad . \quad (26)$$

where v_1, v_2 are the mean velocities over a cross-section of the stream. The principle of the calculation is then simple. There are three equations (26) for the three tubes N, O, Q; and there is the equation of conservation of mass when the streams meet. (The momentum is *not* conserved, for there is a reaction on the tubes.) These four equations suffice to determine the four unknowns, viz. p, v_N, v_O, v_Q, in terms of p_R, p_L, p_H, the densities ρ_g and ρ_f of the gas and fluid, the three empirical constants for the three tubes, ζ_N, ζ_O, ζ_Q, and F_N, F_O, F_Q, the cross-sections of the three tubes at their openings. The algebra need not be set out, for the numerical results depend wholly on the values attributed to the empirical constants; it is given in (11). Here it will suffice to state some of the most important qualitative conclusions, which are confirmed by experiment. These were first stated by Zeuner (12).

The variables considered are p_R, p_L, p_H, v_R, F_N, F_O, F_Q, ρ_g, ρ_f, and S, the volumetric speed, which is equal to v_QF_Q. In each statement the variables not mentioned are supposed constant.

(1) $p_H - p_L$ is proportional to p_R and $v_R{}^2$, so long as p_R is great compared with p_H or p_L.

(2) S is proportional to $\sqrt{p_R}$, to v_R, and to $p_H - p_L$, subject to the same condition.

(3) $p_H - p_L$ depends only on the ratios of the F's, and not on their absolute values.

(4) S is proportional to the F's, if their ratios are constant.

(5) Given one of these ratios, there is an optimum value for the other two, giving maximum S, but the same maximum S can be obtained with different values of the ratios.

(6) S is independent of ρ_g and ρ_f, so long as v_R is constant.

(7) S may be considerably greater than the volume of fluid issuing per second from N, *e.g.* two or three times as great.

Owing to the circumstances in which the pumps are used, the efficiency is seldom important. But the mechanical efficiency reckoned on the basis of the work done in driving the fluid stream appears seldom, if ever, to exceed 25 per cent.

§ (28).—In practice the fluid used is generally steam or compressed air. Ejectors using these fluids are used for vacuum brakes, vacuum cleaners, and grain conveyors. Their great advantage is, of course, their simplicity and freedom from maintenance charges. *Fig.* 18 shows a pump used for railway vacuum brake operation. It will reduce p_L to about 15 cm. of mercury. A more elaborate pump is shown diagrammatically in *Fig.* 19. Here a common supply of steam works two pumps in series. The first consists of the plain nozzle A, the second in the annular gap B, from which the gas is carried into the surrounding annular space C. It is claimed that this pump will attain a pressure of 3 cm. of mercury. Remark should be made that the application of the simple theory to such pumps is extremely precarious, for the assumption that the change of pressure of the gas is infinitesimal is clearly false.

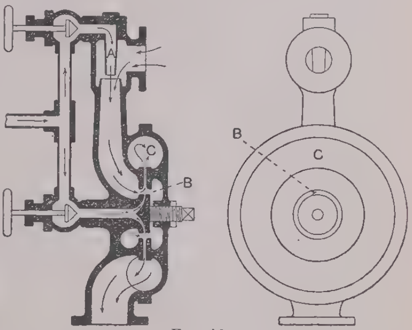


FIG. 19.

Fig. 20 shows a blower used for moving large quantities of air in ventilation under a pressure of a few inches of water. Air or steam is used as fluid; the concentric cones are designed to make the velocity of the gas nearly parallel to that of the fluid, so that the direction of flow of the latter is not disturbed by irregularities in the flow of the former. A somewhat similar arrangement is adopted in the smoke-box of a locomotive, where the exhaust steam is made to create a draught through the boiler flues.

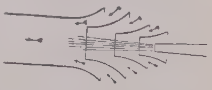


FIG. 20.

§ (29).—The most modern development of the type is the mercury vapour jet pump used in conjunction with "condensation" high-vacuum pumps (*q.v.*). Indeed, as will be seen, the line between vapour jet pumps and condensation pumps cannot be drawn sharply; roughly it may be set at the pressure where the mean free path of the vapour molecules becomes comparable with the dimensions of the tubes; but there is no justification for overlooking the distinction entirely. The construction of these vapour jet pumps is essentially similar to *Fig.* 18, but the apparatus is made of glass; the vapour stream is produced by boiling mercury, and condensing arrangements are provided for returning the vapour to the boiler. A practical form designed by Volmer (13) will reduce the pressure from $p_H = 20$ mm. of mercury to $p_L = \cdot 001$ mm. But it is doubtful whether they will replace generally the rotary aerostatic oil-pump for producing the auxiliary vacuum of high-vacuum pumps. Fragility is their great fault.

B*f*. *Injectors and Ejectors—Liquid Stream*

§ (30).—This type of pump, of which the laboratory filter pump (*Fig.* 21A) is a common example, is often regarded as a mere modification of B*e*, gas or vapour being replaced by a liquid as fluid. But the difference is really greater. If the theory of B*e* is applied to pumps with liquid as fluid, then, even if all plausible corrections are made, the calculated performance is far less (*e.g.* 10 times) than the actual. The error arises in assuming that the gas and fluid are miscible. The flow of the gas into the fluid is not determined simply by the pressure difference, and relative motion of the fluid and gas is possible, even after they are mixed in the exit tube O.

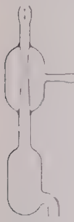


FIG. 21A.

It seems preferable to look at their action from a different point of view. Two processes are involved: first, the entangling of the gas by the liquid stream; second, the conveyance of the entangled gas from the L.P.V. to the H.P.V. During the second process the gas will move relatively to the liquid nearly as if the liquid were at rest relative to the walls. The difference of pressure ultimately obtainable is limited only by the condition that the velocity of the liquid entering the L.P.V. by N is sufficient to carry it out through O against the pressure $p_H - p_L$, and is also greater than the velocity with which the bubbles of entangled gas travel through the liquid in the opposite direction. It is the second process which determines the greatest possible value of $p_H^0 - p_L^0$.

On the other hand, the speed of the pump is determined by the first process. Its nature is obscure; probably the liquid stream carries along a layer of gas on its surface, in virtue of friction and viscosity (cf. § (39)), as would a solid rod travelling with the same velocity. When the liquid breaks into drops in virtue of the inherent instability of liquid jets, this gas becomes entangled between the drops.

On this view the performance of a pump of this type appears quite incalculable. No calculations confirmed by experiment seem to have been based on any view, and few data of performance or of its variation with the construction of the pump seem available.

§ (31).—The filter pump of *Fig.* 21A fed with water at a head of 50 ft. or more will reduce p_L to the vapour pressure of the water. But no measurements of S under varying conditions have been found. It is recorded that the pump is more efficient if placed at the top of a building so high that the exit tube can be made as long as the water barometer.

A variant on the usual design is shown in *Fig.* 21B, which is similar in construction to the Venturi meter. But since the action is improved by a baffle at *b* which breaks up the stream, it is probable that, as suggested, the formation of drops is an important part of the process. A non-return valve, as shown at V, is useful with either of these types to prevent the flow of water into the apparatus if the head becomes insufficient.

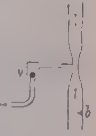


FIG. 21B.

Evacuators of these types are applied outside the laboratory to vacuum cleaning and to grain conveyors. Compressors working on the same principle, but with a different construction, have also important commercial uses; they are known as "trompes." In a very simple form (*Fig.* 22), used for blowing blacksmiths' fires, a stream of water flowing down a pipe with a few holes in it drags with it air from the atmosphere, which is subsequently separated from the water in a closed vessel. A more elaborate form has been developed in America for supplying compressed air to mines where a great head of water is available. The pressure obtainable is a considerable fraction of that corresponding to the head of the water.

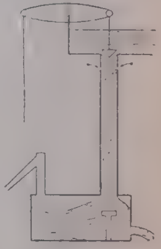


FIG. 22.

B*g*. *Centrifugal Pumps*

(General Reference (11) and (14))

§ (32).—Centrifugal air-pumps are analogous to centrifugal liquid pumps (see "Hydraulics"). They are generally called "fans," and are used as fans or blowers according to the definition of the introduction. The principle is shown in *Fig.* 23, which illustrates the simplest type. The gas entering the circular central aperture O in the housing is whirled round by the rotating vanes, acquires velocity, and issues through B.

Suppose that the conditions necessary for (24) are fulfilled, and that the gas leaves the tips of the vanes with a radial velocity v_1, uniform over the whole surface of the circular cylinder surrounding the vanes. The area of this surface is $2\Pi rd_1 = F_1$, where r is the radius of the vanes and d their breadth perpendicular to the diagram. Then

$$S = F_1 v_1. \qquad . \quad . \quad . \quad (27)$$

If F_2 is the area of the opening, and v_2 is the velocity of exit uniform over the opening,

$$S = F_2 v_2. \qquad . \quad . \quad . \quad (28)$$

If the gas in acquiring its velocity from the vanes preserves its original pressure p_L, then from (25)

$$p_H - p_2 = \tfrac{1}{2}\rho(v_1{}^2 - v_2{}^2) = \tfrac{1}{2}\rho S^2\left(\frac{1}{F_1{}^2} - \frac{1}{F_2{}^2}\right). \quad (29)$$

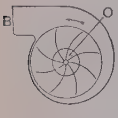


FIG. 23.

But, as in Section B*e*, (24) is not true and allowance has to be made for losses of energy due to friction and to sudden changes in the direction of the gas stream. Further, the velocities are not uniform over the surfaces F_1 and F_2. Again, it is convenient to express $p_H - p_L$ and S in terms of the velocity of the vanes which can be measured directly; it is usual to represent this velocity by v_0, the linear velocity of the tips. It may be assumed that v_1 and v_2 are proportional to v_0. The losses may be then represented by one or more terms proportional to $v^2{}_0$ or to S^2, or to Sv_0, and the general equation for the performance of the pump written

$$p_H - p_L = \alpha v_0{}^2 + \beta v_0 S + \gamma S^2. \qquad (30)$$

The constants α, β, γ are usually regarded as depending on the angles at which the stream of gas strikes the vanes and the housing; they certainly depend on the geometrical quantities characteristic of the pump. Some progress towards calculating them directly from those magnitudes can be made, but some purely empirical constants are always necessary. In designing the forms of the vanes and of the housing such calculations are a useful guide; here reference can only be made to discussions in (11) and (14). It may be noted that in (30), α is always, β usually, positive, while γ is negative.

Three kinds of efficiency are recognised for centrifugal pumps:

§ (33).—(i.) The manometric efficiency $E_{man.}$ is taken as $(p_H - p_L)/\rho v^2$, p_L (usually Π) and p_H being the static pressures of the inflowing and outflowing gas. If v_1 were equal to v_0, the maximum value of $E_{man.}$ would be $\frac{1}{2}$, but since v_1 may be either greater or less than v_0, $E_{man.}$ might theoretically have any value; actually it is seldom if ever greater than 1.

(ii.) The mechanical efficiency $E_{mech.}$. The useful work is generally taken to be given by (9), so that if w is the power exerted at the pump shaft, $E_{mech.} = S(p_H - p_L)/w$. Sometimes the useful work is reckoned by (9*a*); the corresponding efficiency is called the total efficiency, but it is seldom important, as previously explained. $E_{mech.}$ is a true efficiency and can never be greater than 1.

(iii.) The volumetric efficiency $E_{vol.}$, which is taken by some writers to be $S/v_0 r^2$ and by others to be $S/2\pi rdv_0$. The latter quantity would be unity if v_1 were equal to v_0; the former seems to have no general significance, but, being a no-dimensional magnitude, is convenient for comparing similar designs. $E_{vol.}$ is often greater than 1 if the first expression is chosen, sometimes if the latter is chosen.

It is apparent from (30) that the pressure and the efficiencies will vary with S, if S is controlled by changes in the area F_2 or by other changes in the resistance to the flow of the gas. *Fig.* 24 shows typical curves relating the brake H.P. w, the pressure difference $p_H - p_L$, and the mechanical efficiency to the volumetric speed S, the velocity of the vanes being constant. It will be observed that this efficiency is zero for $S = 0$ and for high values of S and has a maximum for some intermediate value. If mechanical efficiency is required the pump must be designed for its special work. If the velocity of the pump is varied over a moderate range, S varies as v_1, $p_H - p_L$ as v^2, and the power expended is v^3. For extreme varieties the "constants" of (30) change.

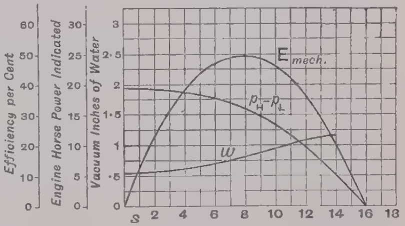


FIG. 24.

§ (34).—Simple centrifugal fans differ in size, in the number and shape of the vanes, and in the shape of the housing surrounding them. This is often divided into a "diffuser," or portion with parallel sides immediately outside the vanes and a "volute," or portion of circular section, outside the diffuser. The cross-section of the volute increases towards the outlet in order to make some use of the kinetic energy (see § (5)). Some fans have inlet openings on both sides of the fan, some only on one. But in their performance, they all have common characteristics; they are all used as low-pressure blowers, the maximum pressure obtained being about 12 in.

of water; p_H/p_L does not exceed 1·03, and assumption (1) of § (26) is justified. In large sizes their mechanical efficiency may reach 80 per cent, but usually it is more nearly 70 per cent and intermediate between that of piston pumps and of Root's Blowers. Over either of these types they have the advantage of simplicity and of being proof against hot and dusty gases if the bearings are suitably protected.

Composite, or multi-stage, centrifugal fans are also common. The fans of successive stages run on the same shaft; the discharge from the circumference of one fan is led by a tube to the central intake of the next. Since $p_H - p_L$ is proportional to ρ by (25) and ρ is proportional to p_L or p_H, we have in successive stages $\delta p \propto p$, or $p_n/p_1 = \kappa^n$, where κ is the range of the simple pump and n is the number of stages. The range of the composite pump is the product of the ratio-ranges of the individual stages. Such multi-stage fans with ten or more stages, each giving $\kappa = 1{\cdot}1$ when driven by a turbine or electric motor at 4000 r.p.m., are used to deliver air to blast furnaces at a pressure of 2½ atmos. They are also used for "super-charging" petrol motors on aeroplanes. In *Fig.* 25 a 3-stage fan of this kind is shown; details of bearings, rings to prevent leakage, and the water-jacket of the outer casing have been omitted. A, B, C are the revolving "impellers," while the parts drawn solid are fixed.

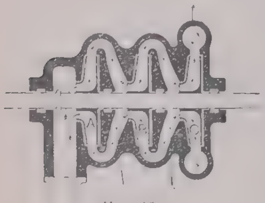


FIG. 25.

Rateau constructed a simple fan, running at 20,000 r.p.m., which gave $\kappa = 1{\cdot}5$; experiments on extremely high speeds have also been made by Parsons and others. But such simple fans seem to have no practical advantage over the composite type.

Bh. Airscrews

(General References (11) and (14))

§ (35).—In type B*g*, the velocity of the gas produced by a rotating solid is perpendicular to the axis of rotation; if the velocity is mainly parallel to that axis, the fan may be called a "propeller," or, better, airscrew. In all that concerns the general relations between the velocity of the solid and the velocity or pressure of the gas, airscrews are indistinguishable from centrifugal pumps. Thus the pressure produced by an airscrew is proportional to the square of its velocity, the volumetric speed to the velocity, and the power to the cube of the velocity. (35) is still true, at least approximately, and similarly defined efficiencies might be employed to state the performance.

The difference between airscrews and centrifugals lies in the connection between the constants of these equations and the geometrical magnitudes. Much more is known of this connection for airscrews, perhaps on account of their importance for other purposes; for this knowledge reference may be made to "Aerodynamics"; since airscrew pumps are not very important, no further account of their theory need be given here.

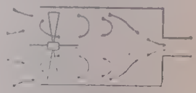


FIG. 26.

§ (36).—Airscrews are largely used for ventilation, either stirring up the air in a room or extracting it into the atmosphere through a hole in the wall. A plain airscrew is very inefficient for the second purpose, since the difference of velocity between the centre and circumference of the screw produces a circulation within the fan itself, as shown in *Fig.* 26. The loss due to this circulation is greater when, as in *Fig.* 26, it is the H.P.V. that is partially closed than when it is the L.P.V. To reduce the loss the centre of the airscrew is often covered with a disc to prevent the return flow; the volumetric speed for a given diameter and velocity is thereby decreased, but the mechanical efficiency is increased.

It is impossible to secure that all the energy given to the gas shall produce axial flow; some inefficient tangential and radial flow is always produced at the same time. In the Rateau screw fan, shown in *Fig.* 27, the tangential and radial flow is greatly reduced by causing the gas to strike the blades (B) with a velocity opposite to that of their rotation. This velocity is imposed on the inflowing gas by the fixed vanes V. The centre of the blades is covered by the fixed disc D. *a* shows a transverse section through the fan, *b* a "cylindrical" section made by a cylinder coaxial with the fan, cutting B and V and developed into a plane. In *b* the motion of the blades B is upwards. The Rateau screw fan resembles in its performance a simple centrifugal.

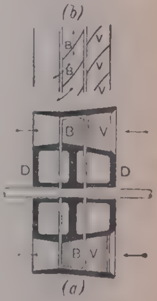


FIG. 27.

Some fans, described as of "mixed flow," are intermediate between centrifugals and airscrews, the flow of gas being partly radial or tangential and partly axial. But they do not differ sufficiently in principle from the many types of pure airscrews and centrifugals, which are also described in makers' catalogues, to warrant special notice. For small powers there seems little to choose between these classes of fans; for larger powers the centrifugal is more suitable; it is also more suitable for the individual members of a composite pump.

Bi. *Thermal Pumps*

§ (37).—The principle of these is sufficiently discussed under "Convection." The chimney of the open fire which ventilates a room and the gas jet in the flue of the chemical fume cupboard are familiar examples of "blowers" of this type. The draught produced by a flame in a flue has also been used to work small wind channels for aeronautical investigation; and generally, if only very small powers are concerned and efficiency is unimportant, chimneys and small fans may be regarded as mutually interchangeable.

C. High-vacuum Pumps

(General References (21), (23))

§ (38).—During the last few years pumps have been invented which will attain pressures definitely lower than those that can be reached with any of the pumps described so far. They depend upon "molecular" processes, that is to say, processes explicable by molecular theory and not by hydrostatic or hydrodynamical theories, which regard a gas as a continuous medium. These processes become important only when the pressure of the gas is below some definite limit, which is usually far below that of the atmosphere. The pumps must therefore be run in series with an auxiliary pump which reduces and maintains the pressure below the limit at which the action of the molecular pump begins; this pressure is of the order of 0·1 mm. As auxiliary pumps, those of type (A*d*) are now usually employed. Further, since the vapour pressure of water is much above the limiting pressure, a drying agent must be used in conjunction with the auxiliary pump; on the other hand, a molecular pump does not distinguish between vapours and permanent gases, and no device is needed to remove from the low-pressure side of the pump any vapours except those which arise from the action of the pump itself.

Two molecular processes have been employed for such pumps, both originally suggested by Gaede. Since the first type was the only member of its class when first invented, it was called by its inventor the "molecular pump." It is convenient to retain the term and confine it to this type, although the second type, invented later, has an equal right to it.

Cj. *Friction Pumps*

§ (39) Gaede Molecular Pump.—The action depends upon the forces between a gas and a solid (or liquid) surface moving relatively to it. At ordinary pressures these forces are determined by the viscosity of the gas, and the influence of the solid boundary enters into the calculation of the flow only through the assumption that v_0, the velocity of the gas at that boundary, is zero and that there is no "slip." But at sufficiently low pressures Kundt and Warburg (15), confirmed by many later observers, showed that the measured flow agreed with that predicted hydrodynamically only if it was assumed that there is some slip, that v_0, the velocity of the gas at the boundary and parallel to it, is finite, and that the force exerted on the gas by the boundary is ϵv_0. ϵ is called the friction coefficient and ϵ/η the coefficient of slip.

From the molecular standpoint the matter appears somewhat differently. The condition $v_0=0$ means that the velocities of the molecules leaving the boundary are symmetrical on either side of the normal. The appearance of slip at low pressures does not mean that this condition is no longer fulfilled. For if the pressure is greater on one side of the normal than on the other, more molecules will arrive at the boundary from the first side: if the molecules leave the boundary equally distributed on both sides, then there will be on the whole a flow of gas from the first side to the second, so long as the distance travelled by the molecules leaving the boundary before they collide is finite. On the other hand, the flow will be less than it would be if the molecules left the boundary with their velocity parallel to it unchanged. Accordingly the condition $v_0=0$ is not inconsistent at low pressures with the hydrodynamical assumption of a finite slip coefficient.

Knudsen (16) has calculated the friction coefficient from such a molecular theory. He assumes that whatever the direction of the incident molecules, the number with any velocity leaving the boundary within a cone of solid angle $d\omega$ making an angle θ with the normal is proportional to $\cos\theta d\omega$ and that the distribution of velocities is Maxwellian. He concludes that $\epsilon=\epsilon_0 . p$, where

$$\epsilon_0=\tfrac{3}{8}\sqrt{\frac{\pi}{2}}\rho_0, \quad . \quad . \quad . \quad (31)$$

and ρ_0 is the density of the gas at unit pressure and the prevalent temperature. (31) has been confirmed by experiments at pressures less than 0·001 mm. Gaede (17) has shown that at higher pressures ϵ_0 is greater, probably owing to the presence of a gas film on the bounding surface.

Consider a layer of gas between two infinite parallel plates, distant h from each other, moving relatively to the gas with velocities v_1, v_2. Let p_1, p_2 be the pressures of the gas at points distant L along the direction of motion. If the pressure is so high that the mean free path is small compared with the distance between the plates, the forces on the gas are due to its viscosity; the relation between p_1 and p_2 is given by the equation similar to that of Poisseuille:

$$(p_1 - p_2) = \frac{6\eta}{h^2} \cdot \mathrm{L}(v_1 + v_2). \qquad (32)$$

But if the distance between the plates is small compared with the mean free path, the conception of a viscosity depending on collisions between molecules becomes insignificant, and the equation must involve only ϵ, depending on collisions with the walls. It is found that

$$\log_e \frac{p_2}{p_1} = \frac{\epsilon_0}{h} \mathrm{L}(v_1 + v_2). \qquad (33)$$

The ratios of the pressures at opposite ends of the plates is a function of the velocities and of the geometrical quantities: it is independent of the pressures. If any geometrical arrangement other than that of parallel plates is used, this proposition is still true, so long as the pressure is sufficiently low, and so long as the velocities are considerably less than the mean velocity of the molecules. If this last condition were not fulfilled the distribution of velocities among the molecules leaving the boundary would be no longer Maxwellian, and ϵ_0 would be greater—which would clearly be desirable for the purpose in view.

§ (40).—The construction of Gaede's pump (18) in which this principle is applied is shown diagrammatically by transverse and longitudinal sections in *Fig.* 28. A is a cylinder rotating in the closely fitting housing B; in the surface of A are cut grooves into which project the obstructions C attached to the housing; the pipes n and m open into the grooves on either side of C. If A rotates clockwise the friction between the rotating cylinder and the gas lowers the pressure at n and increases it at m. The grooves are in series from the middle outwards; m of the middle groove is connected to n of each of the grooves on either side, and so on; m of the outermost grooves are connected to the auxiliary vacuum (H.P.V.) and n of the middle groove to the L.P.V. A is run at about 140 revs. per sec. by a pulley and motor. The axle passes through an oil box which seals the interior of the housing from the atmosphere. The intrusion of oil from the oil box is prevented by an Archimedean screw cut on the axle, which drives the oil backward; this arrangement makes it of great importance that the auxiliary vacuum should be turned on after the pump is started, and turned off before it stops.

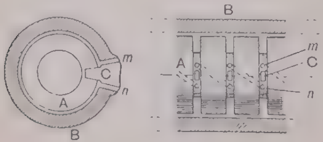


FIG. 28.

The precise calculation of the pressures obtained is very complicated; for there has to be taken into account, besides the driving of the gas from n to m by the friction of the rotating cylinder, the leak of the gas back from m to n past the obstruction C and over the surface of the cylinder between successive grooves. But theory shows and experiment confirms that at a sufficiently low pressure the ratio of initial and final pressures is proportional to the speed of rotation and independent of the pressure, but the ratio falls off when the pressure in any part of the pump rises above that (about 0·02 mm.) at which b is equal to the mean free path. At a speed of 140 revs. per sec. and an auxiliary vacuum of 0·1 mm. $p_{\mathrm{H}}/p_{\mathrm{L}}$ is about 10^5, so that a pressure of 10^{-6} mm. can be obtained. But the ratio varies with the gas, in virtue of the occurrence of ρ_0 in (31); it is less with hydrogen than with air; probably hydrogen formed a large part of the gas with which this measurement was obtained. Lower pressures could be obtained with a better auxiliary vacuum; but the vacuum attainable is definitely limited by that of the auxiliary pump.

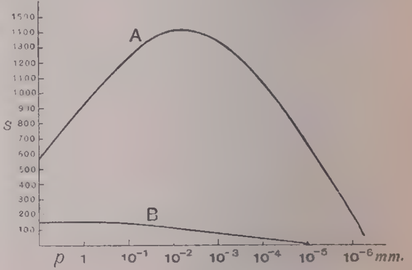


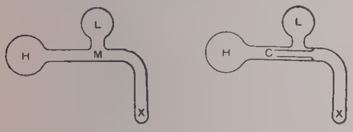
FIG. 29.

An important feature of these pumps is the great speed of pumping. *Fig.* 29 A shows S (in cm.³/sec.) plotted against the pressure (log. scale); for comparison, B shows S for the Gaede rotary mercury pump (*Ab*).

The molecular pump would have made possible modern high-vacuum work; but all its advantages, except one, are possessed by the next type of pump to be considered. This one advantage is that it will remove *all* vapours, while all other high-vacuum pumps leave mercury vapour, which has to be removed by condensation. But the advantage is of little practical importance for most work since the pump will maintain the vacuum only while it is running; if it is to be stopped and the vacuum preserved, some form of tap or trap must be inserted, and such devices always introduce vapours. On the other hand the molecular pump is necessarily expensive and requires skilled attention. Despite its novelty and ingenuity it is already practically obsolete.

Ck. Diffusion Pumps

§ (41) Diffusion Pumps.—In *Fig.* 30 (*a*) let H be the H.P.V. in which is maintained a constant pressure, L the L.P.V. to be evacuated. Let X be a vessel in which some liquid can be heated, while H, but not L, is cooled so as to condense its vapour. If the liquid is heated to a temperature at which its vapour pressure P is large compared with p_H, a continual stream of vapour will pour along the tube XMH, driving the gas before it and condensing in H; if the stream is sufficiently rapid the gas in H will be unable to diffuse back into the tube against it. On the other hand, the gas in L will diffuse out into the vapour stream and be carried by it into H. For this diffusion is not opposed by a vigorous stream in the contrary direction; since L is not cooled, the vapour will not condense in L, and vapour will enter it only at a rate sufficient to replace the gas diffusing out. Accordingly, after some time L will be completely evacuated of gas and contain only vapour. If L is now cooled, the vapour will condense and an almost perfect vacuum be left in L. The vacuum will not be quite perfect because some gas from H will diffuse back against the stream of vapour, however low is p_H and however rapid the stream; but a consideration of the magnitudes involved will show that the residual pressure could easily be made inappreciable.



(*a*) (*b*)

Fig. 30.

Such is the principle of the diffusion pump in its simplest and ideal form. In practice it is impossible to maintain the whole of L (the apparatus to be evacuated) at or above the temperature of the boiling liquid during the evacuation. L as well as H is cooled sufficiently to condense the vapour, and consequently if the simple arrangement of *Fig.* 30 (*a*) were adopted, the diffusion of gas out of L would be opposed by a vigorous stream of vapour entering; if the gas from H could not diffuse against the stream neither could the gas from L; there would be no pumping. Some device, therefore, must be adopted to prevent a stream of vapour entering L.

§ (42) Gaede Diffusion Pump.—The device originally adopted by Gaede (19) was to place in the tube leading to L an obstruction with a very small opening. If the linear dimensions of this opening are small compared with the mean free path of the molecules, the laws of the flow of gas and vapour through the opening are not those of hydrodynamical streaming, but those of diffusion. The flow depends on the partial pressure of the constituents of the mixture and not on their total pressure. Since the partial pressure of the gas in the tube XMH is zero, the gas will diffuse out through the entering vapour in spite of the fact that the total pressure of the vapour is greater in the tube than in L. The problem can be treated exactly by molecular theory. If d is the diameter and σ the area of the opening at M, λ the mean free path, ρ_0 the density of the gas at a pressure of 1 dyne per cm.^{2}, then the volume of the gas, measured at p_L, issuing through M per sec. is given by

$$S = \alpha\frac{\sigma}{\sqrt{2\pi\rho_0}}; \quad . \quad . \quad . \quad (34)$$

where

$$\frac{\alpha}{2} = x^2 - e^{-1/x}(x + x^2), \text{ and } x^2 = \frac{8\lambda^2}{d^2},$$

α attains the maximum 1 when d/λ is small; but σ decreases with d. The maximum value of S, when λ is fixed, is given approximately by $d = \lambda$. This maximum will increase with λ, which, since the vapour pressure of the liquid must be greater than p_H, is limited by p_H. Accordingly the speed of the pump depends greatly on the auxiliary vacuum, and also on the temperature of the liquid. For if P is too small, gas will diffuse back from H; if it is too large, the diffusion of gas from L will be hindered by the opposing flow. The conditions in the pump need therefore careful adjustment. On the other hand, S is independent of the pressure of the gas and dependent only on its nature and temperature: this is the most striking feature of all diffusion pumps. S is greater for the lighter gases; the variation of S with the nature of the gas is the contrary of that for the molecular pump.

§ (43).—Any liquid could be used in a diffusion pump, so long as it could be maintained at the appropriate temperature. Actually mercury is used, for the appropriate temperature is convenient (P = 0·3 mm. at 100° C.);

moreover it is chemically stable and does not wet glass. But its universal adoption is probably due in part to the previous association of mercury with air-pumps—an association based on quite different properties.

The vapour of the liquid used in the pump at the pressure corresponding to atmospheric temperature is left by the pump in L. It is easily removed from the apparatus connected to L by making the connection through a trap cooled in liquid air. The introduction of the cooled trap involves, of course, a continual stream of vapour opposing the diffusion of the gas through L; but at atmospheric temperatures the vapour pressure of mercury is so low that the consequent diminution in the speed of the pump is inappreciable. However, Gaede (19) has pointed out that the existence of this stream causes a slight error, appreciable at the lowest pressures, in the measurement of the pressure in L by a McLeod gauge. Since vapour is streaming from the gauge to L, the pressure of the gas in L is slightly higher than its pressure in the gauge.

§ (44).—The original diffusion pump of Gaede involved complicated glass construction: since it is no longer used, it need not be shown. The maximum value of S obtainable was about 80 cm. and far below that of the molecular pump at the higher pressures. On the other hand S was, as theory predicts, independent of the pressure down to the limits of measurement; at pressures less than 10^{-6} mm. the diffusion was as good as the molecular pump, and no practical limit to the pressure was set by the diffusion of gas from H against the vapour stream.

Moreover, it should be observed that there is nothing in the principle of the pump to limit its use to very low pressures, except the condition that $d=\lambda$: if openings as small as the free path could be obtained at atmospheric pressure the pump would work. Gaede has actually used the pump at atmospheric pressure, taking the pores of an earthenware pot as the openings and steam as the vapour; but since the pores are backed by very fine tubes, through which the gas has to flow before it arrives at the pump, the speed of such a pump is very slow; it is not generally of practical use.

§ (45) **Langmuir "Condensation" Pump.**—A simpler and more efficacious method of preventing the flow of vapour into L is to use the inertia of the stream to carry it past the opening. Thus in the modification shown in *Fig.* 30 (*b*), if the velocity of the stream of vapour issuing at C is as great as the velocity of the molecules in the stream, all the vapour will travel forward till it meets the walls of the outer tube or the gas in H; none will stream towards L and prevent the diffusion of gas from L, although the pressure in the vapour stream, as measured by its density, may be very much greater than the pressure in L. (It will be seen that the construction is similar to the gas injector pump Be, but the principle of action is different. The gas from L diffuses against the hydrostatic pressure; it does not flow with it.)

If the walls of the outer tube were heated by the vapour, the liquid condensing on them would have a vapour pressure greater than that in L; there would be a flow of vapour from the heated walls towards L, which is cool, and this stream would once more hinder the diffusion of gas from L. Accordingly Langmuir (20), who first used this arrangement, laid great stress upon the cooling of the walls struck by the vapour stream; he insisted that the vapour must be immediately condensed to the temperature prevailing in L, so that there should be no flow of the vapour back towards L. On account of the importance attributed to this condensation, he termed his pump a "condensation" pump to distinguish it from Gaede's diffusion pump; but it is equally a diffusion pump in the sense that the gas from L follows the gradient of partial pressure, not that of total pressure. Gehrts (21) has pointed out that Langmuir's principle was anticipated by Magnus (22), who did not see its applications.

It appears, moreover, that though the very efficient cooling of the walls and the complete condensation is necessary to the most efficient working of the pump, it is possible to make a pump of this type with much less efficient cooling. This is achieved in Crawford's parallel jet pump (24). But in its working characteristics this pump resembles the diffusion rather than the condensation pump, and has not the advantages of Langmuir's pump noted below.

§ (46). — The construction of Langmuir's pump in metal (25) is shown in *Fig.* 31; it can also be made without great complication in glass (20). The mercury M is maintained at a temperature of about 100° C. by the expenditure of about 300 watts, supplied electrically or by a burner. The baffle B deflects the vapour stream downwards and against the walls cooled by the water jacket J. The L.P.V. is connected to L; the auxiliary pump to H. If this pump maintains a pressure p_H of 0·01 mm. or less, S is as great as 3000-4000 cm.^{3}/sec. and is, as before, independent of p_L down to the lowest observable pressures. Higher pressures p_H decrease the speed, but the pump will work even if the pressure is nearly 0·1 mm. The speed is independent of the temperature of the mercury, so long as this is above a limit, which is greater the greater is p_H. The great speed of the pump—greater even than the maximum of the molecular pump—and the absence of any need for the accurate control of temperature, are the advantages that have caused condensation pumps to replace wholly the original Gaede type.

Many variations on the original Langmuir design have been made. In some of them (26, 27) the heating of the mercury is effected by an arc maintained between two mercury surfaces inside the pump; in fact this arrangement has been applied to large mercury-vapour current rectifiers (28), so that the rectifier acts as its own high-vacuum pump, only an auxiliary pump being needed. Again, it has been proposed (29, 30, 31) to combine in the same apparatus a mercury-vapour jet pump and a condensation pump, using the same stream of mercury vapour. The two act in series, and the combination will work with an auxiliary vacuum of 10-20 mm.; but the construction is complicated.

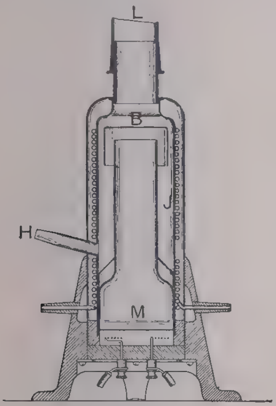


FIG. 31.

However, special reference need be made only to one type of this pump, remarkable for its simplicity. It is found possible to dispense altogether with the inner tube in *Fig.* 30 (*b*) and to make the arrangement of *Fig.* 30 (*a*) act as a condensation pump by merely cooling the walls of the horizontal tube. Since the mercury molecules striking the cooled walls do not rebound therefrom, if the cooled tube is made sufficiently long, all the molecules emerging from its end at M will be moving parallel to the length of the tube and will not enter the side tube. One form of such a pump is described in (32). An even simpler construction is shown in *Fig.* 32 adopted, the whole being made of sheet metal. The limit of p_H at which the pump will work is about 0·015 mm., and somewhat higher than that for the Langmuir type. But the value at which the maximum speed is obtained is not very different; this maximum is about 1500 cm.³/sec., and is set by the dimensions of the inlet tube (see below) rather than by the pump itself. The auxiliary vacuum of 0·01 mm. for this or the Langmuir pump can be obtained by oil-pumps, and presents no difficulty. The simple pump of *Fig.* 32, as well as the Langmuir and other more complicated pumps, is used on the industrial scale in the manufacture of thermionic valves and other high-vacuum devices.

§ (47) HIGH-VACUUM TECHNIQUE.—It has been pointed out that there must be some limit to the pressure reached by a diffusion or condensation pump, determined by p_H and the speed of the vapour stream. When a high-vacuum pump is used the pressure attainable and the speed of pumping are actually limited by factors other than the efficiency of the pump. In the first place, the tubes connecting the pump to the apparatus offer a resistance to the flow of gas. Knudsen (16) has shown that the volumetric speed of a perfect pump is given by

$$S = \frac{1}{\sqrt{\rho_0} \, . \, R}, \qquad (35)$$

where R is a constant dependent of the form of the connecting tube and ρ_0 is the density of the gas at a pressure of 1 dyne/cm.².

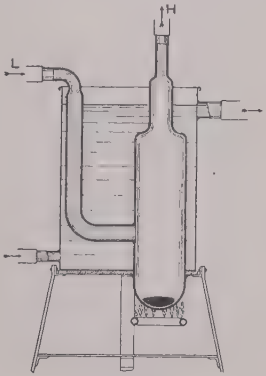


FIG. 32.

For a cylindrical tube of radius r and length L,

$$R = \frac{3L}{4\sqrt{2\pi}}. \qquad (36)$$

If r is expressed in millimetres, L in metres, then for air at 20°,

$$S = 1{\cdot}03\frac{r^3}{L}. \qquad (37)$$

S is 1000 cm.³/sec. for air flowing down a tube 1 metre long and about 1 cm. in diameter. Consequently to make full use of the speed of a condensation pump, connecting tubes not less than 2 cm. in diameter must be used;

if the evacuated vessel has to be sealed off eventually, the speed is often limited by the diameter of the sealing-off constriction. If this constriction is to be sealed off by a blowpipe it is difficult to make it more than 3 mm. in internal diameter; but it is possible by heating the tube very uniformly and by making the temperature gradient along it very steep (*e.g.* by a small electric furnace surrounding it) to seal off tubes 10 mm. or more in diameter.

§ (48).—Secondly, there is an evolution of gas from the apparatus being exhausted. Glass and metals in their ordinary condition give off large quantities of gas when exposed to a vacuum. The gas from glass is chiefly water and CO_2 which has been absorbed from the atmosphere and will be reabsorbed if the glass, having once been freed from gas, is exposed to the atmosphere once more. The gas from metals is largely hydrogen and carbon monoxide, absorbed from flame gases during manufacture and diffusing out from the interior. The evolution is greatly hastened by heat and, in metals at least, by making them the electrode of a discharge, even if it does not cause material heating.

To obtain a high vacuum, it is necessary to heat the glass while the vessel is exhausted to the highest temperature that the apparatus will stand without collapse; about half an hour at this temperature will liberate the gas from the surface, but there is a continual evolution at this temperature which is generally thought to result from an actual decomposition of the glass; this evolution stops when the glass is cooled, but if the cooling is too rapid some of the gas may condense on the glass during cooling and be evolved slowly again. For some purposes it is desirable to enclose the apparatus in a vacuum furnace, so that the external pressure of the atmosphere is removed and the glass can be heated for some time beyond the softening point without collapse. The metal parts must also be heated to near their melting point for several hours; this heating is effected in modern practice, either by making the metal the target of an electron bombardment from an incandescent cathode, or by exciting high-frequency eddy currents in the metal by coils surrounding the apparatus. Much time can be saved by heating the metals in a vacuum before they are introduced into the apparatus.

By long-continued treatment of this nature the evolution of gas can be stopped and a vacuum obtained which is perfect so far as the most delicate manometers can tell, and is maintained indefinitely if the vessel is gas-tight. But if the exhausted apparatus is sealed off from the pump, some gas is always introduced by this operation. For in order to soften the glass it must be heated above the temperature at which an inexhaustible evolution of gas starts. The gas thus introduced can be diminished by heating the sealing-off place to near its softening temperature for some time before sealing, and then completing the sealing as quickly as possible. Again much of the gas (chiefly water vapour) thus introduced disappears, being either absorbed by the glass, or "cleaned up" by a discharge subsequently passed through the vessel. But it seems that, whatever precaution is taken, the most delicate forms of manometer will always detect the presence of some gas in a vessel immediately after it is sealed off.

For further information on these points reference is made to (33), which is the best summary in English of the state of an art which is described in patents rather than in scientific journals. Some important devices are still kept secret.

D. MISCELLANEOUS METHODS OF EVACUATION

§ (49).—It remains to consider some other methods of evacuation which, though they do not satisfy any definition of a pump that would have been acceptable twenty years ago, satisfy, ideally at least, that given at the beginning of this article. Some of them are, and still more have been, of great practical importance; and they do not appear to differ more radically from the older conception of a pump (which implied a mechanical device with moving parts) than the diffusion pumps. Their chief modern use lies in the possibility they provide of evacuating a portable vessel sealed off from all fixed apparatus.

§ (50) CONDENSATION.—The pressure of the gas in a vessel can be reduced by cooling sufficiently any part of its walls. The limiting pressure obtainable by this means is, of course, the vapour pressure of the substance at the lowest temperature available. Savery, when he evacuated the cylinder of his steam-pump by condensing the steam with a jet of water, was using this principle. It has had some more modern applications, *e.g.* when a gas such as air has been removed by displacing it with CO_2 and then condensing the CO_2 in liquid air. The method is also used for compression. Chlorine, CO_2 and SO_2 have been compressed into containers by condensation in place of by compression pumps.

§ (51) CHEMICAL ACTION.—Gas can also be removed by causing it to react chemically with the formation of solid or liquid compounds. Gas analysis by "absorption" with liquid reagents employs this principle, but some developments of it need more special mention. Thus it has been shown (34) that metallic calcium heated to 700° C. will combine with most, if not all, gases, except those of the inactive group, to form solid compounds with low vapour pressures. But some of the compounds, especially the hydride, have considerable dissociation pressures at slightly higher temperatures, and the temperature of the metal must be carefully controlled. The method has its uses in special circumstances (*e.g.* where high vacua have to be maintained away from a laboratory or supply of liquid air). The alkali metals will also combine with all active gases; the combination is usually brought about by the electric discharge. It has long been known that a discharge passed with a cathode of sodium or potassium (more conveniently the alloy of the two) will remove the common gases down to the pressure where the discharge ceases. The latest development in this direction is absorption by heated thorium or zirconium (35).

§ (52) ABSORPTION.—But such chemical methods have been little practised since the discovery of the powerful absorption for gas of charcoal at low temperatures. From the discovery of the method by Dewar (36) to the invention of the molecular pump in 1913, it was the standard method of producing extreme vacua unattainable by liquid piston pumps. A glass or, preferably, silica tube containing a few grams of charcoal is attached to the vessel to be evacuated. The charcoal is heated during the preliminary exhaustion of the vessel, which should be carried to ·001 mm.; the vessel is then disconnected from the pump and the charcoal tube cooled in liquid air. If the vessel is large and the highest vacuum is required, two or more charcoal tubes may be attached, one being sealed off before the next is cooled.

Many experiments have been made on (1) the relative amounts of different gases which charcoal will absorb at different temperatures and (2) the absolute amounts absorbed by charcoal prepared in different ways. A full discussion of the results is beyond the scope of this article, and for fuller information reference may be made to a good summary in (37). As regards (1) it appears that, in general, gases are more absorbed the higher their boiling points, the exception being the inactive gases which are but slightly absorbed. The mass absorbed is proportional to the mass of the charcoal; it increases as the temperature is decreased and as the final pressure of the residual gas is increased; it is doubtful, therefore, whether a really perfect vacuum could be obtained by the method in ideal conditions, but, as with the diffusion pump, the actual limit lies beyond the range of measurement. The rate of absorption decreases greatly as the equilibrium pressure is attained, and, though the speed of evacuation is rapid compared with that of any piston pump down to ·0001 mm., it is probably slower than that of the Langmuir pump at lower pressures.

As regards (2), there is some conflict of evidence which has been only partially removed by the very complete study of the absorption by charcoal which resulted from its use in gas masks during the late war. In general the denser charcoal from the harder woods shows the greater absorption; the shell of the cocoanut and the kernels of some fruits are the best raw materials. The original coking should be at a temperature not exceeding 900° C., and must be followed by some process for the removal of residual hydrocarbons. For this purpose heating in a stream of chlorine at 800° followed by heating at the same temperature in hydrogen has been suggested; but the best modern practice appears to be alternate absorption of air or oxygen at atmospheric pressure and liquid air temperature with "out-gassing" of the absorbed gas by evacuation at 400°-500° C. Some writers maintain that all absorbed gases can be removed by heating to 600° C., others that heating to any temperature over 500° impairs the subsequent absorption. It has been found also that charcoal, activated by special processes, will produce high vacua even at atmospheric temperature.

Absorption, similar to that of charcoal, is displayed by other finely divided solids. In fact, all solids probably absorb some gas at all temperatures, the differences are merely of degree. Of the other solids proposed for practical evacuation, palladium black (which will absorb other gases as well as hydrogen) and finely divided copper may be mentioned. In addition reference may be made to the unusually great absorption of hydrogen by tantalum at atmospheric pressure. Here again reference may be made to (38).

§ (53) ABSORPTION IN THE ELECTRIC DISCHARGE.—In the early study of X-rays it was found that a hard tube often became "harder" by the passage of the discharge through it in consequence of a disappearance of part of the residual gas. This disappearance seems to be a normal accompaniment of the discharge; when it does not take place, or when the contrary process of an evolution of gas occurs, it is because the normal disappearance is obscured by an evolution of gas caused by heating or possibly by some other and distinct action of the discharge.

The facts concerning this absorption of gas are still obscure, and still more obscure the explanation of them. It is certain that the inactive gases are in general less absorbed than others, but whether and to what extent the nature of the electrodes and of the walls determine the absorption is not yet certain. Here reference will only be made to those actions of the discharge in "cleaning-up" gas which are of practical importance. A general reference may be given to (39).

The final evacuation of such apparatus as thermionic valves, rectifiers and X-ray tubes is probably effected by the discharge. However carefully the apparatus is evacuated by pumping, some gas is always introduced in sealing off from the pump. This gas is largely, if not entirely, absorbed by the walls and electrodes before any discharge passes; but during the first few moments of the discharge, which represents the normal function of the apparatus, some further change occurs which makes the "clean-up" more complete and more permanent.

It has long been known that the passage of the discharge between suitable electrodes would promote chemical actions which lead to the removal of gas. An instance is provided by the discharge between electrodes of the alkali metals, which has been already mentioned. Again, it has been found that a discharge passing to an electrode of charcoal would cause the charcoal to absorb at atmospheric temperature as it will absorb without the discharge at liquid-air temperature (40).

§ (54).—But the most practically important process of evacuation dependent on the discharge is that which involves the introduction of phosphorus vapour into the evacuated vessel. It appears to have been discovered first by Malignani (41); it was applied to the evacuation of electric incandescent lamps and has been used for the same purpose continuously since its discovery. At first it appears to have been thought that the action was chemical, but it is now known to be dependent on the passage of a discharge through the mixture of gas and containing phosphorus vapour.

The exact conditions which determine the disappearance of the gas are still obscure, but it may be stated generally that if a discharge is passed through any mixture of gas or vapours containing phosphorus vapour the pressure will be reduced more rapidly and to a lower limit than it would be if the phosphorus vapour were absent. The gas that has disappeared can be restored by heating the walls of the vessel to a temperature at which red phosphorus will evaporate. The latest theory of the action (42) is that the gas is deposited on the walls and covered with a "varnish" of red phosphorus produced by the action of the discharge from the phosphorus vapour, which prevents the re-evolution of the gas so long as the "varnish" remains; it is also supposed that the exceptional electrical properties of the phosphorus vapour are of importance. It is known that sulphur, iodine, and arsenic act in somewhat the same way as phosphorus in this matter.

In an incandescent lamp the necessary discharge passes between the opposite ends of the filament which act as electrodes; the cathode, being incandescent, gives a thermionic emission sufficient to abolish the cathode fall of potential and permit a discharge to pass even when the potential difference between the electrodes does not exceed fifty volts.

In Malignani's original method the phosphorus vapour was introduced by heating a small quantity of red phosphorus in the tube connecting the lamp to the pump just before sealing off. The later practice is to deposit the red phosphorus on the filament or the adjacent supports, whence it is evaporated as soon as the filament is heated. Nowadays it is also usual to deposit on the filament, together with phosphorus, salts such as fluorides or chlorides. Various benefits are attributed to the presence of these salts, but it seems to be agreed that their action is subsidiary to that of the phosphorus, and that the evacuation would not occur unless phosphorus (or one of the other elements mentioned) were present.

By this process of phosphorus evacuation, the use of high-vacuum pumps in lamp manufacture is rendered unnecessary. Even if the pump leaves residual gas at a pressure of 0·1 mm. in the lamp, almost all this gas will be removed in the first few seconds of "burning," and the pressure reduced to less than 0·001 mm. No mercury-pumps are now used in lamp-making; oil-pumps, usually of Class A*d*, are sufficient.

The same method has been applied to other commercial vacuum apparatus, *e.g.* rectifying valves of the old type without an incandescent cathode (43), and, more rarely, to the modern thermionic type.

G. E. C.

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Air Vessel Method of Level Indication. See "Meters, Liquid Level Indicators," § (15), Vol. III.

Aluminium, Atomic Heat of, at Low Temperatures: Nernst's values for, tabulated. See "Calorimetry, Electrical Methods of," § (11), Table VI.

Alundum. The trade name for a tubing composed of fused alumina (Al_2O_3) with a binding of fireclay and used as a protecting sheath for a thermoelement at temperatures up to 1550° C. See "Thermocouples," § (4) (iii.).

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Appold Brake. See "Dynamometers," § (2) (ii.).

Arches. See "Structures, Strength of," § (27)

Archimedean Screw. See "Hydraulics," § (33).

Archimedes' Principle. The resultant of the pressures acting on a body immersed in a fluid is equal to the weight of fluid displaced and acts upwards through the centre of gravity of that fluid.

Atomic Heat, Variation of, with Temperature. See "Calorimetry, the Quantum Theory," § (43).

Autographic Recording Apparatus: For use in Strength Tests of Materials. See "Elastic Constants, Determination of."
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  Dalby's Optical Recorder. § (61) (v.).
  General Methods adopted for Design of. § (59).
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  Riehle Autographic and Automatic Apparatus. § (60).

Automobile Engine, The Vauxhall. See "Petrol Engine, The Water-cooled," § (6) (ii.).

Avogadro's Law. At any one temperature and pressure, equal volumes of different gases contain the same number of molecules. While exact for "perfect" gases only, it is approximately true of real gases. See "Thermodynamics," § (66).

— B —

Bailey Steam Meter. See "Meters for Measurement of Steam," § (20), Vol. III.

Balancing. See "Engines and Prime Movers, Balancing of."
  Balancing of Driving Wheels of Locomotive. § (2) (i.).
  Balancing of Four-cylinder Engines: Inclusion of Valve-gear. § (10) (i.).
  Balancing of Frame Forces. § (2).
  Balancing of Internal Combustion Engines. § (13).
  Balancing of Locomotives. § (12).
  Balancing in Practical Case. § (4).
  Balancing for Primary and Secondary Forces and Couples. § (10).
  Balancing of Reciprocating Masses. § (10).
  Balancing of a Rotor. § (8).
  Balancing of Yarrow Schlick Tweedy Engine. § (11).
  Centrifugal Couple. § (3).
  Couple Closure. § (5).
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  Deductions from Force and Couple Polygons. § (7).
  Force Closure. § (6).
  Four Masses on Four Arms along a Shaft. § (7) (*b*) and (*c*).

Motion of Connecting Rod. § (2) (iii.).

Motion of Mass in a Circle at Uniform Speed. § (2) (i.).

Motion of Mass in a Straight Line at Varying Speed—Bennett's Construction. § (2) (ii.).

Primary Balancing. § (10) (i.).

Salter's Method of treating this Problem. § (8).

Secondary Balancing. § (10) (ii.).

Special Construction for balancing of Four Masses. § (9).

The equation $\mathrm{M}=\dfrac{\text{couple closure}}{d\mathrm{R}w^2}$. § (5).

The equation $\mathrm{M}=\dfrac{\text{force closure}}{\mathrm{R}w^2}$. § (6).

Three Cranks at 120° cannot be designed so that Masses mutually balance. § (7) (*a*).

Ball-bearings: Steps in the Evolution of the Modern Ball-bearing. See "Friction," § (38).

Barnes' Table of Specific Heats of Water at various Temperatures. See "Mechanical Equivalent of Heat," § (7).

Bartoli and Stracciati's Determination of Specific Heat of Water at various Temperatures. See "Mechanical Equivalent of Heat," § (7).

Barus, 1889, compared gas-thermometers with secondary standards of temperature in the range 500° to 1600° and recognised the importance of a uniform temperature distribution about the gas-thermometer bulb for purposes of high-temperature measurement. He introduced the thermo-element in the rôle of intermediary between the gas-thermometer bulb and the temperature to be measured. See "Temperature, Realisation of Absolute Scale of," § (39) (viii.).

Beams, Bending of: Macaulay's Method for several Loads. See "Structures, Strength of," § (10).

Beams: Relation between Load, Shear, Bending Moment, Slope, and Deflection. See "Structures, Strength of," § (12).

Beau de Rochas: Cycle. See "Engines, Thermodynamics of Internal Combustion," §§ (34) and (51).

Beaufoy's Experiments. See "Ship Resistance and Propulsion," § (3).

Becquerel, 1863, compared gas-thermometers with secondary standards of temperature in the range 500° to 1600°. See "Temperature, Realisation of Absolute Scale of," § (39) (iv.).

Bell-Coleman Refrigerating Machine. Used for cold stores and the holds of ships. Air is the working substance used. See "Refrigeration," § (4).

Bend Tests:

Alternating Bend Test beyond the Yield-Point. See "Elastic Constants, Determination of," § (78).

Description of the various kinds of Bend Tests for Metals. See *ibid.* § (29).

Forms of Specimen and Methods of Testing. See *ibid.* § (31).

Bending of Members of a Structure. See "Structures, Strength of," § (6).

Benzene, Latent Heat of Evaporation of, determined by Griffiths and Marshall. See "Latent Heat," § (10).

Bernoulli's Theorem. Along any stream line in a liquid subject only to gravity

$$p+g\rho z+\tfrac{1}{2}\rho v^2=\text{constant},$$

p being the pressure at a point at a depth z below the plane of reference, ρ the density, and v the velocity.

Bjerrum's Calculations from Volumetric Heat Figures. See "Gases, Specific Heat of, at High Temperatures," § (5).

Black Body, invented by Wien and Lummer for the investigation of the laws of radiation from a uniformly heated enclosure: description of modern form of. See "Radiation, Determination of the Constants, etc." I. § (2) (i.), Vol. IV.

Blade-width Ratio for a Screw-propeller is the fraction

$$\frac{\text{Maximum width of blade along its surface}}{\text{Radius of propeller}}$$

See "Ship Resistance and Propulsion," § (41).

Blading in Steam-turbines, Form and Efficiency of. See "Turbine, Development of the Steam," § (3); "Steam-turbine, Physics of," § (6).

Blowers, Theory of. See "Air-pumps," § (1).

BOMB CALORIMETERS

§ (1) **Introduction.**—The laboratory method of determining the calorific value of a fuel is to burn a known weight of a carefully dried sample in a vessel containing oxygen. From the temperature rise of the water in the calorimeter the heating value of the fuel is computed, taking into account certain corrections which are described later. Although the calorific value does not give all the information desired concerning a particular fuel, or determine its suitability for a specified purpose, yet it is generally accepted that the heating value is the most important property to be considered in estimating the value of fuel. Purchasers of large quantities of coal now adopt the heat-unit basis of evaluation, and the technique of combustion calorimetry has been so well developed that a skilled operator can average thirty-five determinations per day.

Two types of apparatus are employed for such tests. In one the fuel sample is burned under normal atmospheric pressure in a calorimeter of the "submerged bell" type,[1] whilst in the other the fuel is burned under high pressure in a "bomb" type of calorimeter. Some authorities prefer the "bell" type to the "bomb" because in it the combustion is carried out in oxygen at nearly atmospheric pressure and, consequently, the conditions resemble those obtained in steam boiler practice. With a "bomb" calorimeter the combustion is almost instantaneous and resembles an explosion in its violence and rapidity.

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[1] See "Coal Calorimeter."

The decomposition products of coal vary somewhat, and it is generally found that the results obtained with the bomb calorimeter are slightly higher than those with the "bell." For scientific work, however, the bomb type is universally used, since under good working conditions the combustion obtained is practically complete. In skilled hands either method gives reliable and concordant results for solid fuels, but the "bomb" is the only method applicable to liquid fuels.

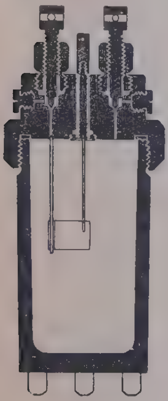

FIG. 1.

§ (2) DESCRIPTION OF A BOMB CALORIMETER OUTFIT.—The calorimetric outfit consists of the following elements:

(i.) The bomb.
(ii.) The calorimeter vessel, stirrer, and constant temperature jacket.
(iii.) The temperature measuring instrument.

(i.) *The Bomb.*—In one of the oldest forms of apparatus — the Mahler-Donkin — the bomb consists of a massive gun-metal cylinder provided with a cover held down by three studs. The cover is provided with a milled-head screw valve for regulating the inlet of oxygen to the cavity inside the bomb. The joint between the bomb proper and its cover is effected by means of a lead washer inserted in a circular groove. The inside of the cover has a projecting ring which registers with this groove when it is screwed down. The bomb is plated inside with gold in order to withstand the corrosive action of the nitric and sulphuric acids produced by the combustion of the fuel. The most satisfactory form of lining is that of platinum, but nowadays it is not much used on the score of expense. Porcelain enamel is also sometimes used for lining the bomb.

The Kroeker type of bomb has a cover screwed on to the bomb (see *Fig.* 1). The bomb is made of steel and has a fixed platinum lining, while the cover is of bronze.

Parr[1] has recently designed a bomb of an acid-proof base-metal alloy which appears very promising.

Some investigators employ a replaceable lining, but in practice it is found difficult to maintain a perfect fit and, consequently, difficulties arise owing to leakage of the products of combustion into the space behind the lining where it corrodes the metal of the body of the bomb.

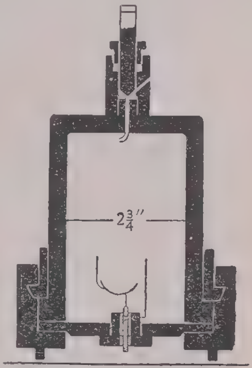

FIG. 2.

The staff[2] of the U.S. Bureau of Mines have developed a form of bomb (see *Figs.* 2 and 3) in which the lid is held in position by a novel form of sealing device. This consists of a tough steel receiving nut and lock so constructed that less than a one-eighth turn with the wrench suffices for sealing. A circular gasket of electrician's solder effects the seal. This locking device is an adaptation of the principle used in the breech locks of artillery. They claim for this design durability, ample strength, and facility of manipulation. Further, when the lock wears out a new one can be substituted without the expense of making and gold-plating a new shell. The shell of the bomb is made of Monel metal which is well adapted to gold plating.

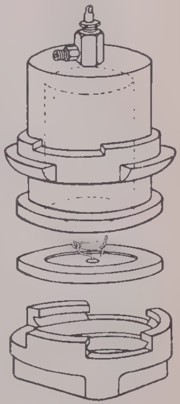

FIG. 3.

Féry has devised a bomb calorimeter in which the heat developed is shown on an indicator. See article on "Calorimetry, Method of Mixtures," § (13) (ii.), "Metallic Block Calorimeters."

(ii.) *The Calorimeter, Stirrer, and Constant*

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[1] "An Acid-resisting Alloy to replace Platinum in the Construction of a Bomb Calorimeter," *Journ. Am. Chem. Soc.*, Nov. 1915, xxxvii. 2515-2522. A test of the above by R. H. Jesse, Jr., *Eighth Int. Cong. Appl. Chem.*, 1912, i. 233, 389.

[2] "A Convenient Multiple-unit Calorimeter Installation," by J. D. Davis and E. L. Wallace, *Bureau of Mines Technical Paper*, 91, 48 pp., Washington, 1918. Abstract in *Engineering*, Jan. 10, 1919.

Temperature Jacket.—In the outfits employed in this country the calorimeter, stirrer, and jacket are similar to those employed in apparatus for ordinary calorimetric experiments by the Method of Mixtures (see § (4)).

At the U.S. Bureau of Mines[1] a form of apparatus has been developed which is especially adapted for combustion calorimetry (see *Fig.* 4).

The calorimeter is made of heavy sheet brass reinforced at the top and middle by brass bands (see A). A tubular stirrer well is soldered to the calorimeter as shown in B. An electrode is fastened to the bottom of the calorimeter, but insulated from it, which makes contact with the bomb plug when this is placed in position.

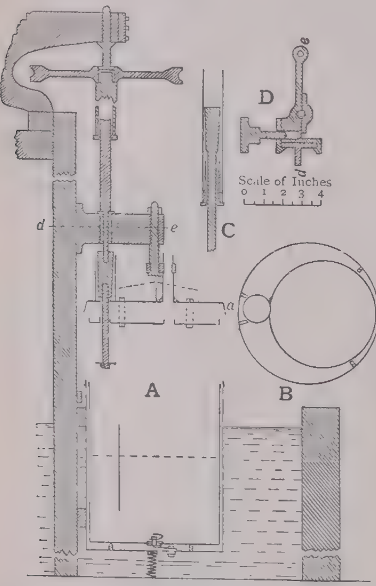

Fig. 4.

Details of Calorimeter: A, vertical section; B, horizontal section through *ab* showing tubular stirrer well; C, shaft clutch; D, horizontal section through *de* showing lid bracket.

The calorimeter is supported in its jacket on three ivory studs. The jacket is a cylindrical vessel provided with a cover of brass ground to a water-tight fit. This cover is provided with a thin sheet-brass water seal (see *Fig.* 4) fixed to the cover proper by means of three thin insulating rods of ivory in such a manner that when the calorimeter is in place and the cover brought down snugly the water seal is in contact with the surface of the water in the calorimeter, thus serving as an effectual seal against evaporation into the space between the calorimeter and its jacket.

Soldered to the jacket are two heavy brass lugs, by which it is held to the vertical brass T-bar of the frame in such a manner as to permit the jacket to slide vertically. The jacket is supported by a heavy helical brass spring bearing against the bottom of the tank, and of such strength that the jacket when charged is held vertical with its top slightly above the surface of the tank water against an adjustable stop fixed to the T-bar.

The stirrer shaft arrangement is also shown in *Fig.* 4. The upper part of the shaft on which the driving wheel is mounted consists of a thick-walled brass tube, into which the lower part of the shaft telescopes, the latter being provided with a conical piece (C) screwed to the end which engages a receiver at the lower end of the shaft tube after the manner of a conical friction clutch when the whole calorimeter is lowered into place. The lower bearing of the stirrer shaft is carried by the lid bracket (D), which is held to the vertical T-bar of the frame by a clamp which permits of raising and lowering the lid and clamping in any desired position.

The calorimeter cover has tubular outlets (not shown in *Fig.* 4) for the thermometer, the stirrer shaft, and electrical leads, so that the jacket and its cover may be totally immersed in the tank water during an experiment.

In the equipment of the Bureau six such outfits are mounted in one thermostatically controlled constant temperature bath.

(iii.) *The Temperature Measuring Instrument.*—With the majority of bomb calorimeter outfits mercury thermometers are employed for the measurement of the temperature rise of the water. Such instruments have the advantages of simplicity, cheapness, and moderate accuracy. They have the disadvantages of considerable lag and lack the sensibility required for work of the highest precision. The mercury thermometer, moreover, possesses the serious drawback that the mercury often sticks in the bore, particularly with a falling meniscus. This trouble can be somewhat alleviated by tapping the stem, and some observers utilise a miniature electric buzzer for this purpose.

The thermometer should preferably be a solid stem type with its scale divided to ·02° C., and the scale divisions should extend down to the bulb to avoid uncertainty as to the magnitude of the emergent column.

For work of the highest precision a calorimetric resistance thermometer is really essential, and a description of suitable instruments for the purpose will be found in the article on "Resistance Thermometers," § (6 (i.)).

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[1] Davis and Wallace, *loc. cit.*

§ (3) METHODS OF CONDUCTING A TEST. (i.) *Calibration of the Apparatus.*—It is necessary to determine the heat capacity of the bomb and its fittings by experiment, since it is rarely possible to calculate this constant from the specific heat of the materials employed in its construction. There are two standard methods of effecting this calibration: (*a*) By an electrical method based on the input into the calorimetric system of a known amount of heat measured as electrical energy. (*b*) By burning substances of known heating value.

(*a*) The electrical method is capable of considerable accuracy, since electrical energy can be measured with high precision. In practice, however, the method suffers under the disadvantage that it involves elaborate equipment and is time-consuming. Further, it is by no means easy to reproduce with it the same conditions as prevail during a combustion test. The electrical method is generally adopted in standardising laboratories, but for the purpose of a works laboratory the second method is to be preferred.

(*b*) To calibrate a calorimeter by means of standard substances, such as naphthalene or benzoic acid, it is advisable to make about half-a-dozen combustions, sufficient amounts of the standard being used to produce about the average temperature rise obtained in tests with coal. This method, besides being simple and easy of application, tends to minimise errors such as those due to thermometer calibration, cooling correction, heat input from stirring, etc. Dickinson[1] has recently redetermined the heat combustions of the following substances: naphthalene, benzoic acid, and sucrose or cane sugar, with a view to their adoption as standards in calibration work. His results are summarised in Table I. together with those of previous observers.

It appears from a comparison of the values given by different observers for the same substance that benzoic acid is the most suitable in view of the close agreement of the results obtained.

Dickinson found naphthalene to be a convenient material to work with, but care was necessary in handling since a gram briquette would lose about 1 milligram in weight per hour by sublimation. Sucrose did not seem so well adapted as benzoic acid for standardisation purposes. It has a smaller heat of combustion and frequently fails to ignite.

Dickinson suggests that the higher results given by other observers for sucrose may be due to the fact that they may not have corrected for the heat generated in the firing wire before the sample ignites. With sucrose a greater length of fuse wire has to be used than with other materials on account of its lower inflammability.

Iron wire is frequently employed because it burns instead of only melting, and is therefore more certain to ignite the sample. The heat of formation of iron oxide is about 1600 calories per gram of iron. The sucrose specimens required about 3 cm. of wire weighing 132 mg. per metre. Hence the correction for the heat liberated in its combustion amounted to about 2 calories per centimetre. Naphthalene ignited readily with 1 cm. of wire.

He also corrected for the small amount of nitric acid formed from the nitrogen contained in the oxygen. The amount is nearly proportional to the heat liberated in the combustion and to the percentage of nitrogen present, and was determined by titration after each combustion. The heat of formation of HNO_3 from $N+O+H_2O$ is about 230 calories per gram of acid. As the oxygen employed contained from 0·3 to 0·5 per cent of nitrogen, the correction to be applied for the heat of formation of HNO_3 was usually about 1 part in 1000.

TABLE I

Authority.	Naphthalene.	Benzoic Acid.	Sucrose.	Date.
Stohmann, Rodatz, Hertzberg	..	6315	..	1887
Berthelot, Vieille .	..	..	3962	1887
Berthelot, Lunginin	..	6322	..	1888
Berthelot, Recoura	..	6345	..	1888
Stohmann, Kleber, Langbein	9628 9619	6322	..	1889
Stohmann, Langbein	..	..	3955	1892
Atwater and Snell .	..	..	3959	1903
Fischer and Wrede	9641	..	..	1904
Fries	.. ..	6334 6318	.. ..	1907 1910
Fischer and Wrede	..	6325	3952	1909
Wrede	9633	6323	3952	1910
Roth	9643	..	..	1910
Leroux	9631	..	..	1910
Dickinson . . .	9612	6323	3945	1910 1912

The data in the above table are expressed in terms of the 15° calorie.

§ (4) PREPARATION OF THE TEST SAMPLES.[1] (i.) *Solid Fuels.*—It is necessary to convert the coal into a small briquette or tabloid for the purposes of test. If the attempt is made to employ the sample in powder form the force of the explosion usually blows some of it against the internal walls of the bomb and it escapes combustion.

Bituminous fuels as a rule will form briquettes by pressure alone. In cases where insufficient tarry matter is present in the natural fuel, just sufficient of a 1 per cent solution of gum arabic may be used to make the particles of fuel adhesive. For half a gram of fuel, three drops of such solution are enough. The briquettes must be heated in an air bath to 110° C. for at least four hours to expel the last traces of moisture thus introduced before testing in the calorimeter.

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[1] "Combustion Calorimetry," *Bull. Bur. Stds.*, 1915, xi. 253.

[1] *The Times Engineering Supplement*, Feb. 23, 1917.

(ii.) *Liquid Fuels.*—In weighing and transferring the liquid fuel to the bomb, carriers consisting of small cylindrical blocks of pure cellulose are used, one of these blocks being able to absorb several times its own weight of any ordinary fuel oil. The saturated block, after being weighed, is burned under the conditions and with all the precautions necessary for solid fuel, the only difference being that a rather higher pressure is used. in order to obtain a greater supply of oxygen gas in the bomb. A blank test with the cellulose alone gives the necessary data for the calculations. As liquid fuels contain only traces of acid-forming elements no trouble arises from corrosion, and a bomb provided with a gold lining will last for some hundreds of tests.

As regards the special precautions necessary to obtain correct results when testing liquid fuels, it must be pointed out that the absorbent cellulose blocks sold for this purpose absorb moisture as well as oil, and that it is necessary to dry them before use for one or two hours in the air-bath at 100° C. When saturated with heavy oils of high boiling-point they are also somewhat difficult to ignite, and it is advisable to place a little of the dry unsaturated cellulose in a loose condition around the platinum ignition wire in order to avoid failure of the test from this cause. As the cellulose blocks are large in proportion to their weight and absorbent capacity, a larger platinum dish will be required than for the tests with solid fuel, and the platinum ignition wire should be arranged to hold down the cellulose block lest the explosive violence of the combustion blows it out of the dish.

§ (5) QUANTITY OF OXYGEN REQUIRED.—The quantity of oxygen required is about three times that which will unite with the charge to give complete combustion. Dickinson found that, when the amount of oxygen was much less than two and a half times that required to unite with the combustible charge, there were often cases of incomplete combustion as indicated by a reduction in the total heat liberated, as well as by the occasional presence of a slight amount of soot and by the odour of the products of combustion.

Since the usual pressure employed in routine tests is 20 to 27 atmospheres, or 300 lb. to 400 lb., it is advisable to have a small back-pressure valve inserted in the milled-head screw in the bomb cover in order to avoid a great loss of gas when disconnecting the oxygen supply pipe and gauge from the bomb, after filling the latter with oxygen. At these high pressures the combustion of the coal is practically instantaneous, and the thin platinum wire used for ignition purposes will generally be found fused owing to the temperature momentarily attained. In order to protect the platinum capsule or crucible from the same effect, and from the action of the molten slag produced, it is necessary to line it with thin asbestos board, cut and shaped to fit the crucible or capsule. This asbestos board must be dried and ignited before use in order to remove all matter that might vitiate the results.

§ (6) CALORIMETRY BY COMBUSTION WITH SODIUM PEROXIDE.—Fusion with sodium peroxide is the only way known for finding the heat of oxidation of elements which do not burn in oxygen and which form oxides insoluble in acids. The method is adapted to the determination of the heat formation of the oxides of a metal and also the heat combination of metallic oxides with sodium oxide.

The method is indirect and the heat effect sought is not the observed effect; hence burning in compressed oxygen is preferable where possible. For example, when carbon is burned with sodium peroxide the observed heat (x) is the result of the following reaction:

$$2Na_2O_2 + C = Na_2CO_3 + Na_2O,$$

and x equals the heat of formation of carbon dioxide plus the heat of combination of carbon dioxide with sodium oxide, and less the heat required to separate two atoms of oxygen from two molecules of sodium peroxide; thus

$$x = C + 2O + (Na_2O + CO_2) - (2Na_2O + 2O),$$

so that

$$C + 2O = x - (Na_2O + CO_2) + (2Na_2O + 2O).$$

Moreover, many substances do not give with sodium peroxide sufficient heat to fuse the mixture, and hence some readily combustible substance, such as sulphur or carbon, must be added which gives in many cases the larger part of the total heat effect. Professor Mixter[1] of Yale University, who has made an extensive study of this method, gives the following results obtained by fusion with sodium peroxide for comparison with those by combustion in oxygen. They are:

Reaction.	Sodium Peroxide Method.	Combustion in Oxygen Method.	Reference.
$C+2O=CO_2$.	96·4	94·7	*Amer. Journ. Science*, xxix. 130; also xix. 434
$Ti+2O=TiO_2$.	215·6	218·4	*Ibid.* xxvii. 343
$3Fe+4O=Fe_3O_4$	267·5	265·2	*Ibid.* xxxvi. 55

Both values for $C+2O$ are for acetylene carbon. One reason for the higher value found in the sodium peroxide method is that the carbon and peroxide were mixed in a mortar, thus allowing the peroxide to absorb a little moisture which added to the heat of the fusion. The value 267·5 for $3Fe + 4O$ is derived from the results of fusions of iron, ferrous oxide, ferric oxide, and the mineral magnetite with sodium peroxide, and 265·2 was the result of burning iron in oxygen.

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[1] *American Journal of Science*, 1917, xliii. 27.

Sodium peroxide absorbs water rapidly from the air, consequently it should be exposed as little as possible, as the hydrated peroxide will give more heat with a combustible than the anhydrous. One of two samples, that which gives off the less oxygen when fused, is the better one. The error from the water content is small in good peroxide. Its effect is further diminished when carbon, for example, is added to make a mixture fuse, because in practice the heat effect of the carbon is found for the carbon and peroxide actually used.

Various substances may be added to a peroxide mixture to increase the temperature of the fusion. Mixter has used acetylene carbon, sulphur, and lamp-black. Pure rhombohedral sulphur in fine powder would appear to be the best of the three, but it becomes electrified when shaken in the bomb with the other ingredients and sometimes sticks to the bomb and is not completely oxidised. Sulphide is formed, and occasionally free sulphur is left. When the bomb is much blackened by a fusion with sulphur the heat result is low. Acetylene carbon is the ideal substance to use, but difficult to obtain.

One part of the carbon requires 13 parts of pure sodium peroxide for combustion, and it is best to take about 20 parts in determining the heat effect of the carbon or lamp-black. For the combustion of sulphur double the calculated amount of peroxide should be used. Oxygen is often evolved in a combustion from the action of an acidic oxide on the sodium peroxide, and the heat required to set it free from the peroxide is added to the observed heat. This correction, 1·73 g.-cal. for 1 c.c. of oxygen at 0° and 760 mm., is derived from Beketoff's $Na_2 + O = 100{\cdot}26$ Cal. and de Forcrand's $Na_2 + 2O = 119{\cdot}8$ Cal.

Apparatus employed for the Tests.—The bomb is made of sterling silver whilst the top and fittings are of brass.

The mixture under test is contained in a cup of fine silver supported in the bomb by its upper edge. A fusion in the cup cools more slowly than when in contact with the cold bomb, and hence the reaction is more complete.

The general arrangement of the apparatus is identical with that employed in fuel calorimetry with the addition of bulbs for collecting any oxygen set free by fusion. E. G.

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"A New Method of Ignition for Bomb Calorimeters," Roehrich, *Eighth Int. Cong. Appl. Chem.*, 1912, x. 269.

Bourdon Gauges, Calibration, Adjustment, and Correction for Temperature of. See "Pressure, Measurement of," § (11).

Boyle's Law on the variation of pressure with volume for a constant mass of gas states that

$pv =$ constant (at constant temperature).

See "Thermal Expansion," § (14) (ii.); "Thermodynamics," §§ (5), (59), (66); "Engines, Thermodynamics of Internal Combustion," § (13).

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[1] The above references have been selected from a more extensive list quoted by Dickinson.

Brake Blocks: Frictional Resistance of Brake Blocks. See "Friction," § (35).

Brayton Type of Internal Combustion Engine. See "Engines, Thermodynamics of Internal Combustion," §§ (29) and (52).

Breast-wheel. See "Hydraulics," III. § (47) (i.).

Bridge, Strength of: Example of Estimation of. See "Structures, Strength of," § (26).

British Thermal Unit (B.T.U.). The quantity of heat required to raise the temperature of 1 lb. of water 1° Fahr. See "Heat, Conduction of," § (2); "Thermodynamics," § (2).

B.T.H. Steam Meter. See "Meters for Measurement of Steam," § (18), Vol. III.

Bulging, Drifting, Flanging, and Flattening Tests for Copper and Brass Tube. See "Elastic Constants, Determination of," § (33).

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Cadmium, Specific Heat of, at various Temperatures; tabulated, with the atomic heat. See "Calorimetry, Electrical Methods of," § (10), Table V.

Calibration Corrections necessary in the Graduation of the Capillary Tube of a Thermometer. See "Thermometry," § (3) (*a*).

Callendar, maker in 1887 of a platinum resistance thermometer, the resistance of a particular specimen of platinum wire being directly determined at various temperatures up to 600° C. See "Resistance Thermometers," § (2).

Callendar's Equation, applied to the expansion of steam; and Tables.

$$v-b=\frac{\mathrm{RT}}{p}-c,$$

where $c=c_0\mathrm{T}_0{}^n/\mathrm{T}^n$. See "Thermal Expansion," § (24); "Thermodynamics," § (61); "Steam engine, Theory of," § (9).

Callendar and Barnes' Method of determining Mechanical Equivalent of Heat. See "Mechanical Equivalent of Heat," § (5) (iii.).

Callendar and Nicolson. Paper on exchanges of heat between steam and cylinder wall (*Min. Proc. Inst. C.E.*, 1897, cxxxi.). See "Steam-engine, Theory of," § (10).

Callendar and Swann. Determination, by the continuous flow method, of the specific heats of air and carbon dioxide at atmospheric pressure at 20° C. and 100° C. See "Calorimetry, Electrical Methods of," § (13).

Caloric Theory, Rumford's Attack on. See "Mechanical Equivalent of Heat," § (1).

Calorific Values of Fuels. See "Engines, Thermodynamics of Internal Combustion," § (68), Tables II., III., IV., and V.

Calorimeter:

An apparatus for the measurement of heat. Used in experiments by the method of mixtures, in which the substances under investigation are mixed. See "Calorimetry, Method of Mixtures," § (5) (i.).

Bunsen's Ice: an instrument in which the heat given out by a body in cooling from some higher temperature to 0° C. is obtained by observing the contraction which takes place in the change from ice to water produced by the heat given by the body. The observed volume change is converted into calories by assuming a value for the mass of mercury drawn into the instrument by the addition of one mean calorie of heat. See "Calorimetric Methods based on the Change of State," § (2).

Bunsen's Ice, Modifications of. See *ibid.* § (3).

Coal. See "Coal Calorimeter."

Constant of Bunsen's Ice: the mass of mercury drawn into the instrument by the addition of one mean calorie of heat: values summarised and tabulated. See "Calorimetric Methods based on the Change of State," § (2), Table I.

Dewar's Liquid Air and Hydrogen: a calorimeter based on an analogous principle to the steam calorimeter in which one of the liquefied gases is employed as calorimetric substance. See *ibid.* § (6).

Differential Steam, for the determination of specific heats of gases at constant volume. See *ibid.* § (5).

Gas. See "Gas Calorimeter."

Joly's Steam: an instrument in which the heat necessary to raise the temperature of a body from the air temperature to 100° is measured by determining the weight of steam which must be condensed into water at 100° to supply this heat. See "Calorimetric Methods based on the Change of State," § (4).

Liquid Hydrogen. See *ibid.* § (8).

Liquid Oxygen. See *ibid.* § (7).

Metallic Block Types of. See "Calorimetry, Method of Mixtures," § (13).

CALORIMETRY

Calorimetry is concerned with the measurement of energy in the form of heat. It constitutes one of the most difficult branches of exact measurements owing to the fact that a perfect non-conductor of heat does not exist.

The common method of measuring quantities of heat is by utilising the different effects of heat on materials such as the change of temperature or the change of state, but in recent years another method has come into extensive use, known as the Electrical Method. In this a definite and easily measurable amount of electrical energy is converted into heat, and the resulting change of temperature or state observed. The electrical method has many advantages when measurements of the highest precision have to be made on account of the facility with which the heat supply can be controlled.

In the brief review given in the following pages the appliances employed in heat measurements will be described, and then the theories which have been advanced to correlate the thermal data with other physical constants.

CALORIMETRY, ELECTRICAL METHODS OF

§ (1) **General.** — The electrical method of calorimetry was first employed by Joule with a view to the determination of the mechanical equivalent of heat (J). Subsequent work by Professors E. H. Griffiths, Schuster, Gannon, Callendar, and Barnes showed that the method was one capable of the highest precision for the determination of J. The article on the determination of the mechanical equivalent[1] of heat should be consulted for details of the method as applied to the determination of heat capacity of water and its variation with temperature.

In passing it might be mentioned that the calibration of bomb calorimeters is frequently carried out by electrical methods in which an equivalent amount of heat to that obtained in combustion is generated in the bomb and its amount measured by observations of the watts dissipated, the procedure being identical with that followed in methods for determining J.

In specific heat determinations the great convenience possessed by the electrical method lies in the fact that it permits of the determination of true specific heats, *i.e.* the specific heat over a very narrow range of temperature, and consequently it has been of immense service in determinations of the variation of atomic heats with temperature.

§ (2) **Specific Heat of Liquids by Electrical Methods.**—It is obvious that any of the appliances which have been devised for the evaluation of J are also applicable for the determination of the specific heat of liquids, and further that they would give data of the highest order of accuracy. There are, however, certain difficulties in practice.

Both Callendar and Griffiths applied their electrical methods for this purpose: the former determined the specific heat of mercury and the latter that of aniline.

§ (3) **Specific Heat of Mercury.**—The apparatus employed by Callendar,[2] Barnes, and Cooke for the determination of the specific heat of mercury is shown diagrammatically in *Fig.* 1. The calorimeter differs from that employed for the determination of J in that the flowing mercury is the conductor in which heat is generated electrically and not a fine platinum wire stretched along the axis of the tube as in the case of the J apparatus. A steady stream of mercury flows through the fine capillary tube and is heated by a carefully controlled electric current. The difference of temperature between the inflow and outflow is observed by means of a differential pair of platinum thermometers. The inflow and outflow tubes AB and CD are exactly similar, about 2 cm. internal diameter and 25 cm. long. They are connected by the fine flow tube BC of 1 mm. in bore and 1 metre in length, coiled up in the form of a short spiral 2·5 cm. in diameter. The inflow and outflow tubes are provided with two side tubes, one pair for conveying the current, and the other pair for the mercury flow.

A practical advantage possessed by the continuous flow method is the fact that the heat loss from the walls can be determined by making experiments with different rates of flow, but keeping the rise of temperature constant.

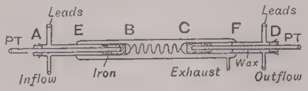

Fig. 1.

(i.) *Methods of determining the True Mean Temperature of Outflow.*—By far the most important practical detail in this method is the device adopted for obtaining the true mean temperature of the outflowing liquid. If a thermometer were merely inserted in the outflow tube, leaving a free space all round for the circulation of the liquid, it is evident that the heated liquid would tend to flow in a stream along the top of the outflow tube, and that the thermometer might indicate a temperature which had little or no relation to the mean temperature of the stream. It is easy to make an error of 20 per cent in this manner. A fairly uniform distribution of the flow might be secured by making the space between the thermometer and the outflow tube very narrow. But this leads to another difficulty in the case of mercury. As the space is narrowed the electrical resistance is increased, and an appreciable quantity of heat, which cannot be accurately estimated, is generated in the vicinity of the thermometers.

The difficulty was overcome in the mercury experiments by fitting the inflow and outflow tubes with soft iron cylinders, 6 cm. long, turned to fit the tubes and bored to fit the thermometers. The soft iron had a conduc-

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[1] See "Heat, Mechanical Equivalent of."

[2] *Phil. Trans.* A, 1902; *Phys. Rev.*, 1902, xv.

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tivity about ten times that of mercury for both heat and electricity. The heat generated by the current in the immediate vicinity of the thermometer bulbs was so small that the watts might fairly be calculated from the difference of potential between the iron blocks at the middle points of the bulbs. The mercury stream was forced to circulate in a spiral screw thread of suitable dimensions cut in the outer surface of the blocks, which prevented the formation of stream-lines along one side of the tube, and secured uniformity of temperature throughout the cross-section of the outflow tube. The high conductivity of the iron also assisted in securing the same result.

A precisely analogous device for averaging the outflow temperature was applied in the water calorimeter. The bulb of the thermometer was fitted with a copper sleeve of high conductivity, on the outside of which a rubber spiral was wound to fit the outflow tube as closely as possible. The accuracy of fit was found to be much more important in the case of water than in the case of mercury. The reason of this is that the thermal conductivity of water being 10 or 15 times less than that of mercury the accurate averaging of the outflow temperature is more dependent on the uniformity of the spiral circulation and the complete elimination of asymmetric stream-lines.

In order to obtain a perfect fit for the sleeves with their spiral screws it was necessary that the bore of the outflow tube should be as nearly uniform as possible and accurately straight. It was most essential that there should be no constriction at the points of junction E and F with the vacuum-jacket, and that the external portions of the tubes AE, FD should not be of smaller bore than the portions inside the vacuum-jacket, though it would not matter much if they were a little larger.

(ii.) *Correction for Variation of the Temperature Gradient in the Flow Tube.*—The elementary theory of the elimination of the heat loss in the steady-flow method of calorimetry assumes that, if the electric current and the flow of liquid be simultaneously varied in such a manner as to keep the rise of temperature the same, the heat loss by radiation, etc., will remain constant. The experimental results of Callendar and Barnes show that this condition is very closely satisfied in the method, and they calculated all the results of the investigation on this assumption. It was noticed, however, that there were small systematic divergences in the experimental verification for the small flows which, though amounting only to a few parts in 10,000, received careful examination as possible indications of constant errors.

So long as the distribution of temperature throughout the apparatus is accurately the same for the same rise of temperature, whatever the flow, the heat loss must also be identical. But if there is any systematic change in the temperature distribution with change of flow, then there must be a corresponding systematic difference in the heat loss, which will lead to constant errors in the calculation if no account is taken of it. A possible source of error of this type is loss of heat by conduction along the outflow tube. When the flow is large, the heated liquid passing along the tube will keep it nearly at a uniform temperature, so that the gradient in the outflow tube will be small and the conduction loss correspondingly minute. As the flow is diminished, supposing the temperature of the outflow to remain the same, the gradient in the outflow-tube must increase in proportion to the reciprocal of the flow, since the radiation loss remains nearly the same. The conduction loss will vary directly as the gradient, or inversely as the flow, for a given rise of temperature.

A small error of this kind, due to conduction, was detected at an early stage in the mercury calorimeter, owing to the large mass of mercury in the flow tube, the small rate of the flow, and the relatively high thermal conductivity of the liquid. It was practically eliminated by filling the greater part of the outflow tube from the end of the vacuum-jacket with paraffin wax, leaving only a small passage for the outflow of mercury. This made the conduction loss very small, and nearly independent of the flow.

§ (4) Variation of the Specific Heat of Mercury with Temperature.—The value of the specific heat of mercury in terms of water was calculated from the experimental data, taking the value of J equal to 4·1891 for a thermal unit at 15·5°, which was the temperature recommended by Griffiths at the Paris Congress in 1900.

The experimental results lead to the expression

$$S_t = S_0 - 1{\cdot}074 \times 10^{-5}t + {\cdot}00385 \times 10^{-5}t^2,$$

or $$\frac{S_t - S_0}{S_0} = -{\cdot}000321t + {\cdot}00000115t^2,$$

where $$S_0 = {\cdot}033458.$$

This gives for the temperature coefficient at any temperature t the expression

$$\frac{d}{dt}\left(\frac{S_t}{S_0}\right) = -{\cdot}000321 + {\cdot}00000230t,$$

and for the average change per degree at 50° the value $-{\cdot}0000069$.

The data obtained in the experiments are summarised in Table I., p. 35.

§ (5) Specific Heat of Aniline.—Professor E. H. Griffiths [1] determined the specific heat of aniline over the range 15° to 50° C. by means of an apparatus similar in its essential features

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[1] *Phil. Mag.*, Jan. 1895; *Proc. Camb. Phil. Soc.*, 1895, viii. part 4.

TABLE I

SPECIFIC HEAT OF MERCURY AT DIFFERENT TEMPERATURES

Mean Temperature.	Specific Heat.	Mean Temperature.	Specific Heat.
2·85	·033435	32·41	·033141
2·93	·033440	36·59	·033121
4·42	·033405	45·00	·033050
18·37	·033291	53·39	·032997
24·52	·033224	65·22	·032929
31·68	·033156	83·89	·032818
32·14	·033151		

to that shown in *Figs.* 5 and 6 of the article "Heat, Mechanical Equivalent of."

For experimental work in calorimetry aniline has various points in its favour. It has a low vapour pressure at ordinary temperatures, is a good electrical insulator, and has a low heat capacity.

On exposure to light it becomes discoloured, but no information is available to show whether this affects the thermal capacity.

For the variation of the specific heat with temperature Griffiths obtained the following equation:

$$S_1 = 0{\cdot}5156 + (t-20)\times{\cdot}0004 + (t-20)^2\times{\cdot}000002.$$

The agreement between this formula and the experimental results will be seen from the table below:

TABLE II

Temperature.	S_1 Experimental.	S_1 Formula.
15°	·5137	·5137
20	·5155	·5156
25	·5175	·5176
30	·5198	·5198
35	·5221	·5221
40	·5244	·5244
45	·5268	·5269
50	·5294	·5294
52	·5304	·5305

In the course of this work it was observed that the volume heat, *i.e.* the specific heat multiplied by the density, was practically constant over the range of temperature investigated as shown by the following results:

TABLE III

Temperature.	Specific Heat, S.	Density, *d*.	S × *d*.
15	0·5137	1·0257	·5269
20	0·5156	1·0218	·5269
30	0·5198	1·0130	·5267
40	0·5244	1·0040	·5267
50	0·5294	0·9955	·5270

§ (6) SPECIFIC HEAT OF OILS.—In some experiments in which it was desired to determine the specific heat of oils over a wide range of temperature the apparatus shown below (*Fig.* 2) was employed by the writer.

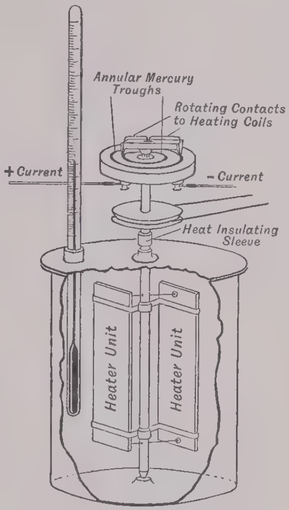

FIG. 2.

At room temperatures the oils were exceedingly viscous, and consequently it was necessary to employ somewhat unusual methods of ensuring that the contents were well mixed. The heating coils were arranged in the form of two flat paddles so that they were in continuous rotation through the oil; suitably disposed baffles further assisted the mixing of the contents of the calorimeter. It was necessary to lead the current in and out of the calorimeter by means of two annular troughs of mercury into which contact bars from the heating coil dipped.

§ (7) SPECIFIC HEAT OF LIQUIDS USED FOR REFRIGERATORS.—Osborn [1] has developed an apparatus suitable for the determination of specific heats and latent heats of the liquids commonly employed in refrigeration work, such as ammonia, CO_2, SO_2, methyl-chloride, and ethyl-chloride. Such determinations present greater experimental difficulties than are met with in work on liquids at ordinary pressure, since these materials have a vapour pressure varying from 1 to 70 atmospheres at the temperatures at which the thermal properties are of importance in engineering work. Consequently, in the design of apparatus for experiments of this character great attention has to be paid to details of construction.

(i.) *The Calorimeter.*—Briefly the arrangement is as follows: The material to be investigated is enclosed in a calorimeter with thick metallic walls of known thermal capacity (*Fig.* 3). Heat is applied electrically and the jacket is maintained at the same temperature by the usual adiabatic arrangement.

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[1] *Bull. Bur. Stds.*, 1917, xv. 133.

An air space between the polished nickel surfaces of calorimeter and jacket furnishes thermal insulation. Two tubes extend from the top of the calorimeter through the jacket and liquid to the outside air, terminating in valves. One of these tubes is intended for connection to pressure-indicating apparatus and the other for the introduction and removal of the material to be investigated.

Care was taken to avoid having heavy metal connections across the air space and by suitably distributing those connections which are necessary over the calorimeter surface, the part of the thermal leakage due to lead conduction was considerably minimised. Thermoelements indicate relative surface temperatures of the jacket calorimeter, 10 junctions being distributed upon each surface. This permits of control over the thermal leakage and the correction for such leakage as could not be avoided.

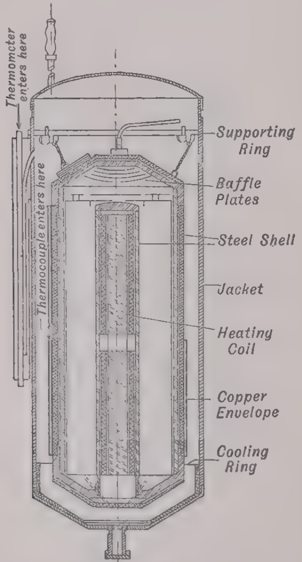

FIG. 3.—Section of Calorimeter and Jacket.

Thermojunctions placed upon the connecting tubes indicate the temperature of these tubes at several points relative to a point on the jacket and in this way the temperature of the vapour expelled during vaporisation experiments could be found.

The inside of the central tube in the calorimeter is accessible at the bottom for the introduction of the heating coil and thermometer. Upon the outside of this tube are fastened 12 radial vanes of tinned iron about 0·3 mm. thick, extending to within about 1 mm. of the surrounding cylindrical wall. These vanes are for the purpose of promoting the distribution of heat within the annular space containing the material under investigation. The vanes extend just above the top of the central tube. At this place are two flat circular baffle plates, separated about 2 mm. by three small steel studs. The lower plate is united to the tops of the radial vanes with tin. A central hole in the lower plate and several holes in the upper one between centre and outside furnish a tortuous passage for vapour coming from below. These two plates are intended to intercept any large drops of liquid which might be thrown up by vigorous boiling, should it occur, and also act as a thermal shield for the top of the calorimeter. A second set of four baffle plates of spherical contour separated by about 2 mm. are attached to the inside surface of the conical part of the calorimeter top. Each plate has a central hole and four slots at the edge so as to avoid trapping gas or liquid, but these passages are so sized and spaced that the main path through the plates is very tortuous, so as to make difficult the passage of liquid particles from below in a current of vapour being withdrawn through the outlets in the top. The entire inner surface of the steel shell and of the various plates within were all tinned, using pure block tin.

(ii.) *Method of Experiment.*—Two distinct methods of experiment were employed. In the first method the heat, added to a fixed amount of the substance under test confined in the calorimeter under saturation conditions, together with the resulting change in temperature, are measured. By using data for the specific volumes of the two phases and the latent heat of vaporisation, the heat lost in the vaporisation of the liquid is estimated and can be allowed for; thus the specific heat of the liquid when kept saturated is found.

In the second method the calorimeter is kept full of liquid at a constant pressure. The heat, added to the variable amount in the calorimeter, and the resulting change in temperature are measured. A correction for the heat withdrawn in the expelled liquid is determined by special experiments. By use of the data for variation with pressure of the latent heat of the liquid, obtained from separate measurements, made with the same apparatus and material, the corrections for pressure variation are applied, and thus a second determination of the specific heat of the saturated liquid is obtained.

As a final result, the specific heat σ, in joules per gram per degree centigrade, of liquid ammonia, kept saturated, at the temperature θ, is given in the range $-45°$ to $+45°$ C. by the equation

$$\sigma = 3{\cdot}1365 - 0{\cdot}00057\theta + \frac{16{\cdot}842}{\sqrt{133-\theta}}.$$

The two curves in *Fig.* 4 show the results graphically.

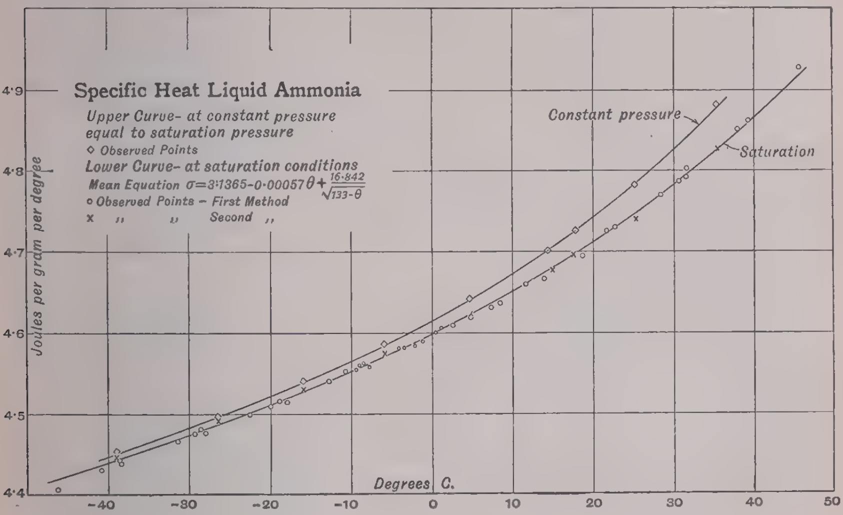

FIG. 4.

§ (8) SPECIFIC HEATS OF SOLIDS BY ELECTRICAL METHODS.—Very little work has been done on the determination of the specific heat of solid substances by the electrical method except for the metals. The method, of course, lends itself admirably to the determination of the specific heats of good thermal conductors, but with poor conductors special devices must be adopted to ensure uniformity of temperature throughout the material under test.

§ (9) GAEDE.—Gaede[1] appears to have been the first to measure specific heats in this manner. In his experiments the specimens formed their own calorimeters. These were machined to a cylindrical form and a deep central core bored out. Into this was thrust a copper core, wound with a properly insulated heater of constantan ribbon and a resistance thermometer of fine copper wire. Thermal contact between the core and the walls of the well was secured by filling the intervening space with mercury, using a thin steel shell when necessary to avoid amalgamation. This calorimeter was suspended in a thermostat and heated through an accurately measured temperature interval of about 15° by a measured quantity of energy supplied electrically.

Very few particulars of the investigation have been published, and the data obtained are summarised in Table IV.

§ (10) METALS.—Professor E. H. Griffiths and Dr. Ezer Griffiths studied[2] the specific heats of the metals aluminium, tin, copper, cadmium, zinc, lead, and silver over the range −160° to +100° C.

TABLE IV

MERCURY.		PLATINUM.		STEEL.	
Temperature.	Specific Heat.	Temperature.	Specific Heat.	Temperature.	Specific Heat.
17·1	·03326	17·5	·03128	16·8	·1064
31·8	·03311	32·0	·03146	32·3	·1082
47·2	·03302	47·8	·03167	47·1	·1099
61·9	·03292	62·2	·03180	62·0	·1114
77·0	·03282	77·2	·03193	76·7	·1129
92·6	·03273	92·2	·03205	91·9	·1144

LEAD.		ANTIMONY.		TIN.	
Temperature.	Specific Heat.	Temperature.	Specific Heat.	Temperature.	Specific Heat.
18·3	·03054	17·1	·05025	16·8	·05398
32·3	·03075	33·0	·05056	32·5	·05446
47·1	·03091	47·2	·05082	46·8	·055486
61·6	·03103	62·4	·05103	62·1	·05534
76·9	·03112	77·3	·05116	76·9	·05580
92·0	·03125	92·5	·05132	92·1	·055623

ZINC.		CADMIUM.		COPPER.	
Temperature.	Specific Heat.	Temperature.	Specific Heat.	Temperature.	Specific Heat.
17·3	·0922	17·1	·05483	16·7	·0911
32·4	·0929	32·2	·05535	32·7	·0919
47·2	·0936	47·1	·05566	47·0	·0924
62·2	·0941	61·8	·05594	61·9	·0931
77·3	·0946	76·9	·05629	76·4	·0935
92·7	·0949	92·2	·05655	92·3	·0940

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[1] *Phys. Zeitschr.*, 1902, iv.

[2] *Phil. Trans. Roy. Soc.* A, 500, 1913, ccxiii. 119; *Roy. Soc. Proc.* A, 1914, lxxxix. 561; *Phil. Trans.* A, 518, 1914, p. 319.

(i.) *Temperatures* 0°-100°.—The apparatus employed for determinations in the range 0° to 100° C. is shown in *Fig.* 5.

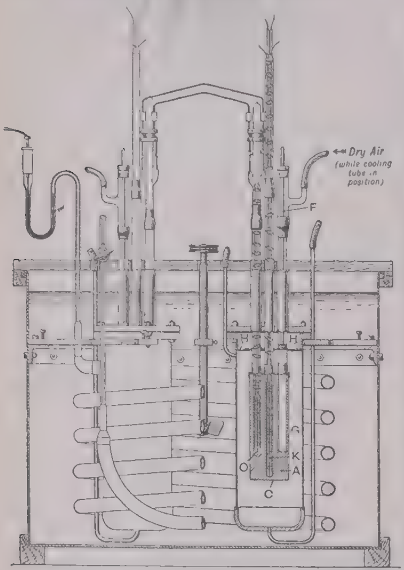

FIG. 5.

Two similar blocks of the metal under test were suspended in two brass enclosures immersed in a constant temperature bath. In the central hole of each block was a heating coil whilst the coaxial holes contained resistance thermometers connected differentially.

The third hole was used for the purpose of cooling the block below the surrounding temperature by the insertion of a thin-walled tube containing ether and connected to a water pump.

In the experiments one of the blocks was heated through a range from one degree below the temperature of the enclosure to one degree above the temperature of the enclosure by a measured supply of electrical energy.

This temperature interval was measured on a resistance bridge in the usual manner. Since the two resistance thermometers were adjusted to close equality and made of the same sample of wire, the balance point on the bridge wire was practically at the centre of the wire at all temperatures when the blocks were in temperature equilibrium with the enclosure. Hence no auxiliary coils were required in the Wheatstone's bridge circuit beyond the equal ratio arms.

The energy supplied to the heating coils was measured by balancing the potential difference at its ends against the E.M.F. of a series of cadmium cells in series, the current through the coil being adjusted until balance was obtained. The resistance of the heating coil was determined for the particular value of the current passing.

(ii.) *Low Temperatures.*—The experiments were continued at low temperatures but with a modified form of apparatus shown in *Fig.* 6, as it was very difficult to obtain any constant temperature baths in the region from −80° to −180° C.

In this apparatus a constant temperature enclosure was obtained by the use of a thick-walled copper box surrounded by a coil of piping through which cooled air circulated. The Joule-Thomson effect of cooling was utilised in a direct manner. Air was compressed to a pressure of 2000 to 3000 lbs. per sq. in., and then entered into the interchanger by the pipe A, *Fig.* 6. This interchanger BB was constructed of ordinary solid drawn copper tubing ⅛ in. bore coiled in the form of flat spirals. Successive layers of the coil were separated by strips of cardboard and the entire coil packed

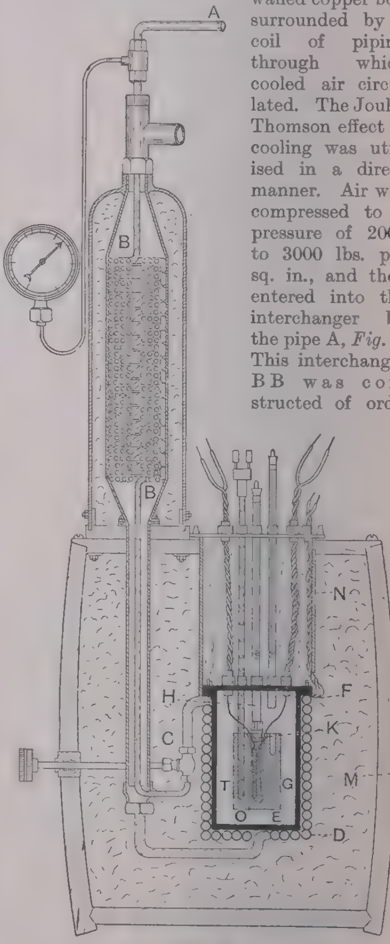

FIG. 6.

around with heat-insulating material. From the interchanger coils the air was carried to the valve C, by means of which an observer controlled the flow, excess of air being discharged at the safety-valve on the compressor. After expansion the air circulated through the coil of lead tubing D, and then back over the surface of the interchanger coils. On the exterior surface of the thick-walled copper enclosure E was wound a layer of insulated copper wire F, which served as a resistance thermometer. Variations in the temperature of the walls of this enclosure were rendered visible by the movements of a galvanometer spot. By controlling the flow of air the oscillations of the spot could be kept within narrow limits and, under normal conditions, the oscillations did not exceed a hundredth of a degree in amplitude.

The interior of the wooden vessel M was packed with slag wool,[1] the passage for the withdrawal of the copper enclosure being kept clear by a cylindrical tube of cardboard N. The space between the top of the enclosure and the outer lid was filled by wrapping felt matting around the glass tubes and leads.

The block of metal G was suspended within the enclosure by a single glass tube H. The centre hole contained the heating coil O, of manganin wire wound on a mica rack and immersed in a light paraffin, usually petrol. The heating coil was fixed to a short taper plug of copper K, which closed the central hole. The resistance of the coil was about 26 ohms. A platinum thermometer was inserted in the cylindrical hole T, the annular gap between the stem and the walls being closed by a packing of asbestos thread. The differential arrangement employed in the previous experiments was abandoned as it would have required too long a time to obtain the equilibrium conditions.

(iii.) *Method of Experiment.*—In these experiments the practice was to heat the material through a small temperature interval from below the surroundings to an approximately equal interval above, and observe the rate of rise during this period. The method of experiment was such that a direct determination of the temperature of the enclosure was not required. An experiment was conducted as follows:

The temperature of the enclosure was lowered progressively by utilising the full supply from the compressor and controlling the flow so as to produce a steady pressure drop through the valve of 120 to 150 atmospheres.

The temperature of the block would fall at a steady rate by radiation and convection to the enclosure walls, and when its temperature had nearly reached the desired point the cold air circulation around the enclosure was stopped.

Its temperature would then rise rapidly by conduction from without and soon pass that of the block which, in consequence of the slow transmission of heat by radiation and convection, would lag behind that of the walls. The temperature of the enclosure walls would then be maintained steady at about three degrees higher than that of the metal block.

Some time had to elapse before the conditions were sufficiently settled to justify the commencement of an experiment.

The first group of readings consisted of observations of the rate of rise of temperature of the block by radiation, etc., the transits of the temperature being observed across successive equal intervals (of about $\frac{1}{20}$th of a degree), the time between successive transits being of the order of 50 seconds.

The electrical supply was then switched on, and, after allowing a little time for the setting up of a steady gradient, transits every fifth of a degree were taken.

When the temperature had risen two or three degrees above the surroundings the electrical supply was switched off and observations of temperature and time continued.

The temperature would then fall steadily under the influence of radiation, etc., the rate of cooling being observed in precisely the same manner as the rate of rise of temperature before the electrical supply was switched on.

If σ is the rate of rise or fall due to radiation for 1° C. difference in temperature between the block and the surroundings, then assuming Newton's law to be valid for the loss or gain by "radiation" (an assumption which was fully justified by the experimental results), we have the expression

$$\frac{\partial\theta}{\partial t}=\sigma(\theta-\theta_0)$$

for the rate of rise or fall under the influence of "radiation" alone. Hence, plotting $\partial\theta/\partial t$ against θ, the straight line joining the two groups will cut the temperature axis at $\theta=\theta_0$ which determines the temperature of the surroundings.

For the rate of rise under the combined effect of the electrical supply and radiation we have the equation

$$\frac{\partial\theta}{\partial t}=\frac{E^2}{RMS}+\sigma(\theta-\theta_0),$$

where E^2/R is the electrical supply per second in thermal units, MS the thermal capacity of the block including that of the resistance coil, etc.

Plotting the observed rates of rise on the same scale as the "radiation" observations, it is obvious that the straight line thus obtained should be parallel to the line joining the two groups of "radiation" observations, since the tangent of the angle made with the θ axis is equal to σ. For $\theta=\theta_0$ the "radiation" term vanishes, hence, if $\partial\theta_N/\partial t$ denotes the value of the ordinate at this point, then

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[1] It is probable that wool in its natural state would have been a better insulator at these low temperatures, since the grease in the wool prevents it from absorbing moisture.

$$\frac{\partial\theta_N}{\partial t} = \frac{E^2}{RMS},$$

from which S can be obtained. The results obtained are summarised in Table V. and shown graphically in *Fig.* 7, where T is the absolute temperature and C_p is the atomic heat, *i.e.* specific heat multiplied by the atomic weight:

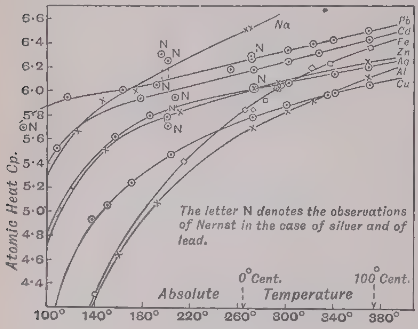

FIG. 7.

TABLE V

Abs. Temp.	Specific Heat.	C_p.	Abs. Temp.	Specific Heat.	C_p.
		COPPER			
138	0·07766	4·94	301·6	0·09230	5·87
149·5	0·07942	5·05	336·7	0·09365	5·95
171·4	0·08235	5·23	340·5	0·09387	5·97
204·9	0·08593	5·46	370·5	0·09521	6·05
273·1	0·09088	5·78	..	..	..
		ZINC			
145·5	0·08421	5·50	323·6	0·09412	6·15
211·1	0·08898	5·82	370·5	0·09521	6·22
273·1	0·09176	6·00	396·5	0·09570	6·26
294·6	0·09265	6·06	..	..	..
		SILVER			
158·1	0·05210	5·62	301·5	0·05613	6·05
187·4	0·05362	5·78	340·5	0·05680	6·13
273·1	0·05560	6·00	370·6	0·05737	6·19
		CADMIUM			
108·3	0·04907	5·52	301·5	0·05554	6·24
181·8	0·05287	5·94	327·6	0·05616	6·31
273·1	0·05475	6·15	370·8	0·05714	6·42
		LEAD			
118·6	0·02867	5·94	301·5	0·03053	6·32
166·3	0·02903	6·01	324·1	0·03073	6·36
254·4	0·02989	6·19	340·5	0·03102	6·42
273·1	0·03020	6·25	370·6	0·03127	6·48
		SODIUM (ANNEALED)			
123·3	0·2466	5·67	273·1	0·2829	6·51
155·2	0·2586	5·95	305·9	0·2910	6·69
179·3	0·2610	6·02	322·4	0·2952	6·79
210·5	0·2707	6·23	340·9	0·3019	6·94
270·6	0·2826	6·51	..	..	..

TABLE V—*continued*

Abs. Temp.	Specific Heat.	C_p.	Abs. Temp.	Specific Heat.	C_p.
		SODIUM (MOLTEN STATE)			
373·9	0·3234	7·44	400·6	0·3205	7·37
376·2	0·3232	7·43	411·6	0·3189	7·34
390·1	0·3217	7·40	..	..	..

§ (11) NERNST AND LINDEMANN.[1]—These observers made a series of point to point determinations at very low temperatures, using a calorimeter developed by Eucken.[2] A piece of metal of suitable size was shaped into a hollow cylinder and a loosely fitting core made for the same. On the core was wrapped a platinum wire, properly insulated, to serve as a resistance thermometer and also as electric heater. The core was placed in the cylinder and paraffin poured into the crevices to improve the thermal contact (see *Fig.* 8). The whole was suspended *in vacuo*, and the specific heat over small temperature intervals determined from measurements of energy supplied electrically and of the temperature rise resulting therefrom. Nernst and Lindemann applied the same method to poor heat conductors. For such materials the design of calorimeter is shown in *Fig.* 9. The wire was wound on a silver tube projecting into a silver vessel, the high conductivity of the silver assisting the equalisation of the temperature through the mass. Some of the data for pure metals obtained by Nernst are given in Table VI.

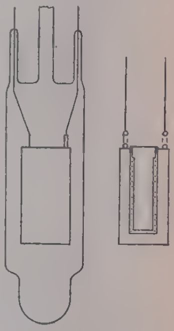

FIG. 8.

§ (12) COPPER.—Harper[3] studied (i.) the specific heat of copper over the range 15° to 50° C. The specimen was in the form of copper wire, which also served as its own thermometer and heater. The wire was 50 metres in length and 2·5 mm. in diameter; it was compactly coiled into a number of flat spirals separated by mica plates. The coil was suspended *in vacuo*, and heated with a measured quantity of energy supplied electrically, the resulting temperature rise being measured by the change of resistance. The

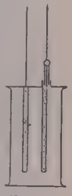

FIG. 9.

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[1] *Journ. de Physique*, 1910, [4], ix.; *Sitzungsber. Berl. Akad.*, 1910, i. 247, 262; *Ann. d. Phys.*, 1911, [4], xxxvi. 395.

[2] *Physik. Zeitschr.*, 1909, x. 586.

[3] *Sci. Paper Bur. Stds.*, 1914, No. 231.

results of 27 determinations between 15° and 50° are represented by the equation

$$S=0{\cdot}0917+0{\cdot}000048(t-25)° \text{ calories}_{20} \text{ per gram degree.}$$

4·182 joules is taken as equal to one 20° calorie.

(ii.) *Comparison of Data by Various Observers.*—Harper[1] has tabulated the data given by various observers for the specific heat of copper. In order to compare the results at one definite temperature the coefficient 0·000044 has been used in reducing results obtained at the various temperatures to the 50° C. value, the formula

$$S=So+0{\cdot}000044t$$

being assumed as valid for values of t from 0° to 100°.

This coefficient is the mean of those given by recent observers employing electric heating and a point to point method as shown in Table VII.

TABLE VII

Observer.	Coefficient. Calories per Gram Degrees.
Gaede	0·000040
E. H. Griffiths and Ezer Griffiths	0·000044
Harper	0·000048

TABLE VI

NERNST'S VALUES AT LOW TEMPERATURES

Al	T.	32·4°	35·1°	83°	86°	88·3°		
	C_p	0·25	0·33	2·41	2·52	2·62		
Cu	T.	23·5°	27·7°	33·4°	87°	88°		
	C_p	0·22	0·32	0·54	3·33	3·38		
Zn *	T.	40°	60°	80°				
	C_p	1·77	3·15	4·09				
Ag	T.	35°	39·1°	44·2°	52·6°	65°	77°	85·3°
	C_p	1·58	1·90	2·36	2·85	3·74	4·07	4·37
Pb	T.	23°	28·3°	37·3°	80°			
	C_p	2·96	3·92	4·41	5·69			

* In the case of zinc the actual figures are not recorded, and the above figures have been obtained from the curves given in the paper.

It will be observed from a comparison of the data given in Table VIII. (p. 42) and shown graphically in *Fig.* 10 that there is substantial agreement between the results of observers using the electrical method both as regards the absolute value of the specific heat and its temperature coefficient. It is very improbable that there is any systematic error common to all since the three methods differ radically in detail.

§ (13) SPECIFIC HEAT OF GASES BY ELECTRICAL METHODS.—The method of electrical heating for the determination of the specific heat of gases at different temperatures has been developed by Callendar and his associates.

Gases present greater practical difficulties than either solids or liquids, since it is necessary to take great precautions to ensure uniformity of temperature in the gas stream

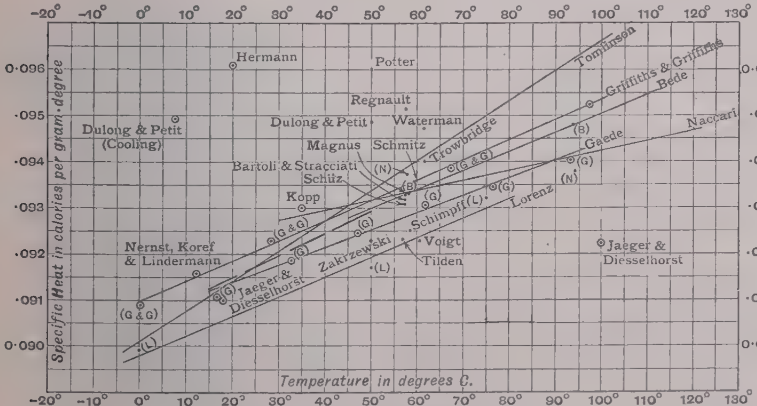

FIG. 10.—Specific Heat of Copper.

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[1] *Sci. Paper Bur. Stds.*, 1914, No. 231.

TABLE VIII

THE SPECIFIC HEAT OF COPPER

Year.	Name.	Temp.	Result.	Calc. Value at 50°.	Remarks.
1817	Dulong and Petit	0-100	0·0949	$0{\cdot}094_9$	..
	" "	0-300	0·1013	..	..
1819	" "	5-10	0·0949	0·097	Method of cooling.
1831	Potter	(50°)	0·096	0·10	Mean of expts. over diff. temp. ranges.
1834	Hermann	20°	0·0961	$0{\cdot}097_4$	..
1840	Regnault	15-100	0·09515	$0{\cdot}094_8$	Very pure Cu; Hammered Cu 1·5 per cent lower than soft Cu.
1843	"	"Room"	0·0886	0·09	Method of cooling. (Results considered worthless by Regnault.)
1856	Bede	15-100	0·09331	$0{\cdot}093_0$	Pure copper. Results expressed within experimental error by the formula $c=0{\cdot}0910+0{\cdot}000046t$.
	"	16-172	0·09483	..	
	"	17-247	0·09680	..	
1864	Kopp	20-50	0·0930	0·094	Commercial copper wire.
1881	Lorenz	0°	0·08988	..	..
	"	50°	0·09169	$0{\cdot}091_7$	..
	"	75°	0·09319	..	..
1884	Tomlinson	(50°)	0·09332	0·093	Mean from expts. in diff. temp. ranges, $c=0{\cdot}09008+0{\cdot}0000648t$.
1887	Naccari	17-99	0·0937	$0{\cdot}093_4$	..
	"	17-171	0·0938	..	..
	"	17-253	0·0951	..	..
	"	17-321	0·0957	..	..
1891	Zakrzewski	– 100-0	0·08514	..	Bunsen ice calorimeter.
	"	0-100	0·09217	$0{\cdot}092_2$	..
1892	Schüz	15-100	0·0931	$0{\cdot}092_8$	Copper wire.
	"	– 78-15	0·0903	..	..
1892	Le Verrier	0-360	0·104	..	..
	"	360-580	0·125	..	..
	"	580-780	0·09	..	..
	"	780-1000	0·118	..	..
1893	Richards and Frazier	0-1000	..	..	$c=0{\cdot}0939+0{\cdot}00003556t$.
1893	Voigt	21-100	0·923	$0{\cdot}91_7$	..
1895	Waterman	23-100	0·09471	$0{\cdot}094_2$	Very pure Cu, drawn.
1895	Bartoli and Stracciati	15-100	0·0932	$0{\cdot}092_9$	Very pure Cu.
1898	Trowbridge	23-100	0·0940	$0{\cdot}093_5$	..
	"	– 181·4-11	0·0868	..	..
1900	Behn	– 186-18	0·0796	..	Drawn copper, 0·5 per cent Sb and Ag. Final result of these expts. and others, not separately reported, $c=0{\cdot}0913+0{\cdot}0000676t-0{\cdot}0_6583t^2$.
	"	– 79-18	0·0883	..	
1900	Jaeger and Diesselhorst	18°	$(0{\cdot}091_0)$	0·092	Very pure copper. Method (in principle a method of cooling) not suited to accurate measurements for good conductors. Results considered of no value by Jaeger and Diesselhorst.
	" "	100°	$(0{\cdot}092_2)$	0·090	
1900	Tilden	15-100	0·09232	0·0920	Pure Cu. Expts. showing phosphor raises and tin lowers sp. ht. of Cu.
1902	Gaede	16·7°	0·09108	0·0925	Very pure copper. Electric heating, point to point method. Results combined in formulas $c=0{\cdot}09111+0{\cdot}00005002(t\text{-}17)-0{\cdot}0_6 1555(t\text{-}17)^2$ within ±0·045 per cent. $c=0{\cdot}09122_5+0{\cdot}0000384(t\text{-}17)$ within ±0·09 per cent. Units, 17° calorie, hydrogen scale.
	"	32·7°	0·09189	0·0926	
	"	47·0°	0·09244	0·0926	
	"	61·9°	0·09310	0·0926	
	"	76·4°	0·09346	0·0923	
	"	92·3°	0·09403	0·0922	
1903	Schmitz	– 190-20	0·0798	..	Very pure copper. 17·5° calorie, hydrogen scale.
	"	– 190-0	0·0793	..	
	"	20-100	0·0936	0·0932	
1904	Glaser	30-1005	0·10884	..	Pure copper. Procedure such that error of 10 per cent quite probable.

TABLE VIII—*continued*

Year.	Name.	Temp.	Result.	Calc. Value at 50°.	Remarks.
1910	Magnus	15-100	0·0933	$0{\cdot}093_0$	..
	,,	15-238	0·09510	..	..
	,,	15-338	0·09575	..	..
1910	Richards and Jackson	– 190-20	0·0789	..	20° calorie, hydrogen scale.
1910	Schimpff	– 190-17	0·0786	..	Copper 99·91 per cent pure. Results expressed in formula $c=0{\cdot}091996+0{\cdot}0000457090(t\text{-}17)-0{\cdot}0_6 6066(t\text{-}17)^2$.
	,,	– 79-17	0·0880	..	
	,,	17-100	0·0925	0·092	
1910	Nernst, Koref, Lindemann	2-22	0·09155	0·0932	Calorie equal to 4·188 joules.
1911	Koref	– 190-83	0·0720	..	Commercial drawn copper. Calorie equal to 4·188 joules.
	,,	– 77-0	0·0876	..	
1911	Nernst and Lindemann	– 233°	$0{\cdot}012_1$	..	Calorie equal to 4·188 joules.
	,, ,,	– 213°	$0{\cdot}029_4$	..	..
	,, ,,	– 193°	$0{\cdot}046_5$	..	..
	,, ,,	– 173°	$0{\cdot}059_0$	..	..
1911	Nernst	– 249·6°	0·0035	..	(Assuming ice-point = 273·1° K.) Electrolytic copper. In terms of a calorie equal to 4·188 joules.
	,,	– 245·4°	0·0051	..	
	,,	– 239·7°	0·0084	..	
	,,	– 186·1°	0·052	..	
	,,	– 185·1°	0·053	..	
1913	Griffiths and Griffiths	0°	0·09088	0·0931	Very pure electrolytic copper. Electric heating, point to point method.
	,, ,,	28·4°	0·09230	0·0932	
	,, ,,	67·3°	0·09387	0·0931	Results expressed by formula $c=0{\cdot}09088\,(1+0{\cdot}0005341t-0{\cdot}0_6 48t^2)$.
	,, ,,	97·4°	0·09521	0·0931	
1914	Harper	15-50	..	0·0929	Results expressed by formula $c=0{\cdot}0917+0{\cdot}000048\,(t\text{-}25)$.

The units in terms of which the above results are expressed are not all the same, but the differences need not be taken into account in making comparisons. In every case the difference between the unit employed and the 15° calorie or the 20° calorie (which differ from each other by about one part in a thousand) is less than the probable experimental error.

Callendar and Swann[1] applied the continuous flow method to the determination of the specific heats of air and carbon dioxide at atmosphere pressure at 20° C. and 100° C.

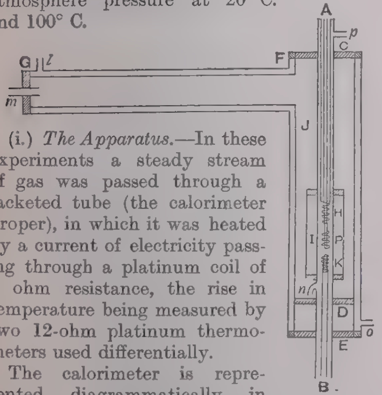

FIG. 11.

(i.) *The Apparatus.*—In these experiments a steady stream of gas was passed through a jacketed tube (the calorimeter proper), in which it was heated by a current of electricity passing through a platinum coil of 1 ohm resistance, the rise in temperature being measured by two 12-ohm platinum thermometers used differentially.

The calorimeter is represented diagrammatically in *Fig.* 11, the heating coil and platinum thermometers being situated in the tube AB, which is jacketed by the tube J. The tube FG formed the heater in which the gas attained the desired temperature, and being double-walled could be steam-heated or water-cooled. This tube was packed with tightly fitting discs of copper gauze. The gas under test entered at *m*, was heated up to the required temperature and entered the space round the calorimeter proper. It next passed through the tube *n*, into the calorimeter, and finally emerged by the tube *p*.

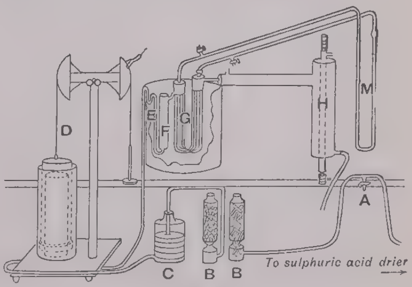

FIG. 12.

A, Teepiece open to atmosphere through constricted opening to allow excess of gas to escape; BB, towers containing solid KOH and $CaCl_2$; C, cotton wool dust filter; D, automatic pressure regulator; E, throttle; F, tube packed with gauze to bring gas to the desired temperature; G, fine metal tubes for measurement of flow; M, oil gauge.

The general arrangement of the apparatus will be understood from *Fig.* 12.

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[1] *Phil. Trans.* A, 1910, ccx. 199.

(ii.) *Theory of Method.*—If C is the electric current, E the potential difference between the ends of the heating coil, $\delta\theta$ the rise in temperature of the gas, Q the rate of flow of the gas in grammes per second, J the mechanical equivalent of heat, and S the specific heat of the gas at constant pressure, the elementary theory of the experiment gives

$$CE = JSQ\delta\theta + h\delta\theta,$$

where $h\delta\theta$ is a term representing the heat loss by radiation, etc.

A similar experiment with a rate of flow about half the above value, and with the electric current adjusted so that the rise in temperature was about the same as before, gave a second equation, so that h could be eliminated and S determined.

The largest currents of gas through the apparatus were of the order of 0·5 litre per second. The rate of flow was kept constant by an automatic pressure regulator. It was measured by passing the gas through 16 fine metal tubes arranged in parallel, and observing the pressure difference between their ends, the mean pressure, and the temperature. The expression giving the rate of flow in terms of these quantities was found by a series of experiments in which the gas was pumped into a reservoir of about 50 litres capacity, and then allowed to discharge through the apparatus. By means of a special device, the times taken for certain quantities of gas to pass through the apparatus were recorded automatically while the gas was actually flowing, so that the initial fluctuations were avoided.

The value of the electric current was obtained by measuring the potential difference set up at between the ends of a standard resistance coil in terms of cadmium cells. The heating effects of the leads of the heating coil were determined by experiments made under the exact conditions of the main experiments.

The rise in temperature in the main experiments was about 5° C., and it was measured to 0·001° C. Thus the specific heats were measured practically at single temperatures instead of over large ranges.

The validity of assuming the heat loss for a given rise in temperature to be independent of the rate of flow of the gas was tested by experiment. The matter was also examined from a theoretical standpoint, and corrections were calculated and applied where the assumptions made in the elementary theory were such as to lead to errors of more than about one part in 10,000. The corrections were small, only amounting to one or two parts in 1000.

Full details of various other precautions are given in the original paper, and the mean of a large number of observations gave the following results:

Air

0·24173 cal. per gram degree at 20° C.
0·24301 „ „ „ 100° C.

Carbon Dioxide

0·20202 cal. per gram degree at 20° C.
0·22121 „ „ „ 100° C.

The several determinations agree in each case to about 1·5 parts per 1000, and the mean results are probably correct to one part in 1000.

The values of the specific heats obtained are greater by about 2 per cent than the corresponding values found by Regnault and by later investigators who have employed methods similar in principle to that of Regnault, but it has now been established that Regnault's method gives values which are low by about this amount.

§ (14) SPECIFIC HEAT OF STEAM.—Brinkworth[1] developed the same method for the determination of the specific heat of steam at atmospheric pressure between 104° C. and 115° C.

(i.) *Outline of the Method.*—Steam is generated in a boiler and thence led to one limb of a U-tube pressure regulator. The pressure of the steam forces the mercury down in this limb of the U-tube and up in the other limb in which the adjustment of the supply of gas to the large ring burner, used for heating the water in the boiler, is made. After passing the regulator the steam, now maintained at a constant pressure, is led between the walls of the jacket surrounding the calorimeter proper, thence through a separator and a throttle into the space enclosed by the double-walled jacket, whence it passes down the calorimeter flow-tube to a condenser. During the passage of the steam through the flow-tube it is heated by means of an electric current passing through a platinum heating coil, and its temperature is measured on a platinum resistance thermometer. Another temperature measurement is made when the supply of electrical energy is cut off, and the difference between these two temperatures gives the rise in temperature of the steam.

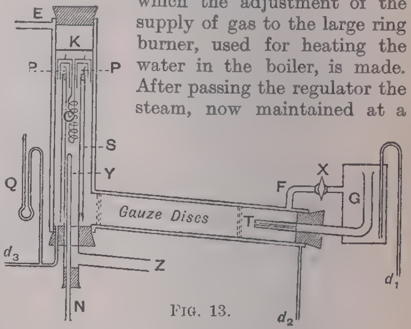

FIG. 13.

d_1, d_2, and d_3, drains for condensed water.

(ii.) *Calorimetric Arrangements* (*Fig.* 13).—The calorimeter proper consisted of a glass tube Y, about 50 cm. long, in which the heating coil C and the thermometer N were fixed. This tube was jacketed by another glass tube S, which enclosed the length occupied by the heating coil and thermometer.

The calorimeter flow-tube and its surrounding glass sheath were carried on a split rubber cork wound with omega tape and fixed, making a steam-tight joint, into a space enclosed by a double-walled brass jacket. The lower part of this jacket communicates with a double-walled side tube and the steam entering at E passes between the jacket walls to F, which communicates with the steam separator G. It then enters the inner portion of the side tube through the throttle T. This tube is tightly packed with gauze discs; thence the steam passes up the tube S into the top of the flow-tube at P, and descending past the heating coil C and the thermometer N flows away into the condenser at L.

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[1] *Phil. Trans.* A, 1915, ccxv. 383.

On its passage through the tightly packed gauze discs in the side-heating tube the steam was heated up to the temperature of the jacket.

In *Fig.* 13, which represents diagrammatically the arrangement thus described, it will be noticed that the cylindrical space inside the double walls of the main jacket is divided into two compartments by a disc K. This disc of brass was soldered to the inner jacket tube about 5 cm. from the upper end. Its function is to prevent the steam in the tube S from impinging on the rubber cork closing the upper end of the tube, and thus being cooled.

Any slight cooling due to the steam striking the lower split cork is of no importance, since the steam would be warmed again during its passage up between the flow tube and the surrounding jacket. The whole of the jacket, the separator, and the connecting tubes were heavily lagged with felt. A novel feature of the apparatus is the "spiral" method of mixing the steam, in which use is made of a number of circular discs punched to fit the thermometer tube and then cut along a diameter, bent, and soldered together to form a continuous spiral round the thermometer (see *Fig.* 14). This method of mixing is found to be a great improvement on the gauze method previously employed.

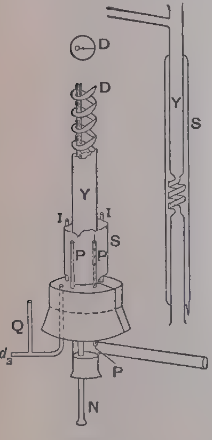

FIG. 14.

The usual equation for the continuous flow method is

$$EC = SQd\theta + hd\theta,$$

where EC is the electrical energy supplied, S the required specific heat if expressed in joules per gram C., Q the rate of flow of the steam, $d\theta$ the rise of temperature of the steam, and $hd\theta$ is a term representing the heat loss. If this loss is independent of the flow, then a linear relationship exists between the values of $EC/Qd\theta$ and $1/Q$. This is found not to be strictly the case, and another term depending on the flow is inserted in the fundamental equation which thereby becomes: $EC = SQd\theta + (h + k/Q)d\theta$. By the employment of three rates of flow, adjusting E in each case so that $d\theta$ remains the same, h and k can be eliminated and S measured.

The value obtained for the specific heat was 0·4856 cal. per gram degree at 104·5° C. and 760 mm. pressure; then, assuming a linear variation with temperature as experimentally determined, this corresponds to a value 0·4878 at 100°, both expressed in terms of the calorie at 20° C.

(iii.) *Effect of Impurities in the Steam.*—Steam in the immediate neighbourhood of the saturation point is liable to carry small particles of water in suspension, which cannot be evaporated completely by a moderate degree of superheat if any impurities, such as salt in solution, are present. Since 1 mg. of water requires more than half a calorie to evaporate it, and the heat required to raise the temperature of 1 gram of steam 10° C. is only 5 calories, it is necessary that the initial steam should not contain more than 1 in 100,000 of water if the specific heat is to be found correct to 1 in 1000 over a range of 10° C.

The rise of the boiling-point θ produced by x gram-molecules of salt per gram of water is approximately $1000x$° C. The proportion of suspended water remaining unevaporated at any degree of superheat θ' will be $1000x/\theta'$. The quantity evaporated in heating the steam from θ' to θ'' will be $1000x(\theta''-\theta')/\theta'\theta''$. This will produce an apparent increase of the mean specific heat of the steam over the range $\theta''-\theta'$ equivalent to $1000Lx/\theta'\theta''$, where L is the latent heat of evaporation. It was found that this extremely simple and convenient reduction formula fitted the results obtained over different ranges of temperature with extraordinary precision, and reconciled apparent discrepancies which had previously been attributed to errors of observation.

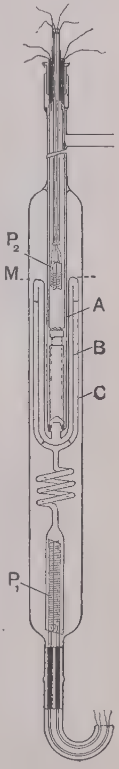

FIG. 15.

§ (15) DETERMINATION OF THE SPECIFIC HEAT OF AIR AND OTHER GASES AT ROOM AND LOW TEMPERATURES BY THE CONTINUOUS FLOW ELECTRICAL METHOD.—Scheel and Heuse[1] have determined the specific heat of air and other gases at +20°, −78°, and −183° by the continuous flow method. The air was directed in a steady stream through a pipe in which it received a known amount of heat by means of a heating coil.

Fig. 15 shows the glass calorimeter in the form in which Scheel and Heuse used it. The gas, which is brought to a steady temperature, enters the calorimeter from below and flows through a spiral, then through two glass jackets C and B, and finally reaches the inner tube A which contains the heating coil. The temperatures of the in- and outflowing gases are determined by the resistance thermometers P_1 and P_2. The whole is surrounded by a vacuum and a glass jacket silvered inside and is contained in a bath at constant temperature. As the air at first flows through the jackets C and B before it reaches the real calorimeter, it absorbs the greater part of the heat given off to the inner vacuum according to the principle of counter current and thus assists the insulating action of the jacket. This greatly reduces the loss of heat, but does not entirely prevent it. The temperature of the outflowing gas is measured in the transverse section M.

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[1] *Ann. d. Phys.*, 1912, xxxvii. 79.

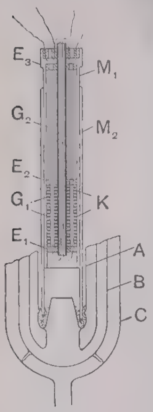

FIG. 16.

The heating coil is shown in *Fig.* 16 and consists of constantan wire K and is wound in two sections on a glass pipe.

In order to distribute the heat equally the wires are bound together and wound round the fine copper gauze G_1 as far as space permits. The wire coils are supported above and below by perforated ivory strips E_1 and E_2. In order to mix the gas thoroughly, the packing of copper wire gauze G_2 is introduced above the coil, which in its turn is fastened with an ivory strip E_3. The wires which conduct the current to the heating coil are led up through the inner glass tube and connected above E_3 with the wires to the source of the current and to the voltmeter. The heating coil and apparatus for mixing are contained in a brass pipe M_1 which fits into a second brass pipe M_2. M_2 is fastened with sealing-wax into an annular groove on the inner pipe A.

When working at room temperature the calorimeter was placed in a large water bath well stirred. At low temperature, on the other hand, it was placed in a vacuum vessel which contained a mixture of CO_2 snow and alcohol, or liquid oxygen. In each case the gas, before entering the calorimeter, passed through a pipe which was contained in the same bath as the calorimeter.

The same investigators,[1] when studying helium and some other rare gases to low temperatures, modified the apparatus so as to employ a closed circuit.

The results they obtained are summarised in Table IX. They also calculated out the corresponding values for the ideal gas state;

TABLE IX

Temperature.	C_p in Watt-Secs. per Gram Degree.	C_p in Gram Cal. at 15° per Gram Degree.	C_p.	C_v.	C_{po}.	C_{vo}.	*k*.	k_o.
			HELIUM					
+ 18	5·278	1·26$_0$	4·993	3·008	4·993	3·008	1·660	1·660
− 180	5·22	1·24$_5$	4·934	2·949	4·934	2·949	1·673	1·673
			HYDROGEN					
+ 16	14·26	3·403	6·860	4·875	6·860	4·875	1·407	1·407
− 76	13·23	3·157	6·365	4·379	6·364	4·379	1·453	1·453
− 181	11·08	2·644	5·330	3·338	5·320	3·335	1·597	1·595
			NITROGEN					
+ 20	1·044	0·249$_2$	6·983	4·989	6·969	4·984	1·400	1·398
− 181	1·071	0·255$_6$	7·162	4·879	6·718	4·733	1·468	1·419
			OXYGEN					
+ 20	0·914	0·218$_3$	6·98$_3$	4·98$_9$	6·97$_0$	4·985	1·399	1·398
− 76	0·898	0·214	6·86	4·84	6·81	4·83	1·416	1·411
− 181	0·956	0·228	7·30	5·04	6·90	4·91	1·447	1·404
			AIR (FREE FROM CO_2)					
+ 20	1·008	0·240$_6$	6·965	4·972	6·953	4·968	1·401	1·400
− 76	1·018	0·243$_0$	7·04	5·02	6·99	5·01	1·401	1·396
− 181	1·046	0·249$_6$	7·23	4·99	6·85	4·86	1·450	1·408
			CARBON MONOXIDE					
+ 18	1·048	0·250$_2$	7·006	5·011	6·991	5·006	1·398	1·396
− 180$_2$	1·084	0·258$_7$	7·244	4·922	6·743	4·758	1·472	1·417

Where C_{po} and C_{vo} are calculated values, *k* and k_o the ratios C_p/C_v and C_{po}/C_{vo} respectively.

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[1] *Ann. d. Phys.*, 1913, xl. 473.

these results are inserted for comparison alongside the others.

The values at constant volume are obtained from the experimental numbers at constant pressure by means of the expressions deduced by a combination of the ordinary thermodynamical equations with D. Berthelot's equation of state.

§ (16) VARIATION OF THE SPECIFIC HEAT OF AIR WITH PRESSURE OVER THE RANGE 1 TO 1200 ATMOSPHERES.—In 1914 Holborn and Jacob[1] made a new series of measurements of the specific heat of air high pressures by an electrical method. The calorimeter employed is shown in *Fig.* 17. The castings used in this construction were made of nickel steel of high tensile strength. The air enters the calorimeter at the bottom through a small spherical piece c_1 and leaves at the top through a similar arrangement c_8. Between these two points there is a nickel steel pipe c_5 with semicircular ends c_3 and c_6, into which the wide pipes c_2 and c_7 lead for the entrance and exit of the current of air. Connections between the various pipes are made by flanges fitted with packing rings.

A heating coil q_1 through which the air flows is enclosed within the pipe c_5; from here the air passed through the annular spaces l_1 and l_2, its direction being changed twice before it leaves the calorimeter. The spaces l_1 and l_2 are enclosed within three nickel walls. The direction of the air current is indicated by the arrows in the figure.

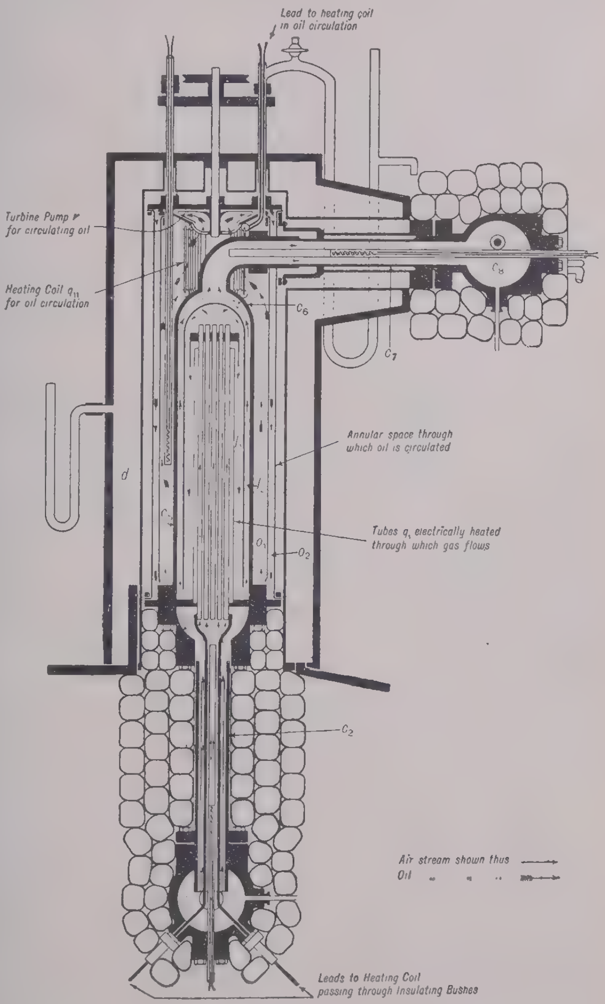
Lead to heating coil in oil circulation
Turbine Pump r for circulating oil
Heating Coil q_{11} for oil circulation
c_6
c_8
c_7
d
Annular space through which oil is circulated
Tubes q_1 electrically heated through which gas flows
o_1
o_2
c_2
Air stream shown thus
Oil
Leads to Heating Coil passing through Insulating Bushes

FIG. 17.

The outer space of the calorimeter is also divided into two cylinders o_1 and o_2, through which oil from the small turbine r is driven in the direction of the arrows. The oil can be heated when desired by means of the resistance coil q_{11}. d is a steam jacket surrounding the calorimeter and the exit pipe, leaving the entrance pipe quite free. This latter pipe is protected against heat loss by a wrapping of twisted silk and from the action of the incoming air by a glass pipe inserted in it. The heater q_1 consists of a group of 60 split nickel tubes about 4 mm. inner diameter and 4·5 outer diameter. These are fastened to two porcelain plates and are held together, one behind the other, by clips. The air passes through and around the tubes; current is supplied to this heater by wires insulated with glass beads. In the diagram the black part represents metal and the shaded parts insulating material.

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[1] *Zeitschr. Vereines Deutsch. Ing.*, 1914, lviii. 1429.

The results at a temperature of 59° are represented by the empirical formula

$$10^4 C_p = 2413 + 2{\cdot}86p + 0{\cdot}0005p^2 - 0{\cdot}00001p^3,$$

where p is the pressure in atmospheres.

The value 0·2413 obtained for the specific heat at 59° and one atmosphere pressure agrees very satisfactorily with those obtained by Swan (·2412 at 20°) and Scheel and Heuse (·2409). It of course differs from Regnault's value, which is now known to be low.

The following table gives the values obtained for the specific heat at a temperature of 59° C. and for various pressures measured in kilogram per sq. cm.:

TABLE X

p.	Holborn and Jakob.		Lussana.	Joule and Thomson.	Vogel.	Noell.
	(Obs.)	(Calc.)				
0	..	·2413	..	..	..	..
1	·2415	·2416	·2370	..	..	..
25	·2490	·2485	·2711	·2481	·2480	·2493
50	·2554	·2556	·3061	·2557	·2543	·2568
100	·2690	·2694	·3675	·2721	·2664	·2701
150	·2821	·2819	·4198	·2919	·2770	·2812
200	·2925	·2925	..	·3150	·2853	·2893

For comparison purposes the results of Lussana and the values calculated from the throttling experiments of Joule and Thomson, Vogel (1911), and Noell (1913) are inserted. These were obtained from Linde's formula

$$C_p = C_0\left(1 - \frac{d\delta}{dT}\right),$$

where C_0 is the specific heat at pressure 0 and δ the cooling when the pressure is reduced by throttling from p to a vanishingly small value.

E. G.

CALORIMETRY, METHODS BASED ON THE CHANGE OF STATE

§ (1) THE METHOD.—In this class of calorimetric appliances the quantity of heat to be measured is determined in terms of any one of the following: (i.) The mass of ice melted; (ii.) steam condensed; (iii.) liquid hydrogen or oxygen vaporised.

Such measurements do not require an accurate measurement of small temperature changes of the calorimetric fluid, and taking into consideration the state of thermometry half a century ago when this method was introduced, this fact was unquestionably of real advantage. Further, the temperature of the surrounding atmosphere can have but little effect upon the indications of the calorimeter since the initial and final temperatures are the same. In recent years calorimetric thermometers have been so highly developed that the measurement of small temperature changes presents no serious difficulty, and the technique of temperature control of the jacket has been greatly facilitated by the use of electrical heating, so that the inherent difficulties associated with the method of calorimetry now under consideration more than counterbalance the advantages it offers: consequently, it is very little used at the present day.

The Bunsen ice calorimeter and the Joly steam calorimeter are classical examples of this method of calorimetry.

Dewar has applied the same principle to specific heat determinations at very low temperatures, and obtained data concerning the mean specific heat of materials between liquid hydrogen and liquid nitrogen temperatures.

§ (2) BUNSEN'S ICE CALORIMETER.—In this instrument the heat given out by a body in cooling from some higher temperature to 0° C. is obtained by observing the contraction which takes place in the change from ice to water produced by the heat given by the body.

The observed volume change is converted into calories by assuming a value for the constant of the ice calorimeter, *i.e.* the mass of mercury drawn into the instrument by the addition of one mean calorie of heat.

Numerous determinations of this constant have been made and the values are summarised in Table I.

TABLE I

Authority.	K in Mgms.	Reference.
Bunsen . .	15·41	*Phil. Mag.*, 1871, xli.
Schuller and Wartha .	15·44	*Wied. Ann.*, 1877, ii.
Than . .	15·42	*Chem. Ber.*, 1877, x.
Velten . .	15·47	*Wied. Ann.*, 1884, xxi.
Zakrzevski .	15·57	*Bull. l'Acad. Cracovie*, 1891.
Staub . .	15·26	Inaug. Diss. Zürich, 1890.
Dieterici . .	15·49	*Ann. Phys.*, 1905, xvi.
Griffiths (Ezer)	15·49	*Proc. Phy. Soc. London*, 1913, xxvi. I.

It might be remarked that many of the above values are based on observations of the heat imparted to the calorimeter by a small quantity of water contained in an envelope whose thermal capacity was comparable with that of the contained water. Since in the majority of cases no attempt has been made to vary the conditions and thus detect systematic errors, all the values are not entitled to the same weight. For example, the figure given by Bunsen is the mean of two experiments repeated under precisely similar conditions: the glass envelope weighed 0·2 gm., contained 0·3 gm. of water, and a platinum sinker (weight 0·5 gm.) was also attached.

Dieterici varied the quantity of water from 0·6 gm. to 2 gm., and consequently his determination is entitled to greater weight than the others. Griffiths' value was obtained by supplying a known quantity of heat measured as electrical energy. The heat was supplied by a manganin coil wound on a mica rack which fitted the interior tube of the calorimeter (see *Fig.* 2), and the results are based on the electrical units of E.M.F. and resistance. The conditions were varied. Thus the rate of energy supply in the fastest experiments was more than seven times that in the slowest, and the probable error by the method of least squares was less than 0·1 per cent.

Bunsen employed his ice calorimeter to determine the latent of fusion of ice as follows:

Known weights of water at a known boiling temperature were introduced into the inner tube of the calorimeter and the contractions observed.

In a separate experiment a known weight of ice at 0° C. was contained in a bulb, the rest of the space being filled with mercury. The ice was melted to water, the temperature being maintained at 0° C. Mercury was drawn into the bulb to occupy the space left by the ice in melting, and from the additional weight of mercury the contraction was obtained. He found that the melting of 1 gram of ice caused a contraction of 0·0907 c.c.

From the results of the two sets of experiments he calculated the latent heat of water to be 80·025 calories, which is 0·3 per cent higher than the value obtained in recent direct determinations.

The method is not a good one for determining the latent heat of water, since the calculations depend on the difference of the specific volumes of ice and water.

§ (3) BUNSEN'S CALORIMETER AND ITS MODIFICATIONS. (i.) *Description.* — A cylindrical test-tube A is fused into a larger glass bulb B, as shown in *Fig.* 1. The bulb B is furnished with a glass stem CD, which terminates in an iron collar D. This stem is filled with pure boiled mercury, which occupies the bulb to the level β. The remainder of the bulb above β is filled with pure boiled water. A calibrated narrow glass tube S, furnished with a millimetre scale, is fitted into a cork with fine sealing-wax, and then passed through the mercury in the collar D, and made fast in the mouth of the tube CD, so that it becomes filled with mercury; and by adjusting the cork in the mouth of the tube CD the extremity of the mercury column in the scale tube S can be placed at any convenient point. By methods, which are described in most text-books on practical physics, a mantle of ice is formed around the lower part of the tube A.

(ii.) *Precautions in Use.*—In the original manner of performing the experiment the instrument, as described above, was placed in a vessel containing pounded ice or snow, the top of the tube A and the tube S alone projecting above the ice. It is, however, generally found that there is always a small difference in the freezing-point of the ice in the instrument and that of the ice outside. If the temperature of the outside ice is higher, then there will be a slow melting of the ice in the instrument, which will cause a continuous creep of the mercury meniscus towards the instrument. If the freezing-point of the ice outside is lower than that of the ice in the instrument, then there will be a slow freezing of the water, causing the meniscus to creep away from the bulb. This creep generally amounts to 2 or 3 centimetres per hour, and is sufficient to make it very difficult to obtain trustworthy measurements. A slight addition to the instrument will almost eliminate the creep, reducing it to about a tenth its normal value. This addition, suggested by Boys, consists in placing the instrument in an empty vessel, the top of which is closed by a cork through which the tubes A and S pass. This vessel is surrounded by the pounded ice. Owing to air being a bad thermal conductor, heat can now only very slowly pass between the instrument and the surrounding ice. Another method of securing the same conditions has been employed by Callendar, who constructed the bulb B in the form of a vacuum vessel. The vacuum being a worse conductor of heat than air, the passage of heat from the surrounding ice to the instrument is in this way reduced yet more. When this device is used, the process of cooling the instrument down to zero will take a considerable time, unless a stream of ice-cool water is passed through the tube A.

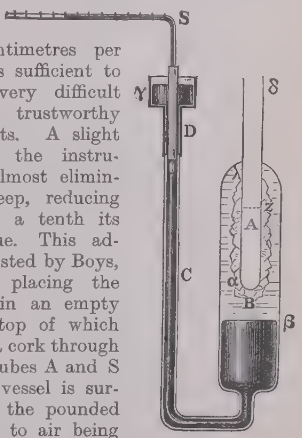

FIG. 1.

The writer adopted the plan illustrated in *Fig.* 2, which shows the instrument as set up for the determination of the constant of the calorimeter by the electrical method. The calorimeter is suspended within a transparent cylindrical vacuum vessel, the mouth of which is closed by a rubber cork as shown.

(iii.) *Variable Density of Ice.*—The objection most frequently quoted against the use of the ice calorimeter for work of precision is that the density of ice is not a constant quantity.

The experimental evidence on this point is briefly summarised below:

Nichols[1] reviews the work of previous investigators on the density of ice and describes his own experiments. He concludes that the density of ice mantles, determined by weighing in petroleum, is 0·91615 ± 0·00009. This result agrees with the mean value deduced from different methods by Plücker and Geissler, Kopp and Bunsen (for a similar variety of ice) to four places of decimals.

His experiments on the causes of the variations in density of artificial ice were not completed. The method was to freeze the ice mantle around the inner tube of a calorimeter by pouring in a mixture of CO_2 and ether. The unfrozen water was shaken out as completely as possible, and the adhering water frozen, the remaining space being then completely filled with mercury. The weight of the mercury, together with that of the ice, gave the data for the computation of the density of the ice mantle.

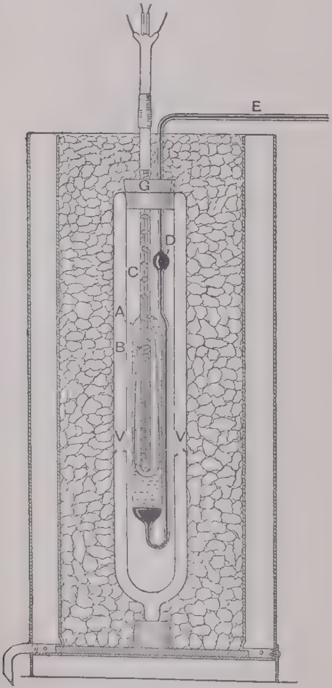

FIG. 2.

Although the results were consistent among themselves the absolute value was subsequently found to be erroneous on account of the deformation of the glass vessel under the weight of the contained mercury.

Nichols at first thought the discrepancy—amounting to 1 per cent—between his value and Bunsen's was due to the much lower temperature at which the mantle was formed in his own experiments. He therefore made some determinations with mantles frozen by means of alcohol at −5° to −10° as refrigerant in the manner devised by Bunsen. The measurements appeared to indicate that the mantles formed by the use of alcohol at −5° to −10° were less dense than those formed by means of CO_2 and ether at −70° by at least 1 part in 1000, and further that one of the latter mantles decreased in density by nearly this amount after standing 24 hours in an ice bath.

The use of CO_2 and ether resulted in a very rapid formation of ice, and the mantle, when a certain size was reached, invariably became filled with a network of fine cracks.

Vincent[2] later took up the subject, and also investigated the coefficient of cubical expansion of ice.

He prepared the ice by means of a freezing mixture, and appears to have obtained a different density for each sample prepared. Table II. summarises all his results:

TABLE II

Experiment.	Density Ice at 0°.	Weight assigned to the Experiment.
1	0·916335	3
2	0·915460	2
3	0·916180	2
4	0·915540	1
	0·916060	2
Weighted mean	0·9160	

His values for the coefficient of expansion of the above samples are consistent, and no connection between variation in density and expansion can be traced. Vincent's mean value for the density is 1 part in 5000 less than the mean of the results of Plücker and Geissler, Bunsen and Nichols.

The experiments of Leduc in 1906 suggest one cause of the variations in density which had been observed by previous workers. Leduc took extreme precautions to get rid of all traces of dissolved air in the water used for manufacturing the ice samples. He condensed the steam from boiling water under oil to obtain air-free water.

The results of these experiments indicated that the density of ice at 0° was not less than 0·9172, and as greater efforts were made to remove traces of gases the values obtained for the density increased. He concluded that the density of gas-free ice at 0° would probably be 0·9176.

It is of interest to note that Leduc considers that ice made from water which has been merely boiled, as in the case of the Bunsen calorimeter, still contains about 1 c.cm. of gas per litre at atmospheric pressure.

Another possible cause of the variations in density is the strains set up in the ice block on formation and which disappear in the course of time.

§ (4) JOLY'S STEAM CALORIMETER. (i.) *The Method.*—In the steam calorimeter devised by Joly[3] in 1886 the heat necessary to raise the temperature of a body from the air temperature to 100° is measured by determining the weight of steam which must be condensed into water at 100° to supply this heat.

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[1] *Phys. Rev.*, 1899, viii.

[2] *Phys. Rev.*, 1902, xv.

[3] *Proc. Roy. Soc.*, 1886, xli. 352.

The instrument, especially in its differential form, has been found very useful for special purposes. But the experience of most users indicates that the condensation method is more troublesome to use than the method of mixtures for the determination of the specific heats of solids and liquids. In the hands of Joly, however, the steam calorimeter has produced data of fundamental importance concerning the specific heat of gases at constant volume.

One disadvantage of the condensation method is the fact that less than 2 milligrams of water is deposited per calorie, and consequently it necessitates accurate weighing.

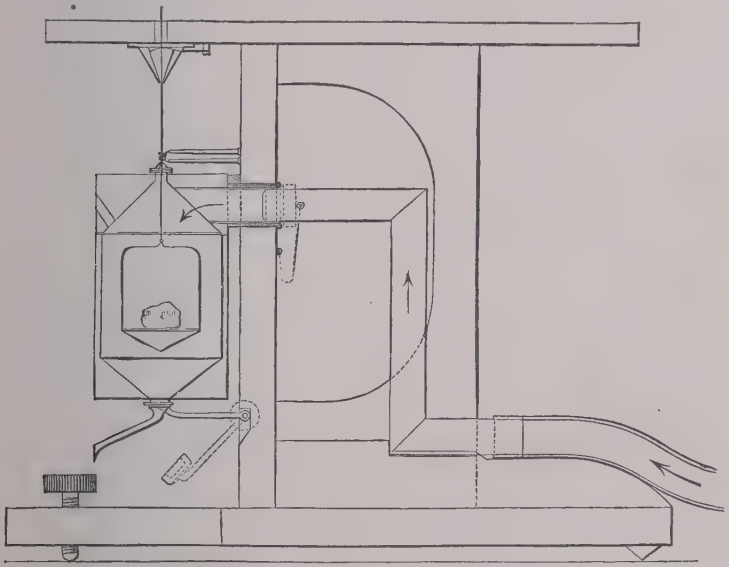

FIG. 3.

(ii.) *The Apparatus.*—The simplest form of Joly's apparatus consists essentially of a steam chamber of thin metal in which is suspended from the arm of a balance a small platinum pan (*Fig.* 3), carrying the substance under test.

Steam can be turned on to this chamber, as indicated, and escapes through a pipe at the base.

It is essential to arrange the inlet valve so that the steam can be admitted rapidly for reasons which are explained later.

(iii.) *Method of Experiment.*—The substance is weighed with air in the chamber and the temperature carefully noted. Steam in the meantime is got up in the boiler, and is suddenly admitted, so that the whole chamber becomes at once filled with saturated vapour. Condensation immediately begins on the substance and the resulting water is caught in the pan, weights being added to the other pan of the balance so as to restore equilibrium. During the process of weighing, the steam is passed through very slowly (by opening an escape tube leading from the boiler) into the calorimeter, so as to avoid disturbance of the pan.

After four or five minutes the substance has generally attained the temperature of the steam, and the condensation is completed. The pan then ceases to increase sensibly in weight, and the equilibrium of the balance is maintained permanently. A very slow increase of three or four milligrammes per hour (due to radiation) is, however, noticed. Let θ_2 be the temperature of the steam and L its latent heat. If w is the increase of weight the quantity of heat given out by the condensation is wL, and this is expended in raising the substance and the pan from θ_1 to θ_2. If W be the weight of the substance, and s its specific heat, the heat acquired by the substance will be $Ws(\theta_2 - \theta_1)$, and that acquired by the supporting pan will be $k(\theta_2 - \theta_1)$, where k is the thermal capacity of the pan, that is, the quantity of heat necessary to raise its temperature 1° C. Hence we have

$$Ws(\theta_2 - \theta_1) + k(\theta_2 - \theta_1) = wL.$$

The quantity k is determined by a previous observation, and the temperature θ_2 is found either directly, by a thermometer inserted in the steam-chamber, or by means of Regnault's tables and a reading of the barometer.

For extreme accuracy a small correction is still necessary. The weight W of the substance is found in air at θ_1, and the weight w is found when the substance and pan are in steam at θ_2. The weight of steam per cubic centimetre at 100° is little more than half that of air at ordinary temperatures; for this reason the weight w is greater than the weight of vapour condensed by excess in weight of a volume v of air at θ_1 over the same volume of steam at θ_2, where v is the volume of substance and pan together. The difference of weight of a cubic centimetre of air at 15° C. and a cubic centimetre of steam at 100° is ·000636 gram, according to Regnault; hence the correction to be applied to w is ·000636v.

This correction being applied, the weight of water condensed is determined, but it must be remembered that the weighing is made in steam; and, if extreme accuracy be desired, it is still necessary to multiply by the factor 1·000589, in order to reduce the weighing to vacuum. The actual weight in a vacuum of the water condensed will therefore be

$$1{\cdot}000589(w-0{\cdot}000636v),$$

so that s is determined from the equation

$$(\mathrm{W}s+k)(\theta_2-\theta_1)=1{\cdot}000589(w-0{\cdot}000636v)\mathrm{L}.$$

Reference should be made to § (6) of the article on "Latent Heat" for values of L.

In order to avoid the condensation of steam on the suspending wire, where it leaves the steam chamber, it passes, not through a small hole in the metal, but through a small hole pierced in a plug of plaster of Paris. Without the plaster the steam condenses on the metal and forms a drop at the aperture through which the suspending wire passes, and destroys the freedom of motion of the wire and prevents accurate weighing. With the plaster of Paris plug no such drop collects, and the weighing can be performed with accuracy. In his later experiments, Professor Joly placed a small spiral of platinum wire around the suspending wire just outside the aperture, and by passing an electric current through the spiral, sufficient heat is produced to prevent condensation on the suspending wire in the neighbourhood of the aperture. Besides accuracy in weighing, a point of prime importance is the rapid introduction of the steam at the beginning of the experiment. When the steam first enters the calorimeter, partial condensation occurs by radiation to the cold air and the walls of the chamber. Some of the condensed globules may fall upon the substance and lead to an error in the value of s. If the steam enters slowly this error may be large, and it is therefore important to fill the chamber at once with steam. This necessitates a good supply of steam and a large delivery tube, but when the chamber is well filled with steam a very gentle afterflow suffices. If the steam supply be cut off, the weight of condensed vapour slowly diminishes. This arises from the distillation over to the colder walls of the chamber, and if the steam be again turned on the weight increases.

The error arising from the deposition of condensed globules on the pan during the initial stages of the experiment is somewhat counterbalanced by radiation from the steam to the substance.

§ (5) The Differential Steam Calorimeter for the Determination of Specific Heats of Gases at Constant Volume.—In the differential[1] form of the steam calorimeter the correction for the weight of steam displaced by the pan is eliminated. In this form (*Fig.* 4) two similar pans hang in the steam chamber, one suspended from each arm of the balance so as to counterpoise each other. The thermal capacity of the pans can be made equal, so that the term with k as a coefficient does not appear in the equation, and the radiation error will also disappear, as it will cause equal condensation on the two pans.

The chief use of the differential form is, however, its application to the calorimetry of gases. For this purpose the pans are replaced by two spherical shells of copper, one containing the gas at a known temperature and the other empty. The spheres are furnished with small pans, or "catch-waters," to collect the water resulting from condensation. Greater condensation occurs on the sphere which contains the gas, and the excess gives the quantity of heat required to heat the contained mass of gas from θ_1 to θ_2. This determines the specific heat of the gas at constant volume. The great advantage of the differential calorimeter is that any source of error common to the two spheres is eliminated, and the gas or other substance enclosed in one of them merely bears its own share of error and not that also of the containing sphere. Thus the effect is practically the same as if the gas were contained in a vessel of zero thermal capacity in the single steam calorimeter form.

The spheres employed by Professor Joly were of copper and about 6·7 cm. in diameter, the one containing the gas being made to stand a safe working pressure of about 35 or 40 atmospheres. If at the beginning of the experiment this space is filled with air at about 22 atmospheres at θ_1 the pressure will rise to about 30 atmospheres at θ_2. In one experiment[2] the weight of air contained was 4·2854 grams. The condensation observed as due to the air was 0·15217 grams. This required a correction to compensate for the difference in weight of the spheres. The corrected value was 0·11629, the range of temperature, $\theta_2-\theta_1$, being 84·52° C. In a series of six experiments the mean precipitation per degree centigrade was 0·018004.

The following corrections are also necessary:

(*a*) Correction for the thermal expansion of the vessel, and the consequent work done by the gas in expanding to this increased volume.

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[1] J. Joly, *Proc. Roy. Soc.*, 1889, xlvii. 218.

[2] *Phil. Trans.* A, 1891, clxxxii. 98.

(*b*) Correction for the dilatation of the sphere under the increased pressure of the gas as the temperature rises.

(*c*) Correction for the thermal effect of stretching of the material of the sphere. (Wires are generally cooled by sudden extension, but the cooling of the copper in this case is too small to merit consideration.)

(*d*) Correction for displacement or buoyancy arising from the increased volume of the sphere, both in the air at θ_1 and in the steam at θ_2.

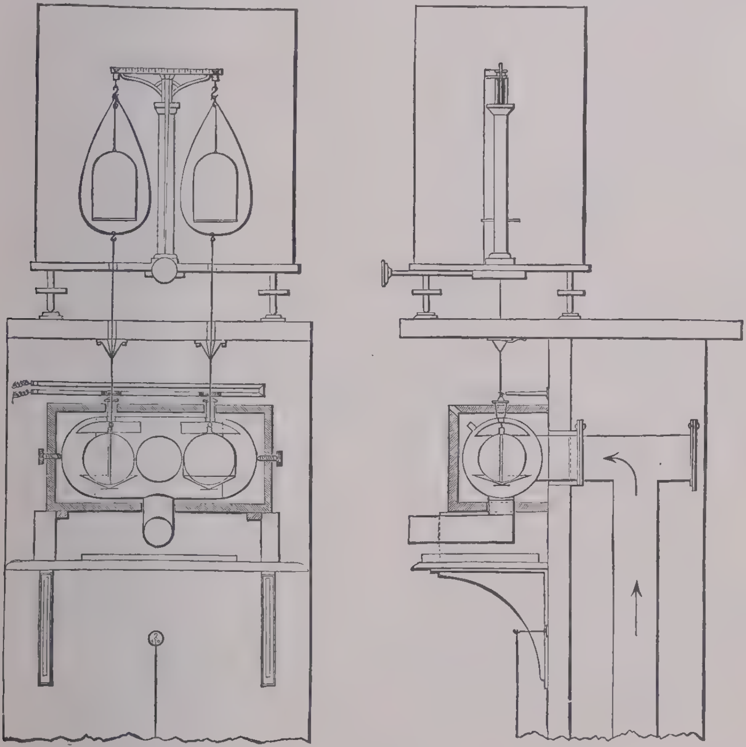

FIG. 4.

(*e*) Correction for unequal thermal capacities of the spheres.

(*f*) Reduction of the weight of the precipitation to vacuum.

Professor Joly's experiments show that in the case of air and carbonic acid the specific heat increases with the density, but with hydrogen the opposite seems to be the case.

For air the specific heat at constant volume at a mean pressure of 19·51 atmospheres, and a mean density of 0·0205, was found to be 0·1721. For carbon dioxide, the change with pressure is shown by the following table:

TABLE III

Pressure in Atmospheres.	Density.	C_v.
7·20	0·011530	0·16841
12·20	0·019950	0·17054
16·87	0·028498	0·17141
20·90	0·036529	0 17305
21·66	0·037802	0·17386

The mean result of the experiments on hydrogen gives a specific heat 2·402.

§ (6) DEWAR'S LIQUID AIR AND HYDROGEN CALORIMETER.—Dewar [1] has devised a calorimeter based on an analogous principle to the steam calorimeter in which he employs one of the liquefied gases as calorimetric substance. Whilst Joly's calorimeter depends upon condensation on a cold object, Dewar's calorimeter depends on the evaporation as a means of absorbing heat from the hot object. Instead of weighing the quantity of gas evaporated, he determines the volume of gas given off from the liquid which, of course, is at its boiling-point. Now the choice of liquefied gas to be employed as calorimeter substance is determined mainly by two considerations:

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[1] *Proc. Roy. Soc.* A, lxxvi. 325; *Roy. Inst. Proc.*, 1904, xvii. 581.

(*a*) The quantity of gas given off by evaporation on the absorption of one calorie of heat, and

(*b*) The range of temperature available through which substance is cooled.

The table below summarises the data for some of the possible gases:

TABLE IV

Liquid Gases.	Boiling-point.	Liquid Volume 1 Gram at Boiling-point in c.c.	Latent Heat in Gram-calories.	Volume of Gas at 0° and 760 mm. per Gram-calorie in c.c.
Sulphurous acid	+10·0°	0·7	97·0	3·6
Carbonic acid .	−78·0	0·65 (solid)	142·4	3·6
Ethylene . .	−103·0	1·7 ,,	119·0	7·0
Oxygen . .	−182·5	0·9 ,,	53·0	13·2
Nitrogen . .	−195·6	1·3 ,,	50·0	15·9
Hydrogen . .	−252·5	14·3 ,,	125·0	88·9

It will be observed that oxygen gives off 13·2 c.c. per calorie whilst ethylene gives only 7. Hydrogen gives off 88·9 c.c. and is particularly advantageous but for the fact that the manipulation is difficult. Although nitrogen is a little better than oxygen, it is preferable to use the latter for the following reason. The boiling-point of air is below that of oxygen. Even if there is no layer of cold oxygen or gas on the surface of the liquid oxygen, the air coming in contact with it through the neck of the calorimeter would still remain gaseous; but if liquid nitrogen is used as calorimetric substance, air, being heavier than nitrogen but having a higher boiling-point, would, in falling down the neck of the calorimeter, come in contact with the cold gaseous nitrogen and be condensed.

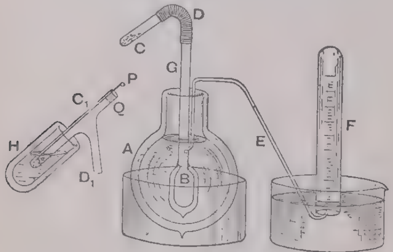

FIG. 5.

§ (7) THE LIQUID OXYGEN CALORIMETER.—It consists essentially of a large vacuum vessel A (*Fig.* 5) capable of holding two or three litres, into which is inserted the calorimeter, a smaller vacuum vessel B of 25 to 50 c.c. capacity, which has been sealed on to a long narrow tube G, projecting above the mouth of A, and held in its place by some loosely packed cotton-wool. From the side of this narrow tube, either before or after passing out of A, a branch tube E is taken off to enable the volatilised gas from the calorimeter to be collected in the receiver F, over water, oil, or other suitable liquid. To the extremity of the projecting tube G a small test-tube C, to contain the portions of material experimented on, is attached by a piece of flexible rubber-tubing D, thus forming a movable joint, which can be bent so as to tilt a few of the small pieces of substance contained in C into the calorimeter, and which afterwards assumes a position of rest somewhat like that in the diagram.

With care it is possible to tilt a single piece at a time from C into B, but an improved form of this receptacle is shown at C_1D_1. In it, P is a wire movable through the cork Q, fitted into the mouth of the test-tube C_1, attached by a branch through the stiff rubber tube D_1 to the end of G, as before. At the end of the wire P is a hook, by which one piece of the substance at a time can be pulled up and dropped into B. When no other arrangements are made, the portions of matter experimented on are at the temperature of the room; but when lower temperatures are required initially, a vacuum vessel H, containing either solid carbonic acid, liquid ethylene, air, or other gas, can be placed so as to envelop the test-tube C or C_1; or if higher temperatures are required, the surrounding vessel may be filled with the vapour of water or other liquids.

Now, when a quantity of liquid air has been undergoing volatilisation for a time, as the nitrogen evaporates more quickly than the oxygen the boiling-point rises slightly. Two points require attention in consequence of this: first, the maintenance of a constant temperature of the liquid air during any one series of experiments; next, the prevention of a tendency for the calorimeter B to "suck back" some of the already volatilised gas. Hence the exterior vessel A should be filled with a large quantity—some two litres—of *old* liquid air, containing a high percentage of oxygen, and the calorimeter itself should be filled with some of the *same* fluid. This will maintain very closely the constant temperature required. When any "sucking back" seems to be taking place, the calorimeter should be emptied and filled anew from the larger flask A. The tube between the calorimeter and the gas receiver should be of the size of wide quill tubing, and its lower end should be so arranged below the surface of the liquid in the collecting vessel as to give no resultant pressure. With such precautions, results may easily be obtained correct to within 2 per cent.

The instrument having been set up and filled with liquid air, an experiment is conducted by tilting up the little test-tube, previously cooled or heated, thereby dropping into the calorimeter a portion of any substance previously weighed. The substance in this way falls from the temperature of the room to that of liquid air. The heat given up by it volatilises some of the liquid, which is carried off by the branch tube and measured in the graduated receiver F. Immediately preceding or following this observation, a similar experiment is made with a small portion of a selected standard substance, usually lead. The quantity of lead is so chosen as to produce about the same volume of gas in the receiver as that supplied by the portion of substance experimented on. By this means the circumstances of the two observations are made as similar as possible, and thereby many sources of error are eliminated.

§ (8) **Liquid Hydrogen Calorimeter.**—In 1913 Dewar[1] further developed the method so as to adapt it to the range of temperature between the boiling-points of liquid nitrogen and hydrogen: from −196° to −253° C., a range of only 57°.

The liquid hydrogen calorimeter is a glass cylindrical bulb vacuum vessel A (*Fig.* 6) of 50 c.c. capacity, silvered, with $\frac{1}{2}$ cm. slit. On the neck is sealed a glass tube B. This projects through the brass coned fitting cap F of an ordinary slit silvered vacuum vessel in which it is supported. A side delivery tube, provided with stopcock D, is sealed near the top of B. A short length of rubber tubing on the neck of F makes a gas-tight joint with B. To minimise splashing, and to reduce the impact of the falling pieces, a thin strip of German silver or lead E, bent out near the top into a shoulder about 1 cm. square, stands centrally in the calorimeter A. The strip is cut from a thin tube of about the same diameter as the calorimeter neck. A short length of the tube is left above the shoulder, and supports the strip by fitting loosely into the neck of A. Some such device is essential in the use of this form of the liquid hydrogen calorimeter.

The calorimeter in its turn is immersed in liquid hydrogen in the supporting vacuum vessel C, the neck of the calorimeter being 8 to 10 cm. below the liquid hydrogen surface. The vacuum vessel C is only slightly wider than the lower part of A, and is provided with a coned cap F, whereby it is also supported and completely immersed in a wider vacuum vessel G containing liquid air. G is also fitted with a brass coned cover, fitting vacuum-tight on to the cap F on C. Both caps are pierced by two thin tubes, one for fitting on to the filling syphons, the other, bent at right angles, serving for connecting to the exhaust in the case of the liquid air vessel, and in the case of the liquid hydrogen to the stopcock leading the evaporating hydrogen through the upper part of the apparatus.

This arrangement thus charged only needs a little liquid air sucked in every one and a half hour. The liquid hydrogen vessel will not need replenishing for at least four hours. The level of the liquid hydrogen in the calorimeter does not fall 1 cm. in six hours with constant use. The bulk of the materials added roughly compensates for the volume of the liquid hydrogen evaporated.

It is important that this level should not materially change, since, after striking the shoulder, bodies move more slowly, are deflected on to the cold wall, and low results are obtained due to the longer cooling of the materials in the vapour before being immersed in the liquid hydrogen.

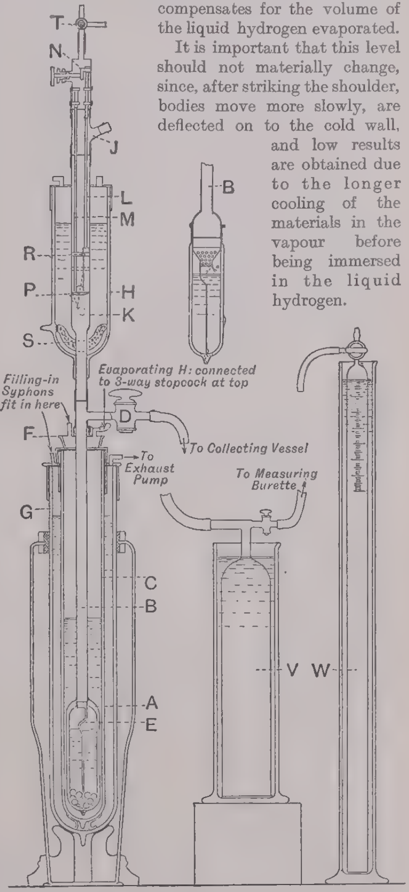
T
N
J
L
M
R
P
H
K
S
B
Evaporating H: connected to 3-way stopcock at top
Filling-in Syphons fit in here
D
F
To Collecting Vessel
To Exhaust Pump
To Measuring Burette
G
C
B
V
W
A
E

Fig. 6.

The isolation of the calorimeter was such that less than 10 c.c. of hydrogen gas was evaporated from it per minute. The whole apparatus is supported between the cork-lined spring jaws mounted on a heavy metal base on which the outer vacuum vessel rests.

The cooling vessel H is connected by an india-rubber tube to the top of the calorimeter. It consists of an ordinary cylindrical silvered vacuum vessel, 20 cm. long and 7 cm. wide, with a central axial open tube K sealed in below. This tube passes through the liquid in the vacuum vessel. It has the same diameter below as the neck tube of the calorimeter. Near the top of the central tube a side tube J, of about the same diameter and some 3 cm. long, serves for the introduction of the weighed pieces of material, which are all cooled previously to the temperature of liquid air, and then fall on to a thin metal disc P fitting loosely the tube K, where they remain about 15 minutes. P is supported by being hinged to two thin ebonite rods, L and M, fixed to a brass fitting cemented on to the top. The rod L is not fixed directly to the disc but to a metal ring R. From the ring R two thin vertical steel wires are connected freely to two points on the circumference of the pan below. This rod and the attached ring can be given a vertical motion by a crank N in the top fitting, thereby tipping the pan and releasing the piece of material, which then falls freely down into the calorimeter. A high vacuum is maintained by a cross-tube S, opening to the annular space, filled with charcoal.

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[1] *Proc. Roy. Soc.*, 1913, lxxxix. 158.

At the temperature of boiling nitrogen, the convection currents in the central tube of such a vessel, when connected to the calorimeter below, have no serious effect on the temperature in the tube at a reasonable distance from the bottom, provided the central tube be not wide. With a larger pattern vessel the width of the central tube was increased to 2·2 cm., and even here the difference was under 3° at the level of the pan. These temperatures were measured by a small helium thermometer, consisting of a 4-c.c. bulb to which was sealed a small mercury manometer of fine capillary tubing.

The hydrogen evaporating from the liquid in the vacuum vessel C, in which the calorimeter is immersed, is employed in the interval of observations to maintain a hydrogen atmosphere through the neck of the calorimeter and the connected measuring tubes. Risk of solid air in the calorimeter neck is thus obviated. A simple arrangement of a three-way cooling vessel allows this to be manipulated.

The vessel V consists of a glass tube 8 cm. in diameter and 40 cm. long, open at the bottom and provided with a wide T-piece at the top. The tube is immersed to the neck in water in a glass cylinder, and is counterpoised by a weight and cord running over a pulley just above. It is thereby readily raised during the time gas is being evaporated from the calorimeter; this ensures that no back pressure is produced. One arm of the T-piece is open and connects to the stopcock D on the calorimeter neck; the other is provided with another small stopcock and connects to a 200-c.c. gas burette W similarly immersed in water. This latter stopcock is closed while the gas evaporated during an experiment is collected.

These arrangements are necessary to secure the minimum impediment to the evaporating hydrogen, which is usually evolved in less than 10 seconds, any temporary back pressure being fatal to concordant results. At least 15 seconds are allowed for collecting the gas given off, and even longer, in some cases, with badly conducting bodies.

As far as possible the materials used were cast in the forms of spheres about 8 mm. diameter.

In the case of liquid bodies, the mould was first cooled by liquid air. Frequently liquids were frozen into solid cylinders in thin glass tubing, and pieces cut off after removing the glass mould. The metallic materials were in some cases fused into buttons of convenient weight in an exhausted quartz tube. The lead, however, of which many pieces were required, was cut from rod, and subsequently squeezed in a small spherical mould.

Volatile bodies were weighed at a low temperature on a light German silver pan supported by a thin platinum wire suspension from the balance pan about 2 cm. above the level of liquid air contained in a wide deep vacuum vessel. Some materials would not make coherent bullets or cast sticks, and these were filled into very thin-walled cylindrical metal capsules.

In order to obtain consistent results it is necessary to employ exactly the same procedure in each test, but with this apparatus Dewar was able to obtain results which rarely varied among themselves by more than 2 or 3 per cent.

The data thus obtained for the mean specific heats of 53 elements at about 50° abs. are summarised in Table V.

TABLE V

Element.	Atomic Weight.	Specific Heat	Atomic Heat.
Lithium	7·03	0·1924	1·35
Glucinum	9·1	0·0137	0·125
Boron	11·0	0·0212	0·24
Carbon (Acheson graphite)	12·0	0·0137	0·16
Diamond	12·0	0·0028	0·03
Sodium	23·0	0·1519	3·50
Magnesium	24·4	0·0713	1·74
Aluminium	27·1	0·0413	1·12
Silicon, fused, elec. cruc.	28·4	0·0303	0·86
,, crystallised	28·4	0·0271	0·77
Phosphorus (yellow)	31·0	0·0774	2·40
,, (red)	31·0	0·0431	1·34
Sulphur	32	0·0546	1·75
Chlorine	35·45	0·0967	3·43
Potassium	39·15	0·1280	5·01
Calcium	40·1	0·0714	2·86
Titanium	48·1	0·0205	0·99
Chromium	52·1	0·0142	0·70
Manganese	55	0·0229	1·26
Iron	55·9	0·0175	0·98
Nickel	58·7	0·0208	1·22
Cobalt	59·0	0·0207	1·22
Copper	63·6	0·0245	1·56
Zinc	65·4	0·0384	2·52
Arsenic	75·0	0·0258	1·94
Selenium	79·2	0·0361	2·86

TABLE V—*continued*

Element.	Atomic Weight.	Specific Heat.	Atomic Heat.
Bromine	79·96	0·0453	3·62
Rubidium	85·1	0·0711	6·05
Strontium, impure	87·6	0·0550	4·82
Zirconium	90·6	0·0262	2·38
Molybdenum	96·0	0·0141	1·36
Ruthenium	101·7	0·0109	1·11
Rhodium	103·0	0·0134	1·38
Palladium	106·5	0·0190	2·03
Silver	107·93	0·0242	2·62
Cadmium	112·4	0·0308	3·46
Tin	119	0·0286	3·41
Antimony	120·2	0·0240	2·89
Iodine	126·97	0·0361	4·59
Tellurium	127·6	0·0288	3·68
Caesium	132·9	0·0513	6·82
Barium, impure	137·4	0·0350	4·80
Lanthanum	138·9	0·0322	4·60
Cerium	140·25	0·0330	4·64
"Didymium"	142	0·0326	4·63
Tungsten	184	0·0095	1·75
Osmium	191·0	0·0078	1·49
Iridium	193·0	0·0099	1·92
Platinum	194·8	0·0135	2·63
Gold	197·2	0·0160	3·16
Mercury	200	0·0232	4·65
Thallium	204·1	0·0235	4·80
Lead	207	0·0240	4·96
Bismuth	208	0·0218	4·54
Thorium	232·5	0·0197	4·58
Uranium	238·5	0·0138	3·30

The interesting fact discovered in the course of this investigation was that the atomic heats were periodic functions of the atomic weight, and the curve resembled, generally, the well-known Meyer atomic volume for the solid state.

E. G.

CALORIMETRY, METHOD OF MIXTURES

§ (1) INTRODUCTION.—Calorimetric apparatus assumes the most diverse form, each type characterised by certain features, which adapts it especially for a particular class of measurement.

For the determination of the mean specific heat of a substance over a range of temperature or for the determination of the heat of combustion of a fuel the Method of Mixtures is a convenient one to employ and is probably the best known of all calorimetric methods.

§ (2) THE METHOD OF MIXTURES.—The principle of this method is to impart the quantity of heat to be measured to a known mass of water contained in a vessel of known thermal capacity, and to observe the rise of temperature produced; from which data, as explained in detail farther on, the quantity of heat can be calculated. This method is the simplest of calorimetric methods, but is not the most accurate. Heat is lost in transferring the hot object to the calorimeter, and although it can be minimised by arranging that the transfer takes place rapidly, it cannot be eliminated or even accurately allowed for. Some heat is lost when the calorimeter is raised above the temperature of its enclosure and before the final temperature is reached. This can be roughly estimated by observing the rate of change of temperature and assuming that the heat loss is directly proportional to the duration of experiment and to the average excess of temperature. The accurate determination of this correction is of fundamental importance in this method and a detailed discussion of it will be given. It is always desirable to diminish the loss of heat as much as possible by polishing the exterior of the calorimeter to diminish radiation, and by suspending it by non-conducting supports inside a polished case to protect it from draughts. It is also very important to keep the surrounding conditions as constant as possible throughout the experiment. This may be secured by using a large water-bath around the apparatus, but in experiments of long duration it is advisable to use an accurate temperature regulator. The method of lagging the calorimeter with cotton-wool, which is often recommended, diminishes the heat loss considerably but renders it very uncertain, and should never be used in work of precision, since the bad conductors take so long to reach a steady state that the rate of loss depends on the past history more than on the temperature of the calorimeter at the moment. A more serious objection to the use of lagging of this kind is the danger of its absorbing moisture. The least trace of moisture in the lagging may produce serious loss of heat by evaporation.

Regnault about 1840 made a careful study of the Method of Mixtures, and by skill and attention to detail he obtained by means of it a valuable series of thermal data. It is only within comparatively recent years that any material improvements on Regnault's apparatus have been effected.

§ (3) THEORY OF METHOD OF MIXTURES.—It will be assumed that the cases under consideration are those of solids and liquids. The determination of the specific heats of gases requires especial consideration and is dealt with in a separate section.

Let M = mass of heated body,
θ = temperature of heated body at the moment of its immersion in the water of the calorimeter,
W = mass of water employed,
t = temperature of water when the body is immersed in it,
T = temperature when the thermal equilibrium is established between the body and the water, *i.e.* when temperature of water ceases to rise (or sink if the body was cooler than the water).

Now if s = mean specific heat of water between T and t, and S = specific heat of the body between θ and T, we get

$$MS(\theta - T) = Ws(T - t),$$

and

$$S = \frac{Ws(T-t)}{M(\theta - T)}. \quad . \quad . \quad . \quad (a)$$

This gives the calorimetric equation for the method of mixtures in its very simplest form; for practical working several corrections are necessary.

(i.) For W we must substitute another magnitude W_1 such that $W_1 = W +$ mass of water, having the same heat capacity as containing vessel, stirrer, and all parts whose temperature is seriously affected:

I.e. if μ = mass of metal reservoir, stirrer, etc.,
and s_1 = mean specific heat of this metal between T and t,

$$W_1 = W + \mu\frac{s_1}{s},$$

and in this case equation (a) becomes

$$S = \frac{W_1 s(T-t)}{M(\theta - T)}. \quad . \quad . \quad . \quad (b)$$

Sometimes the substance has to be supported in a receptacle, and if we put a = water-equivalent of such receptacle and taking $s = 1$ within the limits of the experiment, we get

$$S = \frac{W_1(T-t)}{M(\theta - T)} - \frac{a}{M}. \quad . \quad . \quad . \quad (c)$$

Where the results are to be as accurate as possible, equation (c) takes a more complicated form. In addition to the magnitudes already represented, we have to take account of the thermal capacity of the thermometers, etc.

In addition to these corrections dependent on the nature of the various parts of the apparatus, there is the cooling correction for the loss of heat from the calorimeter to its surroundings.

§ (4) MODERN APPARATUS FOR THE METHOD OF MIXTURES.—The complete outfit required for experiments with the method of mixtures consists of the following elements, which will be considered individually:

(i.) The calorimetric vessel and the device for mixing the contents.

(ii.) The jacket enclosing the calorimeter.

(iii.) The thermometer for measuring the temperature rise of the water.

(iv.) The appliance for heating or cooling the charge (in specific heat determinations).

§ (5) CALORIMETER AND STIRRER. (i.) *The Calorimeter.*—The calorimeter is usually made of pure copper, nickel plated and polished so as to reduce radiation to a minimum.

The use of a vacuum jacket as a calorimeter is not to be recommended for ordinary work. Undoubtedly it is possible by means of it to reduce the magnitude of the cooling rate, but this advantage is more than counterbalanced by the disadvantages of this form of container due to its brittleness, the lag of the portions of the glass walls above the surface of the water, the difficulty of ascertaining its heat equivalent and of keeping it constant.

Under certain circumstances the use of water or other calorimetric fluid becomes impracticable. Hence some investigators have employed as calorimeter thick-walled metallic cups and trusted to the high coefficient of heat conductivity of the metal to equalise the temperature. These specialised forms of calorimeters are described later.

(ii.) *The Stirrer.*—Stirrers vary considerably in design according to the specific purpose for which they are required. A typical form is illustrated in *Fig.* 1; here a screw is employed for stirring the contents. The vessel is constructed with an ∞-shaped cross-section, the stirrer being contained in a tube which is connected with the main tube at the top and bottom. By this arrangement it is possible to ensure a steady circulation through the calorimeter, and it is advisable to direct the stream as smoothly as possible by suitably curved passages.

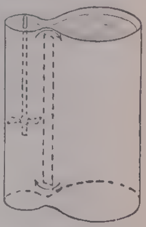

FIG. 1.

In the design of such a calorimeter particular attention must be given to the provision of wide passages for the circulation of the water and care taken to avoid as far as possible dead spaces. It might be remarked that this method of stirring has been found to be the most reliable for comparison baths for mercury thermometers and is much superior to a plain screw in a vessel of liquid.

White [1] adopted a somewhat similar form of calorimeter for his experiments on the specific heat of silicates at high temperatures. The charge, contained in a platinum crucible, was dropped directly from the furnace into the calorimeter.

It will be observed from *Fig.* 2 that the cover W is in actual contact with the water to ensure temperature equilibrium. The annular space under the cover permits the water to vary somewhat in amount without overflowing or failing to wet the cover, while the weight of the cover (70 grm.) prevents it from being floated out of the place when the water is high. An approximate preliminary adjustment of the amount of water is of course necessary, and is very easily obtained. Evaporation through the joint is about 7 mg. an hour, which is not likely to produce appreciable error. An oil seal gives very little trouble, and is used where maximum accuracy is desired.

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[1] "Some Calorimetric Apparatus," *Phy. Rev.*, Dec. 1910, xxxi. No. 6.

Complete enclosure of the calorimetric vessel, with the possibility of quick opening and closing, and thorough circulation without much heating, are the main requirements.

In *Fig.* 3 a totally different type of stirrer is shown where the contents are mixed by a flat paddle of annular form which has a reciprocating motion in the liquid. In order to avoid pumping air in and out of the calorimeter by the reciprocating motion two small rubber bellows are fixed as shown with the end of the stirrer rod attached to the mid-point. The compression of the one bellows is compensated for by the extension of the other. This device is due to Professor E. H. Griffiths.

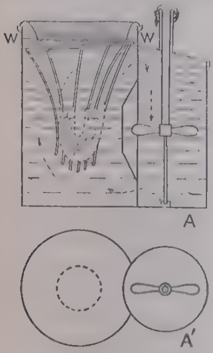

FIG. 2.

A, vertical section; A′, horizontal section. The position of the crucible is shown by the dotted lines.

The method of stirring demands the most careful consideration in any arrangement that may be adopted. Vigorous stirring of the water is necessary to keep the calorimeter at a uniform temperature throughout while the heat is being rapidly supplied to it. There are two distinct functions of a stirrer, mechanically more or less incompatible: First, the whole mass of liquid in a calorimeter must be circulated, so that there are no stagnant portions; and second, there must be thorough mixing of the different portions of the liquid in order that a measurement of its mean temperature may be obtained with a thermometer of convenient size.

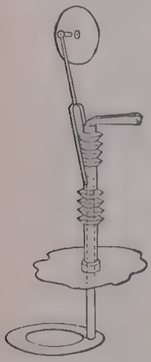

FIG. 3.

The reciprocating paddle form of stirrer mixes thoroughly the smaller portions of the liquid but produces little positive circulation, so that certain portions of the liquid may be left nearly stagnant. The screw propeller type of stirrer, on the other hand, properly applied, produces a rapid circulation of the whole mass of liquid, but may not mix so thoroughly the different portions, *i.e.* it permits of definite stream lines. Such a stirrer can be mounted on a very small shaft and in such a way as not to promote evaporation.

The tendency at the present day is to employ a small propeller driven continuously by a belt from a small electric motor, but for occasional work a hand-operated stirrer is preferred on account of its simplicity.

The common type of stirrer, which is standard on combustion outfits, consists of a copper annulus of such diameter that it loosely fits the inside of the calorimeter and is oscillated up and down by a rod. This form of stirrer is not to be recommended for the stem moves in and out of the liquid, promoting evaporations, which may be a serious source of error.

The paddle form of stirrer shown in *Fig.* 3 avoids this defect. Another simple paddle form of stirrer is illustrated in *Fig.* 4.

In the top edge of the calorimeter two small notches are cut at opposite ends of a diameter. The stirrer consists of a thin horizontal copper rod bent to the form shown and resting in the notches. Two thin copper plates are soldered to the rod while a small glass rod is cemented to form a handle. By giving the handle a to-and-fro motion the paddles move backwards and forwards through the liquid and thus mix it up fairly thoroughly.

(iii.) *Evaporation from Exposed Liquid Surface.* Experience has shown that one of the greatest sources of error in the Regnault form of calorimeter is the heat loss by evaporation from the exposed water surface. The evaporation of 10 milligrams of water consumes 6 calories, and since a calorimeter determination seldom involves a quantity of heat greater than 6000 calories an irregularity in the evaporation rate amounting to 10 milligrams introduces an error of one part in a thousand.

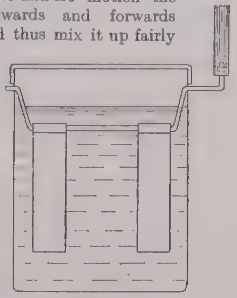

FIG. 4.

Observations with an open calorimeter of standard form have shown that a rate of evaporation of 10 milligrams per minute is not unusual, and this involves a heat loss as great, on the average, as that from the cooling by radiation, etc., besides being far more uncertain.

Consequently, in all precision work the calorimeter should be closed by a fairly tight-fitting cover so as to check the loss by evaporation.

Another precaution to be observed in this connection is to prevent the possibility of the evaporation of small amounts of water from the outside of the calorimeter during the course of the experiment, as the error due to this cause may be especially serious on account of the fact that the evaporation may be completed before the cooling correction is determined and consequently no allowance made for this heat loss. Two possible sources of such error should always be guarded against.

(*a*) The presence of a minute drop of water on the outside of the calorimeter due to lack of care in filling.

(*b*) Absorbed moisture if the surface is hygroscopic. A tarnished copper surface 100 cm. square in area will absorb 7 milligrams of water in a saturated atmosphere at ordinary temperature, whilst a surface of polished nickel of the same size and under the same conditions will not absorb as much as 0·1 milligram. Hence it is always advisable to provide the calorimeter with a lid, although it will be found that to close the calorimeter effectively the addition of a lid complicates the construction.

Even with the simplest form of apparatus a substantial improvement is produced by providing a lid of very thin metal having a slot to allow of its being removed for the introduction of the hot body without disturbing the thermometer.

§ (6) THE JACKET SURROUNDING THE CALORIMETER.—White[1] during the course of his extended researches on the specific heats of the silicates has devoted much attention to the design of the calorimetric apparatus employed.

One form of jacket for maintaining a constant temperature enclosure around the calorimeter which he has described is shown in *Fig.* 5. The jacket is shown in section and also in top view, partly open. Water is held up in the two halves of the cover, and in the upper section of the chamber wall, by atmospheric pressure. On leaving the propeller the water divides and passes across through the three upper passages, and then returns through the lower space. Its circulation is directed by the partitions P and Q, of which Q runs nearly the whole length of the tank outside the chamber, as shown by the dotted line. The chamber is opened by moving the covers aside; their down-turned ends then move in the troughs left at the ends of the tank. The covers slide upon the stout rod T. The pulley for the calorimeter stirrer, and the whole jacket stirrer, are borne on one half cover. This half is clamped firmly in place during an observation; moving aside the other fully exposes the calorimeter opening.

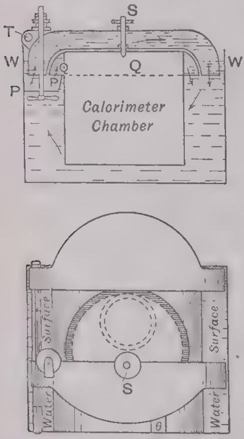

FIG. 5.

T, rod on which the top slides; S, stirrer pulley; *θ*, groove for passage of thermoelectric thermometer; W, water level.

(i.) *Method of supporting the Calorimeter.*—Heat transfer between the calorimeter and its jacket may take place in four ways—by conduction, convection, radiation, and evaporation. The object of applying the cooling correction is to eliminate this heat loss from the final result.

Whilst in practice it is not necessary to study these effects separately, it might be remarked that under ordinary conditions the greater portion of the heat transfer is due to convection and air conduction, the two together constituting about 80 per cent of the total.

It is advisable to reduce the transfer by thermal conduction through the supports of the calorimeter to a minimum since it constitutes an uncontrollable source of error.

Consider, for example, the case of a calorimeter supported within the enclosure on a sheet of cork or rubber. When the calorimeter and enclosure are at two steady temperatures the heat transfer is, by the laws of conduction, proportional to the temperature difference; if, however, the temperature of the calorimeter is changing rapidly the rate of transfer is not even approximately proportional to the temperature difference.

The following is a discussion of this source of error by Dickinson:[2]

Since the conductivity of such materials is always small, compared with that of the metallic sheets in contact with them, the temperatures of the surfaces may be taken, for the purposes of this discussion, as approximately the same as the measured temperatures of the calorimeter and the jacket respectively. The distribution of temperature in such a layer and the rate at which heat is leaving the calorimeter at any time may be then determined from the following considerations:

A sheet of material of thickness c bounded by plane surfaces x_0 and x_1 is initially at temperature θ_0. If x_0 and θ_0 are each taken as 0 for convenience, and the temperature of one of the surfaces x_1 is then caused to rise from θ_0 to θ' in such a way that $\theta=\theta'(1-e^{-\alpha t})$, the temperature distribution in this plate is given by the following equation:[3]

$$\theta=\frac{x}{c}\mathrm{F}(t)+\frac{2}{\pi}\sum_{1}^{\infty}\left[\frac{(-1)^m}{m}\sin\frac{m\pi x}{c}\left(\mathrm{F}(t)-\frac{m^2a^2\pi^2}{c^2}\int_0^t\mathrm{F}(\lambda)e^{\frac{-m^2a^2\pi^2}{c^2}(t-\lambda)}d\lambda\right)\right].$$

Where a^2 is the thermometric conductivity of the material, F(t) is the temperature of the face $x_1=c$, taken here as $\theta'(1-e^{-\alpha t})$.

The point of interest in this discussion is the rate at which heat is leaving the calorimeter at any time, as this determines the value of the emissivity for the portion of the surface in question. If θ' and c are each made unity, and the above expression for θ is differentiated with respect to x, the following expression for the temperature gradient at any point in the material is found :

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[1] *Loc. cit.*

[2] "Combustion, Calorimetry, and the Heats of Combustion of Cane Sugar, Benzoic Acid, and Naphthalene." *Sci. Paper Bur. Stds.*, 1914, No. 230.

[3] Byerly, *Fourier's Series and Spherical Harmonics*, p. 110.

$$\frac{d\theta}{dt}=1-e^{-\alpha t}$$

$$+2\sum_{1}^{\infty}(-1)^m \cos m\pi x(e^{-\alpha t}-e^{-m^2\pi^2a^2t})\frac{\alpha}{m^2\pi^2a^2-\alpha}.$$

The surface $x=1$ is the surface in contact with the calorimeter, so that substituting this value of x and the appropriate values for a^2 and α, the above expression gives the temperature gradient in the material in contact with the calorimeter, which is proportional to the factor k for this portion of the surface.

As an example showing the effect of this kind of distribution of material, suppose that the calorimeter rests on a sheet of ebonite 1 cm. thick and that the temperature in the calorimeter rises quite approximately according to the relation $\theta-\theta_t=(1-e^{-\alpha t})(\theta_1-\theta_i)$, where $\alpha=0{\cdot}03$[1] and θ, θ_i, and θ_1 represent, respectively, the temperature at any time, the initial, and the final temperature. The value of a^2, the thermometric conductivity, for ebonite is approximately 0·001 in C.G.S. units. These quantities substituted in the above equation show that after 60 seconds the rate of heat loss is 2·75 times its final value, after 5 minutes the rate is 1·13 times the final value, and only after 10 minutes does it come to within 1 per cent of its final value. If the area in contact with such a sheet were a considerable part of the whole area of the calorimeter, the error introduced from this cause evidently would be a very serious one. Such a distribution of material as here discussed will also have an effect on the heat capacity of the calorimeter.

This discussion shows that all non-conducting supports should be negligibly small, or, since the thermometric conductivity $a^2=\mathrm{K}/c\rho$, the absolute conductivity K divided by the specific heat (c) and density (ρ), the material used for them should have a small density and specific heat. A form of support should therefore be employed, in which the smallest possible mass of insulating material is used, with the smallest possible area in contact with the calorimeter. The mass of such supports can readily be made negligible compared with that of the calorimeter.

(ii.) *Supports and Space between Calorimeter and Jacket.*—To reduce errors due to the above cause to a minimum Dickinson[2] employed the following arrangement of supports for his calorimeter. The supporting pieces (three in number) are each made up of a brass cone soldered to the bottom of the jacket, and a small ivory tip about 2 mm. in diameter cemented into the end of the cone and resting against small plates (one with a hole, one with a slot, and the third plane) on the bottom of the calorimeter. The thermal conductivity of the ivory tips is small, and their total mass is not over 0·1 gm., so that their effect on the cooling rate is too small to be significant. The brass cones, while they have a considerable mass, have a heat conductivity so great compared with the amount of heat which they can receive by radiation, convection, etc. (about 0·0001 calorie per square centimetre per second per degree temperature difference), that their temperature is at all times measurably that of the jacket, hence their effect is entirely negligible, both as regards cooling rate and heat capacity.

(iii.) *Heat Conduction along the Stirrer.*—The steel stirrer shaft which enters the calorimeter should end just above it in a thin rubber sleeve, which should fit tightly over it and tightly within a larger steel piece coupled to the driving shaft. It is evident that since the heat conductivity of steel is many times greater than that of the hard rubber sleeve, the temperatures of the two metal parts will remain very nearly the same as the temperatures of the calorimeter and the jacket respectively. The heat capacity of the rubber sleeve, some of which should be added to that of the calorimeter, is insignificant.

§ (7) METHOD OF CALCULATING THE COOLING CORRECTION. (i.) *Rumford.*—Rumford was the first to introduce a method of correcting for the heat loss from the calorimeter. His procedure was to make a preliminary experiment to ascertain approximately what the rise of temperature would be and then to cool the calorimeter half this number of degrees below the temperature of the surrounding atmosphere before the next experiment.

For example, let

$$\text{Temperature of atmosphere}=t^\circ,$$
$$\text{Approximate increase}=2\theta.$$

The calorimeter is cooled to $(t-\theta)^\circ$, and the heated body then introduced : the maximum temperature will be approximately $(t+\theta)^\circ$, and Rumford's idea was that the amount of heat gained by the calorimeter during the time its temperature was below t° will exactly compensate for the amount lost by it while its temperature was above t°. This is approximately true, but not quite so, owing to the fact that the rate of increase of temperature diminishes very rapidly as the heated body and the water approach thermal equilibrium : thus, it may happen that the rise of temperature from $(t-\theta)^\circ$ to t° will occur in less than 20 seconds, while the rise from t° to $(t+\theta)^\circ$ will occupy over 100 seconds.

(ii.) *Arithmetical Method of computing Heat Loss.*—A far more accurate, but not nearly so easy a method of correction is the following: Cool the calorimeter several degrees below the enclosure and take very careful readings at intervals of about 20 seconds before and after the introduction of the hot body, and also after the establishment of thermal equilibrium between the hot body and the water. Let θ, θ_1, θ_2, . . . θ_n be the temperatures of the calorimeter at the beginning and at the end of m periods of, say, 20 seconds each before the introduction of the hot body.

Let t, t_1, t_2 t_3, . . . t_n be the temperatures for n periods of equal duration after the introduction of the hot body up to the establishment of thermal equilibrium between the hot body and the water.

Let T, T_1, T_2 . . . T_r be the temperatures for r similar periods after the establishment of thermal equilibrium : t_n and T are virtually the same. The temperature t should not be taken from the reading of the thermometer, but should be calculated as follows:

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[1] Value found by experiment by Dickinson for his calorimeter.

[2] *Loc. cit.*

$$i = \theta_m - \frac{\theta_m - \theta}{m}.$$

During the first interval after the introduction of the heated body the mean temperature of the calorimeter has been $t+t_1/2$, and if we put v for the change in the temperature of the calorimeter due to its surroundings we get for the first interval

$$v_1 = a\left(\frac{t+t_1}{2} - x\right),$$

where a and x have to be calculated. For the second interval we have

$$v_2 = a\left(\frac{t_1+t_2}{2} - x\right),$$

where a and x have the same values as in the expression for v_1. Finally,

$$v_n = a\left(\frac{t_{n-1}+t_n}{2} - x\right),$$

so that between t and t_n

$$\Sigma v = a\left(\frac{t+t_n}{2} - t_1 + t_2 + \ . \ . \ . \ t_{n-1} - nx\right).$$

The values of a and x are found in the following way: Let ϕ represent the mean temperature of the calorimeter before the introduction of the hot body,

$$\therefore \phi = \frac{\theta + \theta_1 + \theta_2 + \ . \ . \ . \ \theta_m}{1+m},$$

so that the mean value of $v = a(\phi - x)$ between θ and θ_m; and putting ϕ_1 for mean temperature after establishment of thermal equilibrium,

$$\therefore \phi_1 = \frac{\mathrm{T} + \mathrm{T}_1 + \mathrm{T}_2 + \ . \ . \ . \ \mathrm{T}_r}{1+r},$$

and the mean value of $v^1 = a(\phi_1 - x)$.

$$\text{Now } v = \frac{\theta - \theta_m}{m}, \text{ and } v' = \frac{\mathrm{T} - \mathrm{T}_r}{r}.$$

From the two equations $v = a(\phi - x)$ and $v' = a(\phi_1 - x)$

we get
$$a = \frac{v - v'}{\phi - \phi_1}$$

and
$$x = \frac{v\phi_1 - v'\phi}{v - v'},$$

from which Σv can be calculated, and hence the corrected value of the change of temperature of the calorimeter.

(iii.) *Graphical Method of deducing the Heat Loss.*—A graphical method of computing the cooling correction is due to Rowland.

(*a*) *Rowland's Method.*[1]—Instead of finding the number of heat units lost by the body while the temperature of the body is rising to its maximum value, the heat loss can be accounted for if the temperature is determined which the body would have attained if there had been no loss. The correction is obtained as follows: First make a series of measurements of the temperature of the water of the calorimeter, before and after the charge is dropped in, together with the times. Then plot them graphically on a large scale as in *Fig.* 6. Five or ten centimetres to a degree are sufficient. *nabcd* is the plot of the temperature of the water of the calorimeter, the time being plotted horizontally and the temperature vertically. Continue the line *dc* until it meets the vertical line *la*. Draw a horizontal through the point *l*. At any point *b*, of the curve, draw a tangent and also a vertical line *bg*; lay *lf* equal to *eg*, and draw the line *fhk* through the point *h*, which indicates the temperature of the atmosphere of the vessel surrounding the calorimeter. Draw a vertical *jk* through the point *k*. From the point of maximum *c* draw a line *jc* parallel to *dm*; where it meets *kj* will then be the required point. Hence, the rise of temperature, corrected for all cooling errors, will be *kj*.

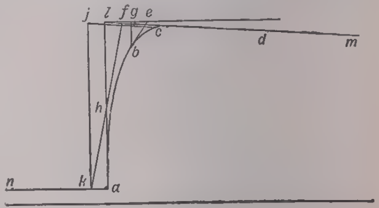

FIG. 6.

This method, of course, only applies to cases where the final temperature of the calorimeter is greater than that of the air; otherwise there will be no maximum.

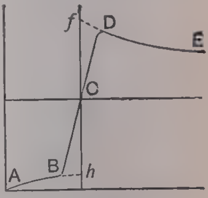

FIG. 7.

(*b*) *Ferry's Method.* — In the following modification by Ferry of Rowland's method this temperature can be obtained to a fair approximation by a simpler graphical construction.

Let C represent the temperature of the surroundings, and let a body at a temperature below these be given a quantity of heat H such that its temperature rises to a value above C. The way in which the temperature changes before the heat H is added is represented by the line AB in *Fig.* 7. The line BD shows how the temperature changes while the body is absorbing the heat H. From B to C the body is, in addition, receiving heat from the surroundings, and from C to D is losing heat to the surroundings. The line DE indicates the temperature changes due to radiation, etc., alone.

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[1] "On the Mechanical Equivalent of Heat, with Subsidiary Researches on the Variation of the Mercurial from the Air Thermometer, and on the Variation of the Specific Heat of Water," *Proceedings of the American Academy of Arts and Sciences*, 1880, xv. 75-200; also *Physical Papers*, p. 402.

Through C draw a vertical line. Prolong ED backward until it cuts this vertical in *f*. Prolong AB forward till it cuts the vertical line in *h*. Then the temperature changes are given by *hf*.

To see that the above method of finding the desired temperature is reasonable, consider the following: If the heat H had not been given to the body, it would have continued to rise in temperature in the same way that it was rising from A to B, so that by the time it really attained the temperature indicated by C it would have reached the temperature indicated by *h*; that is, while the body really rose in temperature from B to C the rise in temperature from B to *h* was due to heat from the surroundings, and the rise from *h* to C was due to a part of the heat H. Again, if the body had not been given the heat H, but if it had been at first at such a temperature that as it cooled it reached the temperature indicated by D at the same instant that it really reached that temperature—and thereafter cooled as shown by DE—it would have been at a temperature *f* at the instant when it really was at the temperature C; that is, while the body really rose in temperature from C to D the fall in temperature due to radiation was the fall from *f* to D, so that if there had been no loss of heat by radiation the rise of temperature during this time would have been from C to *f*. If, then, there had been no gain or loss of heat by radiation the body would have risen in temperature the amount indicated by the distance from *h* to *f*.

While the temperature of the body rose from C to D it was really at a lower temperature than if it had been cooling from *f* to D, and so did not really lose as much heat by radiation as has above been supposed. Hence, the point *f* is higher than it ought to be. For a similar reason *h* is also somewhat higher than it ought to be. If the time from B to C is about the same as that from C to D, these two errors will nearly balance each other.

(iv.) *Adiabatic Methods.* — To eliminate entirely the necessity for correcting for the heat transfer between the calorimeter and its jacket, T. W. Richards[1] has devised various forms of calorimeters in which the bath surrounding the calorimeter is kept throughout the experiment at an equal or equivalent temperature. This device has been found to be particularly convenient in experiments on heat of dilution, heat of reaction, and recently it has been adapted for fuel calorimetry work. The precise method adopted for heating the jacket to keep it in step with the calorimeter varies. Richards has used the heat liberated by chemical reaction of the same character as that under test in the calorimeter. For most purposes, however, electrical heating is the more convenient. It is usually necessary to make a few blank experiments to settle the relative values of the current, so the method finds its greatest field of application when a large number of experiments have to be performed. Since the stirrer in the calorimeter generates an appreciable amount of heat, it is convenient to keep the temperature of the jacket at a temperature of a degree or so below that of the calorimeter, so that the residual heat loss just balances that generated by the stirring. Whilst the device cannot give greater absolute accuracy than that in which a stationary jacket temperature is employed, it has the advantage that the initial and final temperatures are stationary and hence more easily measurable with resistance thermometers than would be the case if the temperature were moving.

§ (8) Thermometer for measuring the Temperature Rise of the Water.—The most generally used instrument for the measurement of the temperature rise of the calorimeter is the mercury thermometer, but in work of precision the resistance thermometer is to be preferred.

Experience has shown that the inherent defects of the mercury thermometer limit the possible accuracy to 2 or 3 parts in 1000 for a 2° rise of temperature, while with a suitable resistance thermometer outfit ten times this accuracy may be obtained, but of course it necessitates an expensive equipment and more labour with the observations. For a discussion of calorimetric mercury thermometers reference should be made to the articles on "Thermometers" and "Resistance Thermometers" respectively.

§ (9) Appliances for heating or cooling the Charge. (i.) *Steam Heater.*—Regnault in his extended series of experiments used a steam-jacket heater to bring the charge to 100° before dropping it into the calorimeter. His form of heater consisted merely of a boiler with a tube containing the charge, and on inverting it the heated charge is dropped into the calorimeter. White has modified the Regnault heater to the form shown in *Fig.* 8 which is self-explanatory. He employs electrical heating, since then it is possible to move the apparatus about without danger of premature cooling. The apparatus is manipulated as follows: Just before discharging the temporary outlet is opened, the condenser and thermoelement removed, the opening stoppered, and lastly the heating chamber unstoppered and the charge dropped into the calorimeter. The object of the shallow cup below the chamber filled with condensed water is to shield the chamber against superheating. White points out that the usual practice of closing the upper end of steam heaters with corks is defective, since the ends are left comparatively cold, and consequently errors of the order of a few parts per 1000 may result.

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[1] *Journ. Am. Chem. Soc.*, 1909, xxxi. 1275; Richards and Burgess, *ibid.*, 1910, xxxii. 431; Richards and Rowe, *Proc. Am. Acad. Arts. Sci.* xlix. 173; Richards and Barry, *Journ. Am. Chem. Soc.*, 1915, xxxvii. 993; MacInnes and Braham, "A Calorimeter for Measuring Heats of Dilution," *Journ. Am. Chem. Soc.*, Oct. 1917, xxxix. 2110.

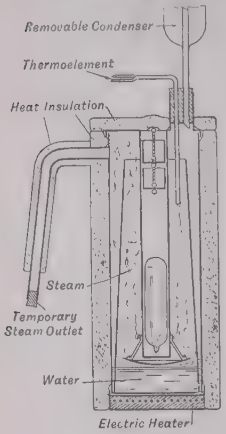

FIG. 8.

(ii.) *Electric Furnace.* — For heating the charge to high temperatures some form of electrical furnace is generally employed, as this permits of the attainment of temperatures up to 1500° C. It is, of course, necessary to ensure that the furnace should give a uniform temperature over the region occupied by the charge, and experience has shown that the simplest method of effecting this is to wind the tube uniformly, and over each end add additional coils capable of independent control; then, by adjusting the relative values of the current in the main circuit and the supplementary coils, a good degree of uniformity can be obtained. It might be remarked, however, that the ratio of the currents in the circuits which will give uniformity at one temperature may not necessarily apply to another, and consequently separate experiments are necessary to determine the best values for each point.

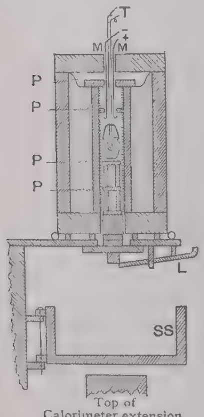

FIG. 9.—Arrangement of platinum-wound Furnace for Experiments between 500° and 1500° C.

P, partitions to shield against the cooling effect of the ends of the furnace; SS, swinging shield; L, latch for dropping out the furnace bottom; MM, heavy wires for the dropping current; T, thermoelement.

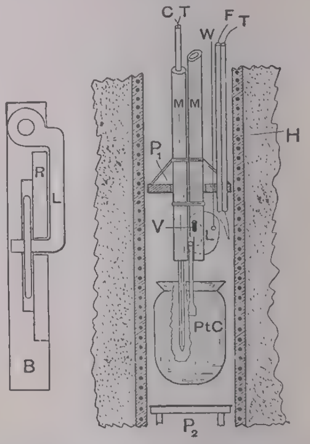

FIG. 10.—Apparatus of Marquardt Material and Platinum for automatically dropping the Platinum Container, PtC.

L, latch; V, its fulcrum; MM, tubes, about ·8 mm. in diameter; W, wire, whose pull unlatches the bail of the container; CT, charge thermoelement; FT, furnace thermoelement; P_1, P_2, shielding partitions; H, furnace winding. The platinum tube around CT in the container is supposed to be cut away. L is 2·3 mm. thick. MM were covered with sheet platinum, which was part of the equipotential shield.

In his work on the specific heat of silicates White employed a furnace with internal platinum winding. This has the advantage of permitting of the attainment of higher temperatures than is possible with a winding on the exterior of the tube, and also diminishes the lag between the coil and the chamber, so that equilibrium is obtained with greater rapidity. This furnace (*Fig.* 9) is mounted on a stout iron plate with an air space beneath. An opening of 5·5 cm. is cut through the furnace bottom and plate, which is closed by a plug of fireclay carried on a movable iron plate. This is held up against a large plate by the latch in such a way that a quick pull on the latch causes the plug to fall away without tipping. Upon the block and inside the furnace chamber is a pedestal made up of three fireclay partitions each faced on both sides, except on the side facing the crucible, with reflecting discs of platinum foil, and supported by a light frame made by grinding away as much as possible from the thin porcelain tube. This pedestal has two functions. It protects the crucible from the cooling effect of the furnace bottom and it also supports the crucible during the period necessary to reach the constant temperature. Platinum is very soft at high temperatures, and the fine wires which support the crucible for an interval of 1 second, which elapses between the fall of the crucible and the pedestal, would have to be very large if they were to hold for any length of time. At the higher temperatures the uppermost partition is apt to stick to the bottom of the crucible, hence it is tied by platinum wire to the plate below, the weight of which is enough to pull it away. Above the crucible another platinum partition is suspended. When the pedestal falls from the furnace it is caught in a light wooden box, which can be pushed out of the way, and at the same time switches off the furnace current.

The simplest method of supporting the charge in the furnace is to use a loop of platinum wire and fuse it by a strong current when it is desired to drop the charge into the calorimeter. This method was used successfully by Harker [1] in his experiments on the specific heat of iron.

White found some difficulty with the fused wire method of making a release at temperatures above 1000° C. owing to the arcing which occurred across the terminals after the wire was melted. This difficulty could no doubt have been overcome by making the current fuse a length of copper wire outside the furnace the same time as the supporting wire inside. He, however, devised a mechanical drop for releasing the charge. This was made of Marquart composition, which is obtainable in the form of tubes and plates. The complete arrangement is shown in *Fig.* 10. Stout platinum wire of 1·2 mm. in diameter was operated automatically to release the container as the wooden shield of the furnace was swung on one side. This was found to be generally satisfactory, but many troubles were encountered owing to insufficient rigidity of the supports, combined with a too rapid swing of the shield.

§ (10) THE SPECIFIC HEATS OF SILICATES AT HIGH TEMPERATURE. — With the above-described apparatus White [2] has made an extensive investigation of the heat capacity of various silicates at high temperatures. The data obtained are summarised in Table I., from which the "instantaneous" atomic heats have been calculated by the procedure described below:

TABLE I

INTERVAL MEAN ATOMIC HEATS

	0-100°.	0-300°.	0-500°.	0-700°.	0-900°.	0-1100°.	0-1300°.	0-1400°.
Silica glass	3·708	4·272	4·627	4·870	5·049	..	..	..
Quartz	3·755	4·359	4·784	5·112	5·217	5·308	..	..
Cristobalite	3·784	4·689	4·876	5·042	5·163	5·276	5·351	5·388
Anorthite	4·079	4·596	4·926	5·144	5·322	5·472	5·638	5·736
Andesine	4·012	..	4·857	5·086	5·263	..	..	..
Albite	3·960	4·479	4·805	5·030	5·207	5·346	..	..
Microcline	3·971	4·474	4·801	5·031	5·200	5·332	..	..
Microcline glass . . .	4·073	4·591	4·926	5·160	5·337	5·516	..	..
Pseudo-wollastonite . .	4·290	4·758	5·050	5·256	5·409	5·534	5·646	5·697
Mag. sil. amphibole . .	4·090	4·624	4·952	5·182	5·354	5·496	..	..
Mag. sil. pyroxene . .	4·103	4·647	4·997	..	..	..	0-1250°	..
Diopside	4·175	4·697	5·021	5·252	5·425	5·560	5·649	..

TABLE II

"INSTANTANEOUS" OR TRUE MEAN ATOMIC HEATS, THAT IS, HEATS AT DIFFERENT TEMPERATURES

	0°.	100°.	300°.	400°.	500°.	600°.	700°.	800°.	900°.	1000°.	1100°.	1200°.	1300°.
Silica glass . .	3·33	4·05	4·95	5·17	5·35	5·48	5·58	5·68	5·75	..	..	..	..
Quartz . . .	3·37	4·1	5·1	..	5·9	..	5·46	5·58	5·66	5·72	..	..	..
Cristobalite . .	..	..	..	..	..	5·46	5·55	5·62	5·67	5·72	5·77	5·82	5·86
Anorthite . . .	3·74	4·39	5·22	5·43	5·58	5·69	5·82	5·95	6·04	6·14	6·31	6·54	6·82
Andesine . . .	..	..	..	..	5·53	5·66	5·78	5·89	..	..	..	..	..
Albite . . .	3·61	4·28	5·10	5·31	5·46	5·59	5·71	5·83	5·91	5·97	..	..	..
Microcline . .	3·64	4·27	5·09	5·30	5·47	5·61	5·72	5·79	5·86	5·92	..	..	..
Microcline glass .	3·73	4·38	5·22	5·44	5·61	5·75	5·85	5·95	6·11	6·34	..	..	..
Wollastonite . .	..	..	..	..	..	..	..	..	6·11	..	..	..	..
Pseudo-wollastonite	3·98	4·58	5·32	5·50	5·65	5·77	5·87	5·95	6·02	6·10	6·18	6·26	6·33
Diopside . . .	3·82	4·49	5·32	5·52	5·69	5·83	5·94	6·03	6·10	6·17	6·24	..	..
Mag. sil. amphibole	3·73	4·42	5 26	5·46	5·62	5·76	5·87	5·96	6·04	6·13	..	..	..
Nernst - Lindemann formula for silica glass	..	..	4·95	..	5·35	..	5·55	..	5·67	5·71	5·75	..	5·80

Quartz at 550°, 6·3.

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[1] "The Specific Heat of Iron at High Temperatures," *Proc. Phys. Soc.* xix.; *Phil. Mag.*, Oct. 1905.

[2] "Silicate Specific Heats," second series, *Am. Journ. of Science*, Jan. 1919, xlvii.

For convenience the values are given as atomic heats, but can, of course, be readily converted back into specific heats by the use of the data given in Table III.

TABLE III

MEAN ATOMIC WEIGHTS, OR MOLECULAR WEIGHTS DIVIDED BY THE NUMBER OF ATOMS, USED AS MULTIPLIERS TO REDUCE SPECIFIC HEAT TO MEAN ATOMIC HEAT

Silica	20·1
Calcium metasilicate	23·27
Magnesium metasilicate	20·12
Diopside	21·70
Anorthite	21·45
Andesine	20·84
Natural albite	20 33
Natural microcline	21·23

The method of experiment gives the mean specific heat over a wide range of temperature and is not suitable for giving with accuracy the true specific heat, if this changes rapidly with temperature, as is the case at very low temperatures. At high temperature, however, the relation between specific and temperature is practically linear, hence it is possible to calculate the "instantaneous" or true specific heats with fair accuracy, as follows:

If the interval specific heat is sufficiently well expressed by polynomial equations with 5 constants, $A+B\theta+C\theta^2$, etc., where θ is centigrade temperature, the total heat from 0° C. up is $A\theta+B\theta^2+C\theta^3$, etc., and the true specific heat at any temperature, which is the differential of the total heat, is $A+2B\theta+3C\theta^2$, etc., so that the quantity which must be added to the mean specific heat to get the true heat is

$$B\theta+2C\theta^2+3D\theta^3+4E\theta^4.$$

But in a series of 4th-degree polynomials each first difference is

$$BP+2CP\theta+DP\left(3\theta^2+\frac{P^2}{4}\right)+EP(4\theta^3+\theta P^2);$$

each 3rd difference is $6DP^3+24EP^3\theta$, where P is the temperature interval between each two successive values in the series. It follows at once that by subtracting $\frac{1}{24}$ of the 3rd difference from the 1st, and then multiplying by θ/P, the difference of true and interval heats is obtained. The method is exactly equivalent to obtaining a series of 4th-degree equations and thus computing the true specific heat, but is much easier.

For quartz and silica glass the values of the *interval* specific heat to 100°, 300°, and 500° satisfy the expressions:

Quartz

$$0{\cdot}1685+0{\cdot}000194\theta-0{\cdot}00000011\theta^2.$$

Silica Glass

$$0{\cdot}1670+0{\cdot}0001890\theta-0{\cdot}000000125\theta^2.$$

§ (11) LOW TEMPERATURE APPLIANCES.—Nernst, Lindemann, and Koref[1] in their experiments at low temperatures cooled down the substance under test in a quartz vacuum vessel, through which passed a tube open at both ends, as shown in *Fig.* 11. This tube was surrounded by liquid air or a mixture of alcohol and solid CO_2. The device is operated as follows: As soon as the equilibrium of temperature has been obtained it is placed over the calorimeter. A slide is opened and the contents, suspended on a thread, are let down into the calorimeter.

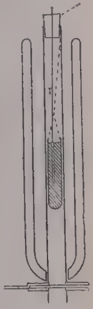

FIG. 11.

Instead of the expensive quartz vessel the following simple device may also be used (*Fig.* 12). In a large test tube A is placed a tube J, somewhat enlarged at the bottom and closed at both ends by means of rubber stoppers. Inside of it is a silver vessel, with the substance and the thermoelement suspended by a thread. The test tube is immersed in the constant temperature bath. The time required to obtain the equality of temperature may be shortened by passing through a slow current of dry hydrogen, which flows into the outer vessel through a small channel in the lower cork and then into the air through the cotton-wool. To bring the substance into the calorimeter the whole device is brought into the neighbourhood of the calorimeter, which is then opened. The inside tube J is rapidly removed from the test tube, the lower cork taken away, and the other one slightly lifted so as to allow the container to drop into the calorimeter. The whole manipulation takes about three seconds. In that time the substance is only slightly warmed up. With liquid air, for instance, it is claimed that the heat loss is hardly 1 per cent.

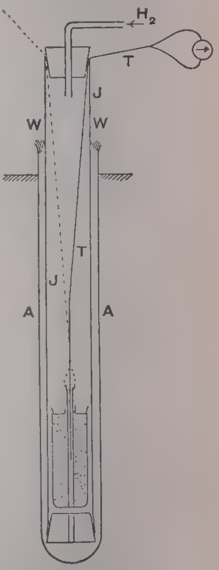

FIG. 12.

§ (12) THE THERMAL UNIT AND THE VARIATION OF THE SPECIFIC HEAT OF WATER WITH TEMPERATURE.—It is customary to express heat quantities in terms of the heat capacity of water for 1° change of temperature, and since this heat capacity is now known with accuracy in terms of the primary units—centimetre, gram, second—it is possible to express heat quantities in "ergs" by the use of an appropriate factor.

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[1] Nernst, Lindemann, and Koref, *Königliche Preussische Akademie der Wissenschaften zu Berlin, Sitzungsberichte*, 1910; Koref, *Annalen der Physik*. 1911 (4), xxxvi. 49.

For a long time it was not fully realised that the specific heat of water varied with temperature, and consequently the practical heat unit varied with the range of temperature through which the water was heated when employed as calorimetric medium. Even when it became apparent that the specific heat could not be regarded as constant the data published as to the magnitude of the variation were so conflicting that the experimenters were compelled to neglect them in the reduction of their observations.

In later years the work of Rowlands (1879), Bartoli and Stracciati (1891), Griffiths (1893), Ludin (1896), Callendar and Barnes (1902), has proved conclusively that the form of the specific heat-temperature curve is that shown in *Fig.* 13, and that there is a minimum value at 35° to 40° C.

Hence the *calorie* is only fully defined when the particular degree (1° C.) of temperature is specified through which unit mass of water is heated. Professor E. H. Griffiths has pointed out that the calorie between 17° and 18° C. is equal to the mean calorie between 0° and 100°. So that the specific heat of water between 17° an 18° C. might be considered as a suitable unit for calorimetry.

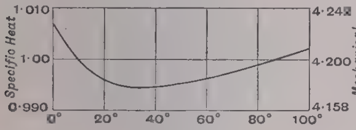

FIG. 13.

Callendar adopts 20° C., and suggests that the calorie be defined *not* as the heat required to raise one gram of water from 19·5° to 20·5°, but as the *mean* value between 15° and 25° of the quantity of heat required to raise one gram of water 1° C.

Whilst it would be a great convenience if experimenters would adopt a common unit, it is of greater importance that they should state the unit employed when the accuracy of their work is such as to justify taking into consideration the variation in the heat capacity of water.

§ (13) METALLIC BLOCK CALORIMETERS. (i.) *Nernst.*—A calorimeter in which the usual vessel of liquid is replaced by a heavy copper block has been employed by Nernst, Lindemann, and Koref for specific heat determinations. The block is hollowed out and the material dropped into its interior. Owing to the good thermal conductivity of copper it has practically the same temperature throughout, and the temperature changes of the block are determined by means of thermoelements. To obtain heat insulation the block is inserted into a Dewar vacuum vessel. The arrangement is shown in *Fig.* 14. The block K weighs about 400 grams. It is desirable to have good thermal contact between the copper block and the inner surface of the glass vessel which has to be made as thin as possible. To ensure this the block is fixed in the vacuum vessel by means of Wood's fusible alloy.

In the diagram T are the thermojunctions. The junctions of the thermoelements are inserted into thin-walled glass tubes which are fixed in holes in the block by means of fusible alloy. Good contacts between junctions and the tubes are also obtained by the use of alloy. The other junctions are in a ring-shaped copper block C by which the vacuum vessel is closed at the top. Through the middle of the block passes a glass tube R, through which the substance is dropped into the calorimeter. This tube can be closed by means of a slide as shown. The whole apparatus is submerged in a constant temperature bath, usually ice or solid CO_2, and the temperature thus maintained constant. To keep the apparatus water-tight a sheath of copper foil is soldered round as shown. To prevent the heated air from rising up in the calorimeter, which means a loss of heat after the substance has been dropped in, some cotton-wool is placed on the top of the silver vessel so as to just close the hole in a copper block after the material has been dropped in.

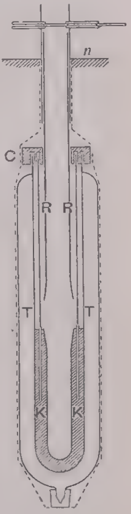

FIG. 14.

In their experiments the temperature changes of the calorimeter usually amounted to from 3° to 6°, giving a galvanometer deflection of 45 to 90 divisions. The apparatus must be calibrated by means of a substance whose specific heat is exactly known, and this was attained by using water for high, and lead for low, temperatures. The calibration had to be repeated from time to time in order to eliminate small errors.

(ii.) *Féry.*—Féry[1] has applied the metallic block method to the design of a bomb calorimeter and constructed a direct reading instrument. The bomb in which the combustion takes place is a nickel-lined, thick-walled vessel. It is supported by two discs of constantan, the disc being soldered to the bomb and also to the nickel-plated surrounding vessel B (*Fig.* 15). The constantan supporting disc forms with the iron of the bomb the hot junction of a thermocouple, the cold junction being made between the discs and the outer vessel B. The E.M.F. generated by the temperature of the bomb is measured by the millivolt calibrated to read direct in calories on the assumption that the same weight of fuel is always burnt.

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[1] *Engineering*, Sept. 20, 1912; *Electrician*, Sept. 13, 1912.

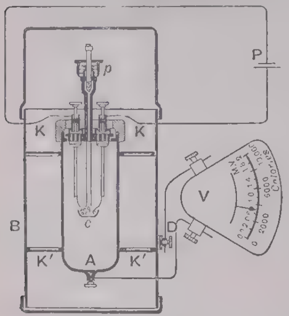

FIG. 15.

A, nickel-lined bomb of iron 3½ kilos. in weight; K, K′, two constantan discs soldered to bomb and to nickel-plated copper surrounding vessel B; C, quartz crucible.

The instrument has not yet been developed to a stage when it can be used for routine tests, several sources of error which influence the readings not having been eliminated. One important factor is the pressure of the oxygen in the bomb. Should the pressure be below that for which the instrument has been calibrated, combustion will become slower and heat loss due to radiation, etc., will be greater than that under normal conditions. A few typical curves illustrating this are shown in *Fig.* 16.

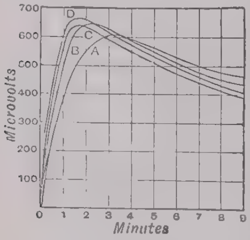

FIG. 16.

Curve A, pressure of gas = 100 lb. per sq. inch; curve B, pressure of gas = 150 lb. per sq. inch; curve C, pressure of gas = 200 lb. per sq. inch; curve D, pressure of gas = 250 lb. per sq. inch.

§ (14) SPECIFIC HEAT OF GASES BY THE "METHOD OF MIXTURES."—When defining the specific heat of a gas it is necessary to specify the conditions under which the heating takes place, since the change of volume with the rise of temperature is considerable under constant pressure, and the thermal equivalent of the external work done during expansion is a large fraction of the whole heat supplied during the change of temperature. Hence in the case of a gas it is customary to speak of two specific heats: (1) at constant volume, and (2) at constant pressure.

The earliest investigators to study the specific heat of gases were Lavoisier and La Place, who employed a calorimetric method based on the measurement of the quantity of ice melted. Later, Delaroche and Berard made some careful experiments in which a uniform current of gas, heated at 100° C., by passing through a tube surrounded by a vapour-jacket, was cooled by passing through a spiral contained in the calorimeter. The method was essentially that of mixtures, and most of the subsequent investigators adopted this method with various modifications to meet special requirements. Consequently, the published data are confined to the mean specific heat over a wide range of temperature. More recently Callendar and his associates have developed the method of electrical heating which permits of the determination of the *true* specific heat.

Amongst the workers employing the method of mixtures Regnault stands pre-eminent. He brought to bear upon the subject his unique skill and experience of calorimetric measurements, with the result that the data he obtained were accepted, almost without question, for the following half-century. There is, however, no doubt that the results given by Regnault were a little low, about 2·5 per cent, due to an inaccuracy in his method of determining the heat conducted into the calorimeter along the pipe through which the gas flowed. Without describing in detail Regnault's apparatus its essential features may be briefly reviewed.

The gas was contained in a large reservoir, heated up by passing through a long spiral immersed in an oil bath, and thence led to the calorimeter. Care was taken to ensure a uniformity of flow of gas through the calorimeter under constant pressure, and independent experiments were made to ensure that the gas leaving the calorimeter had cooled to this temperature. Since it was assumed that the temperature of the gas entering the calorimeter was the same as that of the heater bath, precautions have to be taken to avoid loss of heat by the gas in passing from the bath to the calorimeter, and at the same time prevent as far as possible conduction of heat from the bath to the calorimeter along the connecting tube.

The correction for the heat carried along this tube, which was made of low conductivity material, was deduced from observations of the change in temperature of the calorimeter without the gas flowing. This change of temperature is due to the combined effect of the conduction and the rate of heating or cooling of the calorimeter due to the difference in temperature between it and the surroundings.

If $\Delta\theta$ is the observed rate of change of temperature per unit time, then $\Delta\theta$ is equal to $A - B\theta$, where θ is the excess of the temperature of the calorimeter over that of the room.

The term A corresponds to the heat conducted through the connecting pipe from the heater to the calorimeter, and $B\theta$ to the heat loss by radiation, etc., from the calorimeter. Each of these terms corresponded to about 5 per cent of the total energy supplied by the gas per minute. Regnault measured the constant A and B by noting the rate of rise of temperature of the calorimeter before and after the gas had passed through it. Swann[1] has pointed out that an error arises in assuming that the heat conduction through the pipe is the same when gas is flowing as when no gas is flowing; in fact the hot gas keeps up the temperature of the pipe in the vicinity of the heater and reduces the temperature gradient. The result is that less heat is conducted from the heater into the pipe when the gas flows through than when no gas is flowing. Of course a great deal of heat is conducted by the pipe into the calorimeter when the gas is flowing, but the greater part of this comes from the gas itself. The fact that the average temperature of the pipe is higher when the gas is flowing also results in a greater radiation loss from the pipe. The error acts in the same direction as the other.

Swann made some experiments to verify the above suggestion, and by attaching thermo-junctions to a metallic tube he showed that the gradient was affected by the flow along it and the results were of the magnitude required to account for the difference between his results and those of Regnault.

In Regnault's time there was a lack of knowledge concerning the variation of the specific of water with temperature, which, together with uncertainty as to the absolute scale of temperature, might also cause an error of 1 per cent in his results.

§ (15) Variation of Specific Heat with Temperature and Pressure (over Moderate Ranges).—Regnault's observations cover the temperature interval from $-30°$ to $210°$ and pressures from 1 to 12 atmospheres. He found that the specific heat of the gases, air, oxygen, and hydrogen were independent both of the temperature and the pressure within the limits of the observations.

The specific heat of CO_2, on the other hand, showed a well-marked increase with rising temperature. Regnault's work was repeated by Wiedemann, who confirmed his results.

Witkowski investigated the specific heat of air at low temperatures from $+100°$ to $-170°$, and found that the specific heat was independent of the temperature but increased with pressure. He worked up to a maximum of 70 atmospheres. The variations with pressure increased as the temperature was lowered.

The method of mixtures is not suitable for the accurate determination of the pressure and temperature variation of the specific heat of a gas.

More recent work by observers employing the electrical method has supplied data which supersede those obtained in the above-described investigations.

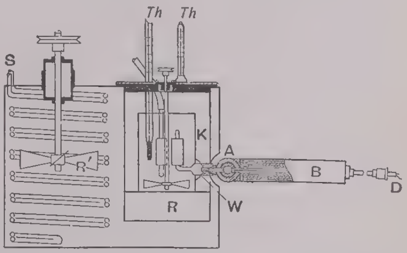

Fig. 17.

§ (16) Specific Heat of Gases at High Temperatures.[2]—Holborn and Austin,[3] and later Holborn and Henning,[4] have investigated the specific heat of gases up to 1200° C. Their method is identical in principle with that of Regnault's, but a special type of heater was necessary for bringing the gas to the high initial temperature. Their apparatus is shown diagrammatically in *Figs.* 17 and 18.

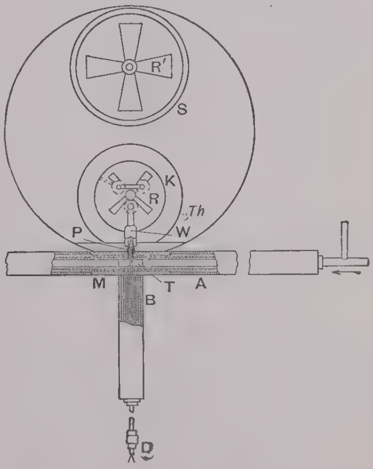

Fig. 18.

(i.) *Arrangement of Apparatus. The Heating Tube.*—The gas was heated electrically in a nickel tube A, about a metre long and of 1 mm. wall thickness, on which was wound a coil of nickel wire. The windings were insulated from the tube by asbestos. The gas was introduced at one end and was heated in the coarse nickel filings with which the tube was filled according to the plan of E. Wiedemann. At three-fourths of its length the tube was closed by a disc silver-soldered in place, and the gas was led out through a side tube into the calorimeter. In this way the influence of the cool end of the tube was eliminated. Otherwise it was found out that the gas in passing through the cool portion gave up so much of its heat that its temperature varied in a marked degree with the rate of flow.

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[1] "Note on the Conduction of Heat along a Pipe through which Gas is flowing in its Relation to Measurements of the Specific Heat of Gases," *Phil. Mag.*, Jan. 1913.

[2] See also article "Gases, Specific Heat of, at High Temperatures."

[3] *Phys. Rev.* xxi. No. 4.

[4] *Ann. der Physik*, 1907, xxiii. 809.

Opposite the outlet tube W a second nickel tube B was joined to the heating tube and through this a platinum platinum-rhodium thermoelement T was introduced. This passed through the heating tube A, which at this point was kept free from nickel filings by the dividing wall M on one side and a disc of wire net on the other. The thermojunction lay in the outlet tube W, 1 cm. from its free end. In this space a thin silver band bent in the form of a screw was inserted to prevent radiation of the thermojunction to the cool calorimeter. The thermoelement consisted of wires 0·25 mm. in diameter, which were insulated through the greater part of the tube B with thin porcelain tubes. The hot junction of the thermoelement, which was hardly thicker than a single wire, was left bare.

Special care was taken that the end of the thermoelement should not come in contact with the tube wall. In one portion of the work this was attained by supporting the flexible end of the element on a bit of mica of the same width as the diameter of the tube.

The platinum band, which was intended for the protection of the thermoelement from radiation from the tube wall, also served to protect the quartz from the nickel oxide which was carried along with the gas current in minute quantities from the filings in the heating tube. Otherwise this after a time became opaque and disintegrated.

A secondary heating coil of nickel wire was placed on the tube B, to compensate for the loss of heat by conduction through the two side tubes and for the loss of one turn of wire on the main coil where the side tubes were attached.

(ii.) *The Calorimeter.*—The calorimeter K of about 0·5 litre capacity was made of pure silver 0·5 mm. thick (*Figs.* 17 and 18). In its centre were situated three silver tubes 1·5 cm. in diameter, filled with silver filings and connected by 0·5 cm. silver tubes. These absorbed the heat from the gas as it passed through. That the gas actually issued from the calorimeter at calorimetric temperature even when heated to the highest point (800°) was made certain by tests with a constantan copper thermoelement.

Later experiments by Holborn and Henning were made by a similar method with a platinum heating tube which extended the temperature range to 1400° C. The calorimeter necessarily gains some heat from the heating tube, and this gain, in the later experiments, was partly compensated by surrounding the calorimeter with a jacket maintained at a much lower temperature.

This compensation was found necessary at high temperature in order to prevent an excessively rapid rise of temperature of the calorimeter: but although it reduces the apparent magnitude of the correction required, it does not diminish the actual amount of heat transferred and does not reduce the uncertainty of the correction. The magnitude of the effect at high temperatures may be judged from the fact that it was found necessary, in the experiments at 1400° C., to maintain the jacket at as low a temperature as 40° C. by passing a stream of cooling water through it in order to prevent the calorimeter rising above 115° C. when no gas was passing. Under such conditions the calorimetric corrections become so uncertain that the probability of systematic errors must increase considerably with rise of temperature.

The rate of increase of the mean specific heat of nitrogen at atmospheric pressure between 840° and 1340° C., shown by the later experiments, was about double that found in the earlier series. Both series of experiments could be represented within the limits of probable error by the linear formula

$$S_{0t} = \cdot 2350(1 + \cdot 00008t).$$

It appears probable, however, that the value of the specific heat at 0° C. given by the formula is too low and that in the case of nitrogen the rate of increase is not uniform, but increases with rise of temperature to some extent.

(iii.) *Possible Sources of Error.*—Since the temperature of the hot gases was determined by a thermocouple near the entrance to the calorimeter, and the time of flow of the gas was only three minutes, there appears to be some doubt whether the couple gave the true mean temperature of the inflowing gas, and also whether the loss by radiation from the couple was properly corrected for. The value of the mean specific heat of air over the range 150° to 270° C. by Holborn and Henning was ·2315. This is about 5 per cent smaller than the probable value over this range. The rate of increase shown by the experiments was within the limits of probable accuracy of the work.

§ (17) Specific Heat of Steam.—Regnault's value, 0·475 for the specific heat of steam at atmospheric pressure over the range 125 to 225° C. was obtained by taking the difference between the total heats of steam, superheated to these temperatures, as observed by condensing the steam in a calorimeter. Since the difference, corresponding to 100° superheat, is only $\frac{1}{14}$th of the total heat measured in either case, it is evident that the method might give rise to large errors. For this reason many writers have preferred to deduce the specific heat of steam theoretically in various ways from Regnault's value of the rate of change of the total heat of saturated steam, namely, 0·305 cal. per 1° C., which, as Callendar[1] has pointed out, is subject to the same source of error in an aggravated form. Thus Zeuner gives $S=0{\cdot}568$; Perry, $S=0{\cdot}306$ at 0° C. to 0·464 at 210° C.: Grindley, 0·387 at 100° C. to 0·665 at 160° C.

A direct measurement of the specific heat of steam by Brinkworth,[2] employing the continuous electric method devised by Callendar, gave $S=0{\cdot}484$ at 108° C. Subsidiary experiments by Callendar in conjunction with Professor Nicolson, by the throttling calorimeter method, enabled the variation of the specific heat with pressure to be calculated. These gave the formula

$$S=0{\cdot}478+0{\cdot}0242p\left(\frac{373}{\theta}\right)^{3.3},$$

where p is the pressure in atmospheres. The approximate constancy of the limiting value 0·478 of the specific heat at zero pressure over the range 0 to 200° C. was verified by calculating the corresponding values of the saturation pressure, which were found to agree accurately with Regnault's observations over the whole range. The theory was also verified by a measurement of the ratio of the specific heats of steam by Makower,[3] which gave values 1·303 to 1·307, agreeing closely with that deduced by Callendar.

The experiments of Lorenz[4] and Knoblauch and Jacob and Linde[5] afforded a remarkable verification of the theory of the variation of the specific heat with pressure. They found the specific heat at 1 atmosphere to be practically constant over the range 100° to 300°, but their value, namely, 0·463, is decidedly lower than Regnault's.

Holborn and Henning[6] in their experiments on the specific heat of steam at atmospheric pressure, improved Regnault's method by employing an oil calorimeter at 110° C so as to avoid condensing the steam in the calorimeter. They determined the ratio of the specific heat of steam to that of air by passing currents of air and steam in succession through the apparatus under similar conditions, and obtained the following values of the ratio for different intervals of temperature:

Temperature Interval.	Ratio/Steam. Air.
110-270°	1·940
110-440	1·958
110-620	1·946
110-820	1·998

In their subsequent series with a platinum heating-tube at higher temperatures they obtained the following ratios:

Temperature Interval.	Ratio/Steam. Air.
115-826°	1·900
115-1180	1·973
115-1324	2·003

The second series appears to make the ratio about 5 per cent lower at 110-820° than the first, which suggests the possibility of constant errors depending on the type of apparatus employed or on the velocity of the gas current. The experiments of Callendar and Swann would make the ratio 2·05 at 100° C. This is higher than any of the values obtained by Holborn and Henning at 1400° C.

Holborn and Henning point out that their results at 1400° C. cannot be reconciled in the case of steam and CO_2 with any of the results of explosion methods. They are 6 per cent to 13 per cent lower than Langen's, which are among the lowest. But, having regard to the fact that the constant-pressure method which they employed appears to give results so much lower than Joly's or Callendar's methods at ordinary temperatures, and that the experimental difficulties increase so greatly at higher temperatures, it does not seem at all improbable that a considerable part of the discrepancy is to be attributed to systematic errors of the constant-pressure method.

§ (18) SPECIFIC HEAT OF CO_2.—The specific heat of CO_2 is of great theoretical interest in view of the considerable increase shown at ordinary temperatures. The table below gives the results obtained by various observers:

Temperature.	Regnault.	Wiedemann.	Swann.	Holborn.
0°	0·1870	0·1952	0·1973	0·2028
100	0·2145	0·2169	0·2213	0·2161
Increase	0·0275	0·0217	0·0240	0·0133

It may also be remarked that the variation of specific heat with density observed by Joly[7] agrees very closely with that calculated by Callendar[8] from the experiments of Joule and Thomson on the cooling effect on expansion.

E. G.

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[1] *Report of B.A. Committee on Gaseous Explosions*, 1908, from which the above is abstracted.

[2] *Phil. Trans. Roy. Soc.*, 1915, ccxv. 383-438.

[3] *Phil. Mag.*, Feb. 1903.

[4] *Forsch. Ver. Deut. Ing.*, 1905, xxi. 93.

[5] *Loc. cit.* pp. 1 and 35; 1906, p. 109.

[6] *Ann. Phys.*, 1905, xviii. 739.

[7] "Calorimetry, Change of State," § 5.

[8] *Phil. Mag.*, 1903.

CALORIMETRY, QUANTUM THEORY

THE VARIATION OF SPECIFIC HEAT WITH TEMPERATURE

§ (1) **THE VARIATION OF ATOMIC HEAT.**—The discovery by Dulong and Petit in 1819 of the empirical law, that the product of the atomic weight and the specific heat is approximately the same for all elements, proved to be of the greatest practical utility to chemists when assigning atomic weight values to newly discovered elements; and further, the simplicity of the law directed attention to the possibility of arriving at it from theoretical considerations of conceivable atomic structures constituting a solid. The mean value for the constant was determined by Regnault as 6·38 with extremes of 6·76 and 5·7. According to the kinetic theory of matter it is easy to see why a relationship of the form discovered by Dulong and Petit should exist. We suppose that the atoms are bound together by interatomic forces tending to bring them to positions of equilibrium about which they oscillate; then in this case the total energy of an atom is half-potential and half-kinetic; for the principle of equipartition of energy is assumed to be valid. Now in the case of a monatomic gas the energy is all kinetic, and proportional to the absolute temperature. Therefore the atomic heat should be half as great in the gaseous state as the solid state. The kinetic energy of a gramme molecule of a monatomic gas is $\frac{3}{2}$RT, where R is the gas constant which has the value 1·985. Hence, on the supposition that a monatomic solid body is built up of atoms each with 3 degrees of freedom, the energy content is 3RT, and from this the atomic heat at constant volume is obtained by differentiation with respect to T giving for the atomic specific heat the value 3R or 5·955.

It might be remarked in passing that the above equation, according to Boltzmann, is applicable to the case of crystals which have at the points of their space-lattice molecules of any degree of complexity, provided that the internal forces acting on each atom are proportional to the distance of the latter from its equilibrium position, or more generally are linear functions of the change of its co-ordinates.

Hence the Dulong and Petit generalisation is consistent with the atomic theory of matter and the equipartition theory of energy.

For nearly a century, however, the exceptions to the law—carbon, boron, and silicon—proved to be an enigma which defied solution. As far back as 1872 Dewar[1] and Weber,[2] working independently, showed that as the temperature increased the specific heat of carbon, whether as diamond or as graphite, continued to increase. Weber concluded that the specific heat of diamond is tripled when the temperature is raised from 0° to 200°.

Dewar's experiments showed the specific heat of carbon between 30° C. and the boiling-point of zinc (918° C.) was 0·32.

Some three years later Weber[3] published results of further experiments, and proved that from 600° C. upwards the specific heat of carbon ceased to vary with increase of temperature and became comparable with that of other elements. Further, the difference between the specific heat of different modifications disappeared. By plotting his results Weber showed that the specific heat temperature curve was of the form of an old English ʃ. He found a point of inflection for diamond at about 60° C., and that for graphite 0° C.

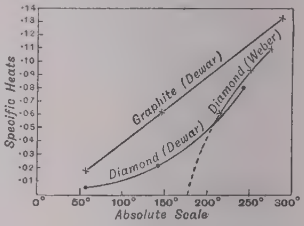

FIG. 1.

Recent researches have shown that the curve obtained by Weber is typical of all materials when the range of temperature investigated is sufficiently large.

By his development of the technique of low temperature research Dewar was able to pursue the subject to still lower temperatures, and the results for carbon obtained up to 1912 are shown graphically in *Fig.* 1. More recent research by various investigators employing the electrical method has shown that the general form of the atomic heat temperature curve closely resembles the curve of magnetisation of a ferromagnetic substance under a steadily increasing magnetic force, with its very gradual beginning, its subsequent rapid rise, and its final asymptotic approach to a limiting value. A few typical curves are shown in *Fig.* 2, the C_v curve being obtained by calculation from C_p and the value of $C_p - C_v$.

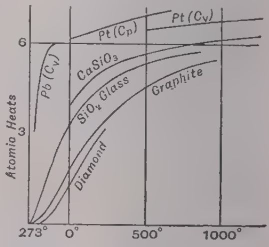

FIG. 2.

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[1] *Phil. Mag.*, 1872, xliv. 461.

[2] *Ibid.* p. 251.

[3] *Phil. Mag*, 1875, Ser. 4, xliv. 285.

§ (2) THE QUANTUM THEORY EXPLANATION.—No satisfactory explanation of these facts was forthcoming until the development of the Quantum theory and its application to the problem of specific heat by Einstein,[1] Nernst and Lindemann,[2] Debye,[3] and others.

These physicists developed formulae connecting the specific heat of a solid at constant volume with the gas constant R, the quantum constant h, the frequency ν, and the absolute temperature T. Debye's formula involves a quantity ν_m, the maximum value of the frequency which can occur.

In the formulae quoted below β is written for the quantity h/R.

For further details reference should be made to the article "Quantum," Vol. IV.

The specific heat formulae found are

$$\mathrm{C}_v=3\mathrm{R}\left(\frac{\beta\nu}{\mathrm{T}}\right)^2\frac{\epsilon^{\beta\nu/\mathrm{T}}}{(\epsilon^{\beta\nu/\mathrm{T}}-1)^2}, \quad . \quad (\mathrm{E.})^4$$

$$\mathrm{C}_v=\frac{3\mathrm{R}}{2}\left[\frac{(\beta\nu/\mathrm{T})^2\epsilon^{\beta\nu/\mathrm{T}}}{(\epsilon^{\beta\nu/\mathrm{T}}-1)^2}+\frac{(\beta\nu/2\mathrm{T})^2\epsilon^{\beta\nu/2\mathrm{T}}}{(\epsilon^{\beta\nu/2\mathrm{T}}-1)^2}\right](\mathrm{N.\&L.}),$$

$$\mathrm{C}_v=3\mathrm{R}\left[\frac{4\pi^4}{5}\left(\frac{\mathrm{T}}{\beta\nu}\right)^3-\frac{3\beta\nu/\mathrm{T}}{\epsilon^{\beta\nu/\mathrm{T}}-1}\right.$$
$$-12\frac{\beta\nu}{\mathrm{T}}\sum_{n=1}^{n=\infty}\epsilon^{-n\beta\nu/\mathrm{T}}\left(\frac{1}{n\beta\nu/\mathrm{T}}+\frac{3}{n^2(\beta\nu/\mathrm{T})^2}\right.$$
$$\left.\left.+\frac{6}{n^3(\beta\nu/\mathrm{T})^3}+\frac{6}{n^4(\beta\nu/\mathrm{T})^4}\right)\right]. \quad (\mathrm{D.})$$

In the last expression ν is written for ν_m for simplicity.

Specific heat is measured at constant pressure. Thus to compare with the theory the experimental results need correcting by means of the formula

$$\mathrm{C}_p-\mathrm{C}_v=\frac{wa^2e\mathrm{T}}{\rho},$$

where a is the coefficient of cubical expansion, e the coefficient of volumetric elasticity, w the atomic weight, ρ being the density.[5]

§ (3) EXPERIMENTAL TEST OF FORMULAE.—E. H. Griffiths and Ezer Griffiths[6] tested the above formulae by means of their experimental data for the metals over the range 120° C. abs. to 400° C. abs.

They found that no one of the formulae was capable of representing exactly the experimental results over the entire range of temperature, even when the values of ν were chosen so as to bring the calculated values of the atomic heat into coincidence with the experimental values at one temperature (about 125° C. abs.).

The results are given in Table I.

TABLE I

COMPARISON OF THE EXPERIMENTAL RESULTS WITH THE FORMULAE OF EINSTEIN, NERNST AND LINDEMANN, AND DEBYE

Column I.—T, absolute temperature.

Column II.—C_p, from the smoothed curves through experimental points.

Column III.—C_p, calculated value from Einstein's formula (E.).

Column IV.—C_p, calculated value from Nernst and Lindemann's formula (N. and L.).

Column V.—C_p, calculated value from Debye's formula (D.).

The values of $\beta\nu$ are given at the top of the columns.

T.	C_p(obs.).	C_p (E.).	C_p(N. & L.).	C_p (D.).
ALUMINIUM.		$\beta\nu=292.$	$\beta\nu=385.$	$\beta\nu_m=385.$
35°	0·33	0·12	0·37	0·35
80	2·27	2·18	2·46	2·35
140	4·26	4·26	4·26	4·28
200	5·14	5·12	5·10	5·10
250	5·53	5·49	5·47	5·47
300	5·81	5·75	5·74	5·74
380	6·13	6·02	6·01	6·01
COPPER.		$\beta\nu=222.$	$\beta\nu=285.$	$\beta\nu_m=286.$
33·4°	0·54	0·36	0·82	0·71
88	3·38	3·60	3·74	3·70
120	4·58	4·52	4·58	4·57
200	5·44	5·46	5·47	5·47
280	5·80	5·78	5·80	5·80
360	6·02	5·96	6·01	6·01
ZINC.		$\beta\nu=160.$	$\beta\nu=210.$	$\beta\nu_m=210.$
30°	0·95	0·84	1·30	1·13
80	4·09	4·33	4·33	4·33
130	5·31	5·32	5·31	5·31
200	5·78	5·76	5·77	5·77
280	6·02	6·00	6·01	6·00
360	6·21	6·19	6·19	6·19
SILVER.		$\beta\nu=157.$	$\beta\nu=207.$	$\beta\nu_m=207.$
35°	1·58	1·40	1·80	1·64
85	4·42	4·56	4·53	4·53
120	5·20	5·23	5·21	5·21
200	5·84	5·73	5·78	5·78
280	6·01	6·01	6·01	6·01
360	6·16	6·16	6·16	6·15
CADMIUM.		$\beta\nu=112.$	$\beta\nu=143.$	$\beta\nu_m=144.$
50°	3·46	4·02	4·10	4·10
115	5·60	5·59	5·60	5·60
160	5·87	5·85	5·86	5·86
200	5·99	5·97	5·99	5·99
360	6·39	6·33	6·34	6·34

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[1] *Ann. d. Phys.*, 1907, xxii. 180-800.
[2] *Berlin Sitzungsber.*, 1911, p. 494.
[3] *Ann. d. Phys.*, 1912, xxxix. 789.
[4] The letters E., N. & L., D. denote Einstein, Nernst and Lindemann, and Debye respectively.
[5] See "Thermodynamics," § (48).
[6] *Phil. Trans. Roy. Soc.* A, 214.

TABLE I—*continued*

T.	C_p(obs.).	C_p(E.).	C_p(N.& L.).	C_p(D.).
LEAD.		$\beta\nu=68.$	$\beta\nu=92.$	$\beta\nu_m=94.$
23°	2·96	3·03	3·06	2·94
80	5·72	5·75	5·65	5·64
120	5·93	5·93	5·91	5·91
200	6·10	6·13	6·13	6·13
280	6·28	6·28	6·28	6·28
360	6·45	6·45	6·45	6·45
SODIUM.		$\beta\nu=119.$	$\beta\nu=152.$	$\beta\nu_m=152.$
50°	3·50	3·87	3·98	3·95
120	5·62	5·62	5·64	5·64
200	6·17	6·02	6·04	6·02
320	6·78	6·36	6·36	6·36
360	7·32	6·43	6·43	6·43
IRON.		$\beta\nu=286.$	$\beta\nu=376.$	$\beta\nu_m=376.$
50°	0·98	0·65	1·10	0·98
140	4·28	4·28	4·29	4·29
220	5·45	5·24	5·21	5·21
300	6·03	5·62	5·61	5·61
380	6·37	5·82	5·82	5·82

Note.—C_v was obtained by calculation from the formulae and the calculated value of C_p-C_v added.

It will be observed that, generally, near the boiling-point of liquid hydrogen, about 30° abs., Einstein's formula gives values which are too low; from Nernst and Lindemann's the values are too high; while Debye's formula gives values which are in fair agreement for Al, Ag, Zn, and Pb, and, in the case of other metals, it agrees with the experimental values better than either Einstein's or Nernst and Lindemann's. At liquid air temperatures all three formulae give values which are too high.

It is of but little use to calculate the appropriate values of ν from the elastic constants of the metals, since these constants are considerably influenced by the nature of the previous heat treatment and of the temperature. But it might be remarked that the values calculated from the elastic constants are in accordance with those required by the atomic heat results as shown by the data in Table II.

TABLE II

COMPARISON OF FREQUENCIES OBTAINED BY CALCULATION FROM THE PHYSICAL CONSTANTS WITH THE VALUES ASSUMED IN DEBYE'S FORMULA.

Frequencies $\times 10^{-12}$

Metal.	Al.	Fe.	Cu.	Ag.	Cd.	Pb.
ν_m (atomic heat) . .	8·2	8·0	6·7	4·5	3·5	1·9
$_m$ (elastic constants)	8·3	9·7	6·8	4·4	3·5	1·5

The above comparison of the frequencies shows that the values obtained from the specific heat equations are in fair agreement with those calculated from the elastic constants.

§ (4) APPLICATION OF DEBYE'S AND EINSTEIN'S FORMULAE TO THE NON-METALS.—Pure metals afford the most reliable data for testing heat theories, but comparisons with the available experimental data for complex substances such as crystalline salts (NaCl, KCl, KBr, AgCl) and diamond are of great theoretical interest.

One difficulty in making such comparisons is uncertainty in the value of C_p-C_v owing to the lack of data concerning the elastic constants. As data were not available for calculating C_p-C_v from the thermodynamical relationship,

$$C_p-C_v=\frac{wa^2eT}{\rho},$$

Nernst and Lindemann [1] obtained approximate values for C_p-C_v by a different procedure.

TABLE III

Absolute Temperature.	C_p-C_v.	
	a^2ewt.	$C_p{}^2TA$.
	ALUMINIUM	
32·4°	0·24	0·21
35·1	0·26	0·23
50	0·29	0·26
80	0·31	0·28
100	0·34	0·30
	COPPER	
23·5	0·15	0·14
27·7	0·16	0·15
50	0·18	0·16
70	0·20	0·17
90	0·22	0·18
110	0·23	0·19
	SILVER	
35	0·21	0·25
40	0·23	0·27
60	0·25	0·29
80	0·27	0·31
100	0·30	0·34
130	0·32	0·36
	LEAD	
23	0·35	0·34
28	0·39	0·37
37	0·43	0·40
50	0·47	0·43
80	0·51	0·46
100	0·55	0·49

They based their method of calculation on Gruneisen's observation that the coefficient of expansion is proportional to the atomic heat, and obtained the approximate relationship

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[1] *La Théorie du rayonnement et les quanta*, 1912, p. 265.

$$C_p - C_v = C_p^2TA,$$

where A is a constant characteristic of the substance which can be deduced from measurements of the compressibility and coefficient of expansion made at one temperature.

It is possible to test the validity of this formulae by comparing the data obtained from it with those given by the thermodynamical equation in the case of the metals Al, Ag, Pb, and Cu (see Table III.).

Nernst tested the formulae of Nernst and Lindemann and of Debye on the data for diamond, and Table IV. summarises the comparison.

TABLE IV

Diamond $\beta\nu = 1860$ for Debye's.
$= 1940$ for Nernst and Lindemann's.

Observer.	T.	C_p.		Difference.	
		Obs.	Calc. (Debye).	Obs.-calc. (Debye)	Obs.-calc. (N. and L.).
Nernst .	88°	0·028	0·049	−0·021	+0·022
„ .	92	0·033	0·058	−0·025	+0·024
„ .	205	0·618	0·61	+0·008	0·00
„ .	209	0·662	0·66	+0·002	+0·01
„ .	220	0·722	0·74	−0·018	=0·04
Weber .	222	0·76	0·75	+0·01	−0·02
Dewar .	243	0·95	0·925	+0·025	−0·02
Weber .	262	1·14	1·10	+0·04	−0·02
„ .	284	1·35	1·32	+0·03	−0·02
„ .	306	1·58	1·54	+0·04	−0·01
„ .	331	1·84	1·82	+0·02	+0·01
„ .	358	2·12	2·07	+0·05	+0·04
„ .	413	2·66	2·61	+0·05	+0·11
„ .	1169	5·45	5·49	−0·04	+0·04

It will be observed that Debye's formula gives an approximate representation of the experimental results. The Nernst and Lindemann formula gives values which are too low between 88° and 92° abs. In the case of Debye's the differences exceed the probable error of experiment, the general trend being for Debye's formula to give values which are too large at low temperatures and too small at high temperatures. In fact the decrease of atomic heat with decreasing temperature is more rapid than that given by Debye's formula. Ewald made experiments on the mean atomic heat of diamond between 83·8° and 194·0° abs. and found the value 0·2119, so that the total energy difference between these temperatures is 23·35 calories; the value calculated from Debye's formula is 25·34. Koref, employing the copper calorimeter described in "Calorimetry, Electrical Methods," § (13), found the mean atomic heat of diamond between 193·8° and 270·0° to be 0·864, corresponding to an energy difference of 65·8. Debye's formula gives 61·9 calories for this interval.

Polyatomic Substances.—The metals and diamond are regarded as monatomic structures, whilst the molecules of graphite and sulphur appear to be compounds of several atoms since the atomic heat curve is for these much less abruptly curved towards the temperature axis.

Nernst[1] has attempted to apply the formulae of Debye and Einstein to polyatomic substances such as KCl, NaCl, etc. He assumes the heat vibration to be of two types: first, the vibration of the molecules as a whole in exactly the same way as the atoms of a monatomic body; and, second, the vibration of each atom about its position of rest. The vibrations of the atoms are interpreted as giving rise to the "Reststrahlen" discovered by Rubens. Since at low temperatures the vibrations of the atoms become much more regular, and accordingly the absorption bands narrower and more pronounced, it is assumed that Einstein's function applies to the atomic vibrations.

Hence for the representation of the atomic heat of the salts the expression is

$$2C_v = F_1\left(\frac{\beta\nu_1}{T}\right) + F_2\left(\frac{\beta\nu_2}{T}\right),$$

in which F_1, F_2 are the functions of Debye and Einstein respectively, ν_1 the frequency calculated from the melting-point,[2] and ν_2 that found by Rubens by means of the optical method, since these salts show one very sharp infra-red absorption band.

For details of the comparison with experiment, reference should be made to the original papers already referred to. E. G.

CALORY OR CALORIE:

The 15°.—The quantity of heat required to raise one gramme of water through 1° C. at 15° C. See "Thermodynamics," § (2); "Heat, Mechanical Equivalent of," § (9).

The 17°·5.—For some purposes the range from 15° C. to 20° C. is taken and the calorie defined at 17°·5.

Gramme.—One-hundredth part of the heat required to warm one gramme of water from the melting-point to the boiling-point at a pressure of one atmosphere. See also "Thermodynamics," § (2); "Heat, Mechanical Equivalent of," § (9).

Pound.—A British unit of heat, being one-hundredth of the amount of heat required to raise one pound of water from the melting-point to the boiling-point at one atmosphere. See also "Thermodynamics," § (2).

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[1] *Theory of the Solid State* (London Univ. Press).

[2] Lindemann, *Physik Zeitsch.*, 1910, xi. 609. Using the hypothesis that the melting-point is determined by the fact that at this temperature the amplitudes of the vibrations of the atoms around their positions of rest become commensurate with the atomic distances.

CAMS, KINEMATICS OF. See "Kinematics of Machinery," § (6).

CANAL WALLS AND EFFECT ON STREAM-LINES OF MOVING SHIP. See "Ship Resistance and Propulsion," § (36).

CARBON DIOXIDE, LATENT HEAT OF VAPORISATION OF; determined by Mathias. See "Latent Heat," § (8).

CARBON MONOXIDE, SPECIFIC HEATS OF; tabulated values obtained by Scheel and Heuse. See "Calorimetry, Electrical Methods of," § (15), Table IX.

CARNOT'S CYCLE. See "Thermodynamics," §§ (18), (40); "Engines, Thermodynamics of Internal Combustion," § (5).

For a Perfect Gas. See "Engines, Thermodynamics of Internal Combustion," §§ (5), (6), (23); "Thermodynamics," § (18).

For Steam. See "Steam Engine, Theory of," § (2).

CARPENTIER-HOSPITALIER INDICATOR. See "Pressure, Measurement of," § (19).

CASCADE METHOD OF COOLING; introduced by Pictet. See "Gases, Liquefaction of," § (1).

CAST IRON. See "Elastic Constants, Determination of."

Crushing Strength. § (39).

Effect of Temperature on the Transverse Strength. § (37) (ii.).

Tensile Strength. § (38).

Testing—Transverse Test. § (37) (i.).

CATHODE RAY MANOMETER

SIR J. J. THOMSON suggested the use of Piezo Electricity[1] as a means of measuring sudden pressures, and the method has recently been worked out by Dr. D. A. Keys.[2] Crystals of tourmaline are exposed to the action of the pressure and the electrical charge acquired, which is proportional to the pressure, is measured by a special form of cathode ray oscillograph.

The amount of the charge is measured by the deflexion of a beam of cathode rays which passes between two parallel condenser plates which receive the charge separated on the tourmaline crystals.

The beam is also deflected in a direction at right angles to this electrostatic deflexion by a magnetic field applied parallel to the electrostatic field by means of an alternating current of known frequency, giving a time displacement, perpendicular to the electrostatic displacement. The beam falls on a photographic plate, thus a charge time record is obtained, and since the charge is proportional to the pressure this is equivalent to a pressure time record. The advantage of the method lies in the fact that the inertia of the moving part of the recorder, the beam of cathode rays, is negligible.

The apparatus consists of two parts: (1) the pressure vessel, and (2) the cathode ray oscillograph.

The pressure vessel is shown in *Fig.* 1. It consisted in Dr. Keys' experiments of a brass vessel AB, about 6 in. in diameter and $1\frac{1}{2}$ in. deep closed by a $\frac{1}{2}$-in. steel plate HK, held down by 12 bolts. The walls of the vessel were $\frac{1}{4}$ in. thick. A sparking plug L and a tap N are fitted in the steel cover. DE is a thin lead plate electrically connected to a copper wire which passes through an insulating plug G in the side of the vessel.

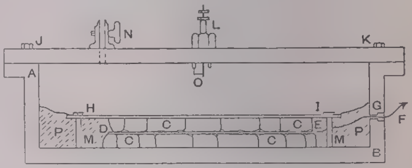

FIG. 1.

Half of the tourmaline crystals, about 1 cm. in thickness, are attached by a little wax to the bottom of the vessel, the other half are attached similarly to a steel plate HI. The lead plate DE separates these two sets of crystals, which are so arranged that all the positive faces are in contact with the lead plate. There are 5 or 6 crystals in each layer, the area of each crystal being about 12 sq. cm. HI is fixed by steel screws to the bottom of the vessel. The lower part of the vessel is filled with vaseline to delay the transfer to the crystals of the heat generated by the explosion.

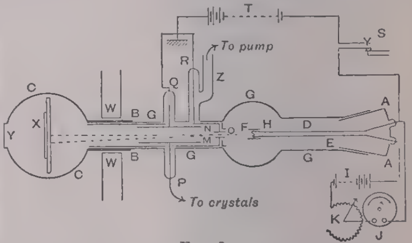

FIG. 2.

When pressure is applied above HI, DE becomes positively charged and the amount of the charge is proportional to the pressure.[3]

The insulated wire EGF conveys the charge to the condenser of the oscillograph. This is shown in *Fig.* 2. The cathode rays are generated by the fine tungsten filament F, which is heated to incandescence by an 8-volt accumulator. The cathode ray tube GG is cemented into a brass sleeve BB which is soldered into a brass cylinder CC—the axis of this cylinder is at right angles to the paper. The rays enter the sleeve BB through a small hole O, pass between the plates MN of a condenser and then between the poles WW of the electromagnet and fall on a photographic plate X in the cylinder CC. The plate slides in a rectangular box within the cylinder and can be moved from outside.[1] A window Y in the cylinder CC closed by a screen of willemite allows visual observations to be made when the plate is drawn aside and thus facilitates adjustment. The electromagnet is excited by an alternating current of known frequency, thus the time scale is fixed. LL is a brass guard tube $\frac{3}{4}$ in. in diameter. This guard tube and one of the condenser plates N are connected to earth and to one pole of a direct current generator supplying a constant potential difference of from 3000 to 5000 volts. The other pole of the generator is connected through a special double-action key S to the tungsten filament; the key also serves to fire the explosive mixture. The second plate M of the condenser is connected to the lead plate of the pressure apparatus. On depressing the key S the cathode potential supplied by the direct current generator is first communicated to the tube; a further motion of the key detonates the charge. The cathode rays are deviated in a direction at right angles to the paper by the magnetic field, and until the charge is fired trace a short vertical line on the photographic plate. The electrification of the condenser plate M due to the pressure produces a deflexion of the rays in the plane of the paper and thus the time-pressure curve is traced on the plate. The displacement of the spot depends on (1) the cathode potential and (2) the potential difference produced by the charge between the plates of the condenser MN; the apparatus requires calibrating for these. For the method of doing this reference should be made to Dr. Keys' paper.

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[1] See article "Piezo Electricity," Vol. II.

[2] "A Piezoelectric Method of measuring Explosive Pressures." *Phil. Mag.*, 1921. xlii. 473.

[3] P. Curie, *Œuvres*, p. 16.

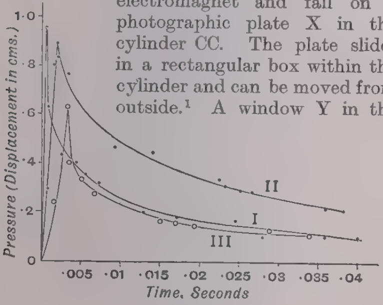

FIG. 3.

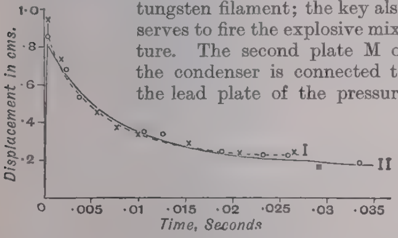

FIG. 4.

Figs. 3, 4, 5, 6, taken by permission from his paper, give the results of some of his experiments; the effect of the addition of air in slowing down the explosion of H_2 and O shown in *Fig.* 3 is very marked. With no air (curve I.) the maximum pressure of about 220 lbs. per sq. in. is reached in less than 0·0002 seconds. With some 60 per cent of air in the mixture (curve III.) the maximum pressure is reduced to less than 60 per cent of its former value and the rise continues for about 0·003 seconds, or some fifteen times as long as previously.

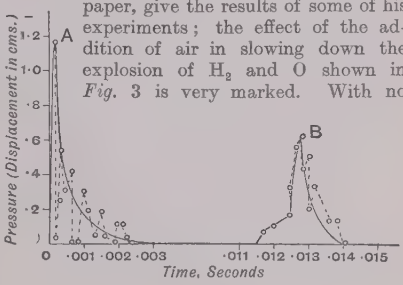

FIG. 5.

The curve A, *Fig.* 5, is a time-pressure curve for the explosion of gun-cotton under sea water; it gives the direct wave, while curve B is due to the wave reflected from the bottom. Again in *Fig.* 6, the effect of altering the distance of the charge from the pressure vessel is shown.

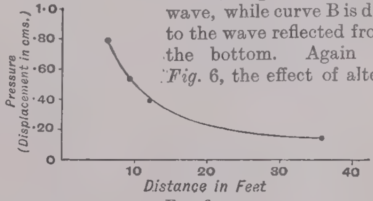

FIG. 6.

CATHODE RAY OSCILLOGRAPH. See Vol. II., "Radio Frequency Measurements," § (45).

CAVITATION. See "Ship Resistance and Propulsion," § (48).

CEMENT:

- Artificial and Natural Hydraulic Cement, Tests for Portland Cement. See "Elastic Constants, Determination of," § (155).
- Characteristic Equations for Tensile Tests on Cement and Mortar. See *ibid.* § (157).
- Setting Time. See *ibid.* § (158)

CEMENT AND SAND (MORTAR) — TENSION TESTING. See "Elastic Constants, Determination of," § (156) (ii.).

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[1] Sir J. J. Thomson, *Rays of Positive Electricity*, 1913, pp. 22-23, Longmans, Green & Co.

Cementing Materials—Classification. See "Elastic Constants, Determination of," § (152).

Centigrade Scale of Temperature: a scale, used in all modern thermometric work, in which the numbers 0 and 100 correspond respectively to the freezing- and boiling-points of water, in each case at pressure of one atmosphere. See "Thermometry," § (2).

Centrifugal Fluid Tacheometers: For measuring number of revolutions per unit time. See "Meters," § (4), Vol. III.

Centrifugal Pumps. See "Air-pumps," § (32); "Hydraulics," § (38).

Channels, Hydraulic Flow in, and Best Dimensions of. See "Hydraulics," §§ (27) and (28).

"Characteristic Curve" for geometrically similar structures of which the corresponding parts are made of the same material, and Non-dimensional Bases. See "Dynamical Similarity, The Principles of," § (44).

Characteristic Equation of State. See "Thermodynamics, Definition of," § (56); "for Ideal Gas," § (57).

Charles' Law on the expansion of gases under constant pressure states that

$$v = v_0(1 + at),$$

v_0 and v being volumes at temperatures 0° and t°, where a is approximately constant (for the more permanent gases) and is called the coefficient of expansion at constant pressure; it has nearly the same value for all the more permanent gases. See "Thermal Expansion," § (14) (i.); "Thermodynamics," § (5); "Engines, Thermodynamics of Internal Combustion," § (14).

Chattock Gauge. See "Pressure, Measurement of," § (27).

Chemical Pumps. See "Air-pumps," § (19).

Chezy Formula for Hydraulic Losses in Pipes. See "Hydraulics," § (25) (i.).

Practical form of same. See *ibid.* § (25) (iii.).

Chronometric Instruments: For measuring number of revolutions per unit time. See "Meters," § (3), Vol. III.

Clapeyron's Equation. See "Thermodynamics," § (41).

Clark Cell, Value of E.M.F. of. See "Mechanical Equivalent of Heat," § (8). See also Vol. II., "E.M.F., Standards of."

Class Variable: a non-dimensional group of terms connecting a class of problems possessing certain geometrical and dynamical similarities, the solutions of which are expressible as functions of the class variable: if the problem can be expressed in terms of a differential equation, the solution is sought for as a power series in the class variable. See "Dynamical Similarity, The Principles of," § (49).

Claude's Modification of Linde's Method for Liquefaction of Gases. See "Gases, Liquefaction of," § (1).

Clausius' Theorem: an advance towards the explanation of the departure of the behaviour of fluids from the laws of perfect gases. See "Thermal Expansion," § (19) (i.).

Clearance in Steam-engine Cylinders. See "Steam Engine, Reciprocating," § (2) (viii.).

Clerk Two-stroke Engine. See "Engines, Thermodynamics of Internal Combustion," §§ (9), (34).

COAL CALORIMETER

Various forms of apparatus have been devised for determining the calorific value of coal. In practically all the selected sample of coal is burned in oxygen; the products of combustion are passed into a known quantity of water at a known temperature, and the rise of temperature is measured.

Fig. 1 is an illustration of the Rosenhain calorimeter as manufactured by the Cambridge and Paul Instrument Company. The instrument consists of two parts, the calorimeter proper containing the water and the combustion chamber in which the sample is burned.

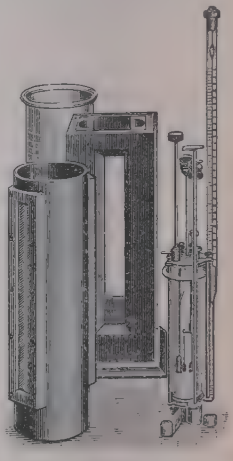

Fig. 1.

The combustion chamber is formed of a glass lamp chimney closed at the top and bottom by metal clamping plates, separated from the glass by rubber washers and held together by clamping screws on three uprights fixed to the lower plate; the wires connected to the electric ignition arrangement pass through the upper plate and also a tube for the supply of oxygen. The whole chamber is immersed in the water of the calorimeter. An aperture in the plate is closed by a ball valve through which the products of combustion can issue into the water, but which prevents the entry of water into the combustion chamber. On the completion of the experiment the valve is raised, allowing some water to enter; this is afterwards forced out by the oxygen and mixed with the rest of the water, thus ensuring that the calorimeter and its contents are brought to one temperature. To reduce the radiation losses the whole instrument is placed within a wooden case with a glass window in one side.

Oxygen from a supply at suitable pressure is passed through a coil of metal piping to bring it to the temperature of the room and then through a wash bottle fitted with a thermometer; from this it enters the ignition chamber; the temperature of the water is read and the coal ignited by connecting the ignition device to a four-volt accumulator; when the ignition is complete the chamber is washed out and the temperature again read. A knowledge of the water equivalent of the calorimeter is required, and this is most easily obtained by carrying out an experiment with a sample of known calorific value; if the temperature range during this experiment is approximately the same as that which occurs in the ordinary use of the instrument, the correction for radiation will be also covered. For accurate work the radiation correction can be found in the manner described in the article on "Fuel Calorimetry."

Coefficient of Performance of a Refrigerating Machine is the ratio of the heat taken in from the cold body to the work spent in the compressor (in heat units). See "Refrigeration," § (3).

Coefficients, Pressure- and Volume-, of Gases. The constant $(p_{100} - p_0)/100p_0 = 1/T_0$ is the "pressure-coefficient" of a gas at the initial pressure stated.

The constant $(v_{100} - v_0)/100v_0 = 1/T_0'$ is the "volume-coefficient" of a gas at the pressure stated. See "Temperature, Realisation of Absolute Scale of," § (18).

Coefficients of Viscosity for Fluids, Experimental Determination of. See "Friction," § (3).

Cold Stores, Refrigerating Machines for. See "Refrigeration," § (4).

Collodion Diaphragm Gauge. See "Pressure, Measurement of," § (23).

Compound Engines. See "Steam Engine, Reciprocating," § (3).

Compressibility, Effect of, on the Motion of a Body through a Viscous Fluid. See "Dynamical Similarity, The Principles of," § (24).

Compressibility of Fluid, Isothermal and Adiabatic. See "Thermodynamics," § (55).

Compression of Gas causes Heating: Explanation on Molecular Theory. See "Thermodynamics," § (66).

Compression Members in Structures. See "Structures, Strength of," § (15).

Compressometers: The Yale, Wisconsin, Ewing, Marten, and Unwin Instruments. See "Elastic Constants, Determination of," § (54).

Compressors, Theory of. See "Air-pumps," § (1).

Concrete:
- Determination of Tensile Strength. See "Elastic Constants, Determination of," § (137).
- Determination of Transverse Strength. See *ibid.* § (139).
- Effect of Vibration, Jigging, and Pressure on the Strength. See *ibid.* § (141).

Condensation, High-vacuum. See "Steam Engine, Theory of," § (8).

Condensation of Supersaturated Vapour as influenced by Surface Tension. See "Thermodynamics," § (54).

Condenser, Steam-engine. The use of a separate condenser was one of great improvements introduced by Watt. See "Steam Engine, Theory of, § (3); "Steam Engine, Reciprocating," § (4).

Conductivity:
- Definition of Thermal. See "Heat, Conduction of," § (2).
- Discussion of Methods of measuring Thermal. See *ibid.* § (9).
- Relation of Thermal and Electrical. See *ibid.* § (10) (i.).
- Values of Thermal, for Solids, Liquids, and Gases. See *ibid.* Tables I. to V.

Conductors of Heat:
- Medium, Methods of measuring Thermal Conductivity of. Cylindrical Method—Wall Method. See "Heat, Conduction of," § (4).
- Medium, Values of Thermal Conductivity of. See *ibid.* Table II.
- Poor, Methods of measuring Thermal Conductivity of. Spherical Shell Method—Cylindrical Shell Method—Wall Method. See *ibid.* § (3) (i.), (ii.), and (iii.).
- Poor, Values of Thermal Conductivity of. See *ibid.* Table I.

Conservation of Energy:
- Mayer's Enunciation of Law of. See "Mechanical Equivalent of Heat," § (1).
- Joule's Experiments on. See *ibid.* § (2).

Conservative Systems of Forces. When the total work done by any system of forces during a series of displacements which bring the bodies acted on back to their original configuration is zero, the system is said to be a conservative one.

Constant Pressure Cycle, Internal Combustion Engines. See "Engines, Thermodynamics of Internal Combustion," § (29).

Constant Volume Cycle, Internal Combustion Engines. See "Engines, Thermodynamics of Internal Combustion," § (32).

Constantinesco System of Power Transmission by Wave Motion. See "Hydraulics," § (66).

Continuous Beams. See "Structures, Strength of," § (11)

Contraction of Area at Fracture. See "Elastic Constants, Determination of," § (18).

Convection of Heat:
  Forced. See "Heat, Convection of," §§ (1), (2), and (3).
  From Fine Wires and Cylinders. See *ibid.* § (2) (ii.) and § (4) (iv.).
  Natural. See *ibid.* §§ (1), (4), and (5).

Cooling Correction, Method of Calculating, in Method of Mixtures: Rumford's Procedure — Rowland's Method. See "Calorimetry, Method of Mixtures," § (7).

Cooling of Body in Quiescent Gas, Rate of; considered by the method of dimensions. See "Dynamical Similarity, The Principles of," § (28).

Cooling of Moving Body in a Gas, Rate of; considered by the method of dimensions. See "Dynamical Similarity, The Principles of," § (30).

Copper, Atomic Heat of, at Low Temperatures; Nernst's values for, tabulated. See "Calorimetry, Electrical Methods of," § (11), Table VI.

Copper and Cuprous Oxide, Emissivity of; determined by optical pyrometer. See "Pyrometry, Optical," § (19).

Copper, Solid and Molten, Emissivity of; determined by optical pyrometer. See "Pyrometry, Optical," § (21).

Copper, Specific Heat of:
  At Various Temperatures; tabulated, with the Atomic Heat. See "Calorimetry, Electrical Methods of," § (10), Table V., § (12), Table VIII.
  Studied by Harper over the range 15° to 50° C. See *ibid.* § (12).

Corliss Valves for Steam Engines. See "Steam Engine, Reciprocating," § (5) (i.).

Cornish Engine. See "Steam Engine, Reciprocating," § (13).

Corresponding States, Van der Waals' Theorem of. See "Thermodynamics," § (60); "Thermal Expansion," § (20).

Counter Instruments: For measuring number of revolutions per unit time. See "Meters," § (1), Vol. III.

Cranes, Hydraulic. See "Hydraulics," § (59).

Crank Effort Diagrams. See "Kinematics of Machinery," § (8).

Critical Constants for Various Fluids, Experimental Values for. See "Thermal Expansion," § (30).

Critical Point, Temperature and Pressure, Definitions of. See "Thermodynamics," § (37).
  For Calculation of Critical Data from Van der Waals' Equation, see also §§ (59) and (60).

Critical Pressure, Refrigeration above the. See "Refrigeration," § (2), *Fig.* 7.

Critical Temperature. The temperature to which a gas must be cooled before it can be liquefied by pressure. If above the critical temperature it cannot be liquefied. See "Thermodynamics," §§ (37), (42); "Liquefaction of Gases."

Crosby Indicator. See "Pressure, Measurement of," § (18) (ii.).

Crusher Gauges. See "Pressure, Measurement of," § (14).

Crystals, Non-isotropic, Expansion of. See "Thermal Expansion," §§ (8), (9).
  Thermal Conductivity of. See "Heat, Conduction of," § (6).

Current Meters. See "Hydraulics," § (12).
  Calibration of Current Meters. *Ibid.* § (13).

Cut-off. The point of the stroke at which the supply of steam to an engine cylinder ceases is called the "point of cut-off." See "Steam Engine, Reciprocating," § (2) (iv.).

Cycle of Operations for Internal Combustion Engines. See "Engines, Internal Combustion," § (2); "Engines, Thermodynamics of Internal Combustion," §§ (2), (28); "Thermodynamics," § (8).

Cycloidal Teeth. See "Kinematics of Machinery," § (9).

Cylinders, Strength of. See "Structures, Strength of," § (31).

— D —

Daimler-Lanchester Worm Gear Testing Machine. See "Dynamometers," § (6) (i.).

Dalton's Law of Partial Pressures in Gaseous Mixtures. See "Thermodynamics," § (62). For proof of Dalton's law on the molecular theory of gases see also § (66).

Damped Harmonic Motion. If the displacement of a point from its equilibrium position is given by an expression of the form $ae^{-\nu t}\sin(nt+\epsilon)$ the motion is said to be damped harmonic motion.

Day, Clement, and Sosman, 1908–1912; comparison of gas-thermometer with secondary standards of temperature in range 500° to 1600°. See "Temperature, Realisation of Absolute Scale of," § (39) (xiv.).

Deeley's Experiments on Fluid Friction. Tables. See "Friction," § (28).

Deflection of Beams. See "Structures, Strength of," § (9).

Deflection Diagram for a Frame. See "Structures, Strength of," § (24).

Degree of Freedom, Definition of. See "Kinematics of Machinery," § (2).

Degree of Freedom, Equipartition of Energy amongst each. See "Thermodynamics," § (66).

Density of Solid, compared with that of liquid, for various substances, tabulated. See "Thermal Expansion," § (31).

Developed Area of a Screw-propeller is the sum of the actual areas of the blades irrespective of shape. See "Ship Resistance and Propulsion," § (41).

Deville and Troost, 1857, used iodine in a bulb of porcelain to compare gas-thermometers with secondary standards of temperature in the range 500° to 1600°. See "Temperature, Realisation of Absolute Scale of," § (39) (iii.).

Diaphragm Pressure Gauge. See "Pressure, Measurement of," § (21).

Diesel Engine. See "Engines, Thermodynamics of Internal Combustion," §§ (31) and (50); "Engines, Internal Combustion," § (15) *seq.*

Dieterici, Experiments of, in determination of Latent Heat of Steam. See "Latent Heat," § (1) (ii.).

Differential Pulley Block — Weston's Principle. See "Mechanical Powers," § (2) (ii.).

Diffusion, Increase of Entropy due to. See "Thermodynamics," § (62).

Diffusion Pumps. See "Air-pumps," § (41). Gaede's. See "Air-pumps," § (42).

Diffusivity:
- Definition of Thermal. See "Heat, Conduction of," §§ (2) (ii.) and (12).
- Thermal, of Soil. See *ibid.* § (12) (i.), Table VI.
- Thermal, of Various Substances. See *ibid.* Table VII.

Dimensional Formula of a Physical Quantity. An expression showing which of the fundamental units enter into the unit of the quantity, with their dimensions, *e.g.*

$$[F]=[MLT^{-2}],$$

F being the unit of force, M, L, T those of mass, length, and time.

Dimensions of Electric and Magnetic Quantities: Electrostatics. See "Dynamical Similarity, The Principles of," § (35).

Dimensions, Homogeneity of, in a Physical Equation: the fundamental principle that all the terms in any equation having a physical significance must necessarily have identical dimensions. See "Dynamical Similarity, The Principles of," § (5).

Disc, Theory of the Sectored, used in radiation pyrometry to cut down the radiation by a definite fraction so that the same galvanometer deflection is obtained for two different temperatures of the radiator. See "Pyrometry, Total Radiation," § (5).

Disc-area Ratio of a Screw-propeller is the ratio of the sum of the actual areas of the blades (*i.e.* the developed area) to the area swept out by the tips of the blades. See "Ship Resistance and Propulsion," §§ (41) and (44).

Displacement Diagrams for Points in a Mechanism. See "Kinematics of Machinery," § (4).

Dobbie-McInnes Indicator. See "Pressure, Measurement of," § (18) (iii.).

Double-acting Engine. See "Steam Engine, Reciprocating," § (2).

Drifting Test of Metals. See "Elastic Constants, Determination of," § (34).

Drop Test of Tyres. See "Elastic Constants, Determination of," § (36).

Drop-Valves for Steam Engines. See "Steam Engine, Reciprocating," § (5) (ii.).

Ductility, Workshop Test for. See "Elastic Constants, Determination of," § (14).

Dynamical Equations in Terms of Displacement, for an Electric Solid. See "Elasticity, Theory of," § (7).

Dynamical Equations in Terms of Stress, for an Elastic Solid. See "Elasticity, Theory of," § (6).

DYNAMICAL SIMILARITY, THE PRINCIPLES OF

I. General Principles

§ (1) **General.**—Scientific research concerns itself with the discovery and expressions of laws as exact relations between physical quantities of different kinds. Involved in the expression for the magnitude of any such quantity are two conceptions—the nature or kind of unit in terms of which it is represented, and the magnitude or number of times the unit occurs in the physical quantity considered. Thus, in the statement that the earth's diameter is 8000 miles, there is implied the nature of the fundamental unit (mile) in terms of which the diameter is measured, and the number (8000) of these units which are present in the physical quantity. If the whole function of scientific research consisted in a comparison of quantities of the same kind then the desiderata in the selection of a unit could be easily defined, and would merely involve such considerations as definiteness, non-susceptibility to secular change, and capability of accurate comparison with the quantities to be expressed in terms of it. Since, however, research in its wider sweep investigates and expresses relations between quantities of such different kinds as forces, temperatures, magnetic moments, etc., it follows that, for simplicity and clearness, not merely must these units be selected to satisfy the above requirements, but only such units must be accepted as express all the quantities concerned in the simplest manner possible in terms of the minimum number of independent units.

§ (2) Space Units.—For mere geometrical relations which are expressions of space truths involving lengths, areas, and volumes, it is clear that only one independent unit is required, which, apart from convenience and form, may arbitrarily be selected as the unit of length, of area, or of volume. If, for example, V, the unit of volume, be taken as the standard dimension, then since the product of three lengths provides a volume, the dimensions of length would be $V^{\frac{1}{3}}$, and that of area $V^{\frac{2}{3}}$. It is obvious that the clearest and, to us, the simplest expressions will be obtained by choosing as our standard unit a length L, giving as dimensions of area L^2 and volume L^3.

§ (3) Kinematic Units.—Passing to laws involving kinematic considerations it becomes at once clear that a new unit must be introduced to provide a measure of motion, a change in length with time; and once more the simplest mode of representation is arrived at by choosing, not a unit for speed or acceleration, and hence expressing the fundamental idea of time in a more or less complicated form, but by selecting a unit for time, say T. Speeds and accelerations are then at once seen to have the dimensions L/T and L/T^2 respectively.

§ (4) Dynamical Units.—But the two units of length and time are not in themselves sufficient to specify quantities which arise immediately we enter the domain of dynamics. At the basis of this section of analysis lies the fundamental idea of force and its representations in terms of mass and acceleration. Before a full expression for the dimensions of dynamical quantities can therefore be set out, either a new unit of force must be introduced in terms of which, by Newton's Second Law, the dimension of mass could be expressed, or a unit of mass must be presumed and the dimensions of dynamical quantities derived from it. Once more for reasons of simplicity we choose the latter. Accordingly, writing M as the dimension of the mass unit, the following table of quantities and their dimensional representation may be drawn up:

Quantity.	Dimension.	Quantity.	Dimension.
Length	L	Force	MLT^{-2}
Time	T	Kinetic energy	ML^2T^{-2}
Mass	M	Impulse	MLT^{-1}
Linear speed	LT^{-1}	Work	ML^2T^{-2}
Angular speed	T^{-1}	Moment of force	ML^2T^{-2}
Acceleration	LT^{-2}	Moment of Inertia	ML^2
Momentum	MLT^{-1}	Density	ML^{-3}

§ (5) Homogeneity of Dimensions in a Physical Equation.—Since the mathematical formulation of any physical law is a statement of equality or relationship between physical quantities necessarily of like nature, since indeed a force cannot be added to a mass but only to another force, nor a temperature equated to an electric charge but only to another temperature, it follows that all the terms in any equation having a physical significance must necessarily have identical dimensions. In the expression for the velocity acquired by a particle *in vacuo* dropped from rest under gravity, for example,

$$v^2 = 2gh,$$

the dimensions of the left-hand side are

$$(LT^{-1})^2 = L^2T^{-2},$$

while on the right-hand side they are

$$LT^{-2} \times L = L^2T^{-2},$$

the constant being of course non-dimensional. This fundamental principle serves not merely as a check on the intelligibility of any formula derived either on experimental or on theoretical grounds, but can be turned to much greater advantage by providing *a priori* information regarding the form which the result of any investigation ought to take. It will become apparent as the subject develops that there exists here a potent weapon for a preliminary analysis of any proposed problem. Particularly is this true in the realms of physics and engineering.

The method that may be adopted will become apparent from a few simple dynamical illustrations. Let us assume that the time of oscillation of a pendulum *in vacuo* is the problem for analysis, in order to determine how the period depends on the length of the suspension, the mass of the pendulum bob, and the value of gravity. Without a preliminary analysis along the present lines, and failing a complete mathematical investigation, it would appear at first sight that a complete experimental study of the problem would involve a threefold series of experiments corresponding to variations in length of suspension, mass of bob, and gravity. Consider the physical factors upon which the period might possibly be dependent. They are, mass of bob m, length of suspension l, value of gravity g, the form of attachment possibly, and the shape of the bob. The last two factors may or may not enter, but in any case it is not clear how they can be directly introduced by any single convenient symbol. Let us assume then that in all the experiments these are invariant, and therefore from our *a priori* knowledge we affirm

$$t = f(m, l, g),$$

where f represents the function which requires determination. Let us suppose this expanded in a power series in m, l, and g, so that $t = \Sigma A m^x l^y g^z$, where the terms A are constants independent of m, l, and g and functions only of the shape of the bob. This much, however, we can affirm, that each term in the expression must of necessity represent a quantity of the nature of time, and must therefore have the same dimensions as t.

Now the dimensions of

$$m^x l^y g^z \text{ are } \mathrm{M}^x\mathrm{L}^y(\mathrm{LT}^{-2})^z = \mathrm{M}^x\mathrm{L}^{y+z}\mathrm{T}^{-2z},$$

and equating these to the dimensions of t we find

$$x=0, \quad y+z=0, \quad -2z=1,$$
$$\therefore x=0, \qquad y=\tfrac{1}{2}, \qquad z=-\tfrac{1}{2}.$$

Hence

$$t=\Sigma \mathrm{A} m^0 l^{\frac{1}{2}} g^{-\frac{1}{2}} = c\sqrt{\left(\frac{l}{g}\right)},$$

where c is some unknown constant. A preliminary analysis of the problem has now reduced the necessity for a threefold set of experiments to one experiment, and one only, to determine the value of c. We have shown in fact that there are not in reality three variables m, l, and g in the problem, but one single variable $t\sqrt{(g/l)}$ which groups together a whole system of experiments as essentially of the same type. What then is the common principle that runs through the problems of the times of oscillations of pendula of different lengths under different values of g, that it should be possible to group them together? What is the physical or dynamical significance of this grouping?

§ (6) DYNAMICAL SIMILARITY.—In the foregoing analysis we might equally well have taken t to represent the time taken for the pendulum to swing out to a given angle α, then $t=c\sqrt{(l/g)}$, where c is now constant, when α is constant. It follows at once that if two pendula of lengths l_1 and l_2, at different points on the earth's surface such that gravity is g_1 and g_2 respectively, are allowed to swing, then at all times t_1 and t_2 such that $t_1\sqrt{(g_1/l_1)}=t_2\sqrt{(g_2/l_2)}$, the pendula will be displaced by equal angles. A series of photographs on plates of the same size taken of all such pendula at any such series of corresponding times will be identical. The geometrical configuration for all such corresponding times are similar. Systems in motion which can thus be grouped together quite generally as passing through geometrically identical phases for equal values of a non-dimensional grouping of corresponding quantities—in the above case for equal values evidently of $t\sqrt{(g/l)}$—are said to possess dynamical similarity.

§ (7) APPLICATION TO PARTICLE DYNAMICS.—Consider a further example from particle dynamics. A particle of unit mass is projected with velocity V in a field of force such that the acceleration at any point (x, y) is $\mathrm{A}(x, y)$, what information relative to the distance traversed after any time is supplied by the method of dimensions? For this purpose let us write $\mathrm{A}(x, y)=a\,.\,\alpha(x, y)$, where a is the actual acceleration at some special point, say the point of projection and $\alpha(x, y)$ is the function giving, as it were, the law of distribution of acceleration. Under these circumstances since both $\mathrm{A}(x, y)$ and a are accelerations, $\alpha(x, y)$ must be of zero dimensions. Restricting ourselves then to a series of problems for which this law of distribution remains unchanged although the magnitude of the acceleration at any point may vary—*i.e.* the function α remains unaffected, but a varies from problem to problem—we may say that S, the distance traversed after any time t, is a function of V, the velocity of projection, a the measure of the acceleration, and t the time, *i.e.*

$$\mathrm{S}=f(\mathrm{V}, a, t).$$

Writing as before

$$\mathrm{S}=\Sigma \mathrm{A}\mathrm{V}^x a^y t^z,$$

and equating dimensions, we find

$$(\mathrm{L}) \qquad 1=x+y; \qquad \therefore x=1-y,$$
$$(\mathrm{T}) \qquad 0=x+2y-z, \quad \therefore z=1+y,$$
$$\therefore \mathrm{S}=\Sigma \mathrm{A}\mathrm{V}^{1-y}a^y t^{1+y}=\mathrm{V}t\,.\,\Sigma \mathrm{A}\left(\frac{at}{\mathrm{V}}\right)^y.$$

Since y is quite arbitrary and may have any value whatsoever while still satisfying the dimensional conditions, this implies that

$$\mathrm{S}=\mathrm{V}t\,.\,\mathrm{F}\left(\frac{at}{\mathrm{V}}\right),$$

where F represents an arbitrary function. The particular case of constant acceleration along the direction of motion giving the well-known formula

$$\mathrm{S}=\mathrm{V}t+\tfrac{1}{2}at^2$$

is readily seen to be of this form, for it may be written

$$\mathrm{S}=\mathrm{V}t\left[1+\tfrac{1}{2}\,.\,\frac{at}{\mathrm{V}}\right].$$

§ (8) NON-DIMENSIONAL VARIABLES.—It should be particularly remarked that the problem has reduced itself to the determination of the relation between two non-dimensional groups of terms $\mathrm{S}/\mathrm{V}t$ and at/V, the functional relation between the two depending purely on the geometrical conditions of the problem and in nowise on the dynamical principles involved. In the same way the case of the oscillating pendulum was reduced to the determination of the value of the single non-dimensional group of terms, $t\sqrt{(g/l)}$ as a mere number.

In general, it will be seen that for dynamical problems, at most three equations can be obtained from the dimensions of mass, length, and time, although there may be less if any of these be absent. In general, therefore, if there are n quantities upon which the whole functional relation depends, the indices of three of these quantities can be expressed in terms of the others, and there will be left a relation between $n-3$ non-dimensional groups of terms.

§ (9) APPLICATION TO PLANETARY THEORY.—As a further example of a slightly different nature showing the insight here provided into a problem without actually analysing in detail the dynamical conditions, consider what may be deduced from Newton's Gravitational Law, regarding the periodic times of planets.

The attractive force between two planets of masses m_1 and m_2 at distance r apart is, according to Newton,

$$\mathrm{F}=\frac{km_1m_2}{r^2}.$$

Accordingly the dimensions[1] of k, a gravitational constant, are

$$[k]=[\mathrm{F}]\times\left[\frac{r^2}{m_1m_2}\right]=(\mathrm{MLT}^{-2})(\mathrm{L}^2\mathrm{M}^{-2})=\mathrm{L}^3\mathrm{M}^{-1}\mathrm{T}^{-2}.$$

If d be the major axis of the orbit of a planet of mass E moving round a central sun S, then τ, the periodic time of the motion, can depend only on d, E, S, and k,

$$\therefore\ \tau=f(d,\ \mathrm{E},\ \mathrm{S},\ k)=\Sigma \mathrm{A}d^x\mathrm{E}^y\mathrm{S}^z k^u,$$

$$\therefore\ \mathrm{T}=[\tau]=[d^x\mathrm{E}^y\mathrm{S}^z k^u]$$

$$=\mathrm{L}^{x+3u}\mathrm{M}^{y+z-u}\mathrm{T}^{-2u}.$$

Hence equating like dimensions

$$x+3u=0,\quad y+z-u=0,\quad -2u=1,$$

$$\therefore\ x=\tfrac{3}{2},\quad y=-z-\tfrac{1}{2},\quad u=-\tfrac{1}{2},$$

$$\therefore\ \mathrm{T}=\Sigma \mathrm{A}d^{\frac{3}{2}}(k\mathrm{E})^{-\frac{1}{2}}\left(\frac{\mathrm{S}}{\mathrm{E}}\right)^z$$

$$=\frac{d^{\frac{3}{2}}}{\sqrt{k\mathrm{E}}}\cdot\phi\left(\frac{\mathrm{S}}{\mathrm{E}}\right).$$

From which Kepler's Third Planetary Law follows immediately, that the square of the periodic time is proportional to the cube of the major axis of the orbit.

But it is not in the field of pure dynamics that the method here developed receives its most fruitful applications. In almost all branches of physics a preliminary analysis of any problem that requires investigation, provides an insight into the main factors which are at work, and thus generally indicates the direction in which the research should be developed.

In aerodynamics and aero-engineering, for example, we are concerned with the laws of air resistance to bodies of various shapes, and with the rates at which heated bodies such as radiators, etc., will cool in a current of air. Especially during recent years have innumerable problems of this nature arisen which, because of the complex and intricate mathematical expressions for the physical laws involved, have not yet yielded to theoretical analysis. On the experimental side, however, a considerable amount of work has been carried out, frequently without according the results the full interpretation that might have been given them from the present standpoint. It will become evident that only by an analysis along the lines developed here can the full and most reasonable interpretation for any such work be obtained.

§ (10) APPLICATION TO THE MOTION OF FLUIDS.—In dealing with the motion of fluids it is necessary, in the first instance, to set out in detail the defining properties of that medium, quite apart from any theories regarding the molecular constitution of the fluid—liquid or gas. In the selection of such definitive properties, then, we are only concerned with those that are called into action under the circumstances contemplated in the problem. It suffices accordingly to define any fluid as dense, viscous, and elastic in the first instance, omitting for the moment those properties that become evident on the application of heat. Other properties may make themselves apparent in special circumstances, whilst even some of those enumerated above may in other circumstances become of little consequence. If we are concerned, for example, with the dripping of liquid from a tube, the elastic properties of the fluid may be omitted, its viscosity may even play only a minor part, while surface tension enters as an important factor since the problem is clearly affected by the nature of the conditions at the surface between the liquid and the air.

As far as all these properties are concerned it is possible to express their measure, as in the case of purely dynamical quantities, in terms of the three fundamental units of mass, length, and time, from the mere definition of the physical properties they are intended to represent.

Viscosity, for example, is introduced through the coefficient of viscosity μ defined by: μ = viscous force per unit area per unit velocity gradient; from which it follows that the dimensions of μ are

$$[\mu]=[\text{Force}]/[\text{Area}]\times[\text{Velocity gradient}]$$

$$=(\mathrm{MLT}^{-2})/(\mathrm{L}^2)\times(\mathrm{T}^{-1})=\mathrm{ML}^{-1}\mathrm{T}^{-1}.$$

As already indicated, density (ρ) has dimensions ML^{-3}. Surface tension (S), defined as a force per unit length of a curve, has therefore dimensions

$$[\mathrm{S}]=[\text{Force}]/[\text{Length}]=\mathrm{MT}^{-2}$$

§ (11) SURFACE TENSION.—As an illustration of the use of these expressions, consider the question of determining the size of the drops of liquid which fall from a tube of given diameter when the liquid is allowed to flow slowly down the tube under the influence of gravity and capillary force. On general physical grounds — on the basis of our experience in fact—we assume initially that the mass of the drops will depend on the value of the surface tension between the liquid and the air, on the density of the liquid, and possibly on its viscosity; and as far as an initial scrutiny of the question is concerned, these would appear to be the only properties of the fluid that ought to enter. On general grounds, moreover, we are aware that for slow motions viscosity plays a very minor part, and since in the present instance the motion is essentially slow since the fluid is only dripping, viscosity might legitimately be omitted from the discussion, but whether or not this is justifiable is of course definitely a question of experiment. The only other factor that would enter then is the size and shape of the tube. Let the diameter be d, but the shape cannot be introduced by any definite symbol.

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[1] It will be convenient to write symbolically in future—Dimensions of $x=[x]$.

Let the mass of each drop M be presumed a function of S the surface tension, ρ the density, g gravity, and d.

$$\therefore\ M=f(S,\ \rho,\ g,\ d).$$

Taking as a typical term in the expansion of this function $AS^x\rho^y g^z d^u$, and writing it dimensionally, we find

$$M=[M]=[S^x\rho^y g^z d^u]=(MT^{-2})^x(ML^{-3})^y(LT^{-2})^z L^u.$$

Hence equating powers of like dimensions

$$x+y=1,\quad -2x-2z=0,\quad -3y+z+u=0.$$

$$\therefore\ x=1-y,\quad z=-1+y,\quad u=1+2y.$$

The typical term is thus:

$$AS^{1-y}\rho^y g^{-1+y}d^{1+2y}=A\frac{Sd}{g}\left(\frac{S}{g\rho d^2}\right)^{-y},$$

$$\therefore\ M=\frac{Sd}{g}\,\phi\left(\frac{S}{g\rho d^2}\right).$$

From this simple analysis it can consequently be deduced that the problem under discussion must be regarded as the determination of the law of variation of the non-dimensional group Mg/Sd with the non-dimensional group $S/g\rho d^2$, and that since the form of the apparently arbitrary function ϕ can depend on nothing but the shape of the tube, the curve obtained by plotting Mg/Sd against $S/g\rho d^2$ will be characteristic of that shape of tube and quite independent of the actual values of S, d, ρ, etc. Actually, it has been found by experiment that M the mass of the drops is proportional to d the diameter of the tube (Tate's Law). If we now therefore examine the function $(Sd/g)\phi(S/g\rho d^2)$, it is clear that unless the function ϕ be a mere constant M cannot possibly be proportional to d, and consequently on the basis of one set of investigations during which d *alone* is varied, while S, ρ, etc. are maintained constant, it can be deduced that when *all* the quantities are varied

$$M=\frac{ASd}{g},$$

where the constant A may be determined from one experiment.

In the same way, if the drop of liquid be set in vibration, the frequency n will depend on g, ρ, d, and S as before, from which, noting that $[n]$ is T^{-1}, it can easily be shown that

$$n=\sqrt{\left(\frac{S}{\rho d^3}\right)}\cdot F\left(\frac{S}{g\rho d^2}\right).$$

§ (12) VIBRATIONS OF A GRAVITATING LIQUID GLOBE.—A globe of liquid, such as a planet in a fluid state, is held together by its own gravity, and is set into a state of vibration by some external disturbance. What information regarding the frequency of vibration in any of its modes can be derived by the Dimensional Method?

The frequency will clearly depend on ρ the density, d the diameter of the sphere, and k the gravitational constant, assuming that surface tension and viscosity are without influence on the question. Accordingly

$$n=f(\rho,\ d,\ k)=\Sigma A\rho^x d^y k^z.$$

The dimensional expressions for each of the quantities d and k are known and have already been used. Inserting these it follows at once that, $x=z=\frac{1}{2}$; $y=0$.

Hence $$n\propto\sqrt{(\rho k)},$$

and it appears that the frequency is independent of the diameter of the sphere and proportional to the square root of the density.

§ (13) VELOCITY OF WAVES.—By a similar process of reasoning it is easy to establish the formulae for the wave velocity in the case of heavy gravity waves in deep sea, $v^2\propto g\lambda$ where λ is the wave-length, and $v^2\propto S/\lambda\rho$ for small surface-tension waves.

A clear insight into the principles of Similitude that lie at the basis of this method is best afforded by a general treatment of the motion of a viscous fluid, correlating many apparently diverse phenomena, and laying them clear as natural consequences of the operation of the general principle of Dynamical Similarity.

§ (14) MOTION OF A BODY IN A VISCOUS FLUID.—Let it be supposed that a body or system of bodies of given geometrical shape, and size specified by the length l of some portion of the system, is moving with velocity V in a fluid of density ρ and kinematic viscosity $\nu(=\mu/\rho)$, and for the moment it will be presumed that the fluid is inelastic. As a matter of actual experiment it is found that not until the velocity approaches that of sound waves in the medium do its elastic properties make themselves apparent. Then the velocity v at some point of the fluid geometrically fixed with reference to the body may quite generally be written as

$$v=\phi_1(V,\ \rho,\ \nu,\ l),$$

while the slope θ of the stream line at that point is

$$\theta=\phi_2(V,\ \rho,\ \nu,\ l).$$

Applying the principle of Homogeneity of Dimensions and remembering that θ, the slope, is of zero dimensions being a mere number, it is found that

$$v = \mathrm{V}f\left(\frac{\mathrm{V}l}{\nu}\right)$$

$$\theta = \mathrm{F}\left(\frac{\mathrm{V}l}{\nu}\right).$$

The second equation indicates that the shape of the stream lines and the direction of motion at any time depend only on the value of the non-dimensional group $\mathrm{V}l/\nu$, and not on the separate constituents of that group. The magnitude of the velocity likewise depends on $\mathrm{V}l/\nu$ and on V alone. We see in fact that provided a system of bodies are all geometrically similar, their sizes being different, the systems of stream lines generated are likewise similar, provided $\mathrm{V}l/\nu$ is maintained constant throughout the system.

§ (15) REYNOLDS' NUMBER.—As a matter of actual experiment Reynolds,[1] by a brilliant series of investigations, demonstrated that the whole process which takes place during the motion of a fluid does not depend alone on any of the single quantities V, l, ν, but on the group; moreover that for any given problem—in his case the flow of fluid through circular pipes—$\mathrm{V}l/\nu$ is a critical variable, which when it attains a certain value corresponds to a more or less rapid change from one state of flow to another, from the steady so-called stream-line flow to the sinuous and turbulent state of eddy formation. Let us suppose then that eddying has originated, that is to say that the fluid on its passage past the obstacle forms in the neighbourhood of the sides of the latter a system of eddies which are shed periodically.

§ (16) FREQUENCY OF EDDY FORMATION.—Let n be the frequency of these eddies, then clearly it can only be a function of V, l, ν, and ρ apart from the geometrical shape of the obstacles.

$$\therefore n = f(\mathrm{V}, l, \nu, \rho).$$

Remembering that the dimension of n is T^{-1} and applying the method of this article, we easily find

$$n = \frac{\mathrm{V}}{l}f\left(\frac{\mathrm{V}l}{\nu}\right),$$

indicating once more that if $\mathrm{V}l/\nu$ is maintained constant for a series of similarly shaped bodies, for such a system

$$n \propto \frac{\mathrm{V}}{l} \propto \frac{\mathrm{V}^2}{\nu} \propto \frac{\nu}{l^2}.$$

Hence for a given value of $\mathrm{V}l/\nu$, and a given fluid so that ν is also constant, the frequency is proportional to the square of the velocity or inversely as the square of the size.

§ (17) DYNAMICAL SIMILARITY IN THE MOTION OF A VISCOUS FLUID.—What this implies may be summarised by saying that if a series of bodies all of the same geometrical shape be moving in a system of fluids, viscosities ν_1, ν_2, . . . with velocities V_1, V_2, . . . then provided $\mathrm{V}l/\nu$ is maintained constant, photographs of the flow pattern taken on cinematograph films of the same size will all be identical as far as the consecutive geometrical configurations of the stream lines and eddying systems are concerned.

The rates at which the processes unfold themselves will, however, be different, being in fact determined by the expressions for the frequency n, but clearly if the photographs be compared at corresponding times as indicated above, the pictures will be identical. We see in fact that the condition, $\mathrm{V}l/\nu = \text{constant}$, involves for geometrical similarity of boundaries, physical similarity in the motions.

This idea has been seized upon and used to great advantage in aerodynamical and naval architectural analysis, for, as will be immediately seen, it provides the basis for the analysis of full-scale problems in aeroplane, airship, and naval construction, by means of tests on models.

§ (18) RELATION BETWEEN EXPERIMENTS ON MODEL AND ON FULL SCALE.—Such analysis, in general, centres itself round the question of how, from measurements of the forces originated during the motion of the model, to predict the corresponding forces that will be called into play during the motion of a full-scale machine or part.

The details of this will be given in another section ("Aeronautics Scale-effect"),[1] but for the purposes of the present article we may note the main principles at work. If R = resistance of a body of dimension l in its motion through fluid of density ρ and viscosity ν with velocity V, we may say that $\mathrm{R} = f(\rho, \mathrm{V}, l, \nu)$, and applying again the method of the present article we easily determine R in the form

$$\mathrm{R} = \rho \mathrm{V}^2 l^2 f\left(\frac{\mathrm{V}l}{\nu}\right).$$

§ (19) NON-DIMENSIONAL RESISTANCE COEFFICIENT.—Stating the law of resistance in the manner that has been followed in previous analysis as a relation between two non-dimensional groups of terms,

$$\frac{\mathrm{R}}{\rho \mathrm{V}^2 l^2} = f\left(\frac{\mathrm{V}l}{\nu}\right),$$

indicating that the question of the forces resisting a body in its motion through a viscous fluid is most properly represented as a curve showing the variation of the "resistance coefficient" (non-dimensional) $\mathrm{R}/\rho \mathrm{V}^2 l^2$ as ordinate, against Reynolds' non-dimensional number $\mathrm{V}l/\nu$, as a base.

§ (20) CHARACTERISTIC CURVE.—Such a curve will be characteristic of that shape of body, but will be absolutely independent of the "scale" of the problem and of the consistent set of units in terms of which the various factors are expressed. If therefore by experiment on a model of an airship, aeroplane, or any portion of it, values of the resistance coefficient can be obtained for a sufficiently wide range of the single variable Vl/ν the same resistance derivative will be valid for the full scale at the same value of Vl/ν. In actual practice it is extremely difficult, except in rare cases, to attain a sufficiently wide range of this quantity, and methods of extrapolation supported by full-scale experiments have to be resorted to.[1]

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[1] *Phil. Trans.* clxxiv. 935; *Collected Works*, ii. 51.

[1] See Vol. IV.

Fortunately, extrapolation is greatly simplified in practice by the fact that for increasing values of Vl/ν, the resistance tends to increase steadily as the square of the speed, so that $R/\rho V^2 l^2$, and consequently $f(Vl/\nu)$, tends to become constant.

§ (21) Forces during Accelerated Motion.—In the above discussion we have merely considered the question of the forces called into play when the body is in steady motion. Can we derive from first principles any information regarding the nature of the resistance when the body, moving under an acceleration a, say, passes through the velocity v, without our having recourse to the complicated mathematics of fluid motion? Employing the same symbols as previously, but writing v as the velocity not of steady motion but of the body at the instant under consideration, and a its acceleration at that moment, then following the process already adopted we obtain as the expression for the resistance

$$R=\rho v^2 l^2 f\left(\frac{vl}{\nu},\ \frac{al}{v^2}\right),$$

where in addition to the non-dimensional group Vl/ν already obtained, there is now introduced the expression al/v^2 on which the additional resistance depends, not, be it noted, on the acceleration alone, but on the variation of this group.

§ (22) Virtual Mass.—To determine more closely the effect this produces on the resistance, let R = resistance for steady motion at velocity v, and $R+\delta R$ the resistance for an acceleration term al/v^2 small compared to unity, then

$$R=\rho v^2 l^2 f\left(\frac{vl}{\nu},\ 0\right),$$

and

$$\begin{aligned}R+\delta R&=\rho v^2 l^2 f\left(\frac{vl}{\nu},\ \frac{al}{v^2}\right)\\&=\rho v^2 l^2\left[f\left(\frac{vl}{\nu},\ 0\right)+\frac{al}{v^2}f_1\left(\frac{vl}{\nu},\ 0\right)+\ .\ .\ .\right],\\ \therefore\ \delta R&=\rho a l^3 f_1\left(\frac{vl}{\nu},\ 0\right),\end{aligned}$$

indicating that the additional force originated by the accelerated motion is equal to that required to give a mass of fluid $\rho l^3 f_1(vl/\nu_1 0)$ the acceleration a of the body. But ρl^3 is proportional to the mass of the displaced fluid, and consequently it follows that for a given value of vl/ν, the effect of accelerating the motion is to give rise to an apparent increase in mass proportional to the mass of the fluid displaced.

Measurements of this so-called "virtual mass" are of importance in the stability of airships where the mass of fluid displaced is the same as the total weight of the ship, and where consequently the "virtual mass" may acquire a considerable magnitude. It may be remarked that for investigations of this nature the results would naturally be plotted showing the variation of the non-dimensional resistance coefficient $R/\rho v^2 l^2$ either with vl/ν for various values of al/v^2 or conversely, and for a complete investigation the full range, zero to infinity, of al/v^2 must be covered.

§ (23) Experimental Determination of Virtual Mass.—For longitudinal accelerations such experiments have been successfully conducted.[2] A body dropped from rest and allowed to fall in a fluid till it reaches its limiting velocity, gives at the commencement of the fall $v=0$ and a and l finite so that theoretically $al/v^2 \rightarrow \infty$ while, when the limiting velocity is reached $a=0$, and l and v are finite so that a photographic study of the change in motion then provides sufficient data to determine the virtual mass.

Numerous and careful experiments have been conducted to test the accuracy of the deduction that for non-accelerated motion of a viscous fluid the state of affairs depends merely on the value of vl/ν. Reynolds,[3] as already indicated, showed beyond doubt that for the flow of a liquid in a tube the passage from steady to sinuous eddying motion occurs more or less sharply at a definite value of this number. Stanton and Pannell,[4] following up these experiments, verified that for air a similar result held. Since then numerous predictions from model results in aeronautical and marine investigations to full scale have completely justified the deductions. On the other hand, it is clear that the circumstances of the problem must be such as will not involve the origination of properties of the medium presumed non-existent.

§ (24) Effect of Compressibility. — In ballistics, for example, we are concerned with the motion of a body at speeds approaching the velocity of sound, so that one might expect that not merely would energy be lost during the motion in overcoming viscous forces, but also in the production of waves: that is to say, we must now regard the medium as being both viscous and compressive. Now the velocity of sound in a medium of elasticity E, density ρ, and viscosity ν, is given generally by

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[1] For a full explanation of this, see "Aeronautics," Vol. IV.

[2] Cowley and Levy, *Advisory Committee for Aeronautics, R. and M.*, 612.

[3] *Trans. R.S.*, *loc. cit.*

[4] *Ibid.* A., ccxiv. 199-224.

$$V = f(E, \rho, \nu) = \Sigma A E^x \rho^y \nu^z,$$

and equating like dimensions we easily find

$$x = \tfrac{1}{2}, \quad y = -\tfrac{1}{2}, \quad z = 0,$$

$$\therefore V = A\sqrt{\left(\frac{E}{\rho}\right)}.$$

To find the modified expression for the resistance in this case we write

$$R = f(\rho, v, l, \nu, E),$$

and following the normal procedure we derive

$$R = \rho v^2 l^2 f\left(\frac{vl}{\nu}, \frac{v}{\sqrt{E/\rho}}\right),$$

i.e.

$$\frac{R}{\rho v^2 l^2} = f\left(\frac{vl}{\nu}, \frac{v}{V}\right),$$

where V is the velocity of sound in the medium, indicating that for a given value of vl/ν, the resistance coefficient depends solely on the ratio of the velocity of the projectile to the velocity of sound.

§ (25) EFFECT OF ROTATION: PROPELLER: SHELL.—If in addition the body possesses a spin Ω due to rifling, there would be present a term $\Omega l/v$,

$$\therefore \frac{R}{\rho v^2 l^2} = f\left(\frac{vl}{\nu}, \frac{v}{V}, \frac{\Omega l}{v}\right).$$

Two points of importance may be noted. In the first place for such high speeds as would cause the elasticity of the medium to play an important part, the value of vl/ν would in general be so large as already to be well above the range for so-called scale variation for the viscous forces; that is to say, further increase in value of the term vl/ν would have no marked influence on the value of the resistance coefficient. In the second place, problems of practical importance involving the rotation of moving bodies divide themselves under two heads; those in which the term $\Omega l/v$ is of prime importance as in the case of the propeller; and those in which the spin has no appreciable effect on the resistance as in the case of bodies symmetrical about the axis of rotation—shells, bullets, etc. In the latter case the rotation exists only for purposes of stability and directivity, and plays no measurable part in affecting the resistance.[1] Restricting ourselves to the second part, experiments on ballistics conducted prior to and during the war (1914–18) have succeeded in providing a more or less complete representation of the forces on a projectile (shell) for speeds varying up to and well over 2000 feet per second. It is found that plotting $R/\rho v^2 l^2$ the resistance coefficient, against v/V, the curve remains practically straight and horizontal until a speed approaching that of sound is attained, when it rises steeply and turning at the critical speed diminishes slowly. It appears then that in the region of this critical speed properties of the medium enter which up to that point play no part. The most natural non-dimensional base for representation then changes from vl/ν at low speeds to v/V at high speeds.

It may be mentioned in passing that in the case of the screw motion of a body not symmetrical about the axis of rotation, such as a propeller for example, our discussion so far implies that at speeds at which the viscous scale effect due to the presence of the vl/ν term is no longer of importance as a variant, the term $\Omega l/v$ will become the principal non-dimensional base of variation until the speeds developed approach the velocity of sound, when the new factor v/V will begin to exercise an increasing influence. The anticipation that at such high speeds a critical state will set in is fully borne out by experiment.

By the method of this chapter it is evident that the expression for the pitch of the note produced by the rotation of a propeller at normal speeds will be given by

$$n = \Omega f\left(\frac{vl}{\nu}\right),$$

while for excessive speeds

$$n = \Omega f\left(\frac{vl}{\nu}, \frac{v}{V}\right).$$

For such an instrument as an Aeolian harp if d = diameter of wires, the pitch of the note is

$$n = \frac{v}{d} f\left(\frac{vd}{\nu}\right).$$

Actually Strouhal has found experimentally $n \propto (v/d)$ indicating that the function is a mere constant. It should be remarked that a similar expression was found for the period of the eddies produced by the passage of the wind past the wires, thus suggesting the connection between the two phenomena.

§ (26) DEDUCTION OF SAVART'S LAW.—An interesting illustration of the power possessed by the dimensional method of reducing experimental investigation to the necessary minimum is afforded by the investigations of Savart[2] on the notes sounded by similar vessels containing air. The law proposed by him after an elaborate series of experiments over extreme limits, many notes, and numerous shapes of resonating boxes, was that the number of vibrations per second, or pitch of the resonating note, is inversely proportional to the linear dimensions of the vessel. His experiments included tests on boxes in the shape of cubes, prismatic tubes, conical pipes, etc., all of which were set resonating in different ways. This result, to establish which required such an elaborate series of experiments, can be seen to follow at once from dimensional considerations, for, if n be the number of vibrations per second, E and ρ be the elasticity and density of the medium and l a linear dimension of the box, then $n=f(l, \mathrm{E}, \rho)$ and remembering that $[n]=\mathrm{T}^{-1}$, $[\mathrm{E}]=\mathrm{ML}^{-1}\mathrm{T}^{-2}$ and $[\rho]=\mathrm{ML}^{-3}$, it is easily seen that $n \propto 1/l\sqrt{(\mathrm{E}/\rho)}$ where the constant of proportionality depends purely on the shape of the vessel: Savart's Law follows at once.

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[1] A discussion of the performance of propellers as a function of the non-dimensional group $\Omega l/v$ will be found under "Propellers."

[2] *Annales de Chimie*, Paris, 1825, xxix.

§ (27) TEMPERATURE.—In the foregoing discussion it has been presumed that the temperature of the medium remains constant, or at any rate it does not bring into prominence any new property of the medium. It is now proposed to remove this restriction. The conception of temperature is of an essentially different nature from any of those we have already discussed. The scale of temperature is selected to provide some basis for the measurement of "hotness" of a body. Having at our disposal standards of mass, length, and time only, it is manifestly impossible by means of them to determine or reproduce a temperature; for this purpose some other property of matter as affected by heat must be selected to serve as a standard of measurement. Accordingly, it becomes necessary at this stage to introduce a new unit Θ of temperature, and in terms of this and the units already adopted it should be possible to write down with comparative ease the dimensional expressions for the quantities that now arise.

In dealing with the conduction and convection of heat there are two coefficients, characteristic of any particular substance, whose dimensions will be required; they are, a quantity measuring the amount of heat that must be given to raise unit volume of the medium through unit degree of temperature, and a quantity measuring the heat transferred across a section of unit depth and unit area in unit time for unit difference in temperature. These quantities are of course respectively the specific heat (c) and the thermal conductivity (k) of the medium.

Expressing heat entirely in terms of energy units where it occurs, it then follows that

$$[c]=[\text{Heat}]/[\text{Vol.}]\times[\text{Temp.}]=\mathrm{ML}^{-1}\mathrm{T}^{-2}\Theta^{-1},$$

$$[k]=[\text{Heat}]/[\text{Length}]\times[\text{Time}]\times[\text{Temp.}] =\mathrm{MLT}^{-3}\Theta^{-1}.$$

In connection with many problems of convection, the effect of the expansion of the air under the influence of the heated surfaces may attain some considerable importance in the transmission of the heat, and as a consequence the quantity δ is introduced, representing the change in density per unit change in temperature, or

$$\delta=\frac{\partial\rho}{\partial\Theta},$$

$$\therefore [\delta]=\mathrm{ML}^{-3}\Theta^{-1}.$$

§ (28) RATE OF COOLING OF BODY IN QUIESCENT GAS.—As an illustration of the utility of the method of dimensions in the discussion of problems of cooling, consider the rate of loss of heat h of a body of given shape, of size l of one part, in a quiescent atmosphere of density ρ and kinematic viscosity ν. If k and c have the meanings previously attached to them and Θ be the temperature difference maintained between the body and the gas at a great distance, at which the latter is undisturbed by the presence of the body, it will be justifiable to write

$$h=f(l, k, c, \rho, \nu, \Theta),$$

assuming that in this case the changes in density with temperature are not sufficient to affect the question. The modifications introduced if this condition is violated will be apparent, and adjustment easily made.

Imagining the function f expanded as in previous illustrations in the form

$$\Sigma \mathrm{A} l^x k^y c^z \rho^u \nu^v \Theta^w,$$

then $$[h]=[l^x k^y c^z \rho^u \nu^v \Theta^w].$$

The dimensions of h, the rate of heat loss, are those of $(\text{Heat})/(\text{Time})=\mathrm{ML}^2\mathrm{T}^{-3}$.

$$\therefore \mathrm{ML}^2\mathrm{T}^{-3}=\mathrm{L}^{x+y-z-3u+2v}\,\mathrm{M}^{y+z+u}\,\mathrm{T}^{-3y-2z-v}\,\Theta^{-y-z+w};$$

$$\therefore [\mathrm{M}]\ldots y+z+u=1 \quad [\mathrm{L}]\ldots x+y-z-3u+2v=2;$$

$$\therefore [\mathrm{T}]\ldots 3y+2z+v=3 \quad [\Theta]\ldots y+z-w=o;$$

$$\therefore x=1-2u, \quad y=1+2u-v,$$

$$z=-3u+v, \quad w=1-u.$$

This provides a typical term in the expansion in the form

$$lk\Theta\left(\frac{\rho k^2}{l^2c^3\Theta}\right)^u\left(\frac{c\nu}{k}\right)^v,$$

and consequently since u, v, and the numerical coefficient of each term may as far as can here be determined have any value whatsoever, it is to be concluded that

$$h=lk\Theta f\left(\frac{\rho k^2}{l^2c^3\Theta}, \frac{c\nu}{k}\right).$$

Whether or not all the quantities which have been assumed of importance in the determination of h actually do enter is a question to be decided only by experiment, to which we shall return shortly.

§ (29) NON-DIMENSIONAL VARIANTS.—For the moment it can be concluded that the problem of rate of cooling in this instance reduces itself to a consideration of the variation not of h alone, but of the non-dimensional group $h/lk\Theta$ with the non-dimensional variants $\rho k^2/l^2c^3\Theta$ and $c\nu/k$. If we are dealing with the rate of cooling in gases then practically $c\nu/k$, the ratio of the emissivities for momentum and for heat, is a constant for all gases, and consequently this term may be omitted from the function f.

A considerable number of experimental investigations have been conducted on this problem and in all cases it has been found that the rate of heat loss varies directly as the temperature difference Θ. In effect this implies that h/Θ is independent of Θ and that consequently $f(k^2\rho/l^2c^3\Theta)$ above is a constant. Hence

$$h = alk\Theta,$$

indicating that the rate of cooling is proportional to the size of the body and to the conductivity. The coefficient a depends of course on the shape of the cooling body.

If, however, it is illegitimate to assume that the expansion of the gas due to heating is negligible as a factor in heat transmission, but that the expanded portions becoming thereby relatively lighter will tend to stream upwards, and as a consequence the heavier portions downwards, a new term introducing the change in weight of unit volume per unit change in temperature must be considered. Such a factor is of course $g(\partial\rho/\partial\Theta)$ or $g\delta$.
Hence generally

$$h = f(l,\ k,\ c,\ \rho,\ \nu,\ \Theta,\ g\delta),$$

and following the lines of the previous argument

$$h = lk\Theta\phi\left(\frac{\rho k^2}{l^2c^3\Theta},\ \frac{c\nu}{k},\ \frac{lg\delta}{c}\right).$$

For the reasons adduced in the former case, the final form sufficiently approximate for all practical purposes is

$$\frac{h}{lk\Theta} = \chi\left(\frac{lg\delta}{c}\right),$$

provided the problem is restricted to heat transmission in gases. The form of the function χ, depending as it does on the shape of the body under consideration, will now be completely determined graphically from a single set of experiments for which the non-dimensional group $lg\delta/c$ is allowed to vary over a range, for each point of which the non-dimensional term $h/lk\Theta$ is measured. The resulting curve obtained, invariable for all systems of units, will then be characteristic of those special boundaries.

§ (30) Rate of Cooling of a Moving Body in a Gas.—Problems of great practical importance arise in connection with the rate of heat transmission from bodies in motion in the air, as in the case of radiators and air-cooled engines. In these problems the rate of heat transmission will actually depend on the translational velocity V in addition to the quantities already enumerated above. The introduction of this extra symbol, as can easily be verified by an application of the dimensional method, gives rise to the "fluid motion" term Vl/ν in the functional expression. If under these circumstances it can legitimately be presumed that the term brought in by the rate of change of density would not materially affect the rate of heat transmission, an assumption that will be examined shortly, then, for reasons already set forth,

$$h = lk\theta f\left(\frac{c\nu}{k},\ \frac{Vl}{\nu}\right),$$

or, since $c\nu/k$ is constant for a gas,

$$\frac{h}{lk\theta} = \phi\left(\frac{Vl}{\nu}\right).$$

§ (31) Experimental Determination of Unknown Function.—In the case of heat loss from long circular wires past which air is streaming with velocity V, it has been found [1] that h varies approximately as $V^{\frac{1}{2}}$. If for the moment this be accepted as correct, it is possible immediately to determine the form of the function, for the only expression for ϕ such as will make $h \propto V^{\frac{1}{2}}$ is $A\sqrt{(Vl/\nu)}$. If the wire, moreover, is of diameter a, it is quite legitimate to substitute a for l in this term in order that when the wire is of infinite length there may result a finite heat loss per unit length. It follows, therefore, on the foregoing assumption that

$$h \propto lk\theta\sqrt{\left(\frac{Va}{\nu}\right)} \propto lk\theta\sqrt{\left(\frac{aVc}{k}\right)},$$

since $c\nu/k$ is constant.

The most rational method of representing graphically the experimental results referred to above would be by plotting the non-dimensional term $h/lk\theta$ on the non-dimensional base $\sqrt{(Va/\nu)}$ or $\sqrt{(aVc/k)}$, from which, if the above law is exact, a straight line would be obtained characteristic of that particular shape of wire.

§ (32) Dynamical Similarity.—It should be remarked that if Vl/ν and $\theta c/\rho V^2$ are maintained constant from problem to problem, the system of stream lines and state of flow generally will remain geometrically similar, for the velocity v at any point geometrically fixed with respect to the body will be of the form

$$\frac{v}{V} = f\left(\frac{c\nu}{k},\ \frac{Vl}{\nu},\ \frac{\theta c}{\rho V^2}\right),$$

and the argument follows closely that adopted in the case of the motion of viscous fluids, already outlined.

More recent investigations of the rate of cooling of wires in a current of air, by L. V.

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[1] Russell, *Phil. Mag.*, 1910, xx. 591.

King,[1] have shown the presence of another term besides that involving $V^{\frac{1}{2}}$, viz. a term proportional to the square of the linear dimensions of the hot body. King in fact found that a close approximation to the results he obtained for the rate of heat transmission was given by the empirical formula

$$h=(\theta-\theta_0)l\{A\sqrt{Va}+B+Ca\},$$

where A, B, and C are constants for the particular gas and shape of wire—circular in his case. L. F. Richardson[2] has suggested that the presence of the term $c(\theta-\theta_0)la$ indicates that the velocity near the hot body is partly produced by the rising of fluid due to thermal expansion. It follows, therefore, if this is sound, that the additional term ought to be expressible in terms of the appropriate group already referred to, viz. $gl/c\,.\,d\rho/d\theta$.

§ (33) More accurate Determination of Unknown Function.—Writing therefore

$$h=lk(\theta-0_0)f\left(\frac{aVc}{k},\ \frac{ga}{c}\cdot\frac{d\rho}{d\theta}\right),$$

as derived on dimensional reasoning, and equating this to the empirical formula found by King,

$$A\sqrt{Va}+B+Ca=kf\left(\frac{aVc}{k},\ \frac{ga}{c}\cdot\frac{d\rho}{d\theta}\right),$$

it is indicated that the form of the function f must be

$$P\sqrt{\left(\frac{aVc}{k}\right)}+Q+R\frac{ga}{c}\cdot\frac{d\rho}{d\theta},$$

where P, Q, and R are now mere numbers determined solely by the geometrical conditions and universally applicable for any gas once the values of these three numbers have been derived from King's experimental results. The complete formula is consequently

$$h=lk(\theta-\theta_0)\left[P\sqrt{\left(\frac{aVc}{k}\right)}+Q+R\frac{ga}{c}\cdot\frac{d\rho}{d0}\right],$$

giving the relation between the rate of heat loss and the remaining quantities k, etc., which were not presumed variable in the actual experiment.

§ (34) Non-dimensional Groups are the Real Variables.—The real variables in fact in the problem are the dependent non-dimensional quantity $h/lk(\theta-\theta_0)$ and the independent non-dimensional groups aVc/k and $ga/c\,.\,d\rho/d\theta$, while the most rational graphical representation of the experimental results would be obtained by showing the law connecting the variation of this dependent variable with each of these independent variables, the system of curves obtained being then characteristic of this class of problem, and dependent only on the geometrical conditions.

II. Dimensions of Electric and Magnetic Quantities

§ (35) Electrostatics.—The treatment of electrical and magnetic phenomena from the standpoint of the present article presents some points of interest. Considering in the first place electrostatic quantities, we possess at present no means of expressing electric or magnetic charge directly in terms of any of the fundamental units already introduced. Accordingly a consideration of electrostatic quantities must introduce a new unit directly related to the charge. The simplest method of representation is through the quantity K, the specific inductive capacity, and the experimentally known law of force between charged bodies. Accordingly we write

$$\frac{1}{K}\cdot\frac{ee'}{r^2}=\text{force},$$

giving as the dimensions of a charge $K^{\frac{1}{2}}M^{\frac{1}{2}}L^{\frac{3}{2}}T^{-1}$. At once, from the definition of the quantities concerned, it is possible to draw up the following table :

Quantity.	Symbol.	Dimension.
Charge	e	$K^{\frac{1}{2}}M^{\frac{1}{2}}L^{\frac{3}{2}}T^{-1}$
Electric intensity .	F	$K^{-\frac{1}{2}}M^{\frac{1}{2}}L^{-\frac{1}{2}}T^{-1}$
Potential difference .	V	$K^{-\frac{1}{2}}M^{\frac{1}{2}}L^{\frac{1}{2}}T^{-1}$
Current	i	$K^{\frac{1}{2}}M^{\frac{1}{2}}L^{\frac{3}{2}}T^{-2}$
Resistance . . .	R	$K^{-1}L^{-1}T$
Capacity	C	KL
Specific independent capacity	K	K

In the same manner, commencing with the definition of magnetic permeability μ as

$$\frac{1}{\mu}\cdot\frac{mm'}{r^2}=\text{force},$$

m and m' being the strength of two magnetic poles, we can draw up a parallel list of quantities involved in magnetic phenomena, *e.g.* strength of magnetic pole has dimensions $\mu^{\frac{1}{2}}M^{\frac{1}{2}}L^{\frac{3}{2}}T^{-1}$. It would appear at first sight as if these two lists were quite independent and that two separate units, K and μ, for the discussion of electric and magnetic phenomena would require to be introduced, but it is known that magnetic and electric phenomena are closely interwoven, that currents flowing through wires originate magnetic fields in their neighbourhood.

§ (36) Relation between Electrical and Magnetic Units.—We have in fact this connecting link, that if a magnetic pole of strength m is made to thread a circuit *once* in which is flowing a current of strength i, the total work done is measured by

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[1] *Phil. Trans.* A, 1914, ccxiv. 373-432.

[2] *Proc. Phys. Soc.* xxxii. Pt. V. 406. A discussion of the experimental results of numerous workers is given from the standpoint of the present article by A. H. Davis, *Phil. Mag.*, Dec. 1920, xl.

$$\text{work done} = 4\pi mi.$$

Hence the dimension of the product of magnetic strength and electric current are those of work.

$$\therefore (\mu^{\frac{1}{2}}M^{\frac{1}{2}}L^{\frac{3}{2}}T^{-1})(K^{\frac{1}{2}}M^{\frac{1}{2}}L^{\frac{3}{2}}T^{-2}) = ML^2T^{-2},$$

$$\therefore [\mu^{\frac{1}{2}}K^{\frac{1}{2}}] = L^{-1}T = [1/\text{Velocity}].$$

The most natural standard velocity to adopt in such circumstances would be the velocity of electromagnetic waves in the particular medium, and accordingly we have the relation

$$\mu K = A/V^2,$$

where A is some constant which accurate experimental investigations have, in fact, shown to be equal to unity. This simple yet fundamental relation between μ and K enables us at once to reduce the two systems of units to one, either by expressing all the electrical quantities in terms of magnetic units or *vice versa*. We express them here in terms of electromagnetic units as follows:

Quantity.	Symbol.	Dimension.
Charge	e	$\mu^{-\frac{1}{2}}M^{\frac{1}{2}}L^{\frac{1}{2}}$
Electric intensity .	F	$\mu^{\frac{1}{2}}M^{\frac{1}{2}}L^{\frac{1}{2}}T^{-2}$
Potential difference .	V	$\mu^{\frac{1}{2}}M^{\frac{1}{2}}L^{\frac{3}{2}}T^{-2}$
Current	i	$\mu^{-\frac{1}{2}}M^{\frac{1}{2}}L^{\frac{1}{2}}T^{-1}$
Resistance . . .	R	μLT^{-1}
Capacity	C	$\mu^{-1}L^{-1}T^2$
Specific independent capacity	K	$\mu^{-1}L^{-2}T^2$
Strength of magnetic pole	m	$\mu^{\frac{1}{2}}M^{\frac{1}{2}}L^{\frac{3}{2}}T^{-1}$
Magnetic force . .	H	$\mu^{-\frac{1}{2}}M^{\frac{1}{2}}L^{-\frac{1}{2}}T^{-1}$
Magnetic induction .	B	$\mu^{\frac{1}{2}}M^{\frac{1}{2}}L^{-\frac{1}{2}}T^{-1}$
Magnetic permeability	μ	μ
Coefficient of inductance	L	μL

By means of these dimensional relations it is possible once more to group together whole classes of investigations as dependent on the variation of certain non-dimensional groups, thus indicating a rational method of plotting the results obtained.

§ (37) APPLICATIONS.—The following selection of simple results can be derived directly by the method of the present article.

Time taken for a current in a circuit of resistance R and inductance L to fall to a given fraction of its intensity when switched off is proportional to L/R.

A periodic E.M.F. of any form whose amplitude is specified by E and period $2\pi/p$ acting on any system, of resistance R and inductance L, produces a maximum current given by

$$i_{\text{max.}} = \frac{E}{R} f\left(\frac{Lp}{R}\right).$$

For a simple E.M.F. given by E cos pt, it is known that

$$i_{\text{max.}} = \frac{E}{R}\left[1 + \left(\frac{Lp}{R}\right)^2\right]^{-\frac{1}{2}}.$$

Expression for the energy per unit volume of an electromagnetic field of electric and magnetic intensities F and H respectively is

$$\mu H^2 f\left(\frac{KF^2}{\mu H^2}\right).$$

The total energy of a charge e of dimension d moving with velocity v in a medium of constants μ and k is

$$\frac{\mu v^2e^2}{d} f\left(\frac{v}{V}\right),$$

where V = velocity of electromagnetic waves in the medium; consequently the electrical mass m is

$$m = \frac{2\mu e^2}{d} f\left(\frac{v}{V}\right).$$

The radiation R from an accelerated electron is

$$R = \frac{\mu e^2 a^2}{v} \phi\left(\frac{v}{V}\right),$$

where a = acceleration, and the radiation is assumed independent of the diameter of the electron.

III. THE APPLICATION OF THE PRINCIPLES OF SIMILITUDE TO MODELS

§ (38) MODEL EXPERIMENTS AND THEIR FULL-SCALE EQUIVALENTS.—The principles developed in the foregoing paragraphs, serving as they do to group together processes occurring in geometrically similar systems, provide the most natural method of approach to the problem of the relations of models to their full-scale counterparts. The lines of development in the case of the use of models for the determination of the wind forces on aircraft have already been discussed. Two further aspects of the general question will be here treated. What, in the first place, must a model fulfil as regards working conditions in order that its motions and working may be directly comparable with those of the full-sized machine? That is to say, what are the relations between weights, external forces, speeds, etc., that two systems initially geometrically similar may continue during their motion to remain geometrically similar, that the relative positions of the parts of one system after a time t may always be similarly situated to the relative positions of the corresponding parts at time t', where t and t' bear a constant ratio to each other. It is clear that if these conditions can be determined and a model produced to operate satisfactorily while working under them, then the full-scale machine will also operate satisfactorily. The second aspect, dealing with the strength of construction of the model and of the full-scale, will be treated shortly.

§ (39) Condition for continued Similarity in Working of Model and Full-scale.—Corresponding to each part of the system there will be a scheme of equations between the moving forces and acceleration produced, of the form $m(d^2x/dt^2)=X$ for the model, and for the full-sized machine $m'(d^2x'/dt'^2)=X'$, where m and m' are the masses of corresponding parts, occupying corresponding positions x and x' at corresponding times t and t'. X and X′ are the corresponding moving forces. For continued geometrical similitude at time t and t' connected by $t=Tt'$, where T is some constant, there must be $x=lx'$, $m=Mm'$, and $X=FX'$ where l, M, and F are constants determining the scale of the one with reference to the other. Hence, inserting these in the first equation,

$$\frac{Mm'l}{T^2}\frac{d^2x'}{dt'^2}=FX',$$

or

$$\left(\frac{Ml}{T^2F}\right)m'\frac{d^2x'}{dt'^2}=X',$$

and the motions will therefore remain identical if $Ml/T^2F=1$, the same equation then applying both to model and full-scale. Hence the moving forces of model and full-scale must be in the ratio F, where

$$F=\frac{Ml}{T^2}.$$

In the same way, had the corresponding times been eliminated by comparing the systems at corresponding speeds we would have found that for similarity the moving forces must be proportional to MV^2/l.

The following conclusions may accordingly be drawn for a comparison of the working of a model with that of the full-sized machine: Since the weights of the parts, regarded as external forces, are proportional to their masses, $V^2 \propto l$, and the velocity of working must be proportional to the square root of the linear dimensions; external moving forces must bear a constant ratio to the weight and must therefore increase as the cube of the linear dimensions, if model and machine are made of the same material.

IV. Similitude of Structures

§ (40) Structural Strength.—Turning to the question of the relation between the structural strengths of models and full-scale constructions, it is proposed to show that the theoretical basis of model strength tests on structures may be developed in a rigorous yet simple form by an application of the method of Dimensions.

When a homogeneous prismatic strut of length l and flexural rigidity EI, simply supported at the ends, is subjected to axial end thrust F up to a load given by

$$F=\frac{\pi^2 EI}{l^2},$$

the undeflected position is one of stable equilibrium provided the ultimate stress is not exceeded. Beyond this so-called Euler's Critical Load, the straight position is one of unstable equilibrium. If the strut be deflected, as long as the load is maintained the axis will continually undergo change in shape until it ultimately takes up a form of elastica. During this process the yield point of the material may be exceeded and the material may rupture. The strut may thus be supposed to fail for two possible reasons, on the one hand because its shape has been permanently changed from the straight normal position owing to the instability of that position, and on the other hand because the actual material has failed to withstand the stresses originated. This conception is of course not limited to such a simple structure as a strut, for it may be seen that in general a framework of any kind may fail for either of these two reasons. It is proposed to investigate what information may be furnished regarding these two types of failure, rupture and instability, for a structure in general, by an application of the dimensional theory.

Consider the case of a structure of the type of an aeroplane framework for definiteness, although the argument is quite general, where it is supposed that the assumptions made in the ordinary beam theory apply to every part. Let the length of one part, say a bay, be given by l, the area of a particular section by A, moment of inertia I, and the elasticity and density of the material of which it is composed E and ρ respectively. Given these quantities for this one portion, it will be assumed that the shape and geometry of the structure involve an exact specification of how to derive the corresponding quantities for the remaining portions. Let the external load be F, applied in some given manner specified geometrically, and suppose this is sufficient to produce a stress just greater than the yield stress f in the material of the weakest member. F can only depend apart from the geometry and manner of application upon the quantities enumerated above, defining the properties of the material. These are as follows:

Quantity.	Symbol.	Dimensions.
Young's modulus .	E	M/LT^2
Area	A	L^2
Moment of inertia .	I	L^4
Length	l	L
Density	ρ	M/L^3
Yielding stress . .	f	M/LT^2
Gravity	g	L/T^2
Load	F	ML/T^2

Since F, the load which will cause collapse by rupture of the material, must involve these quantities grouped together in such a manner as to make the dimensions of the product identical with those of F, we may write

$$F=\psi(E,\ A,\ I,\ l,\ \rho,\ f,\ g).$$

Applying the method already used frequently in this discussion, we easily find

$$F=\frac{EI}{l^2}\psi\left(\frac{f}{E},\ \frac{A}{l^2},\ \frac{W}{fA},\ \frac{I}{l^4}\right),$$

where ψ for the moment is an arbitrary function. Writing

$$\phi^2=\frac{Fl^2}{EI}$$

so that this quantity may be considered as a *non-dimensional* critical loading coefficient, it follows finally that

$$\phi^2=\psi\left(\frac{f}{E},\ \frac{A}{l^2},\ \frac{W}{fA},\ \frac{I}{l^4}\right).$$

§ (41) Non-dimensional Critical Loading Coefficient.—It is clear from the definition of ϕ that for a given structure where l, E, and I are known ϕ is uniquely determined when F, the breaking load, is found, so that the discussion of collapse by rupture may equally well be centred round ϕ. Let it be clearly understood that the exact form of the function ψ above depends only on the geometry of the structure, including the manner of loading and the law of distribution of material.

Out of the general class of framework embraced in this discussion so far, let there be selected a series of which all members are identical with respect to external shape, differing only in scale. This implies that A, the area of cross-section of any selected part, is proportional to l^2, and I to l^4. For this series, therefore, A/l^2 and I/l^4 are constants depending only on the geometry, and accordingly the expression for ϕ now takes the simplified form

$$\phi^2=\psi_1\left(\frac{f}{E},\ \frac{W}{fA}\right).$$

It is not difficult to give an interpretation to the two non-dimensional quantities f/E and W/fA. The former, being the ratio of the yielding stress to Young's modulus, may be taken as the strain at the yield point, on the assumption that Hooke's Law applies rigorously over the whole range. In the same way W/fA is the ratio of the weight of a particular member, say the weakest, to the maximum tensile or compressive force, as the case may be, which that member is capable of withstanding without yielding, assuming that the section A is taken at the position of collapse.

§ (42) Non-dimensional Variables.—As far as a discussion of the breaking load coefficient ϕ^2 is concerned, it is evident that if a series of geometrically similar structures be selected, two distinct non-dimensional variables, and two only, functions of the material, affect the question. These are the strain at the yield point for the weakest member, and the ratio of the weight of that member to the maximum tensile or compressive force it can withstand at the point of yield. It is equally clear, however, that the so-called geometrical restrictions determining the distribution of the constants of the materials in a composite structure are not absolutely vital to the above discussion, so that violations of the laws determining the selection of the material from member to member would not necessarily vitiate the conclusions arrived at here. For example, one would not be rigorously entitled to vary I, A, and E in certain members, since such a change would clearly upset the geometrical similarity of shape and the distribution of force, with the possible result that the previously weakest member might now not remain such. No such trouble, however, could arise if the selection of the materials was made without changing these factors, but allowing a variation in f, the yielding stress, so long as the previously weakest member always remains so in the series. This is equivalent to an increased latitude in the scope of selection of material.

Generally in experimenting with a model of this nature it is convenient, of course, to have corresponding parts in the model and the full-scale structure composed of identical materials. If, for example, the frame of an aeroplane was under discussion, wires in the model would be composed of the same metal as those in the original; corresponding struts, spars, ribs, etc., of the same wood.

Since the quantities f, E, l, ρ, etc., in the expressions for ϕ^2 all refer ultimately to the same part of the model, to ensure geometrical similarity in the widest sense it follows that the assumption that f/E is a constant is one which is obvious experimentally. Under these circumstances also W/fA, as can easily be verified, becomes proportional to the linear dimension l. The breaking load coefficient ϕ^2, therefore, when the weight of the structure contributes towards rupture, is purely a function of l,

$$\therefore\ \phi^2=\phi_1(l),$$

and the form of the function is determined only by the shape of the various parts, the geometry of the loading and the so-called geometry of the material.

§ (43) Limiting Size of a Heavy Structure.—This expression for ϕ^2 may likewise be supposed solved for l in terms of ϕ^2. For every external load coefficient this equation then determines the size of the structure corresponding to failure. When there is no external load and failure is due to weight alone, $\phi_1(l)=0$, an equation determining the smallest size of a structure that will collapse under its own weight.

When the material is not sufficiently heavy to involve the weight of the structure appreciably as a factor in causing collapse, the term

W/fA may be omitted in the general equation and the expression for ϕ^2 takes the relatively simple form

$$\phi^2=\phi_2\left(\frac{f}{E}\right).$$

§ (44) NON-DIMENSIONAL BASES, AND "CHARACTERISTIC CURVE."—Let us imagine that the weakest member is replaced by another of the same geometrical shape and size but with a different value of f/E, though not so different as to prevent its remaining the weakest member; then a test on each one of these models will give a particular value of ϕ^2, breaking load coefficient, and these when plotted against f/E will give a "characteristic curve" for the structure of the given geometry. For geometrically similar structures of which the corresponding parts are made of the same material, so that f/E is constant, the breaking load coefficient becomes an absolute constant for the series depending for its value purely on the shape. *It follows that a simple test to destruction on a model will suffice under these conditions to determine the breaking load coefficient* ϕ^2, and therefore the breaking load F for any other member of the series of geometrically similar structures.

§ (45) FAILURE DUE TO INSTABILITY.—It is assumed in this section that the failure is not necessarily accompanied by collapse or rupture in the material, but is due merely to permanent deformation of the configuration of the structure. It follows that, in the expression for the critical load, the yield stress does not enter although all the other terms A, l, E, I, ρ, g may do so. Using ϕ^2 as a critical load coefficient defined by $Fl^2/EI=\phi^2$, then

$$\phi^2=\psi_1\left(\frac{A}{l^2}, \frac{W}{EA}, \frac{I}{l^4}\right),$$

obtained by the same method as in the previous section.

On the understanding that the structures in the series are all geometrically similar as regards external shape without reference to material composing them, then A/l^2 and I/l^4 are constants for the series, and

$$\phi^2=\psi_2\left(\frac{W}{EA}\right),$$

where W/EA is the ratio of the weight of a member to the tensile or compressive force required to produce unit strain at some particular section of that member.

If corresponding members of the series are made of the same material, then W/EA is easily seen to be proportional to l, from structure to structure, and as before the critical loading coefficient

$$\phi^2=\psi_3(l)$$

depends on the size only.

§ (46) CRITICAL LOADING COEFFICIENT A CONSTANT FOR COLLAPSE DUE TO INSTABILITY.—When the weight of the structure is negligible as far as its effect in contributing to failure is concerned, the whole instability arising from the external loading, then ϕ^2 is a constant for a series of structures of identical form irrespective of the material of which it is composed, and *a test to destruction on one model suffices for the series.*

It has been found that the critical loading for a uniform prismatic strut of negligible weight under end thrusts and simply supported at the ends is given by

$$F=\frac{\pi^2 EI}{l^2}$$

so that ϕ for this structure is π. For a strut under the same conditions, but with clamped ends, $\phi=2\pi$.

In certain cases of simple shapes of structures, it is obvious that the function ϕ may be derived by calculation, as for example in the above two cases, but for more complicated problems where the calculation is too abstruse a number of points on the characteristic curve may be derived by a series of tests to destruction on models.

In the special cases where ϕ is a constant for the series a single test on one model is clearly sufficient. This, as has been shown, applies with equal force where the collapse is due either to instability or to failure of the material.

The basis of the experimental method thus having been made secure, no scale effect difficulty of the type normally arising in, say, aerodynamic work being present, it should now be possible to proceed to a direct accurate test of failure and its causes under various circumstances.

§ (47) WHIRLING OF SHAFTS.—While the foregoing analysis applies to structures under forces of the static type, a parallel argument can be applied where fluctuating stresses are introduced, or where parts of the structure are in motion. In illustration consider the problem of whirling of a rotating shaft of given outline and section. As before, suppose the geometrical shape longitudinally and cross-sectionally to be given, length l, mass per unit length m, and flexural rigidity of a particular section EI_o: then if q be the speed of rotation, the deflection d at the centre, say, will be a function of l, m, E, I_o, and q, *i.e.*

$$d=f(l, m, E, I_o, q).$$

Hence $$\frac{d}{l}=F\left(\frac{mq^2l^4}{EI_o}, \frac{I_o}{l^4}\right)$$

on equating the dimensions of the typical group. For a shaft of given geometrical outline I/l^4 is a constant dependent on the geometry, hence in varying the size of geometrically similar shafts the ratio of the deflection at the centre to the total length will depend only on the quantity mq^2l^4/EI_o.

§ (48) Non-dimensional Variable.—If a whirling speed exists, then, for some value of q, the ratio d/l will theoretically become infinite, or at least indeterminate, and this can only occur for some value of the variable mq^2l^4/EI_o, say a. Hence we find that the real variable in whirling shaft problems as the size and material of the shaft are varied, but maintaining the geometrical shape constant, is mq^2l^4/EI_o. In actual fact the whirling speed of a straight shaft is known to be

$$q=\frac{\pi^2}{l^2}\sqrt{\frac{EI}{m}},$$

i.e.

$$a=\frac{mq^2l^4}{EI}=\pi^4,$$

and for a homogeneous circular stepped shaft made up of three portions, the two end pieces of length $l/4$ and diameter $d/4$, and the central portion of length $l/2$ and diameter d,

$$\frac{mq^2l^4}{EI_o}=3,$$

I_o being taken for the central portion.[1]

From the foregoing analysis, moreover, it is apparent that shafts of identical geometry but differing in size and material will maintain their identity in geometry under rotation provided they be compared at corresponding speeds q_1 and q_2 given by

$$q_1l_1^2\sqrt{\frac{m_1}{E_1I_1}}=q_2l_2^2\sqrt{\frac{m_2}{E_2I_2}}$$

§ (49) Expansion in Terms of the Class Variables.—It has become apparent in the application of the method of the present article to various branches of physics and engineering that in all cases the investigation has centred itself round the behaviour of some expression represented as a function of a certain non-dimensional group of terms. This quantity, connecting together as it were a whole class of problems possessing certain geometrical and dynamical similarities, may be termed the *class variable.* Many problems in mathematical physics expressible in terms of differential equations would appear then to demand a solution as a function of the class variable, and accordingly an expansion of the solution of the differential equation is sought for as a power series in the class variable. This method has been applied with considerable success to the problem of the crippling of struts, the whirling of shafts of variable flexural rigidity, and to the motion of a viscous fluid.[2] In effect this is simply an analytical method of representing the "characteristic curves" on non-dimensional bases obtained so frequently in the foregoing discussion.

Hy. L.

Dynamical Similarity:

Experimental verification of Rayleigh's conditions for the motion of fluids in contact with solid boundaries. See "Friction," § (16).

Rayleigh's method of determining the essential conditions which must be fulfilled. See *ibid.* § (15).

Dynamics, Particle, Application of Dynamical Similarity to. See "Dynamical Similarity, The Principles of," § (7).

DYNAMOMETERS

§ (1) Fundamental Principles.—The work done by a prime mover or other agent when transmitted by a rotating shaft may be expressed by $PR\phi$, and when by a moving chain, rope, or similar agent, by Pd, where P is the force acting, R the torque arm, ϕ the angular displacement, and d the linear displacement.

A dynamometer is an instrument which is used for the measurement of the force P, or the torque PR; the values of the quantities ϕ and d are usually determined independently.

The work done by the prime mover may either be absorbed by the dynamometer or transmitted to a machine in which it is employed usefully after having been measured.

It is necessary that the instrument adopted should perform its function with accuracy. The whole of the energy developed by the prime mover should be included in the measurement made without neglecting that absorbed by shaft bearings in the dynamometer itself or by gearing or bearings which may be interposed between the prime mover and the dynamometer.

The load should be capable of fine adjustment under running conditions, and this adjustment should preferably be over a considerable range of power in order that the prime mover may be tested from its minimum to its maximum power capacity without the necessity for frequently stopping down.

The construction of a dynamometer should be such that it imposes no force on the prime mover other than that for which it was designed. For example, a dynamometer suitable for testing a petrol motor should exert a pure torque, otherwise the motor bearings will be called upon to take loads, and the shaft to resist bending and shear stresses, additional to those for which they were intended.

It is desirable, in the type of dynamometer in which the power is absorbed, that the inertia of the rotating parts should be small in order that a stoppage may readily be effected if failure of the prime mover should occur, as the energy stored in a heavy rotor revolving at a high speed may be sufficient to cause very serious damage.

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[1] Cowley and Levy, *Phil. Mag.*, 1921, xli. 592.

[2] *Ibid.*

In order that accurate measurements may be made, it is essential that the dynamometer should apply the load extremely steadily, otherwise violent oscillations will be set up rendering the employment of heavy dash pots necessary, which is undesirable.

The variation of load with speed for the same setting of the apparatus should be such that "hunting" of the load is eliminated.

(i.) *Absorption Dynamometers.*—In these the energy generated is converted either into heat by the intermediary of solid or fluid friction, or into electrical energy by means of a generator.

The apparatus in most common use is the fluid friction brake, this being simple in construction, easy to regulate over a wide range, and extremely steady in action.

The solid friction brake for large powers requires considerable experience in manipulation in order to avoid unsteadiness due to violent snatching of the load. In its simplest form for small powers, that of the rope brake, it is a very efficient and accurate brake and runs very steadily if proper precautions are taken.

(ii.) *Transmission Dynamometers*, for the measurement of power transmitted by a shaft, exist in three fairly common forms, viz.:

(1) Torsionmeters, in which the angle of twist of a length of the shaft due to torsional stress is measured either optically or by mechanical means.

(2) Torque-meters, in which the torque load is transmitted by springs, or by hydraulic plungers, the extensions of the former and the fluid pressure set up by the latter being indicated or recorded.

(3) Instruments for measuring the power transmitted by means of a belt on the shaft of the prime mover.

The last vary considerably in form but the object achieved is the same in each case, viz. the measurement of the difference in tension of the belt on either side of the driven pulley.

Another important form of the transmission dynamometer is the traction meter for the measurement of the tractive effort of self-propelled vehicles. Dynamometers have been designed for measuring and recording the tractive effort of locomotives, motor cars, and traction engines and the tractive resistance of railway carriages, wagons, wheels, and ploughs.

§ (2) ABSORPTION DYNAMOMETERS. (i.) *The Prony Brake.*—In its original form this brake consisted of two stout beams of wood clamped upon the shaft or on a pulley fixed thereto. The beams were suitably bored in order to grip a large proportion of the circumference of the shaft, and the nuts on the clamping bolts served as a means of adjusting the frictional resistance between the shaft and the brake. A horizontal arm, usually an extension of the upper beam, supported a load pan on which the weights were hung, a fine adjustment being provided by a jockey weight, the position of which could be adjusted along the beam. The movement of the end of the arm was confined within small limits by rigid stops, one above and one below the arm. *Fig.* 1 shows diagrammatically the arrangement.

Very liberal lubrication of the frictional surfaces was necessary to ensure smooth running, but at best considerable vibration of the arm took place due to the variation in frictional resistance between the brake and the shaft, the end of the arm oscillating violently between the stops and rendering a good balance difficult to obtain.

The torque T exerted by the shaft when the brake is balanced is represented by FR where F is the frictional resistance between the brake surfaces, and R the radius of the surfaces; it is measured by the algebraic sum of the moments of the load, jockey weight, and the brake beams about the centre of the shaft.

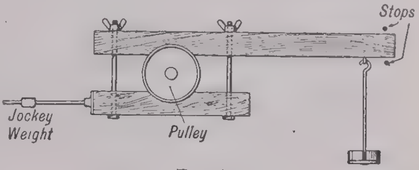

FIG. 1.

In calculating the horse-power developed the factor ϕ is required and is obtained by means of a tachometer or by revolution counter and stop watch.

The rate at which work is absorbed is expressed by

$$\text{Horse-power} = \frac{2\pi TN}{550},$$

where N is the speed in revolutions per second, and T the torque as measured by the brake in pound feet units.

(ii.) *Modifications of the Prony Brake.*—Several improvements and modifications have been made on the original brake as first used by Prony. They consist chiefly in arranging that the grip of the brake blocks shall automatically alter with the value of the coefficient of friction, thus maintaining the total frictional force and therefore the value of T more or less constant.

The Appold brake is shown in *Fig.* 2 and is an example of one of many compensating brakes. The wooden blocks forming the brake are connected together by a steel band provided with a hand adjusting screw at A, the ends of the steel band being connected to the compensating lever at B and C as shown. The end of the lever D is pivoted to the engine frame. The load is supported, as shown, from a hook attached to the steel band, a pointer being provided to indicate when the hook is level with the centre of the shaft.

In the normal position the hook is opposite the pointer and the lever BD is vertical. When the load is lifted, the lever moving about D as centre rotates with the steel band and virtually increases its length, thus slackening the band and partly relieving the load.

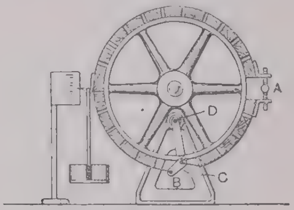

FIG. 2.

If the load falls, due to the coefficient of friction decreasing, the compensating lever shifts round in the opposite direction, tightens the band, and thus increases the grip and lifts the load.

In practice it is found that nearly as much adjustment of the load is required as without the compensating device, but the chief objection to the brake is that when a heavy torque is being measured the lever reacts on the frame with sufficient force to cause an appreciable error in the result obtained if the ordinary method of calculation is adopted.

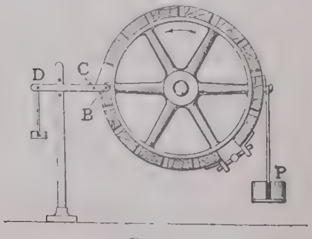

FIG. 3.

A better form of compensating brake, but one perhaps not so well known, is that designed by Mr. Balk and originally used by Messrs. Ransomes Sims and Jeffries. It is shown diagrammatically in *Fig.* 3.

The compensating lever is here outside the disc of the brake wheel, and is connected at B and C to the ends of the strap and the load pan is hung from the point D. Suitable stops are provided for the lever. The weight of the brake adjustment load must be taken into consideration in determining the torque: it must be considered as acting at a radius equal to its horizontal distance from the centre of the shaft, and its moment must be deducted from that of the load P.

Solid friction brakes may be used to absorb greater powers if the wheel is water cooled. The major portion of the energy which is converted into heat is thus quickly disposed of, and one of the chief sources of trouble, namely overheating, is then removed. After a preliminary period of running with water cooling the brake blocks assume a more constant frictional resistance, rendering the brake much steadier to run.

Water cooling of the wheel is very simply effected by making the rim of the brake wheel of trough section, thus enabling it to hold water when revolving by virtue of the centrifugal forces called into action. Water is supplied by a pipe dipping into the trough at the bottom of the wheel, the end of the pipe being turned in the direction of rotation. The outlet pipe is usually flattened at the end which is directed against the stream of water which passes round with the wheel and out through the discharge pipe.

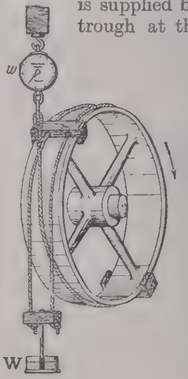

FIG. 4.

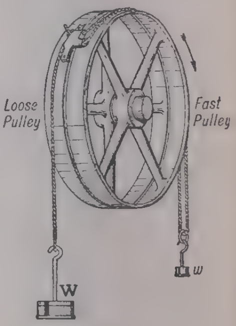

FIG. 5.

The flow is adjusted until a reasonable steady temperature is attained.

(iii.) *Rope Brakes.*—For small powers the rope brakes introduced by the late Lord Kelvin and Professor James Thomson are very successful. The former is illustrated in *Fig.* 4 and the latter in *Fig.* 5.

Lord Kelvin's brake consists of a loop of rope wrapped round the fly wheel of the prime mover, one end of the loop supporting a weight W, the other being held vertically by a spring balance. Wood blocks are secured to the ropes in order to keep them properly spaced to prevent the ropes from rubbing together where they leave the wheel tangentially. The wood blocks are not used as brake blocks.

If the wheel turns in the direction of the arrow the torque is represented by $(W - w)R$, where w is the force indicated by the spring balance and R the radius from the centre of the wheel to the centre of the rope.

When the value of the frictional force changes, the load rises or falls against the action of the spring balance, the brake thus adjusting itself to the new condition.

In using the brake it is advisable to have the ropes and wheel rim either perfectly free from grease, by using new rope and cleaning the wheel rim with petrol, or else thoroughly greased.

If the lubricant is scanty or in patches the weight will hunt, rising and falling regularly, rendering it difficult to obtain a reading of the spring balance with any degree of accuracy.

The brake of *Fig.* 5 consists of fast and loose pulleys mounted side by side on the driving shaft. It is arranged that a rope dipped on the loose pulley hangs down over the fast pulley on one side and over the pulley to which it is fixed on the other.

The frictional resistance between the fast pulley and the rope causes the loose pulley to rotate through a small angle in the direction of rotation of the shaft, and in doing so it lifts a weight suspended by the rope hanging over it. At the same time the rope over the fast pulley, which also supports a weight, is, by the rotation of the loose pulley, slightly unwound from the running pulley, thus reducing its arc of contact, and, therefore, its braking effect. The resistance is thus automatically adjusted.

It should be noted that solid friction brakes, in consequence of the frictional resistance being practically constant over a wide range of speed, cannot be successfully employed for the measurement of power generated by a prime mover such as a petrol motor in which the torque also remains constant over the same range of speed.

In such a case it is extremely difficult to maintain the speed of the prime mover at even an approximately constant value, and the brake needs continuous attention if steady values of the power are to be obtained.

(iv.) *The Froude Water Brake*[1] (*Fig.* 6).—This brake is of the fluid friction type and is very extensively used both commercially and in the laboratory. It consists of a wheel or rotor attached to a driven shaft revolving inside a casing through which water is circulated. The casing is mounted on trunnions which enshroud the shaft at either end but are free from it, and it is free to rotate through a small angle in either direction in bearings offering very little frictional resistance. On both sides of the brake wheel are formed semi-elliptical annular channels divided into 24 pockets by narrow oblique partitions or vanes which are approximately semicircular discs inclined at an angle of 45° to the axis. The vanes on one side of the rotor are inclined at right angles to those on the opposite side, but are also at 45° to the axis. The faces on the inside of the brake casing are formed in the same manner as those of the rotor, the vanes being in the same planes as those on the adjacent rotor face completing the formation of the circular disc inclined at 45°. The number of pockets in the rotor and stator differ by one on each side. The pockets between the rotor and casing thus form complete annular channels of elliptical cross section divided into sections by the vanes. There is only a very small clearance between the faces of the rotor and the casing.

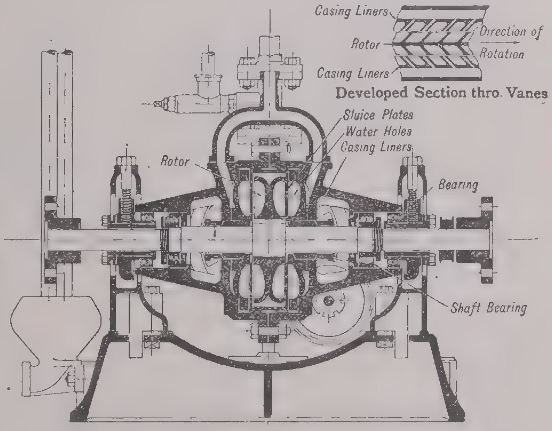

FIG. 6.

The formation of the channels and vanes is shown in detail in the developed section of rotor and casing in *Fig.* 6, the section being made by a cylinder passing through the water holes shown in the principal section.

It will be seen that the vanes are inclined at an angle of 45° to the direction of motion, and that those on opposite sides of the centre line are at right angles to each other.

The difference in pitch of the vanes which may be observed is due to the difference in the number of vanes in the rotor and casing respectively.

If, in the view shown, the rotor be moved until one of its vanes comes into line with one of the vanes of the casing, then the line thus formed is a section across a diameter of an approximately circular disc, one half of which is formed by the rotor and the other half by the casing. The circular disc viewed along the direction of motion has the elliptical appearance shown in the principal sectional view.

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[1] See also "Hydraulics," § (61).

When the shaft is rotated, centrifugal action sets up vortices in the pockets in a plane at 45° to the axis, and the wheel and case are thus urged in opposite directions also at an angle of 45°.

The components of the forces produced are balanced in the direction parallel to the shaft due to the arrangement of the vanes on opposite sides of the centre line, but tangentially they react on the casing. The moment required, therefore, to prevent rotation of the case is equal to the torque on the shaft. The water which is supplied continuously to the brake, when running, passes from pocket to pocket, at the same time rotating at a high rate of speed, finally emerging at a temperature which can be regulated by the supply valve.

The brake is regulated by means of thin sluice gates or plates, inserted between the channels in the rotor and casing. The resistance offered by the brake for a particular setting of the plates varies approximately as the square of the speed, so that the brake is well suited for testing under conditions of constant torque.

The stator case is provided with bearings and packing glands where the shaft passes through it, but the friction of these reacts on the casing and is measured. The case is also provided with a torque arm supported by a spring balance, which in turn is suspended from a hook which can be adjusted in a vertical direction in order that the torque arm shall remain horizontal. The load is hung at the end of the torque arm directly under the spring balance, and the effective torque load is obtained by taking the difference of the load and the spring balance reading.

The effect of the sluice gates is to reduce the effective surface, thus increasing the range of service. The capacity of a dynamometer absorbing 800 horse-power at 800 revolutions a minute can be adjusted to absorb 40 horse-power at 200 revolutions per minute.

(v.) *Electrical Brakes. The National Physical Laboratory Dynamometer.*—In this brake the power developed is absorbed electrically but measured mechanically. The electrical measurement of the power depends in some measure on the accuracy of the instruments employed and on the temperature of the field magnet and armature windings of the generator.

The prime mover under test is coupled direct to the shaft of a generator, the outer frame of which is mounted on bearings offering small frictional resistance, and the torque on this frame is measured. The power generated is carried by very flexible cables to resistance mats by which it is dissipated as heat.

Fig. 7 shows the method of supporting the field magnet frame. The latter is provided with hard steel rings A at either end, these rings being ground truly concentric with the axis of the shaft and securely fixed to the frame.

The rings rest on the sectors B, B, which are also of hard steel ground truly cylindrical on the curved surface with the knife edge as centre. The sectors support the whole weight of the generator, which being thus mounted can oscillate through a small arc with extremely little resistance to motion.

Ball bearings mounted on vertical posts are provided at each end of the generator frame, bearing against the sides of the steel rings to limit the end motion and to take the thrust should the frame inadvertently be moved too far in a longitudinal direction.

The frame is provided with torque arms, one on either side, so that the torque can be measured when the brake is running in either direction.

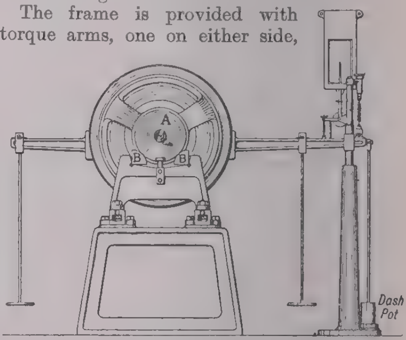

FIG. 7.

The torque arms are balanced, and one is provided in the usual manner with stops to limit the angle of oscillation.

The load is hung from a knife edge on one of the arms, a fine adjustment being obtained by means of a spring which partly relieves the load. It is arranged that extensions of the spring are magnified and indicated on a moving sheet of paper so that the variations in torque of the prime mover under test are recorded. The paper movement is effected by a clock mechanism, and indications of speed are also recorded electrically.

The arrangement of the recording instrument is clearly shown in the figure.

As in the case of the Froude water brake, the bearing friction, and incidentally the brush friction, is measured at the torque arm, so that the method affords an accurate means of measuring power. It has the disadvantages of having a small range of power and speed, and a heavy rotor, but these are somewhat compensated for by the fact that the generator can be used as a motor either for starting an internal combustion engine or for supplying power to a machine whose torque resistance is required to be measured.

It is estimated that, under steady conditions of running, determinations of torque can be made on this dynamometer to an accuracy of 0·1 per cent.

(vi.) *Eddy-current Brake* (*Fig.* 8).—A very successful form of eddy-current brake has been developed by Dr. D. K. Morris and G. A. Lister, and is described fully in a paper to the Birmingham Local Section of the Institution of Electrical Engineers, 1905.

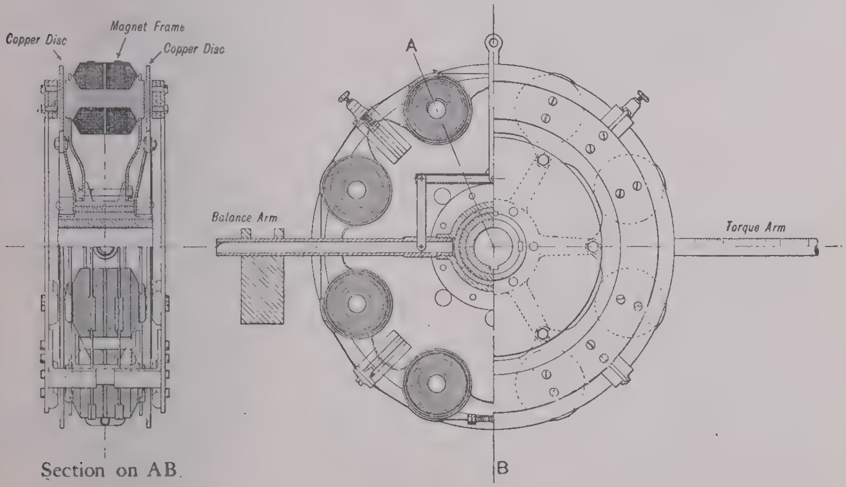

Fig. 8.

The brake was designed to absorb 5 horsepower at 1000 revolutions per minute, and made for use in the Electrical Laboratory of the University of Birmingham.

The apparatus is made for attachment to the shaft of the prime mover in place of the ordinary pulley, and consists of two elements, one in the form of copper discs fixed to and concentric with the shaft, and the other a circular frame also concentric with the shaft and carrying magnetising coils at regular intervals round its periphery.

The magnet frame, or stator, is centred by a revolving bush securely keyed to the shaft and is provided with a torque arm, jockey weight, and stops, in the usual manner.

Two copper discs are provided on the rotor, and are fixed to the revolving bush one on either side of the magnet coils, the axes of which are parallel to the shaft axis, and by this arrangement both electrical symmetry and mechanical balance are assured and the brake is rendered astatic.

The shaft is partly relieved of the weight of the frame by a link suspension attached to an overhead spring balance, and this method of suspension enables the frame to be supported without producing a twisting moment about the shaft centre.

The overhung load on the shaft can be varied as desired by adjusting the tension of the support.

The air gap between the copper discs and the field magnet poles is fixed, and the rigidity of the construction enables it to be made small. External yoke rings are provided and are supported by brackets from the magnet frame, the brackets being clamped in such a manner that the external gaps between the yoke rings and the copper discs can be adjusted.

The limit of the capacity of the brake is determined by the temperature of the magnet coils.

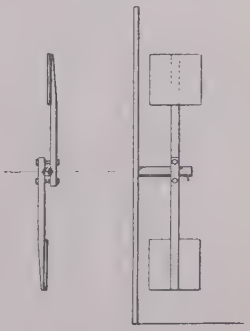

Fig. 9.

(vii.) *Fan Brakes. The Centrifugal Fan* (*Fig.* 9).—The Centrifugal Fan Brake usually consists of two square or rectangular plates held by radial arms in such a manner that they are presented normally to the direction of motion. The brake is more frequently used as a ready means of applying a load to a shaft rotating at a high speed than as a means of measuring torque. It is necessary that it should be calibrated by means of an accurate dynamometer, but the calibration is affected by variations in temperature and pressure of the surrounding atmosphere, cross-currents of air, and the proximity of supports, adjacent walls and floor, and of the prime mover under test.

The load is adjusted either by moving the plates along the radial arms or by fitting plates of a different size, these methods of adjustment having the great disadvantage of necessitating the stoppage of the prime mover under test.

Attempts to render the blades adjustable during rotation have met with little success, owing chiefly to the difficulty of designing the adjusting mechanism to operate against the centrifugal forces acting on the blades.

The torque due to the blades alone, for ordinary speeds and sizes, varies as the square of the speed, the cube of the radius of the blades, the area of the blades, the density of the fluid in which the fan is working, and on a factor depending on the environment.

The fan brake is cheap to manufacture and is usually easy to fit to the shaft of the prime mover. For these reasons it is frequently used commercially as a brake for "running in" and "endurance" tests where the power absorbed is not required to be known with accuracy. It may be used for comparative tests provided the atmospheric conditions and the environment remain constant throughout the tests, but neglect of these factors may effect an error of 10 to 20 per cent in the measured torque.

(viii.) *The "Escargot" Fan Brake for testing Aeronautical Engines.*—In order to reproduce as far as possible the cooling effect produced on an aeronautical engine under flying conditions, it is essential that the engine under test shall work in a current of air projected on it at a speed comparable with the flying speed.

The cooling is effected in the escargot arrangement by a fan brake driven by the engine under test and mounted in a chamber resembling that of an ordinary fan casing.

The intake of the air is in the side of the casing opposite the fan centre.

Rotary or radial engines are usually mounted inside the escargot, *Fig.* 10, being then in the direct path of the incoming current of air.

Vee or vertical engines are fixed on a bed outside the fan chamber, and the air from the discharge orifice is directed on to the engine either from above or from the front.

In the latter arrangement the fan is driven through an intermediate shaft and universal couplings.

For "running in" and "endurance" tests the brake may be calibrated by means of an electric motor, but is open to the objections which have already been summarised under the heading of the "Fan Brake." For tests of greater accuracy it is usual to mount the engine on a floating torque-balance gear by which the reactive torque on the engine may be measured.

When the engine is so mounted it is essential, if accurate results are to be obtained, that the axis of the rocking frame shall coincide with the fan axis, otherwise the moment measured will differ considerably from the true torque according to the position of the fan in the escargot.

§ (3) Transmission Dynamometers. *Torsionmeters.*—In cases where it is desired to keep a check on the power developed by a prime mover, or where the absorption of the power cannot conveniently be effected, a transmission dynamometer is employed. This instrument exists in several forms of torsionmeters, the function of which is to measure the angle of twist in a length of the shaft driven by the prime mover, and from this measurement, by previous calibration or by calculation, the torque is deduced.

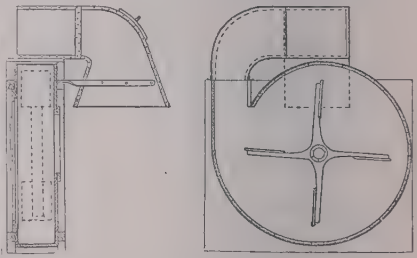

Fig. 10.

(i.) *Hopkinson-Thring Torsionmeter* (*Fig.* 11). —This instrument was designed by Professor Bertram Hopkinson and Mr. Thring for the determination of the power transmitted by turbine shafts, and is used to a very large extent on steamships. The principle of the apparatus is the measurement of the relative angle of twist between adjacent sections of the shaft and the indication of this movement on a fixed scale. The length of the shaft taken up by the instrument is very small, being, for sizes over 8 inches diameter, about three diameters long, and for smaller shafts about four diameters.

The instrument consists essentially of two flanges clipped to the shaft at a measured distance apart. A sleeve enclosing the shaft connects the two flanges, being formed solid with the one but free to rotate through a small angle on the other, by which it is retained concentric with the shaft. The twist of the latter causes relative angular motion between the free end of the sleeve and the flange adjacent to it, and this motion is indicated on a fixed scale by means of a mirror.

The mirror is mounted on trunnions working in bearings fixed to the flange, the axis of the trunnions being radial. The mirror is provided with a short radius arm which is connected to an adjustable block on the sleeve by a thin phosphor-bronze link. The tangential movement of the sleeve block is governed by a micrometer screw in order that the relation between the movement at the radius of the mirror from the centre of the shaft and the spot of light on the scale may be ascertained.

A fixed mirror is attached to one of the flanges, which is adjusted so that the spot of light reflected from it is received at the same point on the scale as that from the movable mirror when there is no torque on the shaft.

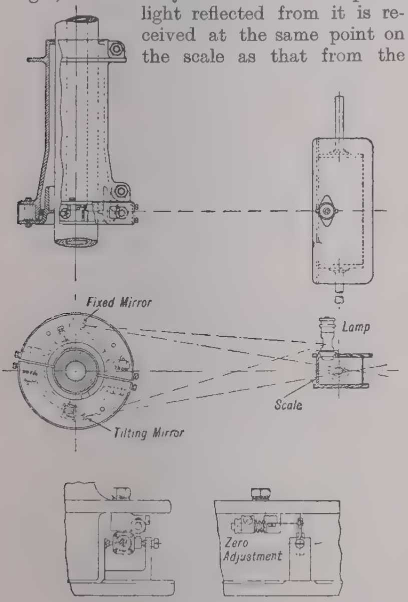

FIG. 11.

An electric lamp fixed just above the scale provides the beam of light which is reflected from the two mirrors on the scale successively as the shaft rotates.

A shift of the apparatus relative to the scale is indicated by an alteration of the position of the beam reflected by the fixed or "zero" mirror, and this can readily be adjusted by moving the scale.

The moving mirror is silvered both back and front so that two reflections, one on either side of the zero, are received from it during one revolution of the shaft. The mean of the two readings may thus be taken whatever the position of the zero.

The instrument may be calibrated directly by applying a known torsion to the shaft and noting the reading of the scale, or by calculation from a knowledge of the torsional rigidity of the shaft and the various leverages and distances involved in the conversion of the twist to the movement of the spot of light on the scale.

The apparatus gives an instantaneous reading of the torsion in the shaft at a particular angular position in the revolution, the angle being fixed by the positions of the mirrors and scale.

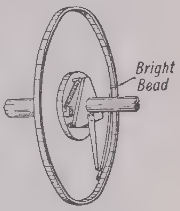

FIG. 12.

The torsion at any other part of the revolution may be obtained by mounting the scale on a circular frame concentric with the shaft and shifting the scale and lamp to the angular position required.

(ii.) *The Ayrton and Perry Torsionmeter* (*Fig.* 12).—This dynamometer is designed for use in the place of the ordinary shaft coupling, which purpose it also serves. The two halves, one on each shaft, are connected by means of helical springs, three or four in number, arranged in a position approximately tangential to the shaft. When power is transmitted, the drive is taken through the springs which extend under the load. The stretch of the springs allows relative angular movement between the shafts, and this is indicated by a lever arrangement which has the ultimate effect of reducing the distance of a bright bead from the shaft centre. The radial position of the bead is observed against a black disc on which a scale is marked.

The apparatus is calibrated directly or by calculation from the stiffness of the springs and the magnification of their extension which has been employed. Calibration against an absorption dynamometer is preferable if it is thought that the arrangement of the springs adopted is likely to give different calibrations statically and dynamically.

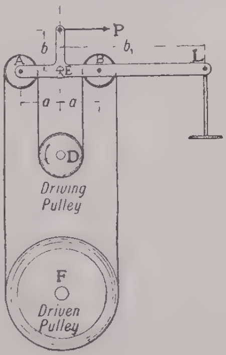

FIG. 13.

§ (4) THORNYCROFT OR FROUDE BELT DYNAMOMETER.—The arrangement of this transmission dynamometer is shown in *Fig.* 13. The pulley D, fixed to the shaft of the prime mover, drives the pulley F by means of a belt which passes in turn round the two pulleys A and B. The latter are supported by a frame ABL which is pivoted at E. It is obvious that the downward force on the pulley A will exceed that on the pulley B by twice the difference in tensions of the tight and slack sides of the belt on either the driving or driven pulley. The frame ABL will therefore tilt downwards at A, but its symmetrical position is restored either by a force at P, or by adding weights to a scale-pan hung on an extension of the arm AB at L. The motion of the end of the lever is confined within small limits by means of stops.

The belt tension difference is expressed by $T = Pb/2a$ and the horse-power developed by $H.P. = 2\pi TRN/550$, where R and N are the radius in feet and speed in revolutions per second respectively of the driven pulley.

The power is not transmitted entirely without loss because of the slip of the belt and the pulley bearing friction. These losses, however, can be determined by independent dynamometer tests.

§ (5) Dynamometer Cars and Traction Dynamometers.—Many instruments have been designed to measure the resistance of vehicles to traction and also the tractive effort of self-propelled vehicles. The pull is usually transmitted through either a small plunger or diaphragm, in which case the pressure set up in a liquid column is recorded by means of a pressure gauge, or through helical or laminated springs the deflection of which is indicated on a scale.

(i.) *The British Association Apparatus.*—Some of the earliest tests in this country were carried out with a special dynamometer, designed for the British Association, for the measurement of the resistance of road vehicles to traction.[1]

The apparatus comprised a castor frame in which could be mounted a single wheel on which the experiments were to be made, a system of levers for transmitting to a small plunger the pull exerted on the wheel, a recording pressure gauge for registering the pull, and a recording speedometer.

The castor frame was of wrought iron and rectangular in shape, and was adapted to take wheels of various widths. The axle of the experimental wheel was mounted on springs, one under each side of the frame. The springs used were of the ordinary laminated carriage type constructed in such a manner that their stiffness could be adjusted by varying the number of plates forming a spring. The frame was loaded by cast-iron weights fitting over the castor frame.

An attachment was made to the levers transmitting the pull by a swivel joint which allowed the frame to oscillate vertically and to follow freely round a curve without affecting the records, but so held it that the wheel always remained vertical. The fulcrum of the lever could be moved to either of four positions such that the pressure on the plunger was equal to, or two, four, or eight times the pull exerted on the frame. The range of the apparatus could thus be varied from 5 to 500 lbs.

The record of pressure and speed was made by pencils on a sheet of paper which was rotated by a roller the motion of which was obtained through gearing from the road wheel.

(ii.) *The Hyatt Instrument.*—This dynamometer has been specially designed for the determination of the draw-bar pull of farm tractors and the tractive resistance of ploughs working under various conditions.

The pull is transmitted from the tractor to the plough through a link or chain and a piston working in a cylinder. The latter contains a rubber bag containing liquid which is compressed by the piston. This pressure unit is hitched in one of three different ways giving different ratios between the pull and the pressure on the rubber bag. The three hitches correspond to maximum pulls of 300, 600, and 1200 lbs. respectively.

The recorder consists of a Bristol recording pressure gauge. It is connected to the pressure unit by a flexible metallic tube which conveys the liquid under pressure from the rubber bag to a Bourdon pressure tube which actuates the pen mechanism. A needle valve, inserted in the pressure tube, may be adjusted to control the flow of liquid and to damp out excessive vibrations of the pen.

The chart is a circular sheet of paper divided by a series of concentric circles across which the pen travels in an approximately radial direction from the centre outwards with increasing pull.

The chart is caused to rotate by a wheel which rolls over the ground and which is connected to the recording instrument by a flexible shaft and suitable gearing. The circumference of the chart is divided into equal parts each representing a distance travelled of 100 feet.

A second pen records the elapsed time on an annular space at the edge of the chart. A clock in the recorder case, fitted with a cam, trips the time pen at one-minute intervals, and fractions of a minute may be estimated by counting the number of smaller spaces over which the pen has travelled. The division is accomplished by dividing the annular space by a number of concentric circles and by causing the time pen to travel from the outer to the inner ring in the minute interval, after which it is again tripped back to the outer ring, its path being practically a radial line.

The recorder is lightly built and is provided with straps by which it is carried by an assistant during a test. It is arranged that the rolling wheel may either be clipped to the tractor or guided by hand.

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[1] British Association Report, 1902.

From the chart obtained, the draw-bar pull, the distance travelled every six seconds, and the speed during the test may be deduced.

(iii.) *The National Physical Laboratory Tractor Dynamometer Car.*—This apparatus was constructed solely for measuring and recording the tractive effort of farm tractors, for which purpose it has been successfully used. The car employed was a heavy four-wheeled lorry trailer, the rear wheels of which were fitted with speeds and band brakes, the latter being operated by a hand wheel from the front of the vehicle. The apparatus was mounted on an extension of the bogie frame carrying the wheels of the trailer.

The pull of the tractor was converted to pressure on a leather diaphragm by means of a bell-crank lever suitably pivoted, the diaphragm being faced on the pressure side with a rubber disc to prevent leakage of the liquid in the diaphragm box through the leather. A flexible hydraulic pipe connected the diaphragm casing to a Bourdon pressure tube operating a pen mechanism, and a record of the pressure was thus obtained on a roll of paper. The latter was caused to move proportionally to the distance travelled by the car, the driving drum being rotated by means of gearing and a flexible shaft from the leading road wheels.

Two other pens were also provided, one operated by a clock in order to give time indications every two seconds, and the other operated through gearing and giving indications of the revolutions of the tractor engine.

The diaphragm box, to which was attached the fulcrum of the bell-crank lever, could be adjusted vertically so that the rope or chain connection to the tractor could be arranged horizontally.

In carrying out a test, the brakes of the car were adjusted, and the car loaded with dead weights, until either the engine of the tractor was pulled up or the driving wheels began to slip. Thus the maximum pull of the tractor on the particular surface chosen for the tests was obtained.

(iv.) *Railway Dynamometer Cars.*—Dynamometer cars have been constructed for traction tests on locomotive engines and tractive resistance tests on railway rolling stock. The car usually takes the form of a special coach, the draw-bar pull and buffer thrust being recorded, with the speed, on a paper-covered drum driven through gearing from one of the road wheels.

A most successful car of this type is that designed by Mr. George Hughes for the Lancashire and Yorkshire Railway Company. The various instruments with which this car is fitted enable complete records of the performance of the locomotive to be taken, including draw-bar pull (or push) speed, acceleration or retardation, and boiler pressure of the locomotive.

Other factors influencing the performance are also indicated on the chart by observers.

Considerable care and experience is required in the design of a car of this kind, and the more salient features of the Lancashire and Yorkshire Railway car are here described for the first time by the courtesy of the Chief Engineer of that Company.

(v.) *The Lancashire and Yorkshire Railway Company's Dynamometer Car.*—The draw-bar pull and the thrust on the buffers of the car are transmitted to two compound springs, one being provided for the pull and the other for the push. Each is made up of a number of independent plates bound together by a pair of buckles at the centre and connected by links at the ends. The link pins can be withdrawn separately as desired, and this enables the number of plates in operation to be adjusted to suit the load hauled, from a single coach to the heaviest train, utilising the full deflection of the springs.

The spring deflection is recorded directly on the record paper without the intervention of levers, showing draw-bar pull above, and buffer thrust below, a datum line.

The external carriage coupling arrangements are standard practice so that the car can be coupled up as readily as any other vehicle.

The draw-bar and buffers are connected to a rigid frame which moves each set of springs in one direction only so that the tightness of the screw shackle between the engine and the car does not affect their indications.

A compensating beam equalises the thrust on the buffers when negotiating a curve.

The draw-bar passes through a friction device which can be brought into operation to take the load off the springs when tests are not being made. The device is also used to absorb exceptional shocks during the last inch of deflection of the springs. A main cross-beam near the centre of the car under-frame forms a foundation for the springs and the instrument table.

The motion of the car is not recorded from the ordinary road wheels but from a special measuring wheel arranged between the leading and trailing wheels of the bogie, so that there is little lateral movement when rounding curves. The measuring wheel is pressed on the rail by a spring and can be raised or lowered as required. The motion is transmitted to the instrument table by worm gearing and flexible couplings forming a positive drive.

The instrument table carries the record paper drums, the speed gear which regulates the relative speeds of the paper and the train, a distance indicator, a work integrator, a recording and a visual speedometer, a standard electrical clock, and an inertia ergometer.

The record paper can be driven at speeds varying from six inches to twenty feet per mile travelled; a counter marks the paper every mile.

The work integrator consists of a disc rotated from the road wheel, and across the face of the disc a roller is moved by the extensions of the springs. The rotation of the roller is therefore proportional to the product of the draw-bar pull and the distance travelled, and this work is continuously recorded on the chart as a serrated diagram, each peak representing approximately 50 horse-power minutes per pair of springs in use. A counter gives the total positive, total negative, or algebraic sum of the horse-power minutes done by the draw-bar, as required.

The speed recorder works on a positive principle, namely, that the distance travelled in a definite time (4 secs.) gives a measure of the mean speed during the four seconds.

(vi.) *The Inertia Ergometer.*—This was introduced by Joseph Doyen, Chief Engineer of the Belgian State Railways, and is a combination of the Desdouits Inertia dynamometer (pendulum) and the Adbank-Abakanowicz integrating roller. The pendulum is mounted on knife edges and swings in the direction of motion of the car. The tangent of the angle of the pendulum with its neutral position is proportional to the algebraic sum of the force of gravity and the force producing change of velocity (acceleration or retardation). The integrating roller multiplies this force by distance and the result is indicated on the record paper as an inclined line. The change in the ordinate of this line represents the change in the algebraic sum of the potential and kinetic energy per unit mass of the train. When coasting free, the loss of energy recorded is due to train resistance, and is therefore a direct indication of it.

By means of a run made up of alternate haulings and coastings the ergometer provides data from which the work done by the prime mover can be calculated. By combining this information with that of the draw-bar integrator the locomotive resistance, train resistance, total resistance, and accelerative effect of the locomotive can be obtained for any speed.

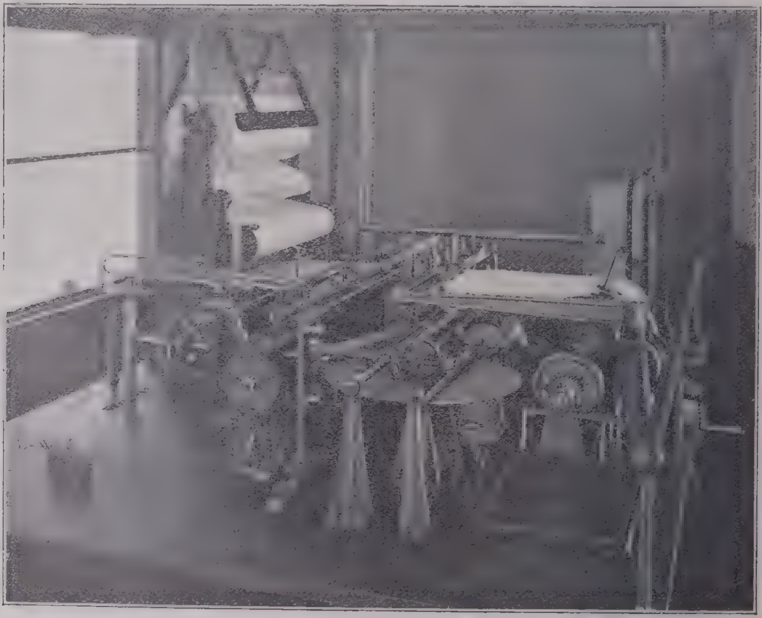


Fig. 14.

Curvature of the track is indicated and has been found to account for many peculiarities in the diagrams obtained. *Fig.* 14 is a general view of the instrument table. The work integrator is shown in the centre foreground; the lever from the draw-bar springs stands up through the central slot in the table and the pendulum lever through the slot on the left. The ergometer drum can be seen to the left of the top of the draw-bar lever. The clock is removed to expose the ergometer to view.

Fig. 15 is a specimen, reduced in size, of the chart obtained for a run of approximately six miles.

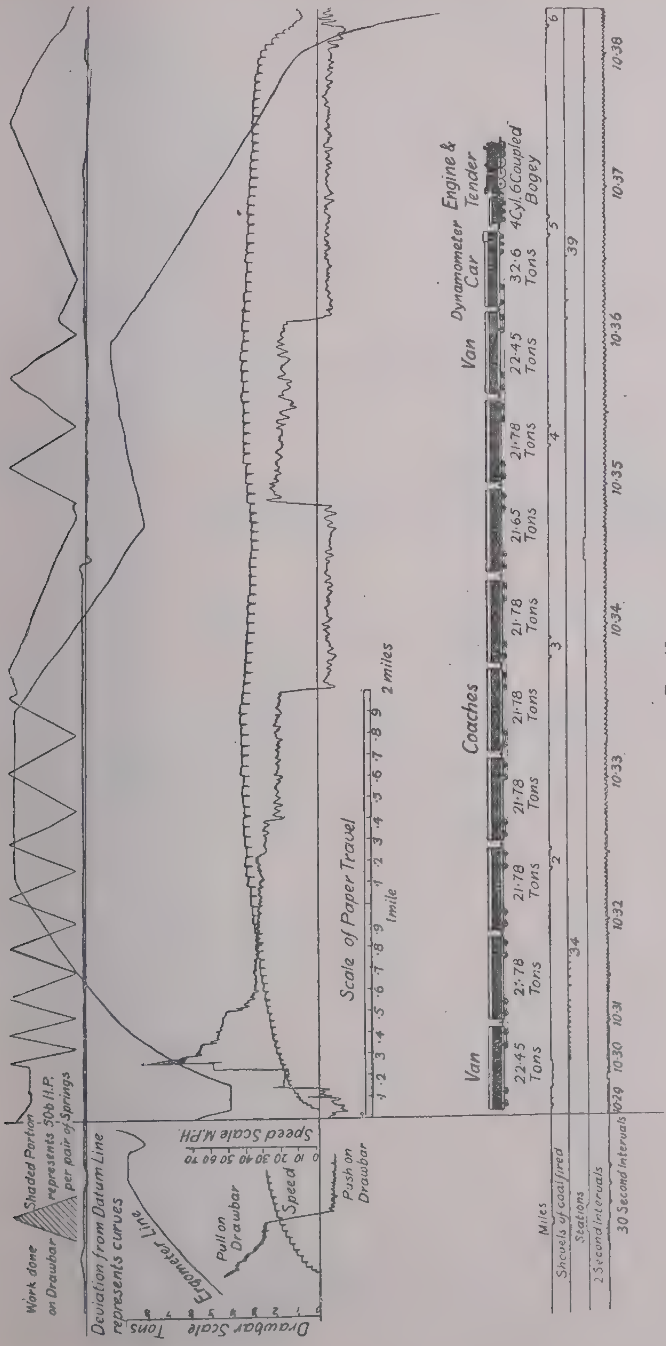

Work done on Drawbar
Shaded Portion represents 50 b H.P. per pair of Springs
Deviation from Datum Line represents curves
Tons
Drawbar Scale
1 2 3 4 5 6 7 8
Ergometer Line
Pull on Drawbar
Speed
Speed Scale M.P.H.
0 10 20 30 40 50 60 70
Push on Drawbar
Scale of Paper Travel
·1 ·2 ·3 ·4 ·5 ·6 ·7 ·8 ·9 1 mile ·1 ·2 ·3 ·4 ·5 ·6 ·7 ·8 ·9 2 miles
Van
22·45 Tons
2:78 Tons
21·78 Tons
21·78 Tons
21·78 Tons
21·78 Tons
21·65 Tons
21·78 Tons
Coaches
Van
22·45 Tons
Dynamometer Car
32·6 Tons
Engine & Tender
4Cyl. 6Coupled Bogey
Miles
Shovels of coal fired
Stations
2 Second Intervals
30 Second Intervals
2
3
4
5
6
34
39
10·29
10·30
10·31
10·32
10·33
10·34
10·35
10·36
10·37
10·38

FIG. 15.

(vii.) *The Swiss State Railway Car.*—Much of the apparatus fitted in this dynamometer car is in many respects similar to that of the Lancashire and Yorkshire Railway Company's instrument. The principal difference between the two cars lies in the method of measuring the tractive effort, which in the present instance is by the use of hydraulic gear. Undoubtedly the hydraulic dynamometer has distinct advantages over the spring type, provided it is carefully designed and accurately made.

The pull of the draw-bar is transmitted to a steel plunger working in a steel cylinder, both elements being ground so accurately as to render the use of packing unnecessary. The cylinder is filled with oil by which the pressure is transmitted to the recording instrument, and it is arranged that there is a very slow leak of oil between the plunger and the cylinder wall in order to provide lubrication for the former and to minimise its frictional resistance.

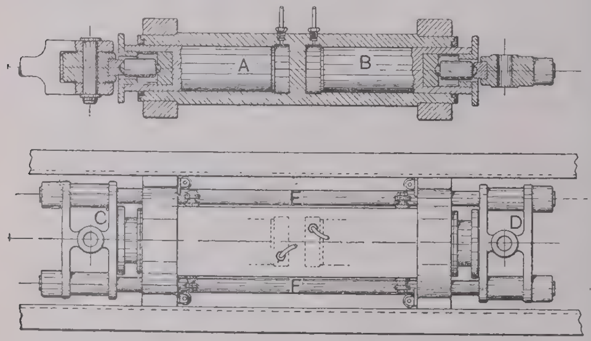


Fig. 16.

The arrangement of the pressure unit is shown in *Fig.* 16. There are two plungers, indicated at A and B, fitting into cylinders which are placed back to back and formed in a solid forging. The draw-bar pull is transmitted to one of the plungers, and the buffer thrust to the other, by means of the crossheads C and D and the rods E and F. The latter are guided by rollers suitably supported by a rigid frame. The clearance spaces in the cylinders, behind the plungers, are connected by pipes to a distributing valve, and from thence to the recording mechanism.

The recorder consists of two small measuring cylinders arranged in tandem and provided with differential plungers opposed to each other. This arrangement provides that by a suitable setting of the distributor valve the resultant area of the plungers exposed to the oil pressure from the main cylinders may be varied in the ratio of 1 : 2 : 3 : thus by rotation of the distributing valve it is possible to select either of three scales for recording the load.

A calibrated helical spring is fitted between the two differential plungers and is compressed by their motion. The compression of the spring is a measure of the resultant end force on the plungers and, therefore, of the difference between the draw-bar pull and buffer thrust, and this motion is transferred directly to the record paper by a rod carrying a style.

During prolonged runs the slow leakage past the main plungers may cause one of them to touch the base of its cylinder, but before such a condition can arise an equalising valve is brought into operation which has the effect of equalising the pressures in the two cylinders and restoring their central positions as shown in the figures. Fresh oil is drawn into the cylinders from a reservoir, through a non-return valve, when required.

§ (6) SPECIAL DYNAMOMETERS. (i.) *The Daimler Lanchester Worm-gear Testing Machine.*—This machine was designed by Mr. F. W. Lanchester for the accurate measurement of the efficiency of worm-gearing. A diagrammatic sketch of the apparatus is given in *Fig.* 17.

The efficiency of modern worm gears for power transmission is of the order of 95 per cent, and, under the best conditions of loading and lubrication, as high as 97 per cent. With such efficiencies, separate determinations of the torques in the worm and worm-wheel shafts by any of the methods already described would be liable to give misleading results unless the accuracy of the measurement in each case was within a small fraction of 1 per cent of the true value. For example, in the case of a gear of 97 per cent efficiency, if the torques could be measured separately to an accuracy of 0·2 per cent, the experimental determinations of the efficiency might range from 96·6 to 97·4 per cent. It is evident, therefore, that the ordinary methods of torque measurement in which the error may be as much as 2 per cent are quite useless for the purpose, and that a method of much greater accuracy is required. The Lanchester machine measures, in effect, the ratio of the two torques, and the over-all efficiency of a worm gear with its bearings can be obtained within an accuracy of 0·2 per cent. Indeed, differences of efficiency due to such causes as change in the temperature of the lubricant can be easily detected.

Referring to *Fig.* 17, the gear box is supported in a cradle A in such a manner that it has freedom of motion through a small angle about two axes at right angles. The worm is driven by a shaft B through the intermediate shaft C, the latter being provided with universal couplings at each end. In the same manner the wheel shaft is connected to the bevel box E through the shaft D and the universal couplings FF. The load is supported by a bracket K fixed to the arm G, the axis of the arm being parallel to, and in the same vertical plane as, the worm-shaft axis.

The load is not fixed directly to the bracket but is carried by a slider from which it is hung: by a screw and nut device the distance of the load from the axis of the arm can be varied.

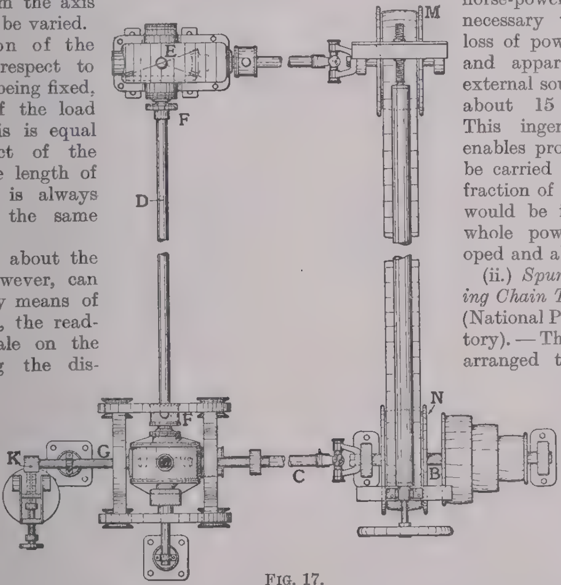


FIG. 17.

The position of the bracket with respect to the wheel axis being fixed, the moment of the load about that axis is equal to the product of the weight and the length of the arm, and is always the same for the same load.

The moment about the worm axis, however, can be adjusted by means of the screw gear, the reading of the scale on the bracket giving the distance of the weight from the axis.

It will be seen, therefore, that assuming the gear efficiency as 100 per cent, and the gear ratio as R, the speed of the worm being R times that of the wheel, the torque on the worm shaft will be 1/R times the torque on the wheel shaft. The load being the same for each torque, the distance of the weight from the worm axis will be 1/R times its distance from the wheel axis.

The efficiency of the gear being less than 100 per cent, the load must be moved farther from the axis of the worm in order to balance the gear box against the two torques when running under load.

The calculated distance of the weight from the worm axis, assuming 100 per cent efficiency, divided by the distance required to produce a balance, gives the efficiency of the gear.

The drive from the worm wheel is taken through the bevel box to the belt pulley M, the latter being of such a diameter that it tends to drive the pulley N on the driving shaft B at about 5 per cent higher speed than is established by the bevel and worm gears.

In other words, it is arranged that the peripheral speed of the pulley M is 5 per cent higher than that of the pulley N.

The belt connecting the two pulleys can be adjusted in tension, over a wide range, an increase in the tension producing an increase in the torque. By this means the pressure between the teeth of the worm and wheel can be made to correspond with the transmission through the box of as much as 150 horse-power, it being only necessary to supply the loss of power in the gear and apparatus from an external source developing about 15 horse-power. This ingenious principle enables prolonged tests to be carried out at a small fraction of the cost which would be involved if the whole power were developed and absorbed.

(ii.) *Spur Gear and Driving Chain Testing Machine* (National Physical Laboratory).—This machine is arranged to measure the difference of the torques of two shafts, together with the measurement of one of the torques separately.

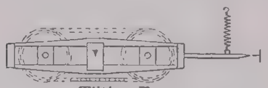
It will be evident that the high degree of accuracy required in the Lanchester machine is not essential in the present machine.

Thus, supposing the true efficiency of a spur gear or chain drive is 97 per cent, and that the determinations of the torque and torque difference are each within 2·5 per cent of their true values, the estimated value of the efficiency of the transmission from the observations may range from 96·9 per cent to 97·1 per cent, which is of a higher degree of accuracy than that obtained by measuring the torque ratio to ± 0·2 per cent.

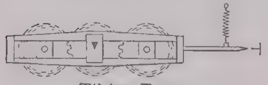
Fig. 18 shows the arrangement of the machine, The frame carrying the gears is entirely supported by horizontal knife edges, and both driving and driven shafts are considerably extended and provided with ball-bearing universal joints at each end. Being supported in this manner, the frame can execute small oscillations in a vertical plane about its neutral position.

The intermediate wheel is rendered necessary in order to secure that the reaction on the frame should be equal to the difference between the torques of the driving and driven shafts, and for this to be the case it is requisite that the shafts should rotate in the same direction.

The first and intermediate gears and the intermediate and the third gears form two pairs under test.



Tilting Frame
Arranged for Chain Drive



Tilting Frame
Arranged for Spur Gears

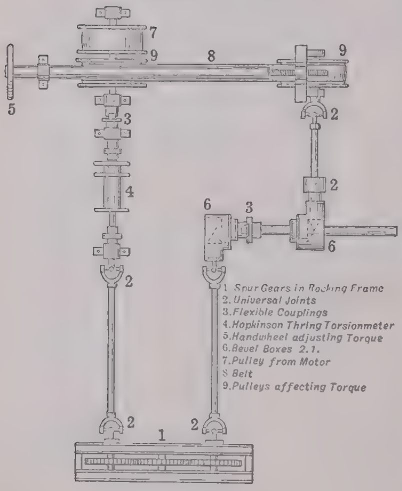

7
9
8
9
5
3
2
2
4
6
3
6
2
2
2
1
1 Spur Gears in Rocking Frame
2. Universal Joints
3. Flexible Couplings
4. Hopkinson Thring Torsionmeter
5. Handwheel adjusting Torque
6. Bevel Boxes 2.1.
7. Pulley from Motor
8 Belt
9. Pulleys affecting Torque

Fig. 18.

When a chain drive is under test no intermediate gear is necessary for the shafts to rotate in the same direction.

If no intermediate wheel were used in the gear test, the reaction on the frame would be equal to the sum of the torques of the two shafts, and the object which it has been attempted to achieve would be defeated.

The frame is provided with an oil dash-pot for damping its oscillations and a torque-arm scale-pan and spring for the torque difference measurement. Rigid stops are provided for the torque arm to limit its amplitude, readings being taken when the arm is level.

A Hopkinson-Thring torsionmeter is used to determine the torque on the driving shaft, and in carrying out a test an observer regulates the torque and maintains it at the desired value, whilst a second observer is employed with the torque measurement on the tilting frame.

The method of increasing the pressure between the gear teeth is on the same principle as that adopted in the Lanchester machine. This secures that only the loss of power needs to be supplied from an external source. The centre distance of the two shafts is made adjustable over a wide range to accommodate varying sizes of gears and lengths of chain drive.

During testing the tilting frame is totally enclosed in a light case and provision is made to spray the lubricant on the gears, the oil being supplied by means of a flexible pipe in such a manner that no control on the frame is exerted.

In testing spur gears the value of the torque T_2 on the last gear can readily be obtained from the observed data. The efficiency of a pair of gears is given, therefore, by $\eta = \sqrt{T_2/T_1}$, where T_1 is the measured torque on the driving shaft.

The efficiency of a chain is given by $\eta = T_2/T_1$.

(iii.) *Spur Gear-box Testing Machine* (National Physical Laboratory), *Fig.* 19.—In this machine the gear box is mounted in such a manner that it is free to oscillate about a mean position on the axis of the driving shaft, and the torque on the frame of the gear box is balanced and measured. The method can only be used in cases where the driving and driven shafts of the unit are coaxial and leave the box at opposite ends: this is usually the form of gear boxes for automobiles for which the apparatus was originally designed.

The unit under test is fixed in a rectangular frame provided with hollow trunnions coaxially with the main shafts. The trunnions are supported in ball-bearings so that the frame and gear box can tilt in either direction, the frame being fitted with a torque arm provided with the usual dash-pot, measuring spring, balance weight-pan, and stops to limit its motion.

The two shafts of the gear box are connected to a prime mover and an absorption dynamometer respectively, or the prime mover may be fitted as a dynamometer, in which case a steadily running brake only is required at the transmission end.

The method of test is very much more accurate than that of measuring the "input" and "output" torques, for in the latter arrangement, when the efficiency of the gear box is of the order of 97 per cent, no exact results can be obtained, because the errors in estimating the two torques directly affect the calculated efficiency.

Consideration of the two cases will show that in the latter, assuming the accuracy of both torque readings as ±1 per cent, the efficiency obtained will vary from 95 to 99 per cent if the actual efficiency is 97 per cent.

On the other hand, using the more accurate method, and assuming the accuracy of the dynamometer as ±1 per cent, as before, and that of the torque measurement on the gear-box frame as ± 2 per cent, the efficiency determination will vary between 96·9 and 97·1 per cent, *i.e.* a variation of only one-twentieth of the first method. In practice the accuracy of the measurement of the torque on the rectangular frame can be determined to about 3 per cent.

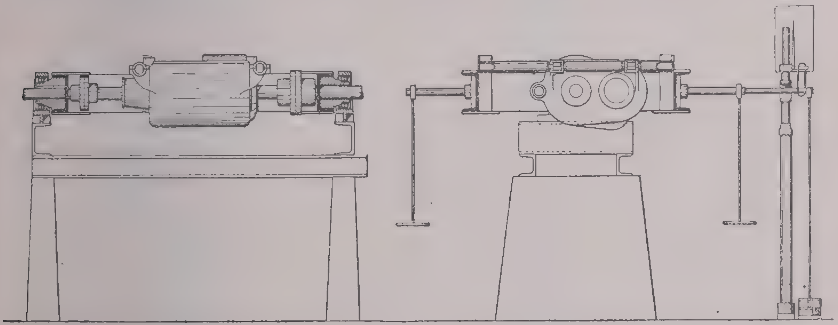


FIG. 19

The apparatus has been used to determine efficiencies under different conditions of lubrication, and differences due to temperature and quality of lubricant have been observed.

§ (7) AERIAL PROPELLER DYNAMOMETERS.—In order to accurately determine the performance and efficiency of airscrews, two dynamometers have been designed and made in the Aeronautics Department of the National Physical Laboratory.

The problem of the design of such an instrument involves the determination of the air speed relative to the propeller, the torque and the thrust, observations of each being made at the same instant of time.

In the two widely different forms described below experiments were made on scale model airscrews about one-sixth full size.

§ (8) WHIRLING ARM DYNAMOMETER.—A sectional drawing of the dynamometer is given in *Fig.* 20. The airscrew shaft is driven through flat coiled springs S, the extensions of the springs, proportional to the torque transmitted, being indicated by the relative displacement of a pointer over a drum which carries a slip of paper. The position of the pointer at any time during an experiment is recorded by passing a series of high-tension electric sparks through the paper so that the magnitude of the torque may be directly calculated from the known calibration of the springs. A pencil or pen could not be used for the indicator because of the difficulty of balancing the centrifugal forces called into play by the high speed of rotation and also the friction of the pen or pencil on the drum. Alternate torque springs are wound in opposite directions so that the forces produced by the tendency to unwind due to centrifugal action counterbalance each other; with this arrangement it was found that the static and dynamic torque calibrations agreed with each other. The airscrew shaft is allowed a small axial movement, and it is so supported that this movement takes place with extremely little friction. The end of the shaft bears against an oscillating lever, being directly controlled by the thrust spring *b*, the tension of which is adjusted by means of the micrometer screw *f* and two adjustable stops, both the latter being insulated electrically from the frame of the instrument.

When the thrust of the airscrew balances the pull of the spring the lever floats between the stops, but when the thrust is too great or too small contact is made with one or other of the stops and an indication is thus given by means of a galvanometer.

Oscillations in the torque springs due to small irregularities in the driving torque are damped by the oil dash-pot K. This consists of a series of concentric discs, alternate discs being attached to the sleeve carrying the inner ends of the springs and to the drum to which the outer ends are fixed. The oscillations are damped by the fluid friction between the discs.

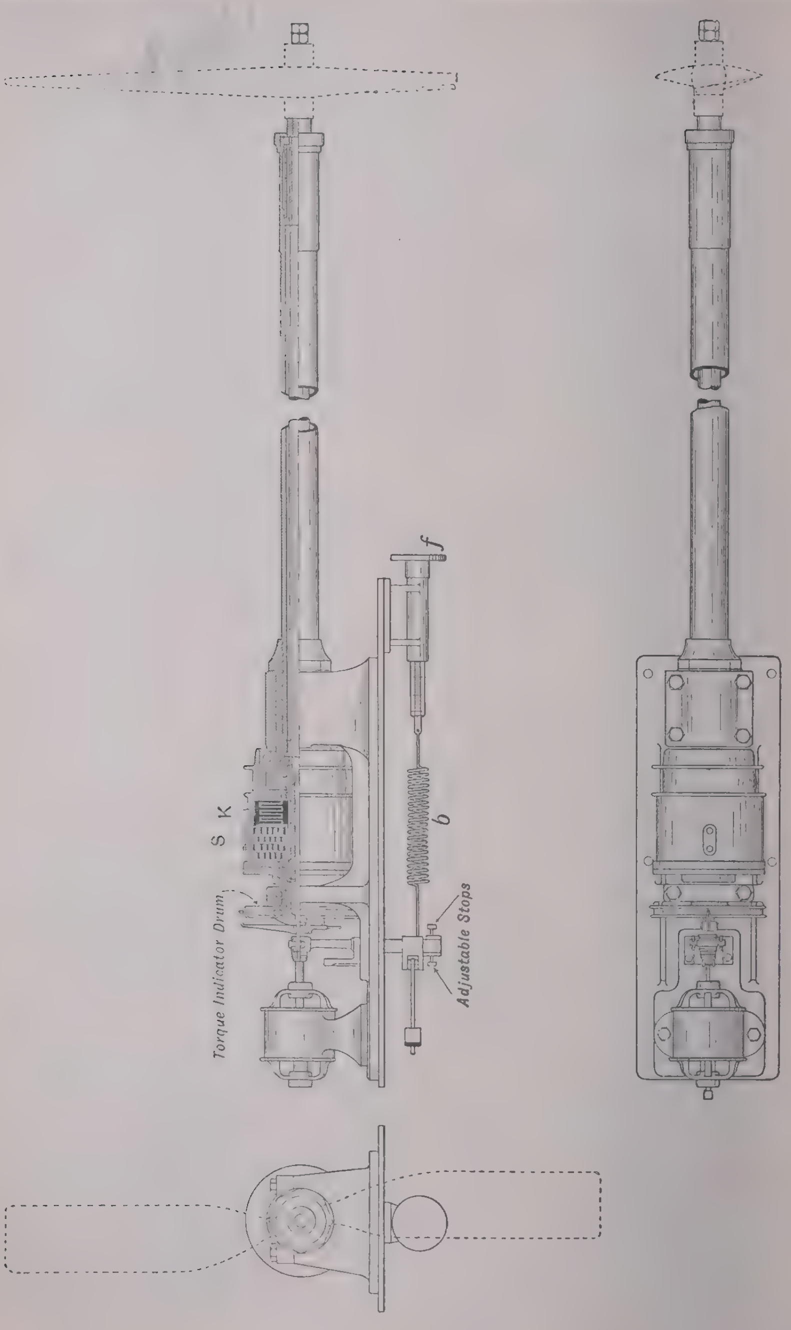


FIG. 20.

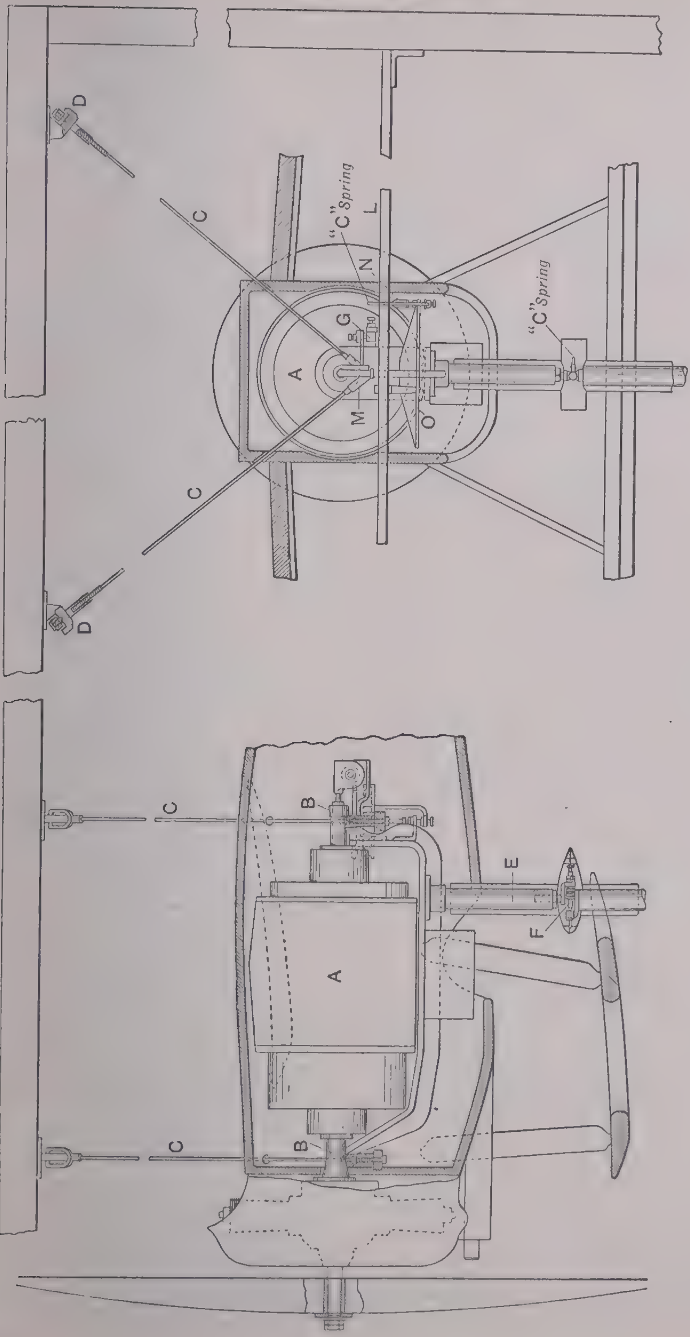


FIG. 21.

The apparatus is mounted on a whirling arm about 30 ft. in radius, and the centrifugal force on the airscrew has no component in the direction of the thrust. Certain forces due to centrifugal action are automatically balanced by suitably placed masses.

The airscrew speed is measured by means of the voltage obtained from a small electric generator driven by the propeller shaft, or alternatively by measuring the time elapsed for each hundred revolutions of the shaft. The speed of the apparatus relative to the air is obtained by means of a Dines tube. All electrical and air connections are taken to the central shaft of the whirling arm, from which they are again taken to an observing table where the speeds of the arm and propeller shaft are controlled.

§ (9) FIXED AERIAL PROPELLER DYNAMOMETER FOR USE IN A WIND CHANNEL.—A sketch of this apparatus is given in *Fig.* 21. It consists of a small electric motor A bolted to a cradle, the ends of which are attached to two hardened steel points bearing in the cups of the "Y" pieces M. These "Y" pieces are rigidly connected to the lower ends of the diagonally arranged wires C, the upper ends of which are supported from the roof of the tunnel by stirrups carrying hardened steel points bearing in the cups D. With this method of support the electric motor is capable, if there is no other constraint, of a swinging motion in the direction of the motor shaft and also of a rocking motion about the axis passing through the points B, B, this axis being parallel to, and slightly below the axis of, the shaft. A spindle arm E, projecting from the underside of the cradle, is connected by a strut and a "C" spring to the top of a spindle clamped to the head of an aerodynamical balance the vertical head of which projects up through the floor of the wind channel. The spindle arm E and the top of the balance arm are enclosed in a guard which is streamline shape surrounding the strut and "C" spring. A revolution counter driven by the motor shaft and in electrical communication with a bell enables the rotational speed of the airscrew to be measured. To avoid any unnecessary constraint of the moving parts of the apparatus the electrical connections to the motor are made through the mercury cups G. The scale of the model airscrew used with the apparatus needs to be such that the motor with its contiguous parts is completely enclosed in a shell of the model aeroplane body K, and such as to ensure sufficient clearance between the surrounding shell and the moving parts of the measuring apparatus. The body is suitably supported by bars from the sides of the channel.

When measuring torque, the rocking axis BB of the motor and airscrew, which is parallel to the airscrew shaft, is fixed parallel to the centre line of the channel by rigidly attaching the "Y" pieces to the bars L. The brackets at the lower end of the arm E and at the top of the balance spindle are adjusted so that the strut which transmits the load to the top of the balance arm is at right angles to the airscrew shaft. A direct calibration of the apparatus may be made by applying a known torque and weighing directly with the balance.

To measure thrust, the "Y" pieces are detached from the bars L so that the motor and airscrew have freedom to swing in a longitudinal direction about the points D at the upper ends of the wires, and the points B of the motor cradle. The brackets on the lower end of the arm E and the top of the channel spindle are adjusted so that the strut F which transmits the thrust to the top of the balance is parallel to the airscrew axis. A strut N, which is held in position with a "C" spring between the bracket O and the cradle and one of the bars L, prevents the motor rocking about the axis BB. The calibration of the thrust apparatus may be obtained by applying a known thrust along the airscrew axis and measuring directly the airscrew balance.

The air velocity in the channel is measured by a tilting manometer. J. H. H.

DYNAMOMETER, BRITISH ASSOCIATION. See "Dynamometers," § (5) (i.).

DYNAMOMETER CARS. See "Dynamometers," § (5).

Lancashire and Yorkshire Railway Co.'s. See "Dynamometers," § (5) (v.).

DYNAMOMETER, HYDRAULIC. See "Hydraulics," § (61).

E

EARTH:

Age of, deduced from Cooling. See "Heat, Conduction of," § (12) (iii.) (*a*).

Foundations, Effect of the Percentage of Water present on. See "Friction," § (34)

Temperature Waves in. See "Heat, Conduction of," § (12) (i.) (*a*).

ECCENTRIC GEAR FOR SLIDE VALVES. See "Steam Engines, Reciprocating," § (2) (iii.).

EDDY CURRENT BRAKE FOR POWER MEASUREMENT. See "Dynamometers," § (2) (vi.).

Eddy Resistance. See "Ship Resistance and Propulsion," §§ (13) and (14).

Eddy Viscosity. See "Mechanical Viscosity, Friction," §§ (10), (11), (12).

Efficiencies of Internal Combustion Engines, Table of Ideal Values. See "Engines, Thermodynamics of Internal Combustion," § (46).

Efficiencies of Petrol Engines. See "Petrol Engine, The Water-cooled," §§ (2)-(4).

Efficiency of a Heat-engine. The efficiency is measured by the ratio $(Q_1 - Q_2)/Q_1$, Q_1 and Q_2 being the amounts of heat taken in and rejected respectively. If T_1 T_2 be the corresponding temperatures on the Absolute Thermodynamic Scale, we have for a reversible engine

$$\frac{Q_1 - Q_2}{Q_1} = \frac{T_1 - T_2}{T_1}.$$

Thus the efficiency of any reversible engine working between temperatures T_1 and T_2 is

$$\frac{T_1 - T_2}{T_1}.$$

See "Thermodynamics," § (20).

Efficiency of Rankine Cycle. See "Steam Engine, Theory of," § (3).

ELASTIC CONSTANTS, DETERMINATION OF, AND THE TESTING OF MATERIALS OF CONSTRUCTION

§ (1) Introduction.—The reasons for the testing of materials of construction are various and depend upon the point of view in which the inquirer approaches the subject.

The designer, in order to proportion the parts of his machine or structure and apply his theoretical calculations, requires to know the physical constants of the materials with which he deals. The aim of the scientific experimenter is to supply this data in order that the theoretical researches in applied mechanics may be applied to the problems with which the engineer is confronted in his practice. Absolute results concerning definite properties of materials form the basis of *scientific testing.*

The material to be used in the construction of any structure can usually be obtained from several sources of supply and of several grades and prices. The engineer wishes first to indicate to the manufacturer the class of material he needs, and for this purpose he supplies him with a specification containing among other things particulars of the properties required. The design is based on certain values for the physical constants, and these values are the real criterion of the suitability of the material for the purpose for which it is intended. The determination of the constants is usually a slow laborious business requiring considerable skill and scientific ability. Approximate tests have been devised, which are only intended to give a comparative indication of the physical properties of the material. The results expected from these tests are inserted in the specification; in some cases they approximate closely to the scientific tests.

It is, obviously, important for the engineer to be assured that the material supplied fulfils the terms of the specification, and for this purpose he selects sample portions of the material and subjects them to the specified tests. *Commercial testing* is therefore carried out to determine whether materials conform to the terms of specifications.

There is a further section of the subject which has been given a great impetus in the last few years, viz. *investigatory testing,* undertaken to determine causes of failure of material in actual practice and to improve processes of manufacture and design of machines and structures.

The science of engineering is advanced by a systematic study of failures probably quite as much as by any other branch of research. In investigatory testing, methods are employed to exaggerate certain properties in order to compare them in various materials with ease and certainty. These same properties can probably be studied by a careful analysis of the absolute results obtained by scientific testing, but the process is difficult and cannot be attended with success without considerable practical experience.

I. Testing Machines

§ (2) General Methods.—There are certain methods of test which are applicable to all branches of test work. The complexity of the machines employed for these tests depends upon the number of purposes for which it is intended to use them. This complexity is contingent upon

(1) The variety of materials it is required to test.
(2) The kinds of straining action needed.
(3) The form and size of the specimens to be tested.
(4) The magnitude of the forces to be exerted and measured.
(5) The accuracy required in the results.

By limiting the requirements, the testing apparatus can be made of great simplicity. For instance, in a transverse testing machine such as is used for checking the quality of cast iron in a foundry, the test pieces can all be of one size, the straining action is always the same (the test piece being broken as a beam), and there is not a great deal of variation of the breaking load. A machine such as this is simple and effective.

Every increase in the requirements adds additional apparatus to the machine. An accessory for one purpose will often interfere with the running of the machine for another purpose, thus causing loss of time in making the necessary adjustments for the experiments in hand, involving great care in the setting of the machine and making it more difficult to handle. It is therefore in the interest of the user to consider carefully the functions which the machine will be called upon to perform, and to limit them to a minimum. In this way increased efficiency is obtained with minimum cost.

The simplest method of testing is by direct loading, and the earlier researches on strength of materials were carried out by this method. Galileo (1638) and Muschenbrock (1729) made many tests on a small scale by this means, and where it can still be applied it is the simplest and best method of testing available. Owing to the difficulty of handling heavy loads it is only suitable for use with weak materials.

When at the beginning of the nineteenth century the demand came for tests on a larger scale, it was necessary to consider other means of applying the force than by direct loading, and within a few years the three arrangements found in modern testing machines were employed, viz.:

(*a*) Load applied by hydraulic press—no weighing device, but load calculated from the pressure on the ram.
(*b*) Load applied by gearing at one end—weighing lever at the other end.
(*c*) Load applied by hydraulic press at one end—weighing lever at the other end.

In 1813 method (*a*) was employed by Brunton & Co. for a chain-cable testing machine. The arrangement for arriving at the load was not capable of giving very great accuracy. It was necessary to make an allowance to the load, which was deduced from the indication of a pressure gauge, for the friction of the packing of the ram or the cup leather, and the amount of this friction was extremely variable.

The Whitworth hydraulic testing machine [1] and the 600-ton testing machine of the Union Bridge Company at Athens, Pa., U.S.A., both use the same principle. The 5000-ton Olsen [2] compression machine of the Geological Survey and the Bureau of Standards, and the 600-ton Olsen compression tester of the Rensselaer [3] Institute at Troy both record the load by the pressure in the hydraulic press, but with these machines the pressure is transmitted to a diaphragm whose area is smaller than that of the main ram. The pressure on the diaphragm is measured by a system of levers. Method (*a*) has been applied to modern testing machines by eliminating the packing and having an accurately fitted ram and cylinder with small clearance. By using oil the slight leakage between the ram and cylinder reduces the friction to a practically negligible quantity.

The evolution of the other two testing machine arrangements, (*b*) and (*c*), from the direct loading method was due to the introduction of the lever. By placing a lever in between the test piece and the load, the amount of dead weight required was reduced by the ratio of the lever arms. This arrangement, although quite suitable for small pieces possessing little ductility, was found to be open to the objection that, as the material stretches, the position of the lever alters and cannot be adjusted.

In 1817 Captain Brown constructed a cable testing machine in which the deformation of the cable was neutralised, and the load applied, by means of gearing at one end, while the load was measured by a lever at the other.

The use of gearing was the method of straining used in America on machines of the lever type. The same arrangement, although common on small machines in this country worked by hand, had not, however, been used on power-driven machines until recently.

Another method of counteracting the deformation of the test piece was applied by Lagerjhelm, of the School of Mines at Stockholm, who conducted experiments in 1825 on a 7-ton machine in which a hydraulic ram neutralised the give of the test piece and applied the load, which was measured by weights acting on a knee lever. About the same time, that principle was also employed by Bramah [4] in the construction of a machine used at Woolwich Dockyard by the late Professor Barlow for some of his experiments on the strength of materials. This method of construction is, however, always associated with the name of David Kirkaldy, who in 1866 erected, in London, his well-known machine of 400 tons capacity. The combination of hydraulic ram and levers has since been in common use in machines of many designs, the chief differences in these designs being in the arrangement of the lever or levers and their position relatively to that of the test piece.

Thomasset,[5] in about the year 1872, measured the load on the lever by means of a diaphragm piston with an extremely limited motion, and balanced the load on the piston by a mercury gauge. Maillard and

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[1] See Peet, *Proc. Inst. Mech. Eng.*, 1898.
[2] *Engineering Record*, Feb. 1909, lix.
[3] *Proc. Am. Soc. Test. Mat.*, 1909, ix.
[4] Barlow, *Strength of Materials*, London, 1851, p. 258.
[5] Lebasteur, *Les Métaux*, p. 52.

Emery a few years later dispensed with the lever and transmitted the whole load to the diaphragm. Emery[1] constructed a machine of 450 tons capacity in which the pressure on the diaphragm piston was measured by transmitting it to a smaller one in which the mercury gauge of Thomasset was replaced by a system of levers and weights.

The first essential requirement of a testing machine is accuracy—the load shown by the machine should be an accurate indication of the real load to which the test piece is subjected. In order to obtain complete accuracy it is necessary that:

(1) Simple means should be available for establishing the truth of the indications of the machine—it should be capable of being easily calibrated. This calibration may alter with use; if of the lever type the knife edges may wear or be displaced, and, on a three-inch fulcrum distance, 1 per cent error in the readings would be caused by a displacement of only 0·03 inches.

(2) The sensitiveness in working should be adequate, *i.e.* small differences in the load should be readily indicated.

Another desirable feature is simplicity—not simplicity resulting from a mere reduction of the attachments for the various tests, but that obtained by ensuring that the varied requirements are carried out without undue complications. This not only reduces the parts liable to alteration with use, but renders the machine capable of rapid and easy manipulation during test, a feature which is especially important in a machine to be used for commercial testing. Other requirements necessary in a machine or commercial testing are:

(*a*) The tests should be accessible from the front of the machine, and the scale and control levers within easy reach of the operator.

(*b*) The test piece should be easily inserted in the machine, *i.e.* the method of gripping should be simple and convenient. For laboratory work it is advisable that the machine should be conveniently arranged to take an autographic apparatus for registering the results, and should be generally adaptable to all sorts of conditions which may from time to time be necessary for varied experimental work.

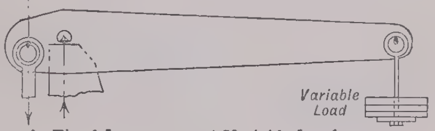
The liability of the testing machine to subject the specimen to momentary shocks should also be considered. The inertia in the lever and weights (in that type of machine) may increase the stress in the test piece beyond that indicated by the scale reading of the jockey weight. The construction of each machine should therefore be understood by the operator, and in skilful hands errors due to this cause can be reduced to a very small amount.

§ (3) Horizontal and Vertical Machines.—Testing machines may be arranged in either of two ways:

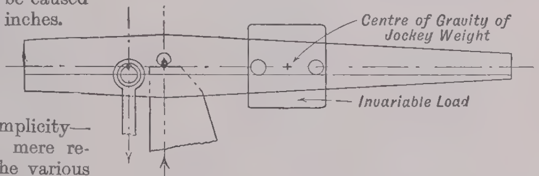
(i.) In which the load is applied horizontally—horizontal testing machines.

(ii.) In which the load is applied vertically—vertical testing machines.

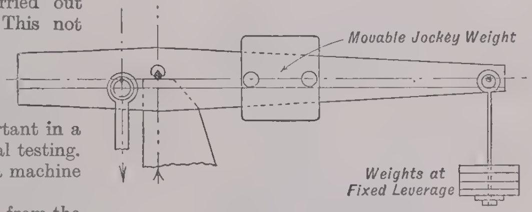
For testing long specimens it is usual to employ a machine of the horizontal type in which it is necessary to support the test piece and the grips which hold it, otherwise forces are applied to the test piece which are not measured by the load-measuring apparatus. The vertical testing machine, however, is more convenient to handle and, except in the circumstances mentioned above, is the arrangement generally adopted.



A. Fixed Leverage and Variable Load



B. Fixed Load and Variable Leverage



C. Movable Jockey Weight combined with a Load at a Fixed Leverage

Fig. 1.

§ (4) Modern Testing Machines.—Modern testing machines usually consist of four main parts:

(i.) An arrangement for applying the load.

(ii.) An adjustment to neutralise the effect of the deformation of test piece so as to provide that there is no movement of the point of attachment of the test piece to the weighing apparatus.

(iii.) A method of measuring the applied load.

(iv.) Holders for connecting the test piece to (i.) and (iii.).

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[1] See Towne, *Inst. Mech. Eng. Proc.*, 1888, pp. 206 and 448.

The various types of testing machine differ in the way in which these requirements are carried out. The first and second parts are usually combined, and the load on the test piece is applied to one end either by (1) a straining cylinder actuated by a pump and accumulator, screw compressor, or oil pump (hydraulic rotary or direct driven); or (2) screw gearing driven by hand or by power (such as an electric motor). Screw power testing machines are divided into two general types—the rotary screw and the rotating nut.

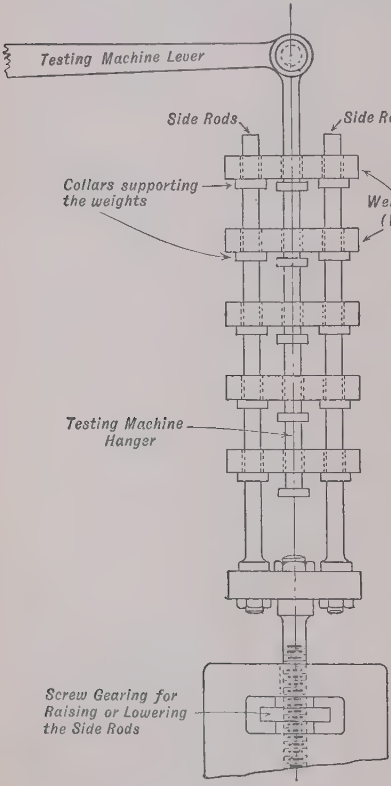
Testing Machine Lever
Side Rods
Side Rods
Collars supporting the weights
Weights (W)
Testing Machine Hanger
Screw Gearing for Raising or Lowering the Side Rods

FIG. 2.

The load or force exerted on the test piece is measured (*a*), more commonly in Great Britain, by a lever or system of levers with weights or a movable counterpoise forming a weighing apparatus;

(*b*) by balancing it with fluid pressure acting on a diaphragm;

(*c*) by measuring the pressure of the oil or water in the straining cylinder;

(*d*) by calculating the load from the angle through which a weighted pendulum is lifted.

Testing machines using methods (*b*) and (*c*) have the advantage that they are cheaper to make than the weighing lever machine, are practically free from inertia forces, and the load is automatically adjusted to the stress. They, however, require very careful attention and calibration, and are considered by the Board of Trade to be undesirable for use when making acceptance tests.

§ (5) TYPES OF TESTING MACHINES. *Lever Testing Machines.*—By far the largest number of machines measure the load by the use of a lever. The loading of the lever is effected either by (*a*) adding separate weights to a scale pan placed at a fixed leverage (*i.e.* fixed leverage and variable load); (*b*) having a single jockey which travels along the beam (*i.e.* fixed load and variable leverage); (*c*) a combination of (*a*) and (*b*).

These arrangements are indicated diagrammatically in *Fig.* 1. With either of the methods the following points must be borne in mind:

(1) The lever during the test will change its position, and in order to secure constant leverage the points of application of the loads must be in a straight line, *i.e.* the centre of gravity of the moving weight and any knife edges employed must all lie in a straight line or, if the moving weight is hung from a knife edge, the knife edge must travel along that line.

(2) Any addition of weights during a test must be carried out without producing shocks on the specimen. In those machines in which weights are added to the scale pan it is usual to carry this out by a mechanical arrangement such as is shown in *Fig.* 2.

In some machines the load is measured by the height to which a pendulum bob is raised. This can be considered as a modification of the lever method. *Fig.* 3 shows how it was employed in the tests described in the British Engineering Standards Association Report No. 55.

It is desirable that, on the same machine, the percentage accuracy of reading the load should be the same with small as with large specimens, and it is therefore usual to arrange modern testing machines so that the same length of scale is equivalent to at least two values of the load. This is carried out either by varying the leverage or varying the weight of the moving poise.

The leverage is varied by providing alternative fulcra. This method is introduced into the Wicksteed vertical machine (see *Fig.* 10A), and consists in providing a third knife edge which can be put in or out of gear. When it is in place, the knife-edge distance is large and the machine is arranged for use with weak specimens, otherwise the full capacity of the machine is available. Thus the same counterpoise may represent say 50 or 100 tons for the same travel of the counterpoise along the lever, according to which fulcrum is used. The method of operating the third knife edge is shown in *Fig.* 4. The three knife edges are in the same line, but the movable support Y for the third knife edge Z is raised so that when it is placed in position by the worm gear there is a slight clearance over the support X, as shown in *Fig.* 4 (B).

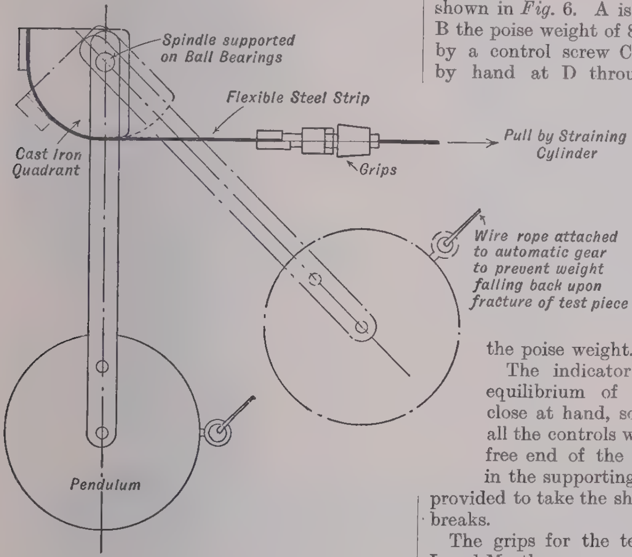
Spindle supported on Ball Bearings
Flexible Steel Strip
Cast Iron Quadrant
Pull by Straining Cylinder
Grips
Wire rope attached to automatic gear to prevent weight falling back upon fracture of test piece
Pendulum

FIG. 3.

The weight of the moving poise is varied in machines supplied by Messrs. W. & T. Avery, Ltd. The counterpoise is split, and for small loads a part of it is detached and used as a separate poise.

In order to obtain a high mechanical advantage, without making the short arm of the lever very small, some machines use a multiple system of levers, each with comparatively small mechanical advantage. It is therefore convenient to divide lever machines into two sections, (1) those using a single lever, (2) those in which the levers are compounded, and to subdivide those sections according as to whether the test pieces are vertical or horizontal and whether the load is applied by a straining cylinder or screw gearing.

§ (6) SINGLE-LEVER MACHINES. (i.) *Vertical.*—*Fig.* 5 shows diagrammatically the simplest arrangement of a single-lever machine, this being the principle adopted by Mr. Wicksteed.[1] It is a vertical machine with a single horizontal lever A on top, to which one end of the test piece B is connected. A movable poise C weighs the load, which is applied to the other end of the test piece by the ram of a hydraulic press (as shown) or by screw gearing. The following are modern examples of machines which use this principle.

(ii.) "*Avery*" *10-ton Electrically Driven Single-lever Testing Machine.*—This machine is shown in *Fig.* 6. A is the weighing lever and B the poise weight of 840 lbs., which is moved by a control screw C. The screw is driven by hand at D through gearing E and F and a hooked joint G which is placed on a line with the fulcrum. The straining of the test piece is produced through worm and spur gearing by means of an electric motor H, whose control is located at I on the standard near to the hand wheel propelling the poise weight.

The indicator J, which shows the equilibrium of the steelyard, is also close at hand, so that the operator has all the controls within easy reach. The free end of the lever plays in a space in the supporting pillar K. Springs are provided to take the shock when the test piece breaks.

The grips for the test piece are shown at L and M; these are provided with attachments for test pieces with either screwed or enlarged ends. The upper grip is attached to the lever, while the lower one, which slides in guides in the standard O, is connected to the straining screw P. The second knife edge Q rests on the standard O.

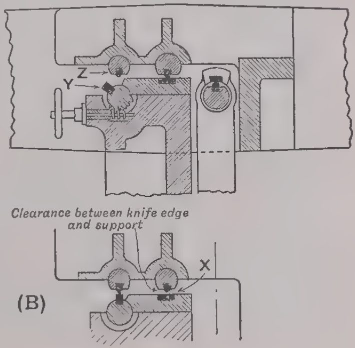

FIG. 4.

(iii.) "*Denisons*" *30-ton Testing Machine.*—In this machine, shown in *Figs.* 7 and 8, the load is applied by a straining cylinder, the pressure to which is supplied by a patent hydraulic rotary oil-pump. This pump has a variable delivery in either direction whilst being continuously driven in one direction. The drive can either be by direct coupling to an electric motor, as shown in *Fig.* 7, or by a belt from a countershaft. A similar pump is used for driving the standard jockey weight in either direction; the speed at which it travels can be varied by means of a lever placed in a convenient position.

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[1] *Inst. Mech. Eng. Proc.*, 1882, p. 384.

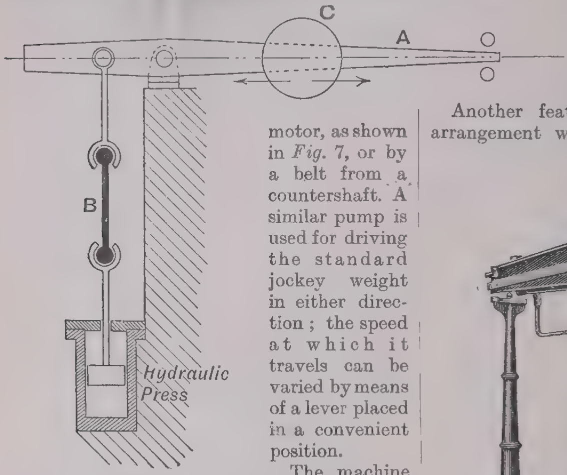


FIG. 5.

The machine is supplied with tell-tale devices, for indicating to the operator the functions of the various parts, as well as all the features which are indispensable for accurate work.

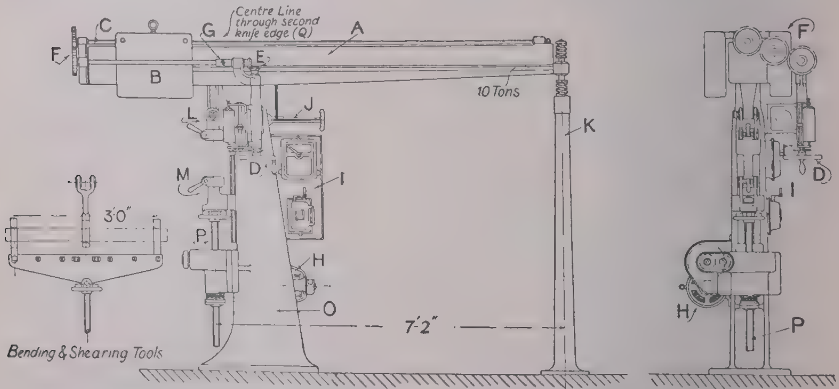


FIG. 6.

(iv.) *Buckton's 100-ton Machine.*—A photograph of the machine of this type which is installed at the N.P.L. is shown in *Fig.* 9. The load is applied by a hydraulic ram and cylinder which are placed in a pit below the floor level. A pressure of 1 ton per sq. in. is supplied to the cylinder by a hydraulic pump and accumulator driven by an electric motor. The counterpoise is driven by hand or mechanically by fast and loose pulleys operated from a suitable line of shafting. The machine seen in the photograph is fitted with alternative fulcra (see § (5)) giving a short knife-edge distance of either 3 or 6 inches, so that the jockey weight of 1 ton in travelling the full length of the scale can be arranged to give either a load of 100 or 50 tons.

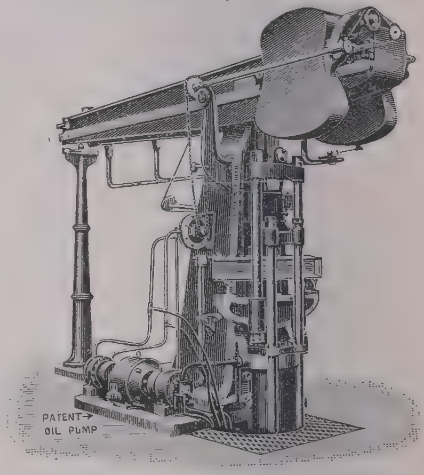


FIG. 7.

Another feature of the machine is the arrangement which has been added at the N.P.L. to prevent the jockey weight accidentally running past the end of the scale in either direction. When a considerable number of tests have to be carried out, it is usual for the operator to measure the extension, contraction of area, etc., and enter up his results, while the counterpoise is running back for the next test. It detracts from his attention if he has to be continually on the alert to stop the counterpoise at the right time. The gear is shown in detail in *Fig.* 10, and operates on the belt-throw-over gear. The shaft A operates the gearing for driving the weight along, placed at the end of the beam, and is worked from the small countershaft (not shown) through bevel wheels B. The striking gear of the countershaft is operated by a shaft C, which has a handle (not shown) on its lower end. A spur gear on shaft A operates a screw D through gearing. This screw is so arranged that it does not rotate but traverses horizontally as the weight is run along the beam. The screw carries a projection E working in a bracket F attached to the striking gear shaft C. Adjustable lugs G and H are fitted on to this projection and are arranged so that when the counterpoise reaches the extreme end of its travel, in either direction, one or other of the lugs G and H will rotate the striking gear shaft and throw the driving belt on to the loose pulley of the countershaft. The lugs can be adjusted so that, for any series of tests, the counterpoise will be stopped at any predetermined position along the beam. A photograph of the gear is given in *Fig.* 10A.

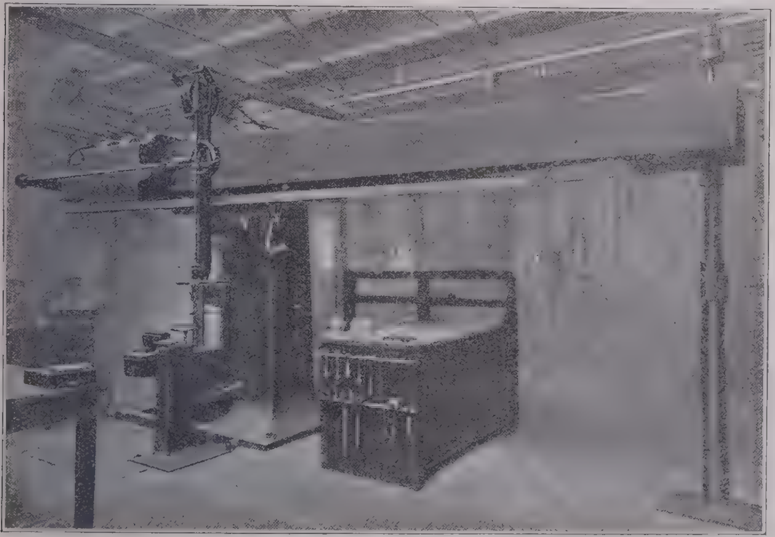


FIG. 9.

(v.) (*Horizontal*). *The Werder Testing Machine.* — The Werder machine, extensively adopted in Germany, is a horizontal single-lever testing machine, in which the load is applied by hydraulic pressure, but is measured by dead weights on a single lever. The type was originally designed by Ludwig Werder in 1852 and exhibited at the Munich exhibition in 1854, since when a considerable number of similar machines have been constructed. The arrangement of the machine is illustrated diagrammatically in *Fig.* 11, from which it will be seen that, (1) only one lever is required for a 500 to 1 leverage, and (2) by an ingenious arrangement, the ram, lever, and weights are all at one end of the machine. It is thus a simple matter to adapt the machine to take test pieces of any length.

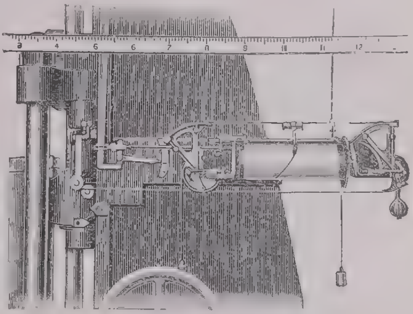


FIG. 8.

The test piece T is held at one end in the framework of the machine A, and at the other pulled by the short arm L_1 of the lever, to which it is connected by a crosshead and long bolts, shown in the diagram by a connecting rod EF. At the end of the long arm of the lever hangs a scale pan W. The central fulcrum of the lever G rests on the end of the ram C, so that, as the test piece extends, the ram moves out, carrying the whole measuring apparatus with it. The arm L is kept horizontal by the help of a level which is fixed to it. *Fig.* 12 shows a sectional elevation illustrating the principal knife edges, and *Fig.* 13 gives a plan of the machine.

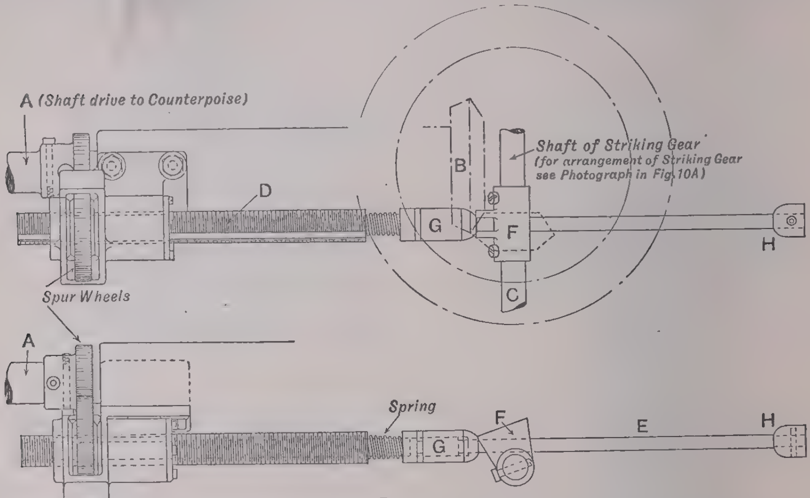

A (Shaft drive to Counterpoise)
Shaft of Striking Gear
(for arrangement of Striking Gear see Photograph in Fig. 10A)
D
B
G
F
H
C
Spur Wheels
A
Spring
F
E
H
G

FIG. 10.

FIG. 10A.

The test piece is pulled from the main crosshead V by four rods (two shown in plan as T_1 and T_2); these rods are connected vertically in pairs by crossheads U_1 and U_2, and pull through knife edges C_1 and C_2 on to a large U-shaped block Q, to which the lever Q_1 is rigidly attached. Block Q has two openings in it, through which the crossheads U_1 and U_2 pass. The horizontal line through C_1 and C_2 passes through the axis of the test piece.

The whole system of rods and crossheads hangs from the ram crosshead at N_1 and N_2 on the traverse bar OO. The block Q hangs from this bar by the knife-edge links P_1, P_2 (the knife edges being in the same line as C_1, C_2), its balance being adjusted by the sliding weight S.

In the middle of the block Q the knife edge *b* is fitted; this bears against a steel face *a* forming part of the ram crosshead. This knife edge is 3 mm. lower than the line of the others. Thus the ram coming out applies a load to the test piece through the block Q, knife edges C_1 and C_2, crossheads U_1 and U_2, and the rods T_1 and T_4. As the knife edge is 3 mm. out of line with C_1 and C_2 a bell-crank lever is formed, the small arm of which is 3 mm. and the long arm 1500 mm., *i.e.* the distance the scale pan is away from the knife edge.

It is clear that the length of the short arm of the lever (3 mm.) cannot be directly checked with sufficient accuracy, a controlling arrangement is therefore provided consisting of a knee lever f_2 having a fulcrum on the frame K and a ratio of 10 to 1. The knife edges on this lever can be accurately measured by ordinary means, and, to test the accuracy of the 500 to 1 ratio, a direct calibration can be made.

The ram C is 11·8 inches in diameter, the knife edge *b* is $14\frac{1}{2}$ inches long, and the two knife edges C_1 and C_2 are together 15 inches long, so that the pressure upon them is about 7 tons per linear inch at full load (100 tons).

The pressure is obtained from two pumps worked by hand or power and placed at the side of the machine—a small one of 0·4 inch diameter, used for the full load, and a larger one, 1·2 inches diameter, for use up to a load of 20 tons.

The arrangement shown is for tension tests; for compression tests the pull is transmitted through links from V to a back plate, or for short specimens the space between V and the back of the cylinder is utilised.

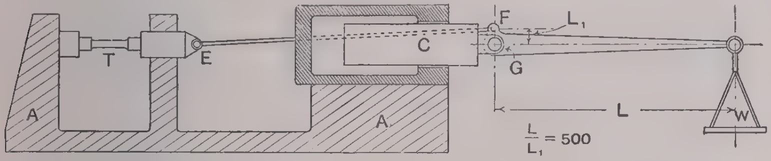


Fig. 11.

§ (7) Compound-lever Machines. (i.) *Vertical Machines with Screw Power.*—This is the arrangement of machine which is largely used in America and is shown diagrammatically in *Fig.* 14. The straining mechanism is of two types — the rotating screw and the rotating nut.

With the rotating screw machines, the pulling screws pass through nuts which are fastened to the pulling head A. The screws are keyed to and rotate with the main gears through bearings in the cover plate, thus moving the head up or down.

In the rotating nut type of machine, bronze nuts are fastened in the main gears and rotate with them, pulling the screws, to which the head is attached, up and down. The gears rest on long pedestals bolted to the base plate of the machine, and in these pedestals keys are fixed which prevent the screws from turning.

The thrust, in each case, is taken on the underneath side of the cover plate by ball or roller bearings. The rotating nut machine usually requires a pit or opening in the foundation for the screws which project below the base line of the machine.

The weighing apparatus consists of a weighing table C, levers D_1 and D_2, and a graduated beam E. For tensile strain there is also a weighing or top head F and two weighing columns G_1 and G_2 attached to the weighing table.

(ii.) *Olsen's 200,000-lb. Testing Machine.*—*Fig.* 15 shows the 200,000-lb. Olsen four-screw compound-lever testing machine of the rotating nut type.

Straining Mechanism.—D, D are the straining screws which are attached to the pulling head C, to which one end of the specimen is attached. They pass through holes at the corner of the platform E and through openings in the levers F and bed-plate G, and enter the driving nuts H and H_1, situated below the latter.

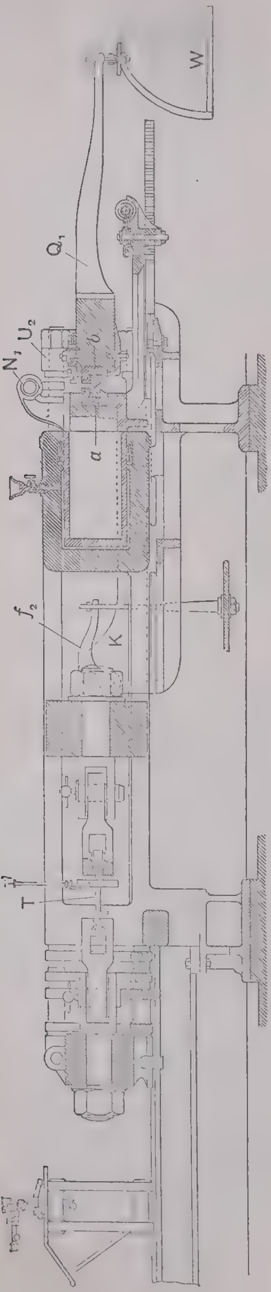
The driving nuts H and H_1, which are made extremely long to ensure a long life, are driven by means of gearing JKMN operated by pulleys and belting. As each screw is fitted with feathers, in longitudinal slots cut through the threads, to prevent them from turning, rotation of the nuts causes the screws to either rise or fall, carrying the pulling head C with them. The thrust of the nuts against the bed-plate G is taken by ball thrust washers.

(iii.) *Weighing Apparatus.*—The weighing mechanism is essentially the same as that of a platform weighing machine. The load on the test piece, applied by the straining mechanism, is communicated to the platform E either directly in a compression test, or through the medium of the upper steel plate B (to which one end of the test piece is wedged) and columns A in the tensile test.

The platform rests wholly on three main levers F so constructed as to act as a single lever, and any pressure on it is transmitted directly to them. The knife edges of only one of these levers are shown in the figure.

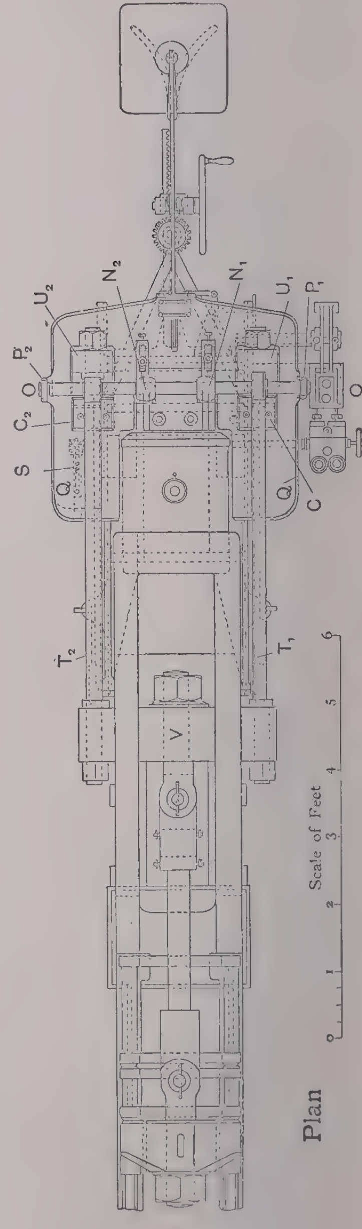
A second multiplying lever F_2 is placed above and parallel to the lever F, and is connected to it by a stirrup J_2. The reaction of the lever F_2 is taken at K on a strong iron framework which is secured to the gear box of the machine by a diagonal brace. This framework also holds the support L_2 for the fulcrum of the graduated steelyard F_3, to which the load is transmitted from lever F_2 by the link *n*.

A cylindrical weight W, which runs on a screw, adjusts the zero of the beam, while a poise *q*, which travels on wheels, balances the



Sectional Elevation

FIG. 12.



Plan

FIG 13.

load applied. The steelyard is graduated up to 100,000 lbs., and an additional weight on the end of the lever adds a further 100,000 lbs.

(iv.) *Horizontal Compound Lever Machines.*—For testing long pieces, a horizontal testing machine is essential. In order to obviate the difficulties introduced by the small knife-edge distance of the Werder machine, multiple lever arrangements are used. A bell-crank lever is adopted to convert the horizontal force into a vertical one, and this is combined with the weighing lever, either with or without the introduction of intermediate levers.

The arrangement of two machines of this type is given in *Fig.* 16.

(v.) *Large Horizontal Machines.*—In order to carry out tests on built-up structural members of a practical size, machines of large capacity are required, and for this purpose no machine of less than 300 tons is of much use. Excluding chain testing machines, the best examples of large horizontal machines built in this country are:

Kirkaldy's machine of 400 tons built in 1866.

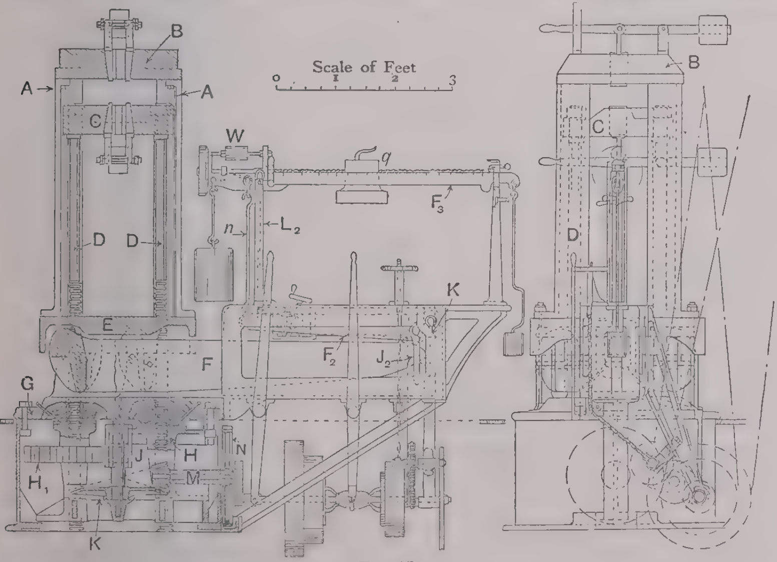


FIG. 15.

The 300 ton machine of the Conservatoire des Arts et Métiers at Paris built by Messrs. J. Buckton and Co. in 1904.[1]

The Avery Machine[2] of 700,000 lbs. installed at Birmingham University in 1909.

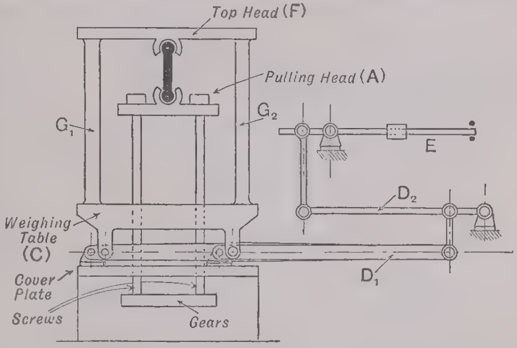


FIG. 14.

(vi.) *The Birmingham University 700,000-lb. Testing Machine.*—This is a universal machine with a capacity of 700,000 lbs. and takes tension bars 28 ft. long and columns 30 ft. long. The span for bending tests is 20 ft., and the wedge grips take a maximum size of 6 × 2½″ flat or 3¾″ diameter.

A diagrammatic view of the machine is given in *Fig.* 17 and more detailed drawings of the straining and weighing mechanism in *Fig.* 18.

The machine has a ram K at one end and the lever system D at the other. The heavy cast-iron bed XX has substantial standards HH at either end whose tops are connected by horizontal columns YY with the distance, of about 40 feet, between them without intermediate support. There is therefore a clear span along the top and sides, enabling heavy test pieces to be lowered in position by the traveller running overhead. To one standard the cylinder L is bolted, while the lever system is fixed to the other.

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[1] *Engineering*, Sept. 2, 1904.
[2] *Engineer*, May 21, 1909.

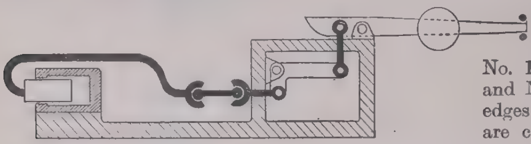


Diagram of Greenwood and Batley's Horizontal Testing Machine

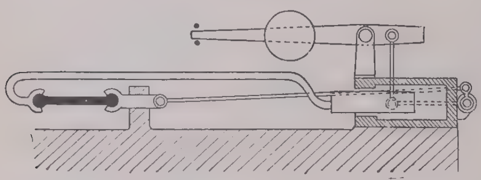


Diagram of Buckton's Horizontal Testing Machine

FIG. 16.

The load is applied by hydraulic pressure. At Birmingham the town pressure of 100 lbs. per square inch is available for low load tests and gives a total thrust of 35 tons. For the full capacity of the machine an accumulator supply of ten times the pressure is used. The diameter of the ram is 32 inches and its stroke 66 inches. The main cylinder L, bolted to the standard, forms part of the machine frame, and the main ram K, having an internal diameter of 20 inches, moves over an auxiliary fixed ram M whose function is to provide for the return of the main ram after the test is completed. The main ram is driven forward by admitting low-pressure water behind the fixed ram. The load on the ram is communicated to the travelling crosshead No. 2 by means of four notched racks N, sliding in the frame of the machine, which are fixed to the head of the ram and into which the crosshead No. 2 is keyed in any desired position.

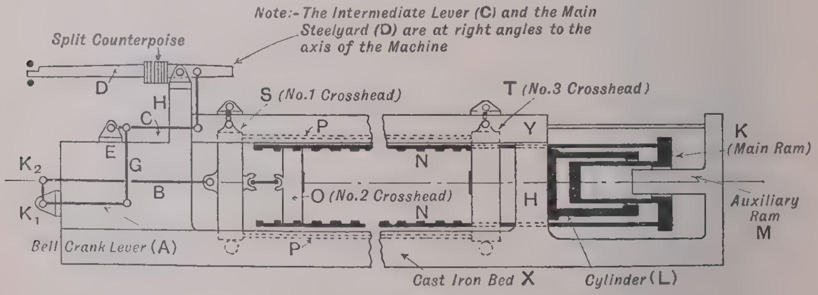


FIG. 17.

One end of the test piece is fixed to the crosshead No. 2 and the other end to the crosshead No. 3 for compression and the crosshead No. 1 for tension. Crossheads No. 1 and No. 3 are suspended from knife edges on the top of the frame Y and are connected by tension rods P, P which, when crosshead No. 3 is in use, transmits the load to the lever system through crosshead No. 1 and the main link BB. The main link BB pulls on the top knife of a bell-crank lever through a hardened steel bearing block Q. Thus, for compression, the load is communicated to the weighing apparatus from the ram by way of (1) sliding racks N, N, (2) travelling crosshead No. 2, (3) the test piece, (4) the floating crosshead No. 3, and rods P, P, (5) crosshead No. 1, and (6) main link B.

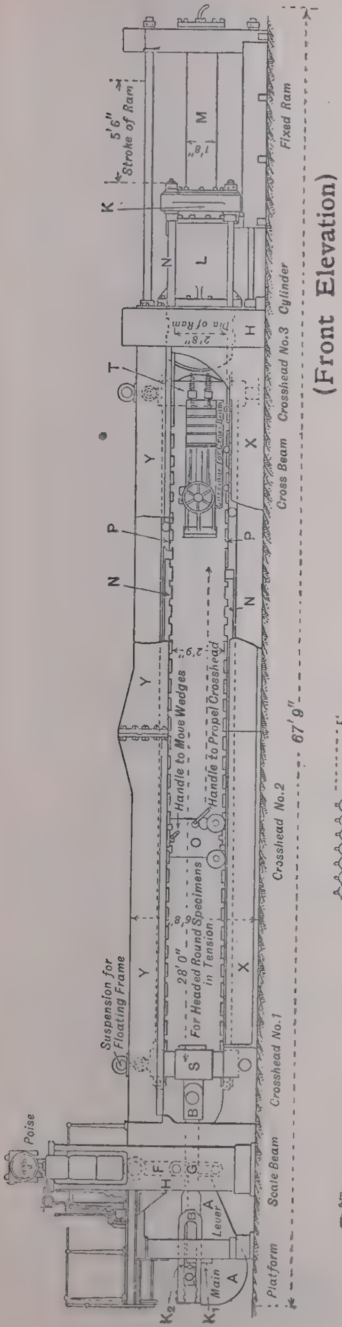
The weighing apparatus consists of a bell-crank lever A whose principal knife edge K_1, 5 feet long, thrusts against a plate fitted in the bed of the machine. The load is applied by the main link B on to a knife edge K_2 similar to the principal knife edge but a short distance above it and forming with it the short arm of the lever. The intermediate lever C and the main steelyard D are at right angles to the axis of the machine. The former has its fulcrum on a substantial casting E fixed to the bed of the machine, and the load from the bell-crank lever is transmitted to it by the link and shackles G, F while the latter has its fulcrum supported by a column H on the side of the machine.

The split counterpoise system (see § (5)) has been adopted in order to obtain an open scale for small loads. In this case it is divided into seven equal poises giving, separately, a maximum load of 100,000 lbs., and when combined the full load of 700,000 lbs.

The counterpoise can be moved by hand or operated mechanically through a screw, and all the movements are controlled by one operator at the lever end of the machine.

§ (8) Diaphragm Machines.—Diagrammatic sketches of diaphragm machines are given in *Figs.* 19A to C. *Fig.* 19A represents the Thomasset machine,[1] constructed about the year 1872, in which the load is communicated through the test piece *t* to a lever *l*, the end of which bears on to a diaphragm *d*. The pressure of the lever on the diaphragm is balanced by fluid pressure, the amount of which is measured by a gauge.

The Maillard machine (*Fig.* 19B) is similar to that designed by Thomasset except that the intermediate lever is dispensed with, the pressure being taken directly on the diaphragm, which is consequently much larger. The Maillard principle is adopted by Messrs W. H. Bailey & Co. in their 5000-lb. Ster-Hydraulic tester, and is a very convenient arrangement for small testing purposes. The



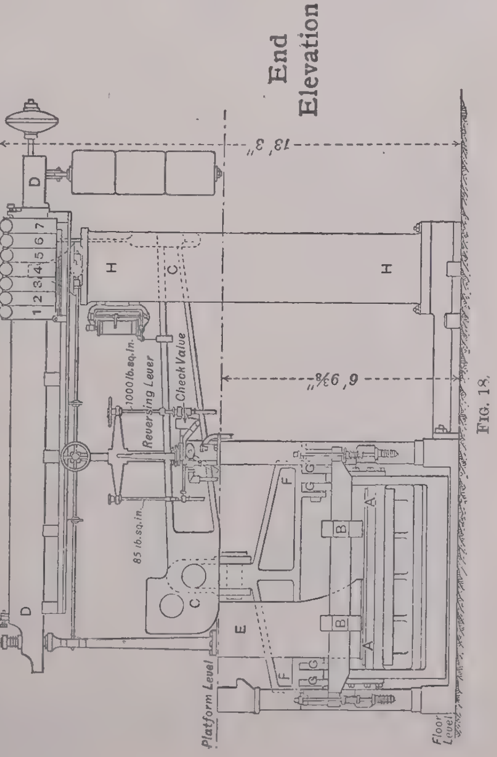


FIG. 18.

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[1] Lebasteur, *Les Métaux*, p. 52.

load is applied to the back of the diaphragm, and the pressure in the diaphragm chamber is measured by a pressure gauge or mercury column.

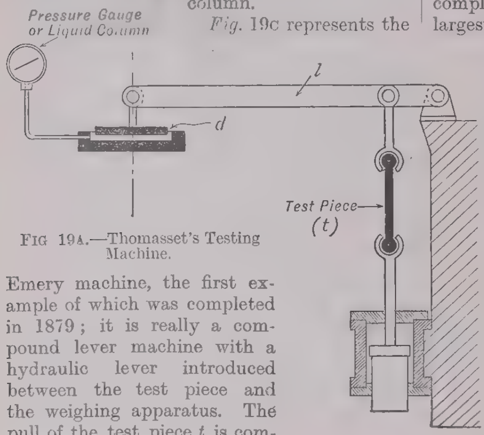

FIG 19A.—Thomasset's Testing Machine.

Fig. 19C represents the Emery machine, the first example of which was completed in 1879; it is really a compound lever machine with a hydraulic lever introduced between the test piece and the weighing apparatus. The pull of the test piece *t* is communicated to a diaphragm D. It is transmitted from this to a reducer R, the pressure on which is balanced by the lever system.

(i.) *Emery Machines.* The first Emery machine owes its origin to the United States Board for testing steel, iron, and other materials, who entered upon their work in 1875. One of the most important actions of the Board was the acquisition of an 800,000-lb. testing machine which they ordered from A. H. Emery of New York. This was completed in 1879, and is one of the largest and most accurate testing machines in the world. Unwin, after inspecting an Emery machine, said: "The merit of the Emery machine was that, while it had been made as much more delicate and sensitive than an ordinary machine as a chemist's balance was in comparison with a grocer's scales, this result had at the same time been obtained by means which rendered the more sensitive machine less liable to injury, less liable to wear, and less liable to get out of order, than the ordinary machine."

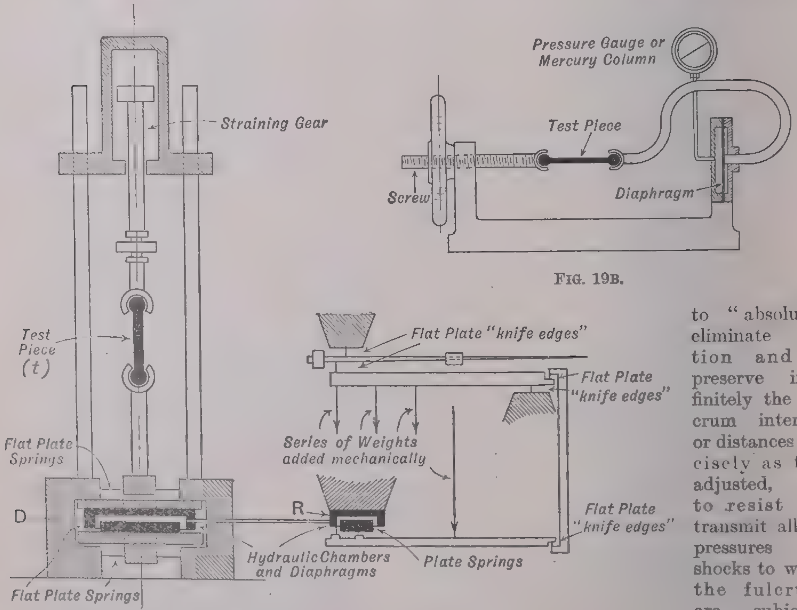

FIG. 19B.

FIG. 19C.—Diagram of Emery Machine.

The principal peculiarities of the Emery machine are: (*a*) An arrangement of hydraulic chambers and diaphragms able to receive large pressures and shocks without injury and to transmit them to a convenient point for measuring and recording.

(*b*) The replacement of knife edges with thin blades of steel. These were introduced to "absolutely eliminate friction and to preserve indefinitely the fulcrum intervals or distances precisely as first adjusted, and to resist and transmit all the pressures and shocks to which the fulcrums are subjected without in the slightest degree impairing their sensitiveness or durability."

Numerous machines have been made in the United States since 1879 on the Emery principle. *Figs.* 20 and 21 show particulars of the more important working parts of a 300,000-lb. vertical Emery testing machine; detailed drawings are given in Mr. Towne's[1] paper.

(ii.) *300,000-lb. Emery Machine* (*Figs.* 20 and 21). *Straining Gear.*—A tension test piece A (*Fig.* 21) is shown between the shackles U and L of the machine. The upper shackle U is attached directly to the ram P of the straining press, which is carried by two main adjusting guide screws M rising from the bed of the machine F. The vertical adjustment is effected by means of the nuts N, which are geared together and are rotated through a shaft and bevel gearing by a handle X.

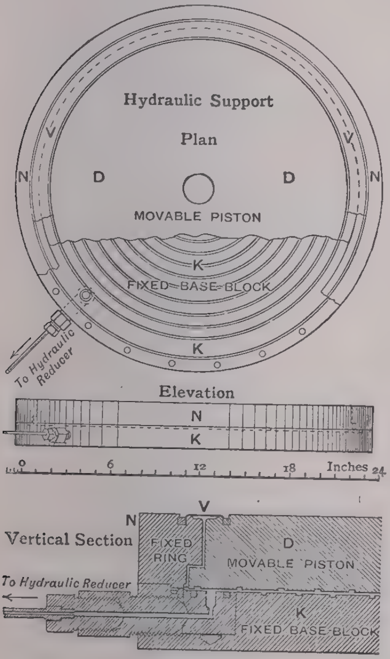

FIG. 20.

The fluid pressure is conveyed to the ram by the pipes T_1 and T_2, which have swivel joints to allow for adjustment of the straining cylinder.

(iii.) *Hydraulic Lever.*—The lower shackle L is secured to a yoke consisting of two blocks H and B connected by four steel bolts W. In this yoke the hydraulic support S is placed between the two beams G_1 and G_2. Thus the load is transmitted directly to the hydraulic support through the yoke and beams; the upper block H acting on beam G_1 for compression and the lower block B communicating the tensile load through beam G_2.

The yoke is secured in its proper position by flat plate springs I which hold it steadily laterally while leaving it free to move vertically through the small range desired. The hydraulic support S (*Fig.* 21) is shown in detail in *Fig.* 20 and consists of a piston D which can move vertically 0·001 inch in a fixed ring and base block (N and K). A space of 0·1 inch is allowed between the piston D and the ring N.

The lower surface of the piston and the upper surface of the base block are turned with annular grooves and are lined with soft sheet brass 0·005 inch thick. The linings form a sac by being interlocked as shown in the diagram. The piston D, while permitted to move vertically, is secured against lateral motion by an annular diaphragm V at its upper surface, and the exposed portion of the working diaphragm or upper portion of the brass sac at its lower face, which acts as a flexible hinge or joint. The sac is filled with alcohol and glycerine or refined kerosene oil, and the load when applied to it is transmitted to a smaller hydraulic chamber or reducer R (*Fig.* 19c), containing a similar thin film of liquid, where it is balanced by the lever weighing apparatus.

(iv.) *Weighing Apparatus.*—The maximum load of the machine is thus reduced to a maximum of 10,000 lbs. on the reducer, and this is all the load that is conveyed to the fulcrum plate of the main lever M (*Fig.* 19c) of the weighing apparatus. Thin pieces of tempered steel are used instead of knife edges, and are forced, under a pressure of about four times the working load, into the pieces that they connect. The relation of the beams, which have a total multiplication of 20,000 to 1, is shown in the figure. The total reduction of load is 600,000 to 1, so that very small weights are all that are necessary to keep the levers horizontal.

(v.) *Calibration.*—The hydraulic support is tested with known loads, applied by a "rating machine," up to the maximum capacity of the Emery machine. The exact weight required on the weigh lever to balance a known load on the machine is thus found by actual test, and the poise weights are carefully adjusted in accordance with this calibration.

The rating machine is a carefully constructed dead-weight loading appliance. The weight was produced from standard weights accurately adjusted at the Bureau of Standards, Washington.

§ (9) MACHINES WITH NO WEIGHING DEVICE.—Machines in this class are those in which the load is applied by a fluid press, and the stress in the specimen is calculated from the pressure on the plunger.

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[1] See Towne, *Inst. Mech. Eng. Proc.*, 1888, pp. 206 and 448.

An objection to this method of test has been that errors are introduced into the results owing to the variability of the friction caused by the ram packing. This objection has now been overcome, in the machine constructed by Messrs. Alfred J. Amsler & Co., by using oil as the fluid together with an accurately fitted ram and cylinder with small clearance, thus eliminating the packing friction.

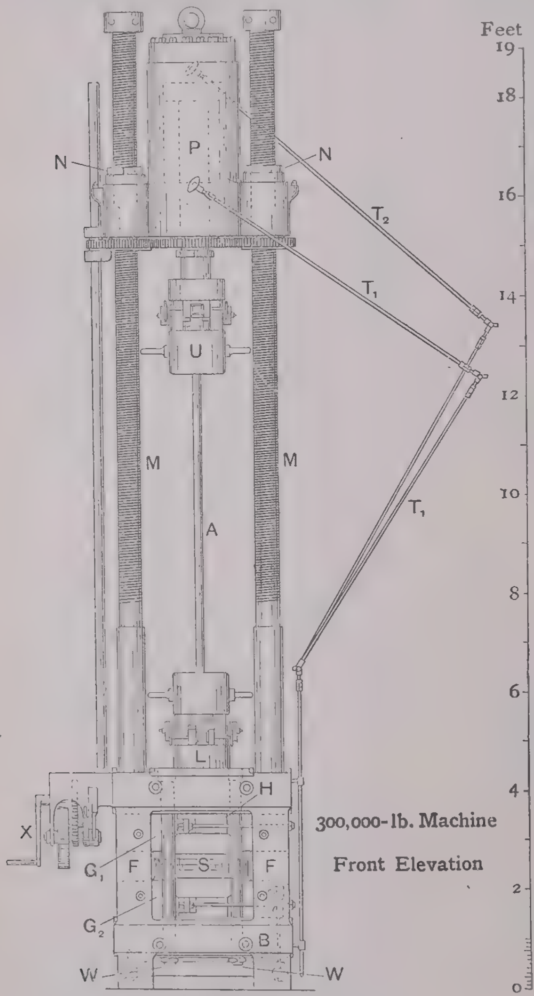


Fig. 21.

(i.) *Amsler Testing Machine.*—A 50-ton universal machine of this make is shown in *Fig.* 22. Each installation consists of three main parts:

(1) The press or actual testing machine P.
(2) The oil pump for producing the pressure C.
(3) The pendulum dynamometer D for measuring the load from the oil pressure in the pressure cylinder.

(ii.) *The Press.*—The press is fixed to the top part of the machine, and the movable cradle carrying the upper grips is freely suspended from the ram by two rods connected by a crosshead which rests on the top of the ram. The bottom of the cylinder of the press is held rigidly to the base of the machine by four round columns, which are made heavy enough to carry the superstructure and the maximum load without deformation.

The base itself does not take any of the load applied to the test bar T, it is required merely as a support for the machine and to raise the working parts to a convenient height above the floor level. It is made in the form of a hollow iron casting with an opening in the front for the recovery of the fractured test pieces.

The ram of the press is fitted in the cylinder with such precision that no collar or cup leather is required to ensure a sufficiently oil-tight working. The small amount of play which the ram has in the cylinder allows a slight percolation of oil to take place between them; this makes the movement of the ram in the cylinder very easy, eliminating friction and allowing an exact measurement of the total load to be made. The ram can be rotated by a handle, so that by occasionally moving it through a small angle the formation of ridges, etc., is avoided.

(iii.) *Oil Pump.*—The press is operated by oil pressure produced by a high-pressure pump (O, *Fig.* 22) driven by an electric motor from a countershaft (for small machines a hand drive can be employed). The arrangement of suction and delivery of the oil is such that if all the valves are kept closed the oil is automatically short-circuited back to the reservoir, which is carried on the pump stand. There are also returns for the oil to the reservoir from the top of the cylinder due to leakage, and for the released oil from under the ram.

Water pressure may be used to work the machine by making arrangements for transforming into oil pressure for working the press.

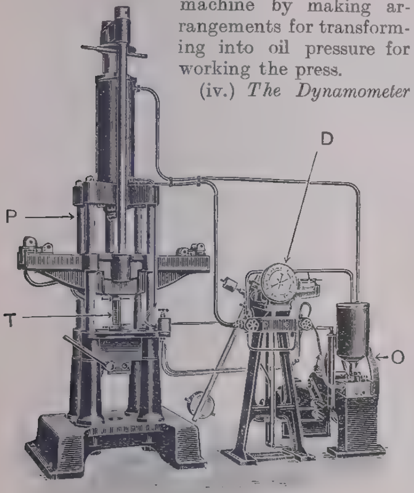


FIG. 22.

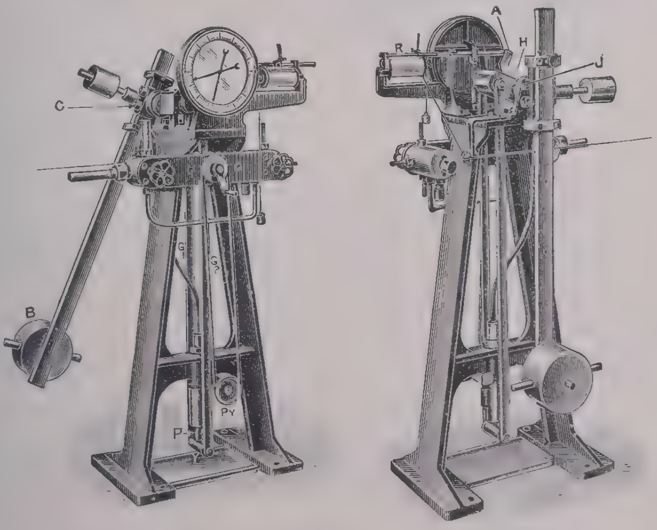
(iv.) *The Dynamometer Apparatus.*—The measurement of the load is made by means of a pendulum dynamometer D. The oil from the high-pressure pump is admitted to the straining cylinder through the dynamometer, so that the actual pressure in the cylinder is shown by the pointer which is operated by the dynamometer.

The pendulum dynamometer is a modification of the dead-weight gauge, which is used for calibrating pressure gauges, the pendulum arrangement being used to automatically balance the load on the plunger, as shown in *Fig.* 23. The oil pressure is applied to a plunger P, which is kept slowly revolving during the test by the pulley P*y*, from which it is transmitted to a yoke Y connecting two vertical rods G_1, G_2. The tops of these rods are attached through a ball-bearing coupling to a short block lever H, which is rigidly secured to a shaft J mounted in ball-bearings in the manometer frame. The depression of the block H by the pressure causes the heavy pendulum B to be deflected until it assumes a position of equilibrium. The point of suspension C of the pendulum can be altered so as to increase the sensitiveness of the machine, the connection to the recording gauge being adjusted at the same time.

The arm and rod A attached to the pendulum operates a pointer on a dial and the dynamometer is also provided with an autographic recording apparatus R.

As a rule each testing machine is accompanied by a pump and separate dynamometer, but it is possible for several machines to use a single pump, if worked in conjunction with an air accumulator, and a separate dynamometer for each machine.

For machines of large power, *i.e.* 300 tons and upwards, Amsler's measure the load by the extension of the columns which carry the top shackle. As these columns extend under the load, they are made to operate a piston in a cylinder. The shapes of the piston and cylinder are so designed that a small relative motion between the two causes a large displacement of the oil which fills the clearance separating them. The displacement of this oil is made to work an indicator which is calibrated in terms of the load.



(*a*) FIG. 23. (*b*)

§ (10) TORSION TESTING MACHINES.—Many universal testing machines of the lever type are fitted with a special shackle for applying torsional stress, as illustrated diagrammatically in *Fig.* 24, in which the torque is applied by a worm and worm wheel A and measured by running the jockey weight B along the lever arm C. It is usually, however, more convenient to use a separate machine, and *Fig.* 25 shows a machine of 250 pounds-foot capacity, made by Messrs. J. Buckton & Co., embodying the method of test mentioned above.

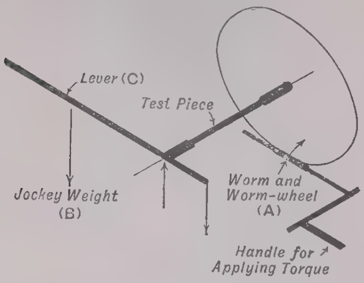


FIG. 24.

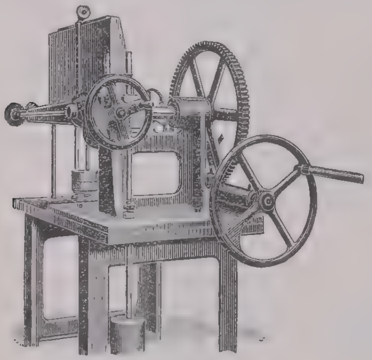


FIG. 25.

A neat little machine, designed by Professor Thurston and made by Messrs. W. H. Bailey & Co., is illustrated in *Fig.* 26. The test piece is placed between two independent jaws, one of which is rotated by hand through a worm and worm wheel. The twist is communicated through the test piece to the other jaw on which a weighted pendulum is attached. The resistance to deflection of the pendulum causes a torque to be applied to the test piece, and the angle of the deflection of the pendulum is a measure of that torque. The history of the test is recorded autographically on a chart attached to a drum which is secured to the jaw which is moved by the gear wheel. The deflection of the pendulum moves a pencil, which is secured to it, over a guide curve fixed to the frame of the machine in such a way that the movement perpendicular to the plane of rotation (that is, the height of the curve) is a measure of the torque, while the length of the curve is proportional to the angle of twist of the test piece.

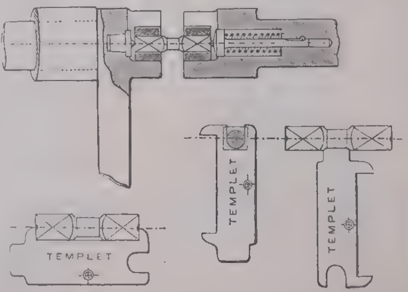
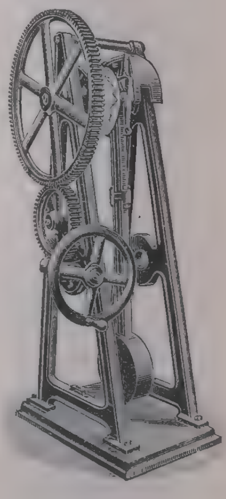


FIG. 26.

The two machines described above are typical of all of the methods employed for torsional testing machines.

§ (11) TRANSVERSE TESTING MACHINES.—Although a transverse testing shackle always forms part of a universal testing set, most testing machine manufacturers supply special machines particularly suitable for testing cast iron by cross-breaking. These machines enable cast-iron foundry bars to be tested rapidly, and give reliable information as to the mechanical properties of different mixtures.

Such a machine, supplied by Messrs. W. H. Bailey & Co., Ltd., is shown in *Fig.* 27. It is suitable for test bars up to 2 in. deep, 1 in. thick, and 36 in. between centres, and has a maximum capacity of 4500 lbs. An upward force is applied to the centre of the beam by means of a screw, turned by a hand-wheel, lifting a central shackle. The reaction is taken on the beam by two end supports and also on the lever near to the fulcrum. The load is balanced and measured by running the counterpoise along the lever while the deflection is obtained from the movement of the screw which applies the load.



§ (12) GRIPS FOR HOLDING TEST BARS.—As much care should be bestowed on the design of the grips for holding the test bars as on any of the other essential parts of the testing machine, if accurate results are required. It is important that these should be so designed that the resultant of all the forces acts along the axis of the bar, for, if this is not done, the amount of the ultimate load is affected.

The usual method of ensuring that this condition is fulfilled is by arranging for some form of spherical or swivelling seating between the test piece and the connection to the testing machine. The principal objection to most of these seatings is that they have no protection from dust, scale, or other small pieces which are apt to be scattered by the test piece on fracture.

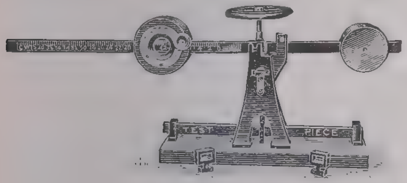


FIG. 27.

In the 100-ton Buckton machine (see § (6)) large hemispherical pieces, H_1 and H_2, are arranged in the top and bottom holders, and to these pieces the special attachments are fitted for test pieces of various kinds.

(i.) *Tension Shackles—Pin Grips.—Fig.* 28 shows the oldest method of holding the ends of plate specimens. A hole is drilled at each end of the test piece for a steel pin A, and the middle part of the plate is reduced in section. The relation of the size of the pinholes and the size of the heads of the test piece to the reduced section is such that the specimen always breaks in the reduced section. This form of holder is one of the best for small specimens, provided care is taken to ensure that the pinholes are accurately on the axis of the reduced part of the test piece, otherwise the stress across the section is not uniform and the specimen tears from the edge where the stress is greatest.

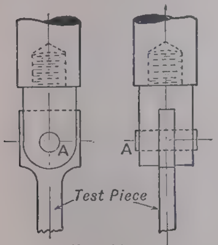


FIG. 28.

(ii.) *Wedge Grips.* — The ordinary method adopted for holding specimens of ductile materials is between two wedges with roughed surfaces, as illustrated in *Fig.* 29, but it does not necessarily ensure that the line of pull is axial. In order to obtain the best results with this method the following points must be borne in mind:

(1) The position of the wedges in the head and of the test piece in the wedges.

Fig. 29 (A) shows a sectional view of the testing machine head with the wedge grips and test piece in position. The wedge grips are bearing their full length against the head and the specimen is bearing for the full length of the wedge grips, and this is the correct condition.

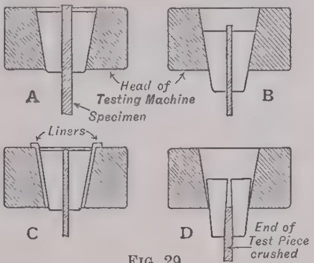
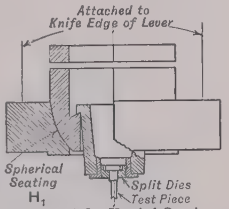


FIG. 29.

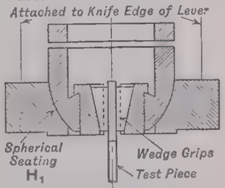
Fig. 29 (B) shows the position of the wedge grips when the specimen is too thin, and while the test piece bears the whole length of the wedge grips, the wedge grips are not bearing their full length against the head. In this case liners are used to keep the wedges back in their proper place, as in *Fig.* 29 (C), although to obviate this difficulty most testing machines are supplied with pairs of wedge grips suitable for various ranges in the thickness of the plate.

If the specimen does not bear for the full length of the wedge grips (*Fig.* 29, D) there is a tendency, on the application of the load, for the end of the specimen to be crushed and a force to be applied to the head which may split it.

(2) The tendency for the wedges to grip the test piece more on one edge than the other.



Arrangement for Headed Specimens in 100 Ton Buckton Machine

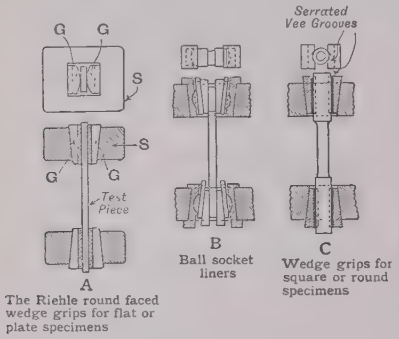


Arrangement for Wedge Grips in 100 Ton Buckton Machine

FIG. 30.

This arises if the specimen is thicker on one side or through want of proper adjustment, when the holders for the wedges are split and separate from the head, as in the Buckton

100-ton machine (*Fig.* 30). To avoid this difficulty Messrs. Riehle Bros. supply wedges with a round face. In *Fig.* 31 (A), G are the round-faced wedge grips contained in the shackle S. Another method for securing proper alignment is a form of ball-socket liner shown in *Fig.* 31 (B).



A
The Riehle round faced wedge grips for flat or plate specimens

B
Ball socket liners

C
Wedge grips for square or round specimens

FIG. 31.

For round or square ductile pieces the wedge grips are provided with serrated V-grooves, as shown in *Fig.* 31 (C).

The ends of the specimen are usually enlarged, as shown in the sketch, but with mild steel or iron this enlargement may be dispensed with.

(iii.) *Self-aligning Grips.* — When testing brittle materials or for accurate experiments the alignment of the specimen is especially of importance. It is usual in these cases to use circular test pieces with the ends to fit special spherical seated holders. *Fig.* 32 shows various methods of machining the test piece and attaching to the shackles.

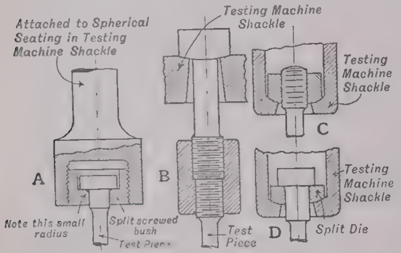
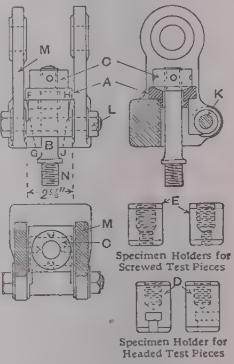


FIG. 32.

(iv.) *Avery 10-ton Shackles.*—The arrangement of Avery's 10-ton shackles is shown in *Fig.* 33. The holder M is attached to a link hanging from the beam of the testing machine by a second link. This holder consists of a forging machined to take a spherical seating A on which the pull of the test piece is taken through a tool steel stem B and nut C. The stem is screwed at N to take specimen holders of various sizes. Two forms of specimen holder are employed, viz. (1) for headed test pieces D and (2) for screwed test pieces E.

The shackle is also arranged for use with wedges. For this purpose the nut, stem, seating, and bolt L are removed and wedges are inserted, which bear along the surfaces FG and HJ. A handle works in the bearing K, and is arranged with lugs which move both wedges up or down at the same time. This facilitates gripping of the specimens and enables tests to be carried out rapidly.



Specimen Holders for Screwed Test Pieces

Specimen Holder for Headed Test Pieces

FIG. 33.

The top holder, for connection to the lever, is shown in the figure. The bottom holder, attached to the ram-straining rod, works on exactly the same principle; a slide is, however, provided for it so as to keep it parallel to the vertical testing machine guides. Both holders are given in the drawing of the testing machine in *Fig.* 6.

(v.) *N.P.L. Self-aligning 10-ton Shackles.*—The shackles given in *Fig.* 34 were designed by the author for tensile testing, more particularly for use in experiments for the determination of the elastic limit and elastic modulus. They consist of a bolt A with a spherical nut B on one end and a holder for screwed or headed test pieces on the other. The nut bears on a spherical seating G, which fits into the shackle of the testing machine. The spherical seating is kept free from dirt by being closed in at the top by a plate E and at the bottom by a piece of wash-leather F, which fits round the bolt. The wash-leather, although forming a dust-proof cover, allows free movement of the nut and bolt on the spherical seating. The space in the bush is filled with vaseline. These shackles have been in constant use and have been found to give every satisfaction.

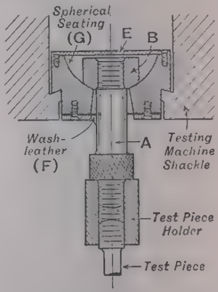


FIG. 34.

(vi.) *Grips for Testing Rope.*—The gripping of ropes or cables requires considerable care in order to obtain reliable results; it is important that the grips should not damage the ends of the rope, otherwise fracture will occur in the sockets.

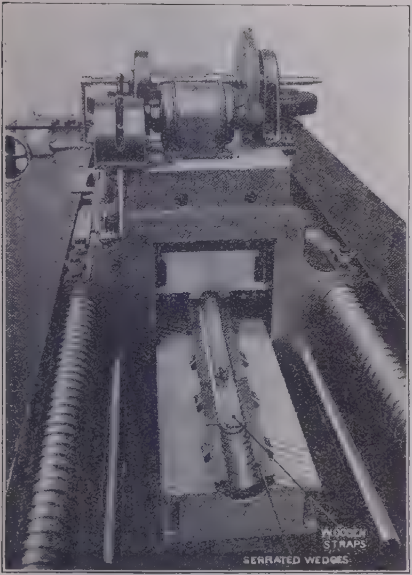


FIG. 35.

The tests are more conveniently carried out in horizontal machines, where there is plenty of available length, than in vertical machines. Serrated wedges (*Fig.* 35) are used for gripping both wire and hemp ropes, and, to prevent them from being damaged, wooden straps of a suitable thickness and length are placed between the grips and the rope. In order to seize the rope over a considerable length these special wedge grips are made very long.

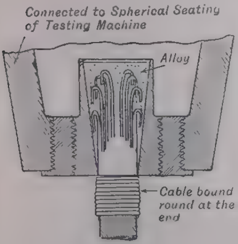

Connected to Spherical Seating of Testing Machine
Alloy
Cable bound round at the end

FIG. 36.

Where only vertical machines are available a satisfactory way of holding wire rope for tensile tests is to use conical dies in the shackles and seal the ends of the wire in them, as shown in *Fig.* 36, with a low melting-point alloy. The wire is first tightly bound with fine wire at about 5 in. from each end, the end strands are then opened, bent over, and tinned. The rope is next set centrally in the dies, with a little fire-clay, and a hard alloy of lead and antimony run in forming a solid end.

For hemp rope and small cables a method which is largely used is to splice an eye on to each end and test by placing a steel pin through the eye. A more reliable method is to roll the ends round specially prepared helically grooved drums and clamp them. Such tools are supplied by Messrs. Tinius Olsen Testing Machine Co. in the United States of America, or Alfred J. Amsler & Co. (Switzerland); they are costly to build, but give accurate results in testing such materials.

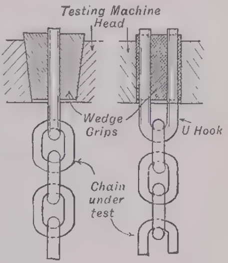


FIG. 37.

(vii.) *Grips for testing Chain in Tension.*—For testing separate chain links or complete chains a piece of steel bent into U form is passed through each of the end links. The open ends of the U-pieces are then gripped in the flat wedges used for testing flat bars (see *Fig.* 37).

Where it is necessary to carry out a considerable amount of chain testing, blocks to suit the various sizes of chain are provided, which fit into the heads of the testing machine GH (*Fig.* 38). By means of these blocks sections of a chain can be tested under a

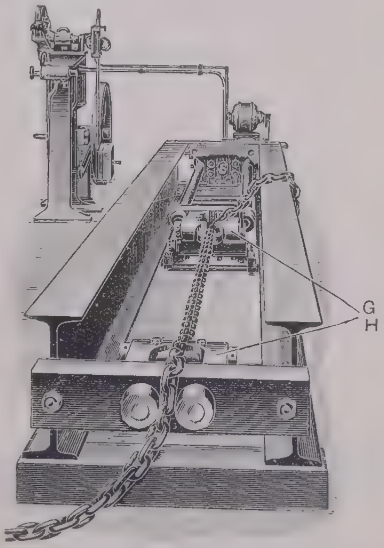


FIG. 38.

proof load; this is not possible with the U-hook method.

(viii.) *Compression Shackles.* — Spherical shackles for compression tests are necessary for accurate work. These either rest on the lower shackle or hang from the upper shackle of the testing machine. Care must be employed in using any of these arrangements to ensure that the axis of the test piece is in line with that of the bearing blocks and testing machine.

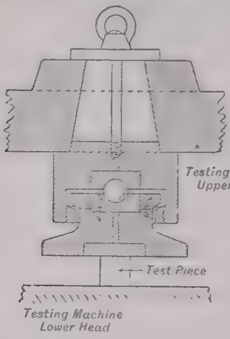


FIG. 39.

Fig. 39 shows a ball-bearing block arranged to fit the upper crosshead of an universal testing machine; this tool allows for free motion in all directions.

(ix.) *Transverse Test Tools.—Figs.* 40A and 40B show two types of shackles for transverse testing. *Fig.* 40A indicates the roller bolster supports and *Fig.* 40B the arrangement adopted with the Buckton vertical testing machine. In the latter method it will be noted that the pressure is applied in halves, at W_1 and W_2, giving a small central span D of uniform bending moment. The object of this method is to reduce local crushing of the test piece T. In both arrangements the standards A, A can be adjusted to graduations on the beam B to give different lengths of span L.

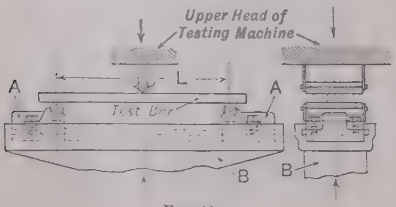


FIG. 40A.

As timber is not homogeneous, four-point loading is usually adopted (*Fig.* 41). This gives a large span over which the bending moment is uniform and in which the beam theoretically bends in the arc of a circle. *Fig.* 42 is a photograph of these shackles fitted to a 10-ton Buckton testing machine.

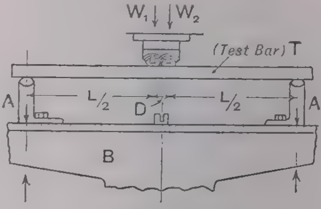


FIG. 40B.

The following points in regard to apparatus for transverse tests should be observed :

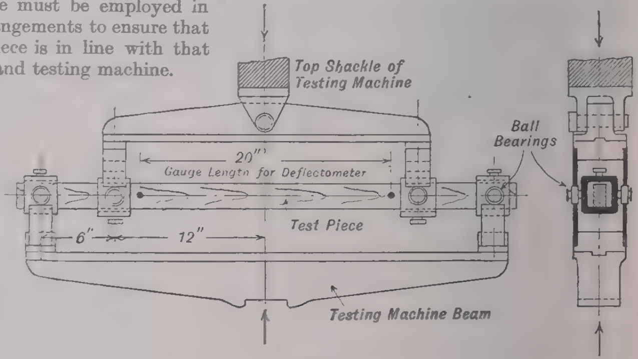

Top Shackle of Testing Machine
Ball Bearings
20''
Gauge Length for Deflectometer
Test Piece
6''
12''
Testing Machine Beam

FIG. 41.

Rocking supports should be used at the ends of specimens.

Metal plates should be provided, if necessary, to prevent a high intensity of pressure under the point of loading or at the supports.

Deflectometers or deformeters should not be attached close to the points of application of the loads.

If deflections are to be observed, it is advisable that the rate of loading should be such that the readings can be taken while the load is being continuously applied. If a stoppage is unavoidable, the time intervals should be constant for the test.

(x.) *Shear Shackles.*—There are two methods of applying a shear stress to materials, one by a torsion test and the other by direct shearing or punching. The torsion formula only holds for perfectly elastic bodies, and if it is used for the breaking stress a figure greater than the real shearing stress is obtained.

Various methods for direct shear are in use, but with these the shear is always accompanied by a certain amount of bending and compression.

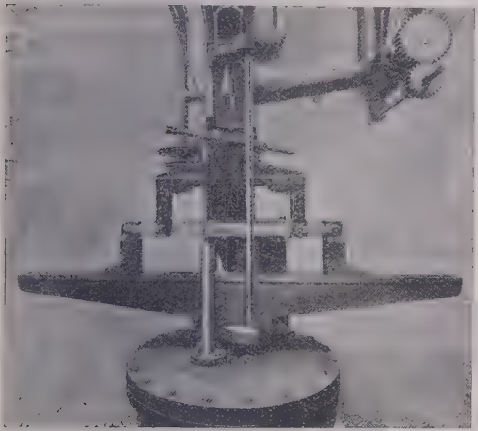


FIG. 42.

(xi.) *Tests of Bar and Plate.*—*Fig.* 43 indicates a shear test tool for round specimens. The specimen A is clamped to a block B and, with the testing machine adjusted for compression testing, the load is applied to the shearing tool C. The test can be arranged so that the specimen is broken in either single or double shear. The dies D, as well as the shearing tool, are made of tempered tool steel ground to an edge. Provision is made for wear of the tools, and they are inserted in such a manner that they can be easily removed when necessary for grinding.

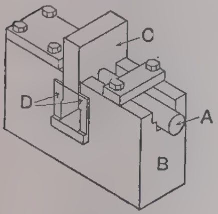


FIG. 43.

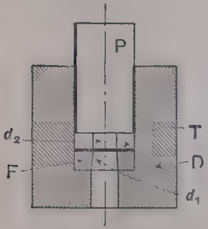


FIG. 44.

A similar method can be used for flat plate, the form of the shearing tools and dies requiring slight modification to that shown in *Fig.* 43.

(xii.) *Test of very Thin Plate.*—An arrangement for shear tests on very thin plates or slices from a bar, used at the N.P.L., is shown in *Fig.* 44. The test piece T is placed in a die D having a hardened steel facing F. A hardened steel plunger P, with a long bearing in the die, is placed on the test piece. The hole in the facing (d_1) is slightly taper (the figure is exaggerated), and the end of the plunger (d_2) is made a good fit in the hole.

The gear is placed between the compression shackles of the testing machine and the load measured, which enables the plunger to push a hole through the test piece.

If t = the thickness of the test piece and W = the load; then the shear stress = $W/\pi d_1 t$.

(xiii.) *Test of Timber in Shear* (see section on timber tests, § (127)).

(xiv.) *Torsion Shackles.*—The easiest way of holding a torsion test piece in the testing machine is to have enlarged ends, either square or round, with two keyways cut in them (*Fig.* 45).

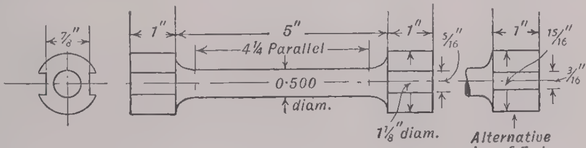


FIG. 45.

§ (13) CALIBRATION OF TESTING MACHINES.—The only satisfactory method of calibrating the readings of a testing machine is by using dead loads through the full range, such as is adopted in the Emery machines, and this method from the point of view of accuracy is above criticism.

Several devices are used to attain the same ends. If dead loads are not directly employed verification depends on a previous calibration of the device used, so that it ultimately depends on a dead-load method.

Next to loading by dead weights the employment of calibrating or proving levers is the most acceptable method. This arrangement is shown in *Fig.* 46. The levers L_1 and L_2, carefully constructed, rest on knife edges in a double chair C_1, which itself rests on the weighing table T_1 of the machine. A single chair C_2 is placed on the other knife edges and forms the fulcrum of the levers; the upper shackle C_2 of the machine is brought down on to this chair. A leverage of ten is generally adopted for calibrating machines up to ten tons capacity, and a leverage of twenty up to fifty tons capacity, which is the maximum to which this method of verification is usually carried.

Other methods of checking are as follows:

(i.) *By a standard test bar* made of steel whose modulus of elasticity is known by previous experiment. A test piece is turned to fit the testing machine, and of such a size that when the maximum load is applied to it the limit of proportionality of load to extension will not be exceeded. An extensometer is fitted to the test piece, and readings of it taken at various loads shown by the testing machine indicator. These readings are then compared with those calculated from the modulus of elasticity, and a verification of the testing machine scale obtained. This method is difficult to carry out except by persons skilled in such work.

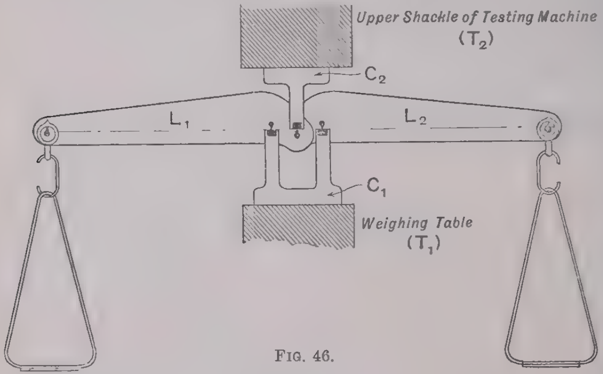


FIG. 46.

(ii.) *With Crushers.*—These are cylinders of copper whose diminution in length with load is known. A series of crushers are loaded in the compression apparatus of the machine with various values of the load, which are noted from the machine indicator. The actual forces exerted are deduced from the diminution of length of the crushers, and these are compared with the indications of the machine.

(iii.) *By a Series of Tensile Test Pieces.*—These are made in duplicate and of varying diameters, so that the largest test pieces will take nearly the full capacity of the machine.

They are all turned out of the same bar of homogeneous material, and one series is broken in the testing machine to be calibrated, and the duplicate series tested under the same conditions in a machine whose accuracy is known.

Both methods (ii.) and (iii.) are simple, but are not available for a greater accuracy than about ±2 per cent.

(iv.) *An improvement of the copper crusher method* has been suggested by Schule and Brunner,[1] who used wood in the form of slabs of small thickness (2 to 4 cm.) as crushers. Wood has the unique property of offering the same resistance to crushing whether tested as cubes or thin slabs. This method dispenses with all measurements of deformation which necessitate the use of instruments of precision, and by selecting wood well dried and without flaws, an accuracy of 1 per cent can be obtained.

Messrs. Schule and Brunner found that the wood of Coniferae is the most suitable owing to its regular structure. Five or six pieces of wood of each cross-sectional area are tested in a standard machine as well as in the machine which is under calibration.

(v.) *With a "Standardising Box."*—This is an instrument supplied by Messrs. Alfred J. Amsler & Co. for quick and accurate checking of testing machines. It is essentially a hollow test bar whose strains under loading are measured by its diminution or increase in volume. It is filled with mercury, and has a horizontal capillary tube, also partly filled with mercury, attached to it on one side, and a micrometer, working a plunger in and out of the box, on the other side. When the instrument is stressed axially the position of the mercury in the capillary tube is altered owing to the change in volume. The position of the mercury is then restored to the zero position by the micrometer. The reading of the micrometer is calibrated under known loads, and this calibration used in conjunction with the standardising box to check the testing machine.

The methods described above are applicable to most testing machines, but with single lever vertical machines a simple test can be carried out quickly and accurately in the following way:

As the load is weighed by a jockey weight, and as there is only one lever, it is only necessary to verify the weight of the jockey and the short arm-length of the lever to ensure that the machine is reading correctly.

(vi.) *Weight of the Movable Counterpoise.*—This can be found by lifting it from the lever with a suitable weighing machine suspended from a crane or, where this is inconvenient, by means of the lever itself, in the following way. First balance the beam and adjust the vernier to zero, then hang up a known weight from the beam at a measured distance from the fulcrum. This raises the beam, and the counterpoise is run along until balance is again restored.

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[1] "A New Method of comparing the Indications of Testing Machines," *Proc. Int. Assoc. of Test. Mat.*, New York, 1912.

If W = weight of counterpoise,
w_1 = hanging weight,
l_1 = distance of hanging weight from fulcrum,
L = distance counterpoise is moved to restore balance,
then $W = w_1 \times l_1/L$.

(vii.) *Distance between the Knife Edges.*—After the beam has been balanced and the vernier adjusted to zero, a heavy weight w_2 is hung in the shackle of the machine and the counterpoise moved forward until balance is restored.

If W = weight of counterpoise,
L = distance counterpoise is moved to restore balance,
w_2 = weight hung in shackle,
l_2 = knife-edge distance,
then knife-edge distance $= l_2 = L \times W/w_2$.

(viii.) *Sensitivity.*—The sensitivity of the machine can be determined by finding the greatest weight which can be hung from the shackles without causing the beam to move after it has been balanced.

J. H. Wicksteed[1] found that with a 100-ton machine in equipoise, but unloaded, so that the dead weight of lever and poise is equivalent to 6 tons on the knife edge, a pull on the shackles applied through a silk thread breaking with a load of 3 or 4 lbs. is sufficient to raise the lever arm through its whole arc.

(ix.) *Zero Error.*—Care should be taken to see that the zero of the machine is in adjustment before commencing a test.

II. Elongation and Contraction of Area

§ (14) Ductility.—For workshop use a bend test gives much information as to the ductility of a material, and requires very little special apparatus to carry it out. The usual measures of ductility, however, are the ultimate percentage elongation or contraction of area at fracture in a tensile test.

§ (15) Distribution of Elongation in the Test Bar.—If a test piece of ductile material is marked out in one-inch lengths and then tested in tension, it is found that it breaks with a neck or waist, and that the increase in length of the section including the neck is considerably more than at any other point along the bar. Observations for some bars are given in Table 1, and results from them are plotted in *Fig.* 47, where the stretched lengths are plotted as ordinates and the original distances along the bar as abscissae. The resulting curve is seen to consist of two parallel branches connected by a S curve, and it will be evident from the curve that the vertical distance between the two parallel branches is the local extension due to the necking of the test piece.

In *Fig.* 48 the same results are plotted in another way. The increases in length in each inch are plotted as ordinates at the centre of each inch length and connected by a curve. It is seen that the rate of elongation at the fracture is very large, and that the curve is nearly symmetrical.

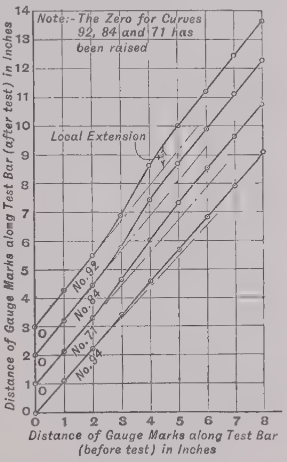

FIG. 47

From the figures in Table 1 the percentage elongations on various gauge lengths can be calculated. Taking the fracture as near to the centre as possible, these results are given in Table 2 and are calculated as follows:

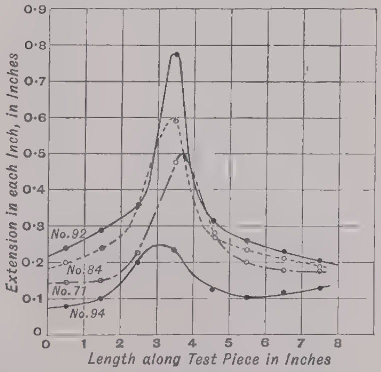

FIG. 48.

If the original gauge length $= L_1$,
the final gauge length $= L_2$,
the elongation $= L_2 - L_1$,
and the per cent elongation $= 100(L_2 - L_1)/L_1$.

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[1] *Proc. Inst. Mech. Eng.*, 1888, p. 454.

It will be evident from the table that the percentage elongation is dependent upon the gauge length, decreasing as the gauge length increases, and that for strictly comparative results the same gauge length should be adopted for bars of the same diameter. Also, for strict comparability, the position of the fracture should be the same in each bar. If the bar breaks near to the end of the gauge length the percentage elongation is low. The formula for calculating the error involved has been expressed by Unwin [1] as follows:

"Let e_1 be the percentage of elongation in a gauge length $L+2X$, and e_2 the elongation in a gauge length $L-2X$, the fracture being at the centre in both cases. Then the elongation in a gauge length L when the fracture is at X from the centre is $e=\frac{1}{2}(e_1+e_2)$."

The results in Table 3, taken from Unwin's paper, show the magnitude of the error. Unless the gauge mark is in the part affected by the local contraction, the effect on the percentage elongation is small.

§ (16) BARBA'S LAW (ELONGATION OF GEOMETRICALLY SIMILAR TEST BARS).—The law connecting the elongation of geometrically similar test pieces was first given by J. Barba [2] in 1880, who observed that similar test bars deform similarly. It follows that, for cylindrical specimens, if the ratio of gauge length to diameter is constant, the percentage elongation is also constant. This is shown clearly by the results on a mild steel and medium carbon steel given in Table 4.

Unwin says [2] that "in plate test bars, not strictly geometrically similar, the percentage of elongation is practically constant, if the ratio of gauge length to square root of cross-section is constant. The form of cross-section within somewhat wide limits, if the area is constant, does not appear to influence the elongation."

§ (17) THE ELONGATION EQUATION. (i.) *Variation with Gauge Length.*—The elongation of a test bar is made up of two parts:

(*a*) The general extension, occurring before the maximum load is reached, which is approximately uniformly distributed along the bar and therefore proportional to the gauge length.

(*b*) A local extension independent of the gauge length occurring after the maximum load has been reached.

If L = the gauge length,
e = the total extension,

then $e=a+bL$, where a and b are constants, the former for the local extension and the latter for the general extension.

The percentage extension

$$=100\times\frac{e}{L}=\frac{100}{L}(a+bL)=100\left(\frac{a}{L}+b\right), \quad (1)$$

i.e. the percentage extension for a given sectional area decreases as L is increased.

(ii.) *Variation with Cross-section.*—Only the local extension is affected by the cross-sectional dimensions of the test piece, *i.e.* the term a in equation (1).

Since the local contraction is proportional to the *linear* dimensions of the cross-section, and the extension is not affected by the form of the cross-section,

$$a=c\sqrt{A}, \quad . \quad . \quad . \quad (2)$$

where A = the cross-sectional area,
c = a constant;

i.e. for dissimilar test bars

$$\%\text{ elongation}=100\frac{e}{L}=100\left(\frac{c\sqrt{A}}{L}+b\right). \quad (3)$$

This formula is true provided

(*a*) The gauge points are not too close to the enlarged ends of the test piece.

(*b*) The length L is not so short as to fall within the area affected by the local contraction.

Equation (3) is in agreement with Barba's law for similar test pieces that, when similarity is maintained, the percentage elongation is constant because, in that case, $\sqrt{A}/L$, and, therefore, e becomes constant.

Unwin [3] gives the following method of least squares for calculating the values of b and c, in the elongation equation, from a series of observations:

This equation is

$$e\%=\frac{c\sqrt{A}}{L}+b.$$

For simplicity, let $\sqrt{A}/L=a$, and suppose the elongations have been observed for n gauge lengths. Then the most probable values of the constants are given by the equations

$$c=\frac{n\Sigma ea-\Sigma e\Sigma a}{n\Sigma a^2-(\Sigma a)^2} \text{ and } b=\frac{\Sigma e\Sigma a^2-\Sigma ea\Sigma a}{n\Sigma a^2-(\Sigma a)^2}.$$

Applying these results to bar 84 (Table 2), whose diameter = 1·000 in. and area = 0·7854 sq. in., and percentage elongation on

$$\left.\begin{array}{ll} 3''=41{\cdot}0 & 6''=31{\cdot}8 \\ 4''=36{\cdot}5 & 7''=30{\cdot}1 \\ 5''=34{\cdot}0 & 8''=28{\cdot}8 \end{array}\right\} \begin{array}{c} \text{we have} \\ \sqrt{A}=0{\cdot}8863 \\ \text{and } n=6. \end{array}$$

L.	$\frac{\sqrt{A}}{L}=a$.	e.	ea.	a^2.
3	·2954	41·0	12·110	·08724
4	·2216	36·5	8·088	·04910
5	·1773	34·0	6·027	·03143
6	·1477	31·8	4·696	·02181
7	·1266	30·1	3·810	·01603
8	·1108	28·8	3·191	·01227
Σ =	1·0794	202·2	37·922	·21798

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[1] "Tensile Tests of Mild Steel," *Inst. Cir. Eng. Proc.*, 1904, clv. 175.

[2] Barba's Law, see Unwin, "Tensile Tests of Mild Steel," *Inst. Civ. Eng. Proc.*, 1904, clv. 179.

[3] "Tensile Tests of Mild Steel," *Inst. Cir. Eng. Proc.*, 1904, clv. 233.

Table 1

Extensions on 1-inch Lengths at Different Distances from the Fracture

No. of Test Piece.	Diameter of Test Piece. Inches.	Yield Stress. Tons/Sq. In.	Ultimate Stress. Tons/Sq. In.	Extension in Inches in each Inch along the Bar.								Total Extension in Inches.
				0-1	1-2	2-3	3-4	4-5	5-6	6-7	7-8	
94	1·000	22·9	47·1	·08	·10	·20	·23	·12	·10	·12	·13	1·08
71	0·999	14·6	22·9	·15	·15	·22	·48	·28	·20	·18	·18	1·84
84	1·000	16·3	29·9	·20	·24	·37	·59	·27	·23	·21	·19	2·30
92	1·000	15·2	24·7	·24	·29	·36	·78	·31	·26	·23	·20	2·67

(The fracture was included between 3-4 in each case.)

Table 2

Percentage Extension on Different Gauge Lengths (including Fracture) on 1-inch Diameter Bars. See Table 1

No. of Test Piece.	Gauge Length in Inches.					
	3.	4.	5.	6.	7.	8.
94	18·3	16·2	15·0	14·3	13·9	13·5
71	32·7	29·5	26·6	25·2	23·7	23·0
84	41·0	36·5	34·0	31·8	30·1	28·8
92	48·3	43·0	40·0	37·2	35·3	33·4

Table 3

Elongation Percentage in 8 Inches, when the Fracture is not at the Centre of the Gauge Length

No. of Bar.	Distance of Fracture from Centre of Bar in Inches.				
	0.	1.	2.	3.	4.
	Fracture at Centre.				Fracture at Gauge Point.
2361	29·2	29·1	28·9	28·1	23·5
2362	29·5	29 5	29·5	29·1	23·9
2363	29·8	29·7	29·8	29·2	24·4
2364	25·3	25·3	25·3	24·4	20·2
2365	29·0	29·0	29·0	28·5	24·1
2390	26·7	26·6	26·6	26·6	23·2
2391	30·5	30·5	29·7	28·3	23·5

Table 4

Tests on Similar Test Pieces

No. of Test Piece.	Diameter of Test Piece. D Inches.	Gauge Length. L inches.	Ratio. L D.	Yield Stress. Tons Sq. In.	Ultimate Stress. Tons Sq. In.	Extension. Per cent.	Reduction of Area at Fracture. Per cent.
B10D	·550	4·6	8·36	28·41	51·69	17·4	38·4
B10E	·420	3·5	8·33	28·88	51·27	17·2	37·5
B10F	·330	2·72	8·24	29·01	51·70	16·8	40·8
A9D	·550	4·6	8·36	21·26	26·52	33·2	70·4
A9E	·420	3·5	8·33	20·51	26·27	32·8	68·7
A9F	·330	2·72	8·24	21·06	26·90	32·2	67·8

$$(\Sigma a)^2 = 1{\cdot}165,$$

$$c = \frac{(6 \times 37{\cdot}92) - (202{\cdot}2 \times 1{\cdot}0794)}{(6 \times {\cdot}2179) - 1{\cdot}165} = 65{\cdot}5,$$

$$b = \frac{(202{\cdot}2 \times {\cdot}2179) - (37{\cdot}92 \times 1{\cdot}079)}{(6 \times {\cdot}2179) - 1{\cdot}165} = 22{\cdot}1,$$

and therefore the elongation equation for this bar of material is

$$e\% = \frac{65{\cdot}5\sqrt{\mathrm{A}}}{\mathrm{L}} + 22{\cdot}1.$$

§ (18) CONTRACTION OF AREA AT FRACTURE.—If a is the initial area of the test bar and A the area at the point of fracture, then the reduction of area is equal to $(a - \mathrm{A})$ and the percentage reduction of area $= 100 \times (a - \mathrm{A})/a$.

If the material is perfectly plastic the percentage contraction of area is equal to the percentage elongation where the latter is calculated on the *final* length of the test bar, *i.e.* $100 \times (a - \mathrm{A})/a = 100 \times (\mathrm{L}_2 - \mathrm{L}_1)/\mathrm{L}_2$. ($\mathrm{L}_1$ and L_2 are the initial and final lengths respectively.) The proof of this is as follows:

The volume of the test piece is assumed to be constant, therefore

$$\mathrm{L}_1 \times a = \mathrm{L}_2 \times \mathrm{A}$$

or

$$\frac{\mathrm{A}}{a} = \frac{\mathrm{L}_1}{\mathrm{L}_2};$$

subtracting each from unity, we have

$$1 - \frac{\mathrm{A}}{a} = 1 - \frac{\mathrm{L}_1}{\mathrm{L}_2}$$

or

$$\frac{a - \mathrm{A}}{a} = \frac{\mathrm{L}_2 - \mathrm{L}_1}{\mathrm{L}_2}.$$

The agreement between elongation and contraction of area is only for the short part of the specimen where there is plastic deformation, and it does not apply to greater lengths.

Professor Elliot [1] has given a method of calculating the constants b and c of the elongation equation ($e = c\sqrt{\mathrm{A}}/\mathrm{L} \div b$) when the elongation is only known on one gauge length together with the contraction of area of the test bar. The method depends upon the assumption of a value for the original length of the contracted region. Taking this as $m\sqrt{\mathrm{A}}$, Elliot found that m varied from 0·9 to 2·0, and adopting the value of 1·37 he obtained very fair agreement with results previously published by Unwin.[2]

III. PROCEDURE IN ORDINARY COMMERCIAL TESTING

Commercial testing is undertaken primarily to ascertain if the material is of the quality required by a specification. These inspection or reception tests have to be made quickly, and are principally directed to giving indications of the strength and ductility of the material.

§ (19) PREPARATION AND SELECTION OF TEST PIECES.—The test pieces must be selected so as to give a reliable indication of the material as a whole. Sampling is inherently imperfect, and is therefore apt to cause difficulties. Samples should always be taken and stamped in the presence of an inspector acting on behalf of the purchaser, and there should be safeguards to avoid heat treatment or other manipulation after selection.

The number of tests should be specified, and, in order to cover errors which are likely to arise, limits should be allowed on the test results, and provision made for re-test if necessary. A usual clause is that, should the test piece fail to conform to the specification, the contractor shall have the opportunity of two further samples being selected by the inspector and tested at the manufacturer's expense. Both of these samples must pass in order for the consignment to be accepted.

Test pieces, if to be taken from a sample, such as a tyre, axle, crank-shaft, etc., should be cut out by cold sawing or drilling, and subsequently prepared by milling, turning, or planing. Should shearing be unavoidable, care must be taken to ensure that the material is not bent, and that the sheared edges are entirely removed before testing. Cutting by oxyacetylene blowpipe should not be allowed, as it alters the structure of the material for from 2 to 3 in. from the cut.

Iron or steel plates are usually tested both in the direction of rolling and across it, and with cast iron special bars are made from each cast for transverse test. For some materials special bars form part of the casting and are afterwards cut off for testing, but it is usual to specify the position of these bars and whether they are to be sand or chill castings.

For special tests some form of heat treatment, such as normalising, annealing, or tempering, may be required.

Annealing means reheating followed by slow cooling. Its purposes may be either to remove internal stresses and induce softness, or to refine the crystalline structure. In the latter case the temperature used must exceed the upper critical range, but not more than by 50° C.

Normalising means heating a steel (however previously treated) to a temperature exceeding its upper critical range (but not more than by 50° C.) and allowing it to cool freely in air. It is desirable that the steel should soak at the required temperature for about 15 minutes.

Hardening means heating a steel to its normalising temperature and cooling more or less rapidly in a suitable medium, such as water, air, or oil. Tempering is to reduce the hardness and increase the toughness, and is carried out by heating the steel (however previously hardened) to a temperature not exceeding its carbon change point.

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[1] "The Relation of the Constants of the Elongation Equation to Contraction of Area," *Inst. Civ. Eng. Proc.*, 1904, clviii. 390.

[2] "Tensile Tests of Mild Steel," *Inst. Civ. Eng. Proc.*, 1904, clv. 175.

§ (20) TENSION TESTS.—The data ordinarily observed in a tensile test for reception purposes is

Yield load.
Maximum load.
Elongation after fracture.
Contraction of area at fracture.

§ (21) YIELD POINT AND ELASTIC LIMIT.—Very often there seems to be confusion as to exactly what is meant by the elastic limit and the yield point. It is a common practice for the elastic limit to be specified when what is actually required is the yield point. The actual elastic limit can only be obtained by the use of delicate strain-measuring instruments (see § (62)); the yield point is slightly higher than the elastic limit, and is the point at which there is an increase in the extension without a corresponding increase in the stress. This point occurs in wrought iron or mild or medium carbon steel, and is usually well marked. With hard steel, bronze, and most alloys it is non-existent, the rate of extension with load increasing after the elastic limit without any sudden jump.

For commercial testing the yield stress is the load per unit area at which there is a sudden visible increase in the extension between the gauge marks on the test piece, or it may be taken as the stress at which the beam or indicator distinctly drops when the load is applied at a uniform rate.

The author prefers to record that "there was no yield point" in cases where these conditions are not observed, but, as some specifications require an arbitrary yield point, it is taken at the point at which the test piece extends approximately 1/200 of the gauge length. With a 2-inch gauge length this, for practical purposes, is the point where the dividers, when set at 2 inches, just fail to "feel" the gauge mark.

Johnson[1] proposes that "in view of the difficulty of determining the true elastic limit an apparent elastic limit be taken as the point on the stress strain diagram at which the rate of deformation is 50 per cent greater than it is at the origin. Under this definition the apparent elastic limit would practically correspond to the yield point in materials having such a point and would give a reasonable value for such materials as cast iron, or hard steel, for which this diagram shows a very gradual curvature away from the straight line. Such a criterion has much merit and would accomplish the following results:

"(i.) It would always fix one and the same well-defined point.

"(ii.) This point would always correspond to so small a permanent deformation as to be, for many practical purposes, the true elastic limit.

"(iii.) It is equally applicable to all kinds of tests, whether on specimens or on finished members or structures, where deformations of any kind can be correctly measured.

"While the 50 per cent increase in the rate of deformation is purely arbitrary, it is not large enough to fix a point having an appreciable permanent set, but it is large enough to fix a well-defined point on the stress strain diagram."

Unwin,[2] however, says that "such a point has no significance," also "that the attempt sometimes made to define arbitrarily a yield point for materials which have no distinct yield point, and, often as manufactured, no elastic limit either, appears to him to be useless and misleading."

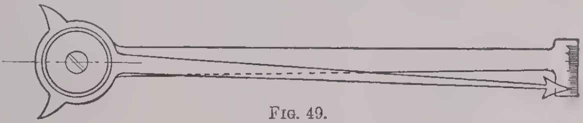

FIG. 49.

It is convenient, where a considerable amount of testing has to be done on the same size of test piece, to use a simplified extension measurer applied to the test piece instead of dividers. G. H. Roberts[3] describes and illustrates an instrument of this kind in which the extension of the test piece is communicated to the pointer on a dial. One division on the dial indicates a movement between the jaws of 0·001 inch. The yield point is seen quite definitely if it occurs, or the load for an extension of 1/200 of the gauge length can be also noted with fair accuracy.

Capp's[4] multiplying dividers answer the same purpose. They are illustrated in *Fig.* 49, and consist of two arms so pivoted that the motion of the points in which the short arms terminate is multiplied about ten times at the scale and pointer forming the ends of the long arms. To use the instrument the points of the dividers are placed in the centre punch marks on the test piece and held firmly in place when the load is applied. The pointer moves across the scale slowly until the yield point, when a sudden and marked increase in the rate is observed.

§ (22) THE MAXIMUM LOAD.—An autographic

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[1] *Materials of Construction*, 1918 edition, p. 10.

[2] *Testing Materials of Construction*, 1910 edition, p. 266.

[3] "Engineering Research," *Inst. Mech. Eng. Proc.*, July 1913.

[4] *Proc. Amer. Soc. Test. Mat.*, 1907, vii. 624.

stress strain diagram shows that, with a test bar which "necks" before fracture, the actual load necessary to break the test piece at the neck is less than the maximum load on the bar before the reduction of area takes place. Nevertheless, the stress obtained by dividing the breaking load by the reduced area is greater than the stress obtained by dividing the maximum load by the original area of the bar, which is the figure required by specifications.

The terms breaking stress, maximum stress, and ultimate stress are applied indiscriminately to indicate the maximum load divided by the original area which is desired by specifications. It is obviously not the maximum stress, although it is calculated from the maximum load. As the term breaking stress is also liable to be misunderstood the author prefers to call it the *ultimate stress*.

§ (23) PERCENTAGE ELONGATION AND CONTRACTION OF AREA.—In specifications the elongation and contraction of area are required in order to obtain a measure of the ductility of the material. Both are empirical values, and the former, as has been shown in § (17), is dependent on the cross-section of the test bar and the gauge length. It has been suggested that the percentage general extension instead of the percentage total extension should be adopted, as this is independent of the form of test bar. It would, however, complicate the calculation of the results, and owing to the general adoption of the other method of expression would doubtlessly cause considerable confusion.

The general extension, if it is required, is obtained either

(i.) From an autographic diagram, as the extension at maximum load, *i.e.* before the necking of the test piece, or

(ii.) From the measurement of the elongation on two-gauge lengths on the same test piece.

Thus if e_1 = elongation on gauge length L_1,
e_2 = elongation on gauge length L_2,
a = total local extension,
B = the general extension per unit length,

then $e_1 = BL_1 + a$,
$e_2 = BL_2 + a$,

and the general extension per unit length

$$= B = \frac{e_2 - e_1}{L_2 - L_1},$$

$$a = \frac{e_1L_2 - e_2L_1}{L_2 - L_1};$$

e.g. with a 30-ton steel, diameter = 1·000 inch,
Extension on 8″ = 2·37 inches (29·6 per cent).
Extension on 3″ = 1·32 inches (44·0 per cent).

$$B = \frac{2{\cdot}37 - 1{\cdot}32}{8 - 3} = \frac{1{\cdot}05}{5} = 0{\cdot}21,$$

$$a = \frac{(1{\cdot}32 \times 8) - (2{\cdot}37 \times 3)}{8 - 3} = \frac{3{\cdot}45}{5} = 0{\cdot}69,$$

i.e. general extension = B = 21 per cent.

§ (24) MEASURING THE EXTENSION.—The gauge length in ordinary commercial testing is marked on the bar before it is tested, by means of either a double centre punch (*Fig.* 50) or a laying-off and per cent gauge (*Fig.* 51). It is, however, nearly as convenient and almost as quick to use a pair of dividers set to the correct gauge length. It is necessary to have a vee block to support the test piece during the marking-off process.

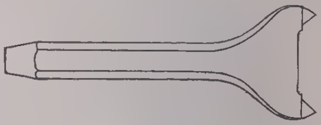


FIG. 50.—Double Centre Punch.

As long as the fracture is not less than a length equal to the square root of the area inside the gauge mark the measure of the elongation is usually accepted as satisfactory, *i.e.* 0·5 inch for a test piece of 0·25 sq. in. area or 0·7 inch for a test piece of 0·5 sq. in. area.

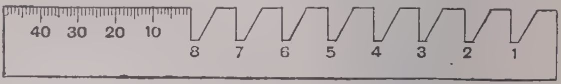


FIG. 51.—Laying-off and per cent Gauge.

Sometimes it is specified that "should a tensile test piece break outside the middle half of its gauge length the test may, at the contractor's option, be discarded and another tensile test shall be made from the same bar, billet, sheet, forging or drop forging." The middle half is defined as shown in *Fig.* 52.

§ (25) METHOD OF REPORTING RESULTS.—It is desirable that the results should be reported in such a manner that the information can be assimilated as easily as possible. It adds greatly to the facility with which the results can be compared and scrutinised if the form of report is always in the same style. The following points are also worth consideration:

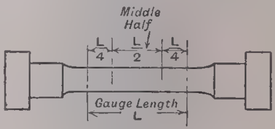


FIG. 52.

(1) The report should contain as far as possible the actual observations.

(2) The heads of information which are purely observational and those which have been reduced by calculation should be kept separate. It is convenient to have the latter on the right-hand side of the form.

A typical form for tensile tests is given in *Fig.* 53, together with the results of a few tests. It is seen that the information which is ultimately required is given in the last five columns.

REPORT.—TENSILE TEST OF MATERIAL CARRIED OUT ON BOILER PLATE SUPPLIED BY A. BROWN & CO., LTD., FOR WHOM THE TESTS WERE MADE

Test No.	Identification Marks.	Original Dimensions.			Yield Load. Tons.	Maximum Load. Tons.	Dimensions at Fracture.				Yield Stress. Tons/Sq. In.	Ultimate Stress. Tons/Sq. In.	Elongation. Per Cent.	Reduction of Area. Per Cent.	Remarks.
		Gauge Length. Inches.	Dimensions. Inches.	Area. Sq. In.			Extension. Inches.	Distance of Fracture from Gauge Mark.	Dimensions. Inches.	Area. Sq. In.					
1201	UE1	2	·563	·2489	3·4	6·57	·67	1·0	·395	·1225	13·7	26·40	33·5	50·8	Slightly laminated fracture
1202	UE2	2	·563	·2489	3·8	7·18	·62	1·1	·380	·1134	15·3	28·85	31·0	54·4	
1203	UE3	2	·563	·2489	4·0	6·77	·62	1·1	·370	·1075	16·1	27·20	31·0	56·8	

FIG. 53.

Example of Test Record and Calculation of Results.—

TENSILE TEST ON BOILER PLATE FOR A. BROWN & CO., LTD.
Material received July 30, 1920

	1.	2.	3.
Test No.	1201	..	..
Identification marks	UE1	..	..
Gauge length	2″	..	..
Dimensions of test piece	0·563″ diam.	..	..
Cross-sectional area of test piece	0·2489 sq. in.	..	..
Yield load	3·4 tons	3·4/0·2489	..
Yield stress	..	..	13·7 tons/sq. in.
Maximum load	6·57 tons	6·57/0·2489	..
Ultimate stress	..	..	26·40 tons/sq. in.
Elongation	0·67″	(0·67/2) × 100	33·5 per cent
Distance of fracture from gauge mark	1·0″	..	..
Dimensions at fracture	0·395″ diam.	..	..
Area at fracture	0·1225 sq. in.	$\frac{\cdot 2489 - \cdot 1225}{\cdot 2489}$	..
Reduction of area at fracture	..	..	50·8 per cent
Remarks	..	..	Slightly laminated fracture

It will be noted that, with the exception of the original area, the observed values are given in column 1 and the calculated figures in column 3. This method simplifies the typing of the report as given in *Fig.* 53.

§ (26) FORMS OF TENSILE TEST BARS.—It is shown by experiment that the form of the tensile test piece has an influence on both the elongation and the strength.

The elongation with ordinary test bars is measured on a gauge length of uniform section. The effect of the enlarged ends is to reduce the local drawing-out of the material. The parallel part of the test piece is therefore always made somewhat greater than that on which the elongation is measured. If the length of the parallel section is reduced so as to form an abrupt change in cross-section of the bar tested under tension, a concentration of stress is caused at the bottom of the groove or notch which is thus formed, while the shoulders formed by the groove prevent free elongation of the metal.

In a material which extends very little, an abrupt change of section will lower the breaking load, while with a ductile material the suppression of the local drawing-out gives an increased tensile stress.

The influence of the gauge length on the percentage elongation has been discussed in § (17).

On account of the considerations stated above, it is essential that definite types of specimen should be specified for commercial tests, in which the results of the tests decide whether the material is to be accepted.

As a result of the work of Barba, the French

Commission recommends the adoption of the relation

$$L^2 = 66{\cdot}67\ A$$

or $L = 7{\cdot}2D$ for circular specimens,
where L = the gauge length,
A = area of cross-section,
D = diameter of the test bar.

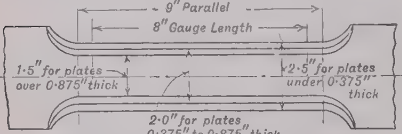
The German Commission recommends the use of the relation

$$L = 11{\cdot}3\sqrt{A}$$

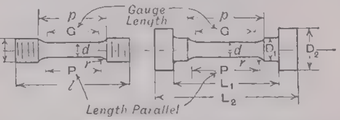
or $L = 10D$ for circular specimens.

Thus while, with a test piece 10 mm. diameter, a gauge length of 72 mm. is recommended in France, a length of 100 mm. is adopted in Germany.

Owing to the practical difficulties in the way of adopting a varying gauge length for different cross-sections of test piece cut from plates and other rolled sections, fixed standards for cylindrical and flat specimens have been prescribed by the American Society for Testing Materials (A.S.T.M.) and the British Engineering Standards Association (B.E.S.A.).


9" Parallel
8" Gauge Length
1·5" for plates over 0·875" thick
2·5" for plates under 0·375" thick
2·0" for plates 0·375" to 0·875" thick

B.E.S.A. Standard Sizes of Test Plates for Plates



	d	G	P	p	l	S	L_1	L_2	D_1	D_2
×	0·977	3·50	4·00	5	8	1·5 B.S.W.	6·25	8	1·25	1·75
×	0·798	3·00	3·375	4	6·5	1·25 B.S.W.	5·25	6·5	1·0	1·5
×	0·564	2·00	2·25	3	4·5	0·75 B.S.F.	4·25	5·25	0·75	1·25
	0·358	1·25	1·50	2	3	0·50 B.S.W.	–	–	–	–
	0·125	0·443	0·625	0·875	1·5	"0" B.A.	–	–	–	–

Radius r should be as large as is consistent with the other dimensions.

× *These are B.E.S.A. Test Pieces*

Standard Forms of Cylindrical Test Bars

FIG. 54.

The sizes selected by the B.E.S.A. are given in *Fig.* 54. Those adopted for cylindrical test bars of forgings (*Fig.* 54) approximately satisfy the law of similarity, where

$$L = 4\sqrt{A},$$

viz. :

Diameter. Inches.	Area in Sq. Inches.	Gauge Length. Inches.
0·564	0·250	2
0·798	0·500	3
0·977	0·750	3½

For plates, both 10-inch and 8-inch gauge lengths were in use before the B.E.S.A. standard was published. A standard width of 2 inches was used for convenience in milling; this gave a cross-sectional area varying with different thicknesses of plate, and thus disregarded the effect of the cross-sectional area on the percentage elongation.

In order to overcome this difficulty the B.E.S.A. standardised test bars having a fixed gauge length of 8 in. and a width varying with the thickness of the plate. In order to lessen the cost of production, three widths only were chosen, viz. :

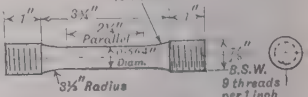
Thickness of Plate. Inches.	Width of Test Bar. Inches.	Gauge Length. Inches.
Under ⅜	2½	8
⅜ to ⅞	2	8
Over ⅞	1½	8

§ (27) ENLARGED ENDS OF TEST PIECES. (i.) *Tensile Tests.*—The design of the enlarged ends of test pieces requires care in order to be sure of satisfactory results. It has already been stated that axial alignment of the test piece is necessary for accurate work and that this is best attained by the use of spherically seated holders.

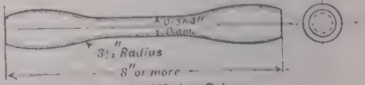
For brittle materials it is essential that the transition from the parallel part of the test piece to the enlarged ends is gradual, and that any part of the enlarged ends which is screwed, or otherwise has a sharp corner, is sufficiently large to prevent fracture at that place. Forms of ends for turned test pieces of cast materials are given in *Fig.* 55.

Test pieces may be tapered inside the gauge length towards the centre, to an amount not exceeding 0·003 in. for turned test pieces and 0·01 in. for sheets or plates, in order to induce fracture near the centre of the gauge length. If this is done the middle half of the gauge length of the test piece should be parallel.

(ii.) *Compression Tests on Metals.*—With the more plastic metals there is no well-defined point of break-down under compression, and it is usual to record the compressive strength at a given unit of deformation. The values which are obtained in this way are only



Test Piece for Screwed Grips



Test Piece for Wedge Grips

FIG. 55.

comparative and do not give results which can be used as a basis for calculating sizes of structures.

With brittle materials the ultimate stress can be obtained with precision, and the results are of value for calculating sizes of compression members.

For metals the test piece is usually a cylinder whose ratio of length to diameter is two.

It is important, especially with brittle materials, that a spherical seated shackle should be provided in order to adjust for inaccuracy of the parallelism of the shackles. A form of shackle is shown in *Fig.* 39. Care should be exercised to ensure that the pressure is applied axially.

§ (28) QUALITY FACTOR.—Various empirical means of estimating the quality of the material from the results of the tensile test have been suggested. They combine the tensile strength with either the percentage of elongation or percentage contraction of area at fracture. The only factor which has been used to any large extent in this country is a modification of one suggested by Wohler:

Wohler's quality factor $= p + c$
and the modified factor $= p + e$,
where $p =$ tensile strength,
$e =$ per cent elongation,
$c =$ per cent contraction of area.

A quality factor must be used with care, bearing in mind the use to which the material is to be put.

§ (29) BEND TESTS.—A rough workshop test of the ductility of metals is given by the bend tests of various types which are now in common use. The test is made by bending a piece of the material either by steady pressure or hammering, the former giving the more concordant results, until an angle of 180° is reached or previous rupture takes place. In the latter case the angle is noted at which fracture occurs on the outside of the bend. A large amount of experimental work has been carried out by Mr. Bernard Firth,[1] but the test has not been so carefully standardised as the tensile test.

The test is also used for ascertaining the effect of punching and drilling on the ductility, by using perforated plate specimens with the hole in the centre of the plate. The diameter of the hole, as recommended by Martens, should be twice the thickness of the plate and the width of the specimen five times its thickness.

§ (30) KINDS OF BEND TEST.—The principal types of bend test in use are:

(i.) Cold bend test (as received).
(ii.) Cold bend test (annealed).
(iii.) Quench bend test or temper bend test.
(iv.) Hot bend test.
(v.) Nick bend test.

(i.) *Cold Bend Test.*—This is the type which is most commonly used and is carried out at normal temperature. To test the ductility at low temperature the test pieces are sometimes cooled artificially.

(ii.) *Annealed Bend Test.*—For the annealed bend test the material is, previously to bending, reheated to a temperature exceeding its upper critical range, followed by slow cooling. The steel is usually very bad if it cracks after annealing.

(iii.) *The quench bend test* is used for determining the effect of alternations of high and low temperature on the ductility on such materials as boiler-rivet steel and stay-bolt iron. Before applying the bend test the material is heated to blood-red and plunged into water at a temperature of 80° F. The colour is judged indoors in the shade.

(iv.) *The hot bending test* is specified to detect red shortness or high sulphur content. The material is bent while at a temperature of about 950 to 1000° C.

(v.) *The nicked bend test* is intended to show whether the material has a crystalline or fibrous structure. The test piece is nicked all round, before bending, with a tool having a cutting angle of 60°. The depth of the notch is 8-16 per cent of the diameter of the test piece. The test piece should be placed so that the bend occurs with the minimum depth.

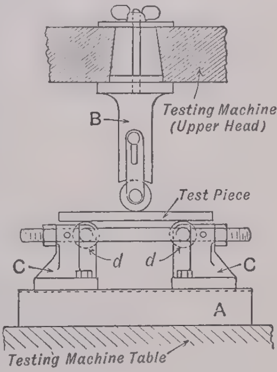


FIG. 56.

§ (31) FORMS OF SPECIMENS FOR BEND TESTS AND METHOD OF TESTING.—Rods are usually tested without machining. Plates should be planed at the sides after shearing, and the corners rounded with a file, otherwise cracks may start at the corners due to the shearing process having weakened the metal. With plates over one inch thick it is especially important that this precaution should be taken.

It is usual to bend bars, unless otherwise specified, round a mandrel of a radius equal to the diameter or thickness of the bar.

A cold bending apparatus is shown in *Fig.* 56, in which a steady pressure is applied during the test. The lower plate A rests on the table of the testing machine while the upper tool B is held in place in the pulling head by wing nuts and plate. The brackets CC are spaced sufficiently apart to allow the upper tool and twice the thickness of the specimen to easily pass between the lower rollers *dd*. The rollers can be changed so as to adapt the apparatus for either round, square, or twisted bars.

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[1] *Inst. Civ. Eng. Proc.* clv. 254.

Where a large number of tests have to be carried out, and the testing machine or press cannot be spared, it is usual to have a special machine used exclusively for bend tests. A hand power machine for specimens up to one inch square is shown in *Fig.* 57.

The correct size of centre pin is first put in the machine, the crank arm is next rotated until the crank arm pin is below the level of the test piece, and then the table is adjusted by means of the small hand-wheel until the top of the specimen rests against the centre pin, which should be at about the centre of the specimen. The graduated ring is set to zero, and power is applied to the crank arm, which revolves until the test piece ruptures or the required angle of bend, read off on the graduated ring, is reached.

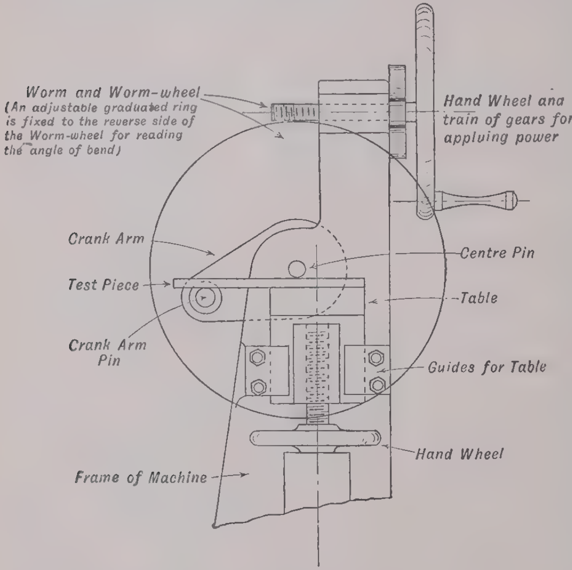
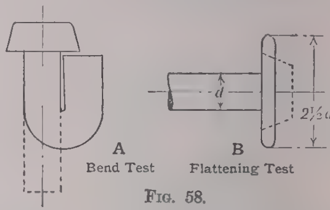

Worm and Worm-wheel
(An adjustable graduated ring is fixed to the reverse side of the Worm-wheel for reading the angle of bend)
Hand Wheel and train of gears for applying power
Crank Arm
Centre Pin
Test Piece
Table
Crank Arm Pin
Guides for Table
Hand Wheel
Frame of Machine

FIG. 57.

A fatigue test is sometimes used as an alternative to the bend test. The test piece is placed on bearings with a span of 6 in., and must stand without fracture twenty blows from a weight (of 1120 lbs.) having a rounded end of 1¼ in. radius and falling 6 in. The test piece is to be reversed after the first and every alternate blow. The height of fall after the twenty blows is increased to 12 in. and the test continued until fracture occurs.

Bend tests are now often replaced by single-blow notched bar tests (see § (98) to § (109)); but these require special testing plant.



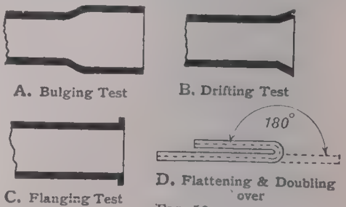
A Bend Test B Flattening Test

FIG. 58.

§ (32) TESTS FOR RIVETS.—Rivets are usually subjected to a bend test and a flattening test. For the bend test they are hammered cold without cracking until the two parts of the shank touch (*Fig.* 58 (A)) and for the flattening test the heads are hammered while the rivet is hot and must flatten without cracking of the edge to a diameter 2½ times that of the shank (*Fig.* 58 (B)).

§ (33) BULGING, DRIFTING, FLANGING, AND FLATTENING TESTS FOR COPPER AND BRASS TUBES.—It is usually specified that brass and copper tubes shall stand a certain percentage increase in the diameter (25 per cent is usual) without crack or flaw when subjected to bulging (*Fig.* 59 (A)), drifting (*Fig.* 59 (B)), or flanging (*Fig.* 59 (C)). Another test which is frequently imposed is flattening and doubling over. It is generally carried out both cold and at red heat. The tube is flattened and doubled over through 180° as shown at *Fig.* 59 (D).

§ (34) DRIFTING TEST OF METALS.—Another workshop test for ductility, which is applied with ordinary shop appliances, is to bore holes of a given diameter near to the edge of the plate and open them by means of a conical



A. Bulging Test **B.** Drifting Test

C. Flanging Test **D.** Flattening & Doubling over

FIG. 59.

drift (taper 1 in 10) until cracking occurs or a definite increase in diameter is reached. The hole is usually ⅝ inch diameter and 1½ inch from the edge of the plate.

The following table gives comparative figures for various sizes of plate:

TABLE 5

Material.	Thickness. Inches.	Original Size of Hole, in Inches.	Enlarged Size of Hole before Cracking, in Inches.
Wrought iron	0·20	0·625	1·00
„	0·25	0·625	1·20
„	0·30	0·625	1·32
Mild steel	0·30	0·625	1·50

§ (35) FALLING WEIGHT TEST (called "drop test" in U.S.A.).—The falling weight test is considered to be a most useful test for rails and is given a prominent place in rail specifications. It is made by placing a piece of the rail 4 to 6 ft. long on supports 3 ft. apart and dropping a heavy hammer, through varying distances, on to it at the mid-point of the span. The results show if the rail is brittle and the deflection gives a measure of the ductility.

TABLE 6

	U.S.A.	British.
Falling weight	Of cast iron with steel striking die having a radius of striking face of 5 in. and length of face of 12 in. Total weight 2000 lbs.	Of cast iron with steel striking face having a radius of 5 in. Weight 2240 lbs.
Anvil	Solid iron casting 15 in. thick and weighing 20,000 lbs.	Weight 26,880 lbs., area of base 45 sq. ft.
Foundation	Anvil supported on 20 springs arranged in groups of five at each corner. The springs are supported by a cast-iron base-plate which is bolted to a substructure with a timber floor in between. The floor to be 18 in. longer than the base-plate.	Anvil supported direct on 25 tons of concrete having an area of 100 sq. ft. No timber or springs between the test piece and foundation.
Supports for test piece (anvil dies)	Pieces of steel having a cylindrical bearing surface of 5 in.	Steel bearers having a radius of 3 in.
Release	A tripping device.	No automatic release allowed.

P. H. Dudley[1] of U.S.A. says: "The physical properties of different sections can be compared and their chemical compositions adjusted from the results obtained on the standard drop-testing machine."

The results are comparative only when carried out under the same conditions.

It is generally recognised that the weight of tup, distance apart of supports, height of fall, radius of striking faces and supports, and form of foundation must be specified. There is a difference of opinion however on this point, more especially in regard to the foundation of the machine.

Particulars of the standard machines usually adopted in the United States of America and in Great Britain are given in Table 6.

The atmospheric temperature is usually reported, and due allowance is made for rails tested at or below 0° C.

The falling weight test is also applied to tyres and axles. For tyres a weight of 2240 lbs. is allowed to fall freely from 10, 15, 20 ft., and upwards on the tread; the tyre being placed in its running position upon a heavy cast-iron anvil supported by a rigid concrete foundation. The tyre must deflect a given amount without fracture.

If d = internal diameter of tyre, as rolled, in inches,
t = thickness of the centre of the tread, as rolled, in inches,

deflection $= d^2/ct^2$,

where c = a constant depending on the breaking stress of the material (see Table 7).

TABLE 7

Value of c.	Breaking Stress in Tons per Square Inch.
45	42-48
50	50-55
55	56-62

§ (36) DROP TEST OF TYRES.—Tyres are subjected to a drop test as well as to a falling weight test. The tyre is allowed to drop freely, in a running position from a specified height, upon a rail fastened to an iron block of not less than two tons weight. The tyre is then turned through 90° and dropped a second time. The height of fall depends upon the diameter of the tyre (see Table 8).

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[1] *Proc. Amer. Soc. Test. Mat.*, 1910, x. 232.

TABLE 8

Height of Fall in Feet.	Internal Diameter of Tyre in Feet.
5	Up to 3½
4	3½ to 4½
3½	4½ to 5½
3	5½ to 6½
2½	Over 6½

§ (37) TESTING CAST IRON. (i.) *Transverse Test.*—Owing to the ease with which it can be carried out the transverse, or cross-breaking test, has been universally adopted as the standard for cast iron. The test is carried out by breaking the test bar as a beam loaded centrally. The strength is measured by the centre load which the bar will carry. The deflection is also usually specified because, when taken in conjunction with the load, it gives an approximate measure of the toughness of the cast iron. It is essential that the form of the test bar should be standardised in order to obtain comparative results, but unfortunately the standards which are in use in various countries are different. Particulars are given in Table 9.

TABLE 9

PARTICULARS OF STANDARD TEST BARS FOR CAST IRON

	English Standard.	American Standard.	German Standard.
Cross-section of bar in inches	1″ wide 2″ deep	1¼″ diam.	$1\frac{3}{16}''$ diam.
Length of bar in inches	40	15	25½
Span in inches.	36	12	23·62
How cast . .	On edge	Vertically	Horizontally

TABLE 10

A COMPARISON OF THE RESULTS ON GREY CAST IRON

Showing decrease of the modulus of rupture as the span increases

Silicon. Per Cent.	Span. Inches.	Mean Centre Load. Lbs.	Modulus of Rupture. Lbs./Sq. In. S_m.	Tensile Strength. Lbs./Sq. In. S_t.	$\frac{S_m}{S_t}$
1·5	12	3000	47,100	25,600	1·84
	18	1935	45,600		1·78
	24	1425	44,700		1·75
2·0	12	2900	45,500	24,370	1·87
	18	1835	43,200		1·78
	24	1265	39,700		1·63
2·5	12	2880	45,200	24,660	1·83
	18	1905	44,900		1·82
	24	1400	44,000		1·78

The general experience, in casting bars of the various standards in use, is that there is a little more difficulty in obtaining sound bars cast to the English section than in the case of either the American or German bars.

Machined bars are generally weaker than unmachined specimens.[1]

If W is the central load, L the span, Z the modulus of the section, and f the stress in the bar.

Then within the elastic limit,

$$\frac{WL}{4}=f\times Z, \text{ or } f=\frac{WL}{4}\times\frac{1}{Z}.$$

Above the elastic limit the stress calculated from the formula varies for the same material with the form of the section, but if the breaking load W_b is substituted for W, a value of f is obtained which is a measure of the quality of the material for bars of similar section, and is called the modulus of rupture.

G. Hailston,[2] in 1914, as a result of a series of experiments on cast iron when tested by cross breaking, finds that whereas the ratio of the breaking loads for $1\times2\times36''$ (span) and $1\times1\times12''$ (span) beams is 1·333 if the formula $W\times L/4=fZ$ is taken as applying up to the point of rupture, the actual value is 1·153 if the pieces are machined, and 1·146 when tested with the skin left on. He further finds that the best test bar to give the most consistent and comparable results, both with breaking load and deflection, is $2\frac{1}{8}\times1\frac{3}{8}\times42''$ cast on the flat and machined down to $2\times1''$ and tested on a span of 36 in. The rate of testing should not exceed 112 lbs. in less than 15 seconds.

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[1] Segundo and Robinson, *Inst. Civ. Eng. Proc.* lxxxvi. 248.

[2] *Iron and Steel Inst. Journ.*, 1914.

The results in Table 10 of tests on grey cast iron by Mathews[1] show that the modulus of rupture decreases as the span increases; the cross-section being circular (1¼ in. diam.) and kept constant.

Table 11, compiled from figures recorded by J. E. Stead,[2] gives a comparison of results obtained on British, American, and German standard bars. His conclusion is that it is desirable that any bar for testing for deflection by transverse stress should not be less than 24 in. long, as, with shorter bars than this, it is very difficult to measure to the required degree of accuracy except with very delicate measuring instruments.

TABLE 11

EXPERIMENTS ON TRANSVERSE STRENGTH

Comparison of tests on English, German, and American Standard Bars

Description.	Dimensions. Inches.	Span. Inches.	Mean Centre Load. Lbs.	Deflection. Inches.	Modulus of Rupture.	
					Lbs./Sq. In.	Tons/Sq. In.
English Standard—Cast on flat . .	1·07 × 1·98	36	2840	0·38	36,500	16·3
German Standard—Cast horizontal .	1·14 diam.	23·62	1210	0·32	49,200	22·0
American Standard—Cast vertical . .	1·24 diam.	12	3230	0·11	51,800	23·1
English Standard—Cast on edge .	1·02 × 2·04	36	3370	0·40	42,900	19·2
German Standard—Cast horizontal .	1·23 diam.	23·62	1660	0·36	53,700	24·0
American Standard—Cast vertical . .	1·25 diam.	12	3380	0·135	52,300	23·3

In commercial work a deflectometer of the type shown in *Fig.* 60 is generally used. It consists of a cast-iron base the bottom of which is carefully planed. This base carries a scale along which a lever moves a vernier reading to 0·001 in. The adjusting screw of the lever is arranged under the transverse tool so as not to be affected by the breaking of the specimen. A simple arrangement which is sometimes adopted is to use a Brown and Sharp lathe indicator under the transverse tool at the centre of the beam.

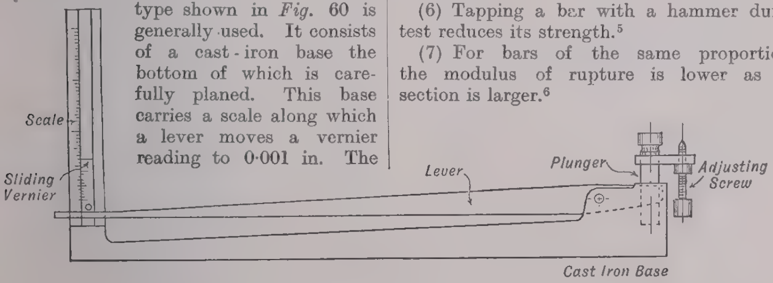

FIG. 60.

The following observations have been recorded as the result of careful experiment:

(1) Square bars, in general, exhibit a slightly higher modulus of rupture than round bars of equal area. Tests by a committee of the American Society of Mechanical Engineers[3] give an average of about 5 per cent higher.

(2) Bars cast horizontally are strongest when the load is applied against the cope face.

(3) Rough bars are stronger than planed bars.[4]

(4) Tumbling in a rattler increases the hardness of the skin and therefore improves the strength.

(5) Bars run with hot metal are weaker and deflect more than bars run with dull metal.[5]

(6) Tapping a bar with a hammer during test reduces its strength.[5]

(7) For bars of the same proportions, the modulus of rupture is lower as the section is larger.[6]

(8) A wide bar gives a higher, and a deep bar a lower modulus of rupture.[6]

With the English standard test pieces

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[1] *Proc. Amer. Soc. Test. Mat.* x. 299.

[2] *Proc. Int. Assoc. Test. Mat.*, 1912, vi. 4.

[3] *Trans. Amer. Soc. Mech. Eng.* xvi. 543 and 1066, xvii. 675.

[4] Segundo and Robinson, *Inst. Civ. Eng. Proc.* lxxxvi. 248.

[5] Millar, *Inst. Civ. Eng. Proc.* lviii. 222.

[6] Unwin, *Testing of Materials of Construction*, 1910 ed., p. 291.

(1″ wide, 2″ deep, 42″ long, and 36″ span) the strength of cast iron is usually specified by its breaking load in cwts., figures of 18, 28, and 38 (cwts.) corresponding to poor, fair, and good cast iron. A usual specification is that the cast iron shall sustain a load of 30 cwts. at the centre with a deflection of not less than $\frac{3}{8}$ in.

A form of report which is suitable for transverse tests is given in *Fig.* 61. It enables the salient points of the results to be seen at a glance, and meets all ordinary cases on the testing of cast iron.

Transverse tests of Materials carried out on....................................

Supplied by..................................for whom the tests were made.

Test Number.	Identification Marks.	Distance between Supports. Inches.	Dimensions.		Deflection at centre in Inches.								Breaking Load at Centre in Lbs.	Deflection at Fracture in Inches.	Remarks.
			Breadth. Inches.	Depth. Inches.	5 Cwt.	10 Cwt.	15 Cwt.	20 Cwt.	25 Cwt.	30 Cwt.	35 Cwt.	40 Cwt.			

Order No.

Reference.

Tested by

Director.

Date.

Fig. 61.

(ii.) *Effect of Temperature on the Transverse Strength of Cast Iron.*—Cast iron loses very little of its strength or toughness up to a temperature of 400° C. The following sets of experimental results by Meyer[1] show this very clearly.

Table 12

Effect of Temperature on the Strength of Cast Iron

Temperature. ° C.	Modulus of Rupture. Tons/Sq. In.	Maximum Deflection. Inches.
18	23·5	0·31
66	22·2	0·30
268	21·1	0·28
297	19·9	0·30
620	12·1	0·55
807	7·6	0·94
18	23·6	0·30
85	21·9	0·28
155	21·3	0·28
370	21·0	0·30
580	16·0	0·57
810	6·2	0·79

§ (38) Tensile Strength of Cast Iron.—It is important when making tensile tests on cast iron to ensure that the pull is axial, as this material is greatly weakened by oblique or eccentric loading. For accurate testing, shackles with spherical seats should be employed (see *Figs.* 33 and 34), the test bar should be cast in one with the work and turned to a suitable size, care being taken to avoid rapid changes of section near the part of the specimen under test.

The size of the bar from which the test piece is taken affects the results obtained, bars of larger section giving, as a rule, lower results.

Tables 13 and 14 give results recorded by J. E. Stead[2] on test pieces prepared from bars of various sizes which have been cast in different ways.

Table 10 gives results obtained by Mathews[3] on cast iron containing various percentages of silicon.

A comparison between the modulus of rupture and the tensile strength is given in Table 15.

§ (39) Crushing Strength of Cast Iron.—The crushing strength of cast iron is obtained on cylinders or prisms in which the ratio of the height to the least lateral dimension is between one and three.

The crushing strength is usually about five times the ultimate tensile stress, and samples of bad, good, and very good cast iron should give a crushing stress of 30, 40, and 50 tons per square inch respectively.

§ (40) Resistance of Cast Iron to Direct Shear and Torsional Shear.—As the result of experiments in direct shear by Platt and Hayward[4] it was thought that the shearing strength of cast iron was about five to six tons per square inch. Izod,[5] however, by using more perfect apparatus, has obtained much higher figures, and these are confirmed by Goodman. The results of these tests, together with the figures obtained from gun-metal and mild steel, for comparison, are given in Table 16.

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[1] *Stahl und Eisen*, xxvi. 1270-71.

[2] *Proc. Int. Assoc. Test. Mat.*, 1912, vi. 4.

[3] *Proc. Amer. Soc. Test. Mat.* x. 299.

[4] *Inst. Civ. Eng. Proc.* xc. 382.

[5] *Inst. Mech. Eng. Proc.*, Jan. 1906.

TABLE 13

TENSILE TESTS ON CAST IRON

Test pieces 0·798 in. diameter, 0·500 sq. in. area

Description.	No. of Tests.	Ultimate Stress. Tons/Sq. In.	Average per cent Variation from Average.	Remarks.
Cast on flat in bars 2 × 1″ . .	4	11·2	1·3	All test pieces turned.
Cast horizontally (1·187″ diameter)	4	12·2	1·8	
Cast vertically (1¼″ diameter) .	4	12·2	1·8	

TABLE 14

TENSILE TESTS ON CAST IRON

Test pieces 0·798″ diameter, 0·500 sq. in. area

Description.	No. of Tests.	Ultimate Stress. Tons/Sq. In.	Average per cent Variation from Average.	Remarks.
Cast horizontally as a test bar (1¼″ diameter)	4	13·9	0·9	Tested with skin on.
Cast horizontally (1·187″ diameter)	6	13·4	3·2	Turned to 0·798″ diameter.
Cast vertically (1·187″ diameter) .	6	13·5	1·8	Turned to 0·798″ diameter.

Only test pieces which broke with a sound fracture are included in Tables 13 and 14.

TABLE 15

COMPARISON OF MODULUS OF RUPTURE AND TENSILE STRENGTH OF CAST IRON

Type of Test Bar and Authority.	Modulus of Rupture. Tons/Sq. In. A.	Ultimate Tensile Stress. Tons/Sq. In. B.	Ratio. A/B.
American Standard (Mathews) .	21·0	11·4	1·84
	20·3	10·9	1·86
	20·2	11·0	1·84
American Standard (Stead) . .	23·1	12·2	1·89
	23·3	13·5	1·72
English Standard (Stead) . .	16·3	11·2	1·46
	19·2	13·9	1·38
German Standard (Stead) . .	22·0	12·2	1·80
	24·0	13·4	1·79

The point of interest in Table 16 is the fact that the shearing strength of cast iron is higher than the ultimate tensile strength. Goodman says that, with the exception of specimens which are defective on account of blowholes or sponginess, he has never had a single instance of a specimen of cast iron giving a lower result in shear than in tension.

Fremont is of opinion that the shear test gives a very useful indication of the quality of cast iron if the sample tested truly represents the material of the casting. He has devised a special test for this purpose.[1] A small piece of the actual casting, 25 mm. diameters and 20 mm. long, is detached by a trepanning tool. The specimen is placed within a block holding a fixed blade, and is cut by a movable blade which is forced into the specimen by a weight

[1] Fremont, *Comptes Rendus*, Dec. 9, 1918.

acting on a lever. The force of rupture is estimated from the weight and leverage. Since the diameter of the hole bored in the casting is only about 1¼ in. diameter the casting is not spoiled by this method of procedure.

Under torsional shear a round bar of cast iron invariably fractures along a helix whose angle is 45°, that is to say, it occurs where the tensile stress is a maximum, and indicates that cast iron has a lower resistance to tension than to shearing. This conclusion confirms the direct shear results of Izod and Goodman. A typical torsion fracture is shown in *Fig.* 62.

TABLE 16

SHEARING TESTS ON CAST IRON

Material.	Ultimate Tensile Stress. F_t. Tons/Sq. In.	Ultimate Shear Stress. F_s. Tons/Sq. In.	Ratio. $\frac{F_s}{F_t}$.	Calculated Coefficient of Torsional Strength. Tons/Sq. In.	Authority.
Cast iron A . .	9·7	14·8	1·52	..	Izod
„ „ B . .	13·4	17·4	1·11	..	
„ „ C . .	11·3	13·9	1·22	..	
„ „ D_1 . .	13·7	16·1	1·18	..	
„ „ D_2 . .	13·5	14·8	1·10	..	
Cast iron . . .	10·9	12·9	1·18	..	Goodman
	11·5	13·0	1·13	16·0	
Gunmetal . . .	12·1	12·5	1·03	..	Izod
	12·4	17·4	1·40	..	Goodman
	13·8	16·0	1·16	21·0	
Mild steel . . .	26·9	21·0	0·78	..	Izod
	23·6	18·9	0·81	..	Goodman

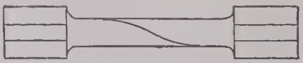

FIG. 62.

§ (41) INFLUENCE OF FORM ON THE PROPERTIES OF MATERIALS.—An abrupt change in the cross-section of a tensile or transverse test bar causes:

(*a*) A diminution of the elongation at the reduced section due to the shoulders of the notch or groove.

(*b*) The stress to be highly concentrated at the periphery of the groove or notch.

There are thus two effects—the suppression of the drawing out of the material causes a gain of strength, while the inequality of the stress on the section of fracture gives a reduction in strength.

With a brittle material, *e.g.* cast iron, the elongation is negligible, and in consequence the effect of a groove is to cause a reduction of strength. On the contrary, a grooved mild steel test piece is stronger than a plain bar of the same material, because the suppression of the drawing out causes an increase in strength which is greater than the effect due to concentration of stress at the groove acting in the opposite direction.

(i.) *The Effect of Perforation on the Strength of Mild Steel Plates.*—In the discussion on a paper by W. Hackney before the Institute of Civil Engineers, Stromeyer[1] describes experiments carried out to determine the effect of perforation on the tensile strength and elongation of boiler plates. The forms of test bars that he used are given in *Fig.* 63, and the results are given in Table 17.

The test pieces are all cut from one piece of plate 12 mm. (t) thick, and the holes are all 24 mm. (d) diameter. The width (b) varies in different bars from 7·6 to 54·0 mm.

(ii.) *Strength of Screwed Bolts.*—The results of some tensile tests on four different kinds of screw threads, made by the author, are given in Table 18. The threads selected were:

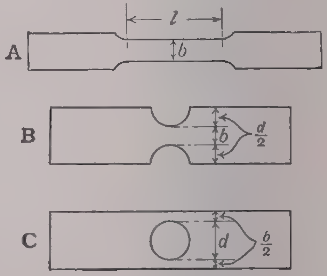

FIG. 63.

(1) British Standard Fine.
(2) British Standard Whitworth.
(3) Sellers Thread.
(4) U.S.A. 60° Sharp Vee Thread.

Four materials were used, ranging in ultimate stress from 24·0 to 52·7 tons per square inch. The bolts were ⅝ in. diameter and were all machined from 1¼-in. diameter bar.

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[1] *Proc. Inst. Civ. Eng.*, 1884, lxxvi. 142.

From the results in Table 18 it is seen that for steel up to 40 tons per square inch the strength (per square inch of section taken at the bottom of the thread) is greater for the bolts than for the plain bar, but that for the 53-ton steel the bolts, with the exception of those screwed with B.S.W. thread, are weaker than the plain bar.

TABLE 17

STRENGTH OF PERFORATED PLATES

Form.	Dimensions. Width. b mm.	Thickness. t mm.	Ratio. $\frac{b}{d}$	Elongation per cent. Of Hole.	In 50 mm.	In 200 mm.	Ultimate Stress. Kilos per sq. mm.	
A	35	12	..	..	..	25½	44·0	
B	46·8	12	1·94	50	26	..	47·4	Mean 47·8
	31·8	12	1·32	40	18	..	48·0	
	18·0	12	0·75	27½	14	..	47·7	
	12·5	12	0·52	21	12	..	48·0	
C	54·0	12	2·25	54	28	..	46·6	Mean 45·7
	44·0	12	1·83	50	26	..	47·0	
	34·0	12	1·42	46	22	..	45·8	
	28·4	12	1·18	41½	22	..	45·0	
	23·3	12	0·97	37½	20	..	48·8	
	18·6	12	0·77	33½	16	..	45·2	
	13·0	12	0·54	25	14	..	45·5	
	7·6	12	0·32	21	10	..	45·1	

TABLE 18

TENSILE TESTS ON ⅝-INCH DIAM. SCREW BOLTS

	Material. A.	Material. B.	Material. C.	Material. D.
Tensile Test on Material—				
Yield stress	15·6	17·6	23·5	25·6
Ultimate stress	24·0	29·5	40·3	52·7
Per cent of extension $L/\sqrt{A}=4$	41·0	39·0	28·0	19·0
British Standard Fine Thread (Root diam. =0·5335 in.)—				
Yield stress	17·8	20·1	24·7	26·7
Ultimate stress	26·3	33·8	46·6	48·6
British Standard Whitworth (Root diam. =0·5086 in.)—				
Yield stress	18·0	20·4	26·0	26·6
Ultimate Stress	27·0	34·0	45·9	54·0
Sellers Thread (Root diam. =0·5069 in.)—				
Yield stress	19·0	21·0	25·5	26·6
Ultimate stress	26·8	34·7	44·9	51·0
U.S.A. Sharp 60° Vee Thread (Root diam. =0·495 in.)—				
Yield stress	17·7	21·3	26·5	25·6
Ultimate stress	28·2	35·3	47·1	47·1

Note.—(1) Stresses are given in tons per square inch. (2) For tests on threads, the area for calculating the stress is taken as that at the bottom of the thread.

Martens[1] has carried out similar tests on ½-in. and 1-in. bolts, using steels of 27·7 and 23·9 tons per square inch respectively. His conclusions are as follows:

(1) Screw threads subjected to plain tension

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[1] *Zeits. Vereines Deutsch. Ing.*, April 27, 1896.

are stronger than plain bars of the same net cross-sectional area, the excess of strength being approximately 14 per cent.

(2) There is no marked difference in the ultimate strength with different forms of threads—the sharp thread is slightly stronger than the others.

(3) Under repeated loads and impact it is probable that the sharp vee threads would develop cracks quicker than the other forms, and that the Whitworth thread would be the last to show this weakness, either with repeated loads using soft material or static loads with high carbon steel.

The two sets of experiments, by different experimenters, described above are in agreement. It will be seen that Martens's supposition with regard to the superiority of the B.S.W. thread with high carbon steels under static loads is shown by the author's tests to be correct.

With regard to Martens's statement of the probable weakness of sharp vee threads under impact, some tests by the author also confirm this conclusion. The tests, of which the results are given in Table 19, were made on 1-in. bolts and broken by a single blow, the energy absorbed in fracture being measured. It will be noted from the table that the sharp vee bolts are the weakest of those tested and the British standard threads the strongest.

TABLE 19

IMPACT TESTS ON SCREW THREADS

(1 INCH DIAMETER)

Kind of Thread.	Diameter at Bottom of Thread. Inches.	Ft.-lbs. absorbed in Fracture.	Per cent of (Energy taken by the B.S.F.).
British standard fine (B.S.F.) . .	0·872	2275	100
Sellers thread (chasing tool) . .	0·838	1921	84
Sellers thread (single pointed tool) .	0·835	1888	83
British standard Whitworth thread .	0·840	2003	88
U.S.A. sharp vee thread . . .	0·780	1599	70

§ (42) INFLUENCE OF TIME ON TEST RESULTS.—General experience shows that elongation and contraction of area are increased by very rapid loading, but that within the ordinary limits of time occupied by a tensile test to fracture, of most materials, the rate of loading does not appear to have any effect on these results. The results from various experimenters, however, are not quite consistent.

(i.) *Effect of very Rapid Loading.*—Maitland [1] found that steel, broken in impact, gave 74 per cent more elongation than the same material broken in the static tensile testing machine.

Blount, Kirkaldy, and Sankey [2] concluded that the impact tensile test gave the ductility in the same terms as the static tensile test, but always with higher numerical values.

On the other hand, Lebasteur [3] was unable to find this difference with steel bars. His tests gave fractures which were the same in appearance in the two cases, and the elongations of similar bars were also nearly identical.

(ii.) *Effect of Variations of Slow Rates of Loading.*—Bauschinger [4] carried out a fairly complete set of experiments under slow rates of loading and showed that:

(1) The effect on the elongation of altering the rate of loading varied according to the material employed. With cast iron the effect was negligible, while with steel, wrought iron, copper, bronze, and brass it was so slight that it was masked by differences in the quality of the test bars. For lead and tin there was a marked increase in the extension with time.

(2) The ultimate stress was in some cases greater when the extension was fast than when it was slow.

Tests on iron and steel given by the Commission des méthodes d'Essai des Matériaux [5] show very little difference in the ultimate extension of the ultimate stress whether carried out in a few seconds or many minutes.

Bottomley,[6] however, when experimenting with soft iron wire, found that, generally, the contraction of area and local contraction were less if the loads were applied more slowly. His researches included a determination of

TABLE 20

Time the Load was allowed to remain on the Wire.	Maximum Load to Fracture.	Elongation.
	Lbs.	Per cent.
Ordinary test in 10 minutes	43·5 to 46	22 to 17
24 hours at 43 lbs. .	49·25	15
84 hours at 43 lbs. .	51·5	14·4
2 months' gradual loading from 40 lbs.	75·25	. .

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[1] "Treatment of Gun Steel," *Inst. Civ. Eng. Proc.* lxxxix. 120.

[2] "Comparison of Methods of Testing Steel," *Inst. Mech. Eng. Proc.*, May 27, 1910.

[3] *Ann. des ponts et chaussées*, 1890.

[4] *Inst. Civ. Eng. Proc.* cvii. 434.

[5] *Méthodes d'essai des matériaux*, i. 344.

[6] Article "Elasticity" in the *Encyclopædia Britannica*.

the effect of time on the breaking load of soft iron wire. He found that the wire, if loaded to just below the ordinary breaking load, increased in strength according to the time the wire was under this initial load.

A summary of the results are given in Table 20.

IV. Measuring Instruments for the Determination of the Elastic Constants of Metals

In order to obtain information as to the elastic constants of materials it is necessary to measure deformations while these materials are being strained elastically.

In commercial testing, where a measure of the ductility only is required, the final amount of deformation is all that is necessary. In a tensile test with a standard 2-in. gauge length and cross-sectional area of 0·25 sq. in., the total elongation may be 0·50 in., and as this can be measured by the aid of dividers to the nearest $\frac{1}{100}$ in. the accuracy of reading is 2 per cent.

Using the same size of test piece and gauge length, the extension at the elastic limit is of the order of 0·002 in., so that for the same percentage accuracy the measurement must be correct to 0·00004 in.; the smallness of this extension can be realised when it is considered that the thickness of a piece of cigarette paper is about 0·001 in. With the best types of mirror extensometer, alterations of length can be accurately measured to $\frac{1}{10}$ of this amount, *i.e.* 0·000004 in.

§ (43) Extensometers.—In any type of strain-measuring apparatus there are certain general principles which it is advisable should be fulfilled.

(i.) If changes in length of the axis of the test bar are to be determined, measurements should be taken on opposite sides of the specimen. Unwin[1] has shown that, if this condition is not satisfied, initial curvature of the bar will cause serious errors in the results, but "if the mean of measurements taken at points symmetrically placed on either side of the bar is adopted, the error due to curvature is nearly eliminated, the lengthening of the distance of one side being compensated by shortening on the other."

(ii.) The apparatus should be directly attached to the test piece at the gauge points only, and after the initial setting it should not be necessary to handle it during the course of the experiment.

(iii.) The instrument should be as light as possible, and designed so as to be rapidly attached to or detached from the test piece without interfering with the application of the load. It is advisable that, if possible, it should be self-contained.

(iv.) The instrument should be arranged so that its accuracy can be determined by a calibrating device, and the attachment to the test piece should be such that the gauge length is capable of determination with sufficient accuracy.

(v.) The mechanical or optical details, and the relative positions of all the parts with respect to the axis of the specimen, should be such that the "constant" of the instrument is the same throughout its range, or that any errors which are involved are so small as to be of no importance.

(vi.) The zero of the instrument should be constant for the same conditions of test; thus if, after straining the test piece, there is no permanent set the instrument should return to its initial position.

Extensions are measured in five different ways:

(i.) With a micrometer screw.
(ii.) By an indicating dial.
(iii.) With a microscope.
(iv.) By a multiplying lever (mechanical magnification).
(v.) By optical magnification.

With methods (i.), (ii.), and (iii.) the deformations are ascertained with little or no magnification, while with instruments employing methods (iv.) and (v.) the strains are considerably magnified before the measurements are made. Some extensometers use a combination of (iv.) or (v.) with (i.), (ii.), or (iii.).

§ (44) Micrometer Screw Extensometer.—These instruments consist of two clamps placed on the test bar at a distance apart which is equal to the gauge length. The movement of one clamp relatively to the other, as the load is applied, is measured by the reading of one or more micrometer screws fixed to one clamp and brought into contact with the other. The accuracy of the instrument depends on:

(*a*) The uniformity of the pitch of the micrometer screw.

(*b*) The constancy of the pressure at the point of contact.

(*c*) The care with which the micrometer screw is operated, because the instrument has to be touched by hand many times during the experiment. Care should be observed in order to ensure that the couple applied by the fingers is perpendicular to the axis of the screw.

Micrometer screw extensometers are largely used in the U.S.A. by students, and give satisfactory results when used with great care. The principal makes of instrument are divided into two sections according as to whether they have one or two micrometers.

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[1] "Measuring Instruments used in Mechanical Testing," *Phil. Mag.*, March 1887; *Phys. Soc. Proc.*, 1887.

§ (45) DOUBLE MICROMETER SCREW INSTRUMENTS. (i.) *Professor R. H. Thurston's*[1] *Extensometer* (1875).—This was the first instrument in which two micrometer screws were used, one placed on each side of the axis of the test piece, and in which electric contact was employed in order to eliminate errors due to variation of pressure.

(ii.) *Henning's*[2] *Micrometer Extensometer.*—This instrument is shown in *Fig.* 64. Two frames A and B grip the test piece by two steel points *h*, *h* and two knife edges *c*, *c*. The lower frame B carries two micrometers *m*, *m*, and the upper frame A is provided with a pair of plugs *g*, *g* which are in line with the micrometers. As the distance between the frames increases with application of the load, the ends of the screws *f*, *f* are brought in contact with the plugs *g*, *g*. The contact is indicated by ringing an electric bell by a weak electric current, or, owing to the uncertainty of bell mechanisms, telephone receivers are sometimes substituted. The extension is read on vertical scales *e*, *e* and the graduations of the micrometer heads *m*, *m*.

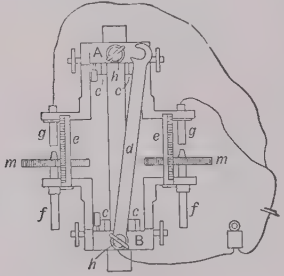

FIG. 64.

The instrument registers to one ten-thousandth part of an inch, and can be used on either round, flat, or square specimens. In order to attach the extensometer to the test piece, the frames A and B are opened. Bars *d* are used for setting the frames A and B at the correct distance apart; they are removed while the test is in progress. An instrument of this type is supplied by Messrs. Tinius Olsen Testing Machine Co., Philadelphia, under the name of The Olsen Standard Duplex Micrometer extensometer.

(iii.) The extensometer invented by C. A. Marshall, of the Cambria Iron Company, differs from Henning's in three small details. The adjusting bars *d* (*Fig.* 64) are omitted and the knife edges *c*, *c* are replaced by a spring and roller attachment which, in connection with a spirit level, assists in the accuracy of the adjustment. The micrometer screws are placed nearer the lower frame than in the Henning extensometer.

(iv.) *The Yale extensometer*[3] is a simplification of the Marshall apparatus. The spring rollers are omitted and a gauge bar introduced to keep the clamps parallel, bring the points of measurement opposite, and gauge the length. The gauge bars are made of different lengths and are removed before beginning to strain the specimen. It reads to ten-thousandths of an inch by micrometer screw.

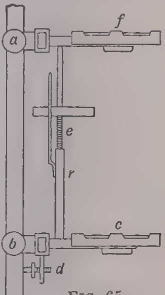

FIG. 65.

§ (46) SINGLE MICROMETER SCREW INSTRUMENT. *Unwin's*[4] *Screw Extensometer.* — This instrument is shown diagrammatically in *Fig.* 65. The two frames are clamped to the test piece at points on a plane passing through its axis by set screws *a* and *b*. The lower frame carries the micrometer screw *e*, on the hardened point of which the upper frame rests. Provided that the frames are at right angles to the axis, the micrometer screw gives the variations in the length between the two points of support on the test piece. To set the frames normal to the axis of the test bar, levels *c*, *f* are attached to them. Level *c* on the lower frame is adjusted by the screw *d*, while the upper frame is set level by means of the micrometer screw *e*. The pressure on the micrometer screw is thus constant, and equal to the weight of the upper frame.

§ (47) SINGLE MICROMETER SCREW COMBINED WITH A MULTIPLYING LEVER. (i.) *Garratt's*[5] *Extensometer.* — In this instrument (shown diagrammatically in *Fig.* 66) there is a mechanical magnification of two, combined with measurement by means of a single micrometer screw. Two pairs of steel plates B and C are attached to the test piece by screws E, E and pivoted together at F. The micrometer M is pivoted to the lower plates and carries a spindle H which passes through a guide K pivoted to the upper plates. The spindle carries a hardened pin on its upper end, and when the test piece extends, this pin, which forms part of an aneroid barometer, moves the aneroid needle. Half of the amount which

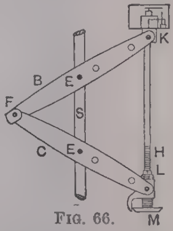

FIG. 66.

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[1] *Materials of Engineering*, ii. 369.

[2] *Trans. Amer. Soc. Mech. Eng.*, 1885, p. 479.

[3] See Johnson's *Materials of Construction*, 1918 ed., p. 80.

[4] *Phys. Soc. Proc.*, 1886, viii. 178

[5] *Inst. Civ. Eng. Proc.* cxxviii. 321.

the micrometer has to be moved to bring the needle to its original position is a measure of the extension.

(ii.) *Riehle improved extensometer*, supplied by Messrs. Riehle Bros. Testing Machine Co., reads the average stretch from two sides of the test bar with one micrometer screw. The arrangement is shown diagrammatically in *Fig.* 67. The two frames A and B are each fixed axially to the test piece by two hardened steel-pointed thumb-screws S, S. The left-hand rod R is rigidly joined to the lower frame B and is pivoted to the upper one A. The right-hand rod swings from a pivot in the upper frame over the micrometer screw in the lower one. The rods are equidistant from the points of attachment to the specimen, so that when elongation occurs the right-hand swinging rod moves away from the micrometer *twice the amount of the elongation.* This extensometer can be used on gauge lengths from 2 in. upwards by having sets of rods for those gauge lengths which are required. Elongations of 0·0001 in. can be easily read, and electric contacts can be employed with the instrument.

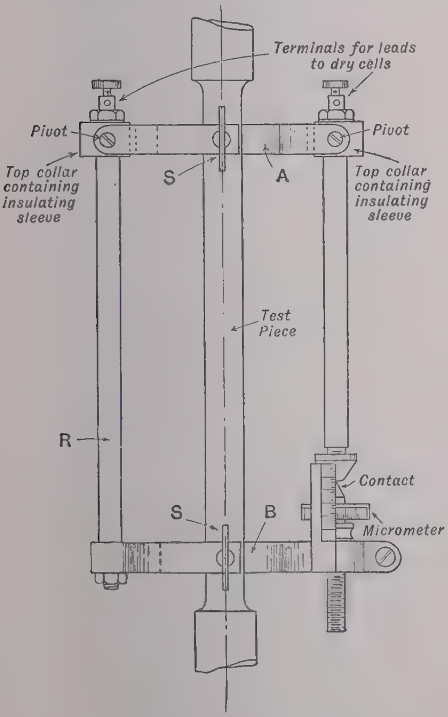
Terminals for leads to dry cells
Pivot
Pivot
Top collar containing insulating sleeve
Top collar containing insulating sleeve
S
A
Test Piece
R
Contact
S
B
Micrometer

FIG. 67.

(iii.) *The Cambridge Extensometer* (*Fig.* 68). —This simple and accurate extensometer is made by the Cambridge and Paul Instrument Co. Ltd., Cambridge, and is especially designed for use as a workshop instrument. It is composed of two separate frames, each of which is attached to the test bar M by hard steel conical points PP and P′P′ arranged in geometric slides so that, after the points are gently driven into the punch marks, they can be clamped in position by the knurled heads R, R. The lower frame carries a micrometer screw H and a vertical arm B at the top of which is a hardened steel knife edge about which the two frames work together. The micrometer screw is provided with a hardened steel point X over which a nickel-plated flexible steel tongue A, forming a continuation of the upper frame, is carried. The tongue and frame form a lever magnifying the extensions of the test piece by five.

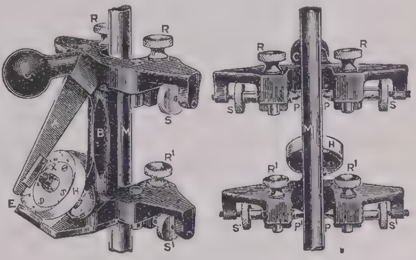

FIG. 68.

In use the vibration of the flexible tongue takes the place of the electric contact, in the instruments previously described, as a delicate means of setting the micrometer screw. The steel tongue is vibrated and the screw turned until the point X just touches the hardened knife edge of the tongue as it vibrates. Readings can be repeated by this means to 0·001 mm. under ordinary conditions of test. The standard instrument is suitable for specimens up to $\frac{3}{4}$ in. diameter on a gauge length of 4 or 2 in.

§ (48) INDICATING DIAL INSTRUMENTS.—This type of instrument consists of one or more calibrated dials with pointers on nicely mounted spindles. These take the place of the micrometer screws in the previous class of extensometers, and are self-indicating for large or small deformations. They are generally used in conjunction with some form of lever magnification.

Messrs. T. Olsen supply a dial extensometer of exactly the same form as Henning's extensometer (§ (45)) except that dials and sliding rods replace the micrometer screws. The sliding rods are attached to the lower frame in place of the micrometer screws, and spindles, carrying drums and pointers, are fixed to the upper frame with their axes horizontal. These take the place of the plugs *g*, *g* (*Fig.* 64). The friction of the sliding rods against the small drums (1 in. in circumference) cause the latter to rotate. This also rotates the spindles and pointers round calibrated discs.

§ (49) DIAL EXTENSOMETERS WITH MECHANICAL MAGNIFICATION. (i.) *University of Wisconsin Wirewound Extensometer.*[1]—This has one dial attached to the upper frame which is operated by a wire arranged to transmit twice the extension of the test piece to the dial drum.

(ii.) *Hurst-Tomlinson Extensometer* (*Combined Lever and Dial*), *Fig.* 69.—This instrument has been especially designed for use on the standard test piece (0·564 in. diameter and 2 in. gauge length); it can be used, however, on specimens up to one inch in diameter. The gauge length of 2 in. is not adjustable, but the instrument can be attached to specimens of greater length than this if required. It is provided with two forked levers, A and B, fitted at the fork with pointed clamping screws which, when tightened on to the test piece, form the pivots of the levers.

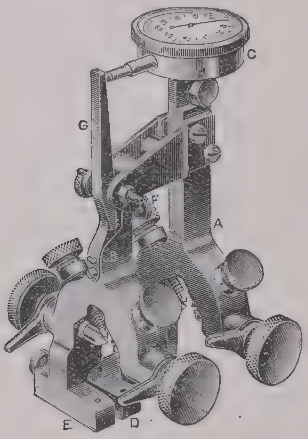

FIG. 69.

The dial indicator C is attached to one end of the upper lever, and the other end, beyond the test piece, has a stout vertical lug D extending downwards from one extremity of the fork nearly to the level of the lower lever. Here the lug bends inwards, and one arm E of the fork of the lower lever extends inwards in the same way.

The two levers are connected by a short length of piano wire which lies in the central vertical plane of the two levers.

This forms an elastic hinge, so that as the test piece stretches the free ends of the levers open away from one another.

This method of connecting the levers also permits the test piece to straighten out slightly under the load, if originally slightly bent, without straining the extensometer.

The lower lever is shorter than the upper one, and it carries at the free end a short vertical screw F with a large milled head.

The end of the screw is rounded and polished, and bears on the end of the shorter arm of a third lever G which is attached by a short length of flexible steel ribbon to a bracket which is screwed and dowelled to the upper main lever.

The end of the longer arm of this third lever bears upwards on the end of the plunger of the dial indicator.

The system of levers is proportioned so that the movement transmitted to the indicator is ten times the extension of the test piece.

By turning the milled head of the screw F the dial indicator can be brought to zero before the load is applied; a light spring maintains the end of the small lever in contact with the point of the screw. While the extensometer is being attached it is necessary for the two main levers to be rigidly locked, with the axis of the clamping points parallel and at a distance of 2 in. apart.

This is accomplished by inserting two locking pins, one of which constrains the axes to be parallel while the other fixes the distance between them.

The first pin is placed in the same horizontal plane as the wire hinge, and its axis intercepts the centre of the wire so that the insertion of this pin leaves the levers only one degree of freedom.

The second pin correctly fixes the relative positions of the two levers.

In order to ensure that the clamping points shall grip the test piece along a diameter and not along a shorter chord, two screws are placed in the forked part of each lever inclined at about 45°.

The axis of these screws intercept the axis of the test piece and the ends are rounded to bear at points on the test piece.

The usual diameter of the reduced part of the test piece is 0·564 in., which gives a cross-sectional area of 0·25 sq. in., and the length of the screws is such that when just tight the ends are 0·282 in. away from the vertical plane through the clamping points.

When attaching the extensometer it is simply held up against the test piece bearing on the ends of the four screws, and the clamping screws are tightened and the points pierce the test piece along two diameters. The extensometer is thus a self-contained marking-out jig. As soon as the instrument is fixed to the test piece the four screws are just slightly slackened so that they are just clear while the test is being made.

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[1] See Johnson's *Materials of Construction*, 1918, p. 81.

One division on the indicator corresponds to an extension of 0·0001 in. and, by estimating, the extension can be readily measured to the nearest 0·00002 in. This instrument is made by Charles W. Cook, Ltd., Manchester.

(iii.) *Riehle Dial Extensometer.* — A similar type of dial extensometer is supplied by Messrs. Riehle Bros. Testing Machine Co. The instrument itself is not quite so elaborate, and requires a special marking-off block which forms part of the outfit.

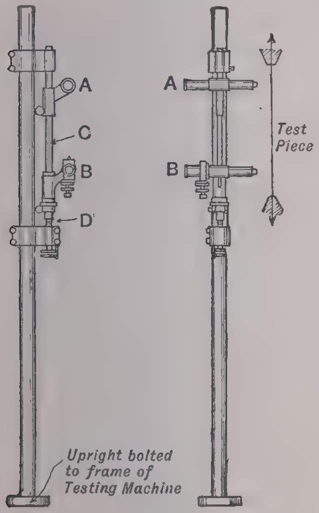

FIG. 70.

(iv.) *Berry Strain Gauge*[1] (*Combined Lever and Dial*). — This instrument measures the deformation on the surface of the test bar or structure and in principle is a pair of beam compasses, one point of which is fixed to the frame of the instrument and the other operating an Ames Dial through a five to one bell-crank lever. The points of the instrument are placed in the gauge marks and an alteration in the gauge length is shown on the Ames Dial.

§ (50) MICROSCOPE READING EXTENSOMETERS. (CATHETOMETERS.)—With this type of instrument the distance between fine scratches on the test piece is measured by sighting with a microscope on the two scratches, and reading off the distance the microscope has been moved along the slide to which it is fixed. The determination of elongations by this method is a lengthy process, and except for special purposes it has been abandoned.

Fig. 70 shows an arrangement of two microscopes which is convenient for measuring the extensions of stranded cables. Microscope A has ordinary cross wires while microscope B is fitted with a micrometer eyepiece. Both instruments are attached to a bar C which can be moved vertically by the screw D. Both microscopes are focussed on scratches on the test piece and the load applied. By means of the screw D the cross wire of the top microscope A is made to coincide again with the top scratch, while the adjustable cross wire is moved in the second microscope to the new position of the bottom scratch. The movement of this cross wire in the calibrated micrometer eyepiece gives the extension.

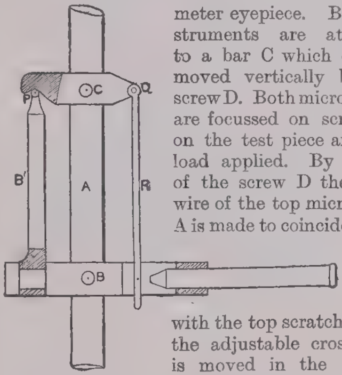

FIG. 71.

§ (51) COMBINED MICROSCOPE AND LEVER EXTENSOMETER. *Ewing Extensometer.*[2]—This instrument has been used for a great deal of the scientific work in this country. It is shown diagrammatically in *Fig.* 71, and in its latest form in *Fig.* 72. The two clips, B and C, are each attached to the test piece A by two pointed screws. Clip B carries the vertical projection B′ (*Fig.* 71) terminating in a rounded point P, which engages with a conical hole in C. The pieces B and B′ are joined to one another in such a way that the test piece may twist a little without affecting the engagement of P with C. When the test piece extends, P serves as a fulcrum for the clip C, and point Q is displaced through a distance equal to twice the extension. This displacement is measured by a microscope attached to B, sighting on a mark on the rod R hanging from the point Q on the upper clip. The readings are taken from a micrometer scale in the eyepiece of

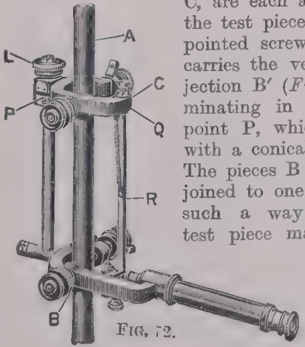

FIG. 72.

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[1] *Engineering Record*, June 11, 1910.

[2] *Roy. Soc. Proc.*, 1895, lviii. 123.

the microscope. The screw (L, *Fig.* 72) serves to bring the sighted mark to a convenient point on the micrometer scale and also determines the scale readings which can be estimated to 0·00002 in. A clamping bar is added by which the clips B and C are held at the correct distance when fixing them on the test piece. This extensometer is made by Messrs. The Cambridge & Paul Instrument Company for gauge lengths of 2 and 8 in. and 10 and 20 centimetres.

An apparatus for marking off the gauge points (*Fig.* 72A) is supplied with the instrument.

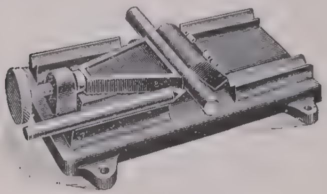

FIG. 72A.

§ (52) MULTIPLYING LEVER EXTENSOMETERS.—(i.) Kennedy [1] designed a simple lever (leverage 100 to 1) extensometer clipped to the test bar at two points.

Martens's [2] lever extensometer is a modification of the Kennedy instrument, and is arranged to take simultaneous readings on two opposite sides of the specimen. It is shown diagrammatically in *Fig.* 73, and consists of two clips D, D carrying the graduated scales C, C, and held on opposite sides of the specimen by a spring S. Diamond-shaped pieces A, A, to which the arms B, B are attached, are pivoted in shallow seats formed in the clips D, D. With a magnification of fifty, readings can easily be made to 0·002 mm.

(ii.) *Kennedy's* [3] *horizontal extensometer* is gripped on the bar at four points in an axial plane and consists of two frames, one fixed to each gauge mark, which rest on each other over the points of attachment to the test bar. A light pointer is provided with two steel points, one resting on each frame. When the test bar elongates the two points move relatively to each other in an axial direction, and this movement is magnified by the end of the pointer which travels over a scale carried by a rod attached to one of the frames.

Other instruments of this type have been designed by Stromeyer,[4] Goodman,[5] Wicksteed,[6] Ashcroft,[7] Col. W. H. Paine,[8] and Dupuy.[9]

§ (53) OPTICAL MAGNIFYING EXTENSOMETER.—For greater accuracy than 0·00002 in. in measuring deformations some form of optical lever is generally employed in which the change of length of the test piece is converted into rotary motion of a mirror which is observed by means of a telescope and scale. Some extensometers employ two mirrors in which readings are obtained from opposite sides of the test bar, while in others the mean extension is measured by the reading from a single mirror.

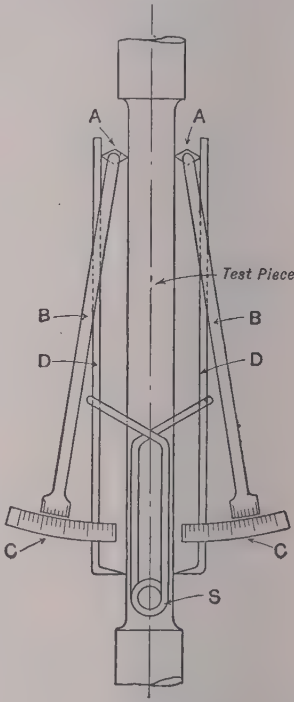

FIG. 73.

(i.) *Two Mirror Apparatus. Bauschinger's Instrument.*[10]—The introduction of mirror extensometers is principally due to Bauschinger, who designed the instrument shown in *Fig.* 74.

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[1] *Inst. Civ. Eng. Proc.* lxxiv., also lxxxviii. 24.

[2] *Handbook of Testing*, i. 546.

[3] *Engineering*, Sept. 12, 1890.

[4] *Inst. Naval Architects, Trans.*, 1886, p. 33, "A Strain Indicator for Use at Sea."

[5] *Engineering*, Sept. 11, 1896.

[6] *Inst. Mech. Eng. Proc.*, 1904, p. 485.

[7] Discussion on Unwin's paper on "Tensile Tests on Mild Steel," *Inst. Civ. Eng. Proc.* xlv. 266.

[8] Martens's *Handbook of Testing*, p. 545.

[9] *Annales des Ponts et Chaussées*, 5th series, xiv. 381.

[10] *Maschine zum Prüfen der Festigkeit der Materialien construirt von Ludwig Werder, und Instrumente zum Messen der Gestaltsveränderung der Probekörper, construirt von Jon. Bauschinger*, München, 1882.

Two clips, *a*, *a* and *b*, *b*, are pressed against the specimen: *c*, *c* are light springs which press outwards against the rollers *d*, *d*. These rollers, which are of caoutchouc, are carried by the clip *b*, *b*, and themselves carry the mirrors *g*, *g*. As the specimen extends the rollers rotate, and these rotations are measured by means of the telescopes E, E and scales *f*, *f*. The results are recorded to 0·0001 mm.

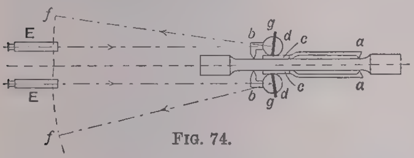

FIG. 74.

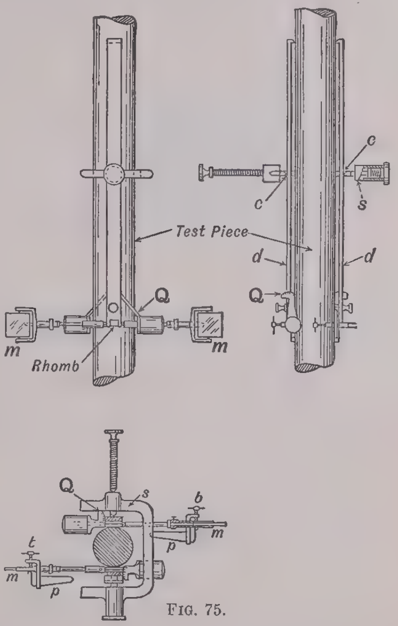

FIG. 75.

(ii.) *Martens's Mirror Extensometer.*—This is an improvement of Bauschinger's instrument and is shown in *Fig.* 75. It is extremely accurate and sensitive and is most adaptable. In this arrangement the multiplying levers of Martens's lever extensometer (*Fig.* 73) are replaced by small mirrors (*m*, *m*, *Fig.* 75) which are attached to the rhombic-shaped pieces of steel acting as fulcra, in such a way that the reflecting surface of each mirror is on the axis of the fulcrum. The two clips *d*, *d* are held on opposite sides of the specimen by a spring *s* which rests in grooves *c*, *c*. Each clip is pointed at one end which is gripped directly on the test bar, while the other end has the rhombic piece interposed between it and the bar. The mirror is mounted in a frame by means of pivots centred in small holes drilled in the glass. In order to adjust the position of the mirror the frame is free to revolve on the axis of the rhomb. An adjusting screw *b* and spring *p* are placed on opposite sides of the mirror to control its

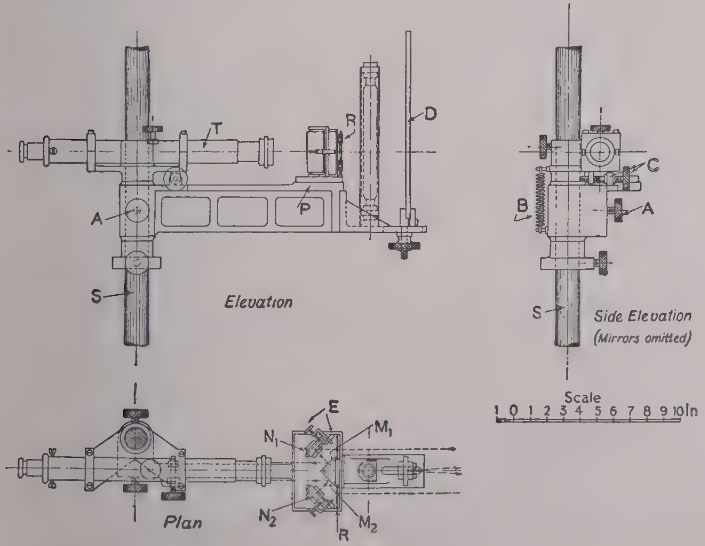

FIG. 76.

position about an axis at right angles to that of the rhomb.

A pointer Q is attached to the rhomb and arranged so that, when it coincides with a mark on the clip *d*, the instrument is set at the proper gauge length.

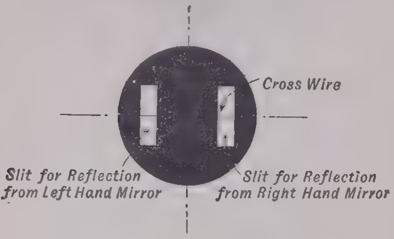
Cross Wire
Slit for Reflection from Left Hand Mirror
Slit for Reflection from Right Hand Mirror

FIG. 77.

Extension of the test piece rotates the rhombs, and the angular rotation is measured by a scale and telescopes as described for Bauschinger's extensometer. Two readings are necessary, one for each rhomb, and two telescopes are usually employed.

At the N.P.L. a "one telescope" arrangement has been in use for some years with complete success. This is shown diagrammatically in *Fig.* 76.

The telescope T is attached to a support S. Immediately under the telescope, and attached to the same support, is a platform P whose height from the ground can be quickly adjusted. It can be locked in any position by a knurled-headed screw A. The telescope arrangement is kept in contact with the platform by means of a spring B, and their relative position in a horizontal plane can be accurately adjusted by means of the pivoted nut and screw C.

The platform carries an illuminated scale D, two fixed mirrors M_1 and M_2, and two mirrors N_1 and N_2 pivoted vertically so that their position can be altered by means of the screw and spring E.

The illuminated scale is reflected from one of the "Martens" rotating mirrors on to the pivoted mirror N_1, and the angle of this is adjusted to bring the reflection into the telescope by means of the fixed mirror M_1. The other "Martens" mirror is made to reflect the illuminated scale on to the pivoted mirror N_2, and this is adjusted to bring the reflection into the same telescope. The difference between the lengths of the two rays is so small that the telescope can, at the same time, focus the scales reflected by each of the Martens rotating mirrors. The mirrors reflect "ghosts," and these are cut out by inserting a mask (*Fig.* 77) in the eyepiece of the telescope, and by having adjustable brass shades (R, *Fig.* 76) fixed in between the mirrors on the platform and the extensometer.

The mirrors used must be absolutely flat, and great care should be taken to see that they are not distorted when they are clamped in position.

A curved scale should be used, but readings can be taken on a straight scale and a correction applied. The illuminated scale is graduated in millimetres and readings of the scale can be estimated to 0·1 mm.

If l = the width of the rhombs,
L = distance from the scale to the axis of the test piece,
x = extension of the test piece,
X = sum of the readings from the two Martens mirrors.

$x/\mathrm{X} = l/4\mathrm{L}$.

Usual values of l and L are 0·180 in. and 45·0 in. respectively. If X is measured in millimetres we have

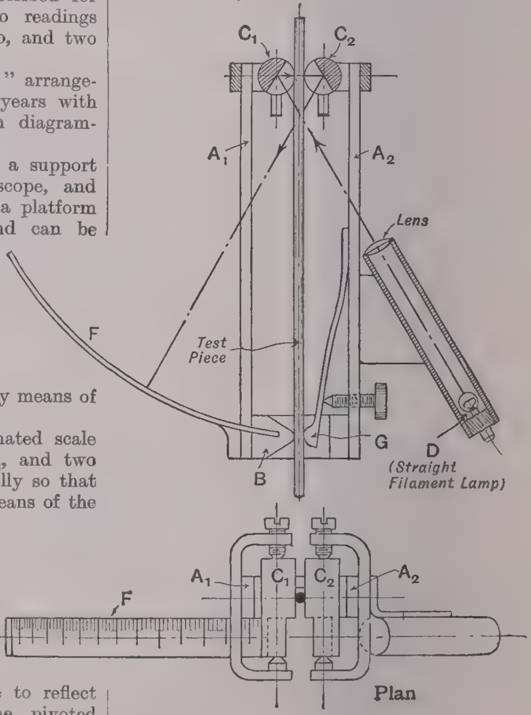

FIG. 78.

$$x = \frac{l \times \mathrm{X}}{4 \times \mathrm{L}} = \frac{0{\cdot}18 \times \mathrm{X}}{45 \times 4} = 0{\cdot}001 \times \mathrm{X} \text{ millimetres.}$$

By providing suitable clips this extensometer can be used on gauge lengths from one inch upwards and with test pieces from 0·03 to 0·8 in. diameter.

An instrument using Martens's method but of slightly different design to that shown in *Fig.* 76 is supplied by Messrs. Alfred J. Amsler & Co., Schaffhausen.

(iii.) *Westinghouse Wire Testing Extensometer.*—This instrument, designed by Lynch & Brace[1] for extensometer tests on small sections such as wire, ribbon, etc., is shown in *Fig.* 78. It consists of two clips A_1 and A_2, fastened at one end to a slotted block B, while the free ends carry hardened rollers, C_1 and C_2, which can turn in pivot bearings.

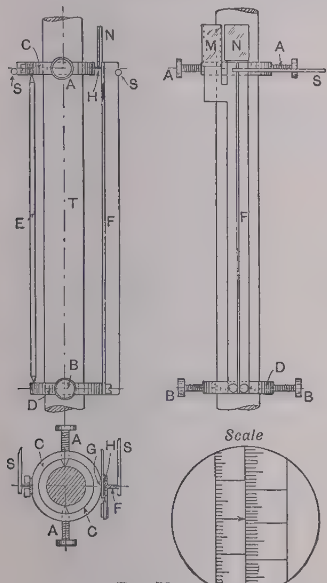

FIG. 79.

One end of the instrument is clamped to the test piece between the spring G and the block B, and the rollers C_1 and C_2 are sprung apart to take the test piece at the other end. Plane mirrors are attached to the rollers, which are arranged so that a spot of light from a lamp D is reflected from the mirror C_1 to mirror C_2, and thence to the curved scale F.

With this extensometer increases of length of 0·00002 in. can be measured.

(iv.) *Single Mirror Apparatus. Morrow's Extensometer.*[2]—This instrument is a combination of Ewing's lever arrangement for making the extensometer indicate the mean strain, with a mirror method of measuring that strain. It is shown diagrammatically in *Fig.* 79. The attachment to the test piece T is by four set screws AA and BB. AA serves as a fulcrum for the top annular ring C, and when the test piece extends, the movement between C and F (which is an extension of the bottom ring D) is approximately twice the extension. So far the principle is the same as that of the Ewing extensometer, but whereas Ewing measures this extension by a micrometer microscope, Morrow uses a "Martens" rhomb H to which a mirror N is attached in a vertical position.

A second mirror M is permanently fixed to F. The images of the scale from both mirrors are seen side by side in the telescope. An index mark is taken on the image of M for reading the movement of N. By this means allowance is made for any movement of the test piece and extensometer, as a whole, relative to the position of the telescope.

With the instrument as used by Morrow the extension of the specimen is obtained to the nearest $\frac{1}{1,200,000}$ of an inch.

Other single mirror extensometers have been designed by Unwin,[3] Martens,[4] and Hartig.[5]

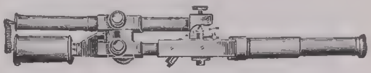

FIG. 80.

§ (54) COMPRESSOMETERS. — Most instruments used for tensile experiments can also be used as compressometers provided that the test piece is of sufficient length to take them. It is, however, often necessary to modify the method of attachment. The conditions applicable to the use of extensometers (§ (43)) apply, with just as much force, to compressometers.

The Yale extensometer (§ (45)), using double micrometer screws, and the Wisconsin (§ (49)) dial extensometer are both used in the U.S.A. for compression tests by a suitable modification of the clamps.

Ewing's compressometer for short blocks (*Fig.* 80) is on the same principle as that used for tension tests. The mechanical multiplication is, however, increased to five times, enabling readings of $\frac{1}{125,000}$ of an inch to be obtained.

Martens's mirror arrangement (*Fig.* 75) has been used by the author as a compressometer on diameters up to $4\frac{1}{2}$ in. by providing additional springs S to hold the clips on to the larger size of test piece.

Unwin[6] has designed an instrument for short blocks combining lever and microscope.

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[1] "Wire Testing Extensometer," *Proc. Amer. Soc. Test. Mat.*, 1919, xix. part ii. p. 696.

[2] *Inst. Mech. Eng. Proc.*, 1904, ii. 469.

[3] *Phys. Soc. Proc.*, 1886, viii. 178.

[4] *Handbook of Testing*, part i. p. 590.

[5] *Civilingenieur*, 1893, part vi.

[6] *Testing Materials of Construction*, 1910, p. 231.

This instrument is shown diagrammatically in *Fig.* 81. The lever L_1, having a knife edge resting on the pillar P, gives a mechanical magnification of 2½. The frame L_2 is fixed to the specimen by four screws, and both frame L_2 and lever L_1 carry silver plates on their ends having a fine scratch on each. The distance between these scratches is measured by the micrometer microscope M. Alterations of this distance with load give the compressive strain as the mean of that on the two sides of the block.

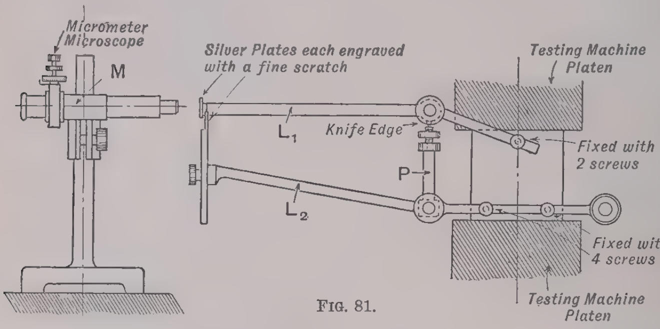

Fig. 81.

§ (55) Torsional Strain Indicators.—For measuring elastic deformations in torsion, in order to determine the coefficient of rigidity, accurate observations are necessary. All instruments for this purpose should be independently fixed on the bar with a definite gauge length.

Where a high degree of accuracy is not essential, a convenient and simple method is to clamp two long pointers on to the specimen at a given distance apart and observe their movement over a fixed scale. The difference between the readings is a measure of the torsional strain.

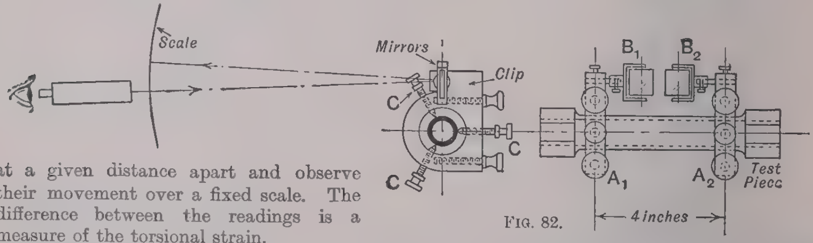

Fig. 82.

(i.) *Porter's Torsion Indicator.*[1]—In this indicator the long pointers of the above instrument are replaced by two rings clamped to the test piece at a gauge length of 3 in. by three set screws. One clamped ring is graduated in degrees and carries on it a concentric ring engraved with a vernier reading to five seconds of angle. The vernier ring is supported on ball bearings and is operated by an arm fixed to the second clamped ring. Provision is made for setting the vernier and for any change of length of the specimen during test.

(ii.) *Coker's Torquemeter.*[2]—The torsional strain is measured by this indicator on a length of 8 in. At one end of the specimen a graduated circular plate is attached to the gauge mark by three set screws. This plate carries an arm and vernier which can be moved round it, and to which is attached a frame carrying a cross-wire and a mirror. At the other gauge mark a chuck, also fixed by three set screws, supports an arm having a micrometer microscope attached to it. This microscope sights on to the cross wire of the vernier plate frame.

When the torque is applied to the test piece the cross wire moves relatively to the microscope, and the amount of this movement is measured by the micrometer eyepiece to about one second of arc. When the strain has exceeded the range of the eyepiece the cross wire can be readjusted to zero by moving the cross-wire arm round the circular plate by means of a tangent screw.

(iii.) *Unwin's Torsion Micrometer.*[3]—This instrument acts on the same principle as that designed by Coker. There are, however, one or two slight differences in detail. With Unwin's method the microscope, attached to a projection from one clip, sights on to a finely divided scale on the other clip at a gauge length of 5 in. The micrometer eyepiece reads to 0·005 degree.

(iv.) *Mirror Torquemeter.*—The method employed at the N.P.L.[4] is shown in *Fig.* 82, and consists of two small clips, A_1 and A_2, each fastened to the test piece by three pointed set screws C. Attached to these clips are the adjustable mirrors B_1 and B_2 of a Martens mirror extensometer. The mirrors are arranged, on a gauge length of four inches, so that they are in the vertical plane passing through the axis of the test piece. The method of measuring the movement of the two mirrors by the use of a single telescope has already been described (§ (53)). The difference between the readings from each mirror at small increments of the torque give, when multiplied by a constant depending on the scale distance, the angle of twist for that increment on a length of four inches.

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[1] *Amer. Soc. Test. Mat.* x. 578.

[2] *Roy. Soc. Edinburgh Trans.* xl. part ii. p. 263.

[3] *Testing Materials of Construction*, 1910, p. 232.

[4] Batson, *Inst. Mech. Eng. Proc.*, March 1917 p. 183.

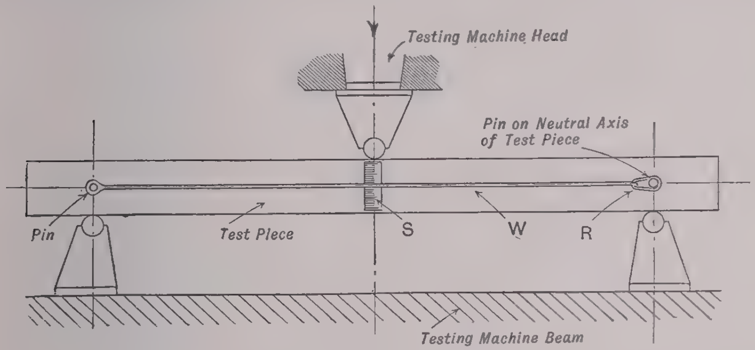

Fig. 83.

§ (56) Deflectometers.—These instruments are used to measure the amount of bending of beams, etc., during a transverse test.

Commercially a multiplying lever deflectometer (*Fig.* 60) is generally employed or a strained-wire arrangement may be used where large deflections are to be measured. This is shown in *Fig.* 83. A fine wire W is strained between two pins fixed on the neutral axis of the beam above the supports and kept taut by a rubber band R. This forms a datum line. A polished scale S is graduated either in mm. or inches and suspended from the neutral axis at the centre of the beam. The observer brings his eye to such a position that the wire and its image in the polished scale coincide, and he then reads the position of the wire on the scale.

For most work an accuracy of 0·001 in. is usually sufficient.

The methods described above do not fulfil some of the essential conditions for accuracy, which are:

(1) The apparatus should be designed to give the relative deflections of points in the neutral axis of the beam. This requirement is not fulfilled by the lever deflectometer.

(2) The mean deflection should be obtained. The apparatus should either provide for taking readings on both sides of the test piece or automatically give the mean deflection. This condition is not accomplished by the strained-wire arrangement.

§ (57) Apparatus for measuring Lateral Strains. (i.) *Coker's Lateral Strain-measuring Apparatus.*[1]—This instrument is shown diagrammatically in *Fig.* 84 and consists of two tubular arms A_1 and A_2 connected by a flexible steel plate B which forms the fulcrum. The steel plate is gripped by two collars CC, one of which carries an adjustable screw L, which bears on the test piece and keeps the instrument from turning. The arms are attached to the test piece T by two screws DD, the pressure on which is provided by the compression of a spiral spring S on the opposite side of the fulcrum.

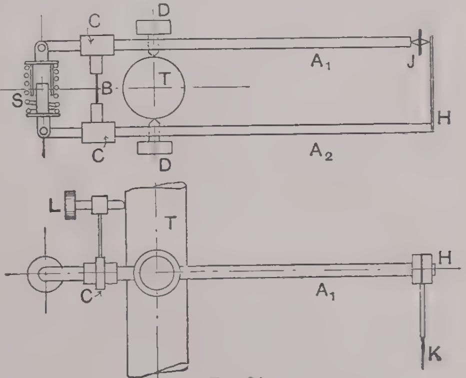

Fig. 84.

The arm A_2 has a projection H, the end of which is opposite to the arm A_1 and is arranged to spring towards it, gripping a knife edge J.

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[1] *Roy. Soc. Edinburgh Trans.* xxv. part i. p. 452.

Any change in the diameter of the test piece causes a relative movement between the end of the arm A_1 and the projection H of the arm A_2. This movement rotates the knife edge J, to which a mirror K is attached. The rotation of this mirror is observed by a telescope and scale and gives a measure of the alteration in the diameter of the test piece.

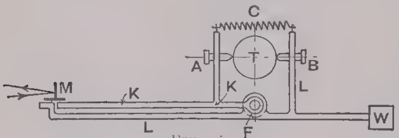

Fig. 85.

The scale distance is arranged so that each scale division represents 0·000001 in.

(ii.) *Morrow's Instrument for measuring the Lateral Contraction of Tie Bars.*—This instrument,[1] shown diagrammatically in *Fig.* 85, also uses optical means of measuring the relative displacement of two arms.

Two arms KK and LL are pivoted at F and two screws A and B grip the test piece T and are pressed inwards by stiff springs C. Alteration in the diameter of the test piece causes arm K to fall relatively to the arm L. This relative motion is measured by the tilt of a mirror M which is supported on three points, two of which rest on arm L and one on arm K. A vertical mirror, not shown, is also fixed to L at the side of the tilting mirror M and serves as an index for that mirror, thus eliminating any errors due to the instrument moving relatively to the observing telescope. The instrument is balanced by a weight W.

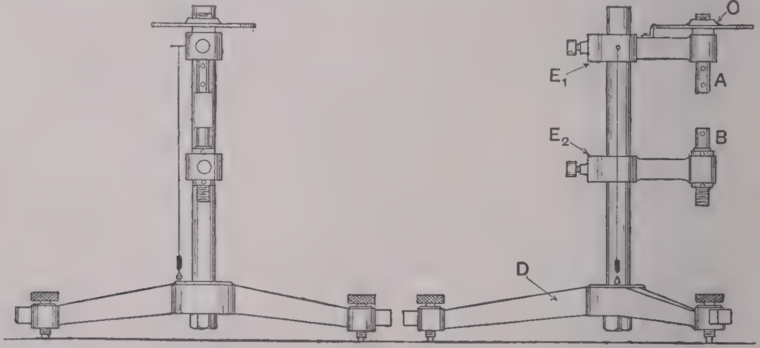

Fig. 86.

§ (58) Calibration of Deformation-measuring Apparatus.—The usual methods of obtaining the constants of extensometers, compressometers, etc., are:

(1) By measurement of the leverage of the instrument. In some types of apparatus this can be carried out successfully. With the Martens mirror extensometer the constant depends on the width of the rhombs and the scale distance. The former can be determined by an accurate measuring machine, and the latter is usually adjusted by setting the scale distance from the test piece by means of a gauge of known length.

(2) By test on a steel test piece whose elastic constants have been accurately determined. The test piece is gripped in the testing machine in such a way that the stress is distributed in the proper manner and readings of the apparatus which is to be calibrated taken at definite increments of the load. These readings are then compared with those calculated from the known elastic constants. If possible, a standardised apparatus is attached to the test piece at the same time as the one whose constants are unknown and a direct comparison made.

(3) By a calibrating instrument. An instrument used by the author is shown in *Fig.* 86 and consists of a stand D carrying two arms E_1 and E_2. These arms are movable along a feather on the stand so that their position can be adjusted. They are bored accurately in line to take two rods A and B. B is fixed, but A can be moved up and down by a micrometer head C. A also works along a feather which prevents rotation.

The apparatus to be calibrated is attached to A and B and its readings compared with the movement of the micrometer head. The pitch of the micrometer screw is 0·025 in. and the head is divided into 25 parts so that one division of the head (about ½ in. long) corresponds to a movement of 0·001 in.

It is essential that the desired reading

[1] Morrow, *Phil. Mag.*, 1903, vi. 417.

should always be approached in the same direction in order to eliminate any backlash in the screw. As the accuracy of the calibration depends upon the accuracy of the screw, this has been determined on a screw-measuring machine and a correction curve obtained for it.

§ (59) AUTOGRAPHIC RECORDING APPARATUS.—The deformation co-ordinate of the curve is always recorded directly from the test piece itself, usually with some form of multiplication. The main difference in the various types of recorders is in the method of obtaining the load co-ordinate. There are two principal ways in which this is done :

(i.) Load recorded by the movement of the counterpoise with or without automatic adjustment.

(ii.) Load co-ordinate obtained by the deformation of a calibrated spring.

The early diagramming apparatuses were applied to pendulum machines. Thurston, in 1876, designed the torsion test apparatus described in § (10), and, in 1877, Abbott constructed a recorder adapted to a machine in which the pull from a hydraulic press is transmitted through the test piece to a pendulum, the angular rise of which gives a measure of the load. An improved apparatus of the same kind was used by Pohlmeyer in 1882, and more recently by the N.P.L. for tests of copper and bronze wires fully described in the British Engineering Standards Association Report, No. 55.

§ (60) AUTOGRAPHIC APPARATUS, RECORDING POSITION OF COUNTERPOISE. — In machines having a moving counterpoise the load co-ordinate is obtained from the position of the counterpoise on the beam by moving a drum or pencil, by suitable pulleys or gearing, from the shaft or screw which drives the counterpoise. The beam is usually kept floating by moving the poise by hand adjustment. By this means, however, the reduction of load at the yield point cannot be obtained and the final reduction, when the test piece is stretching locally, is only imperfectly reproduced.

Riehle Autographic and Automatic Apparatus. — In some American machines an autographic apparatus is provided having an automatic weighing device. This device is designed to move the poise on the beam automatically to balance the load on the specimen. In the Riehle machine the scale beam on rising or falling completes an electric circuit at the top or bottom stop in the beam stand. Each circuit is separate and connected to a magnet.

The driving pulley of the machine turns a horizontal shaft which has a cast-iron disc on its end. This disc in turn drives one of two fibre wheels located equidistant on either side of the centre of the disc. Each fibre wheel has an armature controlled by one of the magnets of the electric circuits mentioned above. Thus when the beam rises and completes the top circuit, one of the magnets attracts its armature, causing one of the fibre wheels to engage with the cast-iron disc and drive the poise along the beam and so balance the load. When the beam drops and hits the lower contact, the armature of the other fibre wheel is attracted by its magnet, which brings this fibre wheel on to the cast-iron disc and drives the poise in the opposite direction.

The screw which drives the poise also, through gearing, drives the recording pencil axially along the drum, so that the reading of the load, which is recorded, corresponds directly to the load weighed.

The other ordinate is obtained by the drum revolving proportionally to the deformation. "Fingers" rest on top of U clamps fastened to the specimen by hardened steel-pointed set screws at the correct gauge distance. The lower "finger" moves downwards as the test piece extends and by means of a rack and pinion converts this motion into a rotary one revolving the drum through mitre wheels. The actual extension is magnified five times. The fingers are so arranged with clamps on telescopic tubing that only the extension between the U clamps is recorded on the diagram.

§ (61) AUTOGRAPHIC APPARATUS USING A CALIBRATED SPRING.—(i.) This method is used in two Wicksteed recorders. In the earlier apparatus [1] the measurement of the stress is entirely independent of the position of the poise on the beam, but is taken as being proportional to the compression of a helical spring acted upon by an auxiliary plunger operated by hydraulic pressure from the straining cylinder of the testing machine. As the load is applied by the straining cylinder it is balanced by running the poise along the beam, and the hydraulic pressure in both the straining cylinder and the auxiliary cylinder increases. The auxiliary plunger therefore compresses the spring, and the amount of the movement of the spring is recorded by a pencil on the recording paper in a direction parallel to the axis of the drum. The drum is also given a rotation proportional to the extension of the test piece and an automatic diagram is thus drawn.

The auxiliary plunger is rotated during the test in order to eliminate the friction as far as possible. The yield and maximum loads are noted from the position of the counterpoise, in order to fix the load scale of the diagram.

(ii.) *The Buckton Wicksteed Patent Spring Balanced Recorder* is entirely automatic, the variations in the resistance of the specimen are accurately recorded. The action of the recorder is shown diagrammatically in *Fig.* 87. The counterpoise W is placed at the end of its travel, thus extending the spring S. As the load W is applied to the test piece an equivalent load is released from the spring S, which therefore contracts. The amount of this reduction in length is proportional to the load on the test piece and is registered on the recording drum R by means of a cord C passing over fixed pulleys P_1 and P_2 and attached to the pencil D of the apparatus.

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[1] Wicksteed, *Inst. Mech. Eng. Proc.*, 1886, p. 27.

The load is thus recorded axially, and the drum is rotated by the movement of the lower pulling head relatively to that of the upper shackle, to give the deformation co-ordinate. As the deformation is not communicated from a fixed gauge length of the test piece, it includes movement in the grips and cannot be taken as the extension of the specimen.

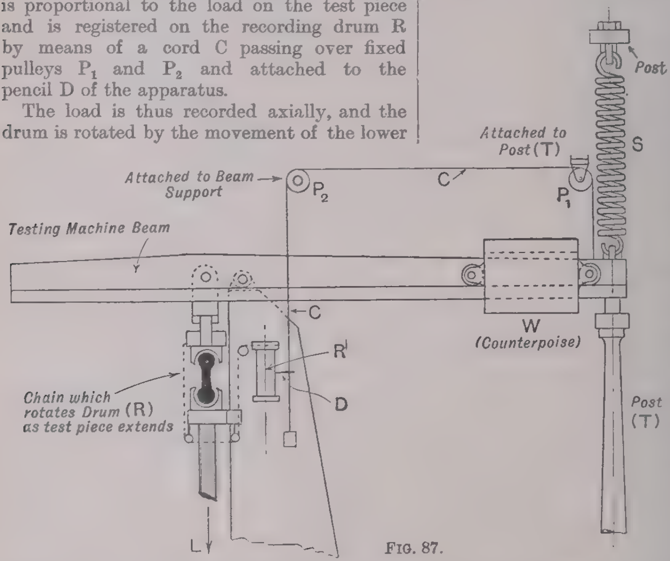

FIG. 87.

(iii.) *Moore's Autographic Recorder* [1] is very similar in action to the Wicksteed spring recorder just described, the only difference being that the spring is placed on the other side of the principal knife edge so that when the poise is run out the spring is compressed. As the load is applied to the specimen an equivalent load is released from the spring, which elongates until equilibrium is established. Moore used a steam-engine indicator; the spring of the indicator is attached to the beam of the testing machine to give the load co-ordinate, and the cord of the indicator is attached to the test piece to record the extension.

(iv.) *Kennedy-Ashcroft Recorder* [2] (*Fig.* 88).—Full details of this recorder are given in Kennedy's paper to the Institution of Civil Engineers, and the following description of the principle of the apparatus is also taken from that paper:

"The test piece '*a*' is placed in the machine in *series* with a stronger bar 'B,' called a spring-piece, and the two, which are connected directly by a simple coupling, are pulled simultaneously, the one through the other. The spring-piece is of a material such that its limit of elasticity occurs only at a load greater than that which will break the test piece. It must also be of material ascertained by previous experiment to be perfectly elastic, so that its extension is strictly proportional to the pull on it, and therefore to the pull on the test-bar. By a simple arrangement a very light pointer '*c*' is made to swing about an axis through an angle proportionate to the extension of the spring-piece, and proportional therefore to the pull on the test-bar. The end of this pointer in its motion always touches a sheet of smoked glass '*d*,' to which is given a travel—in its own plane—proportional to the extension of the test piece, and in this way the diagram is drawn. By an arrangement of differential levers it is assured that the motion of the glass depends solely on the extension between the marked points on the test-bar, so that no amount of extension of the coupling, in the ends of the test piece, or in any other part of the apparatus, can move the glass. The apparatus is also so arranged that the absolute elongation of the spring-piece does not cause any motion of the pointer relatively to the glass."

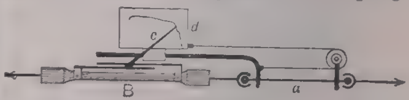

FIG. 88.

(v.) *Dalby's Optical Recorder.*[3]—This apparatus is shown diagrammatically in *Fig.* 89. It is similar to the Kennedy-Ashcroft recorder in that the load is obtained from the extensions of a bar acting as a very stiff spring, but whereas Kennedy uses mechanical means of increasing and transferring these extensions to the diagram, Dalby adopts an optical method and thus obviates the possibility of inertia errors.

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[1] *Proc. Amer. Soc. Test. Mat.*, 1908, viii. 653.

[2] *Inst. Civ. Eng. Proc.*, 1886–87, p. 31.

[3] *Roy. Soc. Proc.*, 1912, lxxxvi.A, and 1913, lxxxviii.A.

The spring-piece W, seen in *Fig.* 90, is hollow and is connected at its upper end to the shackle A of the testing machine. A projection through the shackle is mounted in a light-tight box B. The ray from a point of light Z is reflected by a fixed mirror Q on to a concave mirror M, which again reflects it on to a third mirror N and thence to a photographic plate at F, where it is accurately focussed.

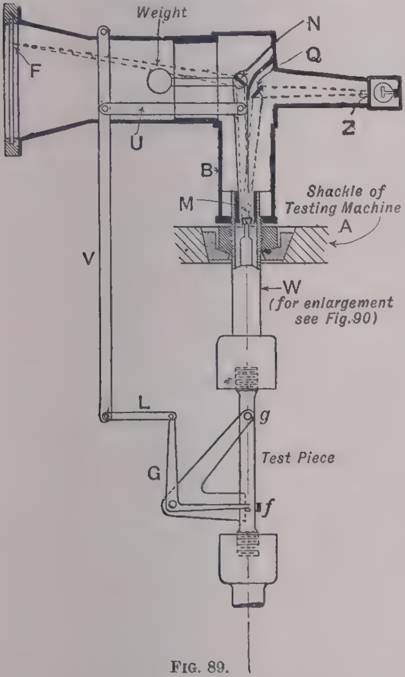

FIG. 89.

The mirror M is supported on three points; two of these, about which the mirror can tilt, are in contact with the hollow spring-bar, while the third rests on a cup on the top of the central rod T. Any stretch of the spring-bar W will cause it to move relatively to the central rod T and thus tilt the mirror M. This tilt will cause the point F to move horizontally across the photographic plate, a distance of 340 times the stretch of the bar.

The mirror N is rotated by the linkwork G, L, V, U proportionally to the stretch of the test piece between the gauge length *gf*, and this movement displaces the point "F" vertically up and down on the photographic plate. The point of light therefore traces on the plate a stress-strain diagram of the test made. After development the relation between load and extension can be measured from the plate with great accuracy.

V. DETERMINATION OF THE ELASTIC CONSTANTS

The complete load-extension diagram is drawn by means of some form of autographic diagram apparatus, but for the accurate determination of the elastic limit and modulus of elasticity it is necessary to determine with extreme accuracy the deformations produced by small loads.

§ (62) THE ELASTIC LIMIT.—The elastic limits in tension and compression obtained from stress strain curves on the first loading of a piece of material such as steel are not constant for the material but depend upon the previous treatment that the material has undergone. They are called the "primitive" elastic limits to distinguish them from the "natural" elastic limits set up in the material when it is subjected to a few alternations of stress.

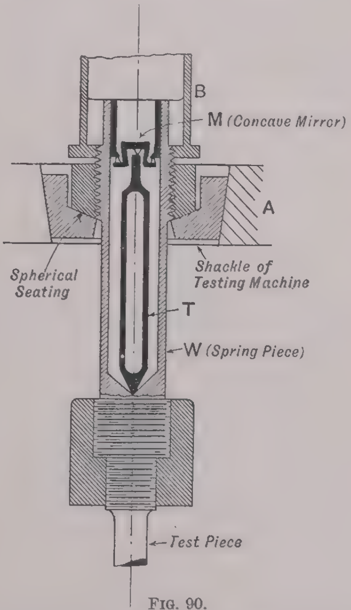

FIG. 90.

The definition of the elastic limit which has been standardised by the British Engineering Standards Association, and is commonly accepted in this country, is that it is the least stress at which Hooke's Law ceases to be exact. It is sometimes called the limit of proportionality or the "P" Limit.

With some materials there is a deviation from Hooke's Law, and a lack of proportionality between stress and strain, even for values of the stress so small that when it is removed the material regains its original dimensions (*i.e.* that there is no permanent set). Hence the elastic limit is occasionally defined as the maximum stress below which the material would fully recover its form upon removal of the load. This may be called the "R" Limit. With most materials the two definitions give substantially the same results, they both require delicate extensometers for their determination, but the method of procedure during the test is slightly different.

In the first case extensometer readings are taken with gradually increasing loads and the results plotted as a load extension diagram. The point at which there is a deviation from Hooke's Law is easily located, and the Modulus of Elasticity can be calculated from the slope of the elastic line. The increments of load taken are usually about $\frac{1}{15}$ of the estimated value of the "P" Limit, and if the value of the "P" limit is known to a first approximation, the increments are made smaller in the neighbourhood of that value and until the test is completed.

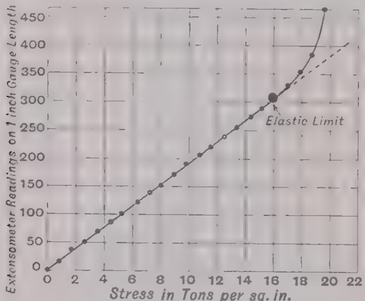

FIG. 91.

The following measurements (Table 21) are taken from a specimen machined from a piece of boiler plate and show how the "P" limit and the modulus of elasticity can be calculated.

The results are plotted in *Fig.* 91 and show that the limit of proportionality is at 16 tons per square inch.

From the elastic line it can be seen that at a stress of 13·3 tons per square inch the extensometer reading is 254; this is equivalent to an extension of 0·001 in. on 1·0 in.

The strain is therefore 0·001 and the modulus of elasticity

$$=\text{stress/strain}=13{\cdot}3/0{\cdot}001.$$
$$=13300 \text{ tons/sq. in.}$$
$$=29{\cdot}8\times10^6 \text{ lbs. per sq. in.}$$

If the readings of the extension and the applied load are known to sufficient accuracy, the limit of proportionality can be ascertained with more precision by calculating the elastic extension for the various loads applied, and plotting the difference between this and the actual extension as ordinates with the loads as abscissae. Table 21 shows this calculation, and the results are plotted in *Fig.* 92.

TABLE 21

EXTENSOMETER TEST ON BOILER PLATE

Diameter of test piece = 0·375 in.; Cross-sectional area = 0·1104 sq. in. Extensions measured on a 1-inch gauge length by a mirror extensometer.

Stress in Tons per Sq. In.	Extension Readings* on a 1″ Gauge Length. A.	Calculated Elastic Extension.* B.	Permanent Set.* A-B.
0·00	0	0	0
0·44	8	8	0
0·89	15	17	− 2
1·33	27	26	1
1·78	35	34	1
2·23	41	41	0
2·68	50	51	− 1
3·13	57	59	− 2
3·57	67	68	− 1
4·01	74	76	− 2
4·46	83	85	− 2
5·35	101	102	− 1
6·24	117	119	− 2
7·14	134	136	2
8·03	150	153	− 3
8·93	169	171	− 2
9·82	188	188	0
10·70	206	206	0
11·59	221	223	− 2
12·48	237	240	− 3
13·38	253	257	− 4
14·27	272	274	− 2
15·17	290	291	− 1
16·05	308	308	0
16·93	330	325	5
17·81	353	342	11
18·73	383	359	24
19·62	461 *	376	85

* 1 unit = 1/254,000 in.; *i.e.* 461 units = 0·0181 in.

Results of tests:

Elastic limit = 16·0 tons/sq. in.
Yield stress = 20·2 tons/sq. in.
Ultimate stress = 28·04 tons/sq. in.
Modulus of elasticity = $29{\cdot}8\times10^6$ lbs./sq. in.
Extension = 36·9 per cent.

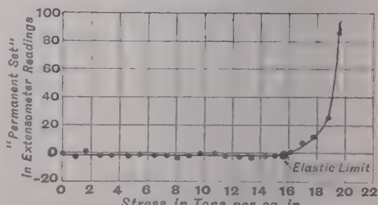

FIG. 92.

It is of first importance that shocks and vibrations should be avoided during the application of the load in an extensometer test, since actions of this kind seriously affect the sensitivity and the accuracy of the instrument.

The second method of procedure, employed to determine the "R" limit, is to apply an increment of the load and upon its release measure the amount of the permanent set, if any. The load is then doubled, and the set on release again measured. A further increase of the load is applied and released, and the operation repeated until a well-defined and increasing permanent set is obtained.

The amount of the permanent set is plotted against the load and an estimate obtained, from the curve, of the point at which the material starts to have a permanent set. From this reading the "R" limit is calculated.

Results show that the actual location of the elastic limit depends on the sensitiveness of the apparatus used, low results being obtained with instruments of high sensitivity. The determination of the "R" limit requires considerable time to carry out, but indicates plastic yielding of the material at an early point when it is properly ascertained.

(i.) *Elastic Limit by Change of Temperature.*—If a material behaves elastically, increasing load produces a cooling of the test piece, but if, however, a permanent set is produced, work is done in internal friction and the temperature of the test piece rises. E. Rasch,[1] of Gross Lichterfelde, has carried out some experiments to determine the temperature-load curve for materials. For temperature measurement he uses thermo-couples of iron-constantan, copper-constantan, or silver-constantan bound to the test piece. Alterations of the temperature are read from the movement of a galvanometer. He finds well-defined points of inflection in his curves. The loads at which these points occur give an elastic limit sometimes called the thermal or "T" limit.

(ii.) *Effect of Overstrain on Elastic Limit and Yield Point.*—Bauschinger found the effect, on the *limit of proportionality*, of overstrain to be as follows:

(1) If the limit of proportionality was exceeded but not the yield point, then the *former is raised* even if the test piece is immediately reloaded.

(2) If the yield point is exceeded, immediate reloading gives a *lower limit of proportionality*, but reloading after a long interval of time may give a *raised limit of proportionality*. James Muir[2] found that this effect can be accelerated by immersion for a few minutes in boiling water.

Bauschinger also found that the *effect on the yield point* was that if the yield point was exceeded, then subsequent reloading gave a *new yield point raised* to the stress to which the bar was loaded in the first place. This effect occurred even if the bar was reloaded immediately, but if there was an appreciable interval of time before the reloading the new yield point might be higher than the maximum stress applied in the first loading.

§ (63) THE MODULUS OF ELASTICITY.[3]—The modulus of elasticity is the number by which the amount of any specified stress or component of a stress within the limits of elasticity must be divided to find the strain or any stated component of the strain which it produces.

There is a modulus of elasticity in tension, compression, and shear.

The modulus of elasticity in tension, denoted by E, is sometimes called the modulus of direct elasticity, or Young's modulus. The value in compression is generally the same as that in tension.

The modulus of elasticity in shear, denoted by C, is called the modulus of transverse elasticity, or the modulus of rigidity.

The moduli C and E are connected by the equation

$$C=\frac{E}{2(1+\sigma)},$$

where σ = Poisson's ratio.

There is also a volumetric modulus of elasticity, sometimes called modulus of elasticity of bulk or modulus of cubic compressibility and denoted by K. It is the number by which the stress upon the exterior of the substance must be divided to give the diminution in volume or cubical strain.

If E and C are known, K can be calculated from the formula:

$$K=\frac{CE}{9C-3E} \quad \text{or } K=\frac{E}{3(1-2\sigma)}.$$

§ (64) MODULUS OF DIRECT ELASTICITY.—An example of the method of determining the modulus of direct elasticity (E) is given in § (62). Some materials, *e.g.* cast iron, mortars, and concrete, have no elastic line and therefore no definite modulus of direct elasticity. It can, however, be considered to be the reciprocal of the slope of the stress-strain curve at zero stress, but where the curvature near the origin is sharp this value of the modulus is of little use except as a comparative value of stiffness.

An example of a stress-strain curve with no definite elastic line is given in *Fig.* 93. The initial modulus of direct elasticity is obtained from the slope of the tangent (OA) to the curve at the origin.

For concrete, an empirical modulus is sometimes used for design calculations. The value is obtained in one of three ways:

(i.) The "tangent" modulus (E_t) is obtained from the slope of the tangent to the stress-strain curve where the ordinate is the working stress—25 per cent of the compressive strength is usual for the working stress—that is, from the tangent CD (*Fig.* 93).

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[1] *Proc. Int. Assoc. Test. Mat.*, 1909, Article vii$_3$.

[2] *Roy. Soc. Phil. Trans.*, 1900, cxciii. 1.

[3] See "Elasticity, Theory of," § (5).

(ii.) The "secant" modulus (E_s) is obtained from the slope of the line OB, where Bb = the working stress of the material.

(iii.) The "chord" modulus (E_c), determined by the ratio of stress to deformation, is given by the chord drawn between two points on the curve, defined by the limits of stress for working loads, *i.e.* from the chord EF (*Fig.* 93).

The tangent and chord methods give moduli of approximately the same value, which are higher than the secant modulus.

Stanton Walker[1] has shown, from a study of curves which he obtained from various samples, that the stress-strain curve is represented by a curve of the type

$$S = Kd^n,$$

where S = unit stress in concrete,
d = unit of deformation,
K = constant depending on strength,
n = an approximately constant exponent,

and the relation between modulus of elasticity and strength of concrete is

$$E = CS^m,$$

where E = modulus of elasticity,
C = constant depending on conditions of test,
S = compressive strength of concrete,
m = an exponent.

Similar equations were suggested by Bach and Morsch.[2]

Effect of Overstrain on the Modulus of Direct Elasticity.—If the limit of proportionality is exceeded the value of E for steel is lowered, in some cases as much as 20 per cent. Recovery, however, is effected by rest or immersion in boiling water.

Marshall[3] showed that if steel is initially strained to a point within the limit of proportionality, a second loading may give slightly higher values of E.

§ (65) VALUES OF THE MODULUS OF DIRECT ELASTICITY (*Young's Modulus*).—For carbon steels the value of E in lbs. per square inch varies between $28{\cdot}5 \times 10^6$ and 31×10^6, with an average value of $29{\cdot}5 \times 10^6$. It is nearly the same in tension and compression, and is practically independent of the carbon content and of the heat treatment.

For low percentage nickel steels (3 to $4\frac{1}{2}$ per cent) the average value of E is $28{\cdot}5 \times 10^6$ lbs. per square inch, while the value of E for high percentage nickel steels (30 to 35 per cent) is low, being about 23×10^6 lbs. per square inch.

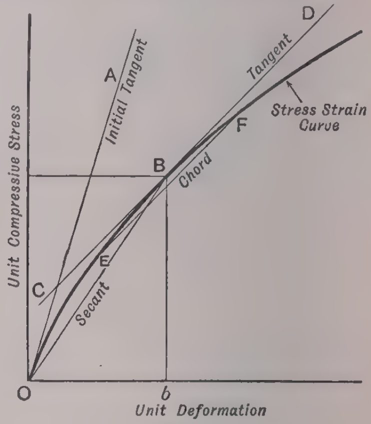

FIG. 93.

With cast iron there is no definite modulus of elasticity. Using a working-stress figure of 10,000 lbs. per square inch, the "secant" modulus for cast iron varies from 12×10^6 to 20×10^6 lbs. per square inch for grey cast iron, and from 20×10^6 to 25×10^6 lbs. per square inch for white cast iron.

Further values of E are given in Table 22.

TABLE 22

VALUES OF THE MODULI OF ELASTICITY

Material.	Value of E. Lbs./Sq. In.	Value of C. Lbs./Sq. In.
Carbon steels	29,500,000	12,000,000
Low percentage nickel steels .	28,500,000	11,200,000
Cast iron (grey) . . .	12,000,000 to 20,000,000	4,800,000 to 8,000,000
Cast iron (white) . . .	20,000,000 to 25,000,000	7,500,000 to 9,500,000
Copper (rolled) . . .	15,000,000	5,600,000
Copper (hard drawn wire) .	17,000,000	6,400,000
Copper (annealed wire) . .	16,000,000	6,000,000
Aluminium	10,000,000	3,800,000
Phosphor bronze . . .	14,000,000	5,200,000
Lead	2,500,000	880,000
Timber (English oak) . .	1,300,000	..
,, (Russian fir) . .	2,500,000 to 3,000,000	..
,, (Christiania spruce) .	1,500,000	..
,, (hemlock) . . .	2,500,000	..

§ (66) MODULUS OF TRANSVERSE ELASTICITY, OR MODULUS OF RIGIDITY (C).—The easiest method of determining the value of C is by means of a torsion test. The strain is measured by some form of torquemeter such as is described in § (55).

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[1] "Modulus of Elasticity of Concrete," *Proc. Amer. Soc. Test. Mat.*, 1919, xix. 512.

[2] Goodrich, *Concrete Steel Construction* (translation of Morsch's *Der Eisenbetonbau*).

[3] *Amer. Soc. Civ. Eng. Trans.* xvii. 62.

A stress-strain diagram is drawn in the same way as for direct stress but having torsional stress as abscissa and torsional strain as ordinate. The value of C is then calculated from the slope of the elastic line. The elastic limit in shear is also easily located from the curve.

A series of observations by the N.P.L. mirror torquemeter (§ (55)) on a piece of a shaft are given in Table 23 and the diagram shown in *Fig.* 94. From this diagram the limit of proportionality in shear is seen to be with a torque of 1350 lbs.-in. corresponding to a stress of 24·0 tons per square inch, and the modulus of rigidity (C) = 11·9 × 10⁶ lbs. per square inch.

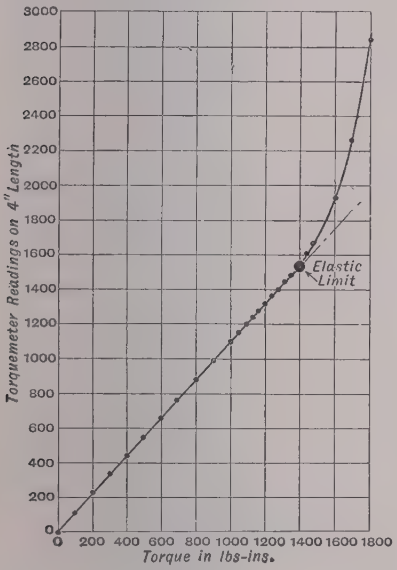

FIG. 94.

$$C = \frac{\text{stress}}{\text{strain}} = \frac{53{,}800}{0{\cdot}00453} = 11{\cdot}9 \times 10^6 \text{ lbs. per sq. in.}$$

Note.—Stress $= 24{\cdot}0 \times 2240$ lbs. per sq. in.
$= 53{,}800$ lbs. per sq. in.

$$\text{Strain} = r\theta/\text{L} \left\{ \begin{array}{l} \text{where } r = \text{radius of the test piece,} \\ \theta = \text{angle of twist on length L in radians.} \end{array} \right.$$

$$= (0{\cdot}25 \times 4{\cdot}15)/(4 \times 57{\cdot}3) = 0{\cdot}00453.$$

For wire the value of C is determined from the time of a single torsional oscillation.

Bauschinger[1] found the value of C for Bessemer steel of carbon content varying from 0·19 per cent to 0·96 per cent to range from 11·9 × 10⁶ to 12·7 × 10⁶ lbs. per square inch.

Platt and Hayward[2] give values varying from 12·3 × 10⁶ to 14·0 × 10⁶ lbs. per square inch.

Average values of C are given in Table 22.

§ (67) POISSON'S RATIO.—Poisson's ratio is the ratio of lateral to longitudinal deformation, and is usually denoted by σ.

An extensometer when used in conjunction with an apparatus for measuring lateral strains, such as are given in § (57), gives the value of σ for any material in the most direct way possible.

TABLE 23

TORSIONAL STRESS-STRAIN OBSERVATIONS

Diameter of test piece = 0·500 inches.
Gauge length = 4 inches.

Twisting Moment. Lbs.-Ins.	Torsional Strain.	
	In Torquemeter Units.*	Angle.
100	111	0° 19′
200	222	0° 38′
300	329	0° 57′
400	442	1° 16′
500	551	1° 34′
600	656	1° 52′
700	769	2° 12′
800	880	2° 31′
900	989	2° 50′
1000	1103	3° 9′
1040	1145	3° 16′
1080	1190	3° 23′
1120	1233	3° 31′
1160	1278	3° 39′
1200	1322	3° 46′
1240	1369	3° 54′
1280	1403	3° 60′
1320	1446	4° 7′
1360	1497	4° 15′
1400	1550	4° 25′
1500	1699	4° 50′
1600	1919	5° 26′
1700	2249	6° 20′
1800	2859 *	7° 58′

* A reading of 2859 = an angle of twist of 7 degrees 58 minutes on a length of 4 inches.

Both instruments are fixed on the test piece at the same time and measurements are obtained, of both lateral and longitudinal strains, for equal increments of load. The results are then plotted as stress-strain diagrams, and the ratio of the slopes of the elastic lines is equal to σ (Poisson's ratio).

The figures in the following table are taken from results obtained by Coker.

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[1] *Civilingenieur*, 1879.

[2] *Inst. Civ. Eng. Proc.* xc. 409.

TABLE 24

MEASUREMENTS OF LATERAL AND LONGITUDINAL STRAINS

Diameter of the test piece = 1·01 inches.
Length under test = 8·00 inches.

Load in Lbs.	Lateral Strain Reading.	Longitudinal Strain Reading.
1,000	0	0
3,000	20	34
5,000	42	66
7,000	63	99
9,000	85	132
11,000	106	165

From the plotted values of these results given in *Fig.* 95 it will be noted that the curves do not pass through the first point, but the remainder of the observations lie on straight lines. The slope of the longitudinal stress-strain curve $= 408{\cdot}7 \times 10^{-10}$, and the slope of the lateral stress-strain curve $= 107 \times 10^{-10}$, therefore $\sigma = 107/408{\cdot}7 = 0{\cdot}262$.

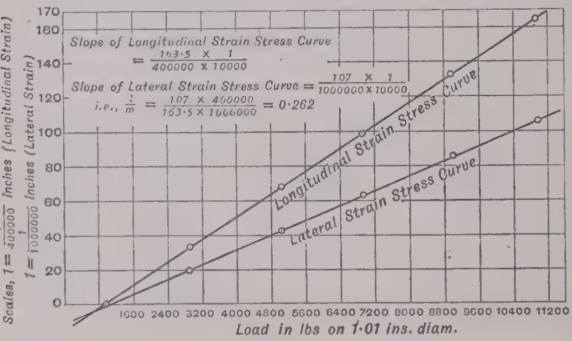

FIG. 95.

Average values of Poisson's ratio for various materials are as follows:

TABLE 25

Material.	Value of Poisson's Ratio.
Glass	0·25
Steel	0·27 to 0·30
Copper	0·33 (0·31 to 0·34)
Brass	0·33 (0·32 to 0·35)
Delta metal	0·34
Muntz metal	0·34
Lead	0·43
Stone	0·20 to 0·34
Concrete	0·08 to 0·18
Cast iron	0·23 to 0·27
Wrought iron	0·27 to 0·29

VI. SPECIAL FORMS OF TEST

§ (68) EXPERIMENTS ON THE REPETITION OF STRESSES.—It has been shown by experiment that materials fail under stresses considerably lower than the ultimate when those stresses are repeated many times.

Stanton and Bairstow[1] have shown microscopically that this deterioration is due to "slip lines being set up at the point of maximum stress in the cleavage planes of the crystals and due to unequal distribution of stress among the crystals." These slip lines broaden out and develop into actual cracks under repetition of stress. This type of failure is sometimes called "fatigue," but is perhaps better described as a "repeated stress failure."

In many cases the stress does not alternate between zero and a maximum in tension; there may be an initial tensile or initial compressive stress. For the purpose of a consideration of this subject compressive stress is taken as a negative tensile stress, and the range of stress as the difference between the maximum and minimum stresses.

Alternating stresses are usually considered as being those in which the maximum and minimum stresses are equal but of opposite sign.

Since 1864, when Fairbairn published results in the *Roy. Soc. Phil. Trans.* on riveted wrought iron girders subjected to repeated stresses, many important researches on this subject have been undertaken. As a result of his work, Fairbairn recommended that the safe repeated stress should be not more than one-third of the ultimate stress.

In 1871 Wöhler published the results of an exhaustive series of repeated stress experiments in direct stress, bending, and torsion which had been carried out during the previous twelve years. A full description of these tests is given in *Engineering* of 1871, and a good account by Unwin.[2] The most important deductions from these experiments on wrought iron and steel are:

(*a*) A stress below the ultimate will fracture wrought iron and steel if it is repeated many times.

(*b*) The range of stress, and not the maximum stress, determines within certain limits the number of repetitions before fracture.

(*c*) For a given maximum or minimum stress the number of repetitions before fracture increases as the range of stress is diminished, and there is a range of stress called the *limiting range* at which the number of repetitions is infinite.

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[1] "Resistance of Iron and Steel to Reversals of Direct Stress," *Inst. Civ. Eng. Proc.* clxvi. 78.

[2] *Testing of Materials of Construction*, 1910, p. 371

(*d*) The limiting range of stress diminishes as the maximum stress increases.

Wöhler's work was continued on the same machines and confirmed by Spangenberg, whose results, however, were not so consistent as the original work by Wöhler.

Results published in 1886 by Baker,[1] who had experimented with soft and hard steel (28 and 54 tons per square inch respectively) and 27-ton rivet iron, were in complete agreement with Wöhler's researches.

A considerable number of experiments on the effect of varying values of the minimum stress on the limiting range of stress have been carried out by Haigh.[2]

In 1915 he reported a series of tests on mild steel to the British Association. These are summarised in Table 26.

TABLE 26

REPEATED STRESS TESTS ON MILD STEEL

Ultimate Static Stress. Tons/Sq. In.	Limiting Minimum Stress. Tons/Sq. In. A.	Limiting Maximum Stress. Tons/Sq. In. B.	Limiting Range of Stress. Tons/Sq. In. B-A.
26·0	+ 4·05	+21·55	17·5
	− 0·88	+20·62	21·5
	− 7·3	+17·20	24·5
	−13·0	+13·00	26·0
	−16·96	+ 6·54	23·5
	−20·10	+ 1·40	21·5
	−21·95	− 2·45	19·5

The results are plotted in *Fig.* 96, from which it is seen that—

(1) When the minimum stress is zero, the range of stress is approximately 21·0 tons per square inch, *i.e.* a tensile stress of 80 per cent of the ultimate stress is sufficient to fracture the material when repeated a considerable number of times.

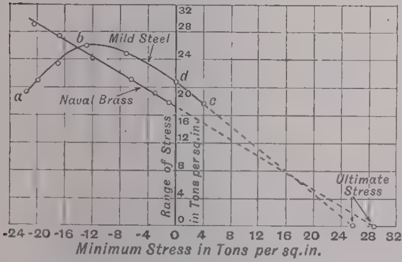

FIG. 96.

(2) When the minimum stress is less than half of the range (*i.e.* from 4·05 to −13·0) the curve approximately follows Gerber's parabola (see § (70) (ii.), otherwise there is a noticeable deviation from the parabolic form.

Results in Table 27 give details of the observations on mild steel when the stresses are alternating (*i.e.* equal tension and compression). These confirm Wöhler's deductions *b* and *c*, given above.

TABLE 27

ALTERNATING STRESS TESTS ON MILD STEEL

Limiting range of stress = 26·0 tons per sq. in.

Ultimate Stress. (Static.) Tons/Sq. In.	Alternating Stresses. (Equal Tension and Compression.)	
	Range of Stress. Tons/Sq. In.	Millions of Cycles to Fracture.
26·0	39·3 *	0·003
	32·7	0·08
	31·3	0·056
	31·3	0·16
	29·9	0·19
	29·9	0·80
	28·8	0·41
	28·7	1·01
	27·5	1·37
	27·5	2·18
	26·4	1·66
	26·4	5·90
	25·4	7·20 †

* 39·3 (range) = +19·65 to −19·65. † Unbroken.

Tests on Naval Brass by Haigh are summarised in Table 28 and plotted in *Fig.* 96, and show that for the brass under consideration the equation connecting minimum stress and range of stress is that of a straight line passing through the point where the minimum stress is equal to the ultimate stress. Haigh says that, as a rule, metals which give a considerable reduction of area at fracture have a high value of the ratio of alternating stress range (with equal tension and compression) to the ultimate stress. This usually varies between 1·20 in ingot iron and the best mild steel and 0·80

TABLE 28

REPEATED LOADING TESTS ON NAVAL BRASS

Ultimate Stress. (Static.) Tons/Sq. In.	Limiting Minimum Stress. Tons/Sq. In. A.	Limiting Maximum Stress. Tons/Sq. In. B.	Limiting Range of Stress. Tons/Sq. In. B-A.
28·7	− 1·0	+16·5	17·5
	− 3·4	+15·8	19·2
	− 6·5	+14·5	21·0
	−12·0	+12·0	24·0
	−17·0	+10·5	27·4
	−20·5	+ 8·5	29·0

---

[1] "Notes on the Working Stress of Iron and Steel," *Amer. Soc. Mech. Eng. Proc.*, 1886.

[2] *Brit. Assoc. Report*, 1915, p. 163; also *Inst. of Metals J.*, 1917, No. 2, p. 55.

in high tensile and tool steels in the annealed condition.

§ (69) BAUSCHINGER'S THEORY OF FAILURE. —Bauschinger's theory of failure due to repeated stresses is the only one which has received serious consideration. He shows that, with bars subjected to cyclical variations of stress, the elastic limits in tension and compression take up new positions, the range between the two limits depending on the material and the stress at the lower limit of elasticity. Thus if the elastic limit is raised in tension by overstrain it is simultaneously lowered in compression, so that for that condition of loading two new limits are set up, which Bauschinger calls the *natural* elastic limits. He showed, further, that the range between these limits was the same in magnitude as the maximum range of stress which could be applied to the material an infinite number of times without causing fracture.

Bairstow,[1] in an important paper communicated to the Royal Society from the N.P.L., has given experimental results which constitute the first strong support of Bauschinger's hypotheses.

In the testing machine used by Bairstow for the purpose of the experiments, cyclical variations of direct stress are automatically produced at the rate of two per minute in such a manner that the extensometer, which is of the Martens mirror type, is fixed to the specimen throughout the test, and in this way the whole history of the progress of fatigue is observed.

When the limits of stress are tension and compression of equal values it is found that, if the range of stress is above a definite value, the stress-deformation curve forms a closed loop, which is called the hysteresis loop, consisting of two parallel straight lines, corresponding to the variation of stress from the limits of stress towards the mean stress, and two curved portions, corresponding to the variations of stress from the mean value to the extreme values (*Fig.* 97). The width of this loop, which is the permanent set of the specimen per cycle, increases as the range of stress increases, but for a definite range of stress tends to a limit which is not greatly exceeded by subsequent repetitions of loading, even when this is the range at which fracture under fatigue eventually takes place. Under these conditions of stress the mean length of the specimen remains constant.

When the limits of stress are unequal the hysteresis loop is formed as before, but is not closed, owing to the fact that the mean length of the specimen gradually changes because of the continued repetition of the same cycle of stress, *i.e.* the change of mean length of the specimen per cycle is the amount by which the hysteresis loop is enclosed. The amount of the permanent extension during the earlier stages of the breakdown becomes considerable as the superior limit of stress approaches the static yield point, and if its value, after the first considerable stretch has occurred, be plotted against the corresponding values of the superior limit, it is found that the curves gradually come into coincidence with the ordinary static "stress-elongation" curve at the yield point. FEABC (*Fig.* 98) shows such a curve, in producing which no cyclical variations of stress are concerned.

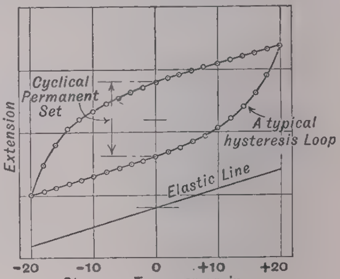

FIG. 97.

The principal conclusions arrived at by Bairstow are summarised thus :

(*a*) The "natural elastic range" is the value to be used in design, and, with equal compressive and tensile stresses, this value is identical with the "Wöhler safe range."

(*b*) The natural elastic range depends upon (1) the material and (2) the lower limit of stress. The elastic range when the lower limit is zero is less than that with equal tensile and compressive stresses (about 15 per cent with axle steel and 6 per cent with Bessemer steel).

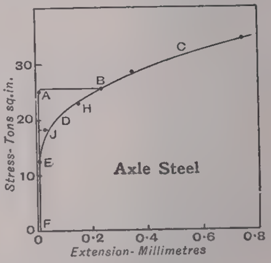

FIG. 98.

(*c*) If a stress-extension curve is plotted, the extension being the value of the permanent extension reached after repeated alternations, it assumes the form found with hard drawn copper wire, which has no yield point but corresponds with the curve FEJHBC of *Fig.* 98 produced back to the "natural" elastic limit.

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[1] *Roy. Soc. Phil. Trans.* Series A, ccx. 35-55.

(*d*) Recovery due to stoppage of the alternations of stress is appreciable, being somewhat rapid for some materials. Recovery reduces the permanent extension at a given load and can be greatly accelerated by immersing in boiling water for a few minutes, as suggested by Muir.[1]

If an inclined line e_1a (*Fig.* 99) is taken to represent the limiting minimum stresses, and the limiting maximum stress corresponding to each limiting minimum stress is plotted on the corresponding ordinate, a curve of maximum stress *ea* is obtained. The vertical distance between the two curves represents the limiting range of stress and it will be observed—

(1) That as the limiting minimum stress increases the range decreases.

(2) That if both the maximum and minimum stress applied to the material fall between the two curves, then the material will not fail under a repeated stress having those maximum and minimum values.

§ (70) FORMULAE FOR REPEATED STRESS TESTS.—Various empirical formulae have been suggested to connect the limiting values of the maximum, minimum, and range of stress.

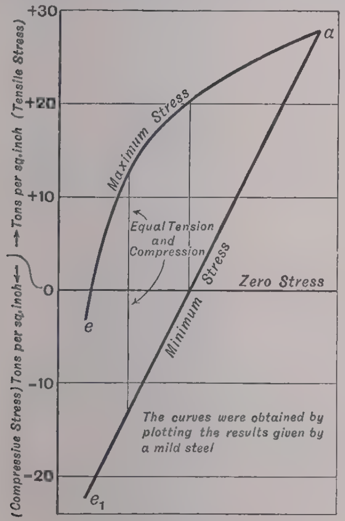

FIG. 99.

(i.) *Weyraugh and Launhardt's Formulae.*—

If $f_{\text{max.}}$ = limiting maximum stress,

$f_{\text{min.}}$ = limiting minimum stress,

f = ultimate static stress,

Z_1 = limiting range of stress when $f_{\text{min.}} = 0$,

$2Z_2$ = limiting range of stress when $f_{\text{min.}} = -f_{\text{max.}}$

Weyraugh[2] suggested the formula

$$f_{\text{max.}} = Z_1 - \frac{f_{\text{min.}}}{f_{\text{max.}}}(Z_1 - Z_2),$$

where the stress is wholly or partly reversed. Launhardt[3] suggested the formula

$$f_{\text{max.}} = Z_1 + \frac{f_{\text{min.}}}{f_{\text{max.}}}(f - Z_1),$$

where the stress is applied without reversal.

Fig. 100 is drawn plotting the values of $f_{\text{max.}}$ from the above formulae assuming the values of $f_{\text{min.}}$ to lie on the inclined line RS and

$$Z_1 = \frac{f}{2} \text{ and } Z_2 = \frac{f}{3}.$$

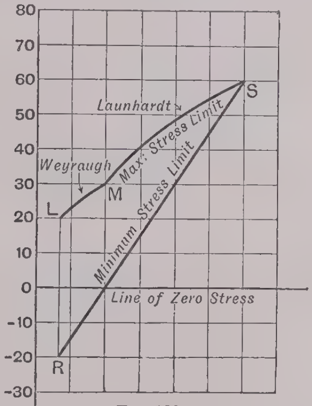

FIG. 100.

The latter have been found to be average values for a variety of materials.

The curved line for the maximum stress limit derived from Weyraugh and Launhardt's formulae is sometimes assumed to be straight, in which case one formula covers the whole field of repeated stresses.

(ii.) *Gerber's Parabolic Relation.*—Gerber showed that, by plotting the limiting minimum stress as abscissa and the corresponding safe range as ordinate, a curve was obtained which, to a first approximation, was parabolic.

Using the previous notation, Gerber's parabolic relation is expressed by the formula

$$f_{\text{max.}} = \frac{f_{\text{max.}} - f_{\text{min.}}}{2} + \sqrt{f^2 - nf(f_{\text{max.}} - f_{\text{min.}})},$$

n being a constant for the material.

§ (71) REPEATED STRESS TESTING MACHINES.—All Wöhler's tests were carried out at a speed of less than 100 reversals per minute, and a determination of the fatigue range occupied a very long time.

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[1] *Roy. Soc. Phil. Trans.*, 1900, cxciii. 1; also *Roy. Soc. Proc.*, 1900, lxvii. 461.

[2] *Inst. Civ. Eng. Proc.* lxiii.

[3] *Zeitsch. des Arch.- und Ing.-Vereins zu Hannover*, 1873; also, *Inst. Civ. Eng. Proc.*, 1880–81, lxiii. 280.

Various machines have been constructed to accelerate the test.

Before describing those machines which are in constant use for this branch of testing, it is advisable to explain the usual procedure which is adopted in the determination of the limiting range of stress.

Starting with an unknown material and a machine giving 2000 reversals per minute, two methods of attacking the problem are used :

(*a*) If the amount of the material is small and only a limited number of test pieces can be prepared, a low range is applied to the first test piece, which is increased by about 30 per cent after ten million reversals. Increments of the same value are added to the stress range after each additional ten million reversals until the test piece fails. (If the test piece is hollow the number of reversals can be reduced to five million.) The range is then known to be between two values (assuming the material to be homogeneous). A second test piece is then inserted in the machine and a range applied slightly higher than the lower of the two previous values. This range is increased by small increments (depending upon the accuracy with which the results are desired) after each ten million reversals, until failure occurs.

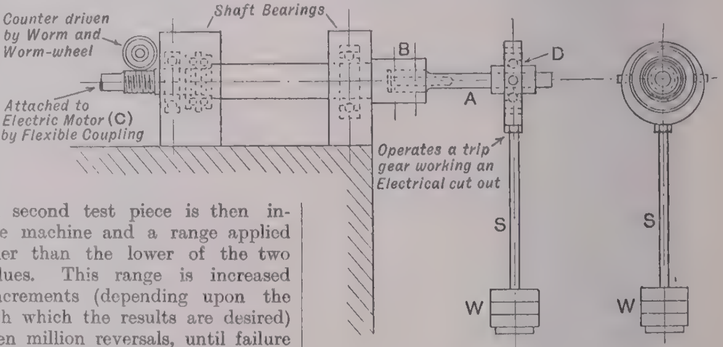
Counter driven by Worm and Worm-wheel
Shaft Bearings
B
D
A
Attached to Electric Motor (C) by Flexible Coupling
Operates a trip gear working an Electrical cut out
S
W
S
W

FIG. 101.

(*b*) Where at least four test pieces are available, the limiting range is approached from the opposite direction. The first test piece has a high range of stress applied to it, and from the number of reversals required to fracture an estimate can be obtained as to the probable range. The next specimen is tested under a range still believed to be on the high side, and this process is repeated until a range is obtained under which the material does not break. A stress-reversals curve can then be drawn, from which the range of stress can be estimated with whatever accuracy is required consistent with the homogeneity of the material.

The method of procedure adopted will depend on

(*a*) The amount of material available.

(*b*) The relative cost of preparation of test piece and running of the testing machine.

(*c*) The importance of the time of duration of test.

With machines of higher speed than 2000 reversals per minute it is usually considered that a larger number of reversals is required in order to be satisfied that, if the test piece does not break, the range applied is within the safe range.

The machines which are in common use can be divided into four main types :

(i.) Rotating cantilever machines.

(ii.) Rotating bar machines with a short length under a constant bending moment.

(iii.) Direct stress machines. Load applied by an unbalanced revolving weight.

(iv.) Direct stress machines. Load applied by the pull of an electromagnet excited by an alternating current.

(i.) *Rotating Cantilever Machines for Reversals of Bending Stress.*—This type of machine has been run successfully at 3000 revolutions per minute and is shown diagrammatically in *Fig.* 101. It consists of a test piece A held in a rotating chuck B driven either direct or through gearing by an electric motor C. The test piece is loaded by weight W on a scale pan S, which is attached to the test piece through the ball bearing D. The connection between the ball bearing and the scale pan is such that the latter can adjust itself in a vertical direction without putting any constraint on the ball bearing.

The number of alternations is recorded by a counter driven by a worm wheel from a worm cut on the machine shaft. The breaking of the test piece usually operates a switch and stops the motor ; by this means the machine can be run continuously with but little attention.

A slight modification of the counting and stopping method is in some cases adopted. The counter is attached to the ball-bearing holder (by which the load is applied) and is driven by the inner race of the ball bearing. The breaking of the test piece causes the counter to cease recording, and the machine continues to run, without a test piece, until the next period of inspection.

A rotating cantilever machine which has

been running successfully at the N.P.L. is shown in *Fig.* 102.

With a bending test the stress distribution over the cross-section is unknown if the elastic limit is exceeded, and the maximum stress only falls on the outer fibres of the test piece. The test piece is often made hollow, as in *Fig.* 103, the part under test being only a thin shell.

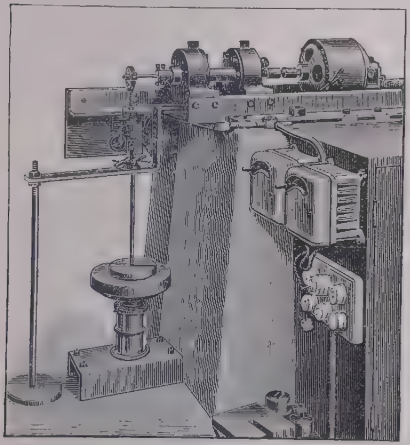

FIG. 102.

Machines of the rotating cantilever or Wöhler type have been used for researches described by Wöhler,[1] Stead and Richards,[2] C. B. Dudley,[3] F. Rogers,[4] Stanton and Pannell,[5] J. O. Roos,[6] C. E. Stromeyer,[7] and A. Martens.[8]

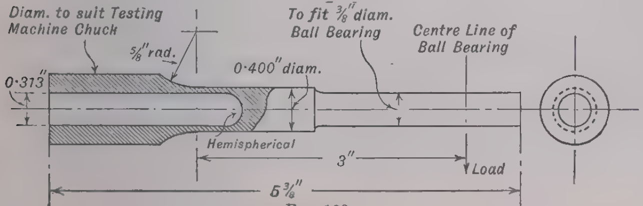
Diam. to suit Testing Machine Chuck
5/8" rad.
0·313"
0·400" diam.
To fit 3/8" diam. Ball Bearing
Centre Line of Ball Bearing
Hemispherical
3"
Load
5 3/8"

FIG. 103.

They are made commercially by Alfred J. Amsler & Co., and Tinius Olsen Testing Machine Co. The latter company call them "White-Souther endurance testing machines."

(ii.) *Rotating Bar Machines.*—This type of machine was first used by Sondericker[9] in 1892. The method of test is shown diagrammatically in *Fig.* 104. The test specimen ABCD is subjected to a uniform bending moment while revolving on its axis. It is supported on swivelling bearings at A and D, and the central portion between B and C is reduced in diameter. The maximum stress ($f_{max.}$) on the test piece is calculated from the formula

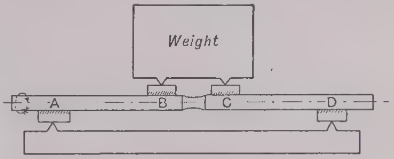

FIG. 104.

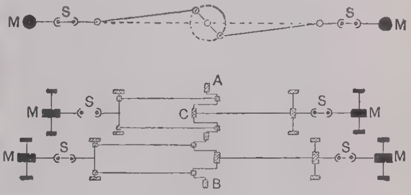

FIG. 105.

$$f_{max.} = \text{Bending moment} \times \frac{32}{\pi d^3},$$

where d = the diameter of the reduced portion. Besides Sondericker this method of test has been used by J. E. Howard[10] and Eden, Rose, and Cunningham.[11]

(iii.) *Direct Stress Machines.*—Load applied by an unbalanced weight.

In 1902 Osborne Reynolds and J. H. Smith[12] described a throw testing machine, for reversals of stress in which simple direct stresses were produced on a test piece by the inertia forces of reciprocating weights driven by a crank and connecting rod from a rotating shaft.

A machine of the same type was designed by T. E. Stanton[13] and used for his work on the resistance of iron and steel to reversals of direct stress. A diagram of the mechanism is shown in *Fig.* 105, which also explains the method of balancing the cranks.

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[1] *Engineering*, ii.

[2] *Iron and Steel Inst. J.*, 1903, No. 2.

[3] *Iron and Steel Metallurgist*, Feb. 1904.

[4] *Iron and Steel Inst. J.*, 1905.

[5] "Experiments of Strength and Fatigue Properties of Welded Joints," *Inst. Civ. Eng. Proc.*, 1911, clxxxviii.

[6] "Some Static and Dynamic Endurance Tests," *Int. Assoc. Test. Mat. Proc.*, 1912, Paper V.

[7] Manchester Steam Users' Assoc., Memo. by Chief Engineer, 1913.

[8] "Fatigue Bending Tests," *Science Abstracts*, 1914, No. 1371.

[9] "Repeated Stresses," *Quarterly Journal of Massachusetts Inst. of Technology*, 1892.

[10] Engineering Record, Sect. 22, 1906; also *Inter. Assoc. Test. Mat. Proc.*, 1909.

[11] *Inst. Mech. Eng. Proc.*, Oct., Dec. 1911.

[12] *Roy. Soc. Phil. Trans.* A, 1902, cxcix.

[13] *Engineering*, Feb. 17, 1905.

Four masses M reciprocating horizontally are operated by four cranks C. There are therefore two independently perfectly balanced systems with cranks set 180° apart on the same shaft AB. The cranks of one system are at right angles to the cranks of the other system, so that the kinetic energy of the moving parts is thus approximately constant. There are four test pieces S, one stressed by each reciprocating mass.

Machines of this type, however, suffer from the disadvantage that the inertia forces vary with the square of the speed. It is therefore necessary to control the speed very carefully if reliable results are expected.

(iv.) *Direct Stress Machines.*—Load applied by pull of an electromagnet excited by an alternating current.

This method has been successfully applied by G. Kapp,[1] B. Hopkinson,[2] and B. P. Haigh.[3]

Haigh has introduced several new features into his latest design. *Fig.* 106 shows a machine installed at the N.P.L. and *Fig.* 107 illustrates the principle adopted.

The test piece S is connected at its upper end to the frame of the machine in such a way that its height can be adjusted to allow for elongation, while the lower end is attached to a frame carrying the armature A of a two-phase electromagnet M_1 and M_2.

The electromagnet is excited by an alternating current from a generator giving a sine wave E.M.F., so that the pull is almost proportional to (voltage/frequency)^2, nearly independent of the air gap, which is small and pulsates with twice the frequency of the electric current.

Two small secondary coils wound on frames are fixed close to the pole faces, and the voltage induced in these coils is measured by a voltmeter and used for calculating the stress.

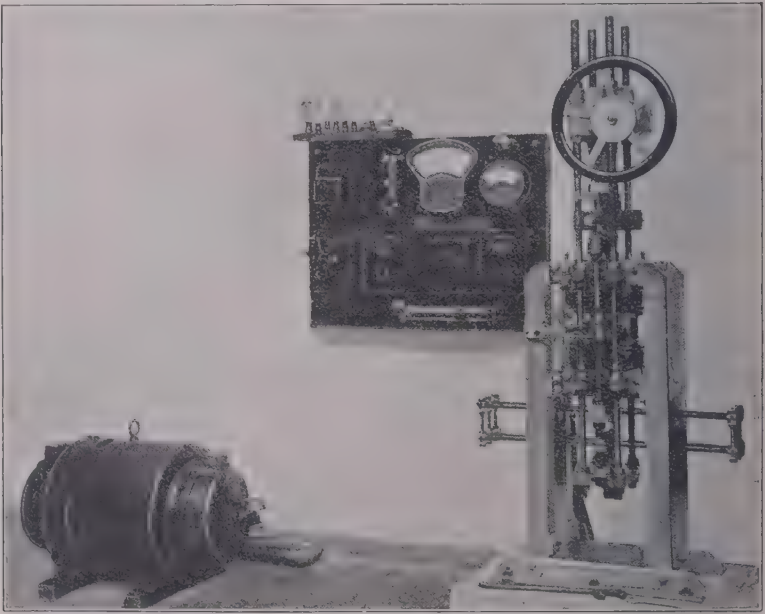

FIG. 106.

In order that the pull of the two magnets shall be of the same amount it is necessary that the air gaps shall be equal. A fine adjustment is provided with the machine to enable the test-piece frame and armature to be raised or lowered during a test until the current in the two coils and therefore the air gaps are equal. The coils are connected to a differential ammeter which reads zero when this condition is fulfilled.

By the use of a choking coil the voltage readings are rendered nearly independent of the frequency over a fairly large range, so that slight alterations of frequency during a test do not alter the range of stress.

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[1] "Alternating Stress Machine," *Zeits. Vereines Deutsch. Ing.*, Aug. 26, 1911.

[2] *Roy. Soc. Proc.* A, Jan. 31, 1912, lxxxvi.

[3] *Engineering*, Nov. 22, 1912; also *Inst. of Metals J.*, 1917, No. 2, p. 55.

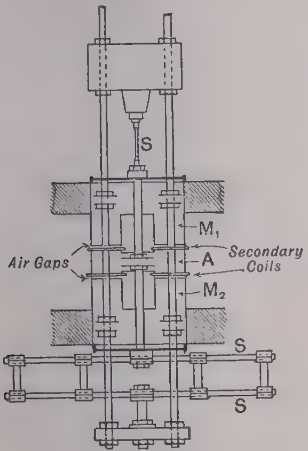

FIG. 107.

The frame carrying the test piece and armature is also attached, below the latter, to standard springs S, S. The stiffness of these springs is adjusted to counterbalance the inertia of the armature, etc., a precaution which is very desirable, as it eliminates a correction which would amount to 5 per cent of the stress range. This adjustment is effected by setting the springs, so that the moving system, without the test piece in place, vibrates in resonance with the magnetic pull.

By extending or compressing the springs an initial pull or push can be applied to the test piece so as to alter the magnitude of the minimum stress.

Thus if f_e = the alternating stress applied electromagnetically,
and f_s = the stress applied by the springs,
then the maximum stress $(f_{\max.}) = f_e + f_s$
and the minimum stress $(f_{\min.}) = -f_e + f_s$.
Range of stress $= Z = f_{\max.} - f_{\min.} = f_e + f + f_e - f_s = 2f_e$,

i.e. the range is not affected by the load applied by the springs.

The voltmeter is calibrated by measuring the range of stress (photographically) by a special form of optical extensometer. This is shown diagrammatically in *Fig.* 108.

§ (72) EFFECT OF SPEED ON THE LIMITING RANGE OF STRESS.—Reynolds and Smith[1] found that the limiting range of stress was smaller at speeds of 1300 to 2500 per minute than at 60 to 80 cycles per minute. Their conclusions have not, however, been confirmed by subsequent experiments.

Stanton and Bairstow,[2] using a machine of the same type, found that a change of speed from 60 to 800 did not seriously affect the range, and later, using a machine of the same type as Wöhler, they found that there was no evidence at all of any reduction in fatigue strength due to a rate of alternations of 2200 when compared with 200.

With respect to Reynolds and Smith's results Stanton says, "It seems probable that the reduction in fatigue strength noted by Reynolds and Smith as due to high rate of alternations is a characteristic of the particular mechanism used for their experiments." Bairstow[3] says, "It may possibly be that the decreased range of stress found by Reynolds and Smith has some relation to the question of recovery, but further experiments are necessary to decide the question, as the effect of the rigidity of the testing machine has not yet been fully investigated. In two instances, at least, low ranges of stress have been traced to natural periods of vibration in the testing machine agreeing approximately with the period of repetition."

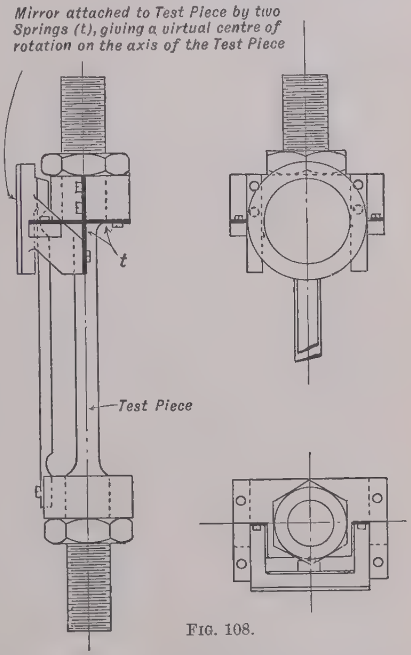
Mirror attached to Test Piece by two Springs (t), giving a virtual centre of rotation on the axis of the Test Piece
t
Test Piece

FIG. 108.

Eden, Rose, and Cunningham[4] found no speed effect between 250 and 1300 revolutions per minute.

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[1] *Roy. Soc. Phil. Trans.* A, 1902, cxcix.

[2] "Resistance of Iron and Steel to Reversals of Direct Stress," *Inst. Civ. Eng. Proc.* clxvi. 78.

[3] *Roy. Soc. Phil. Trans.* Series A, ccx. 35-55.

[4] *Inst. Mech. Eng. Proc.*, Oct., Dec. 1911.

§ (73) EFFECT OF RAPID CHANGES OF SECTION AND SURFACE CONDITION ON THE LIMITING RANGES OF STRESS.—Stanton and Bairstow have shown that rapid changes of section have a marked effect on the resistance of materials to repeated direct stresses.

The standard test piece that they used had a fillet of ⅜ inch, and for comparison with this they made experiments when (1) the fillet was reduced to 0·062 inch, (2) the specimen was screw cut with a Whitworth vee thread, and (3) the corners were left sharp.

TABLE 29

EFFECT OF RAPID CHANGES OF SECTION ON THE LIMITING RANGE OF STRESS

Material.	Percentage Reduction of the Limiting Range of Stress.		
	Specimen Screw cut with a Vee Thread.	Specimen having a Fillet 0·062 in. Radius	Specimen left with perfectly Sharp Corners.
Swedish Bessemer steel No. 3 .	31	32	52
Swedish Bessemer steel No. 2 .	30	28	53
Swedish Bessemer steel No. 1 .	33	32	52
Swedish charcoal iron . . .	32	35	46
Piston-rod steel	33	29	40
Mild steel No. 2	29	28	45
Mild steel No. 1	26	28	36
Wrought iron No. 1 . . .	23	19	40

The results are given in Table 29 and show—

(1) That the resistance of all these forms are, in every case, far below the corresponding maximum limiting resistances of the materials.

(2) That the resistances of forms (1) and (2) are practically the same for any given material, and that the ratio of these resistances to the corresponding maximum limiting resistances does not vary greatly for the different materials.

TABLE 30

EFFECT OF RAPID CHANGES OF SECTION ON THE LIMITING RANGE OF STRESS (USING THE WÖHLER METHOD)

Material.	Ultimate Static Stress. Tons per Sq. In.	Limiting Range of Stress. Tons per Sq. In.			Maximum Reduction in Range. Per cent.
		Fillet ⅝ in. Radius.	Fillet ⅛ in. Radius.	Sharp Corner.	
Nickel chrome shaft No. 1 .	61·9	57·0*	48·0	31·0	46
Nickel chrome shaft No. 2 .	45·2	46·0	..	29·4	36
Boiler Plate	27·9	27·6	..	16·0	42

* 57·0 = +28·5 to −28·5.

(3) That in the case of the specimens having a sudden change of section, the percentage reduction of the limiting range appears to depend on the hardness of the steel, being approximately 52 per cent for the hard steels and 45 to 35 per cent for the wrought irons and mild steels. It is, however, worthy of notice that even under the circumstances supposed to be most fatal to hard steels, *i.e.* a sudden change of section, these steels are very appreciably stronger than wrought irons and mild steels.

The results of some experiments, made by the author using the Wöhler rotating cantilever method of test, are given in Table 30. The standard test piece shown in *Fig.* 103 has a radius of 0·625 inch.

The percentage reduction in the range is seen to agree with the results obtained by Stanton and Bairstow.

The injurious effect of scratches in parts of machinery subject to variations of stress is now recognised. Haigh says that cracks develop from surface scratches under stresses perceptibly lower than the normal. In some experiments carried out by the author the alternate stress range of some aero crankshaft material was reduced from 56·0 (+28 to −28) to 49·0 tons/sq. in. by a sharp scratch 0·003 in. deep.

There seems to be no doubt that many

fatigue failures attributed to faulty material are really due to unsuitable fillets, sharp corners, or surface scratches left during the process of machining.

§ (74) RESISTANCE OF MATERIALS TO COMBINED STRESSES.—Frequent cases of the combination of stresses are met with in engineering practice, and the question of the resistance of materials to those stresses is now recognised as being of considerable importance.

Stress applications may be of three types:

(i.) Simple stress or stress in one direction.

(ii.) Biaxial stress or two stresses acting in directions at right angles to each other in the same plane.

(iii.) Triaxial stress or the application of three stresses at right angles to each other.

With simple stresses, direct tension or compression, the practical constants (modulus of elasticity, elastic limit, yield stress, ultimate stress, Poisson's ratio) are taken as a basis for design. With complex stress distribution the question arises as to the way in which these constants are to be used, and several different theories have been expounded to account for the method of failure of materials under combinations of stresses.

The more important cases of compound stress, considered from an engineering standpoint, are biaxial.

Shear is an example of biaxial stress because it is equivalent to the combination of two equal principal stresses,[1] one compression and the other tension, acting in directions at 45° with the shearing stress. Thus, in *Fig.* 109, the stress on the plane AB, at right angles to the axis, is pure shear, the intensity of which depends upon the diameter of the bar and the torque applied. This shear is a combination of tension along the 45° plane CD and compression along the plane EF. It is well known that mild steel and wrought iron fracture, in a torsion test, across the plane AB, *i.e.* a shearing failure, while cast iron breaks along the plane EF (*i.e.* fails in tension) because it is stronger in shear than in tension.

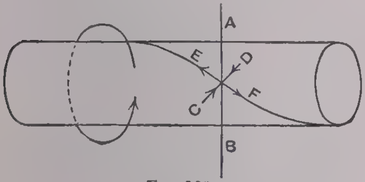

FIG. 109.

Other combinations of biaxial stress are:

(*a*) The crank-shaft which is bent and twisted at the same time. This can be reduced to a case of tension combined with compression owing to the torque producing tension and compression on two planes at right angles as described above.

(*b*) The shell of a steam boiler, which has to resist circumferential and longitudinal stresses, is an example of the combination of two tensions.

(*c*) Plates, concrete slabs or floors supported round the edges and loaded in the centre, are further examples of biaxial stress distribution.

Three-dimensional stress is found in thick steel cylinders under internal pressure accompanied by a longitudinal tension, also in concrete columns with spiral reinforcements, and in hooped guns.

§ (75) CAUSES OF FAILURE IN COMBINED STRESS.—Investigators in the "combined stress" field of research have first of all to decide upon what they consider the point of failure of a material. The elastic theory, being based on Hooke's law, is reliable up to the elastic limit (limit of proportionality or P limit), and therefore, considered mathematically, the elastic limit is the point of failure. This limit is dependent on the previous history of the material, and is therefore variable and indefinite. For experimental purposes the yield point is used, in a good many instances, as the criterion of failure; this is no doubt due to the method of testing which does not allow the tests to be continued to the point of failure.

The most important theories advanced to account for the failure of materials under combined stresses are:

(i.) *Maximum Stress Theory* (Rankine).—This theory is that the material yields when one of the principal stresses reaches a certain amount. (The stress determined by a simple tensile test.) It assumes that a second stress, at right angles to the first, neither weakens nor strengthens the original stress.

(ii.) *Maximum Strain Theory* (St. Venant).—By this theory failure is assumed to occur when the maximum strain reaches a value equal in magnitude to that at the yield point stress of the simple tension or compression experiment.

If a material is subjected to two or three stresses at right angles to each other, the maximum strain theory assumes that the strength is lowered if the stresses are opposite in sign and increased if they are of the same sign.

Thus if S_1, S_2, and S_3 are three stresses at right angles to each other, and e_1, e_2, and e_3 are the unit strains in the direction of each of the respective stresses, also

$E =$ the modulus of elasticity (assumed to be constant)

and $1/\sigma =$ Poisson's ratio;

---

[1] See "Elasticity, Theory of," § (5).

then
$$Ee_1 = S_1 - \frac{1}{\sigma}(S_2 + S_3),$$
$$Ee_2 = S_2 - \frac{1}{\sigma}(S_3 + S_1),$$
$$Ee_3 = S_3 - \frac{1}{\sigma}(S_1 + S_2).$$

For simple tension S_2 and S_3 may be taken as zero and

$$Ee_1 = S_1, \textit{ i.e. } E = \frac{S_1}{e_1} = \frac{\text{stress}}{\text{strain}},$$

which is the expression of the modulus of elasticity.

The maximum strain theory is mathematically correct if it is assumed

(*a*) to hold within the elastic limit.

(*b*) that Hooke's law is absolutely correct.

(*c*) that the material is isotropic.

(*d*) that the effect of temperature is negligible.

By taking the yield point as the criterion of failure, the variation of "E" between the limit of proportionality and the yield point is assumed to be without effect on the results.

(iii.) *Maximum Shear Theory.*—This theory assumes that the condition for initial yielding of a uniform ductile material corresponds to the existence of a specific shearing stress, the intermediate principal stress being without effect.

This theory as originally proposed, by Coulomb in 1776, refers to rupture of the material. It is adopted in Guest's[1] well-known work (1900) where it is used in conjunction with the yield stress. It is approximately verified by W. Scoble[2] and W. Mason.[3]

We can assume that simple tension is a case of combined triaxial stresses where two of the principal stresses are zero. Applying Guest's law, the bar must fail in shear, and in this case the maximum shear occurs in a plane at 45°, and the shear stress intensity is 50 per cent of the tensile unit stress. Seely and Putnam[4] find that the correct ratio of elastic shearing strength to the elastic tensile strength is 0·55 to 0·65, and therefore state that Guest's law which assumes the value to be 0·5 is not a correct statement of the law of elastic breakdown. Becker[5] found this ratio to be 0·56, and the author[6] obtained a ratio of 0·63 to 0·68 for steels with ultimate strengths of 23 and 30 tons per square inch respectively.

Becker,[7] as the result of experimental work, proposes two laws of strength under combined stress, viz.:

(1) That the strength at the yield point follows the *Maximum Strain* theory until the shearing stress reaches the value of the shearing yield point.

(2) After this point failure occurs according to the *Maximum Shear Theory.*

This suggestion appears to fit in very well with existing experimental work.

The maximum shear stress theory has been modified by Perry to include a frictional term proportional to the stress and perpendicular to the plane of shear. He noticed that brittle materials fracture at angles greater than 45° with the cross-section and assumes that this is due to internal friction.

If θ = the angle found experimentally, μ (the coefficient of friction) = tan ϕ, then $\theta = 45° + \phi/2$ for tension and $\theta = 45° - \phi/2$ for compression.

For cast iron the angle (θ) is found to be $54\frac{3}{4}°$, which gives a value of μ of 0·35.

If $\phi = 0$, this theory is the same as the maximum shear theory, and Perry suggests that this is the case for wrought iron and mild steel.

(iv.) *The Maximum Resilience Theory.*—Haigh[8] proposes that the elastic limit of a material under complex stress is reached when the energy per unit volume attains a certain definite value. He bases this view on thermodynamic considerations, and also finds that, when considered from the experimental aspect, the results fit in with this theory better than they do with either the maximum stress, maximum strain, or maximum shear theories.

Mallock[9] considers the volume extension limit or the limit of shear as the fundamental limits of a material, and assumes that the material will fail when either limit has been reached.

§ (76) EXPERIMENTS ON COMBINED STRESS.—Most experimental work has been carried out on ductile materials. Only a few experiments have been made with brittle materials and further research in this direction is needed.

Test on thin tubes in combined tension and torsion have been carried out by Guest,[1] Hancock,[2] Turner,[3] Mason,[4] and Becker.[5]

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[1] "Strength of Materials under Combined Stresses," *Phil. Mag.*, July 1900; and *Phys. Soc. Proc.*, 1899–1901, xvii. 202.

[2] "Strength and Behaviour of Ductile Materials under Combined Stress," *Phil. Mag.*, Dec. 1906, xii. 533.

[3] "Mild Steel Tubes in Compression and under Combined Stress," *Inst. Mech. Eng. Proc.*, 1909, part iv.

[4] *University of Illinois Bulletin*, No. 115, Nov. 10, 1919.

[5] "The Strength and Stiffness of Steel under Biaxial Loading," *University of Illinois Bulletin*, No. 85, April 10, 1916.

[6] Batson, *Inst. Mech. Eng. Proc.*, March 1917, p. 182.

[7] "The Strength and Stiffness of Steel under Biaxial Loading," *University of Illinois Bulletin*, No. 85, April 10, 1916.

[8] "The Strain-Energy Function and the Elastic Limit," *Report of British Association*, 1919, and *Engineering*, Jan. 30, 1920.

[9] *Roy. Soc. Proc.* A, 1909, lxxxii. 26-29, also Dec. 1912, p. 466.

Hancock and Turner used the limit of proportionality as the point of breakdown, Guest and Mason took the yield point, while Becker adopted the "apparent" elastic limit. This is a point between the limit of proportionality and the yield point where the slope of the tangent to the stress-strain curve is 50 per cent greater than it is at the origin. Seely and Putnam[6] also used the "apparent" elastic limit in their experiments.

Guest, Turner, and Becker added internal fluid pressure in order to obtain an additional stress.

Solid round specimens under tension or compression and torsion have been used by Hancock, Scoble,[7] and Smith,[8] while the strength of thick cylinders under internal pressure has been determined by Turner,[9] Cook and Robertson,[10] and Bridgeman.[11]

Crawford[12] used flat steel plates clamped at the edges and subjected to fluid pressure on one side.

§ (77) REPEATED APPLICATIONS OF COMBINED STRESSES.—With repeated stress tests the limiting range of stress in fatigue is a value that can be definitely taken as the one at which the material fails.

Although Wöhler made tests under repeated tension and repeated torsion, his results are difficult to compare owing to lack of specific information regarding the material used in the tests.

Both Turner and Stromeyer[13] have experimented with alternating torsion; the former found that the more ductile materials approximated to the maximum shear stress law, while the more brittle samples were more nearly in agreement with the maximum strain theory.

The only experiments so far carried out on combined torsion and bending were those by Stanton and Batson.[14] A diagrammatic sketch of the arrangement which was adopted is given in *Fig.* 110. In the position shown the cross-section of the specimen at S is subject to a twisting moment WD, and to a bending moment W*d*. When the head had turned through 180° the moments were equal in amount but opposite in sign. When the head had turned through 90° from the position shown the maximum stress was that due to a bending moment WD plus that due to the direct loading, but as in all cases this stress was below the known fatigue limit of the material under reversals of simple bending, its effect was taken to be negligible, and the specimen was assumed to be subject to reversals of the combination of bending and twisting alone.

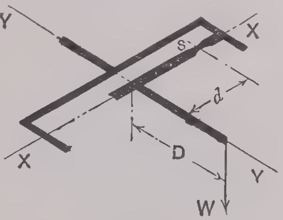

FIG. 110.

§ (78) ALTERNATING BENDING TESTS BEYOND THE YIELD POINT.—The principal objection to the commercial adoption of repeated stress tests is the time and expense of conducting tests in which millions of applications of stress are required in order to obtain the result.

Many machines have been designed in which the test piece is broken rapidly under alternating stresses which exceed the yield point. Such tests do not give information as to relative "fatigue" strength of materials, but have been found useful as indicating mechanical defects, incorrect heat treatment, and brittleness.

(i.) *Arnold Testing Machine.*[15]—In this test, in its latest form, a test piece $\frac{3}{8}$ in. diameter and 5 to 6 in. long is firmly fixed at one end in a vertical position in the vice of the machine. It is bent backward and forward, through a distance of $\frac{3}{8}$ in. on either side of the vertical at a height of 3 in. above the face of the dies, by a slotted steel head fixed to the reciprocating part of the machine. The length of slot is larger than the diameter of the test piece (usually $\frac{3}{4}$ in. with a stroke of the machine of $1\frac{1}{8}$ in.), so that an impact or shock is introduced at each alternation of stress. The standard speed adopted is 650 alternations per minute. The number of alternations to fracture is recorded. Arnold and other investigators improvised shaping or slotting machines to perform the test.

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[1] "Strength of Materials under Combined Stresses," *Phil. Mag.*, July 1900; and *Phys. Soc. Proc.*, 1899–1901, xvii. 202.

[2] *Eng. News*, Aug. 24, 1905, liv., and Sept. 2, 1909, lxii., and *Phil. Mag.*, Oct. 1906, Feb. 1908, and Nov. 1908.

[3] *Engineering*, Feb. 1909, lxxxvii., and July 28, 1911.

[4] "Mild Steel Tubes in Compression and under Combined Stress," *Inst. Mech. Eng. Proc.*, 1909, part iv.

[5] "The Strength and Stiffness of Steel under Biaxial Loading," *University of Illinois Bulletin*, No. 85, April 10, 1916.

[6] *University of Illinois Bulletin*, No. 115, Nov. 10, 1919.

[7] "Strength and Behaviour of Ductile Materials under Combined Stress," *Phil. Mag.*, Dec. 1906, xii. 533.

[8] *Engineering*, Aug. 20, 1909, lxxxviii.; *Inst. Mech. Eng. Proc.*, 1909, part iv.; *Inst. of Metals Journ.*, 1909; *Iron and Steel Inst. Journ.*, 1910.

[9] *Engineering*, Feb. 1909, lxxxvii., and July 28, 1911.

[10] "The Strength of Thick Hollow Cylinders under Internal Pressure," *Engineering*, Dec. 15, 1911, xcii.

[11] *Phil. Mag.*, Jan. and July 1912.

[12] *Roy. Soc. Edinburgh Proc.*, 1911–1912.

[13] Manchester Steam Users' Assoc., Mem. by Chief Engineer for the year 1913.

[14] *Report of the British Association*, Newcastle, 1916.

[15] "Factors of Safety in Marine Engineering," *Engineering*, 1908, lxxxv. 598.

It should be noted that, in his earlier work, Arnold[1] used a test piece $\frac{3}{8}$ in. *square* with the force applied 4 in. above the line of maximum stress. The deflection was $\frac{1}{2}$ in. or $\frac{9}{16}$ in. on either side of the vertical, and the number of alternations was 168 to 266 per minute.

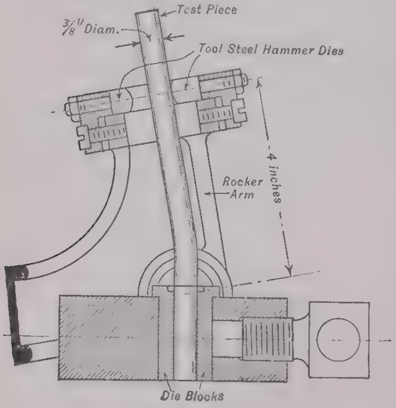

FIG. 111.

(ii.) *Landgraf-Turner Alternating Impact Machine.*—This machine is especially designed and constructed to carry out the Arnold test in order to standardise the conditions employed. In place of the straight line movement of the shaping machine a rocker arm is substituted (*Fig.* 111) to economise space and, by reason of its radius being equal to the free length of the test piece, maintain a constant leverage during the whole stroke.

The number of alternations are recorded by a veeder counter which stops registering when the test piece breaks. The test piece is $\frac{3}{8}$ in. diameter, but, with this machine, the distance from the top of the vice to the striking point of the hammer is 4 in. instead of 3 in. as used by Arnold in his latest experiments.

This machine is manufactured in the U.S.A. by Queen & Co., Philadelphia.

J. B. Kommers[2] of the University of Wisconsin has carried out an investigation with a Landgraf-Turner machine. He summarises his results as follows:

(*a*) A very important factor in a repeated stress test similar to that performed by the Landgraf-Turner machine is the amount of deflection which the specimen receives. When the deflections are less than 0·30 in. the change in the number of cycles required for rupture is very great even for small changes in the amount of deflection.

(*b*) Impact applied to the specimen has *practically no effect* upon the number of cycles required for rupture.

(*c*) At speeds of about 700 cycles per minute the number of cycles for rupture is slightly less than at speeds of about 150, but for small changes of speed this effect is practically negligible. When the deflection is small the results on the same material do not seem to be as uniform as when the deflection is about 0·30 in. or more.

(*d*) The condition of the surface of the specimen has an important effect upon the number of cycles required for rupture.

(iii.) *Sankey Hand Bending Test.*[3]—The principle on which the test is based is to bend backwards and forwards a test piece $\frac{3}{8}$ in. diameter and 4 in. long through a fixed angle until failure, the number of bends and energy required for each bend being recorded. The test is carried out on a machine, manufactured by C. F. Casella & Co., Ltd., and shown diagrammatically in *Fig.* 112. A flat steel spring B has one end gripped in a vice A which forms part of the bedplate, and the other secured to a holder C in which the test piece D is fixed. The test piece is also held in a handle E, 3 feet long, by which it is bent backwards and forwards through an angle of 91½° located by the indicators F and F.

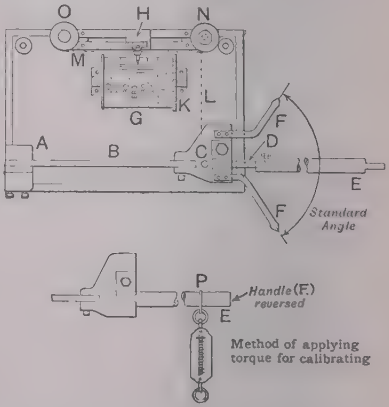

FIG. 112.

The energy required to bend the test piece is measured by the deflection of the spring B, which is recorded on the paper attached to the drum G. The horizontal motion of the pencil H is actuated by the holder C through wires L and M and the multiplying pulley N. The wire is kept taut by a spring box O, but this is sometimes replaced by a weight hanging from the wire M which then passes over a vertical pulley. The motion of the pencil is proportional to the energy required to bend the test piece. The diagram is calibrated by noting the length which corresponds to a measured pull at a known leverage from the point of bending of the test piece. The movement of the pencil holder H moves the drum through one tooth of a ratchet wheel K, which is attached to it so that a diagram is drawn such as is shown on the drum in the illustration.

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[1] *Inst. Civ. Eng. Proc.* (supplement), 1903, and *Engineer*, Sept. 2, 1904, p. 227.

[2] "Repeated Stress Testing," *Int. Assoc. Test. Mat. Proc.*, 1912, ii. part ii. Paper V_{4a}.

[3] *Engineering*, Feb. 15, 1907, p. 209, Dec. 20, 1907, p. 829, and Dec. 27, 1907, p. 882.

The practical value of the Sankey test is stated by Hatfield,[1] who carried out an extensive series with it, to be as follows:

"The energy absorbed in breaking the Sankey test piece is practically proportional to the product of the average bending moment and the number of bends withstood. These two items may be compared roughly to the maximum stress and reduction of area in the tensile test, *i.e.* for conditions of increasing hardness in general, the maximum bending moment increases and the number of bends decreases. It is necessary that we should point out that in a number of instances we have obtained high absorptions of energy with high tensile.

"The maximum bending moment in the Sankey test follows the maximum stress in tension fairly closely, so that the latter values can be predicted within a few tons per square inch from the Sankey values. The variation found in the values of the number of bends in the Sankey are rather wider and of a less predictable nature. In this respect the Sankey test discriminates and emphasises a certain quality of the material in a manner not similarly brought out in other tests, except perhaps to some extent in the Arnold test. From a knowledge of the quality of the steel, its analysis, etc., experience has shown that fairly definite values of ductility in the tensile test, as measured, for example, by the reduction of area, can be expected in conjunction with given values of tensile strength if the steel is free from defects. Similarly the number of bends in the Sankey test should reach such values which with experience of the test may also be fairly definitely established. There are, however, several outstanding exceptions. It certainly appears that samples, which gave Sankey values well below what has been predicted from an average good steel of the same tensile strength, were in nearly all cases also somewhat inferior in reduction of area."

The diameter of the Sankey test piece is $\frac{3}{8}$ in. and its free length $1\frac{3}{4}$ in. Owing to the fact that the machine, as supplied, is not suitable for steels of higher tensile strength than 50 to 60 tons per square inch, Hatfield suggests a test piece 0·3 in. diameter and free length $1\frac{3}{4}$ in. for these steels.

Sankey[2] gives the following constants for inferring static test results from those obtained on his hand bending machine:

Yield point (tons per sq. in.)

$$=\frac{\text{initial bending effort in lbs.-ft}}{\mathrm{C}},$$

where C=2·1 for steels up to 0·3 per cent carbon and 2·7 for medium carbon steel.

Ultimate strength (tons per sq. in.)

$$=\frac{\text{maximum bending effort in lbs.-ft.}}{1{\cdot}54}.$$

Elongation (on gauge length $=4\sqrt{\text{Area}}$) × reduction of area

$$=\frac{\text{number of bends}}{1{\cdot}9}.$$

(iv.) *The Upton Lewis*[3] *toughness testing machine*, manufactured by Tinius Olsen Testing Machine Co. (English agent, Edward G. Herbert, Ltd., Manchester) is very similar in principle to the Sankey machine. It is, however, operated by a motor or belt-drive giving 250 alternations per minute instead of by hand, and the method of measuring and recording the energy absorbed is slightly different. Referring to *Fig.* 113, where the machine is shown diagrammatically, the test piece C is bent backwards and forwards by an arm operated from the adjustable crank M. The energy is measured by the amount of the compression of the springs EF, which is

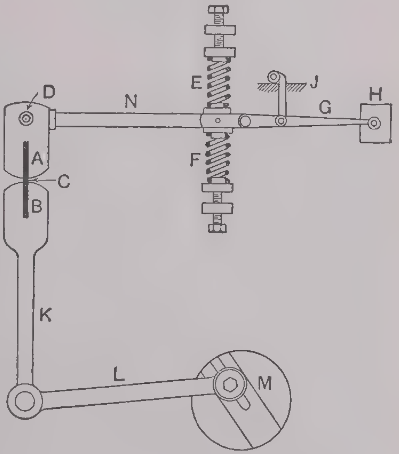

FIG. 113.

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[1] Hatfield and Duncan, *N.E. Coast Inst. of Eng. and Shipbuilders Proc.*, March 19, 1920.

[2] Sankey, Blount, and Kirkaldy, *Inst. Mech. Eng. Proc.*, May 1910.

[3] *American Machinist*, Oct. 17, 1912.

recorded by the bell-crank indicator G marking on a continuous recording paper H. The form of diagram is shown in *Fig.* 113A.

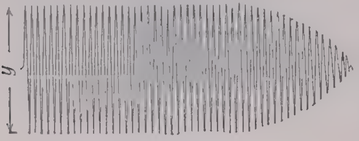

FIG. 113A.

(v.) *Other Investigators and Machines.*—Boudouard [1] used a machine in which one end of the specimen was clamped in a vice and the free end vibrated. The vibrations were maintained electromagnetically, and the oscillations were recorded optically and photographically.

Schuchart [2] tested wire under repeated bending. The wire was gripped in curved-faced jaws and bent backwards and forwards into contact with the curved faces.

Huntingdon [3] applied alternate bending by means of an attachment to a shaping machine.

§ (79) HARDNESS AND ABRASION TESTS.—An examination of the methods employed in hardness testing shows that each of them falls into one or other of two distinct categories. These are:

(i.) Indentation tests, in which the surface of the material under test is permanently distorted by the pressure of a hard steel ball, cone, or knife edge.

(ii.) Abrasion or scratch tests, in which particles of the material whose "hardness" is to be determined are torn away from its surface by sliding contact with some other substance whose corresponding resistance is so high that its surface remains unimpaired by the action.

If each of these methods were a measure of the same definite property of the material which is as characteristic of it as, say, its elasticity, it is evident that the ratio of the results of any two of the methods would be the same for every material tested. Comparisons between the results of these various tests have formed the subject of several researches which have been published during recent years. The general conclusions as summarised by Turner [4] appear to be that, although an approximate agreement may seem to exist between the various methods when applied to the case of relatively pure metals in their cast or normal state, yet when the resistance to deformation is due to tempering or to mechanical treatment no comparison is possible.

That this should be so would seem to follow from the consideration that the resistance which any so-called hardness is supposed to measure is that which the body under test exerts against a complex distribution of stress over its surface which has partially deformed or disintegrated it, and it is evident that its value will depend, not on the stress constants of the material such as its yield point, ultimate tensile and shear stresses, but on intermediate stresses, the precise nature and distribution of which are unknown and whose ratio to the stress constants may not be the same for the same method. If, therefore, such resistance, without qualification, be defined as the hardness of the material in its broadest sense, it is clear that hardness is no more a definite quality of a material than is the strength of a piece of steel of definite dimensions. In the latter case, if the nature, amount, and distribution of the stress are known, its resistance has a definite value which can be calculated. The only difference between this case and that of the hardness test is that, since in the determination of hardness there is no possibility of estimating the stress magnitude and distribution, we are driven more to direct observation of the consequences of such distribution than to a calculation of these consequences from the known characteristics of the material. Mechanical phenomena of this kind are familiar to engineers under other aspects, such as in the case of the resistance of ships and aircraft to propulsion; but whereas in these latter cases the problem is to determine the resultant force exerted by the unknown pressure distribution, in the present case, as in the corresponding one of the resistance of materials to impact, the unknown quality is the ultimate resistance of the material to the unknown stress distribution. In all the cases, however, the practical method of solution is an experimental one, and consists of setting up a similar, or nearly similar, state of stress on a specimen of the material whose behaviour is under investigation, and noting its effects.

H. Le Chatelier says, "The problem therefore seems to be to establish two or three methods of reference for hardness, giving as widely different results as possible, so that hardness (which is an essentially complex phenomenon) may be studied under all its phases. Afterwards, for each particular application the reference method which is most applicable to the conditions may be used."

That this view is now being accepted is indicated by the development, in recent years, of what are called wear tests. For example, there are wear tests for measuring the particular form of disintegration which takes place on the surface of steel rails due to the rolling abrasion of heavily loaded wheels. The characteristics of this kind of wear are the extremely small amount of the relative movement between rail and wheel and the high intensity of the compressive stress at the line of contact. On the other hand, there are wear tests of lubricated surfaces in which the pressure is relatively small and the rate of slipping large, and the same conditions of wear occur, without lubrication, with pins and check gauges.

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[1] *Int. Assoc. Test. Mat. Proc.*, 1912, Paper V_3.
[2] *Stahl und Eisen*, July 1, 1908.
[3] *Inst. of Metals J.*, 1915.
[4] "Hardness," *Iron and Steel Inst. J.*, 1909.

§ (80) Indentation Tests.—The indentation method of determining hardness has been applied in three ways:

(i.) By testing the material with itself.

(ii.) By pressing (statically) a harder material into the material under test.

(iii.) By producing the indentation by dropping a ball- or cone-pointed hammer on to the material and measuring the rebound of the hammer or size of the indentation.

(i.) *The Material tested with Itself.*—Reaumur[1] in 1722 shaped right-angled prisms from two materials which were to be compared and pressed them together. The axes of the prisms were at 90°, and the right-angled edges came into contact, forming a cross. The relative hardness was measured by the depths of the indentations.

A. Foeppl[2] used two cylindrical test pieces of the material whose hardness was required. They were placed one on the other, with their axes at right angles, and were pressed together in a testing machine. Foeppl used the pressure per unit of flattened surface as a measure of the hardness, because he found that the surface of indentation was proportional to the pressure applied.

Haigh[3] has recently re-introduced the test with the substitution of square for cylindrical or triangular prisms. He gives the hardness number as $=L/R^2$,

where L = the load
and R = the length of damaged edge of prism.

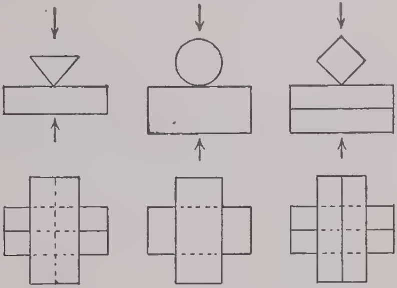
Reaumur, Foeppl, and Haigh's methods are shown diagrammatically in *Fig.* 114.

The necessity for the use of *two* test pieces is probably the reason why this method of test is not commonly applied. There is, however, no limit to the hardness of the material which can be tested, as the test is independent of the use of a harder material as an indenting tool.

(ii.) *The Material tested with an Indenting Tool (statically).*—Many different kinds of indenting tool have been tried and many ways have been suggested to express the relative indenting hardness.

Calvert and Johnson[4] and Kirsch[5] determined the load required to produce a permanent indentation of a given depth, but, whereas Calvert and Johnson used a truncated cone and a depth of 3·5 mm., Kirsch employed a cylindrical plunger of 5 mm. diameter and a depth of 0·01 mm.

The test adopted in 1856 by the United States Ordnance Department[6] was a determination of the volume of the indentation produced by a pyramidal point under a load of 10,000 lbs. A volume of 0·5 cubic inch was taken as unit hardness.



Reaumur (1722) Foeppl Haigh (1920)

Fig. 114.

Middleberg[7] used an indenting tool, in the form of a curved knife edge, for studying the hardness of tyres. The knife edge was $\frac{3}{4}$ inch long, had an angle of 30°, and was formed with an edge curved to 1 inch radius. The reciprocal of the length of the indentation under a load of 6000 lbs. was taken as a measure of the hardness.

Unwin[8] also employed a knife edge, but in his case it was straight and consisted of a piece of hardened and ground $\frac{3}{8}$-inch square steel, $1\frac{1}{2}$ inches in length. Each tool thus had four indenting edges having angles of 90°. The material tested was formed into test bars $\frac{1}{2}'' \times \frac{1}{2}'' \times 2\frac{1}{2}''$, and the knife edge was placed on the bar at right angles to its length, overlapping it by $\frac{1}{2}$ inch on each side. The hardness number was taken as P/H,

where P = the load in tons
and H = the depth of the indentation in inches.

(iii.) *Brinell Hardness Test.*—The method of test devised by J. A. Brinell[9] in 1900 is now extensively employed. A hardened steel ball is pressed under a known load into the material to be tested, and the hardness number is taken as the stress per unit of spherical area. The reason why Brinell used the spherical area is not clear, as there does not appear to be any advantage gained by using it instead of the projected area ($\pi d^2/4$).

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[1] *L'Art de convertir*, 1722, pp. 296 and 299.

[2] *Ann. Phys. Chem.* 63, i. 103-108.

[3] "Prism Hardness," *Inst. Mech. Eng. Journ.*, Oct. 1920.

[4] "Hardness of Metals and Alloys," *Phil. Mag.* 4th series, xvii. 114.

[5] *Mittheilungen des k.k. technologischen Gewerbe-Museums*, Wien, 1891, p. 108.

[6] *Report on Metals for Cannon*, 1856.

[7] *Engineering*, 1886, ii. 481.

[8] *Inst. Civ. Eng. Proc.*, 1897, cxxix.

[9] "Methods of Testing Steel," *Inter. Assoc. Test. Mat. Proc.*, 1901 (Paris), ii. 81.

If P = pressure in kilograms,

D = diameter of the ball used in millimetres,

d = diameter of the indentation in millimetres,

h = the depth of indentation in millimetres (see *Fig.* 115),

then $$h = \tfrac{1}{2}(\mathrm{D} - \sqrt{\mathrm{D}^2 - d^2}) \text{ mm.},$$

and the spherical area of indentation

$$\mathrm{A} = \pi \mathrm{D}h = \frac{\pi \mathrm{D}}{2}(\mathrm{D} - \sqrt{\mathrm{D}^2 - d^2}) \text{ sq. mm.}$$

The Brinell Hardness Number = H = P/A.

The diameter (d) is usually measured by a micrometer microscope. By the use of some forms of Brinell instruments the depth (h_1) of the indentation is taken as the test proceeds (see § (87)). With these instruments the depth which is measured is h_1, and not h (see *Fig.* 115). It should be noted that, owing to the rising of the edges of the Brinell indentation, h_1 is not equal to h, and, as the amount of this side extrusion is not the same with different materials[1] under similar conditions, the ratio of h_1/h is not always constant. It therefore follows that the hardness number obtained in this way is not the hardness number as defined by Brinell.

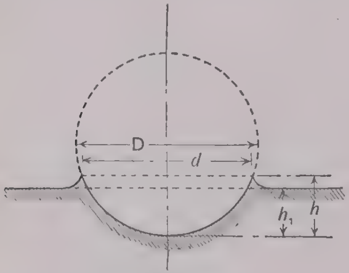

FIG. 115.

It is found that the hardness number varies with the diameter of the ball, and pressure employed. For strictly comparable results fixed values must be used for D and P. The values standardised by Brinell are:

D = 10 mm. and P = 3000 kilograms except for soft materials, when a value of P = 500 kilograms may be used.

(iv.) *Variation of Brinell Hardness with Pressure and Diameter of Ball.*—C. Benedick[2] of Upsala showed that within the range of his tests the value of $(\mathrm{P} \times \sqrt[5]{\mathrm{D}})/\mathrm{A}$ was nearly constant, and that with a ball of diameter $= \mathrm{D}_1$ and load of 3000 kilograms Brinell's hardness number $= (\mathrm{P} \times \sqrt[5]{\mathrm{D}_1/10})/\mathrm{A}$.

In order to allow for variation of pressure, H. Le Chatelier[3] proposed the further modification

$$\mathrm{H} = \frac{\mathrm{P}_1}{\mathrm{A}_1} \times \sqrt[5]{\frac{\mathrm{D}_1}{10}} \times \frac{20{,}000}{17{,}000 + \mathrm{P}_1},$$

where H = Brinell hardness number,

P_1 = load employed in kilograms,

A_1 = spherical area of the indentation (calculated from the diameter) in sq. mm.,

D_1 = diameter of the ball used.

Both the equations suggested by Benedick and Le Chatelier are empyrical and only give approximate values.

E. Meyer[4] in some published results in 1908 showed (1) that $\mathrm{P} = ad^n$, where n is a constant depending on the material, and a is a constant for a given material and given ball diameter; (2) that the mean pressure per unit area ($4\mathrm{P}/\pi d^2$) is constant for a given angle of indentation, whatever the diameter of the ball.

It follows from Meyer's law of comparison that, as d/D and $\mathrm{P}/\frac{1}{4}\pi d^2$ are constant for similar indentations on the same material, P/D^2 is also constant,

where P = pressure,

d = diameter of impression, and

D = diameter of the ball.

This relationship is useful where the piece of material is so small that a pressure of 3000 kilograms cannot be applied with a 10 mm. ball. It is then only necessary to use a smaller ball and a load determined by the above relationship to obtain the standard hardness number required; thus, if a ball 5 mm. diameter ($= \mathrm{D}_1$) is used, the load to be applied is

$$= \frac{\mathrm{P} \times \mathrm{D}_1{}^2}{\mathrm{D}^2} = \frac{3000 \times 5^2}{10^2} = 750 \text{ kilograms.}$$

Some results obtained by Baker[5] and given in Table 31 show remarkably good agreement of the hardness numbers obtained in this way.

TABLE 31

COMPARISON OF BRINELL HARDNESS NUMBERS WHEN USING BALLS OF DIFFERENT DIAMETERS

Steel.	Diameter of Ball in mm.	Diameter of Impression. mm.	Load. kg.	Brinell No.
A	10	6·3	3000	85
	7	4·4	1470	85
	5	3·13	750	87
	1·19	0·748	42·5	86
B	10	4·75	3000	159
	7	3·33	1470	158
	5	2·35	750	163
	1·19	0·567	42·5	158
C	10	3·48	3000	306
	7	2·43	1470	308
	5	1·75	750	310
	1·19	0·411	42·5	311

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[1] Batson, "Hardness Tests," *Inst. Mech. Eng. Proc.*, Nov. 1918, p. 576.

[2] *Recherches physiques et physico-chimiques sur l'acier au carbone*, Upsala, 1904.

[3] *Revue de Métallurgie*, 1906, iii. 689.

[4] *Zeits. Vereines Deutsch. Ing.*, 1908, p. 645.

[5] *Inst. Mech. Eng. Proc.*, Oct. 1918, p. 540.

(v.) *Size of Test Piece and Effect of Time on the Results and the Applications of the Brinell Hardness Test.*—H. Moore [1] found that

(1) The depth of the indentation in a Brinell test should not be greater than 1/7 of the thickness of the test piece.

(2) The centre of the indentation should not be less than 2½ times its diameter from the edge of the test piece.

The time during which the pressure is applied is important. It is found that the time effect is most marked up to about 10 seconds' application of the load, but after about 10 seconds the effect is very small. Consequently it is usual in specifications to draft the Brinell hardness test clause as follows:

"The Brinell hardness test, where specified, shall be made with a 10-mm. diameter ball and a load of 3000 kilograms, *which shall be maintained for not less than* 15 *seconds.* Prior to testing, the skin of the sample shall be removed by filing, proper grinding, or machining at the point to be tested."

Among the most important applications of the Brinell test are the following:

(i.) For rapid control of chemical carbon determinations during iron and steel smelting.

(ii.) For testing finished articles without damaging the same, such as rails, tyres, armour plates, gun-barrels of all kinds, structural steel, etc.

(iii.) For examining the nature of the material in entire or broken parts of machinery where the making of a tensile test bar is impossible.

(iv.) For testing the degree of hardness and softness obtainable by the thermal treatment of any steel.

(v.) For testing uniformity of temper.

(vi.) For ascertaining the effect of the nature and temperature of various hardening fluids.

(vii.) For studying the effect of cold working.

It has been shown by numerous observers (Ast, Breuil, Brinell, Charpy, Dillner, Le Chatelier, etc.) that there is a close relationship between the Brinell hardness number and the ultimate tensile strength of a material.

Dillner [2] finds that, in the case of steel, tensile strength in tons/sq. in. $=C\times$ Brinell hardness number. The values of "C" are as follows:

Brinell Hardness No.	Value of "C." Ball Pressure normal to the Direction of Rolling.	Value of "C." Ball Pressure in the Direction of Rolling.
Below 175 .	0·230	0·225
Above 175 .	0·219	0·206

Unwin [3] deduced the following constants from a table compiled by Hadfield:

$$T=0{\cdot}2H+6,$$
$$Y=0{\cdot}23H-13{\cdot}5,$$

where H is the Brinell number (in kilogrammes/sq. mm.),
T = Ultimate tensile strength in tons per square inch,
Y = Yield stress in tons per square inch.

§ (81) BRINELL HARDNESS TESTING MACHINES.—The Brinell hardness test is quite conveniently carried out in any machine in which a pressure of 3000 kilograms can be accurately applied, such as a universal testing machine having a compression attachment, by the use of a special tool for holding the ball.

Special machines have, however, been developed for the purpose, and these are of two main types:

(*a*) In which the load is applied by oil pressure and measured by a pressure gauge or dead-weight control.

(*b*) Dead-weight or lever machines.

(i.) *Jackman's Oil Pressure Brinell Machine.*—An example of the first type of machine is shown in *Fig.* 116. This is the apparatus supplied by J. W. Jackman & Co., Limited, Caxton House, Westminster, of the Aktiebolaget Alpha (Sweden) design.

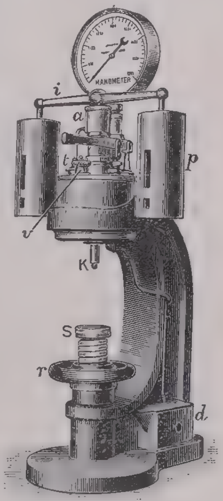

FIG. 116.

The ball K is attached to the downwardly acting ram of a hydraulic press. The test piece is placed on the adjustable table S, and is raised into contact with the ball by the hand-wheel *r*. The pressure is produced by a small hand-pump, and may be read directly in kilograms on a pressure gauge. The machine is also provided with a dead-weight control consisting of a piston, accurately fitted without packing, carrying a cross bar *i*, small cylinder *a*, and weights *p*. The small cylinder is connected to the top of the press so that the intensity of pressure is the same on the piston as on the ram. The maximum pressure is regulated by the weights *p*, and when this pressure is reached the piston rises in the cylinder. The pressure remains constant as long as the piston "floats," forming a small hydraulic accumulator.

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[1] "Investigation on the Brinell Method of determining Hardness," *Inter. Assoc. Test. Mat. Proc.*, 1909, No. 9.

[2] "Researches concerning the Relation between Hardness Number and Tensile Strength," *Engineering*, Nov. 9, 1906, p. 638.

[3] "Mechanical Properties of Materials," *Inst. Mech. Eng. Proc.*, Oct. 1918, p. 432.

The small piston is formed by a steel ball, and the piston rod has a cup-shaped end which presses on the ball. Any leakage of oil past the piston is collected in a receptacle *d*, from which it is returned to the reservoir through the funnel *t*.

The pressure is released by opening a valve *v* which connects the top of the pressure chamber with the reservoir.

(ii.) *Avery's Dead-weight Brinell Machine.*—The arrangement of this machine is shown in *Fig.* 117. The specimen is placed on the weighing platen *s*, and if of any considerable size its weight is "tared off" by moving one of the sliding poises *c* upon the weighing steelyards until the weighing system is balanced. The ball *k* is then brought into contact with the test piece by lowering the screw by which the pressure is applied. This is done quickly by unscrewing the thumb-screw A at the top of the standard, and revolving the screw by the small hand-wheel *d* which is keyed to it. The thumb-screw is then screwed up tightly, to secure the locking plate *f* and prevent the rotation of the screw. The pressure is then applied, by means of a worm and worm-wheel *h* operated by the large hand-wheel *g* at the side of the machine. The weights *p* are "set" for the prescribed load, and the lifting of the steelyards indicates that this load is applied.

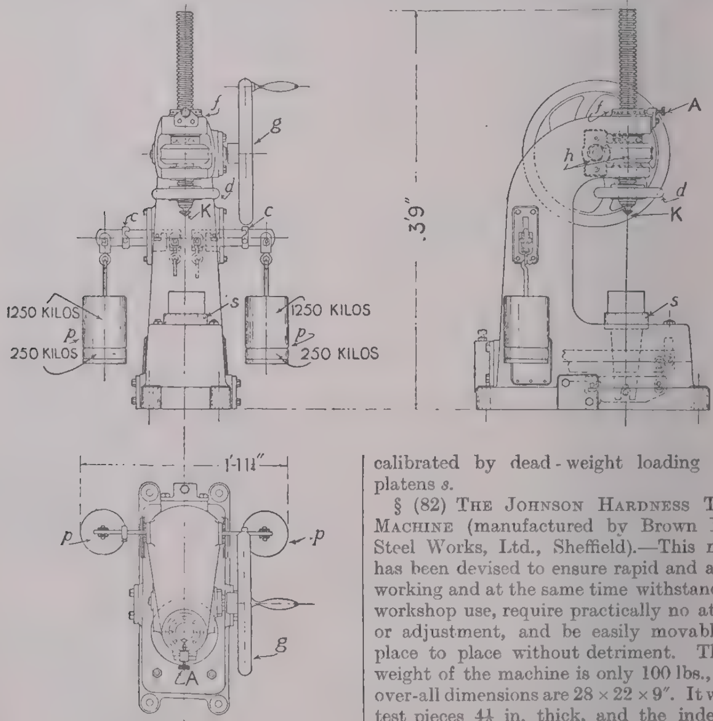

FIG. 117.

The system of levers for measuring the pressure is that adopted by W. & T. Avery, Ltd., Birmingham, in all their platform weighing machines, and is found to be extremely sensitive to small differences of load.

The leverage of the machine can be obtained directly by measurement, but it is also calibrated by dead-weight loading on the platens *s*.

§ (82) THE JOHNSON HARDNESS TESTING MACHINE (manufactured by Brown Bayleys Steel Works, Ltd., Sheffield).—This machine has been devised to ensure rapid and accurate working and at the same time withstand rough workshop use, require practically no attention or adjustment, and be easily movable from place to place without detriment. The total weight of the machine is only 100 lbs., and its over-all dimensions are 28 × 22 × 9". It will take test pieces 4½ in. thick, and the indentation can be made at distances up to 3½ in. from the edge. The arrangement of the machine is shown in *Fig.* 118.

(i.) *Description of the Machine.*—Pressure is applied to the steel ball A by means of the lever B and ball-bearing eccentric C through the single lever D and screw E. The other end of the lever D is held by an eccentric in ball-bearings F, to one of which is attached the lever G and weight H. The weight H is raised by the eccentric F when the pressure on the ball A reaches 3000 kilos, and any further movement of the operating lever B tending to apply further pressure on the ball only results in raising the weight still more.

(ii.) *Method of making Test.*—The specimen to be tested (which must have a smooth *flat* surface) is placed on the table I. The adjusting screw E is then brought down until the steel ball A rests on the prepared surface of the specimen. The lever B is then pulled forward until the weight H rises, allowing the pawl J to fall into the toothed sector K, which will automatically prevent any further movement of the lever B. The load may be kept on for any desired length of time by means of the retaining pawl L, which is moved in and out of position by means of the knurled handle M. In order to release the pressure the lever B is replaced to its original vertical position and the adjusting screw E raised.

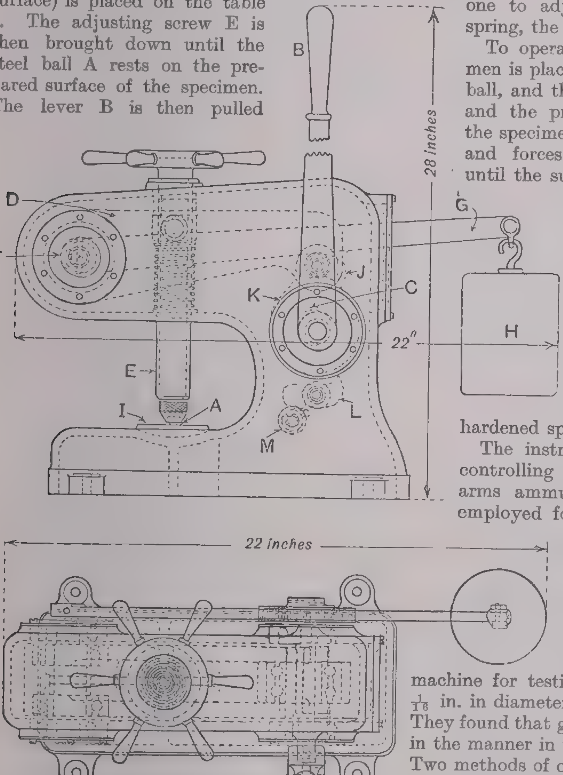

FIG. 118.

§ (83) Brinell Pliers.—For ascertaining the Brinell hardness of small and thin specimens, such as cartridge cases, Rudge Whitworth, Ltd., have devised a hand instrument which they call Brinell pliers. They are shown in *Fig.* 119 and consist essentially of a pair of pliers one nose of which serves as an anvil for supporting the cartridge case C or specimen to be tested, the other nose carrying a cylinder fitted with a piston, at the back of which is a spring. The end of the piston stem projects a short distance and carries a small hardened steel ball B $\frac{3}{64}$ in. diameter. A locking arrangement L enables one to adjust the compression on the spring, the usual pressure being 22 lbs.

To operate the instrument, the specimen is placed between the anvil and the ball, and the plier handles are grasped and the pressure applied. This brings the specimen into contact with the ball, and forces the ball and piston back until the surface of the specimen comes into contact with the face through which the ball projects. Consequently, the actual pressure between the ball and the specimen is quite independent of the pressure exerted by hand.

The diameter of the indentation is measured microscopically and the readings are quite definite, even on hardened spring steel.

The instrument found extensive use controlling the manufacture of small-arms ammunition. It has also been employed for measuring the skin hardness of many varieties of hardened steel articles.

§ (84) Small Machines for testing the Brinell Hardness of Thin Sheet.—Goodale and Banks[1] developed a "Baby" Brinell machine for testing thin sheet, using a ball $\frac{1}{16}$ in. in diameter and a load of 15 kilograms. They found that great care had to be exercised in the manner in which the load was applied. Two methods of operation were tried, viz. :

(1) The test piece was placed on the anvil and the ball pressed on to it until sufficient pressure was exerted to raise the balance beam.

(2) The test piece was forced, under a given load, into contact with the ball. The ball remained in a fixed position.

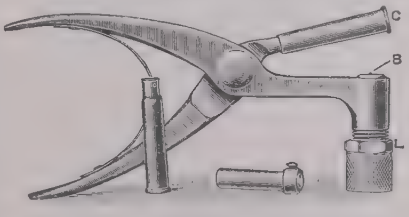

FIG. 119.

The second method gave smaller and more

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[1] "Development of Brinell Hardness Tests on Thin Brass Sheet," *Amer. Soc. Test. Mat.* xix. 758.

consistent results than the first, and was therefore incorporated in the design of their final machine. The load was applied by a dead weight of 15 kilos, and the diameter of the impression was measured with a metallurgical microscope with a 16-mm. objective and a filar micrometer type of eye-piece.

Similar work has been carried out in England;[1] the ball used, however, was 1 mm. diameter.

It is essential for the measurement of small indentations that there should be vertical illumination.

§ (85) The Ludwik Hardness Test.—In order to overcome the variation in the Brinell hardness number with load and size of ball, Ludwik[2] proposed substituting a 90° cone for the ball, and this form of indenting tool is now often used on the Continent for the softer metals. The hardness number is obtained by means of this test in the same way as with the Brinell Test, *i.e.* the test load in kilograms is divided by the surface of the conical indentation in sq. mm., so that if

P = the load,
and d = diameter of indentation,

$$\text{Hardness number} = \text{H} = \frac{\text{P}}{\pi(d/2) \times (d/\sqrt{2})}$$
$$= 0\,9(\text{P}/d^2) \text{ approximately.}$$

With this apparatus the hardness number is the same whatever the load chosen, if the material is homogeneous.

§ (86) Measuring Microscopes for Hardness Tests.—For commercial work the diameter of the indentation is measured to the nearest 0·05 mm., and for this purpose a microscope with a scale in the eye-piece is usually supplied with the Brinell apparatus. To facilitate the reading of impressions made in corners or below the general surface of the specimen, the microscope is sometimes provided with a special adaptable piece.

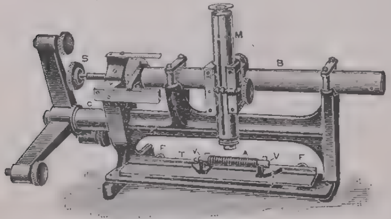

Fig. 120.

For research work an accuracy of 0·001 mm. is essential, and an instrument such as is supplied by the Cambridge & Paul Instrument Co., Ltd., and shown in *Fig.* 120, is necessary for the purpose. It consists of a microscope M clamped to a tube B which is supported by the framework of the instrument and can be traversed by the milled-headed screw S having a pitch of one millimetre. The microscope is fitted with an eye-piece containing cross-wires and is provided with a special focussing mechanism.

§ (87) Depth Indicators for Hardness Tests.—In order to dispense with the use of a microscope the depth of the indentation can be measured during the test by means of some form of depth indicator. Instruments of this type, which can be used in any compression machine, are supplied by Alfred J. Amsler & Co., Schaffhouse,[3] or the Scientific Materials Company, Pittsburgh.

The depth h_1 (*Fig.* 115) is measured relatively to the original surface of the test piece. This excludes the effect of the extrusion round the indentation, and as the amount of plastic flow varies with different metals and different pressures the hardness numbers found in this way are not proportional to the Brinell numbers obtained in the usual way from the diameters.

§ (88) Impact of Dynamic Hardness.—Numerous methods have been devised for measuring indentation hardness by the size of the indentation produced by energy of known amount. This method is complicated by a consideration of the effect of the rebound of the indenting hammer, and there is a difference of opinion as to whether the initial energy of the blow or the net energy absorbed in producing the indentation should be considered in calculating the results.

Edwards and Willis[4] used the initial energy in calculating the results in their research on impact hardness. Unwin,[5] on the other hand, is of opinion "that there should be a single impact and that the energy of rebound should be deducted from the energy due to the height of fall in calculating hardness. He doubted the method of permitting successive impacts till the energy was expended."

A dynamic hardness test was propounded by Martel[6] in 1895. He used a pyramidal point as the indenting tool, and produced the indentation by the fall of a ram on to the tool. Martel found:

(1) That the work of the falling ram was proportional to the volume of the indent, and he therefore expressed hardness as the work required to produce unit volume of indentation.

(2) That for equal energies of blow the volume of the indentation was nearly the same,

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[1] Moore, "A Small Ball Hardness Testing Machine," *Inst. Mech. Eng. Journ.*, Jan. 1921.

[2] "Hardness Tests," *Inter. Assoc. Test. Materials Proc.*, 1908, No. 6.

[3] Primrose, "Hardness Testing," *Inst. Mech. Eng. Journ.*, Oct. 1920.

[4] *Inst. Mech. Eng. Proc.*, May 1918, pp. 335-369.

[5] "Hardness Tests," *Inst. Mech. Eng. Proc.*, 1918, p. 578.

[6] *Commission des Méthodes d'Essais des Matériaux de Construction*, Paris, 1895, Sect. A, p. 261.

when using indenting tools of slightly different form.

The author carried out a series of experiments[1] which confirmed the first of Martel's conclusions. He showed further that, when using a 10-mm. diameter ball, the ratio of the dynamic hardness number to the Brinell hardness number was approximately 1·5 over a range of Brinell hardness from 20 to 680. The dynamic hardness number was taken as being equal to the net energy in kilogram metres divided by the volume of the indentation in cubic centimetres.

The dynamic method is useful for determining the indentation hardness values of metals at high temperatures, under which conditions the standard Brinell Test cannot be conveniently carried out. The indenting tool in the dynamic test is in contact with the specimen for such a short interval of time that it is not appreciably affected by the heat.

§ (89) DYNAMIC HARDNESS TESTING MACHINES.—A moving anvil block is supplied by W. & T. Avery[2] to adapt their Izod pendulum impact machine (see § (100)) for dynamic hardness tests.

§ (90) THE PELLIN HARDNESS TESTING APPARATUS[3] is made in Paris by Ph. & F. Pellin, and is specially designed for determining the hardness of thin materials. The indentation is produced by a falling bar of known weight having at the lower end a steel ball 2·5 mm. in diameter. The release of the indenting tool is arranged electromagnetically.

§ (91) THE AUTO PUNCH, designed by Rudge Whitworth, Ltd., and shown in *Fig.* 121, is a convenient and serviceable form of impact hardness tester. It was originally designed for testing case-hardened surfaces, but has proved valuable in testing materials of every grade of hardness, even pure lead.

It consists essentially of a knurled hollow cylinder C, from one end of which projects a plunger P which ends in an adapter and cap for carrying a ¼-inch hardened steel ball B. The other end of the Auto Punch is closed by a screw-in end, in which is fixed centrally a pawl-operating rod ¼ × 3 in. long, the object of which will appear shortly. Behind the sliding hammer is a spring which, in the case of the 12-inch Auto Punch, requires about 150 lbs. pressure to compress it. A 6-inch Auto Punch spring requires about 40 lbs. The internal mechanism is shown in *Fig.* 121A. The plunger is reduced to about ¼ inch diameter for about 1·7 inches at its inner end. This part of the plunger is small enough to pass right through the hole in the sliding hammer, and will do so when the sliding pawl is moved sideways by the pawl-operating rod already referred to.

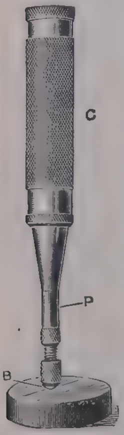

FIG. 121.

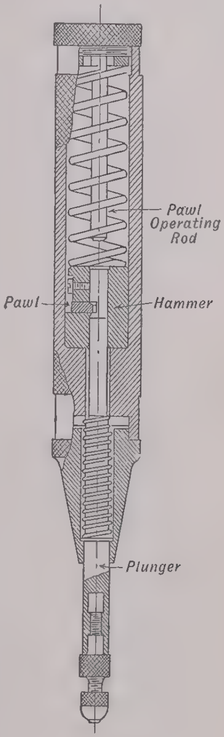

FIG. 121A.

When the Auto Punch is not in use the inner end of the plunger rests on the pawl in the sliding hammer. When the ball is applied to the article to be tested, and the knurled barrel of the Auto Punch is pressed towards the article, the hammer is forced back, compressing the spring, and continuing to compress it until the taper end of the pawl-operating rod presses the pawl to one side, and allows the compressed spring to drive the hammer forward until the front face of the latter strikes the end of the larger diameter part of the plunger. The kinetic energy of the hammer is, of course, *nil* up to the moment of its release, so that however rapidly or slowly the Auto Punch may be worked, the energy of the sliding hammer will be the same. This energy will be used up in deforming and heating the hammer and plunger where these strike one another, also the specimen and ball. Although the energy of the hammer is practically constant, the proportion of the total energy delivered to the specimen varies according to the hardness of the specimen. The softer the specimen, the greater is the proportion of the total energy spent in deforming it.

The diameter of the indentations produced by the Auto Punch follow those produced by the Brinell tests on the same class of material. If a Brinell Standard Block is employed to check from time to time the readings of the Auto Punch, the latter instrument can be used as a quick way of ascertaining the approximate Brinell hardness. In cases where an Auto Punch is used to ascertain whether deliveries are to the specified Brinell hardness, it is wiser to use the Auto Punch as a "workshop" gauge, and to employ the Brinell machine as an "inspection" gauge on all specimens which the Auto Punch shows to be near one or other of the limits of hardness.

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[1] Batson, "Hardness Tests," *Inst. Mech. Eng. Proc.*, Nov. 1918, p. 576.
[2] *Engineering*, xciv. 353.
[3] *Ibid.*, April 20, 1917, ciii. 374.

§ (92) THE SHORE SCLEROSCOPE [1] is a rebound instrument for measuring hardness. A small diamond-pointed hammer about $\frac{3}{4}$ inch long and $\frac{1}{4}$ inch diameter, weighing $\frac{1}{12}$ oz., is allowed to fall freely from a height of 10 inches. The height of rebound of the hammer is measured against a scale, graduated into 140 equal parts, and is taken as a measure of the hardness. The shape of the diamond striking point is slightly spherical and blunt, being about 0·020 inch in diameter.

The instrument is one which depends on the production of a permanent indentation by this point, the rebound being diminished by the work expended in producing the indentation. With rubber no permanent indentation is produced, and the height of rebound is the same as that from a moderately hard steel.

The instrument is provided with an ingenious automatic head by means of which the hammer is lifted and released by air pressure from a bulb.

The Scleroscope and Brinell hardness scales [2] have been shown to be generally closely related throughout the hardness range. The Brinell hardness number divided by six is approximately equal to the Scleroscope number; the ratio appears to increase from 5·5 for soft steels to 8·0 for materials of over 700 on the Brinell scale. The applications of the Scleroscope are therefore similar to those enumerated for the Brinell test in § (80).

The Scleroscope readings have been found to be slightly dependent on the size of the piece tested. This variation can be considerably reduced by clamping the test piece to a firm foundation. Most materials are mechanically hardened by indentation. It is therefore important to ensure that the material is not tested twice on the same place, otherwise high results are obtained.

§ (93) ABRASION OR SCRATCH TESTS.—The abrasion test has been applied in three ways:

(1) As a scratch test.

(2) By drilling or grooving with a hardened steel tool or diamond.

(3) By wearing away with or without an abrasive.

§ (94) THE SCRATCH TEST.—The scratch test has been frequently used in conjunction with an indentation hardness test. In 1722 Réaumur [3] used both methods. As an indication of scratch hardness he tested metals against a bar whose hardness increased from one end to the other; the position on this bar which the metal under test would scratch indicated its hardness. Mohs [4] was the first to give a scratch hardness scale, which is still largely used by mineralogists. Ten minerals were arranged in order from 1 to 10 in such a way that each would scratch the one next below it on the list. Talc was taken as having a hardness of "one" and diamond a hardness of "ten."

(i.) *Turner's Sclerometer.*[5]—In this device a balanced lever is—

(1) Provided at its free end with a diamond point fixed in a vertical pencil.

(2) Loaded with a sliding weight and calibrated in order that the sliding weight can be set to represent known weights at the point.

(3) Supported in such a way that it can be rotated.

The hardness number is the smallest weight in grammes which will produce a scratch just visible to the naked eye on the smoothed and polished surface of the specimen.

A series of scratches are made with diminishing weights, and the hardness is taken as the mean between the least weight which will produce a scratch and the greatest weight which will not produce a scratch.

A. Martens,[6] in order to make the test more definite, defined the scratch hardness number as the load in grammes under which a conical diamond (90°) produces a scratch 0·01 mm. in width.

(ii.) A. L. Parsons [7] further modified the Sclerometer by applying the load on the diamond point by a spring in such a way that the pressure increased as the point was drawn across the test piece. The place at which the scratch commenced was noted, and the corresponding load was taken as the hardness number.

(iii.) *The Quadrant Sclerometer.* — This instrument has been devised in the Rudge-Whitworth Laboratory. The method employed is to apply a hardened edge or series of hardened edges to the surface of the test piece in such a direction that slipping ensues until they "bite" the surface of the article.

The instrument is made in various types for testing either flat surfaces, pins, or the inside of cylindrical surfaces. *Fig.* 122 shows a pin under test.

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[1] "An Instrument for testing Hardness," *American Machinist*, 1907, xxx. 845; also "The Scleroscope," *Amer. Soc. Test. Mat. Proc.*, 1910, x. 490.

[2] Stanton and Batson, "Hardness Tests Research," *Inst. Mech. Eng. Proc.*, Nov. 1916, p. 693.

[3] *L'art de Convertir*, 1722. pp. 296 and 299.

[4] *Grundriss der Mineralogie*, 1822, part i. p. 374.

[5] *Birmingham Phil. Soc. Proc.* v. part ii., 1887.

[6] *Sitzungsberichte des Vereines zur Beförderung des Gewerbfleisses*, 1888, p. 41, and 1889, p. 197.

[7] *Amer. Journ. of Science*, Feb. 1910.

Two specially cut flat rods or files F_1 and F_2 are employed, and the upper one is raised until it makes an angle of about 80° with the lower. The pin P is then placed between them and the "file" lowered until it "bites" the pin. The angle at this point is an index of the scratch hardness of the article.

The theory of the instrument has been worked out by H. L. Heathcote,[1] who has shown that the angle is practically independent of the weight and size of the article to be tested. A soft bar holds the top file up so that it makes an angle of about 70°. A hard bar will not hold the top file up until the angle falls to about 15 to 20°. Hardened and tempered bars give readings between 20 and 70°, according to their surface resistance to abrasion. The instrument can also be employed for measuring the coefficient of friction between one material and another, one being attached to, or substituted for, the upper arm, and the other for the lower.

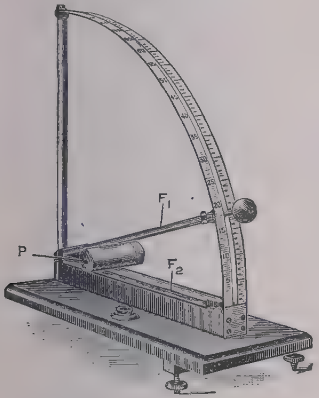

FIG. 122.

§ (95) "DRILLING OR GROOVING" HARDNESS.—S. Bottone[2] measured the resistance to wear as the time required to produce in the material a cut of definite depth with a soft iron disc rotating at constant speed and pressed with constant force.

A. Haussner[3] defined cutting hardness as the resistance, per square mm., to planing, when using a cutting angle of 90°.

Jaggar[4] rotated a diamond point under constant pressure and at a uniform rate, and gave the number of rotations of the point to reach a fixed depth as the hardness. Bauer and Keep[5] both defined cutting hardness as the rate at which a steel drill, running at constant speed and under constant pressure, drills the material.

§ (96) WEAR TESTS. (*a*) Using an abradant or an abrading wheel (pure abrasion).—Felix Robin[6] used cylindrical specimens (50 mm. diameter), which he rubbed under known pressure on papers covered with abrasive powders. Rosiwall[7] defined abrasive hardness as equal to (1/loss of volume), by grinding with an abradant. Behrens[8] used a standard powder and measured hardness by the time required to polish.

Gary[9] determined the resistance to wear by the amount of abrasion caused by a sand-blast, and experimented chiefly with stones, artificial minerals, and timber. W. H. Warren[10] also used the sand-blast on timber.

Jannetaz and Goldberg,[11] and Stoughton and Macgregor[12] measured the comparative loss by grinding. The latter, who also carried out indentation hardness tests, found that the two methods gave different comparative results.

(*b*) Wear by sliding lubricated abrasion.—Derihon[13] constructed a machine in which a specimen was pressed on to the circumference of a polished wheel turning at a speed of 3200 revolutions per minute in an oil bath. The wear was measured by the loss in weight or reduction in diameter in thousandths of a millimetre after 2,000,000 turns of the wheel.

(*c*) Wear by dry rolling abrasion.—Saniter[14] devised a machine for testing the wearing properties of rail steel under dry *rolling* abrasion. In this method the specimen (A), 5 inches long and 0·5 inch diameter, was fixed in a chuck (B, *Fig.* 123) revolving at 4000 r.p.m., and carried near the free end a ball-bearing (C), of inner diameter 1 inch, loaded to produce a pressure of 205 lbs. at the point of contact. The inner ring of the ball-bearing (D, $\frac{11}{16}$ inch wide) was rotated by friction by the test piece, causing the latter to wear. The resistance of the material to rolling abrasion was taken as being inversely proportional to the reduction in diameter in ten-thousandths of an inch after 200,000 revolutions of the test piece.

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[1] *Iron and Steel Inst. Journ.*, May 1914.

[2] "Relation entre le poids atomique, le poids spécifique et la dureté des corps," *Chemical News*, 1873, p. 215.

[3] "Das Hobeln von Metallen," *Mitt. des k.k. technolog. Gewerbe-Museums*, Wien, 1892, ii. 117.

[4] "A Microsclerometer," *Amer. Journ. of Science*, Dec. 1897, iv. 399.

[5] Keep, "Hardness or the Workability of Metals," *Amer. Soc. Mech. Eng. Trans.*, Dec. 1900, abstract in *Iron and Steel Inst. J.*, 1901, i. 496.

[6] *Iron and Steel Inst. J.*, 1910.

[7] *Verhand. k.k. geol. Reichsanstalt*, 1896, xvii. 475.

[8] *Anleitung zur mikrochemischen Analyse*, 1895.

[9] *Baumaterialienkunde*, x. 136.

[10] "Strength. Elasticity, and other Properties of N.S.W. Hardwood Timbers," *Report Dept. of Forestry, N.S.W.*, 1911.

[11] *Assoc. Franc. p. l'Avanc. d. Sc.*, Aug. 9, 1895.

[12] "Hardness Tests," *Amer. Soc. Test. Mat. Proc.*, 1911, xi. 707.

[13] See Nusbaumer on "Notes on Abrasion Tests on Metals," *Inter. Assoc. Test. Mat. Proc.*, 1909.

[14] *Iron and Steel Inst. J.*, 1908, iii. 73; also *Inter. Assoc. Test. Mat. Proc.* ii. No. 9, paper iii./1.

A similar test was used by Stanton and Batson[1] for a series of experiments on hardness tests. The dimensions of the test piece, etc., were made slightly larger than those employed by Saniter.

It was shown that the resistance to rolling abrasion was approximately proportional to the ball hardness number, but that the comparison was not a safe one, as there were frequently cases in which a considerable departure was found from this ratio, *e.g.* manganese steel which was well known to be susceptible to hardening under pressure. The results confirmed Saniter's conclusions, and showed that what was actually measured was the resistance to disintegration of already deformed material, and that this resistance depended on the amount of deformation produced, and had little relation to the material in the unstrained condition. The method, as a means of predicting the relative resistance to wear under conditions of rolling abrasion with heavy loads, was comparatively rapid and gave the information desired.

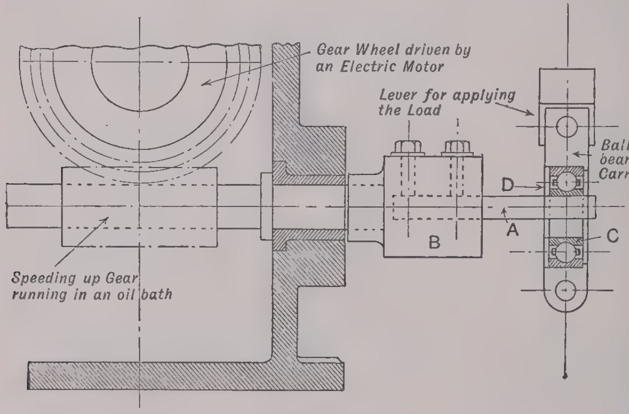

FIG. 123.

(*d*) Wear by dry sliding abrasion.—The Saniter test was carried out with high pressure and *very small* relative motion. Stanton and Batson[1] also carried out a series of tests on dry *sliding* abrasion, in which the amount of relative motion was *large* and therefore corresponding to the wear of pins, collars, etc. This was done by connecting the abrading ring (diameter = D) to the chuck by means of an Oldham coupling, so that both ring and specimen (diameter = d) completed a revolution in the same time, the line of contact remaining fixed relative to the machine. The slip per revolution was then $= \pi(D - d)$ inches. The results indicated that there was very little hardening of the surface of the material under these conditions, and that the Brinell hardness number was not a safe guide in predicting the relative resistances to wear of a miscellaneous selection of steel.

§ (97) IMPACT AND NOTCHED BAR TESTING.—There is no doubt that the tensile test, as usually carried out, does not give all the information about a material it is essential that an engineer should know. A careful examination of a complete load extension diagram, however, reveals differences, such as those due to heat treatment and mechanical treatment, not disclosed by an ordinary tensile test, but the analysis of such diagrams requires considerable experience.

It has, occasionally, been found that a material which has satisfactorily passed the ordinary tensile test fails in practice in a manner which cannot be accounted for by errors in design. Investigation has shown that, in some cases, parts of machinery which have failed in this way have been subjected to shocks. Consequently, various methods of test have been devised to give a definite indication of the shock-resisting properties of materials.

§ (98) NOTCHED BAR TESTS.—Freedom from the tendency to crack at sharp corners, when the variations of stress are considerable, is a property which is very desirable in materials for engineering work. In order to obtain this information about materials "brittleness" tests, in which the test piece is notched so as to limit the plane of fracture and the contraction of area, are employed.

These notched bar tests are made in tension or bending. The test piece is usually broken by a single blow in impact, and the energy of fracture obtained directly from the loss of energy of the striker.

In 1909, Charpy,[2] in a report on impact tests on metals, shows that a static tension test is more efficaciously supplemented by a notched bar bending test than by any other test, and his results seem to indicate that the notched bar bending test gives information regarding the fragility of materials which other tests do not easily furnish. Charpy in the same report shows that materials which give good results with this impact test have stood well in practice, whereas the same class of materials giving low values fail, although the results from the ordinary static tensile test is the same in both cases.

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[1] "Hardness Tests Research," *Inst. Mech. Eng. Proc.*, Nov. 1916, p. 693.

[2] "Official Report on Impact Tests of Metals," *Inter. Assoc. Test. Mat. Proc.*, 1909, No. 7, paper iii./1.

The originator of this method of testing appears to have been M. Barba of Le Creusot, who in 1900 described the results of his experiments on the detection of brittleness in steel by tests on flat specimens, notched with a 45° notch on both sides to 0·35 times the thickness, the radius at the bottom of the notch being 0·2 mm. and the total width of the specimen 30 mm. The specimen was fixed in a horizontal position between jaws, with the notch directly over the edges of the jaws and having a piece of the specimen 25 mm. long projecting from the jaws. This piece was struck by a falling weight of 18 kilograms. Each specimen was provided with a number of notches at spaces of 25 mm. along the specimen, and by making a series of tests at different heights of fall it was possible to predict the energy of blow which would just break the specimen.

It was soon realised that this method of testing revealed marked differences in the behaviour of materials which were not detected by the ordinary tensile test, and other investigators (Le Chatelier, Charpy, Frémont, etc.) carried out experiments, the results of which were communicated to the Congress of the International Association of Testing Materials held at Buda Pesth in 1901.

The report of a commission, appointed at this congress to investigate the notched bar test, was discussed at the congress held in Brussels in 1906, but no advance was made in the matter of standardisation except an expression of opinion that the test gave interesting information. The German Association for Methods of Testing Materials thereupon took the matter up and issued a report in 1907, in which they recommended the Charpy method of testing. This consists in the use of a notched specimen, 30 × 30 × 160 mm., supported horizontally at the ends on knife edges of given form and struck in the centre, opposite the notch, by a pendulum. The notch was formed by drilling a hole 4 mm. diameter in the bar and sawing through from one side, leaving a depth of 15 mm. behind the notch, as shown in *Fig.* 124. The span is 120 mm.

The question of notched bar testing was again brought forward at the International Congress held at Copenhagen in 1909, who recommended the 30 × 30 × 160 mm. Charpy test piece, with an alternative specimen, geometrically similar but one-third of the size, where the larger dimensions could not be obtained. The matter was further discussed at New York in 1912, and the recommendations of 1909 were confirmed. No particular machine for carrying out the tests was selected, but further testing research on apparatus and test pieces of different sizes was suggested as being desirable.

The recommendations of the International Association for Testing Materials have never been fully recognised. For most test work the standard 30 × 30 × 160 mm. test piece has been found to be too large, and difficulties have been experienced with the notch of the geometrically smaller test piece.

Frémont suggests a test piece 10 × 8 × 30 mm., placed horizontally on supports 21 mm. span, with a notch 1 mm. deep and 1 mm. wide (square shape, and made on the broad side at the centre of the length), *Fig.* 125.

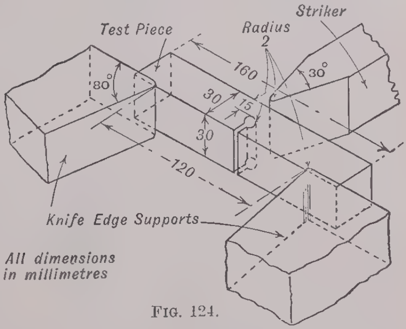

FIG. 124.

In most cases a test piece 10 × 10 mm. in cross-section is used, but the form of notch is varied. Several well-known types of notch are given in *Fig.* 125. The beam test pieces are usually 60 mm. long with a span of 40 mm.

§ (99) NOTCHED BAR TESTING IN BRITAIN.—A considerable amount of work has been carried out, in Britain, on the notched bar test. Yarrows developed a notched bar test in 1902, in which the test bar was broken by more than one blow. Valuable information appears to have been obtained by this test as to the brittleness of steels used for connecting rod bolts.

In 1903, Izod[1] introduced a single-blow notched bar testing machine, in which the specimen is held in a vice at one end and is struck by a falling pendulum at the other. This method is the one which is commonly adopted in this country at the present time.

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[1] *Engineering*, Sept. 25, 1903.

Seaton and Jude[1] carried out some tests in 1904, in which the test piece was broken by a number of blows, the test bar being reversed between successive blows.

In 1908, Stanton and Bairstow[2] and Harbord[3] communicated results of experiments to the Institution of Mechanical Engineers. The main point brought out by Harbord's paper was the variability in the results of individual tests of specimens of the same material. It was shown that two identical test pieces from the same bar of ordinary commercial steel gave results varying from each other by over 60 per cent. This variability has been attributed to—

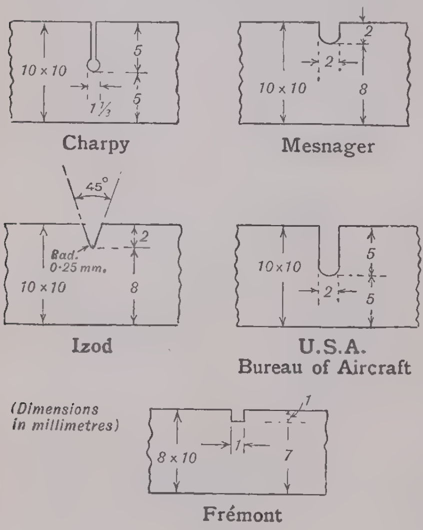

FIG. 125.

(1) The mode of testing.

(2) The heterogeneity of the material under investigation.

Experiments published by Charpy and Cornu Thenard[4] show that by careful selection and heat treatment it is possible to obtain, in the course of experiments in notched bar tests, carried out on bars of steel of different natures or of copper, a degree of uniformity analogous to that obtained by means of tensile or hardness tests. Variability in the results is therefore not due to the mode of testing but to differences in the material under test.

By 1914 there was conclusive evidence that there could be large variations in the resistance of notched specimens of different materials to impact, without any corresponding variation in any of the characteristics brought out by the ordinary tensile test, and there was evidence that this variation was due to correct or incorrect heat treatment.

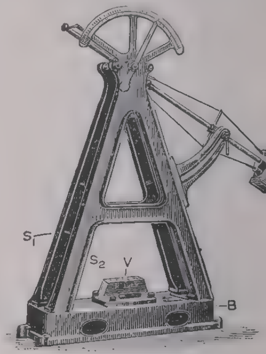

FIG. 126.

§ (100) IMPACT TESTING MACHINES. (i.) *The Izod Test.*—The Izod impact test has of recent years come into great prominence in Government specifications, owing to the great demands made by war conditions.

The original Izod machine had a striking energy of the hammer of 23 ft.-lbs., and the test piece was 2 in. long, $\frac{3}{8}$ths in. wide, and $\frac{3}{16}$ths in. thick, with a vee notch 0·05 in. deep.

The present standard machine made by W. & T. Avery, Ltd., Soho Foundry, Birmingham, has a capacity of 120 ft.-lbs. The cross-section of the test piece is 10 × 10 mm. and it is notched with a 45° vee notch, 2 mm. deep with a root radius of 0·25 mm. The machine is shown in *Fig.* 126, and consists of a heavy base B on to which are bolted two standards S_1, S_2 supporting the pivot of the pendulum. The pendulum P swings on ball bearings and strikes the specimen held in the vice, *cantilever fashion,* its point of contact with the specimen being a hardened steel knife edge. The form and angle of the knife edge is shown in *Fig.* 127. The specimen is gripped in a vice V in such a

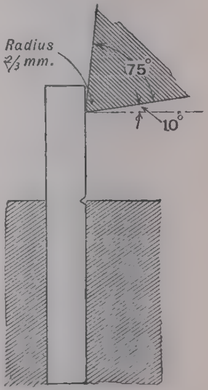

FIG. 127.

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[1] "Impact Tests on Wrought Steels of Commerce," *Inst. Mech. Eng. Proc.*, 1904, pt. iv. 1135.

[2] "Resistance of Materials to Impact," *Inst. Mech. Eng. Proc.*, Nov. 20, 1908, and *Revue de Metall.*, March 1909.

[3] "Different Methods of Impact Testing on Notched Bars," *Inst. Mech. Eng. Proc.*, Oct.-Dec. 1908.

[4] "New Experiments on Shock Tests," *Iron and Steel Inst. Journ.*, No. 2, 1917, p. 61.

position that the bottom of the notch is level with the top of the vice, *Fig.* 128.

The specimen, in being broken, absorbs some of the energy from the pendulum, which is measured by the continued and diminished swing of the pendulum moving an idle pointer over a graduated scale. The pendulum swings less as the resistance of the specimen is greater. The scale is graduated to give the actual energy in foot-pounds absorbed by the blow. The standard three-notch test piece is shown at B in *Fig.* 129.

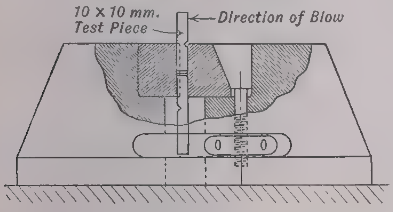
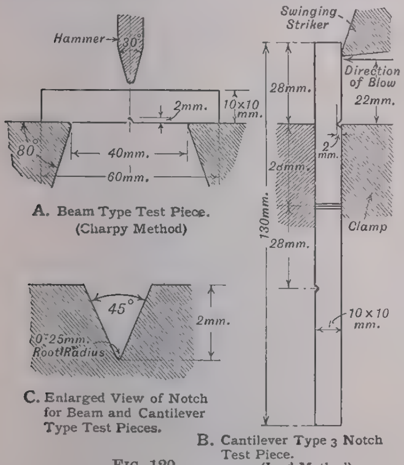


FIG. 128.

(ii.) *The Charpy Pendulum Machine.*—The small Charpy machine has a power of about 30 kilogrammetres (217 ft.-lbs.) and a maximum striking velocity of 5·28 metres per second (17 ft. 4 in. per sec.). The principal part of the pendulum consists of the hammer M, shaped as shown in *Fig.* 130, and suspended at the end of a light hollow bar. The centre is suspended on ball-bearings, and an index hand is mounted with an easy frictional fit and travels over a graduated semicircular dial with the hammer when this rises up after fracturing the specimen. The latter is held in cast-iron supports in the two uprights which are bolted down to a cast-iron bedplate.



A. Beam Type Test Piece. (Charpy Method)

C. Enlarged View of Notch for Beam and Cantilever Type Test Pieces.

B. Cantilever Type 3 Notch Test Piece. (Izod Method)

FIG. 129.

The hammer is raised up for action by hand, and is held up by a catch operated directly by a small lever D. The drop is constant and 1·420 metres (4 ft. $7\frac{5}{8}$ in.) in height. A hand brake B, worked by the hand lever C, enables the hammer to be stopped rapidly after fracture of the specimen. The weight of the falling parts, position of the centre of gravity, and heights of drop counted from the centre of gravity, are all determined experimentally. The angle of rise after fracture is read off from the graduated semicircle. The difference between the height of fall before fracture and rising back after fracture gives the work absorbed. It is, however, necessary in research work to take into account the friction and energy absorbed by the fragments of the test specimen; the latter may be considered to take the same velocity as the pendulum itself. The weight of the test specimen being very light compared with that of the pendulum hammer, the correction is very slight and can be neglected in ordinary practice.

In order to determine the work absorbed by friction the pendulum is caused to oscillate freely and the decrease in the oscillations due solely to friction is noted down. This gives a table of corrections. The required correction can also be arrived at by noting the angle of rise of the pendulum following that causing the fracture.

A larger machine is made, and this has 200 kilogrammetres energy and a striking velocity of 7·8 metres per second.

(iii.) *Guillery Machine* (*Rotary Tup*).—The latest model of this machine is shown in *Fig.* 131. It consists of a flywheel having a "breaking knife" attached to it. This knife is arranged so that it can assume two positions:

(1) Hidden in the rim of the wheel.

(2) Projecting from the rim of the wheel in which position it is required for breaking the test piece.

The position is controlled by "studs" on the casing of the machine and operated by centrifugal force due to the rotation of the wheel.

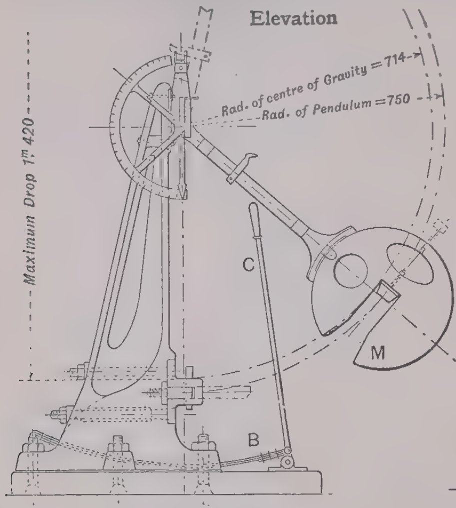
The rotation of the flywheel operates a small centrifugal pump which elevates a coloured liquid in a tube. The machine is designed so that when the liquid is at the top of the tube (and reading zero on the energy scale) there is 60 kilogrammetres of energy in the flywheel. When the flywheel is still, and the liquid is at its lowest level the reading is therefore 60.

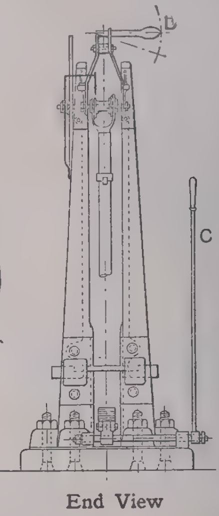
The test bar is placed horizontally on knife edges which have a 40-mm. gap. The front of the machine is covered with a door which is automatically locked when the "knife" is out and the flywheel in motion.

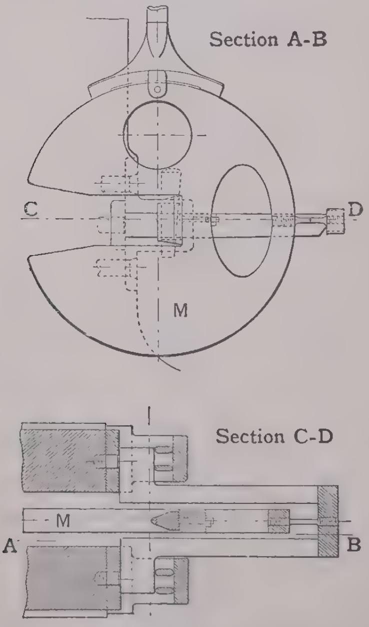
The test is made by rotating the flywheel to a speed slightly in excess of that corresponding to the zero of the energy scale. The gear is then disconnected and the wheel speed allowed to decrease. Immediately the liquid in the tachometer tube reads zero the "out" stud is pressed, the knife springs out and breaks the test piece. The absorption of energy from the flywheel lowers the speed and the tachometer reading gives the energy lost in producing fracture of the test piece. The knife is then set "in" by pressing the "in" stud.

(iv.) *Frémont Machine.* — The Frémont impact machine consists of a hammer of 10 to 15 kilograms, which has on its underside a hardened steel V-shaped striker. The test piece is placed horizontally on knife edges having a gap of 21 mm., so that when the hammer falls from a height of 4 metres the striker hits it midway between the supports and exactly opposite to the notch.

Immediately after the blow of the striker the underside of the hammer, which then only possesses the energy not absorbed by the test piece, strikes against an anvil carrying two housings provided with tempered







Details of Hammer and Knife edges

FIG. 130.

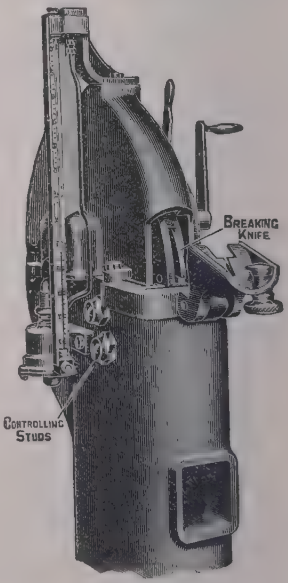


FIG. 131.

steel springs. The springs yield under the blow of the hammer and register by their deflection the energy remaining in the apparatus.

Knowing the initial energy of the hammer, the energy absorbed in fracture of the test piece is found by taking the difference.

(v.) *The Amsler 75-kilogrammetre Pendulum Impact Testing Machine.*—This machine is on the same principle as the Charpy machine, but some of the details are slightly different.

The pendulum is held up by a hook attached to a rope which is wound round a windlass. The tup is released, by detaching the hook by means of a special release, and falling freely swings up on the other side of the machine after having broken the test bar. Instead of swinging back again, as in the Charpy machine, it is held in position by a cord passing round a drum and acting as a brake.

On falling from one side, transverse (bending) tests are made, while tensile tests are made when the pendulum falls from the other side. The energy in the hammer before and after the test is registered on two movable straight scales by the movement of the pendulum. It is so arranged that the second scale raises the pointer on to the first scale and indicates the actual energy absorbed.

TABLE 32

IZOD TESTS ON MATERIALS CORRECTLY AND INCORRECTLY HEAT-TREATED

Materials.	Heat Treatment.	Limit of Proportionality. Tons/Sq. In.	Yield Point. Tons/Sq. In.	Ultimate Stress. Tons/Sq. In.	Elongation. Per cent.	Reduction of Area. Per cent.	Average Izod Figure. Ft.-lbs.
Nickel chrome steel—							
Bar SS1 . .	Correct	40·4	47·8	55·5	28·6	64·0	78·0
Bar SS2 . .	Incorrect	39·6	45·9	54·3	26·5	63·7	9·1
Nickel chrome steel—							
Bar SH1 . .	Correct	34·8	44·0	59·3	26·0	61·9	62·1
Bar SH2 . .	Incorrect	34·8	45·0	60·3	23·0	44·2	8·0
Nickel chrome steel—							
Bar SS3 . .	Correct	26·0	44·0	54·9	26·0	67·0	81·7
Bar SS4 . .	Incorrect	35·2	44·3	54·3	24·5	64·0	14·8
Nickel chrome steel	Correct	32·3	44·3	55·2	26·7	69·5	79·5
	Incorrect	31·9	42·1	53·8	29·4	68·5	15·9

TABLE 33

CHARPY TESTS ON MATERIALS CORRECTLY AND INCORRECTLY HEAT-TREATED (Charpy Method—Izod Notch)

Materials.	Heat Treatment.	Limit of Proportionality. Tons Sq. In.	Yield Point. Tons Sq. In.	Ultimate Stress. Tons Sq. In.	Elongation. Per cent.	Reduction of Area. Per cent.	Energy to Fracture Kg.M.
Boiler plate . .	Correct	18·3	19·1	27·9	42·2	62·8	6·35
	Incorrect	14·2	18·3	26·9	31·6	59·1	0·88
Nickel chrome crank shaft	Correct	27·0	33·0	45·2	21·0	59·2	10·74
	Incorrect	20·0	32·9	44·0	23·5	57·0	2·27

§ (101) FURTHER PARTICULARS OF NOTCHED BAR TESTS.—The leading particulars of the five principal types of notched bar machines are as follows:

Type of Machine.	Striking Energy. Kg.M.	Striking Velocity. Metres/Sec.
Charpy (pendulum) .	30	5·3
Amsler (pendulum) .	75	. .
Izod (pendulum) .	16·6	3·5
Frémont (falling tup) .	20 or 60	8·85
Guillery (rotary tup) .	60	8·85

The notched bar test is principally used as a means of detecting a dangerous condition of microstructure due to faulty heat treatment.

The large variation in the energy absorbed in the Izod test due to alteration of heat treatment is well shown by the figures in Table 32, taken from a paper by Philpot.[1]

Some tests by the author on a piece of boiler plate and nickel chrome crank shaft are given in Table 33, and show the same kind of variability with heat treatment

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[1] "Some Experiments on Notched Bars," *Inst. of Automobile Eng. Proc.*, April 1918.

Bagnall Wild has shown that the brittleness in steel resulting from a high percentage of sulphur and phosphorus is well marked by the Izod test. He says, "Certain steel for making nuts and bolts was found to contain sulphur and phosphorus in excess of 0·1 per cent; such steel invariably failed to give any higher reading than 2 to 3 ft.-lbs. on the Izod, whereas a steel of exactly similar composition and with similar treatment but with sulphur and phosphorus below 0·06 per cent gave an Izod test figure of over 40 ft.-lbs."

§ (102) DIMENSIONS OF STANDARD TEST PIECES.—Although there are several impact testers in Britain of the Frémont and Guillery type, the majority of the test houses use Izod or Charpy machines. The dimensions of test pieces for these machines have been standardised and are shown in *Fig.* 129. It will be noted that in each case the cross-sectional dimensions are 10 × 10 mm. and that the form of notch is the same (the Guillery machine uses the same size of test pieces as the Charpy machine, while the Frémont test piece is smaller and is described in § (98). The form of notch selected, and sometimes called the Izod notch, is a 45° vee, 2 mm. deep, with a root radius of 0·25 mm.

§ (103) THE ANGLE OF THE NOTCH.—Experiments by Thomas[1] at the Watertown Arsenal show that, with mild steel, the angle of the notch does not appreciably affect the results until it has exceeded 45°. The results that Thomas obtained are given in Table 34.

§ (104) ROOT RADIUS AND DEPTH OF NOTCH.—The 10 × 10-mm. test piece in which the notch is formed by a drilled hole 1⅓ mm. diameter, proposed by the International Association of Testing Materials, cannot be produced economically on the majority of alloy steels.

TABLE 34

EFFECT OF VARIATION OF ANGLE OF NOTCH ON THE ENERGY ABSORBED

Angle of Notch. Degrees.	Charpy Impact Values. Ft.-lbs./Sq. In.			Angle of Bend. Degrees.	Brinell Hardness No.
	Maximum.	Minimum.	Average of 6.		
0	304	266	285	15·8	79
15	346	283	316	15·2	77
30	323	284	298	15·0	81
45	382	304	334	29·5	80
60	571	516	539	54·2	80
75	890	818	854	130	78
90	830	806	814	136	78

The form of notch selected as a standard, viz. 45° vee notch, 2 mm. deep and 0·25 mm. root radius, does not suffer from this objection as it can be easily produced even on the hardest steels.

The shape at the bottom of the notch has an important effect on the work absorbed in fracture, which has its least value when the angle at the bottom of the notch is as nearly zero as possible.

Results by Dix,[2] summarised in Table 35, and by the author (Table 36) show this effect very clearly.

TABLE 35

EFFECT OF RADIUS AT THE BOTTOM OF THE NOTCH ON THE ENERGY ABSORBED IN FRACTURE

Nickel chrome steel rod, ⅝″ in diameter

Form of Notch.	Energy absorbed in Fracture (Charpy Machine).	
	Ft.-lbs.	Kg.M.
2 mm. deep (45° vee) sharp	21·6	2·99
2 mm. deep (45° vee) 0·25 mm. root radius	25·0	3·46
2 mm. deep (parallel sides) 1 mm. root radius	40·3	5·57

TABLE 36

0·65 per cent carbon steel, vee notches, 2 mm. deep

Root Radius of Notch in mm.	Energy absorbed in Fracture (Charpy Machine).	
	Ft.-lbs.	Kg.M.
Sharp	4·05	0·56
0·17	6·86	0·95
0·34	8·31	1·15
0·68	13·7	1·89

It has been found that the sharper the notch is made, the better the test discriminates between brittle and tough materials, that is to say, that as the material tested becomes tougher the effect of alteration of the root radius is less.

It is unsatisfactory, commercially, to standardise a dead sharp notch as it is difficult to be always sure that it is produced. A root radius of 0·25 mm. has been selected as it is found that, with a radius of this size, the test is still effective in the detection of faulty heat treatment.

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[1] "Charpy Impact Tests on Heat-Treated Steels," *Amer. Soc. Test. Mat. Proc.*, 1915, xv. 75.

[2] "Single Blow Notched Bar Impact Test as used in the American Industry," *Amer. Soc. Test. Mat. Proc.*, 1919, xix. part ii. 721.

It has also been found that shallow notches increase the energy to fracture of tough materials relatively to that required for brittle materials.

§ (105) SHAPE OF TEST PIECE.—Philpot[1] found that a round test piece can be produced which can be used for acceptance tests on heat-treated steels in place of the standard square specimen. The latter is, however, far the more desirable from the point of view of testing and, by properly rigging up for quantity production, it can be prepared by a pair of straddle mills just as economically as the round test piece.

§ (106) COMPARISON OF TESTS ON DIFFERENT MACHINES.—Philpot[2] has made a comparison of tests in the Charpy (beam) and Izod (cantilever) machines on the standard 10 × 10-mm. test piece with the 45° standard notch, 2 mm. deep and 0·25 mm. root radius. He finds that, where the energy absorbed is less than 70 ft.-lbs., notched bar tests, made in either the Charpy or the Izod machines, give similar values, but where the energy absorbed in fracture is greater than 70 ft.-lbs., there is a tendency for the values from the Charpy machine to be somewhat higher.

Charpy and Cornu Thenard[3] have made a series of experiments, with a steel specially prepared to give consistent results, on "beam" machines of three different types (pendulum, vertical drop, and rotary) and have found that, in regard to energy absorbed in fracture, the results are practically identical when the size of test piece, notch, distance between supports and radii of supports and striker are uniform.

§ (107) EFFECT OF STRIKING VELOCITY ON THE ENERGY TO FRACTURE.—Both Belanger and Frémont have found a variation of the energy absorbed in fracture with striking velocity but in opposite directions. Charpy has found that the influence of the rate of impact is practically negligible within limits which do not exceed those of appliances generally employed in the testing laboratory.

Some tests carried out by the author show that, on a machine of the Charpy type at striking velocities up to 43 ft. per second, increasing striking velocity may either increase, decrease or unalter the energy absorbed in fracture according to the material upon which the tests are made. In any case the effect is not appreciable until after a velocity of 16 ft. per sec. is reached, as is shown by the results in Table 37.

TABLE 37

EFFECT OF STRIKING VELOCITY ON THE ENERGY ABSORBED IN FRACTURE

Material.	Energy absorbed in Fracture.		Striking Velocity.	
	Kg.M.	Ft.-lbs.	Metres/Sec.	Feet/Sec.
Mild steel	4·03	29·1	2·7	9
	3·83	27·7	4·9	16
	2·77	20·0	6·1	20
	2·00	14·5	8·8	29
Iron	2·90	20·9	4·9	16
	2·80	20·2	6·6	21½
	1·47	10·6	13·1	43
Nickel chrome steel . .	4·00	28·9	2·7	9
	4·40	31·8	4·9	16
	5·44	39·3	13·1	43
Medium carbon steel . .	1·03	7·5	2·7	9
	1·05	7·6	6·1	20
	1·05	7·6	13·1	43

§ (108) SLOW BENDING TESTS ON NOTCHED BARS.—It has been shown by Philpot[1] and others that, if a notched bar test piece is broken slowly, the work expended is comparable with that measured by an impact machine (at moderately low striking velocities) and is equally as effective in distinguishing between certain brittle and tough materials.

As an average of a large number of tests Philpot found that the energy absorbed in the slow bending test is about 75 per cent of that given by an impact test (in two or three cases the energies obtained by the two methods are approximately equal), and he attributes this difference to energy dissipated in the pendulum testing machine. The striking velocity in Philpot's impact tests did not exceed 3·5 metres per second, and his conclusions only hold up to that value.

Philpot for his slow bending tests uses an ordinary Brinell machine, and measures the deflections of the specimen for different loads by means of a microscope. The results are plotted and the area of the curve is integrated in order to obtain the total energy absorbed. *The Humphrey Autographic Notched Bar Testing Machine*, made by the Foster Instrument Company of Letchworth, is devised to carry out this class of test rapidly and conveniently and in a manner giving the maximum information as to the material in a minimum time. It is illustrated diagrammatically in *Fig.* 132, and photographically in *Fig.* 133.

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[1] "Some Experiments on Notched Bars," *Inst. of Automobile Eng. Proc.*, April 1918.

[2] *Ibid.*

[3] "New Experiments on Shock Tests," *Iron and Steel Inst. Journ.* No. 2, 1917, p. 61.

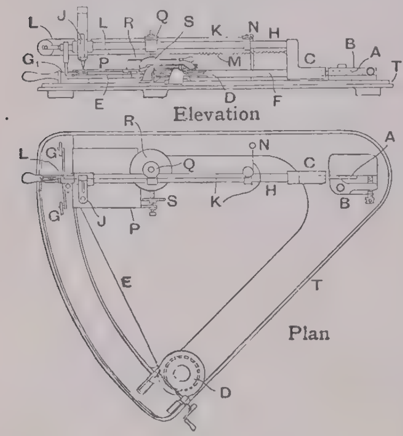

FIG. 132.

The test piece A is gripped in the vice B, so that the jaws of the vice are in line with the notch in the test piece. The socket C is attached on the projecting end of the test piece, and is a reasonably close fit thereon. A bending stress is applied to the test piece by means of the winch D and the wire E, which is attached to the outer end of the spring bar F. The inner end of the spring bar is firmly fixed to the socket C. The outer end of the spring bar is supported by two rollers G and G_1. The socket C carries also a rigid bar H.

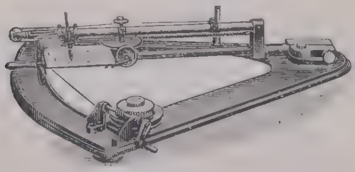

FIG. 133.

The deflection of the spring bar F is a measure of the bending moment applied to the test piece. Since the rigid bar H is not subjected to any bending moment it will move through the same angle as the socket C, and this movement will therefore be a measure of the bending of the test piece. The pen J slides upon the rigid bar H, its position being determined by the thin wires or cords K and L, which are kept taut by a light spring M. One end of the wire K is attached to a post N so that as the rigid bar H moves in consequence of the bending of the test piece, the wire K will draw the pen to the right. The point of the pen marks upon a calibrated chart P carried on the spring bar F, therefore, if the test piece offers no resistance whatever to bending force the pen would draw a horizontal line on the chart, the length of which is proportional to the angle through which the test piece is bent. As, however, the test piece does offer resistance to bending there will be a resulting movement between the spring bar F and the rigid bar H.

The pen, therefore, draws a diagram showing the relation between bending moment applied and angle of bending throughout the duration of the test. The total energy absorbed in the breaking of the test piece can be obtained from the area of this diagram.

The machine can also be fitted with an integrating device so that the energy absorbed can be read directly from the machine. This device is shown in *Figs.* 132-3. The wire K passes round a pulley Q. The pulley Q is attached to a spindle having at its lower end a friction disc R, so that the bending of the test piece which results in the movement of the pen to the right also results in a proportionate rotation of the friction disc R.

The under face of this disc rests upon the edge of the friction wheel, not shown. The spindle of the friction wheel is carried with the chart plate upon the spring bar F. When the bending moment is zero the friction wheel is at the centre of the friction disc, but as bending moment is applied, and there is relative movement between the spring bar F and the rigid bar H, the friction wheel travels towards the circumference of the friction disc, and is consequently rotated. The rotation will be proportional to the product of the bending moment applied and the angle of bending of the test piece, and this device will therefore integrate the total energy absorbed in breaking the test piece. Mounted upon the outer end of the spindle which carries the friction wheel is an index disc S which can be calibrated to read direct in ft.-lbs.

§ (109) TESTS ON SPECIMENS OF DIFFERENT SIZES.—A trustworthy relation has not been found between the energy to fracture in test pieces of different sizes; there is, however, evidence that it is probably different on different materials. This is shown from an examination of the results in Table 38, which gives the results of notched bar impact tests on two different steels with geometrically similar test pieces.

TABLE 38

TESTS ON GEOMETRICALLY SIMILAR SPECIMENS

Dimensions of Specimen in mm.*	Energy to Fracture.					
	Steel A.			Steel B.		
	Kg.M.	Kg.M./Cm.².	Kg.M./Cm.³.	Kg.M.	Kg.M./Cm.².	Kg.M./Cm.³.
10 × 10 × 53·33	4·77	5·96	0·89	1·28	1·60	0·24
30 × 30 × 160	60	8·33	0·42	36·3	5·04	0·25
63 × 63 × 336	296	9·32	0·22	197	6·20	0·15

* The notch for the 10 × 10 test piece was a 45° vee, 0·25 mm. root radius and 2 mm. deep. The notches for the larger test pieces were geometrically similar.

It will be noted that:

(1) The energy absorbed is not proportional to either the square or the cube of the dimensions of the test piece.

(2) With the 10 × 10-mm. test piece, steel A takes 3¾ times the energy that steel B does, but with the 63 × 63-mm. test pieces it is only 1½ times.

There is apparently a scale speed factor in the law of resistance of notched specimens to sudden shock which appears to be of extreme complexity.

§ (110) TESTS ON UNNOTCHED BARS.—With geometrically similar unnotched test bars, either beams or tensile test pieces, the energy absorbed in fracture for the same material is proportional to the volume of the test piece.

Hatt,[1] in 1904, as the result of some experiments in dynamic tension, came to the conclusion that, for steels, there is little difference in the total elongation and the unit work in fracture whether the fracture is brought about in 10 minutes or 0·01 seconds. Stanton and Bairstow [2] in 1908 and Delikhow [3] in 1909 (using a drop hammer type of machine) have also shown that the tensile impact test (*i.e.* dynamic tensile test) gives numerical values which agree with the results of the static tensile test.

Blount, Kirkaldy, and Sankey,[4] in the summary to their paper on steel testing methods, say that "the impact tensile test gives the ductility in the same terms as the static tensile test, namely, elongation and contraction of area, but always with higher numerical values. The breaking stress of the material can be inferred, but must be reduced by a factor in order to obtain the same numerical value as given by the static test; also it only gives the breaking stress. The energy absorbed per cubic inch does not vary greatly with the various types of steel; it is approximately 50 per cent more than that obtained by the static tensile test, and is also no definite criterion of the type of the steel; at any rate, of normal steels containing a small proportion of phosphorus. From the experiments referred to by Breuil [5] it would appear that steels containing an undue proportion of phosphorus give a much smaller energy per cubic inch with impact tensile tests."

§ (111) TESTS UNDER REPEATED BENDING IMPACT.—In 1908 Stanton [6] carried out extensive investigations on the method of testing by repeated impact, and devised a machine to reproduce stress conditions which are met with in certain machine parts in actual use, where the effect of alternating blows is produced on the material at positions where there is a rapid reduction of cross-section. The test bar, 0·500 inch diameter and 6·5 inches long, is supported on knife edges 4½ inches apart. The test piece has a groove turned round the centre of the span 0·05 inch deep, so that the effective diameter at the bottom of the groove is 0·400 inch. The groove is in the form of a vee, whose angle is 55° and root radius 0·01 inch. The test piece is fixed in the machine, where it receives blows at the rate of about 90 per minute from a hammer whose weight is 4·71 lbs., and of which the height of fall is adjustable.

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[1] "Tensile Impact Tests of Metals," *Amer. Soc. Test. Mat. Proc.*, 1904, iv. 282.

[2] "Resistance of Materials to Impact," *Inst. Mech. Eng. Proc.*, Nov. 20, 1908, and *Revue de Métall.*, March 1909.

[3] "Note on the Rupture of Normal Cylindrical Test Samples by Longitudinal Impact," *Inter. Assoc. Test. Mat. Proc.*, 1909.

[4] "Comparison of the Tensile, Impact Tensile, and Repeated Bending Methods of Testing Steel," *Inst. Mech. Eng. Proc.*, 1910, May 27.

[5] *Revue de mécanique*, 1908, p. 537.

[6] "Resistance of Materials to Impact," *Inst. Mech. Eng. Proc.*, Nov. 20, 1908, and *Revue de métall.*, March 1909.

Between each blow the bar is turned through 180° and the test is continued until failure of the test bar. The number of blows to cause failure is recorded by a counter.

A series of tests is usually made on each material with varying energies of blow, and, by plotting the energies of blow as ordinates and the number of blows to fracture as abscissæ, a curve is obtained from which useful information as to both the impact strength and fatigue strength of the material can be obtained. In Stanton's original paper it is clearly stated that when the number of blows is less than 500 the results approximate to those of the single blow impact test, and that, when the number of blows for fracture exceed 100,000, the results are in the order of the fatigue ranges of the materials as determined in a Wöhler test. During the war it has been a common practice to compare materials by the number of blows required to cause fracture when the height of the fall of the tup is kept constant. As the height of fall selected causes fracture after from 4000 to 6000 blows the results cannot be properly interpreted.

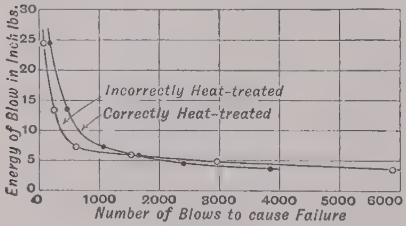

FIG. 134.

The results from tests on steel correctly and incorrectly heat-treated are given in Table 39 and plotted in *Fig.* 134. It will be noted that the incorrectly heat-treated bars are considerably weaker as regards notched bar impact, but slightly stronger under fatigue.

TABLE 39

REPEATED BENDING IMPACT TESTS ON BOILER PLATE CORRECTLY AND INCORRECTLY HEAT-TREATED

Energy of Blow. Inch-lbs.	Number of Blows to cause Failure.	
	Material correctly Heat-treated.	Material incorrectly Heat-treated.
3·53	3886	5871
4·71	2356	2913
5·89	1636	1450
7·35	1043	629
13·32	474	259
24·46	155	61

The test bar previously described is the one adopted by Stanton for his research, and has a notch very similar to the standard Izod notch, which is easily reproduced. A notch 0·05 inch wide and 0·05 inch deep with parallel sides and practically sharp corners has been sometimes used in conjunction with this test.

The Eden-Foster machine, manufactured by the Foster Instrument Company, Letchworth, is a close copy of Stanton's original machine and embodies its principal features.

The illustration (*Fig.* 135) gives a general idea of the external appearance of the machine. The main spindle, not shown in the illustration, projects through the side of the box casting, and is driven by an electric motor with suitable gear or worm reduction. About 0·1 horse-power is required to drive the apparatus.

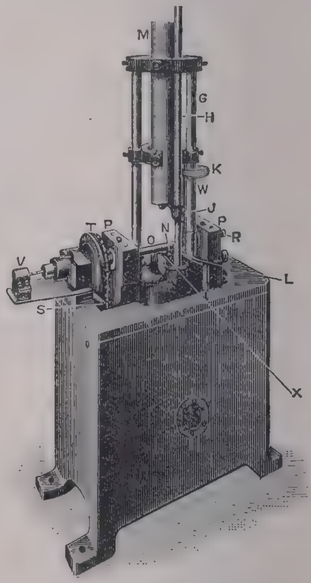

FIG. 135.

The main spindle carries a dog clutch driving a cam, which has a roller bearing on its upper surface and attached to the lower end of the rod H. Fixed on the rod H is an arm J which engages with the lower face of the hammer M; thus, when the rod H rises by rotation of the cam, the hammer M is lifted. The guides for the hammer consist of two sets of three point screws carried by two castings attached to the standard G and its fellow on the opposite side.

Mounted on the standard G is a sleeve W, free to rotate about the standard, but normally held in a fixed position by a spring L. Clamped on the sleeve W is an adjustable catch K. As soon as the arm J has lifted the hammer sufficiently, the spring L causes a partial rotation of the sleeve W, so that, when the arm J again descends, the hammer is held by the catch K. The further descent of the arm J brings its lower inclined face in engagement with the roller N, attached to the sleeve W, in such a manner that the catch K releases the hammer M, allowing it to fall upon the test piece O. The hammer M is furnished with a hardened tool-steel tup. Two hammers are provided to allow for a wide range of tests, the weights being 5 and 2 lbs. respectively. The height of drop, which depends upon the position of the adjustable catch K on the sleeve W, can be varied from 1 to $4\frac{1}{2}$ inches.

The test piece is carried by two hardened steel bushes in the plummer block PP. It is rotated through 180° between successive blows, the rotation being arranged so that it begins and ends entirely between the successive blows.

The revolutions of the test piece are recorded by a counter V, and the number of blows is found by multiplying the counter record by two. When the test piece breaks, it comes in contact with an arm X, and thereby trips the clutch and stops the machine.

The test piece is driven, through a universal joint, by the chain S, free-wheel and clutch T. One end of the chain is attached to the roller, at the lower end of the rod H, which bears on the cam, and the other end carries a suitably guided weight.

All the gear is fixed to the casting which forms the cover of the lower tank casting. The latter is partially filled with oil for lubrication of the surfaces.

§ (112) Effect of Temperature on the Mechanical Properties of Metals.—Many parts of machinery work at temperatures considerably above or below the normal, and it is essential for the purpose of economy and safety in design that the more important effects of temperature on the mechanical properties of materials should be well understood.

§ (113) Testing Apparatus.—The oldest method used for testing at temperatures above that of the air was that of heating the test piece in a furnace, transferring it to the testing machine, and conducting the test very quickly in air. The values obtained by that method were very unreliable, and in recent tests the samples have been immersed in a hot bath for the whole duration of the test.

The medium used in the hot bath can either be a liquid or gas, but it is essential that the heating medium should neither attack nor alloy with the test piece. Suitable materials for liquid baths are given in the following table:

Material.	Temperature used in ° C.
Water	Up to 100
Paraffin	Up to 200
Mineral oil	Up to 350
Nitrate of potassium and nitrate of sodium	350–600
Lead and tin	300–400

Air is nearly always used for the gas bath, but it causes oxidation of the test sample, and for special cases the test may be carried out *in vacuo*,[1] nitrogen,[2] or carbon dioxide.[3] Rudeloff[4] made use of baths of steam (up to 100° C.), naphthalene vapour (200° C.), and naphthylamine vapour (300° C.).

The heating of the bath is carried out in several ways:

(1) By gas jets arranged underneath the test piece, which is held horizontal. This method is adopted by Unwin[5] for a liquid bath, and by Charpy[6] for an air bath.

(2) By gas jets above a horizontal test piece—adopted by Le Chatelier.[7]

(3) By gas jets at the sides. This is used in conjunction with a vertical testing machine by Martens[8] and Bach.[9]

(4) Steam-heating coils are used for tests on 50-feet lengths of copper wire, up to temperatures of 60° C., carried out at the National Physical Laboratory.[10]

(5) Electric heating coils are used by Stribeck,[11] and Hopkinson and Rogers.[12]

For general convenience the last method is to be preferred. Two electric furnaces, successfully used at the National Physical Laboratory, are given in *Figs.* 136 and 137. They are both used in connection with a vertical testing machine.

A platinum furnace for temperatures up to 1200° C. is shown in *Fig.* 136. The heater (a) consists of a platinum strip, 0·75″ × 0·0007″, wound on a fire-clay cylinder (outside diameter 2·25 inches, thickness 0·187 inch, and $13\frac{7}{8}$ inches long), with a pitch of 1 inch. The ends of the strip are clamped in position on the cylinder by metal clips D. A steel case B (about 8 inches diameter and 1/16 inch thick) surrounds the heater, and the space between the two is packed with asbestos and magnesia fluff. The whole is clamped between two end plates C_1 and C_2, on one of which two insulated terminals are fixed; these are connected to the two ends of the platinum heating coil. The furnace, which is slung from the top shackle of the testing machine, takes a current of 15 amperes.

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[1] Rosenhain and Humphrey, *Iron and Steel Inst. J.*, 1913, No. 1.

[2] Hopkinson and Rogers, *Engineering*, 1905, ii. 331.

[3] Bengough and Hanson, *Inst. of Metals J.*, 1914.

[4] *Mitt. a. d. Kgl. Techn. Versuchsanstalten zu Berlin*, 1893, p. 292.

[5] "The Strength of Alloys at Different Temperatures," *Report of Brit. Assoc.*, 1899.

[6] *Bulletin de la Société d'encouragement*, 1896.

[7] *Baumaterialkunde*, 1901, p. 157.

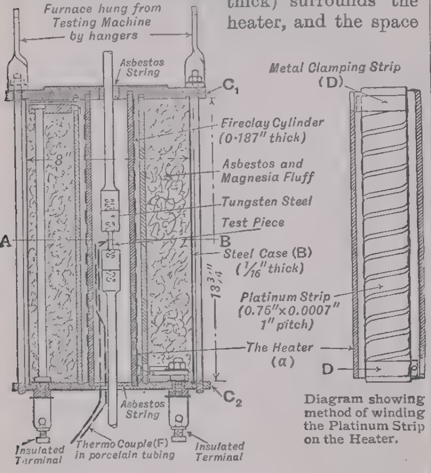
[8] *Mitt. a. d. Kgl. Techn. Versuchsanstalten zu Berlin*, 1890.

[9] *Zeits. Vereines Deutsch. Ing.*, 1900, p. 1749; 1901, pp. 168 and 1477; 1903, p. 1762; 1904, pp. 385 and 1300.

[10] Batson, "Hard-drawn Copper Wire," *Collected Researches, N.P.L.*, 1912, viii.

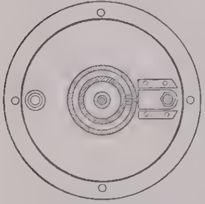
[11] *Zeits. Vereines Deutsch. Ing.*, 1903, p. 559.

[12] *Engineering*, 1905, ii. 331.


Furnace hung from Testing Machine by hangers
Asbestos String
C_1
Fireclay Cylinder (0·187" thick)
8"
Asbestos and Magnesia Fluff
Tungsten Steel
Test Piece
A
B
Steel Case (B) (1/16" thick)
13¾"
Platinum Strip (0.75"×0.0007" 1" pitch)
The Heater (a)
Asbestos String
C_2
Insulated Terminal
Thermo Couple(F) in porcelain tubing
Insulated Terminal
Metal Clamping Strip (D)
D

Sectional Elevation

Diagram showing method of winding the Platinum Strip on the Heater.



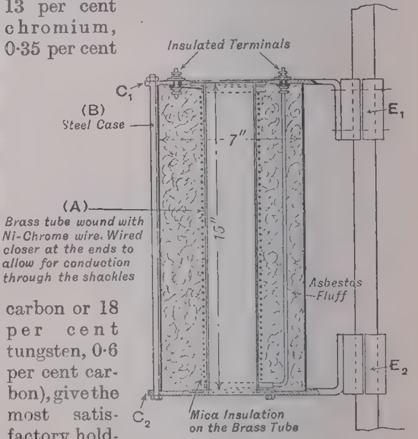
Sec ion on line A-B.

Fig. 136.

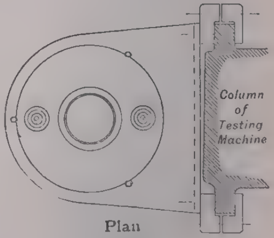
Fig. 137 shows the 600° C. furnace. With this furnace the heater is formed with nichrome wire wound on to a brass tube A (2⅜ inches diameter and 15 inches long). The tube is bound with mica, before winding the wire, in order to insulate it, and over the wire a binding of asbestos string is placed so as to keep the former in position when it expands on rise of temperature. The heater is surrounded by a steel case B, 7 inches diameter, and the space between the two is filled with asbestos fluff. Two steel plates C_1 and C_2 are bolted together, clamping the heater and outer case between them, and are arranged to connect the furnace to the frame of the testing machine at E_1 and E_2. The ends of the heating coil are connected to two insulated terminals on the top plate. No. 18 ni-chrome wire, having a carrying capacity of 11 amperes at 500° C., is used for the heater. The wire is coiled closer at the ends, in order to allow for the conduction of heat through the shackles and give uniform heating over the central 3 inches to 4 inches of the furnace.

In another furnace of the same type the conduction of heat through the shackles is equalised by having the main heating coil wound at constant pitch throughout its length, and supplementing this by two further coils, one wound at each end of the furnace. These coils are wound over the main coil, mica insulation being inserted, and are connected to separate circuits.

§ (114) Arrangement of Testing-machine Grips and Rate of Loading.—It is important that the size of test piece, form of grips, and length of furnace should be carefully proportioned to obtain uniform heating of the test piece. It is found that either high chromium low carbon or high tungsten high carbon steels (*i.e.* 13 per cent chromium, 0·35 per cent carbon or 18 per cent tungsten, 0·6 per cent carbon), give the most satisfactory holders for tests at over 600° C., as they combine a fairly high tensile strength with freedom from scaling. Of the two steels the former scales very little, while the latter, although it scales a little more, has a slightly higher tensile strength at high temperatures.


Insulated Terminals
C_1
(B) Steel Case
7"
E_1
(A) Brass tube wound with Ni-Chrome wire. Wired closer at the ends to allow for conduction through the shackles
15"
Asbestos Fluff
E_2
C_2
Mica Insulation on the Brass Tube

Sectional Elevation



Plan

Fig. 137.

Uniformity of speed in loading is an important condition for strictly comparable results. As the speed of loading increases larger strength values are obtained. Le Chatelier found that with hard-drawn copper wire tested at 250° C., the ultimate breaking strength increased from 11·4 to 21·6 tons per square inch as the duration of the test decreased from 1800 seconds to 20 seconds. Both Le Chatelier and Stribeck have shown that the elongation of copper diminishes with reduction in the speed of loading.

§ (115) TEMPERATURE MEASUREMENT FOR STRENGTH TESTS.—With liquid baths sufficient accuracy can usually be obtained by taking the temperature of the liquid, but it is essential that the test should not be carried out until uniformity of temperature is attained.

With an air-bath there is incomplete conduction of the heat between the surrounding air and the test piece, and it is necessary that the temperature of the sample be directly measured. This is most conveniently carried out by means of a thermo-electric pyrometer, which can either be placed in a hole drilled down the test piece, as was done by Bregowsky and Spring,[1] or laid close against the sample and bound to it with asbestos string.

The thermo-electric pyrometer consists of three distinct parts, viz. :

(i.) The thermo-couple[2] which receives the temperature.

(ii.) The indicator from which the temperatures are observed.

(iii.) Leads connecting the thermo-couple to the indicator.

The thermo-couple consists of two wires, A and B, of different composition. The ends of A and B are joined together and form a circuit. One junction is placed in a known temperature, usually zero, while the other is placed against the material whose temperature is to be ascertained. The difference in temperature between these ends sets up an electromotive force which is proportional to the temperature difference. This E.M.F. is measured and, by means of a calibration, gives the temperature. The cold junction is usually placed in ice in order to ensure a constant temperature. A thermos flask is a convenient receptacle for the ice and cold junction.

Various combinations of metals are used for thermo-couples, according to the maximum temperature for which they are to be used:

(*a*) Base metal thermo-couples.

Iron-constantan (wires) thermo-couples for temperatures up to 600° C. Iron-constantan (rods) thermo-couples for temperatures up to 800° C.

(*b*) Rare metal couples.

Platinum—platinum and 10 per cent rhodium for temperatures up to 1400° C.

Platinum—platinum and 10 per cent iridium for temperatures up to 1400° C.

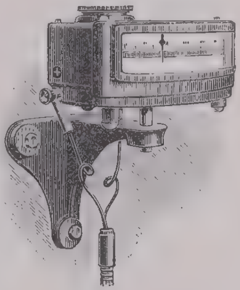


FIG. 138.

It is essential for temperatures above 800° C. that the couple should be protected by quartz or porcelain tubes. A thermo-couple placed in small porcelain tubing is seen at F (*Fig.* 136).

The indicator is usually a sensitive moving coil galvanometer of high internal electrical resistance, having the scale marked in temperature readings which depend upon the type of thermo-couple employed. It is generally an advantage to also have the instrument provided with a scale giving its readings in millivolts.

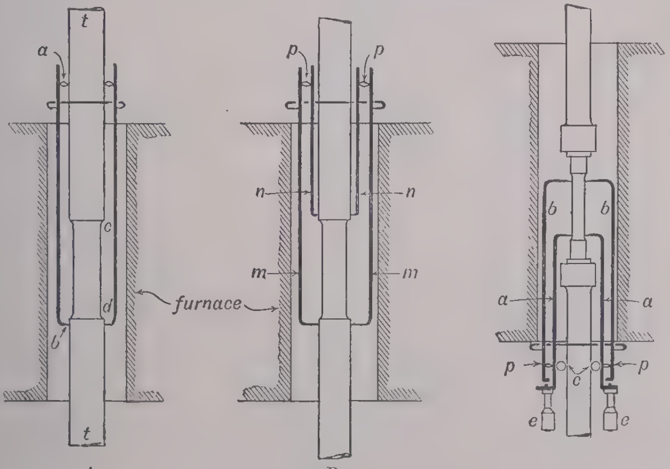


FIG. 139.

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[1] "Effect of High Temperatures on the Physical Properties of some Alloys," *International Association for Testing Materials*, 1912, vii./1.

[2] See "Thermocouples," § (2).

Complete thermo-electric pyrometer outfits are supplied by various firms of instrument-makers. *Fig.* 138 shows the type of indicator which the author has found most suitable for ordinary high temperature test work. It is supplied by the Cambridge & Paul Instrument Company.

§ (116) DETERMINATION OF ELASTIC LIMIT AND YIELD POINT AT HIGH TEMPERATURE.—In the majority of high temperature furnaces the test piece is not visible during the test, so that the yield point, if one exists, cannot be obtained by noting the extension by a pair of dividers, and the attachment of an extensometer is more difficult than with tests at air temperature.

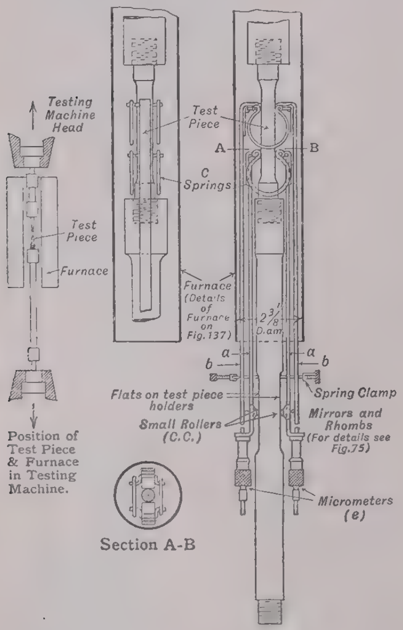
Testing Machine Head
Test Piece
Furnace
Position of Test Piece & Furnace in Testing Machine.
Test Piece
C Springs
A
B
Furnace (Details of Furnace on Fig. 137)
2 3/8 D.am
a
b
a
b
Flats on test piece holders
Spring Clamp
Small Rollers (C.C.)
Mirrors and Rhombs (For details see Fig. 75)
Micrometers (e)
Section A-B

FIG. 140.

In 1890, Martens[1] adapted his mirror extensometer for high temperature work. The test piece *tt* (*Fig.* 139 (A)) was turned down in the centre at *cd*. The extensometer clips were attached at *b* on the lower enlarged end, and were carried out of the furnace for attachment of the measuring rhombs at *a*. The extension was therefore measured on the length *ab*, and correction was made for the extension of the enlarged ends in order to obtain the extension of the gauge length *cd*.

Rudeloff,[2] in 1895, slightly modified Martens' arrangement, and used two pairs of clips, *m* and *n* (*Fig.* 139 (B)), with the mirror rhombs *p* between them. Here again the true elongation has to be obtained by calculation; the length of the thick part, however, is small, and as the whole of the measured length is in the furnace, the observations are uninfluenced by variations of temperature.

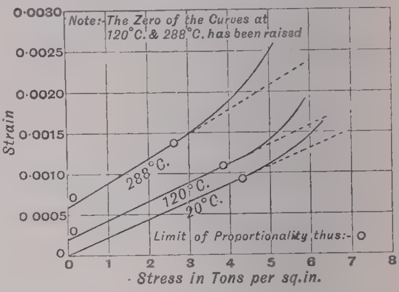

FIG. 141.

Lee and Crowther,[3] in 1914, adapted Rudeloff's arrangement of clips to a horizontal testing machine; they, however, attached them to the reduced part of the test piece.

In *Figs.* 139 (C) and 140 is shown the high temperature extensometer which is proving very useful at the National Physical Laboratory. This is a combination of the best features of both Rudeloff and Lee's extensometers, with the addition of two micrometers for measuring extensions beyond the elastic limit. The clips *aa* and *bb* are attached to the reduced part of the test piece by C springs and protrude from the furnace. The inner clips *aa* are guided on flats on the test piece holders by small rollers *c*. Mirrors and

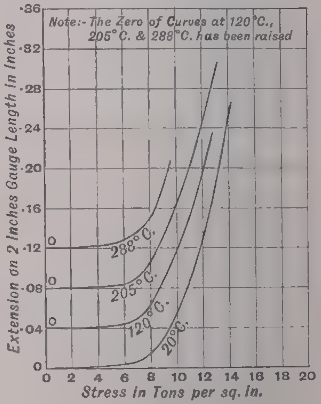

FIG. 142.

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[1] *Mitt. a. d. Kgl. Techn. Versuchsanstalten zu Berlin*, 1890.

[2] *Ibid.*, 1895, pp. 29 and 198.

[3] *Engineering*, 1914, xcviii. 487.

rhombs are placed between the clips, and the whole is clamped together by a special spring attached to notches on the outer clip.

Extension of the test piece causes relative movement between the clips *aa* and *bb*, and therefore rotation of the mirror rhombs *pp*, which is measured in the usual way by a telescope and scale. The relative movement of the clips is also measured by two micrometers *ee* attached to the inner clips and working against the outer ones.

Curves for tests on phosphor bronze with this instrument are shown in *Figs.* 141 and 142. *Fig.* 141 gives the elastic line and limit of proportionality at temperatures of 20, 120, and 288° C., plotted from results with the rhombs and mirrors, and *Fig.* 142 shows the full stress strain curves at temperatures of 20, 120, 205, and 288° C., obtained by using the micrometers.

VII. Results of Tests at Varying Temperatures

§ (117) Results of Tensile Tests. (i.) *Iron and Carbon Steels.*—Strength tests at high temperatures have been carried out since the early part of the nineteenth century.

In 1820 Tremery and Proirier-Saint-Brice[1] showed that, although the tensile strength of hammered wrought iron was 27·6 tons per sq. inch at air temperature, it fell to 5·0 tons per sq. inch at red heat. Fairbairn, in 1856,[2] showed that iron possesses a maximum resistance to fracture at 250° C. and this has frequently been verified for iron and steel. It has also been shown that this maximum resistance is preceded by a minimum, which occurs at about 120° C.

An extensive series of tests were carried out on iron and steel at the Watertown Arsenal[3] in 1888, from which the following conclusions can be drawn:

(1) The change in the ultimate strength is very small up to about 150° C. There is evidence of a slight reduction at from 100 to 120° C., and after 150° C. there is an increase until, at 250° C., a maximum is reached, which is from 10 to 15 per cent higher than the normal. From 250 to 950° C. there is a continuous fall in strength, which at the latter temperature gives a reduction in strength of 80 to 85 per cent of the normal.

(2) The elastic limit decreases with increasing temperature. Up to 350° C. this decrease is directly proportional to increase in temperature, so that at 350° C. the elastic limit is about 70 to 75 per cent of that at normal temperature.

(3) With cast iron the ultimate strength is approximately constant up to 500° C., after which it falls until, at 950° C., it is about 20 per cent of the normal value.

Martens,[4] in 1890, published an elaborate series of tests on the strength of steel at temperatures up to 600° C., and included a determination of the limit of proportionality. He used a paraffin bath for temperatures up to 200° C., a bath of lead and tin for temperatures up to 600° C., and for low temperatures a mixture of three parts by weight of ice and one part of salt. Temperatures up to 400° C. were measured with a mercurial thermometer, and for higher temperatures an air thermometer was employed.

Tests by Lee and Crowther show that the modulus of elasticity of mild steel varies from 29.6×10^6 lbs. per square inch at normal temperature to 12.8×10^6 at 600° C. They also found that the ultimate stress was a maximum at 250° C., but that the stress obtained by dividing the breaking-load by the area at fracture was a minimum at that temperature. The figures given in Table 40 have been taken from the plotted results.

Table 40

Tensile Tests on Mild Steel at Varying Temperatures

Temperature, ° C.	Modulus of Elasticity. Lbs./Sq. In.	Yield Stress. Tons/Sq. In.	Ultimate Stress. Tons/Sq. In.	Reduction of Area. Per cent.
20	29.6×10^6	21·4	28·7	57
110	29.3×10^6	19·4	29·2	..
240	28.3×10^6	16·6	39·0	65
310	27.6×10^6	16·3	37·9	62
445	23.3×10^6	15·4	25·7	32
520	17.7×10^6	12·5	17·3	15
570	14.8×10^6	9·7	12·1	4
600	12.8×10^6	..	..	..

(ii.) *Alloy Steels.*—The behaviour of special alloy steels at high temperature has been studied by Aitchison.[5] A synopsis of results of tensile tests, most of which were made at the National Physical Laboratory, is given in Table 41; these show that the tungsten steels with high percentages of carbon (*i.e.* about 0·6 per cent) have the greatest tensile strength at high temperatures. This strength is nearly equalled by the high chromium steels also containing a high percentage of carbon.

With high chromium and high tungsten steels the percentage of carbon seems to play an important part in their strength at high temperatures. Thus with the 11 per cent chromium steel, reducing the carbon content from 1·0 to 0·4 per cent reduces the tensile strength at 900° C. from 7·5 to 4·8 tons per square inch, while a reduction of the chromium content from 11 per cent to 6·3 per cent, with the carbon content at 1 per cent, has no effect on the tensile strength at that temperature.

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[1] *Annales des Mines*, ii. 513.

[2] *Report of the British Association*, 1856, p. 405.

[3] Johnson's *Materials of Construction*, 1918 edition. p. 759.

[4] *Mitt. a. d. Kgl. Techn. Versuchsanstalten zu Berlin*, 1890.

[5] "Valve Failures and Valve Steels in Internal Combustion Engines," *Inst. Auto. Eng. Proc.*, 1919.

When the temperature exceeds 600° C. the strength of alloy steels falls off rapidly; this is particularly noticeable with nickel chrome steels and nickel steels, between which there is very little difference at high temperature.

TABLE 41

Steel.	Ultimate Strength in Tons per Sq. In. at Temperature ° C.							
	15.	650.	700.	750.	800.	850.	900.	950.
17 per cent tungsten, 0·65 per cent carbon	58	25	17·7	13	10·3	11·1	8·8	6·6
16 " " 0·45 " "	45	. .	15·5	. .	. .	. .	6·3	. .
14 " " 0·70 " "	51	20	15·9	10	7·0	10·0	7·6	. .
12 " " 0·45 " "	48	. .	14·7	. .	7·1	9·6	7·4	. .
11 " chromium, 0·40 " " *	43	14	12·1	8	6·6	. .	4·8	4·0
11 " chromium, 1·0 " "	67	. .	15·1	10	8·5	9·6	7·5	6·5
6·3 " " 0·5 " "	66	. .	17·0	. .	. .	. .	7·0	. .
6·3 " " 1·1 " "	78	. .	18·5	. .	. .	. .	7·5	. .
3 " nickel, 0·3 " "	50	. .	9·4	. .	. .	. .	3·9	. .
3 " " 0·6 " "	. .	. .	11·5	. .	. .	. .	4·5	. .
3 " chromium, 0·35 " "	. .	. .	8·5	. .	. .	. .	4·2	. .
3 " " 0·7 " "	. .	. .	11·7	. .	. .	. .	4·9	. .
Ordinary nickel chromium	55	16	10·5	8	7·0	5·1	4·5	3·0

* Stainless Steel.

(iii.) *Alloys.*—The use of fittings under superheated steam involves temperatures up to 400° C., and the study of brasses, bronzes, etc., up to these temperatures has been undertaken by numerous investigators. Unwin,[1] in 1889, communicated a report to the British Association on tensile tests of delta metal, gunmetal, muntz metal, copper, brass, phosphor bronze, and aluminium bronze at temperatures up to 350° C.

Rudeloff,[2] Bregowsky and Spring,[3] Dewrance,[4] and Law [5] have also carried out a great deal of work on this subject.

The tensile strength at high temperatures of aluminium copper, aluminium copper manganese, and aluminium zinc alloys are given in the reports of the Alloy Research Committee.[6]

The variations, with temperature, in the properties of alloys are fairly regular; there is a fall in the tensile strength as the temperature rises, with change of curvature and sometimes humps in the temperature-ultimate stress curve. Rolled monel metal, an alloy of 70 per cent nickel with 30 per cent copper, exhibits high strength and ductility at temperatures up to 550° C.

Tests at the National Physical Laboratory [7] on hard-drawn copper and bronze wire show that between −20° C. and +60° C. a rise of 1° C. corresponds to a decrease of 0·1 per cent of the breaking-load and an increase of 0·3 per cent in the total extension at 15° C. There is also a slight alteration of the modulus of elasticity with temperatures met with under climatic conditions.

§ (118) TENSILE TESTS AT LOW TEMPERATURES.—These tests are usually carried out by surrounding the test piece with a bath containing a cooling substance, such as:

Ice for temperatures down to . . .	0° C.
Ice and freezing salt for temperatures down to	− 20° C.
Frozen mercury for temperatures down to	− 40° C.
Solid carbon dioxide powder for temperatures down to	− 80° C.
Liquid air for temperatures down to .	− 185° C.

It is essential that the bath should be surrounded with a casing containing down packing or asbestos fluff in order to obtain a uniform temperature.

Rudeloff, in 1895, found that iron and steel gave increased yield stress and ultimate stress as the temperature was lowered, while the extension was generally decreased by cooling. Tests made at Watertown Arsenal [8] gave an increase of about 50 per cent in the yield stress and 35 per cent in the ultimate stress over the normal values, when tested at −185° C., but the extension decreased by 63 per cent.

Cast aluminium light alloys show an increase over the normal in ultimate strength of 9 per cent at −80° C., and 27 per cent at −185° C.

§ (119) NOTCHED BAR TESTS AT VARYING TEMPERATURES.—Charpy [9] has shown that, with steel, the work absorbed in the fracture of a notched bar test piece rises as the temperature is increased and reaches a maximum at about 150° C.; it then decreases and passes through a minimum between 400° C. and 500° C., and rises again until red heat is reached.

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[1] *Report of Brit. Assoc.*, 1889.

[2] *Inter. Assoc. Test. Mat. Proc.*, 1909, vi./1.

[3] "Effect of High Temperatures on the Physical Properties of some Alloys," *International Association for Testing Materials*, 1912, vii./1.

[4] *Inst. of Metals J.*, 1914.

[5] *Iron and Steel Inst. J.*, May 1918.

[6] *Inst. Mech. Eng. Proc.*, 8th Report, 1907, p. 57; 9th Report, 1910, p. 119; 10th Report, 1912, April.

[7] Batson, "Hard-drawn Copper Wire," *Collected Researches N.P.L.*, viii. 1912.

[8] *Engr. Rec.*, liv. 65.

[9] *Inter. Assoc. Test. Mat. Proc.*, 1906.

This work was confirmed by Guillet and Revillon,[1] who gave the temperatures of maximum and minimum resistance to shock as 200° C. and 475° C. They say that there is no particular fragility in a steel broken at blue heat (300 to 325° C.).

Guillet and Revillon carried out their tests on a Guillery 60 Kg.M. machine (see § (100)). The test pieces were heated in an electric furnace to slightly above the temperature required for the test, they were then placed on the anvil and the temperature noted at the time of fracture. The temperature was determined by the use of a thermo-couple inserted in a small hole drilled in the specimen and penetrating up to about 3 mm. from the cross-section to be fractured.

The ends of the test piece were covered with asbestos to prevent the cooling of the extremities when in contact with the anvil. Tests at as close to 100° C. as possible were obtained by heating in boiling water.

Aitchison has shown that with alloy steels the notched bar tests at high temperatures are a reflection of the tensile strength of the steels, being more or less inversely proportional to that property. In all cases the steels gave higher values at elevated temperatures than they did under normal conditions.

§ (120) BALL HARDNESS TESTS AT VARYING TEMPERATURES.—Felix Robin has shown that the curves connecting hardness and temperature are of the same general form as the tensile strength-temperature curves.

Aitchison [2] gives results of hardness tests at high temperatures on tungsten steels of various compositions, ranging from 10 per cent to 18 per cent tungsten, and from 0·2 per cent to 0·7 per cent carbon. Both investigators find a similar regular drop in the hardness as the temperature increases.

In § (88) it is pointed out that a dynamic hardness test is extremely useful for tests at high temperatures. Aitchison notes that his hardness tests were carried out by C. A. Edwards, so that it is probable that the results were obtained by the dynamic method described by Edwards in his paper read before the Institution of Mechanical Engineers.[3]

Hadfield [4] finds that the hardness of steel at the temperature of liquid air ($-185°$ C.) is double that given at normal temperature.

§ (121) ALTERNATING STRESS TESTS AT VARYING TEMPERATURES. — Very little experimental work has been carried out on the effect of high temperatures on the strength of materials under alternating stresses. Unwin [5] conducted some tests at a temperature of 200° C. and found that "the hot bars stood variations of stress rather better than the cold ones."

TABLE 42

NOTCHED BAR TESTS AT LOW TEMPERATURES

Tensile Test of Material at 17½° C.				Charpy Notched Bar Impact Test, on 5 × 5 × 27-mm. Specimens with 45° Vee Notch, 1 mm. deep, and 0·125 mm. Root Radius.		
				Energy absorbed in Fracture in Kg.M.		
Yield Stress. Tons/Sq. In.	Ultimate Stress. Tons/Sq. In.	Extension. Per cent.	Reduction of Area. Per cent.	17½° C.	−40° C.	−80° C.
14	26·1	42·0	61	2·10	0·13	0·10
31	53·1	22·5	44	0·26	0·16	0·19
				Results are the average of five tests.		

Some tests on carbon steels at low temperatures, carried out at the National Physical Laboratory, are given in Table 42.

Tests at the low temperatures on various alloy steels show that, generally, there is an appreciable drop in the energy absorbed in fracture between 17½° C. and −40° C., but that between −40 and −80° C. the drop, if any, is small, and in many cases there is a distinct rise in the value.

Some tests carried out by the author on steel, having a primitive elastic of 21 tons per square inch, ultimate stress of 46 tons per square inch, per cent extension of 26 and per cent reduction of area at fracture of 52, gave the results given in Table 43.

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[1] *Inter. Assoc. Test. Mat. Proc.*, 1909, iii./4.

[2] "Valve Failures and Valve Steels in Internal Combustion Engines," *Inst. Auto. Eng. Proc.*, 1919.

[3] Edwards and Willis, "A Law governing the Resistance to Penetration of Metals when tested by Impact with a 10-mm. Steel Ball," *Inst. Mech. Eng. J.*, 1918.

[4] *Iron and Steel Inst. J.*, 1905.

[5] "Testing of Materials of Construction," 1910 edition, p. 385.

TABLE 43

ALTERNATING STRESS TESTS ON STEEL AT 20° C. AND 250° C.

Range of Stress. Tons/Sq. In.	Alternations before Fracture.	Temperature. ° C.	Remarks.	Limiting Range of Stress. Tons/Sq. In.
+18·2 to −18·2	2,429,400	20	Unbroken	+21 to −21
+19·2 to −19·2	3,299,500	"	"	
+20·9 to −20·9	3,268,400	"	"	
+21·7 to −21·7	1,121,000	"	Broken	
+23·7 to −23·7	3,240,000	"	"	
+24·6 to −24·6	1,996,900	"	"	
+16·5 to −16·5	2,069,000	250	Unbroken	+16·6 to −16·6
+17·0 to −17·0	2,115,600	"	Broken	
+17·2 to −17·2	313,050	"	"	
+18·8 to −18·8	383,750	"	"	

The tests were made in a machine of the Wohler type, the test piece running in an oil-bath heated electrically, and show a reduction of 20 per cent in the limiting range of stress by raising the temperature from 20 to 250° C. They were carried out at a speed of 2000 alternations per minute.

Tests on some rolled aluminium light alloys, by the same method, gave the results in Table 44.

TABLE 44

ALTERNATING STRESS TESTS ON ROLLED ALUMINIUM ALLOYS AT 20° C. AND 150° C.

No.	Ultimate Stress in Static Test. Tons/Sq. In.			Estimated Limiting Range of Alternating Stress in Fatigue. Tons/Sq. In.	
	20° C.	150° C.	200° C.	20° C.	150° C.
A	21·9	. .	. .	+ 6·5 to − 6·5	+5·5 to −5·5
B	26·6	21·1	19·5	+10·5 to −10·5	+7·9 to −7·9
C	28·0	21·5	19·8	+10·2 to −10·2	+5·0 to −5·0
D	24·3	21·4	19·5	+10·2 to −10·2	+8·4 to −8·4
E	22·7	20·3	15·4	+ 9·0 to − 9·0	+6·5 to −6·5

These results show that, whereas the average reduction in static strength by alteration of temperature from 20 to 150° C. is about 17 per cent, the corresponding reduction in the fatigue range is about 30 per cent.

The average limiting elastic range (at 20° C.) for the five alloys (A to E) is about 75 per cent of the static ultimate stress. With steel this value is of the order of 90 per cent.

VIII. TESTS ON MATERIALS OF CONSTRUCTION

§ (122) TESTING OF TIMBER.—Examination of the cross-section of a sawn log of structural timber shows a central pith surrounded by concentric rings which are in turn encircled by the outer bark. Each ring represents the growth of one year in the life of the tree. These annual rings can be roughly divided into two sections, heartwood and sapwood, of which the inner or central portion (heartwood) is usually the stronger in mature trees.

Each annual ring itself consists of two parts, viz. the inner or spring wood and the outer or summer wood, of which the latter is the denser, harder, and stronger. The strength of a piece of timber can therefore be roughly gauged by the proportion of summer wood in the annual rings.

The width of the annual rings indicates the rate of growth of the tree—rapid growth being shown by wide rings—they therefore give useful information as to the uniformity of growth. The rings are usually widest at the centre and become closer near to the bark. Their width and distribution often vary in different trees of the same kind, in various parts of the same tree, and in different parts of the same cross-section.

Timber is usually divided into two classes—hardwood and softwood. The former is derived principally from broad-leaved deciduous trees, while the latter is obtained from evergreen conifers (needle-leaved trees). Exceptions to this classification are yew and long leaf pine, which are hardwoods although coming from conifers, while horse-chestnut, poplar and basswood (broad-leaved trees) are softwoods.

Timber is used by engineers principally for structural purposes, and a knowledge of its behaviour under various conditions of stress is necessary if it is to be used to the best advantage.

(i.) *Density and Specific Gravity.*—The density of timber is largely affected by the percentage of moisture that it contains, by its rate of growth, position in the tree, and percentage of summer wood.

As the density is closely related to the mechanical properties of timber, its determination is of importance. It is usually ascertained by measuring and weighing carefully prepared pieces which, for preference, are made in the form of cubes or prisms. If b, t, and h are the three dimensions of the block in centimetres and W is the weight in grammes,

then the specific gravity $= W/(b \times t \times h)$

and weight per cubic feet $= Wg/(b \times t \times h)$,

where g is the weight of a cubic foot of water in lbs. (62·4).

For comparative results the moisture content must be the same; the density is therefore usually carried out on dried samples. For this purpose they are dried in an oven at 100° C. until of constant weight, and the density calculated from this weight in conjunction with the dimensions (dry).

(ii.) *Percentage of Moisture.*—All the mechanical properties of timber are affected by the moisture content. The strength becomes less as the percentage of moisture increases. A determination of the percentage of moisture is always made from each test piece, and results which do not include this information are valueless.

The determination of the moisture content is made on pieces cut from each sample; in the case of tension, shear, and cleavage tests, these consist of pieces split off adjacent to the failure. From other pieces a disc or cube is taken from as near to the point of fracture as possible.

Discs, etc., are weighed immediately that they are cut; they are then dried in an oven with free circulation, and kept at a constant temperature of 100° C. until they cease to lose weight by further drying. They are then reweighed. The moisture contained is expressed as a percentage of the dry weight of the timber.

If w_w = weight before drying,

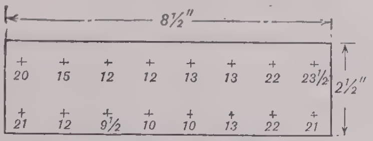
w_d = weight after drying,

percentage of moisture $= 100 \times (w_w - w_d) \div w_d$.

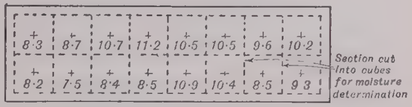
In order to hasten the drying process, the test piece is sometimes cut into small matchsticks or shavings bored from the sample used. If this is done, 12 hours' drying is usually sufficient. Drying at 100° C. will eliminate all except about 2 per cent of the moisture, and this cannot be removed without igniting the sample.

Sections of beams are usually cut into small pieces for a moisture determination of each piece, in order to examine the distribution of moisture throughout the section. The results from such a determination on pieces of spruce and fir are given in *Figs.* 143 and 144.

(iii.) *Rings per inch, per cent of Sapwood, and per cent of Summer Wood.*—These estimations are usually made on the piece cut for percentage of moisture determination. A line, one inch long, is drawn on the cross-section of the disc in a radial direction and passing through a region of average development.

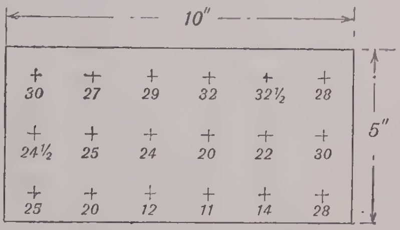


Arrangement and number of annual rings



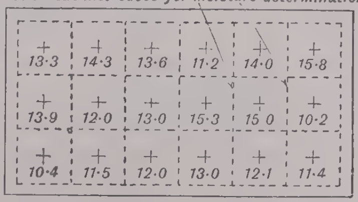
Variation of moisture over the section
Average Moisture = 9·5%

FIG. 143.



Arrangement and number of annual rings.

Section cut into cubes for moisture determination



Variation of moisture over the section
Average Moisture = 12·9%

FIG. 144.

The width of summer wood crossed by this line is obtained by spacing off accumulatively, on a pair of dividers, the width of the summer wood bands for each annual ring in succession. The final distance between the points of the dividers, in hundredths of an inch, is equal to the percentage of summer wood.

The number of rings per inch is obtained by noting the number of annual rings crossed by the measured inch.

The percentage of sapwood is estimated by measuring the amount of the cross-sectional area in which sapwood appears and expressing this as a percentage of the whole area.

§ (123) DRYING TIMBER TEST PIECES.—In order to obtain reliable data from the various tests on timber, the information is usually referred to some standard percentage of moisture. It is considered that timber, if thoroughly dried, will reabsorb 12 to 15 per cent of moisture from the atmosphere. Some experimenters therefore use 12 per cent, but the majority adopt 15 per cent as the standard for comparison. Tests are usually carried out at percentages of moisture above, below, and approximately at the standard, and for this purpose it is necessary to dry samples from the higher to the lower figure.

Two methods of drying are adopted, viz. air drying (or seasoning) and kiln drying, but owing to the long time required for successful air drying, kiln drying is usually resorted to for test work.

An experimental hot-air-drying oven or kiln, which has been successfully used, is shown in *Fig.* 145. The samples are placed on a tray A in a circular vessel B which is heated by a gas burner C, and a current of air is driven through this vessel by a rotary fan D. A complete record of the temperature, which is never allowed to exceed 80° C., is obtained by a recorder which is placed alongside the test samples.

The test pieces are prepared from the samples after drying, and for this purpose the latter are usually left at least ½ in. larger on each of the cross-sectional dimensions and also considerably longer. Before drying in the kiln the percentage moisture is obtained from pieces cut from each end of the sample, and from this information the reduction in weight which is required by the kiln drying in order to reduce the sample to the lower moisture content is calculated. The drying of the test sample is continued until the reduction in weight is obtained.

§ (124) SIZE OF TEST PIECES FOR TESTS ON TIMBER.—Timber tests are divided into two classes :

(1) "Scientific" tests of *small specimens* free from knots or other blemishes and of uniform moisture content.

(2) Tests on *full-size members* of structures—struts, beams, etc., which are not homogeneous and contain serious defects.

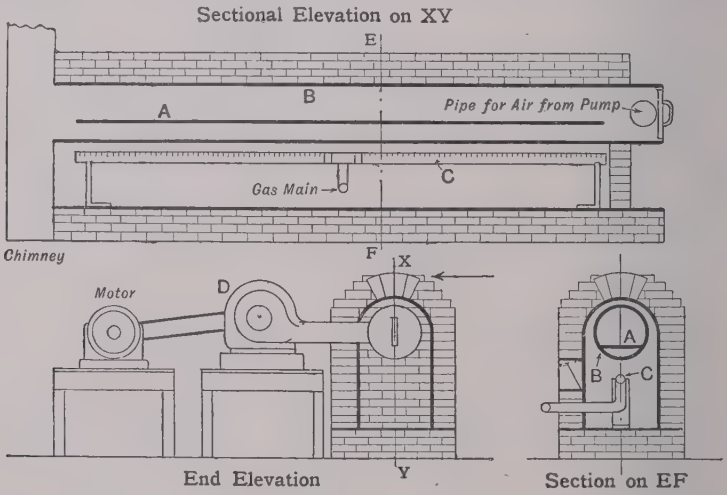


FIG. 145.

Tests are necessary under both conditions. Large sections have the same strength per unit area as small ones when both are of the same proportions and similarly free from defects; but under practical conditions it is found that this law of similarity is not realised, and that tests on small pieces give values for the strength of timber in excess of the strength of large pieces.

§ (125) TENSILE TESTS OF TIMBER.—All comprehensive investigations in regard to the strength of timber include a determination of its tensile strength as it is of scientific importance. The resistance of timber to tensile forces parallel to the grain is greater than under any other kind of loading. In timber structures the tension members are joined at their ends to other parts of the structure, and the weakest point is the joint which is subjected to shear.

It follows that the tensile strength does not govern the strength of the member, and therefore, from a practical point of view, it is of secondary importance.

The heads and shoulders of tensile test pieces have to be very greatly enlarged, otherwise failure takes place either by shearing of the ends along the grain or by crushing of the ends across the grain. The type of test piece adopted is shown in *Fig.* 146. The section to fracture is naturally small in comparison to the ends. Test pieces similar to that shown in the figure, but having the reduced diameter 1·25 inches instead of 0·564 inches, are sometimes used.

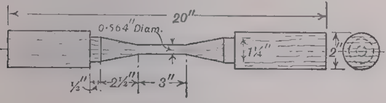


FIG. 146.

§ (126) COMPRESSION TESTS ON TIMBER.—Compression tests along the grain give reliable indications of the quality of samples of timber. Failure usually takes place by shearing along planes inclined about 60 to 75° with the axis, but sometimes by longitudinal splitting. The test pieces (*Fig.* 147) are generally prisms 2 × 2 × 4″ or 3 × 3 × 6″.

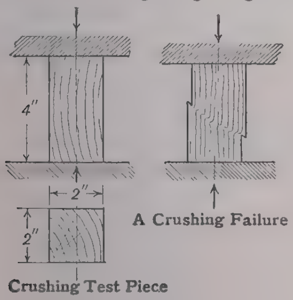


FIG. 147.

For investigatory work it is usual to determine stress-strain diagrams with the aid of a compressometer (§ (34)). These give an initial portion of the curve which is approximately straight, and it is customary from this curve to obtain the modulus of elasticity, and compressive stress at the limit of proportionality. Typical results are given in Table 45.

TABLE 45

COMPRESSION (CRUSHING) TESTS ON TIMBER ALONG THE GRAIN

Test Pieces 2 × 2 × 4 inches

Timber.	Rings per Inch.	Moisture. Per cent.	Limit of Proportionality. Lbs./Sq. In.	Crushing Stress. Lbs./Sq. In.	Modulus of Elasticity. Lbs./Sq. In.	Specific Gravity.
Oak (English)	7	24	2770	5300	$0{\cdot}99 \times 10^6$	0·76
	9	11	2530	5410	$0{\cdot}92 \times 10^6$	0·83
Fir (Russian)	18	14	6020	8940	$2{\cdot}60 \times 10^6$	0·74
Spruce (Christiania)	16	12	3200	4600	$1{\cdot}08 \times 10^6$	0·36
Fir (Columbian)	24	12	7200	9990	$3{\cdot}10 \times 10^6$	0·78

§ (127) SHEARING STRENGTH OF TIMBER.—It is generally acknowledged by all authorities that accurate tests of the shearing strength of timber are difficult to make. Two methods have been adopted:

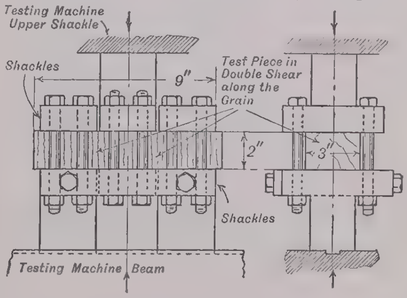

Testing Machine Upper Shackle
Shackles
9″
Test Piece in Double Shear along the Grain
2″
3″
Shackles
Testing Machine Beam

FIG. 148.

(1) With the test piece in double shear as indicated in *Fig.* 148.

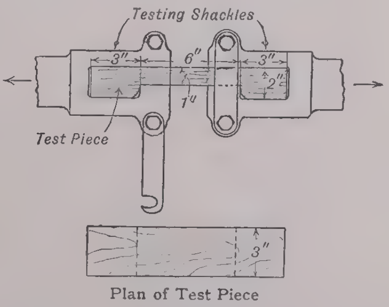


FIG. 149.

(2) With the timber under single shear (*Fig.* 149). This method has been used with success by Warren.[1]

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[1] *Report of the Department of Forestry N.S.W., Australia*, 1911, on "N.S.W. Hardwood Timbers."

The resistance of various woods to shear across the grain has been determined by Trautwine,[1] whose test pieces were cylindrical pins $\frac{5}{8}$ in. diameter subjected to double shear.

§ (128) Bending Tests.—Bending tests are probably the commonest for timber, because of the great use of wood for rafters, joists, beams, and other parts of structures subjected to bending. Long pieces of fairly large section can be tested without testing machines of big proportions.

Test pieces are usually rectangular, are supported at the ends, and loaded either in the centre or at two points as indicated in *Figs.* 40 and 41 (§ (12)). The latter method (called four-point loading) is now generally preferred, as the maximum bending moment is spread over half of the span and is not limited to the point directly under the application of the load as it is in the central loading method.

A sketch showing the direction of the grain at the ends, and the number of the annual rings, should accompany each test, as well as an indication of the position and mode of fracture. The position and character of any defects such as knots should also be specially noted. Failure may take place by tension, compression, or longitudinal shear along the grain.

For the determination of the limit of proportionality and coefficient of elasticity it is necessary to measure the deflection for known increments of the load. Instruments for doing this are described in § (56).

The strength factor usually measured is the modulus of rupture or coefficient of bending strength. Although a fictitious value, it is, nevertheless, a valuable index of the quality of the timber.

The various values obtained from the test data are calculated in the following way:

If P = Total load at the limit of proportionality,
B = Total load at fracture,
l = Span between the supports,
b = Width of cross-section of the test piece,
h = Depth of the cross-section of the test piece,
d = Measured deflection at the limit of proportionality.

(i.) *For central loading* (deflection measured on length $= l$).

Greatest shearing stress $= 3B/4bh$.
Stress at the proportional limit $= 3Pl/2bh^2$.
Modulus of rupture $= 3Bl/2bh^2$.
Modulus of elasticity $= Pl^3/4bh^3d$.

(ii.) *For four-point loading* (where the distance between the loading points $= L$, and the deflection is measured on length l_1 in the section under uniform bending moment, and l_1 is less than L. B and P are the sum of the two loads at the loading points).

Greatest shearing stress $= 3B/4bh$.
Stress at proportional limit $= 3P(l-L)/2bd^2$.
Modulus of rupture $= 3B(l-L)/2bd^2$.
Modulus of elasticity $= 3P(l-L)l_1^2/8bh^3d$.

Fig. 150 shows typical load deflection diagrams for Christiania spruce and Petersburg fir, while *Fig.* 151 gives the load-extension diagrams in tension and compression for the same materials.

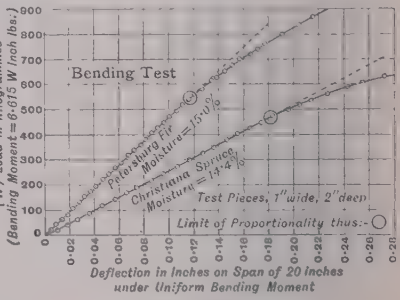

Fig. 150.

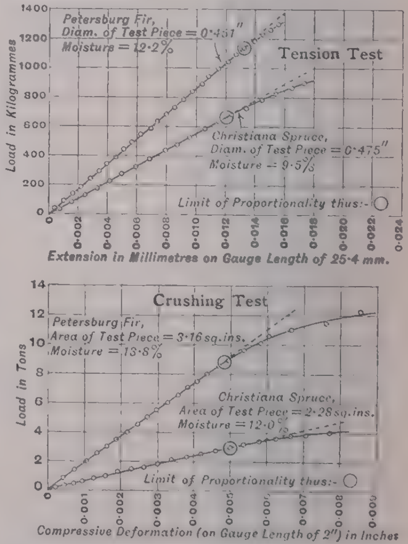

Fig. 151.

§ (129) Toughness Tests.—Toughness is obtained by means of a notched bar impact test in which the energy absorbed by the specimen is measured by the loss of energy in a swinging tup which breaks the material by a single blow. *Fig.* 152 shows the manner in which an Izod testing machine (see § (100)) is utilised to carry out this test, and the form of test piece is given in *Fig.* 153. The test piece is notched with the annular rings as indicated in the sketch, and the top of it, where the tup strikes, is protected by a steel plate.

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[1] *Franklin Inst. Journal*, cix. 106.

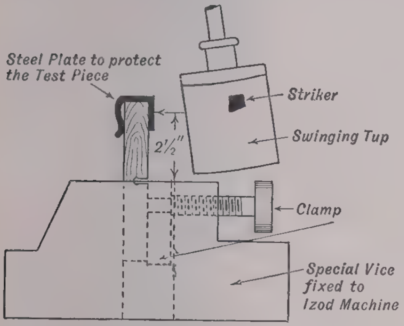
Steel Plate to protect the Test Piece
Striker
Swinging Tup
2½"
Clamp
Special Vice fixed to Izod Machine

FIG. 152.

Impact Bending Test.—In this test the specimen (2 × 2 × 30″) is supported at the ends on a span of 28 inches. The load is applied by dropping a hammer upon the specimen at mid-span, from an initial height of one inch. For succeeding drops the height is increased by 1-inch increments up to 10 inches, and thereafter drops are made at successive increments of 2 inches, until failure. Hammers of 50, 100, and 250 lbs. are used.

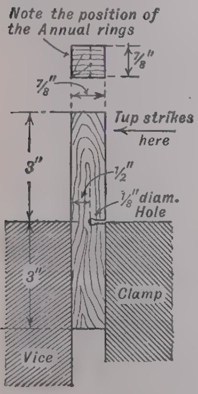

FIG. 153

Deflections of the specimen are measured with each drop, and a curve is drawn connecting height of drop and deflection squared. Deflections are obtained from a drop curve traced on a revolving drum.

If W = weight of the hammer,
H = height of drop at elastic limit,
F = fibre stress at elastic limit,
D = central deflection of specimen at elastic limit,
b = breadth of specimen,
h = height of specimen,
L = span,

then

Fibre stress at elastic limit $= 3\text{WHL}/\text{D}bh^2$.
Modulus of elasticity $= \text{L}^2 \times \text{F}/6\text{D}h$.
Work to elastic limit $= \text{WH}/bh\text{L}$.

§ (130) CLEAVABILITY.—For the resistance to splitting the U.S.A. Department of Agriculture use the test piece indicated in *Fig.* 154, and express the results in pounds per inch of width at fracture. Tests have been carried out to determine the resistance to splitting radially as well as tangentially to the annular rings, and the results (Table 46) show that most hardwoods split more easily along radial planes than along tangential surfaces.

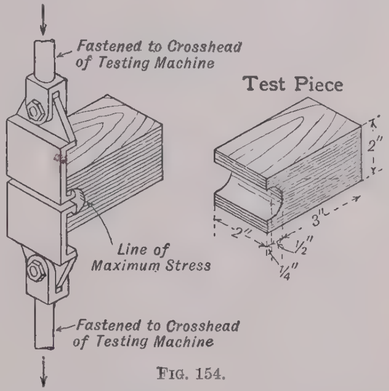
Fastened to Crosshead of Testing Machine
Test Piece
2"
3"
2"
½"
¼"
Line of Maximum Stress
Fastened to Crosshead of Testing Machine

FIG. 154.

TABLE 46

Kind of Timber.	Cleavage Strength in Pounds per Inch of Width.	
	Radial.	Tangential.
Hardwoods—		
Ash	322	316
Elm	297	357
Oak	370	462
Softwoods—		
Fir	138	149
Hemlock	168	151
Yellow pine	155	173
Spruce	120	139

Splitting is related to tensile strength across the grain. A high value is advantageous for timber which has to be fastened by nails or spikes.

§ (131) HARDNESS TESTS ON TIMBER.—The ball hardness test has been applied to timber in two ways:

(1) The first method, adopted by Warren,[1] follows the Brinell test for metals, viz. a ball of fixed diameter is pressed into the timber under a known load maintained for a stated time. The hardness number is calculated from the formula

$$\text{H} = \frac{\text{P}}{\text{A}},$$

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[1] *Report of the Department of Forestry, N.S.W., Australia*, 1911, on "N.S.W. Hardwood Timbers."

where H = the hardness number,
P = the load in kilogrammes,
A = the projected area of the indentation ($\pi d^2/4$).

Warren uses a ball of 20 mm., load of 1000 kilos, and time of 2 minutes. He found that the hardness along the fibre is considerably greater than either parallel or perpendicular to the annular rings.

(2) The second method, largely used in the U.S.A., is to press a ball of 0·444 inch diameter into the timber until it has penetrated 0·222 inch. The load required is taken as a measure of the hardness. The test tool is a steel bar with a hemispherical end. This end projects through a hole in the bottom of a cup-shaped washer. When the tool has penetrated the timber to the correct depth there is binding between the steel bar, washer, and specimen. The load at that instant is noted.

With the Ludwik cone test Warren[1] used the standard 90° cone and a pressure of 400 kilogrammes applied for one minute. The depth was measured by an indicator as the test proceeded.

§ (132) RESISTANCE OF TIMBERS TO ABRASION AND WEAR. — Relative wear has been tested by pressing blocks of the material $2 \times 2''$ under a pressure of 26 lbs. against a table covered with fine sandpaper and revolving at 68 revolutions per minute. A better method was used by Warren,[2] who utilised a sand-blast. The test piece, $3 \times 3 \times 1''$, was clamped against a plate having a $2\frac{1}{2}$-inch hole in it, and together with the plate was rotated about its own axis. A jet of superheated steam under a pressure of 43 lbs. per square inch, carrying sand in it, was projected through an expanding nozzle on to the surface of the rotating test piece. The sand-blast wore away the wood on the exposed diameter of $2\frac{1}{2}$ inches, and the loss of weight of the test piece in two minutes' exposure gave a measure of the wearing quality of the wood. Tests were made—

(*a*) Parallel to the direction of the fibre.

(*b*) Perpendicular to the direction of the fibre and perpendicular to the annular rings.

(*c*) Perpendicular to the direction of the fibre and tangential to the annular rings.

The results showed that the wear was least parallel to the direction of fibre, the ratio in the three directions being $a : b : c = 1 : 2\frac{1}{2} : 4$.

§ (133) HOLDING POWER OF NAILS AND SPIKES IN WOOD.—Nails hold in timber by virtue of the friction developed between the surface of the nail and the fibres of the timber. An investigation into the holding power of nails and spikes in Australian timber was carried out by Warren in 1911, who found that Australian timbers were so hard that it was necessary to first bore a hole slightly smaller than the nail in order to prevent either buckling of the nail or splitting of the timber. Six-inch No. 1 S.W.G. smooth steel wire type nails were driven to a depth of $3\frac{1}{2}$ inches in the timber in holes bored to $\frac{9}{32}$-inch diameter. They were withdrawn by pulling in the testing machine, and the loads to pull from various depths were noted. The results showed that the holding power was approximately proportional to the depth, as shown in Table 47.

TABLE 47

HOLDING POWER OF NAILS

Depth driven in Inches.	Mean Load in Pounds.
$1\frac{1}{2}$	1438
2	2201
$2\frac{1}{2}$	2676
3	3158
$3\frac{1}{2}$	3529

For driving spikes, holes slightly smaller than the diameter of the spike must be drilled. Warren found that $\frac{3}{4}$-inch square section twisted spikes required 30 per cent more load to withdraw them than $\frac{3}{4}$-inch square or $\frac{7}{8}$-inch circular black iron straight spikes.

Hatt,[3] experimenting with $\frac{9}{16}$-inch to $\frac{5}{8}$-inch square smooth spikes and screwed spikes pitch $\frac{1}{2}$ inch, root diameter $\frac{11}{16}$ inch, driven in $\frac{11}{16}$-inch holes to a depth of 5 inches, found that the plain spikes had about 55 per cent of the resistance of the screwed ones. Hatt's results agreed with those of Warren in showing that the holding power was proportional to the depth.

§ (134) INFLUENCE OF CONDITIONS OF TESTS UPON RESULTS. (i.) *The Effect of Moisture.*—The effect of moisture on the strength of wood has been thoroughly investigated by Tiemann,[4] and his results, which are fully given by Johnson,[5] show that there is a large increase in crushing strength, modulus of elasticity, and modulus of rupture with a decrease in the moisture content. Table 48 gives the approximate figures for spruce, chestnut, and pine.

(ii.) *Effect of Temperature of Testing.*—The ordinary variations of temperature of the laboratory (15° C. to 25° C.) are not important, but if greater extremes occur it is advisable that the temperature at which the tests are carried out should be controlled. Tests made at Perdue University show an increase in the strength of red oak ties of from 9 per cent to 17 per cent by testing them at 0° C. instead of 20° C.

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[1] *Report of the Department of Forestry, N.S.W., Australia*, 1911, on "N.S.W. Hardwood Timbers."

[2] *Ibid.*

[3] *Bull.* No. 124, *Am. Ry. Engr. Assn.*

[4] *Bull.* 70, *U.S.A. Forest Service*, 1906, and *Circular* No. 108, 1907.

[5] *Materials of Construction*, 1918 ed., p. 195.

(iii.) *Effect of Speed of Testing.*—Thurston[1] found that 60 per cent of the breaking load given by progressive loading on the ordinary testing machine would break beams if left in place for nine months. Experimenting on the same subject, Johnson[2] determined that it was not safe to assume that the permanent load which timber would carry was greater than 50 per cent of the short time ultimate load as ordinarily found by the testing machine.

TABLE 48

EFFECT ON TEST RESULTS OF VARYING MOISTURE CONTENT

Material.	Per cent Increase in Value by drying from 20 per cent to 10 per cent of Moisture.		
	Crushing Strength.	Modulus of Elasticity.	Modulus of Rupture.
Eastern spruce	62	14	52
Chestnut	59	25	44
Longleaf pine	89	21	47

It is evident, therefore, that the influence of time must be allowed for in testing wood. As a result of work by Tiemann,[3] the U.S.A. Forest Service of Agriculture have standardised fibre strain in testing, *e.g.*:

	Per Minute per Inch.
Bending tests on timber of structural size	0·0007″
Bending tests on small beams . . .	0·0015″
Compression parallel to grain, large test pieces	0·0015″
Compression parallel to grain, small test pieces	0·0030″
Shearing along the grain	0·0150″

The strength of wet or green wood is much more sensitive to changes of speed than is dry wood. A change of the above speeds by 50 per cent may ordinarily be allowed without causing a variation in strength of over 2 per cent.

The modulus of elasticity in bending was found to be practically constant with change of speed.

§ (135) TESTING OF STONE, BRICK, AND CONCRETE.—The important characteristics of stones and bricks from an engineer's standpoint are ease of working, durability, and strength. Durability depends on resistance to

(*a*) Abrasion or attrition.
(*b*) Absorption of water.
(*c*) Alternate freezing and thawing.
(*d*) Fire.
(*e*) Acid.

Stones and bricks are subjected in practice to compression, sometimes to transverse stress and shearing, but not to tension except such as would be caused by wind pressure or other lateral forces. Besides resistance to these forces the specific gravity is an important property. For a given stone the specific gravity and strength increase concurrently, while for some purposes greater stability is given by higher specific gravity.

With concrete, a knowledge of its elastic constants and tensile strength, of its resistance to fatigue and permeability to water, are requisite for the purposes of design.

(i.) *Determination of Specific Gravity.* — In order to determine the specific gravity (including the pores) the sample is dried at 100° C. until it is of constant weight, and, when cool, coated with a thin film of paraffin wax and weighed in air and water.

If W_a = weight in air,
and W_w = weight in water,
the specific gravity $= W_a/(W_a - W_w)$.

A correction for the film of paraffin wax can be made if the results are required to a high degree of accuracy.

The true specific gravity of the stone substance is obtained by grinding the dry stone to a powder and determining the specific gravity of the powder with a standardised Le Chatelier apparatus or similar form of specific gravity apparatus.

The porosity of the material can be obtained by making both the above determinations.

Then if S_s = specific gravity of the stone substance, and the other values are as before:

$(W_a - W_w)$/Weight of a cubic unit of water = Volume of the stone + pores = A.

W_a/(Weight of a cubic unit of water $\times S_s$) = Volume of the stone substance = B.

Then the porosity = (A – B)/A.

(ii.) *Absorption of Water.*—The usual method of obtaining the amount of water which stone, brick, or concrete will absorb is to immerse the specimen in water at approximately 20° C. after it has been dried at 100° C., cooled, and weighed. The immersion is continued for three days, when the surface water is removed and the material is reweighed. The increase in weight divided by the original weight and multiplied by 100 gives the percentage absorption. The process is sometimes hastened by eliminating the long period of soaking and immersing the test piece in water which is slowly raised to boiling-point and kept at that temperature for five hours.

A slight modification is sometimes introduced by weighing the specimen in air (W_a), in water immediately after immersion (W_w), and also in water after three days' immersion

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[1] *Materials of Construction*, part ii.
[2] *Ibid.*, 1918 ed., p. 208.
[3] *Amer. Soc. Test. Mat. Proc.*, 1918, viii. 541.

(W^1_w). Then the absorption in pounds of water per cubic foot of material

$$=\frac{W^1_w - W_w}{W_a - W_w} \times 62{\cdot}37.$$

(iii.) *Alternate Freezing and Thawing.*—This test is made by subjecting the specimen to a temperature of −10° C. to −20° C. for at least four hours after it has been thoroughly saturated with water. It is usual to apply ten alternate freezings and thawings, and then employ the master test of crushing.

(iv.) *Abrasion and Attrition.*—Bauschinger[1] devised an abrasion test for stone which consisted in pressing a 4-inch cube of the material, under pressure of 4 lbs. per square inch, against an iron plate rotating at 20 revolutions per minute. The test pieces were fixed at a radius of 19½ inches, and the abrasive employed was fine emery (No. 3) fed on to the plate at the rate of 2 grammes per revolution.

This test does not differ greatly from the Dorry test for road stones.

The sand-blast test was used by Gary[2] as an additional abrasion test for stone. He gave comparative results by the two methods, together with the crushing strength; these are given in Table 49.

TABLE 49

RESULTS OF ABRASION TESTS ON BUILDING STONES BY GARY

Area of Specimens used on Grinding Table = 7·75 sq. in.
Diameter of Nozzle on Sand-blasting Device = 2·36 inches.

Kind of Stone.	Compressive Strength. Lbs./Sq. In.	Volume of Wear in Cubic Inches per Sq. In.		
		On Grinding Table.	With Sand Blast.	
			Perpendicular to Rift.	Parallel to Rift.
Basalt . . .	38,900	0·042	0·024	0·025
Granite . . .	21,360	0·041	0·037	0·052
Gneiss . . .	21,230	0·079	0·056	0·045
Porphyry . . .	17,840	0·068	0·046	0·036
Graywacke* . .	15,780	0·085	0·059	0·058
Sandstone . .	6,640	0·144	0·155	0·117
Slate . . .	7,480	0·234	0·111	0·082

* A dense sandstone containing rounded or angular particles of quartz, felspar, or slate.

A "rattler" test is used in the U.S.A. for paving brick, and the conditions of the test have been standardised.[3] It consists in placing ten samples of the material in a cylinder together with an abrasive charge consisting of 20 cast-iron spheres of two sizes. The loss in weight is calculated as a percentage of the initial weight of the samples. This test is somewhat similar to the Deval Attrition Test or Lovegrove Rattler Test for road stones.

(v.) *Crushing Tests.*—The crushing strength is usually accepted as being the criterion of the mechanical qualities of brittle materials such as stone, brick, and concrete.

In carrying out this test the kind of bedding employed has a great influence on the results. Unwin[4] has shown that fluidity of the bedding produces vertical splitting of the test piece and a considerable reduction of the crushing strength. The common practice of putting wood or lead layers in between the test piece and the testing-machine compression blocks, in order to allow for irregularities in the surface of the specimen and to distribute the pressure over that surface, is erroneous. In order to obtain plane parallel faces, Unwin's method of applying thin layers of plaster of Paris has been used by the author for a considerable number of tests without obtaining any unsatisfactory fractures.

The information obtained from the test is the stress at which the first crack appears and the ultimate crushing stress. It is essential that the load should be applied axially, and the compression blocks are usually supplied with spherical seatings (see § (12)).

From the character of the final breakdown and shape of the fragments it can be determined whether the load was applied correctly. Photographs of broken concrete test pieces, showing the shearing angles of 45°, which are obtained with the crushing of brittle materials, are given in *Fig.* 155.

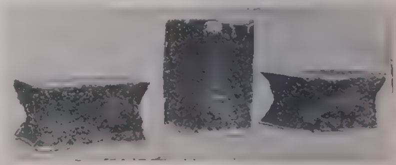

FIG. 155.

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[1] Bauschinger's *Communications*, 1884, x.

[2] *Baumaterialienkunde*, x. 136.

[3] Standard Specification for Paving Brick of the American Society for Testing Materials. Serial Designation = C7-15.

[4] *Report of British Association*, 1887, p. 879.

For geometrically similar test pieces the crushing strength is proportional to the cross-sectional area. Within the limits where bending occurs, the crushing strength becomes less as the height of the test pieces increases. For prisms and cylinders of the same height and cross-sectional area the crushing strength squared is inversely proportional to the circumference of the test piece.

The compressive strength of concrete is reduced, by an excess of water during mixing, to a fractional part of that which it would reach under proper conditions (see Table 50).

TABLE 50

EFFECT OF PERCENTAGE OF WATER ON THE CRUSHING STRENGTH OF CONCRETE

(From *U.S. Survey Bull.*, No. 344.)

Percentage of Water.	Concrete 1 : 2 : 4 by Volume ; Strength in Lbs./Sq. In.			
	Crushing.		Modulus of Rupture (Bending).	
	1 Month.	6 Months.	1 Month.	6 Months.
9·8	2299	3814	391	435
9·0	3547	4808	451	520
7·9	4612	4884	426	496

Crushing test pieces of building stone are usually in the form of cubes of 4-inch sides; for concrete, cubes of from 7- to 10-inch sides are employed; while bricks are generally tested on the flat. The dimensions of the test pieces and, with bricks, direction of crushing are always given with the results.

Representative crushing, bending, and absorption results on stone and brick are given in Table 51.

TABLE 51

STRENGTH OF STONE AND BRICK

	Crushing Strength. Lbs./Sq. In.	Transverse Test. Modulus of Rupture. Lbs./Sq. In.	Absorption. Per cent.
Stone—			
Granite	13,000-25,000	2710-3910	0·4-1·0
Limestone	2,000- 9,100	1160-4660	0·3-6·4
Sandstone	4,900-11,000	360-1320	0·7-8·2
Bricks—			
Vitrified brick . . .	5,000 and over	Over 900	Less than 5
Hard-burned . . .	3,500-4,999	600-900	5-12
Common firsts . . .	2,000-3,499	400-600	12-18
Common	1,500-1,999	300-400	Over 18
Leicester wire-cut . .	3,600-5,300	. .	. .
Fletton	2,600-3,900	. .	. .
White gault, wire-cut . .	2,100-3,100	. .	. .
Aylesford red pressed . .	2,200	. .	. .
London best stocks . .	1,600-2,800	. .	. .
Common stocks . . .	1,250-1,900	. .	. .

§ (136) THE ELASTIC PROPERTIES OF STONE, BRICK, AND CONCRETE.—It has been shown that a linear relation is often not obtained between stress and strain for stone, brick, and concrete (§ (64)), and that for the latter empyrical moduli are used for the purposes of design calculation.

Stanton Walker[1] gives the following particulars of the manner in which different variables affect the strength and modulus of elasticity of concrete :

"(*a*) Both the modulus of elasticity and strength increase within certain limits as the aggregate becomes coarser, although the modulus of elasticity increases less rapidly than the strength.

"(*b*) An increase in the quantity of cement in the batch causes an increase in values of modulus of elasticity and strength. The modulus of elasticity is affected somewhat less by change in the cement content than the strength.

"(*c*) The quantity of mixing water exerts a most marked effect on the modulus of elasticity and strength. An addition of 25 per cent of water to a mixture of normal consistency decreases the modulus of elasticity about 15 to 20 per cent, and the strength about 35 to 40 per cent.

"(*d*) Both the modulus of elasticity and strength increase with the age of the concrete so long as the specimens are kept moist during curing. The strength increases in proportion to logarithms of the age. The modulus of elasticity follows approximately the same relation.

"(*e*) There is no marked difference in the modulus of elasticity and strength of concrete made from high-grade pebbles, crushed limestone, crushed granite, or blast furnace slag.

[1] *Amer. Soc. Test. Mat. Proc.*, 1919, p. 584.

"(*f*) A comparison of specimens stored 14 days in damp sand and 77 days in air with specimens stored 91 days in damp sand shows that the modulus of elasticity and strength are higher for the damp sand storage.

"(*g*) Tests for mixing times ranging from ¾ to 10 minutes show both the modulus of elasticity and strength to be greater for the longer mixing times."

§ (137) TENSILE STRENGTH OF CONCRETE.—Considerable difficulty is experienced in designing suitable end attachments for tensile tests on concrete. Many experimenters cast spherically headed bolts in the centre of the ends of the test piece and pull on these heads by the testing-machine grips; others enlarge the ends so as to form the test piece into a flat dumb-bell shape. Particulars of a few dimensions which have been adopted are given in Table 52.

The tensile strength of concrete is about one-eighth to one-tenth of the crushing strength.

TABLE 52

PARTICULARS OF TENSILE AND COMPRESSION TEST PIECES FOR CONCRETE

Authority.	Tensile Test Piece.	Crushing Test Piece.
Henby *	$2\frac{7}{8}'' \times 3\frac{1}{2}'' \times 21''$ prisms.	$2\frac{7}{8}'' \times 3\frac{1}{2}'' \times 11''$ prisms.
Mills †	$6'' \times 6'' \times 18''$ prisms.	$6''$ cubes.
Hatt ‡	$4'' \times 4''$ (I bar) $17\frac{1}{2}''$ gauge length.	$8''$ diameter $\times 12''$ cylinders.
Woolson §	Square cross-section enlarged ends.	$6'' \times 6'' \times 14''$ prisms.
St. Louis ‖ Testing Laboratory	$8'' \times 8''$ square section (enlarged ends) 5 feet long.	$4\frac{1}{2}''$ diameter, $5\frac{1}{2}''$ deep (cylinders).

* *Assoc. Engr. Soc. Journ.* xxv. 145.
† *Cornell Civil Engineer*, xix. 106.
‡ *West. Soc. Eng. Journ.* ix. 234.
§ *Engr. News*, liii. 561.
‖ Humphrey and Holmes, "Structural-Materials Testing Laboratories at St. Louis, Mo.," *Bulletin* No. 329, *U.S. Dept. of Geological Survey*, 1908.

§ (138) SHEAR STRENGTH OF CONCRETE.—The shear strength of concrete lies between 0·4 and 0·6 of the crushing strength. The shear testing tool is of the same form as that used for metals (§ (12)), but is specially arranged to take a larger test piece. The test pieces are generally cast 5 or 6 inches in diameter and 14 to 18 inches in length, and are tested in either single or double shear.

§ (139) TRANSVERSE STRENGTH OF CONCRETE.—No definite size has been standardised for beams for transverse tests. Withey[1] used beams 6″ × 6″ loaded centrally on a span of 36 inches. The results he obtained on 1 : 2 : 4 concrete, mixed by volume, are given in Table 53.

TABLE 53

	1 : 2 : 4 Concrete—Age 1 Month. Average of 25 to 27 Tests.
Tensile strength—lbs./sq. in. (S_t)	189
Compressive strength—lbs./sq. in. (S_c)	1940
Modulus of rupture—lbs./sq. in. (S_b)	352

$$S_b = 0{\cdot}18S_c = 1{\cdot}86S_t.$$

§ (140) INCREASE OF STRENGTH OF CONCRETE WITH AGE.—The average percentage of strength of concrete at different ages, assuming full strength at one year, is given in Table 54. There is no reliable data for strength of concrete more than two years old.

§ (141) EFFECT OF VIBRATION, JIGGING, AND PRESSURE ON THE STRENGTH OF CONCRETE.—Vibration and jigging methods are useful for getting concrete into place around reinforcing bars and in intricate forms, but do no good, and in some cases are harmful, if applied after the concrete is properly placed in position.

Abrams[2] finds that the vibration of an electric hammer has very little influence on the strength of puddled concrete when applied for periods not exceeding 30 seconds. If a greater time of vibration is employed, there is a steady falling-off in strength. After 45 to 60 seconds the strength is only 90 per cent of that given by the standard method of puddling.

TABLE 54

Age.	Percentage of Full Strength.
7 days	30
1 month	60
2 months	75
3 ,,	85
4 ,,	90
6 ,,	95
9 ,,	98
12 ,,	100

If pressure is applied to concrete after moulding, water is expelled and a drier concrete is produced. This gives increased strength.

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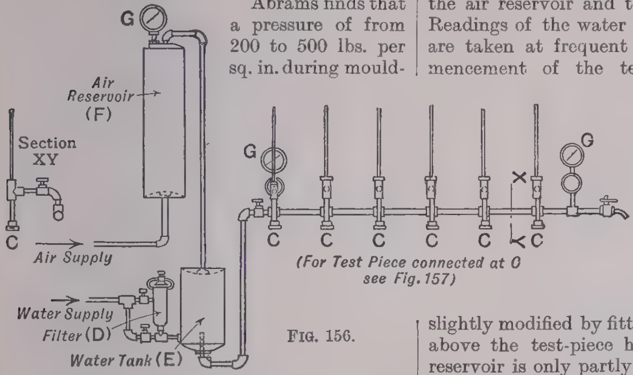
[1] University of Wisconsin, *Bulletin* No. 197.

[2] "Effect of Vibration, Jigging, and Pressure on Fresh Concrete," Structural Materials Research Laboratory, Chicago, 1919, *Bulletin* No. 3.

Abrams finds that a pressure of from 200 to 500 lbs. per sq. in. during moulding increases the compressive strength by 20 to 35 per cent.



(For Test Piece connected at C see Fig. 157)

FIG. 156.

§ (142) PERMEABILITY OF CONCRETE AND MORTAR.—Permeability depends upon minute passage ways in the material which are so connected that water can flow right through. Tests under high heads of water have been made, and show that even cement paste is not absolutely impervious. Concrete and mortar can, nevertheless, be made which, under normal pressures, do not show any dampness on the outside.

Two methods of test are employed:

(1) The amount of water passing through the material is collected and weighed.

(2) The weight of water, which has to be supplied in order to keep a constant head, is measured (*i.e.* the water going into the test piece).

The arrangement of a suitable apparatus for Method 1 is given in *Fig.* 156, and a section of the test piece, together with the can for catching the water, in *Fig.* 157.

The test piece (7½ inches diameter) has annular spaces at the outer edges of both the top and bottom surface painted with a rubber waterproofing paint. This leaves an annular piece 5 inches diameter in the original condition. Rubber washers A, A are placed over the waterproofing, and the specimen is securely clamped between the two cast-iron plates P, P.

The water is applied to the top of the test piece through the pipe C, and any water passing through is caught in the can B. The specimen is soaked in water for 48 hours immediately before the test.

The arrangement (*Fig.* 156) permits of six tests being carried out at once. Each test piece with its holder is attached to a union C. The water passes through a filter D and tank E before reaching the specimen. Air pressure is supplied to the tank E by a compressor, an air reservoir F being inserted to equalise the pressure. Three gauges G are provided for reading the pressure—one on the air reservoir and two on the water pipe. Readings of the water collected in the can A are taken at frequent intervals at the commencement of the test; but the rate of percolation of water diminishes as the test proceeds, and the intervals between the readings can be reduced until it becomes constant.

In order to measure the water supplied (Method 2), the arrangement is slightly modified by fitting a water reservoir K above the test-piece holder (*Fig.* 158). This reservoir is only partly full of water, and the pressure is applied to the top of the water by the compressed air. The amount of water added at regular intervals, to keep the level constant in the gauge glass H, is recorded.

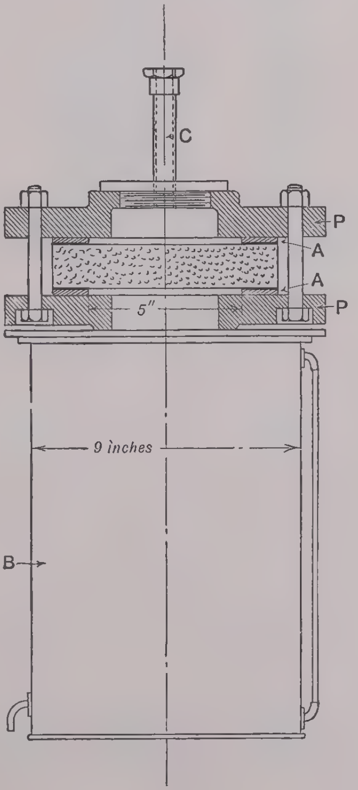


FIG. 157.

With an investigation carried out at the N.P.L. on the permeability of reinforced concrete under a pressure of 8·7 lbs. per square inch, air pressure was replaced by an actual head of water of 20 feet, which was kept constant by adding water to a reservoir as described for the last experiment.

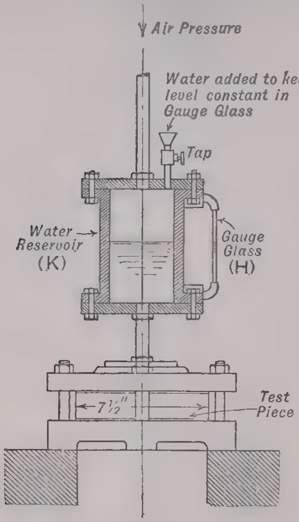


FIG. 158.

§ (143) TESTING ROAD MATERIALS.—There are two methods of estimating the comparative value of materials for road construction. The first is to lay sections of a road with the materials which are to be compared and observe the effect of traffic on them, and the second is to subject the road materials to laboratory tests designed to imitate actual conditions met with on the road. The former method, with great care, will give results, but the process is long and costly. The use of some form of experimental road sections is, however, the only thorough method of testing asphalt carpets. Observations of the behaviour of macadam roads, constructed with rocks whose physical characteristics have been determined by laboratory tests, enables these tests to be used in order to judge the probable adaptability of any rock for use in road construction.

A determination of the relative road-building qualities of different types of rock was first systematically attempted in France. The French School of Bridges and Roads installed a road materials laboratory in 1878, and it was there that the Deval[1] Attrition Machine was designed, and its excellence has led to its general adoption as a standard testing machine.

The testing of road metal was initiated in Germany in 1884 at the first Munich Conference for Establishing Uniform Conditions of Testing. They appointed a committee to draft the methods of testing that were necessary, the work performed by this committee was reported to conferences held in 1886, 1890, and 1893, and definite proposals for testing, including the Deval test, were submitted and approved.

In 1893 Massachusetts Highway Commission[2] founded a laboratory for testing road materials in the Lawrence Scientific School of Harvard University. In 1900 the United States Government, because of the growing importance of road-material investigations, established a laboratory in the Bureau of Chemistry of the Department of Agriculture. This laboratory, transferred in 1905 to the Office of Public Roads, examines, without charge, samples of road material submitted by any citizen of the United States.

The only stone-testing appliance believed to be in existence in England prior to 1911 was the "Rattler" attrition machine, designed and used by E. J. Lovegrove,[3] Borough Engineer and Surveyor of Hornsey.

In 1911 a road laboratory was inaugurated as a division of the Engineering Department of the N.P.L., Teddington, in accordance with a scheme drawn up by the Road Board and approved by the Treasury. The work of the Road Board has now been taken over by the Ministry of Transport (Roads Department).

In order to preserve continuity with the work which has been in progress in other countries, standard types of machines, as used by the United States Office of Public Roads, were adopted.

§ (144) ATTRITION TEST.—The machine used for this test is of the four-cylinder Deval type, which has given satisfactory results in France since 1878. It is shown in *Fig.* 159, and consists essentially of four cylinders, 7½ in. in diameter and 14 in. long, mounted on

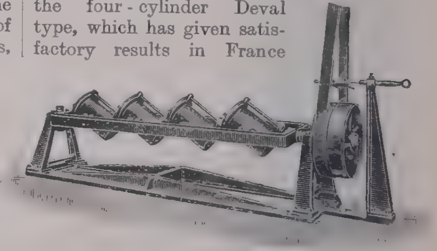


FIG. 159.

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[1] *Ann. des ponts et chaussées*, 1879, and *Bulletin du Ministère des Travaux publics*, 1881.

[2] *Bulletin* No. 44, on *Physical Testing of Rocks for Road Building*, U.S. Department of Agriculture.

[3] Lovegrove, Flett, and Howe, *Road-making Stones, Attrition Tests in the Light of Petrology*.

a frame in such a way that their axes are inclined at 30° to the axis of rotation.

Eleven pounds of rock, numbering as nearly fifty pieces as possible, are placed in one of the cylinders, the cover bolted on, and the machine revolved 10,000 times at a rate of about 30 revolutions per minute. The dimensions of the stones should be such that every stone will pass through a $2\frac{1}{2}$-in. ring but will fail to pass through a 2-in. ring. Only the material worn off which will pass a sieve of $\frac{1}{16}$-in. mesh is considered in determining the amount of wear. This amount is expressed as a percentage of the 11 lbs. used. In addition the French coefficient which is in general use is calculated. This is defined by the relation

$$\text{French coefficient of wear} = \frac{40}{\text{Percentage of wear}}.$$

A wet test is also made, and for this purpose 1·1 gallon of water is placed with the 11 lbs. of stone. With the majority of stones it is found that there is more wear under a wet test than under the dry test, but occasionally the reverse is the case.

An approximate relation has been established between this machine and Lovegrove's Hornsey Rattler.

§ (145) REPEATED BLOW IMPACT (TOUGHNESS TEST). (i.) *Preparation of Specimen.*—The specimen is prepared in the form of a cylinder 1 in. diameter by 1 in. long. A piece of the rock is roughly chiselled to size $1\frac{1}{4}'' \times 1\frac{1}{4}'' \times 3''$. It is ground approximately cylindrical on a large crystolon wheel, and then placed in the chuck of a grinding machine, where it is ground by a small crystolon wheel to exactly 1 in. in diameter. The chuck and stone are then taken to the diamond saw (*Fig.* 160), where the specimen is cut off 1 in. in length.

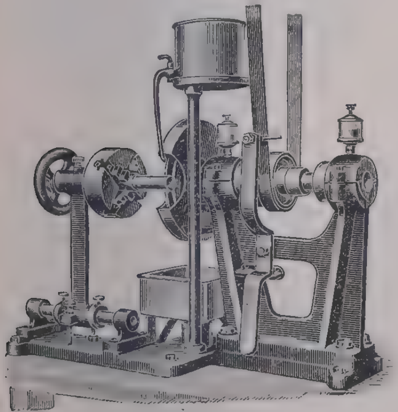

FIG. 160.

(ii.) *The Testing Machine.*—The machine used for this test is known as the Page Impact Machine (*Fig.* 161). The blow is given by a 4·4-lb. hammer H, and acts through a plunger P whose surface of contact with the specimen is spherical, and has a radius of 0·4 in. The blow, as thus delivered, approximates to the blows of traffic, and the spherical end has the further advantage of not requiring great exactness in getting the two bearing surfaces of the test piece parallel; the entire load being applied, at one point, on the upper surface. The test consists of a 0·4 in. fall of the hammer for the first blow, and an increased fall of 0·4 in. for each succeeding blow, until failure of the specimen occurs. The number of blows required to destroy the test piece is used to represent the toughness.

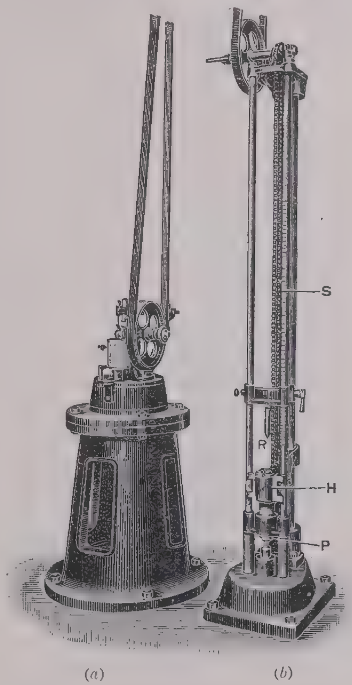

FIG. 161.—(*a*) Repeated Impact Machine for Cementation Tests. (*b*) Page Impact Machine.

A sprocket chain S driven from the shafting is supported on sprocket wheels attached to castings at the top and near to the base of the machine. The chain is provided with small lugs which engage a spring bolt attachment projecting inwards from the top of the hammer. This raises the hammer until it is tripped by a rod R projecting downwards from a crosshead which slides on two rods connecting the upper and lower castings. The crosshead is raised automatically through 0·4 in. after every blow by a lead screw driven by a worm and worm-wheel attached to the same shaft as the upper sprocket wheel. By throwing the lead screw out of gear with the crosshead the latter can be raised or lowered through any desired height or the test can be made with a constant height of fall.

A slightly different method of preparing the test sample is sometimes employed, viz. by the use of a core drill. This consists of a brass core tube having at its lower end a steel ring containing eight small diamonds in its cutting edge, four on the outer edge, and four on the inner edge. The drill arranged to cut a rock core 1 in. diameter runs at 200 to 300 revolutions per minute. Water is supplied to the inside of the drill and the specimen, through a stationary brass ring by a rubber tube connection.

§ (146) Abrasion Test.—The specimen is prepared in the same manner as that for the repeated blow impact test, and in cases where both tests are required the specimens can usually be both cut from the same sample.

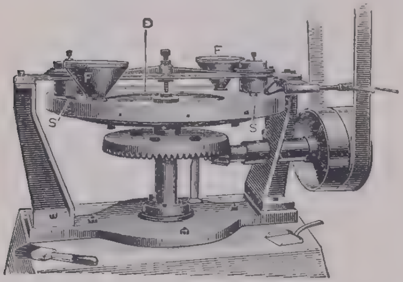

Fig. 162.

The machine used (sometimes called a Hardness Machine) is of the "Dorry" type (*Fig.* 162), and consists of a circular cast steel disc D which revolves in a horizontal plane about a vertical shaft. The specimen S is held with its axis vertical and its lower end pressed with a force of 3·5 lbs. per square inch against the surface of the disc. Standard stand, 30 to 40 mesh, is fed continuously upon the disc through funnels F, and it is important that an even and constant supply of the abrading material should be fed to each sample tested. After 1000 revolutions of the grinding disc, revolving at the rate of about 28 revolutions per minute, the loss of weight of the specimen is found. The test is repeated with the specimen reversed, and the average loss of weight computed from the two runs is used in determining the hardness.

This is given by $H = 20 - \frac{1}{3}W$, where H is the hardness number or coefficient of wear and W is the loss in grammes per 1000 revolutions.

§ (147) Cementation Test.—The cementing value of a rock dust measures the ability of that dust to hold the individual particles together to form a firm impervious road surface.

The processes involved in making a test of the cementation value of a rock are as follows:

(i.) Grinding up a mixture of the rock, coarsely crushed, and water into a stiff paste in a ball mill.

(ii.) Forming briquettes from this paste in a mould under pressure, which are dried after 24 hours.

(iii.) Subjecting each briquette to repeated blows with a small hammer, and measuring automatically the recoil of the hammer after each blow.

(i.) *Ball Mill* (*Fig.* 163).—1·1 lb. of coarsely crushed rock and 0·02 gallon of water are placed in the mill together with two steel shots each 5·1 in. in diameter. The samples are ground in this mill for 2½ hours, the mill revolving at the rate of 2000 revolutions per hour. The resulting paste is then ready to be moulded into briquettes.

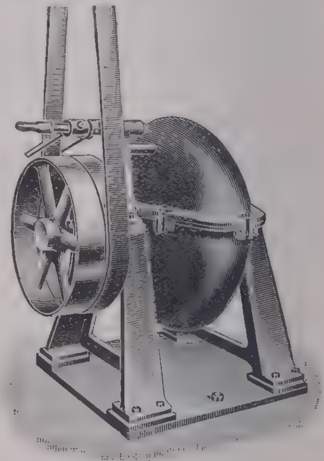

Fig. 163.

(ii.) *Briquette Machine* (*Fig.* 164).—The paste is placed in a mould M and a cylindrical plug P screwed upon it. The mould is in contact with the short arm of a weighted beam W, so that the pressure can be regulated to give a maximum value of 1880 lbs. per square inch. The size of the briquette is 1 in. diameter and 1 in. long.

(iii.) *Repeated Impact Machine* (*Fig.* 165).—After drying for 20 hours in air and 4 hours in a hot-air bath at 200° F., the briquette is allowed to cool for twenty minutes in a desiccator; it is then placed on the anvil of the machine with its axis vertical. A small hammer H is operated by a cam C, so that the effective drop is always 0·4 in., and the force of the blow is transmitted to the specimen S by means of a plunger P working in a vertical guide. This plunger is also connected to the arm of a lever, the other end of which carries a pencil L in contact with paper on a revolving drum, so that any rebound of the plunger after the blow is indicated on the paper. In this way a diagram is obtained which will give the number of blows after which there is no rebound of the plunger, *i.e.* after which the resilience of the specimen is destroyed. Six specimens are made from each sample, and the average result is taken as a measure of the cementing value of the material.

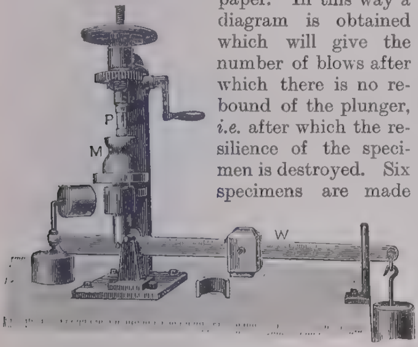

FIG. 164.

Table 55 gives the interpretation of the results from the physical tests on road stones.

§ (148) PETROLOGICAL AND GEOLOGICAL CHARACTERISTICS.—Thin sections are cut from the stone and examined under the microscope to ascertain the nature, quantity, and dimensions of the components which form the stone and determine the correct petrological designation of the material.

TABLE 55

INTERPRETATION OF THE PHYSICAL TESTS ON ROAD STONES

	Attrition Test. Per cent of Wear.		Impact Test. No. of Blows for Failure.	Abrasion Test. Coefficient of Wear.	Cementation Value. No. of Blows for Failure.	Absorption of Water. Lbs./Cubic Ft.	Crushing Strength. Lbs./Sq. In.
	Dry.	Wet.					
Very good . .	2 and under	2 and under	19 and over	19 and over	Over 100	0·10 and under	Over 20000
Good	2·1-2·5	2·1-3·1	16-18	17-18·9	76-100	0·11-0·40	15000-20000
Fairly good .	2·6-3·1	3·2-4·0	13-15	16-16·9	26-75	0·41-1·00	10000-15000
Rather poor .	3·2-4·0	4·1-5·0	8-12	15-15·9	10-25	1·01-3·00	5000-10000
Very poor . .	Over 4·0	Over 5·0	Under 8	Under 15	Under 10	Over 3·00	Under 5000

§ (149) FURTHER TESTS.—Specific gravity, absorption of water, crushing, sand-blast abrasion, and paving-brick rattler tests are carried out in the manner previously described for stones, bricks, and concrete in § (135). It is usual, however, when dealing with road stones, to make the crushing tests on cylinders 1 in. or 2 in. diameter with the length equal to the diameter. The crushing test pieces, when 1 in. diameter, are identical with those for impact and abrasion tests, and can be made with the same apparatus.

§ (150) ENDURANCE TESTS ON MODEL ROADS.—It was recognised that in the case of the various test trial roads laid down in this country, such as those at Sidcup, the best method of road construction would not show measurable wear or deformation until after years of heavy traffic. At the suggestion of Col. Crompton, Consulting Engineer to the Road Board, a model road-testing machine was designed at the National Physical Laboratory in order to provide a more rapid method of comparing the efficiency of methods of construction. This design was approved by the Road Board, and its construction was commenced in 1912, the first run taking place in 1913. The machine (*Fig.* 166) consists of a circular track 30 in. wide, having a mean diameter of 34 feet. The endurance of the centre 24 in. of this track is tested by the rolling on it of eight steel-tyred wheels, 3 in. wide and 39 in. diameter. The wheels are at an angular distance of 45° from each other, and each wheel is rotated by a separate electric motor mounted on a steel arm which

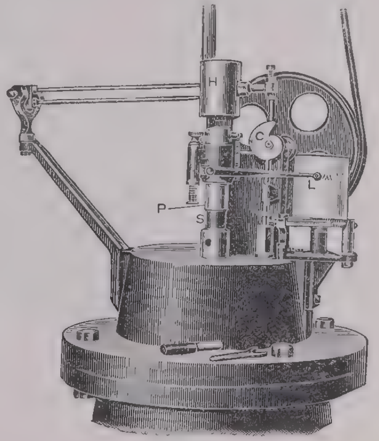

FIG. 165.

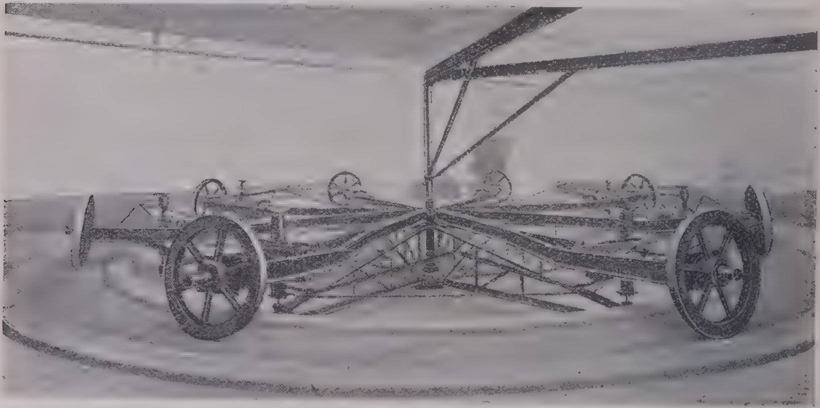

FIG. 166.

revolves round a centre post. The radial distribution of the wheels is such that the whole width of 24 in. is covered in one revolution of the arms, and in order to prevent, as far as possible, the formation of ruts in the tested surface each wheel is moved backwards and forwards radially by a cam mechanism through a distance of 1 in. The eight arms are all hinged to a rotating boss on the centre post, and at their outer ends are connected by spiral springs to eight corresponding cantilevers rigidly fixed to a second rotating boss connected with the former and immediately below it. The pressure of each wheel on the track can be adjusted to any desired value up to one ton.

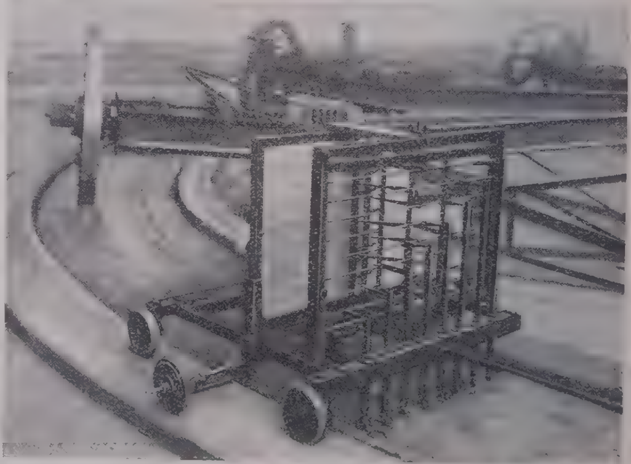

FIG. 167.

In order to investigate the question of the formation of waves a special apparatus (*Fig.* 167) has been constructed for drawing cross-sections of the road at different points round the track and longitudinal sections at the centre of the tracks of the eight rotating test wheels. Plaster casts are also taken to obtain a permanent record of the appearance of the road at various stages during the test.

It has been found that a road becomes too bad for use in three ways:

(i.) By excessive wave formation.

(ii.) By disintegration.

(iii.) By a combination of disintegration and excessive wave formation.

The formation of waves is increased by the road being subjected to heavy traffic after insufficient or unequal consolidation, while disintegration occurs rapidly with change of climatic conditions, if the road consists partly of small or large stones, when these stones are brought to the surface by deformation or wear.

A summary of the more important tests carried out with this apparatus is given in the Sixth Annual Report of the Road Board (1916).

§ (151) BITUMINOUS MATERIALS. — The physical character of bituminous binders and aggregates is determined by microsections, penetrometer and viscosimeter determinations, quantity and quality of the contained bitumen (whether natural or artificial), and the physical and chemical examination of the residue.

§ (152) LIMES AND CEMENTS.—The cementing materials ordinarily used in engineering construction are classified as follows :

(i.) Gypsum plasters.

(ii.) Limes—quicklime, hydrated lime, hydraulic lime.

(iii.) Cements (hydraulic).

§ (153) GYPSUM PLASTERS.—In plaster of Paris, Keene's cement, stucco, etc., the essential constituent is gypsum in a more or less dehydrated state.

Gypsum in its native state is crushed and ground, and then calcined at a temperature of 200° C. The product is then finely ground and screened.

Plaster of Paris is produced when the gypsum is not completely dehydrated. The theoretical composition of gypsum is $CaSO_4 + 2H_2O$ (a hydrous calcium sulphate). Plaster of Paris has the approximate composition of $CaSO_4 + 0{\cdot}5H_2O$. The specific gravity of gypsum is 2·3, of plaster of Paris 2·57, and of completely dehydrated gypsum 2·95.

Keene's cement is practically pure calcium sulphate with a small percentage of calcium carbonate ($CaCO_3$). For the manufacture of this cement the gypsum is procured in as pure a state as possible, and the resulting product is of an exceptionally pure white colour. The small percentage of $CaCO_3$ is introduced by dipping the calcined gypsum into a solution of alum and then burning again. The introduction of this "impurity" produces a slow setting cement which ultimately becomes very hard.

Cement plaster and stucco are calcium sulphates with adulterants which retard setting and increase plasticity.

§ (154) LIMES.—Pure lime (quicklime) is produced by the calcination of nearly pure limestone, at a temperature of about 500° C., in some form of vertical kiln. By this process the carbon dioxide is driven off from the calcium carbonate ($CaO + CO_2 = CaCO_3$). Such limes slake violently on the addition of water, form calcium hydroxide ($Ca(OH)_2$), and increase in volume by about 300 per cent. They harden slowly by absorbing carbon dioxide from the air.

Poor limes have a high percentage of magnesia and slake more slowly.

Owing to the fact that quicklime is insufficiently slaked or mixed in many cases when this is done on the job, this process is sometimes carried out in especially designed and equipped plant, where the operations are conducted more efficiently and with the minimum quantity of water. The resulting product is thoroughly screened and ground and is known as hydrated lime.

Pure hydrated lime should have a specific gravity of 2·08. It is usually tested, by chemical analysis, for fineness and soundness, and sometimes for tensile or compressive strength, in the manner presently to be described for Portland cement.

Hydraulic Lime.—John Smeaton discovered that limestone containing a small percentage of clay, when calcined, produces a lime which hardens by chemical action apart from the absorption of carbon dioxide from the air. This lime slakes in the usual way, and, in addition, hardens under water ; it is therefore called hydraulic lime.

§ (155) HYDRAULIC CEMENTS, of which the best example is Portland cement, are produced by the calcination of chalk and clay or suitable limestone and shales.

In various parts of the world there are deposits in which the mixing of these materials has been carried out by nature. The result of calcination of these natural deposits produces a natural hydraulic cement. The natural rock contains as a rule an excess of carbonate of lime, rendering the resulting cement poor in quality.

Portland cement is an artificially produced cement in which the chalk and clay are accurately proportioned and thoroughly mixed together, before burning to a hard clinker at a temperature of about 750-800° C. The clinker is then ground, and forms the final Portland cement. A finely ground cement makes a stronger mortar than a coarsely ground one, hence the fineness of the cement is a property which it is necessary to specify.

Portland cement is a British invention. It was discovered by Joseph Aspdin in 1824, and owes its name to its resemblance, when set hard, to Portland stone. It sets rapidly, sets under water, and hardens slowly with but little change in volume until it is nearly as strong as stone. The hydraulic property is due to the presence of silicate of alumina.

The British Engineering Standards Association, in their standard specification, define Portland cement as follows : "The cement shall be manufactured by intimately mixing together calcareous and argillaceous materials, burning them at a clinkering temperature and grinding the resulting clinker."

Cementing materials are subject to large variations in the quality. It is necessary therefore to closely control the manufacture by imposing tests to ensure that the properties which it has been found are of first importance should reach a specified standard.

The main properties required in a cement are strength, permanence, and time of setting suitable to the work.

The principal tests which are in general use to determine the suitability of the cement for fulfilling those requirements are:

(*a*) Fineness of grinding.

(*b*) Specific gravity. (This test was deleted from the B.E.S.A. specification in August 1920.)

(*c*) Chemical composition.

(*d*) Strength.

(*e*) Time of set.

(*f*) Soundness or constancy of volume.

In order to obtain uniformity in the testing results, it is necessary that the conditions of test should be exactly the same in each case, and that the personal element should be, as far as possible, eliminated.

This has been provided for in the various specifications for Portland cement, such as those by the British Engineering Standards Association[1] (B.E.S.A.) or the American Society of Testing Materials[2] (A.S.T.M.), in which standard methods for carrying out the tests are described in detail.

(i.) *Sampling.*—It is important that (1) the sample is representative of the whole consignment; (2) a sufficiently large sample is secured to carry out, in duplicate, all the tests required; (3) the storage of the sample is such that the quality of the cement is not affected before the tests are made, and (4) the sample is properly mixed if it is obtained from various parts of the consignment.

Eight pounds of cement are usually sufficient for a sample. The B.E.S.A. specify that each sample shall consist of equal portions selected from twelve different positions in the heaps, bags, or barrels. Samples are usually stored in air-tight tins.

(ii.) *Fineness Tests.*—The fineness of cement for specification purposes is determined by means of the weight of residue (as a percentage of the original weight of the sample) which is left on a sieve after a definite period of sifting. Both the B.E.S.A. and the A.S.T.M. specify that the wire cloth of the sieves shall be woven (not twilled), and that the cloth is to be mounted on frames without distortion. The sizes adopted are slightly different in the two specifications. Particulars are given in Table 56.

TABLE 56

FINENESS TEST

	B.E.S.A. Specification.		A.S.T.M. Specification.
Size of sieve, wires per inch	180 × 180	76 × 76	200 × 200
Diameter of wire, inches	0·0018	0·0044	0·0021
Residue shall not exceed	14 per cent.	1 per cent.	22 per cent.
Weight of sample, grammes	100 grammes.		50 grammes.
Time of continuous sifting	15 minutes on each sieve		Until not more than 0·05 grammes passes through in 1 minute.

Mechanical shakers are sometimes employed in laboratories where a large amount of work has to be carried out. They are not recommended, as very little time is saved as compared with efficient hand sifting, and the results are not so consistent.

It is usually considered that the size of the very fine flour of Portland cement cannot be obtained by the use of sieves. For separating very fine powders, air separators are often used. These are fully described in the Proceedings of the International Association of Testing Materials for 1912.

Abrams,[3] as a result of a series of experiments on the effect of fineness of cement on the strength of concrete, finds that:

(*a*) In general the strength of concrete increases with the fineness of a given lot of cement, but there is no necessary relation between the strength of concrete and the fineness of cement if different cements are used.

(*b*) Fine grinding shortens the setting time, is more effective in increasing the strength of lean mixtures than rich ones, and increases the strength of concrete more at seven days than at times from one to twelve months, *i.e.* fine grinding expedites the hardening of the concrete.

(*c*) If the mixture is "wet," the rate of increase of strength, due to greater fineness, is lowered.

(iii.) *Specific Gravity.*—The weight of cement, or its "apparent" density, decreases with fineness, *i.e.* a cement coarsely ground gives a heavier weight per cubic foot than the same cement ground finely. The "apparent" density of cement is only a comparison of their "real" densities or specific gravities when the cements are of the same degree of fineness.

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[1] *British Standard Specification for Portland Cement*, Report No. 12.

[2] *Standard Specifications and Tests for Portland Cement*, Specification No. C 9-17.

[3] "Effect of Fineness of Cement," *American Soc. Test. Mat. Proc.*, 1919, xix., part ii. p. 328.

The specific gravity of cement is lowered by adulteration, hydration, or under-burning. The differences in the specific gravity are usually so small that it is necessary to exercise great care in making the determination.

The specific gravity of cement should not be less than 3·10. The method of determination which is recommended is by means of a specific gravity bottle. The standard Le Chatelier apparatus is especially designed for use with cement. It is a volumenometer in which the liquid used is benzine, petroleum, or paraffin which has been freed from water by standing over quicklime (water causes hydration of the cement and therefore an alteration of volume).

(iv.) *Chemical Composition.*—The chemical analysis is not so important as the physical tests on cement. It, however, gives valuable indications in the detection of adulteration with considerable amounts of inert material, such as slag or ground limestone. It is also used to determine whether magnesia and sulphuric anhydride are present in excessive quantities.

(v.) *Strength Tests.*—A finely ground cement will take a certain amount of inert material without reduction of strength, therefore a coarsely ground cement will give as high a strength as a finely ground one when the test pieces are made with neat cement. As cement is seldom used neat, a strength test on the neat material is no criterion of its strength when used in practice; it, however, gives information regarding the time of setting and the soundness. Tests on briquettes made with cement and sand (usually 1 : 3 by weight) are consequently specified, as a rule, in addition to those on neat cement.

Cement is never used in tension, yet tensile tests are generally carried out in order to give an indication of strength. This is chiefly on account of their simplicity, rapidity, and cheapness. There is, however, a tendency in some countries for compression tests to be substituted for the hitherto universally adopted tensile method. It is to be noted that the recent strength tests by Abrams on fineness of cement, already referred to, were made on concrete test pieces in compression.

§ (156) Tensile Strength of Cement and Mortar.—Test pieces used for tensile tests are moulded into the form shown in *Fig.* 168, which represents the briquette standardised by the B.E.S.A. The differences between this and the A.S.T.M. standard are only slight.

The form of briquette has a considerable influence on the results. Coker[1] found that the ratio of the maximum stress to the mean stress with the B.E.S.A. briquette is 1·75 approximately, while the American and Continental forms give a value of 1·70 and 1·95 respectively. The intensity of stress is greatest along the sides of the minimum section and least at the centre.

Either single or gang moulds (see *Fig.* 169) are used for preparing the test pieces. The latter permits a number of briquettes to be moulded at one time and are preferred by many laboratories, since the greater quantity of material that can be mixed tends to produce more uniform results.

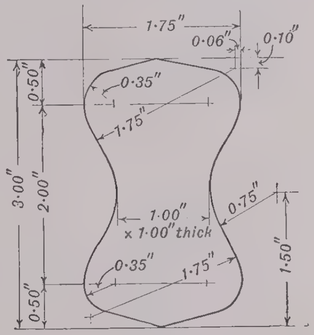

FIG. 168.

(i.) *Neat Cement Tensile Tests.*—The quantity of water used in gauging has a considerable influence on the strength of the briquette, and should be such that the mixture is plastic when filled into the moulds. The amount of water varies with different cements, and it is usual to make trial experiments to find the exact amount of water that is necessary. This varies from 18 to 25 per cent by weight of the cement.

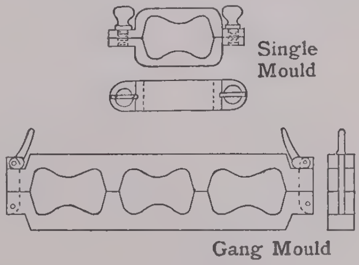

FIG. 169.

The moulds, during filling, rest on slate or some form of non-porous plates. They are filled by using the blade of an ordinary 7½-oz. gauging trowel, and it is usual to specify that no ramming or tamping is permitted.

The temperature of the room and of the mixing water should be as near to 16° C. as it is practicable to maintain it.

The briquettes are kept in their moulds, in a damp atmosphere to prevent them from drying out, for twenty-four hours after gauging. For this purpose a moist chamber is a great convenience and improvement over the method of covering the test pieces with a damp cloth, which is liable to dry out unequally. A moist chamber consists of a slate, concrete or metal lined wooden box covered inside with felt, which is kept wet. The bottom of the box is arranged so as to hold water, and glass shelves for holding the briquettes rest on cleats fixed to the sides of the box.

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[1] "The Distribution of Stress at the Minimum Section of a Cement Briquette," *International Assoc. Test. Mat. Proc.*, 1910–1913, ii., part ii., paper xxviii$_1$.

After the expiration of twenty-four hours the briquettes are removed from their moulds and immersed in fresh water maintained at a temperature of 16° C. This water is renewed every seven days, and the briquettes are left in water until required for testing.

The strength of cement increases considerably with time from setting, consequently the age at which the test pieces are to be broken is specified. The B.E.S.A. specify that six briquettes are to be broken at seven days and six at twenty-eight days after setting, and that the breaking strength shall not be less than 450 lbs. per square inch at seven days, or $x+40,000/x$ lbs. per square inch at twenty-eight days, where $x =$ the actual strength in pounds per square inch at seven days.

The rate of loading has a marked effect on the strength of the briquettes. The latter increases with the rapidity of loading, which has therefore to be standardised. The B.E.S.A. specify a rate of 500 lbs. per minute, while the A.S.T.M. adopt 600 lbs. per minute. A variation from these rates of 100 lbs. per minute introduces an error of about 2 per cent.

The form of jaws used for gripping the test piece is shown on the testing machine in *Fig.* 170. The load must be applied without shock, and care should be observed to see that projecting edges are removed from the specimens to ensure that the briquettes are properly centred in the clips. A deviation of 0·062 inches from correct alignment will decrease the tensile strength from 15 per cent to 20 per cent.

Tension tests of cement briquettes are generally made in small lever testing machines of various types, in which provision is made for applying the load at a steady and definite rate.

Messrs. Adie, London, supply a machine in which the load is applied by a regulated travelling poise. The poise is pulled along the beam by means of a suspended weight. The speed at which the weight descends, and, therefore, that the poise travels along the beam, is regulated by the cock in the plunger of a dash pot. The plunger is attached, through the poise, to the suspended weight by an arrangement of cords and pulleys.

In the Bailey and Reid patent cement tester a cylindrical cistern is hung at the end of the single lever of the testing machine. A small stream of water is allowed to flow into this cistern, enabling the load to be applied in a gradual and almost imperceptible manner. The height of the water in the cistern is indicated by means of a glass tube similar to the gauge glass of a steam boiler, and the graduations, shown on the outside, indicate the load in pounds. A small trigger automatically closes the water-tap when the material is broken.

A further method of applying the load is indicated in the machine shown in *Fig.* 170. This is a compound lever machine with the load applied by lead shot. The briquette is held between the jaws A, the lower jaw being attached to a straining screw S, by means of which the lever L is raised into position between the stops. The load is applied by running lead shot from the container G through a channel H into the bucket C, which is hung on to the end of the lever.

The rate of loading is regulated by means of the adjustable lever N, and the breaking of the test piece automatically shuts off the supply of shot by operating this lever. With this particular machine the leverage is 50 : 1, so that fifty times the weight of the shot gives the breaking load. The levers are floated before testing by adjusting the balance weight W.

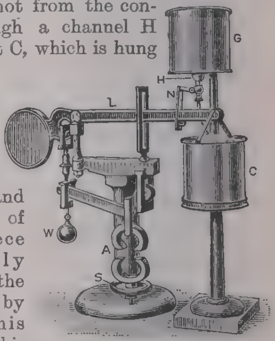


FIG. 170.

(ii.) *Mortar* (*Cement and Sand*) *Tensile Tests.*—Thoroughly washed and dried sand (obtained from Leighton Buzzard in Great Britain), which will pass a 20 × 20 mesh sieve but not a 30 × 30 mesh sieve, is used for tests in which sand is required. The wires for the sieves are 0·0164 inch and 0·0108 inch in diameter respectively.

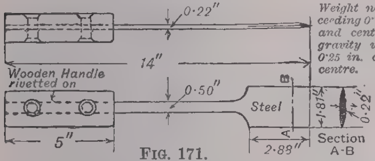
The cement and sand are mixed in the proportion of 1 : 3 by weight for the standard best pieces; about $1\frac{1}{4}$ lb. of cement and $3\frac{3}{4}$ lbs. of sand will make 12 briquettes. The gauging must be made without any excess of water being present. The quantity of water is approximately 10 per cent of the united weights of the sand and cement; the exact quantity required should be determined by a trial mixing.

The gauging is carried out on some form of non-absorbing surface, preferably glass, with a room temperature and water temperature of 16° C.

The method of filling is described by the B.E.S.A. specification as follows:

"The mixture gauged as above shall be evenly distributed in moulds of the form required, each mould resting upon a non-porous plate. After filling a mould a small heap of the mixture shall be placed upon that in the mould and patted down with the standard spatula until the mixture is level with the top of the mould. This last operation shall be repeated a second time and the mixture patted down until water appears on the surface; the flat only of the standard spatula is to be used, and no other instrument or apparatus is to be employed for this operation. The mould after being filled may be shaken to the extent necessary for expelling the air. No ramming or hammering in any form will be permitted during the preparation of the briquettes, which shall then be finished off in the moulds by smoothing the surface with the blade of a trowel." The standard spatula is shown in *Fig.* 171.

Various types of moulding and mixing machines have been devised and are in use in some countries. They are, however, not allowed by the B.E.S.A. specification. Mechanically moulded briquettes give greater strength than hand-moulded ones.



Weight not exceeding 0·75 lb., and centre of gravity within 0·25 in. of the centre.

FIG. 171.

Mortar briquettes are stored in moist air for twenty-four hours and then in water (at 16° C.) until required for testing. They should be broken as soon as possible after being taken out of the water, and should never be allowed to dry. The B.E.S.A. specify that six briquettes are broken after periods of seven and twenty-eight days respectively, at a uniform rate of loading of 500 lbs. per minute (600 lbs. per minute in the U.S.A.). The strength should not be less than 200 lbs. per square inch at seven days after gauging, or $x+10{,}000/x$ lbs. per square inch twenty-eight days after gauging, where x = the actual strength at seven days in pounds per square inch.

§ (157) CHARACTERISTIC EQUATIONS FOR TENSILE TESTS.—Unwin[1] has found that the rate of hardening of cement and cement and sand briquettes follows, very approximately, a simple law.

If y = the strength of the briquette in pounds per square inch at x weeks after gauging, and a, b, and n are empirical constants, Unwin found that $y=a+bx^n$.

n is constant for one cement, and if a be taken as the initial strength after one week,

$$y=a+b(x-1)^n.$$

For tension briquettes the gain of strength is nearly proportional to the cube root of the time of hardening.

Thus for Portland cement in tension $n=\frac{1}{3}$

and $$y=a+b\sqrt[3]{x-1},$$

where a = the initial strength (at 7 days),

b = a constant varying with the rate of increase with time.

These two constants give a clear indication of the character of the cement.

In the B.E.S.A. specification for neat cement briquettes, where the lowest value which is accepted for a is 450 lbs./sq. in., and the minimum value for y is therefore $=450+40{,}000/450=539$,

$$y=a+b\sqrt[3]{x-1},$$

and if $x=4$,

$$539=450+b\sqrt[3]{4-1},$$

therefore $$b=61{\cdot}8,$$

or $$y=450+61{\cdot}8\sqrt[3]{x-1}.$$

§ (158) SETTING TIME.—There is a distinction between setting and hardening. The initial setting is the commencement of the chemical action which occurs when the water combines with the cement; hardening is a much slower process. As a disturbance of the setting process may produce a loss of strength, it is desirable that the initial setting is not interrupted, and that the whole operation of mixing and moulding should be completed before the cement begins to set.

The initial setting time is the time which elapses from the moment water is added until the paste ceases to be fluid and plastic. For the B.E.S.A. specification the time is taken from the moment that the special mould is filled with the gauged cement, and not from the time that water is added to the cement.

The final set is acquired when the material attains a certain degree of hardness.

The B.E.S.A. specification of March 1915 recognised three distinct gradations of time of setting, viz.:

Grade.	Initial Setting Time in Minutes.	Final Setting Time in Minutes.
Quick .	Not less than 2	10 to 30
Medium .	Not less than 10	30 to 180
Slow .	Not less than 30	180 to 420

In the revised specification (August 1920), the medium and slow setting cements are replaced by a grade having a minimum initial setting time of 20 minutes and a maximum final setting time of 10 hours. The quick setting grade is retained, but the minimum final setting time is not now specified—the maximum final setting time is not to be greater than 30 minutes.

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[1] *Testing of Materials of Construction*, 1910 ed. p. 459.

The test block for setting time is made in a special mould (80 mm. diameter and 40 mm. high) and filled with neat cement, mixed and gauged in the manner described for tensile tests.

The time of initial setting is determined empirically by the time taken, after filling the mould, for a weighted needle to cease to pierce the test block completely. The B.E.S.A. specification has decided on a special apparatus for carrying out this test called the "Vicat" needle. The needle is one millimetre square in section with a flat end, and the total weight of needle and attachments is 300 grammes. The apparatus is fitted with a device for measuring the depth of penetration of the needle.

The final setting time is determined empirically by the same apparatus, but using a slightly different form of needle. A needle of the same section as before projects 0·5 mm. beyond a hollowed-out circular cutting edge (5 mm. in diameter). The final setting time is taken as that when the needle makes an impression but the circular attachment fails to do so.

The setting times are affected by the temperature of the mixing water, the percentage of water used, and the temperature and humidity of the air. It is preferable for the tests to be conducted in moist air.

§ (159) Soundness or Constancy of Volume.—A cement which remains perfectly sound is said to be of constant volume. Failure is shown by cracking, swelling, blowing, or disintegration. To ascertain the soundness of Portland cement a rough test is to make a pat of cement ½ inch thick, gauged with 25 per cent by weight of clean water on non-porous material, preferably glass. This is placed in water at 16° C., after twenty-four hours in moist air, and left there for inspection at intervals; it should show no signs of failure.

This test is sometimes accelerated by exposing a pat of the cement for five hours, in an atmosphere of steam, to a temperature of 98° C. to 100° C.

The test specified by the B.E.S.A. is the Le Chatelier test. This is made in the apparatus shown in *Fig.* 172, which consists of a small split cylinder of brass (0·5 mm. thick) forming a mould 30 mm. internal diameter and 30 mm. high. On either side of the split two indicators with pointed ends are attached. The distance from the ends of the indicators to the centre of the cylinder is 165 mm.

The Le Chatelier test has been accepted unreservedly in England, and the International Association of Testing Materials[1] decided to recommend the method as the standard accelerated test for constancy of volume of cements. The method is to be carried out as follows:

"The cement is gauged and filled into the mould on a plate of glass, the edges of the mould being held together. When the mould has been filled it is covered with a plate of glass held down by a small weight, and the whole is immersed in water at 15° C. for twenty-four hours. Any tie or band which has been used to keep the edges of the mould together during setting time is then removed. The distance between the indicator needles is then measured and the mould is placed in cold water, which is raised to a temperature of 100° C. in the course of half an hour and is kept boiling for six hours. The mould is removed from the water, and after it has cooled the distance between the indicator needles is again measured. The difference between the two measurements represents the expansion of the cement. This must not exceed ten millimetres when the cement has been aerated for twenty-four hours, and five millimetres when the cement has been aerated for seven days."

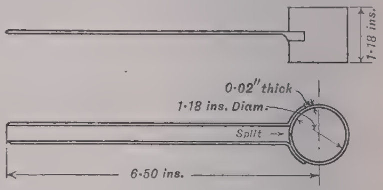

Fig. 172. *Made in Brass*

§ (160) Effect of Storage of Cement on the Strength of Concrete.—Abrams[2] finds that the effect of storage of cement on the strength of concrete or mortar is largely a question of the age at which concrete or mortar is tested. The storage period and the age of the concrete or mortar at test are of greater importance than the exact condition of storage, so long as the cement is protected from direct contact with moisture.

The deterioration of cement in storage appears to be due to absorption of atmospheric moisture, causing a partial hydration, which exhibits itself in reducing the early strength of the concrete and prolonging the time of setting.

Compression tests of concrete and mortar show a deterioration in strength with storage of cement for all samples, for all conditions and periods of storage, and at all test ages.

R. G. B.

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[1] Blount, "On Accelerated Tests of the Constancy of Volume of Cements," *Inter. Assoc. Test. Mat. Proc.*, 1909, 5th Congress, paper x./3.

[2] "Effect of Storage of Cement," Minutes of Spring Meeting of the Portland Cement Association, April 1920, and Structural Materials Research Laboratory, Chicago, *Bulletin* No. 6, June 1920.

Elastic Constants of Metals:

Measuring Instruments for determining. See "Elastic Constants, Determination of," § (4).

Methods of Determination. See *ibid.* § (5).

Elastic Limit:

Definition and Method of Determination. See "Elastic Constants, Determination of," § (62).

Determination of Elastic Limit by Change of Temperature. See *ibid.* § (62) (i.).

Elastic Limit and Yield Point. Method of determination at high temperatures. See "Elastic Constants, Determination of," § (116).

Elasticity, Isothermal and Adiabatic. See "Thermodynamics," § (55).

ELASTICITY, THEORY OF

§ (1) Introductory. — The theory of elasticity is concerned with the small relative displacements of different points in a body which occur under the action of applied forces: the forces may or may not constitute an equilibrating system, but in the great majority of examples which have been treated hitherto they are both in equilibrium and steady, so that ultimately the problem is one of the *statics* of each component particle. Logically considered, the science follows that of rigid dynamics in the process of development by which the scope of mathematical analysis has been extended to embrace more and more of the properties of real bodies. Thus, the theory of the "dynamics of a particle" treats of motion in its simplest form, and can be applied to problems in which it is sufficiently accurate to assume that the displacement of every particle is the same. Rigid dynamics takes account of the additional complexity introduced by rotation, whereby the motion of a constituent particle depends upon its position in the body; but it retains the assumption that the distance between any two particles is unchanging, and is thus equally powerless to deal with such problems relating to the behaviour of real bodies (in which absolute rigidity is never experienced) as the determination of the pressures with which a heavy beam will bear upon three or more supports. It is left for the theory of elasticity to bring problems of this nature, for the first time, within the range of exact calculation.

§ (2) Elasticity defined.—Practically all materials which are employed in construction exhibit in some degree the property of *elasticity*; that is to say, they deform under the action of applied forces, but when the forces are removed they recover their original shape. In the theory of elasticity, as at present developed, this property is regarded as absolute, and the applicability of its results to practice is limited in the main by the extent to which actual materials may be considered to satisfy its fundamental assumption—that their behaviour under applied forces is independent of their previous history. It is doubtful whether perfect elasticity, as thus defined, is exhibited by any actual material.[1] Thus, the behaviour of wood under applied forces is dependent to a considerable extent upon its dryness and temperature, and of rubber upon the forces to which it has recently been subjected: most metals and other crystalline materials possess practically perfect elasticity under small forces, but their behaviour depends in part upon their previous history when the forces exceed certain limits.

§ (3) Stress and Strain.—In the development of the precise mathematical theory, it is found convenient to introduce two new physical concepts, for which we employ the terms "stress" and "strain." To understand these terms, we may consider the motion or equilibrium of that portion of an elastic solid which is contained within the volume of the small parallelopiped indicated in *Fig.* 1. The contained material will in all practical instances be subjected to body forces such as gravity, and to balance these (and also to overcome its inertia, if the material is in accelerated motion), forces must be exerted across the containing faces by the surrounding material. We need not concern ourselves here with the difficult physical problem of explaining the mechanism by which these forces are exerted: it is sufficient for our purpose to remark that the action, whatever it is, must be of a reciprocal nature; that is to say, the force which is exerted upon the contained material, across the face ABDC, by the surrounding material must be equal and opposite to the force which is exerted by the contained material, across the same face, upon the surrounding material. Similar considerations will apply in regard to the other faces.

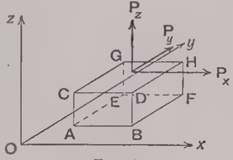

Fig. 1.

Confining our attention to the forces which are exerted *upon* the contained material, let us denote by P the resultant force exerted by the total action across the face ABDC. Whether the elastic solid as a whole be in equilibrium or in motion, the magnitude of P will depend upon the area of this face; but the quantity defined by

$$p=\left[\frac{P}{\text{Area of ABDC}}\right] \quad . \quad . \quad (1)$$

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[1] See "Elastic Constants, Determination of," §§ (14)-(25): "Structures, Strength of."

will tend to a definite (and in general finite) limit as the area is indefinitely reduced, and to this limit we give the term "stress."

The dimensions of a stress can evidently be represented by $[M][L]^{-1}[T]^{-2}$, and it is clear from the expression (1) that it shares many of the properties of a force. It has magnitude and direction, and the stresses acting on a given surface can be resolved and compounded by the vector law: further, from what has been said above, it is really a quantitative expression for the intensity, not so much of the action on any definite portion of the material, as of the mutual action between the two portions of material which are separated by a specified surface.

Just as the idea of "stress" is an extension, for the special purposes of our subject, of the familiar concepts of mechanics, so "strain" is a development of the purely kinematic concept of relative displacement. Reverting to *Fig.* 1, we assert that the material under consideration will be unstrained, whatever its motion as a whole may be, so long as the volume occupied by any definite part of it remains unchanged both in size and shape; the motion of unstrained bodies is thus the province of rigid dynamics. On the other hand, if we consider the material which in one configuration is contained within the parallelopiped ABDCEFHG, and if in a second configuration the same material is contained within a volume of different size or shape, then we may say that the second configuration can be obtained from the first by a process involving strain.

Confining our attention to the edges of the original parallelopiped, we notice that strain may involve a change in one or more of the lines AB, BD, . . ., etc., or in one or more of the angles CAE, CAB, . . ., etc., or in both. Stretching of the material in the direction of the axis Ox will be accompanied by a change in the length of AB. Let A′ and B′ denote the new positions of the points A and B: then the fractional alteration in the length of AB will be given by

$$e=\left[\frac{A'B'-AB}{AB}\right], \quad . \quad . \quad . \quad (2)$$

and while both the numerator and the denominator of this expression tend to zero in the limit, as the length AB is indefinitely reduced, their ratio will tend to a limit which must be finite (if we exclude the possibility of discontinuous displacements, which would involve rupture of the material), but will not in general be zero. To this limit we attach the definite term "stretch": it is clearly a non-dimensional and scalar quantity.[1]

In much the same way, we may measure the change in the angle CAE by the expression

$$\gamma=CAE-C'A'E'. \quad . \quad . \quad . \quad (3)$$

For strict conformity with (2), we ought, of course, to divide the quantity on the right by the original magnitude of the angle. But owing, probably, to the fact that γ, as defined in (3), is already a non-dimensional scalar quantity, it has become customary to dispense with this operation, and to define γ as *the value of the expression* (3) *when the original angle* CAE *is specified to be a right angle.* We thus obtain the concept of a second type of strain, to which is usually given the term "shear-strain," or "slide."

§ (4) Inter-relation of Stress and Strain. Hooke's Law. Principle of Superposition.—Making use of the terms which we have thus defined, we may say that the theory of elasticity is concerned with the determination of the stresses and strains which occur in a body under the action of applied forces, and the fundamental assumption explained in § (2) may be correspondingly expressed by saying that a perfectly definite stress will accompany any given strain, and *vice versa.* We must now refer to another assumption, totally different both in nature and importance, although in the development of the theory it has become almost as fundamental as the assumption of perfect elasticity. This asserts that *the relation between stress and strain is one of direct proportionality.* That the assumption is representative of actual materials was first discovered, in 1678, by Hooke, and the relation is for this reason commonly known as "Hooke's Law."

To give precision to "Hooke's Law," the constant of proportionality requires to be stated, and we must therefore consider what are the possible types of stress and of strain. We have seen in § (3) that the total stress across any imaginary surface in a body can be resolved into components by the vector law. Let us then resolve in directions normal and tangential to the surface. The first component may be termed a "normal stress," since it constitutes an action between the two portions of material which lie on opposite sides of the surface, tending to prevent their relative motion in a direction normal to the surface: if the action tends to prevent their *separation* it is termed a "tensile stress," and if it tends to resist their *approach,* a "compressive stress." The convenience of these terms is obvious, since they describe the stresses which occur on cross-sections of a straight rod under the action of end tension and compression respectively.

For a component stress of which the direction is parallel to the surface it is usual to employ the terms "tangential" or "shear stress." These terms describe an action between the two portions of material lying on opposite sides of the surface, which tends to prevent them from *sliding* relatively to one another, without separation; it will be realised, therefore, that shear stress is brought into action, by solid friction, at the surfaces of two bodies which are in contact and sliding relatively to one another.

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[1] The term "extension" is also used to denote this quantity.

Before proceeding further, it will be convenient to introduce a notation for the component stresses which we have just discussed. Let us consider the action at the face CDHG of the elementary parallelopiped of *Fig.* 1, and let the force exerted on the parallelopiped by the material which lies on the z side of this face be resolved into components P_x, P_y and P_z, parallel to the axes Ox, Oy, Oz respectively. Corresponding to the components P_x and P_y we have tangential stresses, which we may denote by Z_x and Z_y respectively; and corresponding to P_z we have a normal stress, which we denote similarly by Z_z. It will be noticed that in this notation the capital letter defines the face on which the stress in question acts, whilst the *suffix denotes the direction of the force exerted.*

Passing through every point in the material we have three mutually perpendicular faces of the kind just considered, and on each there are three independent components of stress which require symbols. Nine stress-components are thus introduced, namely, three normal stresses, X_x, Y_y, Z_z, and six tangential stresses, X_y, X_z, Y_z, Y_x, Z_x, Z_y. But the last six components can be reduced in effect to three, since we may show that

$$X_y=Y_x, \quad Y_z=Z_y, \quad Z_x=X_z. \quad . \quad . \quad (4)$$

These relations may be proved by considering the equilibrium of an elementary parallelopiped, such as is shown in *Fig.* 2, which requires, *inter alia*, that there shall be no tendency to rotate. A little reflection shows that body forces such as are exerted by gravity, if of finite intensity, have a negligible turning effect, and that the only components of stress which have a tendency to turn the parallelopiped about the axis YY (if the dimensions of the faces are so small that the resultant force corresponding to any stress can be assumed to act at the centre of the face affected) are the components X_z, on the two faces which are normal to the x-axis, and the components Z_x, on the two faces which are normal to the z-axis. The total forces contributed are, for the former stresses, of magnitude (X_z . AC . AD) (we have to multiply the stresses by the areas of the faces upon which they act), and for the latter, of magnitude (Z_x . AB . AC), as shown. Now the first two forces act in opposite directions along lines which are a distance AB apart, and the second in opposite directions along lines which are a distance AD apart: thus the first pair produces a couple of magnitude (X_z . AB . AC . AD), and the second a couple of magnitude (Z_x . AB . AC . AD), and (as is evident from the diagram) of opposite sign. The condition of equilibrium therefore requires that

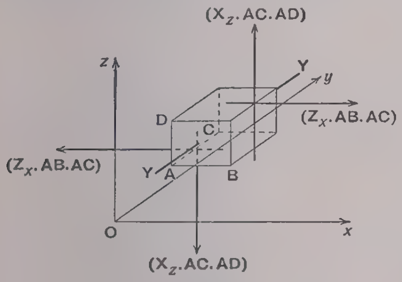

FIG. 2.

$$Z_x . AB . AC . AD = X_z . AB . AC . AD,$$

whence the third of the relations (4) follows directly, and the other two relations by similar reasoning.

It can be shown that the stress-components defined as above, and reduced in number, by the relations (4), to six, are sufficient for representing the most general system of stress which can obtain at any point in a body. By means of certain "formulae of transformation," the stress-components on any other plane through the point can be written down, and it may be shown that in any possible system of stress there will be three planes through any point, mutually perpendicular, on which the stresses are purely normal. Hence, in proceeding to derive the exact stress-strain relations required to give precision to Hooke's Law, we may confine our attention to a system of three mutually perpendicular normal stresses.

Notation is similarly required for the different components of *strain.* In § (3) we considered two distinct types, to which we gave the names "stretch" and "slide." We now introduce the notation e_{xx} for the stretch in the direction of the axis Ox—the double suffix indicating that the strain in question is a relative displacement of two planes, each of which is perpendicular to the axis of x; and in conformity with this notation we employ the symbol e_{yz} for the angle γ defined in (3)—the double suffix here indicating that the strain in question is a relative displacement of planes which are perpendicular to the axes y and z respectively. It is obvious that

$$e_{xy}=e_{yx}, \quad e_{yz}=e_{zy}, \quad e_{zx}=e_{xz}, \quad . \quad . \quad (5)$$

so that corresponding to our six distinct components of stress we have six distinct components of strain, and these may be shown to be sufficient for defining the most general type of strain which can obtain at any point. By means of "formulae of transformation" similar to those which we have noticed as holding for stresses, we can express in terms of e_{xx}, . . ., e_{yz}, . . ., etc., the strain-components corresponding to any other system of axes, and we may prove that in any possible system of stress there will exist three directions through any point which are mutually perpendicular, both before and after strain: the strains in these directions are termed "principal strains" at the point considered, and the directions themselves are termed "principal directions of strain."

Considerations of symmetry show that the principal directions of strain will always coincide, in materials which are *isotropic,—i.e.* which exhibit similar properties in all directions,—with the directions of the three purely normal stresses to which we have referred above; we shall therefore define our stress-strain relations completely if we can write down relations between the three "principal stresses," as they are generally called, and the corresponding "principal strains." We begin by considering the strain system which is involved by a simple tensile stress, of amount

T_1. This stress will evidently be one of the three principal stresses at the point considered, and the other two will be zero; it might be expected, therefore, that two of the three corresponding principal strains will vanish. Experiment shows, however, that the facts are different: the tension T_1 is accompanied by a proportionate stretch in its own direction, together with proportionate *contractions* (*i.e.* stretches of negative sign) in directions perpendicular to this. That is to say, if we choose our axis of x to coincide with the direction of T_1, then corresponding to the stress-system

$$X_x = T_1,\ Y_y = Z_z = 0,$$

we have the strain-system

$$\left.\begin{aligned} e_{xx} = \frac{T_1}{E},\ e_{yy} = e_{zz} = -\sigma e_{xx} = -\sigma\frac{T_1}{E}, \end{aligned}\right\} \quad (6)$$

where E and σ, by Hooke's Law, are constants of the material.

The quantity E is known as *Young's Modulus*. It evidently has the dimensions of a stress, and may in fact be defined as the stress which, acting alone, would involve a stretch of magnitude 1—*i.e.* a doubling of the distance between any two points in the material—measured in its own direction.[1] On the other hand, the quantity denoted by σ is non-dimensional, being the ratio of the lateral contraction $(-e_{yy})$ to the longitudinal extension (e_{xx}): it is known as *Poisson's Ratio*, and the fact that it has finite values in actual materials introduces very considerable complexity into the calculations of elastic theory.

Similar expressions will give the strain-systems which correspond to tensile stresses acting in the directions of Oy or Oz; thus, corresponding to the stress-system

$$X_x = 0,\ Y_y = T_2,\ Z_z = 0,$$

we have the strain-system

$$\left.\begin{aligned} e_{xx} = -\sigma\frac{T_2}{E},\ e_{yy} = \frac{T_2}{E},\ e_{zz} = -\sigma\frac{T_2}{E}, \end{aligned}\right\} \quad (7)$$

and corresponding to the stress-system

$$X_x = Y_y = 0,\ Z_z = T_3,$$

we have the strain-system

$$\left.\begin{aligned} e_{xx} = e_{yy} = -\sigma\frac{T_3}{E},\ e_{zz} = \frac{T_3}{E}. \end{aligned}\right\} \quad . \quad (8)$$

The most general system of stress, as we have seen, will involve three principal stresses at every point, and by a suitable choice of axes it can be written in the form

$$\left.\begin{aligned} &X_x = T_1,\ Y_y = T_2,\ Z_z = T_3, \\ &X_y = Y_z = Z_x = 0. \end{aligned}\right\} \quad . \quad (9)$$

We can at once write down the corresponding strain-system, by means of the relations (6)-(8) above, *if we may assume that each component stress is accompanied by the same strains, whether it acts alone or in conjunction with others.* This assumption cannot be completely justified on *a priori* grounds, but all experimental evidence supports it, and it has become one of the foundations upon which elastic theory has been built up. It is known as the *Principle of Superposition.*

We adopt, therefore, for the strain-system corresponding to (9) the following expressions:

$$\left.\begin{aligned} e_{xx} &= \frac{1}{E}\{X_x - \sigma(Y_y + Z_z)\}, \\ e_{yy} &= \frac{1}{E}\{Y_y - \sigma(Z_z + X_x)\}, \\ e_{zz} &= \frac{1}{E}\{Z_z - \sigma(X_x + Y_y)\}, \\ e_{xy} &= e_{yz} = e_{zx} = 0, \end{aligned}\right\} \quad . \quad (10)$$

and from what has been said above it will be evident that in these equations we have a complete and definite statement of the stress-strain relations, in an isotropic material, of which Hooke's Law is the qualitative expression. It is easy to deduce from them the following alternative forms of the stress-strain relations:

$$\left.\begin{aligned} X_x &= \frac{(1-\sigma)E}{(1+\sigma)(1-2\sigma)}\left\{e_{xx} + \frac{\sigma}{1-\sigma}(e_{yy} + e_{zz})\right\}, \\ Y_y &= \frac{(1-\sigma)E}{(1+\sigma)(1-2\sigma)}\left\{e_{yy} + \frac{\sigma}{1-\sigma}(e_{zz} + e_{xx})\right\}, \\ Z_z &= \frac{(1-\sigma)E}{(1+\sigma)(1-2\sigma)}\left\{e_{zz} + \frac{\sigma}{1-\sigma}(e_{xx} + e_{yy})\right\}, \\ X_y &= Y_z = Z_x = 0. \end{aligned}\right\} \quad (11)$$

§ (5) STRESS-STRAIN RELATIONS IN THE GENERAL CASE.—Equations (11) give the principal stresses in terms of the principal strains. They are frequently written in the simpler form

$$\left.\begin{aligned} X_x &= \lambda\Delta + 2\mu e_{xx}, \\ Y_y &= \lambda\Delta + 2\mu e_{yy}, \\ Z_z &= \lambda\Delta + 2\mu e_{zz}, \\ \text{where}\quad \lambda &= \frac{\sigma E}{(1+\sigma)(1-2\sigma)},\quad 2\mu = \frac{E}{(1+\sigma)}, \\ &\text{and } \Delta \text{ denotes the quantity} \\ &e_{xx} + e_{yy} + e_{zz}, \end{aligned}\right\} \quad (12)$$

which is known as the *dilatation*. Obviously, Δ represents the fractional change of volume which will be produced in an elementary parallelopiped of the material, as the result of the three strains e_{xx}, e_{yy}, and e_{zz} occurring simultaneously, if we may regard these strains as small; for this fractional change is given by

$$\begin{aligned} \frac{\delta V}{V} &= (1 + e_{xx})(1 + e_{yy})(1 + e_{zz}) - 1, \\ &= e_{xx} + e_{yy} + e_{zz}, \end{aligned}$$

if we neglect small quantities of order higher than the first.

By addition of the three equations (12) we have

$$X_x + Y_y + Z_z = (3\lambda + 2\mu)\Delta,$$

---

[1] This is on the assumption that the elastic properties of the material are not impaired by the action of a stress of the magnitude considered; in actual materials, as is stated later (§ (7)), failure of elasticity would occur at a very much smaller stretch.

and if the three stresses X_x, Y_y, and Z_z are equal, and of magnitude T, this equation may be written in the form

$$\Delta = \frac{3T}{3\lambda + 2\mu}, \quad . \quad . \quad . \quad (13)$$

which gives the fractional increase of volume caused by a stress of uniform intensity T in all directions. A stress of this nature, but negative in sign, is produced by the action of hydrostatic pressure; the fractional change of volume is also negative, *i.e.* a contraction; and the quantity $\frac{1}{3}(3\lambda + 2\mu)$, which evidently corresponds in equation (13) to the quantity E in the expression (6) for e_{xx}, is by analogy termed the *Modulus of Compression*, or *Bulk Modulus*, of the material. It is often convenient to have a special symbol for the quantity, although it is not an independent constant of the material, and the symbol usually employed is K: we have, from (12) and (13),

$$K = \frac{1}{3}(3\lambda + 2\mu) = \frac{E}{3(1 - 2\sigma)}. \quad . \quad (14)$$

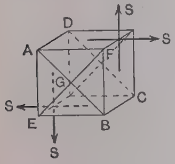

FIG. 3.

Another constant of frequent occurrence, also expressible in terms of E and σ, is the "modulus of rigidity," often denoted by C or N. We may conceive a stress-system such that a small cube of the material is completely free from stress on two opposite faces, and subjected to simple shear, of intensity S, on the other four: the relations (4) show that the shears on one pair of opposite faces must be equal to the shears on the other pair, and we may therefore take *Fig.* 3 as representing the stress-system. If we consider the corresponding stress across the diagonal surface ABCD, it is easy to show, from the condition for equilibrium of either of the two parts into which this surface divides the cube, that it will be purely normal, tensile, and of intensity S; similarly, the stress on the other diagonal plane will be purely normal, compressive, and of intensity S. If, therefore, we take our axes of x, y, and z parallel to EF, BA, AD respectively, we see that the stress-system will be given by

$$\left.\begin{array}{l} X_x = S, \quad Y_y = -S, \quad Z_z = 0, \\ X_y = Y_z = Z_x = 0, \end{array}\right\} \quad . \quad (15)$$

and the corresponding strain-system, by (10), will be given by

$$\left.\begin{array}{l} e_{xx} = \dfrac{1+\sigma}{E} \, . \, S = -e_{yy}, \\ e_{zz} = 0. \end{array}\right\} . \quad . \quad (16)$$

Thus we see that the diagonal EF will lengthen by a fractional amount $(1 + \sigma)(S/E)$, and that the diagonal AB will contract by a like amount. The two diagonals will remain perpendicular, and if dashes indicate positions after strain, we have

$$\tan A'E'G' = \frac{A'G'}{E'G'} = \frac{(1 + e_{yy})AG}{(1 + e_{xx})EG} = \frac{1 + e_{yy}}{1 + e_{xx}}.$$

Now the change produced by strain in the right angle AEB is obviously equal to

$$\frac{\pi}{2} - 2(A'E'G'),$$

and this change is the angle γ of shear strain, or "slide," as defined in equation (3) of § (3). Thus we have

$$\frac{\gamma}{2} = \frac{\pi}{4} - \tan^{-1}\left(\frac{1 + e_{yy}}{1 + e_{xx}}\right),$$

whence
$$\tan\frac{\gamma}{2} = \frac{e_{xx} - e_{yy}}{2 + e_{xx} + e_{yy}}, \\ = \tfrac{1}{2}(e_{xx} - e_{yy}),$$

if we regard the strains as small, and neglect small quantities of the second order. To the same approximation, we may write $\gamma/2$ for $\tan\gamma/2$; we thus obtain, finally,

$$\begin{aligned} \gamma &= e_{xx} - e_{yy}, \\ &= 2(1 + \sigma)\frac{S}{E}. \quad . \quad . \quad . \quad (17) \end{aligned}$$

The "modulus of rigidity," by analogy with the definition of E which has been given above, is defined as the intensity of shear stress required to produce a slide of amount 1. Hence the slide γ which corresponds to a shear stress S is given by

$$\gamma = \frac{S}{C}, \quad . \quad . \quad . \quad . \quad (18)$$

and by comparing (17) with (18) we see that

$$C = \frac{E}{2(1 + \sigma)}: \quad . \quad . \quad . \quad (19)$$

that is to say, the modulus of rigidity C (or N) is identical with the constant μ of equation (12).

The results expressed by (14) and (19) enable us to impose certain limits upon the values which are possible for σ. It is clear that the three moduli denoted above by E, C, and K must all be positive: otherwise, it would be possible to obtain an indefinite supply of energy from elastic material by putting it through an appropriate cycle of stress, and the principle of Conservation of Energy would be violated. It follows that the two ratios K/E and C/E, and hence the quantities $1 - 2\sigma$ and $1 + \sigma$, are necessarily positive: that is to say, σ must lie within the range given by

$$-1 < \sigma < 0{\cdot}5.$$

So far as is known, no material exhibits a negative value of σ; but it is of interest to note that such values are not *a priori* impossible.

If our axes of x and y had been taken parallel to the sides, EB, EA, instead of to the diagonals, EF, BA, of the cube, the stress-system of *Fig.* 3 could have been expressed in the form

$$\left.\begin{array}{l} X_y = S, \quad Y_z = Z_x = 0, \\ X_x = Y_y = Z_z = 0, \end{array}\right\} . \quad . \quad (20)$$

and the corresponding strain system, from what has just been said, would have been given by

$$\left.\begin{aligned} e_{xy}=\frac{S}{\mu},\ e_{yz}=e_{zx}=0, \\ e_{xx}=e_{yy}=e_{zz}=0. \end{aligned}\right\} \quad . \quad . \quad (21)$$

The strains corresponding to shear stress-components Y_z or Z_x could be similarly written down, and by the Principle of Superposition these may be combined with one another, and also with the expressions (11) or (12), which in § (4) were derived on the assumption that X_x, Y_y and Z_z were principal stresses—*i.e.* that X_y, Y_z and Z_x were zero; for the principle asserts that any one stress-component has a definite expression in terms of the strain-components, which will not be affected by the coexistence of any other. Thus, in the general case, where the axes of *x*, *y* and *z* do *not* coincide with the directions of the principal stresses, we have the following relations between stress and strain:

$$\left.\begin{aligned} X_x=\lambda\Delta+2\mu e_{xx},\quad X_y=\mu e_{xy}, \\ Y_y=\lambda\Delta+2\mu e_{yy},\quad Y_z=\mu e_{yz}, \\ Z_z=\lambda\Delta+2\mu e_{zz},\quad Z_x=\mu e_{zx}. \end{aligned}\right\} \quad . \quad (22)$$

The most general expression of Hooke's Law, in isotropic material, is thus seen to involve only two elastic constants.

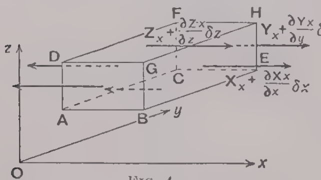
F
H
$Z_x+\frac{\partial Z_x}{\partial z}\delta z$
$Y_x+\frac{\partial Y_x}{\partial y}\delta y$
z
D
E
G
C
$X_x+\frac{\partial X_x}{\partial x}\delta x$
A
B
y
O
x

FIG. 4.

§ (6) DYNAMICAL EQUATIONS IN TERMS OF STRESS.—We have now expressed the purely empirical law of Hooke in a form which is mathematically convenient, and we may proceed to apply our results to the analysis of stress and strain in elastic solids. The motion of any portion must be governed by the ordinary laws of dynamics; that is to say, its acceleration will be determined by the resultant unbalanced force which acts upon it, and by its inertia. If we consider the portion contained within the boundaries of a small parallelopiped, as shown in *Fig.* 4, it is clear that we can express the unbalanced force exerted by the surrounding material in terms of the stress-components defined above. Consider, in the first place, the component of this force which acts in the direction of the axis O*x*. The stress-components which contribute to it are X_x, Y_x and Z_x, acting on the faces shown. The components X_x act on faces whose area is $\delta y \times \delta z$, and which are a distance δx apart, so that the magnitudes of the stresses on the two faces (which clearly tend to pull the parallelopiped in opposite directions) differ by the amount $(\partial X_x/\partial x)\delta x$; hence, their combined contribution to this force is

$$\frac{\partial X_x}{\partial x}\,.\,\delta x\,.\,\delta y\,.\,\delta z.$$

The contributions of the other stresses are given, similarly, by

$$\frac{\partial Y_x}{\partial y}\,.\,\delta x\,.\,\delta y\,.\,\delta z$$

and

$$\frac{\partial Z_x}{\partial z}\,.\,\delta x\,.\,\delta y\,.\,\delta z,$$

and hence we see that the total unbalanced force on the parallelopiped, in the direction O*x*, is given by

$$\left(\rho X+\frac{\partial X_x}{\partial x}+\frac{\partial X_y}{\partial y}+\frac{\partial X_z}{\partial z}\right)\delta x\,.\,\delta y\,.\,\delta z,$$

where ρX is the body-force, per unit volume, acting at the point considered: the substitution of X_y for Y_x is justified by the relations (4) above.

But the mass of the material contained within the parallelopiped is $\rho\,.\,\delta x\,.\,\delta y\,.\,\delta z$, where ρ is the density; hence, if f_x is its acceleration in the direction O*x*, we have as the equation of motion in this direction

$$\rho X+\frac{\partial X_x}{\partial x}+\frac{\partial X_y}{\partial y}+\frac{\partial X_z}{\partial z}=\rho\,.\,f_x,$$

and the equations

$$\rho Y+\frac{\partial X_y}{\partial x}+\frac{\partial Y_y}{\partial y}+\frac{\partial Y_z}{\partial z}=\rho\,.\,f_y$$

and

$$\rho Z+\frac{\partial Z_x}{\partial x}+\frac{\partial Y_z}{\partial y}+\frac{\partial Z_z}{\partial z}=\rho\,.\,f_z \quad . \quad (23)$$

can be obtained similarly.

Equations (23) must obviously be satisfied at every point in a body, independently of any assumption regarding its elasticity. No additional equations are required to define the rotational motion of the parallelopiped, which may ultimately be regarded as infinitesimal, and so treated as a particle. The absence of any resultant turning tendency has been ensured already, by the relations (4).

§ (7) STRAINS EXPRESSED IN TERMS OF DISPLACEMENT.—When the stress-strain relations are known for the material considered, we can express the quantities (other than X, Y and Z) on the left of equations (23) in terms of strain-components; but the equations will still be intractable mathematically, unless we can find a common system of variables in terms of which they may be completely expressed; and the fact that the acceleration components f_x, f_y, f_z may be expressed in terms of the component displacements of the point considered indicates that we should endeavour also to relate the six strain-components to these quantities.

Let *u*, *v*, *w* denote the component displacements of any point (x, y, z) in the directions Ox, Oy, Oz respectively; then

$$f_x = \frac{\partial^2 u}{\partial t^2}, \quad f_y = \frac{\partial^2 v}{\partial t^2}, \quad f_z = \frac{\partial^2 w}{\partial t^2}. \quad . \quad (24)$$

The strain-component e_{xx} denotes the stretch in the direction Ox, and this term was defined in § (3) as the limiting value of the expression (2) when AB is parallel to Ox and indefinitely reduced. We may now write δx for the length AB in the denominator; the numerator is the total increase in the length of AB which results from strain, and this will clearly be equal to the amount by which the displacement of the point B exceeds the corresponding displacement of the point A—*i.e.* to the quantity $(\partial u/\partial x) . \delta x$. Thus, in the limit, when δx is indefinitely reduced, we have

$$e_{xx} = \frac{\partial u}{\partial x},$$

and similarly

$$\left.\begin{aligned} e_{yy} = \frac{\partial v}{\partial y}, \quad e_{zz} = \frac{\partial w}{\partial z}. \end{aligned}\right\} \quad . \quad . \quad (25)$$

The strain-component e_{yz} denotes the slide in the (y, z) plane, *i.e.* the value of γ in equation (3), when AE is initially parallel to the axis of y, and AC to the axis of z. It is the angle by which AC and AE approach one another, and this obviously is the sum of (i.) the angle at which A′E′ is inclined, after strain, to the y-axis, and (ii.) the angle at which A′C′ is inclined, after strain, to the z-axis. By reasoning similar to what has been given above, we may show that the first angle is of magnitude $\partial w/\partial y$ (we shall always be concerned with strains which are very small, and hence it is unnecessary to distinguish between the angle and its tangent) and the second of magnitude $\partial v/\partial z$; hence we have

$$e_{yz} = \frac{\partial w}{\partial y} + \frac{\partial v}{\partial z},$$

and similarly

$$e_{zx} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}, \quad . \quad . \quad (26)$$

and

$$e_{xy} = \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}.$$

Making use of the results expressed in equations (22) and (24)-(26), we can write the equations of motion (23) in terms of the three variables u, v, w, and of constants which are known for the material considered. We thus obtain the result given in equations (28).

The fact that all six of the strain-components can be expressed in terms of the three component displacements u, v, w, indicates that these six quantities are to some extent interconnected; that is to say, if we assign an arbitrary expression to each strain-component, we shall not in general obtain a possible distribution of strain. It is not difficult to see that the reason why such arbitrary expressions are not permissible is that they violate the conditions for continuity of the material after strain.

The necessary and sufficient relations which must be satisfied by the strain-components, in order that the corresponding system of displacement may be a possible one, can be written as follows:

$$\left.\begin{aligned}
&\frac{\partial^2 e_{yy}}{\partial z^2} + \frac{\partial^2 e_{zz}}{\partial y^2} = \frac{\partial^2 e_{yz}}{\partial y \partial z}, \quad 2\frac{\partial^2 e_{xx}}{\partial y \partial z} = \frac{\partial}{\partial x}\left(-\frac{\partial e_{yz}}{\partial x} + \frac{\partial e_{zx}}{\partial y} + \frac{\partial e_{xy}}{\partial z}\right), \\
&\frac{\partial^2 e_{zz}}{\partial x^2} + \frac{\partial^2 e_{xx}}{\partial z^2} = \frac{\partial^2 e_{xz}}{\partial z \partial x}, \quad 2\frac{\partial^2 e_{yy}}{\partial z \partial x} = \frac{\partial}{\partial y}\left(\frac{\partial e_{yz}}{\partial x} - \frac{\partial e_{zz}}{\partial y} + \frac{\partial e_{xy}}{\partial z}\right), \\
&\frac{\partial^2 e_{xx}}{\partial y^2} + \frac{\partial^2 e_{yy}}{\partial x^2} = \frac{\partial^2 e_{xy}}{\partial x \partial y}, \quad 2\frac{\partial^2 e_{zz}}{\partial x \partial y} = \frac{\partial}{\partial z}\left(\frac{\partial e_{yz}}{\partial x} + \frac{\partial e_{zx}}{\partial y} - \frac{\partial e_{xy}}{\partial z}\right).
\end{aligned}\right\} \quad (27)$$

These equations are generally known as the "Conditions of Compatibility for Strain-Components." They can be verified by substitution from (25) and (26), and are obviously independent of any assumption in regard to the properties (other than continuity) of the material.

§ (8) Equations of Motion.—Reviewing the position reached in the preceding paragraphs, we notice that—

(*a*) The equations which express the equilibrium or motion of the material contained within any elementary parallelopiped may be written down from considerations of statics or of dynamics, in terms of its density and of the stresses which act upon its faces;

(*b*) The component velocities and accelerations of the contained material can be expressed in terms of the component displacements u, v, and w;

(*c*) The strains (or changes in the sides and angles of the parallelopiped) may be expressed in terms of the same three quantities, from geometrical considerations alone;

(*d*) An innovation is introduced in the theory of elasticity, by the assumption of relations between stress and strain which enable us to substitute strains for stresses in the equations of equilibrium or of motion, and thereby to express these equations solely in terms of the relative displacements of different points in the body.

The resulting equations of motion, when the body-forces are zero or negligible, may be written in the form

$$\left.\begin{aligned}
(\lambda + \mu)\frac{\partial \Delta}{\partial x} + \mu \nabla^2 u &= \rho \frac{\partial^2 u}{\partial t^2}, \\
(\lambda + \mu)\frac{\partial \Delta}{\partial y} + \mu \nabla^2 v &= \rho \frac{\partial^2 v}{\partial t^2}, \\
(\lambda + \mu)\frac{\partial \Delta}{\partial z} + \mu \nabla^2 w &= \rho \frac{\partial^2 w}{\partial t^2},
\end{aligned}\right.$$

where ∇^2 denotes the operator

$$\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}, \quad . \quad (28)$$

and the "dilatation"

$$\Delta = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}.$$

When the displacements u, v, w, are steady, the terms on the right of these equations vanish; we are then left with "equations of equilibrium."

For some purposes it is convenient to replace the rectangular (or Cartesian) system of coordinates which we have employed above by other systems, such as polar coordinates. For particulars of such systems, reference must be made to the authors cited at the foot of this article: it is sufficient here to state that the motion can always be defined by three variables, between which three independent relations may be found from the equations of motion for the material contained within an elementary volume.

The equations (28) will be satisfied at every point in an elastic body, on the assumption that the relation between stress and strain may be expressed as in § (5) above (*i.e.* that it is linear, and independent of the previous stress-history of the material), *and that the strains occurring in the body are everywhere small.* The latter assumption is necessary both to justify the equations of equilibrium as expressed in terms of stresses, and to give precision to the relationship between stress and strain: fortunately, it imposes no serious restriction upon the extent to which our results may be applied in practice, since it is found that the strains produced in actual materials, by any stress which they are able to sustain elastically, are always extremely small. Remembering this restriction upon the validity of the equations, we may draw a deduction of great practical importance from their form; for it is evident that when any two solutions are combined in any proportion, the resulting expressions for the displacements will also be solutions of the equations. This is, of course, a restatement, in a general and mathematical form, of the "Principle of Superposition" which we have noticed, as an experimentally established law, in §§ (4) and (5) above.

We have said that the general problem in the theory of elasticity is to determine the relative displacements of different points in a given body, produced by forces which are specified as acting either on its surfaces or throughout its volume: forces of the first type are termed *surface tractions*, and of the second type *body forces.* A second form of the problem may now be mentioned, in which the *displacements* of the surface are specified: body forces may also be assumed to act, and in general the specified conditions may include specified tractions at some points of the boundary, and specified displacements at others. The surface displacements or tractions may be specified by their components perpendicular and parallel to the surface, but the displacements can obviously be resolved along any specified direction (and hence expressed in terms of u, v, and w), and the formulae for transformation of stress-components (referred to in § (4) above) enable us to express boundary conditions which consist of specified surface-tractions in terms of the stress components, $X_x, \ldots, X_y, \ldots$ etc., and hence in terms of u, v, and w. Mathematically, therefore, our general problem is to determine the forms of three functions, u, v, w, which must satisfy the equations (28) of motion or equilibrium at every point in the body, and which are subject to certain boundary conditions at the surfaces. If such functions can be found, we are in a position immediately to deduce the strains and stresses which occur at every point in the elastic solid considered, and its behaviour is then completely defined; for Kirchhoff has shown that any solution of the equations of equilibrium, which also satisfies the specified boundary conditions, is unique, and Neumann has extended his theorem to the equations of motion, by showing that a solution of these equations which satisfies specified initial conditions in regard to displacement and velocity is also unique.

For practical purposes, we are often concerned almost entirely with the distribution of *stress* in the interior of an elastic body, and the displacements and strains which accompany the stresses are of little interest. When the surface tractions are specified, it is, therefore, evident that a considerable gain in point of convenience might be expected from methods which would enable us to calculate the stresses directly, without introducing the quantities u, v, w. Much attention has been devoted in recent years to this problem, and methods for the "direct determination of stress" are coming into more and more frequent use. Their principle consists in expressing the different components of stress in terms of one or more common functions, which are determined by characteristic differential equations obtaining at every point, and by appropriate boundary conditions: for details, reference must be made to the authors cited.

§ (9) APPLICATION OF THEORY TO ENGINEERING DESIGN.—The theory, of course, possesses an interest, from the purely mathematical standpoint, which is independent of its practical applications; but for the purpose of the present article it is necessary here to inquire what is its value to the physicist and to the engineer. The immediate answer, from the point of view of the engineer, is that the calculation of stresses is an essential preliminary to the design of structural members of adequate strength. Actual materials, as has been stated above, are elastic only so long as the stresses to which they are subjected lie within certain definite limits: if these limits are exceeded, permanent distortion occurs, by processes of great complexity, about which it is unnecessary to say more here than that they are invariably accompanied by more or less serious deterioration of the materials' capacity for resisting stress. The problem of deciding whether the calculated stress-system will or will not produce this "failure of elasticity" in a given material is one with which the theory of elasticity, in its strict sense, has nothing to do: it is fully discussed in the article on "Theory of Structures," and it is sufficient here to remark that the theory of elasticity supplies the mathematical basis for the calculation of stresses; that the science of Testing of Materials (see article under this heading) supplies experimental data in regard to the strength of the materials of construction; and that the Theory of Structures employs the information thus provided in the practical science of "Design."

§ (10) SOME GENERAL THEOREMS. — Apart from questions of strength, the practical utility of the theory of elasticity will be most easily judged from an account of the problems which have been solved up to the present time by its means. We may notice, in the first place, certain general theorems which have been established. First, it has been shown that the stresses set up by a load which is suddenly applied may be as much as twice as great as those which would be produced by a gradual application of the same load, and that if a load be suddenly *reversed* the stresses may be trebled; this result has, of course, a very great importance for engineering design. Again, the effects of small flaws in materials have been investigated, and it has been shown, for example, that if a member which is subjected to simple tension or compression in one direction contain a small spherical flaw at some point in its interior, the tension or compression in the material will be approximately doubled at certain points on the surface of the flaw; if the flaw have the form of a circular cylinder, with its axis perpendicular to the direction of the tension, the stresses will be trebled.

Again, the effect of an impulsive pressure at the surface of a body has been investigated in general terms, and it has been shown that waves of stress can be propagated in an elastic solid which, if the solid is isotropic, may be of two types, propagated with different velocities. The first is a wave of dilatation, involving an alteration in the volume, but not in the shape, of each element of the material as the wave passes it; such waves are propagated with a velocity $\sqrt{(\lambda+2\mu)/\rho}$. The second is a wave of distortion, involving change of shape, but no change of volume in each element affected; such waves travel with velocity $\sqrt{\mu/\rho}$. Further, it has been shown that a certain type of wave exists which is propagated over the surface of a solid body, and involves practically no disturbance in the interior; its velocity is a little less than that of the waves of distortion just referred to. These results have, of course, an important bearing upon the phenomena which occur in the collision of elastic solids, and in earthquakes.

Light has also been thrown upon the nature of the stresses produced by concentration of load at some point in a body, by means of exact solutions for certain particular examples; and a knowledge of the velocity with which a wave of stress is propagated along a thin cylindrical rod has been used by Hopkinson in devising apparatus for the measurement of the large impulsive pressures which are set up in the detonation of explosives.

§ (11) SPECIAL SOLUTIONS. PRINCIPLE OF ST. VENANT.—Turning now to the consideration of special solutions of the equations of equilibrium and of motion, we may notice that exact solutions have been found for several types of periodic vibration in spheres and circular cylinders, and that the equations of equilibrium have been solved in a form which provides exact knowledge of the stresses produced in a prismatic body of any cross-section, under certain particular systems of loading, of which the general effect is to produce extension, flexure, or twisting; these results, as will be seen later, have been extended to give an *approximate* theory of such actions, *without restriction upon the exact distribution of the loads which produce them*. We also possess exact solutions for the stresses set up in thick tubes, or in spherical shells, by uniform tractions applied to their surfaces, and for the stresses which are produced by the rotation about their axes of certain solids of revolution such as cylinders and thin discs.

In these exact solutions, it is sometimes necessary to assume that a small "auxiliary stress-system" acts at certain parts of the boundary, and the results thus fail to apply exactly to practical problems, in which such systems cannot be assumed to exist: but St. Venant has shown that the discrepancy which thus arises may be, for practical purposes, regarded as involving a slight and unimportant decrease of accuracy, rather than a loss of generality. The principle upon which he bases this conclusion, and which is usually designated by his name, states that any locally applied system of surface tractions, which is itself a completely equilibrating system, has a negligible influence upon the stresses, except at points of the body lying quite close to the region within which the system is applied.

§ (12) PROBLEMS OF THE SPHERE AND OF THE PLANE.—General solutions have also been obtained for the stresses produced by symmetrical distributions of surface traction acting upon circular cylinders or spheres, and the solutions found in the latter case have been applied to problems relating to the form of the earth, such as the dependence of its ellipticity of figure upon the diurnal rotation, and the relative displacements produced by the disturbing attractions of the sun and moon. The latter are analogous to the tidal motion of the sea relative to the land, and for this reason have been called "tides." Lord Kelvin has calculated what he has termed the "tidal effective rigidity of the earth,"—*i.e.* the rigidity which must be attributed to a homogeneous, incompressible solid sphere, of the same size and mass as the earth, in order that tides in a replica of the actual ocean resting upon it may be of the same height as the observed oceanic tides. His calculations led him to interesting speculations as to the constitution of the earth, which do not, however, appear to be entirely supported by evidence obtained from tidal phenomena.

Finally, we may notice that solutions have been obtained for the effects of certain particular distributions of surface traction upon a body of infinite extent, having one plane boundary; it does not appear, however, that these results are of much practical importance.

§ (13) APPROXIMATE RESULTS.—The practical value of the theory of elasticity has been extended less by discovery of exact solutions, such as have been referred to above, than by investigations which have shown that increased generality can be obtained at the cost of some slight loss in accuracy. For the practical purposes of physics or engineering, it will be recognised that generality is of much greater value than absolute accuracy, since a margin of safety has in any case to be provided, to meet such contingencies as faulty workmanship, corrosion, or local damage. We have already referred to the value of St. Venant's principle in this connection: one of its most important applications has been to the theory of such problems as the stresses in beams and thin plates. The exact solutions for the stresses produced by flexure in a uniform beam, to which reference has been made in § (11), show that the resultant action of the stresses occurring on any section of the beam may be expressed in terms of the extension and curvature, at that section, of its strained centre-line (*i.e.* the line through the centroids of cross-sections). It may be shown that St. Venant's principle justifies us in assuming that these expressions will obtain, with reasonable accuracy, even when the external forces are not applied in the manner postulated by the exact solutions, or when the shape and size of the cross-section change at different parts of the beam. Hence, instead of having to consider the motion of each infinitesimal element of the beam, we may write down equations for the motion as a whole of the material contained between two adjacent cross-sections, and we may express these in terms of the relative displacements of the corresponding points of the centre-line. The number of independent variables is thus reduced from three (the co-ordinates of any point in space) to one (the distance of any section, measured along the centre-line, from a fixed point), and a great increase of simplicity in the calculations is thus obtainable.

For an account of the ways in which these principles are applied by engineers, in calculating the deflections which occur in girders under the action of lateral loads, reference may be made to the article on "Theory of Structures." Similar methods enable us to calculate the frequency with which a beam of given dimensions will vibrate, and the results thus obtained are of importance, both in engineering design (where it is necessary to take precaution in advance against the dangers associated with "resonance"), and in the theory of sound. Corresponding simplification has been introduced into the calculations relating to thin plates or shells, and although such problems are necessarily of somewhat greater complexity, it is true to say that many problems can be solved by these approximate methods—with an accuracy which is quite adequate for practical purposes—which would be almost intractable by rigorous analysis. For the physical theory of sound, the late Lord Rayleigh's generalised treatment by approximate methods of problems relating to the vibration of elastic solids has proved particularly fruitful in results.

§ (14) ELASTIC STABILITY.—One class of problem still requires notice. We have referred in § (8) to a theorem of Kirchhoff, that any solution of the equations of equilibrium which also satisfies the specified boundary conditions is unique. If we combine with this the assumption of Hooke's Law, that the strain varies proportionately with the stress, we may conclude that any strained configuration which we can determine will be *stable*, since departure from that configuration must of necessity be accompanied by an increase in the total potential energy of the system. But the theorem of Kirchhoff, being based upon the general equations (28) of motion or equilibrium, depends with them upon the implied assumption, that the strain which occurs in an elastic body, as the result of applied forces, does not affect appreciably the stress which those forces bring into existence at any point. This assumption is in general legitimate: but it fails in some instances when applied to elastic solids, such as thin rods or plates, of which the dimensions are widely different in different directions. For example, if we imagine a straight shaft, which initially rotates about its axis, to deflect into a curved form, then additional stresses will be called into existence as a result of that deflection, since the centrifugal effect of rotation will tend to deflect the shaft still further.

At some definite speed of revolution this distorting effect will exactly neutralise the capacity which the shaft possesses, in virtue of its elasticity, of reverting to the straight form, and hence the deflection, if it occurs, will be maintained. The equilibrium of the straight configuration has in fact become neutral, and the shaft is said to "whirl."

A similar tendency towards "elastic instability" is exhibited by a long straight rod, or "strut," which is subjected to axial compression. At a certain definite value of this end load, the equilibrium of the straight configuration becomes neutral, and the load can hold the strut bent into a curved form. Both of the examples here cited are of the first importance in engineering design, and the reader who desires to have further information about them should refer to the article on the "Theory of Structures."

Elastic instability can also occur in thin circular tubes, when these are subjected either to uniform end compression or to hydrostatic pressure on their external surfaces (as occurs in practice in the flues of steam-boilers), or to torsion. It is then accompanied by the formation of waves, or corrugations, of very regular geometrical types, and the analysis, though difficult and lengthy as compared with that of the problems just described, is tractable and interesting, more particularly as regards the indications which it affords of precautions to be observed in practical work when structural members have to be built up from thin sheets of metal. The development of metal construction for aircraft has, in fact, given to this subject, which was formerly regarded as mainly of theoretical interest, a practical importance comparable with that of the more familiar problems of elastic theory.

§ (15) PHOTO-ELASTICITY AND SOAP-BUBBLE METHODS.—Compared with other branches of mathematical physics, the subject of elasticity is remarkable for the extent to which theory has developed in advance of its experimental verification. Indeed, a little reflection shows that such verification is a matter of the greatest difficulty. No means have been devised for measuring the strains in the interior of an opaque body, and a comparison of the breaking loads given by theory and by experiment fails to supply the required verification of theory, for the reason that fracture in actual materials is preceded by a more or less lengthy period of transition from the elastic to the plastic state, during which the relations between stress and strain fail to obey Hooke's Law (cf. § (9)).

The nearest approach to precise experimental verification which has yet been made employs the property of *photo-elasticity*. It is an experimental fact that an isotropic transparent body—such as glass (which has the merit that it obeys Hooke's Law with considerable precision)—becomes doubly-refracting when stressed, its optical principal axes at any point being coincident with the directions of the principal axes of stress at the point; hence, by the aid of polarised light, it has been found possible to study many two-dimensional systems of stress which are not amenable to mathematical analysis.

In such systems, the stress-components are expressible in terms of a function χ, determined by appropriate boundary conditions, and by the equation

$$\left[\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right]^2\chi=0, \quad . \quad . \quad (29)$$

which must be satisfied at every point of the (x, y) plane lying within the elastic solid considered. In the solutions obtained by St. Venant for the stresses produced in a prismatic solid by torsion or flexure (see § (11)), the stress-components are found to be expressible in terms of a function ψ which satisfies the simpler equation

$$\left[\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right]\psi=0. \quad . \quad . \quad (30)$$

Now equation (30) is satisfied by the normal displacements (when these are small) of an initially flat membrane which is subjected to equal tensions in all directions, and advantage has been taken of this fact by Taylor and Griffith, who have shown how to determine the function ψ, for prisms of any given cross-section, by measurements made on a soap film stretched across a closed boundary of appropriate form. Such experiments are not, of course, to be regarded as a verification of elastic theory, but as a means of extending its results: they are typical of the present trend of its development, which tends more and more in the direction of practically useful extension, even at the cost of some slight decrease in accuracy. R. V. S.

References.—The standard work on the subject is Professor A. E. H. Love's *Mathematical Theory of Elasticity*, which in its third edition (Cambridge, 1920) includes references to practically every paper of importance in the subject which has so far been published. Most of the problems which have been applied successfully to practice are described (in general, with diagrams which serve to show the order of magnitude of the stresses considered) in Dr. A. Morley's *Strength of Materials*, Sir J. A. Ewing's work of the same name, and other engineering treatises of similar scope. The following table of references, for individual paragraphs of the present paper, will serve as an acknowledgment of the use which has been made, in their preparation, of the works of Love and Morley:

§ (8) On the theorems of Kirchhoff and Neumann, cf. Love, *op. cit.* §§ 118, 124; on direct methods, cf. *ibid.* pp. 16, 17, and § 92.

§ (9) Cf. Morley, *op. cit.* chap. ii.

§ (10) Cf. Love, *op. cit.* § 84 and chap. xiii.; Morley, *op. cit.* chap. iii. and Appendix (4th edition); Hopkinson, *Phil. Trans. Roy. Soc.* A, 1914, ccxiii. 437.

§ (11) Cf. Love, *op. cit.* chaps. xii., xiv., xv., v. (for details of the solutions mentioned, in the order given above); also § 89, on St. Venant's principle.

§ (12) Cf. Love, *op. cit.* chaps. xi. and x.

§ (13) Cf. Love, *op. cit.* chaps. xvii.-xxiv. A, and Morley, *op. cit.* chaps. iv.-ix. and xii.-xiv. For details of Lord Rayleigh's method, cf. his *Theory of Sound*, vol. i.

§ (14) Cf. Love, *op. cit.* p. 30, §§ 286, 264-267A, 272 (*d*)-(*e*), and 341; Morley, *op. cit.* chaps. ix., xi., and xiv.

§ (15) Cf. Love, *op. cit.*, § 57 and footnote. Further references have been given by A. R. Low, *Aeronautical Journal*, Nov. 1918. On the method of Taylor and Griffith, cf. *Engineering*, 1917, civ. 655-699. Analytical methods for the solution of equations (29) and (30) have been propounded recently by Bairstow, *Proc. Roy. Soc.* A, xcv. 457-475; c. 394-413.

Electrical Brake — National Physical Laboratory Dynamometer. See "Dynamometers," § (2) (v.).

Electrical Constants. Table of most reliable values for use in determinations of mechanical equivalent of heat. See "Mechanical Equivalent of Heat," § (9).

Electrochemical Equivalent of Silver. See "Mechanical Equivalent of Heat," § (8). See also Vol. II.

Electrolysis, Thermodynamics of. See "Thermodynamics," § (64). See also Vol. II.

Elongation:
  Barba's Law for Geometrically Similar Test Bars. See "Elastic Constants, Determination of," § (15).
  Distribution in a Test Bar. See *ibid.* § (15).
  Variation with Cross-section. See *ibid.* § (17) (ii.).
  Variation with Gauge Length. See *ibid.* § (17) (i.).

Elongation and Contraction of Area. Method of Calculation. See "Elastic Constants, Determination of," § (23).

Emergent Stem, Correction, applicable to thermometers. See "Thermometry," § (9)

Emissivity of a Surface: a term used to denote the ratio of the heat emitted by unit area of the surface to that emitted by an equal area of a "full radiator" at the same temperature. See "Pyrometry, Total Radiation," § (18).

Energy corresponding to a given Wave-length in the Radiation from a Black Body used as a secondary standard of temperature in the range above 500° C. See "Temperature, Realisation of Absolute Scale of," § (41) (iv.).

Energy, Distribution of:
  In the Heat Emission Spectrum of the Metals, and Correction to Optical Pyrometer Readings. See "Pyrometry, Optical," § (17).
  In the Spectra of Platinum, Palladium, and Tantalum, determined by Optical Pyrometer. See *ibid.* § (23).
  In Spectrum of a "Full Radiator" at Various Temperatures, studied Experimentally. See *ibid.* § (2).

Energy, Measurement of, in Thermal Conductivity Determinations. See "Heat, Conduction of," § (9) (i.).

Energy of Ideal Gas and of Gaseous Mixtures. See "Thermodynamics," §§ (57) and (62).

Engine, Heat-. See "Thermodynamics, Definition of," § (1), "Reversibility of," §§ (18) and (19), "Efficiency of," § (20).

ENGINES AND PRIME MOVERS, THE BALANCING OF

§ (1) Preliminary.—An engine is said to be balanced when it runs without vibration.

Masses are added to the moving parts to secure this immunity from vibration, and these masses are called *Balance Weights.*

The importance of keeping the engine still by balancing the moving parts lies in the fact that if it vibrates it communicates its vibration to the foundation on which it is secured, and thus starts a wave which may spread to the surrounding buildings and cause trouble.

The trouble is very much increased if the foundation has a natural period of vibration equal to the periodic time of the vibration of the engine. Then the disturbance produced in the buildings is out of all proportion to the disturbance of the engine itself.

Synchronism of the engine vibration, or a harmonic of it, with the natural period of vibration of the foundation, or a harmonic of it, may thus cause troublesome or even dangerous disturbances in the buildings surrounding the station in which the engine is at work.

The balancing of marine engines when their speed was low and the period of vibration of the ship's hull high was not a pressing problem. But ships grew in size and engines increased in speed so that the natural period of vibration of the hull and the time of revolution of the engine approached synchronism and in some ships found it. Such ships were uncomfortable to travel in. Again the vibration produced by unbalanced engines in destroyers and cruisers was so noticeable that the designed speed in some ships had to be reduced because of the dangerous vibration produced.

The problem of balancing the engines so that they could work at any reasonable speed without producing vibration thus forced itself on the attention of engineers, and solutions exact, approximate, or roughly approximate, had to be found.

The necessity for balancing the moving parts of an engine arises from the manner in which these parts are compelled to move.

Change of speed or change of direction of a moving mass requires the action of an external accelerating force.

The moving parts of a machine are compelled by their mechanical connections to change continuously the speed and direction of their motion, and therefore accelerating forces are in continuous action on them.

These forces have equal and opposite reactions and these reactions ultimately appear as a system of forces acting on the frame of the engine.

The system is complex and changes from instant to instant, but the changes are periodic.

Balance weights are disposed on the moving parts so that, considered as a separate system, they have a resultant action on the frame as nearly as practicable equal and opposite to the resultant of the reactions produced by the motion of the parts.

A part may be balanced in such a way that the reaction supplied by the balance weight is applied directly to the part, and thus no force appears on the frame at all so far as that part is concerned.

A more detailed consideration of the main principles will make the matter clear.

§ (2) Frame Forces and the Balancing of them.—The moving parts of a machine are jointed together and with the frame in such a way that every point in every piece is constrained to move in a definite path. The constrained path is closed and the motion of the point in it is in general periodic.

The first part of the general problem is to find the forces which constrain a mass to move in a path defined for it by its mechanical connections.

The second part of the problem is to find how these forces may be directly or indirectly neutralised and so prevent their reactions acting on the frame.

Let us consider the example of constrained motion furnished by the crank and connecting rod mechanism, because the problems involved in the balancing of its moving parts illustrate the general principles involved in the balancing of any mechanism.

The crank OA (*Fig.* 1) is jointed with the frame at O. The frame being at rest, every point in the crank is compelled to move in a circle about O.

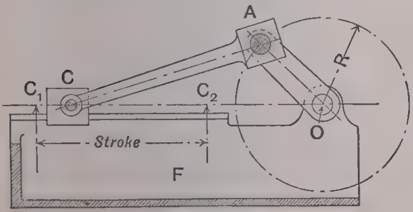

Fig. 1.

The reciprocating mass C is guided in a slide on the frame F and can move therefore only in a straight line.

The crank pin A is connected with the mass C by a rod AC, technically called the connecting rod.

However the mechanism is set in motion the path of every point in every link is defined.

The distance OA is the crank radius. The distance C_1C_2 is the stroke of the reciprocating mass and is equal to 2OA. The end points C_1 and C_2 are called the outer and inner dead points respectively.

The mechanism furnishes three separate problems for consideration.

(1) The motion of a mass in a circular path at uniform speed.

(2) The motion of a mass in a straight path at varying speed.

(3) The motion of a link as defined by the motion of two of its points.

(i.) *Motion of a Mass in a Circle at Uniform Speed.*—Let M (*Fig.* 2) be the mass; R the radius of the circle in which the mass centre moves, and ω the angular velocity of the mass about the centre of the circular path.

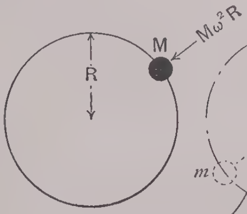

Fig. 2.

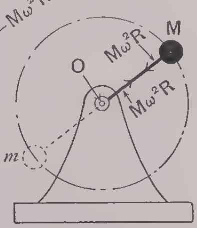

Fig. 3.

Then it is demonstrated in dynamics that the force necessary to cause the motion acts at the mass centre in a direction towards the centre of the circular path and that its magnitude is equal to $M\omega^2R$.

If M is reckoned in pounds the force is expressed in absolute units. If M is reckoned in gravitation units so that $M = W/g$, the force is expressed in lbs.-weight, W being the weight in pounds.

The force acts at right angles to the direction of motion and has therefore no effect on the speed in the path.

This force can be automatically called into existence by connecting the mass to the centre of the path by a radial connector, OM (*Fig.* 3).

When the shaft is turned the mass must move in a circle prescribed by the length of the connecting arm. The necessary constraining force is applied to the mass by the arm. An equal and opposite force is applied by the arm to the frame.

The force acting on the frame is constant in amount but varies in direction. At one instant it tends to lift the frame up, and next it acts to push it down. Placed on the platform of a suitable weighing-machine the weight of the whole apparatus seen in *Fig.* 3 would appear to vary above and below its proper value by the amount $M\omega^2R$.

For example, suppose the weight of the apparatus to be 100 pounds, of which the mass at the end of the arm, 1·5 foot radius, weighs 10 pounds. Then if the shaft were driven by a motor mounted on the apparatus at, say, 1 revolution per second, equal to 2π radians per second, the force in pounds acting on the frame would be 18·35 lbs.-weight.

The scale would therefore show a weight varying between 118·35 pounds and 81·65 pounds.

Increase the speed above that found from the relation $M\omega^2R=100$, and the upward force would be sufficient to lift the whole apparatus momentarily off the platform. For example, at 5 revolutions per second the force is 457 lbs.-weight, and under the action of this force the whole apparatus would jump up and down on the platform of the scale and produce quite a hammer blow.

This is in fact what may actually happen to the driving wheels of a locomotive if the revolving masses put in the wheel to balance forces in the line of stroke are massive enough and are driven at a high speed. This point is specifically considered in § (12) below.

The frame force (*Fig.* 3) may be entirely eliminated by the simple device of extending the radial connector as illustrated by dotted lines and securing to it a mass equal to M at the radius R. Then each mass provides the reaction to the other and the frame is not called upon for any reaction. The arm with its equal pair of masses may be driven at any speed without any tendency to produce vibration of the frame.

On the platform of a weighing-machine the mass could be driven at any speed and no change of weight would be observed.

The tension in the connector rises and falls as the speed rises and falls.

The mass M is said to be balanced by the mass m. It is not necessary that the balance weight m should be equal to M. The added mass has only to require a constraining force equal to the constraining force required by M. If a mass m is added at radius r to balance a mass M at radius R, then equality between the constraining forces is secured when

$$M\omega^2R=m\omega^2r,$$

that is when

$$MR=mr \quad . \quad . \quad . \quad (1)$$

This is the condition for the balancing of two masses on an arm and in the same plane.

It is customary to consider all masses as though they acted at a common radius. The radius usually selected is the Crank Radius. Reduction to crank radius is effected by equation (1).

(ii.) *Motion of a Mass in a Straight Line at Varying Speed.*—Consider the reciprocating mass C (*Fig.* 4). It is compelled to move in the straight path defined by the slide bars. Assume that the crank turns with constant angular velocity. Then the problem is to find what forces are required to produce the motion of the mass C which is defined by the uniform turning of the crank.

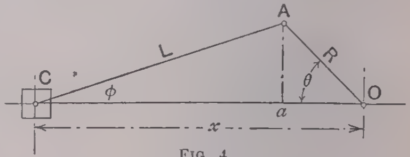

Fig. 4.

The general method of finding the acceleration of C is to form an expression giving the position of the mass in its path in terms of one independent variable. Then differentiate this variable twice with respect to the time. The result of the first differentiation gives the velocity of the mass in its path. The result of the second differentiation gives the acceleration of the mass in its path. The mass multiplied by the acceleration is then the force we are seeking.

The position of the mass C (*Fig.* 4) is defined by the distance x. When the crank angle is θ the value of x is

$$x=Oa+aC=R\cos\theta+L\cos\phi. \quad . \quad (2)$$

Eliminating ϕ by means of the relation $R\sin\theta=L\sin\phi$ and differentiating twice with regard to the time, remembering that $d^2\theta/dt^2$, the acceleration of the crank, is assumed to be zero, the acceleration is found to be

$$A=-\omega^2R\left\{\cos\theta+\frac{RL^2\cos 2\theta+R^3\sin^4\theta}{(L^2-R^2\sin^2\theta)^{\frac{3}{2}}}\right\}. \quad . \quad (3)$$

The instantaneous value of the force is then MA and the reaction on the frame is $-MA$.

The force is seen from this expression to vary in a complex manner. A way of producing an exactly equal and opposite reaction on the frame to balance this is sketched in *Fig.* 5. A mass C_1, equal to the mass C, is made to slide by a connecting rod A_1C_1, equal in length to AC. For this arrangement to work there would be three cranks on the shaft: the centre one OA, and two outer ones

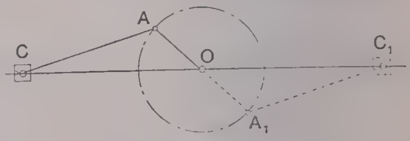

Fig. 5.

in the direction OA_1. The mass C_1 would then be driven by two connecting rods, together equal in mass to the rod AC. The reciprocating masses of an engine constructed in this way would be in perfect balance.

In an actual engine the accelerating force is derived from the fluid pressure in the cylinder. The pressure on the piston has to produce the acceleration of the piston and rod and to transmit force to the crank pin through the connecting rod. The force therefore transmitted to the crank pin is the difference between the total force on the piston due to the steam pressure and that part of the force which has been employed in giving motion to the piston. After about the middle of the stroke, however, where the acceleration changes sign, the pressure on the crank pin is greater than that due to the total fluid pressure by the retarding force, which must be applied to the reciprocating masses by the crank pin.

The acceleration[1] can be found graphically. Bennett's construction, given to the author by G. T. Bennett, D.Sc., of Emmanuel College, Cambridge, in 1902, is convenient for this purpose, and is as follows:

First (*Fig.* 6) find a point B on the connecting rod so that $AB \times BC = AO^2$. This point is found by drawing a perpendicular from O to the rod when the crank is at right angles with the line of stroke.

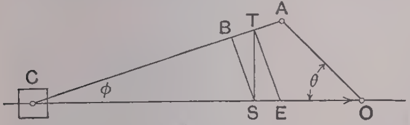

FIG. 6.

Then set out the mechanism with the crank at an assigned angle θ. From B draw BS perpendicular to the rod. From S draw ST perpendicular to the line of stroke. From T draw TE perpendicular to the rod. Then EO represents the acceleration of the mass C to the scale on which AO represents the radial acceleration of the crank pin. The acceleration of C is therefore $M\omega^2R(EO/AO)$ ft. per second per second.

The construction is exact and gives the same value for the acceleration as that which is calculated from expression (3). The proof of the construction is found from equation (1) by eliminating ϕ instead of θ. It is given in Dalby's *Balancing of Engines.*[2]

The expression found in (3) is intractable. An approximation of great utility in practice is found by putting $\cos\phi = 1 - (R^2 2/L^2)\sin^2\theta$ in the process of eliminating ϕ from equation (2). The resulting expression for x differentiated twice then gives for the acceleration

$$A = -\omega^2R\left(\cos\theta + \frac{R}{L}\cos 2\theta\right). \quad . \quad . \quad (4)$$

As close an approximation to the true expression (3) as may be desired can be made by expressing A as a Fourier series.

The general expression then becomes

$$A = -\omega^2R\,(\cos\theta + A_1\cos 2\theta - B\cos 4\theta$$
$$+ C\cos 6\theta \;.\;.\;.)$$

in which A_1, B, and C have the following values, c being the ratio R/L,

$$A_1 = c + \frac{c^3}{4} + \frac{15\,c^5}{128} \quad . \quad .$$

$$B = \frac{c^3}{4} + \frac{3c^5}{16} \quad . \quad . \quad .$$

$$C = \frac{9c^5}{128} \quad . \quad . \quad .$$

See for details a paper by Mr. J. H. Macalpine in *Engineering*, Oct. 22, 1897.

(iii.) *The Motion of the Connecting Rod* (*Fig.* 7).—The end A, called the big end, is compelled to move in a circle. The end C, called the small end, is compelled to move in a straight line.

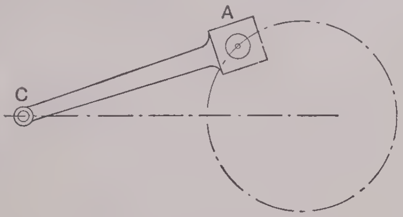

FIG. 7.

The dynamical problem is, assuming the rod to be freed from its connections, what is the instantaneous force which must be applied to the rod to cause it to move the one end in a circle with uniform velocity, the other end in a line passing through the centre of this circle (*Fig.* 7).

The answer to this problem depends upon the mass of the rod and the way the mass is distributed, and on the speed.

There are two important mass points which must be located before the solution can be found. These are the mass centre and the centre of percussion relative to the small end of the rod.

The mass centre is found by balancing the rod on a knife edge.

The centre of percussion H (*Fig.* 8) relative to the small end is found by suspending the rod so that it can oscillate about the small end, and then adjusting a plumb bob to oscillate in time with it. The length of the simple pendulum formed by the bob and its string is the distance from the small end to the centre of percussion required.

The following construction then gives the force corresponding to a given crank angle.

---

[1] See also "Kinematics of Machinery," § (3) (iv.).

[2] *The Balancing of Engines*, W. E. Dalby. Edward Arnold, London, 1906.

Draw the mechanism in the configuration corresponding to the given crank angle (*Fig.* 9).

Apply Bennett's geometrical construction to find EO, the acceleration of the end C. Join EA.

EA is called the acceleration image of the rod. It has this property. Regarded as the connecting rod on a smaller scale, A being the big end and E the small end, the line joining any point on it to O represents in magnitude and direction the acceleration of the corresponding point on the actual rod. Let V be the mass centre, and H the centre of percussion of the rod about C.

Draw Vv parallel to the line of stroke and join v to O; then vO is the magnitude and direction of the acceleration of the mass centre to the scale on which AO is the acceleration of the crank pin A.

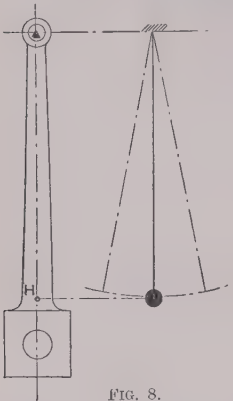

FIG. 8.

The magnitude of the accelerating force is then

$$M\omega^2R\frac{vO}{AO}=R.$$

It remains to find its position.

Draw Hh (*Fig.* 9) parallel to the line of stroke and join h to O. Through H draw a line parallel to hO cutting the line of stroke in J. J is then a point in the line of action of the force. But its direction is given by vO.

Therefore through J draw a line parallel to vO. This is the line of action of the force R, which is competent to produce the motion of the connecting rod at the instant the crank is passing through the angle θ.

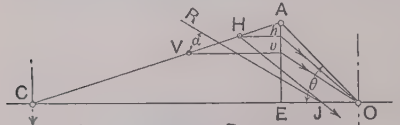

Fig. 9. To find force R competent to produce the motion of the Connecting Rod.

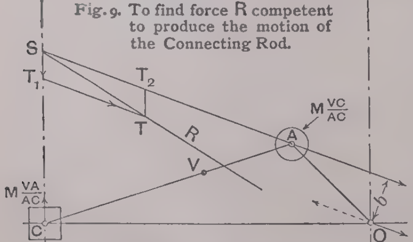

Fig. 10. Frame Reactions.

This force varies in magnitude and direction as the crank angle varies.

The force R is equivalent to an equal and parallel force R acting at the mass centre V, and a couple Ra. The force R at the mass centre produces the linear motion of the rod, and the couple Ra produces the angular motion of the rod about its mass centre V. The combination of this linear and angular motion produces the actual motion.

The proofs of these propositions and constructions are given in detail in Dalby's *Balancing of Engines.*

The next step is to find the frame reactions. Force can only be brought on to the rod at its ends. There is the pull from the crank pin and the reaction at the slide bars.

Neglecting friction the reaction at the slide bars can only be at right angles to the line of stroke.

Therefore at C (*Fig.* 10) set up a perpendicular to the line of stroke to meet R produced in S. Join S to A.

Then if ST represents the force R, the component ST_1 represents the force which must be brought upon the small end of the rod by the slide bars, and the component ST_2 the force which must be brought on to the big end by the crank pin. These two components together produce the actual motion of the rod.

The crank-pin component transferred to the crank shaft gives the couple $ST_2 \times b$, and the equal and parallel force ST_2 acting from the frame to the shaft.

The couple modifies the turning moment on the crank.

The frame reactions are shown by dotted lines in the figure.

The final result is then that the motion of the connecting rod involves forces on the frame like those shown, changing in magnitude from instant to instant, but passing periodically through the same values. These forces tend to cause vibration of the frame.

The connecting rod could be balanced exactly by the method already illustrated in *Fig.* 5. With this arrangement providing that the rods on the right are equal in weight to the rod on the left and similar in form, the resultant of the forces on the frame consequent upon the motion of the rods on the right is exactly equal and opposite to the resultant of the forces on the frame consequent upon the acceleration of the rod on the left.

The forces wax and wane and change their directions in exact unison, so that they neutralise one another from instant to instant, and the frame itself, although under the stress produced by these forces, is never under the action of an unbalanced force and therefore has no tendency to vibrate.

In practice it is usual to eliminate the effect of the rod by treating the problem as though its mass were divided between the crank pin and the reciprocating masses inversely as the mass centre divides the rod.

The share assigned to the crank pin is then balanced as part of the revolving masses; that assigned to the reciprocating masses is balanced with the reciprocating masses.

Considered as a system of two masses the frame forces produced are not exactly the same as the frame forces produced by the actual connecting rod, but while it is impossible to balance the rod except by another rod in the way shown, it is possible to balance these divided masses with ordinary balance weights with sufficient accuracy for most practical purposes.

§ (3) THE CENTRIFUGAL COUPLE.—Referring to *Fig.* 3 it will be seen that the mass M is balanced by the mass *m* placed diametrically opposite to it.

Let *Fig.* 11 be the side view of a shaft carrying a mass M at radius R, and for the moment suppose it to be supported on a pair of parallel knife edges at B_1 and B_2. If the balance weight *m* is placed opposite M at radius *r* such that $MR=mr$, but at a distance A along the shaft as shown in *Fig.* 11, and this is usually necessary in practice, it will still be found to balance M when the shaft is at rest. The shaft will stand in any angular position on the knife edges. It will be in perfect static balance. If now the shaft is put in bearings, say, at B_1 and B_2, its revolution will set up vibrations.

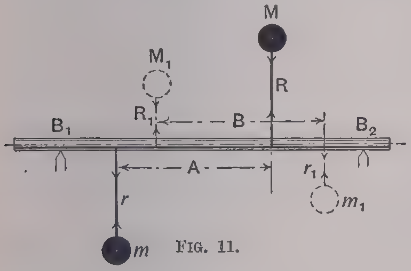

FIG. 11.

Although it is in static balance it is not in running balance. The disturbance is now produced by the couple formed by the reaction of M on the shaft, giving a force Mw^2R, and by the equal and opposite reaction mw^2r on the shaft, these two reactions being separated by the distance *a*. This couple tends to turn the system about an axis perpendicular to the plane containing the forces.

The distance A between the forces is called the ARM of the couple, and the product of one force into the arm is called the MOMENT of the couple.

The couple is called the centrifugal couple, because its forces are the centrifugal forces caused by the rotation of masses.

A couple can only be balanced by an equal and opposite couple acting in its plane. That is, the four forces of the pair of couples lie in one plane, and this plane contains the axis of the shaft.

The moment of the centrifugal couple caused by the mass M and its balancing mass *m* is $M\omega^2RA$.

It may be balanced by a couple $M_1\omega^2R_1B$, the quantities being taken so that

$$M_1R_1B=MRA. \quad . \quad . \quad (1)$$

These couples must act so that they tend to produce rotation in opposite directions.

The dotted lines in *Fig.* 11 show a balancing couple. If masses, radii, and the arm B satisfy the equation above, and if in addition $M_1R_1=m_1r_1$ and $MR=mr$, then it will be found that the shaft may be driven at any speed and there will be no vibration.

At the same time the shaft will stand in any angular position on the knife edges. The masses disposed round it satisfy in fact two separate conditions.

They are in static balance, and they are also in running balance. Static balance implies that the mass centre of all the masses lies on the axis of revolution of the shaft.

Running balance implies that the centrifugal couples of all the masses form a system in equilibrium.

§ (4) THE PRACTICAL PROBLEM.—The problem of balancing an engine presents itself in practice in this way. An engine is designed to fulfil conditions of power, speed, and arrangement, and the result is a crank shaft driven from a number of cylinders. Usually the lines of stroke of the cylinders are contained in one plane, and this plane contains the axis of the crank shaft. If there are *n* cylinders, then there are *n* piston masses, *n* cranks, and therefore *n* revolving masses, and also *n* connecting rods. The accelerations of each of these systems of *n* masses cause frame reactions.

As mentioned above, the connecting rod is eliminated from the problem by distributing its mass between the revolving and the piston masses.

There are now two balancing problems:

(1) To balance the revolving crank shaft masses.

(2) To balance the reciprocating piston masses.

Both fall under one method of solution.

§ (5) SOLUTION OF THE BALANCING PROBLEM.[1]—The initial problem to be solved may be stated in this way. Given a revolving shaft with a number of masses attached to it, find the balance weights which must be added to secure static and running balance.

---

[1] This solution by Professor Dalby was first published in 1899. It is contained in a paper entitled "The Balancing of Engines with special reference to Marine Work," printed in the *Transactions of the Institution of Naval Architects*, 1899, xli. In this paper will be found applications of the method to the balancing of torpedo-boat engines, including with the pistons the reciprocating slide valves as well. The development of the method to include primary and secondary balancing, the balancing of locomotives, and the treatment of many problems arising out of the dynamical conditions involved will be found in *The Balancing of Engines*, by W. E. Dalby, mentioned above.

Solution.—Select a plane at right angles to the shaft in which it is intended to attach one of the balance weights. This plane is called specifically the REFERENCE PLANE. The reference plane may be regarded as a definite plane like a drawing-board, keyed to and revolving with the shaft.

Transfer each centrifugal force $M\omega^2R$ in turn to this plane. Each force transferred gives rise to

(1) an equal and parallel force, $M\omega^2R = F$, acting in the plane;

(2) a couple Fa, where a is the distance through which the force is transferred.

The transference of one force to the reference plane is illustrated in *Fig.* 12. Here the rotation of the mass M produces the centrifugal force $M\omega^2R = F$ on the shaft, and this transferred to the reference plane gives the equal and parallel force F and the couple Fa.

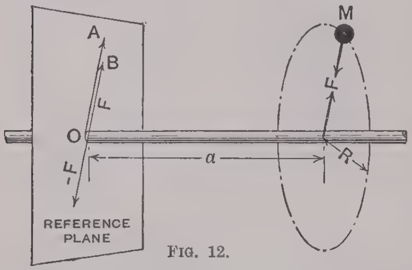

FIG. 12.

For, introduce two equal and opposite forces F at the point O in the reference plane. Then these forces being in equilibrium themselves do not affect the equilibrium of the system. The three forces, F from the mass M, and F and $-F$ introduced at O, may then be analysed into the equal and parallel force F acting at O in the reference plane and the couple Fa.

Draw a line OA parallel to the radius to M to represent the couple Fa. Its length OA is chosen so that $OA = Fa$ and its sense is marked off according to the direction in which the couple tends to turn the shaft about O.

That is, its sense will be from O outwards in the radial direction of the mass for all masses on the right of the reference plane, and from O in the direction opposite to the radial direction of the mass for all masses to the left of the reference plane.

Also draw OB to represent the force.

Then every mass M revolving with the shaft will give rise to two vectors in the reference plane, a force vector like OB pointing always in the radial direction of the mass, and a couple vector OA pointing in the radial direction settled as above.

The next step is to find the resultant of the couples. For this purpose set the couple vectors out in any order to form a polygon. If the polygon closes, then there is no unbalanced couple. If it does not close, the vector required to close it gives in magnitude sense and direction the couple required to balance the couples about O.

The couple closure, as this line is called, is the product of four quantities, namely, ω^2, an unknown mass M acting at a convenient radius R from the shaft and in a plane at a convenient distance d from the reference plane, and is equal to $MR\omega^2 d$.

Select a plane at a distance d from the reference plane where a balance weight may be conveniently introduced.

Select a radius R at which it is convenient to place the balance weight in this plane.

Finally calculate the magnitude of the balance weight from the equation

$$M = \frac{\text{Couple closure}}{dR\omega^2}.$$

The angle at which the radius stands in relation to the other radii on the shaft is given by the direction of the closure line in the polygon.

Then add this balance weight to the system, and transfer it to the reference plane as before, thus adding one more force to the system already there. The couples now balance.

Then set out the force vectors in any order to form a polygon. If this force polygon closes, then there is no unbalanced force. If it does not close, the vector required to close it gives in magnitude sense and direction the force required to balance the forces acting at O.

The force closure is the product of three quantities, namely, a mass, a radius, and ω^2.

Select a radius at which it is convenient to place the balance weight in the reference plane, and then calculate its magnitude from the equation

$$\text{Mass} = \frac{\text{Force closure}}{R\omega^2}.$$

The angle of the radius to the balance weight is given by the direction of the closure in the polygon.

ω^2, being constant for all masses, may have any value in working out the solution. Therefore call it unity.

The addition of the balance weight in the reference plane to balance the forces there has no effect on the balance already produced between the couples. Because a force acting at O has no moment about O, and can therefore be added or removed without introducing a couple into the system.

This is the essence of the solution. The couples are balanced first, and then the forces are balanced without disturbing the balance between the couples.

It is easily shown that if the forces and the couples are separately balanced about any one point along the shaft they are separately balanced about every point along the shaft. Therefore, after the balance weights have been found by the method explained, and have been added to the system, it will be found that both the couple and the force polygons will close for any position of the reference plane along the shaft.

The balance weights found are independent of the position chosen for a reference plane.

This property enables the work to be checked. Having found the balance weights as described, include them in the system, and then select a reference plane at a new place along the shaft.

Ignoring the knowledge that the system is balanced, proceed to find balance weights. It should be found that both force and couple polygon close, showing that no balance weights are required. If the polygons do not close, there has been some error in the original determination of the balance weights. This check on the accuracy of the work is a valuable element of the Dalby Solution.

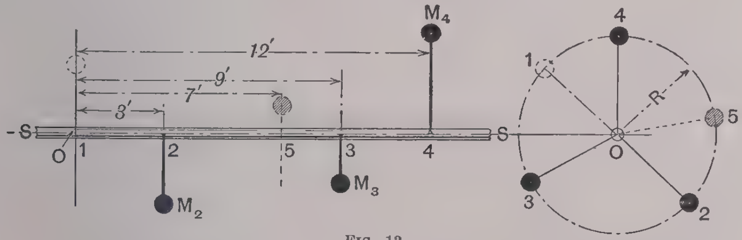

FIG. 13.

End View of Shaft

§ (6) EXAMPLE.—Let there be three masses—$M_2=17$ pounds, $M_3=15$ pounds, and $M_4=7$ pounds—attached at common radius R to the shaft SS (*Fig.* 13) spaced along the shaft as shown, and spaced angularly around the shaft as defined in the end view of the shaft (*Fig.* 13).

The mass centre of each mass revolves in a plane to which the shaft is perpendicular.

Let the problem be to balance these three masses by two balance weights, the one being placed in the plane No. 1 and the other in the plane No. 5.

Choose the reference plane to coincide with the plane of revolution of one of the balance weights. We may therefore choose either plane No. 1 or plane No. 5. Let us choose plane No. 1. Then write on the drawing the distance of each plane in which a mass revolves from the reference plane in the manner indicated in the figure.

Arrange the data in the way shown in SCHEDULE No. 1.

Column 1. Write in the numbers of the planes of revolution.

Column 2. Write in the distances of these planes from the reference plane.

Column 3. Write in the values of M, the mass at common radius R. This mass is equivalent to the centrifugal force when $\omega^2R=1$. The column is therefore headed FORCE.

Column 4. Write in the products Ma calculated from the figures in columns 2 and 3. This product is equivalent to a centrifugal couple when $\omega^2R=1$. The column is therefore headed COUPLE.

SCHEDULE 1

No. of Plane or No. of Crank.	Distance of Plane from Reference Plane.	FORCE. Equivalent Mass at Common Radius, or Centrifugal Force when $\omega^2R=1$.	Couple when $\omega^2R=1$.
	a.	M.	Ma.
No. 1	0	Unknown bal. wt. (14·4 pounds)	0
No. 2	3′	17 ,,	51 lbs.-ft.
No. 3	9′	15 ,,	135 ,,
No. 4	12′	7 ,,	84 ,,
No. 5	7′	Unknown bal. wt. (11·9 pounds)	83 = closure.

Next set out the couple vectors of column 4 in the way shown in *Fig.* 14. Choosing any convenient scale, draw

AB = 51 and parallel to radius 2 (end view, *Fig.* 13).
BC = 135 and parallel to radius 3.
CD = 84 and parallel to radius 4.

DA the closure, measuring 83, is the couple required to produce balance amongst the couples.

The balance weight in plane 5 is calculated from $(R\omega^2=1)$,

$$M=\frac{\text{Couple closure}}{aR\omega^2}=\frac{83}{7}=11{\cdot}9 \text{ pounds.}$$

The angular position of the radius of this balance weight is defined in relation to the others by drawing radius 5 in the end view parallel to the closure of the couple polygon.

This mass is now entered in the schedule as indicated.

There are now four forces acting at O in the reference plane. Starting from any origin, set out (*Fig.* 15)

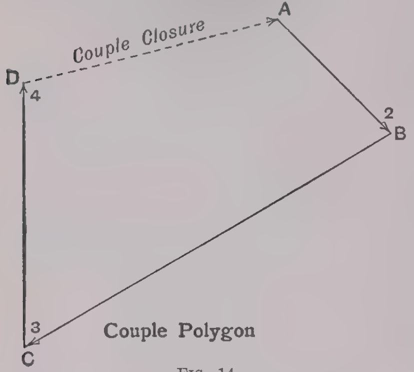

FIG. 14.

$ab=17$ parallel to radius 2 (end view, *Fig.* 13) ;
$bc=15$ parallel to radius 3 ;
$cd=\ 7$ parallel to radius 4 ;
$de=11{\cdot}9$ parallel to radius 5.

ea, the closure measuring 14·4, is the balance weight at common radius required to balance the forces at O. The angular position of the radius of this balance weight is defined in relation to the others by drawing radius 1 in the end view parallel to the closure of the force polygon.

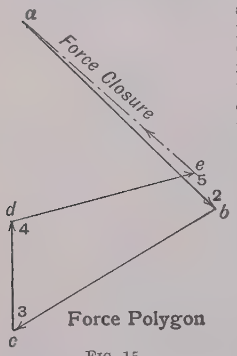

FIG. 15.

These two balance weights added to the system in the planes specified, and in the angular positions defined in the end view, produce a system of five masses in both static and in running balance. The shaft can be driven at any speed when held in a pair of bearings placed anywhere along the shaft without producing vibration of the supports.

Whatever value be given to ω^2R it will be the same for each mass, and therefore will not make any difference in the balance weights found.

§ (7) DEDUCTIONS FROM THE FORCE AND COUPLE POLYGONS.—The balance of a system of *n* revolving masses may be tested by selecting a reference plane at any point along the shaft and then following the method above. Unless the plane has been selected to coincide with one of the planes in which a mass revolves there will be in the reference plane, after the transference of the forces to it, *n* couple vectors, and *n* force vectors. If the masses are in balance the couple vectors set out in any order will form a closed polygon of *n* sides; and the force vectors set out in any order will form a closed polygon of *n* sides. Corresponding sides of these two polygons are parallel, but the ratio of the lengths of a pair of parallel sides is different for each pair. The ratio of a pair is in fact the distance *a* from the reference plane of the mass whose force and couple are respectively represented by the pair of parallel sides.

The balancing of *n* masses amongst themselves is thus conditioned by these geometrical properties of the force polygon and the couple polygon.

If, when the vectors are set out, it should be found that the force polygon closes, but the couple polygon does not close, then the masses are in static balance, but are not in running balance. There is no unbalanced force, but there is an unbalanced couple causing reactions on the frame.

A few important practical limitations can at once be deduced from these polygons.

(*a*) Three masses carried on three arms spaced along a shaft can only mutually balance if the arms are in one plane, a plane containing the axis of the shaft. Seen in end view the arms would all lie in a diameter.

For it is only possible to draw two polygons of three sides each, with corresponding sides parallel, and the ratio of the lengths of each parallel pair different, if the polygons close into a line. It follows that the arms carrying the masses must be in a line so that the middle arm is at 180° with the two outer arms.

Therefore a crank shaft with three cranks at 120° cannot be designed so that the masses mutually balance. If the masses are equal, the force polygon closes and static balance is obtained. But in these conditions the couple polygon cannot be closed.

To balance three masses set at angles with one another a fourth mass must be added to the system equivalent to a fourth crank. The system is then equivalent to four masses on four cranks.

(*b*) Four masses on four arms spaced along a shaft can in general be balanced amongst themselves, because a pair of polygons can be found in which corresponding sides are parallel, and in which the ratio of the lengths of each parallel pair is different.

(*c*) But four masses spaced along a shaft on four arms set at 90° with each other cannot be designed to balance. If the four masses are equal the force polygon closes and there is static balance. But with the angles all right angles the couple polygon cannot be made to close.

Therefore a crank shaft with four cranks set at 90° with each other cannot be designed so that the masses balance. Balance can only be obtained by adding a fifth mass to the system equivalent to a fifth crank. The system is then equivalent to five masses on five cranks, but with the condition that four of the cranks are mutually at right angles.

Many other useful conditions can be deduced from the geometric properties of the force and couple polygons.

§ (8) THE BALANCING OF A ROTOR.—The rotor of a steam turbine is truly turned, and the blading is symmetrical. At first sight it would appear that there is here no balancing problem, because so far as constructional skill can go the mass centre of each elementary disc, of which the length may be imagined to be made up, lies in the axis of rotation.

But owing to slight variations in density, and in the symmetry of the blading, the mass centre of each elementary disc is in general slightly displaced from the axis of rotation. Although these displacements are slight the speed at which the rotor is driven involves accelerating forces which are large enough to cause vibration. So that, after all, there is a rotor balancing problem, and a difficult one, because the displacements of the mass centres of the elementary discs cannot be identified by any method of weighing or measurement.

The problem is in fact similar to that which would be presented to a designer asked to work out the balance weights for a totally enclosed reciprocating engine, with access to the parts denied. All he would see of the moving parts would be the ends of the crank shaft projecting through the bearing at each end, and all he would know of them would be that their motion produced vibration.

Similarly, all he can see in a rotor is an apparently truly turned mass, symmetrically bladed, and all he knows of it is that when driven it produces vibration.

He might by imagination conceive the rotor to be made up of a series of elementary discs, but the position of the mass centre of each disc would be hidden from him without possibility of discovery.

Mr. King Salter, in a paper to the Institution of Naval Architects read at the spring meeting of 1920, has described an ingenious method of solving the problem.

The rotor to be balanced is placed in bearings and is driven by a motor. Being unbalanced it produces a reaction on each bearing. These reactions are separately measured in the horizontal direction. Let R_1 and R_2 be the respective maximum reactions measured in the apparatus when the rotor is driven at ω revolutions per second. Then each reaction may be regarded as produced by a mass M attached to the rotor at radius r. So that the maximum horizontal reaction at each bearing is expressed by

$$R_1 = M_1\omega^2 r_1, \qquad (1)$$

$$R_2 = M_2\omega^2 r_2, \qquad (2)$$

from which

$$M_1 = \frac{R_1}{\omega^2 r_1}, \qquad (3)$$

$$M_2 = \frac{R_2}{\omega^2 r_2}. \qquad (4)$$

M_1 and M_2 can be calculated from (3) and (4) because ω^2 is observed, and r may be put equal to unity or to any convenient constant length.

The next part of the problem is to find the angular position of the radii r_1 and r_2. This is done by the apparatus described in the paper.

The rotor, with its hidden system of unbalanced masses, may now be replaced by a shaft with two known masses attached to it at known angular spacing as illustrated in *Fig.* 17. *Fig.* 16 shows the rotor and the centre line of the bearings at which the reactions are measured. *Fig.* 17 shows the equivalent unbalanced system of two masses as determined in Mr. King Salter's apparatus.

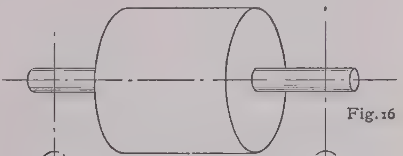
Fig. 16

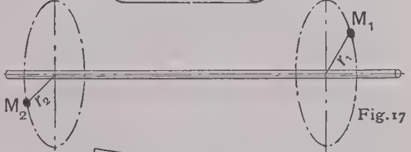

Fig. 17

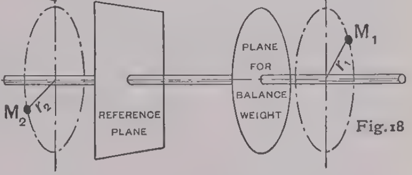

Fig. 18

The two planes in which balancing masses are to be placed are now fixed upon. One is chosen as a reference plane, and then the solution of § (5) is applied to find the actual magnitude and position of the balancing masses to be placed in the two respective planes. The arrangement is shown in *Fig.* 18.

The usual shop method of balancing a rotor is to place it on a pair of level and parallel knife edges and then add balance weights until it will stand in any angular position on these knife edges. This process ensures static balance but not running balance. The unknown unbalanced couple cannot be estimated from experiments on parallel knife edges, and in practice the proper distribution of balancing masses to secure running balance is done by trial and error. After static balance is secured the rotor is driven up to speed, and if the couple is sufficient to produce disturbance the weights added to secure static balance are distributed so that the running balance is improved without interfering with the static balance. Considerable experience is required to get this done satisfactorily. It is advisable to test the balance of a turbine rotor in steam so that the rotor and its blades have expanded to the positions they occupy at the normal running temperature.

A full description of Mr. King Salter's apparatus will be found in the paper cited above. Briefly it consists of a substantial bed with a bearing at each end. Each bearing can slide horizontally through a limited distance, but is prevented from doing so by a spring screwed up against it. When a trial of a rotor is being made the spring is compressed against the bearing so that it just prevents it moving. The strength of the spring is known, so that the maximum force acting horizontally on the bearing in the direction of motion can be observed.

On each end of the rotor is placed a spider or flange, carrying on its periphery a number of radial spokes like the spokes on a steering wheel, but with this difference, that they are free to slide inwards radially, but once pushed in they cannot come back, being held by a spring. When these spiders have been keyed on the end of the rotor, all spokes out to the limit allowed, the rotor is driven at the testing speed by an independent motor, and this motor is then disconnected. The bearings are then slightly relieved so that the rotor can move horizontally under the unbalanced forces. A kind of brake shoe is then gradually approached to the spokes and pushed on to them until the ends are running in a circle. The rotor is then brought to rest, and an examination of the spokes shows the particular radius corresponding to the maximum horizontal displacement, and therefore to the maximum value of the force on the corresponding bearing.

The displacement of the mass centre of a rotor and the angular position of the plane containing the mass centre and the axis of the rotor can be found by direct weighing in the Martin rotor balancing apparatus. The method and the apparatus are described in *Engineering*, December 31, 1920. By the aid of this apparatus a static balance of great accuracy can be obtained. For many rotors static balance may be sufficient, as for the thin disc-like rotors of gyrostats. If the static balancing of a rotor is found to be insufficient, its dynamical balance can be tested and secured by the aid of Mr. King Salter's apparatus.

§ (9) FOUR MASSES. SPECIAL CONSTRUCTION. —Four masses may be set together in mutual balance in an infinite variety of ways. For an infinite number of pairs of four-sided force and couple polygons can be drawn at random to satisfy the geometrical conditions mentioned above.

There is a simple construction, due to Dr. Schubert, and published by the Hamburg Mathematical Society, 1898, which gives the spacing along the shaft, the angular positions, and the masses for mutual balance amongst four masses.

Let the spacing of the cranks along the shaft SS be given as indicated in *Fig.* 19. Choose any pole O. Join points 1, 2, 3, 4, with this pole.

Take any point A on O1, and through it draw a line AB parallel to the shaft SS, and a line AC parallel to the line O3.

Through B draw a line BC parallel to the line O2.

Then OACB is the force polygon corresponding to the position of the pole O and the crank spacing given.

This force polygon defines by the lengths of its sides the relative magnitudes of the masses, and by the directions of its sides the positions of the cranks.

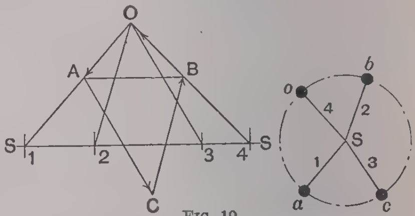

FIG. 19.

Thus in the end view draw crank S*a* parallel to the side OA; S*c* parallel to the side AC; S*b* parallel to the side CB, and S*o* parallel to the side BO.

If now the system be tested it will be found that the couple polygon is closed. The proof of this will be found in Dalby's *Balancing of Engines*, 2nd edition, 1906, p. 50.

The construction may be varied in many ways. For example, the angular position of the cranks in end view may be drawn at random. Then select any pole O and draw lines radiating from it parallel to the crank directions. Then draw across the rays a line SS, and the intersections define the spacing along the shaft.

Then draw a line as AB parallel to the line SS and finish the force polygon. The sides then define the magnitude of the masses.

§ (10) THE BALANCING OF RECIPROCATING MASSES.—We have now to consider the balancing of the reciprocating masses of a multi-cylinder engine of the type where the centre lines of the cylinders lie in a plane containing the axis of the crank shaft.

In the marine type the plane is vertical. In the ordinary land type the plane is usually horizontal, though often vertical. In the V engine the plane is inclined, and there are two systems of parts to be separately balanced, one set in each inclined plane.

The frame reaction caused by the reciprocation of a mass M by a crank turning with uniform velocity is given by equation (3), § (2).

A way of producing an exact equal and opposite reaction on the frame is described and illustrated in *Fig.* 5.

This construction is, however, excluded from the problem before us.

An expression giving values close to the values from (3), and derived from equation (4), § (2), is

$$F = M^2R \cos \theta + M(2\omega)^2 \frac{R^2}{4L} \cos 2\theta. \quad . \qquad (1)$$

The first term of this expression is the projection on the line of stroke of the centrifugal force due to the rotation of the mass M at radius R and at speed ω.

The second term is the projection on the line of stroke of the centrifugal force due to the rotation of a mass M at radius $R^2/4L$ and at speed 2ω.

The greater the length of the connecting rod in relation to the crank radius the smaller the second term.

In the limit, when the rod is infinitely long, the second term vanishes. The crank and slotted bar mechanism partly realises this condition. The ratio of rod to crank is often large enough to make this second term negligibly small. In locomotives the ratio of rod to crank may be 7 to 1, and even more. But in some kinds of marine engines it may be only 3½ to 1. The second term then becomes important.

When the second term is ignored in the balancing the engine is said to be balanced for PRIMARY FORCES and PRIMARY COUPLES only.

If, however, the second term is included in the problem, the engine is said to be balanced for PRIMARY AND SECONDARY FORCES AND COUPLES.

(i.) *Primary Balancing.*—Since the term $M\omega^2R \cos \theta$ is the projection on the line of stroke of the centrifugal force produced by the rotation of a mass M, the conditions of primary balance amongst n reciprocating masses are the same as those for n revolving masses, and may be investigated by assuming that each mass is attached to its own crank pin, so that the n reciprocating masses are replaced by n revolving masses. For if a polygon is closed, then the projections of the sides on any line are algebraically equal to zero. In other words, if the conditions of balance for the revolving masses are satisfied, so too are the conditions for the equivalent reciprocating masses. The Dalby Solution of § (5) may then be applied without change to find the balance weights of the substituted revolving system, and the balance weights found reciprocated in the cylinder plane will balance the reciprocating masses.

The example of § (6) may be restated thus. The reciprocating masses of a three-cylinder vertical engine weigh respectively $M_2=17$ pounds, $M_3=15$ pounds, $M_4=7$ pounds. These masses are reciprocated by cranks spaced along and around the crank shaft, as defined by *Fig.* 13. Find the reciprocating balance weights. The answer is (see Schedule 1), 14·4 pounds reciprocated by crank No. 1; 11·9 pounds reciprocated by crank No. 5; the lines of reciprocation lying in the plane containing the centre lines of the cylinders.

The masses may be designed as lumps of metal reciprocated in properly lubricated guides on the engine frame. They would then be called BOB WEIGHTS.[1] The engine would be described as a three-cylinder engine in which the reciprocating masses, that is the piston masses, were balanced by a pair of bob weights.

Each balance weight may, however, be designed into a set of piston masses and added complete with cylinder and valve gear to the engine. Then the engine would be described as a five-cylinder engine with the piston masses in balance amongst themselves.

The problem of balancing an engine, as stated in § (4), involves the separate solution of two problems. First find the reciprocating balance weights, and add them to the engine either as bob weights or as pistons. Secondly, find the balance weights for the revolving system of the crank shaft and add them to the crank shaft.

Each of these problems is separately solved by the application of the method given in § (5).

The deductions of § (7) apply equally to systems of reciprocating masses. It follows that the fewest number of cylinders in which the pistons may be set in mutual balance is four.

Much has been written about the balancing of four-cylinder engines. In practice, however, even for primary balancing, the valve gear must often be included to get the best results. In particular a four-cylinder torpedo-boat engine becomes with its valve gear a problem in balancing twelve lines of parts. There are some subsidiary problems involved in the complete solution, for which see the worked-out example in Dalby's *Balancing of Engines.*

(ii.) *Secondary Balancing.*—Since the second term of equation (1) above gives a force which is the projection on the line of stroke of a mass M revolving at twice the speed of the main crank at radius $R^2/4L$, the condition that the secondary forces shall balance is that the corresponding force and couple polygons shall close. A schedule is made, as illustrated in Schedule 1, and the work is carried out as in § (5), but with this difference: the end view of the shaft must now show all the crank angles doubled.

Balancing to satisfy the conditions of equation (1) is therefore conditioned by the closing of four polygons, namely, a primary force polygon, a primary couple polygon, a secondary force polygon, and a secondary couple polygon.

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[1] "On the Balancing of Marine Engines," Sir Alfred Yarrow, *Trans. Inst. Naval Architects*, 1892.

Given an engine, there is no difficulty in applying § (5) to test the primary and secondary balance, but it is more convenient to deduce the related conditions analytically.

The following method affords a solution.[1]

The direction of a crank measured from a fixed line is defined by the expression x_1+iy_1, where $i=\sqrt{-1}$. x_1 is measured along the fixed direction. y_1 is measured at right angles to it. Also $x_1^2+y_1^2=1$.

The direction of a crank corresponding to this, starting in coincidence along the fixed direction and revolving twice as fast, is given by

$$x_1^2-y_1^2+i2y_1x_1.$$

If M_1 is the mass at radius R_1 the corresponding force and couple vectors are

Primary force—

$$M_1\omega^2R_1(x_1+iy_1), \quad . \quad . \quad . \quad . \quad (2)$$

Primary couple—

$$M_1\omega^2R_1(x_1+iy_1)a_1, \quad . \quad . \quad . \quad . \quad (3)$$

Secondary force—

$$\frac{M_1\omega^2R_1^2}{L}(x_1^2-y_1^2+i2x_1y_1), \quad . \quad . \quad (4)$$

Secondary couple—

$$\frac{M_1\omega^2R_1^2}{L}(x_1^2-y_1^2+i2x_1y_1)a_1. \quad . \quad . \quad (5)$$

Fig. 20 shows the relations of these quantities for one mass M_1.

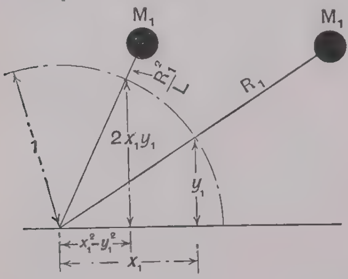

FIG. 20.

For every reciprocating mass there is a set of terms corresponding to these four expressions. The condition that the masses may balance is expressed by the eight simultaneous equations:

Primary forces vanish—

$$(M_1x_1+M_2x_2\ +\ +)=0.$$
$$(M_1y_1+M_2y_2\ +\ +)=0.$$

Primary couples vanish—

$$(M_1x_1a_1+M_2x_2a_2\ +\ +)=0.$$
$$(M_1y_1a_1+M_2y_2a_2\ +\ +)=0.$$

Secondary forces vanish—

$$(M_1(x_1^2-y_1^2)+M_2(x_2^2-y_2^2)+\ +)=0.$$
$$(M_1\,x_1y_1\qquad +M_2\,x_2y_2\qquad +\ +)=0.$$

Secondary couples vanish—

$$(M_1(x_1^2-y_1^2)a_1+M_2(x_2^2-y_2^2)a_2\ +\ +)=0.$$
$$(M_1\,x_1y_1a_1\qquad +M_2\,x_2y_2a_2\qquad +\ +)=0.$$

With the general relation $x^2+y^2=1$ for all x's and y's with the same subscript.

Each of these eight equations has one term for each reciprocating mass.

It is not possible to find a solution of any practical value with fewer than 5 lines of parts, that is, with 5 terms in each line. The engine could be built either as a five-cylinder engine or a three-cylinder engine with a pair of bob weights.

The exploration of the possibilities of balance with engines of different numbers of cranks will be found in the book cited above.

§ (11) THE YARROW SCHLICK TWEEDY MARINE ENGINE.—This is a four-cylinder engine with reciprocating parts balanced amongst themselves for primary forces and primary couples.

Later a four-cylinder symmetrically arranged engine with the reciprocating parts balanced amongst themselves for primary and secondary forces and for primary couples was used, and the method of getting this balance was described in a paper at the Naval Architects in 1900 by Schlick.

Although the eight fundamental equations of the preceding section cannot be simultaneously satisfied for an engine giving only four terms in each equation, yet it is possible to satisfy the first six of them with four terms per equation. Thus a four-crank engine can be set to satisfy the first six equations, and thus it can be designed for primary and secondary balance of the reciprocating forces and primary balance of the couples, but the secondary couples must remain unbalanced.

The symmetrical engine is indicated in *Fig.* 21. A reference plane is taken at the centre of the shaft

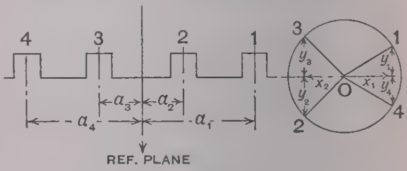

FIG. 21.

and the cranks are spaced so that $a_1=-a_4$. The angle 103 is made equal to the angle 204, and the mass M_1 is taken equal to the mass M_4. With these assumptions, the first six equations give

$$M_2=M_3 \text{ and } a_2=-a_3,$$

and further

$$x_1x_2=-\tfrac{1}{2}, \quad . \quad . \quad . \quad (1)$$

$$x_1^2=-P\pm\sqrt{P^2+Q^2}, \quad . \quad . \quad (2)$$

where

$$P=\frac{a_1^2-a_2^2}{8a_2^2} \text{ and } Q^2=\frac{a_1^2}{4a_2^2}$$

and

$$\frac{M_2}{2M_1}=x_1^2. \quad . \quad . \quad . \quad (3)$$

When the spacings a_1 and a_2 are given, x_1 is calculated from (2), and then x_2 from (1), and the ratio of the reciprocating masses from (3).

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[1] Dalby, "On the Balancing of the Reciprocating Parts of Engines, including the Effect of the Connecting Rod," *Transactions of the Institution of Naval Architects*, 1901.

The solution of the first six equations with four terms per equation is not limited to the symmetrical engine.

The expressions for unsymmetrical engines are complicated. The general solution is given by Lorenz in *Dynamik der Kurbelgetriebe* (Leipzig, 1901), and Professor Ingles, in a paper printed in the *Transactions of the Institution of Naval Architects*, gives curves from which the graphical solution can be found for the proper given conditions.

It will be remembered that in the special construction given in § (9) for the four-crank engine the pole O can be chosen at random. The quantities then satisfy the first four of the eight equations. There is one position of the pole and one only which yields results which satisfy the first six equations. Schlick, in the paper mentioned above, gives a geometrical construction for finding this position for the symmetrical engine. See also study of the problem by G. T. Bennett in a paper entitled "The Balancing of the Four-Crank Engine," read before the International Congress of Mathematicians at Cambridge, August 1912; also a paper by Professor Inglis, *Trans. Inst. Naval Arch.*, 1911, Part 1.

The setting of the four main cranks derived from these six equations and the resulting four reciprocating masses produce balance only amongst themselves. The valve gear with its heavy reciprocating valves has received no consideration. In practice the valve gear must be brought into the problem to secure the best results. A compromise can be made which results in perfect primary balance for forces and couples of all the reciprocating parts, including the pistons and the valves, whilst the condition for secondary balance of the forces is nearly satisfied.

§ (12) THE BALANCING OF LOCOMOTIVES.[1]—Each coupling rod is imagined to be replaced by masses concentrated at the crank pins. The pins share the mass of a rod in the proportion in which they share its weight. Then by § (5) may be found the angular positions and the masses at crank radius to balance the revolving masses of each axle. A balance weight is usually cast with the wheel and is often shaped in the form of a crescent. Let m be the mass of the crescent-shaped balance weight, excluding the mass of the spokes passing through it, and let r be the radius of the mass centre, then by trial and error values of m and r must be found such that $mr = MR$, where M is the balancing mass at crank radius R.

The reciprocating masses are usually balanced by revolving masses placed in the driving wheels. These are combined with the masses required to balance the revolving parts, so that the balance weight seen in a driving wheel is the resultant of two balance weights, namely, one to balance the revolving masses, the other to balance the reciprocating masses. The balance weights added to balance the reciprocating masses are themselves unbalanced in the vertical direction and so produce a varying pressure on the rails. Let W be the static load on a driving wheel, that is, the load measured on a weigh bridge, and let m be the revolving mass at radius r added to balance the reciprocating masses, then the static load is alternately increased and diminished per revolution by the amount $m\omega^2 r/g$ pounds. Omega is here the angular velocity of the wheel in radians per second. It is more convenient to express the angular velocity in terms of the speed of the engine. Let V be the speed of the engine in miles per hour, and D the diameter of the wheel in feet, then the maximum and minimum load on the rail can be calculated from

$$W \pm \frac{0{\cdot}00012 mrV^2}{D^2} \text{ tons.}$$

An average type of coupled engine would have $W = 7\frac{1}{2}$ tons on the driving wheel, and if the reciprocating parts were fully balanced the second term may reach the value of 4 tons at 60 miles per hour even on a wheel as large as 7 feet diameter.

The rail load would then vary between 12½ tons and 3½ tons per revolution. This would be destructive to the permanent way, especially to the cross girders of bridges, but the tractive pull would be even. It is better from the permanent way point of view to leave the reciprocating masses unbalanced. But if this is done the 4 tons variation appears in the tractive pull. In practice a compromise is made and it is usual to balance only ⅔ of the reciprocating masses, leaving ⅓ to vary the tractive pull and throwing the effect of ⅔ on the permanent way.

Having decided what proportion of the reciprocating masses is to be balanced, proportionate masses are supposed concentrated on their respective crank pins. And since the balance weights are going to be added, not as reciprocating masses but as revolving masses, these masses can be included in the schedule of the revolving masses, and § (5) applied to find straight away the resultant balance weights for the driving axle.

When the driving wheel is small there may be difficulty in designing a suitable balance weight for it. The revolving balance weight added to balance the reciprocating masses may then be distributed between the coupled wheels equally or in any proportion thought suitable. The balance weight then seen in any one of the coupled wheels is the resultant of the balance weight required to balance its revolving masses, and the balance weight required to balance the proportion of the reciprocating mass transferred to it.

This distribution of the balance weight required to balance the reciprocating masses also distributes the hammer blow, as the variation of rail load is called. For example, the hammer blow of 4 tons under the driving wheel may be distributed to give 1·3 ton under each of three coupled wheels.

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[1] "Steam Engine, Reciprocating," § (8).

The reciprocating masses of a four-cylinder locomotive may be balanced amongst themselves, as explained in § (10). The possibility is not, however, taken advantage of because the cranks are generally arranged in two 180° pairs at right angles.

It has been pointed out in § (7) (*c*) that this is the one angular spacing at which mutual balance is impossible. If the reciprocating masses are equal there is balance for the forces but the couple is unbalanced. This means that the reciprocating masses produced no variation of the tractive pull, neither is there any variation of rail pressure, so that there is no hammer blow. But the unbalanced couple acts on the engine to make it sway laterally. This swaying can be corrected by the addition of revolving masses in the driving wheel, but these masses, although without effect on the tractive pull, introduce a hammer blow.

If the angles and masses were designed to secure balance there would be no variation of tractive pull, no hammer blow, and no swaying couple. Four sets of valve gear would be required as against the two sets with which the four-cylinder locomotive is usually operated.

§ (13) THE BALANCING OF INTERNAL COMBUSTION ENGINES.—The petrol engine,[1] which before the war was used mainly to drive motor cars, has been developed during the war into a powerful prime mover for aircraft. Dynamically aircraft engines can be distinguished into three types:

Type A. An engine with its cylinders ranged in a line along the crank shaft so that the cylinder centre lines and the crank shaft axis lie in a plane.

Type B. The V engine formed by placing engines of type A to drive a common crank shaft and arranged about it so that the planes containing the centre lines of their cylinders intersect in the axis of the common crank shaft. Thus two four-cylinder engines of type A become an eight-cylinder V engine driving a four-crank shaft. Three four-cylinder engines of type A become a double V engine of twelve cylinders driving a four-crank shaft.

Type C. The radial engine in which the cylinder centre lines radiate from the centre of the crank shaft and all lie in a plane perpendicular to the axis of the crank shaft.

Internal combustion engines used in submarines, in factories, and in motor cars belong as a rule to class A.

The balancing of engines of type A has been considered in sections (5), (6), and (7), and from the results there given deductions can be drawn for a few arrangements found in internal combustion engines, but not yet specifically considered.

The four-crank engine of type A with equal reciprocating masses and with cranks set, the middle pair parallel and the outer pair parallel and at 180° with the middle pair, and with the cranks symmetrically pitched along the crank shaft, is the usual arrangement found in a four-crank motor car engine. With these conditions the reciprocating masses are balanced for primary forces, for primary, secondary, and all higher orders of couples, but are unbalanced for secondary forces.

The unbalanced secondary force may be shown to be

$$\frac{4M\omega^2r^2}{gL}\cos 2\theta \text{ lbs.-wt.}$$

acting vertically at the centre of the crank shaft.

M is the mass in pounds of one set of reciprocating parts, r is the crank radius in feet, L is the length of the connecting rod in feet, and ω is the angular velocity in radians per second. This reduces to very nearly $(5Mn^2r^2/L)\cos 2\theta$, using for g, 32·2. $n=$revolutions per second.

An eight-cylinder V engine formed by setting two four-cylinder engines of the above kind together at an angle of 90° is therefore balanced for primary forces, for primary and secondary and all higher orders of couples, but is unbalanced for secondary forces.

The unbalanced secondary force is the resultant of the unbalanced secondary force from each of the four-cylinder engines. This has just been shown to have the value $(5Mn^2r^2/L)\cos 2\theta$.

Combining these together it will be found that the magnitude of the resultant is very nearly $(7Mn^2r^2/L)\cos 2\theta$, and that this alternating force acts horizontally.

The 6-cylinder engine of type A with equal reciprocating masses and cranks set, the inner pair parallel, the next pair parallel and at 120° with the inner pair, and the outer pair parallel and at 240° with the inner pair, and with the pairs symmetrically pitched along the crank shaft, is balanced for primary and secondary forces and higher orders, excluding orders divisible by 3; and for primary and secondary and all higher orders of couples. Practically it is a perfectly balanced engine. V engines made up of six-cylinder engines of this kind are therefore in almost perfect balance.

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[1] See "Petrol Engine, the Water-cooled"; "Aircraft Engines," Vol. IV.

The radial engine, type C, an equiangular n cylinder radial engine with the connecting rods coupled to one crank pin and with reciprocating masses per cylinder equal to M pounds, is balanced for primary reciprocating forces by a balance weight m opposite the crank pin calculated from

$$m = \frac{nM}{2} \text{ pounds at crank radius.}$$

This is valid for all values of n except 1 and 2.

If further M_1 at crank radius is the equivalent of the unbalanced revolving parts due to the crank arm, crank pin, and the n equal revolving parts of the connecting rods, the actual balance weight to be added at crank radius diametrically opposite the crank pin to balance the reciprocating parts and the revolving parts is $m + M_1$ pounds.

It is shown below that when the number of cylinders exceeds three, the secondary forces along the respective lines of stroke mutually balance.

If there are three cylinders, mutually at 120°, there is an unbalanced secondary force equivalent to that caused by a mass weighing $1\frac{1}{2}\ Mr^2/L$ pounds concentrated at the crank pin of a crank of radius r and revolving twice as fast as the main crank, but in the opposite direction.

These statements are proved as follows.

Let β be the angle between adjacent lines of stroke so that $\beta = 2\pi/n$. Reckoning the crank angle always in the positive direction from any one of the n lines of stroke in the engine, it will be seen from *Fig.* 22 that the crank angle reckoned from the kth line of stroke is $\theta + (k-1)\beta$.

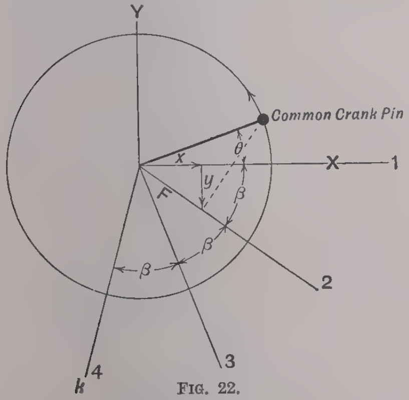

FIG. 22.

The force along the kth line of stroke is therefore

$$F = A\cos\{\theta + (k-1)\beta\} + B\cos 2\{\theta + (k-1)\beta\}$$

$$A = M\omega^2 r \text{ and } B = \frac{M\omega^2 r^2}{L}.$$

The simultaneous values of the force along each line of stroke is found from this equation by substituting in succession $k=1$, $k=2$, to $k=n$.

To find the resultant force on the frame, each of the forces is resolved into components along mutually perpendicular fixed axes and their resultant is determined. The statements made above are easily seen to follow.

These results only apply strictly when the connecting rods are coupled directly to the crank pin. If they are coupled through a floating big end, articulated as it is called, then the expression $A\cos\theta + B\cos 2\theta$ no longer gives the accelerating force in the line of stroke with accuracy.

Whatever be the mechanical arrangement, however, if the engine is balanced the mass centre of all the moving parts remains at rest during the motion. Particular arrangements can therefore be examined from this point of view.

In conclusion a point may be mentioned which is of importance in internal combustion engines. The driving torque on the crank shaft is necessarily accompanied by an equal and opposite reaction on the frame. If the driving torque varies periodically it tends to set the frame in torsional oscillation. This is minimised by increasing the number of cylinders and arranging the sequence of firing so that the torque curve shall be as uniform as possible. The eight-cylinder engine of type A gives a torque curve of least variation. The six-cylinder comes next. The four-cylinder has a large variation per revolution.

The addition of a flywheel, although controlling the variation of speed consequent upon torque variation, has no effect on the torsional reaction on the frame. That must follow every variation of the torque exerted by the crank shaft whether against a resistance or a resistance plus a flywheel. There is no difficulty in securing practical uniformity of torque in a steam engine.[1] W. E. D.

ENGINES, THERMODYNAMICS OF INTERNAL COMBUSTION

§ (1).—IN all fluids the pressure, volume, and temperature are so related that if any two of these be known the value of the third is determined. We may thus take as our independent variables the pressure and volume, or the pressure and temperature, or the volume and temperature. In considering the thermodynamics of internal combustion engines it is in general found most convenient in diagrams to take the pressure and volume as the independent variables.

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[1] "A Comparison of Five Types of Engines," etc., W. E. Dalby, *Trans. Inst. Nav. Arch.*, 1902.

§ (2) Cycles of Operations.—When a definite quantity of any substance is subjected to a series of operations such that it returns finally to its initial condition in all respects, such a series is termed a "Cycle of Operations."

Carnot's conception of a cycle of operations as a convenient means of considering quantitatively the changes undergone by the working substance of an ideal heat engine has proved invaluable in the development of the modern science of thermodynamics; its special merit is that as the working substance returns exactly to its initial state at the end of the cycle, so also does its internal energy, and hence we avoid the problem of discussing the measure of the change of energy of the working substance from its initial to its final state.

In considering the performance of internal combustion engines it is found convenient to divide cycles of operation into two classes, viz. (*a*) perfect cycles, and (*b*) imperfect cycles.

A cycle is said to be perfect when it is of maximum efficiency for the temperature range within which the engine works.

A perfect cycle is reversible, *i.e.* if the engine be worked backwards the heat transferences at each stage are exactly reversed.

A cycle is said to be imperfect when its efficiency is necessarily less than that of a perfect cycle for the same temperature range.

All the cycles of operation employed in actual internal combustion engines are imperfect in this sense; owing to the nature of the operations performed, their maximum efficiency is in every case necessarily less than the conceivable maximum for the range of temperature involved; the reasons for this are fully discussed later.

§ (3) Isothermal and Adiabatic Changes.[1]—When the supply of heat to a substance is so regulated that its simultaneous changes of pressure and volume take place without change of temperature, the changes are termed isothermal.

When simultaneous changes of pressure and volume take place in a substance completely heat-insulated from all external bodies, so that there can be neither loss nor gain of heat, the changes are said to be adiabatic. Expansion takes place, in general, against some external resistance (as, *e.g.*, that offered by atmospheric pressure, or a loaded piston), and hence mechanical work must be done by the expanding substance in order to overcome this external resistance. Anticipating the first law of thermodynamics, it is evident that if heat be supplied to the expanding substance some, at least, of this will be expended in the external work of expansion. On the other hand, if no heat be supplied to the expanding substance, the heat-equivalent of the external work done must be supplied from the store of energy of the substance itself (termed its internal energy), and its temperature will, in consequence, fall as the expansion proceeds. Clearly, then, on a pressure-volume diagram a curve indicating adiabatic changes from any initial condition is always steeper than a curve indicating isothermal changes from the same initial condition.

§ (4) The Two Laws of Thermodynamics. (i.) *The First Law.*—The first law asserts that heat is a form of energy; when heat is converted into mechanical work, or *vice versa*, the quantity of mechanical work done is simply proportional to the quantity of heat converted.

The British Thermal Unit (B.Th.U.) is the quantity of heat that will raise 1 lb. of water at, or near, its temperature of maximum density, through 1° F. This quantity of heat if wholly converted into mechanical energy will perform 778 ft.-lbs. of work. This, the "mechanical equivalent" of heat, is usually styled "Joule's equivalent" and denoted by the symbol J. If the centigrade scale of temperatures be used the value of the mechanical equivalent, then styled the C.Th.U., becomes approximately 1400 ft.-lbs. (See "Heat, Mechanical Equivalent of," § (9).)

(ii.) *The Second Law.*—This puts on record our experience that heat, unaided, is unable to pass from one body to another at a higher temperature. Clausius' enunciation of this law is as follows: "It is impossible for a self-acting machine, unaided by any external agency, to convey heat from one body to another at a higher temperature." (See "Thermodynamics," § (1).)

§ (5) Carnot's Ideal Cycle.—The cycle imagined by Carnot comprises two isotherms and two adiabatics. During the first operation the working substance—which may be any whatever—is caused to expand isothermally from 1 to 2 at a temperature T (*Fig.* 1), receiving during this operation a quantity of heat which may be denoted by H. At some point 2 the heat supply ceases, and the expansion continues adiabatically until a point 3 is reached. At 3 the third operation commences, viz. that of isothermal compression at some lower temperature *t*, to a point 4, such that on further compressing adiabatically from 4, the point 1 is again reached; during

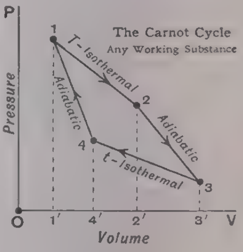

Fig. 1.

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[1] See also "Thermodynamics," §§ (15), (16).

the isothermal compression $\overline{34}$ a quantity of heat, *h*, is abstracted from the working substance. During the isothermal and adiabatic expansions $\overline{12}$ and $\overline{23}$, mechanical work is done *by* the substance, measured by the areas 122′1′1 and 233′2′2 respectively. During the isothermal and adiabatic compressions $\overline{34}$ and $\overline{41}$, mechanical work is done *on* the substance, measured by the areas 344′3′3, 411′4′4 respectively. The complete cycle of operations leaves, therefore, a quantity of "useful work" done *by* the substance represented by the area $\overline{1234}$ obtained by the expenditure of a quantity of heat H, of which a part, *h*, is rejected. By the First Law of Thermodynamics, therefore, the useful mechanical work U (measured, for convenience, in heat-units) must be given by the relation

$$\mathrm{U}=\mathrm{H}-h. \qquad . \quad . \quad . \quad (1)$$

And the efficiency of the cycle, being the ratio of the useful work obtained to the energy expended, is

$$\text{Efficiency}=\frac{\mathrm{U}}{\mathrm{H}}=\frac{\mathrm{H}-h}{\mathrm{H}}=1-\frac{h}{\mathrm{H}}. \quad . \quad (2)$$

This equation expresses the efficiency of the Carnot cycle in terms of the heat received at the upper limit of temperature T, and that rejected at the lower limit *t*.

Moreover, the cycle is reversible; if the engine be worked backwards and an amount of energy U be given to the working substance, it will draw heat *h* from the sink at temperature *t* and give heat H to the source at temperature T, passing through all the stages passed through during the direct process, and being in the same condition in all respects at each corresponding stage in the two.

§ (6) THE CARNOT CYCLE A "PERFECT" CYCLE.—No conceivable engine, whatever its cycle, working between T and *t* can have a greater efficiency than the Carnot engine. For if it be possible, let there be such an engine X using the same "source" of heat at temperature T, and the same "sink" of heat at temperature *t*.

Let X be so coupled to the Carnot engine as to drive it backwards without doing external work; then during each revolution of the coupled engines the following exchanges take place:

Engine X takes heat H′ from the T-source, rejects heat *h*′ to the *t*-sink, and performs mechanical work U = H′ − *h*′; and

The Carnot engine is reversible, and working backwards takes in heat *h* from the *t*-source, and rejects heat H into the T-source in virtue of mechanical work U done upon it, given by U = H − *h*. Thus, on the whole, no mechanical work is done, and H − *h* = H′ − *h*′; hence H − H′ = *h* − *h*′. But since X is the more efficient engine, it does the work U for a less expenditure of heat from the source than that required by the Carnot engine. Thus H′ is less than H, and hence *h*′ is less than *h*. Thus the source at temperature T gains heat H − H′, and the sink at a lower temperature *t* loses an equal amount of heat *h* − *h*′. Hence by means of an inanimate material agency—the two engines coupled—heat is transferred from a body at temperature *t* to one at a higher temperature T, in violation of the Second Law of Thermodynamics. Thus engine X cannot have a greater efficiency, working between T and *t*, than that of the Carnot engine between the same temperatures.

Hence the Carnot cycle is of maximum efficiency, *i.e.* is a "Perfect" cycle.

It immediately follows that, within the same temperature range, all perfect cycles are of equal efficiency, the value of which in terms of heat received and heat rejected is as given in equation (2).

§ (7) KELVIN'S ABSOLUTE SCALE OF TEMPERATURE.—Equation (2) expresses the efficiency in terms of heat quantities; it is next necessary to show how the efficiency may be expressed in terms of the limiting temperatures themselves.

Let A_1E_1 (*Fig.* 2) represent a portion of an isothermal, at temperature T_1, of the working substance, which may be any whatever. Along A_1E_1 let points B_1, C_1, D_1 be taken such that the expansions A_1B_1, B_1C_1, C_1D_1 correspond to the addition of equal quantities of heat, H_1, at the constant temperature T_1; and through A_1, B_1, C_1 draw a series of adiabatics of the substance.

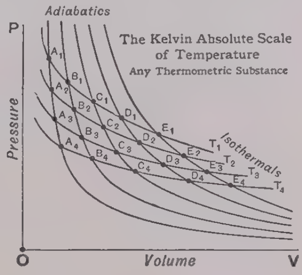

FIG. 2.

Let $A_2B_2C_2$ be a second isothermal of the substance corresponding to some lower temperature T_2.

Then the quadrilaterals A_1B_2, B_1C_2, C_1D_2 are all equal; for each is the useful-work-area, U, of a Carnot engine diagram receiving heat H_1 at the higher temperature T_1, and rejecting heat at the lower temperature T_2. Moreover, since the efficiency is the same for each of these perfect cycles the amount of heat rejected at the lower temperature T_2 is the same for each. Let it be H_2. Hence, also A_2B_2, B_2C_2, C_2D_2

represent the isothermal changes at T_2 due to equal additions of heat, H_2, to the substance. Next suppose there to be drawn a series of isothermals $A_3B_3C_3$, $A_4B_4C_4$ such that the quadrilaterals A_1B_2, A_2B_3, A_3B_4 are all equal; then it is evident that the two series of isothermal and adiabatic curves divide up the whole diagram into quadrilaterals *of equal area.* Now Kelvin defined equal temperature intervals as those between any two consecutive isothermals of this series; as this system of graduating temperatures is independent of the nature of the thermometric substance employed, it is styled the absolute scale of temperature. On this scale it is immediately obvious from the diagram that the efficiency of a Carnot engine working between T_1 and T_3 is twice as great as that between T_1 and T_2; and between T_1 and T_4 three times as great: and, generally, that the efficiency is proportional to the difference between the upper and lower temperature limits *measured in this way, i.e.*

$$\frac{U}{H_1}=C(T_1-T_n), \quad . \quad . \quad . \quad (3)$$

where T_1 and T_n are the upper and lower absolute temperature limits respectively.

The constant quantity C must be independent of T_n, but may be a function of T_1. It is termed Carnot's Function, and its value is immediately determined from the consideration that as equation (3) is to hold whatever T_n may be, let T_n be supposed so reduced (see *Fig.* 3) that the whole of the heat H_1 is converted into useful work U. As U cannot conceivably be greater than this, it can only be inferred that when $U=H_1$ the temperature T_n is the zero of the Kelvin scale; thus $T_n=0$ when $U=H_1$; hence equation (3) gives in this case $1=CT_1$ or

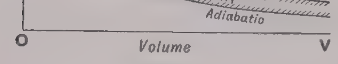

FIG. 3.

$$C=\frac{1}{T_1}. \quad . \quad . \quad . \quad . \quad (4)$$

Thus Carnot's function is the reciprocal of the temperature reckoned from the Kelvin absolute zero.

The efficiency of the Carnot cycle can now be expressed in terms of the absolute temperatures of the limits; for if these be T and t respectively we have from equations (3) and (4)

$$\text{Efficiency}=\frac{T-t}{T}=1-\frac{t}{T}, \quad . \quad . \quad (5)$$

and thus depends only on the *ratio* of the absolute temperatures between which the engine works. From equations (2) and (5) we have at once the important fundamental relation

$$\frac{H}{h}=\frac{T}{t}, \quad . \quad . \quad . \quad . \quad (6)$$

i.e.

$$\frac{\text{Heat received}}{\text{Heat rejected}}=\frac{\text{Abs. upper temp.}}{\text{Abs. lower temp.}} \quad (6')$$

§ (8) TEMPERATURES BY GAS-THERMOMETER.—Anticipating § (14) *et seq.*, we know that for an ideal perfect gas $pv=R\tau$, where τ is temperature as given by the gas-thermometer itself and R a constant; and it is easily shown,[1] by a direct and independent investigation, that with such a gas as working substance, the efficiency of the Carnot cycle is expressed by $1-\tau_2/\tau_1$, where τ_2 and τ_1 are the lower and upper gas-temperature limits respectively. Hence by equations (2) and (6) we have $h/H=\tau_2/\tau_1=t/T$;

$$\frac{\text{Kelvin abs. lower temp.}}{\text{Kelvin abs. upper temp.}}=\frac{\text{Gas-thermometer lower temp.}}{\text{Gas-thermometer upper temp.}}.$$

In the gas-thermometer scale, the temperature interval between the freezing and boiling points of water is divided into 100 equal degrees, and this leads to the zero occurring at 273° below the freezing-point of water. If the same size of degree be adopted for the Kelvin scale, then we shall have $T=\tau_1$ and $t=\tau_2$, and the readings, on either scale, will be identical. As the ordinary permanent gases approximate closely in their properties to those of a perfect gas,—within the temperature range occurring in internal combustion engines,—it is to be expected that the readings furnished by a gas-thermometer will correspond very closely with the temperatures of the Kelvin absolute scale. This is found to be the case, experiments by Joule and Kelvin on the flow of air and other "permanent" gases through porous plugs showing that for all practical purposes the readings of a gas-thermometer sensibly coincide with absolute temperatures on Kelvin's scale.[2]

Callendar (*Phil. Mag.*, 1903) has given tables showing the correction to be made with air, hydrogen, and other gas-thermometers in order to convert their readings to those of the Kelvin absolute scale. The correction is very small, amounting in the constant-volume hydrogen thermometer to not exceeding 1/10th of a degree between −10° C. and

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[1] See "Thermodynamics," § (20).

[2] *Ibid.* § (12).

1000° C. With a constant-volume air thermometer the correction is slightly greater, but still very small. With gas-thermometers of the constant-pressure type the corrections are again slightly greater.

For all thermodynamic investigations on internal combustion engines, however, the readings of a gas-thermometer may be taken as absolute temperatures.

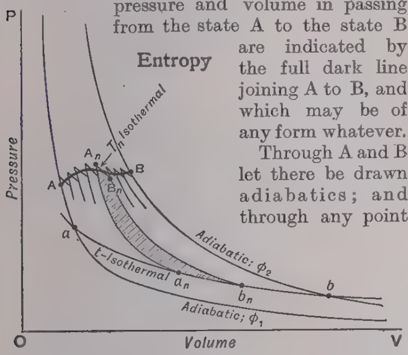

FIG. 4.

§ (9) ENTROPY.—Let AB (*Fig.* 4) be any two points on the pressure-volume diagram of a substance, and suppose that by the addition of heat simultaneous values of its pressure and volume in passing from the state A to the state B are indicated by the full dark line joining A to B, and which may be of any form whatever. Through A and B let there be drawn adiabatics; and through any point below A let there be drawn any isothermal, cutting these adiabatics in *a* and *b* respectively; and let *t* be the absolute temperature corresponding to the isothermal *ab*. Let the path from A to B be divided into a large number of small parts through each of which a small portion of an isothermal is drawn, as A_nB_n, the corresponding temperature being T_n, and through each extremity of each of these small isothermals let adiabatics be drawn cutting the *t*-isothermal ab. as at a_n, b_n. The whole area AB*ba* is thus divided up into a large number of elements, each of which is a Carnot engine diagram receiving heat H_n at temperature T_n during the isothermal expansion A_nB_n, and rejecting heat h_n at temperature *t* during the isothermal compression b_na_n. The work-area $A_nB_nb_na_nA_n$ being denoted by U_n, we have therefore, by equations (2) and (5),

$$U_n = H_n\left\{1-\frac{t}{T_n}\right\},$$

which may be written

$$H_n - U_n = t\frac{H_n}{T_n}. \quad . \quad . \quad . \quad (7)$$

Set out at length, therefore,

$$\{H_1+H_2+H_3+\ .\ .\ .\ \}-\{U_1+U_2+U_3+\ .\ .\ .\ \}$$
$$=t\left\{\frac{H_1}{T_1}+\frac{H_2}{T_2}+\frac{H_3}{T_3}+\ .\ .\ .\ \right\}.$$

But $h_n=(H_n-U_n)$ by equation (1); setting this out at length also we have

$$\{h_1+h_2+h_3+\ .\ .\ .\ \}=\{H_1+H_2+H_3+\ .\ .\ .\ \}$$
$$-\{U_1+U_2+U_3+\ .\ .\ .\ \},$$

and accordingly,

$$\{h_1+h_2+h_3+\ .\ .\ .\ \}=t\left\{\frac{H_1}{T_1}+\frac{H_2}{T_2}+\frac{H_3}{T_3}+\ .\ .\ .\ \right\}.$$

But $h_1+h_2+h_3+\ .\ .\ .\ =h$, the whole quantity of heat rejected by isothermal compression at *t*, from *b* to *a*; so that the result becomes

$$\left\{\frac{H_1}{T_1}+\frac{H_2}{T_2}+\frac{H_3}{T_3}+\ .\ .\ .\ \right\}=\frac{h}{t},\ \text{a constant},$$

whatever H_1, H_2, H_3 may be.

When the number of parts into which the path AB is divided is indefinitely great, each of the quantities H_1, H_2, H_3, etc., is properly denoted by *d*H, and we thus obtain the remarkable and important result

$$\int_A^B\frac{dH}{T}=\frac{h}{t},\ \text{a constant.} \quad . \quad . \quad (8)$$

This is true whatever the form of the path between A and B; it is true wherever A and B be taken on their respective adiabatics; and it is also true wherever the isothermal *ab* be drawn. The result may be expressed verbally as follows: For any working substance let any two adiabatics be drawn; take any point on one and any second point on the other; by means of a heat supply to the substance, varying in any manner whatever, let the condition of the substance be changed from that denoted by the first point to that denoted by the second; divide each very small addition of heat, whatever its source, by the absolute temperature at which it is added. Then the algebraical sum of these quotients is constant in value.

Along any adiabatic $dH=0$ by definition; thus in this case $\int_A^B dH/T=0$.

Hence (*Fig.* 4), $\int_A^B dH/T$ indicates a *quantity* which increases by a constant amount as we pass from one adiabatic A*a* to another B*b*. Clausius gave the name Entropy to the value of the integral $\int_0^A dH/T$ taken from some unknown zero of entropy up to the state A. It is a function only of the state of the body. The zero of entropy is that of a body entirely deprived of heat, a condition unknown to us, but since we are concerned only with the changes of entropy this is immaterial, and equation (8) may be interpreted as expressing the fact that $\int_A^B dH/T$, the difference of the entropies of the two conditions A and B, is a constant. Entropy is usually denoted by the symbol ϕ; if, then, ϕ_1 be that of A and ϕ_2 that of B, we have, starting from any arbitrary zero,

$$\phi_1=\int_0^A\frac{dH}{T},\quad \phi_2=\int_0^B\frac{dH}{T},$$

and hence $$\phi_2-\phi_1=\int_A^B\frac{dH}{T}. \quad . \quad . \quad . \quad (9)$$

Practically entropy is reckoned from some assumed standard condition, as, *e.g.*, for unit mass at one atmosphere of pressure and 0° C. or 100° C. temperature. As along any adiabatic the entropy is of constant value, all adiabatics are also isentropics.

If an element of heat, dH, be added at absolute temperature T to a substance, the corresponding change of entropy is $d\phi$; by equation (8) $d\phi=dH/T$. This may also be written $dH=Td\phi$, and $dH/d\phi=T$. Or, for a finite change,

$$\int dH=\int Td\phi. \quad . \quad . \quad . \quad (10)$$

The value of the entropy is easily determined in any specific case, as is shown later in this article (*vide* § (25)). *Fig.* 5 shows a temperature-entropy diagram; it is usual to take the temperatures as ordinates and entropy as abscissae; let AB be a curve connecting temperature and entropy, and from any points C ($\phi_1 T_1$) and D ($\phi_2 T_2$) on this curve draw ordinates CM and DN respectively; then

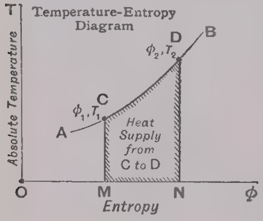

FIG. 5.

$$\int_C^D dH=\int_{\phi_1}^{\phi_2}Td\phi=\text{area MCDNM}.$$

Hence the hatched area measures the heat supply to the substance from condition C to condition D. On the temperature-entropy, or "$T-\phi$" diagram all isothermals obviously appear as horizontal lines, and all adiabatics (isentropics) as vertical lines, as indicated in *Fig.* 6; thus the $T-\phi$ diagram of any engine working on the Carnot cycle, whatever the working substance, is a rectangle as 12341, $\overline{12}$ representing the isothermal expansion at temperature T, $\overline{23}$ the adiabatic expansion from T to t, $\overline{34}$ the isothermal compression at t, and $\overline{41}$ the adiabatic compression from t to T (compare *Fig.* 1). The area M12NM represents H, the heat supplied during $\overline{12}$, while N34MN represents h, the heat rejected during $\overline{34}$; hence the closed area $\overline{12341}$ represents $(H-h)$, *i.e.* U, the heat converted into useful work. And it is immediately obvious from this diagram that the efficiency, U/H, being the ratio of the areas $\overline{12341}$ and M12NM, is also expressed by $(T-t)/T$.

And, generally, the "$T-\phi$" diagram of an engine working on any cycle is represented by an enclosed figure as, *e.g.*, that shown hatched in *Fig.* 7, the area of which represents the useful work performed just as in the case of a *pv* diagram. Through the four extreme points of the boundary of this hatched area let horizontal and vertical lines be drawn as shown. Then the actual engine receives per cycle heat H' represented by the area MABCNM, and rejects heat h', represented by NCDAMN, its efficiency being

$$\frac{H'-h'}{H'},\ i.e.\ 1-\frac{h'}{H'}.$$

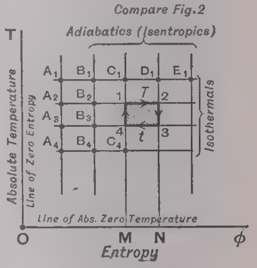

FIG. 6.

Now the temperatures between which this engine works are T and t; and 12341 is the $T-\phi$ diagram of a Carnot engine working between the same temperatures, H being now represented by M12NM, and h by N34MN.

It is obvious that M12NM is greater than MABCNM and that N34MN is less than NCDAMN, that is, that H is greater than H', and h less than h'; thus h/H is less than h'/H'; and, therefore, $(1-h/H)$ is always greater than $(1-h'/H')$; that is, the efficiency of the perfect cycle engine is a maximum, as has already been shown otherwise.

§ (10) ENTROPY IN A COMPLETE CYCLE.—Whatever the cycle of an engine, whether perfect or not, the working substance returns at the end of each cycle to its initial condition in all respects, and thus in a complete cycle there is no change of entropy; or, symbolically, for any complete cycle:

$$\int\frac{dH}{T}=0. \quad . \quad . \quad . \quad (11)$$

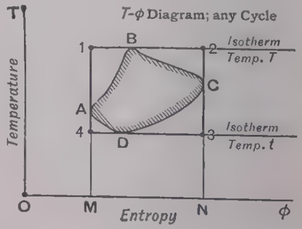

FIG. 7.

In interpreting this equation all heat gained or lost by the substance *as heat*, whatever its source, is to be included. If, for example, the cycle includes processes, as of "unbalanced expansion," any heat generated within the substance itself during the subsidence of its turbulent motions must be included in the summation of equation (11). Any heat so generated is clearly a positive addition to the entropy summation.

If heat acquired by the substance in this, or other analogous manner, be not included in the summation, then it will be necessary to assert that only for any perfect cycle is equation (11) true, and that for an imperfect cycle $\int dH/T$ is negative.

§ (11) TEMPERATURE-ENTROPY DIAGRAMS.—These have proved of much value in studying the action of heat engines using water-steam as the working substance, but have not so far been much used in investigations connected with internal combustion engines.

Among further graphic aids to the study of heat engines may be mentioned the diagrams, introduced by Dr. Mollier in 1904, of total-heat entropy ($H\phi$), and total-heat pressure (Hp); both of these have already been found of service in steam-engine and steam-turbine problems, but they have not yet been employed in connection with internal combustion engines.

§ (12) PERFECT GASES.—For the so-called "permanent" gases, of which (dry) air is the type, experiment has established the following results, termed the Gaseous Laws; they are conformed to by actual gases the more closely as they are raised in temperature the more highly above their condensation-point, that is, the more highly they are superheated. These Gaseous Laws are to be regarded, therefore, as the properties that would be exactly possessed by an absolutely ideal gas.

§ (13) LAW 1 (BOYLE'S LAW).—If temperature remains constant, then the product of pressure by volume is also constant. Hence the isotherms of a perfect gas on a pv diagram are rectangular hyperbolas with the coordinate axes as asymptotes.

§ (14) LAW 2 (CHARLES'S LAW).—Under constant pressure all gases expand at one uniform rate with increase of temperature. As by Law 1, $v \propto 1/p$ when T is constant, and by Law 2, $v \propto T$ when p is constant, it follows that $v \propto T/p$ when both T and p change, *i.e.* $v = R(T/p)$ where R is some constant. This result, combining Laws 1 and 2 in one statement, is usually written

$$pv = RT, \quad . \quad . \quad . \quad (12)$$

and is termed the "Characteristic Equation" of a perfect gas.

If pv and T suffer simultaneous increments Δp, Δv, ΔT respectively, then, as equation (12) always holds, we must have

$$(p+\Delta p)(v+\Delta v) = R(T+\Delta T),$$

i.e.
$$p\Delta v + v\Delta p + \Delta p\Delta v = R\Delta T,$$

and accordingly, in the limit, when the increments are indefinitely diminished,

$$pdv + vdp = RdT \quad . \quad . \quad (12\text{A})$$

is the invariable relation connecting dv, dp, and dT.

Here T is strictly the temperature measured on the absolute scale (§ (7)), but for all practical purposes the reading of a gas thermometer, or a good mercurial thermometer, is sufficiently accurate (*vide* § (8)). From equation (12) it appears that the expression pv/R measures the absolute temperature of a gas, and this property is much used in investigations of the action of internal combustion engines. The value of the constant R is determined in any specific case when the condition of the gas is known at any one instant. Thus for 1 lb. of air at 0° C. (T=273° C. abs.), when p is 14·7 lbs. per sq. in. (=2117 lbs. per sq. ft.), it is found that the volume v is 12·39 cub. ft.; hence from equation (12) $2117 \times 12{\cdot}39 = R \times 273$, whence $R = 96$. Accordingly for air the characteristic equation is

$$pv = 96T. \quad . \quad . \quad . \quad (12')$$

§ (15) LAW 3 (REGNAULT'S LAW).—The specific heat at constant pressure is constant for any perfect gas. This quantity $(\delta H/\delta T)_p$ is commonly denoted by k_p.

Another important mode in which a gas may receive heat is at constant volume. The specific heat at constant volume $(\delta H/\delta T)_v$ is usually denoted by k_v. For air as the result of careful experiment the value of k_p was determined by Regnault[1] as 0·2375 C.Th.U. per lb.; from this the value of k_v is obtained, by calculation, as 0·1689 C.Th.U. per lb.[1]

§ (16) LAW 4 (JOULE'S LAW).—If a gas expands without doing any external work, its temperature remains unaltered. The volume of a gas is always maintained by some external pressure, and if a gas expands mechanical work is necessarily done by it in overcoming this external pressure. If the gas be heat-insulated from all external sources, experience shows that its temperature rapidly falls during expansion; the external work is done at the expense of the internal energy of the gas. When there is no external work done there is no loss of internal energy, and, as Joule's Law shows, no loss of temperature; and this is true whatever the pressure of the gas. Hence it is concluded that the internal energy of a gas is proportional to its absolute temperature. Thus in a perfect gas isothermal curves are also curves of equal internal energy, *i.e.* are "Isenergics."

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[1] These values are now considered to be too low; *vide* § (73); also in actual gases the values are not constant.

§ (17) SYMBOLIC EXPRESSION OF THE FIRST LAW OF THERMODYNAMICS.—In general, when heat is supplied to a perfect gas, the pressure, volume, and temperature all undergo change, though at each instant the characteristic equation $pv=RT$ remains true. Suppose the addition of a small quantity of heat ΔH, to a gas at p, v, T, to cause these quantities to become $(p+\Delta p)$, $(v+\Delta v)$, $(T+\Delta T)$ respectively. The effect may be considered to be produced in two steps, viz. (1) a rise in temperature of ΔT (with corresponding rise in pressure Δp) at constant volume, and (2) an expansion at constant temperature between T and $(T+\Delta T)$ through a volume Δv.

The heat necessary to produce the first step is $k_v \Delta T$, k_v being by definition the quantity of heat necessary to raise the gas through 1° C. at constant volume. For the second step the expansion at constant temperature requires an amount of heat equivalent to the external work done, which lies between $p\Delta v$ and $(p+\Delta p)\Delta v$. And the sum of these two quantities is ΔH. Hence, in the limiting case we have

$$dH \quad = \quad k_v dT \quad + \quad \frac{pdv}{J}, \quad (13)$$

$$i.e.\ \text{Heat supplied} = \left\{\begin{array}{c}\text{Increase of}\\ \text{internal}\\ \text{energy}\end{array}\right\} + \begin{array}{c}\text{Ext. work}\\ \text{done,}\end{array}$$

where J is Joule's equivalent (§ (4)). This is the symbolic form of the First Law of Thermodynamics in the case of perfect gases.

§ (18).—Several fundamentally important results are immediately deducible from this equation. Thus, in isothermal or "isenergic" expansion T is constant; hence $k_v dT=0$; and accordingly in this case

$$dH=\frac{pdv}{J}, \quad . \quad . \quad . \quad (14)$$

so that the heat supplied is the exact equivalent of the external work done, the internal energy of the gas remaining unchanged. And, conversely, if a gas be isothermally compressed, the heat emitted is the exact equivalent of the work done upon the gas in compressing it. Integrating equation (14) we have, denoting by H, the heat supply necessary to change the volume from V_o to v isothermally:

$$H=\int_{v_o}^{v}\frac{p}{J}dv=\int_{v_o}^{v}\frac{p_o v_o}{J}\,\frac{dv}{v}=\frac{p_o v_o}{J}\log_e\left(\frac{v}{v_o}\right). \quad (15)$$

The volume ratio of expansion (v/v_o) is usually denoted by r; hence (15) is also written

Heat supplied = Ext. work of expansion

$$=\frac{p_o v_o}{J}\log_e r \quad \frac{RT_o}{J}\log r \quad . \quad (15')$$

in thermal units, T_o being the absolute isothermal-expansion-temperature.

§ (19) RELATION OF k_v TO k_p.—With the same initial suppositions as in § (17), if the addition of ΔH to the gas produces a rise of temperature ΔT (and corresponding volume increase Δv) at constant pressure p, then by the definition of k_p we have at once $\Delta H=k_p\Delta T$; and, in the limit, $dH=k_p dT$; and the final state of the gas being exactly the same in each case, we may write

$$dH=k_p dT=k_v dT+\frac{1}{J}pdv.$$

But $pv=RT$, and as p is here constant, $p(dv/dT)=R$, *i.e.* $pdv=RdT$; hence

$$k_p dT=k_v dT+\frac{R}{J}dT,$$

so that

$$k_p-k_v=\frac{R}{J}, \quad . \quad . \quad . \quad (16)$$

that is, of course, the difference of the two specific heats is the thermal value of the external work done in raising 1 lb. of the gas through 1° C. at constant pressure. The constant $R=p(dv/dT)$ is the measure of this external work in ft.-lbs.

(i.) *Value of k_v for Air.*—Regarded as a perfect gas, the constant value of k_v for air can now be calculated; for we have seen (equation (12′)) that $R=96$, also $k_p=0{\cdot}2375$, and J has been experimentally determined as 1400 ft.-lbs. per C.Th.U.: hence by (16)

$$0{\cdot}2375-k_v=\frac{96}{1400},$$

whence $\quad k_v=0{\cdot}1689$ C.Th.U per lb.

as already stated.

(ii.) *Ratio of k_p/k_v; Value of γ.*—The ratio of k_p/k_v, always denoted by γ, is of fundamental importance in all thermodynamic investigations; in the case of air, regarded as a perfect gas, we see that

$$\gamma=\frac{k_p}{k_v}=\frac{0{\cdot}2375}{0{\cdot}1689}=1{\cdot}406.$$

Assuming all heat-quantities to be expressed as energy in ft.-lbs.

Equation (13) becomes

$$dH=k_v dT+pdv, \quad . \quad . \quad . \quad (13\text{A})$$

while from equation (16)

$$R=k_p-k_v=k_v(\gamma-1).$$

Hence by equation (12A)

$$RdT=k_v(\gamma-1). \quad dT=pdv+vdp,$$

so that

$$k_v dT=\frac{1}{\gamma-1}\{pdv+vdp\};$$

and accordingly (13A) becomes

$$dH=\frac{1}{\gamma-1}\{vdp+\gamma pdv\}, \quad . \quad . \quad (13\text{B})$$

which may also be written

$$\frac{dH}{dv}=\frac{1}{\gamma-1}\{v\frac{dp}{dv}+\gamma p\}. \quad . \quad . \quad (13\text{C})$$

This equation is sometimes useful in the analysis of actual indicator diagrams of internal combustion engines.[1]

The following formula giving $H-H_o$ readily in work-units, in terms of quantities easily measured from an ordinary indicator (pv) diagram, is also worthy of notice. We have

$$T=\frac{1}{R}pv;$$

hence $$dT=\frac{1}{R}d\overline{pv};$$

and thus equation (13A) becomes

$$dH=\frac{k_v}{R}d\overline{pv}+pdv,$$

i.e. $$dH=\frac{1}{\gamma-1}d\overline{pv}+pdv.$$

Integrating all across from ($p_o v_o H_o$) to (pvH) gives

$$H-H_o=\frac{1}{\gamma-1}(pv-p_o v_o)+\int_{v_o}^{v}pdv. \quad . \quad (13A')$$

Thus $(H-H_o)$ is determined as p, v, p_o, and v_o are directly measurable, and $\int_{v_o}^{v}pdv$ is readily found by planimetric measurement from the indicator diagram.

If the actual expansion (or compression) curve of a diagram can be resumed by an equation of the form $pv^n=$constant (*vide* § (22) *infra*), we have by differentiation

$$v^n\frac{dp}{dv}+nv^{n-1}p=0,$$

i.e. $$v\frac{dp}{dv}=-np;$$

hence in this case equation (13C) takes the very simple form

$$\frac{dH}{dv}=\frac{\gamma-n}{\gamma-1}p. \quad . \quad . \quad . \quad (13D)$$

If, during expansion, the gas loses heat, dH/dv is negative and hence n must be greater than γ.

If, during compression, the gas loses heat, then as $dH=(\gamma-n/\gamma-1)pdv$ and both dH and dv are negative, n must be less than γ.[2]

When $pv^n=a$, a constant, $p=av^{-n}$, and thus equation (13D) may be written

$$\int dH=\frac{\gamma-n}{\gamma-1}a\int v^{-n}dv;$$

on integration and reduction this gives

$$H-H_o=\frac{\gamma-n}{\gamma-1}\,\frac{1}{1-n}(pv-p_o v_o); \quad . \quad (13E)$$

see also equation (35) *infra.*

§ (20) THE ADIABATIC EQUATION OF A PERFECT GAS.—In adiabatic changes no heat enters or leaves the gas, and hence in equation (13) we symbolise this condition by putting $dH=0$. Thus in adiabatic change we have

$$k_v dT+\frac{1}{J}pdv=0, \quad . \quad . \quad (17)$$

which expresses that in this mode of expansion the external work is done wholly at the expense of the internal energy of the gas.

As $p=RT/v$, and $R=J(k_p-k_v)=Jk_v(\gamma-1)$, we may write (17)

$$k_v dT+k_v(\gamma-1)T\frac{dv}{v}=0;$$

that is $$\frac{dT}{T}+(\gamma-1)\frac{dv}{v}=0. \quad (18)$$

Integrating, $\log_\epsilon T+(\gamma-1)$, $\log_\epsilon v=b$, a constant. If T_o and v_o be datum temperature and volume respectively,

$$b=\log_\epsilon T_o+(\gamma-1)\log_\epsilon v_0.$$

Hence

$$\log_\epsilon\left(\frac{T}{T_o}\right)+(\gamma-1)\log_\epsilon\left(\frac{v}{v_o}\right)=0,$$

i.e. $$\log_\epsilon\left(\frac{T}{T_o}\right)\left(\frac{v}{v_o}\right)^{\gamma-1}=0,$$

and thus $$\left(\frac{T}{T_o}\right)\left(\frac{v}{v_o}\right)^{\gamma-1}=1. \quad . \quad . \quad (19)$$

This is the adiabatic equation of a perfect gas in terms of volume and temperature. As $pv=RT$ always, equation (19) may easily be expressed in one of the following three ways:

$$pv^\gamma=p_0 v_0{}^\gamma \;.\;.\;.\; \text{a constant}, \quad . \quad (20)$$

$$T=T_0\left(\frac{p}{p_0}\right)^{1-\frac{1}{\gamma}}, \quad . \quad . \quad (21)$$

$$T=T_0\left(\frac{v_0}{v}\right)^{\gamma-1}. \quad . \quad . \quad . \quad (22)$$

§ (21) EXPANSION TO INFINITY.—If unit mass of a perfect gas be expanded (1) isothermally and (2) adiabatically, from an initial condition p_o, v_o, T_o, to an infinite volume, the external work done being $\int_{v_0}^{\infty}$, pdv is infinite in the isothermal case, by equation (15′). In the adiabatic case, however, we have

$$\text{Work to infinity}=\int_{v_0}^{\infty}pdv=p_o v_o{}^\gamma\int^{\infty}v^{-\gamma}dv$$

by equation (20); that is,

$$\text{Work to infinity}=-p_o v_o{}^\gamma\times\frac{v_o{}^{-\gamma+1}}{-\gamma+1}=\frac{p_o v_o}{(\gamma-1)}. \quad (23)$$

As $p_o v_o=RT_o=k_v(\gamma-1)T_o$, this may also be written

$$\text{Work to infinity}=k_v T_o, \quad . \quad (24)$$

an obvious truth, as in this case the whole internal energy of the gas is converted into external work.

For 1 lb. of dry air at 0° C. and one atmosphere pressure, regarded as a perfect gas, the external work done in expanding to infinity would thus be $0{\cdot}1689\times273=46{\cdot}1$. C.Th.U. $=64{,}540$ ft.-lbs., or 28·8 ft.-tons. As $v_o=12{\cdot}39$ cub. ft. this may also be expressed by saying that 1 cub. ft. at 0° C. and one atmosphere, expanding to infinity in the circumstances here supposed, would perform about $2\frac{1}{3}$ ft.-tons of work.

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[1] Wimperis, *The Internal Combustion Engine*, pp. 64-66.

[2] Ayrton and Perry, *Proc. Phys. Soc.*, 1885.

§ (22) CURVES OF EQUATION pv^n = CONSTANT.—All the curves of this family have a general resemblance to the hyperbola for positive values of the constant index n, the co-ordinate axes being asymptotes. Such curves are frequently of service in the study of heat-engine diagrams as it is often found possible to express actual compression or expansion curves by assigning a suitable value to n.[1] It is thus useful to establish some geometric and thermodynamic properties connected with them.

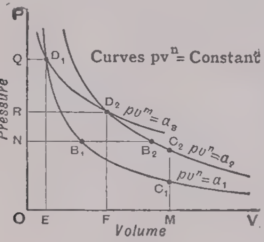

FIG. 8.

Thus (*Fig.* 8), let $D_1B_1C_1$, $D_2B_2C_2$ be two curves whose equations are $pv^n=a_1$, $pv^n=a_2$ respectively.

Property 1.—Let these two curves be cut by any horizontal, *i.e.* constant pressure, line NB_1B_2. Denote NB_1 by v_1, NB_2 by v_2, and ON by p.

Then
$$\frac{pv_2^n}{pv_1^n}=\frac{a_2}{a_1},$$
and therefore
$$\left(\frac{v_2}{v_1}\right)=\left(\frac{a_2}{a_1}\right)^{\frac{1}{n}}, \text{ a constant} \quad . \quad . \quad (25)$$

Hence any horizontal (constant pressure) line cuts the two curves in such manner that the ratio NB_2/NB_1, *i.e.* v_2/v_1, is constant.

Property 2.—Let the same two curves be cut by any vertical, *i.e.* constant volume, line MC_1C_2. Denote MC_1 by p_1, MC_2 by p_2, and OM by v.

Then
$$\frac{p_2v^n}{p_1v^n}=\frac{a_2}{a_1},$$
and therefore
$$\frac{p_2}{p_1}=\frac{a_2}{a_1} \text{ a constant.} \quad . \quad . \quad . \quad (26)$$

Hence any vertical (constant volume) line cuts the two curves in such manner that the ratio MC_2/MC_1, *i.e.* p_2/p_1, is constant.

Property 3.—Let the same two curves be cut by a third curve D_1D_2 whose equation is $pv^m=a_3$. Denote QD_1 by v_1, RD_2 by v_2, ED_1 by p_1, and FD_2 by p_2.

Then
$$p_1v_1^n=a_1,\ p_1v_1^m=a_3.\ p_2v_2^n=a_2,\ p_2v_2^m=a_3.$$

Hence
$$\frac{p_2v_2^n}{p_2v_2^m}\cdot\frac{p_1v_1^m}{p_1v_1^n}=\frac{a_2\times a_3}{a_3\times a_1},$$

i.e.
$$\left(\frac{v_2}{v_1}\right)^{n-m}=\frac{a_2}{a_1},$$

and therefore
$$\left(\frac{v_2}{v_1}\right)=\left(\frac{a_2}{a_1}\right)^{\frac{1}{n-m}}, \text{ a constant.} \quad (27)$$

As this result is independent of a_3, we have the property that all the curves pv^m = a constant cut the pair of pv^n curves in such manner that the value of the ratio (v_2/v_1) is constant.

Lastly, we have
$$\frac{p_2v_2^n}{p_1v_1^n}=\frac{a_2}{a_1}=\left(\frac{p_2}{p_1}\right)\left(\frac{v_2}{v_1}\right)^n=\left(\frac{p_2}{p_1}\right)\left(\frac{a_2}{a_1}\right)^{\frac{n}{n-m}},$$
by equation (27); that is,
$$\left(\frac{p_2}{p_1}\right)=\left(\frac{a_2}{a_1}\right)^{\frac{m}{m-n}}, \text{ a constant.} \quad . \quad (28)$$

This result being also independent of a_3, all the curves pv^m = a constant cut the pv^n pair in such manner that the value of the ratio (p_2/p_1) is constant.

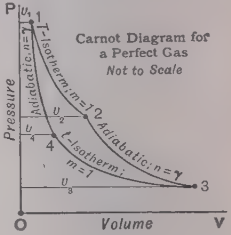

FIG. 9.

§ (23) APPLICATION TO THE CARNOT DIAGRAM.—The diagram of a Carnot cycle engine using a perfect gas as working substance (*Fig.* 9) consists of two isotherms pv = constant, and two adiabatics pv^γ = constant; compare *Fig.* 1.

Here $m=1$, $n=\gamma$; and we have accordingly by equation (27)
$$\left(\frac{v_2}{v_1}\right)=\left(\frac{v_3}{v_4}\right), \quad . \quad . \quad . \quad . \quad (29)$$
or the ratios of isothermal expansion and compression are equal. And from (29) we have immediately also
$$\left(\frac{v_3}{v_2}\right)=\left(\frac{v_4}{v_1}\right), \quad . \quad . \quad . \quad . \quad (30)$$
or the ratios of adiabatic expansion and compression are also equal.

The total expansion ratio is (v_3/v_1); denote this by X, and let r and ρ denote the isothermal and adiabatic expansion ratios respectively. Then as $(v_3/v_1)=(v_3/v_4)(v_4/v_1)$, we have
$$\mathrm{X}=\rho r. \quad . \quad . \quad . \quad (30')$$

Now in this case
$$a_1=p_1v_1^\gamma, \quad a_2=p_3v_3^\gamma,$$
hence by equation (27)
$$r=\left(\frac{p_3v_3^\gamma}{p_1v_1^\gamma}\right)^{\frac{1}{\gamma-1}}=\left(\frac{p_3v_3\times v_3^{\gamma-1}}{p_1v_1\times v_1^{\gamma-1}}\right)^{\frac{1}{\gamma-1}}$$
$$=\left(\frac{\mathrm{R}t}{\mathrm{RT}}\right)^{\frac{1}{\gamma-1}}\times\left(\frac{v_3}{v_1}\right),$$

i.e.
$$r=\mathrm{X}\left(\frac{t}{\mathrm{T}}\right)^{\frac{1}{\gamma-1}}. \quad . \quad . \quad . \quad (31)$$

Hence the isothermal expansion ratio depends upon both the total expansion ratio and the temperature limits. As r is necessarily greater than 1, X must always be taken greater than $(\mathrm{T}/t)^{1/(\gamma-1)}$.

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[1] The value of n is readily determined by logarithmic plotting.

And as by equation (30) $\rho=\mathrm{X}/r$, we have by (31)

$$\rho=\left(\frac{\mathrm{T}}{t}\right)^{\frac{1}{\gamma-1}}. \quad . \quad . \quad . \quad (32)$$

Thus the adiabatic expansion ratio depends only upon the temperature limits. By aid of equations (30), (31), and (32) the diagram can be completely drawn when T, t, R, and one pressure, say p_3, are given. Lastly, from equations (5) and (32) we have

$$\text{Efficiency}=1-\left(\frac{1}{\rho}\right)^{\gamma-1}. \quad . \quad (32')$$

And thus, in the Carnot cycle the efficiency depends only upon the ratio of (adiabatic) compression.

§ (24) Further Consequences of the Equation pv^n = Constant.—The external work done when a gas expands from an initial volume v_0 to any other volume V being $\int_{v_0}^{v} pdv$, if the law connecting p and v be $pv^n=p_0v_0{}^n$, $pdv=p_0v_0{}^n\times v^{-n}dv$ and hence

$$\text{Ext. work done}=p_0v_0{}^n\int_{v_0}^{v} v^{-n}dv;$$

that is

$$\text{Ext. work}=p_0v_0{}^n\left\{\frac{v^{-n+1}-v_0{}^{-n+1}}{-n+1}\right\}$$

$$=\frac{p_0v_0{}^n}{1-n}\left\{\frac{v}{v^n}-\frac{v_0}{v_0{}^n}\right\},$$

which reduces immediately to the simple form,

$$\text{Ext. work}=\frac{1}{1-n}\{pv-p_0v_0\}. \quad . \quad . \quad (33)$$

As $pv=\mathrm{RT}$ and $p_0v_0=\mathrm{RT}_0$, this equation may also be written

$$\text{Ext. work}=\frac{\mathrm{R}}{1-n}\{\mathrm{T}-\mathrm{T}_0\}. \quad . \quad . \quad (34)$$

This result shows that when a gas expands in accordance with the law pv^n = constant the external work done is simply proportional to the change of temperature of the gas. As the initial temperature is T_0, if n be greater than 1 the temperature falls during expansion, whereas if n be less than 1 the temperature *rises* as the expansion proceeds. When $n=1$ equation (34) assumes an indeterminate form, but this is already known to be the isothermal case, and the temperature remains constant during the expansion.

To determine the heat expended we have by aid of equation (13)

$$\mathrm{H}=k_v(\mathrm{T}-\mathrm{T}_0)+\int_{v_0}^{v} pdv=k_v(\mathrm{T}-\mathrm{T}_0)+\frac{\mathrm{R}}{1-n)}\mathrm{T}-\mathrm{T}_0;$$

as $\mathrm{R}=k_v(\gamma-1)$ this reduces to

$$\mathrm{H}=\frac{\gamma-n}{1-n}k_v(\mathrm{T}-\mathrm{T}_0)=\frac{k_p-nk_v}{1-n}(\mathrm{T}-\mathrm{T}_0), \qquad (35)$$

which shows that the heat expended is also simply proportional to the change of temperature of the working gas, and that this expands with an apparent specific heat which is constant and equal to $\mathrm{K}=(\gamma-n/1-n)k_v$; K is negative for values of n between 1 and γ and positive for all other values of n.

In *Fig.* 10 values of $\gamma-n/1-n$, *i.e.* of K/k_v, are plotted against values of n from $n=0$ to $n=2$.

When $n=0$, $\mathrm{K}=k_p$; when $n=1$, $\mathrm{K}=\infty$, and the expansion is isothermal; when $n=\frac{1}{2}(\gamma+1)$, $\mathrm{K}=-k_v$; when $n=\gamma$, $\mathrm{K}=0$, and the expansion is adiabatic; and when n is greater than γ, K increases with n towards k_v as a limit. The curve is a rectangular hyperbola with asymptotes as indicated in *Fig.* 10.

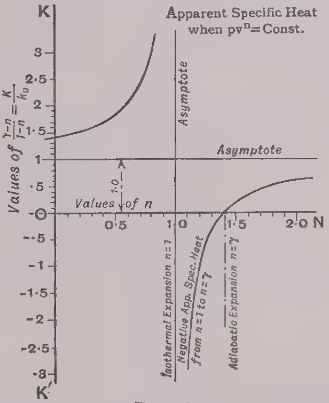

Fig. 10.

It is also to be noted that in expansion of the type pv^n=constant, the change of internal energy, $k_v(\mathrm{T}-\mathrm{T}_0)$, bears a constant ratio to the external work done, $\mathrm{R}/(1-n)\,.\,(\mathrm{T}-\mathrm{T}_0)$. For we have

$$\frac{\text{Internal energy change}}{\text{External work done}}=\frac{k_v(1-n)}{\mathrm{R}}=\frac{1-n}{\gamma-1}. \quad (36)$$

As $(\gamma-1)$ is always positive, the value of this ratio is positive if n be less than 1, and negative if n be greater than 1. When $n=1$, the value is zero and the expansion isothermal; and when $n=\gamma$ the value is -1, and the expansion is adiabatic.

The adiabatic equation (20) is, of course, immediately derived from equation (35) by putting $\mathrm{H}=0$, which obviously requires that $\gamma-n$ shall vanish; thus $n=\gamma$ and the required equation is pv^γ=constant, as already shown.

§ (25) Entropy of a Perfect Gas.—In *Fig.* 11 A_1 and A_2 are any two points on a

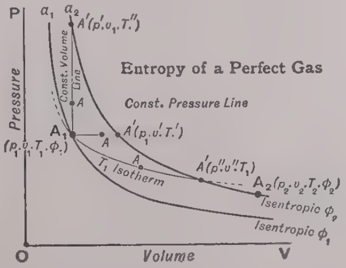

Fig. 11.

pv diagram indicating any two conditions $(p_1, v_1, \mathrm{T}_1, \phi_1)$, and $(p_2, v_2, \mathrm{T}_2, \phi_2)$ respectively of unit mass of a perfect gas. It is required to express $(\phi_2-\phi_1)$ in terms of p_1, v_1, T_1, p_2, v_2, and T_2.

Through A_1 and A_2 draw the isentropics (*i.e.* adiabatics) a_1A_1 and A_2A_2; then $(\phi_2-\phi_1)$ is known if $\int dH/T$ can be calculated for any one path from any point on a_1A_1 to any point on a_2A_2. This can be immediately done if the path be of constant pressure, or constant volume, or constant temperature, and the results in each case will be identical, thus:

(i.) *Constant Pressure Path.*—Through A_1 draw the constant pressure line A_1A', meeting the isentropic a_2A_2 in A'. Let $A\ (p_1vT)$ be any intermediate point; then as $p_1v/p_1v_1=T/T_1$, we have $v=v_1/T_1(T)$, and hence $dv=v_1/T_1(dT)$; thus $p_1dv=p_1v_1/T_1(dT)=RdT$.

Now by equations (9) and (13), expressing work-quantities in heat-units, we have

$$\phi_2-\phi_1=\int\frac{dH}{T}=k_v\int_{T_1}^{T'}\frac{dT}{T}+\int_{v_1}^{v^1}\frac{p_1dv}{T}.$$

or, as $p_1dv=RdT$,

$$\phi_2-\phi_1=k_v\int_{T_1}^{T'}\frac{dT}{T}+R\int_{T_1}^{T'}\frac{dT}{T}=(k_v+R)\int_{T_1}^{T'}\frac{dT}{T},$$

i.e.

$$\phi_2-\phi_1=k_p\log_\epsilon\left(\frac{T'}{T_1}\right). \quad . \quad . \quad (37)$$

But, by equation (21), as A' and A_2 are both on the isentropic a_1A_2,

$$T'=T_2\left(\frac{p_1}{p_2}\right)^{1-\frac{1}{\gamma}};$$

hence (37) becomes

$$(\phi_2-\phi_1)-k_p\log_\epsilon\left(\frac{T_2}{T_1}\right)\left(\frac{p_1}{p_2}\right)^{1-\frac{1}{\gamma}}, \quad (38)$$

which expresses the difference of entropy sought in terms of the pressure and temperature at A_1 and A_2.

As $p_1v_1=RT_1$, $p_2v_2=RT_2$, we have

$$\left(\frac{p_1}{p_2}\right)=\left(\frac{T_1}{T_2}\right)\left(\frac{v_2}{v_1}\right);$$

hence (38) becomes

$$(\phi_2-\phi_1)=k_p\log_\epsilon\left(\frac{T_2}{T_1}\right)^{\frac{1}{\gamma}}\left(\frac{v_2}{v_1}\right)^{1-\frac{1}{\gamma}}, \quad . \quad (39)$$

which expresses the difference of entropy in terms of the volume and temperature at A_1 and A_2.

This may be somewhat simplified; for $k_p-\gamma k_v$: substituting therefore, equation (39) becomes

$$(\phi_2-\phi_1)=k_v\log_\epsilon\left(\frac{T_2}{T_1}\right)\left(\frac{v_2}{v_1}\right)^{\gamma-1}. \quad (40)$$

Or again, as $T_2/T_1=p_2v_2/p_1v_1$, equation (40) may be written

$$(\phi_2-\phi_1)=k_v\log_\epsilon\left(\frac{p_2v_2{}^\gamma}{p_1v_1{}^\gamma}\right), \quad . \quad (41)$$

which expresses the difference of entropy in terms of the pressure and volume at A_1 and A_2.

Equations (38), (39), (40), (41) enable the difference of entropy between A_1 and A_2 to be calculated when any two of the variables p, v, T are known at each point.

As along an adiabatic pv^γ is constant, if the value of this constant be denoted by β, equation (40) takes the simple form

$$(\phi_2-\phi_1)=k_v\log_\epsilon\left(\frac{\beta_2}{\beta_1}\right). \quad . \quad . \quad (42)$$

(ii.) *Constant Volume Path.*—Referring again to *Fig.* 11, consider the change of entropy is proceeding along the constant volume path A_1A' between the two isentropics a_1A_1 and A_2A_2.

In this case $dv=0$, and therefore $pdv=0$, no external work being done, hence we have

$$(\phi_2-\phi_1)=k_v\int_{T_1}^{T'}\frac{dT}{T}=k_v\log_\epsilon\left(\frac{T'}{T_1}\right).$$

But A' and A_2 being on the same isentropic,

$$T'=T_2\left(\frac{v_2}{v_1}\right)^{\gamma-1}$$

and therefore

$$\left(\frac{T'}{T_1}\right)=\left(\frac{T_2}{T_1}\right)\left(\frac{v_2}{v_1}\right)^{\gamma-1},$$

and thus

$$(\phi_2-\phi_1)=k_v\log_\epsilon\left(\frac{T_2}{T_1}\right)\left(\frac{v_2}{v_1}\right)^{\gamma-1}, \quad . \quad . \quad .$$

which is identical with equation (40).

(iii.) *Constant Temperature Path.*—Lastly, consider an isothermal path, as A_1A' (*Fig.* 11) between the two isentropics. Then from equations (15) and (15′) we have at once

$$(\phi_2-\phi_1)=\int\frac{dH}{T}=\frac{H}{T_1}=R\log_\epsilon\left(\frac{v'}{v_1}\right).$$

As before, A' and A_2 being on the same isentropic, we have by equation (22)

$$T_2=T_1\left(\frac{v'}{v_2}\right)^{\gamma-1},$$

whence

$$v'=v_2\left(\frac{T_2}{T_1}\right)^{\frac{1}{\gamma-1}},$$

and accordingly

$$(\phi_2-\phi_1)=R\log_\epsilon\left(\frac{v_2}{v_1}\right)\left(\frac{T_2}{T_1}\right)^{\frac{1}{\gamma-1}};$$

that is, as $R=k_v(\gamma-1)$,

$$(\phi_2-\phi_1)=k_v\log_\epsilon\left(\frac{T_2}{T_1}\right)\left(\frac{v_2}{v_1}\right)^{\gamma-1},$$

which is equation (40) again. Thus, proceeding from A_1 to A_2 by the inter-isentropic paths of constant pressure, constant volume, or constant temperature, we are in each case led to the result that the difference of entropy of A_2 and A_1 is expressed by $k_v\log_\epsilon(\beta_2/\beta_1)$, where β is the adiabatic constant pv^γ.

§ (26) THE INTERNAL COMBUSTION ENGINE "MIXTURE."—All the preceding relations are strictly true only for an ideal perfect gas, which may be defined as a gas of which the absolute temperature is simply proportional to the value of the product *pv*. For all the ordinary "permanent" gases the difference between the actual volume, and the value of RT/*p* is so small that it is unnecessary to take it into account in investigations of the performance of internal combustion engines; in all such investigations it is therefore usual to assume that the effective temperature of the working mixture of gases is given by the value of the expression *pv*/R, with a suitable value assigned to the constant R. Callendar has stated[1] that the experimental evidence available indicates the error of this assumption to be certainly less than 1 per cent for a mixture at 2000° C., the composition of the mixture being known.

All internal combustion engines are essentially air engines wherein the air is very suddenly and very intensely heated by causing chemical action to take place throughout its volume; this is effected by "carburetting" the air, *i.e.* by mixing it with a relatively small quantity of some inflammable gas or vapour in order to produce an explosive mixture, and igniting this explosive mixture at a suitable instant.

Before explosion the mixture consists of air (usually somewhat moist), carburetted with town's gas, blast furnace gas, producer gas, "fuel oil" vapour, kerosene vapour, petrol vapour, benzol vapour, alcohol vapour, acetylene, etc. etc., dependent upon the inflammable agent used, and diluted to a varying degree with some of the exhaust products from the preceding cycle; after complete combustion, and during the working stroke, the mixture consists of nitrogen, steam, carbonic acid gas, and usually some excess oxygen.

§ (27) CYCLES OF INTERNAL COMBUSTION ENGINES.—The very numerous working cycles adopted, or suggested, in actual internal combustion engines are conveniently classified according to the condition in which the working substance receives its heat, thus we have:

Class I.—Cycles of combustion at constant temperature.

Class II.—Cycles of combustion at constant pressure.

Class III.—Cycles of combustion at constant volume.

§ (28) CLASS I.—The Carnot cycle, already fully considered, typifies this class; all heat received is received at constant temperature T, and all heat rejected is rejected at some constant lower temperature *t*. The cycle is perfect, and therefore of maximum efficiency expressed by $(1-t/\mathrm{T})$ (*vide* also equation (32′)).

An indicator diagram drawn to scale for an engine working on this cycle with air between 300° C. and 700° C. only is shown in *Fig.* 12. Although the maximum pressure attained in the case taken is about 525 lbs. per sq. in., the mean effective pressure is only about $11\frac{1}{2}$ lbs. per sq. in., or less than one forty-fifth of the maximum pressure. The engine would necessarily be designed to safely withstand the maximum pressure, and would thus be very heavy in relation to its power output. It has long been recognised, owing to this and other practical drawbacks, that the Carnot cycle is quite unsuitable as an actual working cycle, though within recent years its adoption was seriously proposed by Diesel[2] but soon abandoned in favour of the now well-known constant pressure cycle finally adopted in this type of engine.

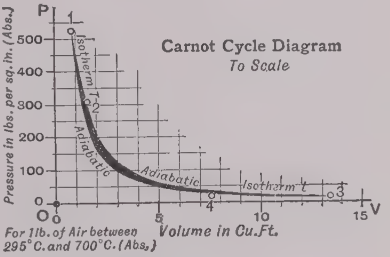

FIG. 12.

§ (29) CLASS II. "*Constant Pressure*" *Cycles.*—In this class are included the American "Brayton" engine (1873), with Simon's modified design, and also engines of the modern highly economical "Diesel" and "semi-Diesel" types.

The Brayton engine included a compressing pump and a working cylinder. The charge of carburetted air, taken in at atmospheric pressure, was first compressed by the pump and delivered into a receiver at a pressure of some 70 lbs. per sq. in. above atmosphere; from this receiver the working cylinder took its charge, the mixture being ignited on entering, inflammation of the contents of the receiver being prevented by the interposition of a fine wire-gauze screen somewhat in the manner of action of the Davy miner's safety lamp.

During the first portion of the working stroke the working cylinder thus received its mixture in an ignited condition at a pressure nearly equal to that in the receiver; at a suitable instant an inlet valve cut off communication with the receiver, the working stroke being then completed by the expansion of the flaming mixture contained in the cylinder. At the end of the stroke the exhaust valve was opened, and during the return stroke the burnt gas was expelled into the atmosphere, thus completing the cycle.

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[1] *Gaseous Explosions Committee*, 1st Report, 1908.

[2] *The Rational Heat Motor*, Eng. ed. by Spon.

A working impulse occurred in every revolution, *i.e.* in every alternate stroke, so that the Brayton engine worked on what is now termed a "two-stroke" cycle; it will also be noted that in this engine the mode of ignition adopted increased not the pressure but the *volume* of the working mixture, the pressure in the working cylinder never quite equalling that in the receiver.

The cycle comprised five operations, viz.:

(*a*) Charging the pump with carburetted air.

(*b*) Compression of the charge into the receiver.

(*c*) Admitting the (ignited) compressed charge to the working cylinder.

(*d*) Expanding after cut-off in the working cylinder.

(*e*) Expelling the burnt gas during the return (exhaust) stroke.

The ideal diagram of such a cycle is drawn by supposing no heating or throttling of the charge to occur during admission to the pump; no loss of the heat developed during compression into the receiver; no throttling of the charge on entering the working cylinder; no loss of heat by the flame to the working cylinder and piston; no back pressure during exhaust; and complete expulsion of all the burnt gas at the end of exhaust, which implies the supposition of no clearance in the working cylinder.

Though these conditions cannot be actually attained, they may be approximated to by skilled design.

The ideal diagram of the Brayton type of engine will therefore have the form shown in *Fig.* 13, and it is convenient to exhibit the diagrams of the pump and of the working cylinder superposed.

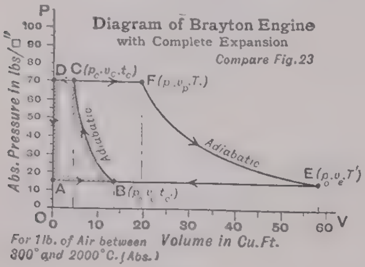


FIG. 13.

AB represents the volume of the pump, AE that of the working cylinder; the carburetted charge is drawn in by the pump at atmospheric pressure AB, and compressed adiabatically along BC to the receiver pressure of about 70 lbs. per sq. in. above atmosphere, and then forced into the receiver at this pressure as indicated by the line CD.

From the receiver it enters the working cylinder at this pressure as indicated by DF, the supply being cut off at F; thence the expansion is adiabatic, and it is first supposed that this expansion is continued until the pressure has fallen to that of the atmosphere, at E. During the return stroke the burnt products are exhausted as indicated by the line EA, and the cycle is then complete.

The area ABCDA represents the work done by the compressing pump *on* the working mixture, while the area DFEAD represents the work done *by* the mixture upon the working piston. The difference, represented by the area BCFEB, accordingly represents the useful work done by the engine per cycle.

The notation adopted is shown in *Fig.* 13; the heat supplied is that corresponding to the line CF, during which the working mixture increases its volume at constant pressure, from v_c to v_p, and is accordingly expressed by

$$H = k_p(T - t_c), \quad . \quad . \quad . \quad (43)$$

while the heat rejected is that during the constant pressure exhaust period EB (regarded as compression at constant pressure), from volume v_e to volume v_o, and is accordingly given by

$$h = k_p(T' - t_o). \quad . \quad . \quad . \quad (44)$$

The efficiency, being the ratio of useful work done to heat supplied, is accordingly $H - h/H$, *i.e.* $(1 - h/H)$, which, by equations (43) and (44), becomes

$$\text{Efficiency} = 1 - \frac{T' - t_o}{T - t_c}. \quad . \quad . \quad (45)$$

But the expansion and compression curves being adiabatics, we have by (22) (Property 1)

$$\frac{v_p}{v_c} = \frac{v_e}{v_o}, \quad . \quad . \quad . \quad . \quad (46)$$

and as the working mixture obeys the law $pv = RT$,

$$\frac{v_p}{v_o} = \frac{T}{t_c} \text{ and } \frac{v_e}{v_o} = \frac{T'}{t_o};$$

and hence by (46)

$$\frac{T}{t_c} = \frac{T'}{t_o}, \text{ and } \frac{t_o}{t_c} = \frac{T'}{T}.$$

Accordingly

$$\frac{T - t_c}{t_c} = \frac{T' - t_o}{t_o},$$

i.e.

$$\frac{T' - t_o}{T - t_c} = \frac{t_o}{t_c}.$$

Thus, by equation (45) the efficiency may be simply expressed in the form

$$\text{Efficiency} = \left(1 - \frac{t_o}{t_c}\right) = \left(1 - \frac{T'}{T}\right). \quad . \quad . \quad (47)$$

Hence when expansion is continued down to atmospheric (*i.e.* pump suction) pressure, the efficiency depends only upon the degree of *compression* BC.

The efficiency may also be expressed in

terms of the equal ratios of adiabatic volume change; for if ρ denotes the ratio of adiabatic expansion v_e/v_p, so that $(1/\rho)$ denotes that of adiabatic compression (v_c/v_o) (by equation (46)), then by equation (22) we have

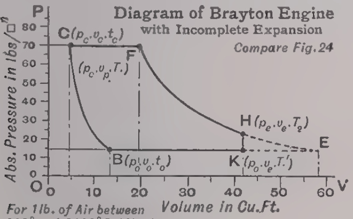
$$\left(\frac{t_o}{t_c}=\frac{v_c}{v_o}\right)^{\gamma-1}=\left(\frac{1}{\rho}\right)^{\gamma-1};$$

whence, by (47),

$$\text{Efficiency}=1-\left(\frac{1}{\rho}\right)^{\gamma-1}, \qquad (48)$$

which expresses the efficiency in terms of the compression ratio alone, and has the same form as for a Carnot cycle. It will be observed that the cycle is *imperfect*, as the efficiency $1-T'/T$ is necessarily always less than $1-t_o/T$.

§ (30) CLASS II. (*continued*).—Actually, it was found to be impracticable to continue the expansion so far that the pressure at release E had fallen to atmospheric, and accordingly the ideal practicable indicator diagram would be as shown in *Fig.* 14. The efficiency is clearly less than in the case illustrated by *Fig.* 13, as, although the heat received remains unchanged, the useful work done per cycle is now reduced by an amount represented by the "toe" of the diagram HEK. As before, the efficiency is expressed by $1-h/H$, where $H=k_p(T-t_c)$. The total heat rejected, h, is, however, now the sum of that rejected during the constant volume period HK, and that rejected during the constant pressure period KB; thus, now



For 1 lb. of Air between 300° and 2000° C. (Abs.)

FIG. 14.

$$h=k_v(T_2-T')+k_p(T'-t_o), \qquad (49)$$

and accordingly (as $\gamma=k_p/k_v$), the efficiency is given by

$$\text{Efficiency}=1-\frac{(T_2-T')\gamma+(T'-t_o)}{T-t_c}. \qquad (50)$$

This expression does not admit of simplification in terms of the temperatures.

We may thermodynamically consider that all the action takes place in a closed working cylinder v_c, being the clearance volume initially filled with a charge of mixture at pressure p_c and temperature t_c, the volume swept through by the working piston being (v_e-v_c) per stroke.

Let the total expansion ratio (v_e/v_c) be denoted by X; the expansion ratio at constant pressure (v_p/v_c) by σ; and the adiabatic compression ratio (v_c/v_o) by $1/\rho$. Then by aid of equations (12) and (22) it is easily found that $T=\sigma\rho^{\gamma-1}t_o$, $T'=(X/\rho)t_o$, $T_2=\sigma^{\gamma}(X/\rho)^{1-\gamma}t_o$, and $t_c=\rho^{\gamma-1}t_o$. Whence, by substitution in equation (50), and reduction, we obtain the result

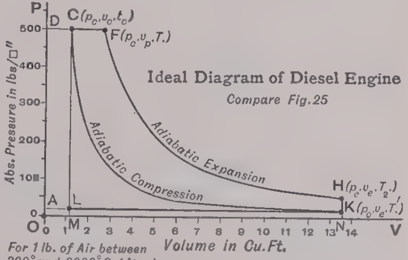
$$\text{Efficiency}=1-\left(\frac{1}{\rho}\right)^{\gamma-1}\times\left\{\frac{\sigma^{\gamma}(X/\rho)^{1-\gamma}+(\gamma-1)(X/\rho)-\gamma}{(\sigma-1)\gamma}\right\}. \qquad (51)$$

§ (31) THE DIESEL ENGINE.—The other important cycle falling within Class II. is that of the Diesel engine in its present form. As already stated, § (28), Diesel originally proposed to use the Carnot cycle, but in a paper read before the Paris Congress in 1900 he announced the cycle finally adopted after extended experiment as comprising the following five operations performed during four strokes of the working piston, the same vessel acting alternately as compression pump and working cylinder (*Fig.* 15):

(1) (1st Stroke).—Suction of air at atmospheric pressure LK.

(2) (2nd Stroke).—Adiabatic compression of this air, KC.

(3) (3rd Stroke; First Portion).—Regulated admission of the carburetting agent (liquid or gaseous fuel) so as to maintain constant pressure during combustion for a portion of the working stroke CF.



For 1 lb. of Air between 300° and 2000° C. (Abs.)

FIG. 15.

(4) (3rd Stroke; Second Portion).—Cut-off of fuel supply and adiabatic expansion of the heated mixture to the end of the working stroke FH.

(5) (4th Stroke).—Opening of exhaust with immediate fall of pressure (and temperature) at constant volume HK, and subsequent expulsion of the burnt gases into the atmosphere during the exhaust stroke KL.

This completes the cycle; it will be observed that one working stroke only occurs in every four, and the cycle is accordingly of the "four-stroke" type.

From *Fig.* 15 it is clear that the areas representing work done on the piston are :

During suction	+LKNM
During compression	−KCMN
During working stroke . . .	+MCFHN
During exhaust	−KLMN
∴ Net useful work per cycle =	+CFHKC

Thermodynamically we may obviously consider that the same mass of air is constantly enclosed within the working cylinder, this being heated at constant pressure during CF, and cooled at constant volume during HK. Thus, in the ideal case, the heat account is as follows :

$$\text{Heat received} = \mathrm{H} = k_p(\mathrm{T} - t_c).$$

$$\text{Heat rejected} = h = k_v(\mathrm{T}_2 - \mathrm{T}').$$

$$\text{Work done} = (\mathrm{H} - h) = k_p(\mathrm{T} - t_c) - k_v(\mathrm{T}_2 - \mathrm{T}').$$

And therefore

$$\text{Efficiency} = 1 - \frac{h}{\mathrm{H}} = 1 - \frac{k_v(\mathrm{T}_2 - \mathrm{T}')}{k_p(\mathrm{T} - t_c)}$$

$$= 1 - \frac{1}{\gamma}\,\frac{\mathrm{T}_2 - \mathrm{T}'}{\mathrm{T} - t_c}. \quad (52)$$

This is immediately deducible from equation (50), which is reduced to the Diesel cycle by putting $t_o = \mathrm{T}'$.

Similarly, referring to equation (51), this reduces to the Diesel case by putting $\mathrm{X} = \rho$; and thus we have for the Diesel cycle :

$$\text{Efficiency} = 1 - \left(\frac{1}{\rho}\right)^{\gamma - 1} \times \frac{\sigma^{\gamma} - 1}{(\sigma - 1)\gamma}, \quad (53)$$

where $1/\rho$ is the ratio of adiabatic compression (v_c/v_e), and σ is the ratio of expansion at constant pressure (v_p/v_c).

This case includes also that of the numerous so-called "semi-Diesel" engines, as, *e.g.*, of the Petter, Blackstone, Ruston, etc., designs.

As the constant pressure expansion ratio, σ, is reduced towards the value unity, the value of the efficiency given by equation (53) continually approaches towards the limiting value $1 - (1/\rho)^{\gamma - 1}$.

This conclusion of theory is apparently realised in practice. In Diesel engines the power is reduced by diminishing the extent of the constant pressure expansion, *i.e.* by reducing σ ; the following figures from tests made by Mr. Ade Clark show the indicated thermal efficiencies obtained :

Engine.	Revolutions per Minute.	Indicated Thermal Efficiency at			
		¼ Load.	½ Load.	¾ Load.	Full Load.
80 H.P. Diesel	160	·375	·412	. .	·389
160 H.P. Diesel	157	·371	·427	·402	·397

The Diesel cycle is clearly of the "imperfect" class.

§ (32) CLASS III. *The "Constant Volume" Cycle.*—In this class is included by far the largest number of actual internal combustion engines, ranging from the early Lenoir and Hugon types to the modern Clerk two-stroke, and de Rochas, or Otto, four-stroke designs. A very large proportion of the enormous number of small internal combustion engines of the present day are worked on the Otto "four-stroke" cycle, while the Clerk two-stroke cycle is extensively adopted in the larger types of stationary gas engines.

The Lenoir Engine.—In the Lenoir gas engine (1860)—now completely obsolete on account of its very low efficiency—the mode of working was evidently inspired by that of the ordinary steam engine. During the first portion of the stroke the working piston drew into the cylinder a charge of coal gas and air at atmospheric pressure ; at an arranged point a slide valve cut off the supply, and simultaneously the charge within the cylinder was ignited, the temperature (and consequently the pressure) suddenly rising greatly ; the working stroke was then completed by the expansion of this mass of heated high-pressure gas ; during the return stroke the burnt gases were exhausted into the atmosphere, thus completing the cycle.

Further following the steam engine, this cycle was caused to occur alternately on each side of the working piston, thus making the engine "double-acting" ; thus a working impulse was obtained in every stroke. The Lenoir engine worked very quietly and smoothly, but was abandoned on account of its very high consumption of gas in favour of more economical types.

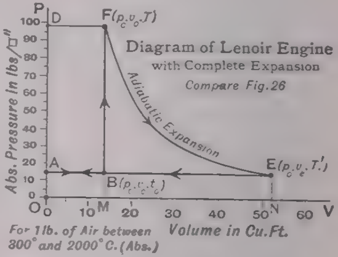


FIG. 16.

The ideal diagram is shown in *Fig.* 16, wherein it is supposed that the cylinder has no clearance and that the expansion is carried so far that the pressure at the end of the stroke is that of the atmosphere. The volume swept through per stroke by the piston is AE ; during the portion AB the carburetted charge is drawn in at atmospheric pressure, doing work on the piston represented by the area + ABMO. At B cut-off occurs, and simultaneously the charge is ignited, explosion takes place, and the pressure suddenly rises as indicated by BF. The heated charge then expands adiabatically, along FE, to atmospheric pressure at the end of the stroke E, doing a further amount of work on the piston represented by the area + MFEN; at E the exhaust opens, and during the return stroke the burnt charge is expelled into the atmosphere, negative work being done upon the piston represented by the area − EAON; this completes the cycle.

The work account is therefore given by ABMO + MFEN − EAON or BFEB; thus the useful work done per cycle is represented by the area BFEB.

Thermodynamically the action is regarded as taking place in a mass of air (regarded as a perfect gas) contained within a closed cylinder having a clearance volume AB, and piston displacement volume BE. Initially, at B, the clearance volume contains air at $(p_o v_o t_o)$; heat is suddenly communicated, raising the pressure, at unchanged volume, to p_c; adiabatic expansion then occurs to E; and during the return stroke heat is abstracted from the air at constant pressure from volume v_e to volume v_o. In this ideal case we have therefore

$$\text{Heat received} = \mathrm{H} = k_v(\mathrm{T} - t_o),$$
$$\text{Heat rejected} = h = k_p(\mathrm{T}' - t_o),$$
$$\text{Useful work done} = (\mathrm{H} - h) = k_v(\mathrm{T} - t_o) - k_p(\mathrm{T}' - t_o),$$
$$\text{Efficiency} = 1 - \frac{h}{\mathrm{H}} = 1 - k_p(\mathrm{T}' - t_o)/k_v(\mathrm{T} - t_o),$$

i.e.
$$\text{Efficiency} = 1 - \gamma\frac{\mathrm{T}' - t_o}{\mathrm{T} - t_o}. \qquad (54)$$

This result may be otherwise expressed in three ways; for by equation (12) $(\mathrm{T}/t_o) = (p_c/p_o) = \Pi$—the ratio of maximum to atmospheric pressure, while $(\mathrm{T}'/t_o) = (v_e/v_o) = \rho$, the ratio of adiabatic expansion. Hence (54) may be written

$$\text{Efficiency} = 1 - \gamma\frac{\rho - 1}{\Pi - 1}. \qquad (55)$$

Again, by equation (22)

$$\mathrm{T} = \left(\frac{v_e}{v_p}\right)^{\gamma - 1}\mathrm{T}' = \rho^{\gamma - 1}\mathrm{T}' = \rho^{\gamma} t_o,$$

as $\mathrm{T}' = \rho t_o$. Hence (54) may also be written

$$\text{Efficiency} = 1 - \gamma\frac{\rho - 1}{\rho^{\gamma} - 1}. \qquad (56)$$

Lastly, as $\rho^{\gamma} = (\mathrm{T}/t_o)$, equation (56) becomes, in terms of the ratio of the explosion temperature to the initial temperature,

$$\text{Efficiency} = 1 - \gamma\frac{(\mathrm{T}/t_o)^{1/\gamma} - 1}{(\mathrm{T}/t_o) - 1}. \qquad (57)$$

In general it was not found practicable to continue the expansion until atmospheric pressure was reached, the more usual case being that in which the pressure at the end of the stroke exceeded that of the atmosphere. The ideal diagram in this case is shown in *Fig.* 17; and it is clear that the heat received, H, is the same as in the preceding case, viz. $\mathrm{H} = k_v(\mathrm{T} - t_o)$; the heat rejected, h, is, however, now the sum of that during the constant volume drop HK, and the constant pressure shrinkage KB, and thus

$$h = k_v(\mathrm{T}_2 - \mathrm{T}') + k_p(\mathrm{T}' - t_o).$$

Hence the efficiency, $1 - h/\mathrm{H}$, is expressed by

$$\text{Efficiency} = 1 - \frac{k_v(\mathrm{T}_2 - \mathrm{T}') + k_p(\mathrm{T}' - t_o)}{k_v(\mathrm{T} - t_o)},$$

i.e.
$$\text{Efficiency} = 1 - \frac{(\mathrm{T}_2 - \mathrm{T}') + \gamma(\mathrm{T}' - t_o)}{(\mathrm{T} - t_o)}. \qquad (58)$$

Denote, as before, the ratio of adiabatic expansion (v_e/v_o) by ρ, and the ratio of explosion pressure to atmospheric pressure by Π. Then $\mathrm{T} = \Pi t_o$; $\mathrm{T}' = \rho t_o$; and by equation (22) $\mathrm{T}_2 = \mathrm{T}(1/\rho)^{\gamma - 1} = \Pi(1/\rho)^{\gamma - 1} t_o$. Accordingly equation (58) may also be written

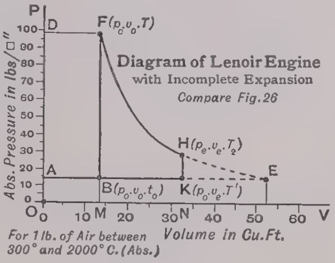


FIG. 17.

$$\text{Efficiency} = 1 - \frac{\Pi(1/\rho)^{\gamma - 1} + \rho(\gamma - 1) - \gamma}{\Pi - 1}; \qquad (59)$$

or again, in terms of the ratio (T/t_o) only,

$$\text{Efficiency} = 1 - \frac{\rho\{\rho^{-v}(\mathrm{T}/t_o - 1)\} + (\rho - 1)\gamma}{(\mathrm{T}/t_o) - 1}, \qquad (60)$$

when $\mathrm{T}_2 = \mathrm{T}'$, $\Pi(1/\rho)^{\gamma - 1} = \rho$, and equation (59) then reduces to equation (55). Also as $\Pi = \mathrm{T}/t_o = \rho^{\gamma}$ when $\mathrm{T}_2 = \mathrm{T}'$, equation (60) then reduces to equation (57). The efficiency is clearly less in this than in the preceding case, since, with the same expenditure of heat, the useful work area is less by the amount KHE, due to the incomplete expansion.

§ (33) CLASS III (*continued*). (i.) *The Free Piston Engine.*—The Lenoir type of engine was superseded by a singular design originally proposed by Barsanti and Mattucci (1857), but first rendered practicable by Otto and Langen in their "Free Piston Engine" of 1867. This essentially comprised a very long vertical cylinder fitted with a heavy *free* piston beneath which a charge of carburetted air was exploded, driving the piston upwards as a projectile is driven from a gun; the heavy piston acquired considerable momentum, and continued its upward motion until the working mixture had expanded to about six times its original volume and to a pressure considerably below that of the atmosphere. During its descent by gravity, and excess atmospheric pressure upon its upper surface, it engaged with, and drove, the crank-shaft through a ratchet and pawl device. The engine was excessively noisy and mechanically unsatisfactory, but the rapid and extended expansion, and subsequent slower cooling, of the working mixture resulted in a considerably increased efficiency compared with earlier types.

An ideal diagram is shown in *Fig.* 18; the charge of carburetted air at atmospheric pressure p_o and temperature t_o is drawn into the cylinder as indicated by AB, and exploded at constant volume v_o (=AB), its pressure rising to p_c and temperature to T as indicated by BF; the piston immediately rises rapidly, the charge expanding adiabatically to E when the piston momentarily stops. The downward (working) stroke now follows under the combined action of gravity and excess atmospheric pressure, the mixture being compressed *isothermally* from E to B—due to the slowness of the working stroke, and finally expelled at constant pressure as indicated by BA. As before, the useful work done is represented by the area BFEB; clearly also the heat supplied, H, is expressed by $H=k_v(T-t_o)$, while the heat rejected, h, is given by $h=Rt_o\log_\epsilon r$, where r is the ratio of isothermal compression (v_e/v_o); *vide* § (18).

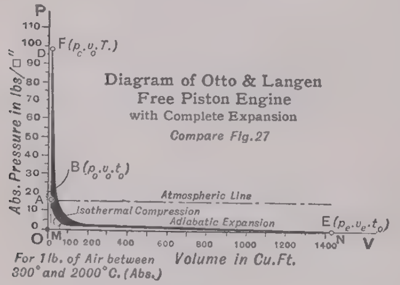


FIG. 18.

Hence the efficiency $=1-\dfrac{h}{H}=1-\dfrac{Rt_o\log_\epsilon r}{k_v(T-t_o)}$,

i.e. as $R=k_p-k_v$,

$$\text{Efficiency}=1-\frac{(\gamma-1)\log_\epsilon r}{(T/t_o)-1}. \quad . \quad (61)$$

This may be further simplified and expressed in terms of the ratio (T/t_o) only; for by equation (22) $(v_e/v_o)^{\gamma-1}=(T/t_o)$, and $\therefore (\gamma-1)\log_\epsilon r=\log_\epsilon(T/t_o)$, and accordingly equation (61) reduces to

$$\text{Efficiency}=1-\frac{\log_\epsilon (T/t_o)}{(T/t_o)-1}, \quad . \quad . \quad (62)$$

which expresses the ideal maximum efficiency of engines of this type.

(ii.) *The Beau de Rochas, or Otto, and the Clerk Cycles.*—Finally there remain to be considered engines working upon the Beau de Rochas, or Otto, "four-stroke," and the Clerk "two-stroke" cycles, which may be considered to include between them all internal combustion engines of the present day. In all these engines combustion is caused to occur at constant volume with previous compression of the working charge.

§ (34) THE CLERK CYCLE.—The first case to be taken is that in which—as in the Clerk cycle—the engine comprises a compression pump and separate working cylinder, the compressing pump taking in a carburetted charge at atmospheric pressure and temperature and compressing this either directly into the combustion chamber of the working cylinder, or into an intermediate receiver from which the working cylinder in turn takes its compressed charge, which is then ignited (exploded) at constant volume, and performs the working stroke by its subsequent expansion.

Six separate operations may be here distinguished, viz.:

(1) Pump suction—Charging the pump with carburetted air.

(2) Pump Compression—Compressing the charge into the receiver, or combustion chamber of working cylinder.

(3) Supply of compressed charge from receiver to working cylinder, when receiver is included.

(4) Explosion of charge at constant volume in working cylinder.

(5) Expansion of exploded charge during working stroke.

(6) Expulsion of exhaust gases at the end of the working stroke.

In considering an ideal diagram for this case the following assumptions are made:

(1) That the charge is compressed adiabatically.

(2) That none of the heat of compression is lost in the receiver.

(3) That the charge is neither heated nor cooled on entering the combustion chamber of the working cylinder.

(4) That the explosion occurs instantaneously, and without any loss of heat to the walls of the combustion chamber.

(5) That the expansion during the working stroke is adiabatic, *i.e.* that no heat exchange takes place with the cylinder walls, or piston crown, during expansion.

(6) That there are no losses by throttling or back pressure.

An ideal diagram embodying these assumptions is shown in *Fig.* 19, wherein it is also supposed that the expansion is continued so far that the pressure has fallen to p_o—that of the atmosphere—a condition not practically attainable, and also that the working cylinder has no clearance; thus the pump volume is AB, and the working cylinder volume AE.

During the pump suction stroke a volume of carburetted air is taken in at atmospheric pressure and temperature represented by AB, doing work +OABK upon the pump piston.

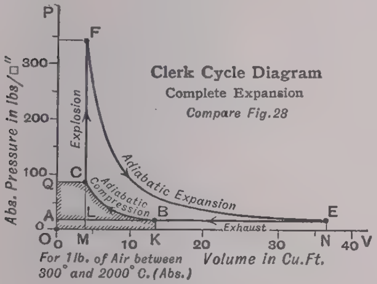


FIG. 19.

This is next compressed adiabatically along BC and delivered into the receiver at the constant pressure p_c along CQ; thus the pump diagram is −OKBCQO.

The charge next enters the cylinder along QC, doing work +OQCM upon the working piston, and is then exploded at constant volume v_c, the pressure instantly rising as indicated by CF, with subsequent adiabatic expansion FE, to atmospheric pressure at E, doing work upon the piston represented by +MFENM. At E the exhaust opens and the burnt gases are expelled at atmospheric pressure p_o, doing work represented by −EAON; this completes the cycle.

The work account is therefore

$$+\mathrm{OABK}-\mathrm{OKBCQO}+\mathrm{OQCM}+\mathrm{MFENM}-\mathrm{EAON}=+\mathrm{BCFEB},$$

and thus the useful work done per cycle is represented by the enclosed area BCFEB.

Thermodynamically the whole action may be conceived as taking place in a closed cylinder of clearance volume AL, LE being the volume swept through by the piston. Commencing at the point C, heat H is added to the charge to an amount given by the equation $\mathrm{H}=k_v(\mathrm{T}-t_c)$; during expansion there is no communication of heat; during exhaust from E to B heat, h, is rejected, given by $h=k_p(\mathrm{T}'-t_o)$; while during compression BC there is no communication of heat.

Hence the efficiency, $1-h/\mathrm{H}$, is expressed by

$$\text{Efficiency}=1-\frac{k_p(\mathrm{T}'-t_o)}{k_v(\mathrm{T}-t_c)}=1-\gamma\frac{\mathrm{T}'-t_o}{\mathrm{T}-t_c}. \quad (63)$$

The efficiency may also be expressed in terms of T, t_o, and the ratio of adiabatic compression $1/\rho=(v_c/v_o)$. For, by equation (22),

$$t_c=\rho^{\gamma-1}t_o;$$

and

$$\mathrm{T}'=\left(\frac{v_c}{v_e}\right)^{\gamma-1}\mathrm{T}.$$

But also

$$\frac{v_c}{v_e}=\left(\frac{v_c}{v_o}\right)\left(\frac{v_o}{v_e}\right)=\left(\frac{1}{\rho}\right)\left(\frac{t_o}{\mathrm{T}'}\right);$$

whence

$$\mathrm{T}'=\left(\frac{t_o}{\rho}\right)^{1-1/\gamma}\mathrm{T}^{1/\gamma}.$$

Thus (63) may be written

$$\text{Efficiency}=1-\gamma\frac{\left(\frac{1}{\rho}\right)^{1-1/\gamma}\left(\frac{\mathrm{T}}{t_o}\right)^{1/\gamma}-1}{\left(\frac{\mathrm{T}}{t_o}\right)-\rho^{\gamma-1}}. \quad (64)$$

As $\rho=(t_c/t_o)^{1/\gamma-1}$, and $\mathrm{T}'=(t_o/\rho)^{\gamma-1/\gamma}\mathrm{T}^{1/\gamma}$, it follows that

$$\mathrm{T}'=\left(\frac{\mathrm{T}}{t_c}\right)^{1/\gamma}t_o, \quad . \quad . \quad . \quad (65)$$

which expresses the relation between the temperatures at the four corners of the diagram.

If, as in previous cases, the total expansion ratio (v_e/v_c) be denoted by X, then

$$\mathrm{T}=\left(\frac{v_e}{v_c}\right)^{\gamma-1}\mathrm{T}'=\mathrm{X}^{\gamma-1}\mathrm{T}';$$

but

$$\mathrm{T}'=\left(\frac{v_e}{v_o}\right)t_o=\left(\frac{v_e}{v_c}\right)\left(\frac{v_c}{v_o}\right)t_o=\left(\frac{\mathrm{X}}{\rho}\right)t_o,$$

whence

$$\mathrm{T}=\left(\frac{\mathrm{X}^\gamma}{\rho}\right)t_o.$$

Hence, as

$$t_c=\rho^{\gamma-1}t_o,$$

we have, on substituting in equation (63) and reducing, the expression for the efficiency in terms of the total expansion and adiabatic compression ratios alone,

$$\text{Efficiency}=1-\left(\frac{1}{\rho}\right)^{\gamma-1}\times\frac{\gamma(\mathrm{X}/\rho-1)}{(\mathrm{X}/\rho)^\gamma-1}. \quad (66)$$

As ρ approaches X in value this expression continually approximates to $1-(1/\rho)^{\gamma-1}$ as a limiting value.

§ (35) THE CLERK CYCLE WITH INCOMPLETE EXPANSION.—Actually it has not been found practicable to continue the expansion so far that the pressure falls to that of the atmosphere; the next case for consideration, therefore, is that in which the pressure at the end of expansion exceeds that of the atmosphere.

An ideal diagram is shown in *Fig.* 20; as before, the heat supplied is $\mathrm{H}=k_v(\mathrm{T}-t_c)$, while the heat rejected is now given by

$$h=k_v(\mathrm{T}_2-\mathrm{T}')+k_p(\mathrm{T}'-t_o).$$

Hence the efficiency, $1-h/\mathrm{H}$, is given by

$$\text{Efficiency}=1-\frac{(\mathrm{T}_2-\mathrm{T}')+\gamma(\mathrm{T}'-t_o)}{\mathrm{T}-t_c}. \quad (67)$$

As $\mathrm{T}_2=\mathrm{T}(1/\mathrm{X})^{\gamma-1}$, $t_c=t_o\rho^{\gamma-1}$, and $\mathrm{T}'=(\mathrm{X}/\rho)t_o$, we have on substitution and rearrangement from equation (67)

$$\text{Efficiency} = 1 - \frac{(\mathrm{T}/t_o)\mathrm{X}^{1-\gamma} + (\gamma - 1)(\mathrm{X}/\rho) - \gamma}{(\mathrm{T}/t_o) - \rho^{\gamma - 1}}, \quad (68)$$

when $\mathrm{T}_2 = \mathrm{T}'$, $(\mathrm{T}/t_o) = \mathrm{X}^{\gamma}/\rho$, and (68) then reduces to the previous case, viz. equation (66).

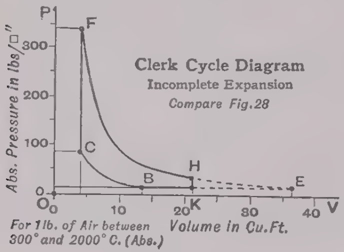


FIG. 20.

§ (36) THE BEAU DE ROCHAS, OR OTTO, CYCLE.—The last and most important of all cycles of internal combustion engines is the "constant volume" cycle first proposed by Beau de Rochas in 1862, and practically realised by Dr. Otto in his famous "Otto Silent" "gas engine" of 1876.

In this cycle there is no separate compressing pump, its function being discharged by the working cylinder itself. The sequence of operations is as follows, "stroke" referring to that of the working piston:

(1) Suction of carburetted charge of air during the whole of the first out-stroke.

(2) Compression of the charge into the combustion chamber of the cylinder during the whole of the first in-stroke.

(3) Explosion of charge at end of first in-stroke.

(4) Expansion of heated charge during the whole of the second out-stroke; this is the "working stroke."

(5) Exhaust of the burnt gases into the atmosphere during the whole of the second in-stroke.

Thus the cycle requires for its performance four consecutive strokes of the working piston, whence the term "four-stroke" cycle; and only one working impulse is obtained for each two revolutions of the crank-shaft.

An ideal diagram is shown in *Fig.* 21. During the first out-stroke the piston draws in the charge at atmospheric pressure, along LK, work being done on the piston represented by the area +MLKN. During the return stroke the volume AK is adiabatically compressed along KC to QC, the work done being represented by −NKCMN.

Explosion is caused to occur at C, and the pressure instantly rises, at constant volume, as indicated by CG; during the second out-stroke the heated gases expand adiabatically along FH, doing work on the piston represented by +MFHNM, and at H the exhaust is opened to the atmosphere causing instant drop of pressure at constant volume HK. During the second in-stroke the burnt gases are expelled along KL into the atmosphere, work being done represented by −NKLM; this completes the cycle. The work account is therefore

$$+\mathrm{MLKN} - \mathrm{NKCMN} + \mathrm{MFHNM} - \mathrm{NKLM} = +\mathrm{CFHKC};$$

thus CFHKC represents the useful work done per cycle.

Thermodynamically the action may be conceived as taking place in the same mass of air always enclosed in the working cylinder, and subjected to the operations indicated by KC, CF, FH, and HK; thus the heat received is $\mathrm{H} = k_v(\mathrm{T} - t_c)$; while the heat rejected is $h = k_v(\mathrm{T}_2 - t_o)$. Hence the efficiency, $1 - h/\mathrm{H}$, is given by the equation

$$\text{Efficiency} = 1 - \frac{\mathrm{T}_2 - t_o}{\mathrm{T} - t_c}. \quad . \quad . \quad (69)$$

This may be simplified and expressed in two other important ways. For by equation (12) and § (22) Prop. 2, we have $\mathrm{T}_2/t_o = p_e/p_o = \mathrm{P}/p_c = \mathrm{T}/t_c$. Hence $t_o/t_c = \mathrm{T}_2/\mathrm{T}$, and also $\mathrm{T}_2 - t_o/\mathrm{T} - t_c = t_o/t_c = \mathrm{T}_2/\mathrm{T}$. Accordingly

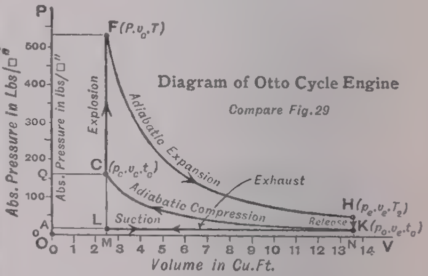


FIG. 21.

$$\text{Efficiency} = 1 - \left(\frac{t_o}{t_c}\right), \quad . \quad . \quad . \quad (70)$$

and thus in this cycle the efficiency depends, in the ideal case, only upon the ratio of the absolute temperatures at the beginning and end of *compression*, and is independent of the explosion temperature. Obviously (70) may also be written

$$\text{Efficiency} = 1 - (\mathrm{T}_2/\mathrm{T}).$$

which expresses it in terms of the ratio of absolute temperatures at the beginning and end of expansion; it is obvious that the cycle is "Imperfect." The second important simplification of (69) is the expression for the efficiency in terms of the (adiabatic) ratio of compression $(1/\rho)$. We have by equation (22)

$$\left(\frac{t_o}{t_c}\right) = \left(\frac{v_c}{v_e}\right)^{\gamma-1} = \left(\frac{1}{\rho}\right)^{\gamma-1};$$

hence by (70)

$$\text{Efficiency} = 1 - \left(\frac{1}{\rho}\right)^{\gamma-1}, \quad . \quad . \quad (71)$$

showing that in the ideal Otto cycle the efficiency depends upon the compression ratio alone.

§ (37) EFFICIENCY FORMULAE.—On comparing equations (32′), (48), and (71) the interesting fact emerges (first pointed out by Callendar) that in the three typical ideal cycles of constant temperature, constant pressure, and constant volume, the efficiency is expressed by the same formula, viz.

$$\text{Efficiency} = 1 - \left(\frac{1}{\rho}\right)^{\gamma-1}, \quad . \quad (71)$$

where $(1/\rho)$ is the ratio of adiabatic compression, and is thus the same, for the same value of ρ, in all three cases.

It must be remembered, however, that in the constant temperature (Carnot) cycle the adiabatic compression raises the temperature of the working substance through the whole range from the lowest to the highest between which the engine works, whence in this case the value of the efficiency is an absolute maximum, as has been shown. In the other two cases the adiabatic compression does not raise the substance from the lowest to the highest temperature, and the expression for the efficiency has a value necessarily less than in the Carnot case, though a maximum in each case for the particular cycle considered; these are, therefore, "imperfect" cycles in the sense as explained in § (2) *supra*.

In the constant temperature cycle all heat is received and rejected at constant temperature; in the cycles of constant pressure and constant volume heat is received at rising, and rejected at falling, temperature; also in the Carnot and Brayton cycles the expansion is "complete," which is not the case in the Otto cycle. Examination of equations (51), (53), and (66) shows the manner in which the value of the efficiency is further diminished by changes in the cycles imposed by practical considerations.

§ (38) TEMPERATURE-ENTROPY DIAGRAMS OF TYPICAL CYCLES.—In *Figs.* 22-29 are shown $T-\phi$ diagrams for the typical cycles of Classes I., II., and III., together with actual numerical values of maximum efficiency for uniform upper and lower absolute temperature limits assumed at 2000° C. and 300° C. respectively. *Fig.* 22 shows the $T-\phi$ diagram for the constant temperature Carnot cycle; as already pointed out (*vide Fig.* 6 and text) this takes the form of a rectangle, and the efficiency—which is independent of the breadth of the rectangle—is here an absolute maximum in value of $1-300/2000=0{\cdot}85$.

§ (39) THE BRAYTON ENGINE.—*Fig.* 23 shows the $T-\phi$ diagram of the constant pressure cycle of the Brayton engine, with complete expansion, as described in § (29) and *Fig.* 13. The entropy along the adiabatic compression is taken as an arbitrary zero; thus on any vertical, as MI in *Fig.* 23, take the point B at the 300° C. level. Then, by aid of equation (21), $t_c=(p_c/p_o)^{\gamma-1/\gamma}$, and taking $p_c=70$ lbs. per sq. in. and $p_o=14{\cdot}7$ lbs. per sq. in., determines $t_c=468°$ C.; this gives the point C on MI and thus determines BC, the isentropic corresponding to the adiabatic compression of *Fig.* 13. Through C draw the curve whose equation is $\phi=k_p \log_\epsilon (t/t_c)$ (see equation (38)), and let this cut a horizontal through 2000° C. in F; then CF on *Fig.* 23 represents the constant pressure expansion line of *Fig.* 13. Through F draw a vertical meeting the curve $\phi=k_p \log_\epsilon (t/t_0)$ in E; then FE is the isentropic corresponding to the adiabatic expansion line in *Fig.* 13, while the curve EB represents the constant pressure compression of *Fig.* 13.

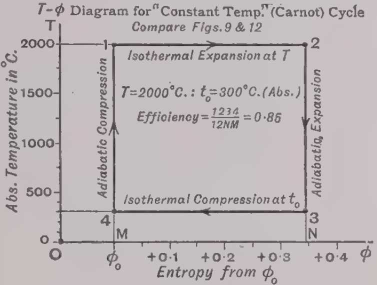


FIG. 22.

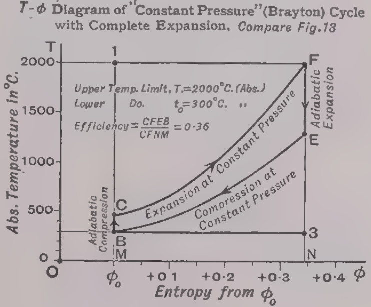


FIG. 23.

Agreeably with equation (10) (see also *Fig.* 5) the efficiency in this case is expressed by the ratio Area BCFE/Area MCFN, which, by direct planimetric measurement—as may be verified by calculation from equation (45)—has here the value 0·36.

The corresponding Carnot engine would take in heat MIFN, and reject heat N3BM, with corresponding efficiency of 0·85. The Brayton engine takes in the smaller quantity of heat MCFN and rejects the larger quantity NEBM, with resultant reduction in the value of its efficiency to 0·36.

§ (40) THE BRAYTON ENGINE WITH INCOMPLETE EXPANSION.—*Fig.* 24 shows the $T-\phi$ diagram of the Brayton engine with incomplete expansion, as shown in the *pv* diagram, *Fig.* 14. The temperature T_2 at release H is taken as 1450° C., and the drop of pressure HK at constant volume in *Fig.* 14 is represented on the $T-\phi$ diagram by the curved line HK calculated from the equation $\phi=k_v \log_\epsilon (T_2/t)$.

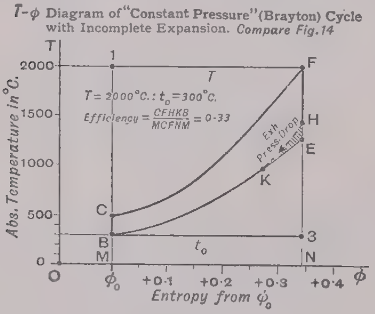


FIG. 24.

Fig. 24 differs from *Fig.* 23 only in the useful heat area being reduced by the amount EHK, which thus represents, in this case, the loss caused by incomplete expansion, and reduces the efficiency from 0·36 to 0·33.

§ (41) THE DIESEL ENGINE.—The $T-\phi$ diagram of the Diesel engine is given in *Fig.* 25. Initially (see *Fig.* 15) $T'=300°$ C., p_o = atmospheric pressure (14·7 lbs.), while p_c is assumed at the value usual in practice, viz. 500 lbs. per sq. in., and T = 2000° C.

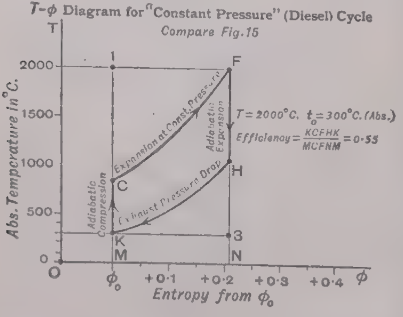


FIG. 25.

Then by equation (21)

$$t_c=(p_c/p_o)^{1-1/\gamma}\times T'=833° \text{ C.};$$

thus the points K and C may be determined, and KC on *Fig.* 25 is then the isentropic corresponding to the adiabatic compression of *Fig.* 15.

Through C the curve $\phi=k_p \log_\epsilon (t/t_c)$ is next drawn, cutting a horizontal through 2000° C. at F, while through K the curve $\phi=k_v \log_\epsilon (t/T')$ is drawn cutting a vertical through F in H. Then FH is the isentropic corresponding to the adiabatic expansion, and HK corresponds to the constant volume pressure drop at release of *Fig.* 15.

For the efficiency in this case, either by calculation from equation (52) or by direct measurement from the diagram of the ratio, we have for the Area KCFHK/Area MCFNM the value 0·55.

§ (42) THE LENOIR ENGINE.—In *Fig.* 26 is shown the $T-\phi$ diagram of the now entirely obsolete Class III. Lenoir engine, with complete expansion, whose *pv* diagram is shown in *Fig.* 16. The diagram is constructed by taking the point B at 300° C. on any vertical and drawing through this point the curve $\phi=k_v \log_\epsilon (t/t_o)$ cutting a horizontal through 2000° C. in F; thus BF represents the constant volume pressure increase due to the explosion (*Fig.* 16). Through F draw a vertical, and through B draw the curve $\phi=k_p \log_\epsilon (t/t_o)$ meeting this vertical in E; then the isentropic FE corresponds to the period of adiabatic expansion, and the curve EB to the constant pressure compression of *Fig.* 16; the point E obviously gives the temperature T′, here 1162° C., as may be verified by direct calculation.

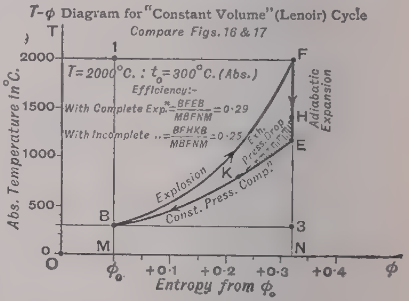


FIG. 26.

$$\text{Efficiency}=\frac{\text{Area BFE}}{\text{Area MBFNM}}=0{\cdot}29,$$

as may also be determined by equation (54).

The dotted line HK on *Fig.* 26 is the modification in the $T-\phi$ diagram due to incomplete expansion (compare *Fig.* 17), the temperature at release H is assumed at 1400° C., and the curve HK is plotted from the equation $\phi = k_v \log_\epsilon (1400/t)$ where ϕ is the defect of entropy below that of H, viz. 0·321. The efficiency, being the value of the ratio Area BFHKB/Area MBFNM, is now reduced to 0·25.

§ (43) Free Piston Engine.—The ideal $T-\phi$ diagram of the Otto and Langen free piston engine is shown in *Fig.* 27, with which *Fig.* 18 may be compared. The initial point is B at 300° C. and the explosion raises the temperature, at constant volume, to 2000° C.; this is represented on the diagram by the curve BF, plotted from the equation $\phi = k_v \log_\epsilon (t/t_o)$, cutting a horizontal through 2000° C. in F. The vertical FE is the isentropic corresponding to the subsequent adiabatic expansion during the rise of the heavy free piston, while the horizontal line EB represents the isothermal compression of the gases during the working down-stroke. The efficiency, either by direct measurement from the diagram of the value of the ratio Area BFEB/Area MBFNM, or by calculation from equation (62), has here the relatively high value 0·66; comparison with the previous diagram (*Fig.* 26) clearly shows the manner in which the efficiency is increased in value as a result of the compression being isothermal instead of at constant pressure.

T-ϕ Diagram of "Constant Volume" (Free Piston) Engine

Compare Fig. 18

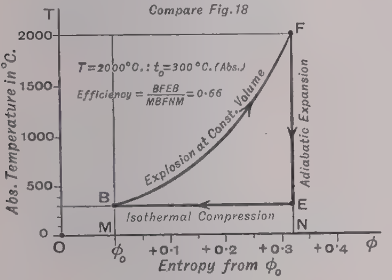
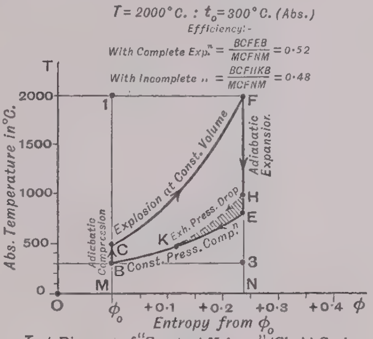


Fig. 27.

§ (44) Clerk Engine.—The $T-\phi$ diagram for the Clerk constant-volume cycle, both with complete and incomplete expansion, is shown in *Fig.* 28; compare *Figs.* 19 and 20. The initial point B is at 300° C. and the compression temperature t_c is assumed at 500° C.; thus BC is the isentropic of adiabatic compression.

The curve CF representing rise of temperature at constant volume (*i.e.* explosion) is next plotted from the adiabatic equation $\phi = k_v \log_\epsilon (t/500)$, and by its intersection with a horizontal through 2000° C. determines F; a vertical through F then meets the curve $\phi = k_v \log_\epsilon (t/300)$ at E, and FE is then the isentropic of adiabatic expansion, while EB represents the operation of compression at constant pressure.

When the expansion is incomplete, assuming that at release the temperature is 1000° C., the curve HK, plotted from the equation $0{\cdot}235 - \phi = k_v \log_\epsilon (1000/t)$, is the $T-\phi$ line corresponding to the pressure drop at release, with corresponding loss of useful work-heat represented by the area EHK.



T-ϕ Diagram of "Constant-Volume" (Clerk) Cycle

Compare Figs. 19 & 20

Fig. 28.

By measurement from the diagram of the ratio Area BCFEB/Area MCFNM, or by calculation from equation (63), the efficiency with the data assumed, when the expansion is complete, will be found to have the value 0·52. In the case, as taken, of incomplete expansion, the efficiency is reduced in value to 0·48.

§ (45) Otto Engine.—In *Fig.* 29 is shown the $T-\phi$ diagram of the "Otto" or "Beau de Rochas" cycle, of which the *pv* diagram is given in *Fig.* 21. As before, the initial point K is taken, on any vertical, at 300° C.; the temperature of compression, t_c, is assumed at 600° C.; hence the vertical line KC is the isentropic of adiabatic compression.

T-ϕ Diagram of "Constant Volume" (Otto) Cycle

Compare Fig. 21

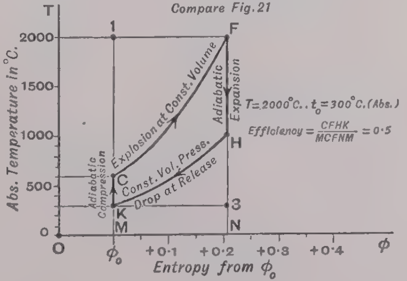


Fig. 29.

From C the curve $\phi = k_v \log_\epsilon (t/600)$ is next plotted, cutting a horizontal through 2000° C.

in F; then the curve CF represents the increase of entropy during the explosion. Through F a vertical is drawn intersecting the curve $\phi = k_v \log_\epsilon (t/300)$ in H; then FH is the isentropic of adiabatic expansion, and the curve HK corresponds to the drop of pressure at constant volume which occurs at release.

The efficiency, either by direct measurement of the ratio Area KCFHK/Area MCFNM, or by calculation from equation (70) has here the value 0·5.

§ (46) IDEAL EFFICIENCIES.—Hence for a temperature range from 2000° C. (abs.) to 300° C., and with the other data assumed in the foregoing paragraphs, the maxima ideal efficiencies in the several cases considered have the following values:

Carnot (constant temperature)	0·85
Otto and Langen free piston (constant volume)	0·66
Diesel (constant pressure)	0·55
Clerk (constant volume), with complete expansion	0·52
Clerk (constant volume), with incomplete expansion	0·48
Otto (constant volume)	0·50
Brayton (constant pressure), with complete expansion	0·36
Brayton (constant pressure), with incomplete expansion	0·33
Lenoir (constant volume), with complete expansion	0·29
Lenoir (constant volume), with incomplete expansion	0·25

§ (47) PRACTICAL CONSIDERATIONS. — Thus the efficiency of the Carnot "Perfect" cycle is substantially greater than that of any of the others, while the Otto and Langen engine has an efficiency—when isothermal compression is realised—second only to that of the Carnot.

For the purposes of practical power production, however, it is not enough that a cycle should be of high ideal efficiency; in order that an engine may be practicable it is necessary, *inter alia*, (1) that its cycle can be effectively performed in a very short time, and (2) that the ratio of mean effective pressure to maximum pressure shall be as high as possible. If (1) cannot be attained the engine can only run slowly, and is thus bulky and weighty in relation to the power developed by it. If (2) is not realised, then since the engine must be designed to withstand the maximum pressure developed it is necessarily weighty and costly relatively to its power output.

These two conditions have, so far, practically excluded all but the Otto, Diesel, and Clerk cycles, and it is of interest to determine in the several cases of § (46), the values of the maximum pressure, the mean effective pressure, and the ratio of these.

The mean effective pressure, p_m, is the average height of the closed figure on the p_v diagram representing the useful work U (ft.-lbs.) done per cycle; if p_m be in lbs. per sq. in., and if V be the greatest, and v the least, volume (in cub. ft.) of the working substance, then

$$p_m = \frac{\mathrm{U}}{144(\mathrm{V} - v)} \text{ lbs. per sq. in.} \quad . \quad (72)$$

The maximum pressure developed will be denoted by P, and in each case it is considered that the engine uses 1 lb. of air, initially at atmospheric pressure p_o of 14·7 lbs. per sq. in. and temperature $t_o = 300°$ C. (abs.), whence, by equation (12′), its volume $v_o = 13{\cdot}6$ cub. ft.

§ (48) MAXIMUM PRESSURES. *The Carnot Cycle; Value of* p_m/P.—The calculation here is conveniently conducted as follows: As T = 2000° C. and $t_o = 300°$ C. we have, by equation (32), for the value of the adiabatic ratios $\rho = (2000/300)^{2 \cdot 45} = 104{\cdot}36$; thus the ratios of adiabatic expansion and compression must *necessarily* each have the enormously large value 104·36. The isothermal ratio r is arbitrary; if it be assumed as 2, then, by equation (30), the total expansion ratio becomes $\mathrm{X} = 2 \times 104{\cdot}36 = 208{\cdot}7$, a value, it is needless to point out, entirely out of the question in any actual engine.

The maximum pressure P is obtained by aid of equation (12) and is given by

$$\mathrm{P} = \left(\frac{\mathrm{T}}{t}\right)\mathrm{X}p_o = \left(\frac{2000}{300}\right) \times 208{\cdot}7 \times 14{\cdot}7$$
$$= \text{about } 20{,}450 \text{ lbs. per sq. in.,}$$

or roundly 1400 atmospheres.

Again, by equation (72),

$$p_m = \frac{\mathrm{U}}{144\mathrm{V}(1 - (1/\mathrm{X}))} = \frac{c(\mathrm{T} - t) \log_\epsilon r}{144\mathrm{V}(1 - (1/\mathrm{X}))}$$

(*vide* equation (12′)); thus

$$p_m = \frac{96 \times 1700 \times \log_\epsilon 2}{144 \times 13{\cdot}6 \times 0{\cdot}995} = 58 \text{ lbs. per sq. in.,}$$

so that the ratio of mean effective to maximum pressure is only 58/20,450, or about 1/353rd part.

Any such maximum pressure as 20,450 lbs. per sq. in. is of course entirely impossible in practice, and it will also be observed that the mean effective pressure is only a small fraction of the maximum pressure. The cycle is entirely impracticable.

The design of an engine is dominated by the maximum pressure to be provided against; if we take this at the value adopted in the modern Diesel engine, viz. about 500 lbs. per sq. in., and also if the upper temperature limit be assumed at the low value of only 800° C. we shall have $\mathrm{X} = 13{\cdot}95$, $\rho = 11{\cdot}06$, $r = 1{\cdot}26$, and $p_m = 6{\cdot}1$ lbs. per sq. in. Thus even here the mean effective pressure is only 1/82nd of the maximum pressure, and has the trifling value of only about 6 lbs. per sq. in. when the Carnot cycle is taken.

§ (49) THE OTTO AND LANGEN "FREE PISTON" CYCLE; RATIO OF p_m/P. — See § (33) and *Figs.* 18 and 27. Here T = 2000° C.,

$t_o = 300°$ C. (abs.), $p_o = 14{\cdot}7$ lbs. per sq. in., and $v_o = 13{\cdot}6$ cub. ft.

The remaining quantities may be thus evaluated:

$p_c/p_o = (T/t_o)$ gives	p_c	$= 98$ lbs. sq. in.,
$p_c v_o^{\gamma} = p_e v_e^{\gamma}$, and $p_o v_o = p_e v_e$ gives	v_e	$= 1419{\cdot}4$ cub. ft.;
and also	p_e	$= 0{\cdot}14$ lbs./sq. in.,
$r = v_e/v_o$ gives	r	$= 104{\cdot}4$,
$U = Jk_v(T - t_o) - ct_o \log_e r$ gives	U	$= 268{,}420$ ft.-lbs.;
then by Eq. (72)	p_m	$= 1{\cdot}326$ lbs./sq. in.,
and $p_m/P = p_m/p_c$ gives	$p_m/P = 1/74$th.	

Thus the ideal cycle in this case requires a total expansion of 104·4 times the initial volume, and gives a mean effective pressure of only 1·326 lb. per sq. in., which is but 1/74th of the maximum pressure developed.

Moreover, in order to approximate even roughly to isothermal compression on the return stroke, it was necessary to run these free piston engines very slowly, and they were in consequence exceedingly cumbrous and furnished but a trifling power relatively to their bulk and weight, notwithstanding their high theoretical efficiency.

A series of tests made by Clerk in 1885 on a 2-h.p. engine with a cylinder 12½ in. in diameter showed that the maximum stroke was 40½ in., and i.h.p. 2·9, with 28 explosions per minute, corresponding to a mean effective pressure of 8½ lbs. per sq. in.; the total expansion ratio attained was, however, only about 6.

The b.h.p. was 2·0, and the mechanical efficiency therefore 70 per cent. The consumption of coal gas was 24·6 cub. ft. per i.h.p. hour, corresponding to an (indicated) thermal efficiency of 0·11. Even this consumption, however, marked a notable improvement upon the results obtained with the earlier Lenoir and Hugon engines; the largest engine made of this very noisy free piston type was only of 3 h.p.

§ (50) THE DIESEL CYCLE; RATIO OF p_m/P.—See § (41) and *Fig.* 15. Here

$$p_o = 14{\cdot}7 \text{ lbs. per sq. in.,}$$

$$v_e = 13{\cdot}6 \text{ cub. ft.,}$$

$$T' = 300° \text{ C. (abs.),}$$

$$p_c = 500 \text{ lbs. per sq. in;}$$

$$\therefore\ v_c = \left(\frac{p_o}{p_c}\right)^{1/\gamma} v_e = 1{\cdot}112 \text{ cub. ft.,}$$

$$t_c = \frac{144 p_c v_c}{96} = 833° \text{ C. (abs.),}$$

$$T = 2000° \text{ C. (abs.),}$$

$$v_p = \frac{RT}{144 p_c} = 2{\cdot}67 \text{ cub. ft.,}$$

$$\sigma = \frac{v_p}{v_c} = 2{\cdot}4,$$

$$\rho = \frac{v_e}{v_c} = 12{\cdot}23,$$

$$\text{and}\quad T_2 = \left(\frac{v_p}{v_e}\right)^{\gamma - 1} T = 1043° \text{ C. (abs.).}$$

Hence (*vide* § (31)), as $U = k_p(T - t_c) - k_v(T_2 - T')$, we have U = 151·67 C.Th.U. of useful work, and therefore, by equation (72),

$$p_m = \frac{151{\cdot}67 \times 1400}{144 \times 12{\cdot}488} = 118 \text{ lbs. per sq. in.}$$

Thus the mean effective pressure has here the high value 118 lbs. per sq. in., and $p_m/P = 1/4{\cdot}24$, a very great advance on previous figures. In actual practice mean effective pressures of 100 to 115 lbs. per sq. in. are ordinarily realised, with an average (indicated) thermal efficiency of 0·4, and revolution speed, in stationary types, of 200 per minute. The Diesel cycle thus satisfies the requirements of actual practice very fully.

§ (51) THE BEAU DE ROCHAS, OR OTTO CYCLE; RATIO OF p_m/P.—See §§ (36) and (45) and *Figs.* 21 and 29. Here also

$$p_o = 14{\cdot}7 \text{ lbs. per sq. in. (abs.),}$$

$$v_e = 13{\cdot}6 \text{ cub. ft.,}$$

$$t_o = 300° \text{ C. (abs.).}$$

t_c is assumed at 600° C. (abs.);

$$\text{whence}\quad v_c = \left(\frac{t_o}{t_c}\right)^{\gamma - 1} v_e = 2{\cdot}489 \text{ cub. ft.,}$$

$$\text{and}\quad \rho = \frac{13{\cdot}6}{2{\cdot}489} = 5{\cdot}47,$$

$$p_c = \frac{96 t_c}{144 v_c} = 160{\cdot}7 \text{ lbs. per sq. in. (abs.),}$$

$$P = \frac{T}{t_c} p_c = 535{\cdot}7 \text{ lbs. per sq. in. (abs.),}$$

$$T = 2000° \text{ C. (abs.).,}$$

$$T_2 = \frac{T}{t_c} t_o = 1000° \text{ C. (abs.),}$$

$$\text{and}\quad p_e = \frac{96 T_2}{144 v_e} = 49{\cdot}0 \text{ lbs. per sq. in.}$$

The useful work done is given by the equation $U = k_v(T - tc/1) - k_v(T_2 - t_0) = 118{\cdot}23$ C.Th.U. whence, by equation (72),

$$p_m = \frac{1400 \times 118{\cdot}23}{144 \times 11{\cdot}11} = 103{\cdot}5 \text{ lbs. per sq. in.,}$$

and the ratio p_m/P has the value 1/4·84, which is of the same order as that obtained in the case of the Diesel cycle.

§ (52) THE CONSTANT-PRESSURE "BRAYTON" CYCLE; VALUE OF p_m, ETC.—See §§ 29, 30, and 39 and *Figs.* 14 and 23; this cycle is practically perfected in the modern Diesel engine, but it is of interest to evaluate p_m, as its ratio to P was, both in theory and practice, very high. As illustrated in *Figs.* 14 and 23 we have here

$$p_o = 14{\cdot}7,$$

$$v_o = 13{\cdot}6,$$

$$t_o = 300° \text{ C.,}$$

$$p_c = 70;$$

whence, by equation (21),

$$t_c = 468° \text{ C.},$$

and therefore

$$v_c = \frac{96 \times t_c}{144 \times p_c} = 4{\cdot}47 \text{ cub. ft.}$$

Again, $v_p = \frac{\text{T}}{t_c} v_c = 19{\cdot}05$ cub. ft.,

and thus $\sigma = 4{\cdot}26$ (*vide* equation 51).

T_2 is assumed at 1450° C., whence, by equation (22), $v_e = 41{\cdot}9$ cub. ft., so that the total expansion ratio $\text{X} = 9{\cdot}4$.

$$p_e = \frac{96 \times t_2}{144 \times v_e} = 23{\cdot}2 \text{ lbs. per sq. in.,}$$

and $\text{T}' = \frac{v_e}{v_o} t_o = 925°$ C. (abs.).

Hence, from § (30), $\text{U} = 126{\cdot}74$ C.Th.U. of useful work, and accordingly, by equation (72),

$$p_m = \frac{126{\cdot}74 \times 1400}{144 \times 37{\cdot}43} = 33{\cdot}0 \text{ lbs. per sq. in.}$$

The effective maximum pressure is $(\text{P} - 14{\cdot}7)$ lbs. per sq. in. $= 55{\cdot}3$, which is only 1·68 times p_m—a very satisfactory feature of the cycle.

A test of a Brayton petroleum engine of about 4 b.h.p., made by Clerk in 1878, showed a maximum pressure in the working cylinder of 47 lbs. per sq. in., a mean effective pressure of 30·2 lbs. per sq. in., and a consumption of petroleum of 2·75 lbs. per b.h.p. hour, corresponding to the very low brake thermal efficiency of only 0·047.

T_2 is assumed at 1400° C. (abs.)., and therefore, by equation (22), $v_e = 32{\cdot}6$ cub. ft.; then

$$p_e = \frac{96 \times 1400}{144 \times 32{\cdot}6} = 28{\cdot}6 \text{ lbs. per sq. in. (abs.),}$$

and $\text{T}' = \frac{v_e}{v_o} t_o = 718°$ C. (abs.).

Hence

$$\text{U} = k_v(\text{T} - t_o) - k_v(\text{T}_2 - \text{T}') - k_p(\text{T}' - t_o)$$
$$= 72{\cdot}67 \text{ C.Th.U. of useful work.}$$

In this case, as the stroke of the piston is AK, we have to write

$$p_m = \frac{1400 \times 72{\cdot}67}{144 \times v_e} = 21{\cdot}7 \text{ lbs. per sq. in.}$$

Thus the ratio of p_m to the maximum effective pressure of $(98 - 14{\cdot}7)$ lbs. per sq. in. has here the satisfactory value 1/3·84; the engine failed commercially, however, on account of its exceedingly great consumption of fuel. Not only is the theoretical efficiency of the cycle low, but even this low efficiency was not nearly attained in practice. The engines were built in sizes of from one-half to three horsepower. An experiment by Tresca on a one-half horse-power engine showed a consumption of 95 cub. ft. of (Paris) coal gas per i.h.p. hour, which is fully eight times as great as that of a modern four-stroke gas engine of moderate power.

§ (54).—Collecting the results obtained in the preceding paragraphs for comparison, we have the following for typical engines, each using 1 lb. of air, and working between the absolute temperatures 2000° C. and 300° C.:

Engine.	Type of Cycle.	Theoretical Efficiency with Conditions assumed.	Maximum Pressure P. Lbs./Sq. In. Abs.	Mean Pressure p_m. Lbs./Sq. In.	Ratio of p_m to (P – 14·7).
Carnot . . .	Constant temperature	·85	20,450	58	·0028
Otto and Langen free piston *	Constant volume	·66	98	1·326	·016
Diesel . . .	Constant pressure	·55	500	118	·243
Otto 4-stroke. .	Constant volume	·50	536	103·5	·200
Brayton . .	Constant pressure	·33	70	33	·600
Lenoir . . .	Constant volume	·25	98	21·7	·260

* With complete expansion.

The modern Diesel engine consumes only about 0·45 lb. of oil fuel per b.h.p. hour, corresponding to a brake thermal efficiency of fully 0·30.

§ (53) THE LENOIR CONSTANT VOLUME CYCLE; VALUE OF p_m.—See *Figs.* 17 and 26 and §§ 32 and 42. Here also

$$p_o = 14{\cdot}7,$$
$$v_o = 13{\cdot}6,$$
$$t_o = 300° \text{ C.},$$
$$\text{T} = 2000° \text{ C.},$$

whence $p_c = 98$ lbs. per sq. in. (abs.).

The relative values and advantages of the various cycles are very fully discussed by Clerk in *The Gas, Petrol, and Oil Engine* (Longmans), vol. i., 1909.

§ (55) LOSSES IN INTERNAL COMBUSTION ENGINES.—In all that precedes it has been assumed that the working substance is dry air—regarded as a perfect gas—and that the conditions are such that each cycle can be perfectly carried out. This is impossible in actual practice, and is due mainly to the following causes:

(1) The working gases lose heat to the surfaces of the combustion chamber, cylinder

and piston crown during and after explosion by radiation and convection.

(2) The unexploded charge is usually heated on entering the cylinder, with consequent expansion and reduction of the mass of charge exploded.

(3) In constant-volume cycles the explosion is never instantaneous, as assumed in theory.

(4) The working substance is not dry air, but a mixture of nitrogen, carbon dioxide, steam, and oxygen, having specific heats which are not constant, but increase with rise of temperature.

(5) The working substance is changed in volume by combustion, so that the volume which is heated and expanded differs from that which is compressed when measured at the same temperature and pressure.

(6) Combustion is not complete when maximum temperature is attained, and is even, in some cases, not complete at release.

(7) Some throttling or "wire-drawing" always occurs during admission, and there is always some degree of back pressure opposing the exhaust.

(8) The working gases lose heat to the cylinder walls during compression.

These several sources of loss may be considered in some further detail.

§ (56) Loss of Heat to Walls during Explosion and Expansion.—The physical properties of cast iron, of which the cylinders of all internal combustion engines, excepting only air-craft engines, are made, renders it necessary that they shall be kept cool either by a water-jacket or, as in many small engines, by a stream of cold air passing over heat-radiating gills formed on the outer surface of the cylinder. In practice it is found that 25 per cent to 50 per cent of the whole heat evolved by the combustion of the working fluid is lost in this way alone. The highly heated gases lose heat to the containing surfaces partly by radiation, and, being in a state of very violent turbulence, also by convection.

The general rule that the better the absorber the better the radiator, and the principle that a perfectly transparent substance, whatever its temperature, could radiate no energy, lead to the conclusion that gases in chemical equilibrium—which then possess nearly perfect transparency—can emit no appreciable radiation. When radiation does take place from gases it appears to arise from the flame due to chemical action proceeding in the gas, and in internal combustion engines any radiation from the flaming mixture is absorbed by the enclosing metal walls of the combustion chamber, cylinder barrel, piston crown, and valve heads, wherein it appears as sensible heat, which is then conveyed by conduction to the jacket and surrounding air. Loss of energy by radiation, in this connection, was first considered by Callendar in 1906. Tests by him on a small petrol engine[1] showed that the loss of heat per cycle could be represented approximately by an expression of the form $a+b\tau$, where τ is the time of one revolution, and a and b are constants. Radiation loss from the burning gases proceeds with very great rapidity near the instant of maximum temperature, and this practically instantaneous loss may be considered as represented by the constant term "a." The second term $b\tau$ represents a loss by radiation and convection-conduction proportional to the time during which the cylinder surfaces are exposed to the burning gases.

§ (57) Radiation from Flames.—R. v. Helmholtz found, when using "solid" flame of about $\frac{1}{4}$ in. in diameter, that a hydrogen flame radiated about 3 per cent, coal gas almost 5 per cent, and CO about 8 per cent of its total heat of combustion. These were very small flames. A large flame radiates more energy per unit of area, since a flame is largely transparent even to its own radiation, and thus radiation is received not only from the surface molecules, but also from all those behind it. Callendar, repeating some of Helmholtz's experiments on a larger scale, found that the radiation from a Bunsen non-luminous coal gas flame 1·2 in. in diameter may amount to as much as 15 per cent of the whole heat of combustion. Experiments made by Julius on different kinds of flame have shown that the radiation is almost wholly due to the CO_2 and H_2O (steam molecules). The explosion of gases in an exhaust vessel, or a gas engine cylinder, differs considerably from any open flame in respect of radiation, as not only is the density of the gas much greater in the closed vessel, but it is also not cooled by mixture with the outside air. Hopkinson (1910) made experiments on the radiation emitted during explosion and subsequent cooling of a mixture of 0·15 coal gas and 0·85 air, by volume, in a closed vessel. He found that the total heat radiated during and after the explosion amounted to over 22 per cent of the whole heat of combustion. The radiation up to the instant of maximum pressure amounted to 3 per cent, and continued at a diminishing rate for a considerable period thereafter. It was still perceptible 0·5 second after maximum pressure, when the gas temperature had fallen to 1000° C.

§ (58).—The conditions existing within a gas engine when working have been vividly described by Clerk, who fitted a cylinder with a stout observation plug of glass. He says: "While the engine is at work, a continuous glare of white light is observed; a look into the interior of a boiler furnace gives a good notion of the flame filling the cylinder of a gas engine." The loss by radiation from the flaming mixture will be greater as the absorbing power of the containing metal surfaces is increased. Hopkinson has coated the interior of an explosion vessel with tin-foil, and compared the results obtained by exploding mixtures of identical composition, firstly with the tin-foil highly polished, and secondly with its surface covered with lamp-black. He found that nearly the same maximum pressure was developed in both cases, but that the fall of pressure during cooling was considerably less with the bright than with the black surface. Further bolometric experiments by Hopkinson consisted in covering a small portion of the inner surface of an explosion vessel with thin copper strip (1) highly polished, (2) blackened, and (3) with the strip protected from direct contact with the flame by a plate of rock-salt. It was found that the rate of increase of temperature of the blackened surface during explosion and the subsequent early stages of cooling greatly exceeded that of the polished strip, the difference between them being roughly equal to the rate of temperature increase observed when the strip was covered by the rock-salt, which transmitted upwards of 90 per cent of the radiant energy to the strip while protecting it against any direct gain of heat by contact with the flaming gas.

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[1] *Proc. Inst. Aut. Eng.*, 1907.

Using a mixture of 0·15 coal gas to 0·85 air, by volume, giving a maximum temperature of 2150° C., Hopkinson estimated from these experiments that the loss of heat by radiation to the enclosing surface up to the instant of maximum pressure, was about 5 per cent of the whole heat of combustion, and that radiation continued during cooling certainly down to 1400° C. If combustion were complete at the instant of maximum pressure, which is also that of maximum temperature in closed vessel experiments, it would follow from the observed greater loss of energy by radiation during explosion to the black than to the bright surface, and the observed equality of the maximum temperatures attained in each case, that the internal energy of the gas at the same temperature would be greater when the enclosing surface was bright than when blackened. Clerk has shown, however, that combustion is never complete at the instant when maximum temperature is attained, and finds that in gas engine practice in general only some 85 per cent of the heat is evolved at this instant, the remaining 15 per cent appearing during a portion, at least, of the expansion working stroke. Combustion is usually practically complete when release takes place; with weak mixtures, or badly-designed combustion chambers, combustion may be markedly incomplete even at release.

§ (59) Cylinder Temperatures.—Although the maximum temperature of the working gases is of the order of 2000° C., the mean temperature at the inner surface of the metal of the cylinder never exceeds a quite moderate value of, at most, some 200° C. above that of the cooling water in large gas engines; in the thin walls of petrol engine cylinders the difference is much less, and is usually below 50° C. Hopkinson placed patches of tin-foil on the inner surface of the combustion chamber of a gas engine cylinder 11½ in. in diameter, and found these were quite unaffected by the successive explosions, although the heat-flow rate is here a maximum, and the melting-point of tin only 230° C. In this connection it is also of interest to record that, using a rich mixture (1 : 9) of coal gas and air, he found at the instant of maximum pressure in a cylindrical explosion vessel about 23½ in. diameter × 27 in. long, large differences of temperature at different points within the vessel. The following are his results:

Mean temperature of the gases (inferred from maximum pressure) . . .	1600° C.
Temperature at centre of volume of the vessel, near point of ignition of the mixture	1900 „
Temperature at 4 in. from the wall of vessel	1700 „
Temperature at 0·4 in. from wall at end	1100- 1300 „
Temperature at 0·4 in. from wall at side	850 „

A rough approximation to the rate of heat-flow per unit area from the gases to the cylinder surfaces is obtained by dividing the mean area exposed to the heated gases into the total heat appearing in the jacket water plus that lost by ordinary external radiation from the engine as a whole.

Comparing in this way a 35 h.p. four-cylinder 4·62 in. diameter × 5·08 in. stroke Siddeley petrol engine running at 930 revs. per minute, with a 40 h.p. single-cylinder 11½ in. × 21 in. Crossley gas engine running at 180 revs. per minute, Hopkinson [1] obtained the following results:

Item.	Crossley.	Siddeley.
C.Th.U. lost per minute to jacket water, and by general external radiation	2250	2300
Percentage of ditto to total heat supply	34·0	40·5
C.Th.U. lost per cylinder per minute	2250	575
Heat-flow in C.Th.U. per sq. in. of exposed surface per minute at—		
In-centre . . .	6·5	7·0
Out-centre . . .	2·05	3·7
Mean	3·2	4·85

[1] *Proc. Inst. Aut. Eng.* vol. iii.

Thus the maximum heat-flow rate did not exceed 7 C.Th.U. per sq. in. per minute, and the temperature grade necessary for this rate through cast iron is only about 100° C. per in. of thickness. As the walls of the Siddeley engine cylinder were only $\frac{5}{16}$ in. thick, Hopkinson points out that the mean temperature of the internal metal surface, where water-jacketed, never exceeded that of the jacket water by more than about 30° C. The walls of the Crossley engine cylinder were almost four times as thick as in the Siddeley, which would correspond to a mean temperature of inner surface of, at most, 125° C. above that of the jacket water, or about 200° C. actual temperature.

§ (60) Piston and Valve Temperatures.—The pistons and valves of internal combustion engines are rarely water-cooled, and consequently become very hot, their heat having to be conducted through some distance before reaching the jacket water. When working at full load, Hopkinson found in the Crossley engine above cited temperatures at the centres of the piston crown and exhaust valve at about 540° C. For the gas engine mixture used by him the temperature of pre-ignition was slightly above 700° C. With oil engines when run at full load pre-ignition usually soon occurs, and is commonly prevented by the injection of a water-spray into the cylinder during explosion.

In the preceding remarks it has been assumed that the internal metallic surfaces exposed to the heated gases are clean; actually the combustion chamber surface is quickly covered with a deposit of carbon, while the working barrel of the cylinder is coated by a thin film of oil. The carbon deposit on the combustion chamber surface and piston crown increases the loss by radiation from the flaming gases; by polishing the surfaces of the combustion chamber and piston of a gas engine a perceptible increase in mean pressure has been obtained. Again, a thick deposit of carbon, especially on the hot piston crown, by its low conductivity may create incandescent points or patches, and thus cause pre-ignition; thus carbon deposit is obviously to be avoided as far as possible.

§ (61) Radiation Losses.—Clerk (Gustave Canet Lecture, 1913) states that radiation losses increase very rapidly with temperature difference, the loss being approximately proportional to the difference of the fourth powers of the absolute temperatures of the gas and inner wall surface respectively—agreeably with Stefan's Law. He states also that radiation increases with increase in the dimensions of the containing vessel; and he points out that Stefan's Law practically limits the temperatures attainable in gas-engine practice.

§ (62) Heat-flow.—Reference may be made to the fifth report of the Gaseous Explosives Committee at the British Association (Dundee, 1912), which contains a valuable resumé of present knowledge relating to heat-flow from the working gases into the cylinder walls of internal combustion engines.

The general conclusions reached are, briefly:

(i.) The rate of heat-flow from gas to walls is greatest at maximum temperature and pressure, and rapidly diminishes as the piston performs its outstroke; the greater part of the heat-flow has occurred in a comparatively short time, and when the piston has moved but little from its in-position. Hence the bulk of the heat lost by the gases to the cylinder passes into the combustion chamber, piston crown, and valve heads, and but little is received by the working barrel. Clerk has calculated that the actual rate of heat-flow in the first $\frac{3}{10}$ths of the outstroke is equal to six times that of the whole stroke, in ordinary gas engines, when working at full load. If pistons be water-cooled, in considering the cooling of the cylinder it is probably sufficient to neglect altogether the heat-flow into the outer half of the working barrel. When pistons are uncooled a water-jacketed barrel is needed mainly to keep the piston cool.

(ii.) The temperature-gradient necessary to maintain the required rate of heat-flow from the inner surface of a combustion chamber to the jacket water rarely exceeds 50° C. per in., and for such surfaces, kept fairly clean, effective cooling presents no great difficulty. At special points, *e.g.* the centre of the piston crown (when uncooled) temperature is high; with a four-stroke engine of 24 in. cylinder diameter a temperature of almost 600° C. may be found here. With large gas engines the necessary great thickness of the combustion chamber walls, and the practical difficulties of ensuring free circulation of jacket water everywhere may result in the formation of high local internal surface temperatures.

(iii.) An important effect of radiation is the greatly increased heat given to cylinder walls when mean pressure is increased by increasing mixture-strength; the metal temperatures and jacket-water temperature are raised in much greater degree than the fuel consumption, and efficiency is diminished. In large engines this sets a practical limit to power output which, if exceeded, results in rapid overheating of the engine.

(iv.) Another important effect of radiation is the greatly increased heat received by the walls from a large than from a small volume of gas; it results from this, that the difference between the efficiency of a large and a small engine is lessened, and also that the difficulty of adequately cooling very large engines is increased beyond that arising from the necessary great thickness of the cylinder walls by the increased amount of heat received through radiation from the greater volume of glowing gas enclosed.

(v.) *Effect of Density.*—The density of the gas in an internal combustion engine of the Otto-cycle type is from four to seven or eight times that of the atmosphere; in Diesel-cycle engines it is considerably greater; increase in density greatly increases heat-flow as compared with an ordinary closed vessel explosion with atmospheric density before ignition. The total heat lost to the jacket water is found to increase with the density, but not quite in simple proportion; the exact relation seems to be complex, and is not ascertained; it is probable that the combined convection and radiation heat losses in a vessel of given form can be fairly well expressed as increasing according to some fractional power of the density.

The question as to the best compression ratio to adopt is closely connected with that of density. When compression is increased by reducing the volume of the combustion chamber, not only is the density of the gas, and therefore the total heat loss, increased, but the area of enclosing surface is also reduced, and hence the rate of heat-loss per unit of area is increased by both causes combined. Though efficiency is increased in theory by increase of compression ratio, this increase may be more than annulled by the increased heat-loss to the enclosing walls, and there is thus a value of the compression for which the efficiency practically attainable is a maximum. But it may easily happen that even before this maximum efficiency is attained the increased rate of heat-flow per unit area of enclosing walls may give rise to cooling difficulties and pre-ignition trouble. The cause of pre-ignition is commonly the overheating of some point or patch of the metal, or carbon deposit thereon, due to excessive heat-flow following increased density. If the enclosing metal surfaces could be kept clean and cool, compression ratios could have much higher values than are at present practicable.

§ (63) Heating of Charge entering Cylinder.—In large gas engines the ratio of surface to enclosed volume of gas is relatively small, while the thickness of the cylinder walls is necessarily great. In order to avoid setting up dangerous stresses in the metal through heating, it is found necessary to keep the jacket cool by arranging for the circulation of an ample supply of cooling water; the temperature on leaving the jackets, in large gas engines and Diesel engines, is usually between 30° C. and 50° C. In very large gas engines the pistons and valve heads are also water-cooled, and in these engines, therefore, the temperature of the whole enclosing surfaces of combustion chamber, piston, etc., is always low. In the much larger class of gas engines with uncooled pistons and valves, the average temperature of the piston crowns and valve heads is very much greater. In the exceedingly large class of small quick-revolution internal combustion engines of the petrol type, the jacket water temperature is usually between 90° C. and 100° C., while in air-cooled engines of this class, the mean temperature of the whole cylinder is considerably above 100° C., and the pistons and valve heads correspondingly higher.

In every case, when a fresh charge is taken into the cylinder, some increase in its temperature is caused by the higher temperature of the enclosing surfaces; in addition, in the three practical cycles of Clerk, Diesel, and Otto, there is always some high temperature burnt gas from the previous cycle remaining in the combustion chamber at the end of exhaust with which the entering fresh charge mixes, and by which its temperature is always raised. From these two causes, the fresh charge is always heated at the beginning of compression, and in Otto cycle gas engines of the smaller type, and petrol engines, a temperature of 200° C. at this point is not unusual. The rise of temperature results in a lessened density of fresh charge taken in, and the mass of fresh charge is further reduced by the presence of residual burnt gases in the combustion space; apart from any effect of this upon thermal efficiency the heat evolved per cycle is lessened, and the power output of the engine correspondingly reduced.

§ (64) The Diesel Cycle; Effect of Heating before Compression.—Consider the case taken in § (50), with $p_o = 14{\cdot}7$ lbs. per sq. in. (abs.), and $v_e = 13{\cdot}6$ cub. ft., the volume at outstroke. But, due to heating on admission, suppose T′ (*Fig.* 15) to be 400° C. (abs.); then at K we have now 13·6 cub. ft. of air at pressure 14·7, and temperature 400° C., *i.e.* the mass of the charge is reduced from 1 lb. to 0·75 lb., and the value of the constant $R' = p_v/T$ is now 72. The volume of the combustion chamber remains unchanged at 1·112 cub. ft., and hence p_c is also unchanged at 500 lbs. per sq. in. (abs.). But $t_c = p_c v_c / c'$ is now increased from 833° C. to 1111° C. (abs.). Hence as, by supposition, the maximum temperature T remains unchanged at 2000° C., we have $v_p = (T/t_c) \cdot v_c = 2{\cdot}002$ cub. ft., whence σ is now reduced from 2·4 to 1·8, and accordingly, agreeably with equation (53) the thermal efficiency increases. Again, $T_2 = (v_p/v_e)^{\gamma - 1} \cdot T = 915°$ C., instead of 1043° C.; the efficiency, by equation (52), is therefore now increased from 0·55 to 0·59. But the heat converted into useful work has been reduced by the reduction of the charge thus:

$U = 0{\cdot}75\{k_p(T - t_e) - k_v(T_2 - T')\} = 93{\cdot}1$ C.Th.U. Hence in consequence p_m is now reduced to $93{\cdot}1 \times 1400/144 \times 12{\cdot}488 = 72{\cdot}3$ lbs. per sq. in. as compared with 118 lbs. per sq. in. in the previous case; thus the output of power of the engine at the same revolution speed has fallen from 1·0 to 0·613. The figures in the two cases are collected together below, for comparison.

In both cases $p_o = 14{\cdot}7$, $v_e = 13{\cdot}6$, $v_c = 1{\cdot}112$, $\rho = 12{\cdot}23_o$ and $T = 2000°$ C.

T′ °C.	c °C.	V_p Cub. Ft.	σ.	Adiabatic Expansion Ratio.	T_2 °C.	Thermal Efficiency.	v. C.Th.U.	p_m. Lbs./Sq. In.	Relative Power Output.
300	833	2·67	2·4	5·1	1043	0·55	151·7	118	1
400	1111	2·002	1·8	6·8	915	0·59	93·1	72·3	0·613

In this cycle increasing the lower limit of temperature T′, increases the compression temperature t_c in the same proportion; hence t_c approaches more nearly to T, and accordingly the constant pressure expansion ratio σ is reduced with consequent increase in the adiabatic expansion ratio and reduction of the pressure drop at the release point H. The efficiency is increased from 0·55 to 0·59, but owing to the smaller expenditure of heat the power output of the engine is reduced in the proportion of 100 to 61·3.

§ (65) THE OTTO CYCLE; EFFECT OF HEATING BEFORE COMPRESSION.—See § (36) and *Fig.* 21. In this case, as by equation (71), the efficiency is $1 - (1/\rho)^{\gamma - 1}$; where ρ is constant in value for the same engine, it follows that the efficiency, in theory, is unaltered by any change in the temperature of the charge before compression. Raising t_o raises t_c in the same proportion, and *if there be the same expenditure of heat, H, as before,* then as $H = k_v(T - t_c)$ we have $T = H/k_v + t_c$, and thus T is also raised, and consequently so also is T_2 agreeably with the relation $T_2/T = t_o/t_c$. In practice, however, in this case the efficiency is *reduced* owing to the greater heat losses resulting from higher temperature of explosion and during adiabatic expansion. In the case discussed in § (51) suppose that, due to heating on admission, $t_o = 400°$ C. (abs.); then the mass of the charge is reduced from 1 lb. to $\frac{3}{4}$ lb., and t_c rises to 800° C. (abs.) since the ratio t_c/t_o is constant. The efficiency is, in theory, unaltered; two suppositions may now be made as to the supply of heat to the working fluid, viz.:

(i.) Suppose the supply of heat to be three-fourths of the quantity in § (51); and

(ii.) Suppose the supply of heat to be only that necessary to cause the working fluid to attain a maximum temperature of 2000° C. (abs.).

Case (i.).—In this case, as three-fourths of the heat is supplied from t_c to three-fourths of the fluid, the *rise* of temperature will be the same as before, and accordingly the maximum temperature attained will be unaltered and $T = 800 + 1400 = 2200°$ C., while T_2 will correspondingly increase from 1000° C. to 1100° C. Thus the working fluid is at a higher temperature throughout the cycle, the heat losses to the cylinder are greater, and hence, in practice, the efficiency is reduced. The useful work done is $U = {\cdot}75k_v\{T - t_c - T_2 + t_o\} = 88{\cdot}67$ C.Th.U., while as a necessary consequence the mean effective pressure is now reduced from 103·5 to $88{\cdot}67 \times 1400/144 \times 11{\cdot}11 = 77\frac{1}{2}$ lbs. per sq. in. Thus, at the same revolution speed, even if the efficiency remained unchanged, the output of the engine would be reduced from 100 to 75; the reduction in output will be actually greater than this, as the efficiency is lessened through the increased heat losses incurred.

Case (ii.).—In this case the efficiency is also, in theory, unaltered, and the temperature of the working substance is only increased by a relatively small amount during the adiabatic compression period; the practical reduction of efficiency may hence be expected to be smaller in this case. The mass of the charge is reduced, as before, to $\frac{3}{4}$ lb., but the heat supplied is also reduced, and is given by $H = {\cdot}75k_v\{2000 - 800 - 1000 + 400\} = 76$C.Th.U. instead of 88·67. The mean effective pressure, at unchanged efficiency, now drops to $66\frac{1}{2}$ lbs. per sq. in., and the engine output is now represented by the number 64·2 only.

Thus in Otto-cycle engines, heating the incoming charge does not, in theory, affect the efficiency, though in practice some loss of efficiency results from increased heat losses to the cylinder. The reduction in the mass of the charge, with consequent reduction in the heat supply, results, however, in a rapid falling off in the output of power.

§ (66) TIME OF EXPLOSION.—In the preliminary simple theory of the internal combustion engine constant volume cycle, explosion is assumed to occur instantaneously; actually a small, but finite, interval of time elapses between the instant of ignition and that at which maximum pressure is attained. The time of explosion in constant volume experiments may be defined as the interval of time between the commencement of increase of pressure and the attainment of maximum pressure, and numerous experiments upon explosions in closed vessels of constant volume by Bunsen, Berthelot, Bairstow and Alexander, Clerk, Grover, Hopkinson, Langen, Petavel, Mallard and Le Chatelier, and others have furnished results showing clearly the mode of dependence of the time of explosion upon the nature and condition of the "carburetted" charge employed.

Constant-volume explosion experiments by Clerk in 1900 made upon electrically ignited mixtures of London coal-gas and air, initially at atmospheric pressure and temperature, contained in a closed cylindrical vessel 7 in. in diameter and 7 in. long internally, furnished the following results :

Mixture $\frac{\text{Volume Gas}}{\text{Volume Air}}$.	Time of Explosion.
	Seconds.
$\frac{1}{4}$	·045
$\frac{1}{5}$	*·042*
$\frac{1}{6}$	·055
$\frac{1}{7}$	·067
$\frac{1}{8}$	·087
$\frac{1}{9}$	·155
$\frac{1}{10}$	·305
$\frac{1}{11}$	·290

Similar experiments by Bairstow and Alexander on mixtures of coal-gas and air exploded at constant volume but in a much larger cylindrical vessel, viz. of 10 in. internal diameter by 18 in. in length, from atmospheric temperature, but with an initial pressure of 55 lbs. per sq. in. (abs.), furnished results from which the following figures have been deduced :

Mixture $\frac{\text{Volume Gas}}{\text{Volume Air}}$.	Time of Explosion (approximately).
	Seconds.
$\frac{1}{7}$	·04
$\frac{1}{8}$	·07
$\frac{1}{9}$	·20
$\frac{1}{10}$	·5
$\frac{1}{11}$	1·0

Petavel, with a 1 : 6 mixture of coal-gas and air, initially at 18° C., and at a pressure of 1136 lbs. per sq. in. (abs.), exploded within a spherical steel bomb of 4 in. internal diameter, observed a maximum explosion pressure of 9508 lbs. per sq. in. attained in ·058 of a second. From these and other results, it appears that for uniform explosive mixtures initially at rest within a closed vessel, the time of explosion, *caeteris paribus*, is (i.) less as the mixture richness is greater, (ii.) greater, with a single point of ignition, as the volume of the containing vessel is greater, and (iii.) independent of the initial pressure of the explosive charge.

As the mixture richness is increased a point is, however, soon reached at which the time of explosion is a minimum ; the following figures from experiments by Clerk, and the Massachusetts Institute, Boston, illustrate this :

Mixture Ratio.	Time of Explosion with		
	Oldham Coal-Gas.	London Gas.	Boston Gas.
$\frac{1}{4}$	·16	·045	·08
$\frac{1}{5}$	·055	*·042*	*·05*
$\frac{1}{6}$	*·04*	·055	*·05*
$\frac{1}{7}$	·06	·067	·06
$\frac{1}{8}$	. .	·087	·06
$\frac{1}{9}$	·08	·155	·08
$\frac{1}{10}$	. .	·305	·11
$\frac{1}{11}$	·17	. .	·14

While with mixtures of 0·68, petrol vapour and air, the Boston experiments gave :

Petrol Vapour.	Time of Explosion.	Petrol Vapour.	Time of Explosion.
Per cent.	Seconds.	Per cent.	Seconds.
1·79	·109	2·78	*·058*
1·96	·091	3·03	·066
2·17	·082	3·23	·067
2·44	·060	3·45	·100
2·63	*·058*		

On the passage of the igniting spark the gas in its immediate vicinity is instantly inflamed with accompanying sudden expansion ; the inflammation very quickly extends outwards in all directions, the inflamed portion rapidly compressing, and thus heating, the uninflamed portion. Parts of the inflamed gas are also projected into the uninflamed volume, and thus cause the general inflammation to proceed at an increasing rate nearly up to the point when complete inflammation is attained. It is clear, therefore, that the time of explosion will be greater as the volume of gas exploded is greater, and also that it will be lessened by having more than one point of ignition, provided the additional ignition points are effective. Further, if a considerable volume of the gas be ignited at once, by a long and powerful spark, or a large flame, the time of explosion will be reduced ; a small separate chamber connected with the main explosion vessel if filled with explosive mixture, and ignited, will project a rush of flame into the main vessel and so much reduce the time of explosion. The shape of the explosion vessel has also a marked influence, and has also the position within the vessel of the point of ignition. This latter point is well illustrated in Bairstow and Alexander's experiments, using a cylindrical vessel 10 in. diameter × 18 in. long, with the ignition point situated at different points within it; the following are the times of explosion observed:

Depth of Ignition Point from Top of Vessel.	Time of Explosion.
Inches.	Seconds.
0	·160
3	·141
6	·177
9	*·105*
12	·123
15	·140
18	·145

The maximum pressure attained was, in each case, roundly 220 lbs. per sq. in. (abs.), and the explosion was most rapid when the mixture was fired from the centre of the cylinder. Though rapid ignition does not cause any increase in the maximum pressure attained, it is yet of importance in internal combustion engines in order to avoid loss of work from a comparatively slow pressure rise upon a rapidly moving piston. Accordingly in large gas engines, and in the petrol engines of racing cars, it is common to find two or even more igniting points per cylinder. In the case of a petrol engine tested by Professor W. Watson,[1] for example, it was found that with single ignition the time of explosion was 0·0055 second, and that this was reduced by almost one-third, viz. to ·0037 second when double ignition was used. Increased power was obtained when using double ignition, the advantage gained being greater as the revolution speed increased, as the following figures show:

Revolutions per Minute.	Horse-power with Single Ignition.	Horse-power with Double Ignition.
1100	18·4	20·8
1600	26·0	29·2

§ (67) TURBULENCE.—At an early date it was observed by Clerk that gas engines would have been impracticable, through necessarily slow running, had the rates of explosion been as great in actual engine cylinders as in closed vessel experiments. In the case just cited, the time of explosion of the petrol air mixture in an actual engine was only ·0055 second, whereas the shortest explosion time obtained in the Boston experiments—albeit with a larger vessel—was ·058 second, or more than ten times as great. In larger gas engines also the same result is observed; *Fig.* 30 is a reproduction from a diagram taken by Humphrey during a test in 1900[2] of a 500 h.p. Premier engine using Mond gas. The cylinder was 28½ in. diameter and the stroke 30 in.; the average revolution speed was 128·05 per minute. The diagram shows that maximum pressure was attained when the piston had moved forward only about $\frac{1}{20}$ of the outstroke, corresponding to a time interval of only 0·026 seconds, which is, in this case, substantially the time of explosion. Thus even in this large cylinder the time of explosion is only about one-half as great as was observed in the closed vessel coal-gas experiments above cited.

This feature has recently received special attention, and Clerk found in some experiments carried out in 1912 that the time of explosion in the same engine diminished with increase in the revolution speed, and that this resulted mainly from the increased rate of inflammation caused by the violent turbulent motion set up by the sudden rush of the fresh charge into the cylinder during the suction stroke, and which persisted during compression; in gas and petrol engines during the suction stroke the average velocity of the entering fresh charge through the inlet valve is from 100 to 120 ft. per second.

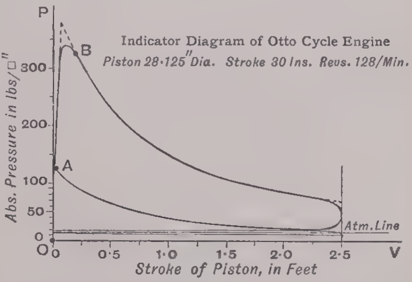

FIG. 30.

Clerk[3] has carried out further experiments showing very strikingly the important part played by turbulence in reducing the time of explosion in actual engines, and thus rendering high revolution speeds practicable. He took indicator diagrams from the same engine: (1) firing the charge in the usual manner, and (2) firing the charge after compressing and expanding it during one or two revolutions of the crank-shaft, thus giving time for the turbulence to largely subside. Comparison of the diagrams so obtained shows at once that the effect of damping down turbulence was to retard the rate of inflammation to a remarkable extent, completely changing the form of the diagram.

Two of the diagrams so taken are shown in *Figs.* 31 and 32; the time of explosion with normal ignition, from A to B,[4] is ·037 and ·033 seconds respectively; while after a third compression of the fresh charge the time of explosion increases to ·092 and ·078 seconds respectively. The cylinder was 9 in. in diameter, with 17 in. stroke, and was run at 180 revolutions per minute under full load conditions with jacket water at 70° C. Two electric igniters were fitted, one in the inlet port at the back of the combustion chamber, the other at the side of the cylinder just clear of the piston, when fully "in." It will be noted that in this case the explosion is more rapid with the side igniter, and that in both cases the time of explosion with partly quenched turbulence is about 2½ times as great as under normal working conditions with undamped turbulence.

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[1] *Proc. I.A.E.*, 1909.

[2] *Proc. Inst. Mech. Eng.*, 1901.

Gustave Canet Lecture, 1913.

[4] Max. *pv* occurs at B.

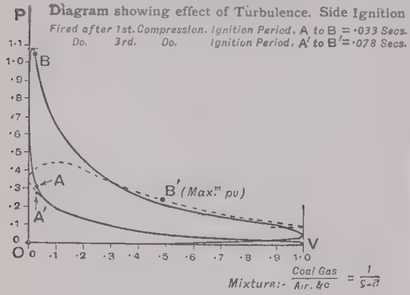

FIG. 31.

Thus, due to turbulence, the time of explosion in the actual engine was only $\frac{1}{4}$th to $\frac{1}{5}$th that observed in the closed vessel explosion experiments of Clerk and Hopkinson, with the mixture at rest, and only about $\frac{1}{8}$th of that observed in Bairstow and Alexander's tests. Hopkinson also carried out tests in 1912 on the effect of turbulence artificially produced by the rotation of a fan within a closed cylindrical vessel 12 in. diameter by 12 in. long; these experiments showed a great reduction in the time of explosion with increase in turbulence. Using a 1 : 9 mixture by volume of coal-gas and air, the following figures illustrate his results:

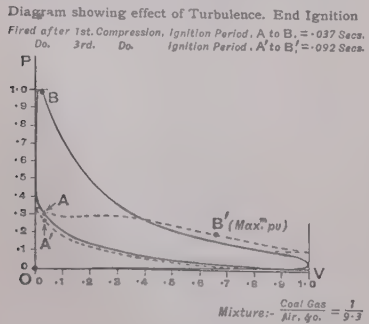

FIG. 32.

Revolutions per Minute of Fan.	Time of Explosion.
	Seconds.
0	·13
2000	·03
4500	·02

§ (68) CALORIFIC VALUES AND SPECIFIC HEATS OF THE WORKING MIXTURES.—As already stated, whatever the fuel used in an internal combustion engine, the working substance after ignition consists of a mixture of nitrogen, steam, carbon dioxide and frequently a small amount of free oxygen. When the chemical formula of the fuel or "carburant" is known, the heat evolved during combustion may be readily calculated. In Table I. some physical constants are given for the substances involved in Internal Combustion Engine practice.

TABLE I

SPECIFIC WEIGHT AND VOLUME OF GASES AT 0° C. AND ATMOSPHERIC PRESSURE

Substance.	Symbol. 1.	Specific Gravity H = 1. 2.	Weight of 1 Cub. Ft. in Lb. 3.	Volume of 1 Lb. in Cub. Ft. 4.
Air	23·0 + 77 N	14·44	·08073	12·387
Oxygen	O_2	16	·08944	11·18
Nitrogen	N_2	14	·07826	12·78
Hydrogen	H_2	1	·00559	178·89
Water (gaseous)	H_2O	9	·05031	19·88
Carbon monoxide	CO	14	·07826	12·78
Carbon dioxide	CO_2	22	·12298	8·13
Methane (marsh gas)	CH_4	8	·04472	22·36
Ethylene (olefiant gas)	C_2H_4	14	·07826	12·78
Acetylene	C_2H_2	13	·07267	13·76
Normal benzene	C_6H_6	39	·21801	4·59

In Tables II. and III. the quantities of heat evolved in the complete combustion of 1 lb., and of 1 cub. ft., of various gases, etc., are given.

§ (69).—From Tables II. and III. it will be noted that a hydrogen-air mixture contracts after combustion to 0·853 of its initial volume, with corresponding reduction in the pressure developed. Further, notwithstanding the very high calorific value of hydrogen *per lb.*, its excessive bulkiness results in a heat evolution of only 164/3·4 = 48 C.Th.U. (app.) per cub. ft.

TABLE II.—APPROXIMATE CALORIFIC VALUES, ETC., OF FUELS PER CUBIC FOOT AT 0° C. AND ATMOSPHERIC PRESSURE

Fuel. 1 Cubic Foot of	The Combustion of 1 Cubic Foot of Fuel requires								Approximate Heat of Combustion per Cubic Foot of Fuel, in C.Th.U.	
	Of Oxygen.				Of Air.					
	Of Oxygen alone. Cubic Feet.	Volume of Mixture in Cubic Feet.			Of Air. Cubic Feet.	Volume of Mixture in Cubic Feet.			Higher Value.	Lower Value.
	1.	Before Combustion. 2.	After Combustion. 3.	Ratio of Volumes. After/Before. 4.	5.	Before Combustion. 6.	After Combustion. 7.	Ratio of Volumes. After/Before. 8.	9.	10.
Hydrogen	0·5	1·5	1·0	0·667	2·4	3·4	2·9	0·853	191	164
Carbon (gas) to CO *	1·0	2·0	2·0	1·0	4·8	5·8	5·8	1·0	164	164
Carbon (gas) to CO_2 *	2·0	3·0	2·0	0·667	9·6	10·6	9·6	0·906	540	540
Carbon monoxide	0·5	1·5	1·0	0·667	2·4	3·4	2·9	0·853	190	190
Methane	2·0	3·0	3·0	1·0	9·6	10·6	10·6	1·0	596	542
Ethylene	3·0	4·0	4·0	1·0	14·4	15·4	15·4	1·0	953	899
Acetylene	2·5	3·5	3·0	0·857	12·0	13·0	12·5	0·961	882	855
Normal benzene	7·5	8·5	9·0	1·06	36·0	37·0	37·5	1·013	2180	2099
Average petrol	10·2	11·2	13·9	1·24	49·0	50·0	52·7	1·054	. .	2580

* As 1 cub. ft. of carbon (as gas) forms 2 cub. ft. of CO, and also 2 cub. ft. of CO_2 it follows that in the formation of *1 cub. ft. of CO*, 82 C.Th.U. are evolved; and in the formation of *1 cub. ft. of CO_2*, 270 C.Th.U. are evolved.

of *mixture* with air at 0° C. and atmospheric pressure.

Hydrogen-air mixtures are very sharply explosive and when hydrogen is present in large proportion in a gaseous fuel, this property frequently causes trouble through pre-ignition. The hydrogen alone would be a comparatively poor and troublesome fuel to employ for power purposes, though its presence is often beneficial in conferring sufficiently prompt inflammability upon dilute mixtures of other gases.

Methane is an excellent gaseous fuel for power purposes; its heat value per cub. ft. of mixture with air is 542/10·6 = 51 C.Th.U. (app.), and thus exceeds that of hydrogen, while there is no contraction of volume after combustion.

With normal benzene (the principal constituent of "benzol") and average petrol there is an increase of volume after combustion. With the ordinary coal-gas-air mixtures used in practice, there is, in general, a contraction of volume after combustion of not exceeding about 3 per cent of the volume before combustion.

Table IV. gives the approximate amount of heat in C.Th.U. evolved by the combustion of 1 cub. ft. at 0° C., and atmospheric pressure of *mixture with air* of various gaseous and other fuels.

Excluding acetylene, which is only employed in a few very special cases, this Table shows that the heat evolved per cub. ft. of *mixture* ranges only from about 48 to 58 thermal units notwithstanding large differences in the calorific values *per lb.* of the various fuels tabulated.

In Table V.[1] average figures are given for the gaseous fuels in common use in stationary gas engines.

The heat evolved per cub. ft. of *mixture* here ranges from 33 to 49 C.Th.U., although the heat evolved by combustion of 1 cub. ft. of *gas* ranges from 59 to 295 units.

In these Tables the volumes of air considered are only such as are just sufficient to ensure complete combustion of the fuel. Generally some excess of air is mixed with the fuel; thus the Continental practice is to use about 20 per cent excess of air with blast furnace gas and producer gas from coke or anthracite, and 40 per cent to 50 per cent excess of air with coke oven gas and town gas.

In large gas engines, in order to avoid undue heating troubles, it is found necessary to so dilute the working mixture that the evolution of heat per cub. ft. of working stroke swept by the piston does not exceed about 28 C.Th.U. for cylinders of up to 20 in. in diameter, and from 20 to 22 C.Th.U. for 30 in. cylinders. With gaseous fuels containing a large proportion of hydrogen, as, *e.g.* coke oven gas, pre-ignition trouble is often experienced. Clerk has entirely overcome this by diluting the fresh charge with from 10 per cent to 20 per cent, by volume, of cooled exhaust gases. By this means, the free oxygen is reduced and replaced by a mixture of carbon dioxide and nitrogen. The total mass of the charge is maintained, while the addition to the inert gases reduces its inflammability, and thus prevents the occurrence of pre-ignition even in sustained heavy load running.

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[1] From Clerk and Burls, *Gas Petrol and Oil Engines*, ii. (Longmans, 1913).

TABLE III

APPROXIMATE CALORIFIC VALUES OF FUELS PER LB.

Fuel. 1 Lb. of	Burnt to	Combustion per Lb. of Fuel.					C.Th.U. of Heat evolved in Combustion of 1 Lb. of Fuel.	
		Lbs. of Oxygen needed.	Lbs. of Air needed.	Weight of Products in Lbs.			Higher Value.	Lower Value.
				Burnt in Oxygen.	Burnt in Air.	Lbs. of Nitrogen.		
	1.	2.	3.	4.	5.	6.	7.	8
Hydrogen . .	H_2O	8·0	34·8	9·0	35·8	26·8	34,170	29,340
Carbon .	CO	1·333	5·8	2·333	6·8	4·466	2,445	2,445
	CO_2	2·667	11·6	3·667	12·6	8·933	8,000	8,000
Carbon monoxide	CO_2	0·571	2·484	1·571	3·484	1·913	2,430	2,430
Methane . .	CO_2 and H_2O	4·0	17·4	2·75 of CO_2 2·25 of H_2O 5·00 Total	18·4	13·4	13,340	12,130
Ethylene . .	CO_2 and H_2O	3·4285	14·914	3·143 of CO_2 1·285 of H_2O 4·428 Total	15·914	11·486	12,180	11,490
Acetylene . .	CO_2 and H_2O	3·077	13·385	3·3846 of CO_2 0·6924 of H_2O 4·077 Total	14·385	10·308	12,140	11,770
Normal benzene .	CO_2 and H_2O	3·077	13·385	3·3846 of CO_2 0·6924 of H_2O 4·077 Total	14·385	10·308	10,000	9,630

TABLE IV

HEAT PER CUBIC FOOT OF AIR-MIXTURES, AND NITROGEN CONTENT, AT 0° C. AND ATMOSPHERIC PRESSURE

1 Cubic Foot of Mixture of Air and	Composition by Volume before Combustion.			Proportion of Nitrogen in Mixture.		C.Th.U. of Heat from 1 Cubic Foot of Mixture (Lower Values).
	Fuel. Cubic Foot.	Oxygen. Cubic Foot.	Nitrogen. Cubic Foot.	Before Combustion.	After Combustion.	
	1.	2.	3.	4.	5.	6.
Hydrogen . .	·294	·148	·558	·558	·654	48·2
CO	·294	·148	·558	·558	·654	55·6
Methane . .	·0943	·1902	·7155	·7155	·7155	51·1
Ethylene . .	·065	·196	·739	·739	·739	58·4
Acetylene . .	·077	·194	·729	·729	·758	65·8
Normal benzene .	·027	·204	·770	·770	·760	56·7
Average petrol .	·020	·206	·774	·774	·735	51·6

§ (70) SPECIFIC HEAT OF GASES.—By aid of these various Tables, the maximum temperature and pressure attained by the explosion of any mixture of known composition could be readily calculated—apart from heat losses to the containing vessel—provided the specific heat of the mixture were accurately known.

For a small range of temperature, as from 0° C. to 200° C., the specific heats of the ordinary "permanent" gases are roughly constant in value, and are given in Table VI.

Consider, for example, 1 cub. ft. of CO-air mixture at 0° C. and atmospheric pressure. From column 6 of Table IV. this evolves on combustion 55·6 C.Th.U. of heat. Also, from Tables I. and IV. its mass is given by $\cdot294/12{\cdot}78+\cdot706/12{\cdot}387=\cdot08$ lb., while from Table VI. it is clear that k_v may be taken at the value 0·17 C.Th.U. per lb. for the mixture. If, then, this 1 cub. ft. of CO-air mixture be exploded at constant volume, with k_v constant in value at 0·17, and no heat loss to the containing vessel, there would be attained a maximum temperature T and pressure p (both abs.), such that $55{\cdot}6=\cdot08\times\cdot17\times(\mathrm{T}-273)$, whence T = 4363° C. (abs.) and

$$\frac{\mathrm{P}}{14{\cdot}8}=\frac{\mathrm{T}}{273}=16,$$

whence $p=236$ lbs. per sq. in. abs.

TABLE V

AVERAGE CALORIFIC VALUES OF THE USUAL GASEOUS FUELS PER CUBIC FOOT AT 0° C. AND ATMOSPHERIC PRESSURE

Item.	Nature of Gas.							
				Pressure Producer.		Suction Producer.		
	Town Gas. 1.	Coke Oven Gas. 2.	Mond Gas. 3.	From Anthracite. 4.	From Coke. 5.	From Anthracite. 6.	From Coke. 7.	Blast Furnace Gas. 8.
Cub. ft. of air required per cub. ft. of gas	5·0	4·0	1·1	1·1	..	0·93	0·92	0·75
Volume of mixture for 1 cub. ft. of gas (A)	6·0	5·0	2·1	2·1	..	1·93	1·92	1·75
C.Th.U. evolved on complete combustion of "A" (lower value)	295	235	89	89	77	77	77	59
C.Th.U. on complete combustion of 1 cub. ft. of mixture (lower value)	49	47	42·4	42·4	..	40	40	33

TABLE VI

AVERAGE SPECIFIC HEATS, ETC., OF GASES BETWEEN 0° C. AND 200° C.

Gas.	Specific Gravity Hydrogen 1. 1.	Density in Lbs. per Cubic Foot at 0° C. and Atmospheric Pressure. 2.	Average Specific Heats in C.Th.U. per Lb.			$\mathrm{R}=\mathrm{J}(k_p-k_v)$ in Ft.-lbs. per Lb. 6.	$\gamma=\frac{k_p}{k_v}$. 7.	k_v for *1 Cubic Foot* of Gas at 0° C. and Atmospheric Pressure. 8.
			k_p. 3.	k_v. 4.	k_p-k_v. 5.			
Dry air	14·44	·08073	·2375	·1689	·0686	96·0	1·406	·01363
Oxygen	16	·08944	·217	·155	·062	86·8	1·403	·01386
Nitrogen	14	·07826	·244	·173	·071	99·4	1·409	·01354
Hydrogen	1	·00559	3·409	2·406	1·003	1404	1·417	·01345
Carbon monoxide	14	·07826	·245	·173	·072	100·8	1·416	·01354
Steam	9	·05031	·49	·37	·12	168	1·32	·0186
Carbon dioxide	22	·12298	·216	·171	·045	63	1·263	·02103
Methane	8	·04472	·593	·467	·126	176·4	1·270	·02088
Ethylene	14	·07826	·404	·332	·072	100·8	1·217	·02598

§ (71) EXPERIMENTAL RESULTS.—The very numerous experiments that have been made upon explosive mixtures of many different kinds all agree, however, in showing that no such high temperature as 4363° C. (abs.) is ever even approached; and that, in general, the maximum temperature (and consequently pressure) realised in constant volume explosion experiments is, roughly, about *one-half* only of that indicated by calculation made in the manner as just illustrated. Thus Table VII. contrasts actual with calculated temperatures and pressures from constant volume experiments by Clerk with Oldham coal-gas-and-air mixtures initially at 290° C. (abs.), and 14·8 lbs. per sq. in.

With reference to the figures given in column 2, which are calculated from the actual maximum absolute pressures attained (column 1) by aid of the formula $\mathrm{T}/t_o=p/p_o$, it may be observed that these do not give the *highest* temperatures existing at the instant of maximum pressure, but are merely *averages*. There is a hot nucleus of gases at considerably higher temperature than those of the portions nearer the cool enclosing walls—as has already been pointed out in § (59); the figures given in column 2 closely approximate, however, to the mean "maximum" temperatures actually attained.

TABLE VII

EXPLOSION OF MIXTURES OF OLDHAM GAS AND AIR

Volume Ratio of Mixture : Gas : Air.	Maximum, Actual.		"Theoretical."		Ratio of Actual to "Theoretical" Temperatures and Pressures. 5.
	Pressure Attained, Lbs. per Sq. In. Abs. 1.	Temperature Calculated from Pressures ° C. Abs. 2.	Maximum Temperature ° C. Abs. 3.	Maximum Pressure, Lbs. per Sq. In. Abs. 4.	
1 : 14	55	1078	2059	105	·524
1 : 13	66	1294	2185	111	·592
1 : 12	75	1470	2331	119	·631
1 : 11	76	1490	2501	128	·596
1 : 9	93	1823	2943	150	·619
1 : 7	102	2000	3607	184	·554
1 : 6	105	2058	4081	208	·504
1 : 5	106	2078	..	..	..
1 : 4	95	1862	..	..	..

§ (72) VARIATION OF SPECIFIC HEAT.[1]—The characteristic equation of an ideally perfect gas is $pv = RT$ . . . (12), where R is the difference of the *constant* specific heats, k_p and k_v, and is therefore also a constant.

But this equation is true for any fluid whereof the specific heat k_v is a function of the temperature only, provided that the difference of the two specific heats remains constant, and such a fluid will obey Boyle's Law exactly at all temperatures; this, however, will only be true if both k_p and k_v are independent of the density of the fluid.

Joly found both for air and CO_2 that k_v appeared to increase slightly with density; thus in the case of CO_2:

At Pressure, Lbs./Sq. In. Abs.	Density (Water = 1).	$k_v =$
106·5	·01153	·1684
320·5	·03780	·1738

an increase of about 3 per cent only. In the case of hydrogen the specific heat appeared to *decrease* slightly with increase of density. The variation with density has, so far, been treated as negligible, and experimentally obtained values of specific heat at various temperatures are resumed by empirically found functions of temperature only.

§ (73) RELATION BETWEEN k_p AND TEMPERATURE.—Regnault examined the relation between k_p and temperature in the case of air and CO_2, and formed the conclusion that between −30° C. and +200° C., the specific heat of air remained sensibly constant, while that of CO_2 increased by nearly 18 per cent.

Mallard and Le Chatelier, and Berthelot, from experiments upon constant-volume explosions of mixtures initially at atmospheric pressure, deduced values of k_v for temperatures up to 2000° C. for air, steam, CO_2, and other gases, and concluded that the specific heat increased considerably with the temperature in all the cases examined by them. It is not clear, however, that combustion had ceased when their measurements were taken.

Early constant-pressure experiments by Holborn and Austin on Air, Oxygen, and Nitrogen, with electrical heating of the gases, temperatures being measured by a thermopile, and heat-quantities by calorimeter, furnished the following results:

TABLE VIII

MEAN k_p FOR AIR, OXYGEN, AND NITROGEN 20-800° C. (ORDINARY)

(Holborn and Austin)

For a Temperature Range from	Mean Value of k_p for		
	Air. 1.	Oxygen. 2.	Nitrogen. 3.
20-440° C.	·2366	·2240	·2419
20-630	·2429	·2300	·2464
20-800	·2430	..	·2497

Thus k_p increases, though but slowly, in each case. Later experiments by Holborn and Austin made at Berlin,[2] using the same method, gave the following results for Nitrogen, Steam, and CO_2, for temperatures ranging from 110° C. to 1400° C.:

TABLE IX

MEAN k_p FOR NITROGEN, STEAM, AND CO_2. 110-1400° C. (ORDINARY)

(Holborn and Austin, 1907)

For a Temperature Range from	Mean Value of k_p for		
	Nitrogen. 1.	Steam. 2.	CO_2. 3.
110- 200° C.	·240	..	·217
110- 280	..	·465	..
110- 400	·242	·467	·229
110- 600	·247	·473	·240
110- 800	·251	·482	·250
110-1000	·254	·494	·258
110-1200	·258	·510	·264
110-1400	·262	·532	·272

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[1] See "Gases, Specific Heat of."

[2] British Association Meeting, 1907.

These results indicate substantial increases in k_p with rise of temperature for all the three gases. Holborn and Austin have also calculated the value of k_p for CO_2 at temperatures from 0° C. to 800° C. from their own results and also from those of Langen's and Mallard and Le Chatelier's experiments. The figures are given hereunder:

TABLE X

VALUES OF k_p FOR CO_2 FROM 0° C. to 800° C. (ORDINARY)

(Calculated by Holborn and Austin)

At Temperature ° C.	Value of k_p from		
	Holborn and Austin. 1.	Langen. 2.	Mallard and Le Chatelier. 3.
0	·2028	·1980	·1880
100	·2161	·2100	·2140
200	·2285	·2220	·2390
400	·2502	·2450	·2840
600	·2678	·2690	·3230
800	·2815	·2920	·3550

Holborn and Austin and Langen's results are concordant, while those of Mallard and Le Chatelier show a much faster rate of increase of k_p with temperature than either of the others.

§ (74) CLERK'S EXPERIMENTS.[1] — In Clerk's experiments both pressure and volume were varied simultaneously, the gas experimented upon being the mixture after explosion in a gas-engine cylinder behind a moving piston.

TABLE XI

APPARENT SPECIFIC HEAT k_v FROM 0° C. to 1500° C. (ORDINARY) PER CUBIC FOOT OF WORKING MIXTURE AT 0° C. AND ATMOSPHERIC PRESSURE

(Clerk)

Temperatures, ° C.		Values of k_v per Cubic Foot.		
		By Experiment.		By Calculation from Equation (73′), in Ft.-lbs. 5.
Ordinary. 1.	Absolute. 2.	In C.Th.U.* 3.	In Ft.-lbs. 4.	
−273	0	..	..	14·0
0	273	·0140	19·6	19·6
100	373	·1409	20·9	21·0
200	473	·0157	22·0	22·2
400	673	·0171	23·9	24·0
600	873	·0180	25·2	25·2
800	1073	·0187	26·2	26·1
1000	1273	·1019	26·8	26·7
1200	1473	·0194	27·2	27·1
1400	1673	·0195	27·35	27·38
1500	1773	·0196	27·45	27·48
∞	∞	(·0200)	..	28·0

* Compare with column 8 of Table VI.

The composition by volume of the mixture after explosion was as follows: Nitrogen, 75·0 per cent; steam, 11·9 per cent; carbon dioxide, 5·2 per cent; oxygen, 7·9 per cent.

Hence from Tables I. and VI. we have for 1 cub. ft. of this mixture at 0° C. and atmospheric pressure:

Constituent.	Cub. Ft. in 1 Cub. Ft. of the Mixture. 1.	Weight, in Lbs. in 1 Cub. Ft. at 0° C. and Atmospheric Pressure. 2.	C.Th.U. per 1° C. Rise of Temperature from 0-200° C. agreeably with Table VI. (at Constant Volume). 3.
Nitrogen	·750	·0587	·010155
Steam .	·119	·0060	·002213
CO_2 .	·052	·0064	·001094
Oxygen .	·079	·0070	·001095
Total .	1·000	·0781	·014557

Thus 1 cub. ft. of the mixture, measured at 0° C. and atmospheric pressure, weighs ·0781 lb. and agreeably with Table VI. should have a mean k_v between 0° C. and 200° C. of ·014557 C.Th.U. or, in work units, 20·38 ft.-lbs.

On referring to Table XI. it will be seen that the value of k_v at 100° C. from actual experiment was 20·90 ft.-lbs.; thus the experimental and calculated results here agree very well.

After explosion the ordinary working stroke was performed, but instead of release taking place near the end of the stroke, both valves were kept closed while the crankshaft continued to revolve through the momentum of the engine fly-wheel, thus alternately compressing and expanding the entrapped mixture. An indicator diagram was taken, recording the changes of volume and pressure undergone by the gases.

The gain of internal energy by the mixture during compression from any point A to B (*Fig.* 33) is equal to the external work (MABN) done upon the mixture in compressing it *minus* the heat lost to the cylinder walls in the interval; while during any expansion period, as BC, the loss of internal energy is equal to the external work (MBCN) done by the mixture in expanding from B to C, *plus* the loss of heat to the cylinder walls.

The external work is readily obtained with considerable accuracy by planimetric measurements of the diagram, while the changes of temperature can be calculated from simultaneous values of p and v taken from the diagram, by aid of the relation $pv/p_o v_o = \mathrm{T}/\mathrm{T}_o$, provided the temperature T_o, at any one point of the diagram be known. The loss of heat to the cylinder walls was estimated by a comparison of the compression curve with the immediately following expansion curve on the assumption that the total heat lost by the mixture during any, the same, part of the stroke is the same during expansion as during compression for the same mean temperature.

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[1] *Proc. Roy. Soc.* A., 1906, lxxvii. 499.

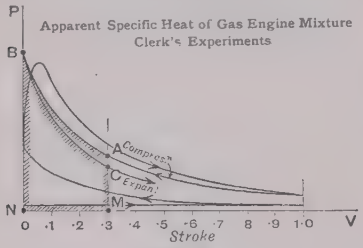

FIG. 33.

Clerk expresses the Apparent Specific Heat, k_v, thus deduced, in ft.-lb. per cub. ft. of mixture at 0° C. and atmospheric pressure; the corresponding values in C.Th.U. per cub. ft. are added in Table XI. His results are very closely resumed by the equation

$$k_v = \left(\frac{d\mathrm{H}}{d\mathrm{T}}\right)_v = a(1 - b\epsilon^{-l\mathrm{T}}), \quad (73)$$

where a, b, and l are constants, and T is absolute temperature in ° C. It will be found that A = 28, $b = \cdot 5$, and $l = \cdot 00186$. For purposes of computation equation (73) is more conveniently written

$$\log_{10}\left(1 - \frac{k_v}{28}\right) = -0\cdot 30103 - \cdot 000808\mathrm{T}, \quad (73')$$

and values calculated from this equation are given in column 5 of Table XI. for comparison with those experimentally determined. The empirical equation (73) makes k_v increase from 14·0 at the absolute zero of temperature, to an asymptotic value of 28·0 when T is infinite.

§ (75).—Integrating equation (73) gives for the increase in internal energy in this case, from temperature T_o to temperature T,

$$\mathrm{H} - \mathrm{H}_o = a(\mathrm{T} - \mathrm{T}_o) - \frac{ab}{l}(\epsilon^{-l\mathrm{T}} - \epsilon^{-l\mathrm{T}_o}). \quad (74)$$

Or, k_v, k_{vo} being values of the apparent specific heats at T, T_o respectively,

$$\mathrm{H} - \mathrm{H}_o = a(\mathrm{T} - \mathrm{T}_o) - \frac{1}{l}(k_v - k_{vo}). \quad (74')$$

By aid of this equation, and of equation (73′), Table XIA has been calculated, giving values of k_v for this gas-engine mixture at every 100 degrees from 0° C. to 2500° C., together with values of the internal energy ($\mathrm{H} - \mathrm{H}_o$), in ft.-lbs. per cub. ft. at 0° C. and atmospheric pressure, relatively to 0° C. as an assumed zero.

TABLE XIA

VALUES OF k_v AND ($\mathrm{H} - \mathrm{H}_o$) FOR GAS ENGINE MIXTURE

(From Clerk's experiments)

Temperature ° C.		k_v in Ft.-lbs. from Equation (73′).	γ from Equation (75A).	$\mathrm{H} - \mathrm{H}_o$ in Ft.-lbs. from Equation (74′)
Ordinary.	Absolute.			
0	273	19·6	1·396	0
100	373	21·0	1·369	2,047
200	473	22·2	1·349	4,202
300	573	23·2	1·334	6,465
400	673	24·0	1·323	8,835
500	773	24·7	1·314	11,458
600	873	25·2	1·307	13,789
700	973	25·7	1·302	16,321
800	1073	26·1	1·297	18,906
900	1173	26·4	1·294	21,544
1000	1273	26·7	1·290	24,183
1100	1373	26·9	1·288	26,875
1200	1473	27·1	1·286	29,568
1300	1573	27·24	1·2845	32,293
1400	1673	27·38	1·2830	35,017
1500	1773	27·48	1·2820	37,764
1600	1873	27·56	1·2812	40,521
1700	1973	27·64	1·2804	43,278
1800	2073	27·69	1·2799	46,051
1900	2173	27·75	1·2793	48,819
2000	2273	27·79	1·2789	51,597
2100	2373	27·83	1·2785	54,376
2200	2473	27·86	1·2782	57,159
2300	2573	27·88	1·2780	59,949
2400	2673	27·90	1·2778	62,738
2500	2773	27·92	1·2776	65,527
3000	3273	27·97	1·2771	79,500
∞	∞	28·0	1·2769	∞

On p. 20 of the *7th Report of the Gas Explosions Committee of the British Association* (1914), a curve is given showing the variation of internal energy with temperature for this mixture. This curve is considered to best resume the most reliable experimental results available; it is reproduced in *Fig.* 33A. For comparison there is shown also the curve obtained by plotting ($\mathrm{H} - \mathrm{H}_o$) against T from the figures given in Table XIA. It will be seen that the two curves agree very well from 0° C. to 1600° C., but that at temperatures above 1600° C. the Committee's curve rises somewhat abruptly above the other. The curve of equation (74′) results from Clerk's figures as given in Table XI., and is a regular curve with equation (74) as its Cartesian equation. It has an asymptote $H=28(T-T_o)-4516$, to which it rapidly approaches, as indicated in the diagram.

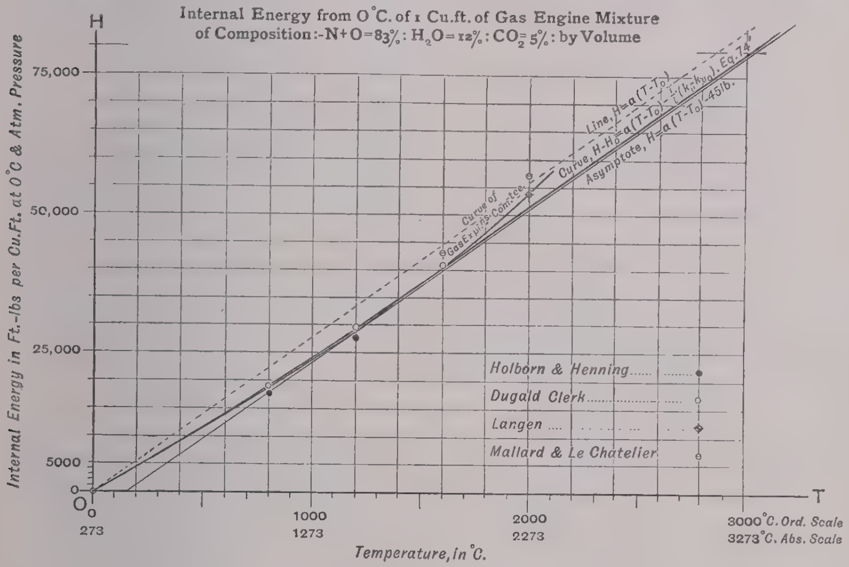

FIG. 33A.

The mean value of k_v between T_o and T is $H-H_o/T-T_o$, and is therefore expressed by

$$\text{Mean } k_v = a - \frac{1}{l}\,\frac{k_v - k_{vo}}{T-T_o}. \qquad (75)$$

Estimation of $H-H_o$ *from the Indicator Diagram.*—From equations (13) and (74′), all quantities being in work-units, we have

$$H-H_o = a(T-T_o) - \frac{1}{l}(k_v - k_{vo}) + \int_{v_o}^{v} p\,dv\,;$$

as $pv=RT$, this may be written

$$H-H_o = \frac{a}{R}(pv - p_o v_o) - \frac{1}{l}(k_v - k_{vo}) + \int_{v_o}^{v} p\,dv. \qquad (74'')$$

As p, v, p_o, v_o, and $\int_{v_o}^{v} p\,dv$ are immediately determinable from the indicator diagram and k_v and k_{vo} are known from, *e.g.*, Table XI. for the values of T corresponding to pv and $p_o v_o$, it is clear that equation (74″) enables $H-H_o$ to be readily ascertained.

If in any specific case the expansion (or compression) curve can be resumed by an equation of the form pv^n=a constant, then in such case we have by aid of equation (33)

$$H-H_o = \left(\frac{a}{R} + \frac{1}{1-n}\right)(pv - p_o v_o) - \frac{1}{l}(k_v - k_{vo}). \qquad (74''')$$

§ (76) VALUE OF γ FOR THE MIXTURE.—Here the relation $pv=c'T$ gives

$$c' = \frac{pv}{T} = \frac{14 \cdot 7 \times 144 \times 12 \cdot 8}{273} = 99 \cdot 25 \text{ ft.-lbs. per lb.}$$

As 1 cub. ft. at 0° C. and atmospheric pressure weighs ·0781 lb., c' per cub. ft. $=99 \cdot 25 \times \cdot 0781 = 7 \cdot 75$ ft.-lbs. Also, heat quantities being expressed in ft.-lbs., $k_p - k_v = c' = 7 \cdot 75$. But $k_p - k_v = k_v(\gamma - 1)$; hence

$$\gamma = 1 + \frac{7 \cdot 75}{k_v}. \qquad (75\text{A})$$

Thus, from Table XI. we have

TABLE XII

VALUES OF γ FROM VALUES OF k_v GIVEN IN TABLE XI

Temperature ° C. Abs.	k_v Ft.-lbs./Cub. Ft.	γ from Equation (75A).
0	14·0	1·553
273	19·6	1·396
373	21·0	1·369
473	22·2	1·349
673	24·0	1·323
873	25·2	1·307
1073	26·1	1·297
1273	26·7	1·290
1473	27·1	1·286
1673	27·38	1·283
1773	27·48	1·282
∞	28·0	1·277

These values of γ are plotted against the corresponding absolute temperatures in *Fig.* 34. For any temperature interval the mean value of γ can be readily obtained either from this curve, or more accurately by aid of equations (75) and (75A). An approximation to the adiabatic curve for the mixture can then be obtained by assuming γ constant at its mean value over a number of successive small ranges of temperature, and drawing a series of curves within each range agreeably with the equation $pv^{\gamma}=\text{constant}$.

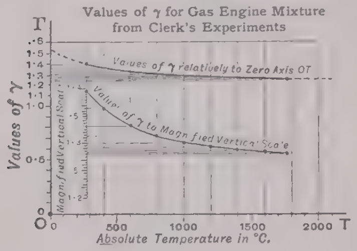

FIG. 34.

From equations (9), (12), (13), and (73) we obtain as the equation of entropy

$$\phi-\phi_0=a\log\epsilon\left(\frac{\mathrm{T}}{\mathrm{T}_0}\right)+c'\log\epsilon\left(\frac{v}{v_0}\right)-ab.\int_{\mathrm{T}_0}^{\mathrm{T}}\frac{\epsilon^{-\iota\mathrm{T}}}{\mathrm{T}}d\mathrm{T}, \quad (75\text{B})$$

whence, putting $\phi-\phi_0=0$, we have for the adiabatic equation

$$c'\log\epsilon\left(\frac{v}{v_0}\right)=ab.\int_{\mathrm{T}_0}^{\mathrm{T}}\frac{\epsilon^{-\iota\mathrm{T}}}{\mathrm{T}}d\mathrm{T}-a\log\epsilon\left(\frac{\mathrm{T}}{\mathrm{T}_0}\right). \quad (75\text{C})$$

The impossibility of expressing the integral $\int\frac{\epsilon^{-\iota\mathrm{T}}}{\mathrm{T}}d\mathrm{T}$ in finite terms renders these two equations unsuitable for arithmetical computation. When $b=0$, equation (75B) reduces, of course, to equation (40).

§ (77) CLERK'S RESULTS.—Referring to these results of Clerk, Callendar[1] remarks: "Since the temperature of a mass of gas when exploded in a closed vessel or in the cylinder of a gas engine is far from uniform, and since the actual distribution of temperature is necessarily somewhat uncertain, it is evident that the variation of the specific heats of the constituents with temperature cannot be *certainly* deduced from a knowledge of the heats of combustion and the effective temperature. . . .

"It is possible, however, by explosion experiments to deduce values of the *apparent* or '*effective*' specific heats which, in so far as they approximate to the conditions existing in an actual gas engine, may be of greater practical utility than the true specific heats. . . . The method of Dugald Clerk in which the specific heat is directly determined from the work done on the charge after ignition, appears to be particularly appropriate for this purpose."

It will be observed that Clerk's results are somewhat higher than those of others; Callendar has stated (*ibid.*) that the values of the specific heats deduced from explosion experiments usually come out higher than those obtained by more direct methods, and considers that the errors incidental to both methods require further investigation.

§ (77′).—According to the kinetic theory of gases, their internal energy is the sum of the translational, rotational, and internal-vibrational energies of their constituent molecules. Of these, the translational motion alone causes the gaseous pressure; the internal-vibrational produces radiation; while the rotational appears to have no external physical effect. Callendar[2] points out also that theory indicates that the energies of molecular translation and rotation should vary directly as the product pv, while the vibrational energy should vary with the temperature agreeably with a formula of the exponential type $\alpha(\epsilon^{\beta/\lambda\mathrm{T}}-1)^{-1}$. This is symbolically expressed by writing

$$\mathrm{H}=\text{internal energy}=\zeta pv+\alpha(\epsilon^{\beta/\lambda\mathrm{T}}-1)^{-1},$$

where ζ, α, β, and λ are constants.

The product pv, though very nearly constant, appears to be expressible as a function of p and T; thus Kelvin (*Math. and Phys. Papers*, vol. i.) found that for the usual permanent gases, including CO_2, an equation of the type $pv=c\mathrm{T}-b(p/\mathrm{T})^2$ will resume the observed facts. Taking this, we have $\mathrm{H}=\zeta c\mathrm{T}-\zeta b(p/\mathrm{T})^2+\alpha(\epsilon^{\beta/\lambda\mathrm{T}}-1)^{-1}$; whence

$$\left(\frac{d\mathrm{H}}{d\mathrm{T}}\right)_p=k_p=\zeta c-2\zeta b\frac{p^2}{\mathrm{T}^3}+\frac{\alpha\beta}{\lambda}\,\frac{\epsilon^{\beta/\lambda\mathrm{T}}}{\mathrm{T}^2(\epsilon^{\beta/\lambda\mathrm{T}}-1)^2}. \quad (\text{A})$$

From these considerations it seems just possible that the extreme closeness with which Clerk's experimental results are resumed by equation (73) may be somewhat more than a coincidence.

§ (78) VARIABLE SPECIFIC HEATS.—In an ideal perfect gas both k_p and k_v are constant, and the quantity R in the characteristic equation $pv=\mathrm{RT}$ . . . (12) being equal to k_p-k_v is also constant. But, as has already been pointed out, equation (12) is also true for fluids for which k_p and k_v are any functions of T, provided only that (k_p-k_v) is constant; and such fluids will obviously obey Boyle's Law, since pv will still be constant when T is constant. Suppose, then, that $k_v=\mathrm{F(T)}$ and $k_p=\mathrm{F(T)}+\mathrm{R}$; if heat $d\mathrm{H}$ be supplied to unit mass of such a fluid we shall have by equation (13) all heat-quantities being expressed in work-units:

$$d\mathrm{H}=\mathrm{F(T)}d\mathrm{T}+pdv. \quad . \quad . \quad (13')$$

If the changes be isothermal, $d\mathrm{T}=0$, and hence whatever F(T) may be, we must have $d\mathrm{H}=pdv=\text{external work}$—as in the case of a perfect gas already considered. As by equation (12) $p=\mathrm{R}/\mathrm{T}/v$, equation (13′) gives at once as the equation of entropy

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[1] *Gaseous Explosions Committee of British Association, 1st Report*, 1908, p. 27.

[2] *Ibid.* p. 32.

$$d\phi=\frac{d\mathrm{H}}{\mathrm{T}}=\frac{\mathrm{F(T)}}{\mathrm{T}}d\mathrm{T}+\mathrm{R}\frac{dv}{v}. \quad . \quad (76)$$

Putting $d\phi=0$ gives as the differential equation to the adiabatic,

$$\frac{\mathrm{F(T)}}{\mathrm{T}}d\mathrm{T}+\mathrm{R}\frac{dv}{v}=0. \quad . \quad . \quad (77)$$

§ (79) VALUES OF K_v.—It is now fully established that k_v for gases is not constant but that it increases with the temperature (and probably changes slowly also with density), and the results of experiment are very commonly resumed by simple equations of the form

$$k_v=\alpha+\beta\mathrm{T}. \quad . \quad . \quad . \quad (78)$$

Thus Clerk has given the following as fairly resuming Mallard and Le Chatelier's experimental results:

Gas.	$k_v=\alpha+\beta\mathrm{T}=$
Nitrogen	$\cdot171+\cdot0000215\mathrm{T}$
Steam	$\cdot3116+\cdot000182\mathrm{T}$
CO_2	$\cdot1423+\cdot0000834\mathrm{T}$
Oxygen	$\cdot150+\cdot0000188\mathrm{T}$

in C.Th.U. per lb., and T in ° C. (abs.).

It may be pointed out here that in all cases where the efficiency of a cycle is expressible by equation (71), any increase in the value of k_v *diminishes* the efficiency, and *vice versa*. For denoting efficiency by η, and putting R/k_v for $(\gamma-1)$, equation (71) becomes

$$\eta=1-\left(\frac{1}{\rho}\right)^{\mathrm{R}/k_v}; \quad . \quad . \quad . \quad (71\text{A})$$

and differentiating all across with respect to k_v gives

$$\frac{d\eta}{dk_v}=\frac{\mathrm{R}}{k_v{}^2}\left(\frac{1}{\rho}\right)^{\mathrm{R}/k_v}\log_\epsilon\left(\frac{1}{\rho}\right)=-\frac{\mathrm{R}}{k_v{}^2}\left(\frac{1}{\rho}\right)^{\mathrm{R}/k_v}\log_\epsilon\rho. \quad (71\text{B})$$

As R, k_v, and ρ are essentially positive, and ρ is always >1, $d\eta/dk_v$ is always negative, and thus $d\eta$ and dk_v have opposite signs.

§ (80) ESTIMATION OF $\mathrm{H}-\mathrm{H}_o$ FROM INDICATOR DIAGRAMS.—When k_v is expressed in the simple form as given in equation (78), we have for the increase in internal energy at constant volume corresponding to a change of temperature from T_o to T,

$$\mathrm{H}-\mathrm{H}_o=\alpha(\mathrm{T}-\mathrm{T}_o)+\frac{\beta}{2}(\mathrm{T}^2-\mathrm{T}_o{}^2). \quad (79)$$

From equations (13) and (79), when v changes, all quantities being expressed in work-units,

$$\mathrm{H}-\mathrm{H}_o=\alpha(\mathrm{T}-\mathrm{T}_o)+\frac{\beta}{2}(\mathrm{T}^2-\mathrm{T}_o{}^2)+\int_{v_o}^{v}p\,dv.$$

As $pv=\mathrm{RT}$ . . . (12) this may be written

$$\mathrm{H}-\mathrm{H}_o=(pv-p_ov_o)\left\{\frac{\alpha}{\mathrm{R}}+\frac{\beta}{\mathrm{R}^2}(pv+p_ov_o)\right\}+\int_{v_o}p\,dv, \quad (79')$$

and all the quantities on the right-hand side of this equation being immediately determinable from the indicator diagram, the value of $\mathrm{H}-\mathrm{H}_o$ is known when using any working mixture for which α and β and R have been previously ascertained.

Whenever the expansion or compression curve can be expressed in the form $pv^n=$constant, equation (79′) by aid of equation (33) takes the simple form

$$\mathrm{H}-\mathrm{H}_o=(pv-p_ov_o)\left\{\frac{\alpha}{\mathrm{R}}+\frac{1}{1-n}+\frac{\beta}{2\mathrm{R}^2}(pv+p_ov_o)\right\}. \quad (79'')$$

Also equation (76) becomes

$$d\phi=\left(\frac{\alpha}{\mathrm{T}}+\beta\right)d\mathrm{T}+\mathrm{R}\frac{dv}{v}, \quad . \quad . \quad (80)$$

the differential equation of entropy, giving on integration

$$\phi-\phi_o=\alpha\log_\epsilon\left(\frac{\mathrm{T}}{\mathrm{T}_o}\right)+\beta(\mathrm{T}-\mathrm{T}_o)+\mathrm{R}\log_\epsilon\left(\frac{v}{v_o}\right), \quad (81)$$

whence, putting $\phi-\phi_o=0$, we have as the adiabatic equation in terms of v and T,

$$\alpha\log_\epsilon\left(\frac{\mathrm{T}}{\mathrm{T}_o}\right)+\mathrm{R}\log_\epsilon\left(\frac{v}{v_o}\right)+\beta(\mathrm{T}-\mathrm{T}_o)=0, \quad (82)$$

which is also sometimes written in the form

$$\left(\frac{\mathrm{T}}{\mathrm{T}_o}\right)^\alpha\left(\frac{v}{v_o}\right)^{\mathrm{R}}\epsilon^{\beta(\mathrm{T}-\mathrm{T}_o)}=1. \quad . \quad (82')$$

From the relation $pv=\mathrm{RT}$, equations (82) and (82′) may also appear as

$$\alpha\log_\epsilon\left(\frac{p}{p_o}\right)+(\alpha+\mathrm{R})\log_\epsilon\left(\frac{v}{v_o}\right)+\frac{\beta}{\mathrm{R}}(pv-p_ov_o)=0, \quad (83)$$

or

$$\left(\frac{p}{p_o}\right)^\alpha\left(\frac{v}{v_o}\right)^{\alpha+\mathrm{R}}\epsilon^{\beta/\mathrm{R}(pv-p_ov_o)}=1. \quad (83')$$

And

$$(\alpha+\mathrm{R})\log\epsilon\left(\frac{\mathrm{T}}{\mathrm{T}_o}\right)+\mathrm{R}\log\epsilon\left(\frac{p_o}{p}\right)+\beta(\mathrm{T}-\mathrm{T}_o)=0, \quad (84)$$

or

$$\left(\frac{p_o}{p}\right)^{\mathrm{R}}\left(\frac{\mathrm{T}}{\mathrm{T}_o}\right)^{\alpha+\mathrm{R}}\epsilon^{\beta(\mathrm{T}-\mathrm{T}_o)}=1. \quad . \quad (84')$$

On putting $\beta=0$, equations (82), (83), and (84) reduce to equations (22), (20), and (21) respectively.

The adiabatic curve may be drawn from a given initial condition $(p_ov_o\mathrm{T}_o)$ by assuming any T and thence determining the corresponding v from equation (82) or p from equation (84); the remaining quantity is then immediately found by aid of the equation $pv=\mathrm{RT}$.

§ (81) OTTO CYCLE WITH VARIABLE SPECIFIC HEAT.—It is of interest to compare the efficiency of the Otto cycle using a fluid of variable specific heat with that obtained on the usual simplifying assumption that k_v is constant, as in §§ (36) and (45); the data assumed are as in § (51), viz. $t_o=300°$ C. (abs.), $t_c=600°$ C. (abs.), $\mathrm{T}=2000°$ C. (abs.), and $p_0=14\cdot7$ lbs. per sq. in. (abs.); see *Fig.* 21.

Instead of air, the working fluid is now 1 cub. ft. at 0° C., and atmospheric pressure, of Clerk's gas-engine mixture as described in § (74), obeying the law

$$144pv=7\cdot754\mathrm{T}. \quad . \quad . \quad (12'')$$

The variation of k_v with temperature is exhibited in *Fig.* 35, plotted from Table XI., and from this curve the following linear formulae are easily deduced for the value of k_v over the four temperature ranges taken:

Temperature Range in ° C. (Abs.).	k_v in Ft.-lbs./Cub. Ft. $a+\beta$T.
300-600	$16{\cdot}8+{\cdot}011$T
600-1000	$19{\cdot}65+{\cdot}00625$T
1000-1500	$23{\cdot}2+{\cdot}0027$T
1500-2000	$26{\cdot}5+{\cdot}0005$T

Then to obtain points on the adiabatic compression curve we have the relations $v=(300/273)\times 1=1{\cdot}0989$ cub. ft., whence equation (82) becomes, using common logs,

$$16{\cdot}8 \log\left(\frac{\mathrm{T}}{300}\right)+7{\cdot}754 \log\left(\frac{v}{1{\cdot}0989}\right)$$
$$+4343\times{\cdot}011(\mathrm{T}-300)=0.$$

Assuming a series of values of T between 300° C. and 600° C., this equation gives immediately the corresponding values of v; and p is then found from v by aid of equation (12″). Proceeding in this way we get

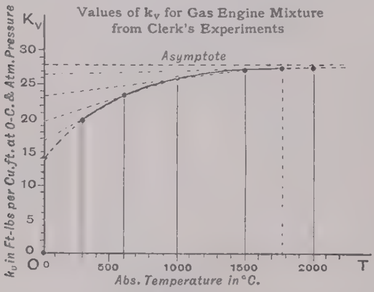

FIG. 35.

T=	300	350	400	500	600
v=	1·0989	0·7330	0·5113	0·2736	0·1599
p=	14·7	25·7	42·2	98·5	202

from which the compression curve can be plotted. The explosion pressure P is obtained from $\mathrm{P}/p_c=\mathrm{T}/t_c$; thus substituting the values $\mathrm{P}=202\times(2000/600)=673{\cdot}3$ lbs. per sq. in.

The adiabatic expansion curve may next be obtained by the same method as that adopted for the compression curve, taking $a=26{\cdot}5$ and $\beta={\cdot}0005$ from 2000° C. to 1500° C., and $a=23{\cdot}2$, $\beta={\cdot}0027$ from 1500° C. to 1000° C.; this furnishes the following figures:

T=	2000	1900	1650	1500	1250	1100
v=	·1599	·1918	·3156	·4413	·8306	1·28
p=	673	532	282	183	81	46

The *pv*-diagram plotted from these results is shown in *Fig.* 36; it will be noted, by comparison with the results obtained in § (51) for the case in which k_v is constant, that working between the same limits of temperature, and with the same temperature of compression, the ratio of expansion with variable specific heat is 6·87, as compared with 5·47, when k_v is constant; while the maximum pressure with variable k_v, due to the greater compression ratio, is also greater than that in the simpler case. In an actual engine the total expansion ratio is fixed: taking this at 5·47, as in the case of § (51), we shall have for the volume during explosion $v_c=1{\cdot}0989/5{\cdot}47=0{\cdot}2009$ cub. ft.; the corresponding compression temperature, from equation (82), will be $t_c=556°$ C. (abs.), and pressure, from equation (12″), $p_c=149$ lbs. per sq. in. abs. (approx.), whence the explosion pressure is now reduced to $\mathrm{P}=(\mathrm{T}/t_c)p_c=536$ lbs. per sq. in. (abs.). The diagram for an expansion ratio 5·47 is shown by dotted lines in *Fig.* 36.

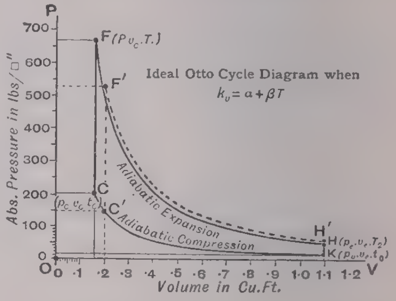

FIG. 36.

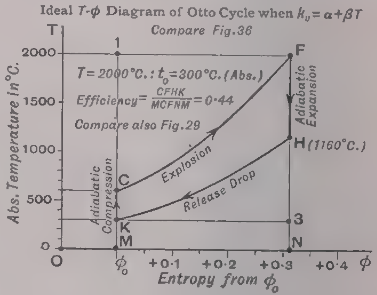

FIG. 37.

§ (82) OTTO CYCLE TEMPERATURE ENTROPY.—The temperature-entropy diagram for the 300°-600°-2000° C. case of *Fig.* 36 is exhibited in *Fig.* 37, the curved portions being obtained by aid of equation (81), using corresponding values of T and v already obtained. The efficiency, being the ratio of KCFHK to MCFNM, may be found either by planimetric measurement of these areas, or by direct calculation of the quantities of heat received and rejected at constant volume from equation (79), the value of T_2 (at H) being taken from the $T - \phi$ diagram; both methods agree in giving 0·44 as the value of the efficiency.

With constant specific heat it has already been shown that for the Otto cycle the efficiency is $1 - (t_o/t_c) = 0·5$ (*vide* equation (70) and § (45)); thus when k_v is expressible in the form $\alpha + \beta T$ and $(k_p - k_v)$ is constant, efficiency is *diminished*.

§ (83) Otto Cycle Ideal Efficiency.—Clerk[1] has calculated by a method of successive approximations values of the ideal efficiency of the Otto cycle when using the mixture of gases whose Apparent Specific Heat is as given in Table XI., and his results are given hereunder; corresponding values of efficiency with constant k_v are given, calculated from equation (71); it will be observed that the efficiency with variable k_v is always *less* than that with k_v constant:

Table XIII

Ideal Efficiencies of Otto Cycle compared. Initial Temperature 273° C. (Abs.); γ for Compression taken as 1·37 (*vide* Table XII.).

$\frac{1}{\rho} =$	Upper Limit of Temperature.		Efficiency with k_v Constant (Equation (71)). 3.
	1873° C. (Abs.). 1.	1273° C. (Abs.). 2.	
$\frac{1}{2}$	·195	·200	·246
$\frac{1}{3}$	·286	·293	·360
$\frac{1}{4}$	·354	·356	·430
$\frac{1}{5}$	·384	·394	·480
$\frac{1}{7}$	·439	·443	·550

§ (84) Efficiency; "Air Standard."—It has already been pointed out (§ (37)) that if $1/\rho$ denote the value of the adiabatic compression ratio in the three ideal cycles of constant pressure, constant volume, and constant temperature, then the efficiency is, in each case, expressed by the same formula, viz.

$$\text{Efficiency} = 1 - \left(\frac{1}{\rho}\right)^{\gamma - 1} . \quad . \quad (71)$$

If the working fluid be air assumed as possessing a constant value of $\gamma = 1·4$, this equation becomes

$$\text{Efficiency} = 1 - \left(\frac{1}{\rho}\right)^{0·4}, \quad . \quad (71')$$

and in this form is termed the "Air Standard" of efficiency, and has been largely employed in the past as representing the theoretical maximum of performance for any given value of ρ. It is clear, however, from the above discussion that when the specific heat is variable, and not constant, that equation (71′) represents an unattainable ideal. Clerk has pointed out that in the cases taken in Table XIII. the "air standard" values are roughly about 20 per cent too high. Much more information is, however, still required before even the apparent specific-heat values are accurately known for the various mixtures of gases and vapours employed in gas and oil engines. When these values become accurately assignable each engine can be separately investigated and the *real* efficiency of its working fluid determined. The proportion borne by the actual thermal efficiency as determined by test to the real efficiency will then give a true measure of the degree of excellence of its performance, and indicate the exact margin remaining for improvement.

On the general question of the values of k_p and k_v for gases and gaseous mixtures reference may be made to the 1st and 2nd Reports (1908–9) of the Gaseous Explosions Committee of the British Association. Clerk (*Proc. I.C.E.*, 1907), however, makes the useful suggestion that if γ be taken at the constant value 1·285 for the expansion curve, and at 1·370 for the compression curve, then for the ranges of temperature met with in gas-engine practice no serious error is introduced.

D. C.
G. A. B.

ENGINES, SOME TYPICAL INTERNAL COMBUSTION

§ (1) The "Internal" Combustion engine is so called because the gaseous or vaporised fuel, mixed with air, is burned or exploded within the working cylinder itself in contrast with earlier types of heat engine in which the working fluid, steam or air, is generated or heated by an external furnace. For the history of the development of the Internal Combustion engine reference may be made to Sir D. Clerk's work on *The Gas, Petrol, and Oil Engine* (Longmans, 1909).

The engineering thermodynamics of Internal Combustion engines are dealt with in the article "Engines, Thermodynamics of Internal Combustion"; in this article some account is given of typical actual engines.

§ (2) The Cycle.—All modern internal combustion engines work on either the four-stroke or two-stroke cycle, with combustion either at constant volume or at constant pressure, or at a combination of both these methods. Of these the constant volume (Otto) four-stroke cycle engines form by far the largest class. A very great deal of attention is, however, now being given to the improvement of the two-stroke cycle engine, which has long been established as a reliable and economical prime mover in large stationary engine practice, and is now rapidly increasing in favour, in single- and two-cylindered designs, in the small quick-speed petrol types used in the propulsion of motor-launches and cycles on account of its great simplicity and relatively low production cost. Its extended adoption in motor vehicles in general is regarded as not unlikely in the near future.

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[1] *Proc. Inst. C.E.*, 1907.

The constant pressure class, typified by the Diesel engine, in both four-stroke and two-stroke designs, includes also the somewhat numerous so-called "semi-Diesel" engines, of which one is described and illustrated later. In these "semi-Diesel" oil engines the fuel admission is, in general, so regulated that combustion takes place partly at constant volume, and partly at (approximately) constant pressure.

§ (3) THE FUEL.—The fuels now successfully employed are also numerous and various. Among gaseous fuels are:—Coal gas; pressure and suction producer gases (Dowson, Mond, National, Crossley, etc.); coke oven gas; blast furnace gas; natural gas (mainly in the U.S.; principally CH_4); and water gas. Of liquid fuels may be mentioned petrol; kerosene (paraffin, or lamp oil); intermediate oils; gas oils; crude oils (filtered and freed from volatile constituents); residual oils; benzol; shale oils; coal tar oils; lignite oils; and alcohol; and mixtures of these, as, *e.g.*, of petrol, benzol, alcohol, and kerosene. Solid fuels are not yet practicable, though, in the laboratory, Diesel engines have been made to run on coal dust, and engines of the Otto-cycle motor-car type on naphthalene.

§ (4) The following have been selected for description and illustration as typical engines exemplifying present-day practice:

A. CONSTANT VOLUME TYPE

(*a*) As a four-stroke Stationary Horizontal Single-cylindered engine of medium power—the 100 Horse-power "National" gas engine.

(*b*) As a four-stroke Stationary Vertical Multi-cylindered engine of medium power—the Two-crank, 300 Horse-power, Tandem Four-cylindered engine of the National Gas Engine Co.

(*c*) As a four-stroke Stationary Horizontal Two-cylindered engine of large power—the Double-acting Tandem Single-crank 2500 Horse-power Nuremberg gas engine.

(*d*) As a two-stroke Stationary Horizontal Single-cylindered engine of small power—the early Clerk gas engine.

(*e*) As a two-stroke Stationary Horizontal Single-cylindered Double-acting engine of large power—the Körting gas engine.

(*f*) As a special two-stroke Stationary Horizontal Single-cylindered Single-acting engine of large power—the Oechelhauser gas engine.

(*g*) As a typical "Paraffin" engine—the Hornsby-Akroyd oil engine.

B. CONSTANT PRESSURE TYPE

(*h*) A description is given, with illustration, of a normal four-stroke Diesel engine, with some account of the two-stroke Diesel, and an illustrated description of a Ruston "semi-Diesel" engine. The large class of small, usually multi-cylindered, quick-revolution internal combustion engines as used for the propulsion of motor vehicles, launches, aircraft, etc., form the subject of a separate section.

§ (5) (*a*) THE 100 H.P. HORIZONTAL "NATIONAL" GAS ENGINE. (i.) *Description.*—A vertical longitudinal section of this single-cylindered engine is shown in *Fig.* 1. Within the "working barrel" or "cylinder liner" AA slides a cast-iron piston BB attached to the crank-shaft by the connecting rod CC. The piston fits the cylinder very accurately, but complete gas-tightness is ensured by six cast-iron spring rings DD. The piston is oiled by means of the lubricator E, and through the hole shown in the piston "skirt" oil is also enabled to reach the "small end" or "gudgeon bearing" of the connecting rod. At the left-hand end of the cylinder is the combustion chamber H, within which are placed the inlet valve I and the exhaust valve K; holes above these valves, filled by easily removable plugs LL, permit them to be readily withdrawn for examination and repair or replacement.

The working barrel of the cylinder, the combustion chamber, and the valve casings are all well cooled by a water jacket WWW; it will be seen that the combustion chamber is a casting separate from that of the working barrel, and that the inlet valve casing is also a separate casting. By disconnecting these three castings all jacket water spaces are exposed, and can be thoroughly cleansed from deposit; as pointed out in the article "Engines, Thermodynamics of Internal Combustion" (§ (62)), adequate cylinder and combustion chamber cooling becomes a matter of increasing importance as the dimensions of gas engines become larger. The arrangement here adopted necessitates making a double joint; of these the inner is made with asbestos, and is screwed up hard so as to be tight under the explosion pressure; the outer joint, having only to maintain jacket water tightness, is made with yielding rubber. The valve-stem guides are also separate castings easily replaceable after wear; in the case of the exhaust valve it will be seen that the design permits close access of cooling water all round the stem; the exhaust valve seat is also so arranged as to be efficiently water-cooled.

The cylinder has a bore of 16 inches with a piston stroke of 22 inches; the piston and valves are not water-cooled. The working barrel, or cylinder liner, MM, is screwed up hard to the combustion chamber casting at the rear (left-hand) end, while at the front end an expansion joint is provided with the jacket casing PP; the working barrel becomes much hotter than the outer casing of the jacket, and it is very necessary to make provision in this way for the difference of expansion thus arising in order to prevent the creation of large internal straining actions with consequent risk of distortion or rupture of castings.

A light cast-iron oil trough RR fitted in the crank-pit prevents waste oil from saturating the engine foundations.

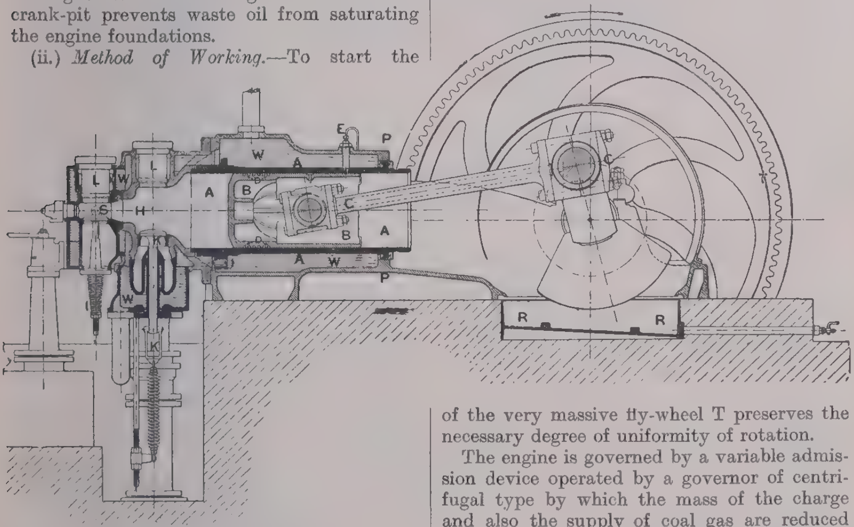

FIG. 1.—National Gas Engine.

(ii.) *Method of Working.*—To start the engine, the fly-wheel is pulled round by hand until the piston has definitely commenced its working stroke; an explosive mixture of coal gas and air is then pumped by hand, through a small auxiliary valve, into the combustion chamber; failing coal gas a small quantity of petrol is used. The explosive mixture thus introduced is then fired by a spark at the ignition plug S obtained by operating the (low-tension) magneto by hand. A working stroke of the piston immediately occurs, and considerable momentum is imparted to the fly-wheel; the subsequent order of operations is as follows:

(1) During the return stroke of the piston the exhaust valve K is lifted and the burnt gases are discharged through the exhaust pipe, and silencing apparatus, into the atmosphere; (2) during the second out-stroke the inlet valve I is open and a fresh charge of gas and air is drawn into the cylinder; (3) the second in-stroke then follows, compressing this fresh charge into the combustion chamber H; (4) at the commencement of the next out-stroke the mixture is ignited by a spark at the ignition plug S, explosion occurs, and the piston is driven forward. This cycle is then repeated continuously so long as the engine runs.

It will be seen, therefore, that the engine works upon the Otto, or four-stroke, cycle, of suction, compression, working, and exhaust strokes, and that the piston accordingly receives *one* working impulse in every *two* revolutions of the crank-shaft; the momentum of the very massive fly-wheel T preserves the necessary degree of uniformity of rotation.

The engine is governed by a variable admission device operated by a governor of centrifugal type by which the mass of the charge and also the supply of coal gas are reduced as the load on the engine is diminished. Below quarter load the governor acts by cutting out ignition.

(iii.) *Details of Working.*—The ratio of compression, $1/\rho$, is about $1/5{\cdot}5$, whence by equation (71′) of the "Engines, Thermodynamics of Internal Combustion," article, the "air standard" efficiency is $1-(1/\rho)^{0{\cdot}4}=0{\cdot}494$; with this value of $1/\rho$, which corresponds to a compression pressure of about 140 lbs. per sq. in. (abs.), pre-ignition is avoided without the necessity of injecting a spray of water into the cylinder at full load running. If p denote the mean effective pressure during the working stroke, in lbs. per sq. in., as ascertained from an indicator diagram, and if

d = Diameter of piston, in inches,
s = Stroke of piston, in inches,
n = Number of revolutions per minute of crank-shaft,

then the work done by the engine is expressed by $\pi/4 \times d^2 \times p \times s/12 \times n/2$ foot-lbs. per minute, and the indicated horse-power (I.H.P.) is 1/33000th of this, whence

$$\text{I.H.P.} = 992 \,.\, d^2snp \times 10^{-9}. \qquad (1)$$

The Brake Horse-power (B.H.P.) is less than the I.H.P. by the power necessary to overcome the frictional and fluid resistances of the engine itself; the ratio B.H.P./I.H.P. is termed the mechanical efficiency of the engine, and is usually denoted by the symbol η; hence, as $\eta = \text{B.H.P./I.H.P.}$, we have from equation (1)

$$\text{B.H.P.} = 992 \,.\, d^2sn\eta p \times 10^{-9}. \qquad (2)$$

The product ηp is termed the "brake mean effective pressure," and is largely used in calculations of the performance of small fast-running internal combustion engines of the petrol motor type with which it is impossible to obtain reliable indicator diagrams under ordinary circumstances.

The total distance, σ feet, covered by the piston in one minute is termed the "piston speed," and is evidently equal to $2n \times s/12$; thus

$$\text{Piston speed} = \sigma = \frac{ns}{6}. \qquad (3)$$

(iv.) *Performance.*—The performance of this 100 h.p. National gas engine has been ascertained by numerous tests in practice, and has been found to be as follows:

Fuel used.	Maximum. I.H.P.	Maximum. B.H.P.	Mechanical Efficiency. η.
Coal gas	111·2	94·2	·847
Benzol	99·5	82·5	·830
Anthracite producer gas	93·7	76·7	·819
Coke producer gas	88·0	71·0	·807

the revolution speed being 210 per minute. It will be noted that at this speed the power required to overcome engine frictional and fluid resistances is 17 I.H.P. From equation (1) the corresponding mean effective pressures are, in lbs. per sq. in.:

Fuel.	$p=$	$\eta p=$
Coal gas	95	80·5
Benzol	85	70·5
Anthracite producer gas	80	65·5
Coke producer gas	75	60·5

while the piston speed corresponding to 210 r.p.m. is 770 feet per minute.

Horizontal single-cylindrical "National" gas engines of this type are built up to a maximum of 185 brake horse-power; for higher powers multiple cylinders are used, in both horizontal and vertical arrangements; one of the latter is next described.

§ (6) (*b*) THE 300 H.P. VERTICAL "NATIONAL" GAS ENGINE. (i.) *Description.*—A transverse section of this four-cylindered two-crank tandem gas engine is shown in *Fig.* 2.

The engine comprises two pairs, each consisting of two cylinders A and B placed one above the other, each pair being connected by a connecting rod C with a crank D; the upper pistons are each 18 inches in diameter and the lower each 17 inches, with a stroke of 18 inches; and the engine runs normally at 300 revolutions per minute, so that the piston speed (equation (3)) is 900 feet per minute.

The upper and lower piston of each pair are connected by a cast-iron sleeve, or distance piece, 4½ inches in diameter, through which passes a long nickel steel bolt by means of which the three pieces are tightly held together; the pistons are not water-cooled but are cast with an internal conical web EE which assists in the conduction of heat from the piston crown—normally one of the hottest parts of an internal combustion engine. Gas-tightness is ensured by the five cast-iron spring rings shown; near the bottom of the skirt of the lower piston is placed a sixth, or "scraper," ring, to prevent the passage past the piston of oil from the crank chamber.

The lower portion of the upper cylinder contains only air, which is alternately compressed and expanded during the running of the engine, and by its "cushioning" action contributes greatly to softness of running by assisting to reverse the motion of the pistons when at the bottom of their stroke; the cranks are also balanced to further aid smoothness in running.

Each cylinder liner with its water jacket casing is a separate casting, but there is a gap at the lower end of each between liner and casing which is fitted with a water-tight packing ring so arranged as to permit any relative movement due to difference of expansion; the lower portions of the cylinder liners are not water-jacketed.

The inlet valves I, I′ and exhaust valves K, K′ open into a common port, and are placed one above the other, the inlets being on top. Both inlet and exhaust valves are of cast-iron, as are also the coned seats in which they rest; the seats are, however, separate rings capable of easy removal and replacement after wear. The inlet valves carry on their stems gas valves M, M′, which have no seats, but slide within a well-fitting cylindrical housing; the gas and air supply passages are formed by the partitioned casing N, N′. The gas valves M, M′ open later and close earlier than the charge inlet valves I, I′; this practice is common to all large gas engines and is adopted to avoid risk of pre-ignition of fresh charge by any residual smouldering or highly heated exhaust gas remaining over from the previous cycle. When the inlet valve opens at the end of the exhaust stroke the combustion chamber contains hot burnt gases from the previous explosion; the first inrush of pure air cools this, and the subsequent supply of mixed gas and air can then safely enter. Towards the end of the suction stroke the gas valve closes, and the final suction is from the lower air chamber N′ only; thus when the charge inlet valve I closes, N′ is filled with pure air in readiness for the commencement of the next suction stroke.

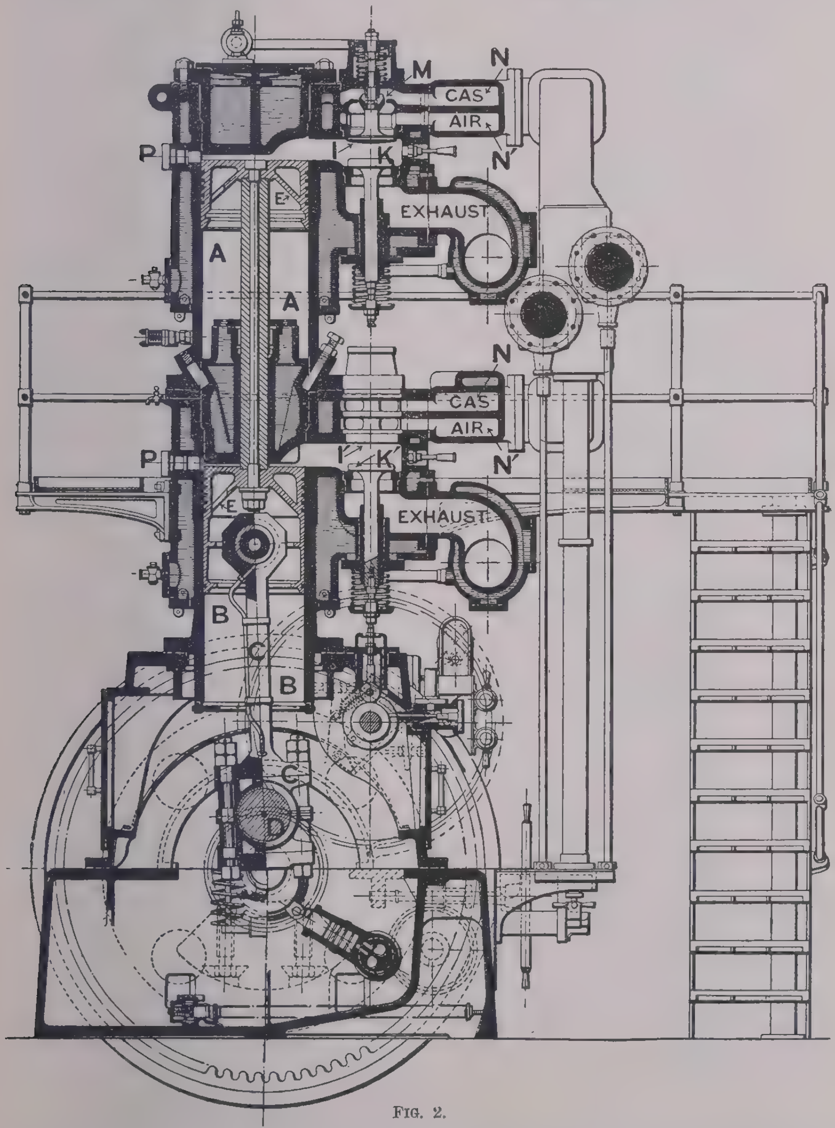

FIG. 2.

A centrifugal shaft governor regulates the supply of mixture to the engine by the "quantity" method, in which the composition of the charge remains sensibly constant, while its mass is varied to suit the load

Forced lubrication is used for all reciprocating and rotating parts, the oil being supplied by engine-driven valveless pumps at a pressure of 20 lbs. per sq. in.; after use the oil returns to the crank casing and drains into a "sump," where it is filtered before passing into the oil pumps by which it is again delivered to the bearings. The lower cylinders are lubricated by the oil which comes from the crosshead or "gudgeon" bearings, while the upper cylinders and metallic-packed joint of the piston rod where it passes between the two cylinders are supplied by a sight-feed lubricator fed by a small pump operated by the engine. Starting is effected by means of compressed air. Ignition is by high-tension magneto, with two "sparking plugs" PP in each cylinder.

The arrangement of the valve-driving shaft, which is gear-driven from the crank-shaft so as to run at half-speed in a four-stroke cycle engine, is clearly indicated in the figure; the cams and rollers by which the valves are operated are of case-hardened steel.

(ii.) *Performance.* — On test, one of these engines developed 404 indicated horse-power, and 362 brake horse-power at 299 revolutions per minute; the power was absorbed by a Heenan and Froude hydraulic brake. These results give the high value ·896 for the mechanical efficiency; to determine the value of ηp, equation (2) must obviously be modified for this case and written

$$\text{B.H.P.} = 2 \times 992 \times 10^{-9} \times sn\eta p \,.\, (\text{D}_1{}^2 + \text{D}_2{}^2 - d^2), \qquad (4)$$

D_1 and D_2 being the respective piston diameters, and d that of the piston rod; all in inches.

For a B.H.P. of 362, this equation gives $\eta p = 57$ lbs. per sq. in. As in this case η has the known value 0·896, we have also for the mean effective pressure, $p = 57/{\cdot}896 = 63{\cdot}6$ lbs. per sq. in.

Tandem single-acting vertical engines of this type are made from 185 to 1000 horse-power; the 1000 H.P. engine has eight cylinders (four pairs) operating four cranks. The cylinders of each pair are 22 inches and 23 inches in diameter respectively; the stroke is 24 inches, and the normal revolution speed is 200 per minute, the corresponding piston speed being 800 feet per minute.

§ (7) Relation of Weight to Power.—In all engines the weight per unit of power developed increases with increase in dimensions, and this is one of the reasons in favour of the adoption in many cases of the multi-cylindered design. The following Table, compiled from actual engines, brings out this point clearly; the rated B.H.P. is given, using blast-furnace gas as fuel; and the engine weights per B.H.P. are given exclusive of weight of fly-wheel. In large horizontal tandem double-acting gas engines the inclusion of the fly-wheel weight adds from 50 to 60 lbs. to the weight per B.H.P. The fly-wheel alone of a 1500 B.H.P. engine of this type commonly weighs about 35 tons.

TABLE I

WEIGHTS PER B.H.P. OF DIFFERENT TYPES OF GAS ENGINES

Rated B.H.P. 1.	Single or Double Acting. 2.	Piston Diameter × Stroke (Inches). 3.	No. of Cylinders and Arrangement. 4.	2-Stroke or 4-Stroke Cycle. 5.	Revolutions per Minute. 6.	Piston Speed Ft./Min. 7.	Weight per Rated B.H.P. (Lbs.). 8.
750	S.A.	18 × 18	6; 3 pairs; vertical	4	300	900	117
1250	S.A.	26 × 24	,, ,,	4	200	800	150
800	D.A.	28 × 34	2; tandem; horizontal	4	130	736	240
110	S.A.	20 × 31	1; horizontal	4	170	567	270
1000	D.A.	32 × 40	2; tandem; horizontal	4	110	733	270
800	S.A.	$51\frac{1}{4} \times 55$	1; horizontal	4	75	470	448
400	S.A.	24 × 30	1; 2 pistons; horizontal	2	130	650 *	179
1000	S.A.	$34 \times 37\frac{1}{2}$	,, ,,	2	125	781 *	179
750	S.A.	$30 \times 37\frac{1}{2}$	,, ,,	2	125	781 *	185
400	D.A.	$22\frac{1}{2} \times 39\frac{1}{2}$	1; horizontal	2	110	725 †	196
600	D.A.	$27\frac{1}{2} \times 48$	,,	2	90	720 †	220
1000	D.A.	$37\frac{3}{8} \times 63$	,,	2	70	735 †	231
1500	S.A.	42 × 51	1; 2 pistons; horizontal	2	94	797 *	269

* The *relative* piston speed is twice as great in these, the "Oechelhauser" engines; see § (11) *infra.*
† Koerting Engines; see § (10) *infra.*

§ (8) (*c*) The Tandem Horizontal Nuremberg Engine. (i.) *Description.*—The design selected as typifying a modern big four-stroke gas engine is that of the 2500 horse-power, two-cylindered, tandem, double-acting, horizontal, single-crank engine of the Maschinen-Fabrik Augsburg Nurnberg A.G.—usually styled, for brevity, the Nuremberg Co.—of which a longitudinal section is given in *Fig.* 3. The two cylinders A, A′ are placed in line, with their pistons mounted on a common piston rod BB, to one end of which the connecting rod C is attached; the whole power of the engine is transmitted by this connecting rod to the single crank D; the crank-shaft carries a massive fly-wheel, by the momentum of which the necessary degree of uniformity of revolution is maintained. This engine is double-acting, that is, each cylinder is closed at both ends, and both sides of each piston receive power impulses. As a single-acting, single-cylindered, four-stroke engine receives one power impulse in every four strokes of its piston, it is clear that in this case as the power impulses are four times as frequent there is a power impulse at every stroke; the engine thus runs with very little variation in its revolution-speed.

(ii.) *Cooling Arrangements.*—British practice so far has generally favoured the single-acting uncooled piston, on the score of simplicity in details, and this practice has limited piston diameters to a maximum of about 26 inches. With a big double-acting engine, water- or oil-cooling of the pistons, and even also of the exhaust valves, becomes necessary. In this case the pistons only are water-cooled, the water being introduced through the hollow piston rod as shown in the illustration.

The cylinders and cylinder covers are, of course, also water-jacketed; the ample water-spaces provided will be noted in the section, and it will be seen that care has been taken to bring the water close up to the valve seatings; it is found that a water pressure of about 15 lbs. per sq. in. above atmosphere suffices to maintain an efficient circulation through the jackets. For the piston rod and pistons, however, it is found necessary—on account of their reciprocating motion—to provide a water pressure of from 45 to 65 lbs. per sq. in. Water from the main is frequently available at this pressure, but when this is not the case a pump is provided, driven by the engine, which delivers the cooling water at the necessary pressure.

The water-cooling system is well arranged throughout; each cylinder is fitted with an open water-tank into which all water drain-pipes discharge in full view of the attendant, and each discharge is provided with a thermometer and a regulating valve so that the temperature of each part can be adjusted independently as desired. To avoid the necessity of shutting each outlet valve when the engine is stopped, a main stop valve is fitted in the supply pipe, and this valve is closed only when the engine is not running.

(iii.) *Lubrication.*—The lubrication of the external bearings is effected from a large oil-tank situated above the engine from which the oil is conducted through pipes of ample diameter fitted with regulating valves, to the various points. Surplus oil is drained away and collects in a sump in the engine house, where it is automatically filtered and then returned to the supply tank by a pump driven by the engine. Rolling levers and eccentrics are grease-lubricated.

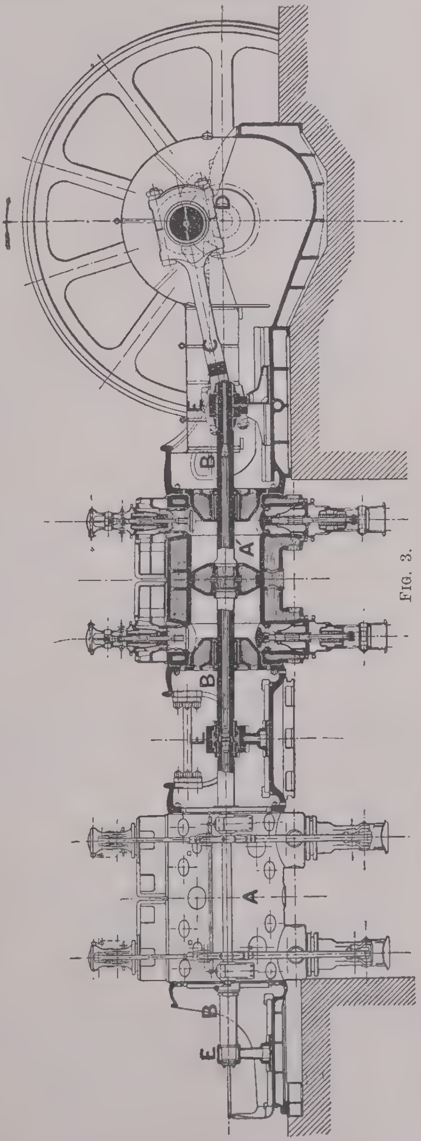

FIG. 3.

For the forced lubrication of the cylinders, piston rod stuffing-boxes, and exhaust valve guides, special oil pumps are provided, and the supply to each point can be independently regulated. The piston rod stuffing-boxes are packed with a series of rings of similar section each in three parts, made alternately of cast-iron and white metal; these are pressed against the surface of the rod by symmetrically disposed circumferential helical springs; lubricating oil under pressure is delivered into the middle of each stuffing-box. Wear is found to be very small, and is confined to the packing rings, which are easily adjusted or renewed.

In illustration of the efficiency of the cooling and oiling arrangements provided in these large engines, it may be stated that a Nuremberg engine of 2000 horse-power working with blast-furnace gas ran day and night for a period of nineteen months. The engine was actually running for 98·3 per cent of the possible working hours, and the 1·7 per cent of stoppages were due to works repairs and in no way to any defect of the engine. At the end of this long period of service the engine was reported as in excellent working order, and continued in operation.

Table II

Test Results from a 1200 B.H.P. Tandem D.A. Nuremberg Engine

Item.	1.	2.	3.	4.	5.	6.	7.
Duration of test, minutes	33·0	28·0	29·0	26·8	25·8	25·8	25·3
Revolutions per minute	106·0	105·8	106·3	106·5	106·1	105·8	105·6
Mean effective pressure, p, in lbs./sq. in. (average of both cylinders)	30·4	42·5	60·0	68·7	71·4	73·1	75·3
I.H.P.	577	807	1146	1312	1359	1383	1427
B.H.P.	280	557	871·5	1037	1115	1147	1186
I.H.P. – B.H.P.	297	250	275	275	244	236	241
Mechanical efficiency	·485	·690	·762	·790	·821	·830	·831
Value of ηp in lbs./sq. in.	14·7	29·3	45·7	54·3	58·6	60·7	62·6
Cub. ft. gas per I.H.P. hour	101	100·6	91·5	86·9	87·2	85·5	84·6
Cub. ft. gas per B.H.P. hour	208	146	120	110	106	103	101·8
Heat value of gas, C.Th.U./cub. ft.	49·2	49·2	49·7	49·8	50·4	49·4	48·8
C.Th.U. per I.H.P. hour	4,969	4950	4548	4328	4395	4224	4128
C.Th.U. per B.H.P. hour	10,245	7174	5969	5480	5353	5090	4968
Indicated thermal efficiency	·285	·286	·311	·326	·322	·335	·343
Brake thermal efficiency	·138	·197	·237	·258	·264	·278	·285

(iv.) *Piston Rods.*—An important detail of construction is that whereby the heavy water-filled pistons are prevented by their weight from causing "ovalling" or undue wear of the lower portions of the cylinders and stuffing-boxes. The long hollow thick-walled tubes of which the piston rod is built up are turned with a slight upward camber so that the rod, when loaded with the pistons, and supported by the three crossheads E, E, E, is exactly straight; arrangements are also provided in addition whereby each piston can be readily adjusted independently to the exact centre of its own cylinder. The whole weight of the pistons and rod is thus borne by the three external crosshead slides, and the pistons accordingly "float" in the cylinders as does the piston rod in the stuffing-boxes; this not only saves wear but also considerably reduces the internal friction of the engine. Gas-tightness of the pistons is ensured by six self-tightening packing rings.

(v.) *Governors.*—The engine is governed on the "quantity" method, by which the composition remains practically constant, while the mass admitted is proportioned to the power output required. Should the quality of the gaseous fuel vary, the ratio of gas to air can be adjusted by hand while the engine is running.

Two ignition plugs are fitted in each cylinder, actuated by small electromagnets forming part of the plugs.

(vi.) *Performance.* — Table II. gives the results of tests on a 1200 brake horse-power Nuremberg engine using blast-furnace gas having a heat value per cubic foot of about 50 C.Th.U.; the pistons were 33·46 inches in diameter (piston rod about 8⅝ inches dia.), with a stroke of 43·3 inches.

With reference to this Table it may be observed that the revolutions per minute, n, are directly counted, and that the value of the mean effective pressure, p, in lbs. per sq. in. is ascertained from indicator diagrams taken from both cylinders, the average value being tabulated. The indicated horse-power may next be calculated, equation (1) being modified to suit this type of engine by writing

$$\text{I.H.P.} = 4 \times 992 \times 10^{-9} \,.\, snp(D^2 - d^2), \qquad (5)$$

D and d being the diameters in inches of the pistons and piston rod respectively.

The brake horse-power is directly measured, and thence the value of the mechanical efficiency $\eta = \text{B.H.P./I.H.P.}$ is known. It will be observed that the frictional and fluid resistances of the engine itself absorb about

260 horse-power when running at 106 revolutions per minute. The brake mean effective pressure, ηp, is now known and tabulated. The total consumption of gas per hour in cubic feet is directly measured, and dividing the figures obtained by the I.H.P. and B.H.P. gives the consumption in cubic feet per hour per I.H.P. and B.H.P. respectively. The heat value of the gas per cubic feet is determined by calorimetric tests, and thence the heat supplied in C.Th.U. per hour per I.H.P. and per B.H.P. is immediately ascertainable.

Finally, as one horse-power hour corresponds to $33,000 \times 60/1400 = 1414$ C.Th.U. per hour converted into mechanical work, the absolute thermal efficiency of the engine is determined by taking the ratio of 1414 to the heat supplied to the engine in C.Th.U. per horse-power per hour.

Thus, at full load, this engine showed an absolute brake thermal efficiency of ·285; that is to say, 28·5 per cent of the whole heat supplied to the engine appeared as useful external work. In one hour the working volume swept through by the pistons is $\pi/4(D^2 - d^2) \times 120n \times s/1728$ cubic feet; and at full load the heat evolved in C.Th.U. per hour is 1427×4128, whence in this case the evolution of heat in C.Th.U. per cubic feet of working stroke swept out by the pistons has the value 22·6 (see the article, "Engines, Thermodynamics of Internal Combustion," § (69)).

§ (9) (*d*) TWO-STROKE ENGINES: THE EARLY CLERK ENGINE. (i.) *Description.*—The low frequency of but one working stroke in four of the Otto cycle has always been regarded as a serious disadvantage, and very numerous attempts have for long been made to increase the impulse frequency without sacrificing the valuable practical advantages of simply and effectively charging, exploding, expanding, and exhausting—combined with high thermal efficiency—which are possessed by the four-stroke engine; no solution has even yet been found which fully satisfies the commercial conditions for both large and small engines.

The first explosion compression two-stroke or "impulse every revolution" engine was invented and built by Clerk in 1878, and the Clerk two-stroke cycle engine, with improvements in detail suggested by experience, is still, particularly for large engines, that which most successfully satisfies everyday requirements.

A section of the second Clerk two-stroke engine is shown in *Fig.* 4; an engine to this design was shown at the Paris Exhibition of 1881. It was of the horizontal, single-cylindered, single-acting type, and comprised a motor, or working, cylinder A containing exhaust ports E, E′, near its outer end, and a displacer cylinder B; within these cylinders respectively work pistons C and D, suitably connected to a common crank-shaft. The crank-pin driving the displacer piston D was about 90° in advance of that operated by the motor piston C.

(ii.) *Method of Working.*—The order of operations is as follows: Near the end of the working (out)stroke the motor piston C overruns the exhaust ports E, E′, and the burnt gases immediately escape thence into the atmosphere. Simultaneously the displacer piston D, being in advance of the motor piston, has passed the end of its out-stroke and has commenced to return; during its out-stroke it has drawn into the cylinder B an explosive mixture of gas and air through the sliding valve H and pipe W. The commencing return of the piston D causes the mixture in B to become slightly compressed before the complete exhaust of A, but not sufficiently to cause any material resistance; in the delivery pipe connecting the two cylinders is an automatic inlet valve; as soon as the pressure from B slightly exceeds that in A, this valve rises and the fresh mixture then enters the combustion chamber G. The return of the motor piston C then causes the automatic inlet valve to close, and subsequently compresses the entrapped fresh charge into the chamber G; this is then fired at the instant of maximum compression, explosion occurs, and the working stroke follows. Thus every out-stroke of the piston C is a working stroke, and the impulses are therefore one per revolution of the crank-shaft, and are accordingly twice as frequent as in the "Otto" four-stroke cycle.

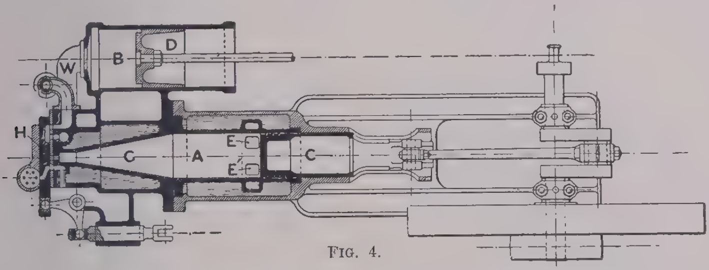

FIG. 4.

(iii.) *Difficulties.*—In the ideal two-stroke motor the operations of charging and exhausting would be performed at least as effectively as in the four-stroke engine, and hence, the working impulses being twice as frequent, a two-stroke engine of given bore and stroke should develop at least twice as much power as the equal four-stroke, with the same thermal efficiency. In practice, however, there are certain fundamental difficulties of the two-stroke cycle which have so far prevented the realisation of this ideal. Thus, in the Otto cycle the inlet valve usually opens slightly before the end of the in-stroke of the working piston and remains open not only throughout the whole suction out-stroke, but also for a short period after its completion; the suction, or "charging" period accordingly continues during about 220° of the crank-shaft revolution. In the two-stroke engine, however, the fresh charge has to be introduced into the motor cylinder while the crank-shaft turns through only about 80°. Hence the duration of the charging operation in the Otto cycle is nearly three times as long as in the Clerk cycle. Though this disadvantage is reduced by providing specially large inlet valves and exhaust port areas, it remains still that the two-stroke engine is not in general capable of being charged so effectively as the four-stroke, and also that more power is absorbed in the charging operation.

A second point is that in the Otto cycle the exhaust valve is open during about 240° of the crank-shaft revolution, and hence the burnt gases have considerable time in which to escape, and are moreover assisted in their exit by the return of the motor piston during the whole exhaust stroke, so that there remains finally only the combustion chamber filled with residual exhaust at, or even slightly below, atmospheric pressure. In the Clerk cycle engine, on the other hand, exhaust has to be accomplished during the very short interval occupied by the motor piston in passing over the last part of its out-stroke and first part of the subsequent in-stroke, so that not only in the combustion chamber but also in the working cylinder there remains some hot exhaust gas. The effect of this is to reduce the quantity of fresh charge that can be introduced into the motor cylinder, *i.e.* to diminish the "volumetric efficiency" of the engine, with consequent diminution in the output of power. Further, the commencement of the introduction of the fresh charge while the exhaust ports are still uncovered causes a loss of unburnt mixture by "short-circuiting" direct through the exhaust ports into the atmosphere, and waste of fuel from this cause is frequently considerable in the very small petrol-type two-stroke motor. In large two-stroke engines, however, this cause of loss is almost completely avoided, (*a*) by providing a combustion chamber of somewhat elongated conical form as shown at G (*Fig.* 4), and introducing the fresh charge at the apex of the cone, and (*b*) by sending first into the motor cylinder a preliminary charge of air only to help in the scavenging (and cooling) of the residual exhaust gases, and following this up by the introduction of a correspondingly rich mixture of gas and air; this practice, initiated by Clerk in 1881, is still followed in all large gas engines. The doubled frequency of working impulses in the two-stroke engine causes necessarily an increase in the mean heat-flow from the cylinder per second, and hence in the design of this type of engine special attention has to be given to the details of the cooling arrangements.

(iv.) *Performance.*—These early Clerk two-stroke engines were constructed in sizes of from 2 to 12 nominal horse-power, and the following test results obtained in 1884 from a series of these engines are still of much interest; the fuel used was Glasgow coal gas, and the figures given are from the usual trials which were carried out on all engines before leaving the manufacturers' works, and thus fairly indicate the performance of the engines in ordinary service.

The mixture delivered by the displacer contained 1 volume of coal gas to 8 volumes of air; on passing through the inlet and mixing with the residual exhaust gases in the motor cylinder it becomes further diluted. The increase of temperature acquired by contact with the exhaust and with the cylinder walls expands the entering fresh gas, a temperature of at least 100° C. being commonly attained before compression commences. The expansion of the entering fresh gases thus caused expels more of the exhaust products through the ports than would correspond to the volume swept through by the motor piston between the closing of the exhaust ports and complete in-stroke. Through "turbulence" mixing occurs to a considerable extent and the net result is the formation of a mixture explosive in every part of it, and of an average composition of 1 volume of coal gas to 9 volumes of other gases; thus the proportion of exhaust gases present is but small; that there is any at all arises from the necessity of preventing any appreciable loss of fresh mixture through the exhaust ports. The mixture employed was a comparatively rich one.

(v.) *Efficiency.*—In these engines the value of the compression ratio, $1/\rho$, was $\frac{1}{3}$, whence the "Air Standard" efficiency is $1-(\frac{1}{3})^{0\cdot4}=0\cdot36$. Regarding this as the highest conceivable efficiency, the degrees of excellence of the engines given in the table are to be estimated relatively to 0·36 and not to unity; thus we have

Absolute nett indicated thermal efficiency	·134	·166	·165	·194	·203
Air standard	·36	·36	·36	·36	·36
Efficiency relative to air standard	·372	·462	·460	·54	·56

so that in the case of the engine of 12 nominal horse-power the relative nett indicated thermal efficiency was 56 per cent of the air standard. Actually, as explained in the previous article, the air standard—obtained on the assumption of constant specific heat—represents an impossibly high ideal, the maximum attainable, with varying specific heat, having the lower value for $1/\rho=\frac{1}{3}$ of only about 0·29; hence the comparison of performance is properly made between an actual thermal efficiency of ·203 and an ideal thermal efficiency of ·290; and thus the actual engine in this case realised no less than ·203/·290, *i.e.* 70 per cent of the attainable ideal.

Table III

Test Results of Clerk Two-stroke Engines in 1884

No.	Item.	Nominal Power.				
		2 H.P.	4 H.P.	6 H.P.	8 H.P.	12 H.P.
1	Diameter of motor piston, ins.	5	6	7	8	9
2	Stroke of motor piston, ins.	8	10	12	16	20
3	Diameter of displacer piston, ins.	6	7	$7\frac{1}{2}$	10	10
4	Stroke of displacer piston, ins.	9	11	12	13	20
5	Revolutions of engine per minute	212	190	146	142	132
6	Mean effective pressure in lbs./sq. in. from motor cylinder diagram	43·2	63·9	53·2	60·3	64·8
7	I.H.P. from motor cylinder diagram	3·62	8·68	9·05	17·38	27·46
8	H.P. absorbed by displacer	0·40	0·80	0·86	1·50	2·00
9	Nett I.H.P. of engine (item 7 – item 8)	3·22	7·88	8·19	15·88	25·46
10	B.H.P. of engine	2·70	5·63	7·23	13·69	23·21
11	Nett I.H.P. – B.H.P.	0·52	2·25	0·96	2·19	2·25
12	Mechanical efficiency $\eta=\frac{\text{B.H.P.}}{\text{nett I.H.P.}}$	·84	·72	·88	·86	·91
13	Approximate calorific value of gas, C.Th.U./cub. ft.	320	320	320	320	320
14	Gas consumption, cub. ft. per I.H.P. hour (item 7)	29·28	24·19	24·23	20·94	20·39
15	Gas consumption per nett I.H.P. hour; cub. ft.	33·0	26·6	26·8	22·9	22·1
16	Gas consumption per B.H.P. hour; cub. ft.	39·4	37·2	30·2	26·6	24·1
17	Absolute thermal efficiency on nett I.H.P.	·134	·166	·165	·194	·203
18	Absolute brake thermal efficiency	·112	·119	·146	·166	·184
19	Compression pressure, lbs./sq. in. (abs.)	53	70	63	64	72
20	Maximum explosion pressure, lbs./sq. in. (abs.)	170	251	210	210	253

§ (10) (*e*) The Körting Engine. (i.) *Description.*—The Clerk two-stroke cycle is very successfully utilised in the special design of large horizontal gas engine made by Messrs. Körting of Hanover, and the De La Vergne Co. of New York; a typical section is shown in *Fig.* 5. The engine is of the single-cylindered double-acting type, and working on the two-stroke cycle, thus furnishes two impulses per revolution of the crank-shaft as in an ordinary steam engine. It comprises a closed, water-jacketed, cast-iron cylinder AA provided with a ring of exhaust ports BB at the middle of its length, and containing a very long piston CC which overruns the ring of exhaust ports in its reciprocations, opening these ports to the one end of the cylinder and the other alternately. The piston is attached to a piston rod which passes through a stuffing box in the cylinder cover and is connected to an external crosshead as in ordinary steam-engine practice; thence the

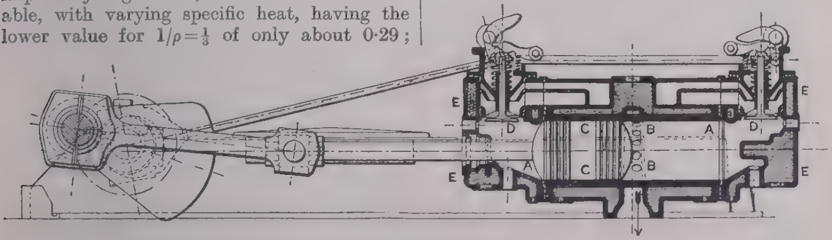

Fig. 5.

power is transmitted to the crank-shaft by a connecting rod as usual. The external crosshead not only keeps the "gudgeon" bearing or "small end" bearing of the connecting rod cool, but also relieves the piston of the lateral thrust due to connecting-rod obliquity, and thus much reduces cylinder and piston wear. Large gas engine cylinders adequately lubricated, and using gas free from dust, show but little wear, a 400 B.H.P. cylinder after one year's working, for example, showing an average wear of only ·013 of an inch. The cylinder is closed by deep cast-steel well water-jacketed covers EE, in which are housed the cages containing the mixture inlet valves DD, normally held up against their seats by helical springs, and opened during the necessary intervals by a simple arrangement of rods and levers operated by a single eccentric on the crank-shaft.

In these large engines, instead of the single "mixture pump" or "displacer" of the Clerk engine, separate double-acting pumps are provided, one of which supplies air only and the other gas only. The displacer crank is placed about 100° in advance of the main crank, as in the Clerk engine, and thus the air pump piston has travelled a short way on its discharge stroke when the inlet valve opens. The gas pump, however, is so arranged that gas is not delivered until somewhat later. Thus a considerable volume of air only flows into the cylinder at first, displacing and cooling the hot exhaust gases, so that when the fresh unburnt gas enters later it mingles with the relatively cool air in the cylinder, and thus risk of pre-ignition is minimised. It is very important in large gas engines to arrange that no explosive mixture shall be formed in chambers or passages; in the Körting engine the gas meets and mixes with the air just above the inlet valve. In small gas engines the gas and air may be mixed in the pump or "displacer," a back-fire into the pump being of little importance; in a large engine, however, in similar circumstances, the result of a back-fire might easily prove a serious matter.

(ii.) *Method of Working.*—The charge having entered the cylinder—first air only and then mixed gas and air—the exhaust products having been thereby displaced, the working piston has closed the ring of exhaust ports by a crank movement of 40° to 45° from the dead centre. Compression then occurs, followed by ignition and subsequent expansion. The air pump valves are so arranged as to deliver a full charge of air at every stroke whether the engine be light or loaded, but the gas delivered by the gas pump varies in amount as determined by the governor. Ignition is electric, and occurs at two points at each end of the cylinder, and means are provided by which the time of ignition can be regulated by hand while the engine is running; poor gases, as *e.g.* blast furnace gas, require earlier ignition than producer gas, and this, in like manner, earlier than coal gas. Where the gas is liable to variation in quality this adjustable ignition arrangement is very useful. Running at light loads, the mixture of gas and air admitted after the preliminary charge of air only being of practically constant composition, a readily ignitible charge always exists at the ignition plugs, and thus regular firing is ensured at light loads.

(iii.) *Performance.*—Körting engines having an aggregate of fully a quarter of a million horse-power have been built, and they compete strongly against the four-stroke type; they are built in sizes of from about 400 B.H.P. with a 22½ inch cylinder and 39½ inches stroke, running at 110 revolutions per minute, to 2000 horse-power from a single cylinder of 43 inches diameter, the stroke being 55 inches, and speed up to 90 revolutions per minute.

The following results were obtained on test with a 600 B.H.P. Körting engine in 1904 (Junge):

Power piston, 29·7 in. diameter—stroke, 55·1 in.

Piston rod, 8·1 in. diameter—revolutions per minute, 80.

Diameter of double-acting air pump, 31·4 in.—stroke, 42·5 in.

Diameter of double-acting gas pump, 27·6 in.—stroke, 42·5 in.

Fuel: producer gas from anthracite.

I.H.P. developed in working cylinder: 845.

B.H.P. of engine: 673.

Fluid resistance I.H.P. of pumps: 88.

Ratio of pump resistance to total I.H.P.: 0·104.

Power—Piston speed: 735 ft. per minute.

Mean effective pressure, p, on power piston: 55·6 lbs./sq. in.

Brake mean effective pressure, ηp: 44·3 lbs./sq. in.

Anthracite burned per B.H.P. hour: 0·8 lb.

Estimating the mechanical efficiency as the value of the ratio B.H.P./Total I.H.P. gives $\eta = 0{\cdot}8$, which is rather low, and is due to the somewhat high pumping resistances which in this particular engine amounted to 10·4 per cent of the total I.H.P.; in later designs the pumping resistances have been much reduced and the mechanical efficiency correspondingly increased. For weight per B.H.P. of this type of engine see Table I. (*supra*).

§ (11) (*f*) THE TWO-STROKE OECHELHAUSER ENGINE. (i.) *Description.*—A second type of successful large two-stroke gas engine working on a modified Clerk cycle is that of Dr. Oechelhauser, of which a diagrammatic section is shown in *Fig.* 6. The engine is of the horizontal, single-cylindered, single-acting type, but the cylinder AA is open at *both* ends and contains two pistons B, B′, working in opposite directions. Just before reaching the extreme "out" position piston B overruns the ring of exhaust ports C, while piston B′ next overruns first a ring of air inlet ports D, and very shortly afterwards a second ring of gas ports E. The burnt exhaust gases immediately escape through the ports C and are assisted in their exit by the charge of fresh air under slight pressure which enters through the ports D. This fresh air mingles with, and cools, the residual exhaust gases, thus minimising risk of pre-ignition and also avoiding appreciable loss of fresh fuel through the exhaust ports when the gas is subsequently admitted through the ports E. The mutual approach of the two pistons eclipses the ports and the entrapped mixture is next compressed between them; at the end of the in (compression) stroke, the mixture is fired electrically as usual, and the expansion (working) stroke then follows.

The double-acting pump H acts on one side as an air pump and on the other as a gas pump; gas and air are separately pumped into the separate reservoirs K and L respectively, wherein they are stored at a pressure of 5 to 6 lbs. per sq. in. above atmosphere.

It will be seen that with this arrangement no gas and air mixture exists anywhere but actually *within* the working cylinder. To further diminish loss of unconsumed gas through the exhaust the maximum quantity of gas and air delivered into the cylinder is only about 0·7 of the cylinder volume. Governing is effected, as in the Körting engine, by reducing the gas charge at light loads.

The crank-shaft has three throws; to the middle throw piston B is linked by the usual connecting rod, while the two side throws are linked by side connecting rods M, M′ to crossheads O, O′, which are coupled up by rods P, P′ to the bridge-piece Q carried on the piston rod R of piston B′; thus the crank-shaft is subjected to a practically simple torque, and the cylinder is not required—as in all other engine designs—to supply the reaction to the actions on the working pistons; as a consequence the engine frame and cylinder can be made lighter than usual.

In the design illustrated the gas and air pump is shown driven directly by the piston rod R, but in later designs the engine dimensions are much reduced and weight diminished by placing the pump at the side of the working cylinder and driving it from a disc crank-pin on the end of the crank-shaft. The cylinder also has been much shortened by the ingenious device of fitting spring rings in each of its ends, which rings bear upon the piston surfaces; in this way the pistons can be arranged to protrude from the cylinder ends by a considerable amount at out-stroke, much as in the case of an ordinary plunger pump. To prevent "canting" of the pistons both are rigidly attached by short piston rods to sliding crossheads guided both vertically and horizontally.

(ii.) *Working.*—The two large pistons alternately issuing from and receding into the working cylinder set up pulsations in the atmosphere of the engine-house, and these may cause troublesome rhythmic vibrations of windows, partitions, etc.; this action is greatly reduced in cases where two engines are installed in the same engine-house with crank-shafts so arranged as to differ in phase by 180°.

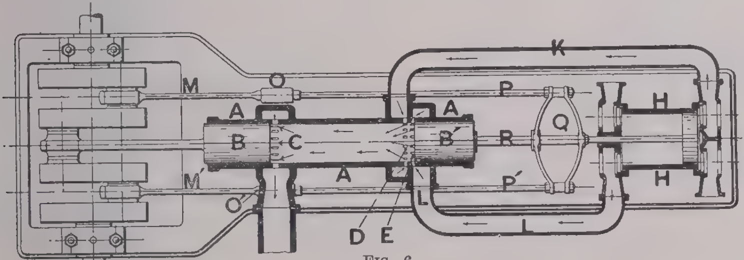

FIG. 6.

The *two* moving pistons cause the rates of compression and expansion of the working gases to be twice as rapid as usual, which is thermodynamically advantageous.

(iii.) *Performance.*—Tests by Professor Meyer in 1903 of a 500 h.p. Borsig-Oechelhauser engine furnished the following results:

DIMENSIONS OF ENGINE

Diameter of cylinder	26·6 in.
Stroke of front piston	37·5 "
Stroke of back piston	37·3 "

Double-acting Air Pump

Diameter of cylinder	44·9 "
Stroke of piston	19·7 "
Diameter of front piston rod . .	3·45 "
Diameter of back piston rod . .	2·78 "

Single-acting Gas Pump

Diameter of cylinder	23·2 "
Stroke of piston	19·7 "

Blower

Diameter of cylinder	65 "
Stroke of piston	37·3 "
Diameter of piston rod	5·9 "

No.	Item.	Number of Trial.				
		VIII.	IX.	X.	VI.	VII.
1	Duration of test, minutes	20	19·95	40	15	19·95
2	Revolutions per minute, average	103	107	106·1	108·2	107·4
3	Mean effective pressure p in lbs./sq. in.	75·0	73·8	69·3	62·3	62·0
4	Total I.H.P. from working cylinder	821	839	780	715	707
5	B.H.P. estimated from blowing cylinder	616·2	626·6	574·8	488	473·8
6	Air pump, mean effective pressure, front, lbs./sq. in.	5·09	5·38	5·12	5·56	6·09
7	Air pump, mean effective pressure, back, lbs./sq. in.	3·36	3·58	3·41	3·73	3·94
8	H.P. absorbed by air pump	68·3	75·2	71·1	79·0	84·5
9	Gas pump, mean effective pressure, lbs./sq. in.	3·53	3·50	3·58	3·73	3·84
10	H.P. absorbed by gas pump	7·7	7·8	7·9	8·5	8·6
11	Total H.P. absorbed by charging pumps	76	83·1	79·1	87·5	93·2
12	Mechanical efficiency of blower	·839	·842	·833	·792	·785
13	H.P. absorbed in engine friction	117·3	117·3	115·4	129·2	129·2
14	Calorific value of gas, C.Th.U./cub. ft. (lower)	221·5	218·5	212·2	218·5	220·3
15	Heat supplied, C.Th.U. per total I.H.P. hour	3660	3637	3616	3703	3682
16	Heat supplied per B.H.P. hour	4872	4870	4910	5420	5480

The fuel used was coke oven gas having the average composition by volume:

Hydrogen	44·7
Carbon monoxide	11·0
Marsh gas (CH_4)	19·5
Heavy hydrocarbons	2·0
Nitrogen	17·5
Oxygen	0·3
Carbon dioxide	5·0
	100

and the average consumption on these tests was 16·8 cubic feet per total I.H.P. hour.

The absolute thermal efficiencies realised were:

Absolute indicated thermal efficiency	·386	·389	·391	·382	·384
Absolute brake thermal efficiency	·290	·290	·288	·261	·258

The mechanical efficiency, taken as the value of the ratio B.H.P./Total I.H.P., varied from 0·67 to 0·75, a somewhat low result; the joint pumping resistances ranged from about 9 per cent at full load to 13 per cent at the lowest test load—calculated relatively to the total I.H.P.

The consumption of lubricating oil in the working cylinder was at the average rate of 1·19 lb. per hour. At full load 4·4 gallons of water were used per total I.H.P. hour; the cooling water temperature on entering was 22° C., and on leaving 42° C.; hence $44 \times 20 = 880$ C.Th.U. of heat were carried off by the cooling water per I.H.P. hour; so that, at full load, the heat expenditure account was roughly:

Heat Expenditure per Total I.H.P. Hour, C.Th.U.

		Per cent.
Converted into mechanical work	1414	39
Carried off in cooling water	880	24
Carried off in exhaust gases and general heat losses	1343	37
	3637	100

(iv.) *Indicator Diagram.*—In obtaining indicator diagrams from an engine it is usual to so connect up the recording drum that the angle turned through by it is always proportional to the distance travelled by the piston along its stroke. In the case of the Oechelhauser engine a special procedure becomes necessary, as the two pistons do not move exactly alike. In the explosion position the middle throw of the crank-shaft—which is driven by the front piston—is at its inner dead centre while the two side throws—driven by the back piston—are at their outer dead centre. When the double working stroke commences, due to connecting-rod obliquity, the front piston travels more rapidly than the back piston, and this continues until nearly half stroke; thereafter the front piston moves more slowly than the back. Thus *Fig.* 7 shows by the inner heavy line the diagram obtained from an engine with reference to the *back* piston; when the back piston has moved through OA the pressure upon it is given by AB. But at the same instant the front piston has described more of its stroke and has reached some point, as C; drawing a vertical through C to meet a horizontal through B in B′, it is evident that B′ is a point on the diagram corresponding to the front piston. In this way the front piston diagrams can be constructed, as shown by the outer full line; midway between is drawn in dotted lines the "corrected" diagram from which the true mean effective pressure is deduced. In some cases the true M.E.P. is as much as 10 per cent greater than that deduced from the single diagram as given by the indicator.

§ (12) DAY'S ENGINE. (i.) *Description.*—The ingenious two-stroke cycle valveless engine invented by Day in 1891 may be briefly referred to here as it is largely used at the present day in very small sizes, as *e.g.* in the (petrol) engines of motor bicycles, launches, etc. The engine is not of high volumetric efficiency and its fuel consumption is, in general, rather heavy, due to the escape of a portion of each fresh charge through the exhaust. Its simplicity of action and relatively low manufacturing cost, however, are resulting in its increasing use in the smallest class of engine.

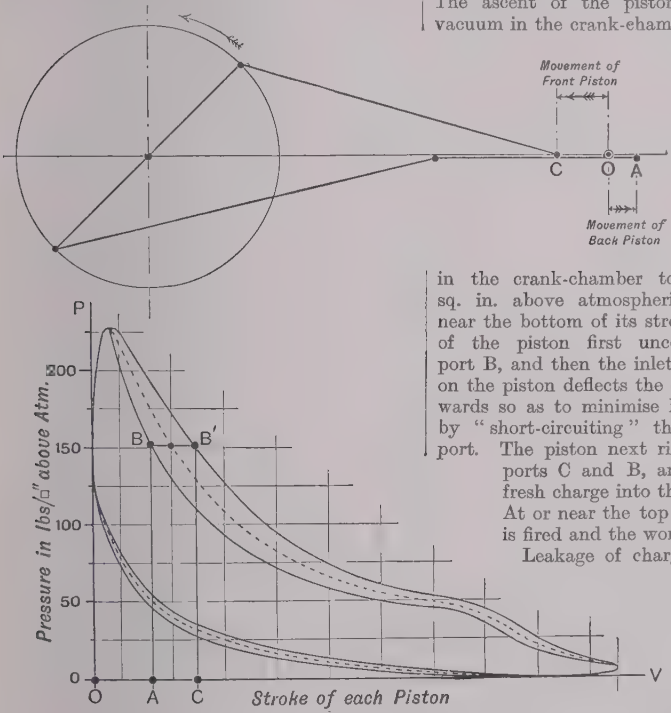

FIG. 7.

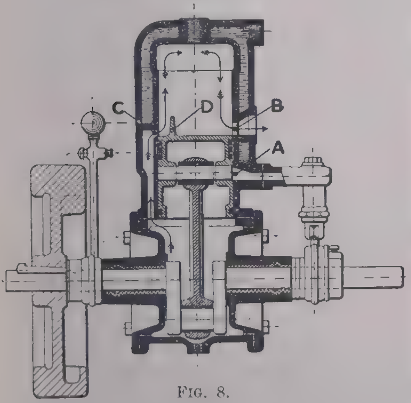

FIG. 8.

A diagrammatic section of the usual "three-port" Day engine is shown in *Fig.* 8. The ascent of the piston causes a partial vacuum in the crank-chamber, and when near the top of its stroke its lower edge uncovers the port A, thus allowing an inrush of carburetted air. The subsequent descent of the piston first closes A and then compresses the charge in the crank-chamber to 3 or 4 lbs. per sq. in. above atmospheric pressure. When near the bottom of its stroke, the upper edge of the piston first uncovers the exhaust port B, and then the inlet port C; the lip D on the piston deflects the entering stream upwards so as to minimise loss of fresh charge by "short-circuiting" through the exhaust port. The piston next rises, cutting off the ports C and B, and compressing the fresh charge into the combustion head. At or near the top centre the mixture is fired and the working stroke follows.

Leakage of charge from the crank-chamber is commonly prevented by making the crank-shaft bearings very long, and grease is sometimes used in these as lubricant.

(ii.) *Performance.*—Experiments on an engine of this type by the late Professor W. Watson and Mr. Fenning in 1910 showed that the proportion of each fresh charge which escaped unburnt through the exhaust port was considerable at low speeds, but diminished as the speed increased, as shown hereunder:

At Revolutions per Minute.	Per cent of Fresh Charge lost.
600	36
1200	20
1500	6

The mean effective pressure was much higher at low than at high speeds, ranging from about $62\frac{1}{2}$ lbs. per sq. in. at 600 r.p.m. to $44\frac{1}{2}$ lbs. per sq. in. at 1500 r.p.m. The volumetric efficiency was only about 40 per cent, and varied but little with speed, the greater loss of fresh charge through the exhaust port at low speeds approximately counterbalancing the larger volume of charge then entering the cylinder.

Owing to the relatively large proportion of burnt exhaust gases in the charge when fired, the "mixture" supplied to this type of engine needs carefully adjusting within somewhat narrow limits in order to obtain regular ignition; otherwise ignition only occurs at every alternate out-stroke, and the engine is said to "four-stroke" in ordinary parlance, the intermediate stroke having only a "scavenging" action. When alternate firing takes place much higher explosion pressures are attained, due to the richer mixture then present. The power output of these small engines, due largely to their low volumetric efficiency, is usually only from about 10 per cent to 30 per cent greater than that of a well-designed four-stroke cycle engine of the same bore, stroke, and speed.

It may be noted that these engines will run equally well in whichever direction they may be started, and this feature is of value when they are used for the propulsion of motor launches, and in other cases where ready reversibility is required.

§ (13) Heavy Oil Engines. *Details of Fuels.*—With the volatile liquids or "oils" of specific gravity less than about 0·76, and flash-point usually lower than the ordinary atmospheric temperature of 15° C., as naphtha, petrol, benzol, etc., the formation of an explosive mixture with air is a simple matter, and the earlier engines accordingly used such "light oils" as they were termed. The problem of readily forming an explosive mixture of uniform composition of the heavier petroleum oils with air proved much more difficult, though it has now been fully solved, very many types of "heavy oil" engine being in everyday service using as fuel "kerosene" ("paraffin" or "lamp oil"), gas oils, intermediate oils, crude oils, residual oils, and even tar oils.

The following table gives some figures obtained from tests of samples of the more usual oil fuels:

Table IV

Description of Fuel.	Specific Gravity.	Composition by Weight, per cent.			Calorific Value in C.Th.U./Lb. (Lower).
		C.	H.	O.	
Crude coal tar	1·05	82·0	7·6	10·4	8,550
Italian refuse petroleum oil	·947	..	..	..	9,800
A heavy Russian crude oil	·938	86·6	12·3	1·1	10,200
A Russian petroleum refuse	·928	87·1	11·7	1·2	10,130
Texas fuel oil	·922	..	..	..	10,120
Blast furnace oil	·920	83·6	10·6	5·8	8,420
Astatki (Russian residual fuel oil) . . .	·906	84·9	14·0	1·1	10,500
An American heavy crude oil	·886	84·9	13·7	1·4	10,000
Benzol (a by-product in coal-gas manufacture)	·88	..	..	..	9,600
A light Russian crude oil	·884	86·3	13·6	0·1	10,800
	·875	..	..	..	10,000
"Russolene" (a refined Russian petroleum) .	·825	86·0	14·0	0	10,600
Broxburn lighthouse oil (from shale) . .	·810	86·01	13·90	0·09	10,300
"Royal Daylight" (a refined "lamp oil") .	·797	85·7	14·2	0·1	11,090
An American "Kerosene" (lamp oil) . .	·796	85·13	14·21	0·66	10,140
A heavy petrol	·760	..	..	..	10,300
A light petrol	·719	85·2	14·8	0	10,250

Figures for petrol are given to enable a comparison to be made with the heavier oils; it will be observed that for the heavy American and Russian petroleum oils the average (lower) calorific value in C.Th.U. per lb. varies but little from a mean of 10,300.

§ (14) (*g*) The Hornsby - Akroyd Oil Engine. (i.) *Description.*—Numerous devices are employed in heavy oil engines to vaporise the charge of oil forming the explosive mixture with air. The type here selected for description and illustration is that in which a combined vaporiser and explosion chamber is formed as a prolongation of the cylinder combustion chamber; of this type the best-known example is the Hornsby-Akroyd engine, of which a longitudinal section is shown in *Fig.* 9.

The engine is of the usual horizontal, single-acting, single- (or double-) cylindered, four-stroke type, but is provided at the combustion chamber end with a partly or wholly uncooled smaller vessel A, termed the "vaporiser" or "hot-bulb," which is constantly in free communication with the cylinder through a relatively narrow neck or "choke" B; the cylinder is also provided with an air inlet and an exhaust valve of the usual type, both cam-operated, which are not shown in the figure.

The oil tank is formed in the engine bed-plate; a centrifugal governor regulates the engine speed by opening a by-pass when the speed increases, thus permitting a portion of the oil delivered by the oil pump to pass back into the oil tank.

(ii.) *Method of Working.*—To start the engine the unjacketed portion of the vaporiser is first heated by a blow-pump for about 10 minutes; the fly-wheel is next turned by hand and the piston performs its suction stroke, drawing, through the inlet valve, a charge of air only into the cylinder. This air enters the cylinder direct, without passing through the vaporiser. Simultaneously a charge of oil is sprayed into the "hot-bulb" by the oil pump shown, and this at once vaporises. On the return stroke of the piston the air is compressed, and a portion passes through the neck B and mixes with the vaporised oil. The mixture is at first too rich to ignite, but the engine is so adjusted that just as compression is completed the correct explosive mixture is reached in the hot-bulb; the heat of the walls then causes explosion and the piston moves outward and performs its working stroke; this is followed by the exhaust in-stroke, and the cycle then recurs. The whole device is eminently simple and has proved very successful, and a great number of internal combustion engines, both of four-stroke and two-stroke type, are now made on the "hot-bulb principle."

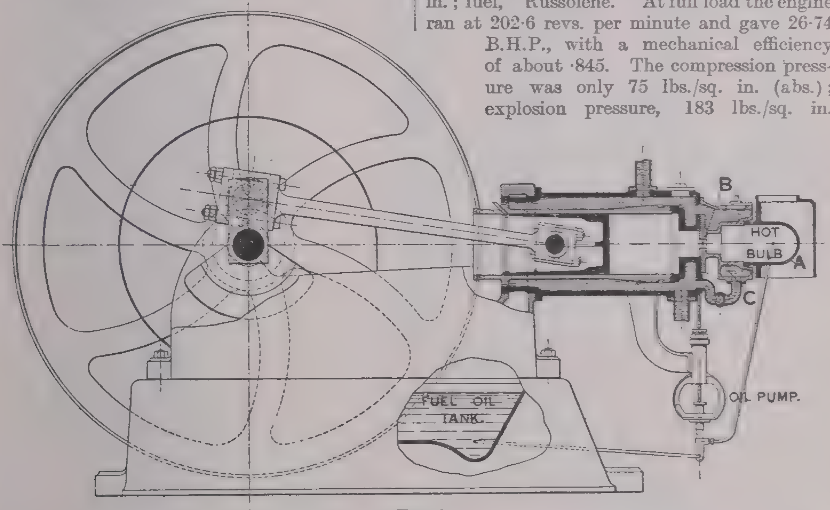

Fig. 9.

After running for a few minutes it is found that the blow-lamp may be removed, the hot-bulb temperature being thereafter sustained by the heat communicated to it from the successive explosions; the ignition then becomes completely automatic. If the vaporiser be allowed to become too hot pre-ignition occurs, and the oil may also be "cracked," with resulting deposition of carbon, which in time chokes up the vaporiser. The temperature is regulated by admitting more or less water to the jacket C which surrounds the neck and part of the vaporiser itself. In many hot-bulb engines, however, pre-ignition at heavy loads is avoided by allowing a few drops of water to enter the chamber during the later stages of compression. On the other hand, the vaporiser may become too cool when the engine is run at light load, and in this case the lamp must be used.

(iii.) *Performance.*—A test of a 25 horse-power Hornsby-Akroyd oil engine made by Professor Robinson in 1898 gave results as follow: Piston diameter, 14·5 in.; stroke, 17 in.; fuel, "Russolene." At full load the engine ran at 202·6 revs. per minute and gave 26·74 B.H.P., with a mechanical efficiency of about ·845. The compression pressure was only 75 lbs./sq. in. (abs.); explosion pressure, 183 lbs./sq. in. (abs.); and mean effective pressure about 44 lbs./sq. in.

The oil per B.H.P. hour was 0·74 lb., corresponding to an absolute brake thermal efficiency of ·185. The best compression pressure for use with any given oil is found by experience; thus "Russolene" was found to permit a higher compression, and to give 15 per cent to 20 per cent more power than "Royal Daylight." Compression may be varied by fitting different-sized vaporisers, and in engines of over about 50 B.H.P. by fitting distance-pieces to the crank-pin end of the connecting rod in addition. These engines run best on kerosenes, and particularly with the standardised Russian oil of ·825 sp. gr. and 30° C. flash-point by Abel close test; they have, however, also been run on crude, gas, and residual oils.

Later tests have furnished still better results; thus in 1908 Professor Robinson obtained from a 32 B.H.P. Hornsby-Akroyd engine using Russolene (cal. value in this case 10,250 C.Th.U. per lb.) a consumption of only 0·613 lb. per B.H.P. hour, corresponding to an absolute brake thermal efficiency of ·226. Single-cylindered engines are constructed up to 185 B.H.P., and with two cylinders to 370 B.H.P.

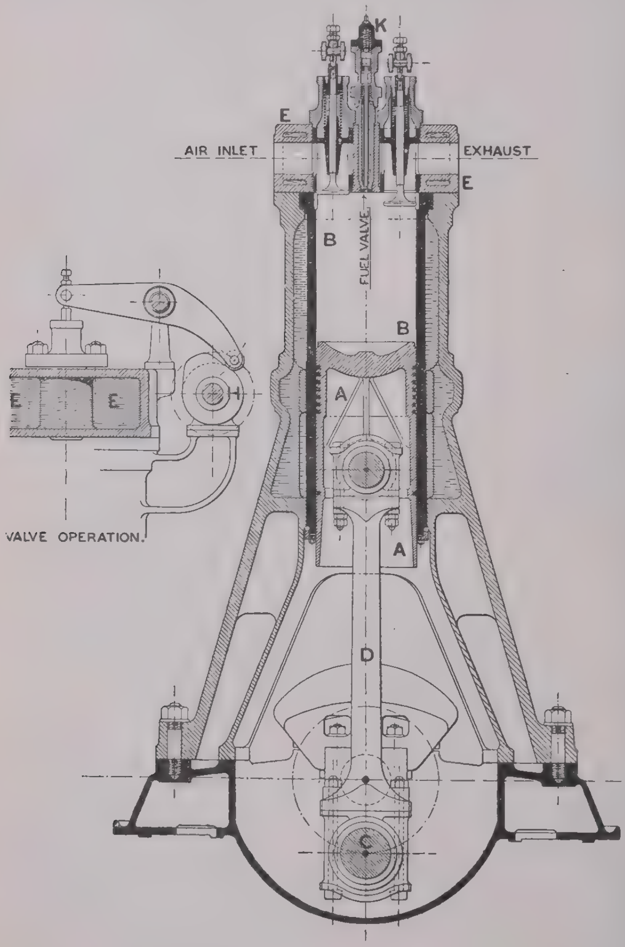

FIG. 10.

§ (15) (*h*) THE "CONSTANT-PRESSURE" CYCLE DIESEL ENGINE. (i.) *Description.*—The leading characteristics of the Diesel engine are: (1) the compression of air only up to the maximum pressure attained in the cylinder, usually from 450 to 500 lbs. per sq. in.; and (2) the regulated admission of the fuel, usually a heavy mineral oil—blown into the combustion chamber by a blast of very high pressure air—in a state of extremely fine spray, during the first part of the working stroke, so that it burns at approximately constant pressure, the ignition occurring automatically from the high temperature developed in the adiabatically compressed air. The cycle has been considered from the thermodynamic standpoint in the article on "Engines, Thermodynamics of Internal Combustion."

A section through a usual type of four-stroke Diesel engine is shown in *Fig.* 10. It will be seen that the engine is of the inverted vertical single-acting type, comprising a long and heavy piston A working within a well water-jacketed cylinder B, and driving the crank-shaft C through the usual type of connecting rod D. In the deep water-cooled detachable cylinder head E are situated the air inlet and exhaust valves (not shown in the diagram), which are of the usual "poppet" or "mushroom" type, normally held up to their seats by springs and opened inwards by cams on an overhead shaft H through the agency of roller-ended rocking levers as indicated. The shaft H is of course driven at half the speed of the crank-shaft. The fuel ignition valve K is also located in the cylinder head, and in recent designs is generally placed in the centre; it is a fundamental difficulty in all internal combustion engines wherein the fuel is sprayed into the combustion chamber at or near the instant of maximum compression to secure a uniform mixture giving rapid and complete combustion, and some of the earlier Diesel designs failed largely from this cause alone. By placing the fuel injection valve in the centre of the cover, and so forming its orifice—and often the upper surface of the piston crown also—as to assist in the instant formation and uniform dispersion of the cloud of spray throughout the combustion chamber space, this difficulty is overcome.

(ii.) *Method of Working.*—The engine is started as follows: By means of a hand lever the crank-shaft is racked round until the crank-pin is just over the top centre; next a starting lever is operated by which a starting cam is brought into working position. The fuel blast reservoir valve and starting reservoir valve being next opened, air from the latter at a pressure of 700 to 900 lbs. per sq. in. enters the cylinder through a small starting valve situated in the cover, and the engine immediately moves off. After a few revolutions under the compressed air from the reservoir the starting lever is moved back, and the engine at once takes up its normal working cycle and continues to run. The full revolution rate is attained at once, but the normal power output is not reached until the cylinder is well warmed up.

In the case of a new engine the air reservoir for starting is sent out fully charged; thereafter its pressure is maintained by a small pump driven by the engine itself.

The charge of fuel is also blown into the cylinder through the fuel injection valve by a blast of air from this reservoir.

The fuel injection valve comprises a needle valve held down on its seat by a spring and lifted by a cam-operated lever. A fuel force-pump delivers the charge of oil fuel into a narrow annular space surrounding the needle which is also in constant communication with the air-blast reservoir during the running of the engine. Hence immediately the needle valve is raised the charge of oil is blown with great velocity into the combustion chamber through an expanding nozzle, in the form of a uniformly diffused cloud of mist which instantly inflames in the adiabatically compressed air where temperature is at this instant from 500-550° C. The fuel is caused to enter the combustion chamber just before the completion of the compression stroke, and at full load the injection is continued during the first 20° to 30° of crank-shaft revolution, the mixture burning at approximately constant pressure during admission. Engine speed is controlled by a governor actuating a by-pass valve in the fuel pump supply whereby a variable proportion of the pumped oil is returned into the suction pipe as the engine load is varied.

The fuel injection needle valve must be regularly cleaned at intervals of about a fortnight; a sticky needle valve may cause pre-ignition through leakage of fuel oil during the compression stroke; such a defective valve, moreover, allows the very high pressure blast air to enter the cylinder in abnormal quantities, and this also may cause rupture of the cylinder through excessive pressure caused by its subsequent further compression by the rising piston. Provision is sometimes made against excessive pressure by fitting relief valves in the cylinder head.

The high-compression and air-blast pressures employed necessitate workmanship of the highest quality in the construction of the Diesel engine, and the necessity of providing against occasional abnormal pressures in the cylinder renders this type of engine somewhat heavy in relation to power output. The high compression also necessitates the fitting of very heavy fly-wheels in few-cylindered four-stroke engines in order to attain the requisite uniformity in rotation of the crank-shaft. For land Diesel engines up to 200 B.H.P. the weights per B.H.P. (including fly-wheels) are, roundly, as follows (compare Table I.):

For single-cylindered engines .	600 lbs.
For two-cylindered engines .	520 „
For three-cylindered engines .	350 „
For three-cylindered engines (without fly-wheels)	275 „

(iii.) *Fuels.*—Experiments have been conducted with a great variety of fuels, including petrol, kerosene (lamp oil), gas oil, crude Russian, American, and German mineral oils, Astatki, shale oils, coal-tar oils, lignite oils, palm and nut oils, castor oil, fish oil, alcohol, coal gas, producer gas, and coal dust. The greatest success has been attained with the kerosenes and heavier petroleum oils, and it is on these, and particularly on the heavy dark-brown crude Texan fuel oil of sp. gr. about ·925, flash point about 85° C., and (lower) calorific value about 10,100 C.Th.U. per lb.—as largely used for firing steam boilers—that most Diesel engines in Great Britain are run. Dr. Allner, in a paper read before the German Gas Association in 1911, stated that the tar produced in large quantities in the coal-gas and coke oven industries can be used successfully as a fuel for Diesel engines, provided a small quantity of a readily ignitable "pilot" fuel, as Texas oil, be injected either just before, or simultaneously with, the admission of the tar to the combustion chamber; the combustion of the pilot fuel starts that of the heavier fuel. Using gas oil as the pilot fuel, Dr. Allner affirms that crude tar may be used in Diesel engines. In general, however, it is held to be a desideratum that the fuel employed should be free from tar, sulphur, and acid impurities. Coal dust, injected with a proportion of "lean gas," has been used — under experimental conditions only—as a fuel, and furnished indicator diagrams of normal type; but the difficulties of utilising any solid substance as a fuel under ordinary conditions of working have yet to be overcome.

(iv.) *Performance.*—The economy of fuel with the four-stroke Diesel engine is very marked; a trial of a two-cylindered, 160 horsepower, four-stroke Diesel engine by Mr. Ade Clarke in 1903 furnished the following results:

Piston diameter, 15·75 in.—stroke, 23·6 in.

Fuel—Texas oil, 0·922 sp. gr.; calorific value, 10,720 C.Th.U. per lb. (higher value).

The outlet temperature of the cooling water, at full load, was 67° C. In the "No load" trial only one cylinder was producing power, the other pumping idly.

TABLE

TEST RESULTS OF TWO-CYLINDER, FOUR-STROKE, 160 H.P. DIESEL ENGINE

No.	Item.	At Load.				
		0.	¼.	⅜.	½.	Full.
1	Duration of trial, minutes	30	60	61	60	60
2	Average revolutions per minute	159	158	158	157	154·5
3	Average mean effective pressure, p, in lbs./sq. in.	43·0	42·1	56·3	69·4	114·0
4	Total I.H.P.	39·6	74·9	100·0	126·8	204·4
5	H.P. absorbed by air compression pump	2·98	2·97	3·0	3·25	3·30
6	Net I.H.P.	36·62	71·93	97·0	123·6	201·1
7	Estimated B.H.P.*	0	35·3	60·4	87·2	164·8
8	Mechanical efficiency, η per cent †	0	47·2	60·4	68·8	80·7
9	Oil per (total) I.H.P. hour, lbs.	·384	·342	·299	·321	·328
10	Oil per B.H.P. hour, lbs.	..	·725	·496	·467	·406
11	Rise of temperature of cooling water in ° C.	57	56	51	43	52·5
12	Temperature of exhaust gases ° C.	..	158	197	240	384
13	Total indicated absolute thermal efficiency	·344	·386	·441	·412	·403
14	Absolute brake thermal efficiency	..	·183	·266	·283	·326

* The B.H.P. was taken as total I.H.P. − 39·6.

† The mechanical efficiency was taken as B.H.P./total I.H.P.

In May 1909 Mr. A. J. Pfeiffer stated before the Inst. of Electrical Engineers that there was then in service a four-cylindered, inverted-vertical, single-acting four-stroke Diesel engine giving 800 B.H.P. at 150 revs. per minute, and that this was about the largest that will run satisfactorily without water- or oil-cooling the pistons and exhaust valves.

§ (16) (*i*) TWO-STROKE DIESEL ENGINES. (i.) *Description.*—Marine Diesel engines are usually of the two-stroke type, single-acting, which possess advantages over the four-stroke of reduced weight and space, lower production cost, and greater simplicity in reversing. There are two principal types of two-stroke, single-acting Diesel engines, in both of which the piston overruns a ring of exhaust ports when near the bottom of its stroke—as in the Clerk engine. In the one type—which is rather the more efficient—air inlet valves are provided in the cylinder head which are opened just after the exhaust ports are overrun, and effect a very complete scavenging action, leaving the cylinder filled with nearly pure air which is compressed by the piston during its return stroke as usual. In the other type, which has been developed largely by Messrs Sulzer Bros., of Winterthur, the air inlet valves are dispensed with, and inlet ports provided in the lower parts of the cylinder which are overrun by the piston shortly after the opening of the exhaust ports; the scavenging is not quite so good, but the engine is of greater simplicity in construction. As in all Diesel engines the fuel is not admitted until nearly the end of compression, any short-circuiting of air through the exhaust ports during its admission is immaterial. In 1911 a four-cylinder, two-stroke, inverted-vertical, single-acting engine of 2400 B.H.P. of Sulzer type was installed in an electric generating station in France.

(ii.) *Difficulties.*—One of the most serious troubles with Diesel engines arises from the loss of compression pressure due to the wear of piston and cylinder walls in the usual design, wherein the piston is directly connected to the connecting rod. This wear is mainly caused by the side thrust due to the obliquity of the rod, and hence in large engines, and especially in marine service, the external crosshead is favoured, as with this the cylinder is relieved of all side pressure. Attention is being given to the production of satisfactory designs of *double*-acting, two-stroke Diesel engines for marine service; the chief difficulty to be overcome is that of providing adequate cooling arrangements.

§ (17) THE RUSTON "COLD STARTING" ENGINE. (i.) *Description.*—Messrs. Ruston, Proctor & Co. have recently produced a valuable design of Diesel type in respect of the high compression of the air and automatic ignition of the fuel, but in which the costly, complicated, and sometimes troublesome air compressor used in the Diesel engine in connection with the fuel injection is completely eliminated.

A section through the cylinder of the 1915 design of "cold starting" engine is given in *Fig.* 11[1]; the engine is of the horizontal, single-cylindered, single-acting, four-stroke type, and is built in sizes ranging from 20 to 170 horse-power.

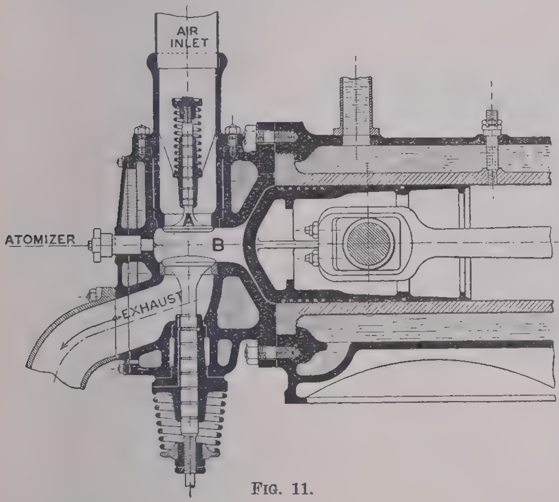

FIG. 11.

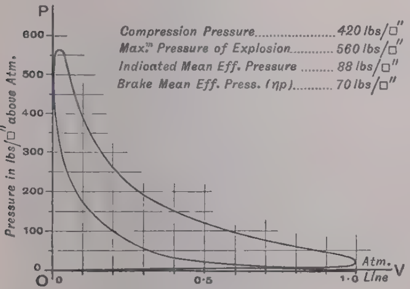

FIG. 11A.

On the suction stroke the piston draws in air only through the valve A, subsequently compressing this into the small explosion chamber B to a pressure of about 430 lbs. per sq. in., which is regarded as giving a sufficiently high temperature to ensure the ignition of any fuel oil. Just before compression is completed the charge of oil is injected directly into the explosion chamber through an "atomiser," whereupon it immediately ignites, and the working stroke then follows; a normal full-load indicator diagram is shown in *Fig.* 11A.

Very perfect "atomisation" of the fuel is essential to high engine efficiency; the Ruston atomiser comprises a spring-closed needle valve and a number of small oil ducts converging to a central orifice. A small fuel force-pump operated by a quick-acting or "steep" cam compels the charge of oil delivered by it to lift the spring-closed needle valve; the charge is by this means caused to enter the explosion chamber in the form of an exceedingly fine spray or "mist"; the lift of the needle valve is of the order of two- or three-hundredths of an inch only.

(ii.) *Method of Working.*—By means of air stored at a pressure of 200-300 lbs. per sq. in. in a reservoir the engine is readily started, and after one or two revolutions picks up its working cycle; starting is thus effected from cold, and the engine may be run on any of the usual grades of fuel oil. When tar oils are employed it is necessary to use about 5 per cent of a more readily ignitable or "pilot" fuel to initiate combustion; this is introduced by a special atomiser fitted with a small needle valve inside a tubular, main fuel needle valve, served by a pilot oil pump. The spring-loaded plunger of this pilot pump is operated by fluid pressure from the main fuel pump in such manner as to absolutely ensure the injection into the cylinder of the pilot igniting charge immediately before that of the main charge. Test results show that, using a fuel of 0·92 sp. gr. and calorific value of about 10,000 C.Th.U. per lb. (lower value), the consumption per B.H.P. hour ranges from 0·48 lb. in the 20 horse-power engine, down to 0·40 lb. in the 170 horse-power size. This corresponds to an absolute brake thermal efficiency of about 0·29 to 0·35, which compares favourably with ordinary Diesel practice.

§ (18) "SEMI-DIESEL" ENGINES.—The great economy of fuel consumption of the Diesel engine has caused many attempts to be made to produce designs in which, without much sacrifice of mean effective pressure and economy, production cost and weight are saved without having recourse to the high-compression pressure used in the Diesel engine, and further, by dispensing altogether with the costly and complicated high-pressure air blast which has so often proved a source of trouble and even danger.

As such designs have been evolved from a study of the performance of Diesel engines they have come to be styled "semi-Diesel," though, as they employ a hot-bulb to ensure ignition of the sprayed charge of oil, they would be more justly described as "Akroyd" engines.

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[1] From Livens on Oil-engines, *Proc. I. Mech. E.*, July 1920.

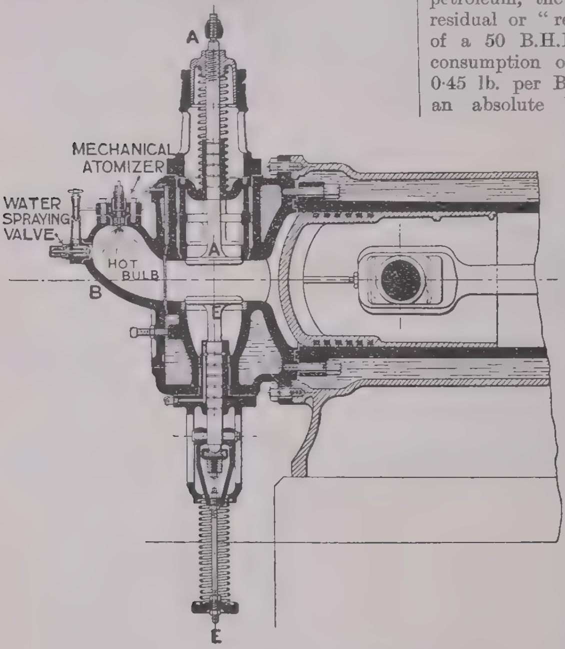

Fig. 12.

(i.) *The C.C. Engine.*—A good example of this class is the "C.C." engine of Messrs. Ruston, Proctor & Co., of which a section through the combustion chamber end of the cylinder is shown in *Fig.* 12; the engine is of the horizontal, single-acting, four-stroke type.

Through the inlet valve A, air alone is drawn during the suction stroke, and this is next compressed into the combustion chamber and hot-bulb B. Nearly at the instant of maximum compression the charge of oil fuel is injected into this hot-bulb by a force-feed pump and atomiser as in the case of the "cold starting" engine described in § (17). The mixture is automatically ignited by the hot-bulb as in the Hornsby-Akroyd engine, § (14), and after performing the working stroke the burnt gases are discharged through the exhaust valve E. The compression pressure employed is only about 275 lbs. per sq. in., and mean effective pressures as high as 95 lbs. per sq. in. are attained at full load. These engines are built in single-cylindered type up to 80 B.H.P. running at 190 revolutions per minute, and in two-cylindered designs up to 130 B.H.P. at 200 revolutions per minute. As fuel, crude petroleum, the ordinary fuel oils, and even residual or "refuse" oils are used. A trial of a 50 B.H.P. engine in 1910 showed a consumption of crude Russian oil of only 0·45 lb. per B.H.P. hour, corresponding to an absolute brake thermal efficiency of ·314.

Starting is effected by compressed air; the time occupied in starting from cold, owing to the necessary heating of the hot-bulb by a blow-lamp, is 10 to 15 minutes. After a short period of running the blow-lamp is removed, and the hot-bulb temperature is then maintained by heat derived from the successive explosions; the ignition is then automatic, as in the Hornsby-Akroyd engine. To prevent pre-ignition "knock" from occurring, a water drip is used at three-quarters full load and above; the water-spraying valve is indicated in *Fig.* 12. After prolonged running at light loads the bulb may become too cool, and recourse must then be made to the heating lamp in order to maintain its temperature. In this class of engine the combustion of the fuel is, in general, partly at constant volume and partly at constant pressure.

D. C.
G. A. B.

Entropy. If in a reversible change a substance receives or loses a quantity of heat Q at an absolute temperature θ, the substance is said to gain or lose an amount of entropy given by Q/θ. See also "Thermodynamics." For Entropy of Fluids see § (33); Entropy of Ideal Gas, § (57); Entropy of Mixtures, § (62); Entropy-temperature Diagram, § (26); "Engines, Thermodynamics of Internal Combustion," §§ (9), (10), (25).

Entropy-temperature Diagram. A diagram in which the condition of a body is represented by the position of a point whose co-ordinates are temperature (ordinate) and entropy reckoned from some standard condition (abscissa). See "Thermodynamics," § (26).

Diagrams for Ammonia and Carbonic Acid. See "Refrigeration," § (2), *Figs.* 5 and 6.

Equation of State, Definition and Use of. See "Thermodynamics," §§ (56), (57); "Thermal Expansion," §§ (21), Table, (22), (27).

Equipartition of Energy, Law of. See "Thermodynamics," § (66).

Ergometer. An instrument for measuring and recording accelerations and retardations. See "Dynamometers," § (5) (vi.).

Ericsson's Regenerative Engine. See "Thermodynamics," § (27).

Escargot Fan Brake. A brake convenient for testing air-screws. See "Dynamometers," § (2) (viii.).

Evacuation. See "Air-pumps."
  By Absorption. § (52).
  By Absorption in the Electric Discharge. § (53).
  By Chemical Action. § (51).
  By Condensation. § (50).
  Miscellaneous Methods. § (49).

Evacuators, Theory of. See "Air-pumps," § (1).

Evaporation under Constant Pressure. See "Thermodynamics," § (29).

Expansion, Area and Volume, Coefficients of, deduced from the linear coefficient. See "Thermal Expansion," § (5).

Expansion, Linear, Coefficient of:
  At very low temperatures, determined by experiment for various substances and tabulated $\lambda=(dl/dT)l$. See "Thermal Expansion," § (7).
  For various substances, determined by experiment and tabulated. See "Thermal Expansion," § (7).

Expansion, Linear, Mean Coefficient of:
  Between 0° and $t°$ C., defined by the equation

$$l_t=l_0(1+\lambda t),$$

  where the distance between two points in the body at 0° C. is l_0 and at $t°$ C. is l_t, and λ is the mean coefficient of linear expansion. See "Thermal Expansion," § (1).
  Fizeau's Interference Method of measuring, depending upon the colours of thin plates. See *ibid.* § (3).

Expansion of Fluids: theoretical considerations accounting for the departure of their behaviour from the laws of perfect gases. See "Thermal Expansion," § (19).

Expansive Working in Steam Engines. See "Steam Engine, Reciprocating," § (2) (ix.).

Extensometers. See "Elastic Constants, Determination of."
  Dial Instruments with Mechanical Magnification. § (49).
  Double Micrometer Screw Instruments. § (45).
  Ewing's Combined Microscope and Lever Extensometer. § (51).
  Indicating Dial Instruments. § (48).
  Instruments combining a Single Micrometer Screw with a Multiplying Lever. § (47).
  Martens's Mirror Extensometer. § (53) (ii.).
  Methods of arranging and General Principles to be fulfilled. § (43).
  Methods of Calibration. § (58).
  Micrometer Screw Extensometers. § (44).
  Microscopic Reading Instruments. § (50).
  Morrow's Single Mirror Apparatus. § (53) (iv.).
  Multiplying Lever Instruments. § (52).
  Using Optical Magnification. § (53).

External Pressure Correction to a Thermometer. See "Thermometry," § (3) (iii.).

— F —

Factor of Safety. See "Structures, Strength of," § (1).

Fahrenheit Scale of Temperature: a scale on which the numbers 32 and 212 correspond respectively to the freezing- and boiling-points of water. See "Thermometry," § (2).

Falling Weight Test—called "drop test" in U.S.A. See "Elastic Constants, Determination of," § (35).

Fan Brakes. See "Dynamometers," § (2) (vii.).

Fixed Points:
  Thermometric. See "Thermodynamics," § (4); "Temperature, Realisation of Absolute Scale of," § (3); "Thermometry," § (3).
  Used as secondary standards of temperature and compared with a gas-thermometer in the range −273° to 0° C. See "Temperature, Realisation of Absolute Scale of," § (31).

Fixed Points, Interpolated:
  On secondary standards of temperature between 100° and 500° C. See "Temperature, Realisation of Absolute Scale of," § (36) (iv.).
  Used as a secondary standard of temperature and compared with a gas-thermometer in the range above 500° C. See *ibid.* § (42) (iii.).
  In range 100° to 500°, compared with gas-thermometer determinations and tabulated. See *ibid.* § (36), Table 10.
  In range 500° to 1600°, compared with gas-thermometer determinations and tabulated. See *ibid.* § (42), Table 13.

Fixed Temperatures :

Used as a secondary standard of temperature and compared with gas-thermometers. See "Temperature, Realisation of Absolute Scale of," § (24).

Used as secondary standards and compared with a gas-thermometer in the range 100° to 500°.

1. Melting-point of zinc.
2. Boiling-points of naphthalene, diphenyl, and benzophenone.
3. Boiling-point of sulphur.

See "Temperature, Realisation of Absolute Scale of," § (35).

Combination of, with a thermal property, used for the realisation of gas-thermometer scales in the form of secondary standards. See *ibid.* § (26).

Flash-point Apparatus. See "Flash-point Determination."

The Abel. § (2).

The Abel-Pensky. § (3).

The Gray. § (5).

The Pensky-Martens. § (5).

FLASH-POINT DETERMINATION

§ (1) Introduction.—The "flash-point" of a substance may be defined as that temperature at which it begins to evolve vapour in such quantity that, on the application of a flame, a momentary "flash" occurs, due to the ignition of the vapour. This temperature is not a definite physical property of the material, but varies with a number of subsidiary factors, such as the rate of heating, the amount of ventilation, the size of the applied flame, and so forth, and is thus dependent on the apparatus used and the conditions under which the determination is made.

Notwithstanding the empirical nature of this "constant" for an inflammable substance, it serves to classify such materials into varying degrees of danger with regard to fire risk, and from this point of view the determination of flash-point has been the subject of legislation and of official regulations.

A short consideration of the phenomena occurring when a sample of oil is slowly heated will serve to distinguish flash-point, burning-point, and ignition-point. If the oil is placed in a small metal basin or crucible and slowly heated, vapour is evolved at a rate depending upon the nature of the oil and its temperature. If a small test flame be applied periodically just above the surface of the oil, no result will be apparent so long as the temperature is well below the value known as the "flash-point" of the oil, but as this is approached the test flame will enlarge when in the neighbourhood of the surface of the oil, and at a temperature a few degrees higher this enlargement becomes more apparent and a flame rapidly travels over the surface of the oil and immediately dies away. This transient ignition of the vapour is termed the "flash," and the lowest temperature at which it is manifest is known as the "flash-point" of the oil. If the heating of the oil and the periodic application of the test flame be continued flashing will also continue, and it will be noticed that the duration of the flash becomes longer as the temperature is raised, until a point is reached when the vapour burns continuously instead of merely flashing; the lowest temperature at which this occurs is known as the "burning-point" of the oil. In general the heat generated by the combustion will rapidly raise the temperature and the combustion will become more violent, and will be maintained without the application of any external source of heating.

If now the experiment is repeated without the application of a test flame, there will be no apparent result until a temperature higher than the burning-point is reached, when the oil will spontaneously take fire and will continue to burn. The lowest temperature at which this takes place is known as the "ignition-point."

The above phenomena are dependent on the accumulation of sufficient vapour in the atmosphere above the oil to form a combustible mixture, and this will be largely influenced by any circulation which is taking place in the surrounding air. If the air is in motion, even to a small extent, some of the vapour will be carried away, and it will be necessary to raise the oil to a higher temperature to secure the same concentration of vapour as would be obtained in perfectly still air. In consequence it is not easy to obtain concordant values for the flash-point, burning-point, and ignition-point by the heating of the oil in an open cup as above described, and other types of apparatus have been designed to reduce the difficulty of making reasonably accurate determinations. In most forms of this apparatus the oil cup is provided with a cover having suitable apertures for the introduction of the test flame and for ventilation purposes. For distinction the two types of test are usually referred to as the "open test" and the "closed test" respectively.

The first apparatus of the "closed test" type was introduced by Sir Frederick Abel in 1876, and was subsequently known as the Abel Flash-point Apparatus. A large number of tests were made to connect the results obtained by the use of this apparatus with those given by the open test then in use. It was found that the mean value of the difference between these results was 27° F. for an oil which flashed at 100° F. on the open test.

Later, in 1880, a modification of this instrument was made by Pensky, who applied a clockwork device to the oil cup cover to perform the opening and closing of the shutter and introduction of the test flame. This apparatus is referred to as the Abel-Pensky Flash-point Apparatus. The provision of this mechanism is probably advantageous from the point of view of ease of operation, but, as will be seen later, it does not tend to any increased accuracy of determination of the flash-point.

The two apparatus above referred to were designed with special reference to the measurement of flash-points over the approximate range 60°-110° F. in connection with the legislation relating to the sale, storage, and transport of illuminating oils; later, however, a demand arose for the determination of the flash-point of oils for lubricating and other purposes, leading to the evolution of such types of apparatus as the Gray and Pensky-Martens flash-point apparatus, which may be employed up to much higher temperatures. The essential difference between these and the earlier forms of apparatus intended for the lower range only is the provision of stirrers in the oil cups.

§ (2) Abel Flash-point Apparatus.—The official specification for the Abel flash-point apparatus will be found in the schedule to the Petroleum Act of 1879 (42 & 43 Vict. c. 47); full details as to the construction and use of the apparatus are also to be found in the several text-books relating to oil and oil testing.[1] The principal dimensions are shown in *Fig.* 1.

An investigation was carried out at the National Physical Laboratory to ascertain the differences existing between the Abel and other types of flash-point apparatus.[2]

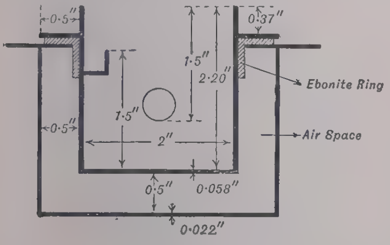

Fig. 1.

An important consideration that arises in the determination of the flash-point is the limit of accuracy attainable. The legal specification directs that the test flame should be applied to the oil every 1° F., and consequently the accuracy of any single determination is limited to 1° F. Departure from the specified conditions gives a different value for the resulting flash-point, as will be seen later. Difficulty, however, arises in the interpretation of a set of results for any one sample of oil, since even when the greatest care is taken individual readings may differ by 1° or 2° F. Furthermore, changes in the barometric pressure produce variations in the resulting flash-point, and Sir Frederick Abel investigated this matter and found that the flash-point was raised by 1·6° F. for an increase of 1 inch in the barometric reading. Hence it is usual to correct all flash-points to a standard barometric reading of 30 in. Corrections for errors of the thermometer, if any, must also be taken into account; the most satisfactory way, therefore, of dealing with a set of observations is to obtain the mean result and apply the necessary corrections for pressure and for the thermometer. The true flash-point is then taken as the next higher whole number of degrees, since the flash-point of a substance as defined in the official regulations cannot be other than a whole number of degrees.

Before intercomparisons between the several types of flash-point apparatus could be carried out it was found necessary to investigate the conditions of use of the Abel apparatus more fully than appeared to have been done previously. Among the points to which attention was given were:

(*a*) Frequency of application of the test flame.

(*b*) Variations in the time of opening of the slide.

(*c*) Variations in the temperature of the water bath.

(*d*) Variations in the depth of the thermometer bulb below the surface of the oil.

(*e*) Size of the test flame.

The results of these experiments led to the following conclusions:

(*a*) Increasing the frequency of application of the test flame raises the flash-point; for example, if the test flame is applied every ½ degree the resulting flash-points are a degree higher, while if applied every ¼ degree the flash-point is raised by over 3 degrees; conversely, a lower flash-point is obtained if the test flame is applied at intervals greater than 1° F.

(*b*) Contrary to the general opinion, the effect of increasing or decreasing the time for which the slide is open does not affect the results except for extreme changes. Thus the provision of an automatic mechanism to secure constancy in this condition is not essential. It is interesting to note in this connection that a number of the Abel-Pensky automatic covers were investigated and it was found that their times of opening differed to a considerable extent. This might well be expected, as it is improbable that the strength of the spring would remain constant under the conditions of use.

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[1] *Petroleum and its Products*, Sir Boverton Redwood, 1906, ii. 550; *Handbook of Petroleum*, J. H. Thomson and Sir B. Redwood, 1906, vi. 82.

[2] *Collected Researches, N.P.L.*, 1912, viii. 19.

(*c*) A change in the temperature of the water bath produces a corresponding change in the flash-point, the general effect being that the flash-point is lowered by 1° F. for each 13° F. rise in the temperature of the water bath above the normal temperature 130° F.

(*d*) and (*e*). The depth of immersion of the thermometer bulb below the surface of the oil and the size of the test flame were seen to play very important parts in flash-point determinations, and the question will be considered later.

§ (3) THE ABEL-PENSKY APPARATUS.—The apparatus adopted in 1880 by the German Government as the official standard design of instrument for flash-point determinations was based upon the Abel apparatus, but modifications were introduced by Pensky, who substituted an automatic device for opening and closing the ventilation holes and applying the test flame in place of the simple slide in the English Abel apparatus. In addition, the dimensions were slightly altered and were expressed in metric units instead of inches, as in the Petroleum Act of 1879. *Fig.* 2 indicates the important dimensions of the Abel-Pensky apparatus in accordance with the German schedule. Very complete details are published in the official specification[1] and will not be further dealt with here.

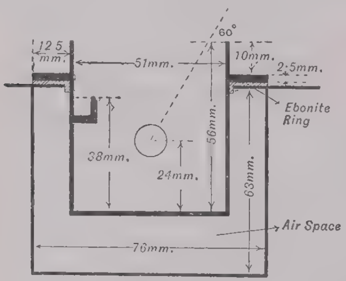

FIG. 2.

The Abel-Pensky apparatus was also adopted by the British Colonies and India under the Indian Petroleum Act of 1889, the German type of apparatus being used although the dimensions of the Abel apparatus were retained. Two modifications have been made to the Colonial type of apparatus for special purposes; these provide for a small additional thermometer and a stirrer in the oil cup for use when obtaining the flash-point of inflammable substances such as rubber solution, metal polishes, etc., which may fall within the scope of petroleum legislation. In both these types of apparatus the effects of variations in the methods of procedure are the same as are described for the Abel apparatus.

§ (4) THE GENERAL THEORY OF FLASH-POINT DETERMINATION.—The general theory of flash-point determination depends on the hypothesis that flashing takes place when the space above the oil contains a definite percentage of oil vapour mixed with air. This condition will be reached for a definite temperature of the oil surface from which evaporation is taking place, and it is generally assumed that it is this temperature which is given by the thermometer and is the temperature taken as the flash-point. The rate of evaporation, however, depends upon the temperature of the surface of the oil, and investigation showed that this temperature differed appreciably from the thermometer reading throughout the course of a determination. Furthermore, the temperature at different points throughout the oil varied by several degrees at any one moment. The temperature distribution depended in part on the form and dimensions of the apparatus and on the relative amounts of heat reaching the oil from different sources. The method of investigation employed was to ascertain these temperature differences by means of differential thermocouples of very fine wire introduced in such a way as to avoid interference with the usual conditions of test. The main source of heat through which the rise of temperature of the oil is derived is of course the water bath surrounding the oil cups through the intermediary of the air space; but it was found that the temperature at the surface of the oil and that of the vapour were materially influenced by the heating derived from the test flame itself. The importance of the size of the test flame is therefore obvious, and as the result of a special series of experiments it was found that an increase of 2° F. in the flash-point was obtained when the size of the oil-burning test flame in an Abel apparatus was decreased to about half the normal diameter. Similar results were found for the gas test flames also in general use. The explanation of this difference is that with the smaller test flame the oil surface does not receive so much heat indirectly, and consequently the temperature of the bulk of the oil has to be raised to produce the same surface temperature; thus the flash-point is apparently higher. For exact work it may be remarked that the ivory bead on the cover of the apparatus is inadequate as a gauge for the adjustment of the size of the test flame. It is therefore preferable to employ a gas jet to which the supply of gas is controlled by a gas meter. In the experimental work to which reference is being made the gas rate adopted was 0·10 cub. ft. per hour.

Investigation of the three types of apparatus, namely, the Abel, and the Colonial and the

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[1] *Die Vorschriften betreffend den Abel'schen Petroleumprober und seine Anwendung*, published 1883 by Carl Hermann, Berlin.

German types of Abel-Pensky apparatus, showed that the temperature distribution in each varied, so that for any definite reading of the thermometer the temperature of the surface of the oil was different in each of the three apparatus. It was further found that the temperature of the surface of the oil at the moment when flashing took place was the same in each of the three apparatus although the thermometer readings differed; the difference in flash-point recorded in the three types of apparatus is therefore completely explained by the consideration of the temperature distribution in the oil cups.

The general results of the investigation showed that the Colonial type of Abel-Pensky apparatus gave a flash-point 1° F. higher than the original form of Abel apparatus, while the German type of Abel-Pensky gave results approximately 4° F. higher; further, these differences are sensibly constant over the range 70°-100° F. Incidentally there is a systematic difference amounting to about 0·5° F. (taking the mean values for a large series of observations) between apparatus of the same type but fitted with oil and gas test flames respectively. As might be expected from the previous remarks, the gas test flame gives the lower flash-point, the flame supplying more heat to the surface of the oil, when adjusted to the specified size.

Further light is thrown upon the mechanism of flash-point determination by the results of a series of tests carried out at the National Physical Laboratory at a later date.[1] The tests in question were carried out on a number of fuel oils, using the Gray and Pensky-Martens apparatus, and also a modified form of apparatus on a much larger scale, which dealt with about 50 gallons of oil instead of the usual quantity. The latter apparatus was designed with a view to ascertaining whether a large quantity of oil would flash at a temperature appreciably lower than the value obtained in the usual types of apparatus. The conditions were varied over a wide range in order to study the effect of different methods of ignition and the importance of ventilation of the vapour space above the oil. Dealing with the latter point, it was clearly seen that combustion of the vapour occurred prior to an actual flash being obtained; this is shown by the enlargement of the applied test flame at temperatures below the flash-point. If the space above the oil is not adequately ventilated it becomes charged with the products of combustion, which may lead to the extinction of the test flame or give an apparently high result. On the other hand, forced ventilation of the surface by the passage of a current of air apparently raised the flash-point, as in this case the oil vapour is swept away before ignition can occur. Consequently the oil has to be raised to a higher temperature in order to evolve vapour fast enough to maintain an inflammable mixture.

A variation of this question was also investigated. In this instance the oil was maintained at a constant temperature and the vapour was allowed to collect for periods of varying length, the test flame being applied after definite intervals had elapsed. Between such applications of the test flame the space above the oil was completely cleared of accumulated vapour by blowing a current of air through the space. The result of such tests was to show that the higher the temperature of the oil the shorter the interval that was necessary for the vapour to accumulate before the flash could be produced. At the lowest temperature at which it was possible to obtain any ignition of the oil vapour a comparatively long period (from 5 to 10 min.) was required before sufficient vapour had accumulated, and in these instances the flash, when it did occur, was generally of a violent nature. With regard to the effect of different methods of ignition, it was found that the most satisfactory results were obtained with a moderate-sized gas flame. Electric sparks or hot wires generally necessitated a higher temperature of the oil before flashing took place, and with the high temperatures a violent, in some cases almost explosive, flash was obtained. Throughout these tests the oil was stirred continuously at such a rate that the temperature throughout the bulk of the oil was fairly uniform, but not so vigorously as to break the surface or to produce splashing. Under these conditions it was found that the temperature in the vapour space immediately above the oil surface was sensibly the same as that indicated by the thermometer immersed in the oil; in other words, the question of temperature distribution met with in the Abel and allied apparatus does not arise here.

The ultimate deduction from these tests was that the Gray and Pensky-Martens apparatus indicate within a few degrees the lowest temperature at which it is possible for a flash to be obtained over this very wide range of conditions; and furthermore the Abel and Abel-Pensky types of apparatus could with advantage be modified by the introduction of a stirrer in the oil vessel and possibly in the vapour space above the oil. A small change in the indications of these apparatus would no doubt result, but the determination of flash-point would become an operation involving considerably less care and would not depend to any marked extent upon the exact form and dimensions of the apparatus employed. In any case the flash-point of a substance must be regarded as an empirical constant, but the modification suggested would render the determination less dependent on any particular design of apparatus.

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[1] "Tests of Fuel Oils," *Collected Researches, N.P.L.*, 1916, xiii. 295.

§ (5) High Temperature Flash-point Determinations.—The limiting temperature at which a flash-point determination can be made in the Abel types of apparatus is about 110° F. if the normal procedure be followed. For higher temperatures than this a modification of the normal procedure may be made in that the water bath itself, instead of being set initially to 130° F., is heated continuously throughout the test until a limiting temperature of about 180° F. is attained. In addition a small quantity of water is placed in the air jacket to facilitate the rise of temperature of the oil in the oil cup. This method is liable to give discordant results if the rate of heating be too rapid, again owing to the unequal distribution of temperature in the oil cup. Two modified apparatus were introduced to overcome this difficulty and at the same time to extend the range of temperatures over which a test may be made. These apparatus are referred to as the Pensky-Martens [1] and the Gray [2] flash-point apparatus respectively.

In both these apparatus the water bath is dispensed with and the oil cup is supported in a cavity in an iron casting heated from below. The Pensky-Martens apparatus has an oil cup similar to that of the Abel-Pensky apparatus, while that of the Gray apparatus is of the same dimensions as the oil cup of the Abel apparatus. The covers in each apparatus differ somewhat in detail, but both are provided with a rotating plate by means of which the ventilation holes are opened and the test flame depressed into the vapour space. Each instrument has a rotary stirrer provided with vanes which agitate both oil and vapour. In carrying out an observation the stirrers are worked continuously between the intervals at which the test flames are applied. The provision of these stirrers ensures that the thermometers indicate the true temperature of the vapour, and in practice it is found that both types of apparatus give sensibly the same flash-point. In carrying out a determination care should be taken that the rate of heating of the oil is slow and regular. The thermometers employed should be calibrated for the degree of immersion obtaining in the apparatus, otherwise it is necessary to apply a correction to allow for the emergent column. This correction is by no means negligible, as under the conditions of immersion in a flash-point apparatus the correction will amount to as much as 15° C. at 300° C.[3]

In carrying out the determination of flash-point of an oil the presence of moisture gives rise to considerable uncertainty, and even a small quantity may prevent the oil flashing until the water has been driven off. The temperature at which this occurs may be well above the normal flash-point of the oil and the resulting flash may be very violent. The drying of oil prior to testing the flash-point is a matter of considerable difficulty, as heating the oil to 100° C. may change it and thereby vitiate subsequent tests. Various methods have been suggested, such as drying with anhydrous calcium chloride or by exposing the moisture-containing oil to a high-tension discharge; neither of these methods is entirely satisfactory in removing every trace of moisture, but undoubtedly helps in preparing an oil for a flash-point determination.

§ (6) Ignition Points.—Although not of importance in connection with the legal aspect of flash-point determination, the ignition points of oils are frequently required in connection with internal combustion engine problems. A number of methods have been devised from time to time to carry out this determination without danger to the operator. The simplest form consists in allowing drops of oil to fall upon a heated iron plate. This method has been modified by H. Moore.[4] In this apparatus the iron plate is replaced by a grooved block of steel which may be heated from below; in the upper part of the block a cavity is provided into which a platinum or nickel crucible fits exactly. The crucible is covered by means of a perforated plate, one hole of which permits the introduction of the substance to be tested while the other serves as an inlet for air or oxygen. The gas employed is heated before admission to the crucible by being allowed to circulate through passages in the steel block. In operation the apparatus is raised to a definite temperature and one drop of the oil under examination is allowed to fall into the crucible. If the temperature is above the ignition point of the sample, combustion, more or less violent, takes place, while below the ignition temperature no explosion or flame is observed. Tests are repeated until the lowest point at which ignition takes place is determined.

W. F. H.

Flash-point Determination at High Temperatures. See "Flash-point Determination," § (5).

Flexible Container Pumps. See "Air-pumps," § (20).

Float Gauges. See "Meters, Liquid Level Indicators," § (14), Vol. III.

Flow in Pipes, Measurement of. See "Hydraulics," § (24).

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[1] A. Martens, "Über die Flammpunktbestimmungen von Schmierölen," *Mitth. k. techn. Versuchsanst.*, 1893, xi. 37-45.

[2] Gray, *Chem. Ind. Soc. J.*, 1891, x. 348.

[3] See "Thermometry," § (9).

[4] *Petroleum Technologist Inst. J.*, 1920, vi. 186.

Fluids:

Motion of, Application of Dynamical Similarity to. See "Dynamical Similarity, The Principles of," § (10).

Thermodynamic, Properties of. See "Thermodynamics, Entropy of," § (33); "Specification of State of," § (35); "Isothermal Expansion of," § (36): "Adiabatic Expansion of," § (38).

Under High Pressures, Experimental Researches on. See "Thermal Expansion," § (18).

Forging Press, Hydraulic. See "Hydraulics," § (57) (i.).

Fottinger Gear for Hydraulic Power Transmission. See "Hydraulics," § (65).

Foundations in Soft Earth. Theory of the supporting power of earth foundations. See "Friction," § (33).

FOURIER'S THEOREM

This relates to the expansion of an arbitrary function $f(x)$ in terms of circular functions of a particular type. Consider, in the first instance, the case where $f(x)$ is itself periodic, *i.e.* its value recurs exactly whenever the variable has a given constant increment a, so that

$$f(x+a)=f(x), \quad . \quad . \quad . \quad (1)$$

for all values of x. The simplest example of this relation is afforded by the circular functions $\cos(2s\pi x/a)$ and $\sin(2s\pi x/a)$, where s is any integer. The theorem is that whatever the form of $f(x)$, subject to certain restrictions, it can be approximated to as closely as we may desire by a series of terms of the above type, thus

$$f(x)=\mathrm{A}_0+\mathrm{A}_1\cos\frac{2\pi x}{a}+\mathrm{A}_2\cos\frac{4\pi x}{a}+\ .\ .\ .$$
$$+\mathrm{B}_1\sin\frac{2\pi x}{a}+\mathrm{B}_2\sin\frac{4\pi x}{a}+\ .\ .\ . \quad (2)$$

Assuming for the present the truth of the theorem, the values of the coefficients are found as follows. To find A_0, which is evidently the mean value of the function, we integrate both sides of (2) from $x=0$ to $x=a$. This gives

$$\mathrm{A}_0=\frac{1}{a}\int_0^a f(x)dx. \quad . \quad . \quad . \quad (3)$$

To find A_s we multiply both sides of (2) by $\cos(2s\pi x/a)$ and then integrate. The coefficient of A_r in the result is

$$\int_0^a \cos\frac{2\pi rx}{a}\cos\frac{2s\pi x}{a}dx$$
$$=\tfrac{1}{2}\int_0^a\left\{\cos\frac{2(r+s)\pi x}{a}+\cos\frac{2(r-s)\pi x}{a}\right\}dx.$$

Except in the case of $r=s$ each cosine goes through a complete cycle of its values at least once within the range of integration, and the integral vanishes. When $r=s$ the result is $\tfrac{1}{2}a$. Hence

$$\mathrm{A}_s=\frac{2}{a}\int_0^a f(x)\cos\frac{2s\pi x}{a}dx. \quad . \quad . \quad (4)$$

By a similar process

$$\mathrm{B}_s=\frac{2}{a}\int_0^a f(x)\sin\frac{2s\pi x}{a}dx. \quad . \quad . \quad (5)$$

For an account of various practical methods of computing the coefficients, and of the mechanical integrators which have been devised to supersede the numerical work, see "Harmonic Analysers," Vol. IV.

There is nothing special to the two points $x=0$, $x=a$, which have been taken as the limits of the above integrations. Any two points at an interval of a period will give the same result. In particular, writing l for the half-period, we have

$$f(x)=\mathrm{A}_0+\mathrm{A}_1\cos\frac{\pi x}{l}+\mathrm{A}_2\cos\frac{2\pi x}{l}+\ .\ .\ .$$
$$+\mathrm{B}_1\sin\frac{\pi x}{l}+\mathrm{B}_2\sin\frac{2\pi x}{l}+\ .\ .\ ., \quad (6)$$

where

$$\mathrm{A}_0=\frac{1}{2l}\int_{-l}^{l} f(x)dx, \quad \mathrm{A}_s=\frac{1}{l}\int_{-l}^{l} f(x)\cos\frac{s\pi x}{l}dx, \ (7)$$

and

$$\mathrm{B}_s=\frac{1}{l}\int_{-l}^{l} f(x)\sin\frac{s\pi x}{l}dx. \quad . \quad . \quad (8)$$

This leads to two particular cases of special importance. If $f(x)$ be an *even* function of x, so that

$$f(-x)=f(x), \quad . \quad . \quad . \quad (9)$$

we have

$$f(x)=\tfrac{1}{2}\{f(x)+f(-x)\}$$
$$=\mathrm{A}_0+\mathrm{A}_1\cos\frac{\pi x}{l}+\mathrm{A}_2\cos\frac{2\pi x}{l}+\ .\ .\ ., \quad (10)$$

with

$$\mathrm{A}_0=\frac{1}{l}\int_0^l f(x)dx, \quad \mathrm{A}_s=\frac{2}{l}\int_0^l f(x)\sin\frac{s\pi x}{l}. \quad (11)$$

Again, if $f(x)$ be an *odd* function, so that

$$f(-x)=-f(x), \quad . \quad . \quad (12)$$

we have

$$f(x)=\tfrac{1}{2}\{f(x)-f(-x)\}$$
$$=\mathrm{B}_1\sin\frac{\pi x}{l}+\mathrm{B}_2\sin\frac{2\pi x}{l}+\ .\ .\ ., \quad (13)$$

where

$$\mathrm{B}_s=\frac{2}{l}\int_0^l f(x)\sin\frac{s\pi x}{l}dx. \quad . \quad . \quad (14)$$

In many applications, especially when the variable x is a space co-ordinate, we are concerned only with a limited range of x, say from 0 to l. As instances we have the vibrations of a string, and the flow of heat through a plate. Outside the above limits the function $f(x)$ may not exist, so far as the physical problem is concerned, but we are at liberty to imagine it continued analytically both ways as a periodic function. We are further at liberty to suppose it continued as an odd or as an even function of x, as is illustrated by the annexed figures. It is to be noticed, however, that in the second case the process of continuation introduces discontinuities at $x=0$, $x=l$, unless $f(x)$ vanishes at these points.

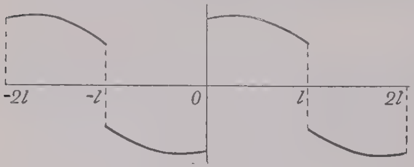

FIG. 1.

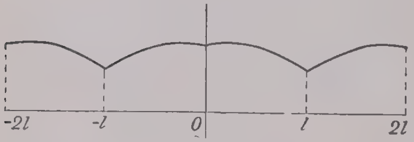

FIG. 2.

Similarly, in the original form (2) of the theorem, if the function is continued with a period a, discontinuities will be introduced at $x=0$, $x=a$, unless $f(0)=f(a)$.

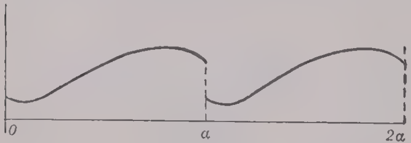

FIG. 3.

A complete statement of the conditions under which the theorem (2) holds is not attempted here. For physical purposes it may be sufficient for the moment to say that the expansion is valid provided $f(x)$ be continuous, and has (within the period) only a finite number of maxima and minima. The mathematical proof, which is necessarily somewhat intricate, must be omitted, but physical arguments of a very convincing kind are easily adduced. Suppose, for example, that x is measured along the circumference of a uniform thin metal ring whose total perimeter is a; and let $f(x)$ denote the initial distribution of temperature. If radiation from the surface be neglected, the theory of conduction of heat indicates that the subsequent process is made up by superposition of the various "normal modes" of approach to a steady state, with arbitrary coefficients, thus

$$\mathrm{A}_0+\mathrm{A}_1 \cos\frac{2\pi x}{a}e^{-\lambda_1 t}+\mathrm{A}_2 \cos\frac{4\pi x}{a}e^{-\lambda_2 t}+\ \ldots$$

$$+\mathrm{B}_1 \sin\frac{2\pi x}{a}e^{-\lambda_1 t}+\mathrm{B}_2 \sin\frac{4\pi x}{a}e^{-\lambda_2 t}+\ \ldots, \quad (15)$$

where

$$\lambda_s=4\pi\kappa s^2/a^2. \quad . \quad . \quad (16)$$

This being granted, the initial distribution $f(x)$ must correspond to $t=0$. Hence $f(x)$ must admit of expansion in the form (2). Arguments of a similar character might be adduced from Acoustics, and other branches of Physics.

The restriction as to continuity, above made, can to a certain extent be dispensed with. Provided the discontinuities are of finite amount, and occur (within the range of a period) only at a finite number of isolated points, the expansion (2) will still hold except at the points of discontinuity. At such a point the value of $f(x)$ is of course ambiguous, but it may be proved that the series on the right-hand side of (2) converges to a definite value which is the arithmetic mean of the values of $f(x)$ immediately to the left and right of the discontinuity. This applies in particular to the discontinuities which may be introduced when a function given over a finite range is continued as a periodic function, in the manner already explained.

Suppose, for instance, that it is required to express in the form (13) the initial temperature of a conducting slab bounded by the planes $x=0$, $x=l$, and, for simplicity, that the initial temperature is everywhere unity. Putting $f(x)=1$ we have

$$\mathrm{B}_s=\frac{2}{l}\int_0^l \sin\frac{s\pi x}{l}dx=\frac{2}{s\pi}(1-\cos s\pi). \quad (17)$$

This is equal to $4/s\pi$ or 0, according as s is odd or even. Thus

$$1=\frac{4}{\pi}\left(\sin\frac{s\pi x}{l}+\tfrac{1}{3}\sin\frac{3s\pi x}{l}\right.$$

$$\left.+\tfrac{1}{5}\sin\frac{5s\pi x}{l}+\ \ldots\right). \quad (18)$$

When in this case $f(x)$ is continued as an odd function there is a sudden change from 1 to -1, or *vice versa*, at $x=0$ and $x=l$. The series has then the value zero, which is the arithmetic mean aforesaid. The figure shows the approximation given by the first three terms of the expansion. It may be added, in leaving this topic, that in the applications to physical problems a mathematical discontinuity (*e.g.* of temperature) is only to be regarded as the idealised expression of a very rapid transition. In practice there is, of course, a limit to

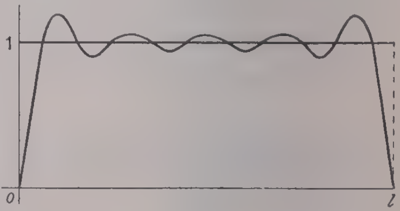

FIG. 4.

the number of the coefficients A_s, B_s which it is convenient to calculate. The rapidity with which the series converges depends on the smoothness or regularity (in a general sense) of the function $f(x)$. It is evident that the coefficients, as determined by (4) or (5), must ultimately diminish without limit as the order s increases, owing to the more and more rapid fluctuations in sign of the circular functions $\cos(2s\pi x/a)$ and $\sin(2s\pi x/a)$, and the consequent more complete cancelling of the positive and negative elements in the integrals. More definite results were given by Stokes. If the function $f(x)$, *as continued*, has only isolated discontinuities, the coefficients ultimately diminish as $1/s$; this is illustrated by (18). If $f(x)$ is everywhere continuous, whilst its derived function $f'(x)$ has isolated discontinuities, the convergence is as $1/s^2$, and so on. The present point has many exemplifications in Acoustics. For instance, the more gradual the initial impulse given to a string, the fainter are the higher harmonics in the resulting note.

The approximate representation of an arbitrary function over a given range by means of functions of a specified type is a problem which naturally admits of solution in various ways. It is of interest to note that Fourier's method is the one which makes the sum of the squares of the errors a minimum. Taking for brevity the sine-series (13), and limiting ourselves, in the first instance, to a finite number of terms, the sum of the squares of the errors is

$$\int_0^l \left\{ f(x) - B_1 \sin\frac{\pi x}{l} - B_2 \sin\frac{2\pi x}{l} - \ . \ . \ . \right. \\ \left. - B_m \sin\frac{m\pi x}{l} \right\}^2 dx. \quad (19)$$

To make this a minimum we must equate the differential coefficients with respect to $B_1, B_2 \ . \ . \ . \ , B_m$ to zero. The typical equation is

$$\int_0^l \left\{ f(x) - B_1 \sin\frac{\pi x}{l} - B_2 \sin\frac{2\pi x}{l} - \ . \ . \ . \right. \\ \left. - B_m \sin\frac{m\pi x}{l} \right\} \sin\frac{s\pi x}{l} dx = 0, \quad (20)$$

whence $$B_s = \frac{2}{l}\int_0^l f(x) \sin\frac{s\pi x}{l} dx, \ . \ . \quad (21)$$

as in (14). Each additional term included in the series necessarily lowers the minimum, and so improves the approximation, as tested by the method of least squares. H. L.

Fourth-power Law, Investigation of, between 1063° C. and 1549° C. See "Pyrometry, Total Radiation," § (4).

Frame is the name given to the rigid structure to which portions of a mechanism are attached and relative to which they move. See "Kinematics of Machinery," § (2).

Framed Structures. See "Structures, Strength of," § (22).

Free Piston, Internal Combustion Engine, Otto and Langen. See "Engines, Thermodynamics of Internal Combustion," §§ (33) and (49).

Freezing Mixtures, Theory of. See "Thermodynamics," § (63).

FRICTION

Introduction.—Friction may be broadly defined as the resisting force which is called into existence at the common boundary of two substances in contact when under the action of some external agency one of the substances slides, or tends to slide, over the surface of the other. The direction of the force of friction is tangential to the surface of contact, and so long as there is no motion its magnitude is equal to the component in the direction of motion of the external force tending to produce sliding.

The phenomenon of friction is common to all substances, solid, liquid, or gaseous, which may be in contact with each other and subject to forces tending to cause relative motion, but the laws correlating the magnitude of the frictional forces produced, with the magnitude of the external forces acting and the state of motion produced by them, differ widely with the nature of the substances. For example, the friction between the wheels of a locomotive and the rails, in virtue of which railroad traffic becomes possible, depends on the weight on the wheels and not on the speed of the wheels, whereas the frictional resistance of the water to the motion of a ship floating on it depends on the speed of the ship and is independent of the pressure of the water.

The term friction has also been extended, with certain restrictions, the necessity for which will be explained below, to include the mutual resistance which different parts of the same substance offer to sliding over each other. That this interpretation must, of necessity, be extended to fluids is obvious from the consideration that in the case of the skin friction of a ship referred to above, the particles of the water in contact with the surface of the ship are at rest relatively to it, and the sliding to which the resistance is due takes place in the body of the water itself. The resistance to motion of the ship is, therefore, due to the shearing resistance of the water in which it moves, and for this reason this shearing stress is commonly called the internal friction of the water. In the case of an elastic solid, however, the conditions of relative motion of its particles due to the action of external forces are more complex. For example, suppose a vertical elastic rod is supported rigidly at its upper end and carries at its lower extremity a mass which executes free torsional oscillations about the axis of the rod. During each half-oscillation a certain amount of the work done by the rotating mass is stored up as potential energy in the rod and this is given out again to the mass on the return movement. It is found, however, that the amplitude of the oscillations diminishes much more rapidly than would be due to the resistance of the air in which it swings, and it is concluded that frictional resistances have existed in the material of the rod to bring this about, *i.e.* the rod has a certain amount of internal friction or viscosity. In this particular illustration we have, therefore, a case in which part of the work done against the resistance to distortion is stored up as potential energy of the material and part is converted into heat. That the two phenomena are essentially different will be seen from the fact that in the elastic distortion the resistance is proportional to the relative displacement, whereas in the case of internal friction, or viscosity, the resistance is proportional to the time rate of relative displacement.

The necessity for the restriction of the use of the term friction to those cases in which the work done against the frictional resistance is converted into heat is, therefore, obvious.

From these considerations it will be seen that a more precise definition of friction than that given in the first paragraph would be as follows. The resisting forces brought into existence at the surface separating two substances in contact, or two parts of the same substance, by the action of any external agency tending to produce relative motion at the surface are denominated frictional forces in all cases in which the work done by them is converted into heat.

In previous works on the subject of friction it has been customary to classify all cases of frictional resistance under one or the other of the following two divisions:

(*a*) Friction between solid bodies in contact; the frictional resistance being subject to certain empirical laws.

(*b*) Friction between solids and fluids or between solids separated by a film of fluid; the resistance being determined from the motion and physical characteristics of the fluid.

In recent years, however, it has become more and more recognised that this broad division of the subject cannot be made for the reason that it is almost impossible in practice to obtain contact between solid surfaces without the intervention of a contaminating film of fluid which renders the phenomenon one in which the nature of the resistance is to a greater or less extent dependent on the characteristics of the contaminating film. The modern tendency is, therefore, to regard the frictional resistance of solids in contact and moving relatively to each other as a limiting case of the friction between the solid surfaces separated by a layer of fluid when the thickness of this layer is diminished to such an extent that its motion can no longer be treated by the laws of hydrodynamics, so that the unknown boundary conditions constitute the main characteristics governing the resistance.

For this reason it is proposed in the present article to commence with the treatment of the internal friction of fluids and the manner in which it is affected by the characteristics of the motion, and then to proceed to a discussion of the force acting at a solid bounding surface of a fluid tangential to the direction of the fluid flow, in the first instance when the conditions at the boundary are known, and finally when these conditions are hypothetical.

The detailed classification of the subject-matter of the present article will therefore be as follows:

Division I. Internal Friction, or Viscosity.

Division II. The Nature of the Motion of Fluids over Solid Surfaces and the Characteristics of the Frictional Forces Consequent on the Motion.

Division III. The Determination Theoretically and by Experiment of the Frictional Resistance offered by Solid Surfaces to the Motion of Fluids over them, in all Cases in which the Resistance is determined by the Motion of the Fluid.

Division IV. The Frictional Resistances between Solid Surfaces separated, or Partially separated, by a Film of Fluid of such a Depth that the Fluid does not Conform to the Laws of Hydrodynamics.

Division V. The Frictional Resistance of Clean Solid Surfaces.

Division VI. The Relation between Friction and Heat Transmission.

I. INTERNAL FRICTION, OR VISCOSITY

§ (1) VISCOUS FLUIDS.—In distinguishing between solids and fluids it is customary to define a fluid as a substance which is incapable of sustaining tangential or shearing stress. This definition, however, is only true in the case of actual fluids when they are at rest. When relative motion exists between neighbouring portions of the same fluid, a measurable resistance to the relative motion can be observed and affords a proof that actual fluids are capable of sustaining shearing stresses; the fluid is said to exhibit the property of viscosity or internal friction. It would appear probable, therefore, that some definite relation exists between this shearing stress in fluids and the rate of distortion of the fluids of which it is the characteristic feature. It was assumed by Newton that, for a fluid moving in parallel layers, the shearing stress at any point at which the velocity gradient in a direction perpendicular to the layers was dv/dy, would be simply proportional to the value of the gradient, *i.e.* that the relation between internal friction and the relative motion would be given by

$$f=\mu\frac{dv}{dy},$$

where μ, called the coefficient of viscosity, is a fundamental characteristic of the fluid.

The definition given by Maxwell is as follows:

The coefficient of viscosity of a substance is measured by the tangential force on unit area of either of two horizontal planes at unit distance apart, one of which is fixed while the other moves with the unit of velocity, the space between being filled with the viscous substance.

The interpretation of the coefficient of viscosity of a fluid, according to the kinetic theory of matter, is as follows:

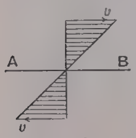

FIG. 1.

Let AB (*Fig.* 1) represent the trace of a plane in the fluid, parallel to the direction of motion, and let there be a definite gradient of velocity at right angles to AB as indicated by the lines parallel to AB representing the magnitude of the velocity of the layers relative to the velocity of AB, so that the molecules immediately above AB are moving faster than those below it. Some of these molecules will cross AB from the upper to the lower side and an equal number will pass upwards to replace them. Thus the layer immediately above AB is continually losing momentum and that below is continually gaining it. The effect is to bring into existence a definite shearing stress on the plane AB which constitutes the viscous drag. The internal friction of fluids consists therefore of a transfer of motion from one layer to the other, but this transfer does not proceed without loss of energy, since the translatory motion of the layers is transformed into heat. This is evident from the consideration that heat motion differs from translatory motion only in the fact that in the former case particles are moving in all possible directions and in the latter in one and the same direction and that a change from uni-directional motion to multi-directional motion cannot fail to take place in a medium which consists of particles which exert actions on each other by the forces of cohesion or collisions.

§ (2) VISCOSITY OF GASES.—In the case of gases in which the molecules are supposed to be outside the sphere of each other's attraction during the greater part of the time considered it is possible by means of the dynamical theory to obtain the value of the coefficient of viscosity in terms of the characteristics of the gas.[1]

We picture the gas as composed of a very large number of molecules, moving with varying velocities in all directions and influencing each other by their collisions. Let us consider a group consisting of N molecules contained in a unit of volume in the form of a cube; then we may suppose, with Joule, that the average velocity at right angles to each face of the cubes is the same, so that if V be the mean resultant velocity and u, v, w the components we have $V^2=u^2+v^2+w^2=3u^2$, for u, v, w are equal.

Thus we may regard the gas as consisting of three groups each containing N molecules and moving with equal velocities $V/\sqrt{3}$ in the three directions at right angles, or as three groups, each containing N/3 molecules, moving with velocity V in each of the three directions in question. In either case the energy of agitation will be the same.

Adopting this latter view it is clear that considering the whole number of molecules N in unit volume of the gas, since one-third of this number will be in motion perpendicular to any two opposite faces of the unit cube containing them, and of these half are moving towards either face and half away from it, therefore the number which will be moving from the upper to the lower side of any one face will be one-sixth of the total. Further, considering all the molecules which pass through this face from the upper to the lower side in unit time, it is evident that these will be limited to the molecules whose distance from the face at the beginning of the time interval was less than the length of the path which they would travel in unit time. Hence all the molecules which cross the face from the upper to the lower side in unit time come from the prism whose base is the face and whose height is measured by the velocity V of the molecules, *i.e.* they all come from a prism of volume V. The number of molecules, therefore, which cross unit area in unit time in this direction is $\frac{1}{6}$NV. It must be remembered also that each of these molecules has only been moving in a direction perpendicular to the face of the unit cube, since its last collision, and that therefore it will only form one of the group during the time which elapses from one collision to another, *i.e.* over the distance known as the molecular free path L, the average distance travelled by a molecule between two collisions.

Now assuming the existence of a velocity gradient perpendicular to the face of the cube whose value is unity, *i.e.* that the velocity of flow at a distance "y" above the unit face is numerically equal to "y" so that according to our definition the friction over the face measures the coefficient of viscosity, we can calculate the friction between the two layers of gas separated by the unit face as follows: The number of particles which pass the face from one side to the other per unit of time is $\frac{1}{6}$NV. These have begun their path towards the face at different depths, but on the average they come from a distance from the face which is equal to the mean free path L, and have described this distance in the unit of time.

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[1] Jeans, *Kinetic Theory of Gases.*

Their mean forward velocity is therefore given by $V=L$. Therefore each molecule carries with it over the face the momentum mL, where m is its mass, and the total momentum carried over in unit time is $\frac{1}{6}NVmL$.

Simultaneously there pass in the opposite direction across the face $\frac{1}{6}$ NV molecules, each of which comes from an average distance from the face of $-L$ and whose mean forward velocity is given by $V=-L$. The total momentum carried over in this direction is therefore $-\frac{1}{6}NVmL$.

Therefore the layer above the unit face loses in unit time the momentum

$$\tfrac{1}{6}NVmL-(-\tfrac{1}{6}NVmL)$$
$$=\tfrac{1}{3}NVmL,$$

and this, by hypothesis, is the amount of the friction exerted on the face, and is the expression for the coefficient of viscosity. Since Nm is the density (ρ) of the gas we may write

$$\mu=\tfrac{1}{3}\rho VL.$$

If, instead of making the assumption that all the molecules have equal speeds, we assume Maxwell's law for their distribution, it can be shown that a more accurate value of the coefficient of viscosity is given by

$$\mu=0{\cdot}30967\rho\bar{V}L,$$

where $\bar{V}$ is the mean value of the speed derived from Maxwell's law. (See Meyer's *Kinetic Theory of Gases*, Appendix iv. p. 451.)

Substituting in these equations the value obtained by Clausius for the mean free path

$$L=\frac{\frac{3}{4}\lambda^3}{\pi s^2},$$

where s is the radius of the sphere of action of the molecules and λ is the volume occupied by a single molecule, we have

$$\mu=\frac{N\lambda^3 mV}{4\pi s^2},$$

and since $N\lambda^3=1, \quad \mu=\dfrac{mV}{4\pi s^2}.$

In this expression for μ there is no factor which depends on the pressure of the gas, a deduction from the theory which led Maxwell to predict that the viscosity of a gas would be independent of its density, and thus that the oscillations of a pendulum in a gas would be equally damped by gaseous friction, however low the pressure might be.

This conclusion was so much opposed to the general opinion of physicists at the time that the experimental verification of Maxwell's prediction, which took place soon afterwards, was perhaps the most important factor in the acceptance by physicists of the kinetic theory. It has been found, however, that when the pressure of a gas is reduced to extremely small values, the viscosity no longer remains constant. This lower limit has been found by Kundt and Warburg to be in the neighbourhood of one-sixtieth of an atmosphere. As would be expected, there is also an upper pressure limit at which the law no longer holds, since at high pressures the general assumptions of the theory are not approximately correct.

A further deduction from the kinetic theory as originally developed by Maxwell and Clausius was that the viscosity was proportional to the square root of the absolute temperature of the gas. Experimental investigation, however, failed to establish this relation, and it was found that the actual law of variation of μ with T was given by $\mu\propto T^n$ where n ranged from its lowest value of about ·7 for hydrogen to about 1·0 for the less perfect gases. On the assumption that n was equal to unity for all gases Maxwell deduced that the molecules must repel each other with a force inversely proportional to the fifth power of the distance between them. This hypothesis, however, had to be abandoned when it became certain that n was a variable quantity. By a re-examination of the fundamental assumptions on which the kinetic theory was based Sutherland was led to the conclusion that, although the attraction between two molecules of a gas is negligible at their average distance apart, yet, when two molecules are passing quite close to each other the force of attraction can bring about a collision which in its absence would not have taken place.

On this hypothesis Sutherland[1] has deduced from the equation of the orbit of a molecule that the effect of the molecular attraction in producing collisions is to diminish the mean free path L in the ratio: $(1+c/T)$, where c is a constant for the gas. It follows therefore, from the expression $\mu=\frac{1}{3}\rho GL$, that since G varies as $\sqrt{T}$ and L varies inversely as $(1+c/T)$, the value of μ will vary as

$$\frac{\sqrt{T}}{1+c/T}.$$

In an experimental investigation into the viscosity of air (see § (5)), it was found by Grindley and Gibson that this relation held with considerable accuracy between temperatures of 0 and 100°, the maximum deviation of the experimental results from the above law being 1·3 per cent.

§ (3)· **The Experimental Determination of the Values of Coefficients of Viscosity for Fluids.** (i.) *Equations of Motion.*—As the majority of the methods used to measure viscosity depend on a relation between the motion of the fluid and the variations of its pressure from point to point, a brief account of the derivation of the equations of motion of a viscous fluid is here given.

Assuming the truth of the Newtonian hypothesis, the equations of motion of the fluid can be obtained as follows: If we imagine three planes to be drawn through any point P in the fluid perpendicular to the axes of x, y, and z respectively, the three components of the stress per unit area exerted across the first of these planes may be denoted by p_{xx}, p_{xy}, p_{xz} respectively; those of the stress across the plane perpendicular to y by p_{yx}, p_{yy}, p_{yz}; and those of the stress across the plane perpendicular to z by p_{zx}, p_{zy}, p_{zz}.

It follows at once that, considering an

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[1] "Viscosity of Gases," *Phil. Mag.*, Dec. 1893.

element $\delta x \delta y \delta z$ having its centre at P, and taking moments,

$$p_{yz}=p_{zy},\ p_{zx}=p_{xz},\ p_{xy}=p_{yx}.$$

Also, if p_1, p_2, p_3 be the principal stresses at P, it can be shown (*a*) that[1]

$$p_{xx}+p_{yy}+p_{zz}=p_1+p_2+p_3\,; \qquad . \quad (1)$$

i.e. the arithmetic mean of the normal pressures on any three mutually perpendicular planes through the point P is the same and equal to p (say); and (*b*) that the values of the stresses in terms of p, the coefficient of viscosity μ, and the rates of distortion are given by the expressions

$$\left.\begin{aligned}
p_{xx}&=-p-\tfrac{2}{3}\mu\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}\right)+2\mu\frac{\partial u}{\partial x}\\
p_{yy}&=-p-\tfrac{2}{3}\mu\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}\right)+2\mu\frac{\partial v}{\partial y}\\
p_{zz}&=-p-\tfrac{2}{3}\mu\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}\right)+2\mu\frac{\partial w}{\partial z}\\
p_{yz}&=p_{zy}=\mu\left(\frac{\partial w}{\partial y}+\frac{\partial v}{\partial z}\right)\\
p_{zx}&=p_{xz}=\mu\left(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x}\right)\\
p_{xy}&=p_{yx}=\mu\left(\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}\right)
\end{aligned}\right\} . \quad (2)$$

The condition for "laminar motion"—motion, that is, in which the fluid moves in a system of parallel planes, the velocity being in direction everywhere the same, and in magnitude proportional to the distance from some fixed plane of the system—is seen to be

$$u=ay, \quad v=0, \quad w=0,$$

from which it follows that the axis of x being taken in the direction of motion and the velocity being proportional to the distance from the plane zx

$$p_{xx}=p_{yy}=p_{zz}=-p, \quad p_{yz}=0, \quad p_{zx}=0, \quad p_{xy}=\mu a,$$

"a" being the rate of distortion.

The stresses in different fluids under similar conditions of motion will be proportional to the corresponding values of μ, but if we wish to compare their effects in modifying the existing motion we have to take account of the ratio of these stresses to the inertia of the fluid. From this point of view the determining quantity is the ratio μ/ρ, which is denoted by the special symbol ν, called by Maxwell the "kinematic" coefficient of viscosity. The equations of motion are obtained by considering the forces acting on a rectangular element having its centre at P. Thus resolving parallel to x, the difference of the normal pressures is $(\partial p_{xx}/\partial x)\delta x\delta y\delta z$. The tangential tractions on the zx faces amount to $(\partial p_{yz}/\partial y)\delta y\delta x\delta z$, and those on the xy faces are $(\partial p_{zx}/\partial z)\delta z\delta x\delta y$. If therefore X, Y, and Z are the components of the external forces per unit mass, we have

$$\left.\begin{aligned}
\rho\frac{\mathrm{D}u}{\mathrm{D}t}&=\rho\mathrm{X}+\frac{\partial p_{xx}}{\partial x}+\frac{\partial p_{yx}}{\partial y}+\frac{\partial p_{zx}}{\partial z}\\
\rho\frac{\mathrm{D}v}{\mathrm{D}t}&=\rho\mathrm{Y}+\frac{\partial p_{xy}}{\partial x}+\frac{\partial p_{yy}}{\partial y}+\frac{\partial p_{zy}}{\partial z}\\
\frac{\rho\mathrm{D}w}{\mathrm{D}t}&=\rho\mathrm{Z}+\frac{\partial p_{xz}}{\partial x}+\frac{\partial p_{yz}}{\partial y}+\frac{\partial p_{zz}}{\partial z}
\end{aligned}\right\}, \quad . \quad (3)$$

where $\mathrm{D}/\mathrm{D}t$ denotes a differentiation following the motion of the fluid, *i.e.*

$$\frac{\mathrm{D}}{\mathrm{D}t}=\frac{\partial}{\partial t}+u\frac{\partial}{\partial x}+v\frac{\partial}{\partial y}+w\frac{\partial}{\partial z}.$$

Substituting the values of p_{xx}, etc., given above, we have

$$\rho\frac{\mathrm{D}u}{\mathrm{D}t}=\rho\mathrm{X}-\frac{\partial p}{\partial x}+\tfrac{1}{3}\mu\frac{\partial\theta}{\partial x}+\mu\nabla^2 u, \quad . \quad (4)$$

etc. .

where $$\theta=\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}$$

and $$\nabla^2=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}.$$

When the fluid is incompressible, these reduce to

$$\left.\begin{aligned}
\rho\frac{\mathrm{D}u}{\mathrm{D}t}&=\rho\mathrm{X}-\frac{\partial p}{\partial x}+\mu\nabla^2 u\\
\rho\frac{\mathrm{D}v}{\mathrm{D}t}&=\rho\mathrm{Y}-\frac{\partial p}{\partial y}+\mu\nabla^2 v\\
\rho\frac{\mathrm{D}w}{\mathrm{D}t}&=\rho\mathrm{Z}-\frac{\partial p}{\partial z}+\mu\nabla^2 w
\end{aligned}\right\}. \quad . \quad . \quad (5)$$

(ii.) *Flow through a Circular Pipe.*—As an example of the applications of these equations we may take the important case of the steady flow of a liquid through a pipe of uniform circular section.

Taking the axis of z to be coincident with the axis of the pipe, and assuming that the velocity is everywhere parallel to z, and dependent solely on the distance from the axis, we have $w=0$, $v=0$, and therefore from (5)

$$\frac{\partial p}{\partial x}=0, \quad \frac{\partial p}{\partial y}=0, \quad . \quad . \quad . \quad (1)$$

i.e. the mean pressure is uniform over each section of the pipe. Again, from (5) we have

$$\frac{\partial p}{\partial z}=\mu\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial z^2}\right), \quad . \quad . \quad (2)$$

where p is a function of z. Transforming into polar co-ordinates r and θ

$$\frac{\partial p}{\partial z}=\mu\left\{\frac{\partial^2 w}{\partial r^2}+\frac{1}{r}\frac{\partial w}{\partial r}+\frac{1}{r^2}\frac{\partial^2 w}{\partial\theta^2}\right\}, \quad . \quad (3)$$

and, since by symmetry w is independent of θ, the last term on the right-hand side vanishes.

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[1] Lamb's *Hydrodynamics* (1916 ed.), p. 569. See also "Elasticity, Theory of," § (6).

Hence the equation may be written

$$\frac{\partial}{\partial r}\left(r\frac{\partial w}{\partial z}\right)=\frac{1}{\mu}\frac{\partial p}{\partial z}r \quad . \quad . \quad . \quad (4)$$

or integrating

$$w=\frac{1}{4\mu}\cdot\frac{\partial p}{\partial z}+\mathrm{A}\log r+\mathrm{B}. \quad . \quad . \quad (5)$$

Since the velocity must be finite at the axis A=0 and, if B be determined on the assumption that $w=0$ at the boundary, *i.e.* there is no slipping at the wall of the pipe ($r=a$ say), we have

$$w=\frac{(a^2-r^2)}{4\mu}\cdot\frac{\partial p}{\partial z}. \quad . \quad . \quad . \quad (6)$$

The flow across any section is therefore

$$\int_0^a w\,.\,2\pi r\partial r=\frac{\pi a^4}{8\mu}\cdot\frac{\partial p}{\partial z}. \quad . \quad . \quad (7)$$

Thus if V be the volume passing in time T so that the flow is V/T, we have

$$\mathrm{V}=\mathrm{T}\frac{\pi a^4}{8\mu}\frac{dp}{dz}.$$

If the fall of pressure along a length l of the tube is uniform and equal to p_1-p_2, this equation may be written

$$\mathrm{V}=\mathrm{T}\frac{\pi a^4}{8\mu}\frac{p_1-p_2}{l}.$$

If, on the other hand, the possibility of a slip at the boundary is not excluded, the most natural assumption to make is that the slipping is resisted by a tangential force proportional to the relative velocity,[1] *i.e.* the boundary condition would be

$$-\mu\frac{\partial w}{\partial r}=\beta w,$$

where β is a slipping coefficient, or

$$w=-\lambda\frac{\partial w}{\partial r}, \quad \text{if } \lambda=\frac{\mu}{\beta}.$$

This determines B in equation above, so that

$$w=\frac{\partial p}{\partial z}\left(\frac{a^2-r^2+2\lambda a}{4\mu}\right).$$

If λ/a is small, this gives sensibly the same law of velocity as in a tube of radius $a+\lambda$ on the hypothesis of no slipping. The corresponding value of the flow is

$$\frac{\pi a^4}{8\mu}\left(1+4\frac{\lambda}{a}\right)\frac{\partial p}{\partial z}.$$

§ (4) EFFECT OF TURBULENCE.—It is clear that any experimental verification of the truth of the law of resistance postulated by Newton will depend upon whether in the relation

$$f=\mu\frac{dv}{dy}$$

a value of μ for any given fluid can be found which has an identical value for all values of the velocity gradient dv/dy. It will be seen later that in those cases of fluid motion in which the motion is turbulent or eddying, the ratio of the shearing stress in the fluid parallel to the mean direction of the flow to the mean velocity gradient perpendicular to this direction is not an absolute constant for the fluid, but depends on the actual values of the velocity of flow and the distribution of the solid boundaries of the flow. In these cases the Newtonian hypothesis breaks down, but its truth under circumstances of streamline flow defined by the condition that the velocity of the fluid at any fixed point is always constant in magnitude and direction has been fully demonstrated. Under these conditions of motion the coefficient μ has been shown to be a physical property of the fluid and is known as the absolute coefficient of viscosity.

§ (5) RESULTS OF EXPERIMENTS. (i.) *Gases.*—Most of the earlier determinations on gases were carried out by the method of noting the damping of the oscillations of a disc in the gas, as in the case of Maxwell's classical experiments.[2] In this method, however, there are considerable mathematical difficulties in determining the motion of the air at the edge of the disc, and in recent work on the subject the method of observing the fall of pressure of the fluid when flowing at a known speed through a channel with parallel walls has been used. An example of this method is seen in the work of Grindley and Gibson on the viscosity of air.[3] The apparatus used consisted of two gas-holders of about 3 cubic feet capacity connected by a length of lead tubing of 0·125 inch diameter, of which part was used as the experimental tube. The ends of the experimental portion are connected to a manometer by which the fall of pressure can be measured. In commencing an experiment one of the gas-holders contains air and the other water, and by admitting water under pressure to the lower part of the gas-holder containing air, the air is forced through the tube at the desired rate. The gas-holders were calibrated so that the volume of air passing through the tube in a given time could be determined. The length of the tube between the vessels, which was about 190 feet, was wound on a central brass cylinder on a helix of $\frac{5}{32}$ pitch and 1 ft. diameter. The cylinder rested on supports in a vessel filled with water and provided with devices for maintaining the temperature at any desired value between 0° C. and 100° C. The experimental part of the tube was about 108 feet long.

By inserting the known values of the flow and value of the pressure gradient in equation (6), § (3), the value of μ was deduced. It was found that in accordance with the results of previous investigators the value of μ was independent of the pressure, and, further, that its variation with temperature closely approximated to the law deduced by Sutherland from theoretical considerations, so that the value could be written

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[1] Lamb's *Hydrodynamics* (1916 edition), p. 572.

[2] *Phil. Trans. Roy. Soc.* clvi.

[3] *Proc. Roy. Soc.* A, lxxx. 114.

$$\mu = \frac{K\sqrt{T}}{1 + c/T},$$

where T is the absolute temperature, and, for the case of air,

$$K = 141{\cdot}8 \times 10^{-7},$$
$$c = 102{\cdot}5.$$

For comparison with the results of other observers which are expressed in the form $\mu = a(1 + bt + ct^2)$, the values of the constants a, b, and c for air were found to be $a = 170{\cdot}2 \times 10^{-6}$, $b = 0{\cdot}00329$, $c = 0{\cdot}000007$.

(ii.) *Liquids.*—In forming a physical conception of the viscosity of liquids it is of interest following Maxwell,[1] to regard the phenomenon, not as an example of the diffusion of matter, but as a limiting case of an elastic solid when the material breaks down under shear. Thus in the case of an elastic solid the shearing stress on any plane is proportional to the space rate of displacement of the material parallel to the plane. The viscous drag is therefore related to the velocity in precisely the same way as the elastic shearing stress to the displacement. We may, therefore, look upon a viscous liquid as capable of exerting a certain amount of shearing stress for a short time and then breaking down and the shear recommencing. If we suppose that the rate at which the shearing stress breaks down is proportional to the shear and is equal to $\lambda\theta$, where θ is the shear given by $\theta = dx/dy$, x being the horizontal displacement, the rate at which shear is supplied is $d\theta/dt$, or $d/dy \,.\, dx/dt$, *i.e.* $d/dy \,.\, v$, where v is the velocity of displacement. We have therefore $\lambda\theta = dv/dy$, and since the shearing stress is given by $f = n\theta$ we have $f = n/\lambda \,.\, dv/dx$, or $n/\lambda = \mu$. The quantity $1/\lambda$ is called the time of relaxation of the liquid and measures the time taken by the shear to disappear when no fresh shear is supplied.

§ (6) EARLY EXPERIMENTS. (i.) *Poiseuille's Method.*—The earliest experimenter on the viscosity of liquids was Poiseuille, who carried out a very extensive series of observations on the flow of water through capillary tubes.[2] In these experiments the outlet end of the capillary tube was connected to a reservoir of water, the pressure in which could be regulated to any desired value by means of an air-pump, connected to the upper cover of the reservoir. Provision was also made for regulating the temperature of the water to any desired value. It was, therefore possible, by using capillary tubes of varying bore and length, to obtain a relation between the rate of discharge, the dimensions of the tubes, and the pressure and temperature of the water. The relation as given by Poiseuille was

$$Q = 1836{\cdot}724(1 + 0{\cdot}0336T + 0{\cdot}000221T^2)\frac{PD^4}{L},$$

where Q is the discharge in milligrammes of water per second, P is the pressure difference between the ends of the capillary tube in millimetres of mercury, and D and L are the diameter and length of the tube in millimetres. It will be noticed that this relation agrees exactly with the results of the motion of a viscous fluid through a pipe of circular cross-section, on the assumption that the velocity at the boundary is zero, *i.e.* the time of efflux of a given volume of water is directly as the length of the tube inversely as the fourth power of the diameter and inversely as the difference of pressure at its ends. As an instance of the high order of accuracy obtained in these experiments it may be remarked that the value of μ for $T = 0$, calculated from the above expression, is in close agreement with the most modern determination. It has been pointed out by Lamb that, if any appreciable amount of slipping at the boundary of the pipes used by Poiseuille took place, a deviation from the law of the fourth power of the diameter would become apparent, and the fact that this was not the case excludes the possibility of such an amount of slipping as has been inferred by Helmholtz and Pietrowski from their experiments on the torsional oscillations of metal spheres filled with water. The question is very fully discussed by Whetham,[3] who concluded that no slipping took place.

(ii.) *Temperature Effect.*—It will be seen that the variation of viscosity with temperature is of the opposite sign from that of gases, and it appears to be a characteristic of all liquids that the viscosity diminishes as the temperature rises. In the case of water the change is fairly rapid, as will be seen from the following table, which gives the results of determinations by Hosking.

Temp.	0° C.	10	20	30	40	50	60	70
$\mu =$	·018	·013	·010	·008	·0066	·0055	·0047	·0040

This rate of variation is in very fair agreement with the temperature coefficient unit determined by Poiseuille and given in the formula above.

(iii.) *Corrections.*—The Poiseuille method, on account of the simplicity of the apparatus required and the ease with which the observations can be made, is still much used, but it should be observed that for accurate work two corrections to the results must be applied.

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[1] "Dynamical Theory of Gases," *Phil. Trans. Royal Society*, clvii.

[2] *Comptes rendus*, 1840–41, tt. xi. xii.

[3] *Phil. Trans. R.S.* A, clxxxi. 559.

In the first place, the difference of pressure between still water on the inlet and outlet sides of the pipe is not an accurate measure of the pressure gradient along the pipe, since some of this pressure difference is required to communicate the kinetic energy of motion to the water. Further, in deriving the equation of flow (§ (3)), it is assumed that the velocity distribution across the pipe has become uniform and that no accelerations parallel to the axis of the pipe are taking place. This condition is not fulfilled near the inlet end of the pipe, as an appreciable length of the pipe from the inlet in the direction of flow is required in which the accelerations die out and the velocity distribution becomes uniform. For these reasons instead of the use of the simple formula derived from equation (7), § (3),

$$\mu = \frac{\pi a^4 \mathrm{T}}{8\mathrm{V}} \cdot \frac{p_1 - p_2}{l},$$

in which T is the time of efflux of the volume V of liquid, $(p_1 - p_2)$ the pressure difference between the inlet and outlet ends of the pipe, and l the length of the pipe, the following formula, in which the corrections explained above are inserted, should be used:

$$\mu = \frac{\pi a^4 \mathrm{T}(p_1 - p_2)}{8\mathrm{V}(l + na)} + \frac{m\rho \mathrm{V}}{8\pi \mathrm{T}(l + na)},$$

where ρ is the density of the liquid under test and n and m are constants. The value of n may be taken as 1·64 in all cases, and provided that the value of the second term in equation is small compared with that of the first, m may be assumed equal to unity. Where high accuracy is required it is necessary that the value of m should be obtained experimentally by a series of viscosity determinations with different rates of flow.

§ (7) MODERN INVESTIGATIONS.—In recent work, however, it has been found that no great difficulty is experienced in making an accurate measure of the pressure gradient along the pipe at a sufficient distance from the inlet, provided that the pipe is made of some substance which can be easily machined. In this method two fine holes are drilled in the walls of the pipe at a known distance apart along the axis, great care being taken to prevent a "burr" being formed in the inner surface of the pipe where the hole passes through the wall. Suitable nipples are screwed into the holes at the outer surface and flexible pipe connections made, one to each side of a sensitive manometer. In this way when the flow is set up an accurate measure of the fall of static pressure of the fluid between the holes is obtained. As it is known that the static pressure is constant across any section of the pipe, the intensity of the surface friction is easily calculated from the formula $\mathrm{R} = pa/2l$, where p is the pressure difference per unit area indicated by the manometer and l is the distance between the holes and a the radius.[1] This method has recently been used at the National Physical Laboratory for the determination of the resistance to flow of thick oils in pipes. As the method of obtaining the pressure gradient in the pipes in these experiments is novel, a brief description of it is given.

The manometer used was one of the Chattock type, the principle of which is that the pressure difference at the two ends of a U-tube is balanced by tilting the tube through a small measured angle so that the "head" due to the difference of level of the fluid in the two vertical arms of the U-tube balances the external pressure difference, and no movement of the fluid through the tube takes place. For the latter purpose a telescope is fixed to the tilting-table, the level of which is always adjusted so that the line of collimation passes through the surface of the manometer fluid, say water or mercury. It is, however, essential for the elimination of unknown forces due to capillarity that the ends of the manometer tube where the surface of the fluid is situated should be cup-shaped as shown in *Fig.* 2, and as the detection of the movement of such a large surface is a matter of some difficulty, the device is adopted of introducing a second liquid, usually oil, which will not mix with the water or mercury, into the horizontal limb of the gauge, and the hair-line of the telescope is focussed on to the meniscus formed by the common surface of the two liquids. This is the usual device adopted for measuring pressure differences due to the flow of gases through pipes when the static pressure of the gas is small. For measuring the pressure drop in pipes conveying liquids and when the static pressure of the liquid is high, the type of manometer illustrated in *Fig.* 2 is more convenient. This consists of a U-tube filled with mercury up to the centre of the lower cups, the space above the mercury being filled with salt water. In order to obtain a sharply defined surface of high sensitivity as an indicator of the movement of the mercury, the right-hand cup is contracted to a small section and a second cup attached to it as shown. The upper end of this second cup is connected to a reservoir containing a transparent oil which will not mix with the salt water. A side connection to the upper cup is connected with the experimental pipe. The gauge is filled so that the separating surface of the oil and water form a meniscus at the extremity of the contracted part of the lower cup, and on this the hair-line of the telescope is focussed. The auxiliary glass-bulb fittings at the sides are introduced for convenience in filling the gauge and making sure that no bubbles of air are left in any part of the connections, and in order to obtain a short-circuiting device by means of which the zero of the gauge can be read off without stopping the flow of liquid in the pipe. For the latter observation the two taps in the experimental pipe connections are shut and the tap in the short connecting pipe opened. By this apparatus the frictional resistance to flow was obtained in the case of thick oils of viscosity ranging up to $20\cdot0 \times 10^{-3}$ in absolute C.G.S. units.

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[1] Clearly $\mathrm{R} \times 2\pi a l$ = total frictional force = difference of thrust $= p\pi a^2$.

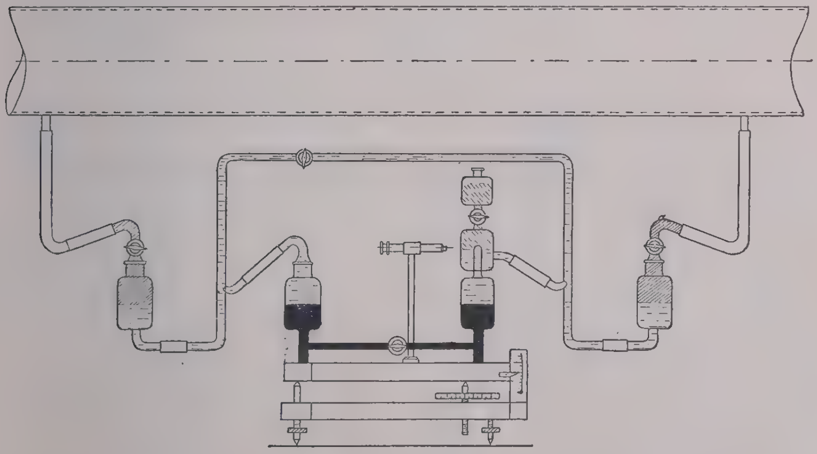

FIG. 2.

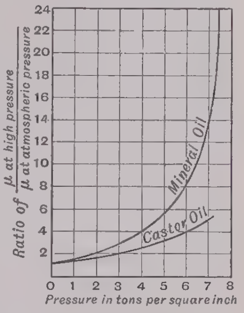

FIG. 3.

§ (8) THE EFFECT OF PRESSURE ON THE VISCOSITY OF LIQUIDS.—In the case of liquids of relatively low viscosity the variation in the value of the coefficient of viscosity with change of pressure is not very marked. The viscosity of water diminishes slightly for pressures of a few atmospheres, and that of benzol and ether increases. Recent researches on the changes of viscosity of certain liquids when the pressure is carried to values as high as 1000 atmospheres have, however, shown enormous increases in viscosity due to high pressures. Thus in the cases of alcohol, carbon bisulphide and ethyl alcohol, Professor O. Faust of Göttingen has found the viscosity of these liquids at pressures of 3000 atmospheres to be more than treble the value at atmospheric pressure. In a recent research carried out at the National Physical Laboratory by Mr. J. H. Hyde,[1] the viscosities of mineral oils at a pressure of 1100 atmospheres were found to have a value exceeding 10 times the value at atmospheric pressure, whereas in the case of a vegetable oil such as rape, the ratio was not higher than 4 to 1. The remarkable character of the rate of rise of viscosity with pressure at the higher pressures will be seen from the curves in *Fig.* 3, which illustrate the results obtained for castor oil and a mineral oil. As this investigation has an important bearing on the constitution of liquids, a short description of the method used at the National Physical Laboratory for measurements of the coefficient of viscosity at high pressures is here given. The apparatus used for these experiments was designed for carrying out the tests by a method suggested by Dr. T. E. Stanton, and a diagrammatic sketch is shown in *Fig.* 4. The instrument consists of a U-tube, the limbs A and B of which are connected together at their lower ends by a large bore tube and at their upper ends by a capillary tube C as shown. The whole is mounted on a frame supported by a knife-edge D, and so arranged that the left-hand side is heavier than the right. The lower half of the circuit is filled with mercury and the upper half with the liquid under experiment. The motion of the frame is governed by the extension of a spring S, to which it is connected through the supporting arm F terminating in a pointer which moves over a finely divided scale. In making an experiment, the tilting frame is set in such a position that the horizontal tubes are level and the scale is adjusted so that the pointer is at zero. A tilt is given to the frame by moving the pointer vertically upwards, the taps T and P being open. The tilt causes a flow in the U-tube, and when this flow has ceased the taps are closed and the frame restored to its original position with the pointer at zero, the tension of the spring being adjusted to support the frame in this position. The head of mercury in the circuit tending to cause flow round it can be calculated from the motion which has been given to the pointer and from the distances DR and AB. If now the tap T be opened, the liquid will flow through the capillary tube from A to B and the mercury from B to A. The spring is so designed that the displacement of the frame due to the flow of mercury from one vertical limb to the other is such that the head of mercury producing the flow remains constant, and it is clear that the fall of the frame as indicated by the movement of the pointer over the scale gives a measure of the volume of liquid which passes from one side to the other. In this way the whole of the data required for the calculation of the coefficient of viscosity are available when the dimensions of the capillary tube are accurately known, and the value of μ is obtained from the formula (see § (3))

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[1] *Proc. Roy. Soc.* A, xcvii.

$$\mu = \frac{\pi \rho h a^4 T}{8 V l},$$

where ρ is the density of the liquid, h the head of liquid, a the radius of the capillary tube, T the time in passing the volume of liquid V through the tube, and l the length of the tube.

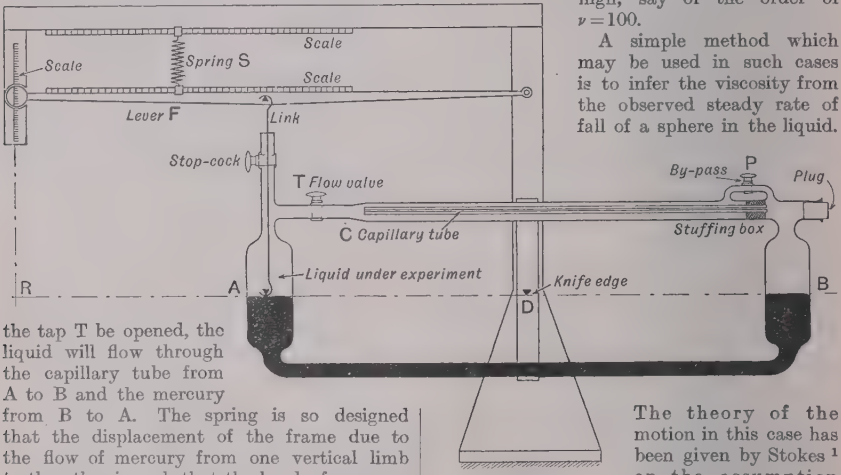

FIG. 4.

It will be seen that the above formula requires a knowledge of the variation in the density of the liquid with pressure before the value of the coefficient of viscosity can be obtained. In the paper by Mr. J. H. Hyde referred to, a simple method is described by means of which the densities of liquids can be obtained up to pressures of 1500 atmospheres. It may be remarked that the construction of the high-pressure viscosity apparatus presented considerable practical difficulty when the pressures used were of the order of 1000 atmospheres, necessitating the use of high-tensile steel for the parts subjected to pressure. For details of these and further numerical data the original paper may be consulted.

§ (9) LIQUIDS OF HIGH VISCOSITY.—The method described above for the estimation of the viscosity of fluids, by observing the known gradient of pressure and the rate of flow of the fluid through a pipe of known dimensions, has obvious practical limitations when the viscosity of the fluid is very high, say of the order of $\nu = 100$.

A simple method which may be used in such cases is to infer the viscosity from the observed steady rate of fall of a sphere in the liquid.

The theory of the motion in this case has been given by Stokes[1] on the assumption that the inertia terms in the equations (5) may be neglected, i.e. that

$$\frac{\partial u}{\partial t} = -\frac{1}{\rho} \cdot \frac{\partial p}{\partial x} + \nu \cdot \nabla^2 u, \text{ etc.}$$

On this assumption the external force acting on the sphere is given by

$$P = 6\pi\mu a U,$$

where a is the radius of the sphere and U the velocity of fall. It has been pointed out by Lord Rayleigh that the assumption on which this solution is based limits the velocity of fall to extremely low values, since the relation will not hold with fair accuracy unless Ua is small compared with ν. The motion at higher speeds than this has been investigated by Williams.[2]

Another method which may be employed for liquids of high viscosity is the oscillating disc method first used by Maxwell. This appears to be well suited for the difficult but practically important case of molten glass.

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[1] *Collected Papers*, iii. 1.
[2] *Phil. Mag.* xxxvi. 354.

§ (10) The Viscosity of Solids.—As pointed out in the introduction to this article, the frictional resistance which one part of a solid body offers to the sliding of the other part over it is made up of two distinct phenomena: (1) the elastic distortion of the substance in which the work done is stored up as potential energy in the molecules of the body, and (2) the viscous distortion, the energy of which is converted into heat. It has been remarked by Lord Kelvin[1] that this distinction is not rigidly correct, since even in the absolutely perfect elasticity of volume presented by homogeneous crystals dissipation of energy is an evitable result of every change of volume, because of the accompanying change of temperature and consequent dissipation of heat by conduction and radiation. It is, however, recognised that the loss of energy due to this cause is small compared with the whole loss of energy which occurs in many cases of the vibration of metals, so that the statement above may be taken as sufficiently accurate for most purposes. The usual method of studying the phenomena of the viscosity of metals is by noting the rate of damping of the torsional oscillations of long rods carrying a heavy mass at one end and fixed at the other end. The damping of the oscillations is said to be more rapid in glass than in most of the elastic metals such as copper, iron, silver, aluminium, but on the other hand the damping in the case of zinc and india-rubber is more rapid than in glass. In the experiments described by Lord Kelvin in the paper referred to, it was found, as would be expected, that the loss of energy in a vibration was greater the greater the velocity, but that the variation with speed was not nearly proportional to the velocity of deformation as in the case of fluids. It would appear, therefore, that the damping is not altogether the effect of viscous resistances of the ordinary type which are proportional to the rates of strain.

§ (11) The Internal Friction of Fluids in Turbulent Motion. (i.) *Eddying Motion.*—Before entering upon a discussion of the characteristics of the internal friction of fluids when the general motion is eddying or turbulent, a brief description of the methods of velocity estimation under these conditions is desirable for the reason that a measurement of the mean rate of flow of the fluid through any fixed element of surface taken over an appreciable time is a matter of fundamental importance in the practical determination of the frictional resistance. Since by definition a fluid in turbulent motion consists of a mass of eddies, it might be supposed that any determination of the kind under consideration would be meaningless as defining any physical condition of the fluid, and this is probably true in such cases as those in which eddies, relatively large in size and slow in period, are thrown off from the projecting edges of bodies immersed in fluids moving relatively to them. It has been found, however, that in the majority of cases of fluid motion in which turbulence is known to exist either from the fact that the critical speed[2] has been exceeded, or from the observations on the resistance to flow being greatly in excess of those due to streamline motion, the eddies are apparently of such small dimensions and of such high periodicity that any appreciable variation of the forces produced on an immersed body by the turbulent motion of the fluid over it cannot be detected by ordinary methods. For example, if in a parallel channel through which air is in motion above the critical speed, a small open-mouthed tube be placed with its axis parallel to the axis of the channel and its other end connected to a sensitive manometer as shown in *Fig.* 5, it will be observed that if sufficient precautions are taken to eliminate external disturbances and irregularities of the mechanism causing the flow, the reading of the manometer remains perfectly steady. Further, if the pressure in the tube be accurately measured it will be found that its value exceeds that of the fluid itself by the quantity $\frac{1}{2}\rho v^2$, where ρ is the density of the fluid and v is the mean speed of the fluid which would exist over the area occupied by the mouth of the tube if it were removed. The pressure of the fluid itself is usually called the static pressure, and in the case of a fluid in motion, either streamlike or turbulent in character, the static pressure at any point is the pressure at the boundary of any smooth solid surface containing the point and parallel to the direction of flow at that point. It is evident that the above relation between the dynamical pressure at the mouth of the tube and the static pressure affords a convenient method of determining the value of the mean speed of a fluid at any point in it.

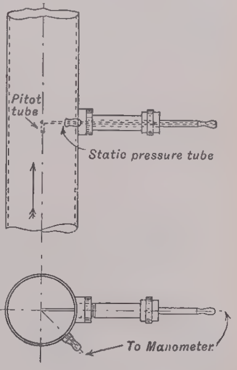

Fig. 5.

(ii.) *The Pitot Tube.*—In the case of the parallel channel, since it is known that the pressure is independent of the radius (see § (3)), all that is necessary is to measure the difference in pressure between the open-mouthed tube facing the current, commonly called a Pitot tube, and that at a hole in the walls of the channel at the same cross-section, and equating this to $\frac{1}{2}\rho v^2$, the velocity can be calculated. In this way the velocity distribution over the cross-section of a parallel channel may be determined and the total flow calculated by graphical methods. Comparison of the total flow so estimated and that given by a discharge meter at the outlet of the channel has shown that the method can be relied upon to a high degree of accuracy. For cases in which the flow does not take place in a channel with parallel walls, it is necessary, for the purpose of obtaining the static pressure, to introduce an artificial boundary as close to the mouth of the Pitot tube as possible. One method of doing this is shown in *Fig.* 6, which is an illustration of the standard instrument used for velocity measurement at the National Physical Laboratory. Careful experiments[1] with an instrument of this type moving in free air have shown that the velocity estimation obtained from it when the pressure differences are measured on a manometer of the Chattock type have a limit of accuracy of one-tenth of one per cent. In the use of the instrument it is, of course, necessary that the density of the fluid at the point considered should be known, and in the case of a compressible fluid where considerable differences of density exist this may involve another experimental determination of some difficulty.

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[1] *Proc. R.S.*, 1865, xiv. 289.

[2] See § (71), (iii.).

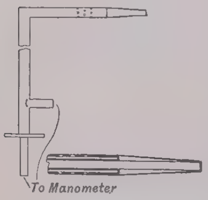

Fig. 6.

The original use of the Pitot tube seems to have been for the purpose of measuring the distribution of velocity in rivers and canals. In this form it consists simply of a glass tube bent through a right angle and held vertically in the current, the height of the column of water inside the tube above the surrounding surface being noted. By this means the use of a static pressure tube is avoided, but it is obvious that the velocity estimations must be of only an approximate nature.

(iii.) *The Critical Velocity.*—It was first shown by Osborne Reynolds that, when a fluid was in motion through a parallel channel, there existed a critical value of the mean speed of flow at which the character of the motion changed from one of steady streams parallel to the axis of the channel to one of turbulence in which the whole of the fluid was broken up into a mass of eddies. The causes of the change in the type of motion will be discussed in greater detail in the subsequent sections of this article, but for the present purpose it is sufficient to remark that the change was found to coincide with a change in the law of frictional resistance to flow through the channel, the streamline motion corresponding to a resistance varying as the first power of the speed, and the turbulent motion corresponding to a resistance varying nearly as the square of the speed. In the latter case, although the mean motion at any point when taken over a sufficient time is parallel to the sides of the channel, it is made up of a succession of motions crossing the channel in different directions. It is evident that in this case if we are to adhere to the definition of the coefficient of viscosity as the ratio of the shearing stress to the rate of distortion, *i.e.* that $f=\mu(dv/dy)$, and, further, if v is taken to express the mean motion taken over a sufficient time, then since f is known to vary as a power of the velocity greater than unity μ must be a function of the velocity and must be held to include the momentum per second parallel to the plane of shear, which is carried by the cross-streams through the plane.[2]

(iv.) *The Two Viscosities.*—If, however, we regard the above relation as expressing the instantaneous value of the intensity of the resistance at a point in the fluid, we must realise that dv/dy is the instantaneous value of the rate of distortion, a quantity which we have no means of measuring directly, and that then μ is independent of the motion and a physical property of the fluid. It appears therefore that, as pointed out by Osborne Reynolds, there are two essentially distinct viscosities in fluids. One is a physical property of the fluid and is a measure of the instantaneous resistance to distortion at a point moving with the fluid, and the other is a mechanical viscosity arising from the molar motion of the fluid and given by the relation $f=\mu'(dv/dy)$, where v is the mean motion at a point taken over a sufficient time, and μ' is a function of v and probably also of the distribution of the solid boundaries of the fluid.

That these characteristics are independent of each other, apart from the fact of the dependence of the existence of the mechanical viscosity on the physical viscosity, is shown by the striking fact that when the motion of a fluid is such that the resistance is as the square of the velocity, the magnitude of the resistance is independent of the character of the fluid in all respects, except that of its density.

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[1] *Report of Advisory Committee for Aeronautics*, 1912–13, p. 35.

[2] Reynolds, *Scientific Papers*, ii. 236.

§ (12) MEASUREMENT OF THE MECHANICAL VISCOSITY.—In a research carried out at the National Physical Laboratory in 1911,[1] the characteristics of the mechanical viscosity of a fluid as affected by the speed and the dimensions of the channel in which the flow took place were investigated. The fluid used was air, which was forced through cylindrical pipes at speeds above the critical, and the distribution of mean velocity was determined by means of a Pitot and static pressure-tube device of the kind described above. In order to simplify the investigation it was desirable that the resistance to flow should vary exactly as the square of the velocity of flow, in which case, as mentioned above, the motion would be entirely independent of the physical viscosity of the fluid. This condition was secured by a suitable roughening of the internal surface of the brass pipes used for the experiments.

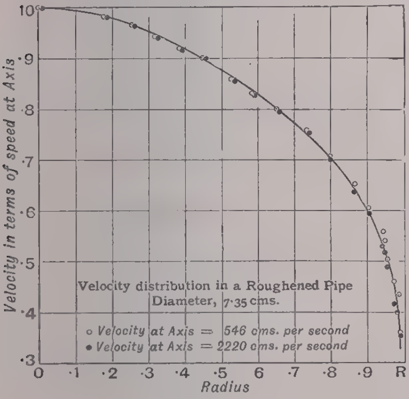

FIG. 7.

The distribution of mean axial speed across the section of the pipe was then measured, and a typical curve of distribution is shown in *Fig.* 7. It will be seen that the distribution of mean axial velocity in the case of the turbulent motion is approximately parabolic from the axis up to a comparatively short distance from the walls, *i.e.* the equation to this part of the velocity curve can be written

$$v = v_c - \mathrm{A}r^2, \quad . \quad . \quad . \quad (1)$$

where v_c is the velocity at the axis, r is the radius at which v is measured, and A is a constant.

It was also found that for any section of the pipe the static pressure of the fluid was constant for all values of the radius, so that for any cylinder of fluid of radius r between any two sections distant l apart the shearing stress on the outer surface would be given by

$$f 2\pi r l = (p_1 - p_2)\pi r^2$$

or

$$f = \frac{(p_1 - p_2)r}{2l}, \quad . \quad . \quad . \quad (2)$$

where $(p_1 - p_2)$ is the fall of static pressure between the two sections, and f the intensity of shearing stress. The shearing stress in the fluid is therefore proportional to the radius.

But from (1) it is seen that

$$\frac{dv}{dr} = -2\mathrm{A}r,$$

so that the relation $f = \mu'(dv/dr)$ becomes $(p_1 - p_2)/2l = -2\mu'\mathrm{A}$.

Hence μ', which is the mechanical viscosity, is constant across the pipe up to within a relatively small distance from the boundary.

The next step in the investigation was to determine the dependence of μ' on the rate of flow through the pipe. By taking a series of distributions of axial velocity at different rates of flow and plotting the values of v/v_c on a radius base, it was found that all the points lay on the same curve, indicating that in equation (1) $v/v_c = 1 - (\mathrm{A}/v_c) \, . \, r^2$ the value of A/v_c was constant, *i.e.* that A was proportional to the speed at the axis, and therefore that the value of dv/dr for any radius was simply proportional to the velocity of flow. It follows that, since the shearing stress is proportional to the square of the speed of flow, μ' must be proportional to the first power of the speed.

Finally, a series of experiments were made with the object of determining the effect of the dimensions of the channels on the mechanical viscosity. For this purpose two pipes of radii a_1, a_2 were prepared, in which the surface roughnesses were geometrically similar, so that the intensity of the surface frictions was exactly proportional to the squares of the speeds of flow. On determining the velocity distributions in these pipes, and plotting the values of v/v_c on a base of r/a, where "a" is the outside radius of the pipe, it was found that all the points fell on the same curve, whose equation was

$$\frac{v}{v_c} = 1 - k \, . \, \frac{r^2}{a^2},$$

where k was a constant.

Now, if f_{s1}, f_{s2} are the values of the *surface* frictions in the two pipes, and f_1, f_2 the values of the internal fluid friction at corresponding radii r_1 and r_2,

$$\frac{f_{s1}}{f_{s2}} = \frac{{v_{c1}}^2}{{v_{c2}}^2}, \quad \text{and} \quad \frac{f_1}{f_{s1}} = \frac{r_1}{a_1} \quad \frac{f_2}{f_{s2}} = \frac{r_2}{a_2},$$

we have therefore

$$\frac{f_1}{f_2} = \frac{{v_{c1}}^2 r_1 a_2}{{v_{c2}}^2 a_1 r_2} = \frac{{\mu_1}'(dv/dr)_1}{{\mu_2}'(dv/dr)_2} = \frac{{\mu_1}' 2 v_{c1} r_1}{{a_1}^2} \cdot \frac{{a_2}^2}{{\mu_2}' 2 v_{c2} r_2}$$

or

$$\frac{{\mu_1}'}{{\mu_2}'} = \frac{v_{c1} \, . \, a_1}{v_{c2} a_2},$$

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[1] Stanton, *Proc. Roy. Soc.* A, lxxxv.

i.e. the ratio of the mechanical viscosities in the two pipes is proportional to the product of the speed of flow and the diameter of the pipe. It appears therefore that the coefficient of mechanical viscosity for flow in parallel channels in which the friction varies as the square of the speed may be written $\mu' = kvd$, where v is the speed of flow, d is the linear dimension of the channel, and k a constant depending on the nature of the fluid used in the experiments.

Since the dimensions of μ' are [M/LT] it is evident that the dimensions of k are M/L^3, *i.e.* that k is proportional to the density of the fluid used, as would be expected from the statement in paragraph (10).

§ (13) EDDY VISCOSITY.—The characteristics of the coefficient of eddy viscosity have also been investigated by Mr. G. I. Taylor in connection with atmospheric phenomena.[1]

Considering the interchange of momentum between consecutive horizontal layers of air in a steady wind due to the movement of the eddies in it, let U_z and V_z be the average horizontal components of wind velocity at a height z parallel to perpendicular co-ordinates x and y, and u', v', w' the components of eddy velocity, so that the three components of velocity are $U_z + u'$, $V_z + v'$, and w', then the rate at which x-momentum is transmitted across any horizontal area is

$$\iint \rho(U_z + u')w' dxdy. \quad . \quad . \quad (1)$$

Mr. Taylor then proceeds to consider a particular case of disturbed motion, supposing the fluid incompressible, and the motion to take place in two dimensions x and z.

If originally the fluid is flowing parallel to the axis of x with velocity U_z, then after disturbance the rate at which momentum leaves a layer of thickness δz is

$$\left[\frac{\partial}{\partial z}\iint \rho(U_z + u')w' dxdy\right]\delta z = I\delta z. \quad . \quad (2)$$

But since U_z is constant over the plane xy and there is no resultant flow across a horizontal plane

$$\iint \rho U_z w' dxdy = 0,$$

and therefore

$$I = \rho\frac{\partial}{\partial z}\iint u'w' dxdy = \rho\iint\left(u'\frac{\partial w'}{\partial z} + w'\frac{\partial u'}{\partial z}\right)dxdy. \quad (3)$$

Further, since the motion is confined to two dimensions $\partial(U_z + u')/\partial z - \partial w'/\partial x =$ twice the vorticity at the point x, y, z, and since every portion of the fluid retains its vorticity throughout the motion, this must be equal to twice the vorticity which the fluid at the point x, y, z had before the disturbance set in. This is equal to the value of dU_z/dz at the height z_0 of the layer from which the fluid at the point x, y, z originated, and therefore

$$\frac{dU_z}{dz} + \frac{\partial u'}{\partial z} - \frac{\partial w'}{\partial x} = \left[\frac{dU_z}{dz}\right]_{z_0}. \quad . \quad (4)$$

From this relation, together with the equation of continuity,

$$\frac{\partial u'}{\partial x} + \frac{\partial w'}{\partial z} = 0, \quad . \quad . \quad . \quad (5)$$

equation (3) becomes

$$I = \rho\iint\left(-u'\frac{\partial u'}{\partial x} + w'\frac{\partial w'}{\partial x} + w'\left[\frac{dU_z}{dz}\right]_{z_0} - w'\frac{dU_z}{dz}\right)dxdy$$

$$= \rho\iint\frac{\partial(w'^2 - u'^2)}{\partial x}dxdy + \rho\iint w'\left(\left[\frac{dU_z}{dz}\right]_{z_0} - \frac{dU_z}{dz}\right)dxdy. \quad (6)$$

The first term of this expression vanishes when a large area is considered, but the second term does not vanish.

Expanding $[dU_z/d_z]_{z_0}$ in powers of $(z_0 - z)$ we have

$$\left[\frac{dU_z}{dz}\right]_{z_0} = \frac{dU_z}{dz} + (z_0 - z)\frac{d^2U_z}{dz^2} + \ldots,$$

and therefore

$$I = \rho\iint w'\left[(z_0 - z)\frac{d^2U_z}{dz^2} + \frac{(z_0 - z)^2}{2}\frac{d^3U_z}{dz^3} + \ldots\right]dxdy, \quad (7)$$

and if $(z_0 - z)$ be of such a magnitude that the change in dU_z/dz in that distance is small compared with dU_z/dz itself,

$$I = \rho\frac{d^2U_z}{dz^2}\iint w'(z_0 - z)dxdy, \quad . \quad (8)$$

and the rate at which the x-momentum leaves a layer of thickness dz is

$$\rho\frac{\partial^2 U_2}{\partial z^2}\delta z\iint w'(z_0 - z)dxdy. \quad . \quad (9)$$

The effect of the disturbance is therefore to reduce the x-momentum of a horizontal layer of thickness dz at the rate $\rho(\partial^2U_z/\partial z^2)\delta z \times$ (average value of $w'(z_0 - z)$ per unit area). The same effect would be produced on a layer of thickness dz by a viscosity equal to $\rho \times$ (average value of $w'(z_0 - z)$) if the motion had not been disturbed.

According to Mr. Taylor's theory, therefore, when we wish to consider the disturbed motion of layers of air we can take account of the eddies by introducing a coefficient of eddy viscosity equal to $\rho \times$ average value of $w'(z_0 - z)$, and supposing that the motion is steady

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[1] *Phil. Trans. Roy. Soc.* A, ccxv. 11.

$(z_0 - z)$ is the height through which the air has moved since the last mixture took place.

The average value of $w'(z_0 - z)$ can be expressed in the form $\frac{1}{2}(\overline{wd})$, where d is the average height through which an eddy moves before mixing with its surroundings and w' roughly represents the average vertical velocity in places where w' is positive.

The movement of the air on the earth's surface as affected by the eddy viscosity may be investigated as follows.

As before, assuming the air incompressible, the equations of motion (see p. 345) may be written

$$\left.\begin{aligned}\frac{Du}{Dt} &= X - \frac{1}{\rho}\frac{\partial p}{\partial x} + \frac{\mu'}{\rho}\nabla^2 u\\ \frac{Dv}{Dt} &= Y - \frac{1}{\rho}\frac{\partial p}{\partial y} + \frac{\mu'}{\rho}\nabla^2 v\\ \frac{Dw}{Dt} &= Z - \frac{1}{\rho}\frac{\partial p}{\partial z} + \frac{\mu'}{\rho}\nabla^2 w\end{aligned}\right\} \quad . \quad . \quad (1)$$

with the usual notation.

Assume a constant pressure gradient G acting in the direction of y. The remaining forces are gravity and those due to the earth's rotation.

Hence
$$\left.\begin{aligned}X &= -2\omega v \sin\lambda\\ Y &= 2\omega u \sin\lambda\\ Z &= -g\end{aligned}\right\}, \quad . \quad . \quad . \quad (2)$$

where ω is the angular velocity of the earth's rotation and λ is the latitude, and

$$p = k - g\rho z + Gy, \quad \text{where } k = \text{constant}.$$

Assuming the motion to be horizontal, equations (1) become

$$0 = -2\omega v \sin\lambda + \frac{\mu'}{\rho}\cdot\frac{d^2u}{dz^2}, \quad . \quad . \quad . \quad (3)$$

$$0 = -2\omega u \sin\lambda - \frac{G}{\rho} + \frac{\mu'}{\rho}\cdot\frac{d^2v}{dz^2}. \quad . \quad (4)$$

Eliminating u we have

$$\frac{d^4v}{dz^4} + 4B^4v = 0, \quad \text{where } 2B^2 = \frac{2\omega\rho\sin\lambda}{\mu'}, \quad (5)$$

or $$v = A_2e^{-Bz}\sin Bz + A_4e^{-Bz}\cos Bz, \quad (6)$$

or substituting in (4)

$$u = A_2e^{-Bz}\cos Bz - A_4e^{-Bz}\sin Bz + \frac{G}{2\mu'B^2}. \quad (7)$$

At great heights, therefore, $v = 0$ and

$$u = \frac{G}{2\mu'B^2} = \frac{G}{2\omega\rho\sin\lambda}, \quad . \quad . \quad . \quad (8)$$

i.e. u is independent of μ' and is the value of the velocity calculated from the pressure distribution, and is called the gradient velocity.

The values of A_2 and A_4 are found as follows:

Assuming that at the surface

$$\frac{du/dz}{dv/dz} = \frac{u}{v},$$

this becomes by substitution from the above relations

$$-\left(\frac{A_2 + A_4}{A_2 - A_4}\right) = \frac{A_2 + (G/2\mu'B^2)}{A_4} = \frac{A_2 + Q_G}{A_4}, \quad (9)$$

where Q_G is the gradient velocity.

Again, if the wind at the surface be deviated through an angle α from the gradient wind in such a way that if one stands facing the surface wind the gradient wind will be coming from the right if α be positive. Then

$$\tan\alpha = -\left(\frac{v}{u}\right)_{z=0} = \frac{A_4}{A_2 + Q_G},$$

and we have

$$\left.\begin{aligned}A_2 &= \frac{-\tan\alpha(1 + \tan\alpha)}{1 + \tan^2\alpha}Q_G\\ A_4 &= \frac{-\tan\alpha(1 - \tan\alpha)}{1 + \tan^2\alpha}\,.\,Q_G\end{aligned}\right\} \quad . \quad . \quad (10)$$

Now the surface wind

$$\begin{aligned}Q_s &= \sqrt{(u^2 + v^2)_{z=0}}\\ &= \sqrt{A_4^2 + (A_2 + Q_G)^2}\\ &= \frac{Q_G}{1 + \tan^2\alpha}\sqrt{\tan^2\alpha(1 - \tan\alpha)^2 + (1 - \tan\alpha)^2}\end{aligned}$$

or $$Q_s = Q_G(\cos\alpha - \sin\alpha). \quad . \quad (11)$$

This relation has been verified by direct observations made by means of pilot balloons on Salisbury Plain by Mr. G. M. B. Dobson.

Again, if H_1 be the height at which the direction of the wind coincides with that of the gradient wind, putting $v = 0$ we have

$$A_2 \sin BH_1 + A_4 \cos BH_1 = 0$$

or $$\tan BH_1 = -\frac{A_4}{A_2},$$

and substituting for A_2 and A_4

$$\tan BH_1 = \frac{1 - \tan\alpha}{1 + \tan\alpha} = \tan\left(\alpha - \frac{\pi}{4}\right). \quad (12)$$

Since α is positive and less than $\pi/4$, the smallest positive value of H_1 is given by $BH_1 = 3\pi/4 + \alpha$.

The height H_2 at which the wind velocity first becomes equal to the gradient velocity is given by $u^2 + v^2 = Q_G{}^2$, or

$$l^{-BH_2} = \frac{(1 + \tan\alpha)\cos BH_2 - (1 - \tan\alpha)\sin BH_2}{\tan\alpha}. \quad (13)$$

The relation between BH_2, BH_1, H_1/H_2, and α are given in the following table:

$\alpha°$.	BH_2.	BH_1.	$\frac{H_1}{H_2}$.
0	·78	2·35	3·0
10	·91	2·53	2·8
20	1·04	2·70	2·6
30	1·20	2·88	2·4
45	1·44	3·15	2·2

Mr. Dobson gives 800 metres/300 metres $= 2·66$ as the observed value of H_1/H_2 for

$\alpha=20°$, which is in very close agreement with the above calculation.

Further, assuming a value for α of 20°, we have from the above table $BH_1=2{\cdot}7$, and substituting in the relation

$$B=\sqrt{\frac{\omega\rho\sin\lambda}{\mu'}},$$

$$\frac{\mu'}{\rho}=H_1^2\frac{\omega\sin\alpha}{2{\cdot}7^2} \quad . \quad . \quad . \quad . \quad (14)$$

Putting $\omega=0{\cdot}000073$ and $\lambda=50°$ N., we have for the South of England

$$\frac{\mu'}{\rho}=H_1^2\times0{\cdot}77\times10^{-5}.$$

Taking the values of H_1 from Mr. Dobson's paper, we obtain the following table:

Wind.	H_1 in Metres.	$\frac{\mu'}{\rho}$.
Strong . . .	900	62×10^3
Moderate . .	800	50×10^3
Light . . .	600	28×10^3

II. The Nature of the Motion of Fluids over Solid Surfaces tangential to the Direction of Flow and the Characteristics of the Frictional Forces consequent on the Motion

§ (14) Resistance to Motion over a Solid Boundary.—The relations between the resistance encountered by a solid body moving through a fluid in which it is completely immersed, or by a fluid in moving over a fixed solid surface, can be broadly divided into two classes. The resistance is either proportional simply to the relative speed of surface and fluid, or it is proportional to a power of the relative speed which is in the neighbourhood of 2.

(i.) *The Critical Velocity, Osborne Reynolds' Law.*—It is found that these two classes of resistance correspond to two definite states of internal motion of the fluid. When the elements of the fluid follow one another along lines of motion which lead in the most direct manner to their destination, the resistance is proportional to the relative velocity simply. When the particles of the fluid eddy about in sinuous paths the most indirect possible, *i.e.* the fluid is in turbulent motion, the resistance varies nearly as the square of the speed. The transparency of most fluids renders it difficult to determine when a fluid is in steady or turbulent motion. Thus in the case of water passing through a glass tube it is quite impossible by visual examination of the flow to determine which of the two states of motion is taking place. A simple method of demonstrating the nature of the motion of fluids and of the manner in which one state of motion will pass into the other was devised by Osborne Reynolds in 1883.[1] In these experiments a straight tube of glass 5 feet long and provided with a trumpet mouthpiece was placed in a water tank 6 feet long, 18 inches deep, and 18 inches broad in the manner shown in *Fig.* 8, and provided with a stopcock to regulate the rate of flow of the water through it. A small reservoir of coloured water was placed above the tank with an outlet into the tank in such a position that a stream of coloured water could be delivered at a very slow speed a few inches in front of the trumpet mouthpiece, with the result that a streak of the colour could be drawn into the pipe when flow through it took place. The flow of colour was regulated by a clip on the india-rubber pipe connection. It was found that when the cock was slightly opened and a streak of colour allowed to enter the pipe with the water, no mixture of the two fluids took place, the streak exhibiting itself as a thin band parallel to the axis of the pipe. As the discharge cock was gradually opened further it was observed that for some definite value of the velocity of flow, which was always the same for the same temperature of the water, the whole of the colour band extending downstream from a point at some distance from the mouthpiece would suddenly break up and mix with the surrounding water, so that this part of the tube became full of a mass of coloured water. By further increasing the rate of flow the point at which the breakdown occurred moved back towards the mouthpiece, but it was not found possible by increasing the flow to the greatest extent possible to cause the break-up of the stream to take place at the mouthpiece. The significance of this effect will be referred to later.

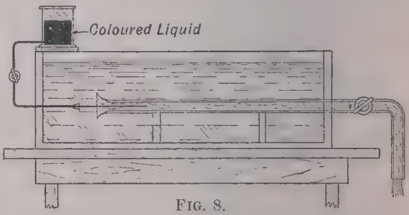

Fig. 8.

As regards the conditions which determined the break-up of the steady motion, it was observed by Reynolds, from an examination of the equations of motion of the fluid, that if the motion be supposed to depend on a single velocity parameter U, say the mean velocity along the tube, and on a single linear parameter c, say the radius of the tube, then the accelerations would be expressed in terms of 2 types; in one of which U^2/c^2 is a factor and in the other $\mu U/\rho c^4$ is a factor. The relative values of these terms depend on the value of the ratio $c\rho U/\mu$, and it would appear, therefore, that if the eddies were due to one particular cause the birth of the eddies would coincide with some particular value of this ratio.

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[1] *Phil. Trans. Roy. Soc.*, 1883.

To test the accuracy of this conclusion, a large number of observations were made in the apparatus shown in *Fig.* 8 to determine the value of the velocity at which the steady motion broke down in pipes of different diameters.

For the purpose of obtaining the effect of a variation in the value of μ, the temperature of the water in the tank was varied between the values 5° C. and 22° C.

The results of the experiments fully corroborated the conclusions drawn from the equations of motion, the law of the critical point being given by the equation $v_c d\rho/\mu = 300$, where v_c is the critical velocity of flow, *i.e.* the discharge per unit of time divided by the area of the pipe; d is the diameter of the pipe, and μ and ρ the viscosity and density of the fluid, all measured in some self-consistent system of units.

(ii.) *Effect of the Boundary.*—It was noticed, however, that the critical velocity was much higher than had been expected in pipes of the sizes used, since resistances varying as the square of the velocity had been found at much smaller velocities than those at which the eddies appeared in the tank experiment. Further, it was observed that the critical velocity was very sensitive to disturbance of the water before entering the tubes, and it was only by the greatest care as to the uniformity of temperature of the tank and stillness of the water that consistent results were obtained. This showed that the steady motion was unstable for large disturbances long before the critical velocity was reached. As it appeared probable that the cause of this phenomenon was dependent on the boundary condition, the following experiment to show the effect on the motion of an elimination of the solid boundaries was devised. A glass tube 5 feet long and 1·2 inch in diameter, having its ends slightly bent up as shown in *Fig.* 9, was half filled with bisulphide of carbon and then filled up with water and both ends corked. The bisulphide was chosen as being a limpid liquid but little heavier than water and completely insoluble, so that the surface between the two liquids could be clearly distinguished. When the tube was horizontal the surface of separation extended along the axis of the tube. On one end of the tube being slightly raised the water would flow to the upper end and the bisulphide to the other, causing opposite currents along the upper and lower halves of the tube. It was found that when one end of the tube was raised quickly by a certain definite amount, waves showed themselves at the surface of separation which presented the appearance of wind waves. Further, it was noticed that after the expiration of some days a skin formed slowly between the bisulphide and the water, and that when this was formed a repetition of the tilt did not result in the formation of the waves but in the production of eddies, below and above the surface of separation. It would appear, therefore, that there is a critical velocity independent of the boundary action, and that the introduction of a boundary condition alters materially the nature of the motion. This conclusion was confirmed by observing the effect of the wind on a surface of water calmed by oil drops. It was noticed that as the sheet of oil on the surface of the water drifted before the wind, there was distinct evidence of eddies in the water below the oil at some distance from the windward edge, but that without oil there was no indication of eddies, thus indicating that the boundary condition introduced by the oil was the cause of the eddies. There appeared, therefore, to be no doubt that the break-down of the steady motion in the straight pipes was due to eddies thrown off from the solid boundary, and that, consequently, eddies produced by any other cause such as a disturbance in the water would also tend to bring about the same result. Further, it follows that there must be another critical velocity which would be the velocity at which previously existing eddies would die out and the motion become steady. It was decided by Reynolds to test this conclusion by allowing water in a high state of disturbance to enter a tube and observing the motion at a distance from the inlet considered sufficient for the eddies to have died out if at all.

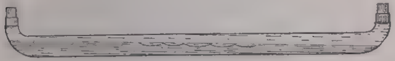
FIG. 9.

Obviously the colour band method was useless for the purpose of settling this question, as the effect of adding colour to a mass of water in turbulent motion was to render the whole of it uniform in colour. It was decided finally to investigate the changes in frictional resistance with speed, as it was thought that the speed at which the law of resistance changed from that of the square to that of the first power of the speed might be sufficiently well marked to define a critical speed. For this purpose lead tubes of about 16 feet in length and a half-inch and a quarter-inch diameter were connected to a supply of water in such a way that the water entered the tubes in a high state of turbulence. The last 5 feet of the tube was connected to a manometer by small pipes leading from two holes, one at each extremity of the experimental length. The water flowing from the tubes was passed through a special meter, so that all the data for calculating the speed of flow and the frictional resistance were available. It was found that the critical velocity at which the motion changed from one type to another was very clearly marked, both by the violent disturbance of the water in the manometer when the critical velocity was reached, and also by the marked change of slope of the curve of resistance of which the ordinates and abscissae were the frictional resistance and the speed of the water.

As was anticipated, the value of the critical speed thus found was considerably lower than that obtained by the method of colour bands, thus indicating that the critical speed in the latter case had a fictitious value. From the friction experiments it was found that the real critical speed was given by the relation

$$\frac{v_c d\rho}{\mu}=2000.$$

(iii.) *Reynolds' Index Law.*—Reynolds' method of presenting the results of the experiments was by means of plotting not the observed values of the resistance and speed, but the logarithms of these numbers. It was found that when this process was carried out for any particular pipe the resulting curve consisted of two straight branches, the lower one corresponding to observations below the critical being inclined at 45° and the upper one at a slope to the axis of pressure of n to 1. These curves are shown in *Fig.* 10 for the case of the two lead pipes used.

It was evident, therefore, that the law of resistance for speeds about the critical was not, as had been supposed, of the form

$$R=Av^2+Bv,$$

but of the form $R=Cv^n$.

It will be seen later that this relation, known as the index law of resistance, is only approximate, but over the range covered by Reynolds' experiments its accuracy is fairly high.

A general law of resistance for pipes of all dimensions and rates of flow was obtained by Reynolds as follows. From the logarithmic plotting it appeared that $R/R_c=(v/v_c)^n$ where R_c and R are the frictional resistances per unit area at the critical velocity and above it. But since by Reynolds' discovery the value of v_c is always given by $v_c d\rho/\mu=k$, and R_c from the streamline theory is known to be equal to $4\mu v_c/d$, we have, by substitution,

$$\frac{R\rho d^2}{4\mu^2 k}=\left(\frac{vd\rho}{k\mu}\right)^n,$$

a relation which, according as $n=1$ or a value in the neighbourhood of 2 depending on the nature of the surface of the pipe, expresses the law of resistance for all sizes of pipe and conditions of flow of the fluid. The experiments of Reynolds on which these conclusions are based were carried out in smooth lead pipes of 1·27 and 0·62 cm. diameter, and through a speed range of from 3 to 706 cm. per second for the 1·27 cm. pipe, and from 7 to 469 cm. per second for the 0·62 cm. pipe. Further, a careful examination of Darcy's very extensive experiments on water flowing in pipes ranging from 1·2 cm. to 50 cm. in diameter did not show any systematic deviation whatever from the general relation laid down, and with very few exceptions the agreement with the Reynolds formula was within 2 or 3 per cent.

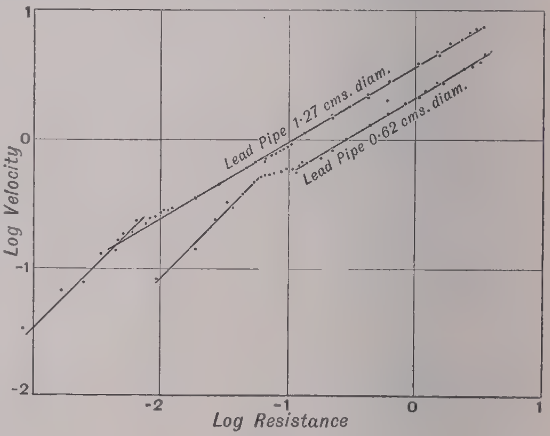

FIG. 10.

III. The Determination by Theory and Experiment of the Frictional Resistance of Surfaces to the Motion of Fluids over Them

§ (15) Resistance to Motion of a Solid.—The analytical investigation of the resistance encountered by a solid moving through a fluid is one of considerable difficulty. In the early study of the problem, in order to simplify the treatment, the fluid was assumed frictionless. As this method led to the conclusion that the resistance was *nil*, in order to obtain results more in agreement with the known facts it was assumed that a surface of discontinuity existed in the fluid whose boundaries consisted partly of fixed plane walls and partly of surfaces of constant pressure. In this way the resistance of a plane lamina in a fluid, inclined to the direction of flow, was obtained by Lord Rayleigh[1] in the form $R=\rho AV^2(\pi \sin \alpha/4+\pi \sin \alpha)$, where V is the velocity of flow and α the angle which the lamina makes with the direction of flow. In this expression the variation of the resistance with the square of the speed has been verified by experiment, but in practice the value of the numerical factor is modified by the existence of a "suction" at the rear of the lamina, which renders the calculated value too small. Broadly stated, the position as regards the possibility of solving problems in fluid resistance by the aid of mathematical analysis is that cases in which the motion of the fluid is "steady"—*i.e.* streamline in character—are amenable to mathematical treatment, but that cases of turbulent motion are intractable, because no analytical method of treating the motion of a fluid which consists of a mass of eddies has yet been evolved. Since in practically all cases of importance the motion in the neighbourhood of the solid surface is turbulent in character, our knowledge of the resistance to the motion of solid bodies through fluids is largely empirical.

§ (16) PRINCIPLE OF DYNAMICAL SIMILARITY.[2] — Considerable progress has, however, been made in the prediction of the resistances of such large bodies as ships and aircraft by the carrying out of experiments on scale models of these bodies in accordance with the principles of dynamical similarity. Thus it has been shown by Lord Rayleigh[3] that the relation obtained by Reynolds for the case of the flow of water in parallel channels is only a particular case of a general law of the resistance of bodies of geometrically similar shape immersed in fluids moving relatively to them, under the assumption that this resistance depends only on the linear dimensions of the body and on the velocity, density, and kinematic viscosity of the fluid. It is further assumed that the geometrical similarity extends to those surface irregularities which constitute roughness. If this be the case, the expression for the resistance may be written

$$R=kV^a l^b \rho^c \mu^d, \quad . \quad . \quad . \quad (1)$$

where R is the resistance per unit area of the body, l is the linear dimension of the body, V is the velocity, ρ the density, and μ the viscosity of the fluid, k being a dimensionless constant.

Now the dimensions of R are $[M/LT^2]$, and the dimensions of the right-hand side of equation (1) are $(L/T)^a(L)^b(M/L^3)^c(M/LT)^d$, and by the principle of dynamical similarity these must be identical, *i.e.* the indices of the respective units must be the same. Equating these we have

$$\left.\begin{aligned} c+d&=1 \\ a+d&-2 \\ a+b-3c-d&=-1 \end{aligned}\right\}, \quad . \quad . \quad (2)$$

or $$c=1-d, \quad b=-d, \quad a=2-d,$$

so that $$R=k\rho V^2\left(\frac{\mu}{\rho Vl}\right)^a,$$

or generally $$R=\rho V^2 F\left(\frac{Vl}{\nu}\right), \quad . \quad . \quad . \quad (3)$$

where F is a function of the one variable (Vl/ν), ν being the kinematic viscosity of the fluid.

It should be observed that the demonstration of the truth of this relation is a matter for experimental investigation, as it is quite possible that some characteristic of the fluid or of its motion may have been overlooked in the initial assumptions, such as, for instance, the elasticity of the fluid. Comparing this relation with the experimentally determined relation of Osborne Reynolds (§ (14), (iii.)), which may be put in the form

$$\frac{R}{\rho V^2}=4k^{1-n}\left(\frac{\nu}{Vd}\right)^{2-n},$$

we see that when $n=1$, *i.e.* when the flow is streamline in character,

$$\frac{R}{\rho V^2}=\frac{4\nu}{Vd},$$

i.e. $$F\left(\frac{Vd}{\nu}\right)=\frac{4\nu}{Vd},$$

and that when $n=1\cdot75$ as in the case of the turbulent flow of water through smooth pipes,

$$F\left(\frac{Vd}{\nu}\right)=4k^{-\cdot75}\left(\frac{\nu}{Vd}\right)^{\cdot25}.$$

A somewhat more general treatment of the problem of the conditions for similarity of motion in fluids was given in 1873 by Helmholtz,[4] of which the following sketch may be of interest. Considering two fluids of densities ρ_1, ρ_2, and kinematic viscosities ν_1, ν_2, the conditions under which the motions of the two fluids are similar are determined thus. Taking the equation of motion of the first fluid as

$$-\frac{1}{\rho_1}\frac{dp}{dx}=\frac{du}{dt}+u\frac{du}{dx}+v\frac{du}{dy}+w\frac{du}{dz} -\nu_1\left\{\frac{d^2u}{dx^2}+\frac{d^2u}{dy^2}+\frac{d^2u}{dz^2}\right\}, \quad (1)$$

and two similar equations, and writing

$$\nu_2=q\nu_1, \quad \rho_2=r\rho_1,$$

then in order that these equations may be transformed into the equations for the second fluid

$$-\frac{1}{\rho_2}\frac{dP}{dX}=\frac{dU}{dT}+U\frac{dU}{dX}+V\frac{dU}{dY}+W\frac{dU}{dZ} -\nu_2\left\{\frac{d^2U}{dX^2}+\frac{d^2U}{dY^2}+\frac{d^2U}{dZ^2}\right\}, \quad (2)$$

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[1] Rayleigh, *Scientific Papers*, i. 291.

[2] See article "Dynamical Similarity."

[3] *Philosophical Magazine*, 1899, xlviii. 321.

[4] *Wissenschaftliche Abhandlungen*, i. 158.

under the given conditions of similarity

$$U=nu,\quad V=nv,\quad W=nw,$$

it will be seen that multiplying (1) by n^3/q the required transformation is effected if the changes in the scales of length, time, and pressure are

$$X=\frac{q}{n}x,\quad T=\frac{q}{n^2}t,\quad P=n^2rp. \qquad . \quad . \quad (3)$$

It follows, therefore, from the linear scale relation, that if l, L are corresponding dimensions of two tubes in which fluids of densities ρ_1, ρ_2, and kinematic viscosities ν_1, ν_2, are flowing in order that the two motions may be similar, $L=(\nu_2 vl/\nu_1 V)$, *i.e.* VL/ν, must have the same value for each. Again, from the pressure scale relation it follows that for similar motion the value of $P/\rho V^2$ is the same for each fluid. The investigation of Helmholtz leads, therefore, to the same condition for similarity of motion as that deduced from the dimension method of Lord Rayleigh. It has been pointed out by Sir George Greenhill that the interpretation of Proposition 32, book ii., in Newton's *Principia* leads to the same conditions.

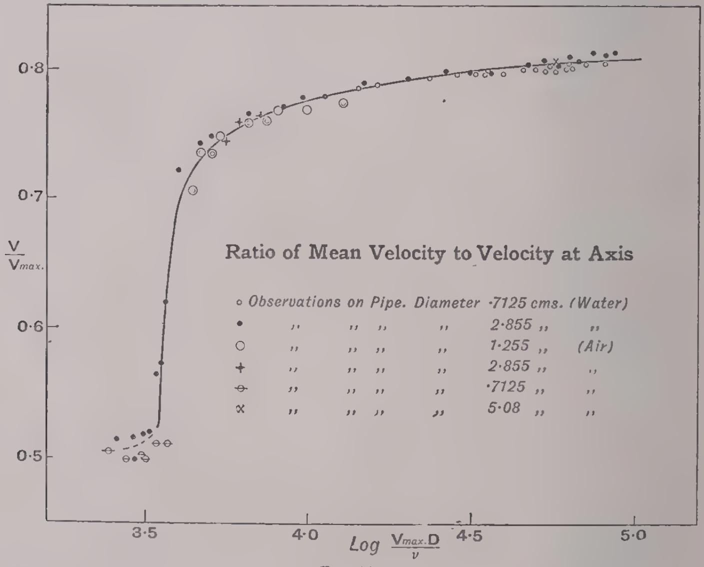

FIG. 11.

§ (17) EXPERIMENTAL VERIFICATION OF THE RAYLEIGH FORMULA.—An extensive series of investigations were carried out at the National Physical Laboratory[1] during the years 1910–1915 to test the accuracy of the assumptions on which the Rayleigh resistance formula is based. These experiments consisted of determinations of the intensity of the surface friction of fluids moving over geometrically similar solid surfaces, so as to include the widest range in the velocities of flow, the linear dimensions of the surfaces, and the nature of the fluids used as could be obtained.

The most convenient method of securing these conditions, both as regards range in the value of the above characteristics and accuracy in the measurements of the velocity of flow of the fluid and the frictional forces consequent upon it, was obviously that previously adopted by Reynolds, consisting in experiments on the flow through parallel pipes of circular cross-section.

Adopting this method of working, a very complete series of experiments were carried out with air, water, and thick oils as the fluids and with velocities of flow ranging from 30 to 6000 cm. per second. For accurate comparison, as pointed out by Lord Rayleigh, the surfaces of the tubes should have been precisely geometrically similar as regards surface roughness, but as this condition could not be fulfilled the experiments were all made on commercially smooth-drawn brass tubes. From the general agreement of the results obtained with different pipes it was not apparent that slight irregularities in this respect had any marked effect on the resistance. The velocity of flow was in all cases taken to be the mean velocity across the section of the pipe. In the case of the pipes of small diameter this was most conveniently estimated by passing the whole of the discharge through a meter and measuring the rate over a given time. This method of procedure was not possible for the large pipes, and in these cases the value of the velocity at the axis of the pipe was measured by means of a Pitot tube and then multiplied by the ratio of the mean speed to the maximum speed. As this ratio was not constant for different speeds and dimensions of pipe, it was necessary to carry out a special investigation on the law of its variation. This was done for the large pipes by fitting the Pitot tube with a micrometer head so that it could be traversed across the pipe, and a series of observations made of the velocity at different distances from the axis. The value of the ratio was then found by plotting and mechanical integration in the usual way.

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[1] *Phil. Trans. Roy. Soc.* A, ccxiv.

The values of the ratio and its variation with the value of $V_{\max.}d/\nu$ are shown in *Fig.* 11, where $V_{\max.}$ is the velocity at the axis. It will be seen that for low values of $V_{\max.}d/\nu$ the variation is considerable, but that when $V_{\max.}d/\nu$ exceeds 100,000 it becomes small.

In all cases the value of the surface friction was calculated from the pressure drop along the pipe between two sections sufficiently far removed from the inlet of the pipe for the velocity distribution across the pipe to have become uniform. The value of the pressure drop was, in the great majority of cases,

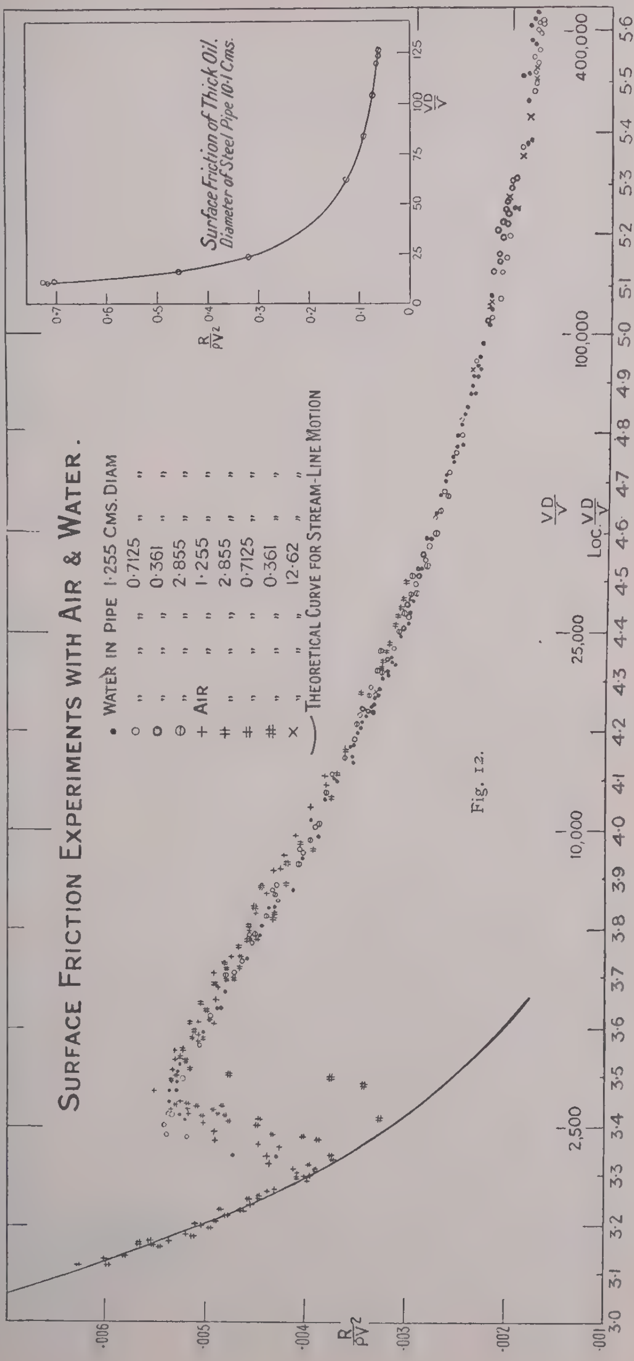

Fig. 12.

measured by connecting two holes in the wall of the pipe, one at each section, with a tilting manometer of the Chattock type, by means of which pressure differences as low as one-thousandth of an inch of water can be accurately measured. For the experiments with water flowing through small pipes at speeds from 5000 to 6000 cm. per second, where the pressure drop to be measured was of the order of 50 lbs. per sq. in., specially calibrated Bourdon gauges were employed. These tests gave the highest value of vd/ν obtained in the experiments—430,000—and the frictional resistance was as high as 18 grammes per square centimetre. The whole of the results are shown plotted in *Fig.* 12, in which the ordinates are the values of $R/\rho v^2$, and the abscissae the corresponding values of vd/ν, where R is the surface friction per unit area, ρ the density of the fluid, and v the mean velocity of flow. It will be seen that the curve consists of two branches connected by a narrow vertical band over which the points are somewhat irregularly spread. This band indicates the region of the flow between the break-down of the steady motion and the establishment of complete turbulence. The left-hand branch, corresponding with speeds below the critical value, is in very good agreement with the critical curve obtained from the hydrodynamical formula for steady flow,

$$R = \frac{8\mu v}{d}.$$

For values of the speed of flow above the critical it will be seen that the inclination of the curve to the horizontal becomes gradually less as the value of vd/ν increases, indicating that the law of resistance tends to become one in which the friction varies as the square of the velocity, and therefore independent of the dimensions of the channel and the temperature of the fluid. Throughout the whole of the range covered by the experiments above the critical speed, *i.e.* from $(vd/\nu) = 2500$ to $(vd/\nu) = 470{,}000$, with the exception of a few individual determinations due possibly to errors of observation, the variation of $R/\rho v^2$ for either fluid in any of the five pipes from its mean value does not exceed 2·0 per cent, so that the accuracy of the assumptions in the derivation of the Rayleigh law of resistance is fully demonstrated.

There remained the comparison on the same basis of the results of previous experimenters on the flow of fluids in pipes with those obtained at the National Physical Laboratory. These included the very elaborate investigations of Darcy[1] on the water mains of Paris, a very complete series of tests on the flow of water through drawn brass tubes by Saph and Schoder,[2] and the results of researches by Brix[3] and Stockalper[4] on the flow of air through lead and iron tubes.

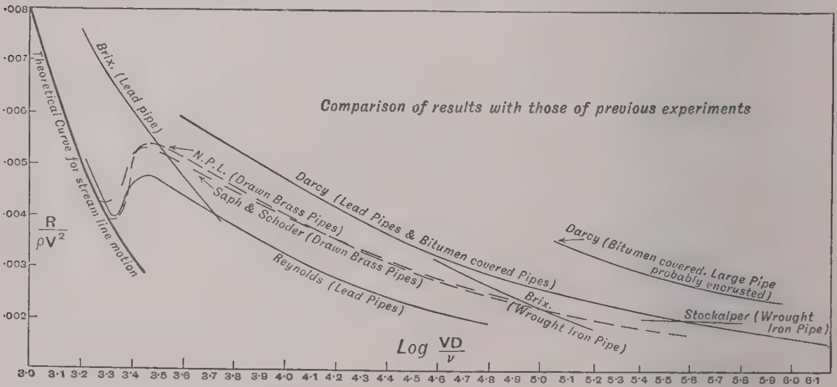

FIG. 13.

Unfortunately in some of these observations the temperature had not been recorded, so that the values of ν could not be calculated with accuracy.

The reduced results were plotted in the same manner as in *Fig.* 12, and the mean curves through them are shown in *Fig.* 13, in which the curve for the National Physical Laboratory experiments is also indicated. In the case of Darcy's experiments the curve for the largest pipe is distinct from that of the others, presumably indicating greater roughness of the internal surfaces, but the general characteristics of the curves are what would be expected, For Reynolds' experiments on two smooth lead pipes a mean line has been drawn which indicates a lower resistance than that found in the National Physical Laboratory experiments, but the agreement is quite satisfactory as regards the character of the curve and the position of the critical point. The most satisfactory comparison, however, is possible with Saph and Schoder's results, since the nature of the surfaces in the two researches was probably identical. It will be seen that the two curves are nearly coincident throughout the whole range of the experiments, and indicate a very satisfactory agreement between the results of the two researches. For the comparison in the case of air flow the amount of experimental data available was not large, and there are some discrepancies which are difficult to account for. These, however, are confined to one series of observations on a small pipe and may be due to an inaccuracy in the determination of its diameter. For the larger pipes the agreement is fair.

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[1] *Comptes rendus*, xxxviii.

[2] *Proc. Am. Soc. Civ. Eng.*, 1903, li. 253.

[3] *Phil. Mag.*, 1909, xvii. 395.

[4] *Revue Univ. des Mines*, vii. 257.

It has been shown by Dr. C. H. Lees[1] that the relation between $R/\rho v^2$ and vd/ν indicated by the curve in *Fig.* 12 may be expressed algebraically in the form

$$\left(\frac{R}{\rho v^2}-0{\cdot}0009\right)\left(\frac{vd}{\nu}\right)^{0\cdot35}=0{\cdot}0765$$

or

$$R=\rho\left[0{\cdot}0765\left(\frac{\nu}{vd}\right)^{0\cdot35}v^{1\cdot65}+0{\cdot}0009v^2\right].$$

It may be remarked from the final expression for R written above, that the frictional resistance will for all velocities be approximately proportional to a power of the velocity between the 1·65th and the 2nd, and that as the speed and diameter increase or the kinematic viscosity decreases the law of variation of the friction will approximate more and more closely to that of the square of the speed. This is in agreement with the experimental results obtained in the 1·26-cm. pipe referred to above. It is shown by Lees that if for any reason a single power formula for the frictional resistance is desired the most accurate value of n in the formula

$$R=Cv^n$$

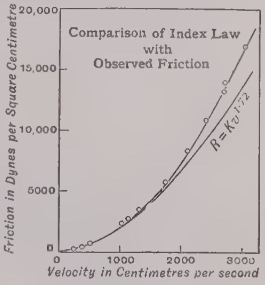
is given by

$$n=2-29{\cdot}7\Big/\left\{85+\left(\frac{vd}{\nu}\right)^{0\cdot35}\right\}.$$

It is also evident that as the resistance becomes more and more proportional to the square of the speed, due to increase of the diameter or velocity, the effect of change of temperature becomes less important. Thus it has been observed by Maw for water that when the resistance varies as the 1·8th power of the speed it decreases 0·6 per cent per degree centigrade, and when the resistance varies as the 1·9th power it only decreases 0·25 per cent per degree.

§ (18) THE ACCURACY OF THE INDEX LAW.—The determination in these experiments of the frictional resistance of a 1·255-cm. pipe when the velocity ranged from 22 cm. per second, corresponding to the first commencement of eddying motion, to 3150 cm. per second, made possible a check, within these limits, of the accuracy of the well-known index law of resistance due to Reynolds and Froude.

The results for the 1·255-cm. pipe were taken, and according to Reynolds' method the logarithms of the friction and velocity from $v=40$ to $v=100$ cm. per second were carefully plotted. The points so obtained were found to be on a straight line whose slope was 1·72 to 1. Assuming a law of resistance $R=Cv^n$ where n had this value, C was determined from the low speed observations used for the evaluation of n, and a series of values of R were calculated up to a speed of 3200 cm. per second. Plotting these calculated values and those actually obtained in the experiments, the two curves in *Fig.* 14 were obtained, from which it will be seen that by the use of the index law the resistance is underestimated by 5 per cent at 1000 cm. per second, by 8·5 per cent at 2000 cm. per second, and by 15 per cent at 3000 cm. per second.



Flow of Water in Pipe
1·255 Centimetres Diameter

FIG. 14.

In order to show the manner of variation of n throughout the whole range of velocities obtained, the values of n were taken out by the Reynolds method at four different stages and were found to be:

Velocity per sec. (cm.) . .	58	258	900	2250
Value of n from plotting .	1·72	1·77	1·82	1·92

Similar results, showing a gradual increase in the value of n as the velocity of flow increased, were obtained by the reduction of the observations for the 0·712 and 1·255 cm. pipes, and it was, therefore, fully demonstrated that an index law for surface friction cannot be devised which will express the facts with high accuracy, except over a comparatively small range in the value of vd/ν. The importance of a realisation of this fact when predicting the skin friction of large bodies moving in fluids from observations on small-scale models of them in the same fluid will be obvious.

§ (19) THE EXPERIMENTAL DETERMINATION OF THE FRICTIONAL RESISTANCE OF THIN PLATES. (i.) *Froude's Experiments.*—As previously mentioned (§ (16)), the results on the frictional resistance to the flow of fluids in pipes, plotted in *Fig.* 12, are only applicable to cases in which the velocity of the fluid at a given distance from the boundary is constant over the whole length considered. They are, therefore, not applicable to the prediction of the frictional resistance of any surface which includes that part of it over which the particles of fluid are being gradually retarded—*i.e.* the part in the neighbourhood of the leading edge of the surface such as, for example, the immersed surface of a ship. For the prediction of the skin friction of ships special experiments are necessary, and a large series of very important observations was carried out by Mr. William Froude in 1872–74.[1] These experiments were made in a water tank 278 feet long, 36 feet wide at the top, the depth of the water being 8 feet 9 inches. The boards of which the frictional resistance was determined were about $\frac{3}{16}$-inch thick, 19 inches deep, the top edge being $1\frac{1}{2}$ inches below the surface. The length varied from 1 foot 6 inches to 50 feet.

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[1] *Proc. R.S.*, 1914, A, xci.

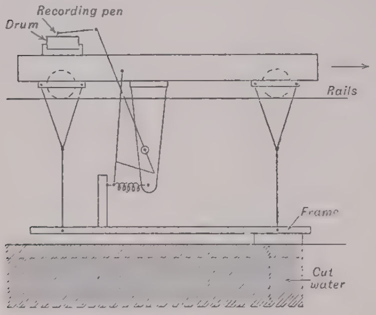

FIG. 15.

The experimental apparatus (*Fig.* 15) consisted of a carriage running on rails fixed to the walls of the tank and capable of being drawn along at a uniform rate. A parallel motion suspended from the carriage carries the board, which is provided with a lead keel and a cut-water. The frictional resistance of the board is taken by the spring shown, the extension of which is magnified and transmitted to a recording drum on which a line is drawn whose distance from the base is proportional to the resistance. The drum is connected by gearing to the carriage-wheels so that the abscissa of the pencil trace is proportional to the distance moved through, and from a second record of time intervals on the paper made by means of an electric clock, the data for calculating the variation of resistance with speed are available.

In this way a series of resistance-velocity curves (*Fig.* 16) was obtained for each length of board, and from these a set of resistance length curves (*Fig.* 17) for given speeds could be deduced. In the first set of experiments the latter curves did not pass through the origin, but above it, showing that the resistance was not zero for zero length. This was found to be due to the bluntness of the bow and stern of the board. On shaping these to knife-edges it was found that the curves passed through the origin. It was found, however, that the curves were not straight but at all speeds were concave to the base line, indicating that the resistance did not vary as the length but at a less rate. This result is due to the fact that the portions of the surface down-stream from the leading edge are in contact with water whose motion relative to them is less than in the neighbourhood of the leading edge, and the frictional resistance of these portions is, therefore, less than that of the other portions nearer the edge.

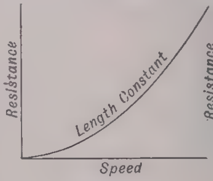

FIG. 16.

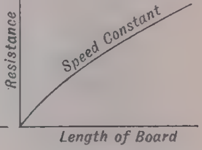

FIG. 17.

It was found that for a given plank the resistance varied as v^n where n was constant for the particular plank, but in different planks depended on the length and quality of the surface. The value of n is found by the Reynolds method of logarithmic plotting previously described. Very approximately the results could be expressed by the formula $R = fSv^n$, where S is the area of the immersed

TABLE I

Length in feet .	2		8		20		50	
	n.	*f.*	*n.*	*f.*	*n.*	*f.*	*n.*	*f.*
Varnish . .	2·00	0·0041	1·85	0·0046	1·85	0·00393	1·83	0·0037
Paraffin . .	1·95	0·00426	1·94	0·0036	1·93	0·00318	..	..
Tinfoil . . .	2·16	0·00208	1·99	0·00284	1·90	0·00331	1·83	0·00364
Calico . . .	1·93	0·0102	1·92	0·00754	1·89	0·00685	1·87	0·00639
Fine sand . .	2·00	0·0090	2·00	0·00625	2·0	0·00534	2·00	0·00488
Coarse sand . .	2·00	0·0110	2·00	0·00714	2·0	0·00588	..	..

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[1] *British Association Reports*, 1872–74. See also "Ship Resistance," § (8).

surface and v is the speed in feet per second. The value of f was found to depend on

(1) The quality of the surface.
(2) The length of the surface.
(3) The temperature of the water.
(4) The density of the water.

The values of n and f deduced from the curves are given in Table I.

§ (20) ESTIMATION OF THE FRICTION OF PLATES FROM THE CHANGES IN MOMENTUM OF THE FLUID PASSING OVER THEM.—The experimental determination of the frictional resistance of the surfaces of thin plates immersed in a fluid by means of a direct measurement of the amount of the tangential force experienced by the plate itself, as in the case of Froude's experiments, is attended with considerable difficulties which render any accurate investigation of the *variation* of the intensity of the resistance almost impossible. These are due to the smallness of the force to be measured, the difficulty of obtaining a thin plate of the required rigidity, and the uncertainty of the large corrections which have to be made in taking account of the resistances of the supporting spindles. For this reason considerable attention has been given to the possibility of an estimation of the frictional resistances from the changes in the characteristics of the fluid which are the necessary effects of the forces exerted, *i.e.* to the changes of momentum of its molecules. One experimental method of this nature has already been discussed in some detail in describing the common device for measuring the frictional resistance of the surface of a parallel channel from the fall of pressure of the fluid between two given points in its axis. The essential feature of this method, however, is that it is only the changes in molecular momentum, or pressure, which are measured, and that the molar momentum of the fluid entering and leaving the two sections is the same. This method is, therefore, inapplicable, as has been previously mentioned, to any case in which the accelerations of the fluid due to the first impingement of the streams against the solid body have not died out, and is, consequently, valueless for predictions of the frictional resistance of air or water craft.

An extension of the method to take into account the changes in molar momentum has recently been attempted in an investigation of the frictional resistance of thin plates in a current of air at the National Physical Laboratory, and, as the results appear to be promising, a brief description of the manner in which the problem has been attacked and the results obtained will be given.

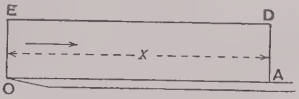


FIG. 18.

In *Fig.* 18 let O be the leading edge of the plate, OA the length whose frictional resistance is to be determined, OE and AD the traces of planes through O and A perpendicular to the direction of flow of the air, and ED the trace of a plane parallel to the plate and at such a distance from it that the flow along ED is unaffected by the presence of the plate. Let V_0 be the velocity of the air impinging on the edge of the plate and supposed uniform over OE, and let V be the velocity of the air in any layer of thickness dz parallel to the plate and distant x from O, *i.e.* in the section AD. Then, considering the changes of momentum in the layer of unit width perpendicular to the plane of the paper passing though the section AD, we have

Change in molecular momentum $=(p-p_0)dz$ where p is the pressure at AD.

Change in molar momentum $=$ (mass passing through the extremity of the layer in AD) $(V_0-V)=\{\rho V(V_0-V)\}dz$.

Therefore the total change of momentum in the layer is

$$\{p_0-p+\rho VV_0-\rho V^2\}dz, \quad . \quad . \quad (1)$$

so that if R is the mean frictional resistance per unit area and we consider a strip of the plate of unit width perpendicular to the plane of the paper and x is the distance OA, we have for the frictional resistance of strip

$$xR=\int_{AD}\{p_0-p+\rho VV_0-\rho V^2\}dz. \quad . \quad (2)$$

Hence if the values of p_0, p, and V_0V for the various layers are known the value of the frictional resistance can be determined by graphical integration. Now p and p_0 are the static pressures of the fluid, and a separate determination of their values or even of their difference is a matter of some experimental difficulty. The expression (1) can, however, be thrown into a form suitable for experimental evaluation if written as follows:

$$xR=\int_{AD}\left\{(p_0+\tfrac{1}{2}\rho V_0^2)-(p+\tfrac{1}{2}\rho V^2)-\frac{\rho}{2}(V_0-V)^2\right\}dz. \quad (3)$$

Here the expression $(p+\frac{1}{2}\rho V^2)$ is the total head of the fluid and is the actual pressure which is maintained in a Pitot tube facing a current of fluid. The determination of the skin friction is therefore seen to involve the measurement of the difference in pressure of two Pitot tubes, one placed in the section OE and the other at various points in AD successively, together with the variation of the velocity with distance from the surface at the section AD. As the third term in (2) is relatively small, the accuracy required in the determination of V is not high.

In the practical application of the above formula, it should be noted that one essential condition is that the reduction in the value of the total head $(p+\frac{1}{2}\rho V^2)$ of the air passing the plate should be entirely due to the frictional resistance of the plate and not affected by the presence of the walls of the wind channel in which the plate is suspended. In all experimental determinations, therefore, the plate should be removed and a measurement made of the fall, if any, in the value of the total head, by noting the difference in the pressures of the two Pitot tubes. If any difference is found to exist, this must be used as a correction to the subsequent readings when the plate is in position.

In the experiments at the National Physical Laboratory a pair of fine Pitot tubes were attached to the arm of a rocking-lever in such a position that their axes were in the same straight line parallel to the axis of the plate. The rocking-lever was operated by a micrometer screw so that the distance of the Pitot tube axis could be set to any desired position. The position of the leading tube was 2 inches in front of the leading edge of the plate and that of the back tube 2 inches behind the back edge of the plate.

Readings of the pressure differences in the tubes and the difference in pressure between the back tube and the static pressure of the fluid at the back tube were taken at successive intervals of ·025 inch from the surface of the plate up to the distance at which the differences in pressure were negligible, *i.e.* the total pressure head of the air flowing past the plate was unaffected by the presence of the plate. The differences in the values of the total heads and of $\frac{1}{2}\rho(V_0-V)^2$ were then plotted on a distance base and the areas of the resulting curves determined by a planimeter. The values of the above integrals multiplied by the width of the plate b are given in Table II., in which are also tabulated the measured resistances of the plate when attached to a sensitive balance arm and weighed directly.

TABLE II

Wind Speed. Ft.-sec.	(1). $b\int_{AD}\{(p_0+\frac{1}{2}\rho V_0^2)-(p+\frac{1}{2}\rho V^2)\}dz$ in Dynes.	(2). $b\int\frac{\rho}{2}(V_0-V)^2dz$ in Dynes.	(1), (2). Dynes.	Measured Resistance of Plate by Direct Weighing.
30	1390	90	1300	1160
50	3980	310	3670	3470

It will be seen that the results given by the method described are probably in good agreement with the results of direct weighings, when consideration is taken into account of the falling off in resistance at the long edges of the plate.

A research on the distribution of the intensity of skin friction is in progress at the National Physical Laboratory. The results obtained show that the "wake" effect in skin friction is much smaller than has recently been supposed, and in this respect tend to agreement with the older experiments of W. Froude.

§ (21) EFFECT OF CURVATURE OF SURFACES ON FRICTIONAL RESISTANCE.—It may be remarked that the experiments of which the results are plotted in *Fig.* 12 were all made on straight lengths of pipe, and that there is some evidence that curvature of the streams in the direction of flow has a marked effect both on the value of the critical velocity and the law of resistance with speed after the critical velocity has been exceeded. This effect was apparently found in Grindley and Gibson's experiments on a long length of pipe 0·32 cm. internal diameter which was coiled round a drum of 36 cm. diameter. In these experiments the critical velocity was found to be as low as 60 cm. per second, corresponding to a value of vd/ν of about 130. For velocities between 60 and 1200 cm. per second the law of frictional resistance was of the form

$$R=Cv^{1\cdot25}.$$

It may be remarked that even at the highest speed attained in these experiments the Reynolds criterion $vd/\nu=2000$ was not attained, so that it is quite possible that in the event of observations at still higher speeds being made the results would have agreed with that part of the curve of *Fig.* 12 above the critical.

It is possible, of course, that the true value of the critical speed found in these experiments might be due to other causes than the curvature of the pipe, but in view of the accuracy in the reduction of the value of the coefficient of viscosity for air from the observations there is a strong probability that such was the real cause of the effect noted.

An interesting series of experiments illustrating the effect of curvature of parallel boundaries have been made by Mr. A. Mallock.[1] In these experiments the fluid was placed in the annular space between two concentric cylinders, one of which revolved, the other being stationary. The frictional resistance of the surface was measured by measuring the moment on the stationary cylinder which could be either the inner or the outer one.

Three distinct sets of conditions were tried, *i.e.* (1) outer cylinder revolving, inner cylinder stationary; (2) inner cylinder revolving, outer cylinder stationary; (3) repetition of series (1) with an annulus of different width.

The arrangement of the apparatus for the tests on series (1) is shown in *Fig.* 19.

The inner cylinder A is suspended by a torsion wire attached by a gimbal ring to the shaft B, which carries a divided circle read by a telescope T. E is the outer cylinder carried on the shaft F, which is driven by a motor provided with a speed regulator. A is filled with water and E is water jacketed, the temperatures of both being observed and the temperature of the fluid in the annulus taken as the mean of these. The cylinder A has its floor about half an inch from the lower edge of the wall, and the space so formed is filled with air. Further, by means of a ring of the same diameter as A fixed to the floor of the outer cylinder, and by filling the annular space between the ring and cylinder with mercury, there is comparatively no drag on the water in the annulus due to the bottom of the outer cylinder.

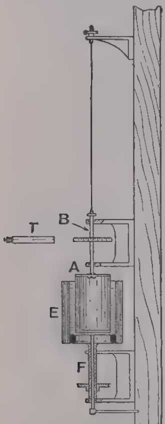


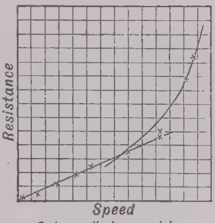
FIG. 19.

In the case of the first series of experiments, with the outer cylinder revolving and the inner one stationary, the results were similar to those obtained in straight pipes, *i.e.* the friction was proportional to the speed up to a certain critical velocity, at which the motion broke down and steady readings of the friction were impossible to obtain, the conditions becoming steady again at higher values of the speed. This is shown in *Fig.* 20. The results, however, differed fundamentally from those obtained in straight pipes in that

(*a*) The critical velocity was not proportional to the value of the coefficient of viscosity, *i.e.* its value did not conform to the theoretical condition $v_c l/\nu$ constant.

Thus, comparing the results obtained with water at 50° C. and 0° C., the viscosity of water at 50° C. is only one-third of its value at 0° C., and it would be supposed therefore that the critical velocity at 50° would be one-third of its value at 0°. It was found, however, that instability began at the former temperature at a speed only 11 or 12 per cent less than at the latter.

(*b*) It was found that at speeds above the critical the frictional resistance varied as a power of the speed of the order of 2·4, whereas in Reynolds' and Froude's experiments with flat boards and pipes the exponent was about 1·8.



Outer cylinder revolving
Inner ,, fixed

FIG. 20.

Values of the coefficient of viscosity deduced from the low-speed experiments in series (1) were found to be consistently higher than the true values. The experiments of series (3), in which the width of the annulus was only half an inch, gave values of μ, which were nearer to the true value. Previous experiments by Mr. Mallock, by the same method but in which the distance was only about $\frac{1}{8}$ in., gave values of μ closely approximating to the true value, so that there is no doubt that the discrepancy is due to the thickness of the annulus. The results are shown in *Fig.* 21.

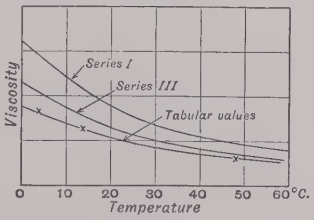
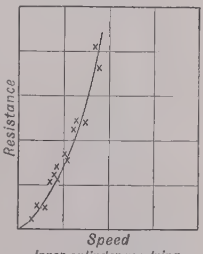


FIG. 21.

In the experiments with the inner cylinder revolving and the outer cylinder fixed, the motion was essentially unstable at all speeds, and no value of the coefficient of viscosity could be deduced. The results from this method are shown in *Fig.* 22, and indicate that the conditions of flow approximate more closely to the eddying flow of fluid



Inner cylinder revolving
Outer ,, fixed

FIG. 22.

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[1] *Phil. Trans. Roy. Soc.*, 1896, p. 14.

in straight pipes in that the friction varies as a power of the speed of about 1·8, and the measured value of the resistance per unit area at 10 feet per second was roughly that obtained by Mr. Froude in his experiments on planks.

§ (22) Possibility of an Increase in Resistance caused by a Reduction in Viscosity.—In the practical application of the results of the experiments embodied in the curve of *Fig.* 12, it must be remembered that there was one important condition which was fulfilled in every case, which was that the observations were not taken until the distribution of speed across the section of the pipes had become uniform, *i.e.* all accelerations parallel to the axis of the pipes had ceased. It was found by trial that this condition was not fulfilled until a distance along the pipe from the inlet of about 100 diameters was attained. The results of the experiments are therefore not applicable to the prediction of the intensity of surface friction of the part of the tube in the neighbourhood of the inlet, when the layers of fluid in the neighbourhood of the surface are gradually reduced in speed owing to the frictional drag of the surface.

Further, although it might be expected that, in the case of solid surfaces of shapes other than those of parallel channels, the phases of flow would correspond with those observed in pipes, *i.e.* there would be a phase of streamline flow in which the resistance would vary as the first power of the relative speed, a critical velocity at which this form of flow would break down and a subsequent condition of turbulent motion in which the resistance would vary approximately as the square of the speed, it cannot be assumed that the form of the resistance curve will be similar to that of *Fig.* 12. For example, throughout the whole range of the turbulent motion covered in *Fig.* 12 the value of $R/\rho v^2$ falls continuously as vd/ν increases. Suppose this increase is due to an increase in v from v_1 to v_2, ν remaining constant. Then it is clear that R_2/R_1 increases at a less rate than $(v_2/v_1)^2$, or $R_2/R_1=(v_2/v_1)^n$, where n is less than 2. Suppose, on the other hand, that the increase in vd/ν takes place by a reduction in the value of ν from ν_1 to ν_2, v remaining constant. Then evidently R_2 is less than R_1 and we conclude that throughout the whole range of turbulence covered in *Fig.* 12 a reduction in viscosity is accompanied by a reduction in frictional resistance. Very conclusive evidence has, however, been found that in some cases of the resistances of bodies immersed in fluids the value of $R/\rho v^2$ increases with increase of vd/ν. It is evident that, in such cases, since the increase in vd/ν may take place by a reduction in the value of ν, v remaining constant, and therefore R_2 being numerically greater than R_1, a reduction in viscosity will be accompanied by an *increase* in the frictional resistance. Further, since in such cases $R_2/R_1=(v_2/v_1)^n$, where n is greater than 2, the criterion of such an effect will be a resistance varying as a power of the speed greater than 2. The following are some examples.

(*a*) *Friction in Smooth Pipes.*—As was mentioned in the description of Reynolds' experiments, it is usually found that over the region intermediate between turbulent and steady motion the friction conditions are so unsteady that consistent readings at any given speed are impossible. In some observations with a small pipe of $\frac{1}{8}$ in. diameter, at the National Physical Laboratory in 1913, it was found, however, that the change from streamline to eddy motion took place very gradually, and that it was possible to obtain a series of consistent readings of friction and speed over the range of speed between the two conditions of motion. The results are plotted in *Fig.* 23, in which the ordinates are the losses of head in feet of water over the fixed length of pipe under observation and the abscissae are the corresponding values of the discharge in gallons per second, *i.e.* the curve shows the variation of the surface friction with the speed of flow. In the lower curve is shown the value of n in the relation $R=kv^n$ obtained from the upper curve, in the ordinary manner by logarithmic plotting. It will be seen that at very small values of the velocity the friction is proportional to the speed, and that at high speeds it varies nearly as the square of the speed. Between these points indicated by AB and CD in *Fig.* 23 is the section BC representing the flow at intermediate stages of

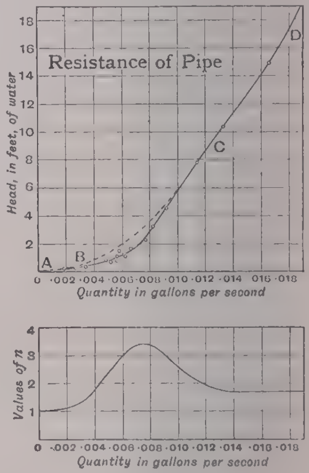

Fig. 23.

turbulence, where, as shown by the lower curve, the resistance varies as a power of the speed which may be as high as 3·5. This evidence confirms the general reliability of the portion of the curve in *Fig.* 12 in the neighbourhood of the critical point.

(*b*) *The Resistance of Spheres.*—At very small velocities the total resistance of a sphere is given by Stokes' formula,

$$\frac{R}{\rho v^2 l^2}=3\pi\frac{\nu}{vl},$$

where l is the diameter of the sphere and v its velocity relative to the fluid at a great distance away from it. Experiments at higher values of vl/ν have been made by M. Eiffel[1] in a wind-channel and by Mr. Shakespeare[2] by dropping weighted spheres down a tower and observing the terminal velocity. Mr. Shakespeare states that over the range of his experiments the resistance increases more rapidly than the square of the speed and more rapidly than the square of the diameter. The experimental results are shown plotted in *Fig.* 24, which, it will be seen, bears a marked resemblance to that for resistance in pipes with the exception that the part corresponding with values of n greater than 2 occupies a greater proportion of the whole experimental range. The part of the curve JK (*Fig.* 24) corresponds with the centre portion of *Fig.* 12.

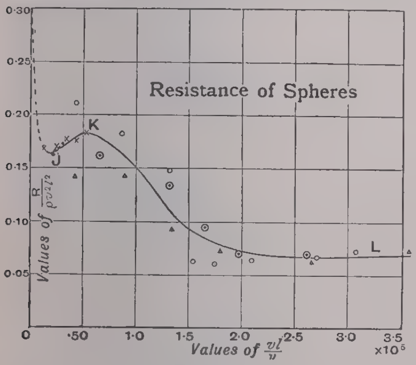

FIG. 24.

(*c*) *The Resistance of Cylinders.*—This subject has received considerable attention owing to the importance of a knowledge of the resistance of wires in aircraft rigging. A very full account of the experiments will be found in the *Report of the Advisory Committee for Aeronautics*, 1912–13, p. 126, and in *Fig.* 25 the mean curve of the results is reproduced. It will be seen that that part of the curve over which $R/\rho v^2 l^2$ increases with the value of vl/ν, thus indicating a diminution of resistance with increase of viscosity, is now a large fraction of the whole curve, and constitutes perhaps the most important evidence of the effect.

It would appear, therefore, that the existence of particular states of flow in fluids, in which an increase in viscosity is accompanied by a reduction in resistance, has been fully demonstrated. The precise explanation of the phenomenon is not clear, but there is a strong probability that the increase in viscosity has the effect of reducing partial and already existing turbulence, and thus procuring a reversion to the conditions of streamline flow.

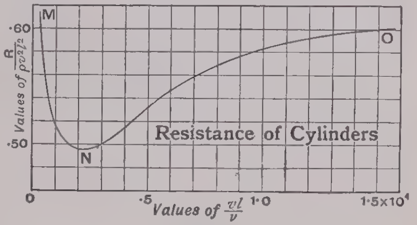

FIG. 25.

§ (23) DIRECT MEASUREMENT OF FRICTIONAL RESISTANCE FROM VELOCITY SLOPE AT THE BOUNDARY.—From the preceding discussion of the practical methods which can be used for the measurement of skin friction in turbulent motion, it will be seen that any accurate determination of the intensity of skin friction on a small area of a surface over which the variation is considerable is a matter of considerable difficulty if not altogether impossible. Up to the present, success in the measurement of skin friction has only been attained in those cases in which the accelerations of the fluid in the direction of flow have died out, such as the flow of fluids in long pipes. Unfortunately the problems of skin friction, which are of the greatest importance in practice, such as the determination of the frictional resistance of ships and aircraft, are those in which the velocity of the fluid is continually changing, and it does not appear that the solution of these problems can be hoped for unless the conditions at the boundary of a fluid in turbulent motion are known, and several attempts to investigate these conditions have recently been made. Such advances as have been made in the knowledge of the boundary conditions of a fluid in turbulent motion have been obtained, as would be expected, from a study of the conditions of flow in parallel pipes. In such flow it is generally accepted that when the speed is below the critical, the layer of fluid in contact with the boundary is at rest relative to it, as any slipping of finite amount would be detected in a variation from the Poiseuille law of the relationship between the diameter of a pipe and the time of efflux of a given volume of fluid. At speeds above the critical, observations near the walls have shown that the mean velocity falls rapidly as the solid bounding surface is approached, and it has been suggested that at the walls there may exist a thin layer in which the flow is laminar in character, in which case, if there is no slipping, the frictional resistance would be determined from the slope of the velocity curve in the surface layer and the coefficient of viscosity of the fluid. Several attempts have been made at the National Physical Laboratory to obtain evidence as to the truth of this assumption, and a series of experiments were carried out in 1920,[1] the results of which afford strong evidence that such a layer exists. The method adopted was to set up a condition of turbulent fluid motion over a surface of which the frictional resistance could be accurately determined and to measure, by means of a very fine Pitot tube, the velocity of the fluid at a point as near the wall as possible. The difficulties of using a Pitot tube under such conditions were: (*a*) that the pressure indications of the Pitot tube might be considerably affected by an interference in the flow in the neighbourhood of the boundary due to the presence of the tube itself; and (*b*) that when such a tube was placed in a current of fluid in which the variation of velocity across the mouth of the tube was very great (amounting in the extreme cases when the tube touched the walls to several hundred per cent of the mean speed), it was by no means certain that the value of the speed deduced from the pressure in the Pitot tube would be the actual speed at the geometrical centre of the tube. The Pitot tube, which was finally adopted for the work, was one of which the external dimensions at the orifice were 0·1 × 0·8 mm. and the internal dimensions 0·05 × 0·75 mm. This was set up in a pipe of 0·714 cm. diameter, through which a current of air was passed by means of a centrifugal fan, and observations of the Pitot tube pressure were taken over as wide a range in the mean rate of flow through the pipe as possible. At the same time the corresponding slopes of the static pressure gradient down the pipe were accurately determined. From the latter measurements the values of the surface friction of the walls for different rates of flow were calculated. The Pitot tube observations were then repeated for other distances of the tube from the walls. For each distance a curve of speed variation with surface friction was plotted, and from these curves it was possible to scale off, for any given value of the surface friction, the speeds corresponding to the different distances of the Pitot tube from the walls and so plot a curve showing the variation of speed with position of Pitot tube. These curves are shown in *Fig.* 26. In the same figure are shown in dotted lines the velocity slopes which would exist at the boundary if the flow were laminar. These are calculated from the known frictional resistances and the values of the coefficient of viscosity for air.

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[1] *La Résistance de l'Air.*
[2] *Brit. Ass. Rep.*, 1913.

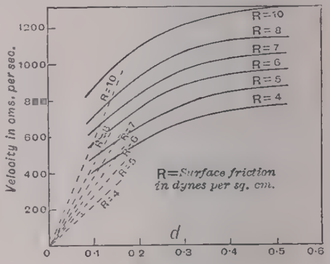

FIG. 26.

To show the relation between surface friction and the speed at a point near the wall, the corresponding observations were reduced by the Reynolds method of logarithmic plotting previously described, as shown in *Fig.* 27. These plottings show that the relation between surface friction and speed at a point is of the same form as for surface friction and mean flow, *i.e.* $R = Cv^n$, but that the value of n diminishes as the boundary is approached until when the geometrical centre of the Pitot tube is ·075 mm. from the wall the value of n is 1·16. It would appear, therefore, that at this distance the eddy motion has nearly disappeared.

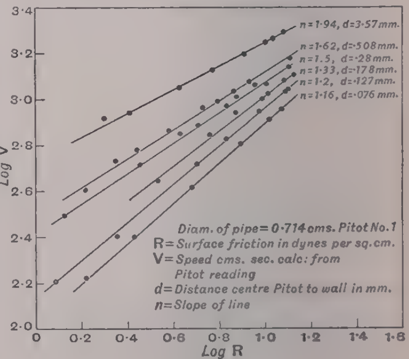

FIG. 27.

On examining the curves in *Fig.* 26, and assuming the errors mentioned under (*a*) and (*b*) above are negligible, the following characteristics will be noted.

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[1] *Proc. Roy. Soc.* A, xcvii. 413.

(*c*) The slopes of the tangents to the velocity curves at distances of ·075 mm. and above from the walls are very much smaller than would be the case if the flow were laminar at these distances from the wall.

(*d*) The speeds calculated from the observations at a distance of ·075 mm. and less from the walls are appreciably higher than would exist in laminar flow with the measured surface friction. For example, with a frictional resistance of 10 dynes per sq. cm. the calculated speed at ·075 cm. from the walls was 795 cm. per second. In laminar flow with this value of the surface friction the speed would be 443 cm. per second.

It appeared, therefore, that if in these experiments it could be assumed that the speed indicated by the Pitot tube was the speed of the fluid at its geometrical centre the streamline motion, if it existed, was confined to a region of less than ·05 mm. from the boundary, and, further, the observations were not inconsistent with a finite amount of slip at the boundary. It was evident, therefore, that the proof of the existence or otherwise of streamline motion at the boundary would involve a much closer exploration of this region than had hitherto been possible. As any further reduction in the cross dimensions of the Pitot tube would have rendered it unworkable, a modification of the shape of the tube was considered, and it was decided to try experiments with a tube in which its boundary adjacent to the wall of the pipe was removed and its place taken by the wall of the pipe itself. This modification is shown in *Fig.* 28. As the space occupied by this tube was considerably less than in the four-walled one, it was decided to fit a Pitot tube of this form to a pipe of 0·25 cm. diameter. The reason for choosing such a small pipe was that the critical velocity in it would be so high that it would be possible to take a series of observations at about 0·1 mm. from the wall at speeds of flow below the critical, in which cases the actual distribution of velocity across the tube was known to be parabolic, and compare these values with those deduced from the Pitot tube readings. In this way the relation between the geometrical centre of the Pitot tube and its effective centre could be determined, and the possibilities of error pointed out in (*a*) and (*b*) above would be avoided.

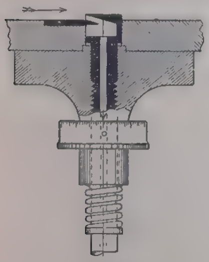

FIG. 28.

A series of tests were accordingly made at a given value of the mean rate of flow below the critical, and at different distances of the geometrical centre of the modified Pitot tube from the wall. From the equation of the distribution of velocity in streamline flow $v=v_c(1-r^2/a^2)$ the values of r were calculated for each value of v calculated from the observations.

Points were then plotted whose ordinates were the values of $(a-r)$ and whose abscissae were the corresponding distances of the geometrical centre of the Pitot tube from the wall.

This process was repeated for different values of the mean flow all below the critical value, and the mean curve through all the plotted points was taken to be the calibration curve for the Pitot tube. This curve is shown in *Fig.* 29. It will be seen that for openings of the Pitot tube of the order of 0·3 mm. the assumption that the calculated speed is that which exists at the geometrical centre of the orifice is not greatly in error, but that when the opening is of the order of 0·075 mm. the interference with the flow is so considerable that the calculated speed is that which exists at the edge of the Pitot tube further from the wall.

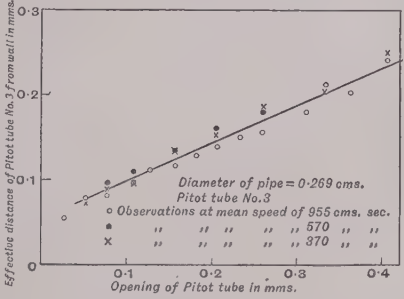

FIG. 29.

A series of observations were then made all at a speed above the critical and with the centre of the tube at different distances from the wall. The speeds calculated from these observations are shown plotted in two methods in *Fig.* 30. On the left-hand side of the figure the abscissae of the points are the distances of the centre of the Pitot tube from the wall, and on the right-hand side the abscissae are the "effective distances" as scaled off from the calibration curve of *Fig.* 29. Assuming that this calibration curve can be applied to conditions of flow above the critical, which seems reasonable in view of the fact that the amount of turbulence at the distances in question must from the evidence of *Fig.* 27 be very small, the curves on the right of *Fig.* 30 show the velocity distribution near the wall when the readings of the Pitot tube are corrected for interference.

It will be seen that the effect of the correction for interference is that at distances of the order of 0·05 from the wall the slopes of the velocity curves approximate fairly closely to the values which would exist in laminar flow. Further, there is no indication of the existence of any slip at the boundary. The conclusion was, therefore, that very definite evidence had been found of the existence at the boundary of a fluid in turbulent flow, of a finite layer of fluid in laminar motion and having zero velocity at the boundary. This evidence was supported by similar observations made on a large pipe of 12·7 cm. diameter, and very large rates of flow, and, although a rigid demonstration was out of the question, there seemed no reason to doubt that in turbulent motion the boundary condition is given by

$$\mathrm{L}t_{x=0}=\mu\frac{dv}{dx}=\mathrm{R}.$$

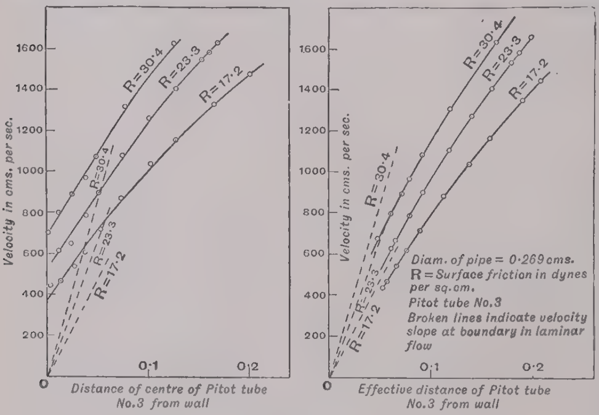

FIG. 30.

§ (24) THE PREDICTION OF THE RESISTANCE OF BODIES MOVING IN FLUIDS, FROM EXPERIMENTS ON SCALE MODELS OF THEM. (i.) *General Considerations.*—From the expression for the resistance of a body moving in a fluid at speeds below the velocity of sound in the fluid $R_1=\rho_1 v_1^2 l_1^2 F(v_1 l_1/\nu_1)$, it is obvious that the resistance of a body S_1 of given shape moving in a fluid of density and kinematic viscosity ρ_1, ν_1 at a speed v_1 may be predicted from the measured resistance of a scale model of the body S_2 moving in a fluid of density and kinematic viscosity ρ_2, ν_2 at a speed v_2, provided that l_2, ν_2, and v_2 are all chosen so that

$$\frac{v_2 l_2}{\nu_2}=\frac{v_1 l_1}{\nu_1},$$

for then

$$R_1=\frac{\rho_1 v_1^2 l_1^2}{\rho_2 v_2^2 l_2^2}R_2,$$

and, since all the terms on the right-hand side are supposed known, R_1 is determinable. If, therefore, the linear dimensions of S_1 are such that no experiment for the purposes of estimating its resistance can be made upon it, the comparison made above opens up a possibility of predicting this resistance from an experiment on a small scale model of it in a laboratory.

In practice the use of this method is limited by the fact that the necessary conditions to be fulfilled in the model test are nearly always impossible to realise. For example, if it is required to predict the resistance of an airship 200 feet long at 60 feet per second and a model of the ship to a scale of $\frac{1}{10}$ is available for the experiments, then supposing the test to be made in air, we have $\nu_1=\nu_2$, $l_1=10l_2$, and therefore $v_2=10v_1=600$ feet per second, a condition which could not be realised in a laboratory experiment. Suppose, however, the *model* experiment be made in water. Then we have $\nu_1=13\nu_2$, $l_1=10l_2$, and since $v_1 l_1/\nu_1=v_2 l_2/\nu_2$, we have $v_2=46$ feet per second. This condition, again, is one which could hardly be realised in water tanks with submerged models, but it will be clear that the possibilities of a prediction are greatly increased. If we could obtain a fluid whose kinematic viscosity was only 1 per cent of that of air, the model test could be made at a speed of 6 feet per second and the prediction could probably be made.

Unfortunately no such fluid is available, so that a direct solution of the problem is not to be hoped for. The importance of scale model tests, however, is not thereby negatived, since a study of the variation in the value of $R/\rho v^2$ over the range in which scale model tests are possible often renders invaluable help in the prediction of the manner in which it will probably vary outside this range. This will be seen from a study of the curves of variation of $R/\rho v^2$ with vl/ν in *Figs.* 12, 24, and 25. In each of these cases it will be seen that for high values of vl/ν, which are those of practical importance, the curve appears to be tending to a definite limiting value, which may be used, with probably fair accuracy, as a basis of prediction in design.

(ii.) *Ship Resistance.*[1]—One of the most important applications of model experiments is the prediction of the resistance of ships, since it is of fundamental importance to know at what speed a ship of given lines will be propelled by engines of a given horse-power, and also to have some reliable means of estimating the benefit which may be derived from a change of lines. The problem of the prediction of ship's resistance is, however, complicated by the fact that a considerable proportion of the total resistance of a ship may be due to the formation of waves on the surface of the sea and that this component cannot be estimated by the application of the law of comparison $vl/\nu = \text{constant}$. A law of comparison for both friction and wave-making may be derived by the ordinary dimensional reasoning, remembering that, in addition to the velocity, density, and viscosity of the fluid, and the linear dimensions of the solid, the acceleration of gravity must also be taken into account, *i.e.* we have

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[1] See "Ship Resistance," § (4).

$$R = kl^a\rho^b v^c \nu^d g^e \quad . \quad . \quad . \quad (1)$$

or the dimensional equation

$$\left[\frac{ML}{T^2}\right] = \left(L\right)^a \left(\frac{M}{L_3}\right)^b \left(\frac{L}{T}\right)^c \left(\frac{L^2}{T}\right)^d \cdot \left(\frac{L}{T^2}\right)^e, \quad (2)$$

from which

$$\left.\begin{aligned} b &= 1 \\ c + d + 2e &= 2 \\ a - 3b + c + 2d + e &= 1 \end{aligned}\right\} \quad . \quad . \quad . \quad (3)$$

or $\quad b = 1, \quad a = 2 - d + e, \quad c = 2 - d - 2e,$

from which

$$\begin{aligned} R &= kl^{2-d+e}\rho v^{2-d-2e}\nu^d g^e \\ &= k\rho v^2 l^2 \left(\frac{gl}{v^2}\right)^e \left(\frac{\nu}{vl}\right)^d \end{aligned}$$

or

$$R = \rho v^2 l^2 F\left(\frac{gl}{v^2}, \frac{vl}{\nu}\right). \quad . \quad . \quad . \quad (4)$$

In order, therefore, that the total resistance of a ship may be accurately predicted from the measured resistance of a scale model of it, the following relations must hold:

$$\left.\begin{aligned} \frac{vl}{\nu} &= \frac{VL}{N} \\ \frac{v^2}{lg} &= \frac{V^2}{Lg} \end{aligned}\right\}, \quad . \quad . \quad . \quad (5)$$

where the capital letters refer to the ship and the fluid in which it is immersed and the small letters to the model and the fluid in which it is towed. The condition is therefore $N/\nu = VL/vl = (L/l)^{\frac{3}{2}}$.

Assuming $L/l = 25$, it would be essential, for the condition to be fulfilled, that the model should be towed in a fluid whose kinematic viscosity is less than 1 per cent of that of sea-water, which, of course, cannot be realised. For this reason the method adopted in practice is to separate the wave-making resistance from the frictional resistance in the following manner. It is clear from equation (4) that if friction be neglected the wave-making resistance of a ship is given by

$$R_w = \rho v^2 l^2 F\left(\frac{gl}{v^2}\right).$$

If, therefore, we determine the wave-making resistance r_w of a scale model of a ship under the condition that the quantity gl/v^2 is the same for the ship and the model, we have

$$\frac{R_w}{r_w} = \frac{V^2L^2}{v^2l^2} = \frac{L^3}{l^3}, \quad . \quad . \quad . \quad (6)$$

i.e. the wave-making resistances of ship and model are in the ratio of the respective displacements. The procedure adopted is to carry out a test in a water-tank of a scale model of the ship at a speed given by $v^2/l = V^2/L$ and determine its total resistance r. By means of tables of frictional resistances based on Froude's tests on the frictional resistance of thin planks the value of the frictional resistance r_f of the model at a speed v is calculated and we then have $r_w = r - r_f$. The wave-making resistance of the full-sized ship is then given by equation (6)

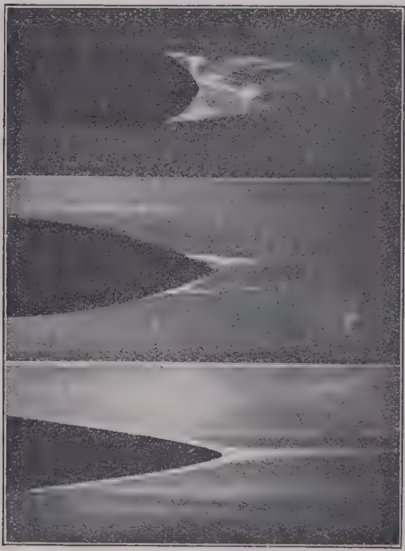
$$R_w = r_w\left(\frac{L}{l}\right)^3.$$

The frictional resistance of the ship is then calculated from the available data and its value R_f is added to R_w to make up the total resistance of the ship.

The objection to the use of data obtained from experiments on thin plates, for the calculation of the skin friction of ships, will be obvious from the consideration that the distributions of the fluid motion in the two cases are widely different. This is particularly marked in the case of the ship's stern owing to the marked tendency of the streams after passing the midship section to separate from the lines of the ship, leaving a wake of eddies at the stern moving at comparatively low speed. This effect is well brought out in the photographs shown in *Fig.* 31. These photographs represent the flow of water round small model balloons of varying fineness of tail. In order to render visible the flow in the region of the tail the model was given a thin coating of colour which would gradually dissolve away as the water flowed past and by the extent of its admixture with the water streams would give an indication of the motion. It will be seen that in each case there is a dead water region at the tail, the extent of which increases with the bluntness of the tail, and since it also increases with the speed of flow, it is probable that the region will be large even in the case of a fine-tailed balloon of normal size and at normal speed. Corresponding effects will take place in the case of a ship unless the lines are particularly fine.

§ (25) THE ESTIMATION OF THE FRICTION OF THE TIDES.—From the curve of *Fig.* 12

it will be seen that for very high values of vl/ν the value of $R/\rho v^2$ tends to a definite limit whose value is not far removed from ·002. It would appear from this that in the case of rivers and canals an approximate estimate of the frictional resistance to flow may be made by assuming that the frictional resistance of the bottom is equal to $\cdot 002\ \rho v^2$. An interesting calculation has recently been made by Mr. G. I. Taylor[1] which indicates that this value may also be applied to the frictional resistance caused by the movement of the tides over the bottom of the sea. Assuming the frictional resistance per unit area to be given by $k\rho v^2$, the energy dissipated per square centimetre per second is $k\rho v^3$. Now the tidal stream at any time t after it has attained its maximum velocity may be taken roughly as $v = V \cos 2\pi t/T$, where V is the maximum tidal stream and T is the semidiurnal tidal period of 12 hrs. 25 mins. The average rate of dissipation of energy over each square centimetre is, therefore, equal to the mean value of $k\rho V^3 \cos^3 2\pi t/T$. Applying this to the case of the Irish Sea, taking $V = 2\frac{1}{4}$ knots, Mr. Taylor finds that the mean rate of dissipation of energy is $k \times 650{,}000$ ergs per sq. cm. per second. By an independent calculation of the rate at which energy flows into the Irish Sea, Mr. Taylor shows that it is possible to find the approximate value of k. Taking the southern entrance to the Irish Sea between Arklow and Bardsey Island, from the known data of the depth of the channel, strength of the current, and range of tide in mid-channel, the mean rate at which energy is transmitted across this section is $6 \cdot 4 \times 10^{17}$ ergs per second. The flow through the North Channel is shown to be negligible compared with this value. In order to arrive at the loss of energy by frictional drag it is necessary to subtract the work done against the moon's attraction by the water. This is estimated by Mr. Taylor to be 110 ergs per sq. cm. per second. Taking the area of the Irish Sea to be $3 \cdot 9 \times 10^{14}$ sq. cm. the energy transmitted is 1640 ergs per sq. cm. per second, and subtracting the 110 due to the work done against the moon's attraction, we have 1530 ergs per sq. cm. per second. Equating this to the above value obtained from the frictional resistance $650{,}000\,k$, we have $k = 0024$. This is in good agreement with the limiting values of k given by the curves of *Fig.* 12.



⟶ Direction of Flow.

FIG. 31.

§ (26) THE FRICTION OF THE WIND ON THE EARTH'S SURFACE.—Assuming that the skin friction of the wind on unit area of the earth's surface can be expressed in the form $F = k\rho Q_s^2$ where Q_s is the velocity of the wind near the surface of the ground, the value of k has been calculated by Mr. G. I. Taylor[2] from his theory of eddy motion and certain meteorological data obtained at Salisbury Plain. Referring to the sketch of this theory given in § (13) above, it will be seen that if F_x and F_y are the components of the friction of the wind along axes parallel and perpendicular to straight isobars and u and v the components of the wind parallel to these axes, then

$$F_x = \int_0^\infty \mu' \frac{d^2u}{dz^2} dz = \mu' \left[\frac{du}{dz}\right]_{z=0}$$

$$\text{and } F_y = \mu' \left[\frac{dv}{dz}\right]_{z=0}, \qquad (1)$$

where μ' is the coefficient of eddy viscosity. The total skin friction is evidently

$$F = \sqrt{F_x^2 + F_y^2}.$$

Now from the equations of § (13) it follows that

$$\left.\begin{aligned} \left[\frac{du}{dz}\right]_{z=0} &= \frac{2\tan\alpha}{1+\tan^2\alpha} \cdot Q_G \frac{\frac{3}{4}\pi+\alpha}{H_1} \\ \left[\frac{dv}{dz}\right]_{z=0} &= \frac{2\tan^2\alpha}{1+\tan^2\alpha} \cdot Q_G \frac{\frac{3}{4}\pi+\alpha}{H_1} \end{aligned}\right\}, \qquad (2)$$

and hence

$$F = 2\mu\, Q_G \sin\alpha \frac{\frac{3}{4}\pi+\alpha}{H_1}, \quad . \quad . \quad (3)$$

so that if, following the ordinary convention, we write $F = k\rho Q_s^2$ we have

$$k = \frac{F}{\rho Q_S^2} = \frac{2\mu}{\rho} \sin\alpha \,.\, \frac{\frac{3}{4}\pi+\alpha}{H_1} \cdot \frac{Q_G}{Q_S^2}, \quad . \quad (4)$$

[1] "Tidal Friction in the Irish Sea," *Phil. Trans. Royal Society*, ccxx. 1.

[2] "Skin Friction of the Wind on the Earth's Surface," *Proc. R.S.* A, D, cxxxvii. 196.

From the data furnished by Mr. Dobson's experiments on Salisbury Plain :

Wind.	$\frac{\mu'}{\rho}$	a.	H_1. Metres.	Q_G.	Q_S.	k.
Strong .	62×10^3	20°	900	1560	950	0·0022
Moderate	50×10^3	$21\frac{1}{2}$°	800	910	590	0·0032
Light .	28×10^3	13°	600	460	330	0·0023

It will be seen, therefore, from reference to the curve in *Fig.* 12 that the limiting value of the ratio $R/\rho v^2$ obtained in laboratory experiments on pipes when the velocity or the linear dimension is large, may also be applied with fair accuracy to the calculation of the skin friction of the wind on the earth's surface.

§ (27) THE FRICTIONAL RESISTANCE OF SURFACES SEPARATED BY A THIN LAYER OF FLUID. (i.) *Cylindrical Surfaces, Tower's Experiments.* — It was discovered by Mr. Beauchamp Tower in 1883 in the course of some experiments made for the Institution of Mechanical Engineers,[1] that in the case of a lubricated bearing resting on a revolving shaft, the surfaces of the shaft and bearing were completely separated by a thin film of oil under pressure, and that the component of the total fluid pressure in the direction of the load was numerically equal to the load.

Tower's apparatus consisted of a half-bearing of brass resting on a shaft 4 in. in diameter, the lower part of the journal being always immersed in oil. Arrangements were made to measure the frictional resistance of the bearing, the speed of the shaft, and the temperature of the oils for various conditions of loading. Further, by inserting pressure holes at various points in the surface of the bearing and connecting these to a manometer it was possible to obtain the distribution of the fluid pressure over the whole surface of the bearing. A typical series of results are given in Table III.

TABLE III

EXPERIMENTS ON JOURNAL 4 INCHES DIAMETER, 6 INCHES LONG

Lower Surface of Journal Immersed in a Bath of Rape Oil at 90° F. Chord of Arc of Contact of Brass 3·92 inches

Load in Lbs. per Sq. In. of Projected Area of Brass.	Coefficient of Friction for Speeds as below.* Revolutions per Minute.							
	100.	150.	200.	250.	300.	350.	400.	450.
573	..	·00102	·00108	·00118	·00126	·00132	·00139	..
520	..	·000955	·00105	·00115	·00125	·00133	·00142	·00148
415	..	·00093	·00107	·00119	·00130	·00140	·00149	·00158
363	..	·00084	·0096	·00110	·00122	·00134	·00147	·00155
258	·00107	·00139	·00162	·00178	·00195	·00213	·00227	·00243
153	·00162	·00200	·00239	·00267	·00300	·00334	·00367	·00396
100	·00277	·00357	·00423	·00503	·00576	·00619	·00663	·00714

* The definition of the coefficient of friction as tabulated above is—Coefficient of friction multiplied by the total load = moment of friction divided by the radius of the journal.

From an examination of the results it can be shown that the frictional resistance as given by the product of the coefficient of friction and the load was fairly independent of the load and was nearly proportional to the velocity and, further, that it varied considerably with the temperature of the oil film.

(ii.) *Osborne Reynolds' Theory.*—After the publication of the results of these experiments the matter was taken up by Osborne Reynolds, who realised that the film might be thick enough for the unknown boundary condition to disappear, in which case the equations of hydrodynamics could be applied to the motion. As the result of a somewhat intricate investigation,[2] he obtained an approximate solution of the problem of the half-bearing, the results indicating that it was essential for the maintenance of the film that there should be a difference between the radius of the bearing and that of the journal, the two surfaces having a given eccentricity depending on the load, conditions of running and the nature of the lubricant. It was further shown by the theory that this position of nearest approach was not in the line of action of the load, but that the relative position of the two surfaces would be somewhat as shown in *Fig.* 32, the position of nearest approach being on what was termed by Mr. Tower the "off" side of the line of load, *i.e.* the side of recession of the journal surface from the line of load as distinguished from the "on" side or side of approach.

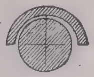


FIG. 32.

Thus, starting from the condition of no load, the distribution of the vertical and horizontal components of the pressure of the oil film would be as indicated by the curves in *Fig.* 33. In this case the position of nearest approach HG is vertically above the centre of the journal, the pressure of the film relative to the pressure at this position being positive on the on side and negative on the off side, and the distribution being such that the total vertical component is zero and the total horizontal component equal to the horizontal component of the friction.

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[1] *Proc. Inst. Mechanical Engineers*, 1883–84.

[2] *Phil. Trans. Roy. Soc.*, 1886, Part I.

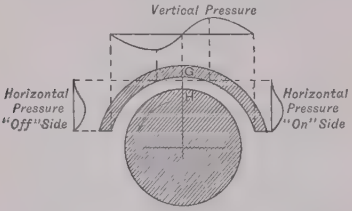

FIG. 33.

On applying a vertical load it is essential for equilibrium that the positive vertical component of the oil pressure should exceed the negative component by the amount of the load imposed, and consequently the bearing tilts over into the position shown in *Fig.* 34, in which the resultant of the vertical pressure of the oil and the friction of the bearing surface exactly balances the load. As the load increases it appears from the theory that the position of nearest approach moves up to the left extremity of the bearing and that on further increasing the load it moves back again towards the central position. Further, when the load is in such a position that the least separating distance HG is almost half the difference in radii of bearing and journal the angular position of HG is about 40° to the off side of the centre line. At this position the pressure on the oil film is everywhere greater than at A and B, the extremities of the bearing and the effect of any additional increase in the load is to produce a negative pressure at A which in practice will produce rupture of the oil film.

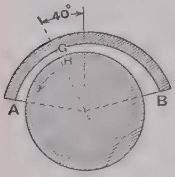

FIG. 34.

It is fairly obvious, therefore, that this condition is the one which obtained in Mr. Tower's experiments at the instant when "seizing" occurred, and may be regarded as determining the maximum load on the bearing. It was possible, therefore, to obtain a check on the accuracy of the theory by calculating the distribution of pressure from the equations when the distance of nearest approach of the surfaces was half the difference of the radii, and comparing this theoretical distribution with the one actually observed by Mr. Tower. This comparison has been carried out in *Fig.* 35, the full curves indicating the theoretical distribution of pressure on an angle base and the crosses the observations. It will be seen that the agreement is very satisfactory.

The mathematical theory of friction under the conditions obtaining when plane or cylindrical surfaces supplied with a lubricant move relatively to each other has also been developed by Sommerfeld, Michell, and Harrison. As the treatment of the problem of the cylindrical bearing by Harrison is simpler than that of Osborne Reynolds, owing to the fact that he has assumed that the bearing is a complete cylinder instead of a half-cylinder and leads in the case of two dimensions to an exact solution, a brief account of the analysis is given here.[1]

(iii.) *Harrison's Theory.*—In forming the equations of motion of the film the effect of gravity and of the inertia of the fluid can be neglected compared with the internal stresses arising from the shearing of the liquid. Also, on account of the thinness of the film, its curvature can be neglected so that the same equations hold whether the surfaces are plane or cylindrical. In the equations x is measured along the moving surface in the direction of motion, y being normal to this surface. The motion is steady and assumed two dimensional.

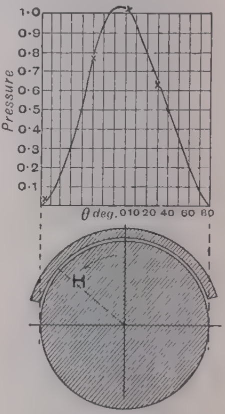

FIG. 35.

If u, v be the component velocities at any point in the liquid, p the pressure, the equations of motion are (see p. 345)

$$\frac{\partial p}{\partial x}=\mu\nabla^2 u, \quad . \quad . \quad . \quad . \tag{1}$$

$$\frac{\partial p}{\partial y}=\mu\nabla^2 v, \quad . \quad . \quad . \quad . \tag{2}$$

where μ is the coefficient of viscosity, and the equation of continuity is

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0. \quad . \quad . \quad . \quad . \tag{3}$$

The boundary conditions are

$$\left.\begin{array}{ll} u=\mathrm{U},\ v=0, & \text{where } y=0 \\ u=0,\ v=0, & \text{where } y=h \end{array}\right\}, \tag{4}$$

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[1] *Transactions, Cambridge Philosophical Society,* xxii. No. 3, p. 39.

where h is the variable distance between the surfaces and is a function of x. Since the surfaces are nearly parallel, v will be small compared with u and the rate of variation of u in the direction of x will be small compared with its rate of variation in the direction of y.

Accordingly equations (1) and (2) become

$$\frac{\partial p}{\partial x} = \mu \frac{d^2u}{dy^2}, \quad . \quad . \quad . \quad (5)$$

$$\frac{\partial p}{\partial y} = 0. \quad . \quad . \quad . \quad . \quad (6)$$

From (6) it is seen that p is independent of y and (5) is then integrable, giving

$$u = \tfrac{1}{2}\mu \frac{\partial p}{\partial x} y(y-h) + \mathrm{U}\left(\frac{h-y}{h}\right). \quad . \quad (7)$$

Now from (3)

$$\int_0^h \frac{\partial u}{\partial x}\partial y = -\Big[v\Big]_0^h = 0,$$

therefore, substituting the value of $\partial u/\partial x$ from (7) and integrating, we have

$$\frac{\partial}{\partial x}\left(h^3 \frac{\partial p}{\partial x}\right) = 6\mu \mathrm{U}\frac{\partial h}{\partial x},$$

or

$$h^3 \frac{\partial p}{\partial x} = 6\mu \mathrm{U}(h - h_1), \quad . \quad . \quad (8)$$

where h_1 is the value of h for which $\partial p/\partial x = 0$.

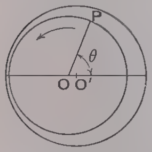

FIG. 36.

Let *Fig.* 36 represent a section through the shaft journal, and let O be the centre of the journal of radius a and O′ the centre of the bearing, $a+\eta$ its radius, and let $\mathrm{OO'} = c\eta$ where $c < 1$.

Then the value of h can be written $h = \eta(1 + c\cos\theta)$ where θ is the angle POO′.

Hence, writing $x = a\theta$, $h_1 = \eta(1 + c\cos\theta_1)$, we have

$$\frac{dp}{d\theta} = \frac{6\mu \mathrm{U}ac(\cos\theta - \cos\theta_1)}{\eta^2(1 + c\cos\theta)^3}, \quad . \quad . \quad (9)$$

from which

$$p = \mathrm{C} + \frac{6\mu \mathrm{U}a}{(1-c^2)^2\eta^2}\Bigg[\frac{c\sin\theta}{(1+\cos\theta)^2}\Big\{-(1-c^2)(1+c\cos\theta) + \tfrac{1}{2}(1+c\cos\theta_1)(1-c^2+3(1+c\cos\theta))\Big\} + (1-c^2)^{-\frac{1}{2}}\{2(1-c^2) - (1+c\cos\theta_1)(2+c^2)\}\tan^{-1}\left\{\sqrt{\frac{1-c}{1+c}}\tan\frac{\theta}{2}\right\}.\Bigg] \quad (10)$$

Now p can only have one value for any given value of θ, and therefore the coefficient of $\tan^{-1}\{\sqrt{(1-c)}/(1+c)\tan(\theta/2)\}$ in the above equation must be zero, *i.e.*

$$3c + (2 + c^2)\cos\theta_1 = 0. \quad . \quad . \quad (11)$$

This equation determines the points at which $(dp/d\theta) = 0$, and substituting this value of θ_1 in equation (10), we have

$$p = \mathrm{C} + \frac{6\mu \mathrm{U}ac\sin\theta(2 + c\cos\theta)}{\eta^2(2+c^2)(1+c\cos\theta)^2}. \quad . \quad (12)$$

It will be seen from this that the positions of maximum and minimum pressure are equidistant from the point of nearest approach, and the one value rises above the value of the pressure at that point by as much as the other value falls below it. The distribution of the pressure for values of C of 0·1, ·3, and ·5 is shown in *Fig.* 37. The total vertical force on the journal due to the pressure acts downwards through O and is given by

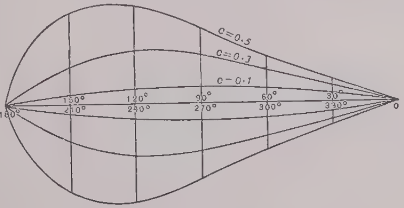

FIG. 37.

$$\mathrm{R} = \int_0^{2\pi} p\sin\theta a d\theta = \frac{12\pi\mu \mathrm{U}a^2c}{\eta^2(2+c^2)(1-c^2)^{\frac{1}{2}}}. \quad (13)$$

The total force due to the viscous drag on the surface of the journal must by symmetry act through O′ and is given by

$$\mathrm{S} = \int_0^{2\pi} f\sin\theta a d\theta.$$

Now

$$f = -\mu\left(\frac{\partial u}{dy}\right)_{y=0} = \left[\mu\frac{\mathrm{U}}{h} + \tfrac{1}{2}h\frac{\partial p}{\partial x}\right]_{y=0} \text{ by (7)}$$

$$= \frac{\mu \mathrm{U}}{\eta}\left[\frac{4}{1+c\cos\theta} - \frac{6(1-c^2)}{(2+c^2)(1+c\cos\theta)^2}\right],$$

or

$$\mathrm{S} = \frac{4\pi\mu \mathrm{U}a}{\eta c}\left[2 - \frac{4-c^2}{(2+c^2)(1-c^2)^{\frac{1}{2}}}\right]. \quad (14)$$

Again the couple exerted on the journal is

$$\mathrm{M} = \int_0^{2\pi} fa^2 d\theta = \frac{4\pi\mu \mathrm{U}a^2(1+2c^2)}{\eta(2+c^2)(1-c^2)^{\frac{1}{2}}}. \quad (15)$$

Now if f' be the viscous drag on the surface of the *bearing*

$$f' = -\mu\left(\frac{du}{\partial y}\right)_{y=h} = \mu\left[\frac{\mathrm{U}}{h} - \tfrac{1}{2}h\frac{\partial p}{\partial x}\right].$$

Hence on the bearing the corresponding forces and couples are R′ acting upwards through O′ and equal to R

$$\mathrm{S'} = \frac{4\pi\mu \mathrm{U}a}{\eta c}\left[1 - \frac{2(1-c^2)^{\frac{1}{2}}}{2+c^2}\right], \quad . \quad (16)$$

$$\mathrm{M'} = \frac{4\pi\mu \mathrm{U}a^2(1-c^2)^{\frac{1}{2}}}{\eta(2+c^2)}. \quad . \quad . \quad (17)$$

Now S and S′ are not equal and opposite, but being of a smaller order than R and R′ may be neglected. In fact it has been shown by Sommerfeld that a closer approximation would establish their identity. The inequality of M and M′ is, however, an essential characteristic of the equality of the force systems (R, M), (R′, M′), R acting at O and R′ at O′. Taking different values of c the ratio of M to M′ is

$c=0$	·1	·4	·6	·9
M/M′ 1	1·03	1·51	2·69	13·8

The coefficient of friction for the journal is given by

$$\lambda=\frac{\mathrm{M}}{\mathrm{R}a}=\frac{\eta(1+2c^2)}{3ac},$$

and that for the bearing λ′ by

$$\lambda'=\frac{\mathrm{M}'}{\mathrm{R}'a}=\frac{\eta(1-c^2)}{3ac}.$$

An appreciation of these facts is of value in the practical determination of the coefficients of friction of bearings in laboratory experiments, for if, as is almost universally the practice, the friction on the bearing is determined it is evident that the actual coefficient of the journal is considerably greater than the value so determined unless the speed be sufficiently great to make c comparatively small.

The fact that the frictional resistance of a journal running at constant speed increases slightly as the load is increased may be illustrated by calculating the values of M and R from equations (13) and (15) for given values of c and assumed values of μ, U, a, and η and plotting the results. This has been done in *Fig.* 38 for a bearing and journal in which

$\mathrm{U}=500$ cm. per sec.,
$a=2{\cdot}5$ cm.,
$\eta={\cdot}005$ cm.,
$\mu-1$,

the calculated values of R and M being as follows:

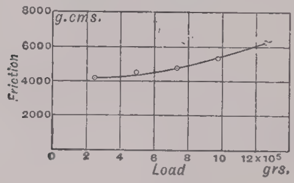

FIG. 38.

c.	R in Grammes.	M in Gramme Cm.
·1	240,200	4084
·2	480,900	4325
·3	722,300	4735
·4	961,500	5319
·5	1,232,500	6164

It will be seen that by increasing the load in the ratio of 5 to 1 the friction is increased by 47 per cent.

The variation in the amount of the frictional resistance as the speed is increased, the load remaining constant, may be shown by calculating the values of U and M from equations (13) and (15) for different values of c, R being constant.

This has been done for the same bearing, journal, and lubricant as in the previous example, taking the value of R as 1,232,500 grammes when $c={\cdot}5$.

The calculated values of U and M are as follows:

c.	U Cm. p.s.	M Gramme Cm.
·5	500	6,164
·4	637	6,660
·3	855	7,970
·2	1285	10,900
·1	2085	16,720

The results are shown in *Fig.* 39, in which the ordinates are the values of M and the abscissae those of U. It will be seen that at the low speeds an increase in the speed of about 30 per cent only results in an increase in the frictional resistance of about 8 per cent, but at the high speeds the friction is practically proportional to the speed.

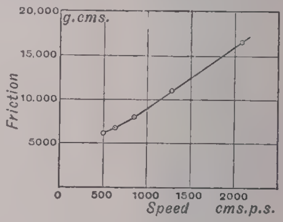

FIG. 39.

It should be noted that the above deductions from Harrison's theory—the ratio of the couple on the journal to that on the bearing and the variations of friction with load and speed—are based on the assumption of a film of lubricant which is continuous round the journal and in which the pressure distribution is symmetrical about a horizontal section through the axis, as in *Fig.* 37. No experimental investigation of the pressure distribution in a lubricated bush appears to have been made, but from some preliminary observations at the National Physical Laboratory it would appear that such a symmetrical distribution is not commonly obtained in practice.

§ (28) FRICTIONAL RESISTANCE OF FLAT INCLINED SURFACES. (i.) *Michell's Theory.*—The hydrodynamical theory of the pressure distribution between two flat inclined surfaces separated by a film of fluid has been worked out by Michell,[1] who has obtained a complete solution in three dimensions for a slide block of finite length and width. The results obtained have recently been applied with great success to the design of thrust blocks of marine engines, and an outline of the analysis may be given here. Let the plane rectangle ABCD in *Fig.* 40 represent the lower surface of a rectangular slide block moving with velocity U over the plane xOz, the space between the planes being filled with a liquid of viscosity μ. U is taken as positive in the direction xO, and the planes AC, xOz are assumed to intersect in the line Oz and the planes AC, xOy the line AB. The width of the block AD is taken as π. Let h = distance between the planes at the point x, y and let $\theta = xOB$, the angle between the planes. Put

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[1] *Zeitschrift für Mathematick*, 1905, lii

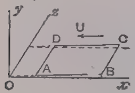

FIG. 40.

$$c=\sin\theta, \quad OA=a, \quad OB=b, \quad h=cx.$$

The boundary conditions are, if p = pressure of liquid at x, y,

$p=0$ when $x=a$ or $x=b$ for all values of z.
$p=0$ when $z=0$ or $z=\pi$ for all values of x.

Then, precisely as in the case of the complete cylindrical bearing (see p. 377), it follows that between the planes AC and xOz, p as a function of x and z must satisfy the equation

$$U\frac{\partial h}{\partial x}=\frac{\partial}{\partial x}\left(\frac{h^3}{6\mu}\cdot\frac{\partial p}{\partial x}\right)+\frac{\partial}{\partial z}\left(\frac{h^3}{6\mu}\cdot\frac{\partial p}{\partial z}\right)$$

or
$$\frac{\partial^2 p}{\partial x^2}+\frac{3}{x}\cdot\frac{\partial p}{\partial x}+\frac{\partial^2 p}{\partial z^2}-\frac{6\mu U}{c^2x^3}=0. \quad . \quad (1)$$

Assuming a solution of the form

$$p=p_1+p_2+p_3+\ .\ .\ .\ p_m,$$

where
$$p_m=\frac{w_m \sin mz}{mx} \quad . \quad . \quad . \quad (2)$$

and w_m is a function of x only, we have

$$p=\Sigma\frac{w_m \sin mz}{mx}.$$

Writing $mx=\zeta$, $24\mu U/\pi c^2=k$, and equation (1) as

$$\frac{\partial^2 p}{\partial x^2}+\frac{3}{x}\cdot\frac{\partial p}{\partial x}+\frac{\partial^2 p}{\partial z^2}-\frac{6\mu U}{c^2x^3}\cdot\frac{4}{\pi}\left\{\sin z+\frac{\sin 3z}{3}+\ .\ .\ .\ \frac{\sin mz}{m}\right\}=0,$$

the sum of the series of sines being $\pi/4$ for all values of z between o and π, we have

$$\left.\begin{aligned}\frac{\partial p}{\partial x}&=m\frac{\partial p}{\partial \zeta}=m\Sigma\left\{\frac{1}{\zeta}\cdot\frac{\partial w_m}{\partial \zeta}-\frac{w_m}{\zeta^2}\right\}\sin mz\\ \frac{\partial^2 p}{\partial x^2}&=m^2\frac{\partial^2 p}{\partial \zeta^2}=m^2\Sigma\left\{\frac{1}{\zeta}\cdot\frac{\partial^2 w_m}{\partial \zeta^2}-\frac{2}{\zeta^2}\cdot\frac{\partial w_m}{\partial \zeta}+\frac{2w_m}{\zeta^3}\right\}\sin mz\\ \frac{\partial^2 p}{\partial z^2}&=-\Sigma\frac{m^2 w_m}{\zeta}\sin mz\end{aligned}\right\}, \quad (3)$$

the differentiation of the series term by term with respect to z being permissible because $p=0$ when $z=0$, and $z=\pi$.

The coefficient of $\sin mz$ in equations (3) now becomes

$$\frac{m^2}{\zeta}\left\{\frac{\partial^2 w_m}{\partial \zeta^2}+\frac{1}{\zeta}\cdot\frac{\partial w_m}{\partial \zeta}-\left(1+\frac{1}{\zeta^2}\right)w_m-\frac{k}{\zeta^2}\right\}, \quad (4)$$

and since every such coefficient must vanish,

$$\frac{\partial^2 w_m}{\partial \zeta^2}+\frac{1}{\zeta}\cdot\frac{\partial w_m}{\partial \zeta}-\left(1+\frac{1}{\zeta^2}\right)w_m-\frac{k}{\zeta^2}=0. \quad (5)$$

The complete solution of (4) is seen to be

$$w_m=A_mI_1(\zeta)+B_mK_1(\zeta)-k\left(1+\frac{\zeta^2}{3}+\frac{\zeta^4}{5\,.\,3^2}+\frac{\zeta^6}{7\,.\,5^2\,.\,3^2}+\ .\ .\ .\right) \quad (6)$$

or

$$w_m=A'_mI_1(\zeta)+B'_mK_1(\zeta)-k(\zeta^{-2}+3\zeta^{-4}+5\,.\,3^2\zeta^{-6}+7\,.\,5^2\,.\,3^2\zeta^{-8}+\ .\ .\ .\ ),$$

where $I_1(\zeta)$ and $K_1(\zeta)$ are the Bessel functions of ζ. Hence the value of p_m will be

$$p_m=\frac{\sin mz}{mx}\left\{A_mI_1(mx)+B_mK_1(mx)-\frac{24\mu U}{\pi c^2}\left(1+\frac{(mx)^2}{3}+\frac{(mx)^4}{5\,.\,3^2}\ .\ .\ .\right)\right\} \quad (7)$$

or

$$p_m=\frac{\sin mz}{mx}\left\{A'_mI_1(mx)+B'_mK_1(mx)-\frac{24\mu U}{(mx)^2\pi c^2}(1+3(mx)^{-2}+5\,.\,3^2(mx)^{-4}\ .\ .\ .\ )\right\},$$

or writing for convenience

$$L_1(mx)=\frac{4}{\pi}\,\frac{1}{mx}\left(1+\frac{(mx)^2}{3}+\frac{(mx)^4}{5\,.\,3^2}+\ .\ .\ .\right)$$

$$L_2(mx)=\frac{4}{\pi}\,\frac{1}{(mx)^3}\left(1+3(mx)^{-2}+5\,.\,3^2(mx)^{-4}.\ .\ .\right),$$

we have
$$C=\frac{6\mu U}{c^2},$$

$$p_m=\sin mz\left\{A_m\frac{I_1(mx)}{mx}+B_m\frac{K_1(mx)}{mx}-CL_1(mx)\right\} \quad (8)$$

or

$$p_m=\sin mz\left\{A'_m\frac{I_1(mx)}{mx}+B'_m\frac{K_1(mx)}{mx}-CL_2(mx)\right\},$$

the first form being suited for calculation when mx is small, the second when mx is large.

The coefficients are determined from the condition that p_m vanishes when $x=a$ or b for all values of z.

The intensity of pressure at any point x, y having been determined, the total pressure

$$P=\int_0^\pi\int_a^b p\,dx\,dz \quad . \quad . \quad . \quad (9)$$

may be found by arithmetical or graphical summation. The friction per unit area at any point x, o, z, precisely as in equations on p. 377, is given by

$$f = -\mu\left(\frac{\partial u}{\partial y}\right)_{y=0} = \mu\frac{\mathrm{U}}{h} - \frac{h}{2}\cdot\frac{\partial p}{\partial x}, \quad . \quad (10)$$

where U is the velocity of the fluid in the direction of x at x, y, z.

The resistance of the whole block is therefore given by

$$\int_0^\pi\int_a^b\left(\mu\frac{\mathrm{U}}{h} - \frac{\partial p}{\partial x}\cdot\frac{h}{2}\right)dxdz$$

$$= \pi\int_a^b \mu\frac{\mathrm{U}}{cx}dx - \int_0^\pi\int_a^b\frac{\partial p}{\partial x}\cdot\frac{cx}{2}dxdz$$

$$= \pi\int_a^b \mu\frac{\mathrm{U}}{cx}dx - \frac{c}{2}\int_0^\pi\left\{(p_b - p_a) - \int_a^b pdx\right\}dz$$

$$= \frac{\pi\mu\mathrm{U}}{c}\log\frac{b}{a} + \frac{c}{2}\cdot\mathrm{P}. \quad . \quad . \quad . \quad . \quad (11)$$

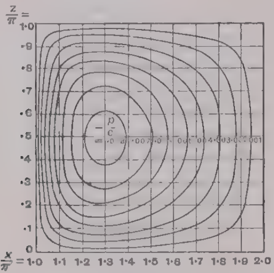

FIG. 41.

(ii.) *The Results of the Application of the Theory to Special Cases. Case I. Square Block—length of side* $=\pi$, $b=2a$.—The distribution of pressure over the block is shown in *Fig.* 41, in which the lines of equal values of p/C, *i.e.* equal values of $pc^2/6\mu\mathrm{U}$, are shown. For tables of the values of the Bessel functions required and the method of obtaining the values of the coefficients, reference may be made to the original paper. The mean pressure is found by arithmetical summation to be $\bar{p} = (\mathrm{P}/\pi^2) = -\cdot 0213(\mu\mathrm{U}/c^2)$, and the position of the resultant pressure is $\cdot 42\pi$ from the rear end of the block. The nominal coefficient of friction

$$= -\frac{\mathrm{F}}{\mathrm{P}} = 10{\cdot}83c.$$

Case II. Slide Block of Width equal to one-third of its Length, $b=2a$, as before.—The distribution of pressure is shown in *Fig.* 42, the mean pressure is given by

$$\bar{p} = \frac{\mathrm{P}}{3\pi^2} = -\cdot 00155\frac{\mu\mathrm{U}}{c^2},$$

and the position of the resultant pressure is at ·39 of the length of the block from the rear end.

The coefficient of friction is $143c$, or thirteen times greater than in the case of the square block.

Case III. Slide Block of Infinite Width.—Assuming $b=2a$ as before, the mean pressure is $\bar{p} = -\cdot 0506\,(\mu\mathrm{U}/c^2)$, the resultant pressure acts at 1·433 of the length of the block from its rear end, and the coefficient of friction is 4·86 C, or less than half of that of the square block.

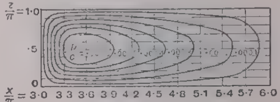

FIG. 42.

The distribution of pressure along the middle line $z=\pi/2$ in this case compared with that for the square and rectangular blocks is shown in *Fig.* 43, in which the influence of the transverse flow in reducing the pressure at the side is clearly brought out.

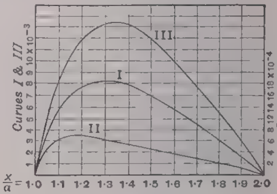

FIG. 43.

The practical application of the essential conditions to secure perfect lubrication between two flat surfaces, indicated by the theory, has led to very great improvements in the design of the thrust blocks of marine engines. Previously collar-thrust bearings of the type shown in *Fig.* 44 were in use. These consisted of collars forming part of the shaft in contact with thrust washers sliding in grooves in the thrust block and transmitting the thrust of the propeller to it. Oil was admitted to the surfaces in contact through radial grooves cut in the thrust washers, but as the two surfaces were coplanar it is evident that no separating oil film could be formed, with the result that the maximum pressure which could be used without seizing was of the order of 60 lbs. per sq. inch. The size and cost of the thrust blocks of large marine engines were therefore very great. The first thrust bearing in which the theoretical condition of an inclination of the surfaces was fulfilled was made by Mr. A. G. M. Michell in

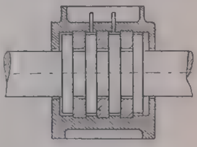

FIG. 44.

Australia in 1905, and since that time the development of this type has been very rapid. *Fig.* 45 shows a bearing on this principle designed by Mr. Newbigin for Messrs. Bellis & Morcom of Birmingham for a steam turbine. In this A is the collar on the shaft, B are the segmental rubbing blocks, the backs of which are cut away to allow of them tipping through the necessary angle to the plane of rotation. C are spacing blocks between which the loose rubbing blocks are fitted. D is the case holding the spacing and rubbing blocks together. E is a washer, concave on one face and flat on the other, to provide means of automatic adjustment. F is the cross-piece with a convex face on to which the washer fits and having holes through which bolts pass to transfer the thrust to the frame of the turbine. The oil is supplied through a hole in the centre of the shaft S. This bearing was tested under a total load of 1500 lbs. or 500 lbs. per sq. inch at a speed of 1750 revolutions per minute, under which conditions it ran continuously at a temperature of 124° F. Owing to the smallness of the angle of inclination of the surfaces it is not practicable to have them fixed, and the blocks are pivoted about an axis passing through the line of action of the resultant of the oil pressure and the friction. In this way the block automatically sets itself to the correct inclination. Designed in this way thrust-bearings have been constructed which have run continuously with a pressure as high as 3000 lbs. per square inch.

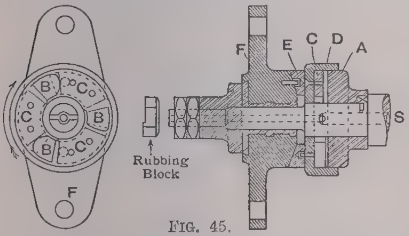

FIG. 45.

IV. THE FRICTIONAL RESISTANCE OF SOLID SURFACES SEPARATED, OR PARTIALLY SEPARATED, BY A FILM OF FLUID OF SUCH SMALL DIMENSIONS LATERALLY THAT THE RESISTANCE IS NO LONGER DUE TO THE SHEARING STRESS IN THE FLUID ALONE

§ (29) OILINESS.[1] (i.) *Lubrication.*—In the cases of the frictional resistance of surfaces separated by a film of fluid whose motion can be predicted from the equations of hydrodynamics, such as those discussed earlier in this article, it is, of course, obvious that the resistance is entirely independent of the nature of the surfaces and dependent on the characteristics of the fluid, so that it may be assumed that, in the case of two bearings and journals of identical dimensions and speed and loaded in such a manner that the relative position of journal and bearing is the same for each, but supplied with different lubricants, the frictional resistances are proportional to the coefficients of viscosity of the two lubricants used. It has long been known, however, by practical engineers that trouble in running machinery caused by hot bearings can be cured by the use of certain oils whose viscosity was of the same order as that of the oil used in the hot bearing. Thus in the case of the heating up of the crank-pins of gas-engines lubricated by mineral oil it is common practice to substitute castor oil for the mineral oil, with the result that the friction diminishes to its normal value and smooth running results. Now the value of μ for castor oil does not differ appreciably from that of a good lubricating mineral oil, so it is evident that the former possesses some lubricating property apart from its viscosity. Although considerable attention has been devoted to the study of this property of good lubricants, no definite relation has yet been found between it and the other physical properties of the oil, and the property itself is somewhat vaguely referred to as "oiliness." There seems, however, to be no doubt that the determining factor of oiliness is the chemical composition of the lubricant and that those oils which contain a large proportion of unsaturated hydrocarbons are the best lubricants. An explanation of the relative superiority as regards lubricating value of animal and vegetable oils over mineral oils is suggested by Langmuir's theory of adsorption. According to this theory, the fact that animal and vegetable oils spread upon water whereas mineral oils do not is due to the presence of an active group in the molecule in the former case and its absence in the latter.[2] The soluble glycerine ester end of the rod-like molecule tends to dissolve in the water, but the insoluble hydrocarbon end refuses to do so, with the result that the molecule stands on end and the surface of the water is covered with a layer of closely packed molecules of the animal or vegetable oil.[3] On the other hand, the mineral oils with hydrocarbon groups at each end of the molecule are inert, and spreading does not take place. Similarly, when a liquid whose molecules contain active groups is in contact with a solid surface adsorption takes place and the molecules orient themselves into the surface layer of the solid in the same manner. We may therefore regard the lubrication of surfaces in apparent contact as being in the extreme case a condition in which the surfaces are separated by a layer of oil one molecule thick, resembling a piece of velvet firmly glued to the metal with the pile outwards. Two such velvet-clad surfaces will presumably be able to glide over each other with little friction, and that lubricant will be the most efficient in which this spreading action over the metallic surfaces is the most complete. It is evident that the action here presumed will depend on the chemical composition of the surface of the bearing, and this affords an explanation of the well-known fact that in cases of lubrication in which, owing to the circumstances of the motion and loading, perfect or "film" lubrication cannot take place, the frictional resistance depends on the nature of the metallic surface, with the result that "anti-friction metals," as they are called, are much used. Mr. Archbutt has recently found that under the same conditions of speed and nature of lubricant the lining of a bronze bearing with white metal enabled double the load to be carried.

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[1] See article "Lubrication."

[2] H. S. Allen, *Proceedings of the Physical Society*, Nov. 20, 1919.

[3] See also Hardy, "Spreading of Fluids on Glass," *Phil. Mag.* xxxviii. 49.

(ii.) *Relation between the Lubricant and the Surface—Deeley's Experiments.*—The lubrication of surfaces under the conditions mentioned above and the determination of the coefficients of friction between surfaces of different metals in contact when lubricated with various oils have been investigated by Mr. R. M. Deeley, who has constructed a special machine for the purpose. In this machine (*Fig.* 46) three steel pegs each $\frac{5}{32}$ inch in diameter symmetrically placed in a circle of 7 cm. diameter rest on the flat surface of a disc of metal which can be slowly rotated. These pegs are carried by an upper disc which can be loaded as desired and which actuates a spindle to which a spiral spring and recording index are attached. When the lower disc is rotated the upper disc is carried with it by means of the frictional resistance between the surfaces of the peg and the lower disc, the index recording the value of the torque, until slipping occurs. A pawl and ratchet are attached to the gear to prevent the index from moving back along the scale when slipping takes place. The movable disc on which the pegs rest lies in a circular dish which can be filled with oil. The mechanical details of the machine will be clear from *Fig.* 46. In order to obtain clean surfaces of the disc and pegs these were ground under water with flour of carborundum. They were then polished with fine wet emery cloth, rubbed well in water with a cork to remove as much emery as possible, dried with clean blotting-paper, and finally heated to get rid of all traces of moisture. As soon as the surface was dry it was wetted with the lubricant to be tested, placed in the machine, and the static friction coefficient determined in the manner described above.

As illustrations of the effect of the nature of the metals in contact and of the lubricant employed, the following results of experiments made by Mr. Deeley may be quoted. The pegs were made of mild steel. One of the discs used was of cast iron and the other was a lead-iron alloy of the following composition:

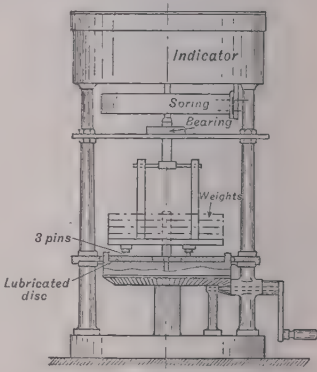

FIG. 46.

Copper	85·88
Tin	5·71
Lead	4·00
Zinc	4·19
Iron	0·10
Nickel	0·12
	100·00

The static coefficient of friction is taken as the mean frictional resistance of the peg divided by the load, and the efficiency of the lubrication is taken as 100 − (static coefficient × 100). Table I. gives the results obtained.

From the results it will be seen that the efficiency of lubrication of mild steel on cast iron is higher than for mild steel on the alloy for all the lubricants tested, but that the amount of the difference varies with the nature of the oil tested, the advantage of the cast iron varying from 0·5 per cent in the case of a mineral oil used for clocks to 12 per cent in the case of a mineral oil used for typewriters. Rape and olive oils give the best results and the mineral oils the worst.

It is of interest to note that those oils which according to the experiments have the lowest coefficient of friction are those which are generally acknowledged by engineers to be the best lubricants.

The general conclusions which Mr. Deeley[1] draws from his experiments are as follows:

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[1] *Phys. Soc. Proc.*, 1919, discussion on Lubrication.

“‘Oiliness’ appears to be an effect produced by the lubricant upon the metallic surfaces with which it is in contact rather than a property depending on any particular physical property of the lubricant. It would appear that the unsaturated molecules of the lubricant enter into a firm physico-chemical union with the metallic surfaces, thus forming a friction surface which is a compound of oil and metal. This solid surface would also appear in the case of metallic surfaces to be much more than one molecule thick, the oil either penetrating some little distance into the metal and altering its physical properties or as a result of abrasion forming a paste of metal plus oil between surfaces covered by oil layers one molecule thick.”

TABLE I

Description of Oil.*		Mild Steel on Cast Iron.		Mild Steel on Lead Bronze.	
		Static Coefficient.	Efficiency.	Static Coefficient.	Efficiency.
H.B. clock oil	M	0·271	72·9	0·275	72·5
Bayonne oil	M	0·213	78·7	0·234	76·6
Typewriter oil	M	0·211	78·9	0·294	70·6
Victory Red oil	M	0·196	80·5	0·246	75·4
F.F.F. cylinder oil	M	0·193	80·7	0·236	76·4
Manchester spindle oil	M	0·183	81·7	0·262	73·8
Castor oil	V	0·183	84·7	0·159	83·1
Valvoline cylinder oil	B	0·143	85·7	..	..
Sperm oil	A	0·127	87·3	0·180	81·1
Trotter oil	A	0·123	87·7	0·152	84·8
Olive oil	V	0·119	88·1	0·196	80·4
Rape oil	V	0·119	88·1	0·136	85·4

* A = animal oil; V = vegetable oil; M = mineral oil; B = blended oil.

§ (30) THE N.P.L. GEAR EFFICIENCY EXPERIMENTS. (i.) *Lanchester's Apparatus.*—A series of experiments on lubrication, in which the contact of the sliding surfaces and the distribution of the lubricant was probably of the nature of that presumed in the foregoing discussion, has recently been carried out at the National Physical Laboratory. These experiments consisted of tests of the efficiency of transmission of power through a worm-gear when using different kinds of oils, and were carried out for the Lubricants and Lubrication Inquiry Committee of the Department of Scientific and Industrial Research. The testing machine was specially designed by Mr. F. W. Lanchester for the accurate measurement of the efficiency of power transmission through a worm-gear. A sketch of the machine is given in *Fig.* 47 and an enlarged view showing a section of the gear-box and the oil circu-

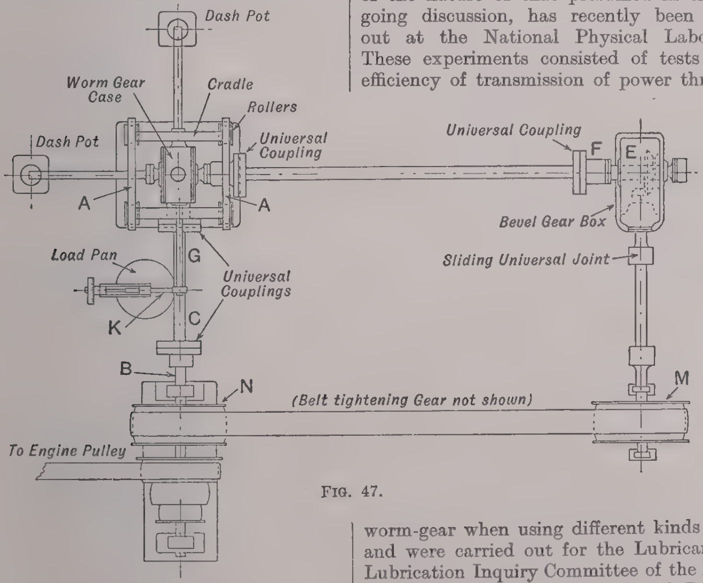

FIG. 47.

lating system in *Fig.* 48. It will be seen from *Fig.* 47 that the gear-box is supported in a cradle A in such a manner that it has freedom of motion through a small angle about two axes at right angles. The worm is driven by the shaft B through the intermediate shaft C, the latter being provided with universal couplings at each end. In the same manner the worm-wheel shaft is connected to the bevel box E, through the shaft D, and the universal couplings F, F. The load is supported by a bracket K fixed to the arm G, the axis of this arm being parallel to and in the same vertical plane as the worm-shaft axis. The load is not fixed directly to the bracket but is carried by a slider, and by means of a screw and nut device the distance of the load from the axis of the arm can be varied. The bracket is permanently fixed to the arm and thus the moment of the load about the worm-wheel axis is equal to the load multiplied by the length of the arm and is always the same for the same load. The moment about the worm-shaft, however, can be adjusted by means of the screw-gear, there being a scale on the bracket giving the distance of the centre line of the load from the axis of the arm. It will be seen, therefore, that if the efficiency of the gear is 100 per cent and the gear ratio 3 to 1 so that the speed of the worm is three times the speed of the worm-wheel, the torque on the worm-shaft will be one-third of that on the worm-wheel shaft, and therefore the distance of the load from the worm axis will be one-third of its distance from the wheel axis. As the efficiency of the gear is less than 100 per cent, it will be necessary to move the load farther from the worm axis in order to balance the gear-box. The calculated distance of the load from the worm axis, assuming 100 per cent efficiency, divided by the distance of the load to obtain a balance of the gear-box is the efficiency of the gear. The drive is taken through the bevel-box to the belt pulley M, the latter being of such a diameter that it tends to drive the pulley N keyed to the driving shaft B at from 3 to 5 per cent higher speed than it is actually making. In this way, by the frictional slipping of the belt over the surface of the pulley keyed to the shaft B, all the power transmitted through the gear, with the exception of that lost in friction of the bevel-gear and shaft-journals, is returned to the driving-shaft. The system constitutes, therefore, a power circuit, and all that is necessary to be supplied

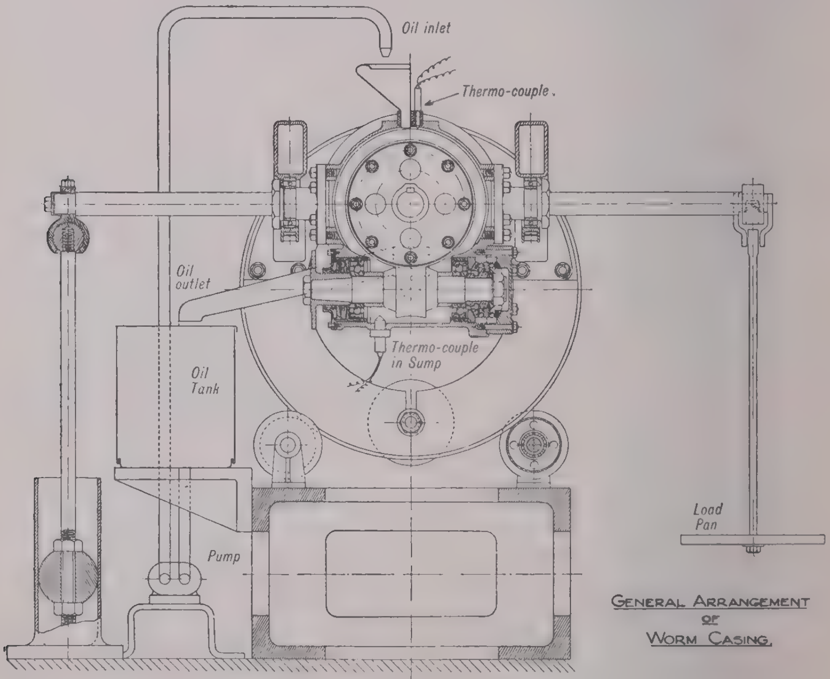

FIG. 48.

from outside sources is the total loss due to the friction of the whole machine. This loss is supplied by a belt drive to the cone pulley P from a 15-h.p. petrol motor. It is arranged that the tension in the belt connecting the pulleys M and N can be adjusted over a wide range, an increase in the belt tension causing an increase in the pressure between the teeth of the worm and worm-wheel, and therefore an increase in the torque. In the machine described the pressure between the teeth could be increased to a value corresponding with a transmission of 100 h.p.

Fig. 48 shows in detail the method of supporting the gear-box. It is held by means of the ball-bearing CC in the cradle A, which can itself rotate through a small angle by rolling in the ball-bearing rollers B and B.

The lubricant contained in the tank shown in *Fig.* 48 is fed into the top of the gear-box by means of a rotary pump, and leaves by means of the pipe K, the hole in the side of the box being about on a level with the top of the worm. The temperature of the oil entering and leaving the gear is taken by thermocouples. It was estimated that in the experiments the absolute accuracy of the efficiency obtained was within 0·2 and 0·3 per cent of the true value, and the agreement of the readings was within 0·1 per cent. A difference corresponding to 0·1 per cent in the efficiency could easily be detected when the apparatus was running smoothly. The gears tested were of the Hindley type, cut by the Daimler Company on Mr. Lanchester's principle, by means of which a high efficiency of transmission is secured even under extremely heavy loading of the lubricated surfaces. In the tests to be described the mean pressure on the surface ranged from 1½ to 2 tons per square inch, and efficiencies of 97 per cent were obtained.

A brief description of the results obtained with various lubricants may be given.

In carrying out the tests it was found that starting with the lubricant at the normal temperature of the room the heat developed in friction at the surfaces of the worm and wheel was such that the temperature of the oil in the receiver gradually increased, thus enabling a consecutive series of observations to be carried out at gradually reduced values of the viscosity.

(ii.) *Experimental Results.* — A complete series of efficiency tests at temperatures of the lubricant ranging from 15° C. to 75° C. were carried out on the following samples of oil:

Mineral	FFF Cylinder. Bayonne. Victory Red.
Animal	Trotter (a substitute for lard oil).
Vegetable . . .	Rape. Castor.
Fish	Sperm.

The more important physical properties of these oils will be seen from the following tables giving the results of determinations made at the National Physical Laboratory:

TABLE II

DENSITY AND VISCOSITY AT 20° C.

	Viscosity / Density	Density.	Viscosity.
Castor . .	7·8	·95	7·5
Rape . .	·99	·91	·90
Trotter . .	·98	·91	·89
Sperm . .	·38	·88	·34
FFF Cylinder	22·1	·89	19·6
Bayonne. .	1·8	·90	1·6
Victory Red.	12·9	·941	12·2

TABLE III

VARIATION OF μ/ρ WITH TEMPERATURE

Temp. ° C.	Castor.	Rape.	Trotter.	Sperm.	FFF Cylinder.	Bayonne.	Victory Red.
5	..	2·02	2·42	..	..	12·7	..
10	..	1·50	1·56	1·13	..	6·40	..
15	12·75	..	..	..	39·39	..	23·15
20	7·84	·99	·98	·38	22·1	1·8	12·9
30	4·32	·68	·61	·27	8·6	·93	4·5
50	1·08	·35	·29	·15	2·4	·37	1·0
70	·49	·17	·16	..	·94	·18	·38
	..	·12	·11	..	·49	..	..

TABLE IV

VALUES OF FLASH-POINTS, SURFACE TENSIONS, AND SPECIFIC HEATS

	Flash-point ° C.	Surface Tension, Dynes per Cm.	Specific Heat, C.G.S. Units.
Castor . .	400	37·6	·508
Rape . .	405	36·6	·488
Trotter . .	353	38·3	·483
Sperm . .	300	38·3	·493
FFF Cylinder	500	36·7	·476
Bayonne. .	375	36·1	·460
Victory Red	274	38·5	·423

(iii.) *The Critical Temperature.*—In the case of all the mineral oils tested it was found that when a certain temperature of the oil had been reached, called the "critical temperature," the running of the gear became decidedly unsteady, and a marked increase in the rate of fall of efficiency with temperature was observed. Experiments beyond the critical temperature were continued until it was considered that the test could not be carried further without injuring the gear. In the case of animal and vegetable oils, no critical stage was reached at temperatures below 75° C., beyond which the tests were not carried.

(iv.) *Conclusions.*—The general nature of the results at a mean pressure on the teeth of $1\frac{1}{2}$ ton per square inch are indicated by the curves in *Fig.* 49, of which the ordinates are the values of the efficiency of transmission and the abscissae the values of the temperature of the lubricant.

It will be seen that at atmospheric temperature the efficiencies of the four fixed oils are appreciably higher than those of the mineral oils, but that in all cases the efficiency is remarkably high, varying from 93·5 for the FFF Cylinder to 95·5 for the fixed oils. When the temperature exceeds 30° the extreme range in the efficiency is not so large as at 20°, and this relative performance is maintained up to about 47° C., at which the critical point of the Bayonne oil is reached and its efficiency falls rapidly. The critical point of the Victory Red oil is not so well defined, but would appear to be approximately 50° C. The performance of the FFF Cylinder, however, remains remarkably constant up to a temperature of 17° C., when a very rapid break in the behaviour of the oil was noted, and the efficiency fell, as in the case of the other mineral oils. The behaviour of the Sperm oil was somewhat abnormal, as the efficiency appeared to form continuously from 20° to 60° and then remained steady. There was, however, no sign of a critical point having been reached. There can be no doubt that the relative superiority of the fixed oils is due to the absence, within the range of temperature obtained, of a critical point. The fact that the critical points of two of the mineral oils were reached at a temperature of 50° C. would have an important bearing on their practical value as lubricants, since such temperatures are not uncommon in internal combustion engines and turbine lubrication.

In *Fig.* 50 are exhibited the results of the same series of tests plotted on a base of absolute viscosity of the lubricant instead of temperature. It will be seen that throughout there is no indication that the frictional resistance of the gears is even approximately proportional to the viscosity of the lubricant employed, as would be the case if the surfaces were separated by a film of oil of measurable thickness. This conclusion is strikingly verified in the comparison of the results for Castor and Trotter oils. At 40° C. the viscosity of Castor oil is approximately six times that of Trotter, but in the tests at this temperature the frictional resistances were approximately the same. As regards the effect of pressure, it was found that there was an appreciable increase (2 per cent) in efficiency obtained by raising the pressure from 1·5 to 2·0 tons per square inch, and further there was an increase of 1 per cent in the efficiency by raising the speed of the worm-shaft from 500 to 1500 r.p.m.

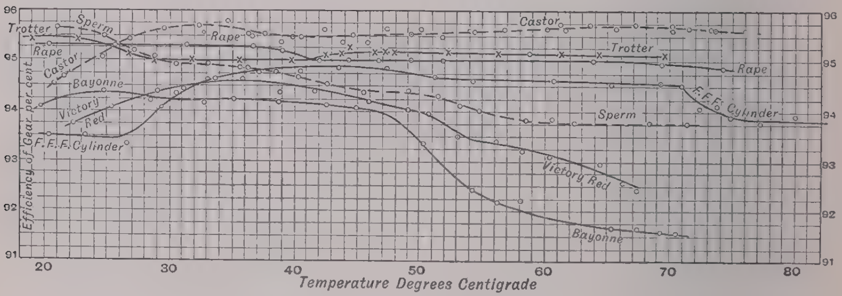

FIG. 49.

(v.) *Effects of Mixture.*—As practical experience showed that the lubricating value of a mineral oil could be increased by adding a certain proportion of animal or vegetable oil to it, some experiments on various mixtures were made. It was found that the addition of Rape oil in any proportion to the mineral oils did not appear to increase the efficiency very appreciably—an increase of 0·2 per cent was noted—but the critical temperature of the oil to which the addition of Rape was made was raised considerably. This effect of raising the critical temperature could be obtained with the addition of as small a quantity of Rape as $2\frac{1}{2}$ per cent, and increasing this amount up to 25 per cent did not appear to make any very marked improvement as regards changing the critical point. The general effect of varying the percentage of Rape oil added to a mineral oil is shown in *Fig.* 51. Experiments were also made on the effect of adding Castor oil in various proportions to a mineral oil, and similar results to those observed in the case of Rape were obtained. To determine whether the beneficial effect of adding the fixed oil to the mineral oil was due to the fatty acids

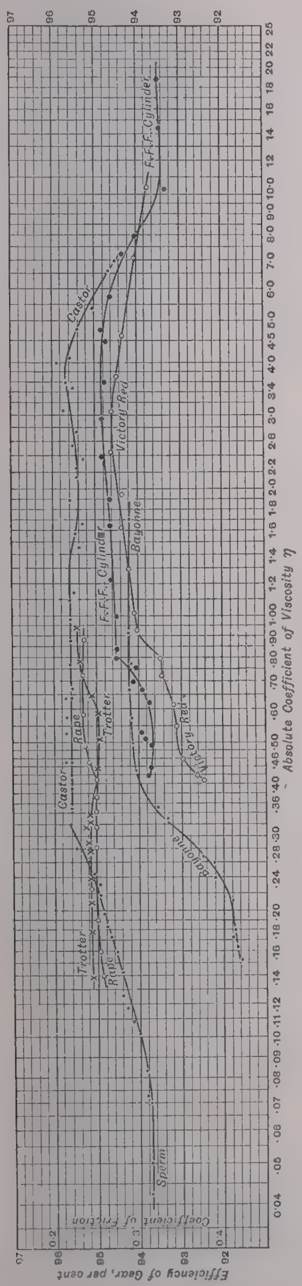

FIG. 50.

contained in the fixed oil, the following comparative tests were made:

(1) Tests on Bayonne oil alone.

(2) Tests on Bayonne oil with 10 per cent of acid Rape added.

(3) Tests on Bayonne oil to which 10 per cent of Rape oil, from which the fatty acids had been removed, was added.

(4) Tests on Bayonne oil to which 1·0 per cent of Rape oil fatty acids had been added.

The results are shown in *Fig.* 52. It will be seen that the effect of adding 1 per cent of fatty acids to the Bayonne is to produce a marked improvement in the efficiency and to raise the critical point from 47° C. to 57° C. The addition of the Rape oil from which the fatty acids had been extracted produced, on the other hand, practically no improvement. The general results of the tests may therefore be taken as confirmatory of the hypothesis that the fatty acids in the fixed oils are the main factors in the relatively greater lubricating efficiency of these oils.

A series of experiments was also undertaken to determine the effect of adding deflocculated graphite to the oils. This is a special preparation of graphite in a finely divided state which when mixed with oils forms a colloidal mixture called "Oildag," from which the graphite cannot be separated by mechanical means. The general effects of adding Oildag to the lubricants was as follows:

Trotter and Rape.—General lubricating efficiency unaffected by the addition.

Castor.—A small rise (about 0·5 per cent) in the efficiency due to the addition was observed.

Bayonne.—The temperature efficiency curve did not show the characteristic critical point, the efficiency falling gradually with rise of temperature up to 80° C.

FFF Cylinder.—A small increase of efficiency (0·2 to 0·4 per cent) due to the addition, and a rise of the critical point from 72° C. to 90° C. was noted.

Victory Red.—The critical period was smoothed out and a rise in the efficiency of about 0·5 per cent was noted.

The general results showed that the addition of the Oildag was to reduce appreciably the differences between the mineral oils and to bring their lubricating efficiencies to nearly the same value.

Oildag appears, therefore, to have the effect of rendering an inferior mineral oil as good a lubricant as a superior one in that it not only raises the lubricating value of an inferior oil, but considerably reduces the rate of fall of efficiency which usually occurs at temperatures above the critical value.

(vi.) *Tests on the Addition of Flaked Graphite to the Lubricants.*—The results of the use in the testing gear of a mechanical mixture of oil and graphite in a finely divided state was also tried. In these experiments the graphite was maintained in suspension in the oil by the churning action of the pump and gears, but when the test was stopped it settled out and was deposited on the surfaces of the gear and casing.

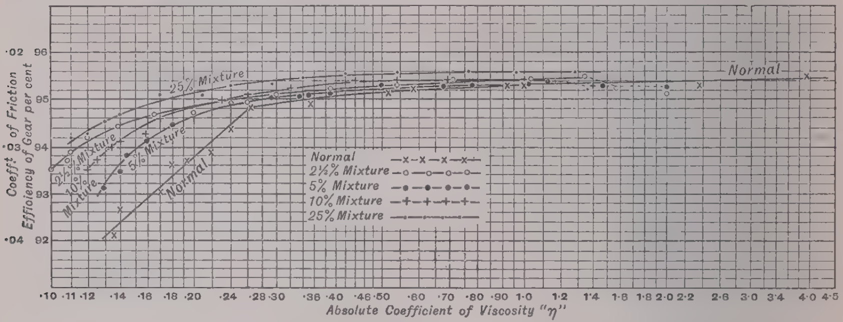

FIG. 51.

The results of the tests showed that the effect varied with the nature of the oils used. In the case of the Trotter and Bayonne oils a marked improvement in efficiency due to the addition of the graphite was found, but the Castor and Victory Red oils were not materially affected. In the FFF Cylinder oil the critical point was raised by 18° C.

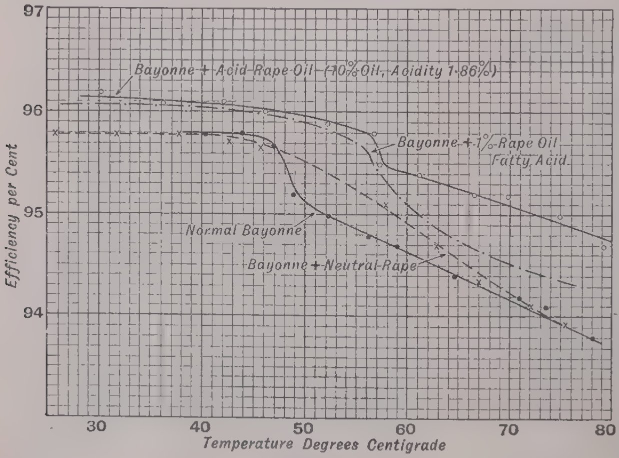

FIG. 52.

It was observed, however, that the rate of wear of the worm-wheel was considerably greater with the flaked graphite than when normal oils or normal oils with Oildag were used.

The results broadly indicated that the addition of graphite to the oils tested has a small but appreciable beneficial effect on their lubricating properties, but it is not certain that this would be the case with all oils.

V. THE FRICTION OF DRY SOLID SURFACES

§ (31) LAWS OF FRICTION.—As would be expected from the imperfect knowledge of the molecular and molar actions which take place at the surfaces of separation of solid bodies in contact, the nature of the surface action is still obscure and, in consequence, the so-called laws of solid friction are largely empirical. Further, it is now recognised that these laws are only rough approximations owing to the fact that the experiments upon the results of which they are based were, for the most part, made on surfaces which were not clean and were usually contaminated by the presence of some fluid moisture, the presence of which materially affected the frictional resistance.

In the case of two solid bodies in contact over a surface the component of any external force tending to cause relative motion of the two bodies by sliding over this surface is exactly balanced by the frictional resistance between the surfaces up to a certain limiting value of the external force, and if this value is exceeded, sliding of the bodies takes place.

The fundamental law of solid friction is that this limiting value of the frictional resistance is a definite fraction of the normal force with which the surfaces in contact are pressed together.

Coefficient of Friction: Angle of Repose.—The value of this fraction is called the coefficient of friction f, and depends on the nature of the surfaces. For example, in *Fig.* 53 let XY be the trace of the surface of contact, and let P, inclined at an angle ϕ to the normal, be the resultant force exerted on one body by the other body.

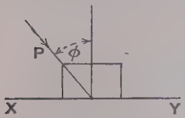

FIG. 53.

Then the force with which the bodies are pressed together is P cos ϕ and the lateral force tending to make them slide over each other is P sin ϕ. Now by the law of solid friction P sin ϕ cannot exceed P cos ϕ without sliding taking place, and therefore at the instant of sliding

$$P \sin \phi = fP \cos \phi$$

or

$$f = \tan \phi.$$

It follows, therefore, that the greatest angle of obliquity of the resultant pressure to the normal to the surfaces is the angle whose tangent is the coefficient of friction. This angle is called the angle of repose, and a knowledge of its value for various substances is important in the design of engineering structures. The following table is commonly used by engineers, and is based on that drawn up by General Morin after very extensive investigations:

TABLE I

	f.	ψ.
Dry masonry and brickwork	0·6-0·7	31°-35°
Timber on stone . . .	About 0·4	22°
Iron on stone . . .	0·7-0·3	35°-16·6°
Timber on timber . .	0·5-0·2	26°·5-11°
Timber on metals . .	0·6-0·2	31°-11°
Metals on metals . . .	0·25-0·15	14°-8°·5
Masonry on dry clay .	0·51	27°
Masonry on wet clay .	0·33	18°
Earth on earth . . .	0·25-1·0	14°-45°

When once sliding has taken place the frictional resistance may retain the value which it had at the instant sliding began, or, as is commonly the case, its value may be appreciably reduced and take up a value which depends on the relative velocity of the surfaces. It will be clear, therefore, that between the surfaces of solids in contact there may be two kinds of friction, (1) the frictional resistance before sliding takes place, and which may have any value up to the limiting resistance depending on the normal pressure and the coefficient of static friction, and (2) the frictional resistance when sliding, in which the coefficient of friction may be smaller than the statical coefficient, and which may also depend to some extent on the motion.

§ (32) STATIC FRICTION.—The subject of static friction is of considerable importance in the theory of the stability of engineering structures, and has received much attention on account of its wide application in civil engineering practice. In the design of structures consisting of masonry and brickwork, it is assumed that the pieces are in contact over the plane surfaces which constitute the joints, and that the mortar or cement used for bedding the surfaces together is incapable of resisting any forces other than compression and friction. It is obvious, therefore, that an essential condition for the stability of the structure is that the obliquity of the pressure should at no joint exceed the angle of repose for the materials.

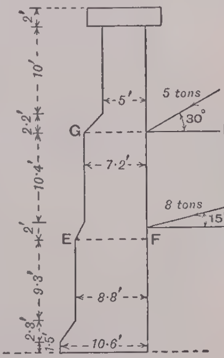

FIG. 54.

As an illustration of the application of the data of Table I. the stability of the buttress in *Fig.* 54 may be calculated. This is supposed to be made of strong brickwork, weighing 112 lbs. per cubic foot of the dimensions given in the cross-section, and of a uniform width of 5 feet. The buttress has to sustain two inclined thrusts applied to it at the points indicated, the upper one being the thrust of a roof principal which is 5 tons, inclined at an angle of 30° to the horizontal, and the lower one the thrust of an arch which is 8 tons, inclined at an angle of 15° to the horizontal. It is evident that the joints which are most liable to slide are GH, where the thrust of the roof is applied, and the joint immediately below the point of application of the arch-thrust which is at EF. If these joints are secure the stability of the others need not be investigated. Considering the joint GH, the lateral force tending to cause sliding is 5 cos 30 = 4·32 tons. The weight of the buttress above GH is 21·6 tons, which, added to the vertical component of the thrust, gives a total normal pressure on the joint of 24·1 tons. The maximum resistance to sliding may be calculated from the value of f for brickwork, which from the Table in § (31) is seen to be ·6, and is therefore 14·4 tons, or more than three times the actual lateral force. The joint is therefore safe.

Again, taking the joint EF, the total lateral force on the buttress above EF is

$5 \cos 30 + 8 \cos 15 = 4{\cdot}32 + 7{\cdot}75 = 12{\cdot}07$ tons. The weight of the buttress above EF is 44·3 tons, which, added to the vertical components of the two thrusts, gives a total normal pressure on the joint of 48·8 tons. The maximum resistance to sliding is therefore $\cdot 6 \times 48{\cdot}8$ or 29·3 tons, or more than twice the lateral force due to the thrust. The buttress is therefore safe as regards lateral movement. To complete the investigation it would, of course, be necessary to consider the resistance of the buttress to overturning or crushing at any joint.

§ (33) THE FRICTIONAL STABILITY OF EARTH.[1]—A structure of earth, whether produced by excavation or embankment, preserves its figure partly by the friction between its grains and partly by means of their mutual cohesion. It is by cohesion that a bank of earth is enabled to stand with a vertical face for a few feet below its upper edge, whereas friction alone would make it assume a uniform slope.

The cohesion of earth is, however, gradually destroyed by the action of air and moisture, so that its friction alone is the only force which can be relied upon to produce permanent stability. It is therefore customary to treat the stability of a mass of earth as arising wholly from the mutual friction of the grains, and to take as the basis of all investigations on stability the principle that the resistance to displacement by sliding along a given plane in a loose granular mass is equal to the normal pressure exerted between the parts of the mass on either side of that plane multiplied by a specific constant. This constant is the coefficient of friction of the mass and is the tangent of the angle of repose.

Now in a granular mass any plane whatever may be considered as a plane joint, and hence it follows that the condition for the stability of a granular mass is that the direction of the pressure between the portions into which it is divided by any plane should not at any point make with the normal to that plane an angle greater than the angle of repose.

Again, it follows, from a consideration of the distribution of internal stresses in a solid, that the plane at any point of it on which the obliquity of the pressure is greatest is perpendicular to the plane which contains the axes of greatest and least pressure, so that the pressure of greatest obliquity and the greatest and least pressures are all parallel to one plane. The relation between them may therefore be obtained from the stress ellipse of a body subject to pressures parallel to one plane, the equation of which is $P^2 = a^2 \sin^2 \alpha + b^2 \cos^2 \alpha$, where P is the stress on a plane whose normal is inclined to the axis of y at an angle α, and a and b are the maximum and minimum stresses. Thus, in *Fig.* 55, let $OA = a$, $OB = b$, and let zz be a plane whose normal makes an angle θ with OM where OM represents the stress on zz. The angle MOQ therefore represents the obliquity of the stress on zz. Also, if R is the middle point of PQ,

$$PQ = a - b, \quad RP = RQ = RM = \frac{a-b}{2},$$

and evidently the angle MOQ is a maximum when RM is perpendicular to OM, and then

$$\text{Maximum angle of obliquity} = \sin^{-1} \frac{a-b}{a+b},$$

and in that case $OM^2 = (a+b/2)^2 - (a-b/2)^2 = ab$, or stress on $zz = \sqrt{ab}$.

Again, for any value of the obliquity, we have, if $OM = p$,

$$\left(\frac{a-b}{2}\right)^2 = \left(\frac{a+b}{2}\right)^2 + p^2 - 2\left(\frac{a+b}{2}\right) p \cos \theta,$$

or $$-ab = p^2 - (a+b) p \cos \theta,$$

which gives the relation between the maximum and minimum stresses and the stress whose obliquity to the plane on which it acts is θ. Now in the case of a mass of earth whose upper surface is either horizontal or inclined to the horizontal at a definite angle, it is clear that on any plane parallel to the surface, and whose depth below the surface is small compared with the lateral dimensions of the surface, the pressure is vertical and of a uniform intensity equal to the weight of the vertical prism standing on unit area of the given plane.

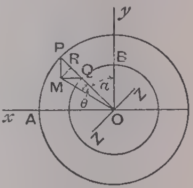

FIG. 55.

Further, it follows that the stress on any vertical plane parallel to the horizontal trace of the first plane must be in a direction parallel to this plane, for considering the equilibrium of a small prism ABCD (*Fig.* 56) whose centre is at O and whose faces are parallel to XOX and YOY, it is evident that the forces exerted by the other parts of the mass on the faces AB and CD are directly opposed and that they are independently balanced. The forces on AC and BD are therefore independently balanced, which cannot be the case unless their direction is parallel to YOY. Stresses related in this manner are said to be conjugate stresses, and it is evident that their obliquities to the planes on which they act are the same.

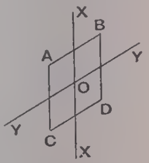

FIG. 56

If, therefore, we consider in *Fig.* 55 another stress p' which is conjugate to p, *i.e.* $\theta' = \theta$, we have

$$\frac{a+b}{2} = \frac{p+p'}{2 \cos \theta'}, \text{ and } \frac{a-b}{2} = \sqrt{\left(\frac{p+p'}{2 \cos \theta}\right)^2 - pp'}.$$

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[1] See Rankine's *Applied Mechanics*, p. 212.

Also, if ϕ be the *maximum* obliquity, it follows that, since

$$\sin\phi = \frac{a-b}{a+b},$$

$$\therefore \frac{(p+p')^2}{4pp'} = \frac{\cos^2\theta}{\cos^2\phi},$$

i.e. p and p' are the roots of the quadratic

$$P^2 - 2P\cos\theta + \cos^2\phi = 0,$$

or
$$\frac{p'}{p} = \frac{\cos\theta - \sqrt{\cos^2\theta - \cos^2\phi}}{\cos\theta + \sqrt{\cos^2\theta - \cos^2\phi}}.$$

Applying this condition to the equilibrium of a mass of earth in which one conjugate plane is a plane parallel to the surface of the earth and at a depth h below it, it is evident that the pressure per unit area of this plane is $wh\cos\theta$, where w is the weight per cubic foot of the earth and θ is the inclination of the surface to the horizontal. The pressure on the other conjugate plane, which is vertical, is, as we have seen, parallel to the surface, but as regards its magnitude it may be either the greater or the smaller of the two conjugate pressures; *i.e.*, calling this conjugate pressure p, the value of p may be either

$$wh\cos\theta\,\frac{\cos\theta - \sqrt{\cos^2\theta - \cos^2\phi}}{\cos\theta + \sqrt{\cos^2\theta - \cos^2\phi}}$$

or
$$wh\cos\theta\,\frac{\cos\theta + \sqrt{\cos^2\theta - \cos^2\phi}}{\cos\theta - \sqrt{\cos^2\theta - \cos^2\phi}}.$$

To find which value to take in any special case, recourse is had to a statical principle known as Moseley's Principle of Least Resistance, which is stated as follows.[1] If the forces which balance each other in or upon a given body or structure be distinguished into two systems called respectively active and passive, which stand to each other in the relation of cause and effect, then will the passive forces be the least which are capable of balancing the active forces consistently with the physical condition of the body or structure.

This proposition may be regarded as self-evident, since the passive forces being caused by the application of the active forces to the body or structure will not increase after the active forces have been balanced by them, and will therefore not increase beyond the least amount capable of balancing the active forces.

In a mass of earth loaded with its own weight the gravity of the mass is evidently the active force and the lateral pressure the passive force, and, therefore, the latter will have the least value which is consistent with the conditions of stability. In the case, therefore, of a bank of earth with a plane upper surface, the pressure parallel to the steepest declivity on a vertical plane at a point distant h below the surface is

$$wh\cos\theta\,\frac{\cos\theta - \sqrt{\cos^2\theta - \cos^2\phi}}{\cos\theta + \sqrt{\cos^2\theta - \cos^2\phi}},$$

or, if the ground surface be horizontal,

$$wh\frac{1-\sin\phi}{1+\sin\phi}.$$

To find the resultant pressure against a vertical plane of depth H below the surface it is evident that, since the pressure is directly proportional to the depth, we may apply the ordinary rules of hydrostatics, from which the centre of pressure is two-thirds of the total depth from the surface and the resultant force is

$$\frac{wH^2}{2}\cdot\cos\theta\frac{\cos\theta - \sqrt{\cos^2\theta - \cos^2\phi}}{\cos\theta + \sqrt{\cos^2\theta - \cos^2\phi}}.$$

§ (34) CASE OF THE SUPPORTING POWER OF EARTH FOUNDATIONS. (i.) *Theory.*—In the case treated above the conjugate pressure was caused solely by the vertical pressure due to the weight of earth above the point considered, and its value was therefore the smaller of the two conjugate pressures.

It is evident, however, that this conjugate pressure may be increased beyond this least amount by the application of the pressure of an external body—for example, the weight of a building founded on the earth. In this case the conjugate pressure will be the least which is consistent with the vertical pressure due to the weight of the building, and if that conjugate pressure does not exceed the greatest conjugate pressure consistent with the weight of the earth above the stratum on which the building rests, the mass of earth will be stable. The most important case in practice is when the surface of the ground is horizontal.

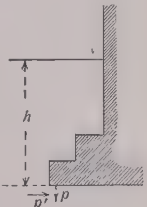

FIG. 57.

Thus, in *Fig.* 57, let p be the intensity of the pressure on a horizontal stratum due to the weight of the building. Then the horizontal conjugate pressure p', being the passive force, is given by

$$p' = p\frac{1-\sin\phi}{1+\sin\phi}.$$

The effect of p' is to cause an upward thrust on the earth immediately above the footings of the foundations, and calling this p'', then since p'' is also a passive force,

$$p'' = p'\frac{1-\sin\phi}{1+\sin\phi}.$$

Now $p'' = wh$, where w is the weight per cubic foot of the earth and h is the depth of the foundation below the surface of the ground, and therefore

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[1] Rankine's *Applied Mechanics*, p. 215.

$$p=p''\left(\frac{1+\sin\phi}{1-\sin\phi}\right)^2=wh\left(\frac{1+\sin\phi}{1-\sin\phi}\right)^2.$$

It is evident that, if h has a less value than that given by this relation, there will be a tendency for the building to sink into the ground by forcing the earth upwards.

h, therefore, is a minimum value to the depth of the foundations.

Again, if the depth is such that there is a tendency for the earth pressure to overcome the pressure due to the weight of masonry, as in the case of the floor of a dock before the water is admitted, then p'' is the active force, and

$$p'=p''\left(\frac{1-\sin\phi}{1+\sin\phi}\right)$$

or $$p=p''\left(\frac{1-\sin\phi}{1+\sin\phi}\right)^2=wh\left(\frac{1-\sin\phi}{1+\sin\phi}\right)^2.$$

The value of h determined from this relation gives a maximum which cannot be exceeded without danger of the masonry being forced upwards.

In applying this formula it must be remembered that by hypothesis the lateral extent of the mass of earth is large compared with the depth. When this is not the case, as in the region at the back of a retaining wall, the frictional resistance of the wall will affect the pressure distribution. It has been shown by Boussinesq[1] that at a distance x from the face of the retaining wall, if θ be the angle of friction between wall and earth, and $a^2=1-\sin\ \phi/1+\sin\ \phi$, then for x less than ah the

$$\text{Horizontal pressure}=\frac{w(h+x\tan\ \theta)a^2}{1+a\tan\ \theta}.$$

$$\text{Vertical pressure}=\frac{w(h+x\tan\ \theta)}{1+a\tan\ \theta}.$$

The ratio of the pressures is a^2 as in Rankine's theory. When $x=0$ the horizontal and vertical pressures become $wha^2/1+a\tan\theta$, $wh/1+a\tan\theta$, and the tangential force on the face is evidently $wha^2\tan\ \theta/1+a\ \tan\ \theta$.

(ii.) *Experimental Verification.*—The limits of accuracy in practice of the Rankine formula has been the subject of several very careful experimental investigations, and it has been found that provided the material is perfectly free from moisture the accuracy is remarkably high. Thus in some experiments by Wilson[2] on dry sand, the following results were obtained from direct observations of the ratio of the pressures, and by calculation of the ratio from the observed angles of repose of the sand.

TABLE II

DRY SAND

	Ratio of Pressures.		
	Maximum.	Minimum.	Mean.
By Rankine theory	0·339	0·296	0·320
By experimental measurement of the pressures	0·336	0·305	0·319

In these experiments the values of the angle of repose were obtained in two ways, one by placing the sand in a heap and subjecting it to vibrations until the surface had taken up a definite slope, and the other by placing the sand in a box with a sliding end which was opened gently after tapping. The extreme variation in the value of the observed angle of repose by these methods was from 30°·0 to 32° 53′.

The pressures were obtained by placing the sand in a cast-iron cylinder 11½ inches in diameter, 14 inches deep, provided with movable pistons at the top and bottom ends, the pistons being compressed between the platens of a 100-ton testing machine. In this way pressure equal to that due to a depth of 1341 feet was obtained. The instrument used for measuring the pressure consisted of two steel discs of 3 inches diameter, separated by a ring, and containing mercury, the displacement of which due to the pressure was noted. By placing this alternately parallel to, and at right angles to, the direction in which the external pressure was applied, it was possible to determine the ratio of the two externally applied pressures which gave the same displacement of the mercury and, therefore, the same pressure on the gauge. From the fact that the relation between the load and gauge indications for any position was a linear one, it followed that the ratio of the two loads was equal to the ratio of the horizontal and vertical pressures for the same load.

§ (35) EFFECT OF MOISTURE ON THE STABILITY OF EARTH.—It is well known that the effect of moisture is to increase the cohesion and hence the stability of earth and sand. This is due to the surface tension of the film of water separating the grains, the amount of the traction depending on the amount of moisture. It was found by Wilson that, commencing with sand initially dry, the force between the grains increased with the percentage moisture up to a certain point and then decreased, but so long as the interstices were not entirely filled with water there was still some tractive force, and hence the ratio of horizontal to vertical pressure should be less than that for dry sand. On carrying out a series of experiments on sand containing varying percentages of moisture in the apparatus described above, Wilson fully verified these conclusions, the ratio of the pressures corresponding to the varying degrees of moisture being as follows:

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[1] *Minutes of Proceedings Inst. C.E.* lxv. 214.

[2] *Proc. Inst. Civil Engineers*, 1901-2, cxlix.

TABLE III

WET SAND

	Percentage of Water.			
	0.	6.	12.	17.
Mean ratio of horizontal to vertical pressure	·319	·221	·212	·280

It is of interest to note that the minimum value of the ratio is obtained when the interstices are half filled with water. It is clear, therefore, that in the design of masonry dams or retaining walls to withstand earth pressure, the use of Rankine's formula with a value of the angle of repose determined from experiments on the dry material will give a result which is on the safe side.

§ (36) THE KINETIC FRICTION OF DRY SOLID SURFACES.—As mentioned above, when once sliding has commenced between the surfaces of two bodies in contact, there is in the majority of cases a sudden fall in the amount of the frictional resistance so that the coefficient of kinetic friction for two substances is usually less than the coefficient of static friction. Jenkin and Ewing[1] have shown that for hard substances such as steel on steel the two coefficients are equal, but when one or both surfaces are of relatively softer material, such as brass or greenheart, the coefficient of kinetic friction is less than that of static. They also found that when the two coefficients differ, the change, at low speeds at any rate, is not instantaneous at the instant at which relative motion begins, but takes place gradually and without discontinuity. Cases have been noted by Kimball in which the coefficient of kinetic friction is greater than that of static friction. Leather on cast iron appears to be an instance of this. A marked falling off in the value of the coefficient of kinetic friction as the relative speed of sliding is increased appears to be a universal characteristic of dry sliding surfaces. There is also in many cases a further progressive fall in the value of the coefficient with time which is probably due to the abrasive action of the surfaces tending to produce greater smoothness.

(i.) *Experiments on Brake Blocks.*—Both of these characteristics are noticeable in the results of Captain Galton's[2] experiments on the frictional resistance of cast-iron brake blocks on steel waggon wheels, as will be seen from Table IV.

TABLE IV

Relative Velocity of Brake and Wheel in Miles per Hour.	Coefficient of Friction.			
	First 3 Seconds.	5 to 7 Seconds.	12 to 16 Seconds.	24 to 25 Seconds.
60	·062	·054	·048	·043
50	·10	·07	·056	..
40	·134	·10	·08	..
30	·184	·111	·098	..
20	·205	·175	·128	·07
10	·32	·209	..	..
5	·36	..	..	..

A similar reduction in resistance due to increase of speed was found in Captain Galton's experiments on the frictional resistance of locomotive driving wheels on steel rails. These results are shown in Table V.

TABLE V

Speed in M.P.H.	Coefficient of Friction.	Speed in M.P.H.	Coefficient of Friction.
10	·11	38	·057
15	·087	45	·051
25	·08	50	·04

It may be remarked that similar speed effects in the case of locomotive driving wheels on steel rails was found by M. Poirée, but the actual values of the coefficients were about twice those given in the table. This difference was probably due to the state of the atmosphere during the trials, which is known to have a considerable effect on the frictional resistance. A common set of rules for driving wheels in different conditions is as follows:

Rails very dry—frictional resistance 600 lbs. per ton.
Rails very wet—frictional resistance 550 lbs. per ton.
Rails in ordinary English weather—frictional resistance 450 lbs. per ton.
Rails in foggy weather—frictional resistance 300 lbs. per ton.
Rails in frosty and snowy weather—frictional resistance 200 lbs. per ton.

(ii.) *Friction of Woven Fabrics.*—In recent practice the use of metal as a material for the friction surfaces of the brake blocks of railway carriages and the clutches of motor vehicles has been in many cases discontinued owing to the discovery that woven fabrics can be made which, when used as linings for brakes and clutches, give a higher coefficient of friction with greater dissipation of heat without burning and sparking, and have a much greater durability than can be obtained with metals.

One of these materials is known as the

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[1] *Phil. Trans. R.S.* clxvii., Part ii.

[2] *Engineering*, xxv. 469.

Ferodo fabric, which consists of layers of woven asbestos bonded together in a special manner.

In an extensive series of tests at the National Physical Laboratory the following results were obtained:

Material.	Pressure Lbs./Sq. In.	Work absorbed in Ft.-lbs./Sq. In. per Minute at 1000 Ft. per Minute.
Ferodo fibre .	60	39,000
Ferodo bonded asbestos compressed	60	18,000
Standard . .	50	15,500

The value of the coefficient of friction obtained and the work absorbed with Ferodo fibre at different speeds are given in Table VI.

It is claimed that the average life of Ferodo blocks on the cars and trailers of the London Electric Railway is respectively 12,000 and 22,000 miles as against 8000 for cast-iron blocks. With cast iron at £6 per ton, the economy per block per 1000 miles is given as 0·75 pence. As many as 600 brake applications are made by each train per day, the average speed, including stops, being 17·17 miles per hour, and the average mileage per set of blocks per car being 16,416.

Table VI

Tests on Ferodo Fibre

Pressure Lbs./Sq. In.	Coefficient of Friction.			Work absorbed in Ft.-lbs./Sq. In. per Minute.		
	Speed (Ft. per Minute).			Speed.		
	600	2900	5500	600	2900	5500
16	·73	·64	..	..	..	..
27	..	..	·56	..	..	80,000
29	·70	·63	..	12,000	..	..
39	..	·64	..	..	70,000	..

The deceleration averages 5 feet per second per second as against 3 feet per second per second with metal block. With this increased deceleration services have been speeded up to $1\frac{1}{4}$-minute headways during "rush" hours. The more rapid braking of the trains has also led to a higher percentage of coasting minutes, the tests showing that coasting is as high as 38 per cent of the total running time.

Notwithstanding this high rate of braking, there has been an entire freedom from accidents to passengers, and there is also a very considerable improvement in the tyres, the surfaces of which are highly polished, and there is an entire absence of cutting and scoring. The economy in this respect is shown by the fact that the mileage of motor wheels provided with Ferodo brake blocks is approximately 35,600 per $\frac{1}{16}$-inch radial wear of the tyre compared with 7000 in the case of metal blocks.

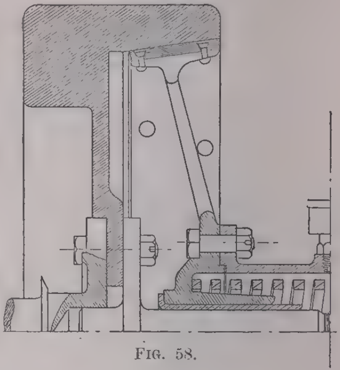

Fig. 58.

An illustration of the application of the fabric to a clutch is shown in *Fig.* 58.

§ (37) The Transmission of Power by Friction. (i.) *Belts.*—One of the most convenient methods of the distribution of power from a central supply to a number of small machines is by means of belt or rope drives from pulleys on a line of shafting driven by the prime mover, to corresponding pulleys on the various machines. When the pulleys are at rest and no power is being transmitted, the tensions on the two sides of the belt are equal; but when the driving pulley commences to rotate, slipping of the surfaces of the pulleys over the surface of the belt is prevented by friction, and rotation of the driven pulley consequently takes place. The relation between the tensions on the two sides of the belt and the coefficient of friction of the belt on the surface of the pulley may be found as follows:

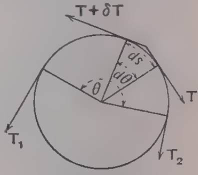

Fig. 59.

Let θ in *Fig.* 59 be the whole arc of contact of the belt over the pulley. Consider any element ds of the arc of the belt of which the tensions at the ends are T and $T+\delta T$. Then if $d\theta$ be the angle subtended by ds at the centre, we have resolving along the radius

$$(T+\delta T)\sin\frac{d\theta}{2}+T\sin\frac{d\theta}{2}=R,$$

the normal pressure of the element of belt ds on the surface of the pulley. Thus $R = T\delta\theta$.

Also resolving perpendicular to the radius

$$(T + \delta T)\cos\frac{d\theta}{2} - T\cos\frac{d\theta}{2} = \text{Friction} = \mu R = \delta T.$$

We have therefore $dT = \mu R = \mu T d\theta$, whence by integration over the whole arc of contact $T_1/T_2 = e^{\mu\theta}$.

For leather belts working on iron pulleys μ has been found to range from 0·3 to 0·4. For wire rope running at the bottom of a grooved pulley $\mu = 0{\cdot}15$. A common value for θ in the case of leather belts is 2·5 radians. Taking $\mu = 0{\cdot}3$ this gives $T_1/T_2 = 2$, or the tension on the tight side is twice the tension on the slack side. For approximate calculation of the size of a belt to transmit a given horse power $T_1btV/2 \times 33{,}000 = \text{H.P.}$ transmitted where V is the velocity of the surface of the pulley in feet per minute, b is the breadth and t the thickness of the belt in inches, and T_1 is the maximum working tension of leather, which may be taken as equal to about 320 lbs. per square inch.

(ii.) *Effect of Slipping.*—Belts are not used in cases in which a very exact velocity ratio between the driving and driven shafts is essential, on account of the tendency of the belt to slip over the surface of the pulley. There is also a definite amount of slipping which follows as a consequence of the elasticity of the belt. The amount of this slipping may be estimated as follows. The actual mass of the belt which passes any fixed point either on the tight side or the slack side in unit time is the same. Let l_0 be the length of this mass when the tension is zero, and let b, t and E be the breadth, thickness, and modulus of elasticity of the belt. Also let T_1 and T_2 be the tensions on the tight and slack sides of the belt. Then on the tight side the length passing any fixed point in unit time will be $l_0(1 + T_1/btE)$ and on the slack side the corresponding length will be $l_0(1 + T_2/btE)$, and it is evident, therefore, that these numbers will represent the velocities of the belt on the tight and slack sides, *i.e.* the velocities on the two sides will be in the ratio of $(1 + T_1/btE)$ to $(1 + T_2/btE)$. Further, it is easy to show that the velocity of the surface of the driving pulley must be equal to that of the tight side of the belt, and that the velocity of the surface of the driven pulley must be equal to that of the slack side of the belt. Thus the belt comes on to the surface of the driving pulley at tension T_1 and leaves it at a lower tension T_2. It has therefore contracted while in contact with the pulley, and it is evident that the nature of this contraction must be a slip against the frictional force exerted by the pulley, with the result that the belt continually slips over the surface of the pulley to the point where it first makes contact. In the same manner the belt comes on the driven pulley at a tension T_2 and extension takes place gradually from the point at which contact begins to the point at which it leaves.

Now the work delivered to the driving pulley is $(T_1 - T_2)\,bt \times$ (velocity of surface of driving pulley) and the work communicated to the driven shaft is $(T_1 - T_2)\,bt \times$ (velocity of surface of driven pulley), *i.e.* the efficiency of power transmission $= (1 + T_2/btE)/(1 + T_1/btE)$, or approximately $1 - (T_1 - T_2)/btE$.

Ex.—The value of Young's modulus for leather is approximately 20,000 lbs. per sq. in. Assuming $T_1 = 2T_2$, which will be the case when the coefficient of friction is about 0·3 and that the maximum working tension of a leather belt is 320 lbs. per sq. in., we have

Efficiency of transmission $= 1 - {\cdot}016 = 98{\cdot}4$ per cent.

The amount of slip is therefore 1·6 per cent.

(iii.) *Friction Drive.*—Another instance of the transmission of power by friction is seen in the attempts which have been made to substitute a friction drive for the ordinary spur gear arrangements commonly adopted in motor cars to obtain variable and reverse speeds. This has been successfully carried out in the well-known "G.W.K." motor car, in which the friction gear takes the form of a metal disc driving a fibre-faced wheel at right angles to it, the speed variation being obtained by varying the distance of the centre of the fibre-faced wheel from the axis of the metal driving disc. Reversal of direction can, of course, be obtained by moving the driven wheel to the opposite side of the disc axis.

In the case of the "G.W.K." car the driving disc is fixed axially, while the driven wheel, which is capable of being moved along its shaft to obtain the various gears, is pivoted at one end and pulled towards the driving disc by means of a spring at the other end. This is so arranged that the contact pressure between the two friction elements is lightest on top gear and increases as the gear ratio is lowered. This method has an obvious advantage over one in which the pressure between the wheels is constant at all speeds, since the amount of the pressure must be fixed to give an adequate frictional resistance at low speeds, and this means that the top gear on which 90 per cent of the running is done is used with a pressure between the wheels about three times in excess of what is necessary.

The coefficient of friction between the fibre material and the polished steel disc used in the "G.W.K." car is about 0·5. It is found in practice that it is safe to allow a peripheral pull on the driven wheel of 100 lbs. for each inch width of the frictional material irrespective of speed.

It is claimed by the makers of the "G.W.K." car that frictional transmission, in the case of cars up to 10 or 12 h.p., has considerable advantages over the ordinary gear-box in that it is much more simple to drive and the cost of replacements due to unskilful handling and ordinary wear is much smaller.

§ (38) Rolling Friction.—The resistance to the motion of wheels and rollers over a smooth plane has sometimes been attributed to the inaccuracies of the surface, and it has been supposed that if these were removed, the resistance would entirely disappear, provided that both roller and plane were made of elastic materials and the load on the roller were not sufficient to cause permanent deformation of either surface. It was no doubt some hypothesis of this kind which led the earlier experimenters on the subject to denote the relations which they obtained by the term "laws of resistance to rolling."

The researches of Osborne Reynolds[1] have, however, shown that in all cases in which surfaces roll over each other, an appreciable amount of slipping of the surfaces over each other takes place and it is the frictional resistance to this slipping which constitutes the resistance to rolling.

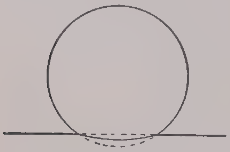
Fig. 60.

This may be seen as follows: Since no material is perfectly hard, when a heavy roller rests on a surface, the weight of the roller will cause it to indent the surface, and the surface of the cylinder will flatten out as shown in *Fig.* 60. It is evident, therefore, that when rolling takes place the distance rolled through the roller in one revolution will not be the circumference of its undisturbed surface. For example, suppose that an iron cylinder rolls on an indiarubber surface across which lines have been drawn at intervals of 0·1 inch, and that as the cylinder rolls across these lines the surface of the rubber extends so that the intervals become 0·11 inch, closing up after the cylinder is past. Then the cylinder measures its circumference on the extended plane and the actual distance rolled through will be one-tenth less than the circumference. Hence if, following Reynolds, we agree to call the distance which the roller would roll through if there were no extension or contraction, its "geometrical distance," then, in the case above, the cylinder rolls through less than its geometrical distance. On the other hand, if we have an indiarubber roller rolling on a steel surface and the surface of the roller extends 10 per cent in passing over the iron surface, it is evident that the roller will pass over a distance in one turn 10 per cent greater than its circumference. It must not be supposed, however, that, if the roller and the plane are of the same material these effects will balance each other. In the case of the flat surface the effect of the materials surrounding a depression will be to stretch the material in the depression still further, whereas in the case of the rounded surface with a small flat on it the material surrounding the flat will compress the material in the flat and decrease its lateral expansion. The magnitude of this latter effect will, of course, depend on the smallness of the diameter of the roller. There are thus two independent causes which affect the progress of a cylinder which rolls on a plane, the relative softness of the materials and the diameter of the roller. These two causes will act in conjunction or in opposition according to whether the roller is harder or softer than the plane. Thus an iron roller on an indiarubber plane will roll through less than its geometrical distance, and an indiarubber roller on an iron plane will roll through a distance more than, less than, or equal to its geometrical distance, according to the relation between its diameter and softness.

(i.) *Slipping and Rolling.*—The precise nature of the slipping action which takes place during rolling is somewhat complicated and is best studied after a preliminary consideration of the relatively simpler cases of the deformation of a soft elastic material between two parallel plates which approach and recede from each other without tangential motion.

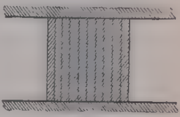
Fig. 61.

Let *Fig.* 61 represent the section of a block of indiarubber between the two plates when in the non-compressed state, and suppose the section to be marked with a series of vertical lines at equal intervals apart. Let the plates approach each other, compressing the rubber, which expands laterally. If there were no friction between the rubber and plates the section would remain rectangular as in *Fig.* 62 and the lines would still be equidistant. If, however, there be a frictional resistance between the rubber and plates which resists the lateral expansion, the section will bulge in the middle as shown in *Fig.* 63. The effect of the friction on the spacing of the lines will be that up to a certain distance, such as *or*, from the centre of the section, the friction will be sufficient to prevent slipping and, there-

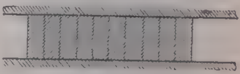
Fig. 62.

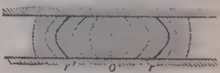

Fig. 63.

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[1] See *Phil. Trans. R.S.* clxvi.

fore, up to this point the ends of the lines will preserve their original distances apart. Beyond the extremity of *or* slipping will commence and will go on increasing to the edge of the section. From *r* outwards, therefore, the distance between the ends of the lines will continually increase.

If now the surfaces gradually separate again, the lines between *o* and *r* will assume the same forms which they had at corresponding points of the compression, but since the portion beyond *or* has been extended by the compression, it will have to contract and the friction between the surfaces will oppose this contraction. Hence the lines beyond *or* which during compression were curved outwards will gradually straighten and curve inwards as shown in *Fig.* 64.

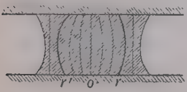

FIG. 64.

It is important to note that during these two actions the smaller the coefficient of friction the greater will be the expansion of the lines during compression, and that as the work spent in friction during separation depends on the amount of this expansion, the work will obviously increase up to a certain point as the coefficient of friction diminishes.

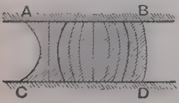

FIG. 65.

In both of the above cases it will be seen that the directions of slipping on opposite sides of the centre are opposite to each other. In the case of a roller, however, it is evident that the material immediately in front of the centre of the surface of contact is being compressed and that behind it is being expanded. This action may be approximately represented in the case of the two plates with rubber between them by supposing the upper plate AB to have been first inclined towards C, so that the material under A was compressed and then inclined towards D so as to raise the end A, thus causing a compression under B and an expansion under A. The sectional lines will therefore assume the form shown in *Fig.* 65, and it is clear that the slipping on each side of the centre now takes place in the same direction. This shows that in the action of rolling the whole of the slipping is in the same direction and tends to oppose the rotation. It is this slipping against the frictional resistance between the surfaces which gives rise to the resistance to the motion of the roller, and hence, as Reynolds showed, the correct denomination of the resistance is "rolling friction." The analogy of the action of the indiarubber between the two plates with the case of a cylinder rolling in a plane surface is not, however, exact, as there is the important difference that in the latter case it is not the entire surface of a bar which is being compressed and expanded but a portion of a continuous surface in which, whatever lateral extension may exist immediately under the roller, must be compensated by a lateral compression immediately in front and behind it.

The nature of the deformation caused by an iron roller moving over an indiarubber plane may be shown as follows. In *Fig.* 66, which represents a section through the cylinder and plane surface, the lines on the indiarubber are supposed to represent lines initially vertical and at equal distances apart. The motion of the roller is towards B, and *or* and *or′* limit the surfaces on which there is no slipping. C and D are the limits of the surface of contact and beyond these points the rubber is laterally compressed owing to the lateral extension of the material under the roller. The lines in this region are, therefore, less than their natural distance apart. From D to *r* the material is being compressed, slipping is taking place, and the lines are convex outwards. From *r* to *r′* there is no slipping, but from *r′* to C expansion with slipping is taking place and the lines are concave outwards. It is clear, therefore, that the distance apart of the lines from *r* to *r′* will depend on the amount of the lateral compression from D outwards, and it is quite possible that this may be such that the distance between the lines from *r* to *r′* may be equal to the natural distance, in which case, as we have seen, the roller will roll through its geometrical distance whatever the actual slipping between D*r* and C*r′*. According to Reynolds this is what actually takes place when an indiarubber roller rolls on an iron plane. The actual slipping is obviously equal to the difference between the intervals between *r* and *r′*, and the intervals at D or C, and will always be greater than the loss of geometrical distance rolled through.

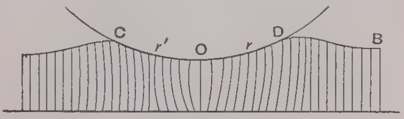

FIG. 66.

From the foregoing examination of the action of rolling, the importance of the distinction between the real and apparent slip in all cases of rolling contact is clearly brought out, the apparent slip being defined as the difference between the circumference of the roller and the distance moved through by its centre in one revolution.

An explanation is also offered of the well-known fact that in general the lubrication of roller bearings is not attended with any beneficial effects in the way of increase of efficiency, but may actually cause a loss of efficiency. Thus referring to *Fig.* 66 it is clear that if the coefficient of friction is very great r and r' will coincide with D and C and there would be no slipping, and hence no work would be spent in friction. On the other hand, if the coefficient of friction were zero, r and r' would coincide with O and there would be no friction and no work spent in overcoming it. Therefore the work spent in friction is zero for two values of the variable which is the coefficient of friction, and since it is positive for all intermediate values it must pass through a maximum value. Hence for some position of r and r', *i.e.* for some value of the coefficient of friction, the work spent in friction is a maximum.

The above conclusions have been verified by Osborne Reynolds by direct experiment, for the details of which reference may be made to the article cited. In particular it may be mentioned that the experiments showed that a hard roller on a soft surface rolls short of its geometrical distance, whereas a soft roller on a hard plane rolls more than its geometrical distance, and that when both roller and plane are of equal hardness the roller rolls through less than its geometrical distance.

§ (39) The Practical Application of Rolling Friction in the Design of Mechanism.—In recent years the loss of energy due to friction of the journals and thrust collars of mechanism has been greatly reduced by the use of ball and roller bearings. Contrary to expectation the insertion between the cylindrical bearing surfaces of mechanism of one or more rings of hardened steel balls running in hardened steel grooves or races so arranged that the sliding action is replaced by the rolling of the balls in the races has proved highly successful under heavy loading. As the effect of a design of incorrect type is likely to prove disastrous in practice, a brief account of conditions to be fulfilled and errors to be avoided may be gathered from the following illustrations of ball bearings which have been tried in practice with varying degrees of success.[1]

(i.) *Ball Bearings.*—One of the earliest types is shown at A, *Fig.* 67. This was formed of a hardened steel sleeve with a V or rounded groove fixed to the shaft, the outer races consisting of two conical rings screwed into the casing by means of a fine thread, with the idea that the races could be adjusted for wear. Since the wear of the race takes place only on the loaded side it is obvious that any adjustment of this kind would be fatal to true running of the shaft.

This fault was remedied in the design B in which the races consisted of two conical rings screwed on the shaft, the outer stationary portion being made flat. In this case the adjustable ring was always rotating and, therefore, the wear was fairly even all round and adjustment was possible, although, as experience has shown, quite unnecessary.

Later two plain races as shown at C, with a cage to keep the balls in position, were found to be successful, the final evolution of the race being shown at D where the races are grooved to a radius of about $\frac{9}{16}$ that of the ball.

In the design of ball thrust bearings the races of the early types were made with 45° grooves as shown at E. The running of this type was very unsatisfactory, with much scratching of the balls. The substitution of a flat surface for one of the grooves as at F was an improvement but not altogether satisfactory. It was found, however, that balls running between two flat rings with a cage G to hold the balls ran quite satisfactorily. This design was still further improved from the weight-carrying point of view by substituting grooves of slightly greater radius than the ball, as in H. For certain purposes, however, the flat race is to be preferred, since, if there is any chance of the shaft getting out of line with its housing, there is less likelihood of trouble in heating and vibration. A high degree of accuracy in the manufacture of balls for ball bearings is very essential. For example, in the case of balls half an inch in diameter, Professor Goodman has found that a distortion of one-thousandth of an inch corresponds with a load of about 130 lbs. of the ball. Hence, if one of the balls in a thrust bearing is one-thousandth of an inch larger than the others, that ball will have about 130 lbs. more load upon it than the others, and probably failure will occur.

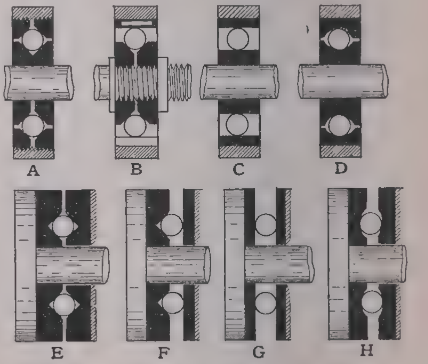

Fig. 67.

In the Skefko ball bearing (*Fig.* 68) the surface of the outer ring is spherical, with the centre at the axis of the shaft. There are two grooves on the inner ring, which can be tilted to one side to allow the balls to be slipped into position, which is a considerable advantage. There is the further advantage that such a bearing can be used on a shaft which is moving out of truth. These bearings are particularly applicable to the case of motor cars and aeroplanes where the framing is not very rigid.

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[1] Goodman, *Proc. Inst. Aut. Eng.*, 1913–14, viii.

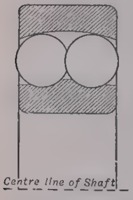

Fig. 68.

(ii.) *Safe Working Load in a Ball Bearing.*—From the results of his experiments, Professor Goodman has been led to adopt the following formula for calculating the maximum working load on a ball bearing:

$$W=\frac{knd^3}{ND+Cd},$$

where W = maximum working load in lbs.,
n = the number of balls in the bearing,
d = the diameter of the ball in inches,
N = the revolutions per minute,
D = the diameter of the ball race in inches; the diameter being taken from the point of contact of the ball with the inner race in a journal bearing, and from the centres of the balls in a thrust bearing.

The constants C and k are as follows:

	C.	k.
For thrust bearings—		
Flat races .	200	500,000
Hollow races .	200	from 1,000,000 to 1,250,000
For journal bearings—		
Flat races .	2000	1,000,000
Hollow races .	2000	from 2,000,000 to 2,500,000

A comparison between the friction of an ordinary white metal bearing and that of a ball bearing is shown in *Fig.* 69. It will be seen that in the case of the white metal bearing the starting effort is very much greater than the normal resistance after it has been running for a considerable time. With ball bearings the reverse is the case, and this, apart from the limiting ratio of the resistances, is an additional reason for the use of ball bearings in machinery which stops and starts frequently.

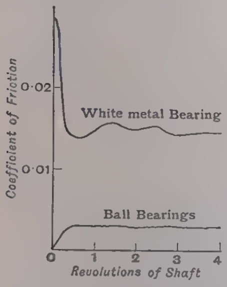

Fig. 69.

For very large bearings, in which the loads are correspondingly high, it is customary to replace the balls by hardened steel rollers. The difficulties in obtaining satisfactory running are greater in these bearings than where balls are used, on account of the end thrust of the rollers brought about by very slight inaccuracies in the machining. A slight amount of end thrust in a roller bearing may increase the frictional resistance tenfold. Various devices to obviate end thrust, such as the insertion of a hardened steel ball between the ends of the roller and the face of the housing, have been tried, but without any great success.

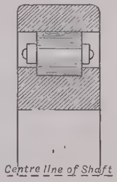

Fig. 70.

One of the most successful roller bearings which have been recently brought out is illustrated in *Fig.* 70. In this the roller runs in a hardened steel rectangular groove ground in the sleeve with great precision, and after an extensive series of tests on it Professor Goodman states that this is the only roller bearing which, in his experience, is practically free from end thrust.

VI. The Relation between Friction and Heat Transmission in the Case of the Motion of a Fluid over a Surface having a Temperature differing from that of the Fluid

§ (40) Heat Transmission. (i.) *Theory.*—The heat interchange between a solid surface and a fluid in contact with it, for a given difference of temperature between them and apart from radiation effects, will evidently be proportional to the rate at which the fluid particles are carried up to the surface, *i.e.* to the diffusion of the fluid in the neighbourhood of the surface. This diffusion may be either (1) molecular, *i.e.* the diffusion of the molecules in a mass of fluid at rest, or (2) molar diffusion, by means of the movement of small portions of the fluid in the form of eddies. The transmission of heat by molecular diffusion constitutes what is known as the thermal conductivity of the fluid, and that by molar diffusion is usually denominated heat convection. Now in the case of a fluid moving over a surface in laminar or streamline motion, there is no molar diffusion between adjacent streamlines or laminae, and, therefore, any transmission of heat to or from the surface in a direction normal to the streamlines must take place by thermal conductivity. On the other hand, when the motion becomes turbulent, it is evident that the eddies constitute the mechanism by means of which the heat is transmitted.

In § (25) it has been shown that in the motion of a fluid over a surface there is always a thin layer at the surface which is in laminar motion when the general motion of the fluid is turbulent, and if the surface be hotter or colder than the fluid it is clear that the heat transmission between layer and surface will take place by means of conductivity, and between the outside of the layer and the mass of the fluid, by eddy motion or convection. In Part I. of this article dealing with "Viscosity," §§ (1), (2), it has been shown that in the case of laminar motion the transfer of momentum between adjacent layers which constitutes the phenomenon of viscosity is effected by the internal diffusion of the fluid, and further that in the case of turbulent motion the corresponding phenomenon of mechanical viscosity is due to the mass diffusion brought about by the eddy-making and turbulence. It would appear, therefore, that the mechanism by which the transfer of momentum which constitutes frictional resistance and that by which the transfer of heat is brought about is essentially the same whether the motion be laminar or turbulent.

The probability of this effect was first pointed out by Osborne Reynolds[1] in 1874 in a paper on the heating surfaces of steam boilers. In this paper Reynolds described an experiment in which, by blowing air through a hot metal tube, the probable accuracy of his theory was demonstrated by the fact that the temperature of the issuing stream of air was approximately independent of the speed of the air current. The method of reasoning leading to this conclusion may be stated as follows. Consider the case of a fluid moving through a parallel pipe of circular cross-section with mean velocity V_m, and let the inner surface of the pipe be maintained at a uniform temperature T_s. Then, neglecting the thermal conductivity effect in the thin layer at the boundary, and assuming all the transmission to take place by eddy motion, the ratio of the momentum lost by skin friction between any two sections distant dx apart, to the total momentum of the fluid, will be the same as the ratio of the heat actually supplied by the surface to that which would have been supplied if the whole of the fluid had been carried up to the surface.

Thus, if dp is the fall of pressure between the sections,
dT the rise of temperature between the sections,
W the weight of fluid passing per second,
V_m the mean velocity of the fluid,
a the radius of the pipe,
T_m the mean temperature of the fluid between the sections.

Then, by the above relation,

$$\frac{(dp)\pi a^2}{W/g \,.\, V_m} = \frac{W dT}{W(T_s - T_m)}.$$

The heat lost per unit area of the pipe is

$$\frac{\sigma W/g \,.\, dT}{2\pi a dx},$$

where σ is the specific heat per unit mass at constant pressure, and if R denote the skin friction per unit area,

$$R = \frac{\pi a^2}{2\pi a} \cdot \frac{dp}{dx}.$$

Hence if Q be the heat transmitted per unit area,

$$Q = \frac{R\sigma(T_s - T_m)}{V_m}. \quad . \quad . \quad . \quad (1)$$

It may be noted that there may be appreciable divergence between the above value of the heat transmission and that actually observed, for, as Reynolds pointed out, "ultimately it is by conductivity that the heat passes from the walls of the pipe to the fluid, so that there will probably be in the result a coefficient $F(k/\nu)$, where k is the coefficient of thermal conductivity of the fluid, the form of which must be determined by experiment."

(ii.) *Experiments.*—A series of experiments were carried out by Dr. T. E. Stanton in 1895 with the object of testing the accuracy of the above relation.[2] The fluid used was water, which was circulated through thin copper tubes heated on the outside by means of a steam jacket. The mean temperature of the tube was estimated from the value of its coefficient of expansion and the increase in length, the value of the surface temperature being then calculated from the known heat transmitted and the thermal conductivity of copper.

In making the experiments it was arranged that the total rise of temperature should not exceed a few degrees, and should be small compared with the range of temperature between metal and water. In this way, in a series of experiments in which V_m was varied, the value of $f(k/\nu)$ referred to above would be approximately constant. Further, since it was known that $R = cV_m{}^n$, writing the relation (1) in the form

$$\frac{Q}{T_s - T_m} = Dv^{n-1},$$

where D is constant, the value of $n - 1$ could be obtained by the usual method of logarithmic plotting.

On making this determination for several sizes of tubes, and with as large a range of speed of flow as could be obtained, it was found that the value of n was practically identical with that found in the frictional determinations (*i.e.* n varied from 1·82 to 1·86), and hence the truth of the relation (1) in its most important practical aspect was fully demonstrated. The fact that the heat abstracted from a hot metal surface by a fluid flowing over it is, for a given difference of temperature between fluid and metal, nearly proportional to the speed at which the fluid moves over the surface, has been of great importance in the design of steam engine condensers, the radiators of internal combustion engines, and the heating surfaces of steam boilers. The application of the Reynolds theory to the case of the design of surface condensers for steam engines has shown[1] that for a surface condenser of given area of cooling surface and supply of cooling water, in order to obtain the greatest velocity of flow, the length of the tube should be as great as possible, and that the most efficient design will be secured by using small tubes of as great a length as is consistent with the extreme limits of frictional resistance to flow allowed. This condition can be easily fulfilled in practice by arranging the tubes in separate compartments, through which the cooling water circulates in series, and thus the *effective* length of the tube can be increased to any desired amount.

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[1] *Proceedings, Manchester Literary and Philosophical Society*, 1874.

[2] *Phil. Trans. Royal Society*, Series A, cxc.

It may be remarked that, if any means existed of determining the thickness of the layer in laminar motion at the boundary, a check between theory and experiment would be found as follows. Consider first the transmission of heat by eddy motion between the mass of fluid in the interior of the pipe and the outer surface of the layer in laminar motion, which may be assumed to have a velocity U.

Then equation (1) becomes

$$Q = \frac{R\sigma(T_1 - T_m)}{V_m - U}, \quad . \quad . \quad . \quad (2)$$

where T_1 is the temperature of the outside of the layer.

Again, considering the transmission through the surface layer, we have

$$Q = \frac{k(T_s - T_1)}{t}, \quad . \quad . \quad . \quad (3)$$

where T_s is the temperature of the surface, t the thickness of the layer, and k the coefficient of thermal conductivity,

and
$$R = \mu \frac{U}{t}. \quad . \quad . \quad . \quad . \quad (4)$$

Combining (2), (3), and (4), we have

$$Q = \frac{R\sigma(T_s - T_m)}{V_m} \cdot \frac{1}{1 + p(\sigma\mu/k - 1)}, \text{ where } p = \frac{U}{V_m}. \quad (5)$$

If, therefore, the value of p were known, it would be possible, by experiments with a fluid flowing over a surface for which the value of R was always known, to obtain a check on the theory.

Unfortunately, no definite information on the value of p is at present available. From an examination of the criterion, due to Lorenz, for the steady motion of fluid between two planes moving tangentially to each other, Mr. G. I. Taylor has arrived at the conclusion that in the case of a pipe u/V_m should become nearly constant as the velocity is increased, and independent of the nature of the fluid. As regards its value, the criterion of Lorenz indicates that it may be approximately 0·38. Alternatively the value of p may be calculated from equation (5) by substitution of the known values of R, $(T_s - T_m)$, V_m, μ, and k in carefully made experiments.

Table I

Soenneken's Experiments with Water flowing in Drawn Brass Tube 192 cm. long, 1·7 cm. diameter

Velocity of Flow in Cm. p.s.	Heat transmitted, Calories per Sq. Cm. p.s.	Temperature.		Friction, Dynes per Sq. Cm.	$\frac{k}{\mu\sigma}$.	Value of $\frac{R\sigma(T_s - T_m)}{V_m}$.	Value of p in Equation (5).
		Surface.	Fluid.				
132·2	1·17	20·65	11·97	57·6	·117	3·78	·30
132·2	1·58	75·15	67·72	45·3	·347	2·54	·32
100	1·44	26·28	13·25	35·0	·120	2·56	·30
100	1·63	67·30	56·63	28·5	·290	3·04	·35
61·7	0·726	23·35	12·66	16·2	·112	2·81	·36
61·7	0·413	70·35	66·50	11·8	·341	2·74	·42
Stanton's Experiments with Water flowing in Copper Tube 46 cm. long, 0·736 cm. diameter							
296	4·43	28·2	15·93	29·8	·128	12·35	·26
296	5·08	51·65	39·65	26·0	·217	10·5	·29

Such a series of experiments have been carried out by Dr. A. Soenneken[1] in the case of water flowing through heated metal pipes, the results of which are given in Table I., together with certain of Stanton's results.

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[1] Stanton on "The Efficiency and Design of Surface Condensers," *Proc. Inst. C.E.*, 1898–99, cxxxvi.

[1] Soenneken, König. Tech. Hochschule Munich, 1910.

It will be clear from the calculated values of p in the above table that the use of the values of 0·38 derived from the theory of Lorenz will not give results greatly in error. The low values of p found from Stanton's results are probably due to the fact that they were obtained from short lengths of pipe in which the ratio of length to diameter was only 34, and that on this account the mean thickness of the surface layers would probably be appreciably less than that finally obtained at a considerable distance from the inlet.

Reverting to equation (5) it will be seen that if the value of $\sigma\mu/k$ for any particular fluid is approximately equal to unity, the agreement between the heat transmission given by equation (1) and that actually observed should be fairly close if the assumptions on which the theory is based are correct.

Now for air $k=1{\cdot}6\mu C_v$,

where C_v is the specific heat at constant volume, and since

$$\sigma=1{\cdot}4C_v,$$

we have

$$\frac{k}{\sigma\mu}=1{\cdot}14,$$

so that experiments with air should furnish a rough demonstration of the accuracy or otherwise of the extension of the Reynolds theory given above.

In some experiments made by Mr. J. R. Pannell[1] at the National Physical Laboratory in 1912 the results given in Table II. were obtained.

Table II

Flow of Air in heated Brass Pipe 4·88 cm. diameter, 61 cm. long

Velocity of Flow in Cm. p.s.	Heat transmitted, Calories per Sq. Cm. p.s.	Temperature.		Friction, Dynes per Sq. Cm.	Value of $\frac{R\sigma(T_s-T_m)}{V_m}$.	Ratio of $\frac{\text{Calculated Heat}}{\text{Observed Heat}}$.
		Surface.	Fluid.			
1180	·0205	37·4	22·5	5·14	·0155	0·76
940	·0162	36·2	22·7	3·18	·0109	0·68
2188	·0369	43·0	26·2	14·9	·0267	0·73

It will be seen that the agreement between calculated and observed heat is not satisfactory, so that assuming that no serious errors were made in the observations on which the figures in the table are based, it would appear that the assumptions on which the extended theory is based are inconsistent with the actual facts. The one whose validity may reasonably be questioned is the discontinuity between the layer at the boundary and the eddying fluid which is implied in the reasoning leading up to the theoretical equation (5).

It will be clear from the state of motion in the neighbourhood of the boundary which is indicated in the curves of *Fig.* 27 that no such discontinuity exists and that the two motions shade off into each other gradually. It is evident, therefore, that the matter requires further investigation before any reliable prediction of the *amount* of the heat transmission can be made from a knowledge of the frictional resistances of fluid and surface.

§ (41) Effect of Surface Roughness on Heat Transmission.—Apart from the practical value of the theory of heat transmission outlined above in giving an accurate measure of the improvement to be effected in heat transmission by increasing the *speed* of flow, the relation also indicates a possibility of increasing the efficiency of the transmission by increasing the coefficient of friction between fluid and surface. It is well known that by roughening the surface over which fluid is in motion the frictional resistance can be increased two or three fold. For example, in Mr. W. Froude's experiments on the resistance to the towing of planks in water, it was found that the resistance of a surface similar to that of sandpaper was more than double that of a varnished surface under the same conditions. In experiments at the National Physical Laboratory the frictional resistance of brass pipes has been increased threefold by serrating the inner surface by a series of sharp ridges produced by a cutting tool. It may be mentioned, however, that the application of the Reynolds theory to the case of rough surfaces is not *a priori* demonstrable, since the precise nature of the modification of the fluid motion as the surface roughness is increased is a matter for speculation, and it is by no means certain that an increase in surface roughness corresponds with an increase in the turbulence which is the main factor in the heat transmitted.

For the purpose of testing the applicability or otherwise of the Reynolds theory to the case of surface roughness, a series of experiments were made at the National Physical Laboratory in 1917 on two pipes $1\frac{1}{8}$ inch diameter, of which one was in the smooth drawn condition in which it was received and the surface of the other was roughened by machining with a screw-cutting tool so as to form a double series of sharp ridges intersecting each other. The depth of the serrations was about 0·01 inch.

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[1] Pannell, *Report Advisory Committee Aeronautics*, 1916–17, viii. 22.

In making the experiments, each pipe was set up in a vertical position and a current of water circulated through it by means of a centrifugal pump provided with speed regulation, so that the mean velocity of flow could be varied from about 15 cm. per second, slightly above the critical value, to about 90 cm. per second. The outside of the pipe was wound with Eureka wire connected to the supply mains through a variable resistance and so arranged that the temperature of the pipe could be maintained at the desired value above the temperature of the water. The mean surface temperature of the pipe was estimated from the extension of the pipe measured by an extensometer specially designed for the purpose. For the estimation of the initial and final temperatures of the water, thermo-junctions embedded in thin brass tubes were placed with their axes in the axis of the pipe at the extremities of the experimental length and connected to a potentiometer. The estimation of the flow of water through the pipe was obtained from a calibrated meter. The frictional resistance of the pipe was found from the fall of pressure along the experimental length.

In order to compare the relative effects due to surface roughening, the ratio of the intensities of surface friction of the rough and smooth pipes have been plotted (*Fig.* 71) on a base of speed of flow through the pipe, and on the same diagram is plotted the curve showing the corresponding ratio of the heats transmitted per unit area per degree difference of temperature. It will be seen that throughout the whole range of speed considered the heat transmitted from the roughened pipe exceeds that transmitted by the smooth pipe by an amount greater than that by which the friction has been increased by the roughening, so that the application of the Reynolds theory to the case of surface roughening was fully demonstrated. In applying these results to practice, it is important to make sure that the conditions of turbulence are similar to those which existed in the experiments described above. For example, from the characteristics of the motion of a fluid at the inlet of a pipe referred to in § (6) (iii.), it would appear probable that had the experimental length of pipe in the above experiments been placed at the inlet to the pipe instead of some considerable distance from it, the effect of roughness on the heat transmission would not have been so marked owing to the absence of turbulence in this particular region. In order to investigate this more closely, a further research was carried out in which the heated surface represented the gills of an air-cooled internal combustion engine. The practical importance of this investigation will be obvious from the fact that if an improvement in heat transmission of the order of that obtained in the roughened pipe could be produced in the cooling surfaces of aircraft engines by artificial roughening, the possibility of considerable increase in efficiency and saving of weight would be opened up.

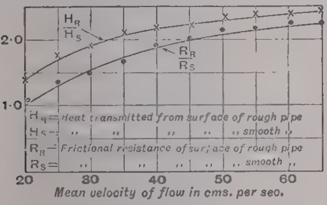

FIG. 71.

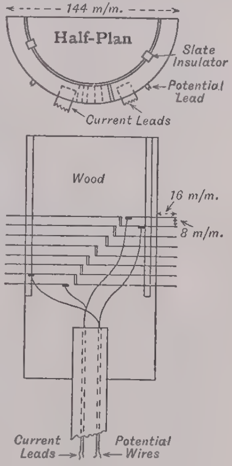

FIG. 72.

For this purpose the apparatus shown in *Fig.* 72 was constructed. This consisted of eight copper discs of approximately the dimensions and spacing of the gills of an air-cooled engine surrounding a cylinder of wood, and set up in a wind channel through which air could be circulated at speeds up to 60 feet per second. The method of making an experiment consisted in heating the gills by a current from a storage battery and measuring the amount of current and fall of potential along them. In this way the mean surface temperature of the gills and the amount of heat abstracted by the air could be calculated. Observations of the velocity and temperature of the cooling air were taken at the same time. The thickness of the copper gills was 0·055 mm., the external and internal diameters being 146 and 114 mm. and the pitch 8 mm.

The roughening was made by a special pair of steel dies by means of which the surfaces were corrugated into serrations 0·5 mm. deep and 1·8 mm. pitch. A series of observations was first made on the smooth gills. These were then taken out of the wind channel, roughened by the dies and replaced, and the experiments repeated under the same conditions of wind speed, and surface temperature as before.

The results of the two sets of observations are shown in *Fig.* 73, the ordinates of the plotted points being the heat abstracted in calories per second per degree difference of temperature between gill and air current, and the abscissae the mean wind speed in cm. per second.

It will be seen that, in complete contrast with the results of the pipe investigation, the roughening had practically no effect on the heat transmission, thus indicating that for these particular surfaces the dimension in the direction of flow was so limited that the convection of momentum to the surface was not more intense for a rough than for a smooth surface. The anticipation that the conditions as regards turbulence in this case were probably similar to those existing at the inlet of a parallel channel appeared therefore to be justified. It is probable that considerable improvement in the design of the cooling surfaces of aircraft engines and radiators may yet be effected by a suitable increase in their frictional resistances, but further research is necessary before the direction in which it may be looked for is apparent.

T. E. S.

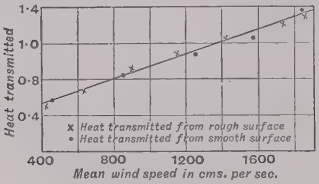

Fig. 73.

Friction and Angles of Repose. Table of Coefficients. See "Friction," § (30).

Friction and Heat Transmission, Stanton's Experiments. See "Friction," § (39).

Friction Drives in Mechanisms. See "Friction," § (36).

Friction of Curved Surfaces in Fluids. See "Friction," § (20).

Friction of Dry Solid Surfaces:
Kinetic. See "Friction," § (35).
Static. See *ibid.* § (30).
See also "Lubrication."

Friction of Locomotive Driving Wheels. See "Friction," § (35).

Friction of the Wind on the Earth's Surface. See "Friction," § (25).

Friction, Static, Frictional Stability of Structures. See "Friction," § (31).

Frictional Drag of the Tides. See "Friction," § (24).

Frictional Resistance of Cylinders in Fluids. See "Friction," § (21).

Frictional Resistance of Fluids increased by reduction in the viscosity of the fluid. See "Friction," § (21).

Frictional Resistance of Smooth Planks moving through a Liquid. See "Friction," § (18); "Ship Resistance and Propulsion," §§ (8)-(12).

Frictional Resistance of Solid Surfaces in Fluids measured by the velocity slope at the boundary. See "Friction," § (22).

Frictional Resistance of Surfaces separated by a Thin Layer of Fluid. See "Friction," § (26).

Frictional Resistance of Thin Plates in Fluids. Experimental determination from changes of momentum of the fluid. See "Friction," § (19).

Frictional Resistance of Woven Fabrics. See "Friction," § (35).

Frictional Stability of Earth. See "Friction," § (32).

Froude Belt Dynamometer. See "Dynamometers," § (4).

Froude Water Brake. See "Dynamometers," § (2) (iv.).

Froude's Experiments on the Resistance of Ships. See "Ship Resistance and Propulsion," § (4).

FUEL CALORIMETRY

§ (1) Introductory.—The measurement of the calorific value of fuels is a branch of calorimetry which has been systematised in order that it may be practised by operators not possessed of the knowledge or experimental aptitude of the trained physicist and chemist. Large numbers of determinations of the calorific value of fuels are made daily, with a reasonably high degree of accuracy and concordance by a host of semi-skilled operators. The latter, however, merely follow instructions in the use of methods which involve the scientific application of fundamental principles of physics and chemistry to the solution of the problem of measuring exactly the heat evolved in the complete oxidation of readily combustible matter. The physical and chemical principles applied in Fuel Calorimetry are essentially identical with those applied in the measurement of the heat evolved in chemical reactions in general, and the methods employed are simply specialised adaptations to a restricted field of the general methods employed by the physicist in tackling the varying problems of calorimetry presented in the pursuit of physical, chemical, and physiological researches. The broad principles and general methods of

Calorimetry have been dealt with in the article under that heading.

§ (2) Methods and Conventions of Fuel Calorimetry.—In this article the specialised methods applied to the measurement of the heat evolved in the combustion of fuel will alone be considered. These methods are, however, also of rather divergent descriptions, owing to the physical differences between the various "fuels." The physical differences referred to chiefly concern the question of the phase or state of the fuel, *i.e.* whether it is solid, liquid, or gaseous at ordinary temperatures. For instance, the calorific value with which fuel calorimetry is concerned is: for methane, the heat evolved by the combustion of *gaseous* methane; for alcohol, the heat evolved by the combustion of *liquid* alcohol; and for sugar or sulphur, the heat evolved by the combustion of *solid* sugar or sulphur. While the supplemental heat attributable to both condensation and solidification is included in the calorific value of a fuel such as sulphur, and that of condensation in the calorific value of alcohol, neither is included in the calorific value of methane, or other substance which is in the gaseous state at ordinary temperatures. Similarly the calorific value as usually determined and accepted for fuel expresses the heat resulting from their combustion when the products of combustion are in the state or phase in which they normally occur at ordinary temperatures. That is to say, any carbon dioxide produced by the combustion of a fuel is assumed to remain in the gaseous state, whereas silicon dioxide or silica would be assumed to be in the solid state, and water in the liquid state.

There is, however, a convention by which the water resulting from the combustion of hydrogen and its carbon compounds in fuels is sometimes considered to remain wholly in the gaseous state among the products of combustion, and the calorific value realised under this convention is termed the *net* calorific value. It is less than the common or *gross* calorific value by the latent heat of vaporisation of the water resulting from the combustion of the fuel. In the common measurements of the net calorific value of a fuel, a further deduction is made from the gross value of the sensible heat of the condensed water in cooling from 100° C. to ordinary temperatures. The figure thus obtained is the conventional net calorific value of the fuel, but the former figure, which differs from the gross calorific value only by the latent heat of vaporisation of the water resulting from the combustion, is the true thermodynamic net calorific value which should be used in calculations of the efficiencies of internal combustion engines, flame temperatures, etc. Except in the case of uncombined hydrogen, the net calorific value of a fuel is a figure which has little real significance, because the uncondensable products of combustion, such as carbon dioxide, nitrogen, and excess of oxygen, pass away at the same temperature as the uncondensed water vapour. In most uses of fuel, this temperature is considerably higher than that of the atmosphere, and the sensible heat of the uncondensable products of combustion as well as the latent heat of vaporisation of the water accompanying them has to be determined for the particular temperature at which they are passing away whenever the question of the thermal efficiency of a particular application of a fuel is investigated.

In considering the calorific value of many of the complex commercial fuels, it is necessary for comparative purposes to bear in mind that a certain component, *e.g.* naphthalene, may be available in more than one state or phase. For instance, in certain mixed fuels there may be a small proportion of gaseous naphthalene; in others a more considerable proportion of molten naphthalene or of naphthalene in solution, while finally naphthalene may be in the solid state as a component of a mixed solid fuel, or indeed in that state be used as a fuel without admixture. The calorific value of a substance such as naphthalene, which is available in three states, will differ according to its phase and its latent heat of vaporisation, of liquefaction, or heat of solution in a particular medium which serves as the liquid vehicle for it. The greater part of the combustibles to which technical fuel calorimetry is commonly applied are, however, complex mixtures which are used in the one state, viz. solid, liquid, or gaseous, in which they usually occur, and it is the calorific value of the fuel in that one state with which fuel calorimetry ordinarily is concerned.

When the fuel is a solid, whether a simple substance, such as carbon, sulphur, phosphorus or silicon, or a complex mixture of cellulosic or hydrocarbon material such as wood, coal, etc., its calorific value is ordinarily determined by measuring the heat evolved on the rapid or almost instantaneous combustion in oxygen of a known weight of the fuel. When the fuel is a liquid which is not very readily volatile, such as the higher alcohols, oil, tar, etc., its calorific value is determined in substantially the same way. When the fuel is a gas, however, though the same method is applicable, it is generally preferred to burn the fuel at a uniform measured rate of consumption for an extended period, and to measure the heat evolved during the time in which a known quantity of the gas is burned. If provision is made for the continuous consumption of the gas at a uniform rate, and for the calorimetric device to furnish a continuous record of the fluctuations in the evolution of heat from the combustion of the gas, it is possible to produce a calorimeter which automatically registers any variations in the calorific value of the gas over a long period. The usual methods of gas calorimetry are, therefore, distinct from the usual methods of the calorimetry of solid and liquid fuels, though there is some overlapping of the two methods in the calorimetry of the more highly volatile liquid fuels, such as motor-spirit, ethyl-alcohol, ether, etc. The continuous calorimetry of a supply of solid fuel, such as coal, furnishing automatically a register of fluctuations in the calorific value of that fuel, is a problem which awaits attack, and of which a satisfactory solution would be of the utmost value in industry. Even in regard to oil fuel, continuous calorimetry is practically unknown, though the problem does not present the same difficulties as that of solid fuel, and a satisfactory solution of it is not far to seek.

There are a number of points in fuel calorimetry to which consideration must be given in the case of almost all types of fuel calorimeters and of all classes of fuel. They may be classified broadly as follows: (*a*) Sampling the fuel; (*b*) Weighing or measuring the portion of the sample on which the determination is made; (*c*) The supply of oxygen for combustion; (*d*) Choice of medium for reception of the heat evolved; (*e*) Thermometry, or the measurement of the rise of temperature of the medium; and (*f*) Incidental heat losses and corrections.

§ (3) SAMPLING THE FUEL.—In general the sampling of fuel for determination of its calorific value follows the course adopted for sampling it for chemical analysis, but as many more samples of fuel are now taken for calorimetry than for analysis, and often by persons not trained in general methods of sampling, a few words on it may appropriately be said here, though, strictly speaking, sampling does not fall within the scope of calorimetry as such.

There are few materials more difficult to sample properly than coal and coke, owing chiefly to the unequal distribution of earthy matter, slate, etc., in the coal as won and supplied to the consumer. Where coal or coke passes through a mechanical breaker which reduces all lumps to a relatively small size, the procedure of sampling a large bulk is greatly simplified, and if ½-cwt. portions are drawn from the outlet of the breaker at intervals corresponding with the passage through it of 5 or 6 tons, and these ½-cwt. portions are subsequently mixed, further broken, quartered, the quarter broken smaller and again quartered, a finely broken small sample is ultimately obtained from which the final sample for the determination of calorific value may be ground. This procedure answers well on gas works, etc., where coal is frequently broken before carbonisation, and coke before sale for household use, but the breaking of coal lowers its value for most other uses and this aid to proper sampling is not generally available. In such cases larger portions, representative of both the large and the small coal, must be taken from each waggon-load, or 5 to 10 ton lot, and these portions mixed to form an average sample, a quarter of which must be roughly broken and dealt with as before. Many elaborate directions have been issued for the sampling of coal and coke deliveries, but a consideration of the particular circumstances will generally suggest the best method if it is borne in mind that the first bulk sample must be large and must be thoroughly mixed before the first quartering. Gain or loss of humidity in the course of sampling must also be avoided.

The sampling of liquid fuel is relatively an easy matter, though, with heavy petroleum oils and certain classes of tar and tar oils, stratification and deposition of solid matter are apt to occur in storage tanks, and warming by means of steam coils and stirring are needed before the sample is drawn from the tank. Liquid fuels frequently contain varying proportions of water, partly in solution and partly in suspension, and on this account a sample drawn from a single barrel is seldom representative of a consignment of many barrels. Barrels should be rolled about to agitate the contents thoroughly before they are sampled.

The sampling of gaseous fuel varies greatly in method according to the circumstances of its origin and the object of the determinations of calorific value. Obviously if all that is needed is an average sample of the output of a producing plant, or of the delivery from storage vessels, different methods will be followed from those adopted if samples are required with a view to ascertaining the extreme fluctuations in calorific value of the gas produced or delivered. It may be said generally, however, that fuel gas supplies vary more in calorific value than might be expected from general knowledge of the diffusion of gases, and of processes for generating fuel gas. Even town gas supplies, derived from works where facilities for mixing and storing appear ample, quite commonly show fluctuations of 2 to 3 per cent on either side of the mean calorific value. Other fuel gases, such as acetylene produced in generators for welding and cutting purposes, are apt to vary still more in calorific value, partly owing to variations in the purity of the raw material used, and partly owing to effects incidental to the rate of generation, etc. Fortunately a continuous supply of fuel gas is generally available for calorimetry, and it is a relatively easy matter with the types of gas calorimeter commonly in use to make a number of determinations, and thereby to ascertain the fluctuations in value of the gas, as well as to eliminate errors due to the taking of casual samples.

§ (4) Weighing or Measuring the Sample.—The portion of either a solid or a liquid fuel used for a calorimetric determination will usually be weighed, but a gaseous fuel is commonly measured and the calorific value per unit weight calculated, if required, from the specific gravity of the gas. The utility of determinations of calorific value of coal, coke, etc., is largely relative to that of other lots of similar fuel, and since these fuels contain more or less free water according to the extent to which they have been exposed to rain (or underground water) or stored under cover, it is essential that the determinations of calorific value should be made on a dried sample. If the water given up in drying is determined, the calorific value of the particular lot of coal or coke in the condition in which it was when sampled can be readily calculated. Unless expressly stated otherwise, it is customary to express the calorific value of a solid fuel in terms of unit weight of the dried material. Since many coals, when finely ground and dried, are extremely hygroscopic, the precautions usually taken in weighing hygroscopic substances must be observed when weighing out the portions for calorimetry. Fuel oils, as a rule, contain very little water, but when certain grades of tar, crude petroleum, etc., are being examined it is necessary to determine the water present, which, moreover, frequently varies greatly in different samples taken from the same bulk.

Gaseous fuel is usually measured saturated with water at atmospheric temperature and pressure, and the calorific value of unit volume so measured is corrected to refer to unit volume in one or other of two standard conditions, viz.: (1) at 0° C., 760 mm. and dry, which are the normal conditions for gas measurements in scientific work, or (2) at 60° Fahr., 30 inches and saturated with water vapour, which are the normal conditions for the sale of gas in this country and generally for technical and industrial measurements of gas. The volume of gas in these conditions is only one ten-thousandth less than its volume at 15° C., 760 mm. and saturated, which are the conditions in which gas is measured for commercial and technical purposes in most continental countries.

§ (5) Supply of Oxygen for Combustion.—The oxygen required for the combustion of the fuel in the calorimeter is obtained in most calorimeters for gaseous fuel through a supply of atmospheric air, but with solid and liquid fuel it is difficult to ensure complete and rapid combustion unless the oxygen is supplied in a more highly concentrated form. In the early calorimeters the oxygen was furnished by potassium nitrate or a similar salt, which was intimately mixed with the fuel, but with the advent of commercial supplies of oxygen compressed in steel cylinders other sources of concentrated oxygen were almost entirely displaced in calorimetry by the oxygen cylinder. Without the latter the Berthelot bomb type of calorimeter could not have come into general use. It is possible, however, to dispense with compressed oxygen if it is not available, by using a peroxide as a source of oxygen. Sodium peroxide is thus used in the Roland Wild fuel calorimeter, but there is in this case a liberation of heat from the reaction between the products of combustion and the reduced peroxide, for which a deduction has to be made from the total heat measured by the calorimeter. The uncertainty as to this deduction with fuels of a widely different ratio of carbon to hydrogen militates against the more general employment in industrial calorimetry of this convenient type of calorimeter.

§ (6) Medium for Reception of the evolved Heat.—The heat evolved by the combustion of the fuel is absorbed as far as possible by water in nearly all the more exact types of calorimeter. The high specific heat of water and its universal availability are the chief factors in its favour, but it has certain disadvantages which have led to the adoption in a few calorimeters of other liquids, or of air or other gas, for the absorption and measurement of the heat evolved on the combustion of the fuel. It may be said broadly, however, that the calorimeters in which water is displaced by another liquid or by a gas are of special and restricted applicability. Certain semi-automatic and recording calorimeters dispense with water, but the majority of them need calibrating against a water calorimeter owing to uncertainty as to the specific heat of the liquid or gas which is used instead of water. The water or other medium is contained in a glass or metal vessel or tube, by which and the combustion chamber or burner a portion of the heat evolved is retained. It is customary in bomb and other non-continuous calorimeters to determine experimentally the mean specific heat of these parts of the calorimeter, and to reckon their heat absorption in terms of the weight of water which would show the same rise of temperature if the same number of heat units were imparted to it. This is commonly known as the "water-equivalent" of the calorimeter, and has to be determined for each instrument. Its determination by present methods introduces an uncertainty in the results of bomb calorimetry of nearly 0·2 per cent. In calorimeters of the flow type in which a heat equilibrium becomes established, the analogous uncertainty in the results may be reduced to less than 0·1 per cent.

§ (7) THERMOMETRY.—The rise of temperature caused by the combustion of the fuel in the calorimeter is usually measured by mercury thermometers, either used directly or used as reference standards to check the records of some other thermometric device. Thus the degree of accuracy attainable in fuel calorimetry generally depends ultimately on the sensitiveness of the mercury thermometer, which when all sources of error are considered and corrections applied cannot be much higher than 0·005°, and in most cases even with high-class thermometers is not, in reality, higher than 0·01°.

The errors in thermometry in connection with bomb calorimetry have been fully discussed by G. N. Huntly in the *Journal of the Society of Chemical Industry*, 1910, xxix. 917, and in the *Analyst*, 1915, xl. 41. Briefly, the capillary tube errors may amount to, but will not exceed, one unit of the divisions if the thermometer is used without special calibration, but this may be reduced to 0·01 division or 0·001° by calibration in special cases. External and internal pressure and temporary depression of zero errors are negligible in the conditions of use of thermometers in bomb calorimetry. The other errors can best be eliminated by determining the water equivalent of the calorimeter with the same weight of water at different temperatures, and therefrom plotting the apparent water equivalent as a function of the temperature. In determining the water equivalent, a pure substance such as benzoic acid or pure cane sugar of which the calorific value is accurately known is burned in the calorimeter.

The errors in thermometry in connection with flow and other calorimeters in which the rise of temperature measured is more than 10° comprise many of the foregoing, and others, such as that for the varying length of stem exposed to the atmosphere, which may be largely eliminated by appropriate design of the calorimeter. While the exposure of the thermometer stem to the air of the room may conceivably in certain calorimeters account for an error of as much as 0·1°, it is very rare that the error is more than one-fifth of this amount, owing to the fact that the thermometer stem is usually so situated that its temperature is more nearly that of the bulb than that of the air of the room. The correction which would otherwise be appropriate for the exposed stem of the thermometer becomes largely merged in the general correction for radiation and other heat losses from the calorimeter itself.

In all calorimetry where readings are made on two thermometers it is essential that the thermometers should be re-standardised at intervals of about twelve months until it is demonstrated thereby that there is no further change of zero through lapse of time. The artificial ageing of thermometers, though of great service, is not always equally effective, and is frequently neglected even with thermometers purporting to be of high grade.

§ (8) INCIDENTAL HEAT LOSSES.—Radiation from the instrument is, with the majority of types of calorimeter, the most important of the incidental heat losses. In the bomb type of calorimeter its effect may be eliminated (*i.e.* reduced to not more than 0·05 per cent) by the application of a formula, deduced from Newton's law of cooling, by Regnault, Pfaundler, and Oussoff. The following observations are needed: T_0, the mean temperature of the calorimeter, and t_0, the final temperature during the initial period; t, the final temperature of the principal or combustion period, and T′, the mean temperature of the calorimeter during the final or cooling period. Then if n is the number of minutes in the principal period, V_0 the mean loss of heat in a minute during the initial period, V the mean loss of heat in a minute during the principal period, V′ the mean loss of heat in a minute during the final period, and S the sum of the minute readings of the thermometer during the principal period, we find for the correction $n\mathrm{V}$ the value

$$n\mathrm{V}_0+\frac{\mathrm{V}'-\mathrm{V}_0}{\mathrm{T}'-\mathrm{T}_0}\left(\frac{t_0-t}{2}+\mathrm{S}-n\mathrm{T}_0\right).$$

For let q be the rate of loss of heat per unit difference of temperature between the calorimeter and the air, T the air temperature, $t_1, t_2 \ldots t_n$ the temperatures of the calorimeter at the ends of the first, second . . . n' minutes; t_n is equal to t the final temperature. The mean temperatures during the first, second . . . nth minutes are $\frac{1}{2}(t_0+t_1)$, $\frac{1}{2}(t_1+t_2) \ldots \frac{1}{2}(t_{n-1}+t)$, and by Newton's law of cooling the loss of heat in each minute is

$$q\left\{\tfrac{1}{2}(t_0+t_1)-\bar{\mathrm{T}}\right\},\ q\left\{\tfrac{1}{2}(t_1+t_2)-\mathrm{T}\right\} \ldots,$$
$$q\left\{\tfrac{1}{2}(t_{n-1}+t)-\mathrm{T}\right\}.$$

The total loss of heat in the principal period is $n\mathrm{V}$. Thus adding these quantities

$$n\mathrm{V}=q\left\{(t_1+\ldots t)+\tfrac{1}{2}(t_0-t)-n\mathrm{T}\right\}$$
$$=q\left\{\mathrm{S}+\tfrac{1}{2}(t_0-t)-\bar{n}\mathrm{T}\right\}.$$

Also since T_0 is the average temperature during the initial and T′ during the final period,

$$\mathrm{V}_0=q\left\{\mathrm{T}_0-\mathrm{T}\right\},\ \mathrm{V}'=q\left\{\mathrm{T}'-\mathrm{T}\right\}.$$

Thus $$q=\frac{\mathrm{V}'-\mathrm{V}_0}{\mathrm{T}'-\mathrm{T}_0},$$

while $$\bar{\mathrm{T}}\doteq\mathrm{T}_0-\frac{\mathrm{V}_0}{q}.$$

Substituting these values we have

$$n\mathrm{V}=n\mathrm{V}_0+\frac{\mathrm{V}'-\mathrm{V}_0}{\mathrm{T}'-\mathrm{T}_0}\left\{\mathrm{S}+\tfrac{1}{2}(t_0-t)-n\mathrm{T}_0\right\}.$$

Huntly has pointed out (*loc. cit.*) that for a given instrument, a given class of fuel, and a constant rate of stirring, this formula may be simplified, because these are the three conditions which determine the shape of the time-temperature curve, and if they are constant $S/T' - T_0$ becomes a constant which can be ascertained once for all.

The foregoing formula eliminates also the losses due to evaporation, and the gain from the development of heat due to the friction of the stirrer. The value of the radiation correction can be reduced to an amount which may be neglected by using in the calorimeter water a few degrees lower in temperature than the surrounding air. This method of equalising the heat transfers to and from the instrument is frequently applied in continuous water-flow calorimeters in order to render unnecessary special corrections for radiation, etc.

The heat of fusion of the ignition wire, and the heat of combustion of any cotton or collodion attached to it, are allowed for, if conditions are kept constant, in the determination of the water equivalent of the bomb calorimeter. A certain amount of nitric acid is produced from the atmospheric nitrogen initially in the bomb, or from nitrogen (if any) in the fuel, and this is fairly constant for a bomb of given capacity used on similar fuels with the same oxygen pressure, etc. It will be responsible for a liberation of 3 to 12 calories, according to circumstances, and is deducted from the observed calorific value in all the more accurate calorimetric work. A little water is put in the bomb before it is closed, and this absorbs the nitric acid formed, which is subsequently determined by titration with standard alkali, and the necessary deduction calculated accordingly. If the fuel contains sulphur, the latter is oxidised in the bomb wholly to the trioxide, which is absorbed by the water therein. It is customary to assume that in the combustion of fuel in grates and furnaces, sulphur is oxidised to sulphur dioxide only, and a deduction is accordingly made from the observed calorific value in respect of the additional heat developed in the bomb from the sulphur. The proportion of sulphur present is ascertained by a determination of the sulphuric acid in the water contained in the bomb. The assumption that sulphur in fuel, *e.g.* coal, when the latter is consumed in a furnace, is oxidised to sulphur dioxide only is now known to be incorrect, as sulphur trioxide is always formed also—more or less according to the furnace conditions, etc. It is, on the whole, better to omit the sulphur correction, as, at least in some uses of a fuel, it is possible that very nearly the whole of the sulphur is burnt to the trioxide.

§ (9) DETAILS OF MANIPULATION. — The principal corrections and sources of error in fuel calorimetry, with the Berthelot bomb in particular, have been enumerated, and it is only necessary in regard to determinations of calorific value with the bomb to refer to one or two points in its manipulation with different types of fuels. It is usually filled with oxygen to a pressure of 25 atmospheres, but higher pressures are used with a view to securing complete combustion of coke, anthracite, and other not readily combustible fuels. The platinum or gold lining of the Berthelot bomb is replaced in the Mahler and other industrial types by an enamel lining, which does not affect the accuracy of the results obtained so long as it remains sound, but it needs renewal from time to time. The capacity of the bombs for industrial use is generally appreciably greater than that of the platinum-lined Berthelot bomb, reaching as much as 750 c.c. as compared with 250 to 600 c.c. for the latter.

§ (10) HIGHLY VOLATILE LIQUIDS.—The determination in the bomb of the calorific value of highly volatile liquid fuels, such as petrol, benzol, and, in particular, the motor-spirits produced by the cracking of heavy mineral oil, etc., is a problem presenting special difficulties. The partial volatilisation of such liquids in the bomb leads to error in the results, due partly to the heat of evaporation of an uncertain proportion of the substance and partly to incomplete combustion of the vapour. Berthelot's method of weighing out the volatile liquid in a small glass bulb which is sealed, or in a tiny thin-glass flask with ground-in stopper, and placing round this in the bomb a known weight of camphor, naphthalene, or other combustible of known calorific value, answers well. The ignition and combustion of the camphor expels the liquid, which is forthwith completely burnt. A less satisfactory alternative, also suggested by Berthelot, is the use of a capsule of collodion, into which the ignition wire passes, for containing the volatile liquid, which is then burned without the addition of camphor, etc. Some workers substitute celluloid for collodion.

The calorific value of highly volatile liquids may also be determined, if their latent heat of evaporation has been ascertained, by combustion of the vapour in a special lamp or burner in one of the forms of flow calorimeter used for gaseous fuels, and this is the best method when large numbers of determinations have to be made on different samples of volatile liquid, *e.g.* motor-spirit.

§ (11) WATER-FLOW GAS CALORIMETERS.—The most widely known of the early forms of water-flow gas calorimeter is that of Junkers, which, however, resembles in principle an instrument designed earlier by Hartley. These calorimeters and subsequent modifications of them consist of an assembly of parallel vertical condenser tubes, down the interior of which pass the products of combustion from a gas burner, while water flows upwards round their exterior. The gas burner is in the centre of the vertical cylindrical space formed by the annular water chamber of the condenser, which water chamber extends inwards at the top and forms a roof over the burner chamber. The gas is burned at a uniform rate, measured by an experimental meter of the wet type, and the rate of flow of the water, which forms a counter current to the gaseous products of combustion, is also uniform and is measured. The difference between the inlet and outlet temperatures of the water flowing through the instrument affords a measure of the calorific value of the gas, since the quantities of water and of gas supplied to it in any convenient interval of time are known. Another form of water-flow calorimeter, designed by C. V. Boys, has the water flowing through a continuous coil of tube, the heat-absorbing surface of which is increased by coils of Clarkson wire or discs joined to it by tin or solder, while the products of combustion pass in a counter current through the annular spaces in which the tube is coiled. The Boys calorimeter is the standard instrument prescribed by the gas referees for ascertaining the calorific value of town gas supplies in the United Kingdom.

Various modifications of the Hartley, Junkers, and Boys calorimeters have been introduced, especially in the United States of America, mostly with the object of eliminating possible sources of small errors, for which corrections can be readily introduced wherever the degree of accuracy of the determination in other respects warrants it. Many of these errors are wiped out automatically if the testings are made in a room of uniform temperature, and with the water supply to the instrument at a very slightly lower temperature than the air of the room. Since there is a contraction of volume on the combustion of most gases, an error is introduced if both the gas and the air supplied for its combustion are fully saturated with water vapour, due to the heat of vaporisation of the water vapour condensed from the lost volume being added to the heat directly resulting from the combustion of the gas. Atmospheric conditions of temperature and humidity in the United Kingdom are, as a rule, such that the results obtained with a gas calorimeter supplied with air which has not been artificially humidified need very little, if any, correction on account of the humidity of the air and the contraction on combustion.

Errors of thermometry, as already mentioned, are common to nearly all types of fuel calorimeters, and can be eliminated in gas calorimetry by the usual precautions and corrections. Errors due to radiation and convection currents vary with the form of the gas calorimeter, and if the calorimeter room is of fairly uniform temperature, can be eliminated by the use of a correction factor, which varies slightly with the room and water temperatures. It can be determined experimentally for a particular set of working conditions.

The errors of the calorimetry proper are commonly of smaller dimensions than the error of measurement of the gas burnt in the calorimeter. This measurement has ordinarily a minimum error of one-third of 1 per cent either way, but it is quite easy to reduce this error to about ±0·2 per cent, which may be taken as the possible error in the calorimetry proper when all ordinary precautions are observed. In special conditions and with exceptional precautions, the aggregate error in gas calorimetry, including the error of gas measurement, need not exceed ±0·2 per cent.

§ (12) Recording Calorimeters.—Calorimeters for producing a continuous record of the calorific value of gas have been made for some years past by Junkers and others, and in the United Kingdom the gas referees are now required by the Gas Regulation Act to prescribe a recording calorimeter for testing continuously the calorific value of the gas supplied to consumers in all the larger towns. As a fact, however, up to the time of writing, no recording gas calorimeter has been approved as of a sufficiently high degree of accuracy and trustworthiness for use in such official testings. The design of an instrument which will meet these conditions is now earnestly engaging the attention of physicists, and several patterns are already on trial. A recording gas calorimeter should embody means for: (1) furnishing a flow of gas to a burner at a known and constant rate, in terms of unit volume at normal temperature, pressure, and humidity, irrespective of fluctuations in the temperature, absolute pressure, and density of the gas; (2) furnishing a flow of water through the calorimeter at a known constant rate, irrespective of fluctuations in its temperature (and of the consequent fluctuations in its viscosity), salinity or hardness, and content of dissolved gases, and (3) measuring the difference in temperature of the water entering and leaving the calorimeter, and recording on a sufficiently open chart the product of it and the factor which for the given rates of flow of water and gas gives the calorific value of unit volume of the gas.

There are minor points, such as the effect of varying humidity of the air supplied for combustion of the gas, which have to be borne in mind in the construction of an ideal recording gas calorimeter. It is a comparatively easy matter to design an instrument which will afford a fairly precise record for, say, 24 hours, at the beginning and end of which period its indications are checked by personal observations. The problem now is to design a recording calorimeter which will automatically make all necessary adjustments and give a trustworthy record throughout a longer period, say 8 or 14 days.

§ (13) CALORIFIC VALUES OF CERTAIN FUELS.—The accompanying table gives typical or average results for the calorific value of samples of various industrial fuels, with the approximate error of the determination by the best methods and operators in the case of a particular example. The margin of difference between the calorific values of different samples of the same class of fuel is, of course, very much larger than this error of determination.

W. J. A. B.

TABLE OF CALORIFIC VALUES

	Average Calories per Kilo.	Error of Determination. Calories.
Anthracite (dry)	8,300	±15
Admiralty Smokeless Steam Coal (dry)	8,250	±15
Bituminous Steam Coal (dry)	8,000	±15
Bituminous Gas Coal (dry) .	7,800	±15
Scotch Splint Coal for blast furnaces (dry) . . .	7,400	±15
Durham Oven Coke (dry) .	7,600	±15
Gas Coke (dry)	7,400	±15
Peat (air-dried, 20 per cent of water)	4,200	±10
Heavy Petroleum, for boiler-firing and Diesels . . .	10,700	±15
Kerosene	11,000	±15
Petrol	11,100	±20
Benzol, motor-spirit grade .	10,000	±20
Alcohol (Methylated Spirit)	6,400	±15

Gases measured at 60° F., 30 in. and saturated.

	Calories per Cubic Foot.	
Coal Gas (South Metropolitan Gas Co., 1921) .	140	±0·30
Town Gas in Great Britain (average), 1921 . . .	120	±0·25
Water Gas, uncarburetted .	75	±0·20
Mond Gas (South Staffordshire)	38	±0·10
Suction Producer Gas, from anthracite	35	±0·10
Producer Gas, from gas-coke	37	±0·10
Oil-Gas, after compression .	275	±0·50

FUELS FOR PETROL ENGINES. See "Petrol Engine, The Water-cooled," § (5).

FUNCTION, UNKNOWN, EXPERIMENTAL DETERMINATION OF, when the non-dimensional variables connected by the unknown function have been discovered by the method of dimensions. See "Dynamical Similarity, The Principles of," § (31).

FUNCTION, UNKNOWN, MORE ACCURATE DETERMINATION OF, by dynamical similarity, in the case of heat loss from long circular wires past which air is streaming. See "Dynamical Similarity, The Principles of," § (33).

FUSION, LATENT HEAT OF: a term used to denote the quantity of heat required to convert unit mass of a solid at the melting-point into liquid at the same temperature. See "Latent Heat," II.

— G —

GAEDE ROTARY PUMP. See "Air-pumps," § (17).

GAMMA, THE RATIO OF SPECIFIC HEATS OF GASES, at constant pressure and at constant volume. See "Engines, Thermodynamics of Internal Combustion," § (19) (ii.); "Thermodynamics," § (15).

GAS CALORIMETER

§ (1) INTRODUCTORY.—To measure the heat of combustion of gas some form of flow calorimeter is usually employed. The gas is burned at a measured rate in a special burner, which is surrounded by a coil of metal tubing through which there is a steady and continuous flow of water. The apparatus is so arranged that the products of combustion as they rise from the burner give up practically the whole of their heat to the water. An increase in the temperature of this water is thus produced, and by measuring this increase and the total flow when the temperatures have become steady, the heat produced can be calculated. In some forms of apparatus air is used in place of water.

It is necessary to wait until a steady state has been reached, for when the apparatus is first started much of the heat is used in raising the temperature of the calorimeter itself. The fundamental quantities which it is necessary to know are the rate of flow of the gas, the quantity of water passing in a given time, and the difference in temperature between the inflowing and the outflowing water. Corrections are required, as described in the article on "Fuel Calorimetry," for the loss of heat by radiation from the calorimeter and for the heat given out by the steam formed by the combustion in condensing to water. The escaping products of combustion, moreover, may be either warmer or cooler than the gas and air entering the calorimeter; they may occupy a smaller volume and are saturated with water vapour. They may therefore contain a small quantity of heat due to combustion, which must be added to that found from the flow of water, or if the temperature of the water is low they may give up heat not due to the combustion, and this must be subtracted from the calorific value found from the flow and rise of temperature of the water.

§ (2) THE STANDARD CALORIMETER OF THE GAS REFEREES.—In *Fig.* 1 is shown the Boys Gas Calorimeter, the standard instrument authorised for use by the Gas Referees. The calorimeter is composed of three parts: (1) The base A, which carries a pair of steatite burners B and a regulating tap. The upper surface of the base is covered by a plate of metal, perforated at the centre and held in place by three centring blocks C. Below the plate is a recess the bottom of which is covered by felt, and six holes Z drilled in the base give communication with the outer air. A tube X of fibre conducts the air to the base of the burners.

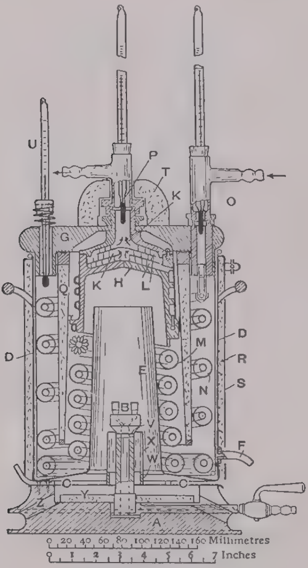

FIG. 1.

(2) An annular copper vessel D, jacketed by felt R and protected by an outer covering S, rests on the blocks C and is centred by them; the interior surface of this vessel forms a chimney E; it is also fitted with a condensed water drip F. Air enters between the blocks C.

(3) The top of this vessel D is closed by the lid G, to which is attached the water circulating system of the calorimeter. The water enters at O through an inlet box which contains the thermometer by which its temperature is measured.

In order to maintain a uniform flow the water-supply comes from an overflow funnel fixed to the wall at a height of about one metre above the base of the calorimeter. A tube leads from this funnel to the inlet pipe of the calorimeter, and in this tube is a diaphragm with a hole about 2·3 mm. in diameter. A second tube is connected to the water-supply through a regulating tap, while a third tube, the open end of which is at a height of about two inches above the bottom of the funnel, is connected to the waste sink. The tap is opened until there is a slight trickle of water through this third pipe; thus a constant head is preserved.

The water circulates through a helix of copper tubing to the bottom of the calorimeter and then up again to the top, where it enters a temperature equalising box H; hence it passes through baffles to the outflow box P; this contains the second thermometer, by which the temperature of the outflowing water is measured before it finally escapes as indicated in the figure. Between the outer and inner coils M and N is placed a brattice or annular partition Q containing cork dust so as to act as a heat insulator; the top of this is closed in an air-tight manner by a wooden ring which prevents the access of moisture to the cork.

The products of combustion rise in the chimney E, then pass down over the coils M between the chimney and the inner surface of the brattice, rising again in the annular space between its outer surface and the inside of the vessel D, and finally escape through a number of holes in the lid, their temperature being measured by a thermometer fitted, as shown at G, for this purpose. Their heat is given up to the water circulating in the coils, and its amount is measured by the rise in temperature of this water as indicated by the difference between the thermometers O and P.

The steam formed by the combustion is condensed by its passage over the coils, and the water of condensation escapes by the outlet F and is measured.

§ (3) METHOD OF EXPERIMENT.—In carrying out an experiment the gas is fed to the burner through a meter and a pressure regulator. The tap of the calorimeter is regulated so that the meter hand makes one revolution in from 60 to 75 seconds. The water is allowed to flow through the calorimeter and the gas lighted; the whole is left for not less than 30 minutes for the temperatures to become steady, the thermometers being read from time to time. The escaping water passes into a change-over funnel, the position of which can be rapidly altered so as to direct the flow either into a measuring flask suitably placed to receive it, or to a waste water sink; during the preliminary interval the water runs to waste. As the meter hand approaches division 75, after the temperatures have become steady, the observer reads the inlet temperature; as the hand passes the 100 mark he shifts the change-over funnel so as to direct the flow into the measuring flask, and starts a stop-watch. As the hand passes division 25 he reads the outlet temperature. At each quarter turn afterwards the outlet temperature is read until fifteen readings have been taken; the hand is then at 75; as it passes division 100 for the fifth time—*i.e.* after four complete turns—the change-over funnel is shifted and the stop-watch stopped. At each turn of the meter except the last, when the hand is between 75 and 100, the temperature of the inlet water is read; thus there are four readings of this temperature. Immediately after the flow has been directed to the waste, the temperatures of the gas and air supply and of the effluent gases are read; the barometer reading, and also the readings of the wet and dry bulb thermometers, used to determine the hygrometric state of the air, are also taken, and the amount of water of condensation which has escaped from the outflow F is noted.

The difference between the mean of the fifteen outlet temperatures and that of the four inlet temperatures gives the rise of temperature of the water; the quantity which has passed is known from the indications of the measuring flask. The product of these two quantities measured in proper units gives the total heat produced—this, as already indicated, needs certain corrections; if the thermometers are divided in centigrade degrees and the measuring vessel in cubic centimetres, the result is in calories. The meters employed under the Gas Referees' regulations allow one-twelfth of a cubic foot of gas measured at a temperature of 60° F. and a pressure of 30 inches of mercury to pass in each revolution, and when in use the time of a revolution should be about one minute. If the pressure and temperature at the time of observation have not these standard values, the meter readings require correction. This is facilitated by a table given in the Notification of the Gas Referees,[1] to which the reader is referred for further particulars, including details as to the application of the various corrections.

The measuring vessel is a flask with a comparatively narrow neck graduated from 1600 to 2400 c.c. In the ordinary use of the calorimeter the amount of water passing in four minutes should be adjusted so as to be sufficient to fill it above the lowest graduation, but insufficient in five minutes to come above the highest division. This flask must be certified by the Referees; the regulations also require that the meter should be checked from time to time, and instructions are given as to the use of the wet and dry bulb thermometers employed to measure the hygrometric state of the air.

§ (4) Recording Calorimeters.—As explained in the article on "Fuel Calorimetry," the Gas Referees are required to prescribe a recording calorimeter for testing continuously the calorific value of gas supplied in the larger towns, and various investigators are at work on the problem. At the present time no instrument has been prescribed. Among the instruments already produced are the Simmance and the Thomas recorder.

§ (5) Simmance's Total Heat Recording Calorimeter. — This instrument consists, as any such apparatus must, of two parts, the calorimeter proper and the recording device; the calorimeter is a flow calorimeter, modified, as will be described shortly; the record is obtained from a curve which gives continuously the temperature difference between the inflowing and outflowing water. If the gas be supplied at a constant known rate corrected for pressure, temperature, and humidity, and the water flow also at a known uniform rate, the temperature difference measures the heat produced by the combustion.

The gas is supplied through a pressure regulator to the meter; it passes through a specially constructed governor placed some three feet above the calorimeter, and then through a vertical iron pipe to the burner, whence it issues at the pressure of the atmosphere in the combustion space. Any change in the density of the gas or in the atmospheric pressure causes a change in the pressure under the bell of the governor, and the movement which occurs operates a valve, thus checking or increasing the flow of gas to the burner. It is claimed that the rate of efflux from the burner orifice is thus regulated and remains constant whatever the density of the gas may be. In a report on the instrument recently issued by the Department of Scientific and Industrial Research [2] it is stated (p. 22) "that the claim that the gravity meter automatically applies a correction for the varying pressure of the gas is substantially true."

Arrangements are made whereby the air supplied for combustion is maintained at the temperature of the cold water and is saturated at that temperature, while the products of combustion escape at a temperature about 1°·5 above this. The water enters the calorimeter tubes under a constant head, and the flow is controlled by a needle valve enabling a fine adjustment to be made. In this manner a uniform flow both of gas and water is secured.

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[1] Metropolis Gas, *Notification of the Gas Referees for the year 1918*. H.M. Stationery Office.

[2] Fuel Research Board, Technical Paper No. 2, *Report on the Simmance Total Heat Recording Calorimeter*, by Thos. Gray and Alfred Blackie. H.M. Stationery Office.

The temperature difference is measured by a differential air thermometer; the inlet and outlet boxes each respectively contain, in addition to the ordinary thermometers, a thin-walled copper box; each of these is connected by capillary copper tubing to one of a pair of aneroid bellows; they thus form the bulbs of a differential air thermometer. The two bellows are rigidly coupled by a connecting rod; thus the position of the rod depends on the difference of pressure of the air in the two bellows, and the rod moves as this pressure difference varies.

Changes in the pressure difference are indicated by a suitable mechanism attached to the rod and recorded by a pen on a moving chart. Before starting the calorimeter, when the whole apparatus is at one temperature, a connection is opened between the bellows so that the pressures in the two are equal; the connection is then closed and the apparatus started; as one of the two copper boxes is heated the pressure of the enclosed air rises proportionately to the rise of temperature, and hence the mechanism indicates the temperature difference between the outflowing and inflowing streams.

If the chart moves uniformly and the water flow is also maintained at a uniform rate, the ordinates of the curve traced measure the total heat produced.

The calorimeter is put into adjustment by determining the calorific value of the fuel in the ordinary way by the aid of the thermometers provided with the instrument; the pen is then set at this value corrected to normal pressure and temperature, and the instrument gives a continuous record.

The report already quoted concludes with the statement that "the variation of the recorder seldom exceeded 2 per cent, notwithstanding the fact that the temperature of the air and water were not artificially controlled, and there is a prospect that the limits of error may be substantially reduced."

§ (6) THE THOMAS RECORDING CALORIMETER.—In this instrument the cooling medium is air, not water; the gas is burned in a closed space; the products of combustion pass downwards through a tube surrounding the burner. The tube is surrounded by a second tube through which a stream of cooling air passes upwards; this tube is open at the top, but is covered by a cap which forces the cooling air to flow back outside the tube, and finally to escape into the atmosphere. The difference of temperature between the incoming and outgoing air is recorded by the instrument, and the rate of flow being known, and also the specific heat of air, the total heat produced by the combustion is obtained.

The temperature difference is measured by two resistance thermometers connected to a special form of Wheatstone's bridge in such a manner that the difference between the two resistances is recorded; the galvanometer of the bridge acts as relay actuating the recording mechanism. The gas to be tested, the air required for combustion, and the cooling air are supplied through three wet meters, which are connected together in such a way that the quantities supplied bear a constant ratio to each other. These meters are partially submerged in water in a large tank, and the air supply is drawn from above the water in the tank; it is thus saturated at the pressure and temperature of the tank. The gas supply passes through a governor which is also submerged in the tank; it is subject, therefore, to the same changes of pressure and temperature as the cooling air.

Since the quantities of gas burned and of air employed to cool the products of combustion to their initial temperature bear a constant ratio to each other, the rise of temperature is a measure of the total heat produced independently of the rate of flow. The gas and the air required for its combustion are supplied in proper quantities by the respective meters and mix in a mixing chamber before they reach the burner.

The apparatus is set by burning gas of which the calorific value has been determined independently; it will then record variations in the calorific power.

GAS-CONSTANT. The ratio of the product of the pressure and volume of any gas to its absolute temperature which is a constant quantity if the gas laws are assumed to hold strictly. Its value if *p* is measured in atmospheres and *v* in cubic centimetres is 82·04. See "Thermodynamics," § (6).

GAS-DISPLACEMENT (HUMPHREY) PUMP. See "Hydraulics," § (42).

GAS ENGINES:

Relation of weight to power in. See "Engines, Internal Combustion," § (7).

Types of. See *ibid.* § (4).

GAS LAWS, DEVIATIONS FROM THE, various methods of expressing. See "Thermal Expansion," § (25).

GAS-SCALE CORRECTIONS: at temperatures of −100°, 50°, 400°, 1000°, comparison of, in thousandths of a degree, tabulated. See "Temperature, Realisation of Absolute Scale of," § (20), Tables 5, 6, 7, 8.

Gas Scale of Temperature:

Correction to. See "Thermometry," § (5).

Measure of Temperature by. See "Thermodynamics," § (4); "Engines, Thermodynamics of Internal Combustion," § (8).

Gas Thermometer, the Actual. See "Temperature, Realisation of Absolute Scale of," § (8).

Gases:

Kinetic Theory of. See "Thermodynamics," § (66). For Expansion of Gases see also § (4) of same article.

Methods of measuring Thermal Conductivity of. Cooling Thermometer Method—Hot Wire Method—Film Method. See "Heat, Conduction of," § (8).

Relation of Thermal Conductivity, Viscosity, and Specific Heat. See *ibid.* § (10) (iii.).

Thermal Conductivity of Mixture. See *ibid.* § (10) (iii.).

Values of Thermal Conductivity of. See *ibid.* Table V.

Gases, Specific Heats of:

By Electrical Methods. See "Calorimetry, Electrical Methods of," § (13).

Determined by the "Method of Mixtures." See "Calorimetry, Method of Mixtures," § (14).

At High Temperatures. See *ibid.* § (16).

Variation of, with Temperature and Pressure (over moderate ranges). See *ibid.* § (15).

GASES, SPECIFIC HEATS OF, AT HIGH TEMPERATURES

§ (1) Introduction.—An increase in the internal energy of a gas may be accompanied not only by an increase in the linear and rotational kinetic energy of the molecules as a whole, but also (except for monatomic gases) by an increase in the intra-molecular energy of vibration of the atoms. This energy taken up in atomic vibration might be expected to increase with the temperature, thus leading to a greater increase of internal energy per degree rise of temperature, or in other words a higher molecular thermal capacity or volumetric heat, at constant volume.

All gases which are not monatomic do in fact show a specific heat increasing with the temperature. The monatomic gases should, according to the kinetic theory of gases, all have a volumetric heat which is independent of the temperature and equal to $\frac{3}{2}$R, where R is the absolute gas constant $=1{\cdot}987$ cal. per gram mol. Moreover γ, the ratio of the volumetric heats at constant pressure and volume, should be 1·66. Both these results have been verified experimentally. Kundt and Warburg found γ for mercury vapour 1·67 and Ramsay found it for argon 1·659 and for helium 1·652. Pier has measured the volumetric heat of argon up to 2000° C. by explosion experiments with oxygen and hydrogen, and up to this temperature could find no sensible variation from the theoretical value 2·98 cal. per gram molecule.

In the case of a perfect gas there can be no variation of volumetric heat with the pressure, since

$$\left(\frac{\partial C_v}{\partial v}\right)_T = AT\left(\frac{\partial^2 p}{\partial T^2}\right)_v \quad \text{and} \quad \left(\frac{\partial C_P}{\partial p}\right)_T = -AT\left(\frac{\partial^2 v}{\partial T^2}\right)_p,$$

and the right-hand side of each equation is zero if pv and T occur in the characteristic equation in the first degree only. In the case of air Witkowski found that Cv and Cp increased with the pressure when the temperature was constant, but the variation only becomes large at very low temperatures, and above 0° C. the volumetric heats are nearly independent of the pressure. For the three diatomic gases N_2, O_2, and CO, which are of importance in connection with internal combustion engines, it may safely be assumed that C_p and C_v are independent of the pressure for the range of temperature 100° C.–3000° C.

It has been suggested [1] that the volumetric heats of CO_2 and H_2O may be dependent to an appreciable extent on pressure as well as on temperature and that they will diminish for any given temperature the greater the density. This is the reverse of what has been observed at low temperatures, and, as shown below, it seems unlikely that, with CO_2 at any rate, it is so to any sensible extent for the temperatures of 1000° C. and upwards with which one is chiefly concerned. In the case of superheated steam, for which the critical temperature is 365° C., the experiments of Knoblauch and Mollier have shown conclusively that the volumetric heats increase considerably with the pressure for temperatures up to 500° C. Probably there is some variation with pressure above this temperature, but the amount is likely to be too small to be of any importance in internal combustion engines: according to Knoblauch and Jacob the variation of Cp between 2 and 8 atmospheres at 500° C. is only about 0·6 per cent, and this will probably diminish rapidly as the temperature gets well above the critical temperature. Since the critical temperature of CO_2 is some 330° C. lower than water, it seems very improbable that its volumetric heats vary appreciably with the pressure at gas-engine temperatures.

§ (2) Methods of Measurement.—Attempts to measure the volumetric heats of air, CO_2, and H_2O at high temperatures have been of three types.

(i.) Constant pressure experiments, in which the gas is heated externally, usually at atmospheric pressure.

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[1] W. T. David, *Phil. Mag.* xxxix. 551.

(ii.) Experiments in which the change of mean temperature of a mass of gas is measured while a known amount of work is done on or by the gas. The change of temperature is calculated from the change of pressure, as measured on an indicator diagram.

(iii.) Explosion experiments, the gases being kept at constant volume.

The principal measurements in the constant pressure class (i.) have been made by Regnault,[1] Wiedeman,[2] Holborn and Austen,[3] Holborn and Henning,[4] and Swann[5]; those of class (ii.) by Dugald Clerk[6]; and those of class (iii.) by Mallard and Le Chatelier,[7] Langen,[8] and more recently by Pier[9] and Bjerrum.[10]

In April 1908 the late Professor B. Hopkinson gave,[11] in his paper on "The Thermal Efficiency of Gas Engines," a curve of internal energy and temperature for the working fluid in his engine compiled from the best data at that time available. In September of the same year the Committee appointed by the British Association to investigate "Gaseous Explosions, with Special Reference to Temperature," made its first report. In this a valuable summary of the position of knowledge at that date as regards specific heats of gases at high temperature is given, together with a careful criticism of the probable accuracy of the various methods of experiment.

(i.) *Constant Pressure Experiments.* — The highest temperature reached in experiments of the constant pressure type is 1400° C., by Holborn and Henning,[1] and it is doubtful whether anything higher could usefully be attempted owing to the very great technical difficulty of carrying out constant pressure experiments at these high temperatures.[12] Since 1908 the results of Swann's experiments on air and CO_2 have been published, and although they were only made at 20° and 100° C. they are important as giving a reliable datum-line figure for the volumetric heats at ordinary temperatures. The Gaseous Explosions Committee accepted Swann's values as correct to 1 per cent, as follows:

	At 20° C.	At 100° C.	
Air		5·0	Cal. per gr. mol.
Carbon dioxide .	6·93	7·76	,, ,,

(ii.) *Clerk's Experiments.*[13]—The experiments of Clerk (class ii.) cover about the same range of temperature as those of Holborn and Henning and gave results about 10 per cent higher for the mean volumetric heat. The accuracy of results from these experiments depends very largely on a correct allocation of heat loss to the cylinder walls between the compression and expansion strokes. Clerk first assumed the loss during the two strokes to be the same except for the difference of mean temperature of the gas, but this is now known to be very far from true, and the difference accounts in part for the high values obtained for the volumetric heat. In order to obtain reliable results from experiments of this type it is necessary to know more of the condition of the surface layers of gas near the cylinder walls during compression and expansion.

(iii.) *Explosion Experiments.*—The majority of attempts to measure volumetric heats at really high temperatures have been by the explosion method, in which a known mixture of gases enclosed in a vessel of constant volume is heated by internal combustion. The temperature after explosion is inferred from the rise of pressure, and from this, and the known heat of combustion of the constituent gases, a value is obtained of the mean volumetric heat of the vessel contents between the initial and final temperatures.

In interpreting the results of experiments of this kind there are several points of importance to consider:

(*a*) The correctness of the volumetric heats by this method depends entirely on an accurate allowance being made for any loss of energy by conduction of radiation up to the point of maximum pressure.

(*b*) The maximum temperature inferred from the maximum pressure is a mean temperature for the volume of gases. The actual temperature range within the gas is very wide: Hopkinson[14] has estimated as much as 1900°–1100° C. when the mean from pressure observations was 1600° C. If the specific heat were the same at all temperatures there would be no change of pressure during an equalisation of temperature, and Mr. S. Lees[15] of Cambridge University has recently shown that, taking the estimated conditions in Hopkinson's experiment, the error from this cause is probably well under 1 per cent and therefore less than experimental errors of measurement.

(*c*) The mean volumetric heats are calculated on the assumption that all the available heat of combustion has been generated in the gas at the moment of maximum pressure. That is to say that all the carbon and hydrogen are then present as CO_2 and H_2O. If any of the gases are still uncombined, or in other words if any of the CO_2 and H_2O are dissociated at the highest temperature reached, and are present as $CO+O$ and $2H_2+O_2$, the effect will be to lower the maximum temperature and give a higher value to the calculated volumetric heat. On this account many of the results from explosion experiments have been called "apparent volumetric heats"; meaning that any effect of dissociation is included in them so that the true volumetric heat will be lower than the "apparent" value.

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[1] *Mémoires de l'Académie*, Jan. 26, 1862.

[2] *Ann. der Physik.*, 1876, clvii.

[3] *Wiss. Abhandl. der Reichsanstalt*, 1905, iv.

[4] *Ann. der Physik*, 1907, xxiii. 899.

[5] *Proc. Roy. Soc.* A, 1909, lxxxii.

[6] *Ibid.* A., lxxvii.

[7] *Ann. des Mines*, 1883.

[8] *Zeit. des Ver. Deutschen Ing.*, 1903, xlvii.

[9] *Zeit. für Elektrochem.*, 1909, xv. 536; 1910, xvi. 897.

[10] *Zeit. für phys. Chem.*, 1912, lxxix.; and *Zeit. für Elektrochem.*, 1911, xvii. 731; 1912, xviii. 101.

[11] *Proc. Inst. Mech. Eng.*, 1908.

[12] For details see "Calorimetry," Method of Mixtures, § (16), Electrical Methods, § (13).

[13] See also "Engines, Internal Combustion," § (74).

[14] *Proc. Roy. Soc.* A, lxxvii. 390.

[15] *Proc. Camb. Phil. Soc.* xx. pt. iii.

It is important to notice that this use of "apparent" volumetric heats is highly unsatisfactory for two reasons: the amount of dissociation and, therefore, the apparent volumetric heat depends not only on temperature but on pressure, and will therefore vary according to the starting pressure of the experiment; and, secondly, the apparent volumetric heat of products of combustion containing both CO_2 and H_2O cannot properly be calculated from the apparent volumetric heats of CO_2 and H_2O found separately. CO_2 and H_2O have a dissociation product, O_2, in common, and the presence of one in a dissociated state will depress the amount of dissociation of the other, and *vice versa*. For this reason the only satisfactory method is that used in some of the more recent experiments, of which results are given below, in which an excess of one of the combining elements is maintained during combustion sufficient to depress the dissociation to a negligible amount. In this way true volumetric heats are obtained.

§ (3) **Results of Experiments.**—Hopkinson has given, in his paper,[1] figures for the mean volumetric heats of air, CO_2, and H_2O up to 800°, 1400°, and 1900° C., based on the work of Holborn and Henning, Holborn and Austen, Langen, and Clerk. These figures Hopkinson considered to give the best compromise between the rather discordant results at high temperatures of Holborn and Henning, and Langen.

Hopkinson's figures are as follows:

Mean Volumetric Heat in Calories per Gram Molecule for Range 100° C. to t° C.

t° C.	800.	1400.	1900.
CO_2 . .	8·9	10·45	11·65
H_2O . .	6·56	7·8	10·0
Air . .	5·03	5·55	5·95

The energy curves obtained from them are shown dotted in *Fig.* 1.

Since the above figures were given by Hopkinson, a number of values for the volumetric heats of N_2, CO_2, and H_2O have been published by Bjerrum, based upon his own and Pier's results of explosion experiments with hydrogen, carbon monoxide, and acetylene. These carry the observed volumetric heat values up to 2400° C., 2700° C., and 3000° C. for N_2, CO_2, and H_2O respectively.

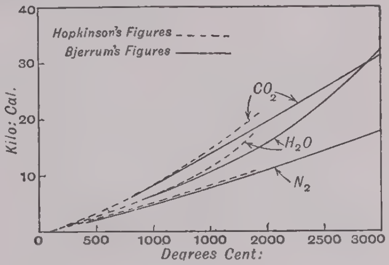

FIG. 1.

The figures given by Bjerrum are as follows:

For Nitrogen [2]

t° C.	Mean Values 0° C.–t° C.	Volum. Heat. Cal. Gram Mol.
200	Holborn and Henning	4·73
630		4·91
1000		5·25
1347		5·31
	Mean Values 18° C.–t° C.	
1519	Pier's experiments recalculated by Bjerrum	5·43
1783		5·58
1951		5·79
2182		5·87
2367		5·93

For Water-vapour [2]

t° C.	Mean Values 110° C.–t° C.	Volum. Heat. Cal. Gram Mol.
620	Holborn and Henning	6·51
1000		6·95
1327		7·40
	Mean Values 18° C.–t° C.	
1811	Pier's experiments recalculated by Bjerrum	7·92
2110		8·54
2377		9·37
2663	Bjerrum	10·00
2908		10·5
3064		10·9

For Carbon Dioxide [3]

t° C.	Mean Values 0° C.–t° C.	Volum. Heat. Cal. Gram Mol.
200	Holborn and Henning	7·48
630		8·6
1000		9·33
1364		9·84
1611	Pier	9·98
1839		10·28
2110		10·47
	Mean Values 18° C.–t° C.	
2714	Bjerrum [4]	10·9

There is no doubt, in the light of Swann's experiments on air, that the values given by Holborn and Henning for nitrogen up to

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[1] *Loc. cit.*

[2] *Zeitschrift für Elektrochemie*, 1912.

[3] *Ibid.*, 1911.

[4] *Zeitschrift für Physik. Chemie*, 1912.

600° C. are too low. Bjerrum recommends the formula $C_v = 4{\cdot}9 + {\cdot}00045\ t$° C. given by Pier, or his own formula[1] based on the theory of quanta, which give practically identical results up to 3000° C. and agree with Swann's value for air at 100° C.

By plotting energy temperature curves from the figures given above, and taking into account Swann's for air and CO_2 at 100° C., the following values for the mean volumetric heats between 100° C. and higher temperatures t° C. have been calculated, and are given as being probably the most accurate at this time available. Where volumetric heats are given by Bjerrum for ranges 18°–t° and 110°–t°, the same figures have been assumed to hold for ranges 0°–t° and 100°–t°. In the higher ranges for water, where Bjerrum gives figures for ranges 18°–t°, the energy has been calculated from Bjerrum's figures as though water were a gas down to 0° C., and 600 cal. subtracted as the energy of the imaginary gas at 100° C.

It has recently been shown[2] that if, using these figures as true volumetric heats and Nernst's constants for the dissociation of CO_2 and H_2O, the actual conditions in an engine cylinder are calculated, then results are obtained which are in excellent agreement with the best experimental engine results.

MEAN VOLUMETRIC HEATS BETWEEN 100° C. AND t° C. IN GRAM CALORIES PER GRAM MOLECULE.

	100° C. up to					
	500°.	1000°.	1500°.	2000°.	2500°.	3000°.
Nitrogen . .	5·17	5·28	5·50	5·75	6·00	6·30
Water-vapour .	6·25	6·94	7·64	8·42	9·71	11·20
Carbon dioxide .	8·25	9·55	10·07	10·50	10·87	10·95

§ (4) CRITICAL COMPARISON OF VOLUMETRIC HEAT FIGURES. (i.) *General.* — Bjerrum has accepted Holborn and Henning's figures up to 1200° C., except as regards their values for nitrogen at low temperatures. This is now known to be too low, and the formula given by Pier and accepted by Bjerrum agrees with Swann's value for air at 100° C., which the B.A. Committee considered correct to within 1 per cent. Hitherto Holborn and Henning's results between 800° and 1400° have been considered as probably too low. Callendar's opinion is that their results, although consistent among themselves, may be subject to systematic errors, making them, over the range 0°–1400° C., as much as 10 per cent too low. This, however, was when Langen's explosion results were the best available for the higher temperatures. The more reasonable view now is to regard the constant pressure results as being confirmed by the recent explosion experiments, and probably accurate to 2 per cent or 3 per cent up to 1200° C.

Above 1200° C. the new figures lie very much below the old, both for carbon dioxide and water. Hopkinson considered that Langen's results were probably too high on account of incomplete combustion and insufficient allowance for heat loss. Part of the difference is probably accounted for, at least in the case of CO_2, by the occurrence of dissociation at the high temperatures in Langen's experiments. In the experiments on which Pier and Bjerrum base their volumetric heat values, precautions were taken to prevent dissociation by having an excess of one of the combining elements present. Bjerrum's experiments were, in fact, designed with a view primarily to measuring the amount of dissociation of CO_2 and H_2O at various temperatures and pressures by comparing the pressures, and hence temperatures, reached with an excess, first of nitrogen, and then of one of the combining elements, present.

§ (5) BJERRUM'S THEORETICAL CALCULATIONS FROM VOLUMETRIC HEAT FIGURES.—Taking a formula given by Einstein, and modified by Nernst and Lindemann, which expresses the distribution of energy in a complex molecule according to the theory of quanta, Bjerrum has developed an expression for the mean volumetric heat of a gas over any range of temperature. Using his own, Pier's, and Holborn and Henning's values for the mean volumetric heats, and evaluating the constants in the formula by means of them, Bjerrum found that the formula can be made to give values for the mean volumetric heat which follow closely the experimental values over the range 0°–3000° C.

In the case of CO_2 some confirmation of the volumetric heat values can be obtained from optical considerations. The formula for volumetric heat involves terms depending on the free period of internal vibration of the molecule. Values of such free periods calculated from observed values of the volumetric heat are found to show very fair agreement with the periodicities at which absorption bands occur in the spectrum of CO_2. Conversely, if the actual wave-lengths for the principal absorption bands are used in the formula, values of the volumetric heat are obtained which only differ by 3 per cent or 4 per cent from observed values up to about 3000° C.

Water exhibits a highly complicated spectrum in the ultra-red region, and this has not yet been sufficiently explored to make it possible to compare the periodicities of actual absorption bands with those calculated from volumetric heat values. It is probable that the complex spectra so far observed are due to the existence of double molecules, and more observations at high temperatures and low pressures are needed before comparison can be made with high-temperature volumetric heat values. Water seems to show, moreover, a rapid increase of volumetric heat above 2000° C., which is not at present understood, and which it is impossible to express in a formula developed entirely from quanta theory. Up to 2000° C. the mean volumetric heat can be closely expressed by a formula of the same type as that for CO_2, but in order to make it follow the observed values above this temperature it was necessary to add to the formula an empirical term involving the fourth power of the temperature.

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[1] *Zeitschrift für Elektrochemie*, 1911.

[2] Tizard and Pye, *Automobile Engineer*, Feb., March, and April 1921.

§ (6) PROBABLE ACCURACY OF EXPERIMENTAL RESULTS.—Of the new explosion experiments under review those by Pier were undertaken primarily with a view to measuring volumetric heats, those by Bjerrum to measuring dissociation. Pier's experiments were all done with initial pressure one atmosphere; Bjerrum's, on the other hand, in order to increase the amount of dissociation, at much lower pressures—about one-seventh of an atmosphere. The point is important in considering the amount of heat loss from the explosion vessel.

The chief cause of uncertainty in all results of explosion experiments lies in estimation of the heat loss during the rise to maximum pressure. Pier neglected this altogether, but fortunately, owing to his method of experimenting, this does not affect the value of his results. Bjerrum did a number of experiments with varying initial pressures, and, on the assumption that heat loss per cent is inversely proportional to pressure, has estimated the heat loss in his own and, where necessary, in Pier's experiments, and has applied corrections accordingly. The magnitude and reliability of his corrections are dealt with under each gas separately.

Pier's method of measuring volumetric heats was to make his explosion experiments comparative, taking as his standard of comparison the monatomic gas argon. The volumetric heat of argon, according to the kinetic theory of gases, should be constant and equal to 2·98 calories per gram molecule. Pier first exploded hydrogen and oxygen with excess of hydrogen and then with excess of argon, but keeping a small excess of hydrogen to prevent any sensible dissociation. By doing experiments at different starting temperatures he was able to obtain figures for the mean volumetric heat of both argon and water-vapour, and to show that argon, at least up to 2000° C., shows no sensible departure from the theoretical value. These results depend on a knowledge of heat loss, which Pier assumed to be zero. Bjerrum has estimated the heat loss in these experiments at 1·8 per cent, and has recalculated Pier's results for the other gases on this basis, and on the supposition that the volumetric heat of argon is constant and equal to the theoretical value. It is impossible, indeed, to conceive of a variation of volumetric heat in a monatomic gas, and this assumption seems justified on experimental as well as theoretical grounds.

(i.) *Nitrogen.*—Pier obtained the ratio between the volumetric heats of nitrogen and argon by comparing the quantities of the two inert gases present with an explosive mixture of oxygen and hydrogen when the same temperatures were produced. In this case the results are therefore purely comparative, and, since heat loss may be taken as being the same under similar conditions of temperature and pressure, Pier's neglect of it does not vitiate his results except in so far as allowance must be made for differences of initial pressure. The magnitude of the loss in these experiments, in which the pressure rise takes place in about one-hundredth of a second, was, according to Bjerrum, only about 2 per cent in any case.[1] So that Pier's comparative results for nitrogen should be accurate to the order of his experimental errors, which were about 1 per cent. On the whole it seems reasonable to expect the figures given by Bjerrum for nitrogen are accurate to within ±1 per cent up to 2000° C. and ±2 per cent up to 3000° C.

(ii.) *Water-vapour.*—The principal experiments on which the figures for water-vapour are based are those of Pier, carried out with initial pressures approximately atmospheric, and for which Bjerrum estimates the heat loss as 1·8 per cent. These carry the values up to 2377° C. Bjerrum has extended this range up to 3064° C. by experiments at low initial pressures (about one-seventh atmosphere), in which he estimates the loss as 14 per cent. This figure he gives with much confidence, but it must be pointed out that a difference of 2 per cent in the allowance for heat loss makes a difference of about 8-9 per cent in the volumetric heat values up to 3000° C. Indeed, in some further experiments with an excess of argon, also by Bjerrum, he finds it necessary to fix the heat loss allowance as 12 per cent in order to bring them into line with the rest of his results. This figure of 12 per cent, he says, was a not improbable one, but does not say why; possibly it was because the argon experiments were at slightly higher initial pressures.

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[1] This figure agrees exactly with that estimated by David (*Proc. Roy. Soc.* A, xcviii. 308) for the heat loss up to maximum temperature, when allowance is made for the difference in maximum temperature and explosion time according to the empirical formula given by him.

In estimating the probable accuracy of the figures for water-vapour, it seems, therefore, that we must allow that there may be an error of ±5 per cent up to 3000° C., and possibly more. Up to 2000° C. the uncertainty is considerably less. It seems likely that the possible error is not more than ±2 per cent or 3 per cent.

(iii.) *Carbon Dioxide.*—In dealing with CO_2 one has the advantage, as with nitrogen, of being able to make the experiments comparative as between two inert gases, as was done by Pier. On the other hand, the rate of combination where carbon is involved is very much lower than with oxygen and hydrogen. Pier found that the combustion of carbon monoxide was too slow to be any use except for mixtures giving a final temperature over 2200° C. He therefore used combustions of acetylene and oxygen, first with excess of nitrogen, then excess of O_2, and finally with CO_2 and O_2. There is no doubt that Pier was getting very considerable loss of heat before the point of maximum pressure, as is clear from the very low value of the heat of combustion of acetylene which he calculated from his experiments.

It is curious that Pier never realised the importance of his heat losses, and was forced to suppose the formation of an endothermic compound of water and CO_2 to explain his heats of combustion of acetylene. There is some evidence that aldehydes are formed during combustion in an engine cylinder, but it is very unlikely that the quantities are as great as Pier supposed. Bjerrum, however, has accepted Pier's figures, being comparative, as substantially correct, and, as mentioned previously, they receive some confirmation from the spectrum of CO_2.

Bjerrum has extended the range of Pier's figures from 2110° C. to 2714° C. by explosions of CO and O_2, first with excess of CO and then of nitrogen. As before, his experiments were done first with a view to dissociation measurements, but in this case he used initial pressures of one-quarter and one atmosphere. Using his results at these two pressures, he estimates his heat loss correction, and gives 10·9 as the mean volumetric heat from room temperature to 2714° C. The combustion time was of the order of one-tenth second as against one-hundredth second for the oxygen and hydrogen experiments, and the correction on the mean specific heat due to heat loss is in consequence as much as 15 per cent of the value given, in spite of the pressures at which he was working being higher than in the case of his experiments on water-vapour.

Pier has given the formula

$$C_v=6\cdot8+3\cdot3\times10^{-3}t-0\cdot95\times10^{-6}t^2+0\cdot1\times10^{-9}t^3$$

as representing his experimental results upon CO_2 up to 2110° C. This formula gives 10·75 for the range up to 2714° C., so that there is good agreement between Bjerrum's and Pier's figures.

An idea of the optical confirmation of the CO_2 figures may be given as follows: The internal or vibrational energy of a triatomic molecule will be shared between the three mutual vibrations of the three atoms, (1) with (2), (2) with (3), and (3) with (1). In the case of CO_2, since the mutual attraction between a carbon and oxygen atom may be supposed greater than between the two oxygens, it is a reasonable assumption that, of the three vibrations, there will be two of high frequency and one of lower. If the three wave-lengths λ_1, λ_2, and λ_3 of the Nernst-Lindemann quanta function which occurs in the volumetric heat formula are evaluated from observed values of the volumetric heat on the above assumption that $\lambda_2=\lambda_3$ the values obtained by Bjerrum are $\lambda_1=8\cdot1$, $\lambda_2=\lambda_3=5\cdot0$. With these constants the formula gives values of the volumetric heat which follow the actual values closely over the range 0°–2000° C., *e.g.*:

Temperature Range.	Volumetric Heat, Cal. Gm. Mol.	
	Observed.	Calculated.
0–200	7·48	7·44
0–2110	10·47	10·43

Now there are actual absorption bands in the spectrum of CO_2 at $\lambda_1=14\cdot7$, $\lambda_2=4\cdot3$, and $\lambda_3=2\cdot7$. If these values are put into the formula and the volumetric heats calculated from purely optical data, the volumetric heats for the same ranges as above are 7·67 and 10·04. The optical data, therefore, give values within 3 per cent or 4 per cent of the observed ones, and on the whole it seems probable that the latest values of the volumetric heat given by Bjerrum up to 2700° C. may be taken as correct to ±3 per cent or ±4 per cent.

D. R. P.

GEAR-BOX TESTING MACHINE (National Physical Laboratory). See "Dynamometers," § (6) (iii.).

GIBBS-HELMHOLTZ EQUATION. See "Thermodynamics," § (64).

GIBBS' THERMODYNAMIC POTENTIALS. See "Thermodynamics," § (51).

GLASSES:

- Suitable for Thermometers, Approximate Percentage Composition of Various, tabulated. See "Thermometry," § (6), Table VI.
- Zero Depressions of Thermometers made of Different, tabulated at 25° C., 50° C., and 100° C. See *ibid.* § (7), Table VI.

GOLD:
Freezing-point of, determined by platinum resistance thermometer and tabulated. See "Resistance Thermometers," § (18).
Solid and Liquid, Emissivity of, determined by optical pyrometer. See "Pyrometry, Optical," § (20).

GOVERNOR FOR STEAM ENGINE. See "Steam Engines, Reciprocating," § (2) (vi.).

GRAMME MOLECULE. The volume of a quantity of gas which has a mass equal to its molecular weight in grammes. It is constant for all gases, and at a pressure of one atmosphere and temperature 0° C. is equal to 22,412 c.c.

GRIFFITHS' DETERMINATION OF MECHANICAL EQUIVALENT OF HEAT. See "Mechanical Equivalent of Heat," § (5) (i.).

GRIFFITHS, EXPERIMENTS ON LATENT HEAT OF EVAPORATION OF WATER, at temperatures of 30° and 40° C. See "Latent Heat," § (2).

GRIFFITHS, LABORATORY FORM OF DISAPPEARING FILAMENT TYPE OF OPTICAL PYROMETER. See "Pyrometry, Optical," § (3).

GROUND STORAGE OF RAINFALL. See "Hydraulics," § (8).

GROUPS, NON-DIMENSIONAL, THE REAL VARIABLES IN PHYSICAL PROBLEMS. See "Dynamical Similarity, The Principles of," § (34).

GUERICKE'S PUMP. See "Air-pumps," § (11).

GUN-RECOIL CYLINDER, HYDRAULIC. See "Hydraulics," § (60) (ii.).

GYROSCOPE

§ (1) DESCRIPTION OF THE GYROSCOPE. INERTIA AND BALANCE.—The gyroscope consists usually of a massive wheel or disc mounted on bearings so as to be free to spin rapidly about its axis of symmetry. The gyroscope as commonly made is symmetrical about an equatorial plane, perpendicular to the axis of symmetry; but this equatorial symmetry is unessential. Any rigid homogeneous solid of revolution may form the spinning element; and, more generally still, the gyroscope may even be irregular in form provided that it is "dynamically symmetrical" about the axis of spin. The conditions necessary for dynamical symmetry are (i.) that the centre of mass O of the gyroscope shall lie on the axis of spin; (ii.) that the axis of spin shall be a principal axis of inertia; (iii.) that the moments of inertia about all axes through O perpendicular to the spin-axis (say, all "diameters") shall be equal. If any body were taken of irregular form with centroid O, and with unequal moments of inertia A, B, C about its mutually perpendicular principal axes through O, and if it were mounted to spin about the C axis, it would differ from the normal gyroscope only in having A and B unequal instead of equal.

In the practical making of gyroscopes the closely approximate symmetry of form is found inadequate to secure these conditions of dynamical symmetry, and a final process of "balancing" is requisite to make the gyroscope run smoothly at high speed. If, in violation of condition (i.), the centroid is not on the spin-axis, but at a short distance from it, the gyroscope is said to be defective in "static balance." With this defect, if the gyroscope is without spin the equilibrium under gravity fails to be neutral (except when the axis is vertical); and when the gyroscope spins about the fixed axis a revolving centrifugal force is thrown on the bearings. A true circular disc, mounted eccentrically on a spin-axis normal to its plane, illustrates the state of bad static balance.

If, in violation of condition (ii.), the principal axis of inertia is not coincident with the spin-axis, but makes a small angle with it, the gyroscope is said to be defective in "dynamic balance." The defect cannot be detected statically; but, when the gyroscope spins, a torque in a revolving axial plane is thrown on the bearings. A true circular disc mounted centrically but obliquely on a spin-axis illustrates the state of bad dynamic balance.

The two defects are both remedied by the addition or removal of small masses of determinable amount and position; and when balanced to run quietly in this way the gyroscope is usually regarded as satisfying all the necessary conditions. But, in violation of condition (iii.), the moments of inertia about different diameters of the gyroscope may be slightly unequal. An elliptical disc mounted centrically and normally on its spin-axis illustrates this defect. The elimination of whirling stress may be regarded as having for its direct consequence the annulment of the product of inertia for every pair of perpendicular planes of which one is axial and the other is normal to the spin-axis; and in consequence the elimination ensures that the conditions (i.) and (ii.) are satisfied. But if the moments of inertia A and B are left unequal, condition (iii.) is violated and the gyroscope is defective. The defect cannot be detected statically, nor by spinning the gyroscope about the fixed axis. It is only when the axis is itself in motion that the uncanonical form of the gyroscope is revealed.

§ (2) THE MOUNTING OF THE GYROSCOPE. DEGREES OF FREEDOM.—In gyroscopic mechanisms each individual gyroscope may have any number of degrees of freedom, up to the limit of six degrees, and these degrees of freedom may arise in an unlimited variety of ways. Most commonly the centroid of the gyroscope remains a fixed point, and only the three angular degrees of freedom about the centroid are left for consideration; and, as the spinning movement accounts for one freedom, there remain only the two degrees of freedom associated with the direction of the gyro-axis z. These two degrees of freedom usually occur as the relative movements of three consecutive members of a chain of pieces. The gyroscope or rotor R (*Fig.* 1) is free to spin about its axis z; and this axis is coincident with the line of bearings of a ring or frame or casing Q, which supports the rotor R. The spinning freedom is thus a rotation about the line z. The casing Q is mounted on a frame or ring P, so as to be free to turn about a line y, intersecting z at O. The rotation of Q relatively to P about y provides the second degree of freedom of R. Finally the ring P is mounted on a fixed base-piece B so as to be free to turn about an axis x, intersecting y at O. The rotation of P relatively to B about x provides the third degree of freedom of R. In the commonest form of apparatus of this sort the axes x and y of the ring P are made perpendicular, and the axes y and z of the casing Q are also made perpendicular, and the fixed axis x is perpendicular to the horizontal base of B and consequently a vertical line. The apparatus usually described as the "Wheatstone gyroscope" is of this type (*Fig.* 1A).

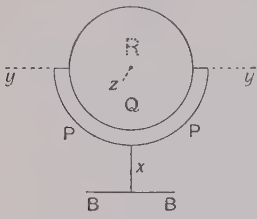

FIG. 1.

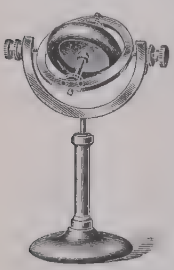

FIG. 1A.—Wheatstone Gyroscope Model in Perspective.

A gyroscope mounted in this way, with complete angular freedom, is sometimes spoken of as a "free" gyroscope. If the hinge-line x is locked the gyroscope has only two degrees of freedom, and is then often described as "constrained." The gyro-axis z is then restricted to move in a circular cone with y as axis and yz as angle. Very specially (and commonly) yz is a right angle, and z is thus restricted to move in a fixed plane normal to y. (In the Wheatstone gyroscope the gyro-axis then turns round a fixed horizontal axis.)

If the hinge y is locked and x is left free then, similarly, the gyroscope has only two degrees of freedom, and z is restricted to describe a circular cone about x (or a plane normal to x).

More generally the gyro-axis z might in any way be left free, by mechanical constraints applied to the casing Q, to move only along a definite route (generating a cone of arbitrary form), and the gyroscope would again have two degrees of freedom.

It is to be observed that the reduction from three degrees of freedom to two degrees may occur without the suppression of the freedom of rotation about x or y. The complete angular freedom of the gyroscope arises from the fact that any angular rotation about O may in general be derived as the resultant of suitable components about x, y, and z. This fails to be true, however, when x, y, and z happen to be coplanar; and this occurs for each of two positions, when the relative rotation of P and Q brings the planes xy and yz into coincidence. For angular velocities about coplanar lines have as their resultant an angular velocity about a line lying necessarily in the same plane. For the Wheatstone mounting (*Fig.* 1A), x and z themselves coincide in these two positions (when the gyro-axis is vertical), and the loss of freedom is often attributed to the identity of the two rotations about the coincident lines; but the identity, though sufficient to account for the loss, is not essential. (The proposal to avoid the danger of this loss of freedom, when three degrees of freedom are to be preserved, by adding an extra member N to the chain of pieces PQR introduces a fresh objection; for the links N, P, Q of the chain BNPQR then have an indeterminate position for any one direction of z, and would move about at random.)

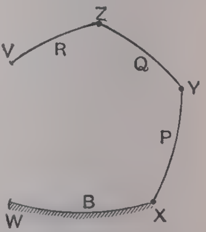

FIG. 2.

The case of a gyroscope with only one degree of freedom, as for x and y both locked, leaving the spinning movement about z as the sole freedom, should be mentioned as the ultimate and simplest form of freedom; but it has no gyroscopic importance.

If abstraction be made of the directions of the three axes x, y, and z of the gyroscopic apparatus, without regard to position in space or any other detail, a unit sphere centred at O and cutting the lines x, y, z at points X, Y, Z gives a completely representative figure on the spherical surface (*Fig.* 2). The fixed point X represents the fixed axis of the base-piece B; the great-circle arc XY represents the ring P; Y represents the axis y connecting the ring P and the casing Q; YZ represents the casing Q; and Z represents the gyro-axis. To complete the figure, any arbitrary line w of the base B gives a fixed point W, and WX represents the base B; and if any line v of the rotor R gives the point V, the arc ZV represents the rotor R. With this open quadrilateral WXYZV, with WX fixed and all the sides of constant length, the complete angular freedom of VZ is given by the rotation of XY about X, of YZ relatively to XY about Y, and of ZV relatively to YZ about Z. And it is when the angle at Y is zero or two right angles that the triangle XYZ degenerates into a great-circle arc, and one degree of freedom is lost; Z having then no movement possible except in the direction perpendicular to XYZ.

§ (3) GYROSCOPIC EXPERIMENTS.—As preliminary to a formal account of the kinematics and dynamics of the gyroscope, it may be convenient to state briefly the results of a number of simple experiments with the Wheatstone gyroscope. When checked by the principles developed later there will appear the reasons underlying these various movements; and there will arise also certain qualifications necessary to some of the statements that seem empirically correct. Familiarity with such experimental results is to some extent serviceable, in lack of more complete information, as a rough working knowledge of practical gyroscopics.

The experiments with the rapidly spinning gyroscope may be briefly described as follows, the degrees of freedom being as stated in each separate case:

(i.) Axis x locked, y free; any forces applied to Q. The gyroscope offers no resistance to the turning of Q about y.

(ii.) Axis x free, y locked; any forces applied to P or Q. The gyroscope offers no resistance to the turning of P and Q about x.

(iii.) Axes x and y both free; B moved about in any manner. Then z retains its original and arbitrary direction unchanged. But if, during the movement of B, x comes to coincide with z (or passes very near z) a violent "kick" occurs and z abruptly changes direction.

(iv.) Axes x and y both free; z horizontal; a weight hung on one end of z and released. Then z remains horizontal, and P and Q rotate uniformly about x at a rate proportional directly to the weight and inversely as the spin of the gyroscope. If the movement of P is "hurried" by a torque applied to P about x the weight rises, and if P is hindered the weight descends.

(v.) Axes x and y both free; z pointing in any direction; a smooth straight-edge is laid perpendicular to z and pressed against the spindle of R. The spindle does not yield to the pressure but moves along the straight-edge in the same sense as if it were rolling along the straight-edge by reason of the spin.

(vi.) Axes x and y free, z horizontal; forces are applied to P as if to turn P about x. The turning about x is resisted and tilting of Q about y occurs. When z reaches the vertical the resistance, after diminishing, disappears and P is freely rotatable in the sense of the spin. If the direction of rotation of P is then reversed a sudden "jolt" occurs, Q begins to capsize, the rotation of P is resisted, and z presently again becomes vertical. The rotation of P is then again unresisted in the sense of the spin.

§ (4) KINEMATICS OF THE GYROSCOPE.—If the gyroscope executes any kind of movement, a description of that movement at any moment may consist in a statement of the component angular velocity Ω about the axis z, usually called the "spin," and the instantaneous rate of movement ω of the axis z. The latter has now come (by an enlargement of the original use of the term) to be called the "precession." The plane in which the axis z moves is the instantaneous plane of precession, and the diameter u of the gyroscope drawn normal to this plane is the instantaneous axis of precession. In the representative spherical diagram (*Fig.* 3) the resultant angular movement of the gyroscope is represented by the spin Ω about Z, together with a velocity ω of Z along (Z) the spherical curve which is the path of Z. If the quadrantal triangle ZTU is completed, then ZT, the tangent to the curve (Z), represents the plane of precession, TU the central plane of the gyroscope, and U the axis of precession. The resultant movement of the gyroscope is compounded of angular velocities Ω about Z and ω about U. The triangle itself has ZU as normal at Z to the curve (Z), and ZU touches at K the curve which is the envelope of ZU and the evolute of (Z). The point K is also the centre of the small circle that osculates the curve (Z) at Z; and ZK ($=\rho$, say) is the angular radius of curvature of (Z) at Z. The movement of the triangle ZTU as a rigid figure is given by the rolling of ZU on the evolute curve, and consists instantaneously of an angular velocity n about K. The components of n about Z and U are $n \cos \rho$ and $n \sin \rho$ respectively, giving velocities $n \cos \rho$ of U along TU and $n \sin \rho$ of Z along ZT. The latter velocity is by hypothesis ω, and therefore $\omega = n \sin \rho$. Hence $n = \omega/\sin \rho$; and the component angular velocities of the triangle ZTU are $\omega \cot \rho$ about Z and ω about U. The gyroscope and the triangle have movements identical in respect of the components about U and differing only in the components about Z.

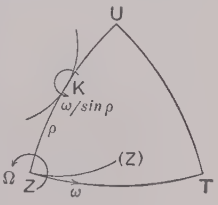

FIG. 3.

§ (5) DYNAMICS OF THE GYROSCOPE.—The angular momentum of the gyroscope is the resultant of the separate angular momenta associated with the two component angular velocities, and is therefore compounded of $C\Omega$ about the axis z and $A\omega$ about the diameter u. The rates of change of these two vectors give rise to two separate torques or couples which, as kinetic reactions, are together equivalent to whatever system of applied forces may be acting on the gyroscope. They may be calculated separately.

The vector rate of change of any space-vector r having angular velocity $\bar{\omega}$ is given by the linear velocity of its extremity, and is compounded of $\dot{r}$ radially and $r\bar{\omega}$ normal to the vector in the plane of its movement. The angular momentum $C\Omega$ about OZ, corresponding to the vector r, thus gives rise to a torque $C\dot{\Omega}$ about OZ and a torque $C\Omega\omega$ about OT. The angular momentum $A\omega$ about OU gives rise to a torque $A\dot{\omega}$ about OU and a torque $-A\omega^2 \cot\rho$ about OT, since $\omega \cot\rho$ is the angular velocity of the triangle about Z.

As the torque about OZ is $C\dot{\Omega}$ it follows that the angular acceleration $\dot{\Omega}$ of the spin Ω is the same as would be produced by the same torque about z if the axis z were stationary. When, however, the gyroscope is supposed to be already set spinning on frictionless bearings, or (in practice) to be supplied with an electrical driving torque counterbalancing the resistances, the applied torque about the axis z is then zero, and hence $C\dot{\Omega}$ is zero and Ω remains constant independently of any movement imposed on the axis z. (This property of a gyroscope is not true if condition (iii.) of § (1) is violated. Unequal values of A and B give rise to fluctuations in the spin Ω.)

It is convenient to represent the torques by equivalent forces, acting at Z perpendicular to OZ, and producing the same moments as those of the torques. These forces are:

$A\dot{\omega}$ along the great-circle tangent ZT at Z.
$A\omega^2 \cot\rho$ along the inward normal ZK at Z.
$C\Omega\omega$ along the normal at Z. (*Fig.* 4.)

The last force $C\Omega\omega$ has direction and sense which, if rotated through one right angle about Z in the sense of the spin Ω, gives the direction and sense of the velocity ω along ZT.

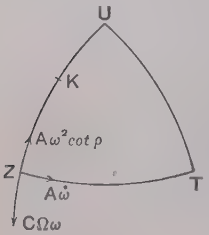

FIG. 4.

Of these three forces (or the torques they represent) it may be noticed that the first two are independent of the spin and of the moment of inertia C, and depend only on A and on the movement of the axis z. They would constitute the whole torque if either C or Ω were zero. A thin rod along OZ credited with the same moment of inertia A would give rise to these same forces; or a single particle of mass A at Z would alone suffice. And the expressions $\dot{\omega}$ and $\omega^2 \cot\rho$ are the components of the acceleration of Z along the tangent ZT and the normal ZK; for $\dot{\omega}$ is the rate of change of the velocity ω of Z, and $\omega^2 \cot\rho$ is the product of the velocity ω and $\omega \cot\rho$ the rate of turning of the tangent ZT about Z. The mass A at Z would thus give the same two forces.

§ (6) THE GYROSCOPIC TORQUE. RULES FOR THE PRECESSION.—The torque $C\Omega\omega$ is known as the "gyroscopic torque." It has OT for its axis and UZ for its plane. In many practical uses of the gyroscope the value of Ω is so large that this torque dominates the rest, and may be taken by itself as sufficiently representative of the whole kinetic reaction. The simple rule then is that the plane of the applied torque and the plane of precession are perpendicular planes through the axis of the gyroscope; and that the torque has a moment equal to the product of the angular momentum $C\Omega$ of the gyroscope and the rate of precession ω. A variety of rules have been proposed as indications of the sense of precession.

(α) The most common statement takes some such form as this: The precession is such that, after a precessional movement continued through one right angle, the axis of the gyroscope would coincide with the axis of the torque, and the sense of the spin would agree with that of the torque. But in practice there is a certain awkwardness and inconvenience in the need to consider a merely hypothetical position of the gyroscope.

(β) A briefer equivalent is to mark the sides of the quadrantal triangle, such as ZTU, with arrows circulating round the triangle and representing the senses of Precession, Spin, and Torque in that (alphabetical) order.

(γ) Rules more readily applied may be stated for *forces*, taken as equivalents of the gyroscopic torque. Thus, as above, if a force perpendicular to the axis of the gyroscope act at a point of the axis, the corresponding force-vector, when swung through a right angle in the sense of the spin, points then in the sense and direction in which the point moves as a consequence of the precession. More realistically the force may be supposed actually applied to the thin smooth spindle of the gyroscope by lateral pressure of a straight-edge; and the precession then makes the spindle slip along the straight-edge; and the sense of travel is *as if* the spindle were *rolling* on the straight-edge by reason of its spin.

(δ) A similar form of statement is true for a force parallel to the axis and supposed to act on the rim of the gyroscope; namely, if the force-vector normal to the wheel is carried round, as if rigidly connected with the wheel, through one right-angle in the sense of the spin, it then indicates the precessional movement of the rim.

§ (7) Kinetic Reaction, Angular Momentum, and Energy.—In regard to the torques of § (5) it should be remarked that $A\dot{\omega}$ and $A\omega^2 \cot \rho$ depend on the (surface component of the) acceleration of Z; and that if that acceleration be itself expressed in terms of any convenient set of components, it is only necessary to multiply each of those separate components by A in order to obtain the required torques. And similarly $C\Omega\omega$ depends on the velocity of Z; and if that velocity presents itself as composed of any set of components it is only necessary to multiply each component by $C\Omega$, and turn it round through one right-angle backwards (*i.e.* against Ω) about Z, in order to obtain the corresponding contribution to the gyroscopic torque.

Of angular momentum it should be borne in mind that, unlike linear momentum, it has only magnitude and direction and is not a located vector. The sum of the momenta of the elements of a gyroscope (with its centre at rest) about any line parallel to the axis is the same as about the axis itself. The gyroscope might be replaced by another equal gyroscope with a parallel axis, inside the same casing, and the gyroscopic torque would not be affected by the change. Further, if the same casing contained several gyroscopes, each spinning on an axis fixed relatively to the casing, the gyroscopic torque for any movement of the casing would be only the same as for a single contained gyroscope with an angular momentum equal to the vector sum of those contributed by the several gyroscopes.

The principle of energy demands that the applied forces should do work on the gyroscope equal to the increase of its kinetic energy. The forces $C\Omega\omega$ and $A\omega^2 \cot \rho$ at Z acting normally to the velocity of Z along ZT, do no work, and the force $A\dot{\omega}$ does work at the rate $A\dot{\omega}\omega$. If the spin varies, the torque $C\dot{\Omega}$ causing the variation does work at the rate $C\dot{\Omega}\Omega$, and the sum $A\dot{\omega}\omega + C\Omega\Omega$ is thus the time-differential of the kinetic energy $\frac{1}{2}(A\omega^2 + C\Omega^2)$.

§ (8) Simple Cases of Motion of a Gyroscope.—The three torques found in § (5), which usually act in combination, may occur separately for special movements of the gyroscope.

(i.) The torque $A\dot{\omega}$ occurs alone if $\Omega = 0$ and $\rho = \pi/2$—*i.e.* if the gyroscope has no spin and its axis is rotating in a plane. The torque $A\dot{\omega}$ also occurs alone, initially, if ω is momentarily zero, with any value for Ω. Hence under the action of a torque the gyro-axis moves initially in the plane of the torque itself. [The direction of motion begins to turn, however, at a rate $(C/A)\Omega$.]

(ii.) The torque $A\omega^2 \cot \rho$ occurs alone if $\Omega = 0$ and ω is constant—*i.e.* if the gyroscope has no spin and its axis describes a cone of any form at a uniform rate. The torque measures the normal pressure on the guiding curve or cone. If ρ also is constant the cone or curve is circular and the torque is constant.

(iii.) The gyroscopic torque $C\Omega\omega$ occurs alone if $\rho = \pi/2$ and ω is constant—*i.e.* if the axis of the gyroscope rotates uniformly in a plane. It may be noticed in this case how, in detail, after a turn of the axis through an angle 2α the time-integral of the torque-vector accounts for the change of momentum. The torque-vector $C\Omega\omega$ moves round one right-angle in advance of the axis. The mean of the vectors ranging over the angle 2α is $C\Omega\omega$ $(\sin\alpha/\alpha)$ along the angle-bisector, and the time is $2\alpha/\omega$; hence the time-integral of the torque gives a momentum $2C\Omega \sin \alpha$ one right-angle beyond the angle-bisector of the gyro-axes. The change of momentum is $C\Omega \times 2 \sin \alpha$ and it has the same direction. In particular when $2\alpha = \pi$, with reversal of the axis, the change of momentum is $2C\Omega$.

(iv.) If ω and ρ are both constant, Z in *Fig.* 3 describes a circle with centre K, and the triangle ZTU rotates uniformly about K as a fixed point. The axis of the gyroscope describes a circular cone of angle ρ. The torque outwards from the axis of the cone is $C\Omega\omega - A\omega^2 \cot \rho$. For the customary rapid spin this is positive in sign; but if ω were rapid also the torque, specially, is zero for the case $C\Omega = A\omega \cot \rho$. This conical steady motion is then a free movement of the gyroscope. If, say, the stationary casing containing the spinning gyroscope is struck a heavy blow, giving an impulsive torque $A\omega$ about a diameter of the gyroscope, the impulse creates suddenly an angular momentum $A\omega$ and there ensues a conical motion of the axis z with $\tan^{-1}(A\omega/C\Omega)$ for the angle of the cone. This motion, being rapid, gives a blurred effect, visually, and is generally spoken of as a "wobble"; but it is nevertheless technically a "steady motion."

The constant torque $C\Omega\omega - A\omega^2 \cot \rho$ necessary in general for the conical steady motion of the gyroscope is linear in Ω but quadratic in ω. Hence, for an assigned value of the torque giving movement on the cone of angle ρ, there is a unique value of the spin for any assigned rate of precession; but two rates of precession for any assigned spin are given by the roots of a quadratic equation, and may be real or coincident or imaginary. The steady motions of a top (which may be regarded as a frameless gyroscope) illustrate these results.

(v.) If the gyro-axis is guided by any smooth constraints, adequate to take any lateral pressures, the torques $C\Omega\omega$ and $A\omega^2 \cot \rho$ are supplied by the constraints, and the agent controlling the precessional movement of the axis has only to supply the torque $A\dot{\omega}$. The gyroscopic torque falls on the rigid constraints, and the agent experiences no "gyroscopic resistance" whatever.

§ (9) GYROSCOPE WITH TWO DEGREES OF FREEDOM ON A ROTATING BASE.—Instead of prescribing directly a movement of Z (*Fig.* 2) the movement prescribed may be that of Y, leaving the movement of Z to ensue as a dynamical consequence. In a simple case YZ may be taken to be a quadrant, and XY also a quadrant, and Y may be made to move uniformly along its path, the great circle with centre X, at the constant angular rate n. The point Z is thus constrained to lie on a meridian, say, of the sphere; the meridian being XZ and rotating at rate n about the pole X. If θ is the polar distance XZ (equal to the angle XYZ) the velocity of Z has components $\dot{\theta}$ along the meridian and $n \sin \theta$ along ZY perpendicular to the meridian. The corresponding components of the gyroscopic torque give forces at Z equal to $C\Omega n \sin \theta$ along the meridian and $C\Omega\dot{\theta}$ perpendicular to it. (It is supposed that Ω and n would agree in sense if the angle θ were reduced to zero.) The acceleration of Z as determined from its component velocities and the local rate of rotation $n \cos \theta$ of ZX at Z (or otherwise) has components

$$\left.\begin{aligned} &\frac{d}{dt}(\dot{\theta}) - (n \sin \theta)(n \cos \theta) \\ \text{and}\quad &\frac{d}{dt}(n \sin \theta) + (\dot{\theta})\,(n \cos \theta), \end{aligned}\right\} \quad . \quad . \quad . \quad (1)$$

i.e. $\ddot{\theta} - n^2 \sin \theta \cos \theta$ along the meridian (2)

and $2n\dot{\theta} \cos \theta$ perpendicular to it. . (3)

The corresponding forces are obtained by multiplying by the moment of inertia A (§ (5)) and are therefore

$A(\ddot{\theta} - n^2 \sin \theta \cos \theta)$ along the meridian (4)

and $2An\dot{\theta} \cos \theta$ perpendicular to it. . (5)

Hence the equation of motion in θ is

$$A(\ddot{\theta} - n^2 \sin \theta \cos \theta) + C\Omega n \sin \theta = 0, \quad (6)$$

with the integral

$$A\dot{\theta}^2 + An^2 \cos^2 \theta - 2C\Omega n \cos \theta = \text{constant}. \quad (7)$$

This integral expresses the kinetic energy as a quadratic function of $\cos \theta$. This case may be compared with simple pendulum motion, with no movement in azimuth, for which the kinetic energy is a linear function of $\cos \theta$; and also with conical pendulum motion, with free and variable movement in azimuth, for which the kinetic energy is a cubic function of $\cos \theta$. The present case is in a sense intermediate. For a pendulum of period $2\pi/p$, with $\theta = 0$ as the downward vertical, and with the vertical plane of swing made to rotate uniformly at the rate n, the equation of motion is

$$\ddot{\theta} - n^2 \sin \theta \cos \theta + p^2 \sin \theta = 0. \quad . \quad (8)$$

The position $\theta = 0$ is a stable position if $p > n$. If $p < n$ the position $\theta = 0$ is unstable, and an inclined position of equilibrium exists given by $\cos \theta = p^2/n^2$, and is stable. The position $\theta = \pi$ is one of equilibrium but is unstable for all values of p/n. (Another simple instance of the same movement occurs in the case of a magnetic needle oscillating in a horizontal plane in the earth's field, with a soft-iron rod horizontal and perpendicular to the needle and rigidly connected with it.)

The equations (6) and (8) are identical if $(C/A)\Omega n = p^2$, and hence the position $\theta = 0$ is stable if $C\Omega > An$. It is unstable if $C\Omega < An$; and a position of equilibrium is then given by $\cos \theta = (C\Omega/An)$. The position $\theta = \pi$ is an equilibrium position but unstable for all values of Ω/n. Hence (*Fig.* 2) when Z is at X the spin agrees in sense with that of the rotation of XY about X, and the position is stable provided $n < (C/A)\Omega$. When Z is at the antipodes of X the senses are opposite, and the position is always unstable. For the first case the spin is customarily so large that the inequality is amply satisfied, and the experimental results are apt to prompt an unqualified statement of stability. The inequality, however, is an essential condition.

The lateral pressure L of the meridian on the gyro-axis is given by

$$L = (2An \cos \theta - C\Omega)\dot{\theta}. \quad . \quad . \quad (9)$$

The force L has a moment $L \sin \theta$ about OX, which has to be supplied by the agent responsible for maintaining the rotation n. He provides therefore a variable torque $L \sin \theta$, which is identically equal to

$$\frac{d}{dt}(An \sin^2 \theta + C\Omega \cos \theta), \quad . \quad (10)$$

the rate of increase of the angular momentum about OX. Further, the agent supplies energy at the rate

$$\tfrac{1}{2}\,\frac{d}{dt}\left[A(\dot{\theta}^2 + n^2 \sin^2 \theta) + C\Omega^2\right]; \quad . \quad (11)$$

and this reduces as it should to $nL \sin \theta$ in virtue of (6) and (7).

If the gyroscope is originally in the unstable position $\theta = \pi$ its axis will pass the position $\theta = \pi/2$ with an angular velocity in θ given by

$$-\dot{\theta} = \left[n^2 + 2(C/A)\Omega n\right]^{\frac{1}{2}} \quad . \quad . \quad (12)$$

and the torque of resistance to the rotation of the meridian at this moment is equal to

$$C\Omega\left[n^2 + 2(C/A)\Omega n\right]^{\frac{1}{2}}. \quad . \quad . \quad (13)$$

For a rapid spin Ω this torque may be considerable even for a value of n too small to show a visible movement of the meridian.

The special conditions $XY=\pi/2$ and $YZ=\pi/2$ involved in the above movement may be removed without alteration of the form of equation (6). The mutual inclination of the lines x and y, y and z may be arbitrary; so that the point Y describes a small circle uniformly about X, and Z keeps at a constant distance from Y. The variable angle XYZ still obeys a rotated-pendulum equation, and may take equilibrium values determined as before. For a rapid spin the stable position of Z is that which gives the smallest value to XZ.

More generally still, if Z is constrained to lie on any rigid curve that is rotated uniformly at a constant rate n about a fixed point X, it is at once apparent that the only positions of equilibrium are those for which the force along ZX is either normal to the curve at Z or else is zero. Hence positions of equilibrium occur at the foot of any normal drawn from X to the curve (Z). The exceptional case of a zero force (giving free movement of the gyroscope, with the guide out of effective action) occurs if $\cos XZ=(C\Omega/An)$; giving all points in which a circle centre X and radius XZ cuts the curve; or more exceptionally if $XZ=0$ or π, which can occur only when the curve passes through X or its antipodes, as in the case first above treated.

It may further and more particularly be shown that the normals give stable positions for a minimum value of XZ when $C\Omega>An$, so that the circle is unreal; and that when $C\Omega<An$, so that the circle is real, the stable positions are given by the minimum values of XZ for points Z outside the circle, and by maximum values of XZ for points Z inside the circle. The other equilibrium positions, reversely, are unstable. For the common cases that arise in present practice n is small compared with Ω, the circle is imaginary, and the minimum values of XZ correspond to the positions of stable equilibrium.

§ (10) General Considerations on Gyroscopic Mechanics.—In considering the rationale of gyroscopic movements the student will be well advised to adhere very strictly to the precise correlation of cause and effect. Even when the spin is large and the gyroscopic torque is dominant the assignment of applied torque and consequent precession as proportional correlatives should be followed in definite detail. In abbreviated verbal reasonings the common use of the word "tendency" is fruitful of much misunderstanding. The "tendency" is usually a movement which is regarded as prevented by a constraint; and as the removal of the constraint would generally be followed by an altogether different movement the latent confusion is extreme.

Of certain well-established statements which must be quoted only with caution and understanding a few may be cited.

"A free gyro tends to keep the axis about which it spins, unaltered in direction." The fixity of direction, in space, of the gyro-axis is not peculiar to the spinning gyroscope, but belongs equally to the non-spinning gyroscope. Indeed any line in any freely balanced body, originally free from rotation, would behave in the same way. The practical effect of the high spin is that the large store of angular momentum demands a much larger value for the time-integral of any disturbing torque to produce an assigned amount of angular displacement; so that the casual disturbing forces, if identical in the two cases, will produce much less angular displacement in the axis of the gyroscope that is rapidly spinning.

"A rapidly spinning gyroscope offers great resistance to any attempt to alter the direction of its axis." It has been seen, § (8) (v.), that if a constraint supplies the gyroscopic torque the agent controlling the movement finds no gyroscopic opposition at all. A gyroscope mounted on a door, so as to spin in the plane of the panel, leaves the door as easy to open or shut as if the gyroscope were not spinning at all. It provides no "resistance," although its axis rotates with the door as the door swings. But a torsional stress (about a horizontal line of the door) is thrown on the hinges. If resistance is desired it can be secured (cf. § (9)) by giving the gyroscope two degrees of freedom relatively to the piece moved by the agent.

"Hurry on the precession and the top will rise in opposition to gravity." This is intended to apply to the slow precession of a top-heavy top leaning at an acute angle from the upward vertical (cf. § (8) (iv.)). The spin and precession circulate in the same sense, and the more general statement of § (6) (γ) covers what is really intended. But as the effect of the forward force applied is (for the case of rapid spin) to produce an angular velocity of erection only, and no sensible increase of the rate of precession at all, the precession is, after all, not "hurried." The meaning seems to be that an attempt is to be made which *would* hurry the precession if the dynamics were *not* gyroscopic. If it is desired actually to hurry the precession the force applied to the axis must be such as to increase the gravity torque.

It may happen on occasion that the effect of "hurrying the precession" is to make it slower, or even to reverse it. A simple example may be given. In the Wheatstone gyroscope the casing Q may be fitted with a weight so that in the absence of spin the axis z is statically vertical for either of two positions—one stable and the other (top-heavy) unstable. If the plane yz is tilted slightly, and the gyroscope spins, a steady motion of slow precession takes place. In the top-heavy case the effect of "hurrying the precession" will be to reduce the tilt, and so leave the rate of precession reduced in the same ratio; and if the precession is so much "hurried" as to annul and reverse the small tilt, then the ensuing precession is reversed in sense. Conversely, in order to produce an actual quickening of the precession it may be momentarily opposed.

The high speed given to gyroscopes in practice, with the consequent dominance of the gyroscopic torque, produces an experimental simplicity of effect which may even obscure the fundamental dynamics. It would appear, *ex. gr.*, that on loading one end of the horizontal axis of the gyroscope of a Wheatstone gyroscope the appropriate rate of precession ensues automatically, and that the axis revolves uniformly in the horizontal plane. If that were so, then neither the extra kinetic energy due to the precession, nor the new angular momentum about the vertical x, can be accounted for; for the attached load is supposed not to have sunk, and its weight has provided no moment about the vertical. If this precise state of motion is to be actually obtained the proper precession must be given initially to the piece P. Without this proviso the point Z (*Fig.* 2) describes in fact not a horizontal great circle but a series of small cycloids with their cusps on the circle, as if generated by a trace-point on a minute wheel rolling below the circle. The mean level of the point Z is below the circle by an amount that accounts for the kinetic energy of the mean rate of precession, and the mean tilt of the axis z gives a component of the spin-momentum about the vertical that is equal and opposite to the angular momentum of the mean precession. The same results in kind hold good if the axis z has a finite slope initially; and the case of a top abandoned with a rapid spin in an inclined position is another form of the same thing. The top appears to execute a steady motion with a precession that is not provided for it; but the actual motion of the axis includes a tremor that involves both a variable precession and a "nutation" in respect of the inclination of the axis to the vertical. (The decay of these rapid tremors, under dissipative forces of friction, etc., is usually very rapid.)

In practical mechanics the attempt is often made to deal with gyroscopic movements by sole reference to the gyroscopic torque, to the complete exclusion of the supplementary torques that are independent of the spin. On occasion the method suffices; but it may lead to meaningless or erroneous results. In the case, § (9), of the gyroscope with an axis free to revolve in a plane that is uniformly rotated, the gyroscopic torque presents a component in the plane itself which no reaction is available to supply. The equation of motion (6) loses all but its last term, and the problem is stultified. It may be further suggested that the dynamical possibilities involved in the use of gyroscopes with moderate spins may presently be developed.

The familiar demand for an "explanation" of the gyroscopic torque of a precessing gyroscope will be found to survive repeated discussions of the phenomenon so long as they are based on the principle of angular momentum; and apparently the demand can only be met (if it need be) by the avoidance of this fundamental sophistication. If the simple case of the movement of § (8) (iii.) is considered, and if the axis of the gyroscope is at the moment pointing east and precessing to the north, and the spin is from zenith to north, then any small portion of the top of the disc is moving northward; but on account of the precession it is located to the west of the meridian both before and after passing the summit, and so has a component acceleration to the west. The bottom of the wheel has similarly an acceleration to the east. Hence the pair of equal forces necessary to account for these accelerations constitute a couple that agrees with the known gyroscopic torque in its plane and its sense (cf. § (6) (δ)). If the mass of the wheel be regarded as replaced by four particles equal in mass and equally spaced round the rim, and two equal particles equally distant from the centre and placed on the axis, so as jointly to preserve the same mass and moments of inertia, the acceleration of each particle may be readily calculated and the forces found in detail. Briefly, if the rim-masses are each m, and the radius a, then $C=4ma^2$. The accelerations of the summit and bottom, to west and east respectively, are $2a\Omega\omega$ (cf. § (9) (iii.)), and all the other accelerations are purely radial. Hence the parallel and opposite forces are of magnitude $2ma\Omega\omega$, with an interval $2a$ between them, and thus give a torque $4ma^2\Omega\omega$, which is equal to $C\Omega\omega$. If the general movement of the gyroscope is considered, in place of this very special steady motion, the additional torques, dependent on the diametral moment-of-inertia A, can be similarly evaluated. The use of the artifice of equivalent particles thus enables the kinetic reaction of the gyroscope to be calculated by appeal to the information supplied by the dynamics of a particle.

G. T. B.

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Hardness and Abrasion Tests: General Considerations. See "Elastic Constants, Determination of," § (79).

Hardness Tests. See "Elastic Constants, Determination of."
  Brinell Test. § (80) (iii.).
  Depth Indicators. § (87).
  Indentation Method. § (80).
  The Ludwik Method. § (85).
  Measuring Microscopes. § (86).
  The Pellin Apparatus. § (90).
  The Shore Scleroscope. § (92).
  The Turner Sclerometer for Scratch Hardness Determinations. § (93) (i.).
  At Varying Temperatures. § (120).

Harker, 1904, comparison of gas thermometer with secondary standards of temperature in range 500° to 1600°. See "Temperature, Realisation of Absolute Scale of," § (39) (xi.).

Harmonic Motion. When the displacement of a point from its position of equilibrium is given by an expression of the form $a \sin (nt+\epsilon)$ the motion is said to be "harmonic."

Harrison's Theory of Lubrication. See "Friction," § (26).

Head Resistance. See "Ship Resistance and Propulsion," § (15).

HEAT, CONDUCTION OF

§ (1) Introduction.—When one end of a metal bar is heated, a change of temperature can very soon be detected along the bar, due to the passage of heat through the metal. This power of transmitting heat is possessed in varying degree by all material substances, and the phenomenon is known as the Conduction of Heat. It is usual to distinguish it from two other agencies by means of which heat is propagated and which are dealt with in separate sections in this work—namely, Convection and Radiation. In the case of convection the heat is transferred by the actual movement of the heated matter, gaseous or liquid, as distinguished from conduction, in which no transfer of the matter itself is apparent. In fluids both conduction and convection seem to be processes of diffusion. In the case of conduction the action is molecular, due to the diffusion through the substance at rest of its molecules, carrying with them their kinetic energy; in that of convection it is molar, brought about by the movement of small portions of the substance as eddies, or in other ways, thus transferring from point to point the kinetic energy of the molecules which go to form the eddy. Both conduction and convection are comparatively slow in their effects. Radiation, on the other hand, is the transfer of energy by etherial vibration and does not depend on the presence of the matter. Its velocity of propagation, whether in the form of waves of light, heat, or those used in wireless telegraphy, is, of course, enormous.

While it is convenient to adopt this classification of the agencies of heat propagation, it should be realised that the phenomena are probably closely related. For example, from the point of view of the molecular theory, it can be supposed that each molecule, as it becomes heated, affects those around it by radiation or by imparting some of its increased vibration to them by direct contact. The phenomenon of conduction might thus be regarded as a process of intra-molecular radiation or convection. If the analysis is carried further by adopting the modern theory of electrons, capable of diffusing through metallic bodies and conductors of electricity, but capable only of vibration in dielectrics, the mechanism of heat conduction is reduced to diffusion in metallic bodies and radiation in dielectrics. It would be out of place here to enter on a full discussion of the mechanism of heat conduction. The theory will be referred to only so far as is necessary to enable us to deal with the practical subjects treated below. These will include a description of some typical methods of measuring conductivity, together with a summary of the results obtained by various observers and also some applications of the theory to conditions of heat flow which are encountered in practice.

§ (2) Definitions.—The first to give a precise definition of conducting power was Fourier, whose monumental work, *Théorie analytique de la chaleur* (1822), forms the basis of all mathematical treatment of the problems of heat flow.

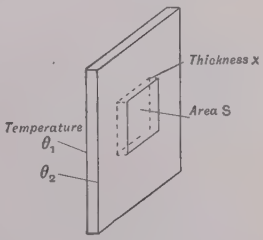

Fig. 1.

(i.) *Conductivity: Steady State.*—A clear conception is, perhaps, best obtained by considering a thin wall of material with parallel faces, one of which is maintained at a temperature θ_1 and the other at a temperature of θ_2 (*Fig.* 1). When the steady flow of heat has been established—that is to say, when the amount of heat flowing into the wall through one face is equal to that flowing out from the other, none being absorbed or given up by the intervening material—it can be shown that the quantity of heat passing through the wall is proportional to the difference in temperature between the faces. The quantity also varies inversely as the thickness x of the wall, and directly as the area S and the time t, so that we have the relation

$$Q = KSt\frac{\theta_1 - \theta_2}{x} \text{ or } KSt\frac{d\theta}{dx}, \quad . \quad . \quad (1)$$

where Q is the quantity of heat and K is a constant depending on the nature of the material of the wall and which is called the "Thermal Conductivity."

It is sometimes convenient to consider the thermal conductivity as being the ratio of the rate of heat flow per unit area to the fall of temperature per unit thickness. The latter quantity $(\theta_1 - \theta_2)/x$ or $d\theta/dx$ is called the "temperature gradient."

Following from the formula above, the unit of conductivity is defined, on the C.G.S. system, as the number of calories per second that would flow through every square centimetre of the surface of a wall of thickness 1 cm. and with a difference of 1° C. between its faces. Another unit commonly used by engineers is the British Thermal Unit[1] per hour, per square foot, per inch thickness, per 1° F. difference. To convert the C.G.S. unit into the other unit it is necessary to multiply by 2903.

In the simple case, which has been taken above, of a wall with parallel faces kept at uniform temperatures, the surfaces of equal temperature in the material, or "isothermal surfaces," will be parallel to the faces of the wall and the lines of heat flow will be straight lines perpendicular to the faces. If, however, the material is not in the shape of a plane wall, or if the conditions of heat supply are different from those assumed, the isothermal surfaces and the lines of flow may be curved and the problem of calculating the heat flow will generally become very complicated. For such cases it is convenient to rewrite the equation in the form

$$Q = Kt(\theta_1 - \theta_2) \times \text{shape factor}, \quad . \quad (2)$$

θ_1 and θ_2 being, as before, the temperatures of isothermal surfaces and the "shape factor" being a constant depending on their shape and relative disposition. The shape factor has been calculated for a few simple cases, such as a sphere heated centrally and a long cylinder heated axially, and approximate formulae have been obtained for some other common forms. These are dealt with in Part II. below.

(ii.) *Diffusivity: Variable State.*—The formulae given above apply only to the flow of heat when the steady state of temperature has been established throughout the body. If, however, the temperature of the body is changing at any point it is obvious that part of the heat inflow (or outflow) will be accounted for by the amount absorbed in raising (or liberated by lowering) the temperature of the body at that point, so that the heat flow will depend partly on the thermal capacity. As a matter of fact, in the equations governing the variable state the constant $K/c\rho$ (or thermal conductivity divided by heat capacity per unit volume) occurs. This constant has been called by Kelvin the "Diffusivity," while Clerk-Maxwell named it the "Thermometric Conductivity," since it measures the change of temperature which would be produced in a unit volume of the substance by the quantity of heat which flows in unit time through unit area of a layer of unit thickness having unit difference of temperature between its faces.

Kelvin's term is the more commonly used, and occasion will be found to refer to it further in dealing with problems arising out of the variable flow of heat.

I. METHODS OF MEASURING CONDUCTIVITY

In general the conductivity of a material is obtained under conditions of steady heat flow, and, as has been indicated above, it can be determined by measuring the heat flowing through a body in a certain time, the temperature at two or more points, and the dimensions of the body. The experimental conditions suitable in particular cases depend mainly on the nature of the material to be tested. It is convenient therefore to consider separately the methods applicable to Solids, Liquids, and Gases, and to subdivide the first mentioned under three main headings, namely, poor, medium, and good conductors. Crystalline or non-isotropic solids are also dealt with separately.

§ (3) SOLIDS: POOR CONDUCTORS (K = 0·00008 to 0·00030). — Under this head come the materials which are used for purposes of heat insulation: for example, in the walls of cold stores and the lagging of boilers. They include such materials as cork, charcoal, kieselguhr, magnesia, slag-wool, asbestos-wool, and so on. Before describing the methods of test, it is as well to get some idea of the structure of the materials and the mode of heat transfer.

The materials are not homogeneous solids in the physical sense, but are loose aggregations of matter with a large number of included air-spaces. Their structure may be either fibrous or, as in the case of cork, cellular, but more often the materials in question are artificial aggregates of small particles or fibres. The transmission of heat through such materials does not take place solely by conduction in the solid particles. The mass of these in a certain volume of the material is relatively small, and their fine state of division gives rise to a high thermal resistance due to the number of point contacts through which the heat has to pass.

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[1] The British Thermal Unit (B.T.U.) is the quantity of heat required to raise the temperature of 1 lb. of water by 1° F. at its temperature of maximum density, 39° F.

In addition to conduction in the solid material there is the transfer of heat by radiation and convection across the air-spaces. That the former is small can be seen by considering an air-space bounded by parallel walls at absolute temperatures of θ_1 and θ_2. The heat loss by radiation across such a space is independent of its thickness, and is proportional to $\theta_1^4 - \theta_2^4$, or where the difference is small to $\theta_1 - \theta_2$. If now a thin partition of conducting material parallel to the walls is interposed between them, it will take up a temperature intermediate between that of the walls (say, the mean temperature), and the radiation from the hot surface to the partition will be approximately one half of that previously passing between the hot and cold surfaces, and similarly, of course, the radiation from the partition to the cold surface. The effect of the partition is thus to reduce the heat transfer by radiation to about one half. Two partitions will reduce the radiation to about one-third, and so on.

If, for example, we take an air-space 3 inches thick, the temperature of one face being 10° C. and of the other 25° C., the heat loss by radiation for surfaces which are "full radiators" would be about twenty times the heat loss that would take place through the air by conduction only — that is assuming all movement of the air could be prevented so that no transfer took place by convection. Nineteen partitions would thus reduce the radiation loss to about the same amount as that due to pure conduction through the air. The equivalent subdivision of insulating materials is far greater than this.

A somewhat similar argument can be applied as regards convection, though the laws are not so well defined as those of radiation. For the case taken above of an air-gap of 3 inches with a temperature difference of 15° C., and supposing the walls vertical, the convection loss would be of the order of seven times the loss which would be due to conduction only. By subdividing the space the convection loss may be reduced, and if the air-gaps are made small enough to render air currents impossible the heat will be transferred solely by conduction.

Thus we see that the best insulating material is one which most nearly approaches in conductivity to air. By subdividing space by small particles of matter we can reduce the heat loss due to convection and radiation until it is not appreciably different from that of pure conduction through the air, but the introduction of solid matter inevitably increases the total of the conduction losses since all solids conduct better than air. Hence it follows that there is a limit of efficiency to be reached by introducing solid material. This is clearly shown in the results given by Petavel for the heat loss through slag-wool insulation of different densities (*Fig.* 2). It will be seen that the minimum value was obtained for slag-wool packed at 10 lbs. per cubic foot.

The conductivity of the class of poor conductors, or insulators, ranges in value from about 0·00008 to 0·00030 C.G.S. units, the best insulators having nearly twice the conductivity of air (0·00005). The values have a positive coefficient with rise of temperature which is frequently of the same order as that of air.

Some typical methods for testing such materials are described below.

(i.) *Spherical Shell Method — Nusselt.* — Theoretically the simplest form of solid, from the point of view of conductivity measurements, is the sphere or spherical shell. The flow of heat from a source at the centre is symmetrical and follows a simple mathematical law. On the other hand the use of a sphere generally gives rise to considerable experimental difficulties and the material has to be made up into a special shape.

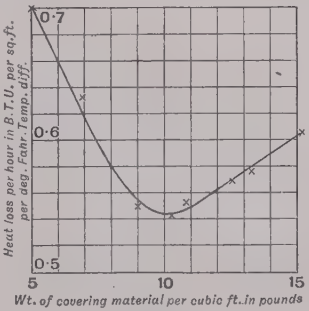

FIG. 2.

However, the method has been successfully adopted by several experimenters, of whom Nusselt[1] may be taken as an example. In his apparatus the shell of material was contained between two spheres of metal. The inner one of copper, 15 cm. in diameter and 1·5 mm. thick, was split into hemispheres which were joined with a bayonet joint so as to give a smooth exterior. The outer sphere was also split into two halves held together by a flanged joint with bolts. It was of zinc 1 mm. thick and either 60 or 70 cm. in diameter. The zinc sphere was coated with paint and its temperature excess above the atmosphere did not exceed 15° C.

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[1] *Forsch. Ver. d. Ing.*, 1909, Hefte 63 and 64.

The space between the spheres was filled with the material under test, and a constant supply of heat was maintained electrically inside the inner sphere. The temperatures at different points in the material were obtained by means of a number of thermocouples of iron—constantan. Care was taken to ensure that the wires of each couple for a few centimetres from the junction were led along the isothermal plane so as to avoid errors due to conduction in the wires themselves. The importance of this precaution was shown by means of an experiment with infusorial earth, in which the junctions of two thermocouples were embedded in the material at equal distances from the centre of the sphere: that is to say, in the same isothermal plane. In one case the wires from the junction were led away along the radius, and in the other at right angles to the radius (*i.e.* along the isothermal plane). The respective readings obtained were 74·9° C. and 116·3° C.

To obtain the temperature distribution in the material, couples were fixed along three radii at right angles to each other, four couples being arranged symmetrically on each radius. In addition to these 12 couples others were fixed at different points so that the distribution of temperature was studied with some care.

When the steady state of temperature had been reached it was sufficient to know the energy supplied to the central sphere, the temperature of two isothermal surfaces. The conductivity is then obtained from equation (2) above, the value of the shape factor being in this case $2\pi ab/(b-a)$, where a and b are the internal and external radii of the shell.[1]

Nusselt used the apparatus for testing a number of insulating materials such as infusorial earth, slag-wool, asbestos, powdered and slab cork, charcoal, sawdust, wool, silk, cotton. The temperature range extended from that of the atmosphere to 500° C. and the results are stated to be accurate to 1 or 2 per cent.

The packing of the material and the arranging of the thermocouples must have been far from easy, and the apparatus does not therefore lend itself to studying the effect of different densities of packing on the conductivity. Nusselt's results are, however, in very fair agreement with those obtained with other observers, as will be seen from Table I.

(ii.) *Cylindrical Shell Method—Lamb and Wilson.*—A cylindrical shell of material with plane ends was adopted by Lamb and Wilson[2] for the purpose of determining the conductivity.

Their apparatus is illustrated in *Fig.* 3. The material was contained in the space between two cylindrical copper pots kept at a definite distance apart by pieces of vulcanised fibre. The inner pot contained a small motor with a fan attached to the axis. A tinplate cylinder, open at the top and with holes at the bottom, was put inside to direct the currents of air over the inner surface of the inside pot, in the direction of the arrows. Energy was supplied electrically to a heating coil within, as well as to the motor; this constituted an internal supply of heat, which maintained the temperature within the pot at any determined upper limit. The motor and heating coil were connected in series, and leads were carried through a small hole in the lid of the pots to measure the current and the potential difference, and thus the power expended on internal heating. The outer pot was immersed in a tank kept overflowing from the water main, the lid of the pot being made into a sort of saucer, into which the incoming water ran. In this way the surface of the outer pot was kept at a uniform and constant temperature.

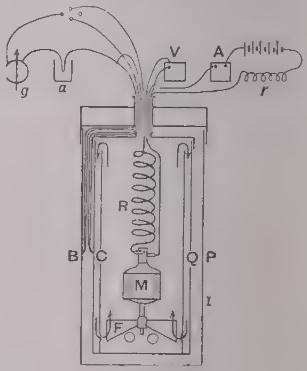

FIG. 3.

The outer pot was 8 inches in diameter and 16 inches high, and there was a clearance of 1 inch between the two pots. The temperatures of the pots were measured by thermojunctions of copper and iron soldered at the points B and C.

Preliminary experiments with couples soldered to different parts of the copper pots had shown that the temperature distribution was constant within 0·5° C., when working with a temperature difference of 25° C., hence the two thermocouples at B and C were relied on to give the temperature difference.

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[1] See "Heat, Conduction of," Mathematical Theory, Eq. (32*a*).

[2] *Proc. Roy. Soc.*, 1899, lxv. 285.

The shape factor in this case was taken as that of a cylindrical shell of the length of the inner pot plus that of the plane ends, taking their areas as the mean of the end surfaces of the inner and outer pots. This gave a value for the shape factor

$$2\pi\left(\frac{l_1}{\log_e(r_2/r_1)}+\frac{r_1^2+r_2^2}{l_2-l_1}\right),$$

where l_1 and l_2 are the lengths, and r_1 and r_2 the radii of the inner and outer pot respectively. The formula is an approximation, but experiments with 2-inch layers gave results consistent with those for 1-inch layers, so that it may be taken as sufficiently accurate.

TABLE I

POOR CONDUCTORS

The thermal conductivity K is given in the Tables below as the number of calories per sq. cm. per sec. per 1 cm. thickness per 1° C. difference in temperature.

Substance.	Temperature. ° C.	$K\times10^4$. C.G.S. Units.	Authority.
Asbestos	0	3·61	Nusselt, 1909.
	100	4·64	
	600	5·67	
	500	1·64-2·14	Randolph, 1912.
	30	2·97	Lamb and Wilson, 1899.
Charcoal	0	1·39	Nusselt, 1909.
	100	1·75	
	30	1·50	Lamb and Wilson, 1899.
„ (dry)	0	1·20	National Physical Laboratory, 1920.
Cork (powdered)	0	0·86	Nusselt, 1909.
	100	1·33	
„ (fine granulated)	0	1·04	National Physical Laboratory, 1920.
„ slab (cork and pitch)	0	1·69	Nusselt, 1909.
	0	1·12	National Physical Laboratory, 1920.
Cotton waste	0	1·31	Nusselt, 1909.
	100	1·64	
	100	1·10	Randolph, 1912.
Eider down	100	1·10	
Kapok (loose)	30	1·22	Lamb and Wilson, 1899.
„ (tight)	30	1·44	
Hair-felt (sheets)	30	1·06	Lamb and Wilson, 1899.
	20	0·8	Niven and Geddes, 1912.
Infusorial earth or Kieselguhr (loose)	0	1·44	Nusselt, 1909.
	100	1·83	
	400	2·19	
	0	1·95	National Physical Laboratory, 1920.
„ „ (baked lagging)	0	1·78	Nusselt, 1909.
	400	3·33	
Kieselguhr and asbestos	100	2·48	Randolph, 1912.
Magnesia asbestos composition	20	1·50	National Physical Laboratory, 1920.
Paper crumpled	30	1·67	Lamb and Wilson, 1899.
Sawdust	50	1·53	Nusselt, 1909.
„ (pine)	30	2·42	Lamb and Wilson, 1899.
„ (fir)	20	1·72	Niven, 1905.
Silk (loose)	0	1·06	Nusselt, 1909.
Slag (loose blast furnace)	50	2·64	
Slag wool (13 lbs. per c. ft.)	0	1·04	National Physical Laboratory, 1920.
	30	1·51	Lamb and Wilson, 1899.
Wool	0	0·92	Nusselt, 1909.
	100	1·39	
„ (density 0·02)	100	1·10	Randolph, 1912.
„ („ 0·2)	100	0·54	

The temperature difference in these experiments was about 20° or 30° C., and the results are summarised in Table I.

As in the case of the spherical shell method, the configuration of the apparatus makes it difficult to secure uniformity of packing, a point of some importance in determining the conductivity of many insulating materials. It is not clear whether the question of heat loss by conduction along the wires leading from the inner pot was considered.

(iii.) *Wall Methods*—(*a*) *Randolph.*—In using the wall method it is assumed that the lines of heat flow are perpendicular to the faces. This is true for the central portion of a wall, but at the edges, unless the leakage of heat laterally can be prevented, the lines of flow will be curved. The "edge effect" thus produced gives rise to troublesome corrections in applying the equation (1) above, and it only becomes negligible if the wall is very large compared to its thickness. The difficulty can, however, be surmounted by the use of the guard-ring principle, the effect of which is to isolate a central portion of the wall where the heat flow is approximately normal to the surfaces.

An example of an apparatus of this kind is that designed by Randolph[1] for measuring the conductivity of insulating materials.

He took a hot plate heated electrically by means of a resistance ribbon wound on mica. The top of this plate is of copper, 7½ inches in diameter and ¼ inch thick, and its temperature is measured by means of couples of platinum and platinum-iridium. The material to be tested, which varied in thickness from ½ inch to 2½ inches, is contained between this hot plate and a water-cooled plate. The latter is in two parts, the central one forming a calorimeter and the outer one a guard-ring. A stream of water is maintained in the calorimeter, and its rate of flow multiplied by the difference in temperature between the inflow and the outflow gives the quantity of heat received by the central disc. A similar stream of water flows through the guard-ring, and its temperature never differed by more than 0·1° C. from the water in the calorimeter. With this small difference in temperature between contiguous portions of the guard-ring and calorimeter, and as there was no metallic connection between the two, there could be no appreciable passage of heat from one to the other. All the heat reaching the calorimeter was thus received directly from the hot plate, and the lines of flow were approximately normal to the surfaces. The conductivity was obtained from equation (1), knowing the temperatures of the hot and cold plates and the thickness of the material.

Randolph used this apparatus for obtaining the conductivity of a number of insulating materials such as asbestos and various asbestos compositions, mineral wool, diatomaceous earth, wool, cotton, eiderdown. A summary of his results will be found in Table I. It is interesting to note that he adopted a standard of compression of the materials which were packed as closely as possible by tapping and were then compressed by 25 per cent. The measurements were easily made with an accuracy of 2 per cent, but variations of 5 per cent were found between different samples of the same material. The cold plate was approximately at atmospheric temperature, while the hot plate ranged from 100° to 600° C. It may be remarked here that the flow calorimeter is not suitable for dealing with small temperature gradients of, say, 10° or 20° C. in the case of insulating materials, since the heat passing through the material would be so small as to give only a minute rise in temperature of the flow. Further, the leakage to or from the surroundings might become comparable with that passing through the test material and thus give rise to considerable errors.

Some of Randolph's values show a tendency to be low, for example, that for wool fibres (at density 0·192) as being 0·000054, or equal to air in conductivity.

(*b*) *National Physical Laboratory.*—In the apparatus described above the heat flowing out through the cold face of the wall of material was measured, but the practice is sometimes adopted of determining the heat supplied to the hot face. This is conveniently done by arranging an electrically heated plate as a sandwich between two exactly similar walls of material. The total energy supplied can easily be measured, and it passes out in approximately equal amounts through the two walls.

Figs. 4 and 5 show an apparatus of this type which is in use at the National Physical Laboratory.[1] The hot plate consists of flat heating units of asbestos-covered wire, or wire wound on mica, which are clamped between sheets of brass or aluminium. The plate is divided into two portions—a central square area A, 3 feet by 3 feet, and a guard-ring B,

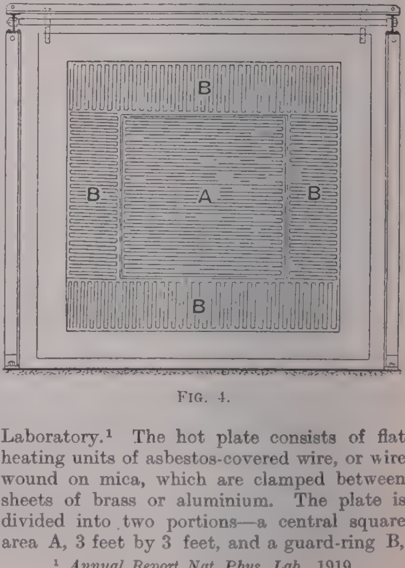

FIG. 4.

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[1] *Trans. American Electrochem. Soc.*, 1912, xxi. 545.

[1] *Annual Report Nat. Phys. Lab.*, 1919.

1 foot wide, surrounding it. These portions are separated by an air-gap, and are kept apart by four small wedges of wood or fibre. They are independently heated, and the energy supplied to each is adjusted so that the temperatures of the contiguous edges are the same. In order to facilitate this adjustment the energy supplied to the top, bottom, and sides of the guard-ring could be separately controlled (see *Fig.* 4). Under these conditions the heat flows away from the central portion in lines perpendicular to the surface. The outer surface of each wall of the material is in contact with an iron plate, which can be maintained at a constant temperature. This is effected by the circulation of water or brine through a coil of square-section lead pipe, which is clamped on to the back of the plate (*Fig.* 5). The plate and coil are contained in a wooden case packed with insulation, which is not shown in the figure. The coil is wound in a bifilar manner, so that inflow and outflow pipes alternate, and thus the temperature is uniform over the whole surface. The cold plates can conveniently be run at any temperature from −25° C. to +15° C. The temperature distribution is studied by means of a number of thermocouples let into grooves in the hot and cold plates. By taking due precautions the plates can be kept uniform in temperature to within about 0·01° C.

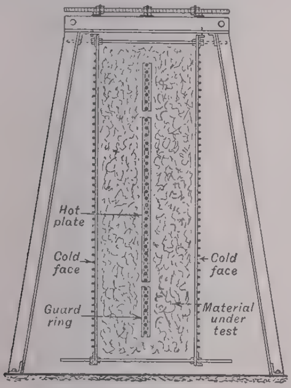

Fig. 5.

By means of this apparatus walls of insulating material have been tested under conditions corresponding closely to those prevailing in cold stores. Some of the results obtained are given in Table I.

§ (4) Solids, Medium Conductors (K=0·0003 to 0·01).—The division between the poor and medium conductors is purely one of convenience. All non-metallic bodies not included in the former class fall into the latter. Examples of the medium conductors are the various kinds of woods and other organic substances, the refractory materials and bricks used in furnace construction, rocks and mineral substances. The limits of conductivity may be said to be from 0·0003 to 0·01 C.G.S. units. No general rule can be laid down as to the change of value with temperature. As may be expected, the more granular materials show an increase in conductivity with rise of temperature, while the homogeneous solids generally show a decrease.

(i.) *Cylindrical Method.*—A number of experimenters have used a solid cylinder of material heated axially to measure the conductivity. The heat is generally supplied by a wire along the axis, and the temperature is measured at two or more points at different distances from the axis. If Q is the quantity of heat generated per unit length of the heating wire, then

$$Q=\frac{2\pi K(\theta_1-\theta_2)}{\log_e(r_2/r_1)}.$$

We may take as examples the experiments of Niven, Poole, Clement and Egy.

(*a*) *Niven.*—The conductivity of a number of materials such as wood, sawdust, sand, plaster of Paris, and garden mould was measured by Niven.[1]

A general view of the apparatus is shown in *Fig.* 6, while the arrangement of the wires for heating and measuring temperature are shown in *Fig.* 7. For the central heating wire CD, platinoid of gauge 34 or 40 was used. The wires AA and BB were of platinum. They were prepared by coating a platinum wire with copper and drawing down till the platinum had a thickness of 0·035 mm. and the copper 0·27 mm. The latter was then dissolved off. The change of resistance of

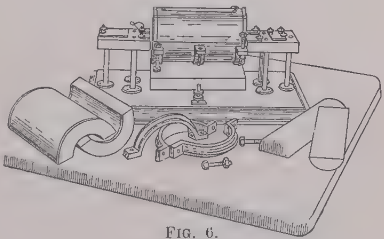

Fig. 6.

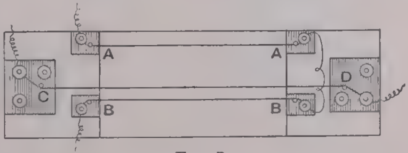

Fig. 7.

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[1] *Proc. Roy. Soc.*, 1905, lxxvi. 34.

these wires gave the difference in temperature between the isothermal surfaces, whose distances from the axis could be accurately measured. Niven experienced considerable difficulty in the use of these fine wires, and eventually abandoned them in favour of thermo-junctions consisting of German silver wire and iron 0·2 mm. in diameter.

When solid material was being tested in this apparatus, it was in the form of two half cylinders. One of these was moved up underneath the wires until they rested on its face, and the other half-cylinder was then fixed on top. For loose materials cylindrical wooden shells were used as containers.

Niven gives an approximate calculation as to the time taken for the flow of heat to reach the steady state. Assuming the temperature to be taken at two points respectively 1 and 3 cm. from axis, and taking sand which has a diffusivity of about 0·0032, the time taken for the temperature difference to reach within 5 per cent of its final value was 1½ hours, and within 1 per cent of its final value 7½ hours.

A summary of Niven's results is included in Table II.

Table II

Medium Conductors

Substance.	Temperature. °C.	$K \times 10^4$. C.G.S. Units.	Authority.
Woods			
Ash	20	3·6	Niven and Geddes, 1912.
Beech	20	3·7	Niven, 1905.
Fir	20	2·64-3·94	
Mahogany	20	3·42	Niven and Geddes, 1912.
Norwegian pine	20	3·18	
Oak	20	5·01	
Teak	20	3·97	
Refractories			
Fireclay Bricks—			
$66\%\ SiO_2 : 29\%\ Al_2O_3$ $4\%\ Fe_2O_3 : 0{\cdot}5\%\ CaO$ burnt at 1050° C.	230-1160	35·0	Wologdine, 1909.
Ditto (burnt at 1300° C.)	230-1000	42·0	
$66\%\ SiO_2 : 31\%\ Al_2O_3$ $1\%\ Fe_2O_3 : 1\%$ Alk burnt at 1330° C.	600	30·5	Dougill, Hodsman, and Cobb, 1915.
	1000	40·5	
$53\%\ SiO_2 : 43\%\ Al_2O_3$ $2\%\ Fe_2O_3 : 1{\cdot}5\%$ Alk	100	16·9	Boyd Dudley, 1915.
	1000	33·9	
Silica Bricks—			
$94\%\ SiO_2 : 1\%\ Al_2O_3$ $2\%\ Fe_2O_3 : 2{\cdot}0\%\ CaO$ burnt at 1050° C.	100-1000	20·0	Wologdine, 1909.
Ditto (burnt at 1300° C.)	150-930	31·0	
$95\%\ SiO_2 : 2\%\ Al_2O_3$ $1{\cdot}1\%\ Fe_2O_3 : 1{\cdot}5\%\ CaO$	400-1200	36·0	Dougill, Hodsman, and Cobb, 1915.
$96\%\ SiO_2 : 0{\cdot}9\%\ Al_2O_3$ $0{\cdot}8\%\ Fe_2O_3 : 1{\cdot}8\%\ CaO$	100	22·0	Boyd Dudley, 1915.
	1000	42·5	
Magnesite Bricks—			
$92\%\ MgO : 5\%\ SiO_2$ $1{\cdot}6\%\ Fe_2O_3 : 1{\cdot}7\%\ CaO$	320-560	151·0	Dougill, Hodsman, and Cobb, 1915.
	700-1400	91·0	
$86{\cdot}5\%\ MgO : 2{\cdot}5\%\ SiO_2$ $7{\cdot}0\%\ Fe_2O_3 : 2{\cdot}7\%\ CaO$	450-830	135·0	Boyd Dudley, 1915.
Diatomite Bricks—	100	3·10	National Physical Laboratory, 1916.
	500	4·61	
Rocks			
Basalt	100	40·0	Poole, 1914.
	400	39·5	
	600	39·5	
	0	31·7	R. Weber, 1880.
	20-100	52·0	Hecht, 1904.
Chalk	..	22·0	Herschell, Ledebour, and Dunn, 1879.

TABLE II—*continued*

Substance.	Temperature. °C.	$K \times 10^4$. C.G.S. Units.	Authority.
		Rocks—continued	
Granite	100	57·0	Poole, 1914.
	500	37·0	
	. .	75·0-97·0	H. F. Weber, 1911.
Lava	16-100	21·0	Morano, 1898.
Limestone	100	50·0	Poole, 1914.
	350	32·0	
Marble	. .	78·0	Hecht, 1904
	. .	82·0	H. F. Weber, 1911.
	. .	71·0	Lees, 1892.
Pumice	50	5·56	Nusselt, 1909.
	. .	6·0	Herschell, Ledebour, and Dunn, 1879.
Slate	94	35·7	Lees and Chorlton, 1896.
		Miscellaneous	
Cement (Portland)	90	7·1	Lees and Chorlton, 1896.
Clay tile	20	17·0	Niven and Geddes, 1912.
Ebonite	35	4·2	Lees, 1898.
Glass (plate)	20	19·2	Niven and Geddes, 1912.
„ (22-oz. window)	35	24·5	Lees, 1898.
Garden mould (dry)	20	4·8	Niven, 1905.
Leather	20	3·29	Niven and Geddes, 1912.
Linoleum	20	3·51	
Naphthalene	35	9·5	Lees, 1898.
Paper	20	3·27	Niven and Geddes, 1912.
Paraffin wax	20	6·65	
Plaster of Paris	20	10·7	Niven, 1905.
„ „ (made up)	20	12·0	B.A. Committee.
Rubber	. .	4·5	Lees, 1892.
Sand (fine)	20	6·5	Niven, 1905.
Sulphur	20	6·15	Niven and Geddes, 1912.
	35	6·7	Lees, 1898.
Shellac	35	5·8	Lees, 1898.
Vulcanite	20	4·21	Niven and Geddes, 1912.

It is not clear from the description given whether any special precautions were taken to guard against "end effect" due to the conduction of heat at the ends of the heating wire and cylinder. Apart from this point the method does not seem to be open to criticism.

(*b*) *Poole.*[1]—This experimenter made use of a cylinder of material for measuring the conductivity of rocks. The cylinder was heated axially by means of a wire dissipating a known amount of energy, and it was contained inside an electric furnace maintained at a constant temperature which ranged up to 600° C. Several modifications of apparatus were used, one of which is illustrated in *Fig.* 8.

The furnace consisted of a copper tube wound with insulated nichrome wire and contained in a lagged case. The specimen cylinder of rock A fitted roughly into the furnace, the ends of which CC were filled with pulped asbestos millboard. The cylinder was 15 cm. long by 3·6 cm. diameter and had a hole bored down its axis. With a soft rock such as limestone, a hole as small as 1·5 mm. diameter could be bored, but with hard rocks like granite or basalt the hole was nearer 4·5 mm. in diameter. In the latter cases the hole was either filled in with cement, through which a smaller hole could then be bored, or the platinum heating wire (0·3 mm. in diameter) was wound with asbestos cord to keep it central in the hole. A constant current was passed through the wire, and the potential drop was measured on the central portion 10 cm. in length. By using a

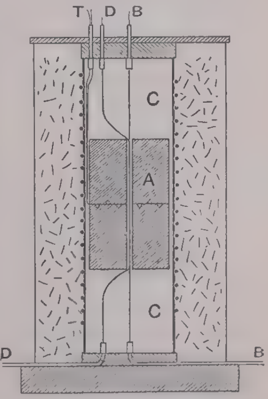

FIG. 8.

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[1] *Phil. Mag.*, 1912, xxiv. 45, and 1914, xxvii. 58.

cylinder relatively long compared to its diameter, and by measuring the energy dissipated in the portion of the wire well within the cylinder, the troublesome end effect found in preliminary experiments was eliminated. The difference in temperature of two isothermal surfaces and the mean of the temperatures of the surfaces were measured by means of couples of platinum and platinum-iridium 0·1 mm. in diameter. The couples were all in the central plane of A perpendicular to the axis, the cylinder being cut in half at this point. The arrangement of the couples is shown in *Fig.* 9, which is a plan of the lower half of the cylinder. The junctions of the couples are fixed in circular grooves cut in the face, the grooves being connected by radial grooves for carrying the wires between the circular grooves and for carrying them out of the cylinder. It will be seen that the couples are arranged so that the mean E.M.F. gives the mean difference in temperature of the circular grooves. The diameters of the latter were respectively 2·61 cm. and 0·725 cm. They are V-shaped, 2·5 mm. deep and 1·5 mm. wide, while the radial grooves were 0·5 mm. deep. The couple wires were cemented in the bottom of the grooves, and the two half-cylinders were cemented together. In the right-hand half-cylinder a groove was cut at a distance from the axis equal to the geometric mean between the radii of the grooves on the opposite face. A single couple, fixed in this groove, thus served to give the mean temperature of the isothermal surfaces. The temperature difference between these isothermals was of the order of 10° C., while the mean temperature of the whole apparatus could be fixed at any point by an automatic control. The measurements were made when the steady state had been approximately reached, and a method is given for correcting for small departures from the steady flow. Poole also made use of a cylinder cut into two parts longitudinally instead of transversely. This has the advantage of enabling the axial groove to be made very small and accurate. A similar arrangement for determining the difference in temperature between isothermal surfaces is adopted, the junctions of the couples being, of course, in a longitudinal plane instead of a transverse plane as before.

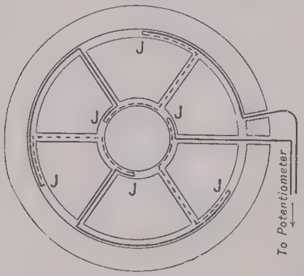

FIG. 9.

Poole made a study of the variation of conductivity of limestone, granite, and basalt by means of his apparatus. He shows that the two former give decreases of conductivity with rise of temperature, while basalt shows a slight rise to about 200° C., above which the conductivity is constant. On prolonged heating all three materials show a permanent fall in conductivity, due apparently to small cracks in the rocks.

(*c*) *Clement and Egy.*—A similar method to those described immediately above was used by Clement and Egy[1] for obtaining the conductivity of fireclay bricks. The specimens were in the form of hollow cylinders heated by a coil of nickel wire wound on a porcelain tube fixed along the axis of the specimen. Two holes were bored parallel to the axis for the insertion of thermocouples.

The method has the disadvantage that specimens were required of special form differing from that in which the material is used, and there would seem to have been some uncertainty in the measurement of the radial distances.

(ii.) *Wall or Slab Methods.*—A number of observers have determined the conductivity of materials of medium conductivity by experiments on walls or slabs. In the important class of refractory materials, the methods adopted by Wologdine, Dougill, Hodsman and Cobb, Boyd Dudley, and Griffiths are described, while for other materials the work of Lees is referred to.

(*a*) *Wologdine.*—An extended investigation of the thermal conductivity of refractory materials of French manufacture was made by Wologdine.[2]

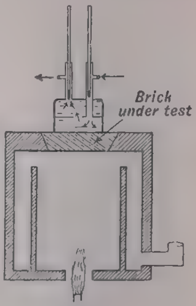

FIG. 10.

A diagram of the apparatus he employed is shown in *Fig.* 10. The specimen under test was specially made in the form of a circular disc, 5 cm. in thickness, with a bevelled edge forming a portion of the walls of a gas-heated muffle. The temperature gradient through the slab was obtained by embedding two platinum, platinum-rhodium couples at a depth of 5 mm. from each face. An additional thermo-element in the air near the hot face gave the furnace temperature.

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[1] *Phys. Rev.*, 1909, xxviii. 71 ; *Bull. Univ. Illinois*, 1909, xxxvi.

[2] *Met. and Chem. Eng.*, 1909, vi. 383, 433, translated by Queneau.

The heat transmitted through the slab was measured by a flow calorimeter in the form shown in the figure.

The design of the calorimeter is the weakest point in the method, since the presence of a cooled area on the upper surface must distort the flow lines from normal. A guard-ring around the calorimeter would have eliminated this source of error.

The couples were calibrated by reference to the freezing-points of copper, common salt, and aluminium, together with the boiling-points of sulphur and naphthalene. With the exception of sulphur, the values assumed for these fixed points are in fair agreement with the values accepted at the present time. For the boiling-point of sulphur the value 455° was taken instead of the generally accepted value 445·5°.

In the case of each material, samples were fired at two temperatures, 1050° C. and 1300° C., and it was found that the conductivity increased with the firing temperature. Thus silica bricks fired at 1300° C. showed a value 50 per cent greater than those burnt at 1050° C. Wologdine also found that thermal conductivity increased with temperature, except possibly in the case of chromite brick, for which the value was nearly constant.

(*b*) *Dougill, Hodsman, and Cobb.*—The thermal conductivities of some fire-clay, silica, and magnesia bricks were investigated by Dougill, Hodsman, and Cobb.[1] Their apparatus was somewhat similar to Wologdine's, but designed to take ordinary sized samples. The heat transmitted was measured by the evaporation of water from a vessel cemented on to the top surface of the brick. This calorimeter was composed of a large vessel, 9 inches long by 4½ wide by 3 inches high, with a plane base. Within the larger vessel was fixed the calorimeter proper, consisting of a chamber 4 inches long by 2 inches wide by 2½ inches high. Steam distilling from this inner vessel passed through a sloping tube to a condenser, the water being collected in a measuring jar.

The usual rate of evaporation was about 5 c.c. per minute. The authors state that it was found advantageous to surround the inner chamber with a non-conducting jacket, and for this purpose sheet rubber was found to be satisfactory.

Probably it would be better if, in this method, the calorimeter was entirely separated from metallic connection with the larger vessel, since the existence of a slight gradient of temperature across the metal base might seriously vitiate the results.

An interesting fact brought out by the investigation is the marked decrease with temperature in the thermal conductivity of magnesia brick.

(*c*) *Boyd Dudley.*—Experiments on fire-clay, silica, and magnesia bricks of American manufacture were carried out by Boyd Dudley.[2] The material under test constituted one wall of a furnace heated by means of by-product coke. By careful regulation of the draught a uniform combustion through a depth of 26 inches was obtained. The bricks composing the wall were 9 inches by 4½ inches, and built so as to produce a flat surface. The heat transmitted through a definite area of the wall was measured by a flow calorimeter 8 inches by 8 inches, fitted with a guard-ring. A period from eight to nine hours was allowed to elapse after starting the fire before observations were commenced.

The temperature gradient through the bricks was obtained by means of a platinum, platinum-rhodium couple inserted to various depths in holes bored to within ¼ inch of the inner face, measurements being made every ¼ inch to within ¼ inch of the cold face.

This method of ascertaining the temperature at various points is simple, but suffers from the disadvantage that the couple wires are perpendicular to the isothermal planes and the readings may be vitiated by conduction along the wires cooling the junction.

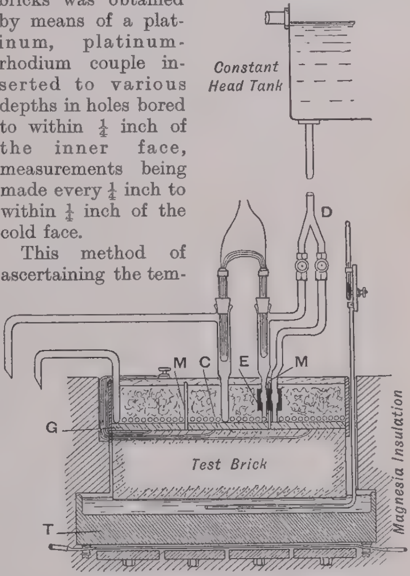

FIG. 11.

Dudley found that the conductivity of magnesite was practically constant between 445° and 830°; which is contrary to the results of Dougill, Hodsman, and Cobb.

(*d*) *Griffiths.*—An apparatus used by Griffiths[3] for measuring the conductivity of slabs of refractory materials is illustrated in *Fig.* 11. The faces of these materials are always irregular and are generally slightly curved, so that it is difficult to obtain a uniform surface temperature. With a view to meeting the difficulty the heat was transmitted to the hot face of the brick by means of molten metal, which adapts itself to the irregularities of the surface. Tin was found to be satisfactory for the purpose at low temperatures. It was contained in the cast-iron tray T, which has a uniformly distributed heating coil attached to its under surface. The depth of the molten metal is about 1 cm., and the brick under test rests on three projections, so that the free surface of the tin around the edges stands at a level of about 2 mm. above the immersed surface of the brick.

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[1] *Journ. Soc. Chem. Ind.*, 1915, xxxiv. No. 9, 465.

[2] *Met. and Chem. Eng.*, 1915, p. 315.

[3] *Trans. Faraday Soc.*, 1917, xii. 109.

The temperature of the molten tin was obtained by means of a platinum-platinum-iridium couple. The details of the construction of this couple are shown in *Fig.* 12. The wires were separately insulated in the quartz tubes, which are enclosed in an outer sheath, the external diameter of which was approximately 5 mm. The bulb B was sealed off in the oxycoal-gas blowpipe after the insertion of the wires, and no difficulty was found in bending the tube into the L form with the wires *in situ*, provided the heating was not unduly prolonged. By rotating the couple the temperature distribution over a wide arc could be explored, and the insulation of the quartz was sufficiently good to prevent any leakage from the heating circuit into the potentiometer.

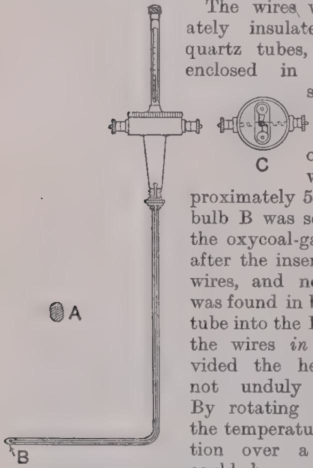

FIG. 12.

The temperature of the cold surface of the brick was determined by several iron-eureka (constantan) thermocouples bedded just flush with the surface. The calorimeter was attached to this surface by means of a thin layer of Purimachos cement.

The construction of the flow calorimeter will be understood from *Fig.* 11. The guard-ring G is separated from the calorimeter proper C by a narrow gap about 1 mm. wide, the space being filled with mica plates M set on edge so as to produce a level surface. The inflowing water divides at D, and flows through adjacent turns of piping in the calorimeter and the guard-ring. Since the temperatures of the two streams are the same, this tends to eliminate any gradient of temperature across the gap separating the calorimeter from the guard-ring. The ratio of flow was adjusted to produce equal temperature rise. Differential couples of six pairs of copper-constantan measured the rise in temperature in the water flowing through the calorimeter. The tube containing the inflowing couple is separated from metallic contact with the calorimeter by a short connecting piece of ebonite E. A steady stream of water is obtained by the constant-level tank shown, and the rate of flow through the calorimeter determined by the time required to collect one litre of water.

In order to detect any systematic errors the experimental conditions were varied by altering the rate of flow, changing the differential couples, and the thickness of slab experimented on.

In experiments at temperatures above 450° the hot face was maintained at a uniform temperature by pressing it against a flat iron plate forming a portion of the wall of a large uniformly heated muffle. In this case the hot-face temperature was determined by two bare nickel-copper, nickel-chromium couples of No. 19 gauge wire cemented on the surface.

Data for the same sample over the same temperature range obtained by two different modes of heating were in close agreement; the molten metal bath method giving more concordant results than the muffle method.

(*e*) *Lees.*—Lees [1] has determined the conductivity of small thin slabs of material by means of a somewhat novel method. His arrangement is shown diagrammatically in *Fig.* 13. A small hot plate is formed of two copper discs C and U, between which is sandwiched a heating coil of platinoid insulated by mica. The thickness of the coil with the insulation was ·11 cm., the thickness of C ·103 cm., and U ·312 cm.

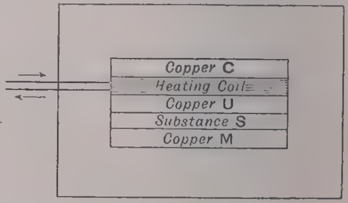

FIG. 13.

S was the substance whose conductivity was to be measured, while M was another copper disc ·32 cm. thick. The contacts of S with U and M were improved by smearing the surface with glycerine and sliding them together. The diameter of all the discs was 4 cm. At opposite ends of a diameter of each copper disc were soldered a platinoid and copper wire which formed a thermojunction for the measurement of temperature.

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[1] *Phil. Trans. Roy. Soc.*, 1898, cxci. 399.

When the discs had been assembled they were varnished to give them the same emissivity, and the pile was suspended in a constant temperature enclosure. The current was switched on in the heating coil, and after a due time had been allowed for the steady state to be reached, the energy was measured and the temperatures of the three copper discs.

The theory may be briefly stated as follows:

Let H = rate of energy supplied to the heating coil,
h = heat loss per second per sq. cm. for 1° excess, of temperature of the discs over that of the enclosure,
v = the excess of temperature over that of the enclosure,
t = thickness of disc,
r = radius of the discs.

Now the heat received per second by the disc M and given up to the air is

$$\pi r^2 h\left(1+\frac{2t_M}{r}\right)v_M.$$

Similarly the heat received by S and given up to the air from its exposed surface or passed on to M is

$$\pi r^2 h\left(1+\frac{2t_M}{r}\right)v_M+\pi r^2 h\frac{2t_S}{r}\left(\frac{v_M+v_U}{2}\right).$$

If K is the thermal conductivity of the disc S, then the heat flowing through the disc is

$$\pi r^2 K\left(\frac{v_U-v_M}{t_S}\right).$$

Assuming that the heat flowing through the disc to be the mean of the heat flowing into and out of the disc, *i.e.* the mean of the first two quantities above, we get

$$K\left(\frac{v_U-v_M}{t_S}\right)=h\left\{v_M+\frac{2}{r}\left(\overline{t_M+\frac{t_S}{4}}\cdot v_M+\frac{t_S}{4}v_U\right)\right\}.$$

Which gives us a relation between K and h. Further, the total heat imparted to the system must equal that given up to the air from all the exposed surfaces, and therefore we get the equation

$$H=\pi r^2 h\left\{v_M+v_C+\frac{2}{r}\left(t_M v_M+t_S\cdot\frac{v_M+v_U}{2}+t_U v_U+t_C v_C\right)\right\}.$$

From these two equations the values of K and h can be determined by measuring H and the temperatures of the discs.

Lees worked out the correction for the conduction of heat along the thermocouple wires and the leads to the heating coil.

He used this method for determining the conductivity, and variation of conductivity with temperature, of a number of substances, such as glass, sulphur, ebonite, shellac, fibre, naphthalene, lard. He also extended this method to apply to liquids in the manner described below (§ (7 ii.)).

§ (5) Solids: Metals (K = 0·015 to 1·00).—The class of metals is distinguished by its high conductivity alike of heat and electricity. This is explained by the supposed dependence of the two phenomena on the diffusion of electrons, and a mathematical relationship has been established between them which, as will be shown below, is partially confirmed by experiment. In the non-metallic solid, on the other hand, the electron is supposed to be capable of vibration only, so that heat conduction is assumed to be due to internal radiation. While this theory explains the known facts better than any yet advanced, it should be pointed out that it fails to account for the fact that some dielectrics have a higher conductivity than the worse conducting metals, *e.g.* the conductivity of quartz parallel to the axis (K = ·029) is greater than that of bismuth (K = 0·016 to 0·019).

The conductivities of metals range from 0·015 to 1·00 C.G.S. units. Practically all the pure metals have a small negative coefficient of conductivity with rising temperature, while the alloys show a positive coefficient.

The high conducting power of metals gives rise to certain modifications in the types of apparatus described above. Thus it is necessary to use a considerable thickness of metal to obtain appreciable temperature gradients, and the measurement of the boundary temperatures is very difficult. This latter point is well illustrated by the early experiments in which a slab of metal was heated on one side by steam or hot water and cooled on the other side by ice or cold water. Under these conditions the temperatures of the faces of the metal were assumed to be those of the steam, ice, or water as the case may be. Acting on this assumption, Clement obtained a value for the conductivity of copper which was 200 times too low, while Péclet, who was aware of the source of error in Clement's work, and took extraordinary precautions to secure efficient stirring, still obtained a value for copper which was about one-fifth of the true value. The difficulty is due to the fact that, even with the most vigorous stirring, there is always in contact with the metal surface a thin film of fluid which is at rest, and thus has a temperature gradient through it. If d is the thickness of the metallic plate (of true conductivity K) and d_1 and d_2 of the stationary films of fluid (of conductivity K′) on each side, it can easily be shown that the effect of assuming the temperatures of the metallic surfaces to be those of the moving liquid on each side is to give an apparent conductivity of the metal equal to the true conductivity divided by

$$1+\frac{K}{K'}\frac{(d_1+d_2)}{d}.$$

In the case of copper and water the value of K/K' is about 700, so that the value of d_1 and d_2 would only have to be $\frac{2}{700}$ of the thickness of the copper to obtain an apparent conductivity one-fifth of its true value.

(i.) *Plate Method. Hall.*—In spite of the difficulties mentioned above, the plate method has been used—notably by Hall,[1] to whom is due an ingenious method of determining the difference in temperature between the two faces of a thin metal plate.

He experimented with a disc of iron 10 cm. in diameter and 2 cm. thick. Each side of the disc was coated with copper to a thickness of 2 mm. By attaching a copper wire to each side a differential thermocouple was obtained the E.M.F. of which gave the difference in temperature between the two surfaces of the disc. Hall had a number of copper wires attached to the copper faces, which were immersed in streams of water at different temperatures. The heat coming through the disc was given by the rate of flow of the cold water (about 20 gm. per second) and the rise in temperature (about 0·5° C.). The difference in temperature between the faces of the iron disc was of the order of 1° C., which corresponded to 10 microvolts of the differential couple. The individual wires gave readings differing by as much as 10 per cent, which was a serious source of uncertainty. Hall obtained values of conductivities as follows :

Cast iron, 0·1490 C.G.S. units at 30° C., temperature coefficient − 0·00075.

Pure iron, 0·1530 C.G.S. units at 30° C., temperature coefficient − 0·0003.

The value for pure iron is in good agreement with recent determinations, while the value for cast iron seems to be too high.

(ii.) *Bar Methods.*—For the reasons which have been indicated above, bars have been commonly adopted instead of plates for the purpose of conductivity measurements.

(*a*) *Forbes.*—One of the earliest methods of determining the conductivity of a metallic bar was that due to Forbes. He took a long bar kept at a fixed temperature at one end. When the steady state of heat flow had been reached, the temperature at a series of points was measured by means of mercury thermometers let into holes in the bar. By plotting these observations and drawing a smooth curve through them, the complete temperature distribution along the bar was determined. The tangent to the curve at any point gave the temperature gradient $(d\theta/dx)$ at that point.

To obtain the heat flowing across the particular cross-section of the bar, Forbes determined the total heat loss from the surface between this section and the cool end of the bar — the two quantities are obviously equal. For this purpose a separate experiment was made on a similar bar which was uniformly heated and left to cool in the same surroundings as the first bar. From the rate of cooling of this bar and a knowledge of its heat capacity, the rate of heat loss per unit area could be determined for any temperature. The data obtained were used for computing the heat loss from the surface of the first bar, measured from the particular cross-section. The temperature gradient having been determined in the first experiment, the conductivity was at once obtained.

Forbes used for the first experiment, which has been called the "static" experiment since it dealt with the steady state of heat flow, a bar of wrought iron 8 feet long and 1¼ inch square section. The hot end of the bar was fixed into any iron crucible which was maintained at the temperature of melting lead or solder. The experimental conditions were varied by having the metal bright polished in one case and covered with white paper applied with a little paste in another case. The surface emissivity in the latter condition was eight times that in the former.

For the second or "dynamic" experiment in which the emissivity was determined for various temperatures, a bar of the same material and cross-section was used, but only 20 inches in length.

Forbes obtained the conductivity of wrought iron for a number of temperatures up to 200° C. He assumed, however, that the specific heat of iron was constant, whereas it increases by some 10 per cent for every 100° C. rise. For this reason Forbes' values are obsolete. When corrected by Tait and Mitchell, they were brought more into line with modern values but were still unsatisfactory. Apparently another source of error arises from the fact that the "dynamic" experiment does not give the true emissivity coefficients applicable to the "static" experiment, owing to the temperature distribution inside the bar being different in two cases. This would not, however, account for the large discrepancies in the experiments, of which no satisfactory explanation has been given. Thus Forbes, using two bars respectively 1¼-inch and 1-inch square section, obtained values differing by some 20 per cent, while Mitchell repeating the experiments on one bar and cooling it in the centre obtained a value differing by 8 per cent.

In spite of the criticism that has been levelled at it, Forbes' experiment is interesting as being the first attempt to determine the conductivity of a metal in absolute measure. His method has been followed, with improvements, by a number of experimenters, notably Mitchell, Stewart, Lees, Callendar, and Nicholson.[1] The two last-mentioned made several important modifications in the method. To avoid uncertainties of surface loss a large bar 4 inches in diameter was used and the loss from the bar was reduced to a quarter by lagging. The end of the bar, which was 4 feet long, was surrounded by a calorimeter and cooled by a stream of water. The rate of flow of the water and the temperature rise gave the quantity of heat passing out at the end of the bar. The heat leakage through the lagging could be computed and was only a small fraction of that absorbed by the calorimeter. The uncertainties of Forbes' method are much reduced by the above-mentioned modifications. The apparatus is illustrated in *Fig.* 14.

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[1] *Proc. Amer. Acad.*, 1898, xxxiv. 283.

Griffiths[2] has also made use of a similar method for measuring the conductivity of a series of aluminium alloys, intended for aeroplane engines, up to a temperature of 350° C. In his apparatus the heat was supplied by a nichrome strip wound on one end of a bar, while on the other end a spiral pipe was wound which acted as a flow calorimeter. The electrical input into the heating coil was found to equal the amount of heat flowing out through the calorimeter plus the amount lost laterally through the lagging, so that the thermal balance sheet was checked. Some of the results obtained by Griffiths are given in Table III.

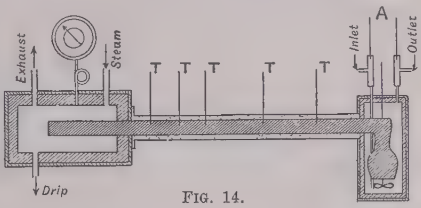

FIG. 14.

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[1] *Encycl. Brit.* 11th Ed. art. "Conduction of Heat."

[2] Advisory Committee for Aeronautics, *Light Alloys Sub-Committee Report*, No. 7, November 1917.

TABLE III

METALS AND ALLOYS

Substance.	Temperature. ° C.	K. C.G.S. Units.	Authority.
		Metals	
Aluminium, 99% Al	−160	·514	Lees,* 1908.
	18	·504	
	18	·480	J. and D.,† 1900.
	100	·492	
	120	·48	Angell, 1911.
Antimony	0	·044	Lorenz, 1881.
	100	·040	
	15	·042	Berget, 1890.
Bismuth	−186	·025	Macchia, 1907.
	18	·0194	J. and D., 1900.
	100	·0161	
Cadmium	−160	·239	Lees, 1908.
	18	·222	J. and D., 1900.
	100	·216	
Copper	−252	4·8	Meissner, 1915.
	−160	1·079	Lees, 1908.
	18	·918	J. and D., 1900.
	100	·908	
Gold	−252	3·5	Meissner, 1915.
	18	·700	J. and D., 1900.
	100	·703	
Iron (pure)	18	·161	
	100	·151	
„ (wrought)	−160	·152	Lees, 1908.
	18	·144	J. and D., 1900.
	100	·143	
„ (cast), 2% C : 3% Si : 1% Mn	54	·114	Callendar.
	102	·111	

* Lees' values at 18° C. may be taken to agree with those of Jaeger and Diesselhorst, except when they are given separately.

† J. and D. signifies Jaeger and Diesselhorst.

Table III.—*continued*

Substance.	Temperature. °C.	K. C.G.S. Units.	Authority.
		Metals—continued	
Iron (cast), 3·5% C : 1·4 Si : 5% Mn	30	·149	Hall, 1898.
Steel, 1% C	−160	·113	Lees, 1908.
	18	·115	
	18	·108	J. and D., 1900.
	100	·107	
Lead	−252	·11	Meissner, 1915
	−160	·092	Lees, 1908.
	18	·083	J. and D., 1900.
	100	·082	
Magnesium	0–100	·376	Lorenz, 1881.
Mercury	17	·0197	R. Weber, 1903.
	30	·0201	Berget, 1900.
	15	·0201	Nettleton, 1913.
Nickel, 99% Ni	−160	·129	Lees, 1908.
„ 97% Ni	18	·142	J. and D., 1900.
	100	·138	
Palladium	18	·168	
	100	·182	
Platinum	−252	·83	Meissner, 1915.
	18	·166	J. and D., 1900.
	100	·173	
Silver	−160	·998	Lees, 1908.
	18	·974	
	18	1·006	J. and D., 1900.
	100	·992	
Tin	−160	·192	Lees, 1908.
	18	·155	J. and D., 1900.
	100	·145	
Tungsten	18	·35	Coolidge.
	0	·383	S. Weber, 1917.
	2100	·273	Langmuir, 1917.
	2100	·342	Worthing, 1914.
Zinc	−160	·278	Lees, 1908.
	18	·265	J. and D., 1900.
	100	·262	
		Alloys	
Aluminium alloys			
1·5 Mg : 2 Ni : 8 Cu : 88·5 Al	100	·39	Griffiths, 1917.
	300	·40	
1 Mn : 8 Cu : 91 Al	100	·36	
	300	·39	
1 Ni : 8 Cu : 91 Al	100	·41	
	300	·42	
2 Fe : 8 Cu : 90 Al	100	·38	
	300	·39	
Brass, 70 Cu : 30 Zn.	−160	·181	Lees, 1908.
	17	·260	
Bronze, 90 Cu : 10 Sn	15	·099	Griffiths, 1917.
	200	·131	
Constantan, 60 Cu : 40 Ni	18	·054	J. and D., 1900.
	100	·064	
German silver or platinoid, 62 Cu : 15 Ni : 22 Zn	−160	·043	Lees, 1908.
	18	·059	
Lipowitz alloy, 50 Bi : 25 Pb : 14 Sn : 11 Cd	−160	·042	
	18	·044	
Manganin, 84 Cu : 4 Ni : 12 Mn	−160	·035	
	18	·053	J. and D., 1900.
	100	·063	
Wood's alloy	7	·032	H. F. Weber, 1879.

(*b*) *Berget.*—Berget[1] made use of a guard-ring method for the purpose of determining the conductivity of a number of metals such as copper, iron, brass, mercury. The arrangement of the apparatus used for the mercury determination is shown in *Fig.* 15. A cylindrical column of mercury was contained in a glass tube AB, which was surrounded by an annular column of mercury which acted as a guard-ring. The surface of the mercury columns was heated by steam, and the base of the outer column rested in an iron plate C which was cooled by ice. The tube carrying the central column of mercury projected through the iron plate into a Bunsen ice calorimeter as shown. The temperature distribution was determined by means of four iron wires which projected into the central column. Each pair of these wires, with the intervening mercury, formed a differential iron-mercury couple which gave the difference in temperature between the ends of the two iron wires. The distribution of temperature was found to be linear when the steady state has been reached. The ice calorimeter gave the quantity of heat passing through the column, and thus the conductivity was obtained.

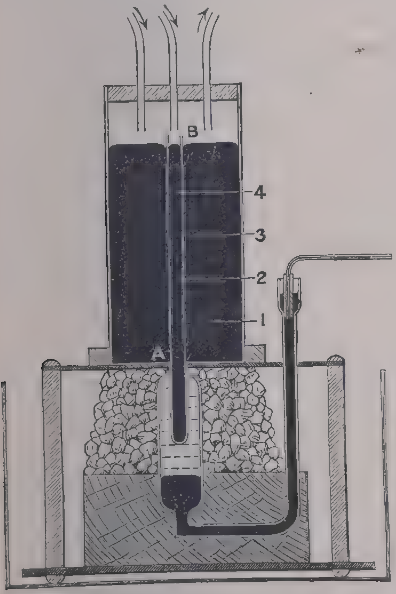

FIG. 15.

Berget's value for mercury of 0·0201 is in close agreement with recent determinations such as that of Nettleton. Having obtained the value for mercury, he measured the conductivity of other metals by comparison. A column of mercury was superposed on a column of metal, each surrounded by a guard-ring, and from a comparison of the temperature gradients the conductivity was deduced.

(*c*) *Lees.*—Lees[2] determined the conductivity of a number of pure metals and alloys over a range of temperature from −180° C. to 30° C.

His apparatus is illustrated in *Fig.* 16.

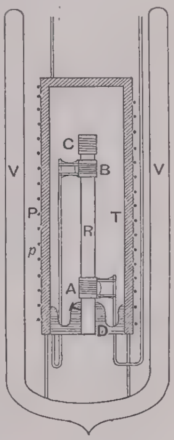

FIG. 16.

R was a rod of metal 7 or 8 cm. long and about 0·5 cm. in diameter. The lower end fitted into a copper disc D, which formed the bottom of a copper cylinder T closed at the top. A, B, and C were three thin brass sleeves, fitting closely to the rod with which good thermal contact was secured by a film of olive oil. Of these A and B carried platinum coils for the measurement of temperature while C carried a heating coil of platinoid wire. The copper cylinder T was placed on a wire frame resting on the bottom of the Dewar flask V. Around the outside of T was wound a platinoid wire *p* of the same resistance as the heating coil C, and whenever the current was switched off from C it was switched on to *p*, so that the rate of heat supply to the apparatus as a whole was constant throughout the experiment. A further heating coil P wound on T enabled the temperature of the apparatus to be raised rapidly if desired.

The experiments were started at liquid-air temperature. The current was switched on to C until a suitable difference of resistance (generally equivalent to about 5° C.) was obtained between A and B. The current was then switched on to *p*, and after five or ten minutes the difference in resistance between the coils A and B was again measured, as well as the actual resistance of the lower one A. The difference in temperature between A and B when the heating current flowed round C, less the mean difference in temperature (before and after) when the current was flowing through *p*, gives the difference in temperature which would be produced if the heating current were continued in C and the surrounding tube T kept at a constant temperature, *i.e.* assuming the rate of rise of temperature of the apparatus was constant which was secured, as explained above.

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[1] *Jour. de Phys.*, 1888, vii. 503.

[2] Lees, *Phil. Trans. Roy. Soc.*, 1908, ccviii. 381.

While the general theory of the experiments was simple, they are remarkable for the skill and care with which Lees explored the various sources of error. Thus the sleeves were of appreciable length and size, but formulae were obtained giving the approximate effect of the dimensions of the sleeves and of the small difference in temperature between the rod and platinum resistance coils on the sleeves. Again corrections were worked out for the effect of the leads to the heating coil C and the platinum coils A and B, while the fact that the observations did not apply strictly to the steady state of temperature distribution was shown to be of small effect.

Lees' experiments over the range from − 170° to 30° C. confirmed generally the conclusions of Jaeger and Diesselhorst—from their work between 0° and 100° C.—that the pure metals show a slight decrease in conductivity with rise of temperature. He did not, however, find that the maximum conductivity always occurred at the lowest temperature.

The electrical conductivity of the metal rods was also measured by Lees, and his conclusions as to the relation between thermal and electrical conductivity will be referred to later (§ (10 i.)).

(iii.) *Electrical Methods.*—Several methods for measuring the conductivity of a bar of metal have been proposed in which heat is supplied by passing an electric current through the bar itself. If the ends are kept at a constant temperature, there will be established throughout the bar a certain distribution of temperature, which will depend on the strength of the electric current, the electrical and thermal conductivity of the material, and the heat loss (if any) from the surface of the bar.

(*a*) *Kohlrausch: Jaeger and Diesselhorst.*—Kohlrausch[1] took the case where all heat loss from the surface of the bar was prevented. Calling

K = thermal conductivity,
λ = electrical conductivity,
θ = temperature at any point,
v = electrical potential at any point,

he obtained the following solution of the differential equations for thermal and electrical equilibrium:

$$\theta = \tfrac{1}{2}\frac{\lambda}{K}v^2 + Av + A',$$

where A and A′ are constants. Hence, by determining the temperatures and potentials at three points, A and A′ can be eliminated, and we get

$$\frac{\lambda}{2K} = \frac{\theta_1(v_2 - v_3) + \theta_2(v_3 - v_1) + \theta_3(v_1 - v_2)}{(v_1 - v_2)(v_2 - v_3)(v_3 - v_1)}.$$

Kohlrausch also showed that if the conductor is of any shape, and is thermally and electrically insulated except for the two areas through which the current is supplied and through which heat is abstracted, the same solution holds. Hence, if holes are drilled in a bar for the purpose of inserting thermometers, the method will still be exact, provided that the temperatures and potentials are measured at the same points.

Jaeger and Diesselhorst carried out Kohlrausch's method into practice. They took cylindrical bars of metal, 25 cm. long and generally about 2 cm. in diameter, with their ends fitted into copper blocks, which were screwed into large water-baths. The bar was surrounded by a cylindrical jacket, kept at a constant temperature by water or steam circulating outside it, the space between the bar and the jacket filled with wadding. There was, of course, a certain amount of heat loss through the wadding, which was estimated by varying the temperature of the jacket surrounding the bar. For the discussion of this and other corrections, reference must be made to the original paper. The temperatures were measured by thermocouples.

The determination of the temperatures and potentials at three points gives the ratio of the thermal and electrical conductivities, and the latter constant being easily determined, the thermal conductivity was obtained.

Jaeger and Diesselhorst carried out a very extensive investigation on a number of pure metals and a few alloys for temperatures between 18° and 100° C. They found that pure metals, with one or two exceptions, showed a slight decrease of conductivity with increasing temperature, and that they obeyed approximately Lorenz's law—that the ratio of the thermal and electrical conductivities divided by the absolute temperature is a constant. Lorenz's constant showed a slight positive temperature coefficient.

Their work represents a most important contribution on the subject of conductivities, and their values have been amply confirmed by subsequent observers.

The method of Jaeger and Diesselhorst has been used by Meissner[2] for measuring conductivities down to a temperature of 20° abs. His work confirmed generally the values of Lees, but he found that both the resistivity (1/K) and Lorenz's constant decreased rapidly as the temperature of liquid hydrogen was approached, showing a reduction at 20° abs. of about 85 per cent on the value at 0° C.

(*b*) *Callendar.*—Callendar[3] has also devised a method of determining the conductivity of a rod of metal which is heated by the passage of an electric current. The ends of the rod are kept at constant temperature by insertion into water-cooled copper blocks. The rod is also surrounded by a jacket at the same temperature, which is taken as the zero of reference. If R_0 is the resistance per unit length of the rod at the zero temperature, α is its temperature coefficient, then the resistance of a small element of the rod at a distance x from the centre is

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[1] *Preuss. Akad. Wiss. Berlin*, 1899, xxxviii. 711. *Abh. Phys. Tech. Reich.*, 1900, iii. 269.

[2] *Deutsch. Phys. Gesell. Verh.*, 1914, xvi. 262.

[3] *Encycl. Brit.* 11th Ed. "Conduction of Heat."

$$R_0(1+\alpha\theta)dx.$$

The heat generated in this element by the passage of a current C is

$$C^2R_0(1+\alpha\theta)dx.$$

As there is a gradient of temperature from the centre of this rod to the ends, heat will pass through the element dx by conduction, and the excess of heat entering the element by conduction over that leaving it will be

$$KA\frac{d^2\theta}{dx^2}dx,$$

K being the conductivity and A the area of the cross-section of the bar. When the steady state has been reached, this excess of heat will be balanced by the heat loss due to radiation and convection from the surface of the element, less the heat generated in the element by the passage of the current. By Newton's law the former quantity will be $Ep\theta dx$, E being the surface emissivity and p the perimeter of a cross-section of the rod. Equating these quantities we get[1]

$$C^2R_0(1+\alpha\theta)=Ep\theta-KA\frac{d^2\theta}{dx^2}.$$

The current C is so adjusted that $C^2R_0\alpha=Ep$, *i.e.* that the loss of heat from the surface of the rod is compensated at all points by the increase of resistance with rise of temperature. The equation then reduces to

$$\frac{d^2\theta}{dx^2}=-\frac{C^2R_0}{KA}=\text{constant}.$$

So that, $2l$ being the length of the rod,

$$\theta=\frac{C^2R_0}{2KA}(l^2-x^2).$$

This gives the temperature at all points of the rod, and it can be shown that the mean temperature is $C^2R_0{}^2l^2/3KA$. Hence, if R_m is the mean resistance per unit length, we get

$$K=\frac{C^2R_0l^2\alpha}{3A(R_m-R_0)}.$$

The distribution of temperature is always nearly parabolic, if the dimensions of the rod are suitably chosen. It is not therefore necessary to set the current to the value $Ep/R_0\alpha$ very accurately, as the correction for external loss is small in any case. The chief source of error is in the measurement of the small change of resistance accurately, and of avoiding stray thermo-electric effects.

Duncan,[2] using this method, found the conductivity of copper at 33° C. to be 1·007, which is about 8 per cent higher than that obtained by Jaeger and Diesselhorst.

(*c*) *Mendenhall and Angell.*—Another example of an electrical method is that suggested by Mendenhall, and described in a paper by Angell,[3] for measuring the conductivity of metals at high temperatures.

If a cylindrical rod is connected at its ends to two heavy electrodes and heated by a constant current, there will be a rising temperature gradient from the ends towards the middle of the rod. This longitudinal gradient will decrease as the middle is approached, and for an appreciable distance on each side of the centre will be negligible. There will be no tendency, therefore, for the heat generated in this portion of the rod to flow towards the ends. The temperature gradients will be radial, and the heat will flow radially and be dissipated at the surface. Consider now a length of rod l within the central zone. Let r be the radius, E the fall of the potential per centimetre, I the current density (which is assumed uniform over the cross-section), θ_2 and θ_1 the temperatures at the axis and circumference respectively.

Then, since the heat generated within any cylindrical surface of radius r passes radially to the surface, we have

$$\pi r^2IEl=-2\pi rlK\frac{d\theta}{dr}.$$

Integrating we get

$$K=\frac{EIr_1{}^2}{4(\theta_2-\theta_1)}.$$

The above formula depends on the assumption that the current density is uniform over the cross-section. This is not strictly the case, since there is a temperature gradient from the axis to the circumference which gives rise to a variation of resistivity. Angell shows, however, that for metal rods of less than 2 cm. in diameter the effect of the variation is negligible. In the actual experiments a hollow cylinder was used, and the formula in this case is

$$K=\frac{EI}{2(\theta_2-\theta_1)}\left\{\frac{r_1{}^2-r_2{}^2}{2}-r_2{}^2\log\frac{r_1}{r_2}\right\}.$$

By measuring the dimensions of the cylinder, the values of E and I, and the temperatures of the inner and outer surfaces, the value of K was obtained.

The apparatus used is illustrated in *Fig.* 17. The metallic rod was 1·2 cm. in diameter and about 15 cm. long. An inner hole was bored through the rod 0·17 cm. in diameter, and this was enlarged, as shown in the figure, except for the central length of 3 cm. The resistance on each side of the central portion being thus increased, the heat generated towards the ends was greater and the portion of the rod with no longitudinal gradient was increased. Thus at 1000° C. the variation in temperature over 1 cm. length in the centre was less than 0·1 per cent.

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[1] The heat developed by the Thomson effect is very small and is eliminated by keeping the two ends at the same temperature.

[2] *McGill Coll. Report*, 1899.

[3] *Phys. Rev.*, 1911, xxxiii. 421.

The temperature inside the rod was measured by a fine platinum, platinum-rhodium couple with a junction at the centre. The measurement of the surface temperature presented great difficulties. A fine thermo-junction was rolled out to a thickness of ·01 mm., and the wires were attached to two springs S as shown. By rotating in opposite directions the rings to which the springs were attached, the connecting wires assumed a diagonal position relative to the rod, and brought the junction in contact with the surface, which was highly polished. In order to get a uniformly radiating surface at the junction, a small piece of the rod material was rolled to a thickness of 0·01 mm., placed over the junction, and held in position by a weighted quartz fibre. The rod and connections were surrounded by a water-cooled enclosure, which could be exhausted.

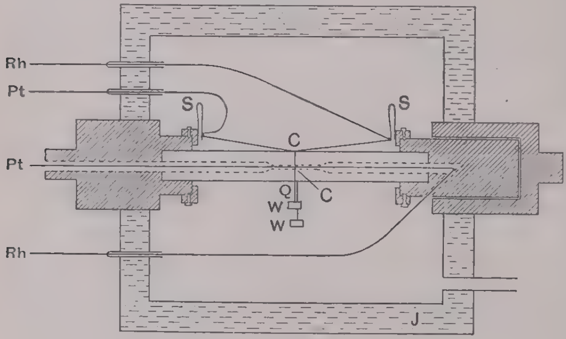

FIG. 17.

For energy supply a welding transformer was used, capable of furnishing 1700 amps alternating at a voltage of 2½. The energy was determined by measuring the current and making a separate experiment to determine the resistance of the central centimetre of the rod. For this purpose the rod was turned down to a small diameter and the volt drop measured on the central centimetre for a known current. The resistance of the original rod at various temperatures could thus be calculated.

Angell used the above apparatus to determine the conductivity of aluminium and nickel. He found that the former increased in conductivity from 0·49 at 100° C. to 1·0 at 600° C., while nickel decreased in conductivity from 0·125 at 300° C. to 0·07 at the critical temperature of 700° C. Between 700° and 1200° the conductivity decreases slowly to a value 0·06.

The method is an interesting attempt to measure conductivity at high temperatures. Its success depends entirely on the accuracy with which the small gradient of temperature between the axis and surface of the rod can be measured. This must be considered as a drawback, having regard to the experimental difficulties in measuring the surface temperature. Further, it is not clear that the longitudinal gradient in the central portion of the rod, though very small, was negligible compared with the radial gradient.

(*d*) *Worthing, Langmuir.*—Somewhat resembling the two methods described immediately above is that due to Worthing [1] for determining the conductivity of lamp filaments to a temperature of 2100° C.

If w is the rate of heat production, and E the rate of radiation per unit area, at any point on the filament, the maximum values being w_m and E_m at the middle of the filament, and if l is the distance measured along the filament, the total length of which is 2L, then he derives the expression

$$K = \frac{2E_m}{r(dT/dl)}\int_l^L \left(\frac{w}{w_m} - \frac{E}{E_m}\right) dl.$$

The full details of the experiments cannot be given here, but they resolved themselves into determining E as a function of T and of l, and w as a function of l. Worthing found, for example, that $E = \sigma T^\beta$, where $\beta = 5{\cdot}35$.

His conclusion as to the conductivity of tungsten was that it varied from ·234 C.G.S. units at 1200° to ·342 at 2100° C. Tantalum showed a change from ·174 at 1400° C. to 0·198 at 1800° C., while carbon gave values of ·0201 and ·0210 at the two temperatures just named.

Langmuir [2] also considered the question of conductivity of tungsten at high temperatures. His value for β was 4·96, and he states that the resistance can be expressed as

$$R = cT^\rho,$$

where $\rho = 1{\cdot}242$. On the basis of these experimental facts, he deduces from Worthing's data that the conductivity of tungsten is nearly independent of temperature and has a value of ·273 at 2100° C., as against ·342 found by Worthing. He also contends that the value of Lorenz's constant is of the order that would be expected from Jaeger and Diesselhorst's value at 18° C., and the temperature coefficient which they found between 18° C. and 100° C. This was contrary to the very large change in the constant which Worthing found over the range 1200° C.-2200° C.

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[1] *Phys. Rev.*, 1914, iv. 535.
[2] *Ibid.*, 1916, vii. 151.

(iv.) *Indirect Methods.*—In the case of metals it is often convenient to determine the conductivity by comparison with some other metal whose conductivity is known.

(*a*) *Wiedemann and Franz.*—As an example, the method of Wiedemann and Franz [1] may be quoted. These experimenters took a bar of metal, one end of which was maintained at a constant temperature, and which was submitted along its length to the effect of cooling by radiation and convection inside a constant temperature enclosure. When the steady flow of heat had been established, the temperatures at three points at equal distances apart were measured.

The temperature distribution along a bar under the conditions indicated is given by (cf. equation in § (5) (iii.) (*b*))

$$\mathrm{KA}\frac{d^2\theta}{dx^2}=\mathrm{E}p\theta$$

If now the temperatures θ_1, θ_2, and θ_3 are measured at three equally spaced points at a distance a apart, it can be shown that

$$a\sqrt{\frac{\mathrm{E}p}{\mathrm{KA}}}=\log_e(n+\sqrt{n^2-1}),$$

where $n=\dfrac{\theta_1+\theta_3}{2\theta_2}$.

So that if we take two bars of different materials, but with the same cross-section A, the same perimeter p, and the same coefficient of heat loss E, and determine the temperatures in each case at three equally spaced points at the same distances apart, we get the ratio of conductivities from the following equation

$$\sqrt{\frac{\mathrm{K}}{\mathrm{K}'}}=\frac{\log(n'+\sqrt{n'^2-1})}{\log(n+\sqrt{n^2-1})}.$$

The rods used by Wiedemann and Franz were half a metre long and 6 mm. in diameter, and in order to secure the same surface emissivity for all the rods, they were electroplated. One end of the rod was heated by steam and the remainder was surrounded by a water-cooled enclosure which could be exhausted if required. The temperatures at three equidistant points were measured by means of a sliding thermocouple.

Wiedemann and Franz drew up a table of relative conductivities of a number of metals. The experiments represented a considerable advance at the time they were carried out (nearly seventy years ago), but some of their results must now be regarded as obsolete, the probable cause being the presence of impurities in their specimens.

There is also the point, to which Péclet drew attention as long ago as 1860, that the assumption of Newton's law of cooling, which only holds approximately for small differences of temperature, must introduce considerable error.

(*b*) *Voigt.*—Another method of comparing the conductivities of two metals is that due to Voigt.[2] He took thin plates of the two materials cut in the form of right-angled triangles, and placed them in contact along their hypotenuses. The lengths of the sides a and b (*Fig.* 18) are chosen so as to be approximately in proportion of the conductivities of the plates. Heat is then applied by means of a copper bar at 70° to 90° C. along the short side b of the good conductor or the long side a of the bad conductor. The whole surface of the composite plate was coated with a layer of elaïdic acid to which wax and turpentine are added. The acid melts at 45° C. and solidifies in crystals yielding a well-defined isothermal curve. The ratio of the conductivities is derived from measurements of the angles which the isothermal makes with the common hypotenuse, being in the proportion of the tangents of the angles. The limit of accuracy of which the method is capable is about 2 per cent.

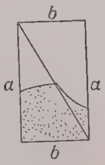

FIG. 18.

Rietzsch [3] used this method for measuring the conductivity of copper containing varying amounts of phosphorus and arsenic. It has also been used for glasses.

(v.) *Periodic Flow Methods.* (*a*) *Ångström.*—The method [4] is of interest in that it differs in principle from those previously described which have all been concerned with the steady state of heat flow. In this case the conductivity was deduced from observation of the periodic flow of heat in a long bar.

If we consider any thin cross-section of a rod the temperature of which is changing, the difference between the heat flowing into and out of this section must be equal to the heat lost from the surface of the section plus the heat required to raise the temperature of the section itself. So that we have (cf. § (5) (iii.) (*b*)

$$\mathrm{KA}\frac{d^2\theta}{dx^2}=\mathrm{E}p\theta+\mathrm{AC}\frac{d\theta}{dt}.$$

If the rod is surrounded by a guard-ring which prevents heat loss from the surface, the first term on the right-hand side vanishes and the equation becomes

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[1] *Ann. de Chimie*, 1854, xli. 107.

[2] *Annal. Phys. Chem.*, 1898, lxiv. 95.

[3] *Ann. der Phys.*, 1900, iv. (3) 403.

[4] *Phil. Mag.*, 1863, xxv. 130.

$$h^2\frac{d^2\theta}{dx^2}=\frac{d\theta}{dt},$$

where h^2 is the diffusivity or the conductivity K divided by the heat capacity per unit volume C.

Now, supposing that a simple harmonic oscillation of temperature is impressed on one end of the bar, it can be shown from the above equation that

$$h^2=\frac{\lambda^2}{4\pi T},$$

where T is the periodic time and λ the wave-length of the temperature oscillations. It follows also that the amplitude of the temperature oscillations at any point at a distance x from the source is proportional to $e^{-2\pi x/\lambda}$. The amplitudes thus decrease as we recede from the hot end and likewise the mean temperature. The temperature distribution at any instant is of the form shown in *Fig.* 19, in which the dotted line represents the mean temperatures.

It is obvious if we determine the complete temperature distribution the wave-length λ can be measured, and thus the diffusivity. From the latter the conductivity may be obtained, if the heat capacity per unit volume is known.

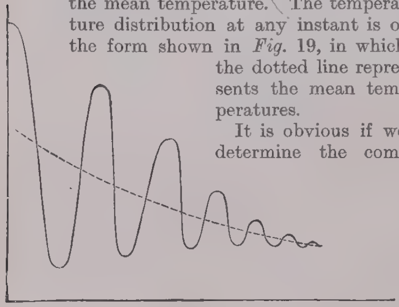

FIG. 19.

It is, however, sometimes convenient to deduce the conductivity from temperature—time observations at two points. The general solution of either of the equations above for a simple harmonic oscillation is of the form

$$\theta=A\sin\left(\frac{2\pi}{T}t+\delta\right).$$

The values of A and δ can be determined experimentally for the two points, and it can be shown that if l is their distance apart

$$h^2=\frac{\pi l^2}{T(\delta-\delta')\log A/A'}.$$

If the temperature oscillations are periodic but are not simple harmonic vibrations, they can be resolved into a number of terms of the form

$$\theta=A_0+A_1\sin(\omega t+\delta_1)+A_2\sin(2\omega t+\delta_2)+A_3\sin(3\omega t+\delta_3)+\ldots$$

where $\omega=\pi/T$.

Again, taking two points, each pair of the terms of the above expression leads to an independent value of h^2,

$$h^2=\frac{n\pi l^2}{T(\delta_n-\delta_n')\log A_n/A_n'}.$$

In practice these series may be limited to the first three or four terms as the coefficients A_0, A_1, A_2, etc., rapidly diminish.

In Ångström's experiments a small section of a bar was enclosed and was submitted alternately to heating and cooling by steam and cold water respectively. The periods of heating and cooling were 12 minutes each, and when they had been continued for some time, the temperatures at each point of the bar became steadily periodic. Mercury thermometers were let into the bar at intervals of 5 cm. for temperature readings.

Ångström used for his first experiments bars 57 cm. long and of 2·4 cm. square section, while for some subsequent experiments he used a bar 118 cm. long and 3·5 cm. thick. His values for the conductivity of copper and iron at 50° C. were

	First Experiments.	Second Experiments.
Copper . .	·910	$\cdot982\,(1-0\cdot001520\theta)$
Iron . .	·163	$\cdot199\,(1-0\cdot00214\theta)$

The values obtained in the second experiments are much higher than those now accepted. The negative sign of the coefficient is confirmed by the work of Jaeger and Diesselhorst and of Lees though the magnitudes are different.

Callendar [1] has applied Ångström's method to the apparatus illustrated in *Fig.* 14. He varied the pressure of the steam in the heater so as to produce approximately simple harmonic oscillations, using periods of 60, 90, and 120 minutes. Callendar found a value for cast iron at 54° C. of $0\cdot1144\,(1-0\cdot00006\theta)$, which is in close agreement with modern values obtained by other methods.

There is no doubt that Ångström's method gives satisfactory results if due precautions are taken. It involves, however, the taking of large numbers of observations and their analysis may be very laborious.

(*b*) *King.*—A method resembling that of Ångström is due to King.[2] His apparatus is shown diagrammatically in *Fig.* 20. A is a wire whose conductivity is to be determined and which is 2·5 mm. in diameter and about 40 cm. long. One end of the wire projects into a heating coil H through which flows a periodic current following a sine law. At two points of the wire are attached the thermocouples T, T, each connected to a short period galvanometer. In each thermocouple circuit is a counter E.M.F. which can be adjusted to balance the thermocouple E.M.F. at its mean value. The lag between the movements of the two galvanometers gives the velocity of propagation of the waves. If experiments are made with the waves of two periods t_1 and t_2 and the velocities are found to be v_1 and v_2, then it can be shown that the diffusivity h^2 is given by

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[1] *Ency. Brit.*, 11th edition, "Conduction of Heat."

[2] *Phys. Rev.*, 1915, vi. 437.

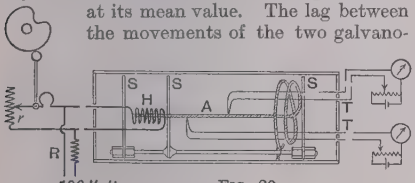

FIG. 20.

$$h^2 = \frac{t_1 t_2 v_1 v_2}{4\pi}\sqrt{\frac{(v_1^2 - v_2^2)}{t_2^2 v_2^2 - t_1^2 v_1^2}}.$$

King carried out experiments with waves of periods of 2 minutes and 5 minutes respectively, and he found values for the conductivity of copper and tin which are in close agreement with those of Jaeger and Diesselhorst.

§ (6) SOLIDS: CRYSTALLINE AND ALEOTROPIC.—In considering the various classes of solids which have been referred to above, it has been assumed that they have been symmetrical as regards the conductivity of heat. Some substances, however, owing to peculiarities of structure, give different values of conductivity in different directions. Thus Tyndall [1] found that, in the case of wood, the conductivity was highest along the fibre and lowest in the direction perpendicular to the fibre and the ligneous layers. As giving some idea of the magnitude of the difference some of his values are quoted in the following table. The figures are only comparative:

Name of Wood.	Parallel to Fibre.	Perpendicular to Fibre and Parallel to Ligneous Layers.	Perpendicular to Fibre and Ligneous Layers.
Oak . .	34	11·0	9·5
Beech . .	33	10·8	8·8
Boxwood .	31	12·0	9·9
Ash . .	27	11·5	9·5
Apple-tree	26	12·5	10·0
Scotch fir .	22	12·0	10·0

Similarly in the case of laminated rocks, such as slates and schists, Jannettaz [2] has shown that the conductivity is highest parallel to the cleavage.

Where such substances can be obtained in sufficient bulk their conductivities can be determined by one of the methods already described. In the case of small objects, however, such as crystals, which often show a marked asymmetry of heat flow, different methods have to be adopted.

(i.) *De Senarmont.*—The first to make a comprehensive study of crystalline conduction was de Senarmont.[3] His method consisted in coating a thin plate cut from a crystal with a film of white wax and applying heat at a point near the centre. As the plate becomes warm the wax melted round the point and the inequalities in conduction in different directions were indicated by the shape of the bounding line of melted wax. The curve in the case of an isotropic substance, such as glass or crystals of the cubic system, was always circular. In the rhombohedral system, with one axis of symmetry, the plates cut perpendicular to the axis gave circles while any section parallel to the axis gave an ellipse. This is illustrated in *Fig.* 21, which gives the curves for a quartz crystal cut perpendicular and parallel to the main axis. Here the isothermal surface due to a point source in the crystal would be an ellipsoid of revolution about the main axis. The rule was found to be general that the thermic and crystallographic axes coincided.

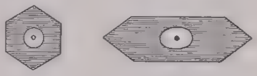

FIG. 21.

In de Senarmont's experiments the heat was supplied by a wire or tube fitting a hole bored in the plate of crystal, or the sun's rays were concentrated on a point by means of a lens. The values obtained were comparative, a circle indicating that the conductivity was the same in all directions, while for an ellipse it could be shown that conductivities along the axes were proportional to the square of their lengths.

(ii.) *Lees.*—A method of determining the absolute values of thermal conductivities of crystals was suggested by Lodge [4] and carried out by Lees.[5] A thin slice of the crystal was placed in contact with the ends of two bars of metal with their lengths in the same straight line, thus forming a composite bar. Brass was chosen because it readily amalgamates with mercury, and the amalgamated surfaces were found to give extremely good thermal contact with the crystal. The bar was packed in sawdust, and one end was heated by steam and the other immersed in cold water. The temperature distribution was obtained by a series of thermocouples attached to the bar, and from this the temperatures of the two faces of the crystal plate could be deduced after making a small correction for the layer of mercury as determined in a separate experiment. The amount of heat passing along the bar and through the crystal was calculated from a knowledge of the absolute conductivity of the brass, which was obtained by an improved Forbes method.

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[1] *Phil. Mag.* 4th series, v. and vi.
[2] *Journ. de Phys.* v. 150.
[3] *Ann. de Chim. et de Phys.*, 1847–48, xxi.-xxiii.
[4] *Phil. Mag.*, 1878, v. 110.
[5] *Phil. Trans.*, 1892, clxxxiii. 481.

Lees found that for quartz the conductivity parallel to the optic axis was 0·0299 and perpendicular to the axis 0·0158.

§ (7) LIQUIDS ($K=0·0003$ to $0·0015$).—The problem of determining the conductivity of a liquid is complicated by the ease with which convection currents are set up in any mass of fluid with a temperature gradient through it. Thus if the liquid at any point is at a higher temperature than the surrounding liquid, it will have a lower density, owing to its expansion with temperature, and will, if below the surface, tend to rise. Should the temperature difference be maintained at the particular point, a continuous current will be established. The laws governing such convection currents are very complicated, and the heat conveyed by them is often large as compared with the amount passing through the liquid by pure conduction or diffusion. In attempting, therefore, to determine the conductivity of the liquid the plan is commonly adopted of eliminating convection. This may be accomplished by taking a column of liquid, supplying heat at a horizontal surface at the top of the liquid, and abstracting heat at a horizontal surface at the bottom of the liquid; or by taking a film of liquid so thin that, independent of its orientation, the transfer of heat by convection is negligible. An alternative method is to regulate the heat loss by convection so that it takes place under well-defined and measurable conditions. Thus, when a liquid is made to flow with streamline motion through a heated tube, a relation can be established between the heat carried off by this forced convection, and the temperature gradients and conductivity in the liquid. Examples of methods applying these principles are described below.

(i.) *Column Method.* (*a*) *Berget.*—The apparatus used by Berget for determining the conductivity of a column of mercury surrounded by a guard-ring has already been described in § (5) (ii.) (*b*). It will be noted that the heat is here supplied at the top of the liquid, and flows down through the liquid to the bottom, which is cooled.

(*b*) *R. Weber* [1] has used a similar method, supplying the heat to the top of a column of liquid from a vessel kept at a certain temperature by oil electrically heated, while the bottom of the column was cooled by a horizontal copper plate standing in ice. The temperature difference was taken at two points, 1 cm. apart, by means of copper-constantan couples. His values are included in Table IV.

(*c*) *Chree* [2] experimented with liquids heated on the surface, and determined the diffusivity by observations of the variable state. He took a cylindrical container 19 cm. in diameter into which liquid was poured to a depth of 5·2 cm. A flat-bottomed dish, 15 cm. in diameter, rested on the surface of the liquid, while a fine platinum wire 6·6 cm. long was adjusted in a horizontal position 2·6 cm. below the surface. This wire, to which were attached copper leads, served as a resistance thermometer. Hot water was poured into the dish, and the rate at which the heat passed through the dish into the liquid below could be obtained from observations of the fall in temperature of the water in the dish after making a correction, obtained by an independent experiment, of the heat lost from the surface of the water. The temperature of the water was initially about 75° C., and immediately after it had been poured into the dish, a temperature rise was indicated by the platinum thermometer, which then remained stationary for several minutes. After this period the thermometer began to rise rapidly, and continued to rise at a decreasing rate towards its maximum, which was not reached for several hours. Chree took temperature time observations, and obtained the point at which the temperature rise was at a maximum rate, generally about ten minutes after the start.

Theory shows that if it were possible suddenly to apply a quantity of heat to the surface of the liquid, the time of maximum rise would be given by

$$t=0·0917\frac{x^2\rho c}{K},$$

x being the distance of the point below the surface, and ρ, c, and K being the density, specific heat, and conductivity of the liquid. The conditions of the actual experiments were different in that the heat was applied at a decreasing rate for an appreciable time, and Chree gives the mathematical expressions applicable to this case. It is interesting to note that in some experiments the water was allowed to remain in the dish for the duration of the experiment, while in others it was syphoned off shortly after pouring into the dish. The results obtained give a very fair agreement, and confirm the correctness of Chree's formula.

The value for the conductivity of water is supported by the more recent determinations by other methods.

(ii.) *Film Method.*—(*a*) *Lees* [3] measured the conductivity of thin films of liquid about 1·2 mm. in thickness. His apparatus was a modification of that already described for determining the conductivity of poor conductors. To the series of discs used in his former experiments, which were 4 cm. in diameter, was attached an ebonite ring E and another copper disc L both 7 cm. in diameter (see *Fig.* 22). The internal diameter of the ebonite ring was 3·7 cm. or slightly smaller than that of the superimposed discs. The ebonite served as a container for the liquid which filled the space between the copper discs M and L. The disc G was of glass 0·28 cm. thick, and its conductivity was determined as previously described. Knowing then the temperature difference between U and M and the conductivity of G, the heat entering M was known. Subtracting from this amount the heat loss from the exposed surface of M, we get the total of the heat passing through the liquid and through the ebonite. The flow in the latter was not rectilinear, and an independent

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[1] *Ann. der Phys.*, 1903, xi. 1047.

[2] *Proc. Roy. Soc.*, 1888, xliii. 30.

[3] *Phil. Trans. Roy. Soc.*, 1898, cxci. 418.

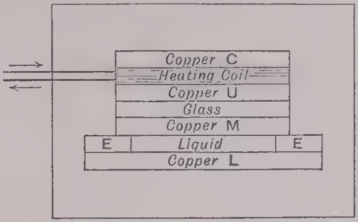

FIG. 22.

TABLE IV

LIQUIDS

Substance.	Temperature. ° C.	$K \times 10^4$. C.G.S. Units.	Authority.
Water and Aqueous Solutions			
Water	0	12·0	H. F. Weber, 1886.
	24	14·3	
	18	12·4	Chree, 1888.
	4	12·9	Wachsmuth, 1893.
	11	14·7	Lees, 1898.
	25	13·6	
	15-34	13·14	R. Weber, 1903.
	0	15·0	Goldschmidt, 1911.
Ammonia solution, 26 per cent	18	10·9	Lees, 1898.
Copper sulphate (s.g. 1·160)	4	11·8	H. F. Weber, 1880.
Potassium chloride (s.g. 1·026)	13	11·6	Graetz, 1885.
Sodium chloride (s.g. 1·178)	4	11·5	H. F. Weber, 1880.
Sulphuric acid (s.g. 1·054)	20	12·6	Chree, 1888.
Zinc sulphate (s.g. 1·134)	4	11·8	H. F. Weber, 1880.
Organic Liquids			
Methyl alcohol (CH_4O)	11	5·2	Lees, 1898.
	47	4·45	
Ethyl alcohol (C_2H_6O)	11	4·6	
	51	3·69	
	0	4·45	Goldschmidt, 1911.
Amyl alcohol ($C_5H_{12}O$)	0	3·45	,,
Formic acid (CH_2O_2)	9-15	6·48	H. F. Weber, 1886.
Acetic acid ($C_2H_4O_2$)	9-15	4·72	,,
	25	4·3	Lees, 1898.
Butyric acid ($C_4H_8O_2$)	9-15	3·60	H. F. Weber, 1886.
Chloroform ($CHCl_3$)	6	3·02	
Benzene (C_6H_6)	5	3·33	
Toluene (C_7H_8)	9-15	3·07	H. F. Weber, 1886.
	14·5	3·42	Goldschmidt, 1911.
Turpentine oil ($C_{10}H_{16}$)	13	3·25	Graetz, 1885.
Olive oil (s.g. 0·911)	6·6	3·92	H. F. Weber, 1886.
Carbon bisulphide (CS_2)	15	5·37	Chree, 1888.
	10-18	2·00	Winkelmann, 1874.
	0	3·88	Goldschmidt, 1911.
Glycerin (s.g. 1·235)	15-34	6·56	R. Weber, 1903.
Paraffin oil (s.g. ·870)	15-34	3·46	
Petroleum oil (s.g. ·789)	15-34	3·82	

experiment was made to determine it having air instead of liquid between M and L. The heat flowing through the liquid was then obtained, and this with the difference in temperature between M and L gave the conductivity of the liquid.

The film of liquid was horizontal in the experiments, and the heating was from the top surface so that convection was eliminated. It is interesting to note that when the apparatus was tilted through 15° the apparent conductivity only increased 1 per cent.

Lees determined the conductivity of water, glycerine, ethyl alcohol, and methyl alcohol, and his values are in close agreement with those of Weber.

Lees also investigated the conductivity of mixtures of water with varying amounts of glycerine, methyl alcohol, ethyl alcohol, acetic acid, sugar, and attempted to connect the conductivity of the mixture with that of the constituents. Plotting percentage composition by weight against conductivity he found that no general law could be deduced. If, however, percentages of volume were taken, similar curves were obtained which gave a conductivity somewhat less than that given by a linear formula .

$$K=\frac{K_1v_1+K_2v_2}{v_1+v_2}.$$

The value was also greater than that calculated assuming a linear formula of resistivity (*i.e.* reciprocal of conductivity). The latter formula is, however, closer to the experimental results than that based on a linear law of conductivity. Lees then obtained a closer approximation for the resistivity formula based on the probable distribution of the molecules of the two substances. From the agreement with experiment he deduces that the thermal conductivity of a substance is not greatly modified when it enters as one constituent into a physical mixture, and that the conductivity of the mixture depends directly on the amounts and conductivities of its constituents. He also experimented with mixtures of lard with varying amounts of finely divided iron, marble, zinc sulphate, and sugar, and found that the same conclusions held.

Lees made further use of the apparatus described above to investigate any change of conductivity on melting. He found that in the cases of the substances naphthylamine, paratoluidene, and sodium hydrogen phosphate ($Na_2HPO_4+12H_2O$) there was no discontinuity at the melting-point, and that the conductivity decreased with rise of temperature both in the solid and liquid state. A decrease of 20 per cent in conductivity was found in the case of calcium chloride ($CaCl_2+6H_2O$) at the melting-point, but owing to its affinity for water the results were not so reliable as for the other substances. It is interesting to note that Barus[1] found a decrease of 15 per cent in the case of thymol at the melting-point (12·5° C.).

(*b*) *Goldschmidt*[2] adapted the "hot-wire" method as used for gases (see § (8) (ii.) below). The liquid was contained in a tube only 2 mm. in diameter, and it is stated that convection was eliminated.

(iii.) Flow Methods.—The first observer to use a flow method for measuring the conductivity of a liquid appears to have been Graetz,[3] but his method is not described here, as those of Callendar and of Nettleton seem to be preferable.

(*a*) *Callendar.*[4]—Consider a long metal tube heated electrically and with a continuous stream of liquid passing through it. Let K be the conductivity of the liquid, c the specific heat per unit volume, v the velocity, and θ the temperature at a distance r from the axis of the tube, and let x be the distance measured along the tube. Then, assuming that the flow of the liquid is linear and neglecting the minute effect of longitudinal conduction, it is obvious that the heat carried off by the liquid flowing through any thin ring in a cross-section of the tube will be equal to the difference between the heat passing into and out of the ring by conduction. This gives the equation

$$\frac{d}{dr}\left(Kr\frac{d\theta}{dr}\right)=vcr\frac{d\theta}{dx}.$$

The solution of this equation will be much simplified if we can assume that the longitudinal gradient $d\theta/dx$ is constant over the cross-section of the tube at any point and is equal to θ'/l, where θ' is the rise of temperature observed in a length l. This will not be true near the inflow end of the tube, where the radial distribution of temperature is rapidly changing, but it will very fairly represent the limiting state, which is attained when the liquid has flowed along the tube for some distance.

Further, the quantity v in the equation above is a function of r and of the viscosity which can be taken as constant for the small temperature changes met with in the experiment. The velocity at any point is then given by

$$v=2V\left(1-\frac{r^2}{r_0^{\,2}}\right)=\frac{2Q\{1-(r/r_0)^2\}}{\pi r_0^{\,2}},$$

where V is the mean velocity, Q the flow in c.c. per second, and r_0 the internal radius of the tube.

Substituting and integrating from the temperature θ_0 of the surface of the tube, we find

$$\theta_0-\theta=\frac{Q\theta'}{2\pi lK}\left\{\frac{r^4}{4r_0^{\,4}}-\frac{r^2}{r_0^{\,2}}-\tfrac{3}{4}\right\}.$$

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[1] *Sill. Journ.* (3), 1892, xliv. 1.
[2] *Phys. Zeitsch.*, 1911, xii. 417.
[3] *Wied. Ann.*, 1883, xviii. 79.
[4] *Phil. Trans. Roy. Soc.*, 1902, cxcix. 110.

The temperature θ_1 at the axis of the tube where $r=o$ is given by

$$\theta_0-\theta_1=\frac{3Q\theta'}{8\pi l K}.$$

The mean temperature θ_m of the flow, allowing for variation of velocity over the cross-section, is given by

$$\theta_0-\theta_m=\frac{11Q\theta'}{48\pi l K}.$$

For the purposes of his experiment, Callendar took a platinum tube through which a current was passed. By measuring the resistance of consecutive sections of the tube, he was able to obtain the value of the gradient θ'/l, as well as the value of θ_0 at any section. θ_0 appears to have been measured at the end of the platinum tube, and the value of θ_m, or the mean temperature of the cross-section of flow at that point, was obtained by ensuring good mixing of the liquid as it left the platinum tube, its temperature being taken by means of a platinum thermometer. These quantities, together with the measurement of the rate of flow, gave the conductivity.

The device used by Callendar for obtaining a constant rise of temperature along the platinum tube is noteworthy. Thus, if R_0 is the resistance per cm. of the platinum tube at the initial temperature, a its temperature coefficient of resistance, and if E is the emissivity of the tube, p its perimeter, A its cross-section, and Q is the rate of flow of liquid, then we have

$$C^2R_0(1+a\theta)=QA\frac{d\theta}{dx}+Ep\theta.$$

If now C^2R_0a is made equal to Ep, the gradient $d\theta/dx$ will be constant. By choosing suitable dimensions for the diameter and wall thickness of the platinum tube it is possible to secure the above condition.

Callendar does not give the full experimental details, but the method seems very promising. It possesses advantages over the film method in that it avoids the difficulty of measuring accurately the thickness of the film, and requires no correction for radiation through the liquid, since all the heat lost by the inner surface of the tube must be absorbed by the liquid itself.

(*b*) *Nettleton.*—An ingenious flow method for determining the conductivity of mercury is due to Nettleton. A slow stream of mercury is forced up a vertical glass tube, of which the top is steam-jacketed, the bottom cooled in ice, and the middle portion surrounded by a constant temperature enclosure. At one point the temperature of the column of mercury will be the same as that of the enclosure, which is taken as the zero of reference. If now θ_1 and θ_2 are the temperatures at distances L/2 above and below this point, it can be shown that

$$\frac{\theta_1}{-\theta_2}=e^{\frac{msL}{2KA}},$$

s being the specific heat of mercury, m the number of grammes passing any section in a second, and A the cross-sectional area of the tube. From the above equation the conductivity is easily obtained.

For full particulars of the method, reference must be made to Nettleton's original papers.[1]

§ (8) GASES (K=0·000015 to 0·00036).—As in the case of liquids, the determination of the conductivity of a gas is complicated by convection and radiation effects. The attempt is generally made to eliminate the former and apply a correction for the latter. A number of observers have investigated the problem, and their methods fall into three classes, viz. the cooling thermometer method, the hot-wire method, and the film method.

(i.) *Cooling Thermometer Method. Kundt and Warburg.*[2]—These observers investigated the rate of cooling of a thermometer, the bulb of which was in an enclosure filled with the gas under experiment. They found that for air the rate of cooling remained constant for pressures between 150 mm. and 1 mm., and for hydrogen, between 150 mm. and 9 mm. It was therefore concluded that within these limits the action of convection currents was negligible, and that the heat loss took place by conduction through the gas and by radiation, after allowing a due correction for the conduction along the thermometer stem. To determine the radiation loss, the enclosure was exhausted as completely as possible, and the rate of cooling was then found to be independent of the shape of the enclosure. This showed that the effect of conduction through any residual gas was negligible. The radiation being thus known, the cooling due to conduction was obtained by difference. If the thermal capacity of the thermometer is measured, the absolute value of the conductivity of the gas could be deduced.

Kundt and Warburg found that the conductivity of hydrogen was seven times that of air, which was in accordance with Maxwell's prediction. Their value for air at 0° C. was 0·0000492.

Stefan[3] adopted a similar method, taking two coaxial cylinders of thin copper with the gas to be tested between them. The inner cylinder served as an air thermometer, its tube passing through the outer cylinder and dipping into a vessel of water. Observing the rate of cooling of the thermometer, Stefan obtained the value for K_0 of 0·0000543 for the conductivity of air.

A value very close to this was obtained by Winkelmann,[4] who experimented with spheres and cylinders on the same lines as Stefan.

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[1] *Phil. Mag.*, 1910, xix. 587; *Proc. Phys. Soc.*, 1910, xxii., and 1913, xxv. 28.

[2] *Pogg. Ann.*, 1875, clvi. 177.

[3] *Journ. de Phys.*, 1873, ii. 147.

[4] *Pogg. Ann.*, 1874, cliii. 497.

(ii.) *Hot-wire Method. Schleiermacher.*—Originally due to Andrews, this method was used by Schleiermacher [1] for measuring the absolute conductivity of a gas. An electrically heated wire is surrounded by a coaxial cylinder containing the gas under experiment. The procedure is very similar to that explained above, except that the wire is kept at a constant temperature, and the rate of energy dissipation is measured. As before, the vessel is exhausted so as to obtain the radiation loss and the end corrections.

The hot-wire method has been adopted by a number of observers, and some of their values for the conductivity of air at 0° C. are given below:

Schleiermacher	0·0000562
Schwarze	0·0000569
Stafford	0·0000447
S. Weber	0·0000568

The latest exponent was Weber.[2] He used a glass tube about 2·5 cm. in diameter with a platinum wire 12 cm. long stretched down the axis. The temperature of the wire varied from 7° to 25° C. above the surroundings. Particular attention was paid to the question of convection by studying the heat loss from the wire in the horizontal and vertical positions, and for pressures from atmospheric down to a few millimetres. The end effect was determined by experimenting with two vessels similar in all respects except their length. For full particulars of the various corrections, reference should be made to the original paper.

(iii.) *Film Methods.*—As in the case of liquids, a thin horizontal film, with the upper surface kept at a higher temperature than the lower, has been used to eliminate convection.

(*a*) *Todd.*—Acting on a suggestion made by Poynting, Todd [3] adopted the film method. The heat transfer across the film being by conduction and radiation and the latter being independent of the thickness, it was possible, by experimenting on layers of different thicknesses, to eliminate the radiation loss.

The apparatus consisted of a plate 30 cm. in diameter, heated by steam in its top surface. The film of air was contained between this plate and a similar plate, which formed the top of a flow calorimeter. The air film was isolated from the surrounding air by a double-walled cylinder of paper, which fitted the two discs closely. The heat reaching the cold plate was obtained from the flow calorimeter in the usual way, while the temperatures of the steam and water in contact with the plates was measured. An appropriate correction was made for the difference in temperature between the steam and water and the respective plates. Todd experimented on films from 1 mm. to 5 mm. in thickness, and obtained a value for the conductivity of air at 55° C. of 0·000057, which is equivalent to 0·0000495 at 0° C.

It would perhaps have been better if it had been possible to provide the calorimeter with a guard-ring and improve its thermal insulation.

(*b*) *Hercus and Laby.*[4]—These experimenters also made use of a film of gas heated on its upper surface. The arrangement of the apparatus is shown diagrammatically in *Fig.* 23. B is a hot plate surrounded by a guard-ring D. C is a water-cooled plate. The function of the plate A is to prevent any heat loss from the upper surface of B, so that all the heat generated in B would pass through the air film to C. For this purpose, A was maintained at the same temperature as B.

All the plates were of copper. A, B, and D each consisted of two sheets of copper clamped together, and enclosing a heating coil of manganin and some thermocouple wires. The latter consisted of silk-covered constantan, which was let into grooves in the plates and insulated by a wax of high melting-point. The constantan wires formed couples with the copper plates and return copper leads. The plate B was supported from D by three I-shaped ivory buttons. The surfaces of B and C, which faced each other, were ground true and silver-plated, and extraordinary care was taken to obtain the mean distance between the plates of careful contouring. The apparatus was made air-tight by a ring of stout rubber clamped to A and C by steel tapes.

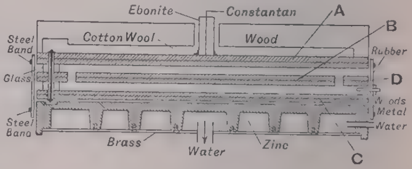

FIG. 23.

The plates were held together near the outer edge by three bolts passing through glass distance pieces.

When making an experiment the temperatures of the plates A, B, and D were adjusted to be approximately equal, corrections being made for the heat transference due to small differences of temperature. The plate A was always kept slightly hotter than B, so as to avoid any possibility of convection currents. The radiation correction was determined by independent experiments on a silvered Dewar flask. It would have been preferable if it had been possible to determine it on the apparatus itself, but the point is not of importance, since the correction amounted to less than 5 per cent.

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[1] *Wied. Ann.*, 1888, xxxiv. 623.

[2] *Ann. der Phys.*, 1917, liv. 325

[3] *Proc. Roy. Soc.*, 1909, lxxxiii. 19.

[4] *Proc. Roy. Soc.*, 1918, xcv. 190.

The thickness of the air-film used was 6·28 mm., and there was a difference of temperature of 20° C. between the plates. The conductivity of air at 0° C. was found to be 0·0000540.

§ (9) Discussion of Methods.—It will be realised from the experiments which have been described above that thermal conductivity is among the more difficult of the physical constants to measure with any degree of precision. When compared, for example, with electrical conductivity, it is noticed that there is nothing corresponding to an "insulator," in the electrical sense, which would enable the heat flow to be easily directed into any desired channel; while the almost instantaneous equilibrium established in an electrical system finds no thermal counterpart. Nevertheless, the divergencies in the values obtained by different experimenters for the same material are often rather surprising. Such divergencies, which are not infrequently of the order of 20 per cent, could probably be minimised by the observance of due precautions. An attempt is made below to set out some of the more obvious of these.

As has been already shown, the determination of absolute conductivity involves the measurement of three quantities, viz. the energy supplied to or passing through a body, the temperature distribution, and the dimensions of the body.

(i.) *Energy Measurement.*—Dealing first with the measurement of energy, the advantages of electrical supply are obvious. It is applicable over a wide range of temperature, it is easy to distribute uniformly, and it presents few difficulties of measurement. Further, the conversion from electrical to thermal units can now be accomplished with considerable precision, in view of the substantial agreement as to the value of the mechanical equivalent of heat.

When an enclosure of any size is to be heated electrically, it is convenient to blow air over a bare resistor element, as was done by Lamb and Wilson, while baffle plates can be used with advantage for the purpose of shielding the walls from radiation and of directing the air flow. When a flat heater is required, care should be taken to secure uniform winding of the resistor, which should be clamped between metal plates, so as to give a flat surface and to reduce inequalities of temperature distribution. For this latter purpose thick plates of copper or aluminium are useful, owing to their high conductivity. If the hot plate is surrounded by a guard-ring, there should be no metallic contact between the two, and the guard-ring heater should be capable of independent adjustment. Except in the case of short resistor elements, such as those of Niven and Poole, the escape of heat by conduction along the leads is not important, but if any correction is to be applied, reference may be made to the method of Lees.

When the body itself is used as a resistor, as with the so-called "electrical methods" applicable to metals, the heavy currents and small potential drops make the energy measurement more difficult (see, for example, Angell's method). It is sometimes desirable to use alternating current to avoid Peltier effects, and the Thomson effect has also to be considered.

An alternative to measuring the electrical energy supplied is to determine the heat passing out of the body. This can be done by the usual devices of calorimetry, such as measuring the ice melted or the water evaporated in a certain time, or, more satisfactorily, by means of a flow calorimeter. Examples of the last-mentioned are seen in *Figs.* 11 and 14. Several important precautions have to be observed with this instrument. The tank for supplying water should be kept at a constant level and temperature. At the inflow or outflow points, where the temperatures are taken, provision should be made for securing thorough mixing of the water so that the mean temperature of the stream may be recorded in each case. This can be done conveniently by means of baffles or plugs of wire gauze inserted in the flow tubes. A metallic pipe, preferably of copper, should be used for conveying the stream inside the calorimeter, but, both at the inflow and outflow, there should be a break in the pipe so as to prevent conduction losses along the metal. For this purpose rubber or ebonite connecting pieces can be used. The rise in temperature between the inflow and outflow can be measured best by a differential arrangement of resistance thermometers or a series of thermocouples.

(ii.) *Temperature Measurement.*—One of the main difficulties met with in conductivity experiments is the measurement of surface temperatures. Attention has already been drawn to the point in the case of metallic surfaces in contact with well-stirred liquids, and it has been shown that enormous errors may occur through assuming that the temperature of the metal surface is that of the mass of the liquid. Similarly in the case of two solid surfaces which are nominally in contact, there will often be a sharp temperature discontinuity due to the interposition of air gaps. The fact is not of great importance where the conductivity of the solid, whose surface temperature is desired, is at all comparable with that of air. Thus, in the case of insulating materials, no appreciable error is introduced by assuming the temperature of the metal plate which forms the boundary of the material is that of the surface of the material itself. More care must, however, be taken in the case of materials of medium conductivity. By using small specimens with good surfaces and improving the thermal contact with a glycerine or mercury film, Lees succeeded in surmounting the difficulty, but if such methods are impracticable it is better to abandon the attempt to determine surface temperature and measure the temperature of isothermal planes within the material. This procedure was adopted by a number of experimenters, *e.g.* Nusselt, Niven, Poole, and Dougill, Hodsman, and Cobb. Coming to metals, the method of Hall for measuring surface temperatures has much to commend it. At higher temperatures, where surface emissivity becomes an important factor, the device of Angell is interesting and appears to have been successful. However, the difficulties of determining the surface temperature of a metal by any contact method are so great that, wherever possible, the temperature should be measured at isothermal surfaces within the metal, which will generally be in the form of a bar.

Another important point affecting temperature measurements is to ensure that the thermal equilibrium has been reached before the final readings are taken. With materials of high diffusivity, such as metals, equilibrium is quickly established, but the process is very slow in the case of insulating materials. If the initial temperature of the material is known, its approximate diffusivity, and the boundary temperatures to which it is to be subjected, it is possible to estimate the time required for equilibrium. A calculation of this kind has been given by Niven for a cylinder of sand heated axially. This procedure is not to be generally recommended, as, apart from the difficulty of such calculations, the data as to the diffusivities of insulating materials is very meagre owing to the fact that the specific heat cannot readily be measured.

The matter, however, can be tested experimentally. Taking, for example, the case in which the temperature of the bulk of the material has to be raised, it will be found that a measurement of the energy supplied to maintain a constant difference in temperature will give an apparent conductivity which will decrease as time elapses, gradually approaching a steady value corresponding to the condition of equilibrium.

As an instrument for measuring temperature distribution the thermocouple is the most generally useful. When employing it, care should always be taken to keep the wires leading from the junction in the isothermal plane for an appreciable distance. Otherwise, as Nusselt has shown, very large errors may occur.

(iii.) *Dimensions and Shape.*—The measurement of dimensions presents few difficulties. It is true that when experiments are made on thin films of gas or liquid, considerable accuracy is demanded, and also that in some cases there may be doubt as to the precise dimension to be measured, as for instance in determining the effective area of a hot-plate surrounded by a guard-ring, or in deciding the distances between two isothermal planes whose temperatures have been taken by thermometers let into holes of appreciable diameter. Generally speaking, however, dimensional quantities are measured more accurately than the other quantities involved in a conductivity determination.

The dimensions should, of course, be appropriate to the material to be tested; thus it is advisable with insulating materials to use large-scale apparatus so as to avoid errors due to local irregularities of structure or packing. The shape is also decided largely by the same consideration, but care should be taken to avoid any shape in which there is likely to be a departure from a rectilinear heat flow, as in such cases the shape factor is extremely difficult to calculate. For this reason objection can be taken to the so-called "box method," adopted by many experimenters, by which an attempt is made to deduce the conductivity from the heat leakage into or out of a cubical box surrounded by a uniform thickness of the material.

§ (10) Discussion of Results.—A summary of results is given in Tables I. to V. The values are mostly those due to the observers whose methods have been described above. In the case of poor and medium conductors wide divergencies are noticeable, which no doubt are largely due to differences between materials nominally the same. With metals, where the chemical composition and physical condition can be defined more precisely, the differences are less marked. In fact the agreement between the values of Lees and of Jaeger and Diesselhorst, using entirely different methods, is very satisfactory. The experimental difficulties due to convection and radiation, in the case of liquids and gases, militate against extreme accuracy. The individual values are not given for gases, Table V. being a summary of the most probable mean values, up to 1914, prepared by Hercus and Laby. It should be mentioned that the recent determinations of S. Weber are higher than those in the table by amounts varying up to 14 per cent.

It is interesting to consider the bearing of the results on the relationship of thermal conductivity to other physical properties. Some of the more important relationships are dealt with below.

(i.) *Thermal and Electrical Conductivity of Metals.*[1]—The ease with which electrical conductivity of metals can be measured has for a long time directed the attention of physicists to the possibility of establishing a relationship between electrical and thermal conductivity, so that the latter could be indirectly determined.

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[1] See article "Electrons."

Forbes was the first to draw attention to the fact that metals can be arranged in the same order as regards their conducting powers for heat and electricity. In 1853 Wiedemann and Franz [1] propounded the law that the ratio of the thermal and electrical conductivities was the same for all metals. Lorenz [2] extended this theory in 1872 by suggesting that the ratio was proportional to the absolute temperature. On the development of the electron theory Drude,[3] and more recently H. A. Lorentz,[4] applying the kinetic theory of gases to the motion of electrons, arrived at the same conclusion as Lorenz. Up to 1900, however, the experimental values were too uncertain to allow any definite conclusion to be drawn. In that year Jaeger and Diesselhorst [5] published the result of their investigation, which gave directly the ratio of the two conductivities for a number of metals and alloys. They found that for seven pure metals—lead, cadmium, tin, nickel, silver, zinc, and copper—the value of Lorenz's constant $K/\lambda T$ was nearly the same over the range 18° to 100° C. The mean value was $2{\cdot}43 \times 10^{-8}$, K being expressed in watts/cm. sec. ° C., $1/\lambda$ in ohms per c.c.[3] and T being the absolute temperature. The few alloys that were tested gave higher values, and iron and aluminium were respectively higher and lower. Lees [6] confirmed the value of Lorenz's constant down to $-170°$ C. for lead, cadmium, tin; nickel showed a slight rise, and zinc, silver, copper showed a falling off in value in the order named. All the alloys tested gave increasingly high values with fall of temperature. Meissner,[6] who carried the investigation of copper, gold, platinum, and lead down to 20° abs., agreed approximately with Lees, but found a very rapid falling off in the constant as the absolute zero was approached. Thus at 20° abs. the value for copper was about one quarter of the value found by Jaeger and Diesselhorst at 290° abs. The work of Onnes and Holst [7] on the conductivity of mercury at 5° abs. confirms this rapid diminution.

TABLE V

GASES

The constant f in the table below is derived from the relation $K = f\eta C_v$, η being the viscosity and C_v the specific heat at constant volume.

Gas.	$K_0 \times 10^5$.		C_p.		γ.		C_v.		$\eta_0 \times 10^4$.		f.
He . .	32·7	2 *	1·255	2	1·667 †		..		1·883	5	2·31
A . .	3·85	2	0·123	1	1·667	1	0·0744	1	2·108	3	2·47
H_2 . .	36·3	9	3·407	5	1·399	4	2·406	2	0·852	12	1·76
N_2 . .	5·14	3	0·244	4	1·401	3	0·175	1	1·673	5	1·76
O_2 . .	5·35	4	0·218	4	1·401	4	0·1531	1	1·925	6	1·79
Air . .	5·22	14	0·239	7	1·402	18	0·1715	1	1·733	31	1·76
NO . .	4·93	2 ‡	0·231	1	1·397	1	..		1·737	2	1·73
CO . .	5·05	3	0·246	2	1·405	2	..		1·677	3	1·72
CO_2 . .	3·25	10	0·2015	6	1·300	13	0·160	3	1·428	12	1·45
N_2O . .	3·34	3	0·220	2	1·317	2	..		1·364	3	1·47
H_2S . .	2·81	1	0·2389	3	1·317	3	..		1·154	1	1·34
SO_2 . .	1·80	1	0·1527	2	1·258	3	0·1061	1	1·204	2	1·35
Cl_2 . .	1·69	1	0·120	2	1·341	3	..		1·256	2	1·50
NH_3 . .	4·71	3	0·531	3	1·306	3	0·390	1	0·949	3	1·23
CH_4 . .	6·76	3	0·592	2	1·316	3	..		1·036	2	1·45
C_2H_4 . .	3·84	3	0·404	2	1·250	4	..		0·940	4	1·27

* The number given after each constant is the number of values of which it is the mean.
† The theoretical value 5/3 is used in preference to the one observed value 1·63 of Behn and Geiger.
‡ Todd's value is not included as the accuracy of the reduction of K from 55° to 0° C. is doubtful.

With regard to temperatures of over 100° C. very little evidence is available. Konno,[8] using a comparative method with an apparatus similar in principle to that of Lees for medium conductors (*Fig.* 13), found that the conductivities of tin, lead, zinc, and aluminium steadily decreased towards the melting-point, where a sharp drop occurred. Taking Tsutsumi's values for electrical conductivity he deduced that Lorenz's law held approximately up to the melting-points. The changes of electrical and thermal conductivity on melting were of the same order, but Lorenz's law did not hold for the liquid state. The conclusions of Angell as to aluminium are widely at variance with those of Konno. While both obtained about the same value for the conductivity at 100° C., the latter gives a value at 600° C. of 0·36 and the former gives a value nearly three times as large. Angell's values of Lorenz's constant for nickel as well as aluminium show rapid increases with temperature.

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[1] *Ann. de Chimie*, 1854, xli. 107.
[2] *Ann. der Phys.*, 1872, cxlvii. 429.
[3] *Ibid.*, 1900, i. 566, and iii. 369.
[4] *Proc. Amst. Acad.*, 1905, vii. 438.
[5] *Abh. Phys. Tech. Reich.*, 1900, iii. 269.
[6] *Phil. Trans. Roy. Soc.*, 1908, ccviii. 381.
[7] *Proc. Akad. van Wet. Amst.*, 1914, p. 760.
[8] *Phil. Mag.*, 1920, ccxxxix, 542.

At still higher temperatures Langmuir gives a value for Lorenz's constant for tungsten of $3{\cdot}49 \times 10^{-8}$ at 1900° C., which means a comparatively small temperature coefficient on the value below 100° C. Worthing, however, does not agree with this value.

The law of Lorenz has therefore only been shown to hold for a certain number of pure metals over the range from about $-100°$ C. to 100° C. or 200° C. Below this region it definitely fails; while above, further evidence will be necessary before it can be accepted. As regards alloys, those tested by Lees and by Jaeger and Diesselhorst gave higher values for Lorenz's constant than the pure metals and large temperature coefficients. Further, Gruneisen [1] has shown that, in the case of copper and iron, small amounts of impurity lower the electrical far more than the thermal conductivity and thus increase the value of the constant. According, however, to recent work in Japan [2] Lorenz's law is obeyed approximately by carbon steels and alloys of iron with nickel, cobalt, and manganese, at ordinary temperatures, and by carbon steels up to 900° C.

(ii.) *Conductivity, Elasticity, and Density of Non-metals.*—Thornton [3] has recently put forward a tentative suggestion as to a relationship of the above-mentioned constants. He shows that for such diverse substances as quartz, flint crown and soda glass, graphite, marble, mahogany, deal, ice, and paraffin wax, the thermal conductivity is proportional to $E\rho$, E being Young's modulus of elasticity and ρ the density. His figures for glass have, however, been criticised by Clarke.[4]

(iii.) *Conductivity, Viscosity, and Specific Heat of Gases.*—Many of the deductions as to the physical properties of gases, based on the kinetic theory, have been confirmed by experiment in a remarkable way. In the case of conductivity, Maxwell's prediction of a value for air of 0·000055 is well within the range of those found by experiment, the mean of which is given by Hercus and Laby [5] as 0·000052, while his value for hydrogen as being seven times that of air has been closely confirmed. Another deduction from the kinetic theory is that the conductivity of a gas is independent of its pressure, and, as mentioned above, this has been found to be true, except for very low pressures where the mean free path of the molecules becomes comparable with the thickness of the gas layer. For these very low pressures Soddy and Berry [6] and Knudsen [7] have shown that the conductivity is nearly proportional to the pressure.

The important relation between the conductivity K, specific heat at constant volume C_v, and viscosity η, of a gas is primarily due to Maxwell, who showed that, assuming molecules are point centres of force repelling each other according to a fifth power law of distance,

$$K = fC_v\eta,$$

the value of f being 2·5.

A considerable amount of experimental evidence is available as bearing on this relation, and a summary of it, due to Hercus and Laby,[8] is given in Table V. The value of C_v has been very seldom observed, but it can be deduced from determinations of C_p and of the ratio of C_p to C_v. The viscosity can easily be measured. It will be noticed that for monatomic gases the value of f agrees approximately with that given by Maxwell. For gases with higher degrees of freedom the value is less than 2·5, being in the case of the diatomic gases 1·75. So far, theory has been unable to account for this value, though various empirical laws have been propounded by Jeans and others.

The determination of the conductivity of a mixture of gases from that of its constituents is sometimes a matter of practical importance. According to the kinetic theory it is given by

$$K = \frac{K_1}{1 + A(p_2/p_1)} + \frac{K_2}{1 + B(p_1/p_2)},$$

K_1 and K_2 being the conductivities, and p_1 and p_2 the partial pressures of the two gases, and A and B being constants the values of which are

$$A = \tfrac{1}{4}\sqrt{\tfrac{1}{2}\left(1 + \frac{m_1}{m_2}\right)}\left\{1 + \left(\frac{\eta_1}{\eta_2}\right)^{\frac{1}{2}}\left(\frac{m_2}{m_1}\right)^{\frac{1}{4}}\right\}^2,$$

$$B = \frac{\eta_2}{\eta_1}A,$$

where η_1 and η_2 are the viscosities, and m_1

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[1] *Ann. der Phys.*, 1900, iii. 43.
[2] *Tôhoku Univ. Sci. Rep.*, 1917, viii.; 1918, vii. 59.
[3] *Phil. Mag.*, 1919, xxxviii. 705.
[4] *Ibid.*, 1920, xl. 502.
[5] *Proc. Roy. Soc.*, 1918, xcv. 190.
[6] *Proc. Roy. Soc.*, 1909, lxxxiii. 254.
[7] *Ann. der Phys.*, 1911, xxxiv. 593.
[8] *Ibid.*

and m_2 the masses of the molecules of the two gases.

S. Weber[1] has, however, shown that these theoretical values of A and B incorrectly give the conductivity of the mixture. The relationship can be expressed in the form above mentioned, but the values of A and B must be determined experimentally for each mixture. The table below, quoted from Weber, shows the difference between the experimental and theoretical values :

Gas Mixture.	Oxygen, Hydrogen.	Argon, Helium.	Hydrogen, Carbon Dioxide.	Nitrogen, Argon.
A theoretical .	2·37	1·62	1·01	0·86
A experimental .	3·06	0·38	2·70	1·07
B theoretical .	0·92	1·16	2·56	1·19
B experimental .	1·38	2·92	0·40	0·95

The variation in conductivity of a gas with temperature has also been considered in the light of the kinetic theory, according to which it should vary as the square root of the absolute temperature. This would give a temperature coefficient of 0·00183. The experimental values for air have varied from 0·0013 to 0·0036. Recently, however, Eucken[2] has shown that for five different gases the ratio of the conductivity to the viscosity is constant for temperatures from − 80° C. to 100° C. As regards viscosity, a formula due to Sutherland has been verified over a wide range of conditions, so that, if Eucken's work is accepted, it can be applied to conductivity. This would give

$$K_\theta = K_0 \frac{273 + C}{\theta + C} \left(\frac{\theta}{273}\right)^{\frac{3}{2}}.$$

For air, taking C = 117, the temperature coefficient would be 0·0029.

II. Application of Theory to Practical Problems

As has already been indicated, the problems arising out of the flow of heat can be considered under two heads, namely, those of the steady state and those of the variable state. In the former the temperature at each point throughout the body remains constant and the flow of heat depends on the conductivity; while in the latter, where the temperature is varying with time, the thermal capacity per unit volume, as well as the conductivity, becomes a determining factor.

§ (11) Steady State—the Shape Factor.—If two isothermal surfaces are considered, differing in temperature by $d\theta$ and at an infinitely small distance dx apart, it has already been shown that the rate of heat loss is given by

$$H = KA\frac{d\theta}{dx}.$$

So that for isothermal surfaces which are at a finite distance apart we have

$$H = \frac{\int K d\theta}{\int dx/A}.$$

For most materials the conductivity K may be taken to be a linear function of the temperature, and hence the value of $\int K d\theta$ becomes the product of the conductivity for the *mean* temperature of the surfaces multiplied by the temperature difference between them.

The quantity $1 \div \int (dx/A)$ depends entirely on the size and shape of the body and the position of the surfaces by which the heat enters and leaves the body, and has therefore been called by Langmuir, Adams, and Meikle[3] the "shape factor." The value of the factor for various shapes is given below, together with the solution of a few typical problems of heat flow.

(i.) *Plane Wall or Slab.*—The shape factor in this case is obviously the ratio of the area A to the thickness x, and if the wall is homogeneous and of mean conductivity K the rate of heat flow through it is given by $KA(\theta_1 - \theta_2)/x$.

If the wall is a composite one, being made up of a number of parallel layers of different materials of conductivities $K_1 K_2$ . . . and thickness $x_1 x_2$ . . . , and if $\theta_1 \theta_2$ . . . are the temperatures of the faces of the successive layers, then

$$Q = \frac{(\theta_1 - \theta_2)k_1}{x_1} = \frac{(\theta_2 - \theta_3)k_2}{x_2} = \ . \ . \ . \ ,$$

where Q is the rate of heat flow through a unit area of the wall.

Eliminating the temperatures intermediate between θ_1 and θ_n we obtain

$$Q = \frac{(\theta_1 - \theta_n)}{(x_1/k_1 + x_2/k_2 + \ . \ . \ . \ )}.$$

$\theta_1 - \theta_n$ is the total difference of temperature through the wall, so that the composite wall of total thickness X is equivalent to a simple wall of this thickness and of conductivity K, such that

$$\frac{X}{K} = \frac{x_1}{K_1} + \frac{x_2}{K_2} + \ . \ . \ .$$

The formula enables one to calculate the heat flow through a composite wall if the conductivities and thicknesses of the components are known and the total temperature drop through the wall.

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[1] *Ann. der Phys.*, 1917, liv. 481.

[2] *Phys. Zeit.*, 1911, xii. 1101; 1913, xiv. 324.

[3] *Trans. Amer. Electrochem. Soc.*, 1913, xxiv. 53.

In many practical problems the surface temperatures of the wall or slab are not known and cannot conveniently be measured, *e.g.* a boiler plate heated on one side by hot gases and cooled on the other by water; or the wall of a dwelling-house or a cold store, the two faces of which are exposed to air at different temperatures. It is desired to calculate the heat transfer from the known temperature of the gas or liquid in contact with each face of the wall. The general treatment of this problem is beyond the scope of the present article, but it may be useful to give an approximate solution of the case of a wall the faces of which are exposed to still air at temperatures not differing greatly.

If θ_1 and θ_4 are the temperatures of air, and θ_2 and θ_3 the temperatures of the faces of the wall, then, since the heat transference between a surface of the wall and the surrounding air is nearly proportional to difference in temperature,[1]

$$Q = E(\theta_1 - \theta_2) = E(\theta_3 - \theta_4) = \frac{K(\theta_2 - \theta_3)}{x},$$

where Q is the rate of heat transfer per unit area.

Eliminating θ_2 and θ_3 we obtain

$$Q = \frac{\theta_1 - \theta_4}{2/E + x/K}.$$

For air temperatures about that of the atmosphere, and for surfaces which are plane and vertical and are full radiators, the value of E is roughly 0·0002 calories per sq. cm. per second per 1° C. difference of temperature. The contributions of radiation and convection towards this value are of the same order of magnitude.

Some applications of the formula can now be considered.

(*a*) If the wall is of thin metal (say, copper), x will be a fraction of a centimetre and K will be nearly 1·0 C.G.S. units. The value of x/K will be negligible as compared with $2/E$, and the heat transference will be governed entirely by emission coefficient of the wall. If the metal surface is polished, its power of absorbing or emitting radiation will be very small, and the value of the emission coefficient E will be much less than that given above.

(*b*) If the wall is of thin paper of conductivity 0·0003 C.G.S. units, x/K would still be small as compared with $2/E$; while, since paper is a good radiator, E will be much greater than for the polished metal, so that we have the apparent anomaly of a thin-paper wall transmitting more heat than a metal wall of the same thickness.

(*c*) If the wall is thick (say 100 cm.), and of good insulating material like granulated cork ($K = 0·0001$), x/K would be large compared with $2/E$, and the transmission of heat would be governed almost entirely by the conductivity of the material of the wall and not by its emission coefficient.

(ii.) *Cylinder or Cylindrical Shell.*—The shape factor in this case is

$$\frac{2\pi l}{\log_e (b/a)}, \text{ or } \frac{2{\cdot}73 l}{\log_{10} (b/a)},$$

where a and b are the internal and external diameters and l is the length, which is supposed to be great compared with the diameter. If the temperatures of the internal and external surfaces are known, the above formula enables the heat transmission to be calculated.

As in the case of a wall, however, it may not be convenient to determine surface temperatures. Taking as an example a steam-pipe covered with insulating material, the inner temperature of the lagging may be assumed to be that of the steam, since there will be no appreciable gradient through the pipe. It is desired to calculate the heat loss, knowing the temperature of the surrounding air. Proceeding in a similar way to that followed in the case of a wall we get

$$Q = \frac{2\pi l K(\theta_1 - \theta_2)}{\log_e (b/a)} = 2\pi b l E(\theta_2 - \theta_3),$$

where θ_1 and θ_2 are the inner and outer temperatures of the lagging and θ_3 that of the air. The elimination of θ_2 gives

$$Q = \frac{2\pi l(\theta_1 - \theta_2)}{((1/K) \log_e (b/a) + (1/bE))}.$$

The heat loss from the bare pipe would be

$$Q = 2\pi l a E(\theta_1 - \theta_2).$$

It does not follow that the heat loss from the bare pipe would always be greater than that from the covered pipe. If the covering were of metal, it is obvious that the reverse would be the case, and even with medium conductors it is possible that the covered pipe would give the greater loss. Péclet,[2] who was probably the first to draw attention to this point, took the case of a pipe 12 cm. in diameter and with insulation varying in thickness from 1 cm. to 12 cm. If the conductivity of the insulation was 0·00012 C.G.S. units, the heat transmission diminished with increasing thickness; if the conductivity was 0·002, or sixteen times as great, the transmission was approximately constant for all thicknesses; and if the conductivity was 0·004 the heat loss increased with the thickness until at 12 cm. it was greater than that for the bare pipe.

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[1] The heat is transferred by radiation and convection. The former is proportional to $(\theta_1 + 273)^4 - (\theta_2 + 273)^4$, and the latter to $(\theta_1 - \theta_2)^{\frac{5}{4}}$. For small temperature differences the heat transferred varies approximately as $(\theta_1 - \theta_2)$.

[2] *Traité de la chaleur*, 1860.

A full treatment of the subject has more recently been given by Porter.[1]

(iii.) *Sphere or Spherical Shell.*—The shape factor is $2\pi ab/(b-a)$, where a and b are the internal and external diameters. The case of lagging surrounding a sphere can be treated in the same way as that for a cylinder, but it is not of much practical importance.

(iv.) *Rectangular Prism or Box.*—Examples of this case are commonly met with in practice, *e.g.* a furnace chamber of cubical or rectangular shape surrounded by walls of uniform thickness. No rigid mathematical solution has yet been obtained for the shape factor, but assuming that the inner and outer surfaces are isothermal, Langmuir, Adams, and Meikle[2] have given an approximate formula.

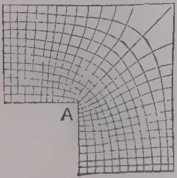

FIG. 24.—Section of Prism showing Flow Lines and Isothermals near a square edge A.

They take first the case of a long hollow prism of rectangular section and uniform thickness. If A is the area of the interior surface and t the thickness of the wall, and if the lines of heat flow were normal to the interior surface, the shape factor would be A/t. The lines of heat flow will, however, be normal only for the central portion of each face. Towards the edges they tend to become radial, though they are not rectilinear. This is illustrated in *Fig.* 24, which shows the lines of heat flow and the isothermals for a portion of the section of the prism. The edge will tend to increase the shape factor, and the amount by which the factor is made to exceed A/t may be regarded as the shape factor of the edge itself. By a series of approximations the shape factor of the edge is shown to be $0{\cdot}477l$, where l is its length.

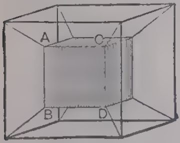

FIG. 25.—Cubical Box. Inner surface shaded. AC and BD, "square" edges. A, B, C, D, "square" corners.

Next, the case of a cubical or rectangular box of uniform thickness is taken. In addition to the edges, which have the same factor as in the case of the prism, there is the effect of the corners. The shape factor of each corner is estimated to be $0{\cdot}151t$, where t is the thickness of the wall of the box. The edges and corners so far dealt with are termed "square" edges and corners (see *Fig.* 25).

If two opposite walls of the box are supposed to approach each other until they nearly touch (see *Fig.* 26), the square edges AC and BD form practically one edge, which is called a "plane" edge and has a shape factor nearly double that of two "square" edges. Similarly the four "square" corners A, B, C, D now form the two "plane" corners AB and CD, for which shape factors are calculated. An example of the case illustrated in *Fig.* 26 is a rectangular plate surrounded by a uniform thickness of material.

It is interesting to note that Langmuir, Adams, and Meikle confirmed their theoretical deductions by experiments based on the analogy of heat flow through walls and the flow of electricity through an electrolyte of the same shape. Elementary cubes, prisms, cylinders, etc., of the electrolyte (copper sulphate) were formed by joining together suitable plates of glass and copper, and the electrical conductivities of the various shapes of electrolyte were compared. In this way the shape factors of a square edge and plane edge were found to be 0·54 and $0{\cdot}93l$ while the theory has given 0·477 and $0{\cdot}952l$. The values of a square corner and plane corner were $0{\cdot}15t$ and $0{\cdot}087t$. The experimental values are accepted in preference to the theoretical, and they were found to hold if the linear dimensions of the inner surface exceed $\frac{1}{5}t$. This limit would be exceeded in most practical cases.

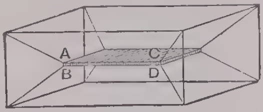

FIG. 26.

ABCD, "plane" edge. AB and CD, "plane" corners.

The author's conclusions as to the shape factor S of a cubical or rectangular box can be summarised as follows:

All interior dimensions $>\frac{1}{5}t$,

$$S=\frac{A}{t}+0{\cdot}54\Sigma l+1{\cdot}2t;$$

one dimension $<\frac{1}{5}t$,

$$S=\frac{A}{t}+0{\cdot}465\Sigma l+0{\cdot}35t;$$

two dimensions $<\frac{1}{5}t$,

$$S=\frac{2{\cdot}73l}{\log_{10}(b/a)};$$

three dimensions $<\frac{1}{5}t$,

$$S=0{\cdot}79\frac{\sqrt{AB}}{t},$$

A and B being the areas of the inner and outer surfaces.

Examples of the last two cases are respectively a small square section rod or a small cube with a great thickness of insulation around them.

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[1] *Phil. Mag.*, 1910, xx. 511.

[2] *Loc. cit.*

Example of Calculation of Shape Factor.—Take an electrically heated oven with 8 inches thickness of insulation, and of inside dimensions 8 × 10 × 20 in., which are all greater than $\frac{1}{5}t$. Then

$$\begin{aligned} \text{A} &= 2\times\ 8\times 10=160 \text{ square inches}\\ &\quad +2\times 10\times 20=400 \quad ,,\\ &\quad +2\times 20\times\ 8=320 \quad ,,\\ &\qquad\qquad\qquad\ \ 880 \quad ,, \end{aligned}$$

$$\Sigma l=4(8+10+20)=152 \text{ inches.}$$

Hence S is the sum of $\frac{\text{A}}{t}=\frac{880}{8}=110$ inches

$$\begin{aligned} +0{\cdot}54\Sigma l &= 0{\cdot}54\times 152=\ 82 \quad ,,\\ +1{\cdot}2t &= 1{\cdot}2\times 8=\ \ 9{\cdot}6 \quad ,,\\ &\qquad\qquad\ \ 201{\cdot}6 \quad ,, \end{aligned}$$

It is interesting to compare this result with two other methods which are sometimes adopted. Calculating the shape factors from the areas of the inner and outer surfaces by means of the formula $\text{S}=(\text{A}/t)$, and taking either the arithmetic or geometric means between the two values, the following results are obtained for the oven:

Arithmetic mean	353·3
Geometric mean	247·4

which differ considerably from the other value.

The formulae which have been summarised above are a useful contribution towards the solution of the problem of calculating the heat losses from rectangular-shaped bodies covered with a uniform thickness of insulation. It must not be forgotten, however, that they apply only where the inner and outer surfaces are isothermal, and that this condition is often not realised in practice.

§ (12) VARIABLE STATE—HEAT DIFFUSIVITY. All problems connected with the variable flow of heat depend on particular solutions of Fourier's fundamental equation,

$$c\rho\frac{d\theta}{dt}=\text{K}\left(\frac{d^2\theta}{dx^2}+\frac{d^2\theta}{dy^2}+\frac{d^2\theta}{dz^2}\right),$$

in which K, c, ρ are the conductivity, specific heat and density of the body, t the time, and θ the temperature at any point whose coordinates are x, y, z. This equation expresses the fact that the rate of heat inflow into, or outflow from, any small element of a body (represented by the right-hand side of the equation) must equal the rate at which the element is gaining or losing heat (left-hand side). It is frequently written in the form

$$\frac{d\theta}{dt}=h^2\nabla^2\theta,$$

h^2 or $\text{K}/c\rho$ being a constant which is known as the diffusivity (see § (2) (ii.) above).

The problems treated below are concerned only with the flow of heat in one dimension. For example, the penetration of heat into a fireproof wall, one face of which is raised to a high temperature, the propagation of heat waves into the earth's interior from the surface, and so on. For these cases Fourier's equation reduces to

$$\frac{d\theta}{dt}=h^2\frac{d^2\theta}{dx^2}.$$

They can conveniently be considered under various heads according to the dimensions of the body, its initial temperature distribution, and the mode of heat supply.

(i.) *Periodic Flow from a Plane Surface into a Semi-infinite Solid.*—Suppose that the heat supply is such as to produce simple harmonic variations of temperature at the surface of an infinite solid, and that θ_0 is the amplitude and T the period of the oscillation. Then the temperature of the surface at any time is given by

$$\theta=\theta_0\sin\frac{2\pi t}{\text{T}}.$$

It can be shown [1] that the general solution of Fourier's equation, which satisfies this boundary condition, is

$$\theta=\theta_0 e^{-\frac{x}{h}\sqrt{\frac{\pi}{\text{T}}}}\sin\left(\frac{2\pi t}{\text{T}}-\frac{x}{h}\sqrt{\frac{\pi}{\text{T}}}\right),$$

where θ is the temperature at a distance x from the surface.[2] It will be seen from the form of this equation that the temperature at any point will vary periodically and that the amplitude of the oscillation will diminish with the distance from the surface. The following characteristics of the wave propagation can be deduced from the equation:

$$\begin{aligned} &\text{Amplitude at any point} = \theta_0 e^{-\frac{x}{h}\sqrt{\frac{\pi}{\text{T}}}},\\ &\text{Velocity of propagation} = 2h\sqrt{\frac{\pi}{\text{T}}},\\ &\text{Wave-length} \qquad\qquad\quad = 2h\sqrt{\pi\text{T}}. \end{aligned}$$

The "lag," which can be defined as the time taken for any crest, trough, or other phase of the wave to reach a point at a distance x from the surface, is equal to $(x/2h)\times\sqrt{\text{T}/\pi}$. Heat will flow into the material during one half period of the temperature oscillation and out of the material during the other half period. It can be shown that the heat flow during a half period is

$$\text{H}=\theta_0\frac{\text{K}}{h}\sqrt{\frac{\text{T}}{\pi}}.$$

(*a*) *Application: Diurnal and Annual Waves in the Earth's Crust.*—The surface of the earth is subjected to daily and annual waves of temperature which, to a first approximation, can be assumed to be simple harmonic oscillations. The square root of the ratio of the periods of the annual and daily waves is about 20($\sqrt{365} = 19{\cdot}1$). So that we can at once deduce from the expressions given above that the wave-length and the lag at any point for the annual wave will be twenty times those for a daily wave. The amplitude of the latter will fall off very rapidly with depth. Taking the case of a daily wave with a range +15° C. to −5° C. in soil of diffusivity ·0050, the amplitude of 20° C. at the surface is reduced to 10° at 8 cm., 1° at 35 cm., and ·02° C. at 80 cm. Since the mean temperature of the soil is 5° C. an amplitude of 10° C. will just be sufficient to take the soil 0° C., so that the freezing temperature will only penetrate to a depth of 8 cm. The lag at this depth will be about 2½ hours. If an annual wave had the same range as the daily wave taken above, the freezing temperature would penetrate twenty times as far and the lag at this point would be 40 days.

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[1] See, *e.g.*, Ingersoll and Zobel, *Introduction to the Mathematical Theory of Heat Conduction* (Ginn & Co.), 1913.

[2] See also "Heat, Conduction of, Mathematical Theory," equation (12).

The mathematical formulae given above apply only to simple harmonic oscillations of temperature. The annual wave, in particular, departs considerably from the simple form, and it may therefore be necessary to analyse it, according to the Fourier method, into a number of simple harmonic components. Those of short period, like the daily wave, rapidly become inappreciable as the depth is increased.

A number of observers have studied the temperature waves in soil for the purpose of measuring its diffusivity, and some of their values are given in the table below:

Table VI

Observer.	Soil.	Locality.	Diffusivity.
Kelvin, 1860 . .	Garden sand	Edinburgh	0·0087
Neumann, 1863 .	Sandy loam	Edinburgh	·0136
Everett, 1860 .	Gravel	Greenwich	·0125
Ångström, 1861 .	Sandy clay	Upsala	·0057
Callendar, 1895 .	Garden sand	Montreal	·0036
Rambaut, 1900 .	Gravel .	Oxford	·0074

The two last-mentioned observers used platinum thermometers, which are much to be preferred to the mercury thermometers with long stems used in some of the earlier work. Callendar points out that water has a very large effect on the diffusivity of the soil. Its presence increases both the conductivity and the heat capacity, but the former to a greater extent, so that the diffusivity is increased. Under very extreme conditions the diffusivity may vary from 0·001 to 0·030.

(ii.) *Insertion of Slab at Temperature θ_0 between two Infinite Blocks of same Material at 0° C.*—If the slab is of thickness $2l$ it can be shown [1] that the temperature at a point at a distance x from the central plane of the slab is given by

$$\theta = \frac{\theta_0}{\sqrt{\pi}} \int_{\frac{-l-x}{2h\sqrt{t}}}^{\frac{l-x}{2h\sqrt{t}}} e^{-\beta^2} d\beta.$$

Values of this integral (the "probability integral"), which is frequently met with in the solution of problems of variable heat flow, will be found in Tables of Mathematical Functions.[2]

As an example we may take the case of concrete, originally at a temperature of 10° C., which is poured into a trench 60 cm. wide cut in the soil. If the temperature of the soil is −5° C. it is desired to know whether the concrete will have time to set before it is frozen.

For this purpose we can calculate how long the freezing temperature will take to penetrate to a depth of (say) 6 cm. below the surface of the concrete, which it is assumed has the same diffusivity as the soil (0·005). Here $l = 30$ cm., $x = 24$ cm., while $\theta_0 = 15°$ and freezing temperature $= 5°$, both relative to the soil, so that

$$5 = \frac{15}{\sqrt{\pi}} \int_{\frac{-54}{2h\sqrt{t}}}^{\frac{6}{2h\sqrt{t}}} e^{-\beta^2} d\beta.$$

The limits of the integral may be called q and $-9q$, and by a process of trial from the table of values it can be shown that $q = 0{\cdot}065$ nearly, so that t is over 4 days.

It is interesting to note that as the conditions of symmetry require that there can be no transfer of heat across the middle plane of the slab, the above solution applies also to the case of a slab of half the thickness one face of which is placed in contact with an infinite block and the other protected from heat loss.

(iii.) *Semi-infinite Solid at 0° with Boundary Surface maintained at θ_0.*—This case differs from the first case discussed above in that the boundary temperature instead of varying periodically is supposed to be suddenly altered to θ_0, and then maintained constant.

It can be shown [3] that the temperature θ at a distance x below the surface is given by

$$\theta = \frac{2\theta_0}{\sqrt{\pi}} \int_{\frac{x}{2h\sqrt{t}}}^{\infty} e^{-\beta^2} d\beta.$$

From this equation it is easy to deduce the

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[1] Ingersoll and Zobel, *loc. cit.* p. 70.

[2] Wellisch, *Theorie und Praxis der Ausgleichrechnung*, p. 257, or Ingersoll and Zobel, *loc. cit.* p. 166.

[3] Ingersoll and Zobel, *loc. cit.* p. 78.

"law of times" according to which the times required for any two points to reach the same temperature are proportional to the squares of their distances from the boundary plane.

As an application of the above equation we may take the case of a mass of concrete (diffusivity ·005) at a temperature of 20° the surface of which is suddenly raised 740° C. It is required to ascertain how long a temperature of 100° C. will take to penetrate a distance of 30 cm. Here θ_0 is 720° and θ is 80°, measured relative to the original temperature, and t is found to be about 36,500 seconds or 10 hours.

(*a*) *Application: Cooling of the Earth.*—The equation above can also be used for the solution of the problem of the time taken for the cooling of the earth to its present condition, neglecting the effect of the earth's curvature. By differentiating the equation it can be shown[1] that

$$t=\frac{\theta_0{}^2}{\pi h^2(d\theta/dx)_0{}^2}.$$

Kelvin assumed the original temperature θ_0 to be 3900° C., $h^2=\cdot01178$, and the surface gradient $(d\theta/dx)_0$ to be 1° C. in 2776 cm., and obtained a value of 100 million years. Even allowing very wide limits of error, this estimate is far below that based on geological considerations. It has been suggested that a probable explanation of the discrepancy between the two estimates is supplied by the recent discovery of the continuous generation of heat by the disintegration of radioactive compounds.

(iv.) *Slab at Uniform Temperature: one Face raised to Temperature θ_0 and kept constant, the other protected from Heat Loss.* — If $l/2$ is the thickness of the slab the relation between time and the temperature of the protected face is shown by Ingersoll and Zobel[2] to be

$$\theta=\theta_0\left\{1-\frac{4}{\pi}e^{\frac{-h^2\pi^2t}{l^2}}+\frac{4}{3\pi}e^{\frac{-9h^2\pi^2}{l^2}}\right.$$
$$\left.\frac{4}{5\pi}e^{\frac{-25h^2\pi^2t}{l^2}}+\ldots\right\}.$$

Generally it is only necessary to take account of the first two or three terms of this expression. As in the other case of a slab dealt with above, the equation also gives the temperature rise of central plane of a slab of twice the thickness (*i.e.* l) each face of which is suddenly raised to and maintained at θ_0.

As an example of the above conditions we may take a wall of concrete backed by a wooden casing or a floor of masonry or concrete on which is piled some poorly conducting but inflammable material. It is desired to know how soon a fire on one side of the wall would cause a dangerously high temperature to penetrate to the other side. It follows at once that the effectiveness of the wall or floor in preventing fire depends on a low value of the diffusivity rather than the conductivity, and that the time taken for a dangerous temperature to penetrate to the rear face will vary as the square of the thickness (cf. law of times above).

The solution given requires that the face of the wall should be suddenly raised to a temperature and maintained there. In practice this condition would not be realised, but Ingersoll and Zobel show that a solution may be arrived at by the device of imagining a layer to be added to the outside of the wall, the surface of which is suddenly raised to a temperature θ_0'. By suitably choosing the added thickness and the temperature θ_0', a very close approximation to the temperature rise of the actual surface of the wall can be obtained. For most purposes, however, a sufficiently close solution is given by using the mean value of θ_0.

For further examples of practical problems depending on the variable state, reference may be made to a standard textbook, such at that of Ingersoll and Zobel, from which many of the solutions given above have been drawn.

TABLE VII

DIFFUSIVITY OF METALS AND OTHER SUBSTANCES

Diffusivity $(h^2)=k/c\rho$

Metals.	Diffusivity.	Other Substances.	Diffusivity.
	C.G.S. units.		C.G.S. units.
Aluminium . .	0·826	Air	0·179
Antimony . .	0·139	Asbestos wool .	0·0035
Bismuth . .	0·068	Brick (fire) . .	0·0074
Brass . . .	0·339	Brick (building).	0·0050
Cadmium . .	0·467	Charcoal . . .	0·0020 to ·0030
Copper . . .	1·133	Coal	0·002
Gold . . .	1·182	Concrete . . .	0·0056
Iron (wrought) .	0·173	Cork (granulated)	0·0020 to ·0030
Iron (cast) . .	0·121	Ebonite . . .	0·0010
Lead . . .	0·237	Glass	0·0057
Magnesium . .	0·883	Granite . . .	0·0155
Mercury . . .	0·033	Ice	0·0112
Nickel . . .	0·152	Kieselguhr . .	0·0020 to ·0030
Palladium . .	0·240	Limestone . .	0·0092
Platinum . .	0·243	Slag wool . .	0·0020 to ·0030
Silver . . .	1·737	Snow (fresh) .	0·0033
Tin	0·407	Soil	See Table VI.
Zinc	0·402	Water . . .	0·00143

F. H. S.

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[1] Ingersoll and Zobel, *loc. cit.* p. 89.

[2] *Loc. cit.* p. 108.

HEAT, CONDUCTION OF: MATHEMATICAL THEORY

Conduction is the name given to the process by which heat is transferred from one portion of a body to another portion immediately adjacent to it, across the common boundary or interface, in consequence of inequality of temperature. It is to be distinguished from Radiation, which may take place between bodies which are at a distance apart, with no material connection, and from Convection, in which the transfer is promoted by currents in the substance.

The points of a body which at a given instant have an assigned temperature θ lie on a surface, called an "isothermal" surface. If we imagine a series of such surfaces to be drawn for equal small intervals of θ we obtain a complete mental picture of the instantaneous distribution of temperature. In general the configuration of these surfaces changes from instant to instant. When on the other hand it is permanent we have what is called a "stationary" state.

The mathematical theory of conduction starts with the assumption that in an *isotropic* substance the flux of heat across a surface element drawn (in any direction) through an internal point P is in the direction of diminishing temperature, and is proportional to the space-rate at which the temperature falls in the direction of the normal. Hence if δS be the area of the element, and δn an element of the normal to it, the (positive or negative) amount of heat which crosses in time δt, in the direction of δn, is

$$-k\frac{\partial\theta}{\partial n}\delta S\,.\,\delta t, \quad . \quad . \quad . \quad (1)$$

where k is a coefficient called the "thermal conductivity." The value of k depends of course on the thermometric scale adopted, and on the nature of the substance. It varies also to a slight extent with θ, but this is not important unless the range of temperatures be considerable, and is usually ignored in the mathematical theory. It is evident that the flux is greatest in the direction of the normal to the isothermal surface through P, and zero in tangential directions.

The simplest problems are those of linear flow, where the isothermal surfaces are parallel planes, as when the faces of a large flat plate or slab are subjected to given changes of temperature, uniform over each. Taking the axis of x normal to these planes we calculate the amount of heat gained in time δt by unit area of a stratum of thickness δx. The amount which enters the rear face is

$$-k\frac{\partial\theta}{\partial x}\delta t,$$

whilst an amount

$$-k\left(\frac{\partial\theta}{\partial x}+\frac{\partial^2\theta}{\partial x^2}\delta x\right)\delta t$$

leaves by the front face. There is thus on the whole a (positive or negative) gain of amount

$$k\frac{\partial^2\theta}{\partial x^2}\delta x\delta t.$$

To find the consequent change of temperature we divide by the thermal capacity of the portion of the stratum considered, viz. $\rho c\delta x$, where ρ is the density, and c the specific heat per unit mass. Hence

$$\frac{\partial\theta}{\partial t}\delta t=\frac{k}{\rho c}\frac{\partial^2\theta}{\partial x^2}\delta t$$

or

$$\frac{\partial\theta}{\partial t}=\kappa\frac{\partial^2\theta}{\partial x^2}, \quad . \quad . \quad . \quad (2)$$

where

$$\kappa=\frac{k}{\rho c}. \quad . \quad . \quad . \quad . \quad (3)$$

It is this quantity κ, rather than k, which determines the rapidity with which temperature changes ensue, under assigned conditions, in a body of given dimensions; κ is therefore called the "thermometric" conductivity. It appears from (2) that the dimensions of κ are those of the square of a length divided by a time. Hence in geometrically similar bodies of the same material the times in which analogous changes of temperature take place are proportional to the squares of the linear dimensions.

In a stationary state we have $\partial\theta/\partial t=0$ and, therefore, from (2), $\theta=A+Bx$, the graph of which is a straight line. Thus if the two faces $x=0$, $x=l$ (say) of a slab be maintained at given temperatures θ_0, θ_1, we have

$$\theta=\theta_0+(\theta_1-\theta_0)\frac{x}{l}. \quad . \quad . \quad (4)$$

The simplest example of a variable temperature is that of a solid bounded by the plane $x=0$ and extending to infinity in the direction of x positive, when the surface is subject to a periodic variation

$$\theta=\theta_0\cos\omega t. \quad . \quad . \quad . \quad (5)$$

The work is shortened if we replace (5) by $\theta=\theta_0 e^{i\omega t}$ and afterwards reject the imaginary part of the result. Assuming $\theta=ue^{i\omega t}$, where u involves x only, we find on substitution in (2)

$$\frac{d^2u}{dx^2}=\frac{i\omega}{\kappa}u, \quad . \quad . \quad . \quad (6)$$

whence

$$u=Ae^{(1+i)mx}+Be^{-(1+i)mx} \quad . \quad (7)$$

provided

$$m^2=\frac{\omega}{2\kappa}. \quad . \quad . \quad . \quad . \quad (8)$$

Since in our case u must not become infinite with x, A must vanish, and putting $x=0$ we find $B-\theta_0$. Hence, retaining only the real part,

$$\theta=\theta_0 e^{-mx}\cos(\omega t-mx). \quad . \quad . \quad (9)$$

The fluctuations of temperature are propagated inwards without change of the period $(2\pi/\omega)$, with the wave-velocity ω/m or $\sqrt{(2\kappa\omega)}$, but diminish in amplitude as they proceed, according to the law e^{-mx}. In the space of a wave-length $(2\pi/m)$ the ratio in which the amplitude is diminished is $e^{-2\pi}$ or 1/536. This example is of interest as illustrating the way in which the daily and seasonal variations of temperature at the earth's surface are modified as they penetrate into the ground. It appears from (8) that the more rapid fluctuations have less penetration than the slower. Thus at a certain depth the alternations of summer and winter may be still sensible, whilst the daily variations are imperceptible. The progressive change of phase is also to be noticed. At a depth π/m the phase is inverted, the temperature being (for instance) highest in winter and lowest in summer.

If we start with an initial distribution

$$\theta=B\sin mx \quad . \quad . \quad . \quad (10)$$

in a solid unlimited both ways, we have, assuming $\theta=u\sin mx$, where u involves t only,

$$\frac{du}{dt}+\kappa m^2 u=0. \quad . \quad . \quad . \quad (11)$$

Hence, determining the constant so as to agree with (10) when $t=0$,

$$\theta=Be^{-\kappa m^2 t}\sin mx. \quad . \quad . \quad (12)$$

This is a simple example of a "normal mode" of decay of inequalities of temperature in a body left to itself. The smaller the scale $(2\pi/m)$ of the inequalities the more rapid is the process of smoothing out. The result may be adapted to the case of a slab bounded by the planes $x=0$, $x=l$, by making $\sin ml=0$, or $m=s\pi/l$, where s is any integer. The normal modes corresponding to $s=1, 2, 3, \ldots$, respectively, may be superposed; thus

$$\theta=B_1 e^{-\lambda_1 t}\sin\frac{\pi x}{l}+B_2 e^{-\lambda_2 t}\sin\frac{2\pi x}{l}+ \ . \ . \ . \ (13)$$

where
$$\lambda_s=\frac{s^2\pi^2\kappa}{l^2}. \quad . \quad . \quad . \quad . \quad (14)$$

The coefficients B_s may be determined so that (13) shall represent, for $t=0$, any arbitrary initial linear distribution of temperature in the slab (*v.* "Fourier's Theorem"). As t increases, the successive terms in (13) gradually diminish, each more rapidly than the preceding one, so that the first component is the last to survive in appreciable amplitude.

To investigate the propagation of heat in a uniform *bar* it is necessary to take account of the loss by radiation from the sides. The usual assumption is that the time-rate of loss per unit area of the surface is $E\theta$, where θ is the excess of temperature above that of the environment, and E is a constant, called the "emissivity," whose value depends on the nature of the surface. The loss in time δt to an element δx of the bar, from this cause, is $Ep\delta x\delta t$, where p is the perimeter of the cross-section. This leads to the equation

$$\frac{\partial\theta}{\partial t}=\kappa\frac{\partial^2\theta}{\partial x^2}-h\theta, \quad . \quad . \quad . \quad (15)$$

where
$$h=\frac{Ep}{\rho c S}, \quad . \quad . \quad . \quad . \quad (16)$$

S denoting the sectional area.

In a stationary state we have

$$\frac{\partial^2\theta}{\partial x^2}=\frac{h}{\kappa}\theta, \quad . \quad . \quad . \quad (17)$$

whence
$$\theta=Ae^{mx}+Be^{-mx}, \quad . \quad . \quad (18)$$

if
$$m^2=\frac{h}{\kappa}. \quad . \quad . \quad . \quad (19)$$

Thus if the ends ($x=0$, $x=l$) be maintained at the constant temperatures θ_0, θ_1 respectively, we find

$$\theta=\frac{\theta_0\sinh m(l-x)+\theta_1\sinh mx}{\sinh ml}. \quad . \quad (20)$$

If the bar is infinitely long we have the simpler formula

$$\theta=\theta_0 e^{-mx}. \quad . \quad . \quad . \quad (21)$$

To investigate the case of a very long bar whose extremity $(x=0)$ is subject to a given periodic variation, we assume in the first instance

$$\theta=\theta_0 e^{i\omega t} \quad . \quad . \quad . \quad (22)$$

as the prescribed terminal temperature. The equation (15) then gives

$$\frac{\partial^2\theta}{\partial x^2}=\frac{h+i\omega}{\kappa}\theta. \quad . \quad . \quad . \quad (23)$$

To solve this we introduce auxiliary constants r and ϵ, such that

$$\frac{(h+i\omega)}{\kappa}=r^2(\cos 2\epsilon+i\sin 2\epsilon), \quad . \quad . \quad (24)$$

that is
$$r^2=\frac{\sqrt{(h^2+\omega^2)}}{\kappa}, \quad \tan 2\epsilon=\frac{\omega}{h}. \quad . \quad . \quad (25)$$

Then, writing for shortness

$$\alpha=r\cos\epsilon, \quad \beta=r\sin\epsilon, \quad . \quad . \quad (26)$$

we have
$$\frac{\partial^2\theta}{\partial x^2}=(\alpha+i\beta)^2\theta, \quad . \quad . \quad (27)$$

whence
$$\theta=\{Ae^{(\alpha+i\beta)x}+Be^{-(\alpha+i\beta)x}\}e^{i\omega t}. \quad . \quad (28)$$

Since θ is to remain finite for $x=\infty$, we have $A=0$, and, putting $x=0$, we find $B=\theta_0$, by (22). Hence taking the real part

$$\theta=\theta_0 e^{-\alpha x}\cos(\omega t-\beta x), \quad . \quad . \quad (29)$$

corresponding to the prescribed oscillation $\theta_0\cos\omega t$ at $x=0$. This contains the theory of an important experimental method.

The simplest case of three-dimensional flow of heat is that of symmetry about a point. If θ be a function of r, the distance from the origin, and t, only, the flux outwards across a spherical surface of radius r in time δt is

$$-k\frac{\partial\theta}{\partial r}4\pi r^2\delta t.$$

A spherical stratum whose inner and outer radii are r and $r+\delta r$ therefore gains heat to the amount

$$4\pi k\frac{\partial}{\partial r}\left(r^2\frac{\partial\theta}{\partial r}\right)\delta r\delta t.$$

Since the thermal capacity of the stratum is $4\pi r^2\delta r\,.\,\rho c.$, the gain is also expressed by

$$4\pi\rho c r^2\delta r\,.\,\frac{\partial\theta}{\partial t}\delta t.$$

Hence
$$r^2\frac{\partial\theta}{\partial t}=\kappa\frac{\partial}{\partial r}\left(r^2\frac{\partial\theta}{\partial r}\right), \quad . \quad . \quad . \quad (30)$$

where $\kappa=k/\rho c$ as before. The equation may also be written

$$\frac{\partial}{\partial t}(r\theta)=\kappa\frac{\partial^2}{\partial r^2}(r\theta). \quad . \quad . \quad . \quad (31)$$

In a stationary state $\partial\theta/\partial t=0$, and therefore

$$\theta=\mathrm{A}+\frac{\mathrm{B}}{r}. \quad . \quad . \quad . \quad . \quad (32)$$

In the case of a spherical shell of internal and external radii a and b respectively, and temperatures θ_1 and θ_2, we have

$$\theta_1=\mathrm{A}+\frac{\mathrm{B}}{a}$$

$$\theta_2=\mathrm{A}+\frac{\mathrm{B}}{b}.$$

Hence
$$\mathrm{B}=(\theta_1-\theta_2)\left(\frac{ab}{b-a}\right).$$

Thus
$$\theta=\mathrm{A}+\left(\frac{ab}{b-a}\right)\frac{\theta_1-\theta_2}{r}.$$

And if Q denote the outward flux per second,

$$\mathrm{Q}=-\kappa\frac{d\theta}{dr}4\pi r^2.$$

$$=\kappa\,.\,4\pi\left(\frac{ab}{b-a}\right)(\theta_1-\theta_2). \quad . \quad . \quad . \quad (32a)$$

In a sphere which is complete to the centre we must have $\mathrm{B}=0$; the only stationary condition is then one of uniform temperature. The second term in (32) would correspond to the case of a steady source of heat at the centre.

When a solid sphere has an arbitrary initial (symmetrical) distribution of temperature, whilst its surface is maintained at a constant temperature (which we may take as zero), the procedure is to ascertain the various "normal modes." Assuming that θ varies as $e^{-\lambda t}$, we have from (31)

$$\frac{\partial^2(r\theta)}{\partial r^2}+\frac{\lambda}{\kappa}r\theta=0. \quad . \quad . \quad . \quad . \quad (33)$$

Hence, putting $\lambda/\kappa=m^2$,

$$r\theta=\mathrm{A}\cos mr+\mathrm{B}\sin mr. \quad . \quad . \quad . \quad (34)$$

Since θ must be finite at the centre, $\mathrm{A}=0$; and since it is to vanish at the surface ($r=a$) we must have $\sin ma=0$, or $ma=s\pi$, where s is integral. The normal modes are therefore of the type

$$\theta=\frac{\mathrm{B}_s}{r}e^{-\lambda_s t}\sin\frac{s\pi r}{a}, \quad . \quad . \quad . \quad (35)$$

where
$$\lambda_s=\frac{s^2\pi^2\kappa}{a^2}. \quad . \quad . \quad . \quad . \quad (36)$$

By superposition of the modes $s=1, 2, 3, \ldots$ it is possible to represent the result of any symmetrical initial distribution. The most persistent mode is that for which $s=1$.

When the surface is not maintained at constant temperature but radiates into surroundings of temperature zero, the total flux outwards will be $\mathrm{E}\theta\,.\,4\pi a^2$. The condition to be satisfied for $r=a$ is therefore

$$-k\frac{\partial\theta}{\partial r}=\mathrm{E}\theta. \quad . \quad . \quad . \quad . \quad (37)$$

Hence, referring to (34), with $\mathrm{A}=0$, and putting

$$\frac{\mathrm{E}}{k}=h, \quad . \quad . \quad . \quad . \quad . \quad (38)$$

we find
$$\tan ma=\frac{ma}{1-ha}. \quad . \quad . \quad . \quad (39)$$

This equation determines the admissible values of ma. The roots are determined graphically by the intersections of the curves

$$y=\tan x, \quad y=\frac{x}{1-ha}. \quad . \quad . \quad (40)$$

It appears on drawing the loci that if $ha<1$ the lowest positive root lies between 0 and $\frac{1}{2}\pi$, whilst if $ha>1$ it lies between $\frac{1}{2}\pi$ and π. The corresponding most persistent normal mode is

$$\theta=\frac{\mathrm{B}}{r}e^{-\kappa m^2 t}\sin mr. \quad . \quad . \quad . \quad (41)$$

In the case of symmetry about an *axis*, the consideration of the flow of heat in and out of a cylindrical shell of radius r and thickness δr leads to the equation

$$r\frac{\partial\theta}{\partial t}=\kappa\frac{\partial}{\partial r}\left(r\frac{\partial\theta}{\partial r}\right). \quad . \quad . \quad . \quad (42)$$

The condition for stationary temperature is that $d\theta/dt$ should vanish, and hence we must have $r(d\theta/dr)=$ constant, or

$$\theta=\mathrm{A}+\mathrm{B}\log r. \quad . \quad . \quad . \quad . \quad (43)$$

For a solid cylinder we must have $\mathrm{B}=0$ unless there is a source of heat at the axis. In the case of a pipe whose inner and outer radii are r_1, r_2, we find

$$\theta=\frac{\theta_1\log(r_2/r)+\theta_2\log(r/r_1)}{\log(r_2/r_1)}. \quad . \quad . \quad (44)$$

The general equation of conduction in three dimensions is obtained by calculating the flow of heat in and out of a rectangular element of volume $\delta x\delta y\delta z$. As in the case of linear flow the gain of heat in time δt due to the two faces perpendicular to Ox is found to be

$$-k\frac{\partial^2\theta}{\partial x^2}\delta x\,.\,\delta y\delta z\,.\,\delta t.$$

Adding the gains due to the remaining pairs of faces we have

$$\frac{\partial\theta}{\partial t}=\kappa\left(\frac{\partial^2\theta}{\partial x^2}+\frac{\partial^2\theta}{\partial y^2}+\frac{\partial^2\theta}{\partial z^2}\right). \quad . \quad (45)$$

From this the particular cases (30) and (42) might be derived by transformation of co-ordinates. The expression in brackets is often denoted by $\nabla^2\theta$. It gives a sort of measure of the extent to which the average temperature in the immediate neighbourhood of (x, y, z) exceeds the temperature at this point itself. Thus the excess of temperature at an adjacent point $(x+\xi,\ y+\eta,\ z+\zeta)$ is, to the second order,

$$\frac{\partial\theta}{\partial x}\xi+\frac{\partial\theta}{\partial y}\eta+\frac{\partial\theta}{\partial z}\zeta+\tfrac{1}{2}\left(\frac{\partial^2\theta}{\partial x^2}\xi^2+\frac{\partial^2\theta}{\partial y^2}\eta^2+\frac{\partial^2\theta}{\partial z^2}\zeta^2\right.$$
$$\left.+2\frac{\partial^2\theta}{\partial y\partial z}\eta\zeta+2\frac{\partial^2\theta}{\partial z\partial x}\zeta\xi+2\frac{\partial^2\theta}{\partial\xi\partial\eta}\xi\eta\right). \quad (46)$$

The average values of ξ^2, η^2, ζ^2 over a spherical region of small radius r having its centre at $(x,\ y,\ z)$ are each equal to $\frac{1}{5}r^2$, whilst the averages of the remaining functions in (46) are zero, since they are as much negative as positive. The average excess is accordingly

$$\nabla^2\theta\,.\,\tfrac{1}{5}r^2. \quad . \quad . \quad . \quad . \quad (47)$$

Reversing the sign, the expression $-\nabla^2\theta$ is called the "concentration" of θ at the point (x, y, z).

The condition of stationary temperature is

$$\nabla^2\theta=0. \quad . \quad . \quad . \quad . \quad (48)$$

This is identical in form with the equation satisfied by the gravitational or the electric potential in free space. Consequently many of the theorems of Attractions and Electrostatics have their analogues in the present subject. For instance, when the surface of a solid of any form is maintained at a uniform temperature the temperature (when stationary) must be uniform throughout the interior.

So far, isotropic substances have been in view. In a *crystalline* structure the flux of heat across any surface is not determined solely by the temperature gradient in the direction of the normal. The natural extension of the previous assumption is that the fluxes per unit area and per unit time across surfaces perpendicular to x, y, z respectively are

$$\left.\begin{aligned}u&=-\left(k_{11}\frac{\partial\theta}{\partial x}+k_{12}\frac{\partial\theta}{\partial y}+k_{13}\frac{\partial\theta}{\partial z}\right)\\v&=-\left(k_{21}\frac{\partial\theta}{\partial x}+k_{22}\frac{\partial\theta}{\partial y}+k_{23}\frac{\partial\theta}{\partial z}\right)\\w&=-\left(k_{31}\frac{\partial\theta}{\partial x}+k_{32}\frac{\partial\theta}{\partial y}+k_{33}\frac{\partial\theta}{\partial z}\right)\end{aligned}\right\} \quad . \quad . \quad (49)$$

The usual method leads to

$$\rho c\frac{\partial\theta}{\partial t}=-\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}\right) \quad . \quad (50)$$

or $$\frac{\partial\theta}{\partial t}=\kappa_1\frac{\partial^2\theta}{\partial x^2}+\kappa_2\frac{\partial^2\theta}{\partial y^2}+\kappa_3\frac{\partial^2\theta}{\partial z^2}+2\lambda_1\frac{\partial^2\theta}{\partial y\partial z}$$
$$+2\lambda_2\frac{\partial^2\theta}{\partial z\partial x}+2\lambda_3\frac{\partial^2\theta}{\partial x\partial y}, \quad (51)$$

where $\kappa_1=\dfrac{k_{11}}{\rho c},\quad \lambda_1=\dfrac{\frac{1}{2}(k_{23}+k_{32})}{\rho c}$, etc. etc. (52)

By a special choice of co-ordinate axes this equation can be brought to the simpler form

$$\frac{\partial\theta}{\partial t}=\kappa_1{}'\frac{\partial^2\theta}{\partial x^2}+\kappa_2{}'\frac{\partial^2\theta}{\partial y^2}+\kappa_3{}'\frac{\partial^2\theta}{\partial z^2}. \quad . \quad (53)$$

The new axes may be called the principal axes of conductivity.

If we further imagine the body transformed by an homogeneous strain, writing

$$\frac{x'}{x}=\sqrt{\left(\frac{\kappa}{\kappa_1{}'}\right)},\quad \frac{y'}{y}=\sqrt{\left(\frac{\kappa}{\kappa_2{}'}\right)},$$
$$\frac{z'}{z}=\sqrt{\left(\frac{\kappa}{\kappa_3{}'}\right)}, \quad (54)$$

where κ may be chosen arbitrarily, we get

$$\frac{\partial\theta}{\partial t}=\kappa\left(\frac{\partial^2\theta}{\partial x'^2}+\frac{\partial^2\theta}{\partial y'^2}+\frac{\partial^2\theta}{\partial z'^2}\right), \quad . \quad (55)$$

which has the same form as for an isotropic medium. Hence results obtained on the hypothesis of isotropy can be transformed so as to be applicable to the more general case. For instance, the isothermal surfaces due to a source of heat at the origin are, in the case of isotropy, the spheres

$$x'^2+y'^2+z'^2=\text{const.} \quad . \quad . \quad (56)$$

In the crystalline case they are the ellipsoids

$$\frac{x^2}{\kappa_1{}'}+\frac{y^2}{\kappa_2{}'}+\frac{z^2}{\kappa_3{}'}=\text{const.} \quad . \quad . \quad (57)$$

The corresponding investigation for conduction in two dimensions gives the theory of de Senarmont's classical experiments on conduction in crystalline plates. H. L.

HEAT, CONVECTION OF

§ (1) Convection Currents.—The article on "Heat, Conduction of," has been concerned mainly with the transference of heat through bodies the parts of which, except for possible molecular and electronic diffusion, are at rest relative to each other and to their bounding surfaces. Where such relative motion occurs and is associated with temperature gradients, heat is conveyed by the moving matter and is then said to be transferred by "Convection." The phenomenon is practically limited to fluids, and it occurs so readily in them that, as was shown in the article on "Heat, Conduction of," the main difficulty in measuring the conductivity of a gas or liquid is the suppression of these internal currents.

It is usual to distinguish two types of convection currents, namely "natural" and "forced." The former are essentially gravity currents which are caused by differences in density in the body of a fluid due to differences in temperature, as, for example, those caused by the cooling of a hot object when brought into a still atmosphere. The latter are due to extraneous causes, and they may modify or completely obscure the natural currents. An example of "forced" convection is the cooling of a hot object by wind or artificial blast.

The subject of convection has long engaged the attention of physicists. Thus over 200 years ago Newton propounded a law[1] to the effect that the rate of cooling of a hot body in a stream of air is proportional to the difference in temperature between the body and the air. The subject is, however, still in a comparatively undeveloped stage; nor is this to be wondered at when it is realised that the problem of calculating the heat conveyed by a moving fluid involves a combination of the Fourier equations of conduction with those of hydrodynamics. The simplifying assumptions necessary to obtain solutions have, as a general rule, restricted seriously the usefulness of the results.

§ (2) FORCED CONVECTION. (i.) *Stream-line Flow: Boussinesq's Theory.*— Forced convection has been considered theoretically by a number of mathematicians. One of the most important contributions was that of Boussinesq,[2] who was the first to treat fully the cooling of a heated body by a stream of fluid when the flow is not turbulent. It was assumed that the fluid was inviscid and incompressible. Subject to these and other simplifying assumptions, Boussinesq obtained a generalised solution according to which the heat loss is directly proportional to the temperature difference (the full solution is referred to in subsection (iii.) below). He[3] also solved a number of special cases, such, for example, as strips and cylinders with their long axes at right angles to the flow. For the latter the heat loss is found to be proportional to the temperature difference and to the square root of the velocity multiplied by the diameter or breadth. Independent proofs of several special cases, including those just mentioned, are also given by Russell.[4]

(ii.) *Verification.*—There is a considerable body of evidence available as bearing on Boussinesq's conclusions. Dealing first with the proportionality of convection loss and temperature difference, this may be regarded as a general law, of which Newton's law is a special application. The latter deals with the total heat loss from a body, due to radiation as well as convection. The radiation loss is well known to be proportional to $T_1^4 - T_0^4$, T_1 and T_0 being the absolute temperatures of the body and its surroundings, or to $T_1 - T_0$, where the difference in temperature is small, for in this case $T_1^4 - T_0^4$ is approximately equal to $4T_0^3(T_1 - T_0)$. Accepting Boussinesq's conclusion, it follows at once that the total heat loss (by convection and radiation) will be proportional to the temperature difference when the difference is small, and it will also be proportional for large differences in temperature if the wind velocity is sufficiently great to make the radiation loss negligible in comparison with the convection loss. These are limiting conditions which have been shown to apply to Newton's law. Mitchell,[5] for example, experimenting on the cooling of a copper sphere 5 cm. in diameter, found that at least within the limits of temperature differences up to 200° C. and wind velocities up to 1666 cm. per second, Newton's law held, and that it was accurate for a temperature difference which increases with increasing velocity of the air current. Compan,[6] using a sphere 2 cm. in diameter, verified the law up to 300° C., while King[7] showed that for fine wires the convection loss was still very nearly proportional to the temperature difference, even when the latter was as large as 1200° C.

As regards Boussinesq's other conclusions, relating to the effect of velocity and linear dimensions, a reference may be made to experiments on cylinders, since these have covered the range from the finest wires to large pipes. In general, the procedure in such experiments has been to measure the energy input required to keep the body at a constant temperature. If, then, the emission coefficient of the surface is known, the heat loss by radiation can be calculated and the convection loss determined by difference.

The case of wires has been very fully investigated, notably by Kennelly,[8] Morris,[9] and King.[10] The last-named, in particular, conducted a most exhaustive research, both theoretical and experimental. Boussinesq's approximate solution for the rate of heat loss by convection from a cylinder was

$$h = A\theta\sqrt{Vd},$$

where θ is the temperature difference, V the velocity of the stream, d the diameter of the wire, and A is a constant whose value is $\sqrt{32c\rho k/\pi}$. Here c, ρ and k signify as usual the specific heat, density and conductivity. King carried the mathematical analysis further and showed that for values of Vd greater than 0·0187 cm.²/sec.

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[1] This law is often taken to apply to natural convection, though Newton expressly said that it was given for a body "not in still air but in a uniform current of air."

[2] *Comptes Rendus*, 1901, cxxxiii. 257.

[3] *Journ. de Math.*, 1905, i. 285.

[4] *Phil. Mag.*, 1910, xx. 591.

[5] *Trans. R.S. Edin.*, 1901, xl. (1), 39.

[6] *Ann. de Chimie phys.*, 1902, xxvi. 482.

[7] *Phil. Trans. R.S.*, 1914, ccxiv. 373.

[8] *Trans. A.I.E.E.*, 1909, xxviii. 363.

[9] *Electrician*, Oct. 4, 1912, p. 1056.

[10] *Loc. cit.*

$$h=\theta(\mathrm{B}\sqrt{\mathrm{V}d}+\mathrm{C}),$$

B and C being constants of value $\sqrt{2\pi c\rho k}$ and k respectively. He conducted experiments on wires from 0·003 to 0·015 cm., using velocities ranging from 17 to 900 cm. per second and temperature differences from 200° to 1200° C. The wires, which were of platinum, were fixed at the end of a whirling arm and their temperatures were determined by their resistances as measured on a special bridge. For full particulars as to the various corrections, reference should be made to the original paper. The above formula was confirmed, B and C being found to be nearly constant and to be in reasonable agreement with the theoretical values.

For larger cylinders the work of Hughes[1] may be referred to. He experimented on cylinders of 0·43 to 15·5 cm. in diameter and with velocities from 200 to 1500 cm. per second. The cylinders were fixed across a wind tunnel and were heated internally by steam, the amount condensed giving the energy dissipated at the surface. The wind velocity was measured by a Pitot tube. The experiments gave

$$h\propto d^m\mathrm{V}^n,$$

value of m being 0·57 and n varying with diameter from 0·55 for small diameters to nearly 1·0 for large diameters. Hughes's results seemed to fall intermediate between those of King for wires, where the convection loss depends approximately on $\sqrt{\mathrm{V}}$, and those of Reynolds,[2] Nicholson and others for large bodies where the convection loss is proportional to V.

As bearing on this point it may be mentioned that Compan, using a sphere 2 cm. in diameter, verified within narrow limits of temperature and velocity Boussinesq's theory that the heat loss varied as $\sqrt{\mathrm{V}}$. Langmuir,[3] too, experimenting on a flat surface with wind velocities up to 410 cm. per second, gives the convection loss as proportional to $\sqrt{\mathrm{V}}$. Mitchell, on the other hand, with his sphere of 5 cm. diameter, seems to have found that the convection loss was directly proportional to V up to a speed of 750 cm. per second, when the air motion becomes turbulent.

(iii.) *Principle of Similitude.*—The correlation of experimental results obtained under widely different experimental conditions can be conveniently considered in the light of the principle of similitude. In a short but highly instructive paper Rayleigh[4] has indicated the possibilities of applying this principle to the problems of convection. He took Boussinesq's case of the heat loss from a hot object at constant temperature, which is immersed in an incompressible fluid moving with velocity V, the fluid being first supposed to be inviscid. He postulates that the heat loss in unit time will depend on the following quantities, the dimensions of which are given in brackets:

l the linear dimensions of the body (L).
θ the temperature difference (Θ).
V the stream velocity (LT^{-1}).
C the heat capacity per unit volume ($\mathrm{ML}^{-1}\mathrm{T}^{-2}\Theta^{-1}$).
k the conductivity of the fluid ($\mathrm{MLT}^{-3}\Theta^{-1}$).

Assigning arbitrary indices to the above quantities the heat loss per unit time ($\mathrm{ML}^2\mathrm{T}^{-3}$) may be written

$$h=l^v\theta^w\mathrm{V}^x\mathrm{C}^yk^z,$$

whence we have by mass $1=y+z$,
by length $2=v+x-y+z$,
by time $-3=-x-2y-3z$,
by temperature $0=w-y-z$.

Solving these equations in terms of one of the unknowns (say x), we get

$$v=1+x,\quad w=1,\quad y=x,\quad z=1-x.$$

Hence substituting and collecting indices

$$h=kl\theta\left(\frac{\mathrm{VC}l}{k}\right)^x.$$

Since x is undetermined and $\mathrm{VC}l/k$ of zero dimensions, any number of terms of this form may be combined and all that can be concluded is that

$$h=kl\theta\mathrm{F}\left(\frac{\mathrm{VC}l}{k}\right),$$

where F is an arbitrary function of the one variable $\mathrm{VC}l/k$. The above equation agrees with Boussinesq's general solution.

The omission of viscosity from the discussion is, of course, a serious departure from practical conditions, and Rayleigh proceeds to consider the effect of introducing a factor ν^u in his original equation, ν being the kinematical viscosity of dimensions ($\mathrm{L}^2\mathrm{T}^{-1}$). Proceeding exactly as above and solving in terms of u and x he gets

$$h=kl\theta\left(\frac{\mathrm{VC}l}{k}\right)^x\left(\frac{\mathrm{C}\nu}{k}\right)^u.$$

Here x and u are both undetermined and the variables in the brackets are of zero dimensions. Hence the conclusion is

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[1] *Phil. Mag.*, 1916, xxxi. 118.
[2] *Proc. Lit. Phil. Soc. Manchester*, 1874, xiv. 9.
[3] *Trans. Amer. Electrochem. Soc.*, 1913, xxiii. 323.
[4] *Nature*, 1915, xcv. 66; see also "Dynamical Similarity, Principles of," §§ (27)-(32).

$$h=kl\theta\mathrm{F}\left(\frac{\mathrm{VC}l}{k},\ \frac{\mathrm{C}\nu}{k}\right).$$

$\mathrm{C}\nu/k$ appears to be a constant for any particular gas (see Table V., article, "Heat, Conduction of"), and not to vary greatly from one gas to another, so that for the same gas, or approximately for all gases, the second term of the function becomes a constant, while the first can be written $\mathrm{V}l/\nu$ or $\mathrm{V}d/\nu$ for a cylinder of diameter d with its long axis perpendicular to the flow. The formula for this latter case then becomes

$$h=kl\theta\mathrm{F}\left(\frac{\mathrm{V}d}{\nu}\right).$$

If, now, the rate of heat loss per unit length is plotted against $\mathrm{V}d$, a constant relation should be obtained for all cylinders.

Davis[1] has considered the data, applying to various sizes of cylinder, and tested them in the above manner. He took the conditions of Hughes's experiments, namely, a cylinder at 100° C. surrounded by air at 15° C., and for values of $\mathrm{V}d=500$ and 1000 calculated the heat loss for fine wires, using King's formula and constants. The results are shown in *Fig.* 1, together with Hughes's values and some results obtained by Carpenter[2] for 5 cm. steam pipes. It will be seen that the extrapolation of King's formula gives remarkable agreement with the curve for larger cylinders. The velocities required to give the value $\mathrm{V}d$ of 500 with a thin wire of 0·003 cm. would be too great to allow this extrapolation to be confirmed experimentally.

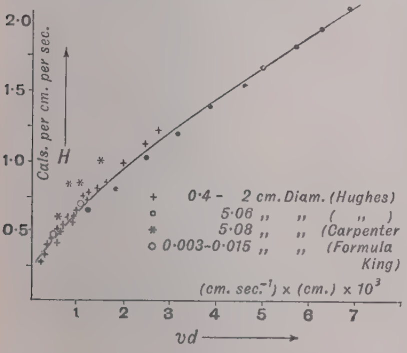

FIG. 1.

(iv.) *Turbulent Flow.*—Very little experimental work has been done on the heat loss due to turbulent flow, except in the case of pipes. Osborne Reynolds,[3] dealing generally with the subject, states that convection in this case is due to two causes, namely, natural internal diffusion and the visible eddy motion which mixes the fluid up and continually brings fresh particles into contact with the surface. He deduces the formula

$$h=(\mathrm{A}+\mathrm{B}\rho\mathrm{V})\theta,$$

A and B being constants and V and ρ the velocity and density of the fluid. A is small, so that h is approximately proportional to V. Stanton[4] has verified Reynolds' theory experimentally and finds that h varies as V^n where n is a little less than unity.

The effect of the change from stream-line motion to turbulence on convection loss does not seem to have been investigated. Russell[5] has obtained theoretically an approximate formula for stream-line motion in a pipe, according to which the convection loss varies as $\sqrt{\mathrm{V}}$. The critical velocity at which eddy motion occurs has been shown by Coker and Clement[6] to vary directly as the viscosity and inversely as the diameter. It is useful to note that these variables are associated in the same way (*i.e.* $\mathrm{V}d/\nu$) in Rayleigh's similitude formula.

§ (3) CONCLUSIONS ON FORCED CONVECTION.—(i.) The heat loss by forced convection from a hot surface is proportional to the temperature difference between the surface and the ambient fluid. This has been shown by Boussinesq from hydrodynamical reasoning, by Rayleigh from the principle of similitude, and it is confirmed by a considerable mass of experimental evidence.

(ii.) For stream-line flow the heat loss is a function of the product of velocity and linear dimensions. This has been shown theoretically by Boussinesq and Rayleigh.

The work of King on fine wires makes the heat loss proportional to ($\sqrt{\mathrm{V}d}$ + constant). The results on larger cylinders, spheres, and flat surfaces are conflicting, some observers finding that the heat loss varies as $\sqrt{\mathrm{V}}$ and others that it varies as V.

(iii.) For turbulent flow in pipes the heat loss is proportional to the velocity.

§ (4) NATURAL CONVECTION. (i.) *Early Results.*—The gravity currents set up in a fluid in the proximity of a hot object have been investigated both theoretically and experimentally. The first work of importance was that of Dulong and Petit, to whom are due an elaborate series of experiments on the cooling of hot bodies. Their researches, published in 1817,[7] are practically confined to the heat loss from thermometer bulbs surrounded by constant temperature enclosures.

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[1] *Phil. Mag.*, 1920, xl. 692.

[2] Quoted in *Trans. Amer. Electrochem. Soc.*, 1913, xxiii. 324.

[3] *Proc. Lit. Phil. Soc. Manchester*, 1874, xiv. 9.

[4] *Phil. Trans. R.S.*, 1897, cxc. 67.

[5] *Phil. Mag.*, 1910, xx. 591.

[6] *Phil. Trans. R.S.*, 1903, cci. 45.

[7] *Ann. de Chim. et Phys.*, 1817, vii.

By exhausting the enclosure and observing the rate of cooling of the bulb the heat loss due to radiation was determined. Experiments were made with the same bulb and enclosure when the latter was filled with gas at varying pressures. As a result the following empirical formula was deduced for the convection loss:

$$h = mp^{0{\cdot}45}\theta^{1{\cdot}233},$$

where m is a constant depending on the shape and position of the body, p the gaseous pressure, and θ the temperature difference.

The work of Dulong and Petit was supplemented by that of Péclet[1] on the value of m for bodies of simple geometrical shape. He measured the rate of cooling of hollow metal cylinders, spheres, and prisms filled with water. He used bodies ranging from 5 to 30 cm. in diameter and 5 to 50 cm. in length. Great care was taken by means of elaborate stirring devices to keep a uniform temperature inside the vessels, and they were placed inside a large water-jacketed cylinder about 80 cm. in diameter and 100 cm. high.

According to Péclet the heat loss by convection is not proportional to the area of the surface but increases more slowly for bodies of fairly large size. Thus for spheres he makes the heat loss proportional to $(1{\cdot}778+0{\cdot}13/r)$ and for horizontal cylinders $(2{\cdot}058+0{\cdot}038/r)$, where r is the radius in metres. Formulae are also given for vertical cylinders and vertical walls, the latter being of the form $a+b/\sqrt{\mathrm{H}}$.

(ii.) *Lorenz's Theory.*—After Péclet, the heat loss from a vertical surface freely exposed to air was investigated mathematically by Lorenz.[2] He made a number of assumptions regarding the upward air streams in the vicinity of the plate. Thus the temperature of the air on crossing the horizontal plane through the lower edge of the plate is supposed to be that of the air at an infinite distance: it is assumed that the air moves from the bottom to the top of the plate at a constant speed and that horizontal currents, which might be due to expansion, are negligible. On this basis Lorenz arrives at the formula

$$h=0{\cdot}131\theta^{\frac{5}{4}}\rho^{\frac{1}{2}}\left(\frac{cgk^3}{\eta \mathrm{HT}}\right)^{\frac{1}{4}},$$

where H is the height of the plate, θ the temperature difference between the plate and the air, the latter being at a temperature T, and c, ρ, k, η are respectively the specific heat, density, conductivity, and viscosity of air, and g the gravitational constant. h, the heat loss, is expressed in cals. per sq. cm. per second. It will be observed that the exponent of θ, namely 1·25, agrees closely with the 1·233 found by Dulong and Petit. The formula gives the heat loss as varying with height in the ratio $1/\mathrm{H}^{\frac{1}{4}}$ (for a further discussion of "height effect" see § (4) (v.) below).

(iii.) *Verification.*—As bearing on Lorenz's formula, the experiments of Langmuir[3] on a vertical disc of diameter 19·1 cm. may be referred to. The disc was heated by a resistor element in contact with the back surface and was embedded in a cylinder of larger diameter, consisting of insulating material, the exposed surface of the disc being flush with the insulation. The temperature of the disc was determined by a thermocouple, and the watts required to maintain it at a constant temperature were measured. In order to reduce the radiation to a minimum the surface was of highly polished silver, the emission coefficient of which, calculated from Hagen and Ruben's formula, varied from about 1·7 per cent at 50° C. to 3·5 per cent at 600° C. of that of a black body. By this means the radiation correction, even at 600° C., was kept as low as 20 per cent. The convection loss was found to fit a formula

$$h=0{\cdot}0000466\theta^{\frac{5}{4}},$$

so that, as regards the dependence of heat loss on temperature difference, Lorenz's deduction was confirmed. The numerical constant given by his formula, taking H the average height of the disc to be 15 cm., was 0·0000485. Seeing that Lorenz's assumptions only applied for small temperature differences and that he took no account of the temperature variations of conductivity, viscosity, and density, it is surprising that his formula holds so well up to 600° C.

Langmuir also investigated the effect of orientation of a flat surface, using the silvered disc above referred to, and found that the heat loss from a flat surface with face upwards was slightly greater than from the vertical surface, while the latter was considerably greater than that for a horizontal surface with face downwards. The results are shown graphically in *Fig.* 2. Apparently in all three cases the heat loss varied as $\theta^{\frac{5}{4}}$.

Some experiments have also been made at the National Physical Laboratory[4] on large vertical plates. Experimenting, first, with a plate freely exposed to the atmosphere, the convection loss was found to be $0{\cdot}0000475\theta^{\frac{5}{4}}$ calories per sq. cm. per second, which agrees approximately with Langmuir's value. The plate used was of polished aluminium, 126 cm. square, and the temperature difference varied up to 100° C.

The convection transfer across enclosed spaces between vertical plates was also investigated. It was found that, for air spaces of 1·2 cm. and over, the transfer was independent of the thickness and varied as $0{\cdot}0000294\theta^{\frac{5}{4}}$. Here θ is the temperature difference between the hot and cold plates, and since the temperature of the enclosed air is intermediate between that of the plates it would be expected that the coefficient would be lower than that for the plate in the open atmosphere—in fact, of the order of one-half. The subdivision of the enclosed space by a horizontal partition into two equal spaces was found not to effect appreciably the heat loss per unit area.

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[1] *Traité de la chaleur*, 1860, trans. by Paulding. Van Nostrand Co., 1904.

[2] *Ann. der Phys.*, 1881, xiii. 582.

[3] *Trans. Amer. Electrochem. Soc.*, 1913, xxiii. 299.

[4] Report on "Heat Transmission by Convection and Radiation," published by the Food Investigation Board (1921).

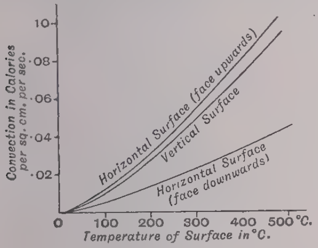

FIG. 2.

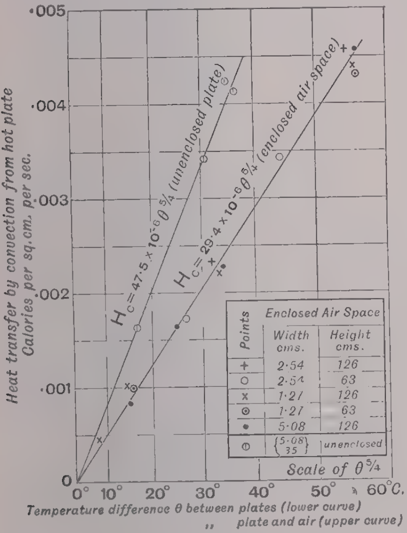

FIG. 3.

The results of the experiments are illustrated in *Fig.* 3.

(iv.) *Langmuir's Experiments and Theory.*—So far all the experimental work quoted has tended to confirm the $\theta^{\frac{5}{4}}$ law closely up to temperatures of 600° or 700° C. Langmuir,[1] however, extending the work of Ayrton and Kilgour on fine platinum wires nearly to the melting-point of platinum, found a considerable increase in the exponent of θ. He used wires ranging in diameter from 0·004 to 0·050 cm. The wires gave rather abnormal coefficients for Callendar's formula, connecting temperature and resistance, namely 0·0035 for a and 1·72 for δ, as against 0·0039 and 1·50 for the purest platinum. The temperatures of the wires were calculated from the resistance, using the parabolic formula to 1100° C. and from 1100° to the melting-point a linear formula. It was estimated that the temperatures were obtained within 20° at 1000° C. and some 50° at 1750° C. The length of wire used was about 50 cm., and a measurement of the current and of the volt drop, on the length of the wire unaffected by end cooling effects, served to give both the energy supplied to, and the resistance of, the length in question.

For the radiation correction the figures of Lummer and Kurlbaum as to the emissivity of platinum at various temperatures were used. For the smallest wire of ·004 cm. diameter the loss by radiation at 1630° C. was only 8 per cent of the total loss, while for the largest wire, ·05 cm., it amounted to 38 per cent. Langmuir found that for air the exponent of θ in the Dulong and Petit formula increased from 1·24 to 1·53 at 1430° C., and similar increases were found for nitrogen, carbon dioxide, and hydrogen. As a result of the experimental work on wires and flat discs Langmuir has put forward his "film theory," which seeks to explain in a simple but comprehensive way the phenomena of convection. He points out that, according to the kinetic theory, the viscosity of a gas increases with the square root of the absolute temperature, and that the thermal conductivity also increases rapidly with temperature, while the driving force of the convection currents, which is proportional to the difference in density between the hot and cold gas, only increases slowly with temperature. He expresses the opinion that there is a stationary film of gas of definite thickness in contact with any hot object at high temperature, and that free convection consists essentially of conduction through this film and can be calculated from the ordinary laws of conduction. Taking first the case of a flat surface, the rate of heat loss per sq. cm. through the film of thickness B would be given by

$$h = \frac{1}{\mathrm{B}}\int_{\mathrm{T}_0}^{\mathrm{T}_1} k d\mathrm{T}.$$

k is not a constant for the big ranges of temperature in question, and in order to evaluate the integral it is necessary to express k as a function of T. This is done by using Sutherland's formula for change of viscosity with temperature as combined with the relation between viscosity, specific heat, and conductivity (see § (10) (iii.) of article, "Heat, Conduction of"). By this means the values of $\int_{T_0}^{T_1} k\,dT$ were calculated for a series of temperatures. The value of B can then be obtained from the experiments on the plane surface. Knowing the rate at which the surface is losing heat by conductivity through the film, it was found that B was independent of temperature and has a value about 0·45 cm. In the case of wires the thickness of the film depends on the diameter of the wire. Making the assumption that not only the temperature but the temperature gradient at the outside of the film is the same in all cases, it can be shown that the diameter b of the film round a wire of diameter a is related to B, the thickness of the film for a flat surface, by

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[1] *Phys. Rev.*, 1912, xxxiv. 401.

$$b \log_e \frac{b}{a} = 2B.$$

Langmuir's observations seem to indicate that b for any particular wire is independent of temperature, and the mean of all his observations on wires gave B=0·43 cm. The agreement between the value of B as deduced from experiments on a flat plate and on wires of various diameters is remarkable, and as yet no satisfactory explanation is forthcoming. It should be pointed out, however, that the whole theory built up by Langmuir is based on certain deductions as to the change of conductivity of the gas up to high temperatures on which no direct experimental evidence is yet available. Further, the use of a hot-wire anemometer indicates the existence of very appreciable air streams within a distance of ·45 cm. from a plane surface, so that the theory cannot, at any rate, be accepted as a literal interpretation of the phenomena of convection.

(v.) *Principle of Similitude.*—As in the case of forced convection the principle of Similitude finds a suggestive application to natural convection. Richardson[1] and Davis[2] have recently shown how to apply the Rayleigh method to this case. Taking a set of variables similar to that adopted by Boussinesq in his hydrodynamical treatment of natural convection we get

$$h = k\theta l \mathrm{F}\left(\frac{C^2 A \theta l^3}{k^2}\right).$$

Here h, k, θ, l, and C have the same significance as in § (2) (iii.) above; A depends on the acceleration of the convection current, and is clearly proportional to $g(\delta\rho/\rho)$ where $\delta\rho$ is the change in density due to unit change of temperature, ρ being the density of the gas. The dimensions of A are $LT^{-2}\theta^{-1}$.

The expression given above is equivalent to that deduced by Boussinesq. It takes no account, however, of the viscosity of the gas, but this can be done in precisely the same way as that given for forced convection. The result is, as before, the introduction under the functional symbol of another variable $C\nu/k$, which, for the same gas and for moderate temperature ranges, is a constant, and does not therefore alter the equation given above.

It is probable that the form of the function is far from simple, but, from the experimental evidence available, certain conclusions can be drawn. For example, taking the variation of heat loss with temperature, a wide range of experiments has shown that it varies as θ^n, where the value of n is about 1·25. Thus Dulong and Petit, Péclet, Compan give the value as 1·23; Langmuir and the National Physical Laboratory find 1·25; while Lorenz on theoretical grounds arrives at a value 1·25.

If the temperature and size of the object alone are variable, as would be the case in air under atmospheric conditions, then, since C, A, and k remain constant, the equation becomes

$$h = k\theta l \phi(\theta l^3),$$

and since the index of θ is $\frac{5}{4}$, we have

$$\phi(\theta l^3) = (\theta l^3)^{\frac{1}{4}},$$

and

$$h \propto \theta^{\frac{5}{4}} . l^{\frac{7}{4}}.$$

Hence in general

$$h \propto \theta^{1\cdot 25} l^{1\cdot 75} k^{\cdot 5} C^{\cdot 5} A^{\cdot 25}.$$

It is interesting to consider the bearing of this expression on the case of vertical surfaces of different heights. The heat loss per unit area is given by

$$\frac{h}{l^2} \propto \frac{l^{1\cdot 75}}{l^2} \propto l^{-\cdot 25}.$$

This agrees with Lorenz's solution (see § (4) (ii.) above). Actually the variation seems to be even less, as was shown by some recent experiments by Griffiths[3] at the National Physical Laboratory. He took a wall 8 ft. high, and divided it horizontally into twenty-five separate elements, which were maintained at the same temperature by independently heated coils. The heat dissipation from each element was determined and gave the result shown in *Fig.* 4.

It will be observed that the heat loss per unit area decreases rapidly to a minimum value (at about 50 or 60 cm. from the bottom), then increases slightly and reaches a steady value. That the lower elements swept by colder air should lose more heat is to be expected, but that there should be a minimum is somewhat surprising. The only plausible explanation appears to be that stream-line motion of the air persists up to a certain height, beyond which turbulence sets in. A further series of experiments at the National Physical Laboratory[1] on vertical cylinders, varying from 4·5 to 263 cm. in height, showed the heat loss per unit area to be practically independent of the height above a limit of about 50 cm. It was found, too, the value of n in the formula $h \propto \theta^n$ had a mean of about 1·25, but that it showed a tendency to increase with height—as appears in the following table :

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[1] *Proc. Phys. Soc.*, May 1920.

[2] *Phil. Mag.*, 1920, xl. 692.

[3] Report on "Heat Transmission," Food Investigation Board.

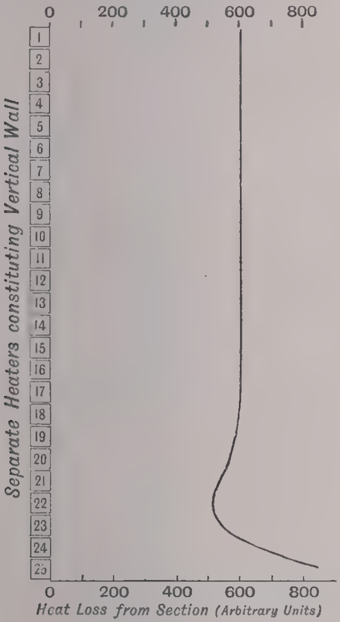

FIG. 4.

Height of Cylinder in cm.	Value of n.	Height of Cylinder in cm.	Value of n.
4·6	1·23	58	1·25
8	1·17	88	1·28
15	1·18	176	1·30
29	1·25	263	1·34

The value of n for the largest cylinder is nearly $\frac{4}{3}$. It is interesting to note that, from considerations of Similitude, the index $\frac{4}{3}$ means that the heat loss per unit area is independent of the height, and, as stated above, experiment has shown this to be the case.

The law of variation of convection loss with pressure of a gas can also be deduced from principle of Similitude provided that the variation of heat loss with temperature is known. Thus if the value of n is taken as $\frac{5}{4}$, we get

$$h \propto \theta^{1 \cdot 25} C^{\cdot 5},$$

and since C, the heat capacity per unit volume, is proportional to the density and therefore to the pressure, we have

$$h \propto p^{\cdot 5}.$$

This is in fair agreement with Dulong and Petit's experimental value of 0·45 for the index, while if their value of n is taken (1·23) the agreement is even closer, giving an index of 0·47 instead of 0·5.

The above examples serve to indicate the use which may be made of the principle of Similitude in the treatment of the problems of natural convection.

§ (5) CONCLUSIONS ON NATURAL CONVECTION.—(i.) The heat loss from a hot surface is approximately proportional to $\theta^{\frac{5}{4}}$ where θ is the temperature difference between the surface and the ambient fluid.

This law represents with fair accuracy the results obtained experimentally for different shapes of surface and for values of θ up to 600° C., and it is in accordance with the mathematical solution given by Lorenz for a vertical surface. For very large surfaces the index shows a tendency to increase.

(ii.) The heat loss has been shown experimentally to vary as $p^{\cdot 45}$ where p is the pressure of the gas.

Neither this law nor the preceding one can be deduced generally by hydrodynamical reasoning or from the principle of Similitude, but they are mutually consistent with the solutions obtained on these lines.

(iii.) Péclet has obtained empirical formulae for the effect of shape and size on the heat loss. For any body of linear dimensions over about 40 cm. the scale effect is unimportant, *i.e.* the rate of heat loss per unit area is approximately the same, being about 0·000045 $\theta^{\frac{5}{4}}$ cals. per sq. cm. per sec.

F. H. S.

HEAT, MECHANICAL EQUIVALENT OF

§ (1) HISTORICAL.—The principle of the Conservation of Energy states that in all its forms energy remains a constant quantity, however many transformations it undergoes. In other words, if energy is made to pass from any condition such as that of matter in motion into any other condition such as molecular or electrical energy, the numerical value of the resulting effect depends simply on the quantity of energy so transformed, not on the method of transformation, the materials, time, or any external conditions.

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[1] Report on "Heat Transmission," Food Investigation Board.

The demonstration of the truth of this principle by experimental evidence has engaged the attention of many investigators during the past eighty years, but the foundations of our belief may be said to have been laid by Newton when he enunciated his third law, "Action and reaction are equal and opposite."

Professor Tait[1] has pointed out that it is a matter for regret that Newton's own explanation of the terms *action* and *reaction* have been so little considered and discussed by succeeding generations. There are two entirely distinct senses in which these words may be used. Action in the one sense is a force only, and to this interpretation attention is and has been almost universally directed.

Newton's second interpretation of his third law is of great importance. It is as follows:

"If the activity of an agent be measured by the product of the force into its velocity and if similarly the counter activity of the resistance be measured by the velocities of its several parts, whether these arise from friction, adhesion, weight, or acceleration, etc., then activity and counter activity in all combinations of machines will be equal and opposite."

It should be noted that by the *velocity* Newton meant the component velocity in the direction of the force. This interpretation tells us that the kinetic energy of a system is increased by an amount equal to the work done in producing motion where the only resistance is that due to acceleration. Where the work is done against friction, however, the visible energy of the system suffers decrease. The principle of the conservation of energy, therefore, could not be regarded as established unless it could be shown that the visible energy thus apparently destroyed is proportional to the heat developed against friction.

The idea that heat is a form of motion and therefore of energy is no modern contrivance.

Before the time of Newton, Lord Bacon[2] stated that "the only conclusion that he could draw from the whole of his facts is a very general one, viz. that heat is motion."

He based this conclusion on a consideration of several means by which heat is produced or made to appear in bodies: as the percussion of iron, the friction of solid bodies, the collision of flint and steel, etc. "In all these examples heat is produced, or made to appear suddenly, in bodies which have not received it in the usual way of communication from others, and the only cause of its production is a mechanical force or impulse or mechanical violence."

Until the beginning of the nineteenth century, however, this pregnant suggestion of Bacon's appears to have attracted little or no attention. Natural philosophers were retarded in their progress by their belief in the theory of caloric. The existence of an imponderable, indestructible fluid termed caloric was practically assumed in all discussions on natural phenomena.

Count Rumford was the first to question publicly the popular caloric theory, when in 1798 he gave an account of his experiments. He placed a hollow gun-metal cylinder beneath a blunt steel borer and observed that after the cylinder had made about a thousand revolutions its temperature had risen from 60° to 130° F., while the scaly matter abraded by the friction weighed only 837 grains troy. "Is it possible," he writes, "that such a quantity of heat as would have caused 5 lbs. of ice-cold water to boil could have been furnished by so inconsiderable a quantity of metallic dust merely in consequence of a change in its capacity for heat?"[3]

The Calorists, however, were not convinced. Even when Rumford proved that the capacity for heat of the solid was the same as that of the dust, they said that, although the heat required to change the temperature of equal masses was the same, yet the solid metal contained a greater quantity of heat than the dust.

Rumford answered that if the heat were rubbed out of the material, a time must come when all its heat would be exhausted, whereas there was no evidence that such was the case. He also proceeded with further experiments in which the metal was immersed in water, and if we work out the results of those experiments we find that 940 foot-pounds of work would raise one pound of water through 1° F. His final argument was as follows:

> "In reasoning on this subject we must not forget to consider that most remarkable circumstance, that the source of heat generated by friction in these experiments appeared evidently to be *inexhaustible*.
>
> "It is hardly necessary to add, that anything which any *insulated* body, or system of bodies, can continue to furnish *without limitation* cannot possibly be a *material substance*. It appears to me to be extremely difficult, if not quite impossible, to form any distinct idea of anything capable of being excited and communicated in the manner in which the heat was excited and communicated in these experiments, except it be motion." He adds, "I am very far from pretending to know how or by what means or mechanical contrivance that particular kind of motion in bodies which has been supposed to constitute heat is excited, continued, and propagated."

Rumford's work was of the highest value. As Professor Tait remarks, it was throughout free from that *a priori* style of reasoning which had hitherto been so fatal to the progress of natural science. Had Rumford shown that the heat developed by the solution of a powder in acid was equal to that developed by the solution of the same mass of the solid, he could have claimed the sole credit of having established the doctrine of the non-materiality of heat.[1]

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[1] *Recent Advances in Physical Science*, pp. 32-34.

[2] Black's *Lectures on Chemistry*, i. 31-32.

[3] Rumford's *Complete Works*, i. 478.

Almost immediately after the publication of Rumford's paper, Sir Humphry Davy proved experimentally that two pieces of ice may be melted by rubbing them together, and thus he gave conclusive proof (although there is evidence that he himself at the time did not perceive it) that heat is not a form of matter, and therefore his experiments are historically of the first importance. In a second series of experiments he so contrived that the friction between the lumps of ice took place in the exhausted receiver of an air-pump. He says:

> From this experiment it is evident that ice by friction is converted into water, and according to the supposition its capacity is diminished; but it is a well-known fact that the capacity of water for heat is much greater than that of ice; and ice must have an absolute quantity of heat added to it before it can be converted into water. Friction, consequently, does not diminish the capacities of bodies for heat.

It was not, however, till 1812 that he enunciated this proposition: "The immediate cause of the phenomenon of heat then is motion, and the laws of its communication are precisely the same as the laws of the communication of motion"[2]; and, on reflection, it seems extraordinary that the publication of the works of Rumford and Davy produced so little effect, and that their conclusions should have been regarded merely as ingenious hypotheses until the time of Joule.

It would be impossible in any historical summary, however brief, to omit the name of Dr. Julius Mayer (1842), as he was the first to employ the phrase, *the mechanical equivalent of heat*, and thus enunciate in a distinct form the law of the Conservation of Energy.[3]

It is doubtful, however, if Mayer has deserved all the credit which was in consequence at one time assigned to him. The data on which his conclusions were based were not sufficient, while certain of his assumptions were undoubtedly erroneous. Mayer denied on more than one occasion that heat depends on motion, and yet he has been called by some "the discoverer of the modern theory of heat." Nevertheless, he, as it happens, rendered a service to the cause of science by distinctly, although perhaps over boldly, enunciating a theorem of the highest importance. In this case no evil consequences resulted from the premature publication, for the work of Joule and of Colding (already partially accomplished) afforded satisfactory and complete experimental evidence of the truth of the proposition advanced by Mayer.[4]

§ (2) JOULE'S WORK.—The position at the time of the advent of Joule may be summarised as follows:

Although not generally apprehended, the principle of the Conservation of Energy had, in its main features, been enunciated by Newton. Rumford and Davy had demonstrated that heat is not matter; but their conclusion was not generally accepted. The school of Calorists still existed, and although the general principle of Conservation of Energy had been enunciated by Mayer, his data were so scanty, and often erroneous, that his conclusions carried but little conviction to men of science. What the situation demanded was rigorous experimental proof that apparent disappearance of energy, when work was done against friction, was invariably accompanied by the generation of a proportional amount of heat, and that this quantity of heat was independent of the manner in which the work was done, or the nature of the materials employed. Hence the singular importance of Joule's experiments.

The actual numerical values obtained by him are, we now know, somewhat inaccurate, but, at the same time, marvellously near the truth when we consider the conditions under which he worked, and more especially the unsatisfactory state of thermometry in his time.

It is not so much the *accuracy* as the *variety* of the evidence supplied by Joule which claims our admiration. He did not content himself with the examination of some single case of transformation of energy. He investigated the heat caused by the friction between solids, and also that developed in overcoming fluid resistance, not only in the case of one fluid but of different fluids.

Again, the *manner* of doing work was varied from the descent of weights to that performed by an electric current in overcoming resistance.

§ (3) EXPERIMENTAL METHODS CRITICISM.—Tables I. and II. give the results of the chief determinations from 1842 to 1920 of the *Mechanical Equivalent*, *i.e.* the number of units of watts required to raise 1 gramme of water 1° C.

The values are all given in kilogramme-metres and therefore no exact comparison is possible unless the value of g is known at each place where the observations were conducted.

If the English ones are multiplied by $981{\cdot}24 \times 10^2$ and the French by $980{\cdot}94 \times 10^2$, the result will give the value in ergs with a sufficiently close approximation for the purposes of a rough comparison, for it must be remembered that the temperature scales differ considerably, as also the mean temperature of the range investigated.[5]

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[1] *Recent Advances in Physical Science*, p. 42.

[2] *Elements of Chemical Philosophy*, 1812, pp. 94-95.

[3] *Ann. Ch. Pharm.* xlii. 233.

[4] See *Phil. Mag.*, 1864, ii. 151.

[5] Since the heat required to raise 1 gramme of water 1° C. depends somewhat on the temperature of the water, this must be specified before any exact comparison can be made (see § (7)).

Table I
Direct Methods *

Date.	Observer.	Method.	Result.
			Kilogramme-metres.
1843	Joule	Friction of water in tubes	424·6
„	„	Electromagnetic currents	460
„	„	Decrease of heat produced by a pile when the current does work	442·2
1845	„	Compression of air	443·8
„	„	Expansion of air	437·8
„	„	Friction of water in a calorimeter	488·3
1847	„	„ „ „	428·9
1850	„	„ „ „	423·9
„	„	Friction of mercury in a calorimeter	424·7
„	„	Friction of iron plates in a calorimeter	425·2
1857	Favre	Decrease of heat produced by a pile doing work	424·464
„	Hirn	Friction of metals	371·6
1858	„	„ „	400-450
„	Favre	Friction of metals in mercury calorimeter	413·2
„	Hirn	Boring of metals	425
1860-61	„	Water in friction balance	432
„	„	Escape of liquids under high pressure	432, 433
„	„	Hammering lead	425
„	„	Friction of water in two cylinders	432
„	„	Expansion of air	440
„	„	Steam engines	420-432
1865	Edlund	Expansion and contraction of metals	428·3, 443·6
1870	Violle	Heating of a disc between the poles of a magnet	435
1875	Puluj	Friction of metals	425·2, 426·6
1878	Joule	Friction of water	423·9
1878	Rowland	Friction of water between 5° and 36°	429·8, 425·8
1891	D'Arsonval	Heating of a cylinder in a magnetic field	421-427
1892	Miculescu	Friction of water	426·84
1897	Reynolds and Moorby	Friction of water, mean capacity 0° to 100°	426·27

Table II
Indirect Methods

Date.	Observer.	Method.	Result.
			Kilogramme-metres.
1842	Mayer	By the relation of $J=p_0V_0a/c_p-c_v$ for gases	365
1857	Quintus Icilius	Heat developed in a wire of known resistance	399·7
„	Weber	Heat due to electric currents	432·1
„ „	Favre Silberman	Heat developed by zinc on sulphate of copper	432·1
„	Bosscha	Measure of E.M.F. of a Daniell's cell	432·1
1859	Joule	Heat developed in a Daniell's cell	419·5
„	Bosscha	E.M.F. of a Daniell's cell	419·5
„	Lenz-Weber	Heat developed in wire of known resistance	396·4, 478·2
1867	Joule	„ „ „ „	429·5
1878	Weber	„ „ „ „	428·15
1888	Perot	By the relation $L=\tau(v_2-v_1)(dp/dt)$	424·63
1889	Dieterici	Heat of electric currents	432·5
1893	Griffiths	„ „ „	427·45
1894	Schuster and Gannon	Electric current, E. and C. being known	427·19
1899	Callendar and Barnes	„ „ „ „	426·52

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* Preston's *Theory of Heat*, with the exception of experiments completed since 1893.

Those who may wish for further information concerning the work of other observers mentioned in Tables I. and II. should consult the summary given by Professor Ames in the *Rapports présentés au Congrès International de Physique*, Paris, 1900, tome I.

The mean of the values given in Table I. =429·1.

The mean of the values given in Table II. =423·15.

Mean of all=427·16.

The extreme values are 368 and 488 kilogramme-metres.

It is evident that no certain conclusion can be drawn from values differing to so great an extent. The only safe procedure is that of selection. In making such a selection we should be guided by the answer to the following questions:

(*a*) Are the temperature determinations sufficiently accurate? The answer will lead to wholesale rejection, especially in the earlier experiments.

The difficulty and importance of temperature measurements were not sufficiently appreciated until within very recent times, and, unfortunately, an error in thermometry is, as a rule, a fatal one; for each thermometer has its own peculiarities and special causes of error; thus, no later increase in knowledge enables us to correct results unless the actual thermometers have been preserved and the conditions under which they were used fully recorded.

Fortunately in two of the most important cases (viz. Joule's and Rowland's) the thermometers actually used have been preserved.

In the former, however, our information is not complete, for we are not sufficiently acquainted with the exact conditions under which their readings were observed by Joule. In the latter, a restandardisation has been accomplished under Rowland's own direction, and thus the corrections can here be applied with far greater certainty. In both cases, as will be shown later, the results as originally published have, in consequence, undergone modifications which from our modern standpoint are considerable.

(*b*) Has the author given us sufficient data to enable us to judge the probable accuracy of all the various measurements involved by his method of experiment?

In this respect, also, the earlier determinations are at a disadvantage as compared with the later ones, for the importance of full information concerning the details of physical measurement has only been generally recognised in recent times.

(*c*) Are we certain that the energy of the bodies under observation has undergone no modification during the experiment in consequence of molecular changes?

If we could accurately determine both the kinetic energy expended in hammering a nail and also the heat developed, it is not certain that the resulting value of the constant would be correct; for the condition as regards density, strain, etc., of the nail (and possibly of the hammer-head) might have undergone alteration and, in consequence of new molecular conditions, have gained, or lost, in energy.

Now, we know that no permanent shearing strain can exist in a fluid, and if the external pressure is unaltered, the density will have undergone no change except that due to change of temperature. Hence, conclusions drawn from observation of work expended in heating a liquid are, *caeteris paribus*, of leading importance.

It is not necessary to enter into further details of the considerations which have led to the selection of the experiments which are about to be discussed. Suffice it to say that a careful study of the writings of most of those authors who are mentioned in these tables has led to the selection from Table I. of the work of Joule, Rowland, and Reynolds and Moorby, and from Table II. the determinations of Griffiths, Schuster and Gannon, and Callendar and Barnes.

The results obtained by Rowland, after the revision of his thermometry, should be considered as of leading importance in the estimation of the numerical value of the constant; while the indirect methods of Griffiths, and above all of Callendar and Barnes, enable us to trace the changes in the capacity for heat of water, and thus render it possible to make a comparison of the values obtained by the different observers.

If their results are expressed in gravitational units a comparison thereof is rendered difficult in the absence of knowledge as to the local value of *g*. Throughout the remainder of this article (with the exception of the final conclusions) the values will be given in the C.G.S. system, and the mechanical work done expressed in *ergs* or its multiple the Joule (1 Joule$=10^7$ ergs).

A further advantage of this method of measurement is that we can express in the same units the work done by an electrical current or by mechanical means. If our electrical units are correct, then when the ends of a conductor whose resistance is 1 ohm are maintained at a potential difference of 1 volt, the work done per second by the current must be 10^7 ergs. Hence, if J be the constant giving the relation between the work done (in Joules) and the heat developed (*i.e.* the mechanical equivalent), then $J=10^7/H$, when H is the number of thermal grammes C.° developed per second.

By whatever method the work is done, the Mechanical Equivalent expressed in this scale is the number of Joules required to raise 1 gramme of water through 1° C. Hence this number will also represent the capacity for heat of water at a given temperature, and the phrases "the Mechanical Equivalent" and "the capacity for heat of water" are transferable.

Whatever method of determination is adopted, the experimenter has two distinct sets of measurements to perform:

(1) The accurate determination in ergs of the change in the mechanical system;

(2) The measurement of the quantity of heat generated by the complete conversion of that number of ergs into the form of heat

It is in the second of these measurements that the chief difficulties have presented themselves.

The measurement of the quantity of heat by the rise in temperature of a known mass of water is the method which has been almost universally adopted from the time that calorimetry became a branch of science. The choice is an unfortunate one, for not only is the accurate measurement of temperature amongst the most difficult of all measurements, but also the material selected, viz. water, is apparently capricious in its behaviour.

Another method is by observation of the quantity of heat required to change the physical condition of a body, as, for example, a given mass of ice. The latter is one which possesses the great advantage that it is independent of temperature measurements. Unfortunately, however, the density of ice cannot be regarded as invariable, and the magnitude of the unit thus obtained is in some respects inconvenient.[1]

Whatever method is adopted, it should be borne in mind that the true primary unit is the heat equivalent of one erg.

In order that the weight of the evidence in favour of the final conclusion may be rightly estimated, it is advisable to study in some detail the works of the selected observers above referred to.

§ (4) DIRECT EXPERIMENTS. (i.) *Joule.*[2]—The mechanical method finally adopted by Joule in 1878 consisted in stirring water by means of a paddle which was rapidly turned by hand-wheels, shown at *d* and *e* (*Fig.* 1); the vessel was suspended by a vertical shaft *b*, which also carried a large fly-wheel *f*. The mass of the water and the water-equivalent of the calorimeter were carefully determined, the rise in temperature was noted on a mercury-in-glass thermometer, and the work consumed in heating the water was measured by a dynamometer, which consisted of an arrangement for balancing the moment acting on the suspended calorimeter (owing to the revolution of the paddle) by a moment produced by the tension of the cords fastened tangentially to the calorimeter. The cords passed over pulleys and supported weights *k*. If this moment is constant and is called M, and if the number of revolutions per second of the paddle is N, the work done per second is $2\pi MN$.

In order to reduce the metallic friction as far as possible, the base of the calorimeter rested on a hydraulic supporter, which consisted of two concentric vessels *v* and *w*, the space between them being filled with water. The three uprights attached to *w* pressed against the base of the calorimeter and reduced the pressure on the bearing at *o* nearly to zero.

Joule's calorimeter had a water equivalent of 313·7 grammes of water at 15·5° C.; the mass of water used in an experiment was about 5124 grammes, each experiment lasted 41 minutes, and the observed rise in temperature was about 2·8° C. The mean of his results gave 772·65 foot-pounds at Manchester, as the quantity of work required to raise the temperature of one pound of water 1 degree F. on his mercury-in-glass scale at 61·69° F. Changing to the centigrade scale and to the C.G.S. system, Joule's result may be stated as follows: the quantity of work required to raise the temperature of 1 gramme of water 1 degree centigrade on his mercury-in-glass thermometer at 16·5° is $4{\cdot}167 \times 10^7$ ergs.

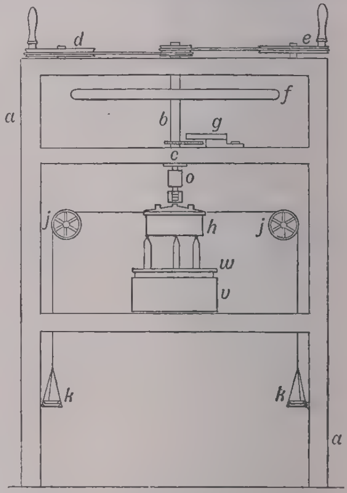

FIG. 1.

In 1895 Professor Schuster[3] compared Joule's thermometer with a Tonnelot thermometer which had been standardised in terms of the nitrogen thermometer of the Bureau International at Sèvres; and in this way was able to recalculate Joule's value for the mechanical equivalent. Rowland also, when reviewing this experiment of Joule's, called attention to certain errors in the determination of the water-equivalent of the calorimeter, whose value had, in consequence, been underestimated by nearly 1 part in 1000. The corrected result is given by Schuster as follows: the capacity for heat of 1 gramme of water at 16·5° C. Paris nitrogen scale is $4{\cdot}173 \times 10^7$ ergs.

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[1] See Appendix II.

[2] *Scientific Papers*, i. 632-657.

[3] *Phil. Mag.*, 1895, xxxix. 477-506.

This value differs by 1 part in 400 from determinations made by Rowland and others. In fact the thirty-five observations from which Joule calculated his final value give results which differ by more than 1 per cent from each other, and his thermometer did not permit the most accurate reading of temperature. The cause of these discrepancies is to be found in three conditions of the experiment: irregularity of the stirring, which was done by hand, incomplete correction for loss of heat by radiation, and insufficient knowledge of the variations in the readings of mercury-in-glass thermometers caused by variations in the conditions under which they are used and standardised.

(ii.) *Rowland.*[1]—In the years 1878 and 1879 Professor Rowland of the Johns Hopkins University, Baltimore, performed a series of experiments for the determination of the capacity for heat of water at different temperatures. His method was in principle the same as that of Joule, although devised independently.

By means of a petroleum engine, a specially designed paddle-wheel was turned at a rapid rate (200 to 250 revolutions per minute) in a calorimeter which was suspended by a wire and which was prevented from turning, when the paddle was revolving, by means of a moment applied by weights. The water-equivalent of the calorimeter was 347 grammes, and the rise in temperature was about 0·6 per minute, and was observed on different mercury-in-glass thermometers. These thermometers had all been compared with a constant volume air-thermometer, and the readings were all finally reduced to the "absolute scale"; the results of Thomson and Joule's experiments on the expansion of air through a porous plug being used to make the necessary corrections to the air-thermometer temperatures. While the temperature was rising through any range, *e.g.* from 8° to 30° C., readings were made on all the instruments for each degree or half degree, and in this way any one experiment gave a great number of determinations of the capacity for heat. The work was calculated for 10° intervals, and one-tenth of that required to raise the temperature of one gramme of water from $(t-5)°$ to $(t+5)°$ was called the specific heat at $t°$ C. All corrections for losses due to radiation, variations in speed of paddle, etc., were carefully considered and made.

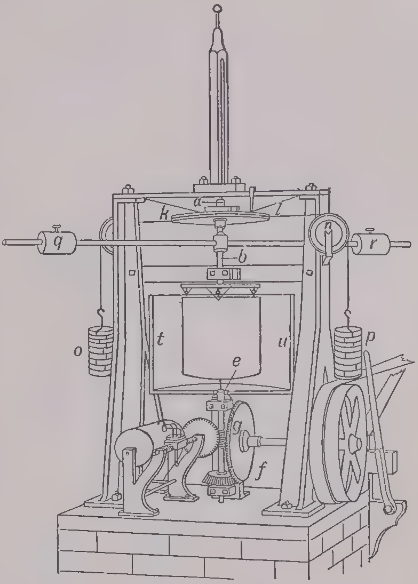

FIG. 2.

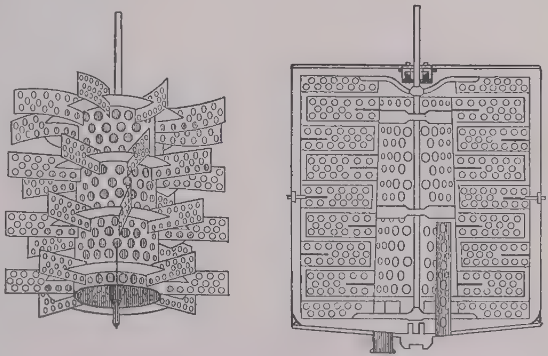

FIG. 3.

FIGS. 2 and 3.—*ab* is a vertical shaft supporting calorimeter and suspended by a torsion wire.

Axis of paddle passed through base of calorimeter and was connected with shaft *ef*, which was kept in uniform rotation by the driving engine.

o and *p*, weights attached to silk tapes passing round wheel *kl*, the couple acting on calorimeter being thus measured (corrections being applied for the torsion of the suspending wire). The moment of inertia could be varied by means of the weights *q* and *r*. A water-jacket *ut* surrounded the calorimeter and was used for the estimation of the radiation.

Rowland's method of using his mercury thermometers was different from that which has been universally adopted within the past few years owing to the efforts of Professor Pernet of Zürich, and of

[1] *Proc. American Academy*, No. 1880-81.

MM. Chappuis and Guillaume of the Bureau International, and the scale of his air-thermometer in terms of other thermometers used by later observers was not known. In 1897, therefore, a series of comparisons was undertaken at the Johns Hopkins University between Rowland's thermometers, three Tonnelot mercury thermometers standardised at the Bureau International, and a Callendar-Griffiths platinum thermometer. The result has been a recalculation of Rowland's figures. In the following table the values for the specific heat are given as Rowland first published them, and as recalculated by Day,[1] and by Waidner and Mallory.[2]

Professor Pernet has also endeavoured to recalculate Rowland's values from a careful study of Baudin thermometers of the same glass and construction as those of Rowland. His figures are almost exactly 1 part in 400 less than those determined by Day and by Waidner and Mallory, and in such a case as this one must have the greater confidence in the direct comparisons.

TABLE III

Temperature.	Rowland's Original Values, Absolute Scale.	Recalculated by Day, Paris Hydrogen Scale.	Same reduced to Paris Nitrogen Scale.	Recalculated by Waidner and Mallory, Paris Nitrogen Scale.
° C.	Ergs.			
5	$4{\cdot}212 \times 10^7$			
10	4·200	$4{\cdot}196 \times 10^7$	$4{\cdot}194 \times 10^7$	$4{\cdot}195 \times 10^7$
15	4·189	4·188	4·186	4·187
20	4·179	4·181	4·180	4·181
25	4·173	4·176	4·176	4·176
30	4·171	4·174	4·174	4·175
35	4·173	4·175	4·175	4·177

Rowland took pains to vary all the conditions of his experiments as much as possible, running his engines at different speeds, using different thermometers, carrying his observations over different ranges and making in all thirty series of observations. Therefore great weight must be given to his determinations. The only criticisms that can be made are that the range of 10 degrees is too large if the capacity for heat at the mean temperature is desired, and that the radiation correction is uncertain at and above 30°. As Rowland himself says: "The error due to radiation is nearly neutralised, at least between 0° and 30°, by using the jacket at different temperatures. There may be an error of a small amount at that point (30°) in the direction of making the mechanical equivalent too great, and the specific heat may keep on decreasing to even 40°."

Professor Ames's criticism on this work is as follows:

> Rowland estimates his possible error at less than 2 parts in 1000; and, now that his thermometric readings have been recalculated, the possible error is probably reduced to less than 1 part in 1000, unless there was a constant or systematic error, which is most improbable. Rowland's method of making thermometric readings is one which is liable to serious error, and it is possible that in the recalculations made by Day, and by Waidner and Mallory, the thermometers were not used in identically the same manner as they were originally. There is no obvious reason, however, for believing, as Pernet does, that there is a systematic error in Rowland's research.

(iii.) *Reynolds and Moorby.*[3]—The experiments of Reynolds and Moorby are also examples of physical work of the highest order. In 1897 they published an account of their determinations of the mean specific heat of water between 0° and 100° C. The apparatus used was of such a nature that it is not possible to convey, in a brief description, any clear idea of the machinery and its connections; therefore attention will only be called to the manner in which the work was controlled and estimated. The general idea is that of a hydraulic brake attached to the shaft of a triple expansion 100 horsepower engine making 300 revolutions per minute.

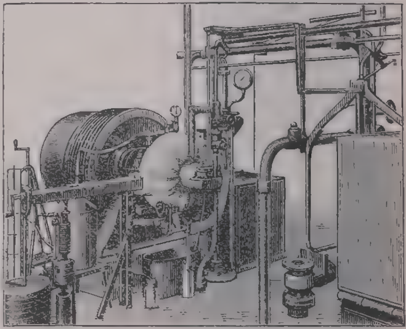

FIG. 4.—This figure shows the Hydraulic Brake lagged with cotton-wool covered by flannel, the brake projecting from it towards the left.

The water enters the brake at, or near, 0° C. and runs through it at such a rate that it issues at, or near, 100° C., the work expended on the water being estimated by means of a dynamometer consisting of a lever and weights fastened to the brake. The whole of the work done being absorbed by the agitation of the water in the brake, the moment of resistance of the brake at any speed is a definite function of the quantity of water in it. Except for this moment the unloaded brake is balanced on the shaft, the load being suspended on the brake lever at a distance of 4 feet from the axis of the shaft. If the moment of resistance of the brake exceeds the moment of this load the lever rises, and *vice versa.* By making this lever actuate the valve which regulates the discharge from the brake, the quantity of water is continually regulated to that which is just required to support the load with the lever horizontal, and thus a constant moment of resistance is maintained whatever the speed of the engines.

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[1] *Phil. Mag.*, 1898, xlvi. 1-29.

[2] *Physical Review*, 1899, viii. 193-236.

[3] *Phil. Trans. Roy. Soc.* A, 1897.

In order to eliminate as many errors as possible, three "heavy trials" were made in succession, followed by three "light trials," each trial lasting 60 minutes, and the *difference* in the two cases, "heavy" and "light," of the "mean work per trial" and the difference of the "mean heat per trial" were taken as equivalent.

In a "heavy trial," the dynamometer was adjusted to a moment of 1200 foot-pounds, and the quantity of water run through in the 60 minutes was about 960 pounds; in a "light trial" the moment was generally 600 foot-pounds, and the quantity of water run through in the 60 minutes was about 475 pounds, although six trials were made with the moment at 400 foot-pounds. As it was only necessary to determine temperatures in the neighbourhood of 0° and 100° C., the results are almost independent of the *nature* of the temperature scale, as all temperature scales must be in agreement at the two standardising points, while the temperature range was so great that an error in actual elevation at either end of it would have but a small effect. Again, the large scale on which the experiments were conducted would tend to diminish the effect of inaccuracies in the measurements of the thermal loss by radiation, etc. The most minute attention was paid to all possible causes of inaccuracy, and there is no apparent constant source of error in the final results.

When their value is expressed in ergs, it becomes $4{\cdot}1833 \times 10^7$; that is, the mean capacity for heat of unit mass of water between 0° and 100° C. is $4{\cdot}1833 \times 10^7$.

Unfortunately the work of Reynolds and Moorby does not afford us much assistance in our efforts to determine the actual value of the heat equivalent. Assuming the validity of their conclusions, we know how many ergs (or primary units) are required to raise the temperature of 1 gramme of water from 0° to 100°; but we are unable to compare their results with the values obtained by Joule and Rowland, unless we know the relation of the mean thermal unit over the range 0° to 100° to the thermal unit at the temperatures covered by the experiments of those observers. It is quite certain that the number of primary units required to raise 1 gramme of water through 1° at different temperatures is not the same; hence it is impossible in the present state of our knowledge to ascertain if Reynolds and Moorby's results are coincident with those obtained by other investigators.

On the other hand, if we assume the validity of the result obtained by Reynolds and Moorby, and compare it with the numbers given by Rowland, we can find the value of the mean thermal unit in terms of a thermal unit at some definite temperature;—a piece of information of great value when we remember that no use can be made of the Bunsen's calorimeter methods in the absence of such knowledge.

§ (5) Indirect Methods.—We now pass to the consideration of indirect methods, many of which were first employed by Joule. At various times he estimated the heat developed by electromagnetic currents, the decrease of heat in a voltaic cell when the current does work, compression and expansion of air, etc.

Attention has already been called to the great variety of experiments performed by Joule, and it would be difficult to overestimate the importance of this variety as evidence that all forms of energy can be expressed as heat. For the purpose of our present inquiry, however, viz. the exact determination of the equivalent, his indirect determinations are of little use.

(i.) *Griffiths*,[1] 1888 to 1893.—The chief objects of this investigation were (1) an examination of the validity of our system of electrical units, and (2) a study of the changes in the specific heat of water, as the only information possessed at the time was that drawn from the experiments of Regnault and Rowland. The former indicated a steady increase from 0° C. upward, while Rowland found a decrease over the range 5° to 30° C.

Professor Ames wrote as follows: "E. H. Griffiths of Cambridge, England, devised a method for the determination of the specific heat of water by the use of the heating effect of an electric current, which is, to a large extent, free from the errors connected with the previous methods in which electric currents were used (*Fig.* 5).

"If a coil of wire carrying a current is immersed in water any one of the three following methods may be used to determine the energy spent in raising the temperature of the water:

(1) Measure[2] E, C, and *t*.
(2) Measure C, R, and *t*.
(3) Measure E, R, and *t*.

"Griffiths used the third method, although for many reasons it is the most difficult. The obvious difficulty lies in the measurement of R; because, unless measured actually during the progress of the heating experiment, it is necessary to know the temperature of the wire and the temperature coefficient of its resistance; and its temperature is *not* that of the surrounding water. Griffiths thought to obviate this difficulty by making a series of subsidiary experiments which were designed to give the difference in temperature between the water and the wire when the former was at a known temperature and an E.M.F. of known strength was applied to the latter. The resistance of the wire was then measured at a known temperature and its temperature coefficient was also measured; therefore, when in the course of a heating experiment the temperature of the water was read, the resistance of the wire could be calculated. Griffiths found also that most rapid and thorough stirring of the water was necessary in order to secure consistent or satisfactory results. He designed a most efficient stirrer which made about 2000 revolutions per minute, the rise in temperature produced by the stirrer alone being in some cases equivalent to 10 per cent of the whole work spent in raising the temperature. The necessary correction, owing to this, was ascertained by a series of preliminary experiments.

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[1] *Phil. Trans. Roy. Soc.* A, 1893; *Proc. Roy. Soc.*, 1894, lv.; *Phil. Mag.*, 1895, xl.

[2] E = electromotive force; C = current; R = resistance; *t* = rise of temperature.

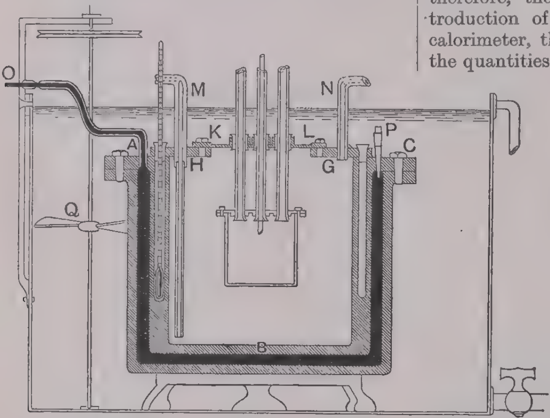

FIG. 5.—Section of Constant Temperature Chamber in which the Calorimeter was suspended by Glass Tubes.

ABC is a large steel vessel with double walls, the annular space (printed black in figure) being filled with mercury which is connected with a gas regulator by the tube D. The steel vessel stood in a large tank filled with water, which was rapidly stirred by the paddle Q. A small stream of water flowed continuously into the tank, the excess passing away at W. The temperature of the incoming water was controlled by the regulator which was governed by the mass of mercury (exceeding 70 lbs.) within the walls ABC. A very constant temperature could thus be maintained within the steel vessel. The space between the calorimeter and the steel walls was thoroughly dried and the pressure reduced to less than 1 mm.

"Griffiths' apparatus (*Fig.* 6) consisted of a platinum wire (diameter 0·004 in. (0·010 cm.), length 13 in. (33 cm.), resistance about 9 ohms) coiled inside a cylindrical calorimeter, 8 cm. in height and 8 cm. diameter, whose water-equivalent was 85. This wire was heated by means of a current from storage cells. The terminals of the wire were maintained at a constant difference of potential by balancing against sets of Clark cells; and, while the temperature of the water contained in the calorimeter was raised from 14° to 25° C., the time varying from forty to eighty minutes, observations of the temperature and time were made every degree. The E.M.F. used varied from that of three to six Clark cells. Experiments were made using different quantities of water; and by taking *differences* in the energy and the heat produced in the different sets, many errors were eliminated, and the water-equivalent of the calorimeter disappeared from the equation. In the end, therefore, the method depended on the introduction of 120 grams of water into the calorimeter, this being the difference between the quantities used in two trials.

"Griffiths measured his E.M.F. in terms of the Cavendish standard Clark cell; his resistance in terms of the 'B.A. ohm of 1892,' which is the 'International Ohm' as defined in 1893; his time by a 'rated' chronometer; and his temperature by a Hicks mercury thermometer which had been compared with a Callendar-Griffiths platinum thermometer and also with a Tonnelot thermometer standardised at the Bureau International. In accordance with the work of Glazebrook and Skinner he assumed the E.M.F. of the Clark cell at 15° C. to be 1·4344 volts, and its temperature coefficient to be $1 + 0{\cdot}00077\,(15° - t)$.

"Later, Schuster called attention to an error of one part in four thousand due to the capacity for heat of the displaced air,[1] but this was neutralised by the fact that there was a slight probable error discovered in the estimation of the E.M.F. of the Clark cells used by Griffiths,[2] reducing the value to 1·4342 volts at 15° C.

"Hence Griffiths' final values are:

15° C. nitrogen scale,	$4{\cdot}198 \times 10^7$ ergs.
20° ,, ,,	$4{\cdot}192 \times 10^7$,,
25° ,, ,,	$4{\cdot}187 \times 10^7$,,

"In criticism of the method, it may be said that using as small quantities of water as Griffiths did, always practically under the same external conditions, there is more opportunity than should be for systematic errors and for errors due to radiation corrections. In this connection reference must be made to criticisms by Schuster[3] and to the reply by Griffiths."[4]

It will be seen from this summary by

[1] *Phil. Trans. Roy. Soc.* A, 1895, clxxxvi.

[2] *Phil. Mag.*, 1895, xl.

[3] *Phil. Trans.* A, 1895, clxxxvi.

[4] *Phil. Mag.*, 1895, pp. 431-454.

Professor Ames that the results of Griffiths' work lead to the conclusion that (assuming the value of J obtained from Rowland's revised experiments) there is at all events no error exceeding 1 in 1000 in the value of our electrical units; but that there is an indication of a possible error of some such magnitude in the electro-chemical equivalent of silver, or in the E.M.F. of the Clark cell, an indication which (as we shall see later) is strengthened by the work of Schuster and Gannon, Kahle, Patterson and Guthe.

A comparison of the curves resulting from Rowland's work and Griffiths' proved that they could not both be correct in their thermometry, and this of course excited suspicion as to any conclusions regarding the value of the electrical units. The revision of Rowland's thermometry was partly due to this discrepancy, and the double restandardisations by different methods were, as we have seen (Table III. *supra*), in agreement with each other, while the resulting corrections caused Rowland's curve of the changes in capacity for heat to be almost parallel with Griffiths' over the range of his experiments. This parallelism did not mean that the value of J, obtained by the assumption of the validity of the electrical units, was coincident with Rowland's, but that their remaining differences were probably due to the nature of the calorimetric determinations, or to some hitherto undiscovered error in the system of electrical measurements.

(ii.) *Schuster and Gannon*,[1] 1895.—These observers also measured the heat developed by an electric current when overcoming resistance; but in this case the work was estimated by observation of E and C, the latter by the use of a silver voltameter (see *Fig.* 7).

The rise in temperature was determined by a Baudin mercury thermometer, which was compared directly with a Tonnelot thermometer standardised at the Bureau International. The calorimeter had a water-equivalent of 27 and the mass of water used was about 1514 grammes. The heated wire was of platinoid, 760 cm. long and of about 31 ohms resistance. The E.M.F. was produced by storage cells, and was constantly balanced against twenty Clark cells. The resulting current was in the neighbourhood of 0·9 ampere, and passed in series through a silver voltameter consisting of a silver plate and a platinum bowl 9 cm. in diameter and 4 cm. deep, whose weight was approximately 64 grammes. An experiment lasted ten minutes, during which about 0·56 gramme of silver was deposited, and the temperature of the water was raised about 2·2° C. All experiments were performed in the neighbourhood of 19·6°. The final result is the mean of six experiments which agree closely with each other.

The result of their investigation gives—

Capacity for heat of water at 19·1° C. on nitrogen scale = $4{\cdot}1905 \times 10^7$ ergs.

These experiments were conducted with the skill and accuracy which we necessarily associate with the name of Professor Schuster: "Nevertheless" (to quote Professor Ames again), "there are several criticisms which may be offered to this research. There was only one voltameter used throughout, and none of the conditions were varied. The radiation corrections were most carefully considered, but no details are given of the stirring or of any correction for it. Griffiths in his investigation insists strongly on the need of thorough, not to say violent, stirring.

"These facts make the final result uncertain to an extent which it is difficult to estimate, but which probably is not large. If,[2] as seems probable from the work of Kahle and Patterson and Guthe, the electro-equivalent of silver is 0·001119 instead of 0·001118, Schuster and Gannon's value for the specific heat at 19·1° becomes $4{\cdot}189 \times 10^7$; and, if a consequent error of one part in a thousand is made in the assumed value of the E.M.F. of their Clark cell, the corrected result is $4{\cdot}185 \times 10^7$."

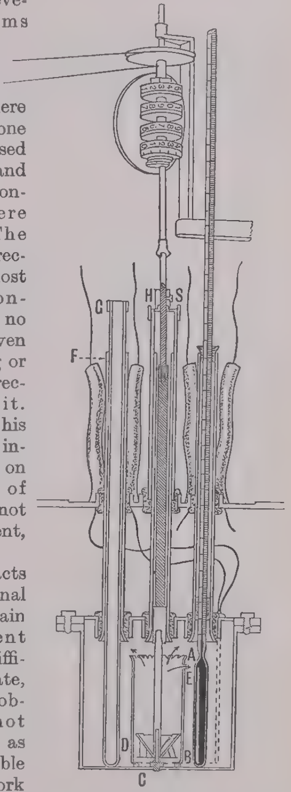

FIG. 6.—Section of Calorimeter showing the stirring arrangements at D by which the water was drawn through the bottom of the cylinder AB, and thrown against the roof of the calorimeter.

The platinum coil is not shown in this section, as it was wound in a horizontal circle placed near the base of the vessel, the rack on which it was wound being supported by the rod indicated by the dotted lines to the right of the thermometer bulb. At the top of the stirrer shaft is shown the counter which recorded the number of revolutions.

Certain other possible causes of slight inaccuracies present themselves. The form of stirrer adopted was a plunging one and the calorimeter was not airtight. It is probable that the correction for the latent heat of evaporation during a rise of temperature exceeding 2° C. might be appreciable. The authors refer to this matter as follows: "Evaporation will produce a certain amount of cooling of the calorimeter, if that is not perfectly enclosed; but unless the rate of evaporation changes with the temperature, the effect will only be a lowering of temperature by a constant quantity. The rate of cooling can only be affected by evaporation in so far as it increases with the temperature, and since for small changes it would vary as a linear function of the temperature, any error due to evaporation is eliminated by the cooling correction."

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[1] *Phil. Trans. Roy. Soc.* A, 1895, clxxxvi.

[2] This is not borne out by the later determinations of the electro-chemical-equivalent. The value 0·001118 is now adopted.

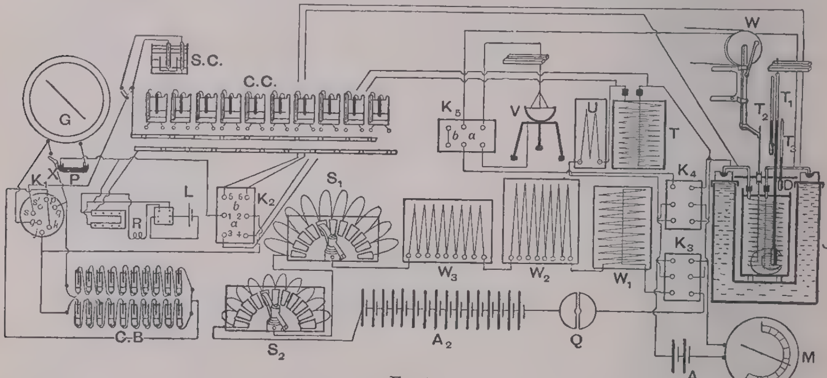

FIG. 7.

The International Committee on Electrical Units and Standards, in their report January 1912,[1] state that they are unanimous in the conclusion "that voltameters in which filter paper is used as a septum lead to results which are too large."

Messrs. Schuster and Gannon state that the silver plate was "protected by filter paper."[2]

It is also necessary to remember that Schuster and Gannon did not trace the value of the secondary unit over any appreciable range of temperature, all their observations being confined to a rise of about 2·2° C. in the neighbourhood of 19° C.

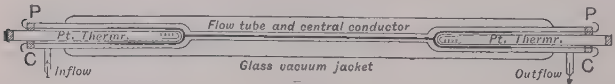

FIG. 8.

(iii.) *Callendar and Barnes.*[3]—The following description is extracted from a report by Professor Callendar to the British Association in 1899:

"The general principle of the method, and the construction of the apparatus, will be readily understood by reference to the diagram of the steady-flow electric Calorimeter given in *Fig.* 8. A steady current of water flowing through a fine tube is heated by a steady electric current through a central conductor of platinum. The steady difference of temperature between the inflowing and outflowing water is observed by means of a differential pair of platinum thermometers at either end. The bulbs of these thermometers are surrounded by thick copper tubes which, by their conductivity, serve at once to equalise the temperature, and to prevent the generation of heat by the current in the immediate neighbourhood of the bulbs of the thermometers. The leads CC serve for the introduction of the current, and the leads PP, which are carefully insulated, for the measurement of the difference of potential on the central conductor. The flow-tube is constructed of glass, and is sealed at either end, at some distance beyond the bulbs of the thermometers, into a glass vacuum jacket, the function of which is to diminish as much as possible the external loss of heat. The whole is enclosed in an external copper jacket (not shown in the figure), containing water in rapid circulation at a constant temperature maintained by means of a very delicate electric regulator.

"Neglecting small corrections, the general equation of the method may be stated in the following form:

$$ECt = JMd\theta + H.$$

"The difference of potential E on the central conductor is measured in terms of the Clark cell by means of a very accurately calibrated potentiometer, which serves also to measure the current C by the observation of the difference of potential on a standard resistance R included in the circuit.

"The Clark cells chiefly employed in this work were of the hermetically sealed type described by the authors in the *Proc. Roy. Soc.*, October 1897. They were kept immersed in a regulated water-bath at 15° C., and have maintained their relative differences constant to one or two parts in 100,000 for the last two years.

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[1] *International Com. on Electrical Units*, Washington Government Printing Office.

[2] *Phil. Trans. Roy. Soc.* A, 1895, p. 422.

[3] *Proc. Roy. Soc.*, 1900; *Phil. Trans. Roy. Soc.* A, cxcix.

"The standard resistance R consists of four bare platinum silver wires in parallel wound on mica frames and immersed in oil at a constant temperature. The coils were annealed at a red heat after winding on the mica, and are not appreciably heated by the passage of the currents employed in the work.

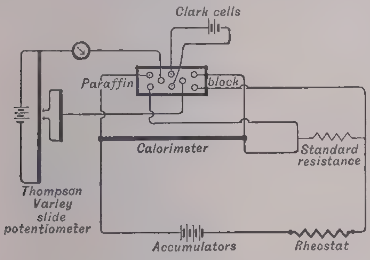

FIG. 9.—Diagram of the Electrical Connections.

"The time of flow t of the mass of water M was generally about fifteen to twenty minutes, and was recorded automatically on an electric chronograph reading to ·01 second, on which the seconds were marked by a standard clock.

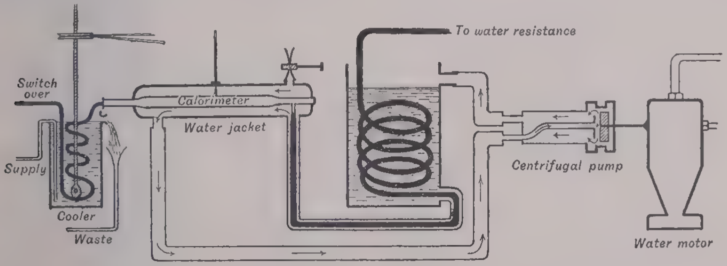

FIG. 10.

"The letter J stands for the number of joules in one calorie at a temperature which is the mean of the range $d\theta$ through which the water is heated.

"The mass of water M was generally a quantity of the order of 500 grammes. After passing through a cooler, it was collected and weighed in a tared flask in such a manner as to obviate all possible loss by evaporation.

"The range of temperature $d\theta$ was generally from 8° to 10° in the series of experiments on the variation of J, but other ranges were tried for the purpose of testing the theory of the method and the application of small corrections. The thermometers were read to the ten-thousandth part of a degree, and the difference was probably in all cases accurate to ·001° C. This order of accuracy could not possibly have been attained with mercury thermometers under the conditions of the experiment.

"The external loss of heat H was very small and regular, owing to the perfection and constancy of the vacuum attainable in the sealed glass jacket. It was determined and eliminated by adjusting the electric current so as to secure the same rise of temperature $d\theta$ for widely different values of the water-flow.

"The great advantage of the steady-flow method as compared with the more common method in which a constant mass of water at a uniform temperature is heated in a calorimeter, the temperature of which is changing continuously, is that in the steady-flow method there is practically no change of temperature in any part of the apparatus during the experiment. There is no correction required for the thermal capacity of the calorimeter; the external heat loss is more regular and certain, and there is no question of lag of the thermometers. Another incidental advantage of great importance is that the steadiness of the conditions permits the attainment of the highest degree of accuracy in the instrumental readings.

"In work of this nature it is recognised as being of the utmost importance to be able to detect and eliminate constant errors by varying the conditions of the experiment through as wide a range as possible. In addition to varying the electric current, the water-flow, and the range of temperature it was possible, with comparatively little trouble, to alter the form and resistance of the central conductor, and to change the glass calorimeter for one with a different degree of vacuum, or a different bore for the flow-tube. In all six different calorimeters were employed, and the agreement of the results on reduction afforded a very satisfactory test of the accuracy of the method."

In 1902 [1] Dr. Barnes published a very full account, together with a discussion of the results.

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[1] *Phil. Trans. Roy. Soc.* A, cxcix.

His final value at 20° C. of the nitrogen scale is $4{\cdot}478 \times 10^7$ ergs. He assumed the Clark cell value as 1·43325 International volts at 15° C. and the true ohm as 1·01358 B.A. units.

§ (6) COMPARISON OF RESULTS :

TABLE IV

CAPACITY FOR HEAT OF WATER PER 1° OF THE N THERMOMETER

Name.	Method.	Standards.	Results.	Temperature.
Joule	Mechanical	..	$4{\cdot}173 \times 10^7$	16·5°
Rowland . . .	,,	..	4·195	10°
			4·187	15
			4·181	20
			4·177	25
Reynolds and Moorby .	,,	..	4·1833	mean calorie
Griffiths	Electrical $\frac{E^2}{R} \cdot t$	Clark cell = 1·4342 International ohm	4·198	15°
			4·192	20
			4·187	25
Schuster and Gannon .	E. C. t	Clark cell = 1·4340 El. Ch. E. of Ag = 0·001118	4·1905	19·1°
Barnes . . .	E. C. t	Clark cell = 1·4333 Ag = 0·001118	4·193	10°
			4·184	15
			4·178	20
			4·175	25

The discrepancy between the values given in the above table is, in reality, much less than would appear from a casual inspection. Before coming to any final decision, it is necessary to study the evidence regarding the variation in the capacity for heat of water, and in this connection to consider also such evidence as we possess which is independent of any determinations based upon the transformation of energy. Again, discrepancies arising from the different values assigned to the Clark cell by the observers must be taken into account.

§ (7) THE VARIATION IN THE CAPACITY FOR HEAT OF WATER DUE TO CHANGES IN TEMPERATURE.—Throughout this section C_t indicates the number of ergs required to raise 1 gramme of water through 1° of the hydrogen scale at a temperature of t° C.

The specific heat σ_t at any temperature t° C. is the ratio C_t/C_θ, where θ is some selected standard temperature. It is evident that the value of σ_t thus ascertained is not affected by inaccuracies in the magnitude of the electrical units, hence the peculiar value of such methods for the determination of its change consequent on change of temperature. Thus, although electrical methods may be of secondary importance when the object is the determination of the numerical values of C_t, they are of primary importance in the attempt to trace changes in the value of σ_t consequent on changes of temperature.

Until the publication of Rowland's work the value of σ_t obtained by Bosscha's reduction of Regnault's experiments[1] was almost universally accepted, and the changes in the specific heats of other bodies than water were, in nearly all cases, dependent thereon.

A study of Regnault's paper,[2] however, will lead to the conclusion that Regnault, with two exceptions, performed no experiments concerning the specific heat of water below 107° C., and these were only made with the view of testing the working of the apparatus, and that Regnault himself attached no importance to them. After discussing his results over the range 107° to 190° C., he stated what would be the nature of the variation between 0° and 100° if deduced by extrapolation of the experimental curve obtained at the higher range.

Bosscha discussed Regnault's experiments, made several small corrections, found an equation which in his opinion closely represented the results over the range 107° to 190°, and then assumed that the expression held good down to 0° C.

It has been necessary to dwell upon this matter, for succeeding investigators who have endeavoured to find the specific heat of various bodies and also the changes therein with change of temperature have, in general, reduced their results to a standard temperature by these extrapolated values. Hence the majority of their conclusions require the revision rendered necessary by our knowledge that the changes in σ_t differ both in magnitude and in direction from the changes at higher temperatures. Unfortunately such revision is rarely possible, as the required data are not usually given, for the observer has had such confidence in Regnault's supposed values that he has considered them unnecessary.

Rowland's conclusion (1879) that, so far from increasing, the value of σ_t decreased with rise of temperature up to about 30° C.

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[1] Bosscha, *Pogg. Ann.*, *Jahrg.* 1876, p. 549.

[2] Regnault, *Mémoires de l'Acad.*, 1847, xxi. 729.

has been confirmed by the work of Griffiths (range 13° to 27°) and Callendar and Barnes (1° to 99°).

The changes in the *specific heat* have been determined by Bartoli and Stracciati [1] and Ludin.[2] In both cases the conclusions were arrived at by the method of mixtures, and are therefore independent of all energy measurements. Bartoli and Stracciati devoted nearly nine years to their investigation; they not only mixed water with water, but also with mercury and several metals. Their thermometry was based on the standards supplied by the Bureau International, and their results are given in a formula containing the third power of t. They find a minimum about 20°. The difficulties of this method are considerable, and a careful examination of their experimental numbers leads to the conclusion that the discrepancies between individual experiments are too great to allow of our attaching much authority to their final values.

A very fine series of determinations by the same method was made by Ludin in 1890. This investigator was able to take advantage of many of the recent advances in thermometric measurements, and his work is of a very high order. Pernet has written a very full criticism of this work, and Ludin in his reply introduces a few corrections.

The following values (Col. II.) of σ_t in terms of σ_{16} are those given by Dr. Barnes in *Phil. Trans. Roy. Soc.* A, cxcix. 252.

TABLE V

I. Temperature, ° C.	II. $\frac{\sigma_t}{\sigma_{16}}$ (Air Scale).	III. $\frac{\sigma_t}{\sigma_{15}}$ (Hydrogen Scale).
5	1·0053	1·0052
15	1·0003	1·0000
25	·9981	·9975
35	·9974	·9967
45	·9976	·9967
55	·9985	·9975
65	·9997	·9986
75	1·0010	1·0000
85	1·0024	1·0018
95	1·0037	1·0033

For the purposes of comparison with the results of other observers these values have in Col. III. been expressed in terms of σ_{15} instead of σ_{16}. It appears that Dr. Barnes' temperature scale is that of the constant-pressure air thermometer. Over the range 0° to 100° C. this scale is in close agreement with that of the nitrogen thermometer; the numbers in Col. III. are expressed in the hydrogen scale, the reduction having been effected by the tables given by M. Chappuis.

The following table summarises the result of the observers above referred to, over the range 0° to 35° of the hydrogen scale:

TABLE VI

VALUES OF THE SPECIFIC HEAT OF WATER REFERRED TO THAT AT 15° C. AS UNITY

Temperature by the Hydrogen Thermometer.	Rowland (Day).	Bartoli and Stracciati (Pernet).	Griffiths.	Ludin.	Callendar and Barnes.
0°	..	(1·0080)	..	(1·0051)	(1·0084)
1	..	..	..	..	..
2	..	..	..	..	..
3	..	1·0059	..	1·0035	..
4	..	52	..	31	..
5	(1·0042)	46	..	27	1·0052
6	1·0036	40	..	23	45
7	31	34	..	19	38
8	26	28	..	16	32
9	23	23	..	13	26
10	19	18	..	10	21
11	14	13	..	8	16
12	10	09	..	6	11
13	07	05	1·0006	4	07
14	03	02	03	2	03
15	1·0000	1·0000	1·0000	1·0000	1·0000
16	0·9996	0·9998	0·9997	0·9998	0·9997
17	93	97	94	97	94
18	90	96	91	96	90
19	86	95	88	95	88
20	83	94	85	94	85
21	81	93	82	93	83
22	79	93	79	93	80
23	76	94	76	92	77
24	74	95	73	92	75
25	72	97	70	93	74
26	71	98	0·9968	93	73
27	69	1·0000	..	94	72
28	69	02	..	94	71
29	68	05	..	95	70
30	67	10	..	96	69
31	67	1·0011	..	97	69
32	67	..	..	98	68
33	67	..	..	99	68
34	67	..	..	1·0001	67
35	0·9969	..	..	1·0003	0·9967

Values in brackets obtained by extrapolation.

For the purposes of our final reduction of the values of the mechanical equivalent to a common standard, the range from 10° to 25° is of special importance, and the agreement between the values obtained by Rowland, Griffiths, and Callendar and Barnes may be regarded as satisfactory (greatest difference 2 in 10,000), especially when we remember the difference in the methods of experiment, which may be summarised as follows:

Rowland.—Mechanical work done against friction of water, the revised results being dependent on the thermometry of the Bureau International.

Beiblätter, 1891, xv. 761.

[2] *Ibid.*, 1897.

Griffiths.—Work done by an electric current, the data being E and R and the thermometers standardised by platinum thermometers and also by the standards of the Bureau.

Callendar and Barnes.—Electrical methods, data being E and C, and the thermometry the differential platinum method.

The methods adopted by Bartoli and Stracciati and Ludin were similar in principle, although differing in detail, hence it might be expected that their results would have been in closer agreement than those obtained by the other observers. Such, however, is not the case. At 10° they differ by 8 parts in 10,000, and at higher ranges by as much as 14 in 10,000.

It appears unlikely that the skill and patience shown by these observers can be exceeded, and it is probable that the method of mixtures presents peculiar difficulties and uncertainties which are absent in the energy determinations.

These considerations have led to the adoption of the following table for the reductions to some standard temperature:

TABLE VII

VALUES OF σ_t OVER THE RANGE 10° TO 25° OF THE HYDROGEN THERMOMETER IN TERMS OF σ_{15}

Temperature.	
10°	1·0020
15	1·0000
20	·9984
25	·9972

By means of the curve thus obtained we can now reduce the values given in Table IV. (*supra*) to a common temperature.

TABLE VIII

CAPACITY FOR HEAT OF WATER AT 20° C. OF THE HYDROGEN SCALE

Joule	$4{\cdot}170 \times 10^7$
Rowland	$4{\cdot}182 \times 10^7$
Griffiths	$4{\cdot}193 \times 10^7$
Schuster and Gannon	$4{\cdot}190 \times 10^7$
Callendar and Barnes	$4{\cdot}185 \times 10^7$

The doubt as to the exact conditions under which Joule used his thermometers compels the omission of his value.

§ (8) ON THE ACCURACY OF THE VALUES OF THE ELECTRICAL STANDARDS ASSUMED BY THE OBSERVERS.—Professor Ames[1] in 1900 wrote as follows:

"In regard to the electrical standards, it must be observed that no meaning can be attached to the 'electro-chemical-equivalent' of silver, unless the construction and use of the voltameter are most carefully specified, and even then there is considerable doubt unless several instruments are used in series. This fact is well shown in the recent work of Richards, Collins and Heimrod at Harvard University, and of Merrill at Johns Hopkins University. The former deduce from a comparison of their porous-jar voltameter with other forms of instruments that the electro-chemical equivalent with their instrument is 0·0011172 gram per sec. per ampere; while Patterson and Guthe with their instrument find 0·0011193. If, however, the same voltameter and the same method of use are adopted in the experiments on the electro-chemical-equivalent and in those on the E.M.F. of a Clark cell, the value of the latter is independent of the value assigned to the former. For this reason Kahle's value of the E.M.F. of the standard Clark cells of the Reichsanstalt (1·4325 volts at 15° C.) is probably correct. The Cavendish Laboratory standard cell has been compared with the German ones; its resulting value is 1·4329 at 15° C., and the later investigations of Patterson and Guthe would reduce this to 1·4327. As Griffiths used the value 1·4342 and as the E.M.F. enters into the equation to the second power, the necessary correction would be almost exactly two parts in one thousand.

"For the method used by Schuster and Gannon, where both the E.M.F. and the current are measured, a correction may be accurately applied to the E.M.F., but not to the current, as it is not known what amount of silver one ampere should deposit in their voltameter; but if we assume that the correction in both cases is 1 in 1000, these results also would be reduced by 2 parts in 1000. The correction assigned is probably in the right direction.

"The cells used by Callendar and Barnes have not been compared with those of the Reichsanstalt, and no 'correction' can be applied with certainty.[2] The figures used above[3] are probably in excess."

Messrs. Ayrton, Mather, and Smith in 1905–7 made a careful redetermination of the ampere.

The inquiry was conducted at the National Physical Laboratory under the supervision of Sir Richard Glazebrook.

A very perfect form of current weigher was constructed, and the authors wrote: "The current weigher has proved to be the most perfect absolute electrical instrument hitherto constructed, and has enabled us to determine the ampere to a very high degree of accuracy."[4] The work was conducted with a skill and care which marks the investigation as a notable contribution to Physical Science.

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[1] *Congrès International de Physique*, Paris, tome i.

[2] Here the value of the current was obtained by measurement of R/E, where R was a standard resistance, and it is somewhat difficult to estimate the probable effect of the change in the equivalent of silver in the absence of certain knowledge concerning the *comparative* value of Callendar and Barnes cells, in terms of the Rayleigh cell.

[3] *I.e.* a correction of 2 parts in 1000.

[4] *Phil. Trans. Roy. Soc.* A, ccvii. 468.

The results afford the data for the determination of the E.M.F. of the Clark cell.

The authors' conclusion is as follows:

"Value for Clark cell at 15° C. $=1{\cdot}432_3$" at 15° C.

The value assumed by Griffiths was 1·4342.

The value assumed by Barnes was 1·4342.[1]

The value assumed by Schuster and Gannon was 1·4340,[2] and the value of the electrochemical-equivalent of silver = ·001118.

§ (9) Final Values.—Assuming the value of E as $1{\cdot}432_3$, and applying the resulting corrections to the numbers in Table VIII. *supra*, the results are as follows, the temperature being 20° C.:

Table IX

Rowland	$4{\cdot}182\times10^7$
Griffiths	$4{\cdot}184\times10^7$
Schuster and Gannon	$4{\cdot}186\times10^7$
Callendar and Barnes	$4{\cdot}181\times10^7$
Mean	$4{\cdot}183\times10^7$

Professor Ames, in his criticism given *supra*, suggests a *further* reduction of 1 part in 1000 in the value obtained by Schuster and Gannon. If we also take into consideration the other uncertainties referred to *supra*, it appears probable that a correction of the order of 1 in 2000 is applicable.

If this is done, the final results are given in the following table:

Table X

Capacity for Heat of Water at 20° C. of the Hydrogen Scale

Rowland	$4{\cdot}182\times10^7$
Griffiths	$4{\cdot}184\times10^7$
Schuster and Gannon	$4{\cdot}182\times10^7$
Callendar and Barnes	$4{\cdot}181\times10^7$
Mean	$4{\cdot}182\times10^7$

In order to obtain the most probable values of C_t at other temperatures, we can proceed as follows:

From Table VI. *supra* we obtain the mean values of C_t/C_{15} given by Cols. I., III., V., over the range 0° to 35°.

From 35° to 100° we must be guided by the observations of Callendar and Barnes. We can thus deduce the value of C_t/C_θ over the whole range. Assuming the value of C_θ at some given temperature, we can then obtain the values of C_t at other temperatures.

A convenient standard temperature for ordinary conditions is 17°·5 C., and the value of $C_{17\cdot5}=4{\cdot}185\times10^7$.

The value obtained by Reynolds and Moorby for "the mean thermal unit" was 4·1833 (*supra*). A study of their tables shows that the actual range was—on the average—from about 1°·3 to 100° C. An inspection of Table XI. shows that the rate of variation of σ_t is very rapid near 0° C. and a probable value of $\sigma_1/\sigma_{17\cdot5}$ is about 1·008.

Table XI

Temperature on H. Scale.	Col. I. σ_t.	Col. II. C_t.
0°	(1·0083)	$(4{\cdot}220)\times10^7$
5	54	·207
10	27	·196
15	7	·188
20	·9992	·182
25	78	·177
30	75	·175
35	74	·174
40	73	·174
45	74	·174
50	77	·175
55	81	·177
60	87	·179
65	93	·182
70	1·0000	·185
75	7	·188
80	15	·191
85	23	·194
90	31	·198
95	40	·202
100	(1·0051)	(4·206)
Mean	1·00033	4·186

This approximate correction will only raise Reynolds and Moorby's value to 4·184. This differs from the mean of the numbers in Col. II. by 1 part in 2000. The correspondence is remarkable, and greatly increases the probability of the accuracy of the conclusions at which we have arrived.[3]

These conclusions may be summarised as follows:

Assuming the Standard Thermal Unit as the energy required to raise 1 gramme of water from 17° to 18° C. on the Paris Hydrogen Scale, or one-fifth the amount required to raise it from 15° to 20° C. on the same scale, then the Standard Thermal Unit $=4{\cdot}185\times10^7$ Ergs.

The following resulting values may be found useful:

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[1] Barnes in *later* tables gives the value of the mechanical equivalent resulting from the assumption that E = 1·43325.

[2] *Phil. Trans. Roy. Soc.* A, 1895, p. 420.

[3] This close correspondence between the value of $C_{17\cdot5}$ and the "mean thermal unit" is in many ways a convenience. For example, we are enabled to express the value of thermal measurements obtained by Bunsen's calorimeter in terms of the standard unit, and that with sufficient accuracy, for Professor Nicholls has shown that the variations in the density of ice sometimes amount to 2 parts in 1000.

Assuming $g = 981$:

1 kilogram-degree C. = 426·6 kilogrammetres.
1 pound-degree C. = 453·6 = 1399·8 ft.-lbs.
1 pound-degree F. = 252·0 = 777·6 ft.-lbs.

Assuming $g = 981·2$, its value at Greenwich, these values become 426·4, 1399·5, 777·5.

In latitude 45°, assuming $g = 980·62$, they become 426·7, 1400·3, 777·9.

APPENDIX I

The following are the values of the constants which have been employed in the final comparison of the values of $C_{17·5}$ found by the selected observers.

True ohm = 1·01358 B.A. units.
Electro-equivalent of silver = 1·1182 per coulomb.
Weston cadmium cell, $C \times R = 1·01830$ at 17° C.
Weston cell temperature coef.,
$E_t = E_{17} - 3·45 \times 10^{-5}(t - 17) - 0·066 \times 10^{-5}(t - 17)^2$.
Clark cell, $C \times R = 1·4323$ at 15° C.
Clark cell temperature coef. between 0° and 30° C.,
$E_t = E_{15} - 1·200(t - 15°) - ·0062(t - 15°)^2$.

APPENDIX II

THE THERMAL UNIT

Professor Rowlands' views on this matter are expressed in the following letter:[1]

JOHNS HOPKINS UNIVERSITY, *December* 15, 1895.

As to the standard for heat measurement, it is to be considered from both a theoretical as well as a practical standpoint.

The ideal theoretical unit would be that quantity of heat necessary to melt one gramme of ice. This is independent of any system of thermometry, and presents to our minds the idea of quantity of heat independent of temperature.

Thus the system of thermometry would have no connection whatever with the heat unit, and the first law of thermodynamics would stand, as it should, entirely independent of the second.

The idea of a quantity of heat at a high temperature being very different from the same quantity at a low temperature, would then be easy and simple. Likewise we could treat thermodynamics without any reference to temperature until we came to the second law, which would then introduce temperature and the way of measuring it.

From a practical standpoint, however, the unit depending on the specific heat of water is at present certainly the most convenient. It has been the one mostly used, and its value is well known in terms of energy. Furthermore, the establishment of institutions where *it is said* thermometers can be compared with a standard, renders the unit very available in practice. In other words, this unit is a better practical one at present. I am very sorry this is so, because it is a very poor theoretical one indeed.

But as we can write our text-books as we please, I suppose that it is best to accept the most practical unit. This I conceive to be the heat required to raise a gramme of water 1° C., on the hydrogen thermometer at 20° C.

I take 20° because in ordinary thermometry the room is usually about this temperature, and no reduction will be necessary. However, 15° would not be inconvenient, or 10° to 20°.

As I write these words I have a feeling that I may be wrong. Why should we continue to teach in our text-books that heat has anything to do with temperature? It is decidedly wrong, and if I ever write a text-book I shall probably use the ice unit. But if I ever write a scientific paper of an experimental nature, I shall probably use the other unit.

E. H. G.

HEAT, TRANSFER OF, BETWEEN CALORIMETER AND JACKET, IN METHOD OF MIXTURES: this may take place by conduction, convection, radiation, and evaporation, and is eliminated from the final result by the cooling correction. See "Calorimetry, Method of Mixtures," § (6) (i.).

HEAT, UNITS OF. See "Thermodynamics," § (2); "Heat, Mechanical Equivalent of," §§ (8), (9).

HEAT-ACCOUNT FOR A REAL PROCESS. See "Thermodynamics," § (44).

HEAT-DROP. See "Thermodynamics," § (38).

HEAT-ENGINE:

A machine which absorbs a certain quantity of heat at a high temperature and discharges a smaller quantity of heat at a lower temperature, the difference between these two quantities being converted into work. See "Thermodynamics," § (1).

Working Substance in. See *ibid.* § (7).

HEAT-FLOW IN CYLINDER WALLS. See "Engines, Thermodynamics of Internal Combustion," § (62).

HEAT TRANSMISSION TO FLUIDS FLOWING IN HOT METAL PIPES. Soenneken's Experiments. See "Friction," § (39).

Effect of Surface Friction. See *ibid.* § (40).

HEAT TRANSMISSION BETWEEN SOLIDS AND FLUIDS FLOWING OVER THEM. Theory of Osborne Reynolds. See "Friction," § (39).

N.P.L. Experiments. See *ibid.* § (40).

HEATING OF CHARGE ENTERING CYLINDER OF INTERNAL COMBUSTION ENGINE. See "Engines, Thermodynamics of Internal Combustion," §§ (63)-(65).

HEAVY OIL ENGINES. See "Engines, Internal Combustion," § (13) *et seq.*

HELE-SHAW PUMP. See "Hydraulics," § (64).

HELIUM, SPECIFIC HEATS OF, tabulated values obtained by Scheel and Heuse. See "Calorimetry, Electrical Methods of," § (15), Table IX.

HELMHOLTZ'S FREE ENERGY OF THERMODYNAMIC SYSTEM. See "Thermodynamics," § (51).

HENNING, detailed investigation over the range 0° to −200° C. of temperature of the comparison between the readings of the hydrogen gas thermometer and the platinum resistance thermometer. See "Resistance Thermometers," § (17).

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[1] *B.A. Report*, Liverpool, 1896.

Henning, Measurement of Latent Heat of Water. See "Latent Heat," § (3).

High-speed Steam Engines. See "Steam Engine, Reciprocating," § (7).

High Temperature Melting-points, Extrapolation of the Scale of the Platinum Thermometer for the Determination of. See "Resistance Thermometers," § (18).

High Vacuum Technique. See "Air-pumps," § (47).

Higher Pairs. For definition see "Kinematics of Machinery," § (2).

Hodgson Kent Steam Meter. See "Meters," Vol. III.

Hoists, Hydraulic. See "Hydraulics," § (58).

Holborn and Day, 1899, comparison of gas-thermometers with secondary standards of temperature in range 500° to 1600°. See "Temperature, Realisation of Absolute Scale of," § (39) (x.).

Holborn and Kurlbaum, modification of disappearing filament type of optical pyrometer by addition of objective and eyepiece. See "Pyrometry, Optical," § (3).

Holborn and Valentiner, 1906, comparison of gas-thermometer with secondary standards of temperature in range 500° to 1600°. See "Temperature, Realisation of Absolute Scale of," § (39) (xiii.).

Holborn and Wien, 1892, comparison of gas-thermometers with secondary standards of temperature in the range 500° to 1600°. See "Temperature, Realisation of Absolute Scale of," § (39) (ix.).

Homogeneous Head of Steam. See "Steam Turbine, Physics of," § (1).

Hooke's Law: a fundamental assumption of the theory of elasticity, which asserts that the relation between stress and strain is one of direct proportionality; this was discovered, by Hooke, in 1678, to be representative of actual materials, and expressed thus: *Ut tensio sic vis.* See "Elasticity, Theory of," § (4).

Horizontal Gas-engine, Typical. See "Engines, Internal Combustion," § (5).

Hornblower's Engine. See "Steam Engine, Reciprocating," § (12).

Hornsby-Akroyd Oil-engine. See "Engines, Internal Combustion," § (14).

Hull Efficiency and Wake Fraction. See "Ship Resistance and Propulsion," § (47).

Humphrey Gas-pump. See "Hydraulics," § (42).

Hyatt Dynamometer. See "Dynamometers," § (5) (ii.).

Hyde's Experiments on the Effect of Pressure on the Viscosity of Liquids. See "Friction," § (7).

Hydraulic Gradient. See "Hydraulics," § (26).

Hydraulic Mean Depth. The depth which the volume of water contained in a pipe would have, if contained in a rectangular channel with a flat bottom of the same area as the wetted walls of the pipe. See "Hydraulics," § (25).

Hydraulic Ram. See "Hydraulics," § (43).

Hydraulic Transmission: Compagne Hydraulic Gear. See "Hydraulics," § (64).

Hydraulically-braked Machines. See "Hydraulics," § (60).

Hydraulically-driven Machines. See "Hydraulics," § (57) *et seq.*

HYDRAULICS

I. Natural Sources of Power in Water

§ (1) Water Power.—During recent years there has taken place throughout the world a great development in the utilisation of the natural resources of water, and the reasons are not far to seek. On the one hand, there has been a growing consciousness that a plentiful supply of this element is the basis of all sanitary science, and on the other, the necessity of substituting, as a source of power, coal and oil fuels because of their increasing costliness by the utilisation of water. These two factors have combined to give a special and increasing importance to the question of water supply.

The problem from the point of view of the domestic supply is, of course, not a new one, having engaged the attention of every civilised nation, ancient or modern, but until the nineteenth century it was solved along the line of least resistance and the employment of high pressures was evaded. The needs of the modern city, added to the demands for an adequate supply of water power, have accordingly given a new direction and impetus to the study and practice of Hydraulics.

§ (2) Source of Water Supply. (i.) *Precipitation or Rainfall.*—Precipitation, embracing as it does the fall of rain, dew, snow, and hail, is the main source of all water supply, and is usually studied under the general term of rainfall. The supply is derived almost entirely by evaporation from the surfaces of the various oceans and seas which cover the greater portion of the earth's surface, and depends upon the radiation from the sun and the capacity of the atmosphere to contain the moisture evaporated.

As might be expected, there exist great irregularities in the evaporation, and therefore in the rainfall of large areas. The continuous current of heated air which ascends in the region of the tropics and flows towards the poles carries with it a full complement of moisture and distributes it when temperature and other physical conditions determine.

The precipitation of the moisture in a district varies greatly with its situation, the configuration of the surrounding country, its altitude, and the direction of the prevailing winds. Where the latter are charged with moisture through crossing large tracts of open water the rainfall on the first high ground encountered will naturally be heavy.

On the other hand, the rainfall of a district is small if the prevailing winds traverse a large tract of land and thus become depleted of their moisture. An example is obtained if a straight line be drawn from the east coast of the United States at latitude 35° to a point on the west coast at latitude 45°, the variations of rainfall with longitude being as follows :

Longitude.	Rainfall.	Longitude.	Rainfall.
75°	60 in.	107°	14 in.
79	45 „	115	15 „
87	40 „	119	15 „
95	30 „	123	50 „
103	17 „	125	100 „

The observed rainfall of the Forth valley above Queensferry affords an excellent illustration of the effect of altitude on precipitation. There the values vary from 28 inches at Queensferry (sea-level) to a maximum of 110 inches at a height of 2300 feet above sea-level at the western end of Loch Katrine some 50 miles distant.

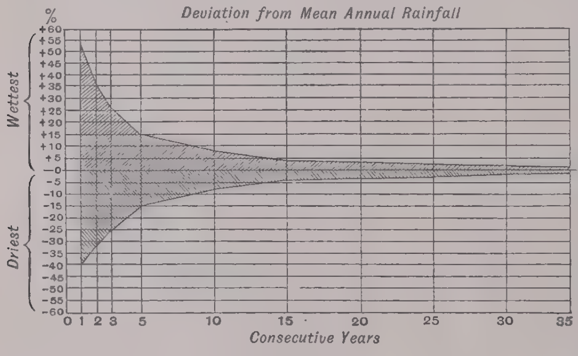

Fig. 1.

These numbers do not represent the extreme precipitation values, which range from practically nothing over the deserts of Africa to as much as 600 inches in the Himalayas.

A striking case of unequal distribution of rainfall is supplied by the rainfall of Western India. At Bombay at sea-level the rainfall is about 76 inches ; at a station situated on the Western Ghats about 60 miles distant 546 inches was actually measured. Farther west but still on the Deccan plateau at an elevation of nearly 2000 feet the precipitation was only about 20 inches, the wind, deprived of the great bulk of its moisture burden, being then comparatively dry.

(ii.) *Annual Variations of Rainfall.*—In addition to the very unequal distribution of rainfall from point to point and area to area, great irregularities occur from year to year over the same area.

If over a long series of years a selection is made of the wettest year on record it may be expected to have a rainfall of 51 per cent above the average, while the driest year will be 40 per cent below. There is thus a range between the two equal to 91 per cent of the mean annual rainfall. When the average rainfall of the two consecutive wettest years is taken it will be about 35 per cent above the mean, and that of the two driest 31 per cent below that amount, with a range, therefore, of 66 per cent of the mean rainfall. If the period be increased to three consecutive years the average for the wettest years will be 1·27 and for the three driest years 0·75 of the mean.

Fig. 1, which is taken from the paper read by Sir Alexander R. Binnie before the Institution of Civil Engineers in 1892, on "Average Annual Rainfall," shows how the deviation from the mean annual rainfall of the average rainfall over a number of consecutive years diminishes as the included consecutive period increases, until after about thirty-five years it approximates to the total mean value.

From this figure it may be observed that it is possible to have a series, say, of five consecutive years in which the rainfall amounts to only 84 per cent of the mean fall of the district, a circumstance which possesses special significance when calculating the possibilities of the supply of water to be obtained.

(iii.) *Seasonal Variations of Rainfall.*—In the temperate climate of the British Isles, where the year cannot be divided sharply into dry and wet seasons, the monthly rainfall forms the best means of comparison between one period of the year and another. *Fig.* 2 has been prepared from the Greenwich records of the years from 1861 to 1891, and shows that February, March, and April are low months of less than 2 in. fall; that January, May, and June are normal months; that the final six months are high months; and that the maximum variation above and below does not exceed 25 per cent of the average monthly value.

The rainfall on the Western Ghats, already mentioned as an instance of irregular distribution over neighbouring areas, is also an excellent example of seasonal variations. The bulk of the rain here falls generally in three periods: the first burst, lasting perhaps fourteen days, occurs early in June; then the big monsoon from about the middle of July to the middle of August, and the final burst during September and October. During eighty days 546 in. has been measured at one of the gauges on these hills about 3000 feet above the sea, the greatest weekly fall being 110 in. Only a few scattered showers, amounting on the average to about 3 in., fall during the dry season, which consists of the remaining months of the year.

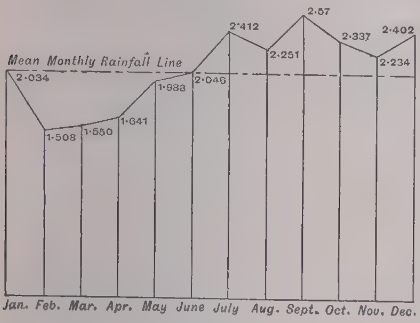

FIG. 2.

§ (3) RAIN GAUGES.[1]—The average rainfall on a catchment area is estimated by rain gauges.

It is obviously impossible to construct a rain gauge which shall in all cases catch all the rain that falls. It is however possible, by a due regard to the selection of a site, to the method of fixing, and to the proper maintenance of and regular attendance on the ordinary standard gauge, to ensure that valuable information is available concerning the rainfall on a given catchment area. As many gauges as possible must be used, and these situated in places representative of the different elevations and conditions to be met with in the district under investigation. In order that the cycle of variation may be complete it is suggested that gauging should extend over thirty-five years or thereby.

§ (4) CATCHMENT AREA.—In estimating the possibilities of a water supply from a certain district it is necessary not only to ascertain the rainfall but also to arrive at the extent of what is known as the catchment area or gathering ground available for the supply. This comprises all the country the drainage of which converges to the point selected as the distributing centre of the supply. The area is usually expressed in acres or in square miles of 640 acres. Its extent may be measured from a topographical map, if such is available, showing the watershed and divide, along which drainage for adjacent areas separate. Otherwise a close approximation may be found on an ordinary map by marking the area boundary as passing midway between the apparent heads of tributaries of neighbouring watercourses draining into different stream systems.

§ (5) VOLUME OF RAINFALL.—The volume of water falling as rain on that area will then be given by the product of the catchment area into the annual rainfall. Since one acre contains 43,560 square feet, a rainfall of 1 in. on 1 acre represents a volume of 3630 cubic feet or 22,622 gallons.

§ (6) RUN-OFF.—While rainfall is the source of all water supply its volume is not the direct measure of the quantity available for a required supply. When rain falls upon the ground a portion sinks into the soil: the remainder runs off the surface into the catchment basin or remains for a time on the surface. The quantity which passes into the ground depends upon the conformation of the surface, and the porosity, depth, and degree of saturation of the material comprising it. Part of the ground water continues to sink and finally finds its way to the nearest stream valley, but part is retained by the roots of vegetation, being converted into cellular fibre or exhaled as vapour from vegetation or from the surface into the atmosphere. If all the water which is consumed by vegetation or which is vaporised from the surface is included in the term evaporation, then the volume of water available for a required supply will be the volume of the rainfall on the catchment area, less the volume of evaporation over the same area. When this quantity is expressed as the volume which passes a given point continuously it is termed the flow.

§ (7) EVAPORATION.—An approximation to the flow may be deduced from the difference between the rainfall and the evaporation if these are known. The latter may be computed on the basis of monthly ratios to rainfall, which vary according to the requirements of vegetation, the capacity and condition of ground storage, and the temperature. The monthly rate of evaporation is represented by the expression

$$e = a + br,$$

where a and b have values which change with the month, from about ·25 and 0·1 in January to 3·0 and 0·3 in July for a and b respectively, while e is the monthly evaporation and r the corresponding rainfall.

A temperature correction may be applied so that the values obtained as above for a drainage area with a given mean annual temperature T can be used to determine those for other latitudes of temp. $t°$. The values determined for the monthly evaporation e are multiplied by a temperature factor expressed by $(\cdot 05\,t - c)$, where t is the mean annual temperature, and c is a constant which reduces the expression to unity when t equals the mean annual temperature T.

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[1] See "Meteorological Instruments," Vol. III.

§ (8) GROUND STORAGE.—The undulations of the surface of the drainage area exercise an important influence upon the distribution of the flow. Evidently the storm run-off will be greater from a hilly country than from flat tablelands, and the rainfall remaining available for ground storage will be correspondingly smaller. Then the degree of absorption and run-off largely depend upon the geology of the drainage area, since the percolosity of the ground determines its storage capacity. Where a rock ledge is at the surface or crops out in banks the depth of the overlying drift is readily ascertainable, otherwise its extent and character is found by making borings, if practicable, to the rock. Considerable attention has also been paid of late years to the influence of flora and culture on rainfall and flow. Forests and vegetation do not probably affect the total rainfall, but they certainly cause it to fall more uniformly. By protecting the ground from the direct heat of the sun evaporation is retarded, the air immediately overhead is kept cooler, thus increasing its power to condense the moisture in heated and saturated air at a higher level and so steadying the rainfall. Wooded country compares favourably with cultivated land, as the many obstructions to the storm run-off in timbered areas result in a larger proportion finding its way into the ground storage. Then the requirement of moisture for tree growth is considerably less than that of crops, very little surplus being available in the latter case for run-off during the growing season. The effect of the ground storage in equalising the flow is shown by the diagrams of *Fig.* 3. The measured monthly rainfall in inches is set up from the base AB, and from the resulting curve is set down the sum of the estimated evaporation and the measured run-off. Where the final ordinates fall below AB a flow from the ground storage takes place, and where the balance is in favour of the rainfall the ground storage benefits. The lower portion of the diagram represents the variation in the ground storage, saturation being complete during only four months.

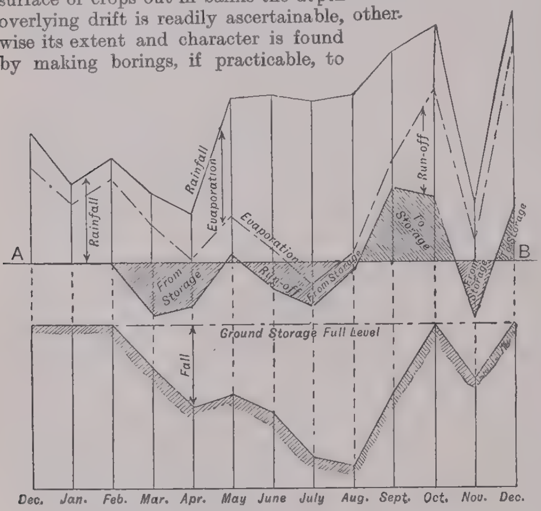

FIG. 3.

§ (9) GAUGING OF STREAM FLOW.—While it is possible, as has been shown, to deduce the total run-off or stream flow from a drainage area from the difference of rainfall and the computed evaporation over that area, the exact determination of the supply available from day to day and month to month can only be obtained by a system of stream gauging. The method adopted will depend upon the volume of water which has to be measured.

(i.) *Notch Measurements.*—When the stream is small, measurement by notch or weir can be used. The weir is generally constructed of planks and the notch is formed in an iron plate screwed to it, and may be either the right-angled triangular type (for small quantities) or rectangular. The weir and face of the notch must be vertical and perpendicular to the direction of the stream, with a clear discharge into the air. If the notch is rectangular the sill must be horizontal and of a length not less than three times the head above the notch measured to still water. *Fig.* 4 shows the arrangement of a standard sharp-edged weir.

For accurate work measurements of the head taken in a flowing stream are inadmissible. To minimise the effect of oscillations of the surface and to avoid the disturbance produced by an immersed object in the stream a well should be made at a sufficient distance behind the weir in the bank of the stream and connected to it by a pipe opening out

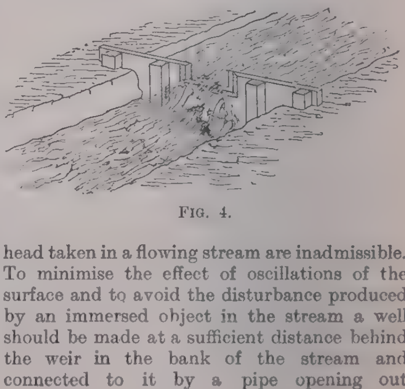

FIG. 4.

flush with the approach channel and at right angles to the direction of flow. The required head is the height of the surface of the water in this well above the level of the sill or the bottom of the vee.

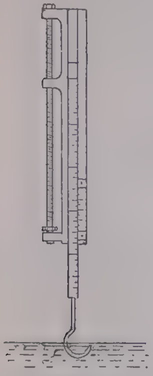

FIG. 5.

This height may be conveniently measured by means of an anchored float carrying a scale and rising and falling in contact with a fixed index whose level is known, or carrying an index which moves over a fixed scale. A more exact method is that of the hook gauge shown in *Fig.* 5.

A hook is attached to the lower end of a divided scale, and its position may be adjusted by means of screw till the point of the hook is just visible at the surface of the water. If the index to the scale has been fixed to show zero when the water is level with the sill, then the head may be read directly on the scale by the aid of a vernier.

(ii.) *Rectangular Notch.*—For heads above 6 in., where the bottom and end contractions are free and where the length of crest is greater than three times the head, values in close agreement with experiment are given by the Francis formula,

$$Q=3{\cdot}33\left(b-\frac{n}{10}H\right)H^{\frac{3}{2}} \text{ cub. ft. per sec.,}$$

where Q is the discharge in cubic feet per second, H is the head in feet, b the breadth of the notch, and n is the number of end contractions, the bottom being supposed to be free in every case.

(iii.) *Triangular Notch.*—Let H be the head of water above the vertex P of the notch (*Fig.* 6), then assuming the velocity of flow at any depth x to be given by $\sqrt{2g\cdot x}$ it can readily be shown that the discharge $Q=4{\cdot}28\,C\tan(\theta/2)H^{\frac{5}{2}}$, where θ is the angle included between the sides of the notch.

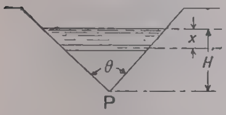

FIG. 6.

A right-angled notch was found by Professor Jas. Thomson to have a coefficient C of the mean value of ·593 with a variation for a range of heads from 2 to 7 in. of less than 1 per cent, and is the form usually adopted for measuring small quantities. The formula for the discharge over a right-angled vee notch therefore becomes

$$Q=2{\cdot}536H^{\frac{5}{2}} \text{ cub. ft. per sec.}$$

§ (10) VELOCITY OF FLOW.—For larger streams or rivers the weir is too cumbrous and costly to be used as a temporary measuring contrivance. It is then necessary to obtain the mean velocity of flow which, multiplied by the cross-sectional area of the stream, gives the discharge Q. Experiment shows that the motion at any point in an open channel is never steady and uniform, and that its velocity, if near the surface, may vary by 20 per cent, and if near the bed, by 50 per cent in a short interval of time. This constitutes the difficulty of obtaining the mean velocity with a reasonable degree of accuracy, and the mean of many observations is required. Two methods may be used: floats and current meters.

§ (11) FLOATS.—These are classified into surface, sub-surface, twin, and velocity rod types, and are illustrated in *Fig.* 7.

(i.) *Surface floats* (*a*) are made of any light material painted to be easily seen, but not projecting more than an inch above the surface in order to minimise the effect of the wind. The wind effect, together with the tendency of the floats to follow any cross-current or surface eddy, render the results obtained very unreliable.

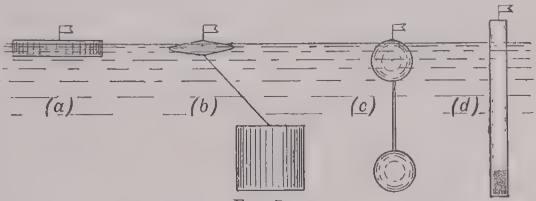

FIG. 7.

(ii.) *Sub-surface floats* (*b*) consist of a small surface float from which a lower float is suspended, the length of connection being adjusted so that the latter may remain at any required depth. In theory the velocity of the lower strata should be indicated, but in practice this is not realised, the relative position of the lower float varying with the direction and velocity of the wind and with the length of connection between the two floats.

(iii.) *Twin floats* (*c*) usually consist of two spheres coupled together by means of wire, the lower one weighted so as to float vertically beneath the surface one and as near the bottom as practicable. With this adjustment the velocity of the float will be approximately the mean velocity of the vertical column of water in which the instrument floats.

(iv.) *Velocity rods* (*d*) are light wooden rods or tin tubes made in adjustable lengths and weighted at one end with lead strips or wire to float upright, the lower end being as near the bottom as practicable. The velocity of the rod is approximately the same as the mean over its depth and probably gives the more reliable measurement of the mean velocity of the stream on the vertical in which it floats.

To carry out the operation of gauging by means of floats it is necessary to select a straight stretch of channel of about 200 feet in length, as nearly uniform as possible in section and containing no obstructions. Two cross-sections are ranged out by wires stretched from bank to bank, if the width will allow, and 100 feet apart; these should be perpendicular to the direction of flow. A third line is usually stretched midway between these two. If the observations are to refer to different depths and different distances from the centre of the stream the wires should be divided up into corresponding sections of, say, 10 feet intervals by attaching tags of different colours. The floats are placed in the water about 50 feet above the upstream gauging section, the *locus* of their passage through the gauging area being noted by line markers and the times of entry and exit recorded. Where the width of channel does not permit of this being done the positions of the floats may be fixed by the use of a theodolite measuring angles from a base line marked out on the bank.

§ (12) CURRENT METERS.—Current meters may consist of an arrangement whereby the velocity is determined either from the revolutions of a revolving part rotated by the current or by a modification of the Pitot tube. Typical examples of the former are:

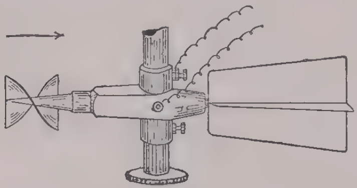

FIG. 8

(i.) *Amsler current meter*, where the revolving part carries helicoidal vanes mounted on a horizontal axis (*Fig.* 8), and

(ii.) *Price current meter*, where a series of conical cups is mounted on arms revolving round a vertical axis (*Fig.* 9). In either case the meter is fitted with a guide vane to keep its axis perpendicular to the direction of the current. The number of revolutions is usually indicated by a make-and-break contact in an electrical circuit.

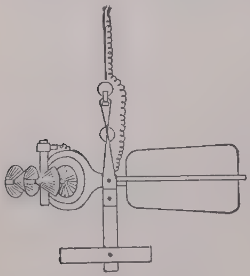

FIG. 9.

(iii.) The *Pitot tube* [1] method is well adapted for use with the higher velocities common in pipe flow to which its use is more confined, and is discussed in the section dealing with that flow.

§ (13) CALIBRATION OF METERS is usually effected by suspending the meter from a travelling carriage, which tows it with a uniform velocity through still water at a depth of about 2 feet. The time over the measured length, and the number of revolutions of the meter, are recorded by means of a chronograph and form a rating table. When in use the meter should be suspended in a similar manner to that adopted during its calibration, as the speed of revolution has been shown to vary with the degree of freedom permitted.

The meter may be used by holding it successively at certain points in a cross-section, or, alternatively, by keeping it in motion during the whole period of its immersion and moving it uniformly from the surface to the bottom of the channel in a series of vertical or diagonal lines. The latter method is not nearly so accurate as the former. Observations taken at points six-tenths or mid-depth in a series of equidistant verticals, the mean velocity in each of these verticals being found by applying a factor, are capable of giving reliable results.

Simultaneously with the velocity observations, soundings should be taken from which the cross-section of the stream can be ascertained.

From the data thus obtained is constructed a rating table showing the relation at a given point between the height of water, referred to some permanent bench mark, and the discharge of the river.

§ (14) AVERAGE WATER SUPPLY AVAILABLE.—Gaugings are not as a rule available over the period of thirty-six years which has been suggested as necessary for complete information, but a ratio between the flow of the stream and the rainfall over the area may be established if one complete year's record of the former is known. By means of this ratio the flow during a sequence of years may readily be deduced from rainfall records, and the average water supply available be determined.

§ (15) NET WATER SUPPLY.—The reliable procedure is to find the year of lowest annual rainfall during a cycle of, say, fifteen years and to establish the daily flow during this year, comparison being made with the year of actual gaugings on a basis of the effective rainfall, after allowance for evaporation has been made. Alternatively, the general mean annual fall, reduced by 20 per cent, may be taken as the equivalent of the three driest consecutive years, which in practice is the largest amount that can safely be relied upon. A further reduction should be made to allow for evaporation, which in the British Isles may vary from 10 to 20 in., and the remainder, subject to any claims for compensation water to be given off to streams previously supplied by the drainage area, becomes the flow available per annum for a proposed supply.

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[1] See § (24) (ii).

§ (16) Maximum Continuous Flow.—The most important consideration in any water supply, whatever the purpose for which it is projected, is not the mean annual or daily flow, but the maximum continuous flow, which can under all circumstances be maintained. It is evident that the natural run-off of a drainage or catchment area, subject as it is to continually varying rainfall, evaporation, and ground storage, will itself be of a constantly fluctuating character. If, therefore, the natural flow is to meet all the demands that are to be made upon it, it follows that these demands must be so regulated as never to exceed the minimum dry weather supply of the drainage area which thus becomes the maximum continuous flow of the projected supply. In such a case all flow in excess of this amount becomes of no direct interest to the engineer and must be allowed to go waste.

§ (17) Storage.—If, however, the requirements of the supply necessitate a greater discharge, or if full advantage of the drainage area is to be secured, the water which is in excess during the wet season or during floods must be impounded for use in the dry season. This is done by the accumulation in reservoirs of some of the excess over the low-flow volume during any or all of the periods when it occurs and the utilisation of this stored supply to maintain a continuous flow, higher than the natural low-flow. Storage is usually essential to any extensive supply and is one of its most important features. It assists to still further conserve natural resources by making use of at least a portion of the large and wasting flood waters, and serves to bring them under control, thereby obviating destruction to property and sometimes to life which may otherwise result.

§ (18) Storage Capacity.—The storage capacity which it will be necessary to provide will depend not only on the proportion of flood water compared to the low flow intended to be stored, but will be principally affected by the duration of the longest period of dry weather, which may not necessarily be a period entirely without rain. In this climate, where rainy days are frequent and general, and the soil is always more or less saturated, the storage provided will depend on the extent of the fall, being greatest where the rainfall is least, and ranges from 120 days' supply in the North of England to twice that amount in the south. In India, where the monsoon forms the principal source of rainfall, and is liable to fail any one year, it is necessary to provide storage to give supply during two consecutive dry seasons.

An excellent example of this is afforded by the Tata Hydro-Electric Power Supply, Bombay, where the total quantity of water required to enable the turbines to give 100,000 h.p. for ten hours daily during nine months, after allowing for loss by evaporation, soakage, and friction inpipes and turbines, is 6700 million cubic feet, and the combined capacity of the two main storage lakes is about 10,100 million cubic feet. The excess capacity is given in order that balances in years of excessive rainfall may make up for occasional short monsoons.

§ (19) Storage Reservoirs.—Generally speaking, the best site for storage is that where the largest amount of water can be stored by means of the shallowest, shortest, and smallest embankment. If natural lakes can be utilised by the construction of a dam across the effluent river it is usually the most economical method. Notable examples of this are given in connection with the water supplies of Glasgow, Manchester, and Dundee. Otherwise a wide and flat valley with impervious strata, preferably rock and ending in a narrow gorge, should be looked for. The dam for the closing of the outlet to the storage area must be perfectly watertight, and of such a construction as will prevent the impounded water passing below, round the ends, or over the top of the work.

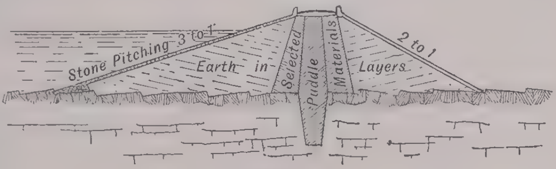

Fig. 10.

(i.) *Earthen Dams.*—Formerly the usual type of dam was the earthen embankment, (*Fig.* 10) with its core of tough, impervious clay or puddle-wall carried to a sufficient depth to ensure perfect union with an impervious stratum. This was backed on either side by harder and less water-tight materials, the inner slopes being covered with stone pitching to prevent the wash of the waves from injuring the earthwork. For embankments of moderate height the inclination of the surface is usually 3 to 1 on the inner or water side, and 2 to 1 on the outer slope.

(ii.) *Masonry Dam.* — The earthen dam, where properly constructed, has been very successful, but the tendency in modern practice is to adopt the masonry dam, at least where the height of the wall exceeds 80 feet. The structure may be of coursed masonry, or of concrete mass, or of concrete reinforced by steel, the necessary condition of stability in each case being a continuous rock foundation.

§ (20) OUTLETS.—The outflow of the water under proper control was usually provided for by iron pipes, or stone or brick culverts carried through and under the earthwork of the embankment. Failures have frequently resulted from the leakage of water along the surface of these outlets, or from the fracture of the outlet caused by excessive local pressures or unequal settlement. The latest and best constructed reservoirs have outlets which are entirely disconnected from the embankment, and consist of tunnels formed of masonry, or reinforced concrete, placed in an adit which has been regularly mined, the line adopted being either round and clear of the end of the embankment or at a considerable depth beneath its lowest point.

For the regulation of the flow the entrance to the culvert or the tunnel is best commanded by a valve tower, which may be provided with sluices on the outside and contain in the interior an upstand pipe connected with the outlet main, and having valves at different levels, so that delivery may take place from near the surface, and therefore be as free from suspended matter as possible.

Syphon outlets are sometimes used where the depth of water does not exceed 27 or 28 feet. The discharge pipe may be carried up the inner slope over the top of the bank and down the outer slope, or it may be laid along the solid ground from the toe of the inner slope round the end of the whole work, as in the case of the tunnel outlet. Valves are necessary, both at the inner and outer ends of the pipe, the latter being at a level sufficiently low to ensure that flow will be maintained against the friction of the pipe. To guard against the accumulation of air at the summit of the pipe, valves suitable for the extracting of air and the charging the pipe with water must be provided.

§ (21) SETTLING TANK.—In order to allow for the settlement of sediment and the interception of floatage a "residuum lodge" or settling tank is formed at the entrance of the storage reservoir. Sluices or valves are provided, so that the discharge from the settling tank may be passed into the reservoir or diverted into a by-channel and thence passed into the waste watercourse. This latter alternative is adopted when it is considered inadvisable, either because of flood discoloration or similar reasons, to permit the water to pass into storage. A device which effects this automatically is the leaping weir. When the flow is normal the catchwater channel, situated just below and at right angles to the supply stream, collects it entirely and conveys it to the reservoir. When in flood, the velocity of flow becomes so great that the water misses the collecting channel and passes on to waste (*Fig.* 11).

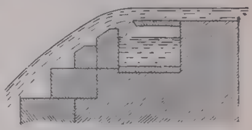
FIG. 11.

§ (22) WASTE WEIR.—Since the heaviest floods experienced often occur after a season of rain, when in all probability the reservoir is quite full, it is necessary to arrange for a waste weir to deal with a quantity of water equal at least to the entire flood of the drainage area. In earthen dams it should be kept distinct from the embankment and formed in a cutting in the solid ground at one of its extremities. The flood water carried off should be conveyed by the waste watercourse to the stream bed below in such a manner as not to injure the stability of the structure of the dam. Where the construction is in concrete or masonry the spillway may form part of the wall.

§ (23) FLOW IN PIPES AND OPEN CHANNELS. In the construction of a distributing system for a proposed supply the important factors to be determined include the volume of water to be passed, the minimum size of pipe or section of channel requisite for its conveyance, and also the conditions involving losses of head and the magnitude of the latter. The choice between pipes and a form of open conduit or tunnel will depend on whether the water is to be conveyed under pressure or not, and largely on the nature of the country to be traversed. Frequently in large works use is made of both methods.

§ (24) VELOCITY OF FLOW IN PIPES.—Many methods have been devised for the measurement of velocity and volume of flow in pipes. The most accurate is that of collecting the discharge during a definite time in a calibrated tank, but this is only suitable when the discharge is small.

(i.) *Chemical Method.*—For low-fall installations, where large quantities of water have to be dealt with, use is increasingly being made of the chemical or titration method, in which a given weight per minute of a chemical is introduced into the supply pipe and the quantity present at some point nearer the exit is afterwards determined and the rate of flow deduced. As recently developed this method is said to give results which are accurate to within 1·5 per cent.

There are also meters of the positive type, where, for example, the filling and emptying of a tank or cylinder is the special feature and the number of repetitions in a definite time are registered on an indicator. Or the volume of water passing through a small turbine may be inferred from the number of revolutions in a given time of its runner after careful calibration by means of the passage of known volumes.

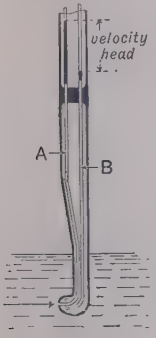

FIG. 12.

(ii.) *Pitot Tube.*—One of the simplest velocity indicators is the Pitot tube. Its essential feature is a vertical tube A (*Fig.* 12) of fine bore with the bottom end bent at right angles facing and open to the flow. The height of the column of liquid in the tube is the measure of the pressure equivalent to the velocity head added to the statical head of the tube water outside. To determine the statical head, the tube may be turned through an angle of 90° about its vertical axis so that its orifice would be parallel to the direction of flow. The illustration shows a second vertical tube B communicating at the bottom end with the casing, which in turn has openings arranged tangentially to the flow. This serves as a statical head indicator, and the difference in level at any instant between the two tubes serves as a measure of the velocity head, and therefore of the velocity itself (see also article "Aeronautics").

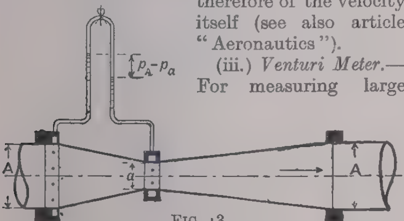

FIG. 13.

(iii.) *Venturi Meter.*—For measuring large quantities of water the simplest and most satisfactory method is that of the Venturi meter, shown in *Fig.* 13. It consists of a pipe of area A, uniformly converging to area a, then a short parallel neck afterwards diverging to its full diameter. It forms a section of the main pipe. The whole quantity of water passes through it and there is no obstruction to flow. The difference of pressure which exists at the inlet to the converging length and at the parallel neck, due to the accelerated velocity at the latter point, is measured as in the Pitot tube by a differential gauge connected to these points. Then it can be shown that the velocity is given by the equation

$$V_A = \sqrt{\frac{2g(p_A - p_a)}{W\{(A/a) - 1\}}} \text{ feet per sec.,}$$

where p_A and p_a are the pressures at the areas A and a respectively and W the weight of 1 cubic foot of the liquid, and the discharge in cubic feet per second by

$$Q = V_A \, . \, A = A\sqrt{\frac{2g(p_A - p_a)}{W\{(A/a) - 1\}}} = K\sqrt{\frac{p_A - p_a}{W}}.$$

The degree of error which may reasonably be expected is only about 1·5 per cent, and the meter registers efficiently at almost any velocity.

§ (25) PIPE LINE LOSSES.[1]—The losses which occur are due mainly to friction in the pipe and to friction and eddy formation at entrance and exit, at valves, sluices, elbows, bends, pipe junctions, and at sudden alterations in the cross-sectional area of the pipe. Much experimental work has been undertaken to determine the laws governing the loss of head resulting from resistance to the flow of water through a pipe, as effected by variations in the velocity of flow, the size of the pipes, and by the degree of smoothness of the internal surface.

(i.) *Early Experiments.*—From early experiments it was deduced that the frictional resistance to the flow of a fluid was:

(a) Independent of the fluid pressure per unit of area.

(b) Nearly proportional to the area of the wetted surface, whatever the form of the cross-section A of the containing channel, that is, to lP, where l is the length and P the wetted perimeter.

(c) Approximately proportional to the square of the velocity v of the fluid.

The frictional resistance F to the motion of a prism of water might therefore be considered equal to $cwlPv^2$; w being the weight of unit volume of the fluid and c a suitable constant.

If such a prism of weight equal to wlA moves under the action of gravity through a distance x along a channel having a gradient i given by the ratio h/l, the energy expended will be $wlAix$ ft.-lbs., where h is the vertical height between the two ends of the pipe.

This is equal to the work done against friction, and therefore

$$wlAix = Fx = cwlPv^2x.$$

Hence

$$i = c\frac{P}{A}v^2, \quad \therefore v = C\sqrt{\frac{Ai}{P}} = C\sqrt{mi},$$

where m is written for A/P and $C = (1/c)^{\frac{1}{2}}$. This is known as the Chezy formula.

The ratio A/P = (area of cross-section/wetted perimeter) is termed the "hydraulic mean depth" and is the depth the volume of water would have if it were contained in a rectangular channel with a flat bottom of the same area as the wetted walls of the channel. Thus the head h necessary to maintain a uniform flow v along a channel of length l and hydraulic mean depth m would be represented by

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[1] See article "Friction," § (14).

$$h=c\frac{v^2l}{m}.$$

The coefficients c or C, as used in the Chezy formula $v=\mathrm{C}\sqrt{mi}$ given above, can only be constant in the case of pipes if h is proportional (1) to v^2 for all values of the velocity and of the pipe diameter, whether large or small, and (2) to the wetted perimeter for pipes of different diameters, and is in addition independent of the roughness of the internal surface.

It is now known that none of these assumptions are warranted, though for a time the compensation of errors obscured the truth. Dubuat, Prony, D'Aubuisson, and others directed their attention to the true relationship between resistance and velocity, while D'Arcy investigated the effects of different diameters of pipes and of varying degrees of roughness, and the investigators in both directions sought to express their results in terms of a binomial function of v.

(ii.) *Reynolds' Index Law.*—Professor Osborne Reynolds evolved a rational formula based on the assumptions that the resistance to flow varied with the diameter, length, and surface conditions of the pipe; with the viscosity and density of the fluid, and with the mean velocity of flow through the pipe, and that it varied with some power of each of these factors. A modification of this rational formula has been adopted by various later investigators, including Unwin, and is represented by the relation

$$h=\frac{flv^n}{d^x},$$

where h as before is the drop of head in feet required to maintain a uniform velocity. The resistance is thus proportional to the nth power of v and inversely proportional to the xth power of the diameter d, while f is a coefficient deduced along with the values of n and x from the results of several experimenters,[1] l being in feet.

Unwin's mean values are as follows:

Surface.	f.	x.	n.
Wrought iron . . .	·0226	1·210	1·75
Asphalted pipes . .	·0254	1·127	1·85
Riveted wrought iron	·0260	1·390	1·87
New cast iron . . .	·0215	1·168	1·95
Cleaned cast iron . .	·0243	1·168	2·00
Old cast iron . . .	·0440	1·160	2·00

The values of f and n increase with the roughness of the surface, and x varies with the surface and increases with the diameter.

(iii.) *Practical Considerations.*—In dealing with practical problems, however, it is often convenient to express the relation between the velocity and the head in the form adopted by Chezy,

$$h=\frac{flv^2}{m2g}, \text{ or } v=\mathrm{C}\sqrt{\frac{mh}{l}},$$

where $f/2g$ has been written for the coefficient C, and Professor A. H. Gibson includes, in his "Hydraulics and its Applications," tables of the values of f and C calculated from the mean results of the formulae of Unwin, Tutton, and Thrupp, showing within the range of velocities common in practice their variation with v and with the pipe diameter in pipes of different types. The internal corrosion of a pipe, by increasing the roughness of its walls and by reducing its effective area, will usually increase the value of f considerably after a few years' use, and must be allowed for when estimating the diameter necessary to maintain a given discharge.

(iv.) *Loss at Entrance.*—This will depend on the form of entrance adopted, the loss of head varying from about ·05($v^2/2g$) ft. with a bell-mouth to $v^2/2g$ ft., where the pipe projects into the reservoir and forms a re-entrant mouthpiece.

Weisbach has shown that with a gate valve in a circular pipe the loss in head due to the presence of the valve when three-fourths open would be represented by ·26($v^2/2g$) ft., and at half opening be eight times as great.

(v.) *Bends and Elbows.*—If loss of head is represented by F($v^2/2g$) the following values of F are approximately correct for losses in pipes of radius r, with bends of radius R, making an angle of 90°:

$\frac{\mathrm{R}}{r}$	Elbow	4	8	12	16	20
F	1·20	·40	·29	·36	·29	·22

(vi.) *Loss at Exit.*—The whole of the kinetic energy of pipe flow $v^2/2g$ is usually dissipated in eddy formation when discharged into another mass of water, and if discharge takes place at a height h above the free receiving surface this height will be included in the loss of head.

(vii.) *Total Loss of Head.*—The total head H, therefore required to maintain a uniform velocity of flow v will be equal to the kinetic energy at exit + the loss by friction in the pipe + the losses at entrance valves, bends, or sudden changes in section of the pipe, and will be represented by an equation such as

$$\mathrm{H}=\frac{v^2}{2g}\left(1+\frac{fl}{m}+\Sigma\mathrm{F}\right).$$

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[1] See "Friction," § (14) iii.

Hence, $$v^2=\frac{2gH}{1+\Sigma F+(fl/m)}.$$

We obtain thus the velocity due to a given difference of head in a pipe of given length and diameter. Since the discharge in cubic feet per second $Q=\text{area}\times v=(\pi d^2/4)v$,

we find $$Q=\frac{\pi d^2}{4}\,\frac{\sqrt{2gH}}{\sqrt{1+\Sigma F+(fl/m)}}.$$

If the length and diameter of the pipe are known, the head necessary to give any required discharge may be determined. Thus

$$H=\frac{8Q^2\{1+\Sigma F+(fl/m)\}}{\pi^2 d^4 g}.$$

§ (26) HYDRAULIC GRADIENT.—If steady flow is to be maintained between two points at different levels through a pipe running full with a velocity v, the total difference of level must be equal to the total loss of head. Thus $\Sigma F(v^2/2g)=H$.

If a longitudinal section be made along the line of the pipe and, if from a horizontal line drawn through the upper surface there is set down a series of vertical ordinates representing at various cross-sections the total loss of pressure per unit volume from the pipe entrance to each section considered, then the curve formed by joining the ends of these ordinates defines the hydraulic gradient for the pipe line. For a straight length of pipe without valves or other obstructions the gradient would be constant and would be represented by a straight line (*Fig.* 14). The slope of the curve giving the hydraulic gradient is termed the virtual slope of the pipe.

§ (27) OPEN CHANNELS.—The formula $h=(flv^2/m2g)$ is generally adopted as a basis for determining the loss of head for a non-accelerated flow in open channels, where $m=(A/P)=$the ratio of the cross-sectional area to the wetted perimeter.

If the slope of the channel be uniform then h/l is a constant, which we denote by i, and to maintain constant flow we must have

$$v=\sqrt{\frac{2g}{f}\frac{h}{l}m}=C\sqrt{mi}.$$

The difficulty has been to determine values of f or C which will be applicable to channels having widely differing physical characteristics.

Bazin deduced for C a value given by $157{\cdot}6/1+(N/\sqrt{m})$ foot units, where N is a quantity which varies with the character of the surface. This gives good results for channels under 20 feet wide and with velocities not greater than 4 feet per second.

The value generally used is that derived from the Ganguillet and Kutter formula:

$$C=\frac{41{\cdot}6+({\cdot}00281/i)+(1{\cdot}8112/N)}{1+\{41{\cdot}6+({\cdot}00281/i)\}(N/\sqrt{m})}\ \text{foot units.}$$

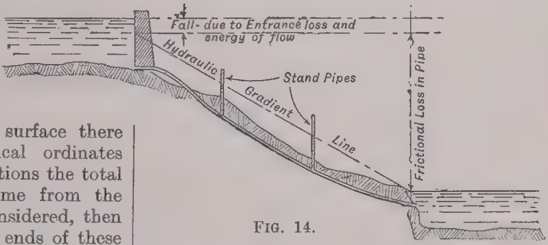

FIG. 14.

N, again, depends on the character of the surface. Probably a rational formula of the type

$$h=\frac{flv^n}{\{A/P\}^x}=\frac{flv^n}{m^x}$$

would most nearly represent the law of channel flow.

Amongst others, Professor Claxton Fidler determined the values of f, n, and x for such a formula from the many experimental results available, and a few examples are given below:

Form of Section.	Material of Surface.	n.	x.	f.
Circular	Smooth neat cement	1·75	1·167	·0000676
Rectangular . . .	Smooth neat cement	1·75	1·167	·0000787
Circular . . .	Cement and sand	1·75	1·167	·0000787
	Smooth brick	1·75	1·167	·0000787
Rectangular . . .	Smooth ashlar	1·75	1·167	·0000904
Circular . . .	Bare metal pipes with riveted joints	1·77	1·18	·0000871
	Rough brickwork	1·80	1·20	·0000977
Rectangular . . .	Rough brickwork or ashlar	1·80	1·20	·0001122
Circular	Lined with fine gravel	1·90	1·33	·0001202
Rectangular . .	Lined with fine gravel	1·96	1·40	·0001521
	Rubble masonry	2·10	1·50	·0002240

§ (28) BEST DIMENSIONS OF CHANNEL.—The best form and dimensions of channel are those which will give the maximum discharge for a given slope and given cross-sectional area. Now $Q = Av = C\sqrt{(A^3/P)i}$. On the assumption that C is constant for a given surface, in order that Q should be a maximum we must find the differential coefficient of this expression and equate it to zero. The following sections give the proportions thus obtained for a few of the common forms of channel and the resulting value of Q on the assumption that A has a constant value.

Trapezoidal Channels.—Let b = the half bottom breadth, d = the depth, $s = \cot\theta$, where θ is the angle of slope of the sides.

Then we have

$$A = 2bd + sd^2, \quad P = 2(b + d\sqrt{1+s^2}),$$

and remembering that A is constant we obtain, on substituting in the value Q from these equations and differentiating, as the condition for maximum discharge, the result that

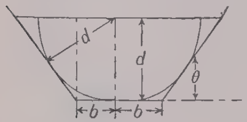

FIG. 15.

$$(1+s^2)d^2 = (b+sd)^2,$$

and in this case the sides of the channel touch a circle of radius equal to the depth of the channel having its centre in the surface.

The maximum discharge in terms of b and s becomes

$$Q = \frac{Cb^{\frac{3}{2}}(2\sqrt{1+s^2} - s)}{\sqrt{2}(\sqrt{1+s^2} - s)^{\frac{5}{2}}}\sqrt{i},$$

and for a rectangular section where s is 0

$$Q = C\sqrt{2i}\,b^{\frac{5}{2}}.$$

Semicircular Channel.—The hydraulic mean depth of any polygonal channel where A is constant is greatest when the sides and bottom of the channel are tangential to a circle having its centre in the water surface. The semicircular section when running full may be regarded as the limiting case, and has a hydraulic mean depth of $d/4$, which is the maximum for properly designed polygonal forms, and therefore gives the maximum discharge for a fixed area.

Circular Sections.—With a circular channel of fixed diameter, but where the water-level subtends a varying angle θ at the centre, the condition of *maximum velocity* can be shown to be $\theta = \tan\theta = 257{\cdot}5°$, and for *maximum discharge* $2\theta - 3\theta\cos\theta + \sin\theta = 0$. $\theta = 308°$ satisfies this latter condition.

Tables showing the relative discharges of trapezoidal channels with varying slopes, and circular channels with different flow levels, are given below:

Slope of Sides.	θ.	Wetted Perimeter. P.	Area. A.	Hydraulic Mean Depth. $A \div P = m$.	Velocity. $v \propto \sqrt{m}$.	Discharge. $Q = A \cdot v \propto A\sqrt{m}$.
			Trapezoidal			
1 to 0	90°	$4d$	$2d^2$	$d/2$	$\sqrt{d/2}$	$2d^2\sqrt{d/2}$
1 in ·5	63	$3{\cdot}47d$	$1{\cdot}74d^2$	,,	,,	$1{\cdot}74d^2$,,
1 in 1	45	$3{\cdot}66d$	$1{\cdot}83d^2$	,,	,,	$1{\cdot}83d^2$,,
1 in 1·5	34	$4{\cdot}21d$	$2{\cdot}11d^2$	,,	,,	$2{\cdot}11d^2$,,
1 in 2	26·5	$4{\cdot}94d$	$2{\cdot}47d^2$	,,	,,	$2{\cdot}47d^2$,,
Depth in terms of $2r$.			*Circular*			
·1	74	$1{\cdot}286r$	$\cdot163r^2$	$\cdot127r$	$\cdot504\sqrt{r/2}$	$\cdot082r^2\sqrt{r/2}$
·3	133	$2{\cdot}318r$	$\cdot792r^2$	$\cdot342r$	·828 ,,	$\cdot656r^2$,,
·5	180	$3{\cdot}141r$	$1{\cdot}571r^2$	$500r$	1·00 ,,	$1{\cdot}57r^2$,,
·81	256	$4{\cdot}480r$	$2{\cdot}726r^2$	$\cdot608r$	1·10 ,,	$3{\cdot}00r^2$,,
·95	308	$5{\cdot}382r$	$3{\cdot}083r^2$	$\cdot572r$	1·07 ,,	$3{\cdot}304r^2$,,
1·00	360	$6{\cdot}28r$	$3{\cdot}14r^2$	$\cdot500r$	1·00 ,,	$3{\cdot}14r^2$,,

§ (29) UTILISATION OF SUPPLY.—The supply of water which has been determined by the methods described and brought under control may either be utilised for the potential energy it contains or distributed for various domestic and commercial purposes. The potential energy of any store of water is measured by the product of the fall H (in feet) which it would make in its descent from a higher to a lower level, multiplied by the weight of water which may be utilised per unit time. If W be the weight of water delivered per second the theoretical energy per second $= WH$ ft.-lbs. and is equivalent to a theoretical horse-power of $(1/550)WH$.

On the basis of a mean annual rainfall of 36 inches, and an average available fall of 50 feet, 240,000 million foot-pounds of energy would be available per annum per square mile of gathering ground. If it was possible to collect and utilise half of this energy throughout a normal working year at an 80 per cent efficiency it would represent a total of 16 h.p. per square mile. Professor A. H.

Gibson estimates the water-power available in the world as exceeding 200 million horse-power, while the amount available and developed in America and Britain has been given as follows:

Brake Horse-power

	Available.	Developed.	Per cent Developed.
	Million.	Million.	
United States .	28·1	7	24·9
Canada (North).	18·8	1·7	9·2
Canada (South).	8·1	1·7	21·3
Great Britain .	·96	·08	8·3
Europe . . .	34·5	6·0	17·4

These numbers representing the available power can only be regarded as very approximate, and so far as they relate to this country include much of the supply necessary for domestic purposes, or which must be conserved for trade and commercial purposes. The daily rate of consumption under this heading varies from about 20 gallons per head of population in Sheffield to 70 gallons in Paisley, and provision requires to be made so that, in the event of the emergency of fire, the supply per hour may exceed this by as much as 50 per cent.

The total available water supply is for commercial and domestic purposes capable of being supplemented by the use of wells and underground reservoirs, which, however, necessitate the use of pumps, and do not add directly to the power resources of the country.

The following are mentioned as a few typical examples of the largest power installations which have recently been made:

Undertaking.	Country.	Working Head (ft.).	No. of Units.	Total Output. B.H.P.
Trollhattan Station	Sweden	106	8	100,000
Keobuk Power Station, Mississippi .	U.S.A.	39	15	210,000
Cedar Rapids Station, Montreal . .	Canada	30	12	129,600
Aura Plant	Norway	2350	6	141,000
Tata Plant, Bombay	India	1650	6	81,000
British Aluminium Co., Kinlochleven .	Scotland	900	11	36,300

§ (30) Tidal Power.—Perhaps the greatest source of water-power is to be found in the daily action of the tides of the ocean. Twice each day, due to the gravitational attraction of the sun and moon, great masses of water become possessed with an energy of position which it is possible to convert into work. While the practicability of such a project is assured, the desirability and expediency becomes entirely a matter of cost. The choice of methods lies between the use of a single tidal basin divided from the sea by a dam in which are placed turbines, having intermittent periods of working separated by more or less lengthy intervals of idleness and equipped with suitable storage plant, and the adoption of a system of duplicate tidal basins capable of working at all states of the tide and therefore without storage plant. There are several options with either method.

(i.) *Single Tidal Basin.*—When a single tidal basin is used water may be impounded through sluices during the rising tide and, when the sea-level has fallen a portion of its range, allowed to operate the turbines at a nearly constant head until low water. If the curve ABC (*Fig.* 16) represents the sea-level on a time base, then *ab* shows the level in the tidal basin, on the supposition that the rate of fall is kept constant and the head is nearly equal to *b*B throughout. The period of working is represented by a_1b_1 and the interval of idleness by the tide period A_1C_1, less a_1b_1. An increase in the working head can only be obtained by means of shortening the period of operation, and the maximum output will be obtained when the head is approximately half the tidal range. An increase in output may, however, be made by utilising both rising and falling tides, the arrangement of the water passages permitting the use of the turbines with a flow from either side of the wall in which they are set, or duplicate inflow turbines are provided.

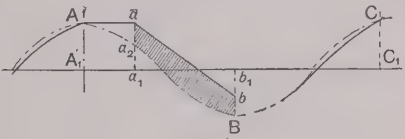

FIG. 16.

The work done per complete tide would then be approximately 50 per cent greater, the idle period shorter, and the storage plant necessary would be correspondingly reduced.

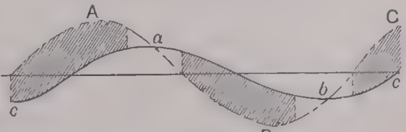

FIG. 17.

Another possible modification is where the water during both rising and falling tides is allowed to flow through the turbines and to adjust its own level. If ABC (*Fig.* 17) repre.

sents the sea-level, the level in the tidal basin would also show a cyclical variation as at *abc*, the working head being the intercept between the two curves. At their intersection points *ab* the head is zero, and for an interval before and after these points the turbines will cease to function. The working period and the possible output per tide is greater with this arrangement, but the variation of head, with all its disadvantages, is very large.

(ii.) *Two Tidal Basins.*—When the installation consists of two tidal basins, power may be developed continuously by allowing flow through turbines to the sea to take place from one called the upper basin during the lower portion of the tidal fall, while the other or lower basin is emptying through its sluices. Flow from the sea through the turbines to the latter occurs during the upper portion of the tidal rise, and meanwhile the upper basin is being filled through the sluices. This is shown diagrammatically in *Fig.* 18. In addition to the cost of providing two tidal basins this method necessitates duplicating turbines, while the output is not larger than in the best of the single-basin systems.

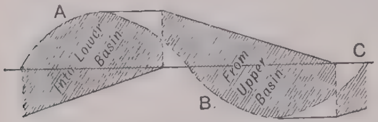

FIG. 18.

Whichever form is adopted the cost per horse-power will vary, other things being equal, with the tidal range. Because of this, and also because of its favourable configuration, much attention has been given to the estuary of the Severn, the mean range of spring tides at Chepstow having the high value of 42 feet and of neap tides of 21 feet, as compared with the average value around the coast of Great Britain of 16·4 and 8·6 feet respectively.

It is estimated that if an area of 20 square miles could be utilised at the spring tidal range of the Severn the average daily output working without storage plant would approximate to 10 million horse-power-hours.

The principal difficulty, however, in connection with any tidal-power scheme lies in the relatively great fluctuations in head. The cyclical daily variations may be provided for and continuous operations ensured, but the great relative differences between spring and neap tides force the choice of either designing for the minimum head and thus utilising only a small proportion of the available energy, with a consequent increase of the power unit cost, or of adopting some equally costly form of storage. While this is so the vast possibilities which await a development of tidal power at a reasonable cost makes it imperative that the investigations at present in progress should be continued until success is achieved.

II. Availability of Water Supplies for Power or other Purposes

§ (31) Pumps.—In order to utilise directly a supply of water there must be in addition to an adequate quantity an available head capable of being converted into power or used in overcoming resistance during its distribution for domestic, trade, or irrigation purposes. When the supply does not possess the requisite head it can be acquired by the raising of the water from a lower to a higher level by means of a properly designed machine, and is termed Pumping. The power of the machine or pump must be sufficient not only to raise the required amount in a given time but also to overcome the various resistances to flow encountered in the process. These consist principally of the friction of the pump and of the inlet and delivery pipes. The nature and magnitude of the pipe line losses have already been dealt with in discussing the loss of head incidental to a gravitational supply. The characteristics, losses, and efficiency of the different types of pumps will now be treated in turn.

§ (32) Scoop Wheels, supposed to have been used by the Chinese in very remote times, are still in operation in the Fen district and also in the lowlands of Holland, where large volumes of water have to be lifted to comparatively low heads of 4 to 6 feet. In some of the recent examples they have been capable of dealing with 300 to 400 tons per minute against a head of 5 to 6 feet, the diameter of the wheel being 30 feet by 5 feet wide, and the number of revolutions 5 per minute. The entrance of the water to the wheel is controlled by a sluice gate which delays the contact of wheel and water until the vanes have nearly reached the bottom of their path. The tail sluice may either be adjustable about a horizontal axis at its lower end or free to take up a natural position as determined by the discharging stream. The efficiency of the plant including the drive varies from 65 to 75 per cent, depending upon the ratio of the wheel diameter to the lift. Generally speaking, the larger the diameter for a given lift H the more efficient is the pump, and the usual proportion is that the diameter is $10\sqrt{H}$.

§ (33) Archimedean Screw.—It is fitting in the case of this obsolete type that as there is a record of its use in Egypt before Hero's time, so the largest and possibly the last installation of importance should be that at Katatbah in 1881. It consisted of ten sets of screws each designed to raise 25 tons of water per revolution against a total head of 12 feet, which, with a speed of 6 revolutions per minute, represented

120 h.p. per screw. The plant was not a success and was soon replaced by another type. In the designed arrangement a helical screw rotated about an axis inclined to the horizontal at an angle less than that made by the surface of the helix, and was contained in a closely fitting tube, the lower end being immersed in water. On rotating the screw, the water tended to run down the surface of the helix by reason of its greater inclination and consequently passed up the tube emerging at the top in the head water. Under favourable circumstances an efficiency of 75 per cent has been reached.

§ (34) RECIPROCATING PUMPS.—This important class of pump consists essentially of a cylinder in which the reciprocating motion of a bucket, plunger, or piston, or a combination of these, is used to lift water directly, by the action of suction or of pressure. On the up or suction stroke of the pump a partial vacuum is formed beneath the plunger or piston, and atmospheric pressure acting on the free surface of the supply produces flow in the suction pipe. A volume of water equal to the displacement of the plunger is admitted to the pump chamber through a foot suction valve which on the down stroke closes automatically while an equal volume is transferred to the pressure side of the delivery valve. Where the arrangement includes both bucket B and plunger *p*, as in *Fig.* 19, the water displaced on the down stroke passes through lift valves *v* in bucket, and a volume equal to the displacement volume of the plunger is discharged through D. On the up stroke a discharge equal to the difference of the bucket and plunger displacement volumes takes place.

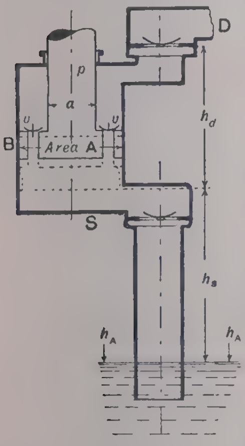

FIG. 19.

When the pump is of the type shown in *Fig.* 20, having a piston and plunger, then during the in stroke a volume of water equal to the volume of the piston displacement is passed through the discharge valve D, but a portion of this is simply transferred to the opposite side of the piston. The actual discharge during this stroke is only equal to the plunger displacement volume. On the out stroke the delivery is equal to the difference between the volumes of the piston and plunger displacements. In each case, therefore, the discharge is divided over the two strokes.

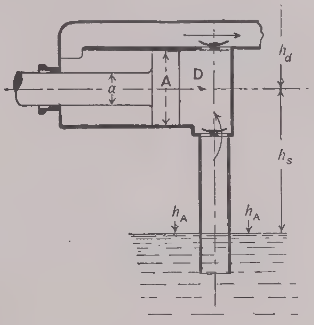

FIG. 20.

Let h_s = head of lowest position of plunger above the lower water surface.

h_d = head of upper water surface above lowest position of plunger.

h_A = atmospheric pressure head.

h_f = head required to overcome friction of suction pipe.

h_a = head required to produce acceleration of suction water column.

The pressure head available to produce flow in the suction column $= h_A - h_s$ and must be not less than $h_a + h_f$.

If the flow is small $h_a + h_f$ will nearly equal 0, and h_s may nearly equal h_A or 34 feet approximately. In practice $h_A - h_s$ represents a head of about 10 feet, and h_s is not greater than 24 feet.

If A = area of cross-section of bucket or plunger in square feet.

L = length of stroke.

W = weight of cubic foot of water in lbs.

Then WAh_s = weight of water displaced during suction stroke through a distance L feet.

WAh_d = weight of water lifted during delivery strokes through a distance of L feet.

Therefore the total work done per cycle

$= WA(h_s + h_d)L$ foot-lbs. $= WALh_L$ foot-lbs.,

where $h_L = h_s + h_d$ = total lift.

With the bucket type pump, suction and delivery takes place during the up stroke, the return stroke being idle. When a plunger is used suction takes place during up stroke and delivery during return. In either case delivery is intermittent and occurs during alternate strokes. Where a bucket pump is converted into the combined bucket-and-plunger type by the enlargement of the bucket rod the work done per cycle remains the same, but a delivery, equal to the difference of the volumes swept through by bucket and plunger respectively, occurs during the suction stroke, while the remainder of the bucket volume is discharged during the down stroke. For equal discharges on the two strokes the area A of the piston or bucket should be twice the area a of the rod or plunger.

To equalise the work done during the two strokes of the cycle, when the pump is vertical and the weight of the reciprocating parts is W_p, the following relations are necessary:

$$W\{(A-a)h_d+Ah_s\}+W_p=Wah_d-W_p$$

and
$$a=\frac{W_p}{Wh_d}+\frac{A(h_d+h_s)}{2h_d}.$$

§ (35) SINGLE- AND DOUBLE-ACTING PUMPS.—Where suction takes place during one stroke only of the cycle the pump is said to be single-acting, though the flow is continuous. This type may be converted into the double-acting type by the duplication of suction and delivery valves, so that suction takes place during both in and out strokes of the piston. If the pressure head against which delivery is taking place is high or the water being pumped contains considerable impurities rendering accessibility to packing desirable, the plunger type of pump is usually adopted; and if the pump is double-acting, outside in preference to central packing is employed. This is shown in *Fig.* 21.

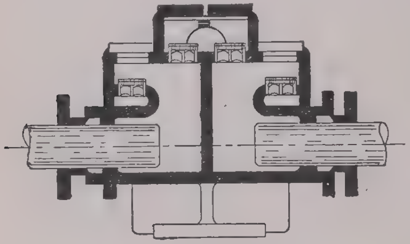

FIG. 21.

(i.) *Variations in Discharge.*—In pump design it is intended that the volume of water dealt with either in suction or in delivery should just equal the volume of the plunger displacement, and it follows that with passages of uniform section the rate of flow will depend on and vary with the velocity of the plunger.

A, B, C, *Fig.* 22, shows on a time base the velocity curve of a plunger driven by an ordinary crank of definite radius, during the suction stroke, and will also represent to some scale the varying velocity of the suction water. C, D, E is the corresponding velocity curve during the delivery stroke, and will therefore be a measure of the discharge velocity and for short intervals of time of the discharge itself.

If the pump be a single cylinder, single-acting machine, the suction velocity will vary from zero at A to a maximum at B, falling again to zero at C and remaining so during the return stroke. On the delivery side there is no velocity during the period from A to C, after which it begins to rise, reaching a maximum at D and thence falling to zero at E.

These fluctuations of velocity may be modified by the introduction of a duplicate pump, drawing from the same suction and discharging to the same delivery pipes, but driven by a crank set at an angle of 90° with the first, the resulting velocities being shown by the curve A_1FBGC_1HD.

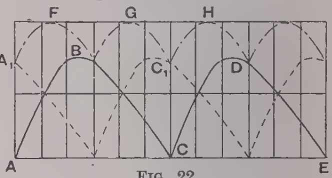

FIG. 22.

(ii.) *Variations of Pressure.*—Since the flow in the suction pipe is dependent on the varying velocity of the plunger, the water in that pipe must be subject to fluctuations of pressure following on the varying accelerations necessary to maintain contact. These accelerations in the flow will be proportional to the piston acceleration a at the same instant. When the pump is driven from a shaft rotating with uniform angular velocity of ω radians per second, by a crank of radius r through a connecting rod of length l, the maximum values of the acceleration, which are equal[1] to $\omega^2 . r(1\pm r/l)$, occur when the piston is at its inner or outer extreme travel positions, that is at the opening of the suction and delivery valves. There must be sufficient force available at these points to produce the required acceleration if flow is to take place and separation of the water column and piston is to be avoided. In addition to the accelerating head h_a there is the force necessary to overcome friction represented by a head h_f. In *Fig.* 23 (a) the

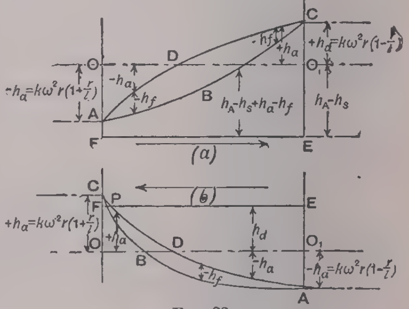
C
$-h_f$
$+h_a$
$+h_d=k\omega^2r(1-\frac{r}{l})$
D
O
O_1
$-h_a=k\omega^2r(1+\frac{r}{l})$
$-h_a$
$-h_f$
B
$h_A-h_s+h_a-h_f$
h_A-h_s
A
F
E
(a)
C
(b)
P
F
E
$+h_a=k\omega^2r(1+\frac{r}{l})$
h_d
$+h_a$
O
D
O_1
B
$-h_a$
$-h_f$
$-h_a=k\omega^2r(1-\frac{r}{l})$
A

FIG. 23.

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[1] See "Kinematics of Machinery," § (3) (ii.).

curve ADC measured from OO_1 as base shows how h_a varies, and ABC shows how the sum of h_a+h_f varies from beginning to end of the suction stroke. The mean pressure head (h_A-h_s) available to balance these resisting forces is represented by FE, and the intercepts between ABC and FE are a measure of the pressure head remaining on the plunger during the suction stroke. If at any instant h_A-h_s is less than h_a+h_f, separation and knocking will ensue.

The corresponding diagram for the delivery side is shown in *Fig.* 23 (*b*). The acceleration h_a is represented by ADC and becomes positive during the latter portion of the stroke. It is then diminished by h_f, the resulting curve being ABC, and the resisting head is then equal to $h_d-h_a+h_f$. If, therefore, at any instant $h_d<h_a-h_f$ separation would result.

If separation does occur, impact between the water column and the plunger will subsequently take place and pressures many times greater than the normal values will be registered. This would happen during the earlier portion of the suction stroke, and if the pressure head resulting be equal to the delivery head, the delivery valve will open and discharge take place directly from the suction to the delivery pipe. It may also occur during the latter portion of the delivery stroke and the oscillations set up may cause the pressure head in the pump cylinder to fall sufficiently to open the suction valve so that direct discharge again results. In either case the discharge would be greater than the plunger displacement and its coefficient be greater than 1. Even though separation does not occur, a rise of pressure which is of the nature of water-hammer takes place at the closing of the valves owing to the elasticity of the water column and the consequent difference of the velocities of the two ends of the column.

(iii.) *Air Vessels.*—The introduction of air vessels on the suction and delivery sections is a means adopted for minimising abnormal pressures. During the first portion of the suction stroke, when normally the pressure behind the plunger is reduced, water flows out of the suction air vessel and the flow through the suction pipe is diminished and the acceleration and the frictional resistances are accordingly also reduced. The pressure behind the plunger available for maintaining contact is thus higher on account of the diminished resistance and acceleration, and the tendency to separate is decreased. During the latter portion of the stroke when the plunger is being retarded the rise in pressure of the water is damped by the surplus flow being absorbed into the air vessel. Similarly during the first portion of the delivery stroke the delivery column is allowed to acquire motion gradually by the flow from the pump chamber being passed in the first instance into the air vessel, and when retardation of the plunger takes place and the delivery column tends to lose contact with it, discharge is supplied from the air vessel.

(iv.) *Air-charging Device.*—In addition to the proper proportioning of the air vessels arrangements must be made for their being kept adequately charged. The volume of air in the delivery air chamber tends to diminish by being absorbed into the continually changing body of water with which it comes into contact, while the opposite holds good in the suction air chamber. In order to equalise these changes some form of automatic air pump is necessary, and a simple apparatus known as the Wipperman air charger is frequently fitted. It is illustrated in *Fig.* 24 and consists of a small chamber H connected by a delivery valve *d* to the air vessel A and by a screw-down valve *k*, normally open, to the pump chamber, while a suction valve *a* gives direct opening to the air. During the outward stroke of the plunger the pressure in H falls to the suction pressure of the pump and the valve *a* admits additional air. On the return stroke valve *a* closes and the delivery valve *d* opens, admitting a fresh charge to the air vessel, where a relief valve prevents overcharging. If necessary, the air inlet may be connected to the suction air vessel, where surplus air tends to collect, instead of to the atmosphere, and is the usual method when an air pump is used.

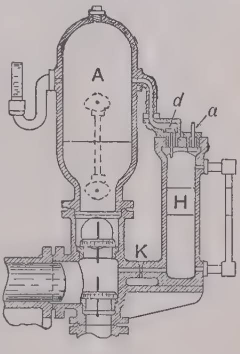


FIG. 24.

(v.) *Pump Valves.*—The operations of suction and delivery, as they have been described, have implied that the control was by means of automatic valves. It has been assumed that during suction the inlet control was by a valve which opened as a consequence of the displacement of the plunger in one direction and closed immediately on the motion being reversed; also that the outlet control was similar but opposite in action, being closed during the suction stroke by the external pressure of the discharge water and opened during the succeeding stroke by the excess of the pressure in the pump chamber over the pressure head of the delivery column. For low pressures and slow speeds the procedure just described is that which is usually adopted. The valves are made of rubber or vulcanite discs working against a perforated grid. They are returned to their seat by the action of a spring or by reason of their own elasticity aided by the pressure of water above them and are shown in *Figs.* 25 and 26. In order to withstand the high pressures which have now become common, metallic valves were introduced. Examples of these are the single mitre type shown in *Fig.* 27 and the double-seated ring valve in *Fig.* 28. The average number of revolutions for pumps using these types of valves is approximately 60 per minute.

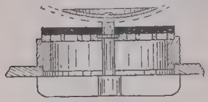


FIG. 25.

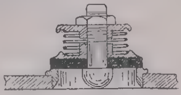


FIG. 26.

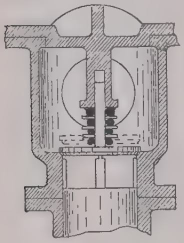


FIG. 27.

The closing of valves is accompanied by a certain amount of shock, the violence of which depends on the kinetic energy stored in the valve and in the following mass of water at the instant of closing. This will vary as the lift, as the weight, and as the velocity of the valve. All three are functions of the diameter, and the lift to be fully effective should, in the case of a ring single-seated valve, be equal to one-quarter of the diameter. By an increase in the number of valves, the diameter, and therefore the lift, the weight, and the velocity may be reduced. Suitable arrangements are illustrated in *Fig.* 29, where (*a*) shows a quadruple-seated suction ring-valve and (*b*) a three-tier Beehive multiple-valve box.

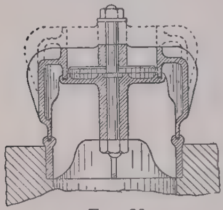


FIG. 28.

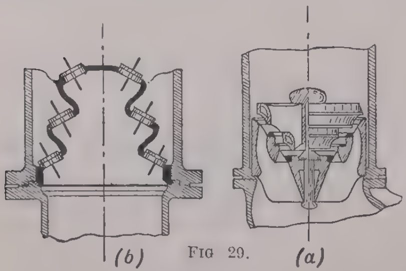


FIG 29.

Where high speeds are necessary for obtaining the required discharge special valves are used which, by their construction, tend to modify the shock resulting from sudden changes in the direction of flow at the valves. Among the most successful of these are the Haste and Gutermuth types. The former is a conical composition valve working on a guide stem carried by the valve box, and opening automatically to afford a free passage of the moving column of water to the discharge end of the pump, and is shown in *Fig.* 30A. The Gutermuth metallic flap, *Fig.* 30B, is formed from a long strip of sheet metal, a portion being coiled several times round a spindle, with its inner end held in a slot. The tension of the flap is adjusted by altering the position of the slot. This form of valve has considerable advantages over the ordinary ring type by reason of its lightness and elasticity, being very sensitive and affording a free waterway without any abrupt changes of direction. A speed of 200 revolutions has been reached by pumps using this type.

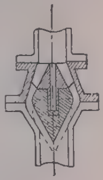


FIG. 30A.

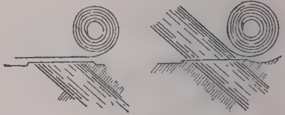


FIG. 30B.

For still higher speeds mechanically operated valves are necessary, and *Fig.* 31 shows one unit of a Reidler Express pump, where the valves are worked from a wrist-plate driven by an eccentric on the main shaft.

The advantages of high speed are not confined to the reduction in the weight and size of the pump when compared to the slow-speed class, but extend to the discharge, where, with very high lifts, it is of the utmost importance to preserve the constant velocity of outflow which follows from the greater number of revolutions.

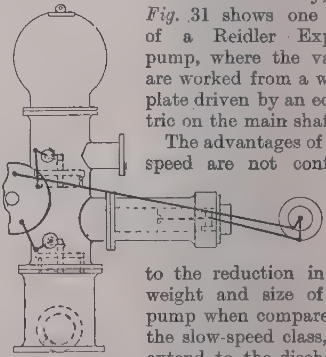
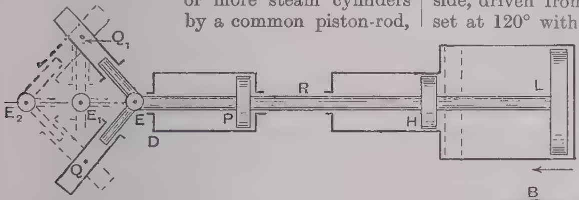


FIG. 31.

(vi.) *Efficiency.*—With reciprocating pumps of the latest types of construction, efficiencies varying from 80 to 86 per cent may easily be obtained, and when the speed of working is low a well-designed pump may have an efficiency of 90 per cent.

§ (36) SPEED VARIATIONS.—The method of equalising the variations in the resistance of the pump has already been dealt with. There are, in addition, variations in the external forces actuating the pump which require consideration.

(i.) *Direct-driven.*—When the pump is steam-driven it may be directly connected to one or more steam cylinders by a common piston-rod, so that the total steam pressure at any instant is directly transmitted to the water. As it is desirable for the sake of efficiency to use the steam expansively, and therefore with varying pressure, the pressure in the pump will have a considerable range. On the other hand, the resistance of the pump is approximately constant, and so it becomes necessary to introduce some form of compensator whereby sufficient energy is stored during the first portion of the stroke to supply the deficiency which exists in the second. In *Fig.* 32 let the ordinates measured from OO_1 to the curve ABCD be the total resultant pressures on the driving rod during the outward stroke, and ACF the more or less constant water cylinder pressures: then the shaded area included above AC represents the excess of energy requiring to be stored, and the area included beneath CF the deficiency which must be met in the stroke. The best-known method of doing so is by the use of the Worthington oscillating cylinders, a diagrammatic arrangement of which is shown in the upper portion of the figure. H and L indicate the high and low pressure steam pistons actuating the pump plunger P through the rod R. To the crosshead E on the extension of R are attached rods which operate plungers in the pair of oscillating cylinders placed symmetrically about the centre line at Q and Q_1. While the crosshead is moving from E to E_1 the plungers are displacing water from their respective chambers into a differential accumulator communicating with an air vessel, and work is stored. In travelling from E_1 to E_2 the outward displacement of the plungers is assisted by the pressure of the accumulator, and work is given out. The work stored and given out during each stroke in the equalising cylinders is adjusted to balance the variation in energy above and below the mean required in the pump chamber.



(ii.) *Flywheel and Shaft driven.*—Another class of pumps receive their motion by means of a crank from a shaft on which is mounted a heavy flywheel, and which is operated directly by a steam engine or driven by belt or gearing from an electric motor. Here the variations of speed and pressure in the prime mover are equalised by the action of a properly designed flywheel. Such pumps are usually constructed with three pump chambers side by side, driven from a common shaft by cranks set at 120° with each other, and are termed three-throw ram plunger pumps. The flow in this class is very continuous, and the type is largely used for boiler feed purposes.

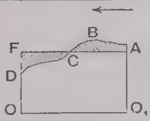


FIG. 32.

§ (37) ROTARY PUMPS.—A pump of the rotary class is valuable for use where lack of space prevents the adoption of an ordinary plunger pump. It is valveless, steady in working, and its discharge is practically continuous. It is adapted for working over a large range of speeds at comparatively low heads, with widely varying discharge, and is frequently adopted for irrigation purposes. Its principal disadvantage lies in the difficulty of avoiding leakage past the rotating surfaces and the consequent loss of efficiency. This type takes various forms. One consists of a two-part cylindrical casing in which revolve a piston wheel and drum; another employs two piston wheels.

The latter is illustrated in *Fig.* 33 by the cross-section of a pump with cycloidal wheels, capable of lifting 27,000 gallons of water per minute to a height of over 30 feet for irrigating rice-fields.

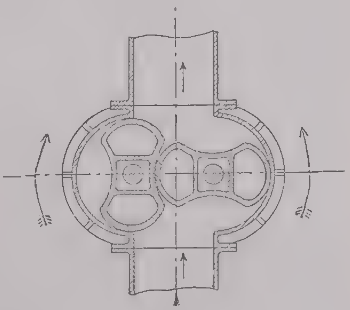


FIG. 33.

The right-hand piston rotates counter clockwise and drives the water from the lower to the upper part of the casing; the motion of the left-hand piston is clockwise with the same result.

An example of the former is the Enke pump, shown in *Fig.* 34, the important feature being the non-contact method of arranging the piston wheel or displacer and the controller drum to avoid leakage and to minimise wear. The displacer consists of three arms F machined to fit accurately between a segment of the casing and a fixed drum D. It is carried at one end by a disc keyed to the driving shaft, and at the other the arms are connected by an annular disc which revolves within a recess in the cover. As the displacer revolves the arms fit into recesses in the periphery of the controller drum C, causing it to turn in another segment of the casing. No attempt is made to secure fitted contact between the tips of the displacer arms and the corresponding recesses, as leakage back of the water is sufficiently prevented without it. An over-all mechanical efficiency of 80 per cent for engine and pump has been obtained with this particular form of rotary pump.

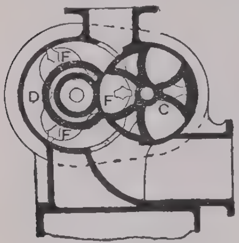


FIG. 34.

§ (38) CENTRIFUGAL PUMPS.—In a centrifugal pump, pressure energy is imparted to a mass of water by the rotation of an impeller wheel. The wheel is formed of a number of curved vanes, and revolves in a suitable casing. When the wheel is charged with water, its rotation produces a forced vortex in the mass of the water contained, with a resulting increase of pressure in a radial direction outward and a tendency to outward flow. If the speed of rotation is sufficiently high the increase in pressure becomes great enough to more than balance the static head against which it operates, and flow takes place. This has the effect of reducing the pressure, thus causing water to rise in the suction pipe and enter the wheel at its centre.

When discussing the factors which govern the working of such a pump, certain assumptions are usually made. These are that the pump runs full, and that every particle of water enters the impeller radially without any tangential or whirl velocity, and leaves with a common velocity in a direction tangential to the periphery at every point of discharge.

It is possible to establish a relationship between the total lift of the water H, that is, the difference of level between the surfaces of the suction and discharge reservoirs, and its whirl velocity w as it leaves the tips of the impeller moving with a linear velocity u_o if it is assumed that the work done on the pump is just equal to the work done by it, and the efficiency therefore equal to unity.

Let Q = volume of water dealt with per second in cubic feet,

W = weight per cubic foot in lbs.,

r_o and r_i = outer and inner radius of impeller,

w_o and w_i = velocity of whirl at r_o and r_i,

ω = angular velocity of the wheel,

U = work done on the water passing through per second in ft.-lbs.

= change per second in its angular momentum multiplied by its angular velocity

$$=\frac{WQ}{g}\cdot(w_o r_o - w_i r_i)\omega \text{ ft.-lbs.},$$

and since it is assumed that $w_i=0$, while $u_o = r_o w$,

$$U=\frac{WQ}{g}(w_o r_o \omega)=\frac{WQ}{g}(w_o u_o).$$

But the work done by the water = WQH, and since this is equal to U, therefore $WQ \,.\, w_o u_o/g = WQH$, and $H = w_o u_o/g$.

All losses due to eddy formation, shock at entrance to or exit from the impeller, and friction in the passages are equivalent to an increase in the head against which pumping has to take place, and must be avoided as far as possible by adopting the form of vane best suited to the conditions of working.

(i.) *Shock at Entry.*—To avoid shock at entry, the direction of the relative velocity of water and vane must be tangential to the surface of the vane at the inner edge, as shown in *Fig.* 35 (*a*), where

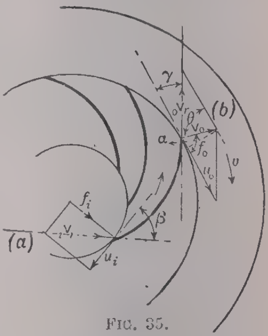


FIG. 35.

β = vane angle at entrance,

u_i = peripheral velocity of vane at entry,

f_i = radial velocity of water at entry $= u_i \tan\beta$,

${}_iv_r$ = relative velocity of water and vane $=\sqrt{u_i^2+f_i^2}$.

If variations in the speed of the impeller occur, the relations expressed above will not be maintained, and shock will result. During its travel through the wheel passages the relative velocity of water and vane remains unchanged, but for the loss taking place in overcoming the frictional resistance of the sides, and its direction conforms to the curve of the vanes.

(ii.) *Shock at Exit.*—At exit the water leaves the impeller at an angle which is the angle of the vane tip, and its absolute velocity v_o and radial component f_o may be determined by a velocity diagram, as in *Fig.* 35 (*b*).

If the mean direction of flow in the casing can be taken, as has been assumed, tangential to the impeller circle and its velocity v, the stream discharging from any passage will make with it an angle θ, and a loss of energy will result, which may be represented by the equation

$$a\,.\,\frac{v^2}{2g}+b\,.\,\frac{v_o^2}{2g}\text{ ft.-lbs.}$$

per lb. of the discharging stream,

where a and b are constants depending on the angle θ and the ratio m of the volumes of the main and discharging streams at their junction.

(iii.) *Loss due to Kinetic Energy of Discharge Flow.*—The casing surrounding the impeller is usually designed to include a volute chamber the cross-sectional area of which increases uniformly from A to B, as shown in *Fig.* 36, and allows a constant velocity of flow v. If free discharge, therefore, is permitted to the upper reservoir at this velocity, a loss equal to the kinetic energy of the discharge stream will occur, and have a value of $v^2/2g$ ft.-lbs. per sec. per lb. of water.

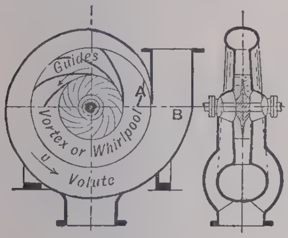


FIG. 36.

(iv.) *Friction Losses in Suction and Delivery Pipes.*—In addition, the frictional losses due to the resistances of the suction and delivery pipes, and which are of the nature of those already dealt with in pipe flow, require to be allowed for.

It follows, then, that the pressure head H_m which must be developed in the pump must be greater than the theoretical value H by an amount sufficient to cover these losses.

(v.) *Increase of Pressure Head throughout the Pump.*—The steps by which the necessary increase of pressure is obtained may now be examined, as well as any modifications of the primary design having for their purpose a reduction in the total losses, and therefore of the total pressure necessary.

The actual velocity of a particle of water during its passage through the impeller may be regarded as made up of two component velocities, one of whirl with the wheel and equal to ωr; the other parallel to the vanes, having a value v_r, which is the relative velocity of the water and vane. The first corresponds to a rotation in a forced vortex with an angular velocity ω, and the equation of motion for points on the inner and outer periphery of the wheel would be

$$p_i-\frac{W}{2g}\omega^2 r_i^2=p_o-\frac{W}{2g}\omega^2 r_o^2,$$

$$\therefore\ \frac{\text{Pressure difference}}{W}=\frac{\omega^2(r_o^2-r_i^2)}{2g}=\frac{u_o^2-u_i^2}{2g}.$$

The second component corresponds to an outward flow parallel to the vanes, of the same volume, with the wheel at rest. In this case

$$\frac{\text{Pressure difference}}{W}=\frac{{}_iv_r^2-{}_ov_r^2}{2g}$$

$$=\frac{f_i^2+u_i^2-f_o^2\operatorname{cosec}^2\gamma}{2g}.$$

The total difference of pressure p_o-p_i, therefore, between the inner and outer edges of the vanes is given by

$$\frac{p_o-p_i}{W}=\frac{u_o^2+f_i^2-f_o^2\operatorname{cosec}^2\gamma}{2g}.$$

This gain of pressure may be increased if some fraction k_v of the kinetic energy $v_r^2/2g$ ft.-lbs. contained in the water as it leaves the impeller is converted into pressure head in the volute chamber, and would equal $k_v v_r^2/2g$ ft.

Further, if the discharge takes place from the volute into a pipe of gradually increasing sectional area, the angle of divergence being in the neighbourhood of 5½°, so that the mean velocity of flow v in the volute is changed into a discharge pipe velocity of v_d ft., the increase of head between the volute and the delivery pipe will be given by

$$\frac{p_d-p_v}{W}=\frac{k_d v^2}{2g}\text{ ft.,}$$

and k_d will approximately equal ·85.

The total difference of pressure through the pump to the delivery pipe will under these conditions equal

$$\frac{p_d-p_i}{W}=\frac{u_o^2+f_i^2-f_o^2\operatorname{cosec}^2\gamma+k_v v_r^2+k_d v^2}{2g}.$$

(vi.) *Vortex or Whirlpool Chamber.*—Since the efficiency of the pump depends very largely on the conversion of the kinetic energy of the water leaving the impeller into pressure energy, many devices have been tried to secure this end. One of these, devised by Professor James Thomson, is the vortex or whirlpool chamber, circular and concentric with, but of larger diameter than, the impeller, and introduced between the latter and the volute. It is shown in lower half of *Fig.* 36. The water on leaving the vanes forms approximately a free vortex, and as the velocity diminishes towards the outside of the chamber the pressure increases, assuming there is no eddy losses, thus:

$$\frac{p_c-p_o}{W}=\frac{v_o^2-v_c^2}{2g}.$$

If the ratio between the inner and outer radii of this chamber equals C, then $v_c/v_o = \mathrm{C}$, and

$$\frac{p_c - p_o}{\mathrm{W}} = \frac{v_o^2(1-\mathrm{C}^2)}{2g} = \frac{(u_o^2 + f_o^2)(1-\mathrm{C}^2)}{2g} \text{ ft.}$$

Owing to eddy formation the gain in pressure is only a fraction k_c of this amount, its numerical value varying from ·4 to ·5. The effectiveness of the device, if full advantage of it is desired, is marred by the necessity of unduly increasing the over-all dimensions of the pump.

(vii.) *Guide Vanes.*—Another device intended to counteract the tendency to instability of motion and the formation of eddies on leaving the impeller consists in the introduction of fixed guide vanes designed to receive without shock the water from the impeller and to direct it by diverging passages either into a vortex chamber, as just described, or into the volute. The upper portion of *Fig.* 36 shows the latter arrangement. The angle of entrance of these guides should be parallel to the path of the water particle as it leaves the impeller and equal to α (*Fig.* 35 (*b*)) if shock is to be avoided. When the pump is required to work under varying conditions the angle should be suited to that of maximum efficiency. When this condition is departed from considerably, the result may be that the guides are a source of loss instead of a gain in efficiency. It will be seen from *Fig.* 35 (*b*) that the entrance angle is represented by

$$\alpha = \tan^{-1}\frac{f_o}{u_o - f_o \cot\gamma}.$$

Since the velocity of the water is at its maximum when leaving the impeller, and as only a portion of its kinetic energy can be recovered though every precaution is taken, it follows that the velocity of discharge should be kept as low as is consistent with maintaining the efficiency of the pump.

Efficiency.—The useful work done by a pump per lb. of water may be taken as represented by $\mathrm{H} + \mathrm{H}_f + v_d^2/2g$ ft.-lbs., where H is the total difference of level between the suction and delivery surfaces; H_f is the equivalent of the friction loss in the suction and delivery pipes; and v_d is the velocity in the discharge pipe. The summation of these may be termed the equivalent head H_m. The work which theoretically must be done per lb. of water when all losses are neglected is $w_o u_o/g$ ft.-lbs., and to this must be added L_h the sum of all the hydraulic losses, and L_m that of all the mechanical losses. The efficiency η of the pump is then given by the ratio:

$$\frac{\text{Useful work done by the pump per lb. of water passing}}{\text{Total work done on the pump per lb. of water passing}},$$

and this is equal to

$$\frac{\mathrm{H}_m}{w_o u_o/g + (\mathrm{L}_m + \mathrm{L}_h)/\mathrm{WQ}},$$

$$\therefore \eta = \frac{\mathrm{H}_m}{u_o(u_o - f_o\cot\gamma)/g + (\mathrm{L}_m + \mathrm{L}_h)/\mathrm{WQ}}.$$

So far as this depends on the angle γ, it is evident that a reduction of γ will increase the efficiency of the pump, and in addition give to the passages a more uniform cross-section. It must not be forgotten, however, if the head H to be pumped against remains the same, that any decrease of w_o effected by a reduction of the angle γ requires an increase of the peripheral speed u_o, with consequently an increase of frictional loss. In practice the value of γ varies from about 15° to 90°, depending on the purpose for which it is designed and the head against which lift takes place, the maximum permissible value increasing with the working head.

Single impeller pumps are used for heads between 6 and 100 feet and have an actual efficiency of over 75 per cent, and the efficiency is well maintained through a fairly large range of speeds. The limit set to the maximum lift of this pump by the high speed of rotation necessary, and the consequently excessive frictional and eddy losses which occur, have been overcome by the introduction of the

§ (39) COMPOUND MULTIPLE CHAMBER PUMP.—This consists of a series of two or more impellers on the same shaft, each pumping water into the central space of the next adjoining, with the exception of the last, which pumps directly into the delivery pipe as shown in *Fig.* 37. The impeller diameters and vane angles are made the same for each chamber, and the total lift is equal to the lift of one stage multiplied by the number of stages. Lifts up to 1500 feet are possible by this arrangement. Guide vanes are almost invariably used with the multiple type, as it is essential for efficiency that as far as possible the kinetic energy of discharge from each wheel should be converted into pressure energy before entering the next chamber. In most cases the impellers are mounted in pairs back to back, with the flow in opposite directions, by which method the end thrust which occurs in single-inlet impeller wheels is conveniently neutralised. For multiple pumps the impeller vanes are of the enclosed type, thereby reducing the leakage of water between the pump-case and the vanes,

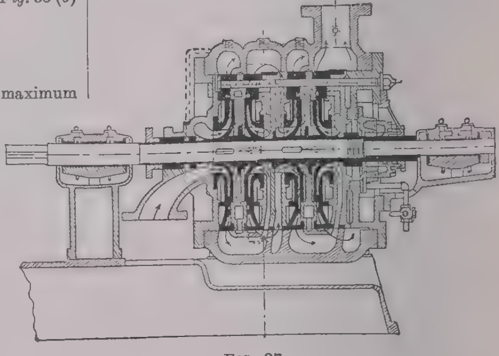


FIG. 37.

and also the disc friction of rotation. A centrifugal pump before starting is charged with water, a foot-valve being provided to allow of the charge being retained. To enable delivery to begin, the conditions of rotation of the wheel must satisfy the equation

$$\frac{p_o - p_i}{W} = \frac{u_o^2 - u_i^2}{2g} = \frac{\omega^2(r_o^2 - r_i^2)}{2g} > H_m.$$

§ (40) OTHER PUMPS.—In the types of pumps already considered the operating force has been applied through the medium of a rigid solid, such as a piston or a revolving wheel. There is, however, an important group where the solid is dispensed with and a fluid in direct contact with the water supplies the motive power. This group will now be considered.

§ (41) PULSOMETER STEAM PUMPS.—This pump is closely related in fact, though not in appearance, to the steam reciprocating pump. There is no piston, and the steam which is the working fluid acts directly but alternately on the surfaces of the water, contained in two pear-shaped chambers cast side by side in one piece. An oscillating valve common to both chambers is placed at the junction of their stems, and when the valve admits steam to one chamber it closes to the other. Under pressure of the steam the water with which the chamber has been charged ready for starting is forced through a foot-valve into the delivery pipe. Condensation of the steam remaining takes place, expedited in some cases by the injection of a fine water spray, and the reduction of pressure which ensues closes the steam inlet valve and the delivery foot-valve while opening the suction valve, and admits water from its inlet opening at the bottom for a fresh charge. The same cycle of operations takes place in the other chamber consequent to the movement of the steam valve, so that delivery in one chamber synchronises with suction in the other.

The use of this class of pump is limited by practical considerations to lifts below about 100 feet, the most efficient steam pressure being from 45 to 60 lbs. per square inch, though a lift of 170 feet has been attained with steam at 100 lbs. pressure per square inch. The capacity of such pumps based on a lift of 20 feet varies with the size from 1000 to 150,000 gallons per hour. Its efficiency is not high, but it is a useful and cheap appliance for pumping of a temporary kind, and it has the great advantage of not requiring any provision for fixing, being suspended by means of a chain or rope at the desired level.

§ (42) GAS DISPLACEMENT OR HUMPHREY GAS PUMP.—The Humphrey gas pump bears a similar relationship to a gas engine working on the four-stroke cycle as the pulsometer does to the steam engine. It is a development in the direction of utilising the force obtained from the combustion of an explosive mixture of gas and air to raise water by direct pressure. The action of the pump is as follows. At the beginning of the power stroke a charge of gas and air, compressed between the end of the combustion chamber C (*Fig.* 38) and the column of water contained in the delivery pipe D continuous with it, is ignited by electrical spark and expands. By this means the water column is set in motion, acquiring momentum, and part of its contents is discharged. Due to the momentum the expansion is continued until the pressure falls to or below atmospheric, when the water valves W connecting to the suction tank ST and the exhaust valves E open by suction effect, the gas inlet valve I being meantime locked shut. Its momentum exhausted, the water column oscillates back while the products of combustion are discharged, the exhaust valve remaining open until closed by impact, after which compression of the remaining air in the combustion space takes place. This is followed by another expansion, during which the gas inlet opens and a fresh combustible charge is taken in, while the exhaust valve is locked shut. The return oscillation closes the inlet valve and compresses the charge until the water column is again brought to rest, when ignition takes place and the cycle starts afresh.

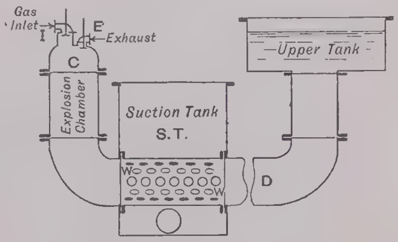


FIG. 38.

If two combustion chambers are provided instead of one and the inlet and outlet valves of the one alternate with those of the other by one complete oscillation, the arrangement corresponds to a two-cylinder gas engine and the discharge is approximately doubled.

The pump in either of its forms may be adapted to work with a suction lift, and can be utilised for operating against a high-pressure head by introducing two air vessels separated by non-return valves as an integral part of the delivery pipe. On test a thermal efficiency of 23 per cent has been obtained, which is equivalent to a consumption of nearly 1 lb. of anthracite per water-horse-power per hour.

A set of five pumps of this type, with a total estimated capacity of 180 million gallons per day and a lift of 30 feet, was constructed for the Metropolitan Water Board for use at their Chingford Reservoir. The rate of working approximates to 12 cycles per minute, and in each of the larger units 10 tons of water are delivered per cycle.

§ (43) HYDRAULIC RAM.—The hydraulic ram forms another of this group, the working fluid in this case being water, and use is made, as in the gas displacement pump, of the water-hammer principle. A valve-box B (*Fig.* 39) is placed in communication with a running stream or supply pipe having a small operating head. A waste valve w when open permits of a flow being set up under the influence of this head until the dynamic pressure on the inner side is sufficient to close it. The effect of the sudden closure of the valve is to cause an increase of pressure great enough to open a delivery valve d communicating with a delivery pipe D through an air vessel A. A portion of the water which escapes is used to compress the air in A, and a portion passes up the delivery pipe before the momentum of the column is absorbed. The pressure in the air vessel reacting initiates an impulse in the opposite direction, the delivery valve closes, and the reduction of pressure in the valve-box which results enables the waste valve to again open. The normal flow resumes with the next oscillation, increasing until the dynamic pressure once more closes the valve. By this method a low-head large flow is enabled to lift a smaller flow through a large head.

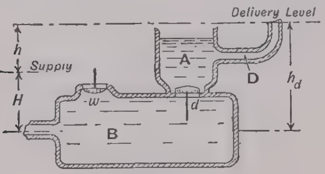


FIG. 39.

The efficiency of the ram depends on what is considered the effective lift of the discharge Q. If this is assumed to be the difference between the levels of the supply intake H and the surface of the storage tank h_d, then the useful work done is represented by $Q(h_d - H)$, while the work done on the ram equals $q \,.\, H$ and $\eta = Q \,.\, (h_d - H)/qH$, where q = water flowing past waste valve.

If, however, the useful lift is regarded as h_d, then

$Q \,.\, h_d$ = the useful work done by the ram, and
$(Q + q) \,.\, H$ = the work done on it, and
$\eta = Q \,.\, h_d/(Q + q)H$.

The values obtained on the first assumption are consistently lower than with the second, but even on that basis the efficiencies where the delivery head does not exceed four times the supply head may be as high as 75 per cent. For a given supply head the efficiency of the ram falls off very rapidly with an increase of the delivery head ratio above the value just given. For small diameters of supply pipe this type of pump gives excellent results, but with larger sizes trouble is apt to arise through excessive shock when the valve is suddenly closed. To obviate this sometimes an air dashpot is fitted to the waste valve spindle, but while effective for this purpose it has the disadvantage of lowering the efficiency of the pump, since slowness of closing, allowing leakage past the valve at the time when the velocity of the waste water is at its maximum, means a larger proportional loss of energy, compared to the whole kinetic energy of the water column. The introduction of mechanical regulation of the valves enables this type to be successfully applied to the pumping of water on a much larger scale and against greater heights than is possible where automatic valves are used.

§ (44) JET PUMP.—The working fluid in the jet pump is also water, but the principle of working is quite different. It is operated by the conversion of the high-pressure energy of a water supply into kinetic energy, and as the velocity is increased the pressure diminishes until it may be that a pressure less than atmospheric is reached. This is effected in the passage of the water through a converging nozzle N (*Fig.* 40), which is surrounded by a concentric chamber C communicating with the lower reservoir by means of a suction pipe. The reduction of pressure at the face of the nozzle due to the issuing jet induces flow in this pipe which combines with the water from the jet and is carried forward into a diverging portion of the discharge pipe D. There the dynamic pressure is partly reconverted into pressure energy sufficient to maintain flow against

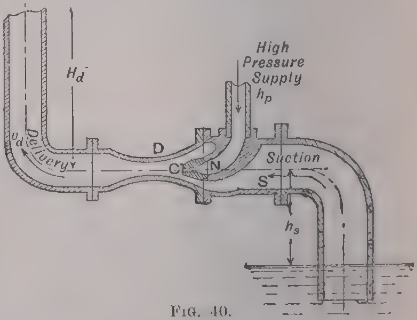


FIG. 40.

the discharge head. The energy contained per lb. of the combined streams is $p_d/W+v_d^2/2g$, and must be equal to H_d+ the losses between the vena contracta and the delivery surface, where p_d, v_d, and H_d are the pressure, velocity, and head in the delivery pipe.

The square of the velocity necessary for delivery varies as H_d and is independent of the suction head h_s, and as the loss of energy is proportional to v_d^2 it will also be proportional to H_d. For a given lift, therefore, the suction head should be made as great as possible.

$$\eta=\frac{\text{The useful work done by the pump}}{\text{The work done on it}}$$

$$=\frac{Q_s\,.\,(H_d+h_s)}{Q_p\,.\,(h_p-H_d)},$$

where Q_s and Q_p are the quantities from suction and pressure supply respectively, and h_p is the high pressure head.

The actual values of η reached do not exceed 30 per cent, and are usually round about 25 per cent.

This low efficiency may be increased by substituting for the single-stage impact between the streams a multi-stage arrangement. By this means the loss of energy due to shock when the high velocity jet meets the low velocity flow is substantially reduced, and the efficiency may be raised to 33 per cent.

When the fluid used for the high-pressure jet is steam the injector becomes the well-known locomotive type associated originally with the name of Giffard and largely used for boiler feed purposes.

Another modification may be mentioned where the above process is reversed and a high-pressure water jet is used to draw away steam from, and to maintain a vacuum in, the exhaust chamber of a steam engine. This combination is termed an Ejector Condenser.

§ (45) Air-lift Pump.—The method of raising water on the aeration principle is another example still of the direct application of a working fluid, and has found considerable favour during recent years especially as applied to artesian wells. It consists primarily of two pipes, one (A, *Fig.* 41) having its lower end submerged in the liquid to be raised and its upper end arranged to discharge into a reservoir at the required height, and the other (a) for conveying air from a compressor to a nozzle n, situated in the submerged opening of the rising main. The air is diffused through the water in the uptake pipe and forms a mixture having a low specific gravity. The pressure of the heavier fluid in the surrounding casing forces the lighter mixture above the supply level and out of the top of the delivery pipe. The difference of pressure thus obtained determines the height to which the water can be lifted, and will vary with the depth of submersion of the pipe. A sketch of three alternative forms of the arrangement is shown in *Fig.* 41.

The air tube may be fitted either concentric internally or externally to the uptake pipe or parallel to it. To the first (1) of these methods the objection is raised that it increases the frictional resistance to the water flow and consequently lowers the efficiency, but it is very convenient in the case of a small bore hole. The second (2) admits of more effective air distribution which is an essential in this type of pump, but where the well is of large diameter the system of parallel pipe (3) has the advantage of being most readily accessible and very flexible. Broadly speaking, the least pressure of air that will give continuous flow is the proper pressure to use. Its approximate value is ·65 lb. per square inch for each foot of lift from the surface of the water.

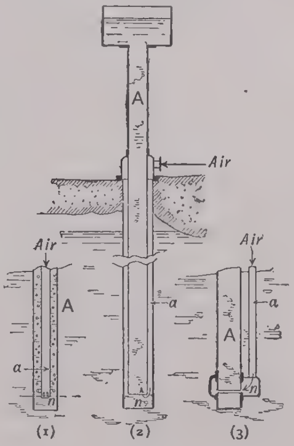


Fig. 41.

The efficiency of this type of pump is low, and reckoned as the ratio of water H.P. to the compressor cylinder I.H.P. does not exceed 45 per cent. If calculated from the indicated power of the prime mover not more than 30 per cent would probably be registered. Against this low efficiency must be set the simplicity of the mechanism of the pump, its ease in setting up and the immunity it possesses from the scouring and choking effects of sand or sediment. These latter considerations probably account for the revival of interest in this ingenious type of pump.

III. The Application of Water Power to Industrial Purposes

§ (46) The Energy of Stored Water.—It has been seen how supplies of water can be collected from natural sources and stored, or made available by artificial means for the purpose of distribution to satisfy the requirements of the consumer. It is with the means taken to utilise the power latent in a supply that it is now proposed to deal.

The capacity for work of a store of water may be made the means of furnishing working forces during the descent of the water from a higher to a lower level, through a properly designed machine, whereby a steady motion of that machine may be maintained against various resisting forces. If Q be the quantity of water in cubic feet per second available, H the total available head in feet, and W the weight of 1 cubic ft. of water, then the Capacity for work or Potential Energy of the water $=$ WQH ft.-lbs. per second, and this is equivalent to H ft.-lbs. per second per lb. of water. The ratio of the useful work done by the water in its descent to the potential energy or capacity for work latent in the water when stored is termed the efficiency of the machine. Machines designed to utilise this potential energy are called Hydraulic Motors or Prime Movers. The work done is due entirely to the loss of head of the water during its descent, but the method of applying the energy will depend on the type of motor selected. In general, it consists of a wheel which is caused to rotate either by the weight of the descending water or by the dynamic pressure arising from a change in direction and velocity of the moving stream. Piston engines, where the water does work in virtue of its static pressure only, form, however, an important class.

When the water enters the wheel at one part only of the circumference the machine is called a Water-wheel; when it enters the entire circumference more or less simultaneously it is called a turbine. For convenience it is proposed to classify the various types into the three main divisions: (*a*) water-wheels, (*b*) turbines, and (*c*) pressure engines, and to consider them in that order.

§ (47) WATER-WHEELS.—In this division the working force is obtained:

(i.) By the weight of the water, producing rotation as in the overshot and breast wheels;

(ii.) By utilising the kinetic energy of a moving stream as in undershot wheels;

(iii.) From the impact of a high velocity jet of water as in the Pelton wheel.

(i.) *Overshot Wheels.*—The construction of this type of motor, which was very general for small powers with heads ranging from 15 to 50 feet, is very simple and is illustrated in *Fig.* 42.

The water is supplied, as near the highest point of the wheel as possible, to a series of buckets formed of shrouded vanes, and escapes when the outer part of the bucket is horizontal, which occurs before the lowest position of the bucket is reached. The useful head is less than the theoretical head H (*a*) by an amount which depends on this discharge position, and (*b*) by the amount required to supply the kinetic energy of the stream. If h is equal to the total head thus absorbed, then $H-h$ will represent that available for useful work and the efficiency possible is $=(H-h)/H$.

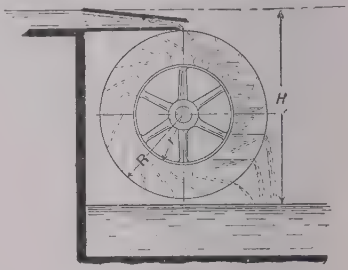


FIG. 42.

Maximum efficiency is obtained when the peripheral velocity of the wheel is equal to one-half the velocity of the inflow water. To prevent loss by shock at entrance to the buckets the vane angle at the tip should be parallel to the relative motion of the water and vane there. This angle is usually arranged to make 25° to 30° with the tangent to the circumference, and as a consequence the bucket retains water for a vertical distance nearly equal to ·8 of the wheel diameter. When working under suitable condition efficiencies up to 80 per cent may be obtained.

(ii.) *Breast Wheel.*—Where the working head ranges between 6 feet and 15 feet the supply is admitted to the buckets at some point situated in the breast of the wheel (*Fig.* 43). Any loss of head due to the premature escape of the water from the buckets becomes proportionately greater, since the head is less, and is prevented by the building of a breastwork of masonry with a minimum clearance between it and the wheel. Precautions similar to those taken in the overshot type are required to prevent shock at entry, and special provision is made of air vents at the inner circumference of the wheel to let the air out as the water rushes into the buckets. Under favourable circumstances an efficiency of 65 per cent may be reached.

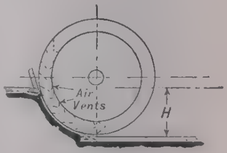


FIG. 43.

The *Sagabien wheel* is a form of breast wheel in which the bucket is replaced by long flat vanes, tangential to a circle concentric with the wheel and making an angle at the outer circumference parallel to the relative velocity of water and vane as shown in *Fig.* 44. The velocity of rotation is proportional to the flow, and the wheel is therefore capable of dealing with large variations of supply. Any increase of the load, however, has the effect of slowing down the wheel and reducing the supply just at the time when an increase of energy is most required. Efficiencies up to 80 per cent have been attained by this form.

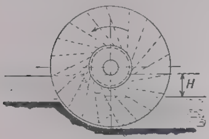


FIG. 44.

(iii.) *Undershot and Poncelet Wheels.*—In the undershot wheel adopted for low heads of 3 feet and under, work is done by the action of a moving stream impinging against a series of radial vanes set round the circumference of the wheel, the change of momentum of the water being a measure of the force applied. The wheel dips into the stream, the tips of the vanes just clearing the bottom of the channel. A maximum efficiency of 50 per cent is obtained with a peripheral velocity of wheel one-half the velocity of the stream, but in practice the efficiency does not reach more than 35 per cent.

A modification of this wheel is the *Poncelet wheel* (*Fig.* 45), where the vanes instead of being radial are inclined backward to make an angle at the tips with the circumference. By this means loss at entrance due to shock is avoided, while if properly designed the loss of energy in the discharge stream is reduced.

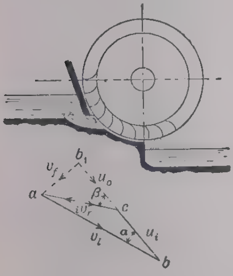


FIG. 45.

If v_i = the absolute velocity of water at entrance,
α = the angle which it makes with the tangent to the circumference at the tip,
u_i = peripheral velocity of vane tip.

(*a*) *To avoid Shock at Entrance.*—Since v_i and u_i are completely represented by *ab* and *cb* respectively (*Fig.* 45), the relative velocity v_r will be represented by *ac*, and the angle β which it makes with u_i should be that between the vane and the circumference.

(*b*) *Minimum Loss of Kinetic Energy to Discharge Stream.*—The relative velocity of water and vane on discharge will also make an angle β with u_o, and if it be assumed that there is no frictional loss in the bucket it will have the same value but be opposite in direction, and will therefore be represented in the figure by *ca*. Since u_o is the same as u_i, the absolute velocity of discharge v_f will be represented by b_1a, and will be a minimum when its direction is perpendicular to u_o, and have a value of $v_i \sin \alpha$. In this case $v_i \cos \alpha = 2u_i$.

The capacity for work of the water at entrance is proportional to v_i^2 and the energy transferred to the wheel to $v_i^2 - v_f^2$,

$$\therefore \text{Efficiency } \eta = \frac{v_i^2 - v_f^2}{v_i^2} = \frac{v_i^2(1 - \sin^2 \alpha)}{v_i^2} = \cos^2 \alpha.$$

α is usually made equal to about 15° and the theoretical efficiency of the wheel is 93 per cent. Again, since $\tan \beta = 2 \tan \alpha$, the value of β is 28·2°.

(iv.) *Pelton Wheel.*—By far the most important of the class of water-wheels is that known as the Pelton wheel, almost invariably adopted for heads over 500 feet. It is of a purely impulse type. The water is supplied to one or more nozzles in which the potential energy measured by the fluid pressure is converted into kinetic energy. The issuing jet from the nozzles is directed on to a series of buckets fixed round the periphery of the wheel. In the first stage of its development the buckets consisted of flat plates, but these were later replaced by hemispherical cups, fixed alternately on each side of the centre line of the wheel and concave to the jet, whereby the theoretical efficiency of the wheel was doubled and made equal to unity. Since then the improvements have been mainly in the direction of evolving a type of bucket which would bring the practical efficiency within a reasonable distance of the theoretical one.

For the cups there has been substituted a series of concave buckets fitted with knife-edge ridges so as to split the jet, and having curved surfaces arranged to deflect it to the sides of the wheel with as little friction as possible. In this manner the central portion of the jet is used with the greatest effect.

These improvements have been so successful that with the designs now in common use efficiencies up to 89 per cent are being obtained. In addition there is a long range of loads within which efficiency is well maintained, having a value of 85 per cent at half load and reaching 80 per cent at one-third of the normal loading.

(v.) *Efficiency of Pelton Wheel.*—This depends on the velocities of the jet and wheel, the angle at which the jet strikes the bucket, and the angle through which it is deflected.

We assume, as a first approximation, that the jet is moving tangentially to the wheel at impact and is deflected through an angle which approaches 180°. The modifications required if the jet is not tangential will be found in books on hydraulics.

Let u be the velocity of the bucket at the point of impact—the centre of the jet,
v the initial velocity of the jet,
c the ratio of the relative velocity after impact to its value before,
γ the angle of deflection of the jet.

Then

Relative velocity before impact . . . $=v-u$,

Relative velocity after impact . . . $=c(v-u)$,

Absolute velocity in tangential direction after impact . . . $=u+c(v-u)\cos\gamma$,

Change of momentum per second per lb. of water striking the wheel . $=v-\{u+c(v-u)\cos\gamma\}$
$=(v-u)\{1-c\cos\gamma\}$,

Work done per lb. per second . . . $=\dfrac{u(v-u)(1-c\cos\gamma)}{g}$ ft.-lbs.,

Efficiency . . . $=\eta=\dfrac{u(v-u)(1-c\cos\gamma)}{gh}$.

To find for what value of u the efficiency is a maximum, assuming c and γ independent of u, we put $d\eta/du=0$, whence we obtain $v=2u$, or the velocity of the wheel at the point of impact is half that of the jet.

In this case

$$\text{Efficiency}=\frac{v^2(1-c\cos\gamma)}{4gh}=\frac{1}{2}(1-c\cos\gamma),$$

since v is the velocity due to a fall through a height h. If there be no friction in the buckets and the jet is deflected 180°, then $c\cos\gamma=-1$ and $\eta=1$. In practice the angle of deflection is about 160° and the maximum theoretical efficiency is about 95 per cent. Other losses bring this down to about 85 per cent.

(vi.) *Design of Buckets.*—The path of the water particle across the bucket is represented in *Fig.* 46 for two positions: (i.) where the jet first impinges on the bucket, (ii.) where the jet is nearly parallel to the tangent to the wheel at the point of impact.

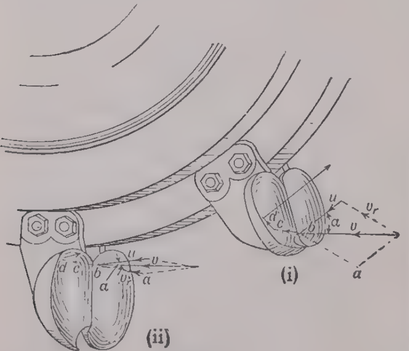


FIG. 46.

In (i.) the direction of the relative velocity at incidence is given by ab, the point of impact being b. It is shown diagrammatically deflected by the curvature of the bucket until at c its tangential direction is reversed, and it finally leaves the wheel at d, where its relative velocity should be parallel to u and equal to it if its absolute tangential velocity is to be zero.

In (ii.) the jet strikes the bucket nearly parallel to the direction of u and a is approximately zero, therefore the relative velocity is tangential and in the direction ab. Its direction is reversed at c and the full deflection angle is completed at d. For zero absolute tangential velocity at that point ${}_2v_r={}_1v_r=v-u=u$, therefore $v=2u$. The width of the buckets is from three to five times the diameter of the jet, the ratio varying inversely as the size of the jet, and the ratio of the wheel and jet diameters should not be less than 10. The number of the buckets should ensure that the jet is continuously intercepted, each bucket being in action until the one following is in a position to receive the jet.

(vii.) *Speed Regulation.*—With any considerable departure from the theoretically correct ratio between the velocities of the wheel and jet there is a substantial reduction in the efficiency, and if the latter is to be maintained the means of regulation adopted should enable this ratio to be kept as nearly constant as possible through wide variations of load. There should also be, in response to load changes, corresponding changes in the quantity of water used, and any retardation of the flow in the pipe line when necessary must be slow and gradual if dangerous increases in the pressure are to be avoided.

Since the wheel as a prime mover has to maintain a constant speed of rotation however the load fluctuates, it follows that for the sake of efficiency the velocity of the jet must also be kept constant. When load is taken off therefore the method of regulation should consist either of diverting, wholly or partially, the jet from the wheel, the total quantity of water used remaining the same, or of diminishing the quantity while still keeping the velocity of flow constant.

The means of applying the first of these forms of regulation is by swivelling the nozzle, which is then fitted with a ball-and-socket joint, or by interposing a deflector-plate between the nozzle and the wheel and so causing the stream partially to miss the buckets at part load. Under these conditions there is an obvious waste of energy and the direct discharge of the jet into the wheelpit may prove troublesome, but there are the advantages that no sudden rise of pressure will occur on change of load, and the flow through the supply pipe will be constant. This method of regulation is seldom adopted in its simple form but frequently as part of a combination regulator.

The use of a simple throttle valve situated in the supply pipe to diminish the quantity of water with the load would result in varying the velocity at the jet and inducing a water-hammer effect on partial closing, besides causing a loss of energy by the obstruction it presented in the pipe.

The arrangements actually in use are of two types:

(*a*) Where the nozzles are rectangular a portion of the jet may be cut off by a sluice sliding across the orifice. The nozzle at A (*Fig.* 47) shows the upper side formed by a flap hinged at *a*, fixed in position by a connecting-link attached at *b* and operated from the governor. The hood regulator in the example B consists of a quadrant pivoted at *c* and worked by a hand-wheel through a rod connected at *d*.

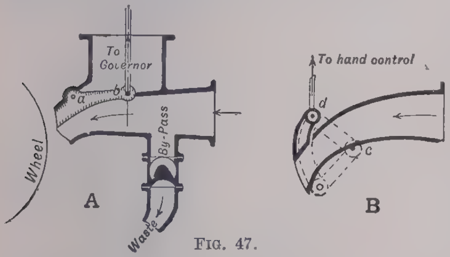


FIG. 47.

(*b*) The section of a circular jet may be partially or wholly reduced by the endways movement of a spear or needle regulator, consisting of a needle of tapering section fitted inside the nozzle axially with the jet as shown in *Fig.*48. When properly proportioned the jet issues as a clear and transparent rod more or less hollow but converging to a solid cylinder of water at a short distance from the nozzle. This needle is a feature of practically all regulators now in use.

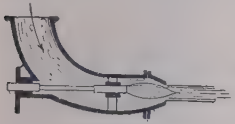


FIG. 48.

The regulator in most cases is worked automatically from the governor, and the operation may be divided into the two stages of obtaining the necessary power and applying it. With high-pressure and large-capacity machines the power required to move the regulator, if directly connected, would be beyond the capacity of an ordinary mechanical governor. Because of this an hydraulic type actuated by water pressure was evolved, but this is now generally replaced by an apparatus depending on oil pressure. The principle on which it acts is illustrated diagrammatically in *Fig.* 49—high- and low-pressure oil supplies, A and B, are connected to a chamber C in which a distributing valve works; the valve passages communicate with the opposite ends of a cylinder containing a piston, the motion of which operates the regulator. When the position of the governor balls changes as a consequence of load variation a rise or fall of the sleeve E occurs; there is an angular movement of the lever EG about G as a centre, a displacement of F and therefore of the distributing valve takes place. Oil under pressure is accordingly allowed to pass to either the top or the bottom of the piston, which moves correspondingly. In addition to its action on the regulator this alters the position of G, and there is a further angular movement of EG, with E_1 as its centre, which tends to restore F to its mid position. When this occurs pressure is cut off and the piston comes to rest. The mechanism is then in readiness to meet any fresh fluctuation of speed. This arrangement, or some modification of it, is practically standard as a means for obtaining power to actuate the regulator. There still remains, however, the difficulty of applying the methods of regulation in such a way as to prevent a rise in pressure following a sudden closing of the valve. This is met in two ways: (1) a by-pass valve (*Fig.* 47 (A)) in the supply pipe is opened temporarily to permit of the passage to waste of the water checked by the sudden closure of the regulator; or (2) regulation is secured at first by the use of a deflector-plate to divert the jet, and afterwards by closing the needle so gradually and slowly as to prevent any serious rise of pressure. When stable conditions are restored the by-pass valve is shut or the deflector is swung clear of the jet.

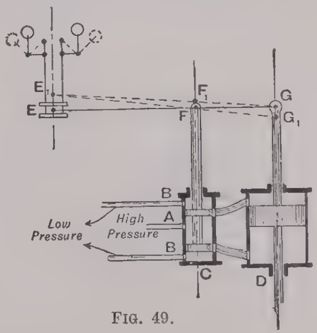


FIG. 49.

This method of regulating by combined needle and deflector is the most common of all, and the application of the motion of the piston to it is shown in *Fig.* 50. HK is a link pivoted at D and K and carrying a pin at H. This pin works in a slot in the end of the needle rod NH, and holds the needle regulator N in any required position against the pressure of a spring S. This spring works in a dashpot J. On the motion of the piston the point D moves to D_1, causing the deflector M to cut into the jet, and the link pin slides along the slot from H to H_1. The pressure of the spring S tends to close the needle suddenly, but its movement is damped by the dashpot fluid so that the closure is slow and only continued until the end of the slot again bears on the pin H.

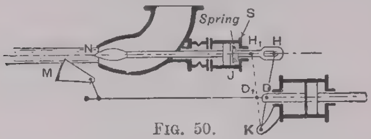


FIG. 50.

(viii.) *Surge Tank.*—If the length of the supply pipe line is considerable in comparison with the total head it may be that the water in the pipe line cannot accelerate with sufficient rapidity for good governing in the case of a sudden demand. This obviously cannot be dealt with by the regulator, and a remedy is found in the provision of a standpipe or surge tank. The standpipe consists of a vertical open pipe with its lower end connected to the supply pipe near its junction with the prime mover; its height is such that under a static pressure equal to that of the supply head the water level is a little below the top. An increase of pressure at the prime mover, due to a sudden closing of the valve, causes the water to rise in the standpipe and absorbs the kinetic energy of the water column in the supply pipe. Any excess produces overflow at the top, so that the maximum pressure possible in the supply pipe will be practically equal to that of the supply head, together with the head equivalent to the energy absorbed in the overflow. In the event of a sudden demand for more power the water in the standpipe, being more easily accelerated, responds readily and supplies the additional quantity required until the water column in the main supply pipe has had time to accelerate.

The larger the area of the standpipe the less will be the amplitude of the oscillations set up in it, but mechanical difficulties and considerations of cost put a limit on the permissible size. A modification designed to effect the same end without the disadvantages of very large diameters has been introduced in the differential surge tank. It consists of the simple standpipe having a diameter approximately equal to that of the supply pipe, and communicating at the top with a large diameter tank, normally through a comparatively small opening A, but in the case of an extremely heavy fall in the load, provision is made for the water to escape by special opening B, and over the top, as shown in *Fig.* 51.

The improvement lies in the throttling effect of the small opening between the upper tank and the standpipe. A sudden demand for power is met in the first instance by the water in the standpipe owing to its easy acceleration, and secondly by the slower acceleration which takes place in the tank. The advantage is indicated in the diagram *Fig.* 51A, where the falls in the surface level of the standpipe water are plotted against intervals of time for both simple and differential types. The limiting height of standpipes is about 200 feet, though there are cases where this has been exceeded.

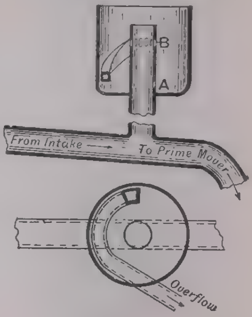


FIG. 51.

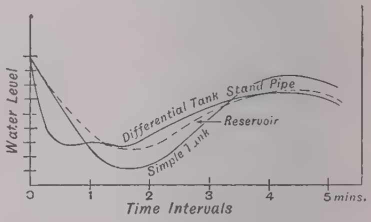


FIG. 51A.

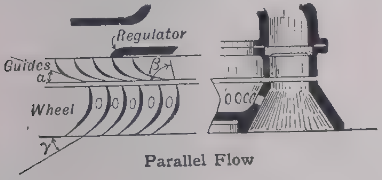
§ (48) IMPULSE TURBINE: GIRARD.—A turbine has already been defined as a water-wheel to which water is admitted simultaneously at all points of its circumference. There is a class which only partly satisfies this definition, namely the Impulse Turbine, a typical example being the Girard. It may be regarded as a Pelton wheel with multiple jets impinging on curved vanes which replace the buckets, and having guide passages to serve as substitutes for the nozzle. The vanes cause a change in the direction of the flow of the water and consequently of its momentum tangential to the turbine. Thus force is exerted and work is done on the turbine shaft.

The pressure of the water throughout the wheel remains uniformly equal to that in the turbine casing and is usually atmospheric. To ensure this the water is prevented from filling the space between any two adjacent vanes by the introduction of ventilating holes which admit air to the wheel passages and confine the stream to the driving side of the vanes.

The general direction of flow may either be parallel to the axis of the shaft or perpendicular to it, and in the latter case may be radially inward or more usually outward. *Figs.* 52 and 53 are diagrammatic representations of the two divisions.

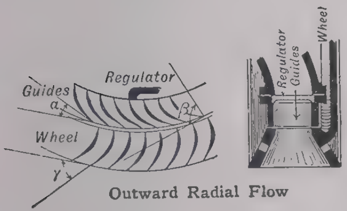
The considerations which influence the design of the bucket in the Pelton wheel hold good in the design of the vanes, and the angles at inlet and outlet are similarly determined. Using the same notation α varies from 12° with large heads and small volumes to 30° with low heads and large volumes, while the angle through which the stream is deflected averages about 135°. Since the effective width of the vane passages diminishes outwards owing to the curvature of the vanes, the sides are splayed out in the direction of discharge, the final dimension being 2·5 to three times the inlet breadth.

The best theoretical speed of wheel is one-half that of the water velocity as it issues from the guide passages, but in practice it is usually about three-fifths. This type of wheel may be used with heads as low as 18 inches; in this case it is necessary that the wheel should be horizontal in order to avoid the relatively large difference of level which would exist between the diametrically opposite vanes of a vertical wheel. With such a low operating head an efficiency of 55 per cent may be reached, but when working more normally with higher heads, values up to 80 per cent are attained, and even with part loads this efficiency is well maintained.



Parallel Flow

FIG. 52.



Outward Radial Flow

FIG. 53.

§ (49) PRESSURE TURBINES. (i.) *Fourneyron or Outward Flow Turbine.*—The first of the real class of turbines is the outward radial flow reaction wheel invented by Fourneyron in 1827. By reason of its cheapness and high efficiency it largely replaced for a time all other forms of water-wheels. The arrangement is shown in *Fig.* 54. The inlet is by the central curved passage F, and the flow is directed by the guide vanes G to the wheel vanes N, where its direction of motion in the plane of the wheel is changed, and discharge takes place at the outer periphery of the wheel.

A device known as the Boyden diffuser, to increase the efficiency by recovering a portion of the dynamic energy of the discharge water, was adopted for a time. It consisted of a fixed casing surrounding the wheel made up of two plate rings, the distance between which increasing radially outward, formed a diverging passage. The benefit derived was small and its use was abandoned.

The Fourneyron turbine was employed for heads up to 350 feet, and gave an efficiency as high as 75 per cent. Owing to the passages through the wheel being of necessity divergent, with a consequent production of eddies, no further improvement was possible. A disadvantage of this type is the difficulty of governing. Any increase in the speed of the wheel due to a reduction in load increases the kinetic energy at discharge, thus lessening the discharge pressure head and tending to increase instead of diminish the flow: a further increase of speed therefore results, and governing is rendered more difficult.

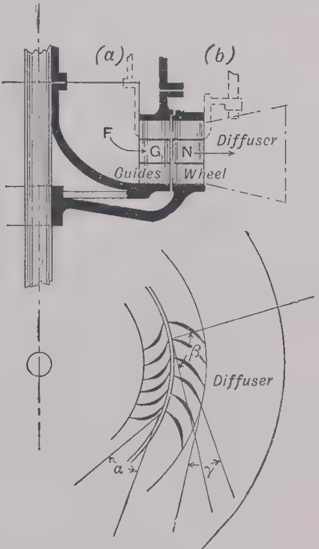


FIG. 54

(ii.) *Jonval Turbine.*—The Jonval turbine, like its prototype the Borda wheel, is of the axial flow variety. A radial section shows the wheel buckets as rectangular, but the vanes which form their radial bounding surfaces usually make an angle of 90° at entrance and curve away to a much flatter angle at discharge. It differs from the Borda wheel in being fitted with radial guide vanes which direct the pressure water into the buckets, and shows a marked improvement not only on it but on the Fourneyron machine as well. The main advantage lay in the fact that the motion of each water particle was confined to one tangential plane, and had practically no radial velocity. Thus the governing was not complicated by pressures due to centrifugal forces. On the other hand, since each particle entered the bucket at a nearly uniform velocity, and since the linear velocity of the entrance points of the bucket vanes varies with the distance of the points from the centre of rotation, it follows that the vane angle for correct design would require to vary with the radius. As this cannot conveniently be done it is necessary for efficient working either to make the radial dimension of the bucket small in comparison with the radius of the wheel or else divide it into several parts, each compartment having its own vane angle. This also facilitates speed regulation since the circular slide, which on a reduction of load or an increase of head is made to shut off a number of the buckets, does so in sections corresponding to the divisions. By this means a fairly high part-gate efficiency may be maintained, varying from 74 per cent with one out of three sections open to 81 per cent with all three sections open.

(iii.) *Francis or Inward Flow Wheel.*—Just as the Fourneyron turbine was superseded by the Jonval, so the latter was displaced by the Francis inward flow turbine. The Francis is directly comparable to the Fourneyron with the direction of flow reversed, but possesses many advantages over it. The inlet is located at the outer circumference of the runner, as shown in *Fig.* 55, and a portion of the supply head at entrance is retained in pressure form to balance the centrifugal pressure of the water in the wheel, being afterwards utilised during its passage through the vanes. As a consequence the velocity of inflow of the water is considerably less than in machines of the impulse type and a lower peripheral speed of runner can be adopted, while the hydraulic friction losses will be proportionally lessened throughout. Apart from ordinary means of regulation common to the various types of turbine, the inward flow form tends to become self-regulating as an increase in speed, due to diminution of load, causes an increase of pressure at entrance and a lessening of the velocity of flow. A Francis turbine is classified as low pressure when the working head is less than 75 feet; the turbine is then installed in an open flume. With heads of from 75 to 180 feet, when a circular casing is used, it is termed medium pressure. A high-pressure turbine employs a head having a range of 150 to 550 feet, when it is provided with a spiral casing. A very great development has taken place in the first of these classes, especially in the direction of high-speed runners, of which *Fig.* 56 is a typical example.

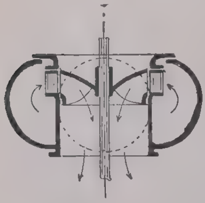


Fig. 55.

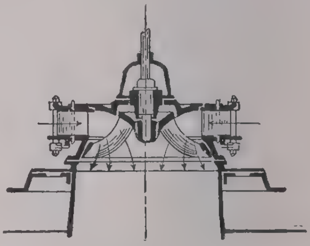


Fig. 56.

As an indication of the magnitude of the installations now becoming general that of the Laurentide Co., Quebec, may be cited, where plant consisting of six units each of 20,000 b.h.p. working with a head of 76 feet at 120 revolutions per minute and having a single vertical runner was laid down in 1915. At the other end of this class may be quoted the Chester Municipal power plant operating with an average head of 7 feet and consisting of 2 units of 415 and 305 b.h.p. and speeds of 50 and 55 revolutions per minute respectively.

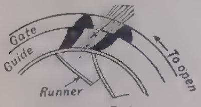
For medium-pressure plants the single-runner type seems to be gaining in favour on account of the high over-all mechanical efficiency obtained. Spiral casings are now being used instead of the circular form, and the largest output per unit is that of the Tallassee Power Co., U.S.A., where 31,000 h.p. is generated per runner at a speed of 154 r.p.m. under a net head of 180 feet, and having a guaranteed efficiency of 90 per cent.

The high-pressure Francis turbine has of late years raised its upper limits and invaded the field previously held exclusively by the Pelton wheel. Heads of 500/600 feet are now not uncommon, and the maximum reached is 745 feet. With high heads in order to ensure freedom from break-down the axial thrust is eliminated where possible by the use of double runners operating back to back, when no special thrust bearing is required. In the case of the single-runner type special balancing methods are necessary.

(iv.) *Suction Tubes.*—The success of the Francis type of turbine has been largely due to the use of the suction or draft tube which permits the plant to be erected at a convenient height above the tail race without loss of head and enables part at least of the energy contained in the velocity of the water as it leaves the runner to be converted into work. This is of the utmost importance in connection with high capacity runners where the discharge velocity of the water may represent 15 to 20 per cent of the total head, the over-all efficiency therefore depending upon its more or less complete recovery. The device which is due to Jonval consists in lengthening the discharge pipe until its lower end is always submerged in the water of the tail race, the surface of which is of course at atmospheric pressure. The pressure at the discharge side of the turbine blades will therefore be less than that at the surface level of the tail race by an amount equal to the difference of the static head between turbine and tail race and may approximate to 25 feet. The area of the suction tube at its connection to the turbine is made equal to the discharge area of the runner and gradually increases towards the outlet end, thereby converting part of the kinetic energy of discharge into pressure head.

(v.) *Vortex Turbine.*—Reference has been made to the spiral casing used in medium and high-pressure turbines. This was an improvement introduced by Professor James Thomson, together with a special form of guide vane. The water is brought tangentially into the large end of the spiral and is then directed by the curve of the casing through a series of movable guides pivoted near their inner end so as to follow the lines of flow in a spiral vortex. The guides are so connected by bell crank levers and links as to move simultaneously when acted on by the governor, and thus shut off water equally from all parts of the wheel. As fitted to the modern Francis turbine the number of guide vanes, which was formerly small, is now nearly equal to the number of wheel vanes; the guides are of a shorter type.

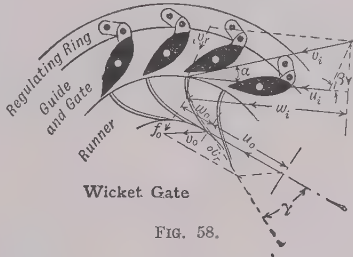
§ (50) SPEED REGULATION.—There are three types of construction employed to regulate the quantity of water admitted to the turbine, namely, (*a*) cylinder gate, (*b*) register gate (outside and inside), and (*c*) wicket gate. These have as their common feature the throttling of the supply to or of the discharge from the wheel, (*a*) being illustrated in *Figs.* 54 and 59, and (*b*) and (*c*) in *Figs.* 57 and 58. Both (*a*) and (*b*), though simple to operate, suffer from the disadvantage that, when controlling the inlet, the entering water after contraction at the edge of the gate expands again to fill the wheel passages. Eddy formation and consequent loss of energy ensue. In (*a*) this defect is sometimes modified by dividing the wheel by diaphragm plates (*Fig.* 54), so that it becomes a multiple wheel. The effects of the gate are thus confined to one compartment and part-gate efficiencies are well maintained.



Register Gate

FIG. 57.

Where throttling of the discharge occurs, the increase of pressure which results at the exit diminishes the effective head and causes a greater loss of kinetic energy. The method (*c*) has practically superseded the others by eliminating impact losses and the formation of eddies by reducing obstructions to the approach of the water to the runner. The wicket gate consists of a number of streamline vanes, each pivoted on its own spindle and receiving movement from a regulating ring to which it is attached by a short connecting link.



Wicket Gate

FIG. 58.

The method of operating the different types of regulators is usually by means of servomotors, and has already been discussed in the case of the Pelton wheel.

§ (51) SPECIFIC SPEED.—The very wide variations in the leading characteristics of head and flow of the natural water-power resources in existence, and the necessity of utilising each under its most favourable conditions, have given rise to an individuality in the design of turbines and made available for further development a great wealth of data. But in order to render the information useful for standardisation purposes it requires to be based on systematic tests and correct design. Consequently a fundamental basis of comparison is necessary. Now if H = head in feet, P = brake horse-power, N = speed in r.p.m., Q = quantity of water in cubic feet per minute, then for any given runner it follows that P varies as $H^{\frac{3}{2}}$, Q as $H^{\frac{1}{2}}$, N as $H^{\frac{1}{2}}$.

If P_1, Q_1, and N_1 are the values obtained when the effective head is equal to unity, then

$$P_1 = \frac{P}{H^{\frac{3}{2}}}, \quad Q_1 = \frac{Q}{H^{\frac{1}{2}}}, \quad N_1 = \frac{N}{H^{\frac{1}{2}}},$$

and these values serve to compare turbines of the same diameter and design.

With machines of different diameters and operating under different conditions, comparison is made by means of a factor known as the "specific speed," which indicates the speed at which a turbine would run when having an output of 1 b.h.p. under a head of 1 foot. Assuming P_1, Q_1, N_1 as above to be the characteristic values of a runner under a constant head of H_1 feet, if now the diameter of the wheel and all its other dimensions are varied proportionally the variation of P_1, Q_1, and N_1 may be determined.

Thus P_1 and Q_1 are both proportional to D^2 while N_1 varies as $1/D$.

Thus if P_1 be the output from a wheel of diameter D_1, D_s the diameter of a wheel which develops 1 h.p. is obtained from the equation

$$D_s^2 : D_1^2 = P_s : P_1, \quad \text{or } D_s = \frac{D_1}{\sqrt{P_1}}.$$

since $P_s = 1$;

and since $N_s : N_1 = D_1 : D_s$,

$$\therefore N_s = N_1 \sqrt{P_1} = \frac{N}{H^{\frac{1}{2}}}\left(\sqrt{\frac{P}{H^{\frac{3}{2}}}}\right) = \frac{N\sqrt{P}}{\sqrt[4]{H^5}}.$$

Where more than one runner per turbine is used the output P is that of each runner.

The use of specific speeds as the basis of comparison enables runners of apparently different characteristics to be compared and development in the direction of the most efficient to take place. As a consequence there has been evolved a slow-speed type of runner using a small quantity of water under a large head, and a high-speed type using a large quantity of water with a low head, an example of the latter being shown in *Fig.* 56.

The prevailing tendency in recent years has been to increase the specific speeds and to develop the high capacity runner, and progress has been so marked that the average efficiency as obtained from published tests has risen during the last twelve years by about 10 per cent, and in several low- and medium-pressure turbines of the Francis type the measured efficiency has exceeded 93 per cent for the former and 92 per cent for the latter.

§ (52) PRESSURE TURBINE THEORY. (i.) *Relations of Vane Angles.*—Let the velocity of the water as it leaves the guides be v_i, making an angle α with the tangent to the inlet circumference of the wheel (*Fig.* 58). Let u_i and u_o be the inlet and outlet peripheral velocities of the wheel; w_i and w_o be the corresponding whirl velocities, and f_i and f_o the radial velocities of the water. Then

$$f_i = w_i \tan\alpha = (w_i - u_i) \,.\, \tan\beta,$$

$$u_i = w_i\left(1 - \frac{\tan\alpha}{\tan\beta}\right),$$

$${}_iv_r = f_i \operatorname{cosec}\beta,$$

$$f_o = (u_o - w_o)\tan\gamma,$$

$${}_ov_r = f_o \operatorname{cosec}\gamma,$$

and if $w_o = 0$, then $v_o = f_o$.

(ii.) *Work done in Runner.*—

The initial moment of momentum $\Big\} = \dfrac{QW}{g} w_i r_i$.

The final moment of momentum $\Big\} = \dfrac{QW}{g} w_o r_o$.

The moment on the runner = change of moment of momentum

$$= \frac{QW}{g}(w_i r_i - w_o r_o).$$

$$= \frac{QW}{g} w_i r_i, \text{ if } w_o = 0.$$

The work done on runner = moment on wheel × angular velocity

$$= \frac{QW}{g} w_i r_i \omega = \frac{QW}{g} w_i u_i$$

ft.-lbs. per second.

$$= \frac{w_i u_i}{g} \text{ ft.-lbs. per lb. of water.}$$

In a moving wheel the energy at entrance = work done per lb. of water + losses through the wheel + head at discharge.

If all losses in the wheel are neglected we have by the equation of energy

$$\frac{p_i}{W} + \frac{v_i^2}{2g} = \frac{p_o}{W} + \frac{v_o^2}{2g} + \frac{w_i u_i}{g} = H.$$

But if $w_o = 0$, then $v_o = f_o = f_i(a_i/a_o) = w_i \tan\alpha(a_i/a_o)$, and

$$\frac{p_i}{W} + \frac{w_i^2 + f_i^2}{2g} = \frac{p_o}{W} + \frac{f_o^2}{2g} + \frac{w_i u_i}{g}$$

$$= \frac{p_o}{W} + \frac{f_o^2}{2g} + \frac{w_i^2}{g}\left(1 - \frac{\tan\alpha}{\tan\beta}\right)$$

$$= \frac{p_o}{W} + \frac{w_i^2}{2g}\left(\tan^2\alpha \frac{a_i^2}{a_o^2} + 2 - 2\frac{\tan\alpha}{\tan\beta}\right)$$

$$= H.$$

$$\therefore w_i = \sqrt{\frac{2g(H - (p_o/W))}{2(1 - (\tan\alpha/\tan\beta)) + (\tan\alpha(a_i/a_o))^2}},$$

and

$$u_i = 1 - \frac{\tan\alpha}{\tan\beta}\sqrt{\frac{2g(H - (p_o/W))}{2(1 - (\tan\alpha/\tan\beta)) + (\tan\alpha(a_i/a_o))^2}}.$$

Since $u_i = w_i(1 - (\tan\alpha/\tan\beta))$, if β is increased and u_i maintained constant, then w_i diminishes and more energy remains in the pressure form at the inlet to the runner. The tendency is therefore to make $\beta = 90°$. If $(H - (p_o/W)) = H_1$ = available head, then with this angle $w_i = \sqrt{gH_1}$ approximately, that is, $w_i^2/2g = H_1/2$, and half the available energy is in kinetic form and the other half in pressure.

Theoretical efficiency

$$\eta = \frac{(WQ/g)w_i u_i}{WQH_1} = \frac{w_i^2(1 - (\tan\alpha/\tan\beta))}{gH_1}$$

$$= \frac{w_i^2(1 - (\tan\alpha/\tan\beta))}{g(w_i^2/2g)\{2(1 - (\tan\alpha/\tan\beta)) + \tan^2\alpha \,.\, (a_i^2/a_o^2)\}}$$

$$= \frac{2(1 - (\tan\alpha/\tan\beta))}{2(1 - (\tan\alpha/\tan\beta)) + (\tan^2\alpha \,.\, (a_i/a_o)^2}.$$

(iii.) *Runner Discharge Angle.*—If the velocity of whirl w_o at discharge be assumed to be 0, the following consideration will determine the discharge vane angle γ:

$$\tan\gamma = \frac{f_o}{u_o} = \frac{f_i a_i}{u_o a_o} = \frac{w_i \tan\alpha}{u_o} \cdot \frac{a_i}{a_o},$$

$$u_o = u_i \cdot \frac{r_o}{r_i}, \quad \text{and if } \beta = 90°, \text{ then } w_i = u_i,$$

$$\therefore \tan\gamma = \frac{r_i a_i}{r_o a_o}\tan\alpha.$$

If the radial flow is constant $a_i = a_o$, and therefore $\tan\gamma = (r_i/r_o) \cdot \tan\alpha$.

(iv.) *Change of Pressure through the Runner.*—The total difference of pressure between the inlet and the outlet circumference of the runner will be equal to the change of pressure resulting from the motion of the water particle in a forced vortex with an angular velocity ω added to the change of pressure due to its motion parallel to the vanes with a variable relative velocity v_r. Thus

$$\frac{p_i - p_o}{W} = \frac{u_i^2 - u_o^2}{2g} + \frac{{}_o v_r^2 - {}_i v_r^2}{2g},$$

$${}_o v_r = f_o \operatorname{cosec} \gamma, \quad {}_i v_r = f_i \operatorname{cosec} \beta, \quad u_o = u_i \frac{r_o}{r_i},$$

$$\therefore \frac{p_i - p_o}{W} = \frac{u_i^2}{2g}\left(1 - \frac{r_o^2}{r_i^2}\right) + \frac{f_o^2 \operatorname{cosec}^2 \gamma - f_i^2 \operatorname{cosec}^2 \beta}{2g}.$$

In *inward flow* runners the difference of pressure tends to increase as the peripheral velocity of the runner increases so that the turbine becomes to a certain extent self-governing. This effect will increase as the ratio $r_i : r_o$ increases. When the direction of flow is reversed as in the *outflow runner* the expression $p_i - p_o/\mathrm{W}$ becomes negative and increases if a sudden reduction of the load causes the speed to increase. The supply of energy, therefore, increases when it is desirable that it should diminish and hunting ensues. This is accentuated as the ratio $r_i : r_o$ is made greater.

The general expressions for efficiency hold good in the case of the *axial-flow turbine*, excepting that the peripheral velocity u_i at inlet equals the peripheral velocity u_o at outlet since $r_i = r_o$, and there is consequently no change of pressure due to centrifugal action.

§ (53) MIXED FLOW OR AMERICAN TYPE TURBINE.—A type of turbine which makes no claim to special efficiency, but which has the merit of lowness of first cost is illustrated in *Fig.* 59. The wheel vanes are curved both laterally and axially, the path of the water through the runner being approximately the quadrant of a circle. A large discharge area is thus secured and the wheel is capable of dealing with large volumes of water, but the extra depth of wheel required at inlet results in a lowering of the part-gate efficiency. A higher peripheral velocity is generally adopted for this type, varying from ·7 of the theoretical velocity of the water with full gate to ·6 at half-gate for maximum efficiencies.

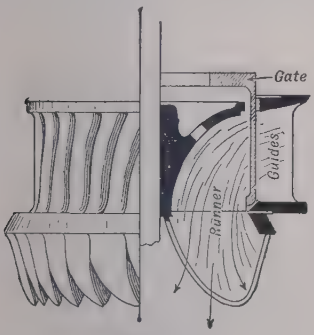


FIG. 59.

A difficulty exists in determining the correct inclination of the vane at various discharge points as the velocity of flow across the outlet area is not equal. This is due to the frictional resistance to flow varying as the length of the path traced out by the water and to the varying centrifugal pressure induced. It is usual in practice to adopt a mean discharge angle on the assumption that the outflow is uniform over the discharge section.

The guides are usually fixed and the regulation is entirely effected by gate or ring sluice.

The best machines of this type when working under their most favourable conditions have an efficiency of about ·8 at full gate, when they compare quite well with the Francis turbine, but are at a decided disadvantage when working at half-gate, the efficiency falling to about ·65.

§ (54) PRESSURE ENGINES.—The class of motor described as pressure or hydraulic engines is important where a supply of high-pressure water is available and intermittent motion at moderate speeds is desired. This is especially the case when the load is more or less constant, though the variation in speed may be considerable. Under such conditions the reciprocating piston engine is to be preferred to any kind of rotary motor.

The two most successful types which have been evolved have three single-acting cylinders set radially, their centre-lines intersecting at angles of 120°. In one of these the cylinders are fixed and are fitted with trunk pistons; the connecting rods act on a single crank pin and drive the crank shaft. The outer end of each cylinder—the power stroke is inwards—can be connected alternately to the pressure supply and the exhaust through a passage controlled by a disc valve which rotates with the crank shaft. This is the well-known Brotherhood engine, and is shown in *Fig.* 60.

In the other type known as the Rigg engine the three cylinders are arranged round and pivoted on a fixed hollow crank pin A. Two ports on the surface of A serve to connect the cylinders in turn as they rotate to the pressure supply and exhaust respectively. The driving shaft B of the machine rotates about a centre which does not coincide with that of A; the distance between these centres gives the throw of the crank.

A disc fixed to the driving shaft carries three pins, C_1, C_2, C_3, and the piston rod ends are connected to these pins. The power stroke is outwards and occurs as the cylinder rotates from D to E. For any position on the semicircle DC_2E the force exerted by the piston on a pin such as C_2 has a moment in the direction of rotation about the centre of the shaft B. The shaft and disc are thus kept in rotation.

In their simple form, however, both types labour under the disadvantage that if the load is reduced the quantity of pressure water consumed remains constant and the efficiency falls at a very rapid rate. For the economical transmission of power whilst obtaining power control at constant speed or speed control with constant power, some method of using the water expansively or varying the piston displacement becomes a necessity.

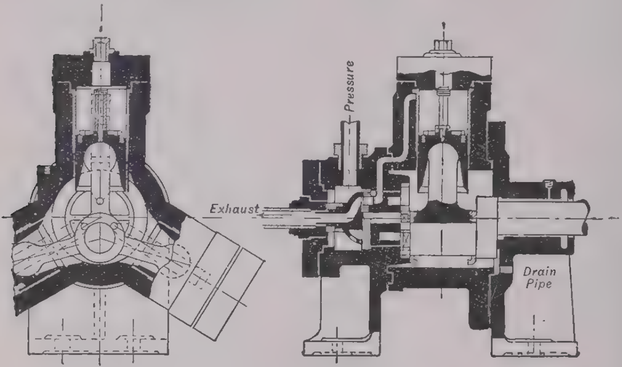


Fig. 60.

Various devices have been tried to overcome the difficulty. These include cutting off the high-pressure supply before the end of the stroke and using an auxiliary low-pressure supply to complete it, and also the use of an air chamber similar to that adopted in pumps to meet inertia effects.

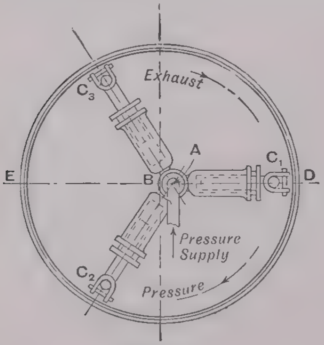


Fig. 61.

The most successful method has been by varying the stroke. In the Brotherhood engine this was effected for a time by the Hastie regulating device, but was afterwards discontinued. In this device the rotation of a cam shaft relative to the hollow crank shaft in which it worked caused a cam to vary the crank radius and adapted the volume of the piston displacement to the demand for power.

With the Rigg type the eccentricity of the fixed crank pin about which the cylinders revolve relative to the driving shaft is capable of adjustment. As the centres diverge from each other the stroke increases, its magnitude being twice the eccentricity, and if the divergence be opposite in sign a contrary direction of rotation results. A supplementary engine or servo-motor operates the change of position of the crank pin and is controlled by the governor.

A full-power efficiency of 80 per cent at speeds from 200 to 300 revolutions per minute has, it is stated, been obtained with this type of motor.

IV. Hydraulic Transmission of Energy and Applications

In the previous sections consideration has been given to the methods by which a supply of water possessing potential or kinetic energy, either inherent or acquired, may be made to do work by setting prime movers in motion. The motion or force so imparted is then usually transmitted through trains of mechanism or machines to where resistance may be usefully overcome.

There are, however, examples which remain to be discussed of the direct application to the working machine of the energy of the water, and also the very important case where advantage is taken of their physical properties in order to use fluids as elements in a train of mechanism.

Under all circumstances it is desirable that the supply of fluid should be brought to the machine with the least possible loss of energy and in the most favourable condition for the operation to be carried out.

§ (55) Capacity of a Pipe Line.—The energy transmitted through a given pipe line varies directly as the velocity of flow and the pressure, but as the frictional resistance of the pipe varies as the square of the velocity the best conditions for power transmission will obviously be low velocities and high pressures. Such conditions render the hydraulic system readily adaptable to machines such as presses, lifts, and cranes in which the motion is comparatively slow and intermittent, but where the force required to be exerted is large.

The over-all efficiency of hydraulic transmission, including losses at the central station and on conversion into work, is generally about 70 per cent, but much will depend on the length of the pipe line. In the pipe line itself the theoretical efficiency may be determined as follows:

Let p = pressure at inlet in lbs. per sq. in.
d = diameter of pipe in feet.
l = length of pipe in feet.
a = area of pipe in sq. ft.
v = velocity of flow in feet per second.
W = weight of 1 cub. ft. of fluid.
Q = discharge in cubic feet per second.
m = hydraulic mean depth.

Energy entering pipe per sec.

$$\begin{aligned}&=H=\text{total pressure}\times\text{velocity}\\&=(144\,pa)v\ \text{foot-lbs.}\\&=\cdot262\,pav\ \text{H.P.}\end{aligned}$$

If h is the head required to overcome the resistance to flow of the pipe, W the weight of a cubic foot, and Q the discharge in cubic feet per minute (see Part I. § (24) (ii.) and (iii.)), then

$$\text{Loss of energy per second}=H_f=h\frac{WQ}{550}\ \text{H.P.}$$

$$=\frac{flv^2}{2gm}\cdot\frac{WQ}{550}\ \text{H.P.}$$

$$=\frac{flv^2}{2gm}\cdot\frac{Wav}{550}\ \text{H.P.}$$

$$\text{Efficiency}=\gamma=\frac{H-H_f}{H}=1-\frac{flv^2}{2gm}\cdot\frac{W}{550}\cdot\frac{1}{\cdot262p}.$$

$$\text{But}\qquad v=\frac{H}{\cdot262pa};\quad g=32\cdot2;\quad m=\frac{d}{4}.$$

Therefore by substituting

$$\gamma=1-\cdot635\cdot\frac{flH^2}{p^3d^5}.$$

It will be observed that the conditions for high efficiency include:

(*a*) A small value of H which may be obtained by keeping down the velocity of flow and preventing it exceeding, say, 4 feet per sec.

(*b*) A high value of p and d. A limit is put to the range of these factors by the consideration that the saving effected by the increased efficiency will ultimately be more than balanced by the additional cost of the greater diameter and thickness of pipe line required, and by the difficulty of preventing leakage at joints. For these reasons p is usually limited to 1200 lbs. per sq. in., and d to 6 in. Multiple pipes are used where the power required is greater than the capacity of such a pipe.

The energy actually transmitted by the pipe line

$$\begin{aligned}&=\gamma H\\&=\left(H-\frac{\cdot635flH^3}{p^3d^5}\right)\text{H.P.}\end{aligned}$$

and this will be a maximum if

$$H=\cdot725\sqrt{\frac{p^3d^5}{fl}}$$

when it will have the value

$$\cdot483\sqrt{\frac{p^3d^5}{fl}}\ \text{H.P.}$$

This gives an efficiency under these conditions of ·66.

Hydraulic power supply departments have been established by many of the large Municipal Corporations, the pressures adopted varying from 700 to 1600 lbs. per sq. in. The water used is generally from the town's public supply, and its initial pressure is in some cases utilised to work an intensifying pump by which a small proportion of the total quantity is forced directly into the high-pressure main. The remainder is passed to a storage tank, from which it is pumped by steam-driven pumps into a main pipe feeding the accumulators which are the reservoirs for the power supply.

For heavy, cumbersome, and slow operations such a supply is pre-eminently suitable, and the system has the merits of cheapness and direct applicability without any intricate mechanism, and of ease in detecting leakage. The principal objections urged against it are the trouble with air-locks and the danger of bursting during frosty weather, necessitating the draining of cylinders when not in operation or external heating at vulnerable points. In spite of these drawbacks hydraulic transmission of energy is now extensively adopted.

§ (56) PIPE-LINE APPLIANCES. (i.) *Accumulators.*—Although the supply of energy from the pumps is designed to equal the over-all demand for power over a considerable interval of time, the demand is usually intermittent owing to the nature of the operations which the motors are called upon to perform. As it is desirable to keep the pumps in continuous operation some form of storage is necessary, and the accumulator was devised by Sir W. G. Armstrong to provide this and at the same time to regulate the delivery pressure. It acts automatically.

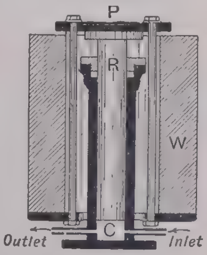


FIG. 62.

Its general form is illustrated in *Fig.* 62

and consists of a vertical cylinder C with inlet and outlet passages at its base. Through its cover a ram R passes and supports at its upper end an external platform P. From this is suspended a load W of some heavy material such as pig-iron or iron scrap.

If L is the displacement of the load W, and A is the area in square inches of the ram, then the potential energy stored in the cylinder when the ram is at its upper limiting position is WL ft.-lbs., and the working pressure p equals W/A lbs. per sq. in.

(ii.) *Tweddell's Differential Accumulator.*—This is a modification of the above arrangement whereby high pressures may be maintained by comparatively small loads, but the storage capacity is low and it is only suitable for supplying single tools. The cylinder is inverted and movable, as shown in *Fig.* 63, and carries the load W on its external surface. The ram is fixed to the base and is of two diameters, D_1 and D_2, where it passes through glands at top and bottom respectively of the cylinder. The inlet water enters where the ram is stepped, and if the difference of areas is equal to A, then, as before, $pA = W$. If $D_2 - D_1$ is made small enough it is possible to obtain high values of p for a moderate weight W.

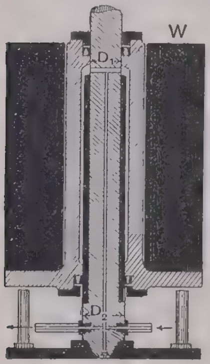


Fig. 63.

(iii.) *Pressure-loaded Accumulators.*—These are used where it is inconvenient to have a heavy dead load. In this case the upper end of the ram works in another closed cylinder and carries a piston subjected on its upper face to steam pressure. Adjustment of the relative areas of the piston and plunger, together with the introduction of a reducing valve regulating the pressure of the steam, ensures the maintenance of the required hydraulic pressure. While the accumulator, speaking generally, has a steadying effect on the pressure, the inertia of the weighted type is productive of considerable shock when the outward flow of the fluid is suddenly stopped. On occasions this may be advantageous, as for example in a hydraulic riveter where the momentary increase of pressure may be utilised to effectively clinch the rivet. To guard against abnormal pressures from this cause a spring-loaded relief valve is often introduced, and operates when the pressure exceeds the normal value by an arranged amount. This precaution is not necessary when the load is applied by the action of steam pressure.

(iv.) *Intensifiers.*—Where the pressure supply from the accumulator is insufficient for the working of a particular machine, a device known as the intensifier is introduced. A common form, as applied to testing machines, is shown in *Fig.* 64, and is similar to an inverted steam accumulator. The ratio of the areas of the piston and ram equals A/a, and is equal to the ratio of the intensifier pressure P to the pressure supply p.

Thus $P = p(A/a)$ lbs. per sq. in. if the friction F of the glands and the weight W of the piston and ram are neglected. If these are taken into account

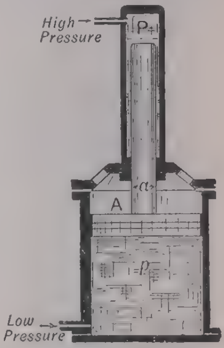


Fig. 64.

$$P = p \cdot \frac{A}{a} - \frac{W+F}{a} \text{ lbs. per sq. inch.}$$

This type only operates in one direction, and if a continuous supply of high-pressure water is required, duplicate intensifiers working alternately must be employed.

§ (57) Hydraulically-driven Machines.—The direct applications of hydraulic power to the working of modern machinery has been extensive and varied. Perhaps the most successful examples are the modifications of the Bramah press as developed in baling presses, plate flanging and heavy forging machines, all of which require a slow but powerful compression, and may be regarded simply as reversed accumulators. The cycle of operations in each case may be divided into stages :

(*a*) An idle stage during which the head is brought up to its work.

(*b*) An initial compression of the material, but full power not developed.

(*c*) The completion of the compression under full power.

(*d*) The sudden stoppage of the head producing inertia effects which may be useful in certain types of machines.

(*e*) The return of the ram to its initial position.

It will be seen that during the first two stages the full value of the pressure, if there is only one supply, is not being utilised though a volume of water equal to the displacement of the ram is used. A loss of efficiency results which is increased if there is only one common inlet and outlet passage to the cylinder, since this passage requires to be refilled with pressure water at the commencement of each operation. As the quantity of water used for a given displacement cannot be varied, the only alternative is to vary the operating pressure from stage to stage. Two different methods of effecting this in the case of the Forging Press are shown.

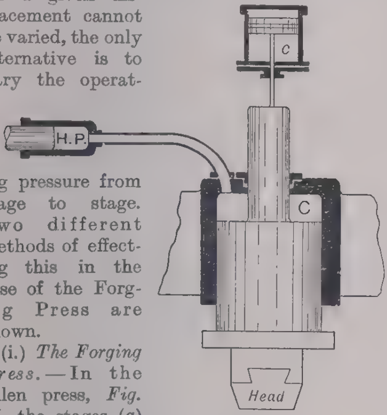


FIG. 65.

(i.) *The Forging Press.*—In the Allen press, *Fig.* 65, the stages (*a*) and (*b*) are carried out in connection with a low-pressure supply, which is cut off when full pressure is required during stage (*c*). Communication is then made with a high-pressure pump (HP) working without valves and imparting a to-and-fro motion to the connecting column of liquid. The inertia of the latter increases the pressure at the end of the working stroke. The water is then exhausted from the cylinder C and the ram and head lifted by steam pressure acting on a piston in the auxiliary cylinder C above.

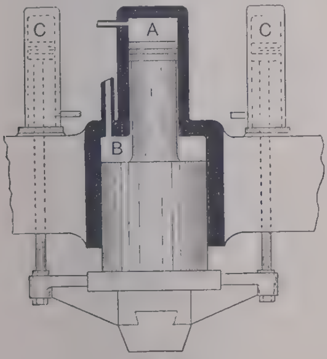


FIG. 66.

The other method is illustrated in *Fig.* 66, which shows how total pressures of three different magnitudes can be obtained by the use of a differential ram. Thus when the high-pressure water is admitted to chamber A, a force is available at the working head sufficient for stage (*a*). With the pressure supply in communication with chamber B, and A cut off, operation (*b*) is carried out, while full power is obtained when both A and B are connected with the supply. The head is brought back and the water exhausted by auxiliary pistons actuated by a low-pressure supply and working in two lifting cylinders C. In machines of this type it is possible to use supply pipes of comparatively small diameters and still maintain a fairly high efficiency. This is due to the low speed of operation and the consequent small amount of energy transmitted.

(ii.) *Riveters.*—The hydraulic riveter is a typical example of water power adapted to use in machine tools. Portable machines are constructed both for light and heavy duties, and are of the "hinged" and "bear" forms. *Fig.* 67 shows the ordinary "fixed-jaw" or "bear" type, with a pressure supply at (*a*) to the main ram A which acts directly on the rivet, and at (*b*) to a central drawback ram B. The body of the machine is arranged to turn by means of a worm and worm-wheel round a cast-steel hanger C, and to permit this a water-tight swivel joint is fitted at D.

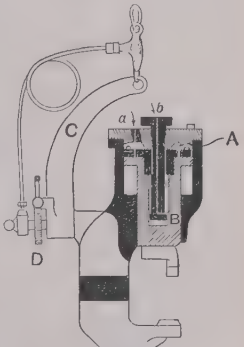


FIG. 67.

The hinged type for use in a restricted space is illustrated in *Fig.* 68, and shows the ram and riveting tool at opposite sides of the hinge. The drawback ram B is here arranged eccentrically.

There is also a class of fixed riveters where the work to be riveted is slung and movable. A gap suitable for the kind of work to be done is left between the jaws, which may be either horizontal or vertical. The largest machines are used for riveting the seams of marine boiler shells, and the gaps may be from 9 to 11 ft. The total pressure exerted in such a tool may be arranged to vary from 25 to 100 tons with an intermediate stage of 75 tons, so that high-pressure water may be saved in

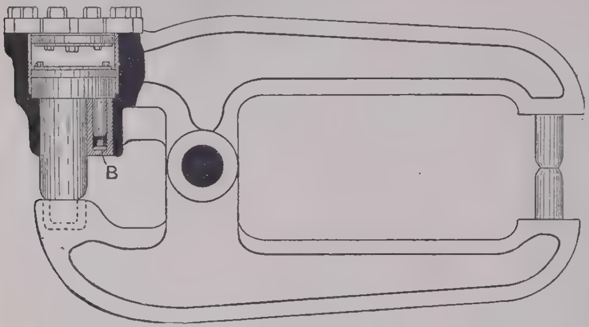


Fig. 68.

the first stages of the operation. A pressure diagram from the cylinder of a hydraulic riveter is shown in *Fig.* 69 and illustrates the use made of the variable power. During the period AB the ram is brought up to its work under low pressure, and during BC with an intermediate pressure the rivet head is formed. Full power is exerted from C to D for the closing of the plate and the clinching of the rivet, the latter operation being assisted by the rise DE, which is an inertia effect consequent on the sudden stoppage of the accumulator ram.

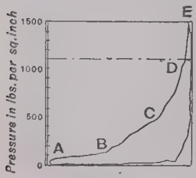


Fig. 69.

§ (58) Hoists and Lifts.—The ordinary direct-acting hydraulic hoist is very similar in its arrangement to the common type of press, but the ram cylinder is sunk in the ground to a depth somewhat greater than the travel of the ram which carries the platform and cage. If only the static load producing direct compression is taken into account, the area A of ram which would be sufficient to support a gross load of W lbs., including useful load and weight of platform and ram, when working with a supply pressure of p lbs. per sq. in., would equal W/p sq. in. This requires to be increased because of the stress induced when acceleration takes place, and also because of the additional force required to overcome friction. It follows then that the area should be made equal to $\{W[1+(a/g)]/p\}+F/p$ square inches, where a is the acceleration and F the friction.

If the travel is great, when the cage is at the top position the ram which acts as a column becomes liable to buckle and the area must be increased to guard against this danger. This increase of area involves a corresponding greater lifting power if the pressure p is retained, and calls for a further modification in the design.

(i.) *Balancing of Hoists.*—Every operation of the direct-acting hoist includes the lifting of the dead load, which is large as compared with the useful load, and the efficiency is therefore low. This is obviated by some means of continuously balancing the weight of the ram and cage. One method is the use of counterbalance weights, attached to the top of the cage by chains or wire cables passing over guide pulleys, which fall as the cage rises. This increases the force necessary to produce acceleration, since the mass is thereby increased, and has the disadvantage that it tends to put the upper portion of the ram in tension, which would result in the cage crashing to the top of the lift shaft should a fracture occur.

Another and better arrangement is shown in *Fig.* 70, where the lift cylinder is supplied from a balancing cylinder C in which there works a hollow ram B. To the interior of the latter is admitted a high-pressure water supply through the central passage A, and the hydraulic pressure on the ram in the balance cylinder due to this would be sufficient to lift the effective load. To balance the dead load the ram may be designed so that an extension of it forms an external platform carrying ring weights inducing a pressure sufficient for the dead load. The illustration, however, shows the balancing force obtained by superposing an inverted cylinder D over the plunger-like extension of the ram and supplying the annulus between with water from a low-pressure tank. During the down journey of the lift this balance water is returned to its tank while the high-pressure water is exhausted to waste and forms the only loss. The saving thus effected may be as high as 75 per cent.

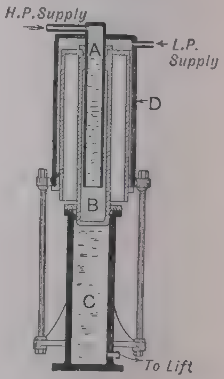


Fig. 70.

Where the displacement required is great, as in lifts installed in public buildings, the suspension type of lift (*Fig.* 71) is used, and a comparatively short stroke of hydraulic ram is multiplied to the requisite travel of the cage by means of a jigger.

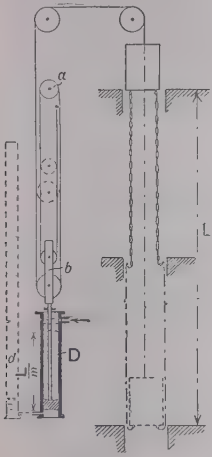


FIG. 71.

(ii.) *Jigger.*—This consists of a system of pulleys arranged in two blocks, the upper one (a) fixed in position, the lower one (b) having the motion of the ram. The wire rope usually employed is passed over the sheaves of the lower and upper blocks alternately, and one end being fastened to the upper block and the other led over guide pulleys, is attached to the top of the cage. The multiplying factor m or the ratio of the motion of the cage to that of the ram is equal to the number of ropes supporting the bottom block. The load to be carried may be distributed over two or more ropes, but the value of m is unaffected.

(iii.) *Balancing of Lifts.*—For any great degree of efficiency the dead weight of the cage must be balanced, and this is done by the use of balance weights suspended by wire ropes carried over guide pulleys and attached, like the lift ropes, to the top of the cage. A considerable variation in the effective weight apart from any change in live load occurs, due to the transfer of rope weight from the ram to the cage side of the suspending pulleys as the cage rises and falls. Two alternative methods of balancing this variation are shown in *Fig.* 71. The simplest is where one end of a balance chain of length equal to half the travel of the lift is fastened midway up the shaft and the other end to the bottom of the cage. When the cage is at its lowest position the weight of the chain is entirely carried by the fixed fastening, and, when halfway up, equally by shaft wall and cage. The weight thus thrown on cage is designed to equal the weight of rope transferred to the ram side of the guide pulleys.

If w_1 be the weight of the balance chain, and w the weight of the suspending rope in pounds per foot run, then in order that the dead load shall remain constant

$$w_1 = w\left(1 + \frac{1}{m}\right).$$

The other alternative is the use of a water column compensator d connected to the bottom of the ram cylinder D, as shown in dotted lines. When the ram is at its highest position the cage is at its lowest and the water-level in d is least. As the cage rises and the weight of suspended rope is diminished water is displaced from D to d, and its head decreases the effective pressure of the ram in a corresponding degree.

If L be the lift of the cage, L/m the stroke of the ram, D and d the diameters of the ram and compensator cylinders respectively, then the height h of the compensator will equal $\mathrm{L}/m(\mathrm{D}^2/d^2)$, and its balancing head will have a total difference of $h + \mathrm{L}/m$ feet. The maximum variation of weight to be balanced will equal $w\mathrm{L}(1+1/m)$, and from this data the diameter d may be determined.

The suspension system necessitates adequate safeguards against the fracture of a rope, and suffers in comparison with the direct lift owing to the inefficiency of the jigger, the loss varying directly as the value of m, but it is a much more compact and convenient arrangement in use.

The usual speed for lifts, having regard to the comfort of passengers, is about 2 ft. per second, but for express service in America a speed as high as 8 ft. per second is not uncommon.

§ (59) CRANES.—The principle of the hoist and jigger is extended to the handling and raising of goods by cranes, the cage being replaced by a hook from which the load is suspended. Separate rams and cylinders are employed to effect through jiggers the operations of lifting, slewing, and racking, each under independent control. In special appliances, such as the hydraulic coal tip, the combination includes a direct-acting hoist, capable of lifting at the dockside a railway waggon weighing 10 to 20 tons to a height sufficient to enable its contents to be discharged through a shoot into the hold of a vessel suitably moored. The main operation of lifting the platform carrying the load is by means of direct-acting rams, while the operations of tipping and working the cranes connected are all performed by hydraulic jiggers.

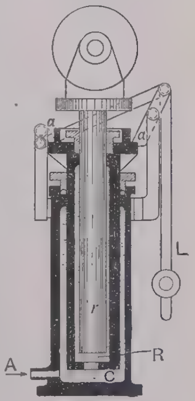


FIG. 72.

An important feature in cranes is the method adopted where the load varies within wide limits in order to economise water at light loads. A differential ram of the type shown in *Fig.* 72 is used, high-pressure water being admitted at A. A solid ram r fits inside a hollow ram R working in a cylinder C. It is arranged that if the load is a light one R may be locked to C by a catch a and the lifting done by r. For heavy loads a is released by an upward movement of the lever L, r and R move together, and the lifting area is then that of R.

§ (60) HYDRAULICALLY-BRAKED MACHINES.—This class of hydraulic machine is the converse of those just described, and has for its primary object the rapid but gradual absorption of energy from a moving body and its dissipation with the least possible shock. Its essentials are contained in the ordinary dashpot used for damping the oscillations of various mechanisms. This device consists of a cylinder fitted with a piston and rod and filled on either side of the piston with water or some more viscous fluid and the ends connected together by constricted passages. Any displacement of the piston is accompanied by a transference of the fluid from one end to the other, and the rate at which this is effected governs the speed of the piston and the vibrations of the mechanism to which it is attached.

A high velocity is imparted to the displaced fluid during its flow through the constricted passage, and energy is dissipated partly in eddy formation, partly in overcoming fluid friction, and partly in overcoming the mechanical friction of the device. It is usually desirable that the resistance should be uniform throughout the piston displacement, and to ensure this the velocity of flow upon which the resistance mainly depends must be kept constant. Since the rate of displacement diminishes as the moving body is gradually brought to rest it is necessary to reduce the area of the connecting passage proportionally.

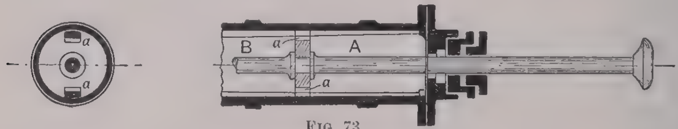


FIG. 73.

(i.) *Buffer Stop.*—A typical example of the use of this on a large scale is the buffer stop for railway work, as shown in *Fig.* 73. The connecting passages are two rectangular slots a, cut in the piston and working over tapered longitudinal strips of uniform width fitted to the cylinder. As the piston is displaced from A to B the area of the passages is proportionately reduced. In this arrangement the piston rod is designed to be in compression, and is continued through the rear end of the cylinder. Without the addition of this tail rod the volume of fluid displaced by the piston would be greater than the volume available on the other side by an amount equal to the volume of the piston rod when closing, and motion would be impossible unless a portion of the fluid was allowed to escape from the cylinder.

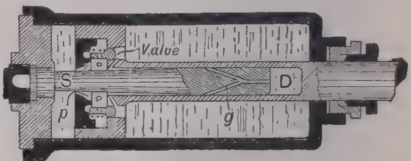


FIG. 74.

(ii.) *Gun-recoil Cylinder.*—Another application is the tension-recoil cylinder for a gun illustrated in *Fig.* 74. Here the annular connecting passage p works over a tapered circular spindle S fixed longitudinally in the cylinder, the area of opening varying with the diameter of the spindle, which also acts as a tail rod to the piston. Since in this case the piston rod is intended to be in tension, the absence of a tail rod would tend to produce a partial vacuum behind the piston and thereby increase the resistance to closing. The piston is returned in readiness for another operation by springs compressed during the forward stroke or by counterbalance weights, though with fixed structures a supply of pressure water, if available, may be used. A secondary dashpot D is introduced to damp the return oscillation, the central spindle then acting as the piston and the escape of the fluid taking place along grooves in its surface marked g. The general equation of energy for this class of machine is as follows :

A = net effective area of piston in square feet,
a = effective area of passages,
L = length of piston displacement in feet,
l = length of connecting passages,
v and V = velocities of fluid through passages and of moving body respectively,
wAL = weight of fluid displaced,
W = weight of moving body,

Work done in bringing body to rest $= \dfrac{WV^2}{2g}$

= Kinetic energy imparted to fluid
+ energy spent in overcoming fluid friction
+ energy spent in overcoming mechanical resistance F ;

$$\therefore \frac{WV^2}{2g} = AL\frac{wv^2}{2g}\left(1+\frac{fl}{m}\right)+F$$

$$= \frac{wV^2}{2g}\cdot\frac{A^3}{a^2}L\left(1+\frac{fl}{m}\right)+F.$$

§ (61) DYNAMOMETER.[1]—This braking mechanism serves not only the purpose of absorbing, but also of measuring the energy available at a rotating shaft. The standard Froude arrangement illustrated in *Fig.* 75 consists of a disc wheel A rigidly connected to the revolving shaft, and has a series of curved recesses or pockets E on both faces. The recesses are semi-cylindrical, with the dividing walls F contained by planes making an angle of 45° with the axis of the shaft and inclined forwards in the direction of motion. The casing B is mounted on, but free to rotate about, the shaft D and has, on its inner faces, pockets which are the complement of those in the disc. Water under pressure is led through the inlet G passing by ducts H in the dividing walls to the clearance space between the two sets of pockets, and thence into the pockets, where a vortex motion of the water particles is set up. The water is thus projected from the outer periphery of the impeller to that of the casing pocket and guided by its semicircular boundary back to the impeller. The clearance between the disc and the casing allows of a small escape of water from the pockets to the chamber C, the amount being controlled by the outlet valve. The change in the moment of momentum of the water about the shaft, which occurs as it passes from the impeller to the casing pockets, produces a moment acting on the casing equal to the moment in the shaft. This moment is balanced and the casing kept stationary by the moment of an external force acting on an arm projecting from the casing, and at right angles to the shaft. The magnitude of this moment is easily measured, and the energy absorbed per minute by the brake is equal to $2\pi \times$ measured moment (ft.-lbs.) $\times$ revolutions per minute of the shaft.

The external force consists of balance weights applied at the extremity of the arm, together with a rider weight sliding along the scale marked on the arm.

The brake is regulated automatically by having the casing connected to the outlet and inlet valves, so that a constant moment is preserved. If the speed of the shaft increases, causing an increased moment on the impeller, the casing receives a slight displacement, and this is made to close the inlet and open the outlet valves. This reduces the mass of water in the brake and, consequently, the moment of the casing. Oscillations due to variations of speed are damped out by a dashpot D connected to the brake arm.

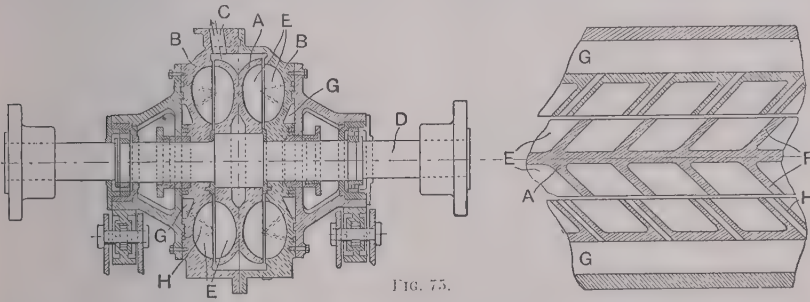


FIG. 75.

§ (62) HYDRAULIC TRANSFORMERS. — The possibility of using hydraulic elements efficiently in a train of mechanism has lately claimed considerable attention. The fluid is not regarded as possessing energy, but is viewed, like belting or gearing, simply as a means of transmitting it. Development has taken place in two distinct directions :

(*a*) The earlier and the commoner is based on the continuous flow or motion of a fluid column as a whole, and assumes the more or less complete incompressibility of the fluid, and

(*b*) the other depends upon the elasticity of the fluid, and utilises its resilience to propagate pressure waves and thus transmit energy from the generator to the point where force is to be applied.

Either system requires as essential elements a pump to impart energy to the fluid and a motor to transform it into work.

§ (63) TRANSMISSION BY FLUID MOTION. *Hydraulic Jack.* — This well-known machine may be taken as the simplest and oldest example of the hydraulic transformer, and is shown in *Fig.* 76. A plunger working in a pump chamber C, and operated by the reciprocation of a hand lever A, pumps water from a reservoir B, through suction and delivery valves v_s and v_d, into the ram cylinder D. The ram forms at its lower end the base upon which the jack stands. Leakage between the ram and its cylinder is prevented by a cup leather which makes a water-tight joint between them. The weight is carried either centrally on the top cover or eccentrically by a projecting claw near the base, and is lifted by the displacement effected by the water pumped into the ram cylinder. Any accumulation of air is removed through a small hole in the cover normally filled by a screw plug. Should the proper lift be exceeded water escapes through the opening O, and lowering is effected by means of a screw-in valve L which allows communication between the reservoir and the ram cylinder. The elements accordingly are a simple force pump and an accumulator.

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[1] See also article on "Dynamometers."

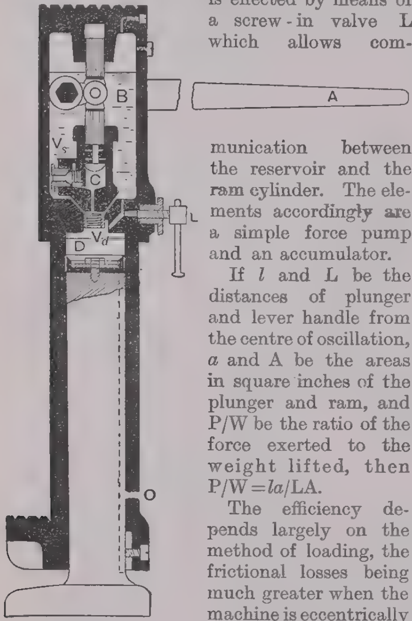

FIG. 76.

If l and L be the distances of plunger and lever handle from the centre of oscillation, a and A be the areas in square inches of the plunger and ram, and P/W be the ratio of the force exerted to the weight lifted, then $P/W = la/LA$.

The efficiency depends largely on the method of loading, the frictional losses being much greater when the machine is eccentrically loaded. These losses being largely mechanical, and not hydraulic, do not increase proportionately to the load, and hence large machines are more efficient than small ones.

§ (64) **Compagne Hydraulic Gear and Hele-Shaw Pump.**—An interesting application of energy transmission is the reducing gear of motor-car and other engines. A successful example is the Compagne Hydraulic Gear, which has as its important feature the well-known Hele-Shaw pump with variable stroke. The pump, which is driven by the prime mover, takes water or oil from a supply tank and delivers it with added pressure and kinetic energy to a hydraulic motor of the constant-stroke type, keyed to a secondary shaft. By varying the stroke of the pump while maintaining the speed, the discharge may be made to vary, and as a consequence the speed of the motor and its shaft.

The principle of the pump, which is of the rotary-plunger class, is similar to but the reverse of the Rigg engine, and is illustrated in *Fig.* 77. Multiple cylinders A with their axes set radially form a monobloc which is coupled directly to the engine shaft. The plungers C working in the cylinders carry gudgeon pins D which engage in and are guided by grooves in a floating ring F which is mounted on ball-bearings and rotates in a housing G. The housing is capable of a transverse motion across the casing, so that the axis about which the ring rotates may either coincide with that of the engine shaft or be eccentric to right or left of it. The stroke of each plunger with respect to its cylinder is equal to twice this eccentricity, and changes its relative direction when the axis of the ring passes through the concentric position. This results in a reversal in the direction of the flow without an alteration in the direction of rotation of the prime mover, and the change from full forward to full reverse discharge is made gradually and without shock. The working fluid is led to the cylinders by ports in a fixed stub-axle B which fits into the hollow end of the engine shaft, and is controlled by a circular rotary valve. It is drawn from the supply tank during the outward or suction stroke of the plunger through the passage H, and by this arrangement the effect of centrifugal force is to reinforce the supply pressure and thus prevent separation of fluid and plunger at high speeds. The interior of the floating ring is always flooded with the fluid, but the space between it and the casing is kept carefully drained to minimise disc friction. The friction of the gudgeon pins in their guides being greater than the resistance between the housing and the ring ball-bearings, causes the ring to revolve with the cylinders. This is an essential feature in the high efficiency of the pump, the frictional resistance being thus reduced to a minimum.

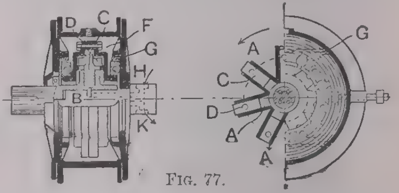

FIG. 77.

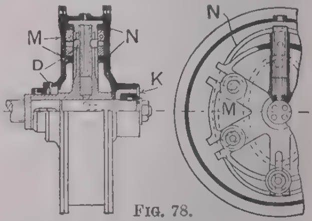

FIG. 78.

Discharge takes place through the passages K to the hydraulic motor, which is of similar construction to the pump, but of the constant-stroke type, and works inversely. It is coupled directly to the driving-wheel axle, and is illustrated in *Fig.* 78. Instead of the floating cage a cam N is used as a track or guide for the ball-bearing rollers M carried by the gudgeon pins D, and it is designed so that each plunger makes two strokes per revolution.

The over-all efficiency of a pump and motor transmitting 37 horse-power is stated to range from 75 to 85 per cent. The pump had a constant speed of 770 revolutions, and the variation in the speed of the motor is given as from 50 to 170 revolutions per minute. It seems likely that the advantages of a continuous-variable gear of this class will be better realised in connection with the transport of heavy loads in the future.

§ (65) Fottinger Gear.—Where the energy to be transmitted and the speed of working are of much greater magnitudes, a suitable form of hydraulic transformer is the Fottinger gear. Such conditions obtain in the case of the power plant for ship propulsion, where there exists the necessity for a high speed of turbine shaft together with a low speed of propeller shaft in order to secure the maximum efficiency of both. This involves some system of speed reduction, which in the case of slow-moving steamers may be as great as 20 to 1, and for fast boats 6 to 1. It is in connection with the latter ratios that hydraulic transformers have been found suitable. The arrangement (*Fig.* 79) consists of an impeller wheel A keyed to the after end of the primary power shaft, and a two-stage rotor wheel R mounted on the propeller shaft. The water passages of the impeller and rotor wheels, with the addition of a short length C of guide passage which is attached to the casing, form a closed circuit and are filled throughout with water. The rotation of the primary shaft sets up a pressure difference between the inlet and outlet of the impeller A as in a centrifugal pump, and flow is induced in the circuit. Part of the energy thus given to the water is absorbed by the motor in its first stage B. The water is then guided by the fixed passages C towards the second stage D, from whence it discharges into the inlet of the impeller A and the cycle is repeated. By a suitable design of the vanes the rate of flow through the rotor is very much less than in the impeller, and the speed of the secondary shaft thereby adapted to the efficient working of the propeller. The transformer is supplied with water at N from a low-pressure supply tank for the purpose of making good any leakage from the wheels.

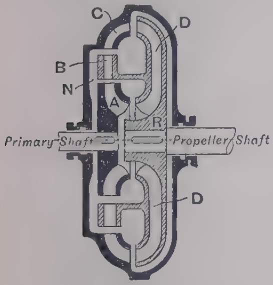

Fig. 79.

Where reversing requires to be provided for, a go-astern transformer similar to that described is incorporated, and the water of the transformer not in use is emptied into a drain tank, from which it is delivered by means of a small centrifugal pump into the supply tank. A manœuvring valve of the balanced piston type controls the opening to this tank of the transformer in action and to the drain tank of the other, simultaneously. The transmission ratio remains constant at all speeds, and a reduction of 6 to 1 has been successfully applied, the energy transmitted being 25,000 horse-power, and the efficiency stated to be 90 per cent.

§ (66) Transmission by Wave Motion.—With this system energy is transmitted from one point to another, which may be at a considerable distance, by means of impressed periodic variations of pressure producing longitudinal vibrations in a fluid column. The characteristics are analogous to those existing when a valve is suddenly closed in a long pipe line containing water in motion, and waves of alternate pressure and rarefaction are propagated throughout the length of the pipe. Where the principle is utilised for the transmission of energy, the pressure wave is initiated in the fluid by the outward stroke of a pump plunger operated by a prime mover or generator. The displacement of the plunger is resisted by the inertia of the fluid, a change of pressure occurs, and a pressure wave travels along the column; elastic deformation takes place and resilient energy is stored. At the distant or outlet end a motor of equal capacity and similar dimensions to those of the pump absorbs the energy contained in the fluid, and if all frictional losses are assumed to be negligible the motor plunger has a displacement equal to that of the pump. The return stroke of the pump is similarly followed by a wave of negative pressure or rarefaction, which on reaching the motor induces the reverse motion of the plunger. If properly synchronised, the further motions of the pump are followed by corresponding movements in the motor, and the energy available at the latter will be equal to the power of the pump diminished by the fluid friction in the pipe line and the resistance of the mechanical elements. The displacement volume of the plungers does not exceed the elastic volumetric deformation of the fluid, and the stroke is accordingly small. To enable, therefore, any considerable energy to be transmitted, the number of strokes per second is made large, and a typical 10-horse-power generator rotates at a speed of 2400 revolutions per minute, giving 40 wave impulses per second to a plunger of $1\frac{9}{16}$ in. diameter and $\frac{29}{32}$ in. stroke, the length of the pipe line being 240 ft. The maximum pressure reached under these conditions is 1500 lbs. and the mean about 750 lbs. per sq. in. The presence of air in the pipe line creates surges and fluctuating pressures, and valves for the release of air are provided both at the pump and the motor.

A capacity chamber communicating with the main pipe, filled with water and fitted near the pump, acts in a similar manner to the air-vessel in an ordinary reciprocating pump and equalises the pressure. Before starting, the pipe line is charged by an auxiliary water-pump, if a gravity feed is not available, to an initial pressure of about 100 lbs. per sq. in. Any free air in the system is allowed to escape, and the pump, driven by an electric or other convenient generator, is then started up.

Fig. 80 shows the arrangement of a portable duplex pump driven from a generator shaft developing 10 horse-power at 40 cycles per second. Two capacity chambers B, with a communicating passage P to equalise pressures, are fitted.

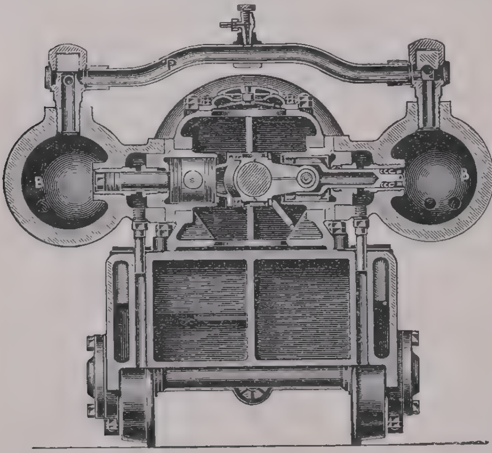

Fig. 80.

The motor may be similar to the pump but working inversely, or the energy may be taken off at various points along the pipe line by motors specially adapted to the tools to be operated.

It is obvious that in long pipe lines flexibility and the prevention of leakage of the fluid are factors of vital importance, and *Fig.* 81 shows a sectional view of the piping which has been specially designed for the relatively high alternating pressures.

The system described is that devised by Mr. G. Constantinesco, who first utilised the principle to procure synchronisation of machine-gun fire with the revolutions of aeroplane propellers by means of his C.C. Interrupter Gear, which enabled 2000 shots per minute to be fired between the blades of a propeller revolving at a high speed without the danger of the blades being struck. For the system as a means of power transmission are claimed the merits of great flexibility and perfect safety, while its efficiency is stated to greatly exceed that of compressed air or electricity. Its application on a practical scale is at present being demonstrated, and may be readily expected to yield important results.

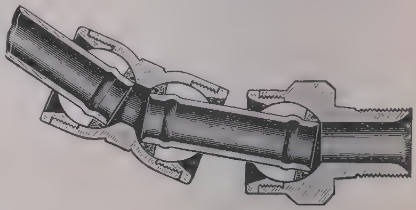

Fig. 81.

Hydrogen, Characteristic Constants of, tabulated. See "Thermal Expansion," § (14) (iii.).

Hydrogen, Separation from Water-gas. See "Gases, Liquefaction of," § (2).

Hydrogen, Specific Heats of, tabulated values obtained by Scheel and Heuse. See "Calorimetry, Electrical Methods of," § (15), Table IX.

Hydrogen, used as Thermometric Substance. See "Thermodynamics," § (4).

Hydrogen Scale of Temperature, Normal, represented by a set of *verre dur* thermometers at the International Bureau, the international standard of temperature. See "Temperature, Realisation of Absolute Scale of," § (30).

Hydroplane. See "Ship Resistance and Propulsion," § (35).

— I —

Iϕ Chart, representation of action of ideal vapour compression refrigerating machine on. See "Refrigeration," § (3).

Ice, Density of, at 0° C., tabulated values assembled by Roth for the determination of a value for use in calculating the latent heat of fusion. See "Latent Heat," § (15), Table IX.

Ice, Latent Heat of Fusion of:
- Determined by A. W. Smith by the Electrical Method. See "Latent Heat," § (16).
- Determined by means of the Ice-calorimeter. See *ibid.* § (15).
- Determined by Regnault by the "Method of Mixtures." See *ibid.* § (14) (i.).
- Determined by various observers and tabulated. See *ibid.* § (17), Table X.
- Investigated by Black in 1762 and by Lavoisier and Laplace in 1780. See *ibid.* § (13).
- Variation with Temperature of. See *ibid.* § (18).

Ice-point on the Kelvin Thermodynamic Scale, Determination of. See "Temperature, Realisation of Absolute Scale of," § (21).

Ideal Gas: Entropy, Energy, Total Heat, and other Properties of. See "Thermodynamics," § (57).

Ignition Point, Determination of. See "Flash-point Determination," § (6).

Image, Effect of Size of, on Indications of Radiation Pyrometer. See "Pyrometry, Total Radiation," § (14) (ii.).

Impact and Notched Bar Testing. See "Elastic Constants, Determination of."
- The Charpy Method. § (98).
- Dimensions of Standard Test-pieces. § (102).
- Effect of Variation of the Angle of the Notch of the Test-piece. § (103).
- Effect of Variation of Root Radius and Depth of Notch of the Test-piece. § (104).
- Effect of Size of Specimen on the Results obtained. § (109).
- Effect of Variation of Striking Velocity on the Energy to Fracture. § (107).
- Experiments by Izod, Stanton and Bairstow, Harbord, and Progress of Impact Testing in Britain. § (99).
- General Considerations and Methods of Test. §§ (98)-(101).
- Results of Izod Tests on Materials Correctly and Incorrectly Heat Treated. § (101), Table 32.
- Slow Bending Tests on Notched Bars. § (108).
- Tests at Varying Temperatures. § (119).
- Tests under Repeated Bending Impact. § (111).
- Use of Round Test-pieces. § (105).

Impact Tests:
- On Screw Threads—Results with Different Forms of Thread. See "Elastic Constants, Determination of," § (41).
- On Unnotched Bars. See *ibid.* § (110).

Impulse Turbines:
- Hydraulic. See "Hydraulics," III. § (45).
- Steam. See "Turbine, Development of the Steam," § (2); "Steam Turbine, Physics of the," §§ (10), (15).

Inclined Plane. See "Mechanical Powers," § (1).

Index Law in Fluid Resistance. See "Friction," §§ (13), (17).

Indicated Thermal Efficiency. See "Petrol Engine, The Water-cooled," § (2).

Indicator, Hopkinson's Optical. See "Pressure, Measurement of," § (19).

Indicator Diagram, Watt's. A curve showing the relation between the volume and the pressure of a substance undergoing thermodynamic change. See "Thermodynamics," § (11).

Indicators, Steam-engine. See "Pressure, Measurement of," § (18).

Injectors and Ejectors:
- Gaseous Stream. See "Air-pumps," § (27).
- Liquid Stream. See "Air-pumps," § (30).

Instability, Failure of Structures due to. See "Dynamical Similarity, The Principles of," § (45).

Instantaneous Centres of Points in a Mechanism. See "Kinematics of Machinery," § (5).

Intensifiers, Hydraulic. See "Hydraulics," § (56) (iv.).

Internal Combustion Engines:
- Losses in. See "Engines, Thermodynamics of Internal Combustion," § (55).
- Loss of Heat to Walls during Explosion and Expansion. See *ibid.* § (56).
- "Mixtures" for. See *ibid.* § (26).

Internal Energy of a Body. See "Thermodynamics," §§ (10), (12), and (30).

Internal Pressure Correction to a Thermometer. See "Thermometry," § (3) (ii.).

Involute Teeth. See "Kinematics of Machinery," § (9).

Inward-flow Turbines (Hydraulic). See "Hydraulics," III. § (49) (iii.).

Iron Oxide, Emissivity of, determined by optical pyrometer. See "Pyrometry, Optical," § (18).

Isentropic Change. A change in the pressure volume and temperature of a body carried out reversibly in such a way that the entropy of the body remains constant. See "Thermodynamics," § (24).

Isopentane, Critical Isotherm of: value of pressure (1) calculated by Dieterici's second equation and (2) observed, tabulated against the volume. See "Thermal Expansion," § (21).

Isothermal Change. A change in the volume and pressure of a body carried out reversibly in such a way that no change in the temperature of the body is allowed to occur. See "Thermodynamics," § (16); "Engines, Thermodynamics of Internal Combustion," § (3).

Isotropic Materials: a name given to substances which exhibit similar properties in all directions. See "Elasticity, Theory of," § (4).

— J —

Jack, Hydraulic. See "Hydraulics," § (63) (i.).

Jaquerod and Perrot, 1905, comparison of gas-thermometer with secondary standards of temperature in range 500° to 1600°. See "Temperature, Realisation of Absolute Scale of," § (39) (xii.).

Jet Propeller for Ships. See "Ship Resistance and Propulsion," § (52).

Jets, Theory of. See "Steam Engine, Theory of," § (12).

Jigger for Hydraulic Lifts. See "Hydraulics," § (58) (ii.).

Joule:

Equivalent. See "Mechanical Equivalent of Heat," § (2).

Experiments on Mechanical Equivalent of Heat. See *ibid.* § (2).

Joule-Thomson Effect. See "Gases, Liquefaction of," § (1); "Thermodynamics," §§ (12), (43), (50), (57).

Joule-Thomson Effect, Inversion of. See "Thermodynamics," § (50).

Joule-Thomson Equation: a thermodynamic equation, providing an additional method of testing a gas equation. See "Thermal Expansion," § (23).

Joule - unit of Work. See "Mechanical Equivalent of Heat," § (3).

Joule's Law. See "Engines, Thermodynamics of Internal Combustion," § (16).

Joy's Valve Gear, Velocity Diagrams for. See "Kinematics of Machinery," § (4) (iii.).

— K —

Kelvin Double Bridge Method of measuring Resistance applied to the Platinum Resistance Thermometer. See "Resistance Thermometers," § (19).

Kelvin's Absolute Scale of Temperature. See "Engines, Thermodynamics of Internal Combustion," § (7).

Kinematics. That part of the science of mechanics which treats of the motions of bodies without reference to the bodies or to the causes which give rise to the motions.

KINEMATICS OF MACHINERY

This subject, as its name implies, deals with the motions of the various parts of machines without reference to either the forces involved or the actual proportions of the parts, other than those dimensions which determine the motion.

A machine consists of one or more *kinematic chains*, each of which consists of a series of members moving in a definite manner. The various members are treated as rigid bodies, any effect due to their elasticity being introduced as a correction. Parts which are deliberately made flexible, such as belts, chains, etc., are not usually considered as members in this sense, but merely as imposing certain constraints on the members on which they act.

§ (1) Degrees of Freedom. Co-ordinates.

(i.) *Definitions.*—To determine the position of a rigid body in space, six quantities are needed. For instance, we may specify the Cartesian co-ordinates of a point on it, and also the three angular co-ordinates, usually called Rodrigue's Co-ordinates, which specify its position with respect to three rectangular axes through that point. Any one of these co-ordinates may vary without affecting the others, giving six different motions, and hence a rigid body is said to have six *degrees of freedom.* Any motion may be specified in terms of the rates of change of these co-ordinates, and any limitation imposed on them is called a constraint, the degree of the constraint being the number of equations between the co-ordinates to which it gives rise, and the body being said to have lost that number of degrees of freedom. For instance, if one point is constrained to lie on a plane, one constraint is imposed and five degrees of freedom remain, whilst if two points are constrained to lie on a straight line, four conditions are imposed and two degrees of freedom are left, which are easily recognised as translation along the line and rotation round it.

If the motion of a body be such that all points in it which initially lie in a fixed plane continue to do so, three conditions are imposed and the motion is referred to as *plane motion*, only three degrees of freedom remaining to be considered.

The rigid structure to which the various parts of a mechanism are attached, and relative to which they move, is called a *frame*.

Two members of a machine which react on one another, either directly or through the medium of a flexible belt or chain, are said to form a *pair*. Pairs may be classified according to the number of degrees of freedom which they allow to one member when the other is fixed.

Lower pairs only permit one degree of freedom. A pin on one member revolving without end motion in a hole in the other which it exactly fits, a block on one working in a slot in the other, and a nut on its screw, are instances of lower pairs. The first two are instances of plane motion, the third is not.

Higher pairs allow two degrees of freedom. A typical instance is a round pin in a slot, where the pin can move along the slot, and can also revolve on its own axis. In plane motion, two surfaces which touch along a line form a higher pair, for both rolling and sliding can occur. This is sometimes taken as a definition of a higher pair, a lower pair being then defined as having contact over a surface. The contact of toothed wheels is thus an instance of higher pairing, and so is the connection of two pulleys by a belt.

In many cases a higher pair can be replaced by an additional member and two lower pairs. It is easy to see that this does not alter the number of degrees of freedom. For instance, a pin in its slot may be replaced by a pin in a block which works in the slot. This has the important practical advantage of reducing wear. A belt can at any instant be considered as a rigid member pin-jointed to the points of contact.

(ii.) *Types of Motion.*—Pairs of higher order are seldom used and are not classified. It is usually a simple matter to determine by inspection whether one or more types of motion are possible to an assemblage of parts, but rules are given in books on mechanism for finding if this be so by counting the number of members and pairs of each type. The simplest method is to count the number of members excluding the frame and to multiply by three (for plane motion), and then to deduct twice the number of lower pairs, which impose two constraints each, and once the number of higher pairs, which impose one each. If several members are connected by the same pin, consider the pin as attached to one of them and imposing two constraints on each of the others. There are thus four constraints where a pin unites three members, six when it unites four, and so on.

For example, consider two rods attached by pin joints to one another and to the frame. Here we have two members each with three degrees of freedom, making six in all, also three joints imposing two constraints each, so there is no degree of freedom left and we have not a mechanism but a frame. If the number of degrees of freedom is negative the frame is said to have redundant members.

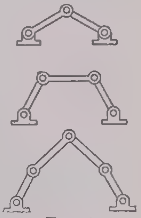

FIG. 1.

Consider next three rods pin-jointed to one another and to the frame. Here we have $3 \times 3 = 9$ degrees of freedom, and $4 \times 2 = 8$ constraints, hence one degree of freedom remains, and we have a mechanism or kinematic chain, usually called the Four Bar Crank Chain, the fourth bar being the frame.

The introduction of a fourth rod would give $4 \times 3 = 12$ degrees of freedom, $5 \times 2 = 10$ constraints, hence two degrees remain, more than one type of motion is possible, and the arrangement is not a kinematic chain.

Consider a crank driving a connecting-rod, the other end of which carries a pin moving in a slot. Here we have, in addition to the frame, two members, each with three degrees, two lower pairs imposing two constraints each, and one higher pair imposing one. Hence there is one degree left. This mechanism, modified by replacing the higher pair by a block in the slot, pin-jointing to the connecting-rod, is called the Slider Crank Chain, and assumes various forms or *inversions* by fixing various members. As described it is the mechanism of the direct-acting engine, fixing the connecting-rod we get the oscillating cylinder engine, fixing the crank we get the rotary engine and the quick return, and fixing the block we get a mechanism known as the pendulum pump, but not much used.

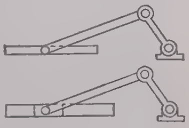

FIG. 2.

Consider a rod with two pins working in two slots in the frame. Here we have one member with its three degrees of freedom, and two higher pairs, leaving one degree. This mechanism, with the higher pairs replaced by blocks and lower pairs, is called the elliptic trammels, as any point in the rod describes an ellipse, and by inversion we get the elliptic chuck and Oldham's coupling.

§ (2) LOCI.—An important branch of the subject is the determination of the loci of points in a mechanism, and the design of mechanisms to trace given loci. The ordinary methods of analytical geometry are employed, but there are special approximate methods used in certain cases where the complete solution is not readily obtained. Much thought was at one time given to the subject of parallel motions, a term used to denote a mechanism which guided a point in a straight line. Such mechanisms are now much less important than they were, but an analysis of one of them (Watt's) will illustrate a method which is very useful. In this mechanism two rods each pivoted at one end have their other ends joined by a link. A portion of the path of one point on the link will then be a close approximation to a straight line.

The arrangement of the mechanism is shown in *Fig.* 3, with a centre line diagram below it. Let AX and BY be the rods, pivoted to the frame at A and B, and coupled by the link XY, which is not far from perpendicular to them when they are parallel to one another. Let Z be a point on the link and let $AX=a$, $BY=b$, $XZ=x$, $YZ=y$. Let AX be rotated through a small angle θ, then X rises $a\theta$ and moves to the left $\frac{1}{2}a\theta^2$, both expressions being correct to the second order in θ.

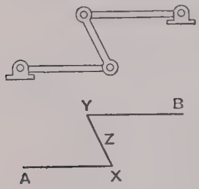

FIG. 3.

Neglecting the effect due to the change in obliquity of the link XY, Y will also rise $a\theta$, and hence BY turns through an angle $a\theta/b$, causing Y to move to the right a distance $\frac{1}{2}b(a\theta/b)^2=a^2\theta^2/2b$. Hence Z moves to the left a distance $(ya\theta^2/2-xa^2\theta^2/2b)/(x+y)$ which is zero if $ax=by$. Z then rises in a path which deviates from a straight line only in terms of the third order.

In many cases loci are plotted by drawing out the mechanism in a succession of positions. The labour may often be reduced by drawing part of the mechanism on tracing paper which is moved into successive positions and the point whose locus is required is pricked through on to the paper below. Models in cardboard, jointed by eyelets or pins, are often extemporised, and for some purposes well-made metal models, with members adjustable in length, are used. This is notably the case with valve gears.

§ (3) DISPLACEMENT, VELOCITY, AND ACCELERATION. (i.) *Graphical Methods.*—In studying the motion of a point it is often found useful to plot the displacement, velocity, and acceleration on a time base. This is especially the case when the mechanism derives its motion from a uniformly rotating shaft. A circle described round the centre of the shaft is divided into a number of equal parts, and the mechanism is drawn out with the driving crank in each of these positions. This determines a series of positions of the point under consideration at equal intervals of time, and, a horizontal line being divided into a corresponding number of equal parts, ordinates are set up to represent the distance of the point from some fixed point on its path. This curve is known as the displacement-time curve. Its form gives us a considerable amount of useful information. It shows the extreme points of the motion, and when they occur, the range, and the time between the two given positions. It may also serve to suggest an approximate formula for the displacement, and can be submitted to harmonic analysis. The curve can be obtained from a model even better than by drawing.

If more definite information as to the velocity is required it may be obtained by graphic differentiation of the above curve, as the slope of the latter is evidently a measure of the velocity. If this process is to be employed the curve must be drawn with great care, and the exact direction and point of contact of the tangent is best determined by laying on the curve a piece of celluloid on which are scratched two straight lines at right angles, and adjusting this till the eye accepts the lines as the tangent and normal at the point under consideration. The tangent being marked on the paper, two points are taken on it, and the difference of their ordinates, interpreted on the displacement scale, divided by the difference of the abscissae, interpreted on the time scale, gives the velocity.

A velocity-time curve can now be plotted. It is difficult to obtain accuracy by this method, and a considerable amount of fairing of the points and redetermination of the tangents will probably be needed before a satisfactory curve is obtained. Hence the desirability of more direct methods given below.

A graphic differentiation of the velocity-time curve gives the acceleration, which can also be plotted on a time base. The accumulation of the errors of two such differentiations renders the need of more direct methods imperative. The chief use of a knowledge of the acceleration is to calculate inertia forces.

In many cases it is useful to plot velocities and accelerations on a displacement base. This is especially so when studying the motion of the piston of a steam engine or of the cutting tool of a slotter or shaper.

(ii.) *Analytical Methods.*—Velocities and accelerations can sometimes be found analytically. For instance, consider the direct-acting steam engine, shown diagrammatically in *Fig.* 4. This consists of a crank CP revolving round a fixed centre C, and a connecting-rod PD which couples the end of the crank to the crosshead to which the piston rod is attached. As the piston reciprocates in a straight line, if we wish to study its motion it will suffice to study the motion of D. Let the length of the crank CP be r, that of the connecting-rod PD be l, and in the position shown let the angle DCP be θ and CDP be ϕ. Then D is to the left of C a distance equal to $r\cos\theta + l\cos\phi$.

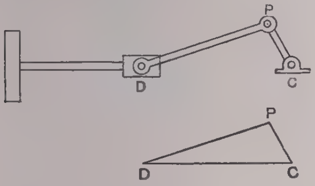

FIG. 4.

Since $r\sin\theta = l\sin\phi$ we may write this

$$r\cos\theta + l\sqrt{1-\frac{r^2}{l^2}\sin^2\theta},$$

and if r/l be not too large a close approximation is

$$r\cos\theta + l\left(1-\tfrac{1}{2}\frac{r^2}{l^2}\sin^2\theta\right)$$

$$=r\left[\cos\theta-\frac{r}{2l}\sin^2\theta\right]+l$$

$$=r\left(\cos\theta+\frac{1}{4n}\cos 2\theta-\frac{1}{4n}\right)+l,$$

where $n = l/r$.

It may be noted that the mid-point of the stroke is a distance l to the left of C, and for most purposes it is convenient to measure from it, omitting the l in this expression.

This gives the displacement of D. Its velocity is, by differentiation, with respect to the time,

$$\omega r\left(\sin\theta+\frac{1}{2n}\sin 2\theta\right)$$

towards the right, where $\omega = d\theta/dt$, the angular velocity of the crank. A second differentiation gives for the acceleration, if we assume ω to be constant, the value

$$\omega^2 r\left(\cos\theta+\frac{1}{n}\cos 2\theta\right).$$

It may be noted that the second harmonic becomes more important with each differentiation. This is a general occurrence.

Where analytical methods are not practicable, the following methods are used.

(iii.) *Velocity Images.*—Consider two points A and B. Let the velocity of A be u, and that of B be v. Take an origin o, draw oa to represent u in magnitude and direction, and ob to represent v. Then by the triangle of velocities, ab represents the velocity of B relative to A. If A and B be two unconnected points, no restriction is placed on ab, but if A and B be two points on a rigid body the only possible motion of B relative to A is one of rotation round A, and hence ab must be perpendicular to AB. oa is the velocity of a and ob that of b, and it is easy to see that the velocity of a point C on AB is oc where c divides ab in the same ratio that C divides AB. The line ab is conveniently called the velocity image of AB, and the image of a point D carried by AB but not in the line AB is found by constructing a triangle abd similar to ABD, od then representing completely the velocity of D.

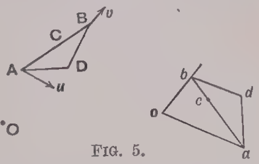

FIG. 5.

If now, in any given problem, we know the velocity of A we can draw oa, and then we can draw a line ab at right angles to AB, on which b must lie. The direction of the line joining o and b is usually given from the knowledge of the direction of motion of b, and the intersection determines b. ob then gives completely the velocity of b. The velocity of b relative to a is given by ab, and the angular velocity of the rod is evidently ab/AB. As an example, consider the direct-acting engine. Referring to *Fig.* 6, where the centre-line diagram is repeated from *Fig.* 4, with the same notation, to determine the velocity of the piston set off op at right angles to CP, to represent ωCP, the velocity of the crankpin on a convenient scale. ω is, as above, the angular velocity of the crank. Draw pd at right angles to PD to represent the velocity of D relative to P in direction. The actual motion of D is along the line DC, and hence we draw od in that direction to cut pd in d. od now represents the velocity of D and pd that of D relative to P, hence the angular velocity of the connecting-rod is represented by pd/PD. (It is easy to see that if DP be produced to meet the perpendicular through C in T, the triangles opd and CPT are similar, and hence CP/CT$=op/od$, and since op represents ωCP, od represents ωCT, *i.e.* the velocity of the piston is ωCT.)

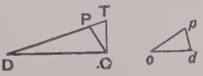

FIG. 6.

As a more complicated case consider Joy's valve gear. The centre-line diagram is given in *Fig.* 7. To the connecting-rod AB of a direct-acting engine a rod CE is attached, E being constrained to move round O_1 by the rod O_1E. To a point D in CE a rod DG is attached, a point F on which is constrained to move round O_2 either by a rod O_2F or by a curved slot in which F must lie. G is attached by the rod GV to V, the end of the valve spindle. The method of velocity images can be applied as follows to find the velocity of *v*. Set off *oa* to represent ωOA, and determine *b* as above. Divide *ab* in *c* in the same ratio as C divides AB. Draw *ce* perpendicular to CE for the velocity of E relative to C, and as its actual velocity is perpendicular to O_1E, draw *oe* perpendicular to O_1E to meet the former line in *e*. Divide *ce* in *d* as CE is divided in D. Draw *df* perpendicular to DF, to meet *of*, perpendicular to O_2F, in *f*, and produce *df* to *g* in the same ratio that DF is produced to G. Lastly, draw *gv* perpendicular to GV to meet a line through *o* parallel to the direction of motion of V. *ov* then represents the velocity of V on the same scale as *oa* does that of A.

(iv.) *Acceleration Images.*—An analogous method is used to determine accelerations, but there is now an important difference. If A and B be two points on a rigid body the relative acceleration is not in general either along or perpendicular to AB, but consists of two components, one of ω^2AB along AB and one of $\dot\omega$AB at right angles to it. These may be called the radial and tangential components; in general we can calculate the radial component when we have drawn the velocity image, and have found ω, but all we know about the tangential component is its direction.

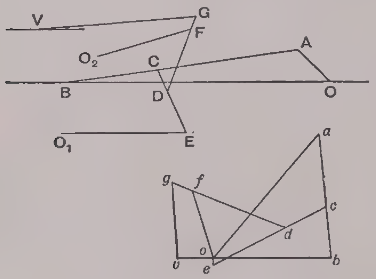

FIG. 7.

As an example consider the direct-acting engine. To determine the acceleration of D, first draw the velocity diagram *opd* as in *Fig.* 6. Then from a fresh origin set off *op* to represent ω^2CP, the acceleration of P in magnitude and direction. The acceleration of D relative to P consists of a component Ω^2DP from D towards P and $\dot\Omega$DP at right angles to it, where Ω is the angular velocity of the connecting-rod. Ω is known from the velocity diagram, being given as *pd*/PD, but $\dot\Omega$ is not yet known. Therefore we set off pd_1 parallel to DP and representing Ω^2DP, on the acceleration scale, and then draw a line through d_1 perpendicular to PD, to represent the tangential component $\dot\Omega$DP in direction. Since the actual acceleration of D is along the line of stroke, a line through *o* parallel to CD meeting dd_1 in *d* determines the point *d*. The line *od* now gives the acceleration of D in magnitude and direction, and dd_1, being the tangential component, represents $\dot\Omega$DP, and hence $\dot\Omega$ is found, if required.

This method of acceleration images can be applied to any system of linkwork. It should be noted that the actual image is *pd*, not pd_1 or dd_1, and that the acceleration of any other point in PD is given by the line joining *o* to the corresponding point on *pd*.

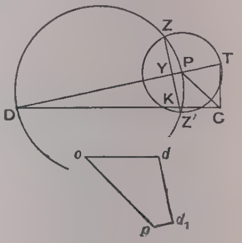

FIG. 8.

A very neat construction due to Klein can be readily derived from the above. Produce DP to meet the perpendicular from C in T, on D as diameter describe a circle, and with P as centre and PT as radius cut it in Z and Z_1. Join ZZ_1 cutting DC in K. The required acceleration of D is ω^2CK. It can be seen readily that CPYK is similar to opd_1d, in the same way that CPT is similar to *opd*, and since PY . PD = PT², the proof follows at once.

§ (4) INSTANTANEOUS CENTRES.—A very useful method of analysing the motion of a mechanism is by the use of *instantaneous centres*. A body moving in a plane may be brought from any one position to any other by a rotation round a certain point. For let A and B be two points in the body in one position and A′B′ the same points in another position. Bisect AA′ and BB′ by lines at right angles meeting in I. Then IA = IA′, IB = IB′, and the triangles IAB, IA′B′ are equal, hence the angles AIB and A′IB′ are equal, and hence so are AIA′ and BIB′, or the rotation round I which carries A to A′ also carries B to B′.

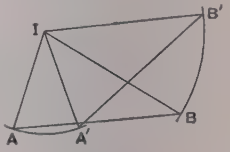

FIG. 9.

If the displacement be made small so that AB and A′B′ are consecutive positions, AA′ and BB′ become the directions of motion of A and B respectively, and I is called the instantaneous centre, and the motion of AB is at that instant a rotation round I. The position of I is found by erecting perpendiculars to the directions of motion of any two points of the body, and this is readily done in many cases owing to the nature of the constraints, *e.g.* if a point moves along a straight slot, I lies on a perpendicular to the slot, and if a point is guided in a circle by a link, I lies on the centre line of the link, produced if necessary. For instance, in the direct-acting engine, the instantaneous centre of the connecting-rod lies at the point I where the centre line of the crank produced cuts a line through the crosshead pin perpendicular to the slide bar, for P is moving at right angles to the line IPC, and D at right angles to ID. The velocity of D may now be found as follows. If ω be the angular velocity of the crank, the velocity of P is ωPC. Hence the angular velocity of the connecting-rod about I is ωPC/PI and the velocity of D is ωPC . ID/PI. By similar triangles this can be proved equal to ω . CT as shown above.

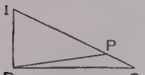

FIG. 10.

To apply the method to more complicated mechanisms we need the proposition that the relative instantaneous centres of three bodies taken in pairs lie on a straight line. This is easily proved. Let the bodies be A, B, and C, and let the instantaneous centre of B relative to C be *Ibc*, and so for the other pairs. Then I_{bc} is a point on C at rest relative to B. Since I_{ab} is the instantaneous centre of B relative to A, I_{bc} considered as a point on B is moving at right angles to $I_{ab}I_{bc}$. Similarly as a point on C it is moving at right angles to $I_{ac}I_{bc}$. This can only happen if the two directions coincide, *i.e.* if I_{ab}, I_{bc}, and I_{ca} are in the same straight line.

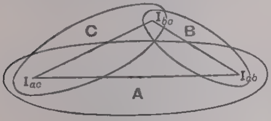

FIG. 11.

In applying the above theorem to a complicated mechanism the notation is improved by omitting the I's and denoting the instantaneous centre of A and B by *ab*. The relative instantaneous centre of two members connected by a pin is at the centre of the pin, and if the instantaneous centre of one of them is already known a line can be drawn through it and the centre of the pin and this line must contain the instantaneous centre of the other. A second pin joint on the member under discussion gives another line, and the centre required lies at the intersection of these two lines.

As an example consider Joy's valve gear. Denote the links by ABCDEFG as shown and the frame by O. Then the instantaneous centres *oa, ab, bc, cd, do, ce, ef, of, eg* can be labelled at once as they are pin joints. *ob* is then found by producing *oa, ab* to meet the vertical through the crosshead pin as in the last example. Since *ob* and *bc* are known, *oc* must be on the line joining them, and it must also be on the line joining *do* and *dc*. This locates *oc*. (Note the cyclic order *ob, bc, co,* and *od, dc, co.*) We now proceed to find *oe* using the lines *oc, ce* and *of, ef*. Since one end of G is moving round *oe* and the other is moving along the line of stroke of the valve, *og* is found at the intersection of *oe, eg* with the vertical through the other end of G. The velocity of the valve is then found from the angular velocity of the crank by following the motion through the mechanism from centre to centre, multiplying the angular velocity by the distance of a point from one instantaneous centre to get its linear velocity, and dividing by the distance from the next to get the angular velocity round it, and so on.

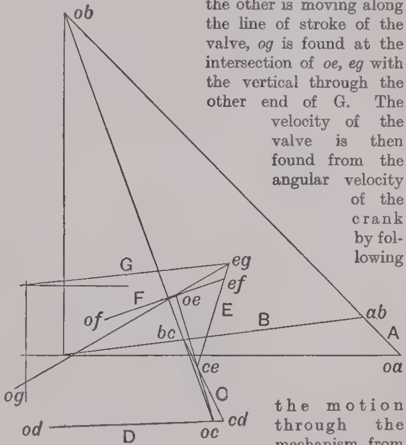

FIG. 12.

This method is in many cases preferable to the method of velocity images, as it can be applied directly to the centre-line diagram of the mechanism, and does not involve the drawing of a second diagram and the use of the parallel ruler.

There is an analogous method for determining accelerations, by using the properties of the centre of no acceleration. To find the latter we draw lines making an angle β whose tangent is $\dot{\omega}/\omega^2$ with the directions of the accelerations of two points on the body, to meet in J, the acceleration of any point P being then ω^2PJ along the line PJ and $\dot{\omega}$PJ at right angles to it. The actual constructions to find J depend on the data, and in general the method of acceleration images is more useful and simpler.

§ (5) CAMS.—When a member of a mechanism is required to have a motion that cannot conveniently be given by means of a linkwork, recourse is had to a *cam*. A cam is a revolving member having sliding contact with a sliding or rocking member called the follower, the former being so shaped as to give the required motion to the latter. Cams may be divided into two classes usually known as edge cams and face cams. The former, as the name implies, consists of a flat disc, the follower bearing against the edge, whilst the latter consists in principle of a disc, one side of which is a formed track against which the follower bears. A cylinder with a groove on its curved surface, in which runs a roller carried by the follower, is evidently equivalent to a face cam. Face cams are easily designed, as the development of the cylinder track must evidently be the displacement-time curve required for the follower.

Edge cams may be divided into two types, called point cams and tangent cams; in the former the motion is imparted to a point on the follower, and in the latter an edge of the follower is always tangential to the cam. In point cams the point may be constrained to lie on a line through the centre of the cam, on a line passing to one side of the centre, or in an arc of a circle about a point on which the follower is pivoted. In a tangent cam the follower may be constrained to move parallel to itself or to swing about a centre.

In all cases the simplest way to set out an edge cam is to imagine the cam at rest and the remainder of the machine revolving round it; the required form of the cam is then found as the locus of the points on the follower in a point cam, or as the envelope of the successive positions of the edge in a tangent cam. Usually in a point cam a roller is used to reduce friction and wear, and in that case the cam as actually made is the envelope of a series of circles whose centres lie on the cam as designed above to give the required motion to the centre of the roller, and whose radius is that of the roller.

§ (6) VALVE GEARS. — These, and the methods used in studying them, form an important branch of the subject, but are not dealt with in this article.

§ (7) CRANK EFFORT DIAGRAMS.—Although involving the idea of force, and hence not strictly belonging to kinematics, mention must be made of the subject of crank effort diagrams. In studying the action of a steam engine it is useful to plot on a crank angle base the effort exerted by the engine in turning its shaft. The method may be reversed and used to study the turning moment needed to drive a machine.

By the principle of virtual work, if u be the velocity of the crankpin and v that of the piston, P the net thrust on the piston, and T the force on the crankpin in the direction of its motion, then $Pv = Tu$, *i.e.* $T = Pv/u$. The plotting of a crank effort diagram involves, therefore, the determination of P and the determination u/v.

The latter is done by means of the velocity image, by instantaneous centres, or by any other method suitable to the mechanism under consideration. In the direct-acting engine it has been proved that $u/v = CT/CP$, hence we have $T.CP = P.CT$, and since $T.CP$ is the moment of the force at the crankpin about the crank, all we have to do is to plot the product $P.CT$ which is equal to it. P is determined from the indicator diagrams, being the difference between the pressures on the two sides of the piston, multiplied by the area of the piston, and corrected for inertia. It is usual to plot not the total P, but the value of P per sq. in. of piston; the same diagram then serves for engines of different sizes, provided they have the same steam distribution. To correct for inertia we require to deduct Wf/Ag lbs. per sq. in. from the difference of pressure shown by the indicator, where W = weight of reciprocating parts, A = area of piston, f = acceleration of piston. The last must be found for a sufficient number of points on the stroke, and this may be done by Klein's construction. It is more usual, because simpler, to use the formula $f = \omega^2 r(\cos\theta + 1/n \cos 2\theta)$ proved above, and to select the points where $\theta = 0°, 45°, 90°, 135°, 180°$, when the expression in the bracket becomes $1 + 1/n$, $1/\sqrt{2}$, $-1/n$, $-1/\sqrt{2}$, $-(1 - 1/n)$ respectively, values which are easily calculated, and which give a sufficient number of points to enable a smooth curve to be drawn. A diagram of net pressure being first constructed by measuring from the top of one indicator diagram to the bottom of the other, the inertia curve is drawn across it, and the corrected pressure scaled off and multiplied by the corresponding value of CT; the product is then plotted on a crank angle base.

The chief use of such diagrams, showing the variation of effort during a revolution, is to enable us to study the resulting fluctuation in speed, and to serve as a basis for the design of flywheels.

In a similar manner diagrams may be drawn showing the effort needed to drive, say, a shaping machine, given the portion of the cutting stroke for which the tool is in contact with the work, and the cutting pressure. We can also find the fluctuation of effort needed to drive an air compressor or hydraulic pump.

§ (8) TOOTHED WHEELS. — An important branch of the subject is the discussion of toothed wheels, both as regards the angular velocity transmitted by a train of wheels and as regards the correct form of curve for the teeth.

The determination of the ratio of the velocities of the first and last wheels in a train is usually a simple matter. If two wheels have n_1 and n_2 teeth respectively and make R_1 and R_2 revolutions respectively, then evidently $n_1R_1 = n_2R_2$. In a simple train of wheels where each one gears into the preceding one and drives the next one in the train directly, the numbers of teeth in the intermediate wheels evidently cancel out, and the velocity ratio transmitted depends only on the number of teeth in the first and last. The intermediate wheels are often called idle or transposing wheels. It should be noted that if there is an even number of wheels the last one revolves in the opposite direction to the first, and if an odd number, in the same direction.

In a compound train there are one or more shafts, each carrying two wheels; motion is imparted to the shaft through one of the wheels and is taken off by the other. For such trains we have the simple rule: Velocity ratio = product of numbers of teeth in drivers divided by product of numbers of teeth in driven wheels. Worm wheels may be included in this rule if we regard a single-threaded worm as having one tooth, a double-threaded one two teeth, and so on.

In epicyclic trains, wheels, known as planet wheels, gear with a central wheel called the sun wheel, whilst their centres are constrained to move in a circle round it. Such trains are easily dealt with by first considering the planet wheels to have their centres fixed, assigning a velocity of rotation to one wheel of the train, and tabulating the velocities of the others, paying attention to sign, and then superimposing on the whole system such a rotation as will reduce to rest the wheel that actually is at rest. The velocity ratio of any two wheels can then be written down by inspecting the table thus modified. Such trains are largely used where a big reduction in speed is needed, and also in change speed gears for cycles and motors.

As regards the number and proportions of the teeth, let O, O′ be the centres of two wheels which are to gear together. Divide OO′ in P, so OP/O′P is the velocity ratio to be transmitted. P is usually called the pitch point, and it is evident that two rough circles of radii OP and OP′ will transmit the required velocity ratio. The actual wheels may be considered as derived from these circles by providing them with teeth. The distance from the point where one tooth cuts the circle to the point where the next one cuts it, measured along the pitch circle, is called the circular pitch, and must evidently be an exact submultiple of both circumferences. If the circular pitch be expressible in inches and fractions, the radii cannot be, since the radius of a wheel of n teeth is $np/2\pi$, and conversely if the radii are expressible in inches and fractions, p, the circular pitch, will be incommensurable. The latter system is the most convenient in practice, and instead of working in terms of circular pitch it is more usual to work in terms of p/π, which is evidently equal to the diameter divided by the number of teeth. It would be convenient to call this the diametral pitch, by analogy, but as it is usually expressed as a fraction with unity as numerator, the custom has arisen of naming only the denominator and calling that the diametral pitch. The diameter of the pitch circle is then found by dividing the number of teeth by the diametral pitch thus defined, and as it is usual to make the height of the tooth above the pitch circle (called the addendum) equal to p/π, the over-all diameter of the blank before cutting is found by adding 2 to the number of teeth and dividing by the diametral pitch. For example, a wheel of 24 teeth, and diametral pitch 4, would have a pitch circle diameter of 6 in. (24/4), an over-all diameter of $6\frac{1}{2}$ in. $(24+2)/4$, the teeth would stand $\frac{1}{4}$ in. above the pitch circle and would be cut slightly more below it, and the circular pitch p would be $\pi/4$ in.

As regards the form of the teeth, the essential condition is that the teeth, which move over one another with a combined rolling and sliding action, should transmit a constant velocity ratio. If this condition is not fulfilled there will be vibration and noise accompanied by loss of power. This condition makes it necessary that the common normal to the two teeth at their point of contact should pass through the pitch point in all positions of the wheels. This may be proved as follows. Imagine the left-hand wheel at rest and the right-hand wheel rolling on it. P is evidently the instantaneous centre, and hence Q considered as a point on the right-hand wheel is moving at right angles to PQ. If the teeth are neither to penetrate one another nor to separate, this involves the common tangent at Q being perpendicular to PQ. (It is evident that the velocity of rubbing is $(\omega_1+\omega_2)$PQ, where the ω's are the angular velocities, and this only vanishes when the teeth are in contact at P.)

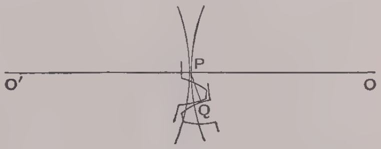

FIG. 13.

It is evident that if the form of the teeth for one wheel is given, that for the other can be determined by imagining one wheel to roll on the other and to force the material of which it is made into the required shape. This can also be done on the drawing-board, by rolling a piece of tracing-paper with a circle to represent one pitch circle on the pitch circle of the other drawn on the paper below, and tracing through the teeth in the successive positions. The required form for the teeth of the upper wheel is found as the envelope of the teeth so traced.

In general, however, wheels are manufactured to standard forms, in series, such that any one of a series of wheels of given pitch will run with any other. For this purpose suitable forms must be chosen. Two systems are usually described in text-books, involute teeth and cycloidal teeth. The former are almost exclusively used now, the disadvantages formerly attributed to them being much less serious than their positive advantages.

Imagine that the two wheels carry circles called base circles whose radii are slightly less than those of the pitch circles, the ratio being the same for both wheels. Imagine an inextensible string attached to these circles, and crossing between them; then as the wheels revolve the string is unwound from one base circle and wound up on the other, keeping taut in virtue of the ratio of the radii. Detach the cord from one base circle and, keeping it taut, unwrap it from the other. A tracing point on it will describe a curve known as the involute of the base circle. Reattach the cord to the other base circle and describe an involute relative to it, using the same tracing point. Now attach the cord to both base circles; the two curves must meet at the tracing point which described them, and must touch, since the cord is normal to both. As the wheels revolve, and the tracing point passes across from one base circle to the other, the involutes remain in contact, and hence are possible forms for wheel teeth. It is easily seen by similar triangles that the cord passes through the pitch point, and as it is the common normal the condition for uniform velocity ratio is fulfilled.

The other form of teeth consists of two parts, the part outside the pitch circle (the face) being an epicycloid formed by rolling a small circle on it, and the part inside (the flank) being a hypocloid formed by rolling a small circle within it. For two wheels to run together the face of one and flank of the other must be generated by rolling circles of the same diameter, and so must the flank of the former and face of the latter. As far as one pair of wheels is concerned these two rolling circles may be of different diameters, but if the wheels are to form a series any pair from which will run together, all the rolling circles must have the same diameter. A proof that such teeth fulfil the necessary conditions is given in Dunkerley's *Mechanism*.

Involute teeth have two very important advantages. Firstly, if the centre distance be varied slightly they still gear together correctly, as long as they are sufficiently far in mesh to run at all, as can be seen by considering the cord and involutes described above. This is not true for cycloidal teeth. Secondly, the rack which gears with an involute wheel has straight-sided teeth, and by means of this property involute teeth can be generated from a cutter with a straight edge, and not merely copied from a formed cutter or former. Hence the accuracy of cutting is more to be relied upon.

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C. H. B.

KINETIC THEORY OF MATTER APPLIED TO EXPLAIN THE GAS LAWS. See "Thermal Expansion," § (15); "Thermodynamics," § (66).

KIRCHHOFF'S LAW OF RADIATION. A law which states that, for radiation of any one frequency, the ratio of the coefficient of emission to the coefficient of absorption is the same for all substances whatever, and depends only on the temperature and the frequency of the radiation considered. See "Radiation Theory," § (4).

KÖRTING TWO-STROKE ENGINE. See "Engines, Internal Combustion," § (10).

— L —

LAG, THERMOMETRIC. The interval of time which elapses between the plunging of a thermometer into a medium at a different temperature and the attainment of the final reading. See "Thermometry," § (10).

LANGMUIR "CONDENSATION" PUMP. See "Air-pumps," § (45).

LATENT HEAT

I. LATENT HEAT OF VAPORISATION

§ (1) EXPERIENCES OF REGNAULT AND DIETERICI. — The determination of the heat of vaporisation of a liquid does not present much difficulty if only approximate values are desired. But, when an accuracy greater than 1 per cent is aimed at, great precautions must be taken to minimise the errors due to the vapour carrying over small particles of liquid.

Since the thermal constants of water have been studied more exhaustively than those of any other substance the methods employed for the determination of the latent heat of steam will be considered as illustrating the procedure in such experiments.

(i.) *Regnault* came to the conclusion that the best method of reducing errors due to the condensation of vapour and heat loss from the pipes, etc., was to work on a large scale.

His method of experiment was to condense steam under saturation pressure in a calorimeter of about 100 litres capacity and observe the temperature rise of the water in the calorimeter. The quantity of condensed water was of the order of 10 litres. The initial temperature of the steam varied from 107° to 187° C., and the flow into the calorimeter was controlled by a throttle valve. It might be remarked that the expansion of the high-pressure steam down to atmospheric pressure in passing through the valve does not alter the total heat of the steam, provided there is no external loss of heat and no change of kinetic energy. If kinetic energy is generated at the valve it is reconverted into heat in the calorimeter.

Regnault's apparatus is now of historical interest only, and a detailed description of it will be found in Preston's *Theory of Heat* (3rd ed.), p. 367.

The results obtained by Regnault appear to be fairly reliable over the range from 100° C. to 200° C., but below 100° C. are vitiated by several sources of error, and in recent years Regnault's values have been superseded by data obtained under more favourable conditions of experiment.

In Regnault's time there was no information available concerning the variation of the specific heat of water with temperature, nor was he aware of the temporary changes of zero of mercury thermometers.

TABLE I

REGNAULT'S VALUES

Number of Experiments.	Temperature.	Latent Heat (Calorie at 15° approximately).
3	120·3	521·7
4	126·8	517·6
11	135·9	511·9
13	145·2	504·9
10	155·5	495·7
5	162·4	491·5
14	175·2	482·3
9	185 6	478·1
4	194·6	471·0

Another source of error in Regnault's apparatus was the uncertainty concerning the heat conducted into the calorimeter along the pipe conveying the steam. Regnault obtained his correction by observing the rate of rise of an exactly similar calorimeter, with similar connections, into which no steam was passed. But the temperature gradient along the pipe which determines the heat flow depends upon the rate of flow of the steam, for this necessarily alters the temperature gradient along the pipe. He consequently overestimated the correction and obtained latent heat values which were too low.

(ii.) *Dieterici* [1] measured the latent heat at 0° by means of a Bunsen ice-calorimeter. The water was contained in the inner tube of the ice-calorimeter and the weight of mercury extruded observed on evaporation of the water.

His observations were very concordant, and the differences from the mean were less than 1 part in 600.

At first he assumed the value 15·44 milligrams of mercury per mean calorie for the constant of the ice-calorimeter, which was the mean of the results of previous observers.

This gave the value 596·80 mean calories for the value of L at 0°.

Subsequent experiments of Dieterici [2] in which water enclosed in a quartz bulb heated to 100° was dropped into the calorimeter gave the value 15·50 [3] for the constant of the calorimeter.

On this basis the value of L at 0° would be 594·83 mean calories.

§ (2) GRIFFITHS' EXPERIMENTS.—The value of the latent heat of evaporation of water at the temperatures of 30° and 40° C. was determined by E. H. Griffiths,[4] whose apparatus is shown in *Fig.* 1.

A known weight of water was put in a glass bulb B with a narrow jet, and this was placed in a small silver flask F to which was attached a coil of silver tube T 18 ft. long. Between the flask and the coil of tube was a spiral of platinum-silver wire which, heated by an electric current, supplied the heat necessary to vaporise the water. The flask, tube, and wire were all enclosed in the calorimeter, which was filled with aniline in the earlier experiments, but later with a special petroleum oil which was non-volatile and a good insulator. The calorimeter, being surrounded by a vacuum and a mercury thermostat jacket, was kept at a constant temperature, never varying as much as 0·01° C. The end of the silver tube passed outside the apparatus and was connected with an air-pump. On working the pump the water was made to issue drop by drop from the bulb under its own vapour pressure so that the rate of evaporation could be kept quite regular. Thus the vapour formed in the flask had to pass up the whole length of the silver spiral tube and issued at the temperature of the calorimeter and free from water mechanically carried over. Special attention was given to the efficient stirring of the oil in the calorimeter.

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[1] *Wied. Ann.*, 1889, xxxvii. 506.

[2] *Ann. Phys.*, 1905, xvi. 593.

[3] This was confirmed by the writer using an electrical method (see § (1) of "Calorimetry, Methods based on the Change of State").

[4] *Phil. Trans.* A, 1895, p. 261.

The equation for determining the latent heat of evaporisation may be put in the following form:

$$ML = Q_e t_e + Q_s t_s + \Sigma q,$$

M being the mass of water evaporated, Q_e the heat per second supplied by the electric current, Q_s the heat generated by the stirrer, t_e and t_s the times during which heat was supplied by these two sources respectively (t_e and t_s being practically equal), and Σq, the total heat received owing to other causes such as radiation and conduction.

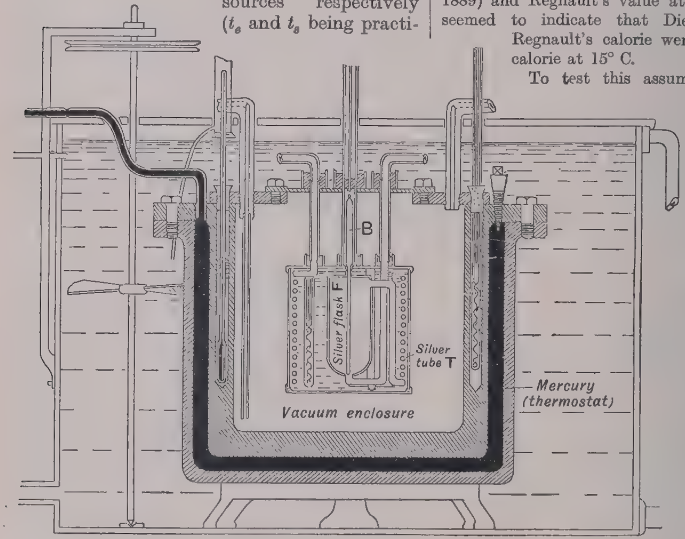

FIG. 1.

Although Q_e was necessarily measured in electric units, it was reduced to thermal units by employing the value of J determined by means of the same method of electric measurement and the same electrical standards. Thus any errors due to uncertainty in the values of the electric units were eliminated. The greatest uncertainty was in the estimation of Q_s, but this constituted only about 1 per cent of the rate of heat supply. The following are the values of L obtained:

TABLE II

Temperature by Nitrogen Thermometer.	Latent Heat (15° C. Unit).	Latent Heat (20° C. Unit).
40·15° C.	572·60	573·51
30·00° C.	578·70	579·25

The advantages of this method over most of the others that have been made use of is that it is practically independent of errors in thermometry and is not affected by changes in the specific heat of water.

The temperature being stationary, the heat capacity of the calorimeter or of its contents does not enter into consideration, and the radiation correction is small and determinate.

Griffiths noticed that his two results at 30° and 40° C. lay very nearly on a straight line joining Dieterici's value[1] at 0° C. (uncorrected value of 1889) and Regnault's value at 100° C., which fact seemed to indicate that Dieterici's calorie and Regnault's calorie were both equal to the calorie at 15° C.

To test this assumption Griffiths persuaded Joly to make a determination of the relation between the latent heat of condensation at 100° C., and the mean specific heat of water from 12° to 100° C. by means of the Joly steam-calorimeter.

This he did; and assuming the calorie at 15° equal to the mean calorie between 12° and 100°, obtained the value 539·3 for the latent heat instead of 536·7 as given by Regnault. Hence either the mean thermal until between 12° and 100° C. was much smaller than the calorie at 15° or Regnault's value was in error.

The result of Callendar and Barnes' experiments[2] on the variation of the specific heat of water, seven years later, proved that the calorie at 15° was very nearly equal[3] to the mean calorie. Regnault's value for the latent heat is now known to be too low. Dieterici's later experiments on the constant of the ice-calorimeter proved that his value for the latent heat at 0° was too high. So the relation between latent heat and temperature over the range 0° to 100° is not a linear one. Griffiths' values at 30° and 40° C. are in close agreement with the theoretical curve.

§ (3) HENNING'S EXPERIMENTS.—*Henning* adopted the same method as Griffiths in measuring the latent heat of steam. He used the apparatus shown in *Fig.* 2. The cylindrical evaporation vessel C was made of copper, of about 1 litre capacity. The seams of the vessel were soldered up and a screw stopper G used for closing the vessel. The cylinder contained a heating coil D and a platinum resistance thermometer E, which were both led through the cover and were electrically insulated and steam-proof. The jacket A surrounding the evaporation vessel contained about 13 litres of colza oil and was kept by electrical heating at about the boiling-point of water. This vapour-jacket, originally devised by Ramsay and Miss Marshall, not only kept constant the heat loss of the inner vessel, but also protected the steam from condensing on the way to the condenser.

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[1] 596·80 mean calories.

[2] *Phil. Trans.* A, 1902, cxcix. 55-148.

[3] The calorie at 15° is very nearly one part in 2000 less than the mean calorie.

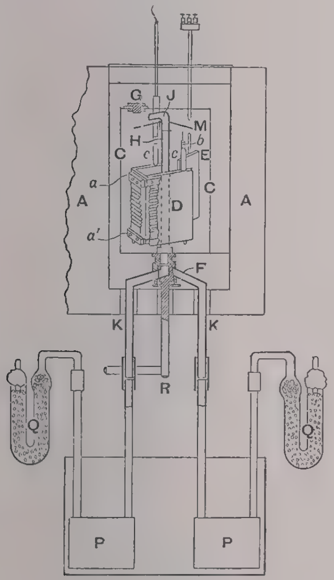

Fig. 2.

The steam developed in C was led through an elbow-joint J into a 5-mm. copper tube H, which was hard-soldered into the bottom of the vessel C; the umbrella-like roof M diverted the drops of water from the end of the steam pipe. The three-way tap F opened to either condenser, according as the handle R of the tap was turned to the right or left. By this means the irregularities which occurred at the beginning and end of the boiling period were eliminated. These irregularities appeared in the temperature measurement and were due to the fact that at the commencement of boiling the temperature in the water was not completely equalised by the rising steam bubbles, and also because the escaping steam raised somewhat the pressure, and at the same time the boiling-point of the water. At the end of the experiment the reverse effect took place for the same reason.

Initially steam was led one way until the heating current was steady and the steam was evolved regularly, then the tap F was turned and the steam led through the second path for a certain time, and condensed in a wide cooling apparatus, while the electrical energy dissipated in the heating current was observed at the same time. At the end of the run the tap was again turned into the first position and the heating current interrupted.

Henning measured the latent heat of steam at six different temperatures between 30° and 100° C. He kept the temperature constant during the measurement by altering the pressure at which the water boiled. The copper condenser P, which was connected by means of rubber tubing to the pipes K, was cooled to a temperature dependent upon the steam pressure of the boiling water. For this purpose water at room temperature, ice, or a mixture of alcohol and CO_2 were employed. Drying-tubes filled with calcium chloride and phosphorus pentoxide could be dispensed with when working at atmospheric pressure. By altering the pressure the boiling-point could be fixed at any desired point and the temperature kept at the required value by the use of a small pump.

Of the 680 gm. of water with which the vessel was filled 200 gm. were evaporated. The rate of evaporation could be raised to 50 gm. in 15 minutes. It was found that at low temperatures the accuracy of the measurements was not so great, on account of the large specific volume of the vapour; the steam had to be evaporated slowly to ensure that no water was withdrawn.

In order to determine the heat loss or gain from the surroundings, Henning made experiments with different rates of energy supply.

The weight of the condensed steam was determined by weighing the condenser; the small amount of water remaining in the steam pipe K (about 10 mgm.) being absorbed by a small piece of weighted filter-paper. Finally, account had to be taken of the fact that at the end of an experiment more steam was in the vessel than at the beginning, because in the interval the volume of water had decreased. This small correction amounted at 100° to +0·06 per cent and at 30° to +0·003 per cent.

He obtained values at six points between 30° and 100° C.

The final result is given by the formula

$$L = 538{\cdot}86 + 0{\cdot}5994(100 - t)$$

for the latent heat of water between 30° and 100°.

These results are expressed in terms of the calorie at 15°, which he takes as equivalent to 4·188 joules.

§ (4) SMITH'S EXPERIMENTS.—*A. W. Smith* employed a current of air for the evaporation of the water.

The calorimeter is shown in *Fig.* 3, and consists of a large test-tube E, closed at the top by a cork. Two glass tubes for the air current passed through this cork, one extending to the bottom of the test-tube, the other just projecting through the cork. A third tube closed at the bottom, and not shown in the figure, contained the heating coil. This was a single heater from a Nernst lamp provided with current and potential leads. The coil was immersed in paraffin oil so as to transmit the heat to the surrounding water.

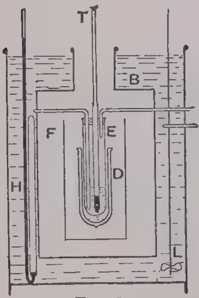

FIG. 3.

This test-tube calorimeter stood within a double-walled vacuum vessel D, the whole being well surrounded with light cotton-wool and held in a cardboard box, which in turn was supported in the middle of the constant temperature bath F. The remaining space around the sides, the top, and the bottom of the chamber was also lightly filled with cotton-wool to prevent convection currents.

The calorimeter was packed and set in position at least one day before an experiment could be performed, so as to obtain constant temperature conditions. The water evaporated was collected in a weighed pair of tubes containing sulphuric acid. These were changed every two hours and the increase of weight observed.

TABLE III

A. W. SMITH'S VALUES

Number of Experiments.	Temperature.	Latent Heat.
4	13·95	588·6
12	21·17	584·7
4	28·06	580·9
2	39·80	573·9

(Results expressed in mean calories on assumption that E.M.F. of Clark cell is 1·434 international volts at 15° C. and J = 4·1836 joules.)

Later Smith[1] made some observations at 100° C. employing a very *slow* rate of evaporation, and found the latent heat to be appreciably higher than when the boiling was rapid. These he explained on the assumption that the steam carried over minute quantities of water.

The value for the latent heat under these conditions was 540·70 mean calories at a temperature of 100° C.

§ (5) COMPARISON OF DATA FOR STEAM.—Callendar[2] has analysed the data obtained by various observers for the latent heat of steam and conveniently expressed them in terms of the *total heat of steam* H. This is defined as the quantity of heat required (1) to raise unit mass of water from 0° C. to the temperature t of the boiler, and (2) to evaporate it at that temperature, the whole operation being performed under a constant pressure p equal to the saturation pressure at the temperature of the boiler. The heat required for this second operation is of course the latent heat of evaporation at saturation pressure and temperature.

Regnault had previously expressed his results between 0° and 200° C. by the simple linear formula

$$\text{Total heat } H = 606 \cdot 5 + 0 \cdot 305t,$$

and this was accepted for the next fifty years without question.

It is now known that the relationship between total heat and temperature is not a linear one, and the results of later investigators have been collected together by Callendar, who has compared them with two formulae.[3] The curves computed from the formulae are shown by solid and dotted lines in *Fig.* 4.

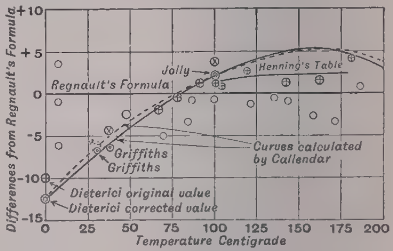

FIG. 4.

Since the object is to bring out the differences from Regnault's formula this is represented by the horizontal straight line marked "Regnault." Observations giving a lower result than Regnault's formula lie below the line, those giving higher results above, and the differences are plotted in mean calories.

Regnault's observations are shown by plain circles (○). The plain circle giving the value 636·7 at 100° represented the mean of thirty-eight experiments, and the corresponding value of the latent heat is 536·2 calories, if Regnault's formula is adopted for the total heat of the liquid. It might be remarked that Regnault was well aware that this result was probably too low on account of the presence of water in the steam, and his formula actually gives a value 0·3 calorie higher.

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[1] *Phys. Rev.*, 1911, xxxiii. 181.

[2] *Properties of Steam* (Arnold), 1920.

[3] See § (6).

The experimental points plotted between 65° and 85° C. represented the results of twenty-two experiments in four groups, some of which nearly reach the line, while others lie 5 or 6 calories below. The mean of all shows a defect from the formula of 2·5 calories, or 0·4 per cent, of the total heat at 75° C.

Combining the mean of these with the observations at 100° C. we should obtain a rate of increase of 0·40 in place of 0·30 for the total heat between 75° and 100° C., as remarked by Griffiths.

In experiments above 100° C. errors due to leakage vitiate Regnault's observations. For he states that the effects of leakage became very troublesome when the pressure was 10 atmospheres, and the joints had to be renewed daily. This is the probable explanation of the low point between 150° and 175°, and Regnault attached no weight to these points in selecting his formula.

Regnault's values between −2° and 16° C. are given in three groups in the neighbourhood of 8° C. His method of experiment was different, and discrepancies of the order of 10 calories were found between successive experiments.

Dieterici's two values at 0° C. differ on account of the change in the value of the Bunsen calorimeter constant. The old value shown thus (⊕) was on the assumption of the ice-calorimeter constant being 15·44 milligrams of mercury per mean calorie. His direct determination of this constant made it 15·50, and the corrected value of the latent heat falls very nearly on the theoretical curve at 0° C.

Griffiths' values (⊙) are in very satisfactory agreement with the theoretical curve.

Henning's values (⊕) are in good agreement with the theoretical curve below 100°, although the two points at 30° and 49° are appreciably higher while that at 100° is slightly lower.

Henning's five points between 100° C. and 180° C. are connected by the wavy line marked "Henning's Table."

This curve was deduced by a graphic process of smoothing, and according to Callendar does not represent the actual observations satisfactorily. It is obviously inadmissible for theoretical purposes, as it involves a discontinuity in the curve at 100° C. Three of the five observations are in very fair agreement with Callendar's theoretical curve, but the other two are lower by about 0·5 per cent, which is less than the probable error of experiment.

A. W. Smith's values (⊗) are systematically higher than both Griffiths' and Henning's. His point at 100° C. was obtained by slow evaporation, and is nearly 1 calorie higher than Joly's value.

He expressed his result in joules per gram, assuming the E.M.F. of the Weston cell to be 1·01888 at 20° C. Callendar has reduced Smith's values to the mean calorie by taking the E.M.F. of the Weston cell to be 1·0183 volts and the mean calorie to be 4·187 joules.

§ (6) FORMULAE FOR VARIATION OF LATENT HEAT OF STEAM WITH TEMPERATURE.—Many empirical formulae have been proposed for the representation of the variation of latent heat of steam with temperature, and of these the most satisfactory appears to be that of Thiesen.

This expression is based on the accepted view that the latent heat must vanish at the critical temperature (t_c). So that $L=L_1(t_c-t)^{\frac{1}{3}}$ where L_1 is a constant representing the value of L when $t_c-t=1$.

The index varies slightly for different substances.

Henning found that he could represent his results for water below 100° C. by a formula of this type, but assumed a critical temperature 9° C. too low.

Jakob and Davis, in reducing specific heat results above 100° C., employed the same form.

Callendar has recomputed these formulae, using the experimentally determined value of the critical point, viz. 374° C., and keeping the same index and the value of L at 100° C. given by the original formulae. Taking the logarithmic form for convenience, Henning formula reduces to

$$\text{Log } L=1{\cdot}96955+0{\cdot}31248 \log (374-t).$$

Davis and Jakob formula becomes

$$\text{Log } L=1{\cdot}96393+0{\cdot}3150 \log (374-t),$$

assuming $L=539{\cdot}3$ at 100° C.

Callendar, basing the constants of the formulae on the observations of Dieterici and Joly for the latent heat at 0° and 100° C. respectively, obtains the following result:

$$\text{Log } L=1{\cdot}97145+0{\cdot}31192 \log (374-t).$$

He, however, prefers for use in computations dealing with dry saturated steam the theoretical formula for the total heat H

$$H=0{\cdot}4772T-SCp+464,$$

where S is the specific heat at saturation pressure p, T the absolute temperature, and C the Joule Thomson cooling effect, *i.e.* the ratio of the fall of temperature to the fall of pressure in a throttling process at constant total heat.

A comparison of the data obtained by computation from the formulae is given in Table IV.

TABLE IV

VALUES OF L

Temp	Callendar.		Henning.	Davis and Jakob.
	Theoretical Formula.	Thiesen Type.		
0°	594·3	594·3	593·7	594·8
20	583·8	584·2	583·6	584·6
40	573·2	573·7	573·1	574·0
60	562·3	562·7	562·2	562·9
80	551·1	551·3	550·7	551·4
100	539·3	539·3	538·7	539·3
120	526·9	526·7	526·1	526·6
140	513·6	513·4	512·8	513·2
160	499·3	499·3	498·7	498·9
180	483·9	484·2	483·6	483·7
200	467·4	468·1	467·4	467·4

In *Fig.* 4 the full line represents the form of the theoretical formula, while the dotted line is the Thiesen type. The latter agrees with the theoretical form within 1 in 1000, intersecting at 160° and again at 260° C.

Davis and Jakob formula agrees with Callendar's within less than 1 in 1000 from 100° to 200°, and within 1 in 5000 from 200° to 230°, and the difference is still very small at 250° C.

Callendar's formula for the total heat, which applies to dry steam in any state, superheated or supersaturated, has been extrapolated by him to 259° C., although the direct experimental evidence does not extend beyond 180° C.

It is reasonable, however, to suppose that the extrapolated values are sufficiently accurate for practical purposes at higher temperatures, because the calculated values of the saturation pressure (which depend on small differences and afford a very severe test of the theory) also agree with observations to within less than 1° C. at 250° C.

§ (7) HEAT OF VAPORISATION OF AMMONIA.—Owing to the extensive use of ammonia in refrigerating plants, a knowledge of its thermal constants is of considerable practical importance. The latent heat has frequently been calculated by thermodynamic formulae from other properties more easily measured, but a number of direct determinations have also been made, and the following summary has been given by Osborne and Van Dusen :[1]

Regnault[2] published a record of twelve experiments saved from the ruins of his laboratory, destroyed during the siege of Paris in 1870. The apparatus consisted of two calorimeters — the first, or evaporation calorimeter, in which the ammonia was allowed to evaporate from a steel container and flow through a chamber containing baffle-plates; and the second, or expansion calorimeter, in which the ammonia vapour from the first calorimeter was allowed to expand to atmospheric pressure. The capacity for liquid ammonia in the first calorimeter was 246 c.c.; but it was filled with various amounts, ranging from 17 to 134 grams in different experiments. In each experiment the ammonia was completely evaporated and all vapour expanded to atmospheric pressure. The observed fall in temperature of the water in the first calorimeter varied from 1·7° to 13°, and in the second it was usually less than 1°. From the data obtained in the first calorimeter Regnault calculated a quantity λ, which is the heat required to change 1 gm. of saturated liquid ammonia at the initial temperature and pressure to vapour at the mean temperature of the experiment and at a pressure equal to the pressure in the expansion chamber of the first calorimeter. He stated, however, that it is better to combine the results of the two calorimeters, and so proceeded to calculate another quantity λ', which is the heat required to change 1 gm. of saturated liquid ammonia at the mean temperature and pressure to vapour at the mean temperature and at atmospheric pressure.

Regnault's results have been variously interpreted by different writers. Holst computed the latent heat of vaporisation from the observation in the first calorimeter and ignored the partial expansion below saturation pressure which occurred there. As a mean result he obtained 296 calories per gram at 12° C. Jacobus[3] computed the latent heat of vaporisation from the observations in both calorimeters and obtained as a mean value 290 calories per gram at 12° C. Landolt and Börnstein[4] give values from Regnault's data, the mean value at 12° C. being 294·5 calories per gram.

Von Strombeck[5] used the same type of apparatus as Regnault and obtained from twelve experiments a mean value of 296·5 calories per gram at 18° C.

Estreicher and Schnerr,[6] according to Landolt and Börnstein, determined the heat of vaporation at the normal boiling-point and obtained a value of 321 calories per gram at −33·4° C.

Franklin and Kraus[7] determined the heat of vaporisation at the normal boiling-point. The apparatus consisted of a Dewar flask containing a liquid bath and a glass evaporating cell, each supplied with a platinum heating coil. The energy required to evaporate a certain volume of liquid ammonia was measured and the mass computed from the volume evaporated, using the value 0·674 for the density. The mean result from three experiments was 341 calories per gram. Recomputed, using more recent data for the density (0·683), the mean value becomes 337 calories per gram. This result is the same as that deduced by Franklin and Kraus from the absolute boiling-point and the molecular elevation by van't Hoff's formula.

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[1] *Sci. Paper Bur. Stds.* No. 315, 1917.

[2] *Ann. Chim. Phys.*, 1871, xxiv. 375.

[3] *Trans. A.S.M.E.*, 1890, xii. 307.

[4] *Phys. Chem. Tabellen*, ed. 1912.

[5] *Jour. Franklin Inst.*, 1891, cxxxi. 470.

[6] *Bull. de l'Ac. de Cracovie*, 1910, p. 345; *Phys. Chem. Tables*, 1912.

[7] *Jour. Phys. Chem.*, 1907, xi. 553.

Osborne and Van Dusen redetermined the latent heat of ammonia by an electrical method. The apparatus[1] consisted of a metal shell with a re-entrant tube containing a heating coil and resistance thermometer; while the annular space contained the liquid ammonia.

The results may be expressed by the formula

$$L = 137{\cdot}91\sqrt{133 - \theta} - 2{\cdot}466(133 - \theta)$$

over the temperature range

$$-42^\circ \text{ to } +52^\circ \text{ C.},$$

TABLE V

HEAT OF VAPORISATION OF AMMONIA IN CALORIES$_{20}$ PER GRAM COMPUTED BY VARIOUS WRITERS AND GIVEN IN THEIR AMMONIA TABLES

Temperature.		Ledoux, 1878.	Peabody, 1889.	Wood, 1889.	Zeuner, 1890.	Mollier, 1895.	Dieterici, 1904.	Wobsa, 1908.
° F.	° C.							
− 40	− 40	335·2	332	322·0	333·0	332·7	..	..
− 22	− 30	330·5	324	316·0	329·9	330·6	..	324·3
− 4	− 20	325·3	316	309·9	325·8	327·2	..	317·0
+ 14	− 10	319·7	308	303·8	320·8	322·3	..	309·0
+ 32	0	313·6	300	297·6	314·9	316·1	309·7	300·4
+ 50	+ 10	307·2	292	291·3	308·0	308·6	298·4	290·9
+ 68	+ 20	300·3	284	284·8	300·1	299·9	285·4	280·6
+ 86	+ 30	293·0	276	278·4	291·3	289·7	272·2	269·4
+104	+ 40	285·3	..	271·9	281·6	278·0	258·3	257·4
+122	+ 50	..	..	265·3	..	..	243·6	244·6
+140	+ 60	..	..	258·6	..	..	227·9	..
+200	+ 93·3	..	..	..	..	..	165·2	..
+250	+121·1	..	..	..	..	..	..	..
+270	+132·2	..	..	..	..	..	..	..

Temperature.		Hybl, 1911.	Macintire, 1911.	Lucke, 1912.	Mosher, 1913.	Holst, 1915.	Keyes, 1916.	Osborne and Van Dusen, 1917.
° F.	° C.							
− 40	− 40	..	..	335·3	334·4	328·5	342·0	331·7
− 22	− 30	325·2	327·9	328·1	327·1	322·5	333·6	324·8
− 4	− 20	318·2	320·8	320·9	319·6	316·0	324·9	317·6
+ 14	− 10	310·7	313·0	313·1	311·8	309·0	315·7	309·9
+ 32	0	302·6	304·4	304·6	303·6	301·4	306·0	301·8
+ 50	+ 10	293·7	295·0	294·8	295·0	293·2	296·0	293·1
+ 68	+ 20	284·2	284·7	284·6	285·9	284·4	285·5	283·8
+ 86	+ 30	274·0	273·5	273·5	276·4	274·8	274·4	273·9
+104	+ 40	263·0	..	261·4	266·2	264·2	262·7	263·1
+122	+ 50	..	..	248·3	255·4	..	250·2	251·4
+140	+ 60	..	..	234·7	243·7	..	236·8	..
+200	+ 93·3	..	..	176·7	195·3	..	181·9	..
+250	+121·1	..	..	..	127·6	..	..	..
+270	+132·2	..	..	..	61·2	..	..	..

Ledoux, *Annales des Mines*, Mémoires, 7th series, 1878, xiv. 205. (Translation by Denton, Jacobus, and Riesenberger, under title of *Ice-making Machines*, D. Van Nostrand Co. (1892), p. 173.)
Mollier, *Zeit. ges. Kälte Ind.*, 1895, ii. 91.
Wood, *Thermodynamics, Heat Motors, and Refrigerating Machines*, ed. 1900, p. 466.
Zeuner, *Technische Thermodynamik*, 1901, ii. 18 (appendix).
Dieterici, *Zeit. ges. Kälte Ind.*, 1904, xi. 24.
Peabody, *Steam and Entropy Tables* (1909), p. 72.
Hybl, *Zeit. ges. Kälte Ind.*, 1911, xviii. 165.
Marks, *A.S.R.E. Trans.*, 1912, viii. 208.
Goodenough and Mosher, *Univ. of Ill. Bull.*, 1913, lxvi.
Holst, *Bull. Assoc. Int. du Froid*, 1915, li.
Keyes and Brownlee, *Thermodynamic Properties of Ammonia* (1910), 1916.
Wobsa, *Zeit. ges. Kälte Ind.*, 1908, xv. 11.
Macintire, *Ice and Refrigeration*, 1911, xli. 44.
Lucke, *Engineering Thermodynamics* (1912), p. 603.

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[1] See *Fig.* 3, "Calorimetry, Electrical Methods," § (7).

in which the latent heat is expressed in joules per gram required to convert saturated liquid into saturated vapour at constant temperature.

If the latent heat is required in calories (20° C.) per gram, and 1 calorie is assumed equal to 4·183 joules, then the expression becomes

$$L=32{\cdot}968\sqrt{133-\theta}-0{\cdot}5899(133-\theta).$$

§ (8) HEATS OF VAPORISATION OF CARBON DIOXIDE, SULPHUR DIOXIDE, AND NITROUS OXIDE.—Mathias[1] determined the latent heats of the above compounds by a method in which the temperature of the calorimeter was kept constant, and measured the heat absorbed by carefully adding concentrated sulphuric acid.

His results for carbon dioxide are of great interest, for they extend from 6·65° to 31·16°, or almost to the critical temperature. The carbon dioxide contained 0·75 per cent of air, which would cause the critical temperature to be rather lower than the true value, 31·35°. The formula deduced by Mathias from his results,

$$L=118{\cdot}485(31-t)-0{\cdot}4707(31-t)^2,$$

gives L=0 at 31°.

So the experiments may be taken to prove the heat of vaporisation does really become zero at the critical point.

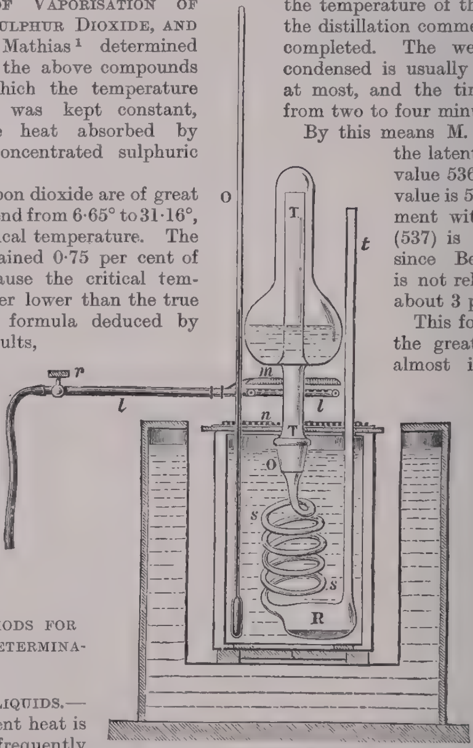

FIG. 5.

II. GENERAL METHODS FOR LATENT HEAT DETERMINATIONS

§ (9) ORGANIC LIQUIDS.—The value of the latent heat is a physical constant frequently required in the case of organic liquids, and Regnault's method is scarcely applicable when the quantity of the material is limited and the experiment has to be conducted with reasonable facility.

(i.) *Berthelot's Method.*—To meet these requirements Berthelot devised the apparatus shown in *Fig.* 5.[2]

The flask containing the liquid under examination is heated by a circular gas burner *l*, burning under a metallic disc *m*. The centre of the flask is traversed by a wide tube TT, through which vapour descends into the calorimeter, where it condenses in the spiral SS and collects in the reservoir R. The calorimeter is placed inside a water-jacket, and is protected from the radiation of the burner by a slab of wood covered by a sheet of wire gauze. By means of this arrangement partial condensation is avoided before the vapour enters the calorimeter, and the error arising from conduction is roughly corrected for by observations of the temperature of the calorimeter before the distillation commences and after it is completed. The weight of the liquid condensed is usually about 20 to 30 gm. at most, and the time occupied is only from two to four minutes.

By this means M. Berthelot found for the latent heat of water the value 536, while the accepted value is 540. The close agreement with Regnault's value (537) is probably fortuitous, since Berthelot's apparatus is not reliable to better than about 3 per cent.

This form of apparatus has the great defect that it is almost impossible to avoid the superheating of the vapour owing to the proximity of the central tube to the flame. The flame, moreover, causes disturbances of the calorimeter and thermometer by its radiation unless carefully shielded.

(ii.) *Hartog and Harker*[3] made a careful study of the Berthelot apparatus, and by modifying the arrangement were able to reduce the two sources of error due to (1) vapour passing over before the liquid actually reached the boiling-point, (2) the disturbance at the end of the experiment, caused by the removal of the boiler from the vicinity of the calorimeter, for it is assumed in Berthelot's form that the correction for the heat given during the condensation period can be obtained by observation of the rate of cooling after the boiler and flame are removed.

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[1] *Jour. de Phys.*, Nov. 1905.

[2] Preston, *Theory of Heat*, 3rd ed., Fig. 99, p. 365.

[3] *Memoirs and Proceedings Manchester Literary and Philosophical Society*, 1895–96, x.; *ibid.*, 1893–94, viii.

Their modification is shown in *Fig.* 6. The boiler consists of a flask A, through which the tube BC passes centrally. The upper end of BC is ground conically to fit into a hollow cap D, which is itself attached by a glass rod to the movable bell E. This bell fits loosely into a rim, which is filled with mercury so as to form a lute. The bell and cup may be thus raised or lowered at will, so as to open or close the valve at C, through which the steam passes downwards through CB into the condensing worm W. At F a side tube is connected with a condenser, if desirable, by means of an india-rubber tube fitted with a clip. The tube at F is kept open during the preliminary period; it is shut just after C is opened, and opened again just before C is closed, so that at no period does the internal pressure exceed that of the atmosphere. The end B of the tube BC is ground into the upper end of the condensing worm, of which the construction differs slightly from that of Berthelot. The steam in the apparatus enters the condensing worm by the straight portion, and not by the spiral. They found this alteration necessary as, otherwise, after the closing of the valve, the air entering the worm tends to drive the condensed water back into BC.

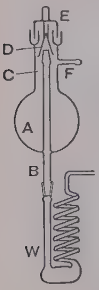

Fig. 6.

The lower half of A was surrounded by a piece of copper gauze bound on with asbestos string. The projecting portion of the tube BC was surrounded by asbestos and by a leaden steam-coil wrapped closely around it.

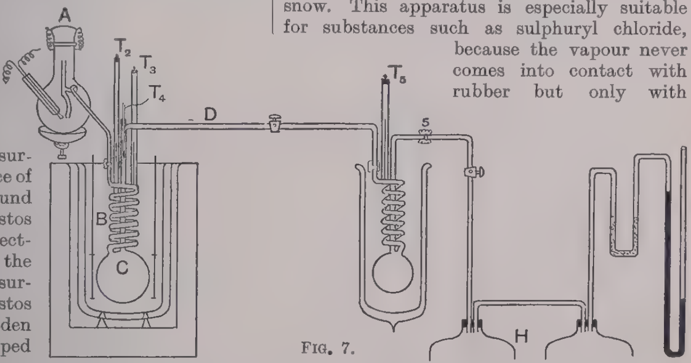

Fig. 7.

The boiler was heated by a small ring burner, of which the flame was kept at a constant height from the moment of lighting till it was extinguished. The gas was passed first through a Moitessier glycerine regulator (which maintained the pressure constant to within half a millimetre of water), and then through a tap fitted with a long handle moving in front of a graduated circle.

These precautions are necessary for accurate measurements, as variations in the height of the flame naturally cause the radiation to the calorimeter to vary. They were able to regulate the amount of this radiation at will; but, of course, too small a flame made the determination too slow, while too large a flame rendered the initial and final corrections too high. The calorimeter and its jacket, and the thermometer, were protected from excessive radiation by means of screens of asbestos board.

(iii.) *Trautz, Dechend, and Vogel.*—To prevent the liquid superheating, Trautz and Dechend[1] used an electrically-heated wire as a "boiling accelerator" when they were determining the latent heat of sulphuryl chloride. With a similar apparatus Vogel[2] later measured the latent heat of isopentane (boiling-point, 27° to 28°). In order to obtain a really constant temperature he did not heat the flask directly by a flame, but placed it in a water-bath which was heated by a small flame enclosed in an asbestos box. The apparatus is shown diagrammatically in *Fig.* 7. A glass spiral B, which ends in a flask C, is connected to the boiling vessel A as shown. This flask is connected by a tube D with a volume of air contained in flasks H of 120 litres content. The pressure in H is measured by means of a mercury manometer. In order to prevent the possibly uncondensed vapour from the flask C reaching H, a second condensation flask with glass spiral is inserted in the circuit and is cooled in a Dewar vessel by means of a mixture of carbon dioxide snow. This apparatus is especially suitable for substances such as sulphuryl chloride, because the vapour never comes into contact with rubber but only with glass; the mercury manometer measuring the pressure is also protected from the chemically active vapours by a U-tube filled with caustic potash.

The calorimeter is filled with 1200 c.c. of water, in which B and C are placed. It is built up of three beakers; the two smaller beakers were silvered inside and stood on pieces of cork, the outer beaker stood in a box covered outside with tin-foil and filled with diatomaceous earth. The wooden lid covering the calorimeter was also covered with tin-foil in order to reduce the radiation. A circular opening in the lid, made for the passage of the parts of the apparatus, was closed during the experiment by three layers of cardboard covered with tin-foil, which were separated from each other by insulating layers of air, and only contained the necessary openings for connecting tubes, thermometer, and stirrer. The stirrer consisted of two ring-shaped brass plates cut out obliquely in the form of an H. The rings, 6 cm. apart, were soldered to two wires, which, in order to obtain better thermal insulation, were connected above to two pieces of glass tubing cemented together. The space between the spiral and the wall of the beaker was almost entirely occupied by the stirrer. Before an experiment the desired pressure was obtained in the artificial atmosphere by means of an air pump.

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[1] *Zeitschr. f. Elektrochemie*, 1908, xiv. 275.

[2] *Zeitschr. f. physikal. Chemie*, 1910, lxxiii. 445.

The amount of heat measured in the calorimeter Q is made up of the latent heat and the heat which the liquid gives off while it is cooling from its boiling-point t_1 down to the temperature t_2. If W is the amount of vapour, L its latent heat,and c the average specific heat of the fluid in the region of temperatures t_1 to t_2, then

$$Q = WL + Wc(t_1 - t_2),$$

and the latent heat of the fluid,

$$L = \frac{Q}{W} - c(t_1 - t_2).$$

The specific heat of the fluid must, therefore, be known in order to obtain the latent heat.

(iv.) *Ramsay and Marshall.*[1] — Sir William Ramsay and Miss D. Marshall employed an exceedingly simple and convenient apparatus for the determination of the heat of evaporation of different liquids at their boiling-points. The apparatus consisted essentially of a glass bulb with a heating coil inside. This was surrounded with a jacket of the vapour of the liquid at the boiling-point. The heat necessary for evaporation was supplied electrically, the quantity evaporated being determined by the loss of the weight of the liquid from the bulb. In their experiments they employed a duplicate set of apparatus connected in series, and obtained the latent heat as a ratio from the known value of the one substance taken as standard. This method is such that the procedure enabled the experiment to be carried out without an exact knowledge of the values of the electrical units, except so far as it was necessary to correct for the unequal resistance coils in the two vessels. In using the method at the present day it would probably be preferable to employ a single unit and determine the energy generated in the heating coil with precision electrical instruments. One advantage of the method is that no correction is necessary for heat loss.

The apparatus is shown diagrammatically in *Fig.* 8. The bulb had a fine spiral platinum wire, the ends of which were attached to stout platinum terminals sealed into the glass; the terminals were gilded and amalgamated. The upper end of each bulb was drawn out into a rather narrow open tube through which the liquid could be introduced, and which could be closed to prevent loss during weighing. This bulb was set up in an ordinary vapour jacketing arrangement provided with side bulb and condenser. The jacket was closed at the bottom by an india-rubber cork through which passed two U-tubes containing mercury. The terminals of the heating coil rested on the inner ends; into the outer ends dipped the wires carrying the current. The cork was protected by a layer of mercury so that it could not come into contact with the liquid. Each lamp was jacketed by a vapour of its own liquid so that the temperature of its contents could be raised to the boiling-point without ebullition taking place until the current was switched on. In performing an experiment the liquid in the side bulb of the jacket was first caused to boil, and the current was not switched on until the contents of the lamp were judged to have reached the temperature of the condensing vapour. If this were so the liquid would pass into tranquil ebullition the moment the circuit was operated. It was generally found advisable to drop into the lamp a little glass capillary tube to provide a starting point for boiling, as most of the liquids showed a great tendency to be superheated and bump. A small correction was necessary for the loss of liquid before the boiling-point was reached, and this was determined by separate experiments.

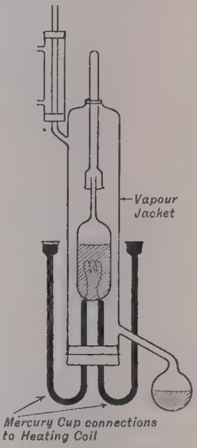

Fig. 8.

The original intention was to use water as the standard of comparison, but so many practical difficulties arose in connection with this that it was decided to adopt benzene.

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[1] *Phil. Mag.*, 1896, xli. 38.

§ (10) LATENT HEAT OF EVAPORATION OF BENZENE.—To obtain absolute value of the latent heat of benzene Griffiths and Marshall[1] employed the apparatus shown in *Fig.* 1.

The procedure was exactly the same as that adopted in the case of water and the following values were obtained:

TABLE VI

Temperature (Nitrogen Scale).	Latent of Benzene (Unit = calorie at 15° C.).
50°	99·14
40	100·71
30	102·30
20	103·82

These figures correspond closely with the linear formula

$$L=107{\cdot}05-0{\cdot}158t.$$

Assuming this formula to hold up to 80·2° C., the boiling-point of the benzene at atmospheric pressure, we get as the latent heat of vaporisation of benzene at its boiling-point

$$L=94{\cdot}34,$$

expressed in terms of the thermal units at 15° C. This does not differ very much from the direct determinations of R. Schiff[2] (93·4) and K. Wirtz[3] (92·9), using the Berthelot type apparatus.

§ (11) DISCUSSION OF LATENT HEAT DATA.[4]—Numerous attempts have been made to connect the value of the latent heat of a substance with its other physical properties.

(i.) *Trouton.*—The best-known generalisation is that of Trouton,[5] which states that

$$\frac{ML}{T}=\text{constant},$$

where L is the latent heat, M is the molecular weight, T the absolute temperature.

This law is true for members of the same chemical group such as the hydrocarbons, etc., but is not true for widely different substances, as Table VII. shows.

The law also applies to metals and inorganic substances with a different constant, a few values of which for representative groups is given below:

Hg, Cd, Zn, Bi *	19·36 to 20·20
P, PCl_3, CO_2, H_2S, CS_2	21·2 to 21·4
S, $SiCl_4$, Hg	21·6 to 22·0
Br_2, HI	23·0
NH_3	23·6

* Van Aubel, *Compt. rend.*, 1913, clvi. 456.

It was shown by Despretz in 1823[6] that some relationship of the form given above by Trouton held, and later Pictet (1876), Ramsay (1877),[7] rediscovered it independently, but it is now generally referred to as Trouton's law although his paper only appeared in 1884.

TABLE VII

	Latent Heat.*	Boiling-point.	$\frac{ML}{T}$.
Benzene	94·4	80·2°	20·65
Toluene	86·8	110·8	20·61
Metaxylene	82·8	138·5	21·03
Water	536·6	100·0	25·64
Alcohol	216·5	78·2	28·09
Acetic acid	97·0	118·5	14·72
Methyl formate	110·1	31·8	21·45
Ethyl formate	94·4	54·3	21·13
Methyl acetate	97·0	57·1	21·53
Propyl formate	90·2	80·9	22·38
Ethyl acetate	88·1	77·15	21·93
Methyl propionate	89·0	79·7	21·99
Propyl acetate	83·2	101·25	22·45
Ethyl propionate	81·8	99·2	22·22
Methyl butyrate	79·7	102·7	21·43
Methyl isobutyrate	75·0	92·3	20·74
Methyl alcohol	261·6	64·7	24·8
Formic acid	120·4	100·6	14·8
Chloroform	58·4	61·5	20·8
Aniline	113·9	184·0	23·2

* Correction required in some cases owing to evaporation of the liquid during the heating to the boiling-point, but correction probably of the same order as the errors of experiment.

(ii.) *Mills.*—In more recent years the theory has been developed a step further by J. E. Mills,[8] who has deduced the expression

$$\frac{L-E_1}{\sqrt[3]{\rho_L}-\sqrt[3]{\rho_V}}=\text{constant},$$

where L = latent heat of vaporisation,

E_1 = heat equivalent of external work.

Hence $L-E_1$ is the internal heat of vaporisation. ρ_L and ρ_V are the densities of liquid and saturated vapour respectively.

The formula is based on the following assumptions:

(1) The molecules are evenly distributed.

(2) The number of molecules does not change during vaporisation.

(3) No energy expended in intra-molecular work.

(4) The force of attraction does not vary with temperature.

(5) The molecular attraction varies inversely as the square of the distance.

In connection with assumption (5) Mills points out that if the other assumptions are correct the fifth follows.

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[1] *Phil. Mag.*, 1896, xli. 1.

[2] *Liebig's Annalen*, 1886, ccxxxiv. 338.

[3] *Wied. Ann.*, 1890, xl. 438.

[4] See also *Stoichiometry*, by Sidney Young (Longmans, Green, 1918).

[5] *Phil. Mag.* xviii. 54.

[6] *Ann. Chim. Phys.*, 1823, xxiv. 323.

[7] Phil. Soc. Glasgow.

[8] *Jour. Phys. Chem.*, 1902, vi. 209; 1904, viii. 383 and 593; 1905, ix. 402; 1906, x. 1: cf. Appleby and Chapman, *Trans. Chem. Soc.*, 1914, cv. 734; Sutton, *Phil. Mag.*, 1915, xxix. 593.

Mills, in testing the formula, obtained values of L over a wide range of temperature by calculation from the thermodynamic equation

$$L=\frac{T}{J}(V_V-V_L)\frac{dp}{dt},$$

where L = latent heat,
T = the absolute temperature,
dp/dt = the rate of increase of vapour pressure per degree.
V_V and V_L = volumes of saturated vapour and liquid.

Of 31 substances examined, 8 gave abnormal results. The measurements in the case of 3 were not altogether satisfactory, while the remaining 5 exceptions (water, 3 alcohols, and acetic acid) were abnormal in several of their physical properties.

From the remaining 23 substances it was possible to calculate 378 values of the constant. Mills found that the mean value was within 2 per cent for 349 cases, and 29 cases of divergencies were possibly due to errors of measurement.

§ (12) SUPPLEMENTARY REFERENCES TO LATENT HEAT DETERMINATIONS.

TABLE VIII

Observer.	Method.	Reference.
Wirtz	Joly steam calorimeter	*Wied. Ann.*, 1890, xl. 438
Wehnelt and Musceleanu .	Electrical method suitable for high temperatures	*Deutsch. Phys. Gesell.*, 1912, xiv. 1032
Jahn	Ice-calorimeter	*Zeit. physik. Chem.*, 1893, xi. 788
Fletcher and Tyrer . . .	Electrical method	*Trans. Chem. Soc.*, 1913, ciii. 517
Kahlenberg . .	Electrically heated platinum wire	*Journ. Phys. Chem.*, 1901, v. 215, 284
Shearer . . .	„	*Phys. Rev.*, 1903, xvii. 469
J. C. Brown . .	„	*Trans. Chem. Soc.*, 1903, lxxxiii. 987
Estreicher. . .	„	*Zeit. physik. Chem.*, 1904, xlix. 597; *Acad. Sci. Cracovie Bull.*, 1910, vii.A 345
Dewar . . .	Metallic spheres of known heat capacity dropped into liquefied gases: volume of gas measured	*Chem. News*, 1895, lxxi. 192
Behn	„	*Drude's Ann.*, 1900, i. 270

III. LATENT HEAT OF FUSION

The latent heat of fusion of any substance is defined as the quantity of heat required to convert unit mass of the solid at the melting-point into liquid at the same temperature.

A variety of methods have been devised to determine this physical constant, the substance usually studied being ice.

§ (13) HEAT OF FUSION OF ICE.—*Black*, in 1762, made a rough measurement and obtained the value 79·7 calories, and to him is also due the credit of first drawing a clear distinction between specific and latent heat.

In 1780 *Lavoisier and Laplace* made some experiments in which they determined the amount of ice melted by a known weight of water cooling through a known interval of temperature. The heated water was enclosed in a thin metal sphere, and it required sixteen hours for the sphere and its contents to reach equilibrium temperature with the ice on immersion in the calorimeter.

Although the method of experiment was obviously unsatisfactory, the value they gave —75 calories per gram of ice melted—was accepted for the next sixty years.

§ (14) DETERMINATIONS BY "METHOD OF MIXTURES." (i.) *Regnault*, in 1842, applied the "method of mixtures" to the problem. His first experiments were made with snow cooled slightly below 0°. The snow was immersed in the calorimeter and the change of temperature of the water observed. The value 79·2 calories was obtained for the latent heat. Other experiments with small blocks of ice gave 79·06 calories.

The following year La Provostaye and Desains[1] published an account of their experiments by a similar method, and gave the value 79·01 calories. Although considerable care was taken to eliminate the usual sources of error in calorimetric work it is not quite certain that sufficient care was taken to remove all traces of moisture adhering to the ice before immersion in the calorimeter. Their value is undoubtedly too low.

(ii.) *Hess*, in 1848, employed the same method, but cooled the ice several degrees below 0° C. to ensure the absence of adhering moisture. This procedure necessitated a determination of the specific heat of ice.

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[1] *Ann. de Chimie*, 3e, 1843, viii. 5.

The value obtained for the latent heat of fusion was 80·3 calories. Two years later *Person* [1] obtained the value 80·0 calories by a similar method.

(iii.) *Bogojawlensky*,[2] in 1904, employed mercury as calorimetric medium. The results of 6 experiments range from 79·41 to 79·91 with a mean value of 79·61.

He also determined the specific heat of ice over the range from −85° C. to −5·5° C. This constant was required for the reduction of the results of the latent heat experiments in which the range of temperature of the ice was from −10° to 0°. The value obtained for the specific heat of ice was 0·52.

(iv.) *Dickinson, Harper, and Osborne* [3] employed both the "method of mixtures" and an electrical method.

Samples of ice were prepared in the form of hollow cylinders and were cooled to a uniform temperature of either −0·72° or −3·78°.

Experiments on 92 samples of ice did not indicate any variation in the latent heat with the mode of preparation of the sample.

Observations on ice contaminated with a mixture of ammonia, sodium chloride, and calcium chloride to the extent of about 1 part in 1000 gave results about 1·4 per cent lower than for pure ice.

The mean of 21 determinations of pure ice gave the value 79·63 calories for the latent heat.

§ (15) DETERMINATIONS BY MEANS OF THE ICE-CALORIMETER.—In 1870 Bunsen devised the ice-calorimeter which bears his name. Knowing the change in volume on the melting of a known quantity of ice, the latent heat of fusion may be determined by experiments with the ice-calorimeter as shown by Bunsen. He observed the weight of mercury drawn in when water at the boiling-point was dropped into the inner tube.

Bunsen found the value 80·025 for the latent heat (see article on "Calorimetry, Methods based on the Change of State," § (2)).

Roth [4] has assembled the results of a number of determinations of the density of ice and deduced a value to use in calculating the heat of fusion from the data of Behn [5] and of Dieterici,[6] both of whom used the Bunsen ice-calorimeter.

The density determinations reviewed are as follows:

TABLE IX

Observer.	Reference.	Density.
Bunseu .	*Ges. Abh.* iii. 475 (corrected value)	0·9168
Zakrzevski	*Ann. der Phys.*, 1892, (3), xlvii. 155	·91658
Leduc . .	*Compt. rend.*, 1906, cxlii. 149	·9176
Nichols .	*Phys. Rev.*, 1899, viii. 21	·91603 to ·91795
Barnes .	*Phys. Rev.*, 1901, xiii. 55	·91649
Vincent .	*Phys. Rev.*, 1902, xv. 129	·9160

He states the mean, excluding Nichols' values, to be 0·9167, but uses Bunsen's corrected value 0·9168, on the assumption that the conditions of producing the ice used in that density determination were nearly the same as those under which the ice-calorimeter is generally used.

Applying this value to Behn's data Roth obtains for heat of fusion of ice 79·69 cal.$_{15°}$, similarly for Dieterici's data 79·60 cal.$_{15°}$.

§ (16) ELECTRICAL METHODS.—A. W. Smith,[7] in 1903, applied the electrical method to the problem.

The method employed by Smith may be described briefly as follows: The ice was broken into small pieces, and cooled several degrees below zero. It was transferred to a calorimeter, containing light oil, also cooled below 0° C. The calorimeter was then warmed slowly by a feeble electric current until the temperature was about 1° C. below zero. A larger current was then applied for a sufficient time to melt the ice, and raise the resulting water to half a degree above zero. The electric energy had then been expended in raising the temperature of the ice and the calorimeter from −1° C. to 0° C., in melting the ice, and in raising the water and calorimeter to 0·5° C. In addition some heat was lost by radiation, conduction, and convection. By suitable arrangements of the experimental conditions all of these quantities were determined.

§ (17) RESULTS OF EXPERIMENTS.—The results obtained by various observers for the latent heat of fusion of ice are summarised in Table X.

§ (18) VARIATION OF LATENT HEAT OF FUSION WITH TEMPERATURE.—Person calculated that the heat of fusion of a body is smaller the lower the temperature becomes. Petterson verified this in the case of ice, and found that for 1° C. lowering of the melting-point of ice the heat of fusion was decreased by 0·59 calorie as against 0·5 predicted by the theory. At −6·5° C. observation gave a value of 76·0 calories.

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[1] *Ann. de Chimie*, 3^{e}, 1850, xxx. 73.

[2] *Schriften der Dorpater Naturforscher Gesellschaft*, 1904, xiii.

[3] *Sci. Paper*, No. 209, 1913. (Contains a summary of previous work.)

[4] *Zs. für phys. Chem.*, 1908, lxiii. 441.

[5] *Ann. der Phys.*, 1905, (4), xvi. 653.

[6] *Ibid.*, 1905, p. 593.

[7] *Phys. Rev.*, Oct. 1903, No. 4, xvii.

TABLE X

Name.	Date.	Experiments.	Temperature Range of Water.	Heat of Fusion.
Black	1762	2	80 · 0	79·7
Regnault . . .	1842	4	16 − 7	79·24
Regnault . . .	1843	13	22 − 11	79·06
Provostaye and Desains . . .	1843	17	24 − 10	79·1
Hess	1848	40	19 − ·7	80·34
Person	1850	6	16 − 5	80·0
Bunsen. . . .	1870	2	100 − 0	80·02
Smith	1903	8	Electrical method	79·59*
Bogojawlensky .	1904	6	Mercury	79·61
Behn (calc. by Roth)	1905	. .	. .	79·69
Dieterici (,, ,,)	1905	. .	. .	79·60
Dickinson, Harper, and Osborne .	1913	21	Electrical	79·63†

* Recalculated by Dickinson, Harper, and Osborne to the 15° calorie and the gram mass.

† Calories at 15° per gram mass.

E. G.

LATENT HEAT, DETERMINATION OF. See "Latent Heat."
  Berthelot's Method. § (9) (i.).
  Hartog and Harker's Method. § (9) (ii.).
  Ramsay and Marshall's Method. § (9) (iv.).
  Trautz, Dechend, and Vogel's Method. § (9) (iii.).
  By different observers not referred to in the text, tabulated references to. § (12), Table VIII.

LATENT HEAT OF VAPOUR. See "Thermodynamics," § (29).

LATERAL STRAINS—APPARATUS FOR MEASURING. See "Elastic Constants, Determination of," § (57).

LEAD, ATOMIC HEAT OF, at low temperatures, Nernst's values for, tabulated. See "Calorimetry, Electrical Methods of," § (11), Table VI.

LEAD, SPECIFIC HEAT OF, at various temperatures, tabulated, with the Atomic Heat. See "Calorimetry, Electrical Methods of," § (10), Table V.

LEADS, COMPENSATING, IN THERMO-ELECTRIC PYROMETER: lead wires of the same materials as those employed in the thermocouple. See "Thermocouples," § (20).

LEAST ACTION, PRINCIPLE OF. If the total energy of a system of bodies in motion remains constant, the sum of the products of the masses, the velocities, and the spaces described is a minimum. Thus $\Sigma m\int v\,.\,ds$ is a minimum.

LENOIR GAS ENGINE. See "Engines, Thermodynamics of Internal Combustion," (§ 32) (i.).

LIFTS, HYDRAULIC. See "Hydraulics," § (58).

LIME, GENERAL MECHANICAL PROPERTIES OF. See "Elastic Constants, Determination of," § (154).

LIQUEFACTION OF GASES

AN important part of the science of *Refrigeration* (*q.v.*) deals with methods of producing cold so extreme as to liquefy air and other so-called permanent gases. This, apart from its application to purposes of research, is now the basis of a considerable industry, which employs the liquefaction of air as a step towards the separation of its constituents, with the object of making commercial use of the oxygen or the nitrogen or both. To liquefy any pure gas the gas must be cooled below its critical temperature,[1] which for oxygen means a cooling below −118° C. and for nitrogen a cooling below −146° C. Temperatures much lower than this have been reached by the methods hereafter described. Hydrogen, whose critical temperature is −235° C., has not only been liquefied (first by Dewar in 1898) but solidified; its melting-point under atmospheric pressure is about −258° C. or 15° absolute. Even helium, the most refractory of all known gases, has been liquefied (first by Onnes in 1908) under conditions that lowered the temperature to within three or four degrees of the absolute zero. Its critical temperature is −268° C. These remarkable achievements became possible through the invention by Dewar in 1891 of the *Vacuum Vessel*, which secures thermal insulation by the use of two glass walls with a very perfectly exhausted space between them. On the inner surface of the outer wall Dewar deposits a film of mercury, which greatly reduces the entry of heat by radiation. In such vessels liquid gases may be decanted and handled with ease, carried about from place to place, and even stored for short periods with no more than a moderate loss by evaporation.

§ (1) REGENERATIVE COOLING.—One way of reaching a very low temperature, originally used by Pictet and called the "cascade" method, which has done valuable service in the hands of Dewar, is to have a series of vapour-compression refrigerating machines so connected that the working substance in one, when cooled by its own evaporation, acts as the circulating fluid to cool the condenser of the next machine of the series, and so on. Different working fluids are selected for the successive machines, so that each in turn reaches a lower temperature than its predecessor. The working fluid in any one machine is evaporated at a temperature which is lower than the critical temperature of the fluid that is used in the next machine of the series. Each machine has its compressing pump, condenser, and expansion-valve. Thus the first machine may use, say, carbonic acid, letting it evaporate at a temperature of − 80° C. or so. The next machine may use ethylene, condensing it in thermal contact with the evaporating carbonic acid, and letting it evaporate at about − 130° C. This in turn will serve for the condensation of such a gas as oxygen after moderate compression in a third machine.

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[1] See article "Thermodynamics," § (37).

But it is in a different way that the commercial liquefaction of air and other gases is actually carried out. The usual process is a regenerative one, first successfully developed by Linde, in which the Joule-Thomson effect of irreversible expansion in passing a constricted orifice (see "Thermodynamics," § (50)) serves as the step-down in temperature, and a cumulative cooling is produced by causing the gas which has suffered this step-down to take up heat in a thermal interchanger from another portion of gas that is on its way to the orifice. The gas to be liquefied is itself the working substance.

Imagine a gas such as air to have been compressed to a high pressure P_A, and to have had the heat developed by compression removed by circulating water or other means, so that its temperature is that of the surroundings. Call this initial temperature T_1. Let the compressed gas at that temperature enter an apparatus in which it expands irreversibly (through an expansion-valve or porous plug or constricted orifice of any kind) to a much lower pressure P_B, at which pressure it leaves the apparatus. If the gas were an ideal perfect gas this irreversible expansion would cause no fall in temperature. In a real gas there is in general a fall from T_1 to some lower temperature T'. The fall $T_1 - T'$ measures the Joule-Thomson cooling effect for the given drop in pressure. In Joule and Thomson's experiments on air it was about a quarter of a degree for each atmosphere of drop in pressure.

The cooling effect of the drop in pressure may be measured by the quantity of heat which would have to be supplied to the gas, per unit of mass, after expansion, to restore it to the temperature at which it entered the apparatus. Call that quantity Q: then

$$Q = K_p(T_1 - T'),$$

where K_p is the mean specific heat of the gas at the lower pressure P_B between these temperatures.

We may define Q as the quantity of heat which each unit quantity of the gas would have to take up within the apparatus if its temperature on leaving the apparatus were made equal to its temperature on entry. It measures the available cooling effect due to each unit quantity of gas that passes through the apparatus.

Suppose now that there is a counter-current interchanger by means of which the stream of gas which has passed the orifice takes up heat from the stream that is on its way to the orifice, with the result that the outgoing stream, before it escapes, has its temperature restored to T_1 or very near it. This is easily accomplished by having, within the apparatus, a long approach pipe or worm through which the compressed gas passes before it reaches the orifice, and round the outside of which the expanded gas passes away, so that there is intimate thermal connection between the two streams. For simplicity we may assume the interchanger to act so perfectly that when the outgoing gas reaches the exit it has acquired the same temperature T_1 as the entering gas. Each unit of it will therefore have taken up a quantity of heat equal to Q as defined above.

Under these conditions the apparatus will steadily lose heat at the rate of Q units for every unit quantity of gas that passes through. If we suppose the apparatus as a whole to be thermally insulated against leakage of heat into it from outside, there will consequently be a continuous reduction of the stock of heat that is held by the pipes and the gas in them. The result is a progressive cooling which constitutes the first stage of the action.

It may help to make the matter clear if we draw up an account of the energy received and discharged by the apparatus. Gas enters at A (*Fig.* 1) under the pressure P_A and at the temperature T_1. Gas leaves the apparatus at B under the pressure P_B and at the same temperature T_1, having taken up, through the action of the interchanger, a quantity of heat equal to Q. The pipes and expansion orifice are not shown in the sketch: they are within the enclosing case, which is assumed to be a perfect non-conductor of heat. During the first stage of the action the stopcock C (which is used at a later stage for drawing off the liquefied gas) is closed, and all the gas

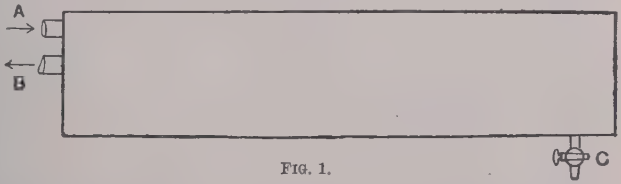

FIG. 1.

that has gone in at A goes out at B; it is only by the entry of gas at A and by its escape at B that energy enters or leaves the apparatus.

Each unit of entering gas contains a quantity of internal energy E_A, and the work that is done upon it as it goes in is P_AV_A. Each unit of outgoing gas contains a quantity of internal energy E_B, and does work, against external pressure, equal to P_BV_B. Hence, for each lb. that flows through, the net amount of heat which the apparatus loses is

$$E_B+P_BV_B-(E_A+P_AV_A), \text{ or } I_B-I_A.$$

But the amount so lost is Q, namely the heat that is required to restore the gas to the temperature at which it makes its exit. Hence for each unit quantity of gas that passes through the apparatus,

$$Q=I_B-I_A.$$

The contents of the apparatus become colder and colder in consequence of this continued abstraction of heat. But it should be noticed that their fall in temperature does not affect the value of Q. We assume that the action of the thermal interchanger continues to be perfect; in that case the exit temperature will still be equal to the initial temperature T_1 however cold the interior becomes in the neighbourhood of the expansion orifice. There will be no change in the value of either I_B or I_A, and consequently no change in Q. The value of Q, as the above expression shows, depends entirely on the conditions at A and at B; with perfect interchange this means that it depends only on P_A, P_B, and T_1. It is independent of any temperature conditions within the apparatus, and therefore remains unaltered as the action proceeds.

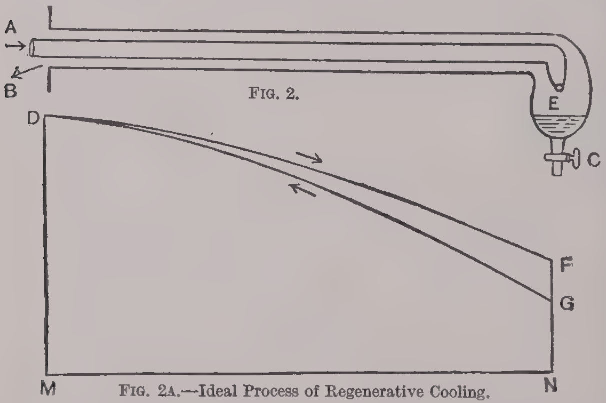

FIG. 2.

FIG. 2A.—Ideal Process of Regenerative Cooling.

This stage of progressive cooling continues until the temperature of the gas at the place where it is coldest, namely on the low-pressure side of the expansion-valve, falls not only below the critical point, but to a value T_2 which is low enough to let the gas begin to liquefy under the pressure P_B. In other words, T_2 is the boiling-point corresponding to P_B. This is the lowest temperature that is reached.

A continuous gradient of temperature has now become established along the flow-pipe within the apparatus from the point of entrance, where it is T_1, to the high-pressure side of the expansion valve, where it exceeds T_2 by the amount of the Joule-Thomson drop. There is also a continuous gradient along the return pipe from T_2, on the low-pressure side of the valve, to T_1 at the exit. The flow and return streams are in close thermal contact, and at each point there is an excess of temperature in the flow which allows heat to pass by conduction into the return, except at the entrance, where, under the ideal condition which we have postulated of perfect interchange, the temperature of both flow and return is T_1.

This state of things is diagrammatically represented in *Figs.* 2 and 2A. There the flow and return are represented as taking place in straight pipes, one inside the other to provide for interchange of heat. Entering along the inner pipe A the compressed gas expands through a constricted orifice E (equivalent to an expansion-valve) into a vessel from which it returns by the outer pipe B. The vessel is provided with a stopcock C, by which that part of the fluid which is liquefied can be drawn off when the second stage of the operation has been reached. In the temperature diagram (*Fig.* 2A) MN represents the length of the interchanger, DM is the temperature T_1 at which the gas enters and leaves the apparatus, GN is T_2, and FG is the Joule-Thomson drop. DF is the temperature gradient for the flow-pipe, and GD for the return.

When this gradient has become established the gas begins to liquefy, the apparatus does not become any colder, and the action enters on the second stage, which is one of thermal equilibrium. A certain small fraction of the gas is continuously liquefied and may be drained off as a liquid through the stopcock C. The larger fraction, which is not liquefied, continues to escape through the interchanger and to leave the apparatus at the same temperature as before, namely the temperature T_1, equal to that of the entering gas. Call this unliquefied fraction q; then $1-q$ represents the fraction that is drawn off as a liquid at the temperature T_2. Since the apparatus is now neither gaining nor losing heat on the whole, its heat account must balance; from which

$$I_A = qI_B + (1-q)I_C,$$

where I_A is the total heat per lb. of the gas entering at A, I_B is the total heat per lb. of the gas leaving at B, and I_C is the total heat per lb. of the liquid leaving at C. In this steady working the aggregate total heat of the fluid passing out is equal to that of the fluid passing in, since the fluid, as a whole, takes up no heat in passing through the apparatus.

Suppose now that the liquid which is drawn off at C were evaporated at its boiling-point T_2, and then heated at the same pressure from T_2 to T_1. Assuming that the specific heat of the vapour may be treated as constant, the heat required to perform that operation would be

$$(1-q)[L + K_j(T_1 - T_2)].$$

But that hypothetical operation would result in this, that the whole of the fluid then leaving the apparatus would be restored to the temperature of entry, namely T_1, since the part which escapes at B is already at that temperature. Hence the heat required for it is equal to the quantity Q as already defined. We therefore have

$$(1-q)[L + K_p(T_1 - T_2)] = Q,$$

from which

$$1 - q = \frac{Q}{L + K_p(T_1 - T_2)}.$$

This equation allows the fraction that is liquefied to be calculated when Q is known. The fraction so found is the ideal output of liquid, for we have assumed that there is no leakage of heat from without, and that the action of the interchanger is perfect in the sense that the outgoing gas is raised by it to the temperature of entry. Under real conditions there will be some thermal leakage, and the gas will escape at a temperature somewhat lower than T_1; the effect is to diminish the fraction actually liquefied.

The fraction that is liquefied is increased by using a larger pressure drop. It is also increased by reducing the initial temperature T_1; thus the output of a given apparatus can be raised by using a separate refrigerating device to pre-cool the gas. Pre-cooling is indispensable if the method is to be applied to a gas in which, like hydrogen, the Joule-Thomson effect is a *heating* effect at ordinary temperatures, but becomes a cooling effect when the initial temperature is sufficiently low.

This principle of regenerative cooling was applied by Linde in 1895 for the production of extremely low temperatures, and for the liquefaction of air, by means of an apparatus shown diagrammatically in *Fig.* 3. It consists of an interchanger CDE formed of two spiral coils of pipes, one inside the other, enclosed in a thermally insulating case. A compressing pump P delivers air under high pressure through the valve H into a cooler J, where the heat developed by compression is removed by water circulating in the ordinary way from an inlet at K to an outlet at L. The highly compressed air then passes on through the pipe BC to the inner worm, and after traversing the worm it expands through the throttle-valve R into the vessel T, thereby suffering a drop in temperature. Then it returns through the outer worm F and, being in close contact with the inner worm, takes up heat from the gas that is still on its way to expand. Finally, it reaches the compression cylinder P through the suction valve G, and is compressed to go again through the cycle. During the first stage it simply goes round and round in this way; but when the second stage is reached and condensation begins, the part that is liquefied is drawn off at V and the loss is made good by pumping in more air through the stop-valve at A, by means of an auxiliary low-pressure pump, not shown in the sketch, which delivers air from the atmosphere to the low-pressure side of the circulating system.

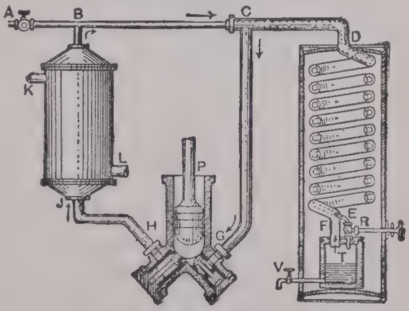

FIG. 3.—Linde's Regenerative Apparatus.

Linde showed that by keeping this lower pressure moderately high it is practicable to reduce the amount of work that has to be spent in liquefying a given quantity of air. He pointed out that while the cooling effect of expansion depends upon the difference of pressures P_A and P_B on the two sides of the orifice, the work done in compressing the air in the circulating system depends on the ratio of P_A to P_B. It is roughly proportional to the logarithm of that ratio, for it approximates to the work spent in the isothermal compression of a perfect gas. There is accordingly a substantial advantage in respect of thermodynamic efficiency in using, for the main part of the working substance, a closed circulation with a fairly high back-pressure.

In working on a small scale it is convenient to accelerate the action by using carbonic acid as a preliminary cooling agent. This is illustrated in the laboratory apparatus of *Fig.* 4. There compressed air enters on the right and passes first through a coil surrounded by carbonic acid which has been cooled by release through an expansion valve from a compressed state. The effect is to pre-cool the air to about −80° C. It then passes on to the regenerator, which is situated in the centre of the apparatus and consists of a long spiral of fine metal tubing outside of which the air streams off after passing through the expansion valve at the bottom. In passing the valve some of the air is liquefied and collects in the vacuum vessel below.

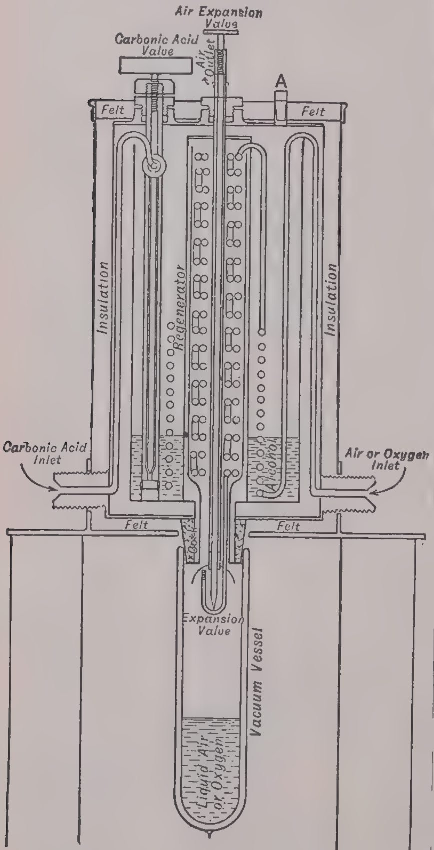

FIG. 4.

Important modifications of Linde's method were made in 1902 and later by G. Claude. It had long been recognised that there would be a thermodynamic advantage if the fluid, instead of expanding irreversibly through a constricted orifice, were made to do external work by expanding in an expansion cylinder or equivalent device. A greater step-down in temperature would then be obtained, for in addition to the Joule-Thomson cooling effect there would be the larger cooling effect due to the energy which the substance loses in doing work. Claude succeeded in overcoming practical difficulties as to lubrication which had prevented the use of an expansion cylinder in very low temperatures, and devised various forms of expansive working, one of which is shown diagrammatically in *Fig.* 5. There only a part of the compressed air, which enters through the central pipe of the counter-current interchanger M, passes into the expansion cylinder D. It expands, doing external work, and is then discharged through a tubular condenser L, in which it serves as a cooling agent to maintain a temperature that is not only lower than the critical temperature of air, but is sufficiently low to make air liquefy at the pressure at which it enters the apparatus. The remainder of the compressed air is directly admitted to the tubes of L and is condensed there, still under the pressure of admission, dropping as a liquid to the lower part of the vessel, from which it can be drawn off through a tap at the bottom.

§ (2) SEPARATION OF THE CONSTITUENTS OF AIR.—After air has been liquefied the constituent gases can be separated by re-evaporation because they have different boiling-points. The boiling-point of nitrogen, under atmospheric pressure, is about −195° C., whereas that of oxygen is −182° C. When a quantity of liquefied air evaporates freely both gases pass off, but not in the original proportion in which they are mixed in the liquid. The nitrogen evaporates more readily, and the liquid that is left becomes richer in oxygen as the evaporation proceeds. This difference in volatility between oxygen and nitrogen makes it possible to carry out a process of *rectification* analogous to the process used by distillers for extracting spirit from the "wash" or fermented wort, which is a weak mixture of alcohol and water.

In the device used for that purpose there is a rectifying column consisting of a tall chamber containing many zigzag shelves or baffle plates. The wash enters at the top of the column and trickles slowly down, meeting a current of steam which is admitted at the bottom and rises up through the shelves. The down-coming wash and the up-going steam are thereby brought into close contact and an exchange of fluid takes place. At each stage some of the alcohol is evaporated from the wash and some of the steam is condensed, the heat supplied by the condensation of the steam serving to evaporate the alcohol. The condensed steam becomes part of the down-coming stream of fluid; the evaporated alcohol becomes part of the up-going stream of vapour. Finally, at the top a vapour comparatively rich in alcohol passes off; at the bottom a fluid accumulates which is water with little or no alcohol in it. A temperature gradient is established in the column: at the bottom the temperature is that of steam, and at the top there is a lower temperature approximating to the boiling-point of alcohol. The wash enters at this comparatively low temperature, and takes up heat from the steam as it trickles down.

A corresponding method was patented by Linde in 1902 for separating the more volatile constituent (nitrogen) from liquid air. In his appliance, the primary purpose of which was to obtain oxygen, there is a rectifying column down which liquid air trickles, starting at the top with a temperature a little under −194° C. or 79° absolute, which is the boiling-point of liquid air under atmospheric pressure. As the liquid trickles down it meets an up-going stream of gas which consists (at the bottom) of nearly pure oxygen, initially at a temperature of about 91° absolute, that being the boiling-point of oxygen under atmospheric pressure. As the gas rises and comes into close contact with the down-coming liquid, there is a give and take of substance: at each stage some of the rising oxygen is condensed and some of the nitrogen in the down-coming liquid is evaporated; the liquid also becomes rather warmer. By the time it reaches the bottom it consists of nearly pure oxygen: the nitrogen has almost completely passed off as gas, and the gas which passes off at the top consists very largely of nitrogen. More precisely it consists of nitrogen mixed with about 7 per cent of oxygen: in other words, out of the whole original oxygen content of air (say 21 per cent) two-thirds are brought down as liquid oxygen to the bottom of the column, while one-third passes off unseparated along with all the nitrogen. The oxygen that gathers at the bottom is withdrawn for use, and in its evaporation it serves to liquefy air which is being pumped in under pressure. The cold gases that are leaving the apparatus, namely the oxygen that is the useful product, and the nitrogen that passes off as waste gas at the top of the column, are made to traverse counter-current interchangers on their way out, in which they give up their cold to the incoming air.

A rectifying column arranged in this way does not completely separate the two constituents, for although it yields nearly pure oxygen it allows a part of the oxygen to escape and does not yield pure nitrogen. In a commercial process for the manufacture of oxygen this is of no consequence. But a modification of the process, introduced later by Claude (*Comptes rendus*, Nov. 20, 1905), enables the separation to be made substantially complete, and yields both gases in a nearly pure state. The modification consists in extending the rectifying column upwards and in supplying it at the top with a liquid rich in nitrogen. A fractional method of liquefaction is adopted which separates the condensed material at once into two liquids, one containing much oxygen and the other little except nitrogen. The latter is sent to the top of the rectifying column, while the former enters the column at a lower point, appropriate to the proportion it contains of the two constituents. Practically pure nitrogen passes off as gas at the top, and practically pure oxygen from the bottom.

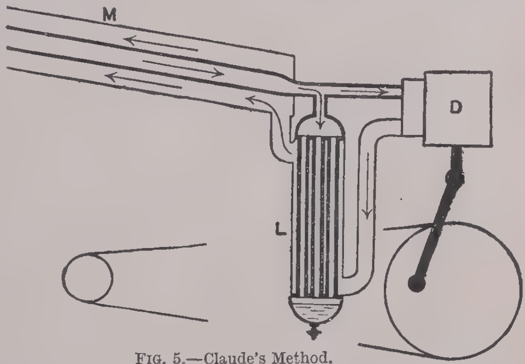

FIG. 5.—Claude's Method.

Fig. 6 is a diagram showing the modified process as carried out by Claude. The counter-current interchangers, which are of course part of the actual apparatus, are omitted from the diagram.

Compressed air, cooled by the interchanger on its way, enters the condenser at A. The condenser consists of two sets of vertical tubes, communicating at the top, where they all open into the vessel B, but separated at the bottom. The central tubes, which open from the vessel A, are one set; the other set form a ring round them and drain into

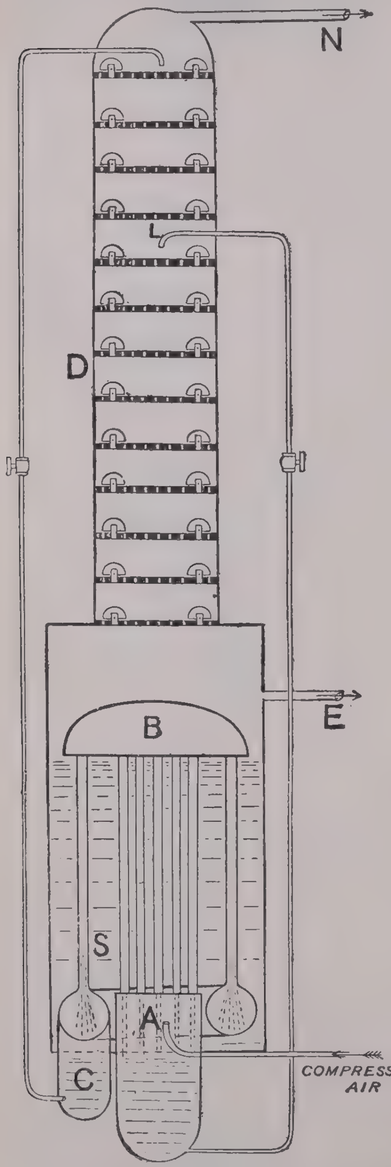

FIG. 6.—Claude's Apparatus for the complete Separation of Oxygen and Nitrogen.

the vessel C. Both sets are immersed in a bath, S, of liquid which, when the machine is in full operation, consists of nearly pure oxygen. The condensation of the compressed air causes this oxygen to be evaporated. Part of it streams up the rectifying column D, to be condensed there in carrying out the work of rectification and consequently to return to the vessel below. The rest of the evaporated oxygen, forming one of the useful products, goes off by the pipe E at the side. The compressed air, already cooled by the interchangers, enters the condenser at A. It first passes up the central group of condenser tubes, and the liquid which is formed in them contains a relatively large proportion of oxygen. This liquid drains back into the vessel A, where it collects, and the gas which has survived condensation in these tubes goes on through B to the outer set of tubes, is condensed in them, and drains into the other collecting vessel C. It consists almost wholly of nitrogen. The liquid contents of C pass (through an expansion-valve) to the top of the rectifying column, while those of A (after also passing through an expansion-valve) enter the column lower down, at a level L which is chosen to correspond with the proportion of the constituents. The result is to secure practically complete rectification, and the second product of the apparatus—commercially pure nitrogen—passes off at the top through the pipe N.

To understand how a rectifying column acts in separating the constituents of air, it is useful to refer to the experiments of E. C. C. Baly (*Phil. Mag.*, June 1900) on the evaporation of mixtures of liquid, nitrogen, and oxygen. Given a mixture of these liquids in any assigned proportion, equilibrium between liquid and vapour is possible only when the vapour contains a definite proportion of the two constituents, but this proportion is not the same as that in the liquid mixture. Say, for example, that the liquid mixture is half oxygen and half nitrogen, then according to Baly's experiments the vapour proceeding from such a mixture will consist of about 22 per cent of oxygen and 78 per cent of nitrogen. With these proportions there will be equilibrium. If, however, a vapour richer than this in oxygen were brought into contact with the half-and-half liquid, part of the gaseous oxygen would condense and part of the liquid nitrogen would be evaporated, until the proportion giving equilibrium is reached. The curve, *Fig.* 7, shows for each proportion of oxygen in the mixed liquid what is, in the vapour, the corresponding proportion of oxygen necessary for equilibrium, in other words what is the proportion in the vapour, when that is being formed by the evaporation of the mixed liquid in the first stages of such an evaporation, before the proportion of the liquid changes. In this curve the base line specifies the percentage of oxygen in any mixture of the two liquids, and the ordinate gives the proportion of oxygen in the corresponding vapour, when the vapour is formed under a pressure of one atmosphere. Much the same general relation will hold at other pressures. It will be seen from the curve that when the evaporating liquid mixture is liquid air (oxygen 21 per cent, nitrogen 79 per cent) the proportion of oxygen present in the vapour that is coming off is about 7 per cent or a little less. This is what occurs at the top of Linde's original rectifying column. The liquid that is evaporating there is freshly formed liquid air, and hence the waste gases carry off about 7 per cent of oxygen. Coming down the column the liquid finds itself in contact with gas containing more oxygen than corresponds to equilibrium. Accordingly oxygen is condensed and nitrogen is evaporated at each stage in the descent, in the effort at each level to reach a condition of equilibrium between the liquid and the vapour with which it is there in contact.

Again, the curve shows that when air, containing 21 per cent of oxygen, liquefies, the liquid that is formed should contain about 48 per cent of oxygen, if its composition is such as will maintain equilibrium with the gaseous air. In the apparatus shown in *Fig.* 6 this condition holds for the contents of the vessel A. The first portions of the air to be condensed trickle down the sides of the central condenser-tubes and are "scrubbed" by the air as it ascends; that is to say, they are brought into such intimate contact with the ascending air that a condition of equilibrium between liquid and vapour is at least closely approached. Accordingly the liquid which collects in the vessel A contains something like 48 per cent of oxygen. By making the condenser-tubes long enough it is clear that little or no oxygen will be left to pass over through B into the other tubes, and the liquid that collects in C will be nearly all nitrogen. It is true, of course, that in the upper parts of the central tubes the liquid that is formed consists largely of nitrogen, but as this trickles down the tube in which it has been condensed there is a give and take between it and the ascending gas, precisely like that which occurs in a rectifying column, and when the liquid reaches the bottom it is nearly in equilibrium with the gaseous air, and therefore contains about 48 per cent of oxygen.

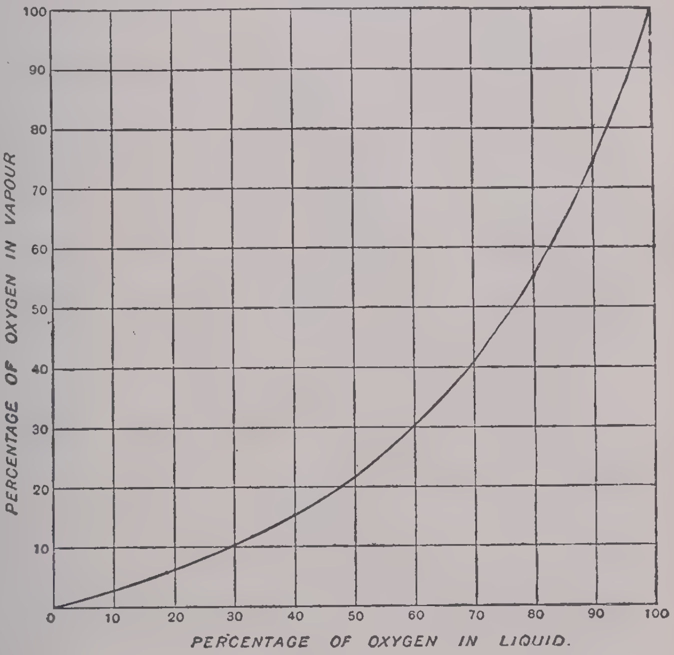

FIG. 7.

When this liquid from A is discharged through an expansion-valve into the rectifying column at L, part of it immediately evaporates, producing an atmosphere which has the composition of air (21 per cent of oxygen). The part of the column which extends above this point reduces the percentage of oxygen in the ascending gas from 21 per cent to practically nil, by means of the liquid from C.

In some plants for carrying out the process of rectification on a large scale it is claimed that nitrogen with a purity of 99·8 per cent is obtained. A Linde plant at Odda, in Norway, separates out the nitrogen from about one hundred tons of air daily for use in the manufacture of cyanamide, a nitrogenous fertiliser which is formed by passing gaseous nitrogen over hot calcium carbide.

Another commercial application of the liquefaction of gases is in the manufacture of hydrogen from water-gas, which consists mainly of a mixture of hydrogen with carbon monoxide. The carbon monoxide is separated out by liquefying it, leaving the hydrogen in the state of gas. The last stage of this separation is effected by using liquid nitrogen as an auxiliary cooling agent. The scheme of a hydrogen apparatus as used by the Linde Company is shown in *Fig.* 8. Water-gas, under compression, enters at W and passes through a counter-current interchanger into the coil B in the condensing vessel K. The vessel K is kept nearly full of liquid carbon monoxide which has come from the coil and has suffered a step-down in temperature by passing through the expansion-valve R. When the compressed water-gas enters the coil B, most of the carbon monoxide in it becomes condensed, but most of the hydrogen remains gaseous. The mixture in the coil passes on into a separating vessel A enclosed in a vessel V and kept very cold by liquid nitrogen which enters through S after being formed in a rectifying tower that is not shown in the diagram. The nitrogen in V is continuously boiling off at a low pressure. This serves to remove most of the residue of carbon monoxide, and nearly pure hydrogen passes off, still under pressure, at H. Both it and the carbon monoxide, which passes off at CO, pass through pipes in the counter-current interchanger by means of which heat is removed from the incoming water-gas.

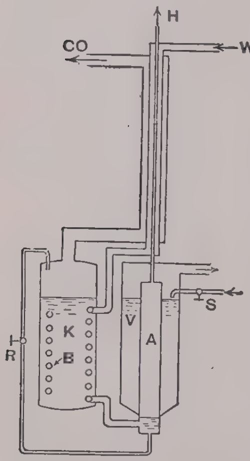

FIG. 8.—Apparatus for separating Hydrogen from Water-gas.

J. A. E.

Liquid, Absolute Expansion of a; determination by a hydrostatic method. See "Thermal Expansion," § (11).

Liquid Columns for Pressure Measurement. See "Pressure, Measurement of" § (3).

Liquid Level Indicators. See "Meters," Vol. III.

Liquid Meters. See "Meters," Vol. III.

Liquid Piston Pumps. See "Air-pumps," § (15).

Liquids:

- Measurement of Relative Expansion of. See "Thermal Expansion," § (10).
- Methods of measuring Thermal Conductivity of. Column Method—Film Method—Flow Method. See "Heat, Conduction of," § (7).
- Specific Heat of, by Electrical Methods. See "Calorimetry, Electrical Methods of," § (2).
- Values of Thermal Conductivity of. See "Heat, Conduction of," § (7), Table IV.

Loading Coefficient, Non-dimensional Critical. See "Dynamical Similarity, The Principles of," § (41).

Locomotive Engines. See "Steam Engines, Reciprocating," § (8).

Locus of a Point in a Mechanism. See "Kinematics of Machinery," § (3).

Logarithmic Decrement: the natural logarithm of the ratio of two successive amplitudes of a point executing damped harmonic motion, given by $ae^{-\nu t}\sin(nt+\epsilon)$. Its value is νT, where T is the time of a complete oscillation which is equal to $2\pi/n$.

Lower Pairs. For definition see "Kinematics of Machinery," § (2).

Lubricants, Mixtures of: the effects on the frictional resistance of mechanism due to mixing small quantities of fixed oils to mineral oils. See "Friction," § (29).

LUBRICATION, BOUNDARY CONDITIONS IN

Lubrication falls into two sharply distinct divisions according as the solid faces are or are not fully separated by a layer of lubricant. In the latter the physical properties of the solid faces play an important part; in the former the layer is thick enough for the lubricant to develop its properties when in mass, and the friction therefore is the internal or viscous friction of the lubricant modified by the form of the bounding solid faces. The friction of "dry" and "greasy" faces constitutes the one division. In it what Osborne Reynolds called "boundary conditions" operate, and the friction therefore might be called boundary friction. The other division includes the so-called "complete lubrication" of journals or slide blocks running in a bath of lubricant so that the rubbing faces are completely floated apart.

The remarkable feature of lubrication, and one which a complete theory must explain, is that these two divisions are opposed to each other in all important characteristics. In flotation static friction is absent, and the resistance to relative motion varies directly with the viscosity, that is to say it depends on the internal friction of the lubricant and the area of the opposed surfaces, and increases as the velocity of relative motion increases. In boundary lubrication there is static friction, the frictional resistance being equal to that in kinetic friction unless the state of the solids themselves is altered by the stresses; and the resistance varies as some inverse function of the viscosity of the lubricant, and is independent of the area and relative velocity.

The relations are so completely opposed to one another as to make it probable that if we could introduce a single pure chemical substance between two clean solid faces so as gradually to increase the thickness of the layer, we should meet with a discontinuity of state such that at a certain critical thickness boundary conditions would disappear to give way to flotation.

Naturally occurring solid surfaces such as the surface of a pane of glass are contaminated by an invisible film of matter condensed from the atmosphere or derived from contact with other substances. The grosser part of this film being of a greasy nature, it reduces the friction of "clean" faces by about 70 per cent. The film may be removed, or rather the grosser part of it, by various methods. For example, glass may be cleaned by washing with soap and water, heating in sulphuric and chromic acids, washing in water and drying in clean dry air. The coefficient of friction μ, of clean faces, that is to say the ratio of the tangential pull to the total normal pressure, is high, being 0·94 for crown glass, 0·74 for hard steel, and 0·50 for bismuth.

Thin films of a given lubricant may be deposited on such clean faces by exposing them to its vapour, or by spreading lubricant over them and then polishing off all excess with linen which has been deprived by prolonged and special treatment of all material capable of being conveyed to the solid face. Such surfaces are what are commonly called "dry" surfaces. The most interesting method of producing "dry" lubrication is by allowing the lubricant itself to spread over the clean surfaces under the influence of molecular attractions. As an example, take the following: a single small drop of pure acetic acid is placed near one corner of a clean plate of glass, say six cm. square, immersed in clean dry air. Nothing obvious to the senses follows. The drop of acid to all appearance remains where it was without change. The whole surface of the plate will, however, now be found fully lubricated as by acetic acid, the coefficient of friction having fallen from 0·94 to 0·41, because an invisible film has spread over it from the visible drop of acid.

Films can be formed in this way only in clean dry air, because water has a singular power of disturbing the relations between the solid and the lubricant. The films are, therefore, subject to evaporation so active as to remove the lubricant as fast as it is spread from the drop, save when, as in the case of acetic acid or tripropylamine, it is very strongly adsorbed by the solid. This is prettily shown by propyl alcohol. In the immediate neighbourhood of a drop of the substance the friction of a glass plate is found to fall to a low value ($\mu = 0·49$), but as one moves away the friction rapidly but gradually rises to the "clean" value. One can picture, but not see, a film continually spreading from the drop of alcohol and as continually being removed by evaporation, so that it vanishes completely a few centimetres from the edge of the drop.

When the surfaces are flooded by the lubricant so that the slider moves in a pool, the thickness of the film which persists between the faces will be determined by capillary forces. The friction of such flooded surfaces is in all cases so far measured equal to that of surfaces covered only by the invisible film just described, and we thus come to the second remarkable paradox of boundary friction, namely, that the boundary friction is independent of the quantity of lubricant on the faces. The following figures will serve to illustrate this:

	Surface flooded with Lubricant.	Burnished Film of Lubricant.
Amyl Alcohol . .	$\mu = ·58$	$\mu = ·59$
Caprylic Alcohol .	$\mu = ·51$	$\mu = ·54$
Heroic Acid . .	$\mu = ·46$	$\mu = ·46$
Caprylic Acid . .	$\mu = ·34$	$\mu = ·34$

A description of the method adopted for making these measurements will be found on the following page.

The third remarkable feature of sliding friction is expressed in what is usually called Coulomb's law—unfortunately so, since the law was clearly formulated more than eighty years earlier by Amontons in 1699.[1] It states that the total frictional resistance is independent of the area of contact and of the relative velocity, being dependent only on the total normal pressure. If we speak of the reaction of the fixed surface—using the word in the sense in which Newton used it—and call the normal component N and the tangential component F, then Amontons' law is expressed by $F/N = \mu = \text{constant}$. This law is characteristic of boundary conditions—that is to say, of "dry" or "greasy" surfaces, and is replaced by a quite different law when the lubricant floats the surfaces completely apart.

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[1] Amontons, *Mém. de l'Acad. Roy. des Sciences*, 1699, 206; Coulomb, *Mém. des Sçavans étrangers*, 1785, x. 163, 333.

In considering the question whether the law is an approximation, and if so within what limits it holds, we will take the two negative statements separately. In 1895 H. Painlevé[1] investigated mathematically the properties of mechanical systems subject to the relation $\mu = F/N$, and came to the conclusion that it leads to certain impossible discontinuities of motion. His method consisted in introducing into the equations of motion the reactions F and N as forces doing virtual work. His paper led to a controversy, carried on by French and German mathematicians, which continued for about fifteen years. From it one conclusion seems to emerge, namely that Amontons' law holds only when the geometrical relations of the surfaces which are compared are such that the normal and tangential reactions, the position of the centre of gravity, and the moment of inertia of the slider have the same geometrical relations throughout.[2] As an example, the value of μ will depend not merely upon the weight of a circular plane disc sliding on a plane surface but also on the position of the centre of gravity with respect to the geometrical centre of the disc. This relation must not be confounded with Michell's[3] conclusion that the coefficient of friction of a plate sliding over a fully lubricated surface depends upon the geometrical position of the load, because the former holds for static as well as kinetic friction, whereas Michell's analysis, like Osborne Reynold's, postulates relative motion.

The measurements of static friction which form the basis of this article were made with a slider having a curved surface which was applied to a plane surface, both faces being highly polished.[4] From the middle of the slider projected a small arm from which a fine thread passed over a light pulley to a pan. When weights were placed on the pan the slider rocked forward and the weight which just failed to cause sliding was used to compute the static friction. Some lubricants appear to abolish static friction, the smallest weight then causing a very slow glide. Ricinolic acid on bismuth is an example.

By a simple modification of the form of the arm, the pull could be adjusted so as to do away with the rocking of the slider. This was not found to alter the friction. Both the weight and the radius of curvature of the slider were altered over a large range without detectable change in the value of μ.

The contact between slider and plate was a circle of unknown but very small area. The weight of the slider was varied from 15 to 170 grammes; the normal pressure was, therefore, in some cases very great.

All measurements were made in a chamber filled with air dried and freed from dust. The slightest trace of moisture was found completely to alter the values.

When both the slider and the bearing have plane surfaces, there must always be uncertainty as to the area of real contact, because, as Burgess[5] proved, even the most carefully trued faces touch only at a few points, being separated elsewhere by a film of air or condensed water vapour. When a spherical surface slides over a plane surface the area of contact varies with the radius of curvature and with the $\frac{2}{3}$ power of the load. The values of μ were found to be strictly independent of the load and curvature, and therefore of the area of the surface of contact. Much more investigation is needed in the interests both of theory and practice; but the experimental evidence so far justifies the statement that Amontons' law is an exact law and not an approximation so long as the physical state and chemical nature of the solid and lubricant remains the same.

The second statement, namely that μ is independent of the relative velocity, was examined by Fleeming Jenkin and Ewing,[6] who found that for hard substances, such as steel on steel, static and kinetic friction were equal; but that they were unequal, the latter being less than the former, when one or both faces were of relatively soft material, such as brass or greenheart. Coulomb and Morin,[7] in 1781 and 1830 respectively, had already found the kinetic friction less than static friction when one or both faces were of wood. It is probable therefore that μ is independent of velocity only so long as the solid faces themselves are rigid enough not to be modified by viscous flow induced by the tangential and normal stresses.

The theory of boundary friction which may be said to hold the field at present is due to Coulomb. He states that friction must be due to the engaging with one another (l'engrainage) of the asperities of the surfaces, and that coherence must play a negligible part, because the effect of coherence would necessarily be proportional to the number of points of contact — that is to say, to the area.

Pushed to the limit, this theory means that truly plane surfaces are sensibly frictionless and that the asperities form frictionless inclined planes, the friction being due solely to the component parallel to the surface of the resistance offered by them. Since the surfaces experimented with were always highly polished, the asperities must have been very small, and the average effect of all might therefore be supposed to be the same for equal areas of sensible size wherever placed. The theory demands that the resistance due to any single area acting alone shall be equal to that of a surface including any number of such areas provided the total normal pressure is constant. If the actual surface is frictionless, the relation of tangential resistance to pressure would seem to follow from Hooke's law. The tangential resistance will be proportional to the change of form of the surface, and this again is proportional to the normal force. There is therefore nothing violent in the assumptions, except that of frictionless surfaces.

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[1] *Comptes rendus*, 1895, cxxi. 112.

[2] Compare, for example, Chamat, *Comptes rendus*, 1903, cxxxvi. 1634.

[3] *Zeits. Math.*, Leipzig, 1905, lii. 123.

[4] For details see W. B. and J. K. Hardy, *Phil. Mag.*, 1919, xxxviii. 32, 49, and *Phil. Mag.*, 1920, xl. 201.

[5] *Proc. Roy. Soc.*, 1911, lxxxvi. A, 25.

[6] *Phil. Trans.*, 1877, clxvii. A, 509.

[7] *Comptes rendus*, 1830, p. 37.

The difficulty of Coulomb's theory lies, not in the supposed action of asperities, because there is no such thing as a plane surface of particulate matter, and we merely have to inquire what constitutes the insensible asperities of a highly polished face, but in the ascription of a negligible part to cohesion, for we know that there exists on the external face of all solids a field of attraction for other forms of matter strong enough to bind to the face a film which cannot in many cases be detached without abrading the solid itself. There can be no manner of doubt but that the force required to cause one of these films to slip over the solid varies directly with the area. Given such a field of force, there will be resistance to the tangential motion of matter over a face, save in the unique case when the equipotential surfaces of the field are plane and parallel to the material face and the motion is parallel to those planes.

The dilemma in which the theory places us is apparent when we consider faces separated by a continuous layer of lubricant. Cohesion and repulsion now operate over the whole area, and the friction is strictly internal friction; why, then, should the frictional resistance not be proportional to the area as it is in the internal friction of homogeneous fluids and in the surface friction of a solid face moving through a fluid. Coulomb hints that it is a question of degree, the resistance being mainly due to asperities, but careful measurements show too close an agreement with theory for an escape to be found in this way.

The scale of the asperities to which friction is due may be estimated from the following facts. The more highly polished is a surface of glass the greater is its friction—thus glass trued and polished to an optical face has sensibly the same static friction as has plate-glass, and both give higher values than ground-glass. The asperities are therefore of insensible dimensions. The friction of polished faces of glass, steel, or bismuth is reduced to a relatively low value by lubrication with an aliphatic acid or alcohol, even when all excess of lubricant is burnished off. The invisible film of lubricant which persists gives the lowest obtainable value for static friction, as was noted earlier, yet it can be at most very few molecules in thickness. When the alcohol or acid is volatile at the temperatures of observation (*e.g.* ethyl or propyl alcohols), the burnished film evaporates off the surface in a few minutes, the friction rising as rapidly to the high value for "clean" faces. The fact that a film so thin gives the best boundary lubrication the particular substance is capable of exerting, proves that the asperities must be of molecular dimensions. We may therefore confidently reject what appears to be the current conception, namely that friction is due to material asperities each of which acts in opposing relative motion like a frictionless inclined plane, and assume that friction is due to the attraction of the molecules of one solid for the other across the interface when the surfaces are "clean," and of the molecules of the lubricant for each other when they are "dry" or "greasy."

The field of force already mentioned as existing at a free face of solid or liquid is due to the unbalanced attractions of the molecules, and the tangential component of this field constitutes the surface tension. Friction of clean faces may be regarded as being due to the mutual reaction of the fields, and, so long as boundary conditions operate, a lubricant reduces friction because it partly masks or "saturates" the fields of the solids.

Greasy films are formed on solid or fluid surfaces because they reduce the potential of the fields of attraction, and this process of condensation of foreign matter on to a surface is called adsorption. A lubricant may therefore also be said to be adsorbed by the solid face, and, other things being equal, the greater the work done by molecular forces in forming the layer the better the lubricant. For an experimental proof of this relation see the *Philosophical Magazine*, 1919, xxxviii. 49, and 1920, xl. 201.

It has been known for a long time that adsorption is determined by chemical constitution. Broadly speaking, those substances which are more active chemically in relation to the particular fluid or solid are more strongly adsorbed by it. Thus acids, alcohols, and esters are more strongly absorbed by the surface of water than are paraffins. It is therefore in accordance with expectation to find that aliphatic alcohols and acids are better lubricants of solids than their related paraffins.

This is illustrated by the following values of μ for glass, steel, and bismuth, there being in each case an excess of lubricant on the faces.

NORMAL PARAFFINS

	C_5H_{12}.	C_6H_{14}.	C_7H_{16}.	C_8H_{18}.	$C_{19}H_{40}$.	$C_{24}H_{50}$.
Glass . .	·71	·69	·67	·65	·41	·33
Steel . .	·48	·45	·43	·41	. .	. .
Bismuth .	. .	·37	·35	·32	·09	·07

NORMAL ALCOHOLS

	CH_3OH.	C_2H_5OH.	C_3H_7OH.	C_4H_9OH.	$C_5H_{11}OH$.	$C_8H_{17}OH$.	$C_{16}H_{33}OH$.
Glass . .	·67	·65	·63	·60	·58	·51	·33
Steel . .	·47	·44	·41	·39	·37	·30	. .
Bismuth .	. .	. .	·34	·30	·27	·25	·17

If these figures are plotted against molecular weight the curves will be found to be straight lines, so that we may write $\mu=\mu_0-\alpha M$, where μ_0 and α are constants and M is the molecular weight.

The effect of the nature of the solid face is unexpectedly simple. Within the limits of error the slope of the curve is independent of the nature of the solid face. That is to say, α is independent of the solid face and a pure function of chemical constitution, the values being:

Normal paraffins . . . $\alpha=·0013$
Normal alcohols . . . $\alpha=·0016$
Normal acids . . . $\alpha=·0043$

The first two acids in the series, formic and acetic acids, give abnormally low values of μ, as might be expected from their other physical properties.

The value of the constant μ_0 is, on the contrary, determined by the nature of the solid faces. It is, as the theory would lead us to expect, a function of the friction of the faces when "clean." Thus we have for the normal paraffins the following values:

	Glass.	Steel.	Bismuth.
Clean $\mu_0=$ . .	·94	·74	·50
Lubricated $\mu_0=$.	·80	·57	·48

These relations may be put in another way. For the normal paraffins, acids, and alcohols the effect of changing from one solid or another is to shift the curves with respect to the axes whilst keeping them parallel to themselves. The general relation between the internal friction (viscosity) and boundary friction of the members of a chemical series is that the former *increases* and the latter *decreases* as the molecular weight increases. For normal paraffins the equations of isotherm are

$$\mu=\mu_0-\alpha M$$

and $\eta=\eta_0+\beta M$,

where η is the coefficient of internal friction and β is a constant. The relations are not so simple as this in other chemical series.

The lubricating action on bismuth of nearly one hundred substances has been measured and relations found which at present have received no explanation. Some of these are illustrated by the following values of μ:

CHAIN COMPOUNDS

Alcohols

Propyl . . .	·34	Isopropyl. . .	·32
Butyl . . .	·30	Isobutyl . . .	·30
Amyl . . .	·27	Glycol . . .	·30
Octyl . . .	·25	Glycerol . . .	·22
Cetyl . . .	·17	Penterythritol .	·40

Acids

Propionic . .	·31	Oleic . . .	·10
Valeric . . .	·28	Ricinolic . .	·02
Caprylic . . .	·19	Lactic . . .	·20
Stearic . . .	·15	Glyceric . . .	·22

Esters

Ethylacetate . . .	·36
Ethylvalerianate . .	·35
Tristearin	·24
Triolcin	·14

Ring Compounds

Benzene . . .	·34	Ethyl benzene .	·32
Naphthalene . .	·29	Toluene . . .	·28
Anthracene . .	·26	Xylene . . .	·30
Phenol . . .	·25	*p.* Cymene . .	·31
Catechol . . .	·39	β Naphthol . .	·38
Quinol . . .	·40	Naphthoic acid .	·39
m-Cresol . . .	·26	Benzoic acid . .	·38
Benzyl alcohol .	·31	Cinnamic acid .	·27
Ethyl hydrocinnamate . . .	·28	Benzilic acid . .	·45
		Salicylic acid . .	·41
Ethyl cinnamate .	·32	Carvacrol . .	·23
di-Pentene . .	·31	Thymol . . .	·24
Iso-cholesterol .	·27	Menthol . . .	·26

Cyclic Compounds

Cyclohexane . .	·31	Cyclohexanol . .	·35
Methyl cyclohexane	·30	Cyclohexanone .	·35

Castor oil . .	·03	Carbon tetrachloride	·40
Water . . .	·33	Chloroform . .	·33

The most interesting is the distinction between ring and chain compounds. The simple ring compounds benzene, naphthalene, and anthracene show the linear relation to molecular weight, and the values are much the same as those for paraffins of the same molecular weight. The similarities, however, end here, for any change in the molecular structure produces opposite effects according as it takes place in a chain or ring. Thus a double bond decreases the lubricating action of a ring compound, but increases that of a chain compound. As examples, compare naphthoic acid with double-bonded oxygen with naphthalene, menthone with menthol, cyclohexanone with cyclohexane, benzoic acid with benzene. As examples of double-bonded carbon, compare cinnamic ester with hydrocinnamic ester, di-pentene, having two unsaturated carbon atoms, with menthol and cyclohexane. Also the more saturated cyclic compounds are better lubricants than the less saturated ring compounds.

When a ring or chain is joined, as in butylxylene, the result is a better lubricant than either.

The esters occupy a quite unexpected position. The simple aliphatic esters are worse lubricants than their related acids and alcohols. The ring esters, on the contrary, are better lubricants than are their related acids (*e.g.* ethyl benzoate and benzoic acid).

Perhaps the most interesting substances are the hydroxy-acids with OH and COOH groups. This conjunction produces a remarkable increase in the lubricating power of a chain compound (lactic acid and ricinolic acid), and almost destroys lubricating action in the case of the ring compounds (salicyclic and benzylic acids).

In the ring compounds the replacement of hydrogen decreases lubricating power in the case of N : O, or ·COOH, and increases it in the case of other groups in the order $C_2H_5 < CH < OH$.

The effect of a second group of the same or of a different kind is to decrease the effect of the first. Compare, for instance, toluene with xylene; catechol, quinol, and cresol with phenol; and methyl cyclohexanol with cyclohexanol. The simpler the group the more effective it is. Compare cymene with toluene or xylene, and benzyl alcohol with phenol.

When the atoms are disposed with complete symmetry about a carbon atom, the result is a very bad lubricant, as we see in carbon tetrachloride and the alcohol penterythritol $C(CH_2OH)_4$.

It will be noticed that no ring compound is a good lubricant. Even cholesterol, with the molecular weight 366, is no exception.

The group SH acts much as OH, thiophenol C_6H_5SH and benzyl-hydrosulphide $C_6H_5CH_2SH$ resembling phenol and benzyl alcohol respectively.

The theory of boundary lubrication outlined above may be developed further by considering the properties of composite surfaces. When a substance, say an oil, spreads on water, under the influence of surface forces a surface is formed whose properties are neither those of a clean surface of water nor of the oil. The water and the oil have in fact reacted on one another. Such surfaces may be called "composite." All naturally occurring surfaces are composite, because the free energy of such a composite surface is less than that of a clean or simple surface. The reaction between the components of a composite surface can be classed as chemical, but in a very restricted sense, because the condition of immiscibility of the components makes the relation of the molecules two-dimensional. The law of average values therefore applies only in a restricted way, so that the characteristic law of chemistry, the law of definite and multiple proportions, ceases to hold.

The partial nature of the reactions and the structure of a composite surface may be illustrated by an example. When an aliphatic acid such as palmitic acid forms the film on water, the film is probably only one molecule thick and the carboxyl groups are attracted by the water so that each molecule stands on end. (See for example Adam, *Proc. Roy. Soc.*, A, 1921.) The film therefore is composed of a layer of carbon chains oriented at right angles to the water face. Orientation to an unknown degree may also be supposed to obtain in the superficial layer of the water itself.

Composite surfaces are formed on solids in a way so similar to that obtaining on fluids (cf. *Phil. Mag.*, 1919, xxxviii. 49) as to make it practically certain that they have the same general structure, namely, marked orientation of the molecules. Consider two such composite faces applied to one another; the orientation may be disturbed by mutual attraction between the molecules of the films but it will not be destroyed. The applied faces now form a region which varies rapidly in constitution along the normal to the interface, and its boundaries are indefinite, for we do not know how far into the solids the molecular pattern characteristic of the region extends.

The film on a solid face, by more or less completely saturating the attractive forces, lessens the capacity for cohesion. Its presence therefore is one, but not the only one, of the causes why solids do not weld when pressed together. It also lessens the frictional resistance of the face.

The thinnest continuous film of foreign matter which can be deposited on a clean surface of water is one molecule thick. It is the film which is formed when the area of the water face is large enough to permit of the substance, *e.g.* oleic acid, spreading as far as it has a tendency to. The thickness of films formed in a similar way on solid surfaces has not yet been directly measured, but analogy justifies us in assuming that if the area is large enough the film will also be one molecule thick and that it can be thickened by contracting the area in reference to the quantity of foreign substance present in it. It must be admitted that the relation of the friction to the thickness of film on each solid face is difficult to follow, but the evidence so far available points to the conclusion that static friction decreases as the layer of the lubricant is increased, but rapidly reaches a minimum when it begins to increase until the critical thickness of the film is reached beyond which flotation occurs. At the critical thickness static friction falls more or less abruptly to zero.

The fact that the layer of lubricant manifests static friction proves that it behaves under tangential stress like an elastic solid. If it be a fluid when in mass the effect of the attractions of the solid faces, combined with the traction, must therefore be to confer on a thin layer both orientation of its molecules with respect to the normal, and also solidity. The orientation of the molecules spoken of above is due to the fact that the attraction of the molecules of the solid faces is not the same for all parts of a molecule of the lubricant. We may suppose, for instance, that the attraction for the $-COOH$ or $-OH$ group is greater than for the remainder of a carbon chain. The orientation of the molecules of the lubricant will therefore be greatest at the solid faces and least midway between them, because it tends continuously to be upset and give way to the random relations of the interior of a fluid. The layer of lubricant, therefore, even if it be only two molecules thick, varies rapidly in structure along the normal, and the peculiar feature of boundary friction is that the stresses occur in a medium which is excessively heterogeneous along one axis, namely, the normal to the solid faces, and homogeneous along the tangential planes.

The friction, whether static or kinetic, is merely an expression of the strain produced in the tangent planes by the tractions, and the peculiar features are due to the heterogeneity of the material which tends to confine the yield point when static friction gives way to kinetic friction to a single tangent plane instead of the yield being distributed throughout the mass as in a homogeneous fluid or elastic solid.

The frictional resistance—the "tangential component of the reaction," as French writers fittingly call it—has its origin in the resistance of the atoms to displacement and, since the traction will tend to produce rotation, in resistance to disturbance of the molecular orientation which is a configuration of minimal potential energy. When the traction reaches a certain value fixed by the nature and configuration of the atoms and molecules of both lubricant and superficial parts of the solids, the tangential reaction reaches the maximum possible and there is "yield" on some particular tangent plane, the position of which is indeterminate owing to the fact that, because of the rotation spoken of above, the distribution of strain along the normal is a function of the intensity of the traction. The common equality between static and kinetic frictions follows from the fact that the former measures the greatest tangential reaction the system is capable of.

The above hypothesis and the limitation imposed by our imperfect knowledge may be illustrated by considering the friction of faces of bismuth in some detail. This metal is highly crystalline and so brittle that a plate a few millimetres thick can be broken like a biscuit. It is readily polished, and in the process of polishing an amorphous layer is formed covering the crystal facets which previously were easily visible to the naked eye. This amorphous layer—which may be called the Beilby layer—is formed, as Beilby showed, by a flowing of the metal, and, like a truly fluid surface, it forms under the influence of surface tension. No polishing powder is needed to polish bismuth—the operation can be performed by rubbing the metal on the skin of the ball of the thumb. The skin then becomes covered thickly with a fine dust of metal, a considerable amount of material being rapidly removed from the surface, and the dust is seen under the microscope to be composed of spheres each ·01 mm. in diameter. The formation of these spheres can be followed under the microscope. At the edge of the plate the flowing surface of the metal is detached in a thin sheet which breaks up into drops under the influence of its own tension. The structure of the dust and its mode of formation are therefore complete evidence of the fluidity of the surface produced by rubbing.

Consider now the structure of the material which is to be the seat of strain when two such polished faces are separated by a layer of lubricant only a few molecules thick. Starting in the mass of the metal we have a magma of solid crystals cemented together; this is followed in order by the Beilby film of unknown but slight thickness, the lubricant, a second Beilby film, and finally a second crystalline mass.

In each crystalline mass the molecules are disposed in a regular lattice. The disposition in the Beilby film is unknown, but the two arrangements are probably incompatible with one another, so that the transition may be expected to be abrupt. In the lubricant the molecules abutting on the bismuth face may be supposed to be more highly oriented than are those in the middle of the layer. The whole region therefore includes four surfaces of discontinuity, namely, at the interfaces between the Beilby films and the crystals on one side and the lubricant on the other, and a layer, namely the lubricant, which varies rapidly in state, the variation being disposed symmetrically about an imaginary surface midway between the solid faces. It is obvious that it is impossible to predict where the yield point will be under tangential stress in so heterogeneous a system, especially as, for reasons given above, the cohesion is a function of the traction.

When the friction is high the metal is torn if any slipping takes place. This occurs when no lubricant is present save perhaps a film of condensed gas, and also when the lubricant is one which fails to reduce the maximum friction —that of clean faces—by 80 per cent. This may be interpreted in two ways neither of which is free from difficulty. The first is that the yield point is at the interface between lubricant and solid, so that the layer of lubricant becomes broken and the solid faces come together.

The evidence, however, goes to prove that tearing of the metal coincides with the inception of slipping when the layer of lubricant must be still intact, in which case we come to the second interpretation, namely, that the yield point when the lubricant is capable of bearing a tangential stress above a critical value is in the metal, possibly at the interface between the Beilby film and the crystals. This second explanation is in better accord with many of the facts, but it leaves the friction to be fixed by the yield point of the metal for all except "good" lubricants. All "bad" lubricants therefore should be indistinguishable from one another—they should be "neutral" substances, as indeed they are for glass. This is not the case for bismuth, as the table shows.

The matter may be left at that. The argument was entered into not because it could be pushed to a conclusion—more investigation is needed to make that possible—but merely to illustrate wherein the difficulties lie.

W. B. H.

Lubrication of Surfaces under High Pressure. See "Friction," § (28).

Ludin's Determination of Specific Heat of Water. See "Heat, Mechanical Equivalent of," § (7).

Lummer and Pringsheim: Experimental Observations of Temperature of "Black Body," tabulated and compared with values calculated from the fourth-power law. See "Pyrometry, Total Radiation," § (3), Table I.
  Investigation of Radiation from a "Black Body" Enclosure over the temperature range 100° to 1300° C. See *ibid.* §§ (2), (3).

M

McNaught's Engine. See "Steam Engines, Reciprocating," § (14).

Magnetic Tachometers: for measuring number of revolutions per unit time. See "Meters," § (7), Vol. III.

Mallock's Experiments on the Friction of Curved Surfaces in Fluids. See "Friction," § (20).

Manometers. See "Pressure, Measurement of."
  Primary. § (2) *et seq.*
  Secondary. § (8) *et seq.*
  Range and Sensitivity of various types. § (28).

Marine Diesel Oil Engines. See "Engines, Internal Combustion," § (16).

Marine Engine, The First. See "Steam Engines, Reciprocating," § (16).

Marine Engines. See "Steam Engines, Reciprocating," § (9).

Mass, Virtual: the apparent increase of mass of a body moving with acceleration in a viscous fluid; proportional to the mass of the fluid displaced. See "Dynamical Similarity, The Principles of," § (22).
  Experimental Determination of. See *ibid.* § (23).

"Maximum Energy Ordinate E_m," variation with temperature of, studied by Lummer and Pringsheim. See "Pyrometry, Optical," § (2) (ii.).

Maxwell's Distribution Law. See "Thermodynamics," § (66).

Maxwell's Method for Deflection of Frames. See "Structures, Strength of," § (24).

Maxwell's Theorem on the Relation between Pressure and Temperature of a Fluid. See "Thermal Expansion," § (28).

Maxwell's Thermodynamic Relations. See "Thermodynamics," § (47).

Mean Free Path of a Molecule: the average value of the length of the path which a molecule traverses between successive collisions. See "Thermodynamics," § (66).

MEASUREMENT, UNITS OF [1]

§ (1) FUNDAMENTAL UNITS.—The fundamental units on which all physical measurements are based are those of mass, length, and time. Two systems of units are at present in use: the Metric system, with the metre as the unit of length and the kilogramme as the unit of mass, and the Imperial or British system, in which the yard is the unit of length and the pound the unit of mass. The system used almost universally for scientific purposes is that in which the centimetre (=one-hundredth of a metre) is the unit of length, and the gramme (=one-thousandth of a kilogramme) is the unit of mass. This system is known as the centimetre-gramme-second (C.G.S.) system.

The choice of units was in the first instance arbitrary, and no system based on actual physical entities, such as the velocity of light, gravitational attraction, etc., is in use.

The most recent values of the metrical equivalents of the fundamental British units are those legalised in the Order in Council of May 19, 1898. The equivalents of the units of length were obtained by Benoît in 1895, and of the units of mass by Broch in 1883, at the International Bureau of Weights and Measures.

The metric standards of length and mass are kept at the International Bureau of Weights and Measures in Sèvres near Paris, and the Imperial units are preserved at the Standards Office of the Board of Trade, Old Palace Yard.

§ (2) UNITS OF LENGTH. (i.) *The Metre.*—The *metre* is defined as the distance, at the melting-point of ice, between the centres of two lines engraved upon the polished neutral web surface of a platinum-iridium bar of nearly X-shaped section called the *International prototype metre.* This is a copy of the original Borda platinum standard—the *mètre des archives*—which was intended to be equal to 10^{-7} or one ten-millionth of the length of the meridian through Paris from Pole to Equator. According to Clarke's figures the correct relation is a quadrant = 1·0007 × 10^7 metres; the mean of the values obtained by Helmert and the U.S. Survey for the mean polar quadrant is 1·00021 × 10^7 metres. The length of the bar as constructed is now taken as an arbitrary standard.

(ii.) *The Yard.*—The *yard* was defined by the Weights and Measures Act, 1878, as the distance at temperature 62° F. between the central transverse lines in two gold plugs in the bronze bar called the *Imperial standard yard,* when supported on bronze rollers so as best to avoid flexure of the bar. The bar is of 1 inch square section, and is 38 inches long; the defining lines are at the bottom of two holes so as to be in the median plane of the bar.

(iii.) *Equivalents.*—

(*a*) *Metric Units.*

Metre . . . 1 m. = 39·37008 in.
= 3·280840 ft.
= 1·093613 yd.
Kilometre . . 1 km. = 0·6213712 mi.

(*b*) *British Units.*

Mile . . . 1 mi. = 1609·344 m.
Yard . . . 1 yd. = 0·914400 m.
Foot . . . 1 ft. = 0·304800 m.
Inch . . . 1 in. = 2·54000 cm.
Nautical mile (English) [2] = 1853·152 m. (Adm.)
= 6080 ft.
= 1·1515 statute mi.

(*c*) *Astronomical Units.*—For astronomical work it is convenient to use larger units than those defined above.

The astronomical unit is equal to the semi-major axis of the earth's orbit.

1 astronomical unit = 1·495 × 10^8 km.
= 9·289 × 10^7 mi.
Parsec . . . = distance at which the astr. unit subtends 1 second (1″).
= 2·06 × 10^5 astr. units approx.
= 3·083 × 10^{13} km.
= 1·9158 × 10^{13} mi.
Light-year . . = the distance travelled by light in 1 year (velocity of light = 2·9986 × 10^{10} cm./sec. = 186,326 mi./sec.)
= 0·31 parsec.

(iv.) *Small Units.*—For measurements of the wave-length of light and X-rays the unit is one ten-thousand millionth metre, and is known as a "tenth-metre" or the Ångström unit.

Ångström unit . . . 1 A.U. = 10^{-10} m.
Micron . . . . . . 1 μ = 10^{-6} m.
Millimicron . . . . 1 $\mu\mu$ = 10^{-9} m.
Mil . . . . . . 1 mil. = 10^{-3} in.

(v.) *Ancient French Units.*—

1 toise = 6 ft. . = 1·9490366 m. = 2·1314918 yd.
1 foot = 12 in. = 0·3248394 m. = 1·0657461 ft.
1 inch = 12 Paris lines . . = 27·069953 mm. = 1·0657461 in.
1 line . . = 2·255829 mm. = 0·0882165 in.

(vi.) *Russian Measures.*—

1 verst = 1·06678 km. = 0·663 mi.

§ (3) UNITS OF AREA.—Measures of area are based on the standard of length.

(i.) *Equivalents.*—

(*a*) *Metric Units.*

Square centimetre : 1 cm.^{2} = ·1550 in.^{2}
= ·001076 ft.^{2}
= ·0001196 yd.^{2}
100 m.^{2} = 1 are.
= 0·0988 rood.
10,000 m.^{2} = 1 hectare.
= 2·4711 acre.

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[1] Most of the information contained in this article has been taken by permission from the *Computer's Handbook* of the Meteorological Office, to which the reader is referred for further details.

[2] See § (18) "Geodetic Measures."

(*b*) *British Units.*

1 in.^{2} . . . . . = 6·4516 cm.^{2}
1 ft.^{2} . . . . . = 929·03 cm.^{2}
1 yd.^{2} . . . . . = 8361·3 cm.^{2}
1 acre . . . . . = 4840 sq. yds.
= 0·4047 hectr.
1 square mile . . = 259·008 hectr.
= 2·59 sq. km.

§ (4) VOLUME (see also "Measurements of Volume," Vol. III.).—The unit of volume is based on the unit of length, but in many cases the legal unit has been defined as the space occupied by a certain weight of a standard liquid—usually water—under standard conditions.

(i.) *Metric.*—An attempt was made by the founders of the metric system to correlate the two units by defining the unit of mass as the mass of a quantity of water which at its temperature of maximum density occupied 1 cubic decimetre; the litre or unit of volume could then be defined as the space occupied by a kilogramme of water at its maximum density or as the space occupied by a cube with side 10 centimetres. The experimental relation now accepted is that 1 kilogramme of water at 4° C., and pressure 760 mm., occupies 1000·027 c.c. In 1872 the unit of mass was redefined as the mass of the International kilogramme in its actual state, and in 1901 the definition adopted for the *litre* was the space occupied by a kilogramme of water at its maximum density and under normal atmospheric pressure (760 mm.).

(ii.) *British.*—In British units the *gallon* is the unit of volume, and is defined as the space occupied by 10 lbs. weight of distilled water weighed in air against brass weights at a pressure of 30 in. and temperature 62° F. Units based on the unit of length are also in common use.

(iii.) *Equivalents.*—
(*a*) *Metric Units.*

1 c.c. = ·0610 c. in.
1 litre = 1000·027 c.c.
= ·03531 c. ft.
= 1·7598 pint.
= ·2200 gal.

(*b*) *British Units.*

1 c. in. = 16·387 c.c.
1 c. ft. = 28·317 litres = 28317 c.c. = 6·22882 gal.
1 c. yd. = 0·7645 m.^{3}
1 pint = ·5682 lit.
1 gallon = 4·5460 lit.

§ (5) MASS, MEASURE OF. (i.) *Metric.*—The *International prototype kilogramme* is the mass of a cylinder of platinum-iridium, which is a copy of the original Borda kilogramme—the *kilogramme des archives*. This was intended to be equal to the mass of a cubic decimetre of pure water at its maximum density (see § (4)).

(ii.) *British.*—The *Imperial standard pound* is the weight *in vacuo* of a platinum cylinder.

(iii.) *Equivalents.*—
(*a*) *Metric Units.*

Kilogramme . . 1 kg. = 2·2046223 lb.
Gramme . . . 1 g. = 15·432356 gr.
Metric tonne . . 1 t. = 1000 kg.
= 2204·622 lb.
= 0·9842 ton.

(*b*) *British Units.*

Pound . . . 1 lb. = 453·59243 g.
Ounce (avoir.) . 1 oz. = 28·3495 g.
Ounce (troy and apothecary) . 1 oz. = 31·10348 g.
Grain . . . 1 gr. = 0·06479892 g.
Ton . . . 1 ton = $1·016047 \times 10^6$ g.

§ (6) DENSITY.—The density of any substance is the mass of unit volume, and is measured in grammes per cubic centimetre, or in pounds per cubic foot. The term specific gravity is occasionally used to denote the density of a substance relative to that of water.

1 g./c.c. = 62·43 lb./c. ft.
1 lb./c. ft. = ·01602 g./c.c.

(i.) *Density of Water.*—Water has its maximum density at 3·98° C. when pressure is 760 mm.; at other pressures the temperature of maximum density is given by the formula $t_m = 3·98 - ·0225(p-1)$, where p is measured in atmospheres.[1]

The density of pure water under one atmosphere for different temperatures is as follows:

Temperature a.	Density g./cc.
268	·99930
273	·99987
277	1·0000
293	·99823
323	·9881
373	·9584

Density of mercury at the normal freezing-point of water = 13·5955 g./c.c.

(ii.) *Density of Dry Air.*—The density of dry air varies with pressure and temperature according to the formula

$$\rho = \rho_0 \cdot \frac{p}{p_0} \cdot \frac{T_0}{T}.$$

For dry air free from CO_2 Regnault obtained the value $\rho_0 = 1292·78$ g./m.^{3} for $p_0 = 760$ mm., $T_0 = 273$, which gives

$$\rho = 348·321\frac{p}{T},$$

where p is measured in millibars. The addition of 0·04 per cent CO_2 increases the value of ρ_0 by 0·021 per cent, and the formula becomes

$$\rho = 348·394\frac{p}{T}.$$

i.e. $\rho = 1201$ g./m.^{3} approximately at 1000 mb. and 290 a.

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[1] Kaye and Laby, *Physical and Chemical Constants*, 1918, p. 22.

(iii.) *Density of Damp Air.*—The density of damp air may be obtained from the density of dry air by means of the formula—

$$\rho=\frac{\rho_D(p-0{\cdot}378e)}{p},$$

where ρ_D is the density of dry air,
p is the total pressure,
e is the vapour pressure.

Hence $$\rho=348{\cdot}394\frac{p-0{\cdot}378e}{T}.$$

§ (7) TIME, MEASURE OF.—The standard of time is derived from the period of the earth's rotation, and the unit of time in both metric and British systems is the mean solar second which is equal to $1/24\times60\times60$, *i.e.* 1/86400 mean solar days.

A true solar day is defined as the interval between successive transits of the centre of the sun's disc over a meridian, but this interval varies throughout the year, and in order that the civil day may be of uniform length, standard time is measured with reference to a "mean sun" which is supposed to revolve uniformly round the earth in a time equal to the average length of the true solar day.

(i.) The *mean solar day* on which the definition of unit time is based is therefore defined as the average interval between successive transits of the centre of the sun across any given meridian.

(ii.) The *tropical or solar year* is the average interval between successive passages of the sun across the first point of Aries (the first point of Aries is the point of intersection of the celestial equator with the ecliptic where the sun crosses the equator from south to north).

(iii.) *The Civil Year.*—According to the Julian calendar the civil year contains 365 days for three successive years, the fourth year containing 366; a further correction is made by which a century year contains 365 days unless divisible by 400, when it contains 366.

The average value of the civil year

$$=\frac{365\times303+366\times97}{400}$$

$$=365{\cdot}2425 \text{ days},$$

and is accordingly approximately equal to the solar year, which contains 365·2422 mean solar days.

(iv.) The *Sidereal Day* is defined as the interval between two consecutive transits of the first point of Aries across any selected meridian, and is therefore equal to the period of rotation of the earth with reference to the fixed stars—the value is 23 hours, 56 minutes, 4·0906 seconds.[1]

(v.) The *Sidereal Year* is the time interval in which the sun appears to perform a complete revolution with reference to the fixed stars.

(vi.) *Equivalents.*—

1 tropical or solar year . . =365·2422166 mean solar days.
1 sidereal year =366·2564 sidereal days.
=365·2564 mean solar days (epoch 1900).
1 mean solar day =86,400 sec.
=0·002737909 mean solar years.
=1·00273791 sidereal days.
=24 hr. 3 min. 56·56 sec. in sidereal time.
1 sidereal day . =86,164·0906 sec.
=0·99727 mean solar days.
=23 hr. 56 min. 4·09 sec. in mean time.
If 1 year . . =360°,
1 mean solar day = 0°59′ 8·33″.
1 week . . =6° 53′ 58″.
30 days . . =29° 34′ 10″.
1 hour . . $=1{\cdot}140795\times10^{-4}$ year.
=0° 2′ 27·847″.
1 minute . . $=1{\cdot}90132\times10^{-6}$ year.
=2·464″.
1 second . . $=3{\cdot}168866\times10^{-8}$ year.
=0·041066″.

Length of the seconds pendulum in London =39·13929 in.

(vii.) *Rotation of the Earth.*—

Relative to a star . $\omega=0{\cdot}00007292$ r./s.
Relative to the sun . 1 hr. =15°.
1° =4 min.

	Revolutions.	Radians.	Degrees, etc.
Sidereal day . .	1	2π	360°
Mean solar day .	1·00273791	6·300388	360·98565°
Hour . . .	$4{\cdot}178075\times10^{-1}$	$2{\cdot}625162\times10^{-1}$	15·04107°
Minute . . .	$6{\cdot}963458\times10^{-4}$	$4{\cdot}375270\times10^{-3}$	15·04107′
Second . . .	$1{\cdot}160576\times10^{-5}$	$7{\cdot}292116\times10^{-5}$	15·04107″

§ (8) STANDARD TIME.—For the British Isles and the greater part of Western Europe (France, Belgium, Spain, and Portugal) Greenwich Mean Time is the standard and is known as G.M.T. or W.E.T. (Western European Time). For other countries a system of zone time has been adopted in which the time is referred to some standard meridian chosen so that the difference between the standard time for the zone and G.M.T. is a whole number of hours or half-hours. Thus zone 0 lies between 7½ W. and 7½ E. and keeps the time of meridian 0°, *i.e.* G.M.T.; zone 1 lies between 7½ W. and 22½ W., and keeps the time of meridian 15° W., *i.e.* one hour behind G.M.T.; zone

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[1] Owing to the "precession" of the earth's axis the true period of the earth's rotation is approximately ·01 secs. longer than the sidereal day.

−1 is between 7½ E. and 22½ E., and keeps the time of meridian 15° E., one hour in front of G.M.T. Some adjustment of the zones is made on account of political boundaries.

(i.) *Local Mean Time.*—In order to convert time in G.M.T. into local mean time add 4 minutes for each degree of longitude for places east of Greenwich, and subtract 4 minutes for each degree for places west of Greenwich.

(ii.) *Apparent Time.*—Time based on the length of the true solar day is called "apparent time," and it is this which is measured by a sundial or sunshine recorder. In order to obtain local apparent time from local mean time a correction must be applied, known as the equation of time. The correction is zero on April 16, June 15, September 1, and December 5, reaches maxima of +16 minutes 21 seconds on November 3, and + 3 minutes 49 seconds on May 14; and minima of −14 minutes 25 seconds on February 12, and −6 minutes 18 seconds on July 26—a positive sign meaning that the value is to be added to mean time to obtain apparent time, and a negative sign meaning that the value is to be subtracted. Accurate values of the equation of time for each day are given in the *Nautical Almanac.*

(iii.) *Sidereal Time.*—If a great circle be drawn from the pole to a star, the angle this hour circle makes with the meridian is termed the *hour angle.* The hour angle west of the first point of Aries, turned into time at the rate of 15° per hour, is known as sidereal time.

(iv.) *Summer Time.*—Since 1916 it has been the practice in most countries of Western Europe to use Summer Time, which is one hour in advance of G.M.T. The period over which summer time extends varies in different countries and from year to year.

§ (9) MEASURES OF ANGLE.—The symbol π is used to denote the ratio of the circumference of a circle to its diameter.

$$\pi = 3{\cdot}14159265,$$
$$\log \pi = 0{\cdot}49715,$$
$$\frac{1}{\pi} = 0{\cdot}318309886,$$
$$\log\left(\frac{1}{\pi}\right) = \bar{1}{\cdot}50285.$$

(i.) *The Radian.*—The unit of measurement for angles is the radian, which is equal to the angle subtended at the centre of a circle by an arc of length equal to the radius.

$$\begin{aligned} \pi \text{ radians} &= 180^\circ, \\ 1 \text{ radian} &= 57{\cdot}29578^\circ \\ &= 57^\circ\ 17'\ 44{\cdot}81'', \\ 1^\circ \quad . \quad &= 0{\cdot}017453 \text{ radians}, \end{aligned}$$

(ii.) *The Point.*—Wind direction is often measured in points where

$$1 \text{ point} = \tfrac{1}{32}\,(360^\circ) = 11\tfrac{1}{4}^\circ.$$

§ (10) VELOCITY, MEASURE OF.—The velocity of a body is defined as the ratio of the distance moved through to the time that is taken; the unit of velocity is such that unit distance is moved through in unit time, and is accordingly fixed by the fundamental units of space and time.

In the metric system velocity is measured in centimetres per second (C.G.S. unit), metres per second, or kilometres per hour; and in the British system in feet per second, miles per hour, or knots. The unit chosen depends to a large extent on the magnitude of the quantities to be measured.

The equivalents of the several units are as follows:

(i.) *Metric Units.*—

Centimetre per second: 1 cm./s. = 0·0328084 ft./s.
= 0·022369 mi./hr.
= 0·019435 knots.
Kilometre per hour: 1 km./hr. = 0·6213712 mi./hr.

(ii.) *British Units.*—

Foot per second: 1 ft./s. = 0·304800 m./s.
Mile per hour: 1 mi./hr. = 0·44707 m./s.
= 1·609344 km./hr.
Knot . . . 1 knot = 1 nautical mile/hr.
= 1·1515 mi./hr.
= 0·51453 m./s.

(iii.) *Velocity of Light.*—The mean value of the velocity of light *in vacuo* is $2{\cdot}9986 \times 10^{10}$ cm./sec. = 186,326 mi./sec.

(iv.) *Angular velocity* is measured in radians per second (1 radian = 57·296°), or in the number of revolutions per unit time.

§ (11) ACCELERATION, MEASURE OF.—The unit of acceleration is such that in unit time the velocity changes by unit amount, and it depends therefore solely upon the units chosen for length and time, and has dimensions, velocity/time, *i.e.* L/T^2.

(i.) *Metric Units.*—The C.G.S. unit is the "gal.," which is 1 centimetre per second per second.

1 gal. = 1 cm./s.^{2} = 0·0328084 ft./s.^{2}
1 leo. = 1 decametre per second per second.
1 milligal. = 10^{-3} cm./s.^{2}
= 10 micron/s.^{2}

(ii.) *British Units.*—

1 foot per second per second = 30·4800 cm./s.^{2}

For acceleration of gravity, see § (17) "Gravity."

§ (12) FORCE, MEASURE OF.—Force is measured by the acceleration produced in unit mass, and unit force is that which would produce unit acceleration in unit mass. Force may also be measured by change of momentum per unit time, and has therefore the dimensions of momentum/time = ML/T^2.

(i.) *Metric Units.*—The unit on the C.G.S. system is the dyne, or the force which accelerates or retards the velocity of a mass of 1 gramme by 1 cm. per second per second. A gravitational unit which is the weight of 1 gramme is also used; it is equivalent to g dynes, and therefore varies with latitude and height above sea-level (see § (17) "Gravity").

(ii.) *British Units.*—The British unit is the "poundal," the force which in one second retards or accelerates the velocity of a mass of 1 lb. by 1 ft. per sec. per sec. The corresponding gravitational unit which is commonly employed by engineers is the weight of 1 lb. $=g$ poundals.

(iii.) *Equivalents.*—

1 dyne . . . = 1 gramme centimetre per second per second.
= weight of ·00102 g.
$=7{\cdot}233\times10^{-5}$ poundals.
1 gramme weight = 980·6 dynes at latitude 45°.
= 981·2 dynes at London.
= 978·1 dynes at the Equator.
= 983 dynes at the Pole.
1 poundal . = 13,825 dynes.
1 pound weight = 32·172 poundals at sea-level in latitude 45°.
$=4{\cdot}45\times10^{5}$ dynes.

§ (13) WORK AND ENERGY, MEASURE OF.—A force is said to do work when its point of application moves in the direction of the force, and the work done is measured by the product of the force and the distance through which the point of application moves.

Energy is measured by the work which a body can do; it may take the form of:

Potential energy or energy of position,
Kinetic energy or energy of motion,
Thermal energy or energy of heat.

(i.) *Metric Units.*—The unit on the C.G.S. system is the erg, which is the work done by a force of 1 dyne when its point of application is moved through 1 cm.; usually a unit 10^7 ergs = 1 joule is used. A practical unit is the kilogramme metre, which depends on the value of g.

(ii.) *British Units.*—The British unit is the "foot-poundal," with a corresponding gravitational unit the foot-pound.

(iii.) *Equivalents* ($g=981$ cm./s.^{2}).—

1 erg . . . . = gramme (centimetre per second)^2.
$=2{\cdot}3731\times10^{-6}$ foot-poundals.
$=7{\cdot}373\times10^{-8}$ ft. lbs.*
1 joule . . . $=10^7$ ergs.
1 kilogramme metre $=10^5$ gramme cm.
$=g\times10^5$ ergs $=9{\cdot}81\times10^7$ ergs.*
1 Board of Trade Unit (B.T.U.) . . = 1 kilowatt-hour.
$=3{\cdot}6\times10^{13}$ ergs.
Foot-pound: 1 ft. lb. = 13825 gram. cm.
$=1{\cdot}3562\times10^7$ ergs.*
Foot-poundal . . $=4{\cdot}2139\times10^5$ ergs.
Foot-ton . . . = 2240 ft.-lbs.
$=3{\cdot}097\times10^7$ g.cm.
$=3{\cdot}0380\times10^{10}$ ergs.*

Note.—The values marked with an asterisk (*) are dependent on the value of the acceleration of gravity g.

(iv.) *Rate of Working.*—The rate at which force does work is measured in units of work per second.

On the C.G.S. system the unit is 1 erg per second or g.(cm./s.)^2/s. The practical unit is the "watt," which is equivalent to 1 joule per second.

In British units the common unit is the horse-power = 550 ft.-lbs. per second.

(v.) *Equivalents.*—

1 watt . . $=10^7$ erg/s.
= 1 joule per second.
$=1{\cdot}34\times10^{-3}$ horse-power.
1 horse-power . = 550 ft.-lbs. per second.
$=7{\cdot}46\times10^9$ erg/s.
= 746 watts.
1 force de cheval $=7{\cdot}36\times10^9$ erg/s.

§ (14) PRESSURE, MEASURE OF. (i.) *Units.*—Pressure is the force per unit area which any liquid or gas exerts on the surface in contact with it. The unit of pressure is that produced by unit force acting on unit area, and on the C.G.S. system is a force of 1 dyne per square centimetre, on the British system a force of 1 poundal per square foot. Units depending on the value of g, such as 1 gramme weight per square centimetre or 1 lb. weight per square foot, are also in common use.

(ii.) *Barometric Pressure. Bar and Millibar.*—After the introduction of the barometer pressure came to be measured as the length of a column of fluid, usually mercury (see "Atmosphere, Physics of," § (1)); and this length was subsequently corrected for variations in the value of g and of temperature. The relation between the "mercury-inch" and the "mercury-millimetre" and the value of pressure in units of force is obtained from the equation

$$l\rho g=\text{pressure in dynes per square centimetre},$$

where l is the length of the column in centimetres, ρ is the density of mercury in grammes per cubic centimetre, g is the acceleration of gravity in cm./s.^{2}

More recently the practice has become established of measuring the pressure of the atmosphere in units of force, and for this purpose the millibar, which is equivalent to a pressure of 1000 dynes per square centimetre, has come into use in the British Isles (see "Millibar"). The normal atmospheric pressure at sea-level is 1013·2 millibars, which differs little from 1 bar. In the United States the "bar" is taken as equivalent to 1 dyne per square centimetre, and the pressure of the atmosphere is measured in kilobars, which are equivalent to the English millibars.

(iii.) *Equivalents.*—

C.G.S.

1 dyne per sq. cm. = 1 microbar (= 1 bar U.S.A.).
= $1{\cdot}45 \times 10^{-5}$ lbs./sq. in.
= $2{\cdot}95306 \times 10^{-5}$ mercury-inches.
= $7{\cdot}50076 \times 10^{-4}$ mercury-millimetres.
1 millibar . . = 1000 dynes per square centimetre.
= 1 kilobar (U.S.A.).
1 centibar . . = 10 millibars.

British Unit.

Mercury-inch . = 1 inch of mercury at 32° F. in latitude 45°.
= 33·8632 mb.
Mercury-millimetre = 1 mm. of mercury at 0° C. in latitude 45°.
= 1·333200 mb.
760 mm. = 1013·231 mb.
1000 mb. = 14·496 lb./in.^{2} } in London.
= 2087·424 lb./ft.^{2} }
1 lb./sq. in. = 68,971 dynes/cm.^{2} = 70·31 gm./cm.^{2}
1 ton/sq. in. = $1{\cdot}545 \times 10^8$ dynes/cm.^{2} = 1·575 kgm./mm.^{2}

Russian Half-lines (normal at 62° F.).

1 half-line = 1·68801 mb. = ·0498 in.
600 half-lines = 1012·804 mb.

The standard "atmosphere" is equivalent to

760 mm. mercury at 0° C., lat. 45°, and sea-level.
= 759·4 mm. mercury at 0° C. in London.
= 29·92 in. mercury at 0° C., lat. 45°, and sea-level.
= $1{\cdot}0132 \times 10^6$ dynes per cm.^{2}
= 14·7 lbs. per sq. in.
= 0·94 tons per sq. ft.

§ (15) TEMPERATURE, MEASURE OF.—Three arbitrary scales are in use for the measurement of temperature, viz. the Centigrade, the Fahrenheit, and the Réaumur. In all the fixed points of the scale are—(1) the temperature of ice when melting under standard atmospheric pressure of 760 mm. (1013·2 mb.); and (2) the temperature of steam from water boiling normally under pressure 760 mm. The scales differ, however, in the numerical values assigned to these two temperatures, and consequently in the size of the degree.

(i.) *Centigrade.*—On the *Centigrade* scale, which is now used almost universally for scientific purposes, the temperature of melting ice is taken as the zero, and the temperature of boiling water as 100°. Temperatures on this scale are denoted by °C. following the value, and for values below the melting-point a negative sign is prefixed.

(ii.) *Fahrenheit.*—On the *Fahrenheit* scale, which is commonly used in the British Isles, the melting-point of ice is 32° F., and the boiling-point of water is 212° F. The scale was originally introduced by Fahrenheit in the early part of the eighteenth century. The zero denoted the lowest temperature then reached, viz. the temperature of a mixture of ice and salt, and the normal temperature of the human body, which was found to be nearly constant, was taken as 24°, the melting-point of ice being 8°. The size of the degree was then reduced to one-fourth, and the fixed points became 0°, 32°, and 96°. The scale was subsequently redefined to agree with the centigrade scale at 0° C. and 100° C.

(iii.) *The Réaumur Scale.*—On this scale the melting-point of ice is indicated by 0° R., and the boiling-point of water by 80° R. The scale was until recently in common use on the Continent, but is gradually being replaced by the centigrade scale.

(iv.) *Absolute Scale.*—The absolute scale of temperature, sometimes known as the thermodynamic or work scale, was originally introduced by Lord Kelvin, and is based on a consideration of the amount of work which can be obtained from a heat engine for a given supply of heat. The zero is such that a heat engine working between any source and a receiver at the zero of temperature would convert all the heat taken in into work (see "Thermodynamics," §§ (21), (22)).

If the size of the degree is taken as equal to that on the centigrade scale the temperature of absolute zero is approximately −273° C.; for degrees on the Fahrenheit scale the temperature of absolute zero is −459·44° F. The centigrade thermodynamic scale is adopted as fundamental by the National Physical Laboratory in view of resolutions of the Fifth General Conference of Weights and Measures, 1913.

The scale of the hydrogen gas thermometer at constant volume is nearly coincident with the work scale. Some of the more recent values for the absolute temperature of the ice point on the hydrogen scale are 273·14 (Callendar, 1903), 273·05 (Berthelot, 1907), 273·06 (Buckingham, 1908), 273·13 (Rose-Innes, 1908).

The name "Tercentesimal" has been suggested for an approximate absolute scale, taking the zero as −273° C., and "Quingentesimal" for a scale in Fahrenheit degrees measured from −459° F.

(v.) *Kilograd Scale.*—A fifth scale recently suggested by Professor McAdie is one in which the zero is taken at −273° C., and the degree is one-thousandth part of 273 centigrade degrees; the normal freezing-point is therefore taken as 1000.

(vi.) *Equivalents.*—The relations between the scales are as follows:

$$\begin{aligned}
\text{C.} &= \tfrac{5}{9}(\text{F} - 32) = \tfrac{5}{4}\text{R} = \text{A} - 273 \text{ (approx.)},\\
\text{F.} &= 32 + \tfrac{9}{5}\text{C} = 32 + \tfrac{9}{4}\text{R} = \tfrac{9}{5}\text{A} - 459{\cdot}4,\\
\text{R.} &= \tfrac{4}{9}(\text{F} - 32) = \tfrac{4}{5}\text{C} = \tfrac{4}{5}(\text{A} - 273),\\
\text{A.} &= \tfrac{5}{9}(459 - 4 + \text{F}) = 273 + \text{C} = \tfrac{5}{4}(218{\cdot}4 + \text{R}).
\end{aligned}$$

The relation between the size of the degrees on the four scales is

$$1^\circ \text{ C} = 1^\circ \text{ A} = \tfrac{9}{5}^\circ \text{ F} = \tfrac{4}{5}^\circ \text{ R}.$$

§ (16) HEAT, MEASURE OF QUANTITY. (i.) *Units.*—The empirical unit for the measurement of quantity of heat is the amount of heat required to raise the temperature of unit mass of water through one degree, and depends on the unit chosen for the measurement of mass and on the scale of temperature.

In physical investigations the unit of heat is the *gramme-calorie*, or the heat required to raise 1 gramme of water through a temperature of 1° C.

The unit of heat in British units, known as the *British Thermal Unit*, is the quantity of heat required to raise the temperature of 1 lb. of water through 1° F.

The unit of heat so defined is not the same at different points of the scale, and for precision the limits through which the temperature is raised must be specified. The unit used by Regnault—the "zero-gramme-calorie"—was the quantity of heat required to raise 1 gramme of water from 0° to 1° C. The 20° C., and 15° C. calorie, are also used to denote the value of the calorie for T = 20° C. and T = 15° C. respectively. More recently Professor Callendar has defined the "mean calorie" as one-tenth of the quantity of heat required to raise 1 gramme of water from 288 a. to 298 a.—15° C. to 25° C.; the same term is, however, sometimes used to denote one-hundredth of the heat required to raise 1 gramme of water from 273 a. to 373 a.

1 British Thermal Unit = 777 ft.lbs. = 252·00 calories.
1 calorie . . . = 0·00397 B.Th.U.
The large calorie, or
major calorie . = 1000 calories.

(ii.) *Dynamical Equivalent of Heat.*—When it became recognised that heat is a form of energy, it became possible to connect the empirical units defined above with the fundamental units of mechanics. The relationship depends on the experimental determination of the mechanical energy equivalent to a certain quantity of heat, a quantity which has come to be known as the Mechanical or Dynamical Equivalent of Heat.

The fundamental relation is

1 mean calorie (273 a.–283 a.) = 4·184 joules.
1 20° calorie . . . . = 4·180 joules.

The reciprocal of the dynamical equivalent usually denoted by A = 0·239.

(iii.) *Capacity for Heat.*—The capacity for heat of any substance in calories (or B.Th. Units) is the quantity of heat required to raise unit mass through 1° C. (or 1° F.).

(iv.) *Specific heat* of a substance is the ratio of its capacity for heat to the capacity for heat of water at some standard temperature. For standard temperature 293 a. the specific heat is numerically the same as the capacity for heat in calories per gram per degree.

(v.) *Specific Heat of Water.*[1]—The value of the specific heat of water at various temperatures is as follows (Kaye and Laby for 20° calorie):

Temperature.	Specific Heat.	Joules.
273 a. = 0° C.	1·0094	4·219
283 a. = 10° C.	1·0027	4·191
293 a. = 20° C.	1·0000	4·180
303 a. = 30° C.	·9987	4·175
313 a. = 40° C.	·9982	4·173
323 a. = 50° C.	·9987	4·175
333 a. = 60° C.	1·0000	4·180
343 a. = 70° C.	1·0016	4·187
353 a. = 80° C.	1·0033	4·194
363 a. = 90° C.	1·0053	4·202
373 a. = 100° C.	1·0074	4·211
393 a. = 120° C.	1·0121	4·231
413 a. = 140° C.	1·0176	4·254
433 a. = 160° C.	1·0238	4·280
453 a. = 180° C.	1·0308	4·309
473 a. = 200° C.	1·0384	4·341
493 a. = 220° C.	1·0467	4·376

The specific heat of sea water at 290 a. = 0·94,
ice at 260 a. = 0·502.

(vi.) *Specific Heat of Gases.*—The specific heat of gases has different values according as the temperature is raised at constant pressure or at constant volume.

Dry air at constant pressure at 293 a. (C_p) . = 0·2417 (Swann, 1909).
Dry air at constant volume (C_v) at 273 a. . . = 0·1715 (Joly, 1891).
Water vapour at constant pressure at 373 a. . = 0·4652 (Holborn and Henning, 1907).
Water vapour at constant volume at 373 a. . = 0·340 (Pier, 1909).

The ratio of the specific heats, usually denoted by κ or $\gamma = C_p/C_v$, is a quantity of some importance.

Velocity of sound in any gas . $= \sqrt{\gamma \cdot p/\rho}$.
For a gas expanding adiabatically $pv^\kappa = \text{const.}$

$$T/p^{(\gamma-1)/\gamma} = \text{const.}$$

For dry air . . . . . $\gamma = 1{\cdot}40$
$(\gamma - 1)/\gamma = {\cdot}286$.

(vii.) *Latent Heat.*—*The latent heat of vaporisation* is the amount of heat required to change 1 gramme of substance from liquid to vapour without change of temperature.

Latent heat of steam at 273 a. = 597 calories.
= 2495 joules/g.
Latent heat of steam at 373 a. = 539 calories.
= 2252 joules/g.

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[1] For a discussion of this quantity and values in terms of a 17·5° C. calorie, see "Heat, Mechanical Equivalent of," §§ (7), (9).

The formula connecting latent heat and temperature given by Smith[1] is

$L_T = 597{\cdot}2 - {\cdot}580(T-273)$, where L is in calories.

$L_T = 2495 - 2{\cdot}43(T-273)$, where L is in joules.

The latent heat of fusion is the quantity of heat required to convert 1 gramme of substance from solid to liquid without change of temperature.

Latent heat of 1 gramme of water = 79·77 calories.
= 333·4 joules/g.

§ (17) GRAVITY, MEASURE OF.—The law of universal gravitation is that every body attracts every other body with a force which varies directly as the product of the two masses and inversely as the square of the distance between them.

(i.) *The constant of gravitation* is the constant G in the law of attraction set out above, and is defined by the equation

$$\text{Force of attraction} = G\frac{\text{mass} \times \text{mass}}{(\text{dist.})^2},$$

$$G = 6{\cdot}6576 \times 10^{-8}\ \text{cm.}^3/\text{g.s.}^2\ (\text{Boys}).$$

(ii.) *The acceleration of gravity* is the acceleration produced in any body by the force of the earth's attraction; as actually measured the acceleration is that due to the earth's attraction minus the centrifugal force of the earth's rotation.

Owing to the fact that the earth is not perfectly spherical in shape, but is more nearly a spheroid, and also on account of the variation with latitude of the centrifugal force of the earth's rotation, and the irregularities in the density of the earth's surface, the formulae giving the variation of g over the earth's surface are complicated.

A formula of the following form is given by Helmert:

$$g = A(1 + B\sin^2\phi)\left(1 - \frac{2h}{R} + \frac{3h}{2R}\frac{\delta}{\Delta} - \frac{h'(\delta-\theta)}{2R\Delta} + y\right),$$

where ϕ is the latitude, R the mean radius of the earth, h = height above sea-level, h' = thickness of surface strata of low density, Δ = mean density of the earth (5·6 × density of water), δ = mean density of surface strata (2·8 × density of water), θ = actual density of the surface strata in the region, y = orographical correction due to neighbouring mountains.

Assuming that $\delta = \theta$ and y is negligible, we obtain

$$g = 980{\cdot}617(1 - {\cdot}00259\cos 2\phi)\left(1 - \frac{5}{4}\frac{h}{R}\right),$$

where $g = 980{\cdot}617$ is the value of gravity at sea-level in latitude 45°.

Putting $R = 6{\cdot}37 \times 10^6$ metres,

$$g = 980{\cdot}617(1 - {\cdot}00259\cos 2\phi)(1 - 1{\cdot}96 \times 10^{-7}h),$$

where h is in metres.

In British units, putting $R = 2{\cdot}09 \times 10^7$ feet,

$$g = 32{\cdot}172(1 - {\cdot}00259\cos 2\phi)(1 - 5{\cdot}97 \times 10^{-8}h),$$

where h is in feet.

The above formula applies to places on the earth's surface at different heights above sea-level, and takes account of the additional attraction of the high ground; for points above the earth's surface the factor $R^2/(R+h)^2 = 1/(1+h/R)^2$, which is approximately equal to $1 - 2h/R$ if h/R is small, replaces $1 - 5h/4R$.

(iii.) *Centrifugal Force of the Earth's Rotation.*—On account of the rotation of the earth the acceleration produced in any body is the resultant of the acceleration produced by the gravitational attraction of the earth, and the acceleration produced by the centrifugal force of the rotation. This latter component is equal to $-R\omega^2\cos^2\phi$, where R is the radius of the earth and ω the angular velocity.

At the equator $R = 6{\cdot}377 \times 10^8$ cm., and since $\omega = 7{\cdot}292 \times 10^{-5}$ r./s.,

$$\therefore\ R\omega^2 = 3{\cdot}39\ \text{cm./s.}^2,$$

hence for latitude ϕ the value of g is diminished by $R\omega^2\cos^2\phi$, *i.e.* by $3{\cdot}39\cos^2\phi$ cm./s.², on account of the rotation of the earth.

§ (18) GEODETIC MEASURES. (i.) *The Nautical Mile.*—According to the definition adopted in England and the United States, the nautical mile is equal to the length of one minute of arc of a great circle on a spherical earth assumed to have the same area as Clarke's ellipsoid (see below).

On the Continent the terms "nautical mile" and "geographical mile" are interchangeable, and both are defined as the mean length of arc of one minute of latitude which varies from 1842·7 m. at the equator to 1861·3 m. at the poles.

Adopting the English definition for nautical mile,

Nautical mile . = 1853·152 m. (Admiralty).
= 6080 feet.
= 1·1515 statute miles.
Geographical mile = 1852 m. (Annuaire du Bureau Central des longitudes).
= 6076·8 feet.

(ii.) *Clarke's Ellipsoid.*—The surface of the planet as determined by "sea-level" is approximately an ellipsoid, known as Clarke's ellipsoid, with axes as follows:

Semi-polar axis . = 6,356,068 m.;
Semi-equatorial axes = 6,378,294 m. and 6,376,350 m.;

and according to Clarke's figures:

1 quadrant = 10,007,000 m.

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[1] *Phys. Rev.*, 1907.

The values of the radii have also been given as follows:

	Equatorial Radius.	Polar Radius.
	m.	m.
Clarke, 1880 . .	6,378,249	6,356,515
Helmert, 1906 . .	6,378,200	6,356,818
U.S. Survey, 1906 .	6,378,388	6,356,909

(iii.) *Geodetic Constants.*—The mean polar quadrant = 10,002 kilometres (determined from a mean of Helmert and U.S. Survey).

Value of g: Equator . $= 978{\cdot}024$ cm./s.^{2}
Lat. 45° . $= 980{\cdot}617$ cm./s.^{2}
London . $= 981{\cdot}19$ cm./s.^{2}
Pole . . $= 983{\cdot}210$ cm./s.^{2}
Mean density of earth . $= 5{\cdot}5$ g./c.c. approx.
Mean density of surface of earth . . . . $= 2{\cdot}65$ g./c.c.
Volume of earth . . $= 1{\cdot}082 \times 10^{21}$ m.^{3}
Mass of earth . . . $= 5{\cdot}98 \times 10^{27}$ g.
$= 5{\cdot}87 \times 10^{21}$ tons.
Area of land (estimated) . $= 1{\cdot}45 \times 10^{18}$ cm.^{2}
Area of ocean (estimated) $= 3{\cdot}67 \times 10^{18}$ cm.^{2}
Mean depth of ocean (Murray) . . . $= 3{\cdot}85 \times 10^5$ cm.
Volume of ocean . . $= 1{\cdot}41 \times 10^{24}$ c.c.
Mass of ocean . . . $= 1{\cdot}45 \times 10^{24}$ g.
Mass of the atmosphere . $= 5{\cdot}33 \times 10^{21}$ g.
Velocity of a point on the equator due to the earth's rotation $= R\omega = 4{\cdot}64 \times 10^4$ cms./s.^{2} or 1037 miles per hour approx.

Length of 1° of Longitude in different Latitudes

Latitude.	Metres.	Nautical Miles.	Miles.
0°	111,307	60·064	69·164
10°	109,627	59·157	68·120
20°	104,635	56·463	65·018
30°	96,475	52·060	59·948
40°	85,384	46·075	53·056
50°	71,687	38·684	44·545
60°	55,793	30·107	34·669
70°	38,182	20·604	23·726
80°	19,391	10·464	12·049
90°	0	0	0

§ (19) Astronomical Constants. (i.) *Sun.*—

Distance from earth . $= 1{\cdot}494 \times 10^{11}$ m.
$= 9{\cdot}282 \times 10^7$ miles.
Mass . . . . . $= 329{,}320 \times$ earth's mass.
Equatorial diameter angular . . . . $= 32' \, 2''{\cdot}36$.
$= 865{,}980$ miles.
$= 1{,}393{,}338$ km.
Mean time taken by light from sun to earth . $= 498{\cdot}2$ seconds.
$= 8{\cdot}3$ minutes (approx.).
Obliquity of ecliptic . $= 23° \, 27' \, 4''{\cdot}04$ (1909).
Mean equatorial solar parallax (Hinks, 1909) $= 8''{\cdot}807$.

(ii.) *Moon.*—

Distance . . . $= 60{\cdot}27 \times$ earth's radius.
Mass . . . . . $= \frac{1}{81{\cdot}53} \times$ earth's mass.
Diameter . . . $= 2163$ miles $= 3481$ km.
Inclination of moon's orbit to ecliptic . $= 5° \, 8' \, 43''$.

E. E. A.

Mechanical Efficiency. See "Petrol Engine, The Water-cooled," § (4).

Mechanical Equivalent of Heat: the amount of work (J) which must be expended in the production of unit quantity of heat, if the work is wholly converted into heat. In the absolute (C.G.S.) system $J = 4{\cdot}1868 \times 10^7$ ergs per gramme-calorie. On the British system $J = 777{\cdot}8$ foot-pounds per B.T.U. This assumes 1400 foot-pounds (in the latitude of London) as equivalent to one mean pound-calorie. If we assume the standard thermal unit to be the energy required to raise one gramme of water from 17° C. to 18° C., then $J = 4{\cdot}185 \times 10^7$ ergs, and assuming $g = 981$, its value is 252·0 kilogramme-metres, or on the British system, 777·6 foot-pounds. See also "Heat, Mechanical Equivalent of," § (9); "Thermodynamics," § (3).

Comparison of Reliable Determinations of. See "Heat, Mechanical Equivalent of," § (6).

Criticism of Discrepancies in Values of. See *ibid.* § (3).

Discussion of Data of Various Determinations and Selection of Reliable Values. See *ibid.* § (3).

Direct Determination Experiments. See *ibid.* § (4) (i.).

Indirect Methods of Determination of. See *ibid.* § (5).

Joule's Method of Determination of. See *ibid.* § (4) (i.).

Summary of Methods of Determining. See *ibid.* § (7).

MECHANICAL POWERS

The mechanical powers are simple pieces of apparatus devised originally with the object of raising a heavy weight by the application of a small force. Let P be the applied force, and suppose that by the motion of its point of application through a distance a, measured in the direction of the force, the weight W is raised a vertical distance b, or more generally a force W is exerted through a distance b.

Then the work done by the force is Pa, that done on the weight is Wb, and, neglecting any losses in the machine due to friction and other causes, these two are equal. Thus

$$Pa = Wb$$

and

$$\frac{W}{P} = \frac{a}{b}.$$

The construction of the machine is such that a is considerably greater than b, so that a large weight is raised by a small power.

Since the displacements a and b take place in the same time the quantities a and b measure also the velocities of the points of application of the weight and the power; the equivalence of work expressed by the equation $Pa = Wb$ used to be known as the principle of virtual velocity, and a/b is spoken of as the velocity ratio of the machine; it measures, as we have seen, the ratio of the weight raised to the force applied.

§ (1) THE INCLINED PLANE.—The inclined plane is one of the most ancient of the mechanical powers known to mankind. It is probable that it was used, in its ancient form, in the raising of large stones such as those used in the construction of the pyramids or of Stonehenge.

If the length of the plane, measured along its surface, be denoted by α, and the height and horizontal distance by β and γ respectively, then the displacement of a body along the plane will be represented by α, the vertical displacement by β, and the horizontal displacement by γ; α, β, and γ also represent the relative velocities in these three directions.

A survival of the ancient use of the inclined plane as a means of raising heavy loads is seen to-day in the railway embankment, by which a railroad is carried up and down hill, the gradient being reduced to a workable minimum by the erection, at enormous expenditure of labour, of long sloping banks of earth.

In the form of the wedge, the inclined plane is commonly employed in a great variety of ways, from the common wedge for splitting timber to its use in apparatus and machinery of precision.

§ (2) PULLEYS.—Combinations of belts or ropes and pulleys are used to transmit rotational motion between shafts, the axes of which may be at any angle to each other. In belt driving it must always be arranged that the point at which the belt leaves one pulley must be in the central plane of the next pulley over which the belt has to pass. If this rule is observed the belt will remain on the pulleys for one direction of drive, but if it is desired to drive in both directions properly placed guide pulleys must be provided.

(i.) *Tackles.*—In a combination in which one axis is capable of moving parallel to itself, the mechanism is commonly termed a "tackle," the object of which is to cause relative displacement of the axes on which the pulleys rotate: the rotation of the pulleys is not essential to the mechanism, their object being purely to minimise friction.

Of the combinations forming "tackles" or "purchases" there are three common systems. In the first, *Fig.* 1, the weight is suspended from the axle of the lowest pulley and the rope passing round it is attached at one end to a fixed beam above, the other end being fixed to the axle of the next lowest pulley: the rope passing round the latter has one end attached to the beam and the other to the axle of the third pulley, and so on. The last rope has a free end by which the force required to lift or lower the weight is applied in a vertical direction.

The velocity ratio of the system is 2^n, where n is the number of pulleys.

In the second system, *Fig.* 2, one rope only is used, and this passes round all the pulleys in turn. Two blocks are provided, each carrying a number of pulleys either on the same or separate axles.

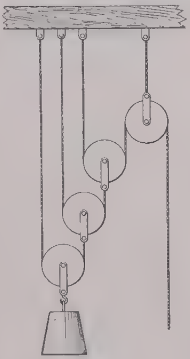
FIG. 1.

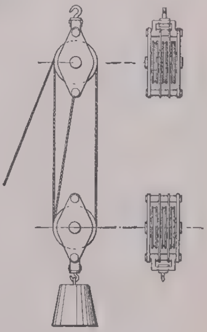
FIG. 2.

The upper block is supported from a beam and the lower carries the weight to be lifted or lowered. The rope is fixed at one end to the upper block and passes successively under the first pulley of the lower block, over the first pulley of the upper block, under the second pulley of the lower block, and so on, travelling in the same direction throughout. At the free end of the rope the force is applied.

The velocity ratio of the system is numerically equal to the number of ropes connecting the upper and lower blocks.

In the third system, *Fig.* 3, all the ropes are attached to the weight, or to a beam supporting the weight. The rope passing over any pulley is attached at one end to the beam, and at the other to the axle of the next lower pulley except in the case of the lowest pulley, where the free end forms the means of applying the force. The first or uppermost pulley is supported by its axle from an overhead beam, and the weight is slung from the lower beam at a point suitably chosen so that all the ropes remain parallel to each other. If there are n pulleys, the velocity ratio is $2^n - 1$.

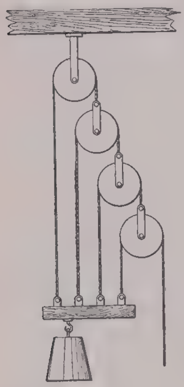

FIG. 3.

(ii.) *Differential Pulleys — Weston's Differential Block.* — The upper block of this tackle carries two pulleys differing slightly in diameter and fixed to the same axle, and the lower block a single pulley.

The chain passes over one of the upper pulleys, then under the lower one, and finally over the second upper pulley on the opposite side of the axle, as shown in *Fig.* 4. A fair length of chain is retained, the ends being joined to form an endless linkage. The pulleys are grooved to receive the chain links so that no slip of the chain over the pulleys is possible.

By this device a very high velocity ratio can be obtained, and is equal to $2D/(D-d)$, where D and d are the diameters of the two upper pulleys respectively.

The Weston block is a modification of the differential wheel and axle, the difference being that in the latter device the ends of the chain or rope are fixed to the large and small axles respectively, the power being applied by rotation of the axles about their common axis.

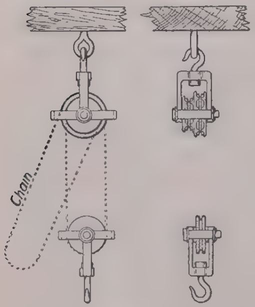

FIG. 4.

§ (3) THE SCREW.—If a strip of rectangular section be wrapped round a cylinder in such a manner that its inclination to a transverse axis is always the same, the curve assumed by the initially straight strip is known as a helix. The projection formed by the strip is commonly called the "thread"; this may be continued round the cylinder any number of times in succession.

The "pitch"[1] of a screw is the distance, measured parallel to the axis of the cylinder, between similar points of two successive turns. If the screw consists of one continuous thread, it is said to be "single threaded." The pitch may, however, be divided into a number of equal parts, and additional threads may be introduced side by side with the original one, thus producing a screw of "multiple pitch."

In practice, the thread form is varied to suit the nature of the work for which it is to be used. Thus, there are threads of rectangular, semicircular, and triangular section, each designed for a particular purpose.

If, instead of imagining the strip to be wrapped on the outside of a cylindrical bar, we think of it as lying on the inside of a hollow cylinder, and imagine the thread form cut in this, we have a hollow screw. This is commonly known as a nut, and can be made to fit exactly on to a bar or bolt having a corresponding thread. If the nut is rotated and the screw held, it will move axially along the screw, or, again, if the nut be rotated, but prevented from axial motion while the screw is free to move axially but is prevented from turning, axial motion of the screw will take place and the combination of nut and screw can be used as a mechanical power.

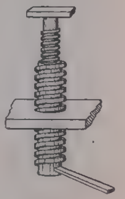

FIG. 5.

If the force causing the motion of the rotating element acts at a radius R, then the velocity ratio of the combination will be $2\pi R/p$, where p is the pitch of the thread or its advance per revolution.

The thread may be wound in a left or right-handed direction, and is named according to the direction in which it requires to be rotated to cause it to advance through its nut.

Theoretically the velocity ratio may be increased indefinitely by sufficiently decreasing the pitch, but in practice this is not possible because of the resulting reduction in the thread strength.

The ratio may, however, be greatly increased by means of the adoption of the differential screw, illustrated in *Fig.* 5. This consists of two screws, the one being formed hollow and acting as a nut for the other. The smaller screw is prevented from rotating. Consideration will show that the resultant vertical displacement of the weight for one revolution of the large screw is equal to the difference in pitch of the two screws, and this may be made very small.

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[1] The term "pitch" is sometimes erroneously used to denote the number of coils per unit length of the cylinder.

The velocity ratio is thus $2\pi R/(p-p_1)$, where p and p_1 are the pitches of the two screws respectively.

§ (4) WORM-GEAR.—The term "worm-gearing" is now almost universally applied to that form of mechanism in which a screw gears with a wheel, the axes of the two elements being at right angles to each other. Before the recent improvements in this form of gearing for power transmission were made the term was often used to include screw, helical, and skew gears.

The two elements, the worm and the worm wheel, are combined in practice in three different forms. That most commonly adopted is the parallel worm type, in which the worm is a truly parallel screw, the wheel being hollowed out to fit the worm.

A second form is known as the "hollow" or Hindley type, in which both worm and wheel have hollow faces, and in a third form the wheel is a true screw with the worm hollow faced to gear with it.

Of the two latter forms the former only has been developed to any great extent.

The gear is used to transmit power from one axis—that of the worm—to a second—that of the wheel—at right angles; the worm drives the wheel, and, as it rotates, that tooth of the wheel which, at the moment, is in gear, is made to move parallel to the axis of the worm, just as a nut, when incapable of rotation, moves axially along a screw when the latter is turned. As this tooth passes out of gear, the next becomes enmeshed with the succeeding turn of the work, and so the motion is continuously transmitted.

In the parallel type it is usual to design the teeth of the worm and wheel to secure contact in a plane through the axis of the former and at right angles to the axis of the latter; the section of the wheel by this plane is made similar to that of a spur gear, having the same diameter and pitch of teeth, and the section of the worm that of a rack to gear with the spur wheel. For power transmission at high loads and speeds, however, it is necessary to obtain satisfactory contact between the teeth of the two elements at all sections parallel to the central plane, and this is accomplished by using a worm-thread form of special section. The relative motion of the two members at all points of contact should be perpendicular to their common surface at both points.

With the Hindley type of gear the teeth may be of any convenient form, as consideration of the accompanying diagrams of the central sections of the gears will show.

It will be seen in the second case that the thread form has not only rotational motion about its own axis but also about the axis of the wheel. The teeth of both elements follow geometrically the same path and a rolling form of tooth section does not therefore need to be considered in this gear. It is essential, however, that the tooth form chosen should render a fair length of worm possible without producing interference between the teeth of the two elements.

In practice the diameter of the worm at the ends does not exceed about 7 per cent the diameter at the throat.

The parallel and hollow-worm types have of late years received considerable attention, and gears of both types have been produced for the transmission of power with an efficiency of 97 per cent.

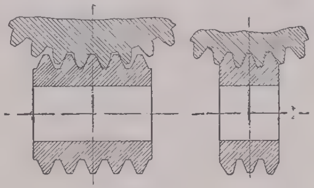

FIG. 6.

The design of a worm-gear depends in a great measure on the work for which it is intended. Thus, for a gear to transmit motion where efficiency of transmission is not important, the gear ratio is only limited by the size of the thread section and by the angle of friction between the gear teeth.

If it is desired that the wheel shall not drive the worm—that is, that the gear shall not be reversible—the angle of the tooth with the axis must be less than the angle of friction. If efficiency is unimportant, the angle may be as small as desired, but if efficiency is a governing factor it should be made as near to the friction angle as possible. In the latter case the maximum limit of efficiency is 50 per cent.

For power transmission, where high efficiency is desired, the worm will be reversible and the worm angle must be made about 40°.

The efficiency is given by the equation

$$\eta=\frac{\tan\ \theta}{\tan\ (\theta+\mu)},$$

where θ is the worm angle and μ the friction angle.

J. H. H.

MELTING-POINTS OF INORGANIC COMPOUNDS studied as fixed temperatures for secondary standards of temperature and compared with gas thermometers in the range 500° to 1600° C. See "Temperature, Realisation of Absolute Scale of," § (90) (iii.).

Melting-points of Metals:

As determined by Day, Sosman, and Allen, tabulated. See "Temperature, Realisation of Absolute Scale of," § (40), Table 11.

Studied as fixed temperatures for secondary standards of temperature and compared with gas thermometers in the range 500° to 1600° C. See *ibid.* § (40) (i.).

Melting-points of Refractory Elements and Compounds, determined by optical pyrometer. See "Pyrometry, Optical," § (24).

Mercury:

Absolute Expansion of, determined by Callendar. See "Thermal Expansion," § (11) (ii.).

Absolute Expansion of, determined by Regnault. See *ibid.* § (11) (i.).

Coefficient of Absolute Expansion of, tabulated from calculations by various methods. See *ibid.* § (11) (ii.).

Influence of Pressure on Thermal Expansion of, tabulated results of P. W. Bridgman. See *ibid.* § (13).

Specific Heat of, determined by Callendar, Barnes, and Cooke by the electrical method. See "Calorimetry, Electrical Methods of," § (3).

Specific Heat of, at Different Temperatures, tabulated. See *ibid.* § (4), Table I.

Metals:

Atomic Heat of, at Low Temperatures, Nernst's values for, tabulated. See "Calorimetry, Electrical Methods of," § (11), Table VI.

Methods of measuring Thermal Conductivity of. Bar Methods—Electrical Methods—Indirect Methods—Periodic Flow Methods. See "Heat, Conduction of," § (5).

Molten, Emissivity of, observed approximately by Burgess with a Féry radiation pyrometer. See "Pyrometry, Total Radiation," § (18) (iv.).

Specific Heats of, Gaede's determination of, by the electrical method, tabulated results. See "Calorimetry, Electrical Methods of," § (9), Table IV.

Specific Heats of, studied by Professor E. H. Griffiths, F.R.S., and Ezer Griffiths over the range −160° to +100° C., by the electrical method. See *ibid.* § (10).

Values of Thermal Conductivity of. See "Heat, Conduction of," Table III.

Metals which can be melted in Air: Tin, Zinc, and Gold, determination of melting-point of. See "Thermocouples," § (22) (iii.).

Metals which require a Neutral Atmosphere when they are melted: Nickel and Cobalt, determination of melting-point of. See "Thermocouples," § (22) (ii.).

Metals which require a Reducing Atmosphere when they are melted: Antimony, Aluminium, Silver, and Copper, determination of melting-point of. See "Thermocouples," § (22) (i.).

Meters. For the article on "Meters" see Vol. III.

Micromanometers. See "Pressure, Measurement of," § (20).

Micrometer Water Gauge. See "Pressure, Measurement of," § (26).

Micropyrometer: an instrument of the disappearing filament optical pyrometer type, devised by Burgess for the determination of the melting-points of minute specimens of materials. See "Pyrometry, Optical," § (4).

Mills, J. E., formulation of the expression giving the value of L_1, the latent heat of vaporisation of a substance, in terms of E_1, the heat equivalent of the external work done, S_L and S_V, the densities of liquid and saturated vapour respectively.

$$\frac{L_1 - E_1}{\sqrt[3]{S_L} - \sqrt[3]{S_V}} = \text{constant.}$$

See "Latent Heat," § (11) (ii.).

Mitchell's Theory of the Resistance of Flat Inclined Surfaces separated by a Film of Fluid. See "Friction," § (27).

Mixtures, Method of: the best known of calorimetric methods. See "Calorimetry, Method of Mixtures," § (2).

Model Experiments and their Full-scale Equivalents. See "Dynamical Similarity, Principles of," § (38).

Model and Full-scale, Condition for Continued Similarity in Working of. See "Dynamical Similarity, The Principles of," § (39).

Modulus of Direct Elasticity (Young's Modulus):

Method of Determination. See "Elastic Constants, Determination of," § (64).

Tabulated Values. See *ibid.* § (65).

Modulus of Elasticity. The ratio of a stress applied to a body to the strain it produces. See "Elastic Constants, Determination of," § (63).

Modulus of Transverse Elasticity (Modulus of Rigidity):

Method of Determination. See "Elastic Constants, Determination of," § (66).

Tabulated Values. See *ibid.* § (65), Table 22.

Molecular Pump (Gaede). See "Air-pumps," § (39).

Molecular Theory of Matter. See "Thermodynamics," § (66).

Molecules, Finite Size and Cohesion of, as affecting Perfect Gas Equation. See "Thermodynamics," § (66).

Mollier's Chart of Entropy and Total Heat. See "Thermodynamics," § (42).
  For Steam. See "Steam Engine, Theory of," § (9).
  For Substances used in Refrigerating Machines and the Use of Oblique Co-ordinates. See "Refrigeration," § (3), *Figs.* 8 and 9.
Molybdenum, a possible substance for use as an outer protecting sheath for a thermo-element, at high temperatures. See "Thermocouples," § (5) (v.).
Moment of Inertia of a Body about a Line. The sum of the products of the mass of each particle of the body into the square of its perpendicular distance from the line,

$$I = \Sigma(mr^2).$$

Monatomic Gas, Energy of. See "Thermodynamics," § (58).
Multiple Expansion Steam Engines. See "Steam Engine, Theory of," § (10).
Multiple Screws for Ships. See "Ship Resistance and Propulsion," § (49).

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Naphthalene and Benzophenone, determination of boiling-points of. See "Temperature, Realisation of Absolute Scale of," § (36).
National Gas-engine. See "Engines, Internal Combustion."
  100 H.P. Horizontal Type. § (5).
  300 H.P. Vertical Type. § (6).
N.P.L. Gear Efficiency Experiments. See "Friction," § (29).
Natterer and Cailletet, experimental researches on the expansion of fluids under high pressures. See "Thermal Expansion," § (18) (i.).
Nernst and Lindemann, formula for specific heat, tested in the case of the metals Al, Ag, Pb, and Cu by comparison with values given by the thermodynamical equation, in tabular form. See "Calorimetry, the Quantum Theory," § (46), Table III.
Neville and Heycock, work on resistance thermometer using a thermocouple. See "Thermocouples," § (22) (iii.).
Newcomen's Engine. See "Steam Engine, Reciprocating," § (10).
Newton's Third Law of Motion, Meaning of Action and Reaction. See "Heat, Mechanical Equivalent of," § (1).
Nichrome Tubes, Cast, used to some extent as outer protecting tubes for thermo-elements. See "Thermocouples," § (5) (iv.).
Nickel Carbon Eutectic (1330° C.), used as standardising point for thermocouples. See "Thermocouples," § (22) (iii.).
Nickel Oxide Surface, Emissivity of, measured by Burgess and Forte with a Féry radiation pyrometer. See "Pyrometry, Total Radiation," § (18) (i.).
Nitrogen:
  Separation from Air of. See "Gases, Liquefaction of," § (2).
  Specific Heats of, tabulated values obtained by Scheel and Heuse. See "Calorimetry, Electrical Methods of," § (15), Table IX.
  Used as Thermometric Substance. See "Thermodynamics," § (4).
Nitrous Oxide, Latent Heat of Vaporisation of, determined by Mathias. See "Latent Heat," § (8).
Nozzle, De Laval's Convergent-divergent. See "Thermodynamics," § (44).
Nozzles:
  Design of. See "Steam Engine, Theory of," § (12).
  Quantity of Steam passing. See "Steam Turbine, Physics of," §§ (2), (3).
Nuclei, Condensation on; experiments of Aitken and C. T. R. Wilson. See "Thermodynamics," § (54).
Number of Molecules in a given Volume of Gas. By Avogadro's law this is the same for all gases; the number in a gramme molecule or Mol is $4\cdot5 \times 10^{25}$.

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Oechelhauser Two-stroke Engine. See "Engines, Internal Combustion," § (11).
Oil-engines. See "Engines, Internal Combustion," § (13) *et seq.*
Oils, Specific Heat of, determined by the electrical method. See "Calorimetry, Electrical Methods of," § (6).
Onnes, Kamerlingh: investigation of the expansion of hydrogen at low temperatures. See "Thermal Expansion," § (18) (v.).
Optical Indicators for High-speed Steam Engines. See "Pressure, Measurement of," § (19).
Osmotic Pressure. The excess pressure which must be applied to a solution in order to make its vapour pressure equal to that of the pure solvent. See "Thermodynamics," § (63).
Otto Cycle used in Gas-engines. See "Engines, Thermodynamics of Internal Combustion," §§ (34) and (51).

Outlets from Reservoirs. See "Hydraulics," § (20).

Outward-flow Turbines (Hydraulic). See "Hydraulics," § (48) (i.).

Oxygen, Specific Heats of, tabulated values obtained by Scheel and Heuse. See "Calorimetry, Electrical Methods of," § (15), Table IX.

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Paddle-wheels. See "Ship Resistance and Propulsion," § (53).

Parsons' Design Coefficient K. A coefficient of importance in the theory of the steam turbine. See "Turbine, Development of the Steam," § (12); "Steam Turbine, Physics of," § (7).

Peltier Effect, Definition and Thermodynamic Theory of. See "Thermodynamics," § (65).

Pelton Wheel. See "Hydraulics," § (47) (iv.).

Periodic Heat Flow:
  Into Infinite Solid. See "Heat, Conduction of," § (12) (i.).
  Along Rod. See *ibid.* § (5) (v.).

Petavel's Optical Indicator. See "Pressure, Measurement of," § (19).

PETROL ENGINE, THE WATER-COOLED

§ (1) Introductory.—The petrol engine has now (1921) reached such a state of development that its performance approaches very closely to that possible with the Otto cycle, using petrol as fuel.

At the present time brake mean pressures of the order of 130 lbs. per sq. in. are not uncommon.

In cases where high speeds can be employed, outputs of 38 brake horse-power per litre of piston displacement have been attained.

As regards thermal efficiency, a number of engines have realised indicated thermal efficiencies, relative to the air standard efficiency for the compression ratio employed, of 67 per cent.

By careful attention to the reduction of mechanical losses, particularly as regards piston friction, the mechanical efficiency of modern engines has been raised to the neighbourhood of 90 per cent even in engines running at moderately high speeds.

In considering the output of an engine in terms of brake horse-power per litre the curve shown in *Fig.* 1 may be useful as giving the brake mean pressure required to produce one brake horse-power per litre at various speeds from 1000 to 5000 revolutions per minute.

For example, at 4000 r.p.m. an engine giving 36 b.h.p. per litre would require to develop a b.m.e.p. of $3{\cdot}25 \times 36 = 117$ lbs. per sq. in.

On examining the conditions governing the power performance of petrol engines it is obvious that for a given speed of rotation the chief factors are:

1. Indicated thermal efficiency.
2. Volumetric efficiency.
3. Mechanical efficiency.

In other words, the brake output of the engine depends on burning the charge to the best possible advantage; burning the greatest possible weight of charge per stroke; and lastly, wasting as little indicated power as possible in moving the parts of the engine and in carrying out the various functions inherent in the cycle of operations.

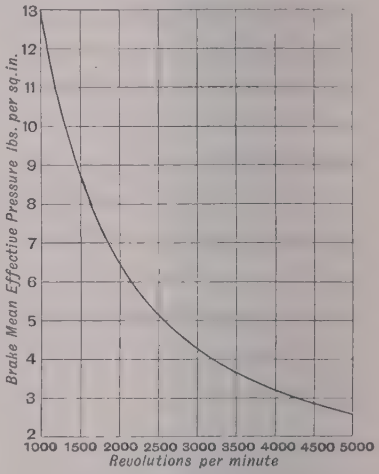

Fig. 1.

§ (2) Indicated Thermal Efficiency.—With regard to the first item, Tizard and Pye[1] have shown that, taking into account dissociation and change in specific heat, the limiting indicated efficiencies possible with the Otto cycle, using a hydrocarbon fuel such as petrol, and assuming no heat loss to the cylinder walls during combustion and expansion, are as given in column 3 of Table I.

This table[2] shows, in column 1, the various compression - expansion ratios considered; column 2, the air cycle efficiency for each of the ratios in column 1; column 3, Tizard and Pye's ideal indicated efficiencies, taking account of losses due to dissociation and change in specific heat but taking no account of losses due to direct passage of heat to the cylinder walls; column 4, the highest attainable indicated thermal efficiencies, assuming "that the combustion chamber is designed to allow of the minimum possible heat loss, that the cylinder is of comparatively large capacity, and that the revolutions are not less than 1500 r.p.m." Column 5 gives the actual indicated thermal efficiencies obtained in a special variable compression engine designed by Ricardo, a description of which will be found in *Engineering*, September 3, 1920.

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[1] See *Automobile Engineer*, February, March, and April 1921.

[2] See "Some Possible Lines of Development in Aircraft Engines," by H. R. Ricardo. The Royal Aeronautical Society, 1921.

It will be observed that the difference between columns 4 and 5, indicating the margin remaining for improvement, is very small.

The values for efficiency shown in Table I., both those which have been calculated and those which were observed, are for a homogeneous mixture of air and hydrocarbon vapour.

TABLE I

1. Expansion Ratio.	2. $E\ \ 1-(1/r)^{0\cdot4}$.	3. $E=1-(1/r)^{0\cdot295}$	4. $E=1-(1/r)^{0\cdot25}$.	5. Observed Results, Variable Compression Engine.
4·0	0·4256	0·336	0·296	0·277
4·5	0·4521	0·359	0·314	0·297
5·0	0·4747	0·378	0·332	0·316
5·5	0·4944	0·396	0·348	0·332
6·0	0·5116	0·411	0·361	0·346
6·5	0·5270	0·424	0·375	0·360
7·0	0·5398	0·437	0·386	0·372
7·5	0·5534	0·449	0·396	0·383
8·0	0·5647	0·460	0·406	...

Either by employing a stratified charge[1] or by the burning of certain fuels, giving by either means a lower mean temperature to the "cycle," a higher efficiency can, both theoretically and experimentally, be obtained.

The theoretical indicated thermal efficiency of the Otto cycle is given by the formula $E=1-(1/r)^{\gamma-1}$, where r is the compression-expansion ratio.

It is evident that the larger the number of expansions the greater will be the theoretical thermal efficiency, and following on this consideration every opportunity is taken in practice to raise the expansion ratio, and therefore necessarily the compression ratio, to the maximum value which the fuel will bear without detonation.

The fuels which can be employed in a petrol engine (including members of the paraffin group, the aromatics, and the alcohol group) vary very considerably in their resistance to detonation. Some data relative to this characteristic are given later.[2]

For any given fuel, however, there is a number of factors which control the maximum compression ratio which can be usefully employed. The most important of these factors is the form of the combustion space. To have the minimum tendency to detonation the combustion space must be compact, of good depth with respect to its diameter, symmetrical and free from shallow pockets or any cavities in which portions of the charge can become more or less isolated.

The location of the igniter is also of considerable importance. It has been shown[3] that when the igniter is situated on the opposite side of the combustion space to the exhaust valve, so that the portion of the charge in the vicinity of this valve is the last to burn, the tendency to detonate is increased, and the limiting useful compression ratio is consequently reduced.

Given a combustion space having the attributes set out above, it was found that the highest compression ratio giving freedom from detonation for any particular fuel was reached by employing two sparking plugs located on opposite sides of the combustion space.

If a combustion space is of such form that a portion of the charge can become in some degree isolated from the main portion, it is found that the tendency to detonation is greater than with a compact space. A reasonable theory appears to be that, since the isolated portion does not become ignited until late in the process of combustion, there is a tendency for it to be raised to a high pressure and temperature before inflammation owing to its compression by the general rise of pressure in the cylinder; consequently, when the flame finally reaches this unignited gas, the conditions are favourable for its detonation.

Apart from the increased tendency to detonation, the existence of cavities or pockets in a combustion space involves thermodynamic losses, owing to the chilling of the charge during inflammation, which may lead to incomplete combustion, and in any case must cause greater direct heat loss to the walls.

The next most important item controlling the compression ratio is the temperature of the charge after compression.

It has been clearly established that if the charging temperature is raised the tendency to detonate is increased. In order to run with freedom from detonation under such conditions, a lower compression ratio must be employed than is possible with a lower charging temperature using the same fuel.

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[1] See *Automobile Engineer*, June 1921.

[2] See *Automobile Engineer*, "The Influence of Various Fuels on the Performance of Internal Combustion Engines," by H. R. Ricardo, February to August 1921.

[3] See *Engineering*, September 3 and 10, 1920.

As being closely connected with the charging temperature, and to a large extent influencing that temperature and therefore the temperature reached after compression, it must be remembered that the charge from the time it enters the cylinder until probably well into the compression stroke is absorbing heat from the cylinder walls, piston, and valve heads.

In this connection the use of aluminium alloys for pistons has had a most important influence in enabling higher compression ratios to be employed, owing to the better heat dissipation from the centre of the piston crown due to the higher conductivity of aluminium as compared with cast iron and steel.

With regard to valve-head temperatures, it is often desirable to employ multiple exhaust valves in order to limit the size of the valve and thus ensure better cooling of the head.

An important factor in reducing the tendency to detonation is the provision and maintenance of an adequate degree of turbulence of the charge. Where a sufficient degree of turbulence is preserved there is less opportunity for any portion of the charge to remain in contact with highly heated surfaces, such as the head of an exhaust valve, for sufficient time to be raised to the critical temperature at which it will tend to detonate.

Apart from this point of view, it is of the utmost importance that a high degree of turbulence should exist in the combustion space at the moment of ignition, for the purpose of ensuring sufficiently rapid spread of inflammation to enable the combustion to be completed in the limited time available in high-speed engines.

If it were not for the existence of turbulence it would be impossible to run engines at the high speeds now common. The time of pressure rise in a stagnant mixture is many times that available for the entire cycle in a petrol engine of even moderate speed.

Turbulence has a further valuable effect in that it tends to ensure uniformity of mixture strength throughout the charge by breaking down zones of rich and weak mixture.

It is obvious that a combustion space which has the attributes already mentioned will be favourable to the maintenance of the initial turbulence arising from the entry of the charge at high velocity past the inlet valve.

§ (3) VOLUMETRIC EFFICIENCY.—With regard to volumetric efficiency, it is important to bear in mind that it is the weight and not merely the volume of the fresh charge which controls the power output from a cylinder of given swept volume.

This being the case, it is obvious that two conditions must be fulfilled to obtain a high volumetric efficiency. In the first place, every endeavour must be made to enable the cylinder to fill up as nearly as possible to atmospheric pressure, and secondly, the temperature of the charge must be kept as low as possible.

In order to fulfil the first condition it is essential that the resistance offered to the inflow of the charge shall be kept low by the use of moderate gas velocities, proper streamlining of the ports and inlet valve head, and the use of a valve-opening diagram in relation to piston velocity which shall give the greatest possible area for flow at the latter end of the charging period with a sharp cut off at closing.

The most difficult part of the first condition to fulfil in practice is the provision of a large inlet area at the end of the charging period, with a rapid cut off, and this difficulty is very greatly accentuated in engines intended to run at high speeds. A method of achieving this result without having resource to abnormal accelerations in the valve operation is the method of masking the first and last portions of the inlet valve travel devised by Ricardo. In employing this device the head of the valve is arranged to act as a piston valve when near its seat, thus giving sharply defined opening and closing points and allowing ample time to seat the valve in a comparatively leisurely manner.

With regard to the second condition, it has been found [1] that the power output steadily falls with addition of heat to the incoming charge at the rate of approximately 1 per cent for every 3° C. rise in inlet temperature. It is clearly desirable, therefore, to limit the preheating of the charge to the minimum which will ensure satisfactory distribution. The term distribution is here used in the sense of uniformity of mixture strength of the charges supplied to the cylinders of a group fed from a common carburettor.

In connection with the subject of preheating the charge before its entry into the cylinder, it must be remembered that petrol and the other inflammable volatile fluids have considerable latent heats of vaporisation. Complete evaporation of the petrol, in a mixture of such proportions as will give just complete combustion, lowers the temperature of the mixture by about 18° C. It follows, therefore, that if suitable arrangements are made to do so, it is possible to improve appreciably the volumetric efficiency by making use of the latent heat effect to cool the incoming charge, and thus to increase the weight of charge taken into the cylinder.

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[1] See *Engineering*, September 3, 1920.

Bearing the above fact in mind, it is clearly desirable from the point of view of power to restrict preheating as much as possible, and to endeavour to make use of the latent heat of the fuel to offset the unavoidable rise in temperature undergone by the charge after its entry into the cylinder and before mixing with the residual exhaust. This rise of temperature due to absorption of heat from the piston, valves, and walls alone is estimated by Ricardo to amount to about 50° C.

In taking advantage of the latent heat of the fuel to increase the volumetric efficiency, it must be remembered that any evaporation which takes place after the inlet valve has closed has no effect on the volumetric efficiency.

The use of a fuel having a high latent heat in conjunction with the minimum of preheating which will give good distribution and ensure efficient combustion, has the additional advantage of lowering the mean and maximum temperatures of the cycle, with a consequent reduction in the losses due to change of specific heat, dissociation, and direct heat loss. It has the effect also of reducing the duty thrown on the cooling arrangements, which is an important advantage in the case of air-cooled engines.

§ (4) Mechanical Efficiency.—With regard to the remaining factor controlling the brake power output from a given engine capacity, namely, mechanical efficiency, it is clear that large rubbing areas, heavy inertia pressures, and high pumping resistances are the chief causes of negative work in a petrol engine.

The greatest loss, amounting to some 50 per cent to 60 per cent of the total mechanical losses, almost invariably arises from the piston, and is chiefly caused by the shearing of large areas of partially decomposed oil film while under pressure. In order to limit the lateral thrust of the piston at high speeds it is clearly necessary to reduce as much as possible the reciprocating mass formed by the piston and small end of the connecting rod. The use of aluminium alloy pistons has enabled designers to fulfil the requirement of providing adequate heat conduction from the centre of the piston crown with a moderate piston weight.

A large source of friction in many piston designs is the use of heavy piston ring pressures. There appears to be no advantage in exceeding from seven to eight pounds per square inch pressure between the ring face and cylinder wall, and these relatively light rings give entirely satisfactory results provided the rings are adequately shielded from the maximum explosion pressure by the provision of a deep top land.

If the top land is of insufficient depth, or, being of adequate depth, there is excessive clearance between it and the cylinder wall, it is found that light piston rings will allow the gases occasionally to blow past. The explanation appears to be that piston rings depend for their gas tightness to a large extent on gas pressure behind the ring forcing the rubbing face of the ring into intimate contact with the cylinder bore. In the case of an insufficiently shielded ring the rapidly applied high pressure which occurs on ignition forces the ring hard against the lower face of the ring groove before the gases have had time to act on the back of the ring. In this condition the ring is clamped against the lower face of its groove and only exerts its inherent spring pressure against the cylinder wall, which pressure is insufficient to prevent the gases from blowing past the rubbing face of the ring.

Next to the piston loss in importance comes the loss due to filling the cylinder with fresh mixture and exhausting the products of combustion. Owing to the fact that the exhaust is released at a substantial pressure, the work thrown on the piston in clearing the cylinder is usually small as compared with the work expended by the moving system in starting the fresh charge into motion and drawing it into the cylinder.

In a well-designed cylinder having ports properly stream-lined and an inlet gas velocity of from 120 to 130 feet per sec., the pumping loss should not exceed from 3 to 3·5 lbs. per square inch pressure on the piston. The pumping losses rise rapidly with increase of gas velocity, and at 200 feet per sec. amount to about 7·5 lbs. per square inch.

The remaining mechanical losses include bearings, camshafts, and auxiliaries, such as magnetos, oil and water pumps, and, in the case of automobile engines, the radiator fan.

The bearing losses increase slowly with speed, the rate of increase being influenced by the loadings thrown on the bearings by the inertia and centrifugal forces and by the increase in oil-shearing resistance with speed.

The losses due to the water and oil pumps can with unsuitable design rise to a high figure at high speeds, but with good design the losses due to bearings, camshafts, and auxiliaries together amount to between 1·75 and 2·5 lbs. per square inch on the piston at speeds of the order of 1500 r.p.m., the former figure applying to a large engine with light reciprocating parts and the latter figure to one with relatively heavy parts.

In aero engines having propeller reduction gearing, a further loss occurs in this gearing. In a good example of epicyclic gear the loss amounts to from 1·5 per cent to 2·0 per cent.

§ (5) Fuels.—With regard to the subject of fuels and their behaviour in petrol engines,

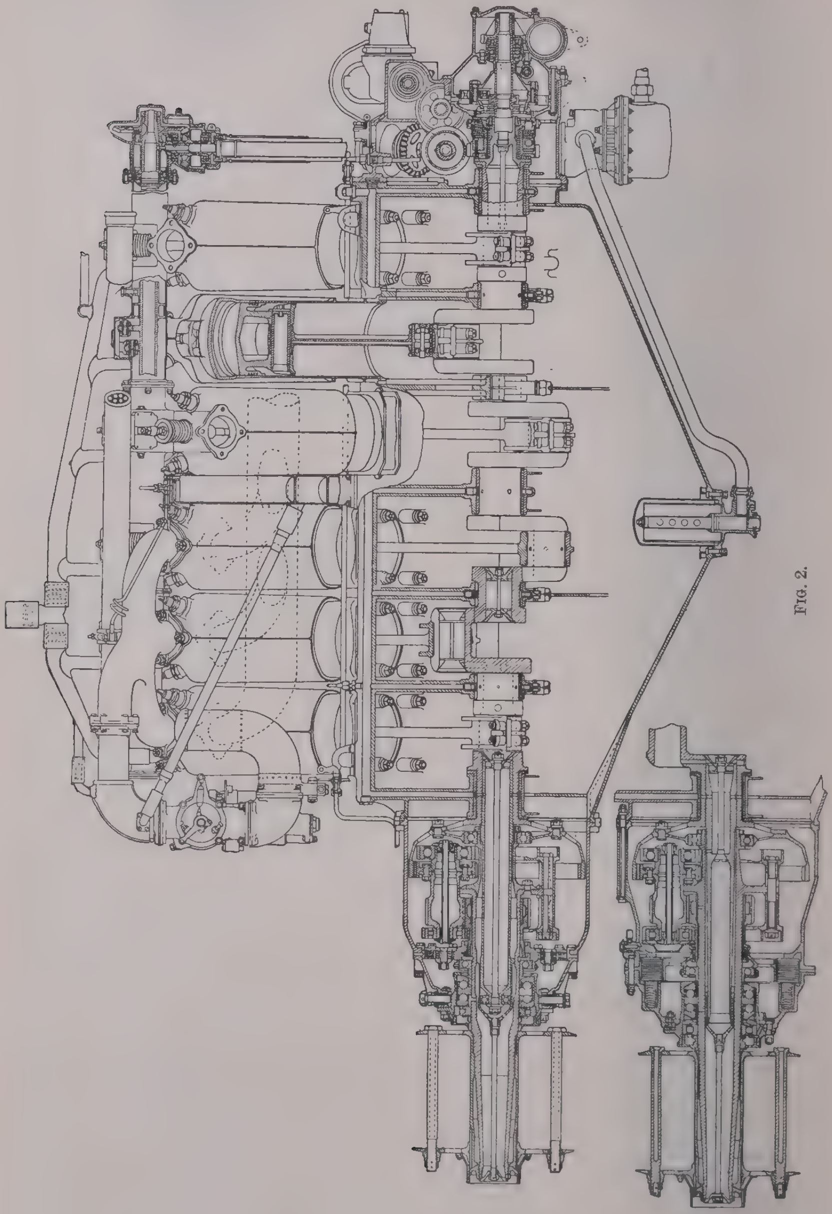

FIG. 2.

the reader may refer to the publications given in the footnote below.[1]

For the purpose of classifying them with respect to their relative resistance to detonation Ricardo has compared a large number of fuels using toluene and an aromatic-free petrol as the basis of comparison. Taking the effect of toluene as represented by 100, and that of aromatic-free petrol as being zero, the following table gives the values for the fuels mentioned :

TOLUENE VALUES

Toluene	+100	Acetone	+ 75
Benzene	+ 66	Cyclohexane . .	+ 30
Xylene	+ 83	Carbon bisulphide .	+ 10
Ethyl-alcohol, 99 per cent	+166	Methyl mercaptan	+5 to 10
		Ether	− 60

§ (6) WATER-COOLED ENGINES. (i.) *The Rolls-Royce Eagle.*—The following descriptions and illustrations will give some idea of the trend of modern practice in water-cooled petrol engine design and of the performances obtained from the engines described.

The first description is that of the Rolls-Royce "Eagle" aero engine, which has been largely used by the British air service.

The "Eagle" engine has been built in eight series, and the present description deals with the Series Eight model. All the series embody the same main features, the differences between the different series lying chiefly in such details as compression ratio, number and size of carburettors, arrangement of induction system, number of magnetos, and ratio of propeller reduction gearing.

This engine has twelve cylinders of 4·5 in. bore and 6·5 in. stroke arranged in two banks of six at an angle of 60°.

The cylinders are all independent of one another and are of the built-up type, having a barrel and head machined from a steel forging, to which a pressed-steel jacket is welded.

Upon the head of the cylinder body sockets are formed into which hoods are welded to form the inlet and exhaust passages and to carry the valve guides. One inlet and one exhaust valve is provided per cylinder.

Projections formed upon the hood serve to carry studs by which the overhead camshaft is attached to the cylinders.

Steel flanges carrying four studs are screwed on to the ends of the hoods and serve for the attachment of the inlet and exhaust manifolds.

The two sparking plug bosses are screwed and welded into the cylinder crown, and are located one on the rear side of the inlet valve and the other on the forward side of the exhaust valve.

The water-jacket is in two parts welded together by a longitudinal seam and attached by welding to the cylinder body by a flange at the lower end of the barrel and projections on the ends of the valve hoods.

A water-inlet pipe is welded to the outer side of the jacket at its lowest point and an outlet pipe at the upper end of the inner side of the jacket.

The cylinders are spigoted into the crankcase and are provided at their lower end with four deep bosses which are tied together, top and bottom, by square flanges all milled out of the solid. The cylinder holding down studs pass through these flanges and bosses, thus providing a light construction and taking the strains due to gas pressures from the cylinder at two points on the barrel.

Both the valves are of the "tulip" type and work in long phosphor bronze guides, tapered externally where they fit into the valve hoods.

Shoulders are provided on the guides to support the stationary spring collars which take a long bearing on the outside of the guide. The ends of the valve stems are provided with hardened steel plugs on which the valve rockers bear, and are provided with a screw thread by which the upper spring collar and locking nut are secured to the stem. A cotter pin passes through the nut, stem, and hardened plug.

Two springs of opposite hand are provided for each valve.

The pistons are of aluminium alloy of the "Zephyr" type and carry four pegged gas rings upon the upper skirt and one unpegged scraper ring upon the lower skirt.

The gudgeon pin is hollow and is supported directly in the aluminium piston bosses, in which it is located endways by a flange at one end and a bar carried by a stud at the other end. One end of this bar engages a recess in the piston and prevents the gudgeon pin from turning.

The connecting rods are of H section, the small ends of both rods and the anchor pin end of the articulated rod being bushed with phosphor bronze.

The articulated rod is anchored by lugs to the leading side of the master rod big end.

The big end is lined with white metal cast directly into the rod, in which three dovetailed grooves are provided to retain the metal. The big end caps are stepped into the main rod and are secured by four bolts. Thick liners are fitted between the cap and rod to facilitate the fitting of the bearing. The small end bearings are lubricated by a pipe strapped along the shank of the rod and leading oil from the big end and anchor pin ends respectively.

The crankshaft is hollow throughout, the bores being closed at the ends by means of conical plugs held in position by bolts. At the forward end a large flange is provided to carry the annular wheel of the propeller reduction gear, and at the rear end a flange is formed to which an extension is bolted. This extension carries the skew-gear wheel by which the auxiliaries are driven.

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[1] See *Proceedings of the Royal Aeronautical Society*, 1921, and *Automobile Engineer*, February, March, April, May, June, July, and August 1921.

The crankshaft is supported in seven plain bearings, comprising bronze shells lined with white metal. The shells have no flanges and are prevented from turning by thick bronze shims, and are located endways by a dowel in the cap and an oil-nipple in the top half.

The main bearings are entirely independent from the lower portion of the crankcase and are supported by steel caps held to the top half by long bolts passing up to the upper surface of the crankcase, thus relieving the aluminium webs of the top half of the case from explosion stresses.

The aluminium alloy crankcase consists of four main parts as follows: A main portion carrying the cylinders and crankshaft, an oil sump, a rear portion containing the gears which drive the camshafts and auxiliaries, and a nosepiece which contains the epicyclic reduction gear and the propeller shaft.

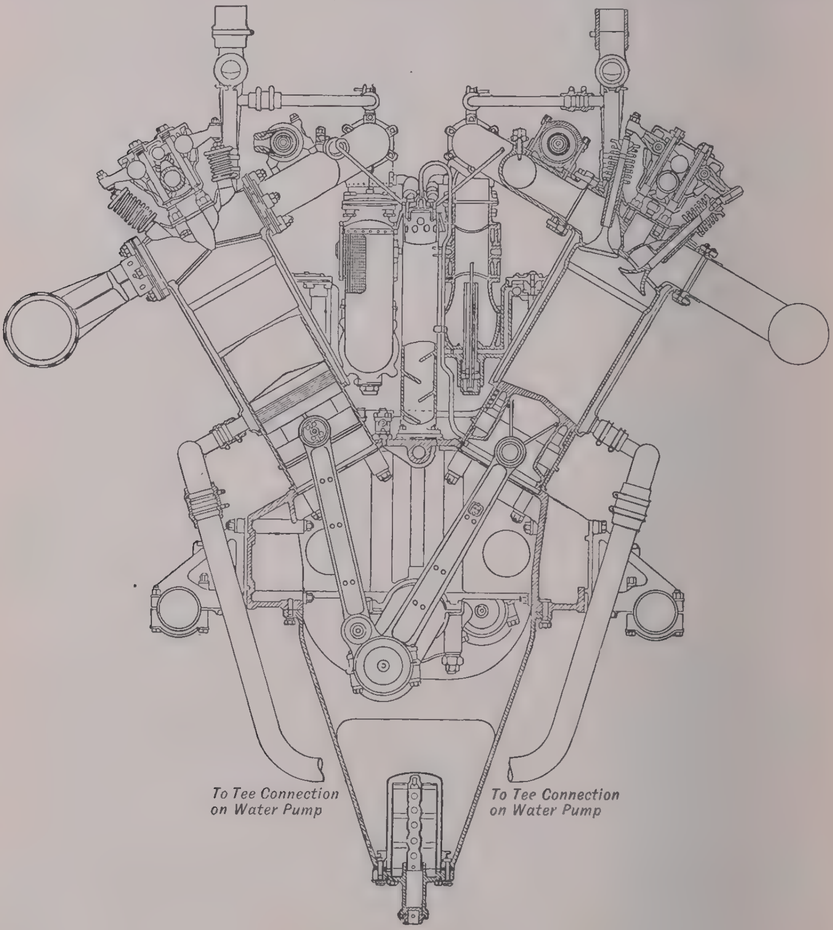
To Tee Connection on Water Pump
To Tee Connection on Water Pump

Fig. 3.

The main portion is deep and well stiffened by transverse webs in which the centre and intermediate webs are supported. The two end main bearings are carried in the end walls of the main portion.

Along the outside of the main portion a longitudinal flange is provided at the crank-shaft level to stiffen the case against horizontal forces.

The engine is supported by four detachable bearer feet, each of which is secured to the main portion of the crankcase by four bolts which are relieved from shear stress by horizontal tongues engaging grooves in the crankcase.

Each cylinder is secured to the crankcase by four studs, which are provided with midway flanges. The studs are screwed into holes in the case and are locked by nuts on the inside of the case. By this means the studs are securely fixed in the case without relying entirely on screw threads in aluminium.

The sump is secured to the main portion by numerous studs, closely pitched, in order to ensure a good union between the two parts so that the sump shall contribute to the stiffening of the main portion. The sump is braced by two transverse webs. A detachable suction filter is provided at the lowest point of the sump.

The rear portion of the crankcase is attached partly to the main portion and partly to the sump.

The main drive for the camshafts and auxiliaries comprises a skew wheel of sixteen teeth, carried by a transverse shaft, from the ends of which one pair of magnetos is driven.

The bevels driving the inclined camshaft drives have twenty teeth and are mounted on the transverse shaft by means of serrations. A spur wheel is mounted upon the hub of the right-hand bevel and engages an idle wheel, which in turn drives a spur wheel of the same size as the first. This last spur wheel is mounted on a second transverse shaft, from the ends of which the second pair of magnetos is driven. An air-pump for supplying air to the petrol tank and the engine tachometer are both driven from this second shaft.

The twenty-tooth bevels engage thirty-tooth wheels carried by short hollow shafts mounted in bearings carried by the main portion of the crankcase.

These short shafts are internally serrated at their upper ends and the serrations engage serrated sleeves attached to the lower ends of the inclined camshaft drive shafts. The upper ends of these inclined shafts are internally serrated and engage the serrated ends of short shafts on the upper ends of which twenty-five tooth bevels are mounted. These bevels engage fifty-tooth bevels secured to the camshafts by cones and keys.

An unusual feature of these engines is the use of a spring drive between the crankshaft and the whole of the auxiliaries and camshafts, in order to protect these parts from any torsional oscillations in the crankshaft.

The spring drive is incorporated between the first skew-gear wheel and the crankshaft extension, upon which the former is rotatably mounted. The oscillations of the spring drive are damped by means of a multiplate brake.

The camshafts are supported in three-part bronze casings, which are mounted on the cylinders by pairs of studs carried by the cylinder heads.

The hollow camshafts with integral cams are supported by five plain bronze bearings in the camshaft casings, the thrust of the driving bevels being taken by ball-thrust bearings.

The valves are operated by rockers carried by the camshaft casing and are provided with rollers to engage the cams.

The valve clearance is adjusted by placing shims between the rocker and the head of the screw which contacts with the valve stem.

The valve timing is as follows: Inlet opens 10° after top centre and closes 54° after bottom centre; exhaust opens 54° before bottom centre and closes 10° after top centre.

Mixture is supplied to the cylinders in groups of three by four Rolls-Royce Claudel Hobson carburettors placed in pairs at the ends of the V formed by the cylinders. The bend of the induction pipe connecting the carburettor to its manifold is water-jacketed.

In order to facilitate starting, the manifolds are provided with nozzles to enable petrol to be sprayed into the induction system by means of a hand-pump.

The carburettors are fitted with Messrs. Rolls-Royce's altitude control, consisting of a fluted needle valve situated between the float chamber and the jet well.

The cylinder cooling water is circulated by a centrifugal pump, which is driven at one-and-half times engine speed by a shaft and a skew-gear wheel, which engages the skew wheel mounted on the crankshaft and operated by the spring drive.

The engine is lubricated under a pressure of from 35 to 45 lbs. per square inch by means of a gear-type pump driven at reduced speed by means of an epicyclic gear.

This gear is driven by a shaft carrying a skew-gear wheel meshing with the crankshaft gear wheel.

Above the pressure pump and coaxial with it a second gear-type pump, of larger capacity than the pressure pump, is provided to scavenge the sump and return the oil to the reserve oil tank.

Two relief valves are provided and connected in series, so that a pressure of about 8 lbs. per square inch is maintained between the valves. This low pressure supply is employed to lubricate the camshafts, timing gears, and propeller reduction gears.

The propeller reduction gear is of the compound epicyclic type, in which an annulus gear, mounted on the forward crankshaft flange, engages planets carried by the propeller shaft, these planets in turn engaging a stationary sun-wheel secured to the nosepiece.

There are three pairs of planets, each pair comprising a wheel engaging the internally toothed annulus and a pinion engaging the sun.

In its original form the sun-wheel was anchored by means of an Oldham coupling, but in order to relieve the gear from any abnormal torque variations Messrs. Rolls-Royce have recently employed a friction anchorage for the sun-wheel, so that this member may rotate slightly when the torque exceeds a certain value. The friction anchorage is provided by means of a multiplate clutch (see power view, *Fig.* 2).

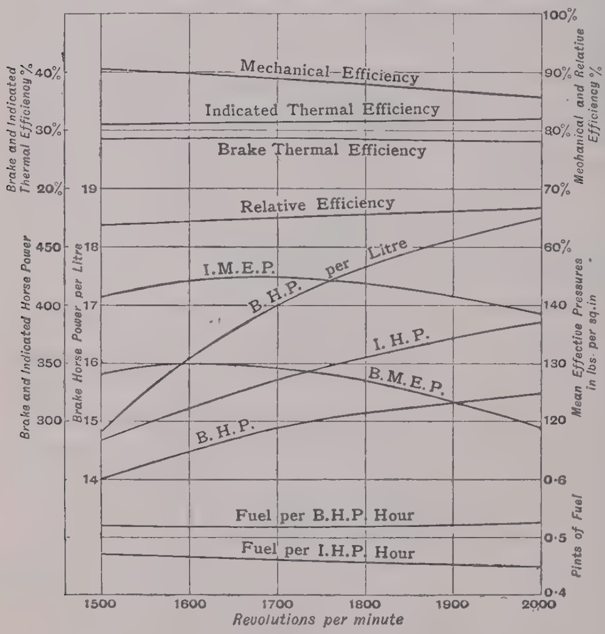

FIG. 4.

A noticeable feature throughout these engines is the use of relatively fine serrations in place of keys, and as spanner holds in place of hexagons and squares.

The graphs (*Fig.* 4) give the brake performance based on the average of several hundred engines of this type.

The indicated performance has been obtained by calculating the mechanical losses, by the method employed by Ricardo,[1] and adding these losses to the brake mean pressure, thus enabling the mechanical efficiency to be arrived at. The mechanical efficiency of the propeller reduction gear has been taken as 99 per cent at 1600 r.p.m., 98·5 per cent at 1800 r.p.m., and 98 per cent at 2000 r.p.m.

(ii.) *The Vauxhall Engine.*—The following description refers to the 30-98 h.p. Vauxhall engine as fitted in the fast touring car chassis produced by Vauxhall Motors Ltd., of Luton.

This engine has been developed to meet the requirements of fast travel on the road, and although capable of propelling a suitable racing chassis at speeds exceeding 100 miles per hour on the track it is in no sense a freak design.

The engine has four cylinders of 98 mm. bore and 150 mm. stroke, giving a piston displacement of 4·5 litres.

The cylinders are of cast iron and form one monoblock casting with all the valves on one side of the engine.

The inlet and exhaust valves are located in a side pocket forming an extension of the combustion chamber, the axes of the valve stems being inclined to the cylinder axis at an angle in order to allow sufficient room for the valve springs, while keeping the combustion chamber as compact as possible with this type of head.

The valves are both of 2 in. diameter in the throat. The material used for the inlet valve is 3 per cent nickel steel case-hardened all over, and for the exhaust valve 14 per cent chromium steel. Over each valve a bronze cap is provided. Valve guides of bronze are employed, these guides having suitable seatings formed on them to carry the upper end of the valve springs. The lower end of the valve spring is held by a light steel cupped carrier which is attached to the valve stem by means of a pair of externally coned cotters, which engage in a recess turned on the valve stem.

In the crown of each cylinder a bronze fitting is provided to carry a sparking plug and to close the opening in the water jacket through which the jacket core sand is removed. This fitting is screwed into the cylinder crown and passes through a large recessed washer which rests on a facing on the jacket and serves as a gland box to contain packing compressed by a gland and nuts carried by the fitting. By the use of this construction a water-tight joint is ensured without straining the walls of the cylinder head.

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[1] See *Automobile Engineer*, July 1916.

On the side of the cylinder block remote from the valves a longitudinal passage is cast which communicates with two passages passing within the water jacket between cylinders 1 and 2, and 3 and 4 respectively to convey mixture to the inlet valves on the other side of the block.

The carburettor employed is an aero type 48 R.A. Zenith and is attached through a water-heated elbow to a facing on the side of the longitudinal passage referred to. An external pipe leads hot water from the top of the cylinder block to the upper side of the elbow.

The pistons used in this engine are Ricardo aluminium alloy slipper pistons[1] having two rings and the gudgeon pin free to rotate in the bosses as well as in the small end of the connecting rod. Slipper surfaces of unequal area are used, the larger surfaces being on the side subjected to thrust during the working stroke.

Considerable advantages are gained by the use of this type of piston, both as regards mechanical efficiency and oil consumption. The reciprocating mass can be kept very small owing to the elimination of much metal and the favourable disposition of that retained. The gudgeon pin is well supported close up to the sides of the small end of the connecting rod, thus removing all danger of bending, with consequent local overloading of the bearing surfaces. Owing to its short length and good support the gudgeon pin can be made considerably lighter than in the conventional trunk type of piston. In addition to small mass a further gain as regards mechanical losses results from the reduction of oil shearing surfaces to the minimum required to deal with connecting rod thrust. There appears to be

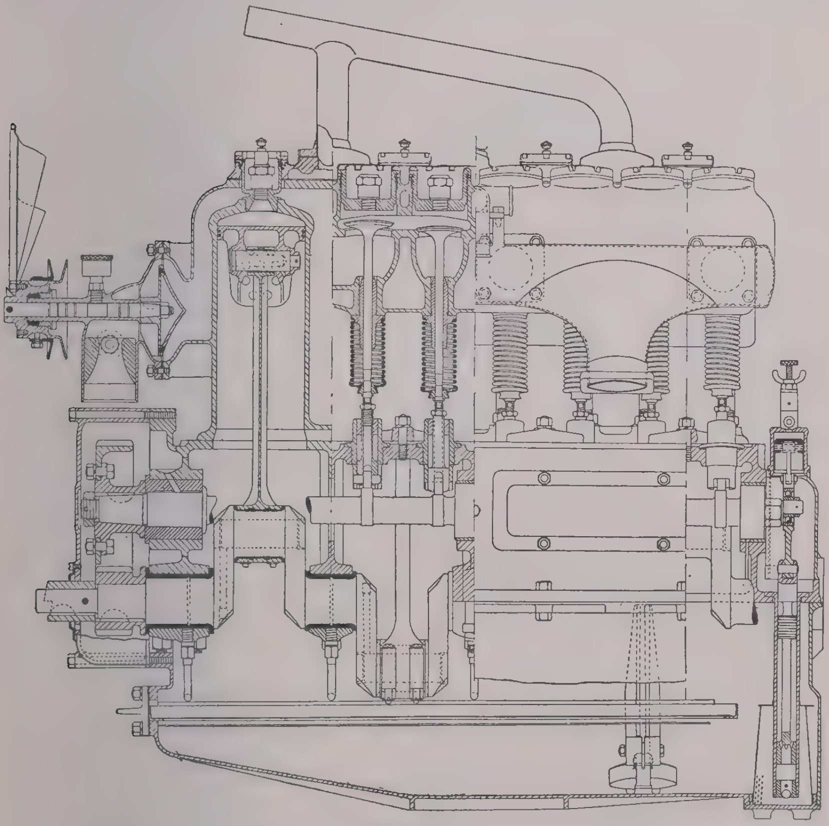

FIG. 5.

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[1] For a full account of this type of piston, together with test results, see *Automobile Engineer*, March 1917 and October 1918.

no doubt, as the result of extensive experiments, that the shearing of oil films between piston and cylinder is responsible for a large part of the total piston friction loss. It must be borne in mind that the oil in question is largely contaminated by carbon and gummy products resulting from the burning of the oil film during the working stroke.

The connecting rods are H section 3 per cent nickel steel stampings machined all over. The small end is bushed with phosphor bronze and the big end is lined with white metal cast direct into the rod. This white metal is retained by two circumferential dovetailed grooves machined in the rod and cap. By casting the white metal into the rod three advantages are gained: in the first place, better thermal contact and consequent heat dissipation are obtained, owing to the absence of the oil film which unavoidably forms between a bronze shell and the rod; secondly, the weight of the shell is saved; and thirdly, the bulk, and therefore the weight of the big end, is reduced. The saving of weight in the big end is of the utmost importance in a high-speed engine, owing to the very serious load imposed on this bearing by centrifugal force due to the rotating mass on the crankpin.

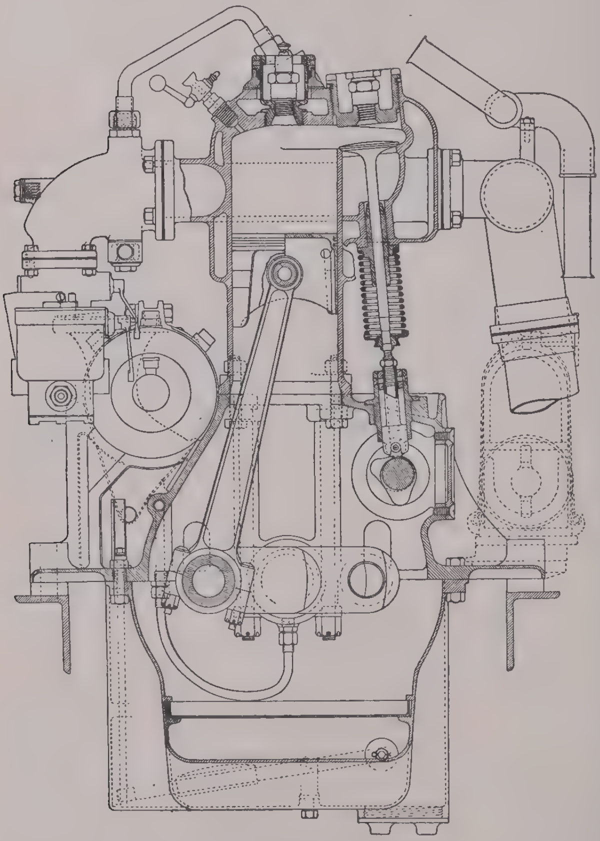


Fig. 6.

At high speeds the centrifugal load is the largest component of the big-end load, and as it is always acting in one direction, this bearing at high speeds obtains no relief to facilitate the ingress of lubricant.

Four bolts are provided to hold the big-end cap to the rod.

The crankshaft is supported in five die cast white-metal bearings supported directly in the crankcase without the use of bronze shells. The crankpins and main journals are hollow, the ends of the bores being filled by caps driven and soldered in position. Holes are drilled in the forward web of each crank throw to enable oil to pass from the hollow journals to the crankpins.

At the forward end of the crankshaft a silent chain pinion is keyed, and beyond the pinion a sleeve carrying claws to engage the starting handle is pinned to the shaft. The flywheel is bolted to a flange formed on the rear end of the crankshaft, and immediately behind this flange an oil thrower is turned upon the shaft.

As will be seen from the sectional drawings, the bearing surface provided for the crankpins and main bearings is generous.

The caps of the main bearings are supported independently of the sump by means of long bolts which pass up to the upper face of the top half of the crankcase, the heads of these bolts being sunk in recesses to leave a clear surface for the cylinder feet. The use of through bolts in this manner is a very desirable practice as it transmits the bearing loads directly to the cylinder feet and avoids threads in aluminium, which are necessary when studs are employed.

The crankcase is of aluminium alloy and is split on the crankshaft axis. The upper half is provided with longitudinal flanges adapted to support the engine and is swelled out on one side to enclose the camshaft. The bolt centre to centre distance between the bearers is 17·75 in., and the depth of the top half of the crankcase from the crankshaft axis to the cylinder feet is 7·0 in. The crankcase as a whole is particularly rugged, and the top half is additionally stiffened by the monoblock cylinder unit.

The sump serves as an oil reservoir and is detachable without disturbing the crankshaft or any of the working parts. A filtering tray is provided in the sump to filter all oil which drains from the cylinders and moving parts, and this tray slides into position and can be withdrawn for cleaning by removing a cover plate at the front of the sump. At the rear end of the sump a large plug is provided to enable a second filter, surrounding the oil pump suction, to be removed for cleaning.

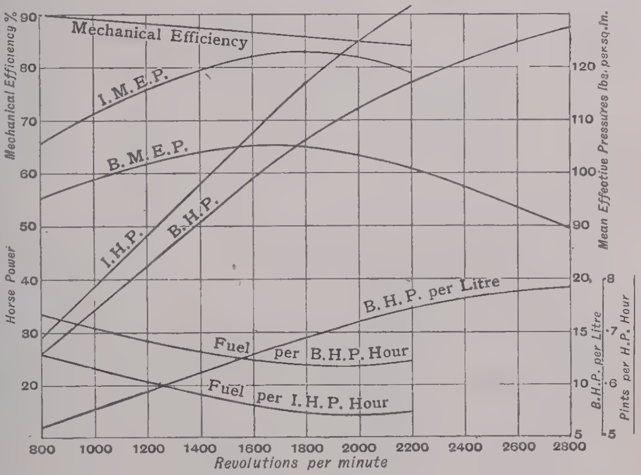


FIG. 7.

The camshaft is mounted in three plain bearings and is driven by a silent chain, the chain wheel being bolted to a centre piece which is mounted on the camshaft by a cone and key. The cams are integral with the camshaft and are of the eccentric base circle type. The cams operate directly on rollers carried by the valve tappets which work in bronze guides held in position in the top half of the crankcase by dogs, each of which holds down two guides.

The clearance between the tappet and valve stem is adjusted by a set screw and lock nut carried by the upper end of the tappet. At the rear end of the camshaft an overhung crankpin is provided to operate the plunger oil pump and the air pump for placing the fuel tank under pressure.

The oil pump consists of a long cylinder carried by the top half of the crankcase and projecting down to near the bottom of the sump. At the lower end of this cylinder a ball suction valve is provided. The plunger has two enlarged portions adapted to form pistons and provided with oil-retaining grooves. The lower of these enlarged pistons is hollow, and has four inclined holes communicating with the hollow and with the annular space contained between the two pistons.

On the down stroke of the pump oil is forced from the underside of the lower piston to the annular space, from which it passes by an external pipe to the rear end of an oil-distribution pipe cast into the side of the top half of the crankcase.

On the up stroke of the plunger, oil flows into the pump cylinder through the ball valve and fills the space below the lower of the piston portions. A certain amount of oil flows back through the inclined holes, but owing to the greater resistance offered by these holes as compared with the resistance offered by the ball valve, a considerable amount of fresh oil is drawn in on each up stroke.

A float carried on one end of a counterpoised rocking lever is provided in the bottom of the sump to operate a rod projecting upwards through the engine-supporting flange to indicate the oil level.

The camshaft and magneto are driven by means of a silent chain, the tension of which is adjusted by arranging the bearings of the spindle of the magneto chain wheel in eccentric housings. The magneto is driven by a laminated spring coupling and is mounted on a platform in such a manner that it can be displaced sideways to accommodate the movements of the spindle when the chain is adjusted. A pulley for a V belt to drive the radiator fan is mounted on the end of the chain wheel spindle.

The carburettor is a 48 R.A. Zenith aero type.

The curves shown in *Fig.* 7 give the brake performance as obtained by Messrs. Vauxhall using a Froud water dynometer. The indicated performance and mechanical efficiency have been arrived at by calculation by the method already referred to.

(iii.) *The Ricardo* 150 *h.p. Tank Engine.*—The two engines already described represent examples of the light type of relatively high speed petrol engine in which advantage is taken of special materials and a high rotational speed for the development of large power outputs for a low weight. As contrasted with this type there is the relatively lower speed engine designed to develop a large torque at low speeds and to run at or near its full load continuously for long periods.

As illustrating this latter type, the writer has selected the Ricardo 150 h.p. Tank Engine, of which the following is a description (see *Figs.* 8 and 9).

In considering this engine it must be borne in mind that its design was in many respects controlled by limitations imposed by the fact that it was required to be interchangeable with the 105 h.p. Daimler engine, previously used in the Tank service, both as regards length at the crankshaft centre line and overall width. The depth below the crankshaft was also limited.

It was also required that a minimum use should be made of special steels and aluminium, so that mild steel had to be employed for such parts as crankshafts, and aluminium only used for pistons and induction pipes.

The bore and stroke are 5·625 in. and 7·5 in. respectively, and at the normal speed of 1200 r.p.m. the engine develops 165 b.h.p.

The six separate cylinders are of cast iron with large openings at the sides, which are covered with steel panels screwed on. This form of cylinder facilitates foundry work and allows the cylinder centres to be brought close together, thus shortening the engine and reducing the bending moment due to the couples arising from each group of three cylinders.

Single inlet and exhaust valves are provided in each cylinder, the inlet valve being arranged above the exhaust and operated by a rocker and push rod. The inlet valve seating is formed in a detachable housing so that the exhaust valve can be changed or ground in by removing the inlet valve and its housing.

As will be observed from the illustrations, very great care has been taken to ensure ample cooling of the exhaust valve by carrying the water space all round the seating. The stem is cooled by extending the water space as close as possible up to the valve head and by the use of a bronze guide which assists in conducting away heat.

The valves are made of 3 per cent nickel steel case-hardened all over. The case-hardening provides a highly carburised surface which resists pitting and enables a phosphor-bronze guide to be employed without risk of seizing.

The inlet valve is "masked," that is to say, it operates as a piston valve during the first and last portions of its travel by closing into a recess formed round the valve seat. The fit of the valve head into this recess is sufficiently close to prevent any substantial flow of gases past the valve when the pressure difference on either side of the head is small.

The advantage of "masking" is that it enables the period between the valve leaving its seat and returning thereto to be substantially lengthened, while employing a normal valve timing as referred to the points of leaving and entering the mask. The lengthening of the period during which the valve is off its seat enables a very rapid cut-off to be obtained while using low accelerations, as the valve can be slowly dropped on to its seat after it has entered its mask.

As a result of this arrangement the valve can be held very nearly wide open until the out dead centre is reached, and the gases can then be rapidly cut off by lowering the valve into its mask. Consideration of the valve-opening diagram in relation to piston velocity will show that the gases have excellent facilities for filling the cylinder at the end of the suction stroke.

The pistons employed on these engines are of a type developed by Mr. Ricardo and known as cross-head pistons, from the fact that the side thrust due to the connecting rod obliquity is taken on a cross-head and guide, thus relieving the cylinder walls from all side loads.

The piston is of aluminium alloy and consists of a head carrying the rings and provided with a tubular extension on its underside. At the lower end of this extension bosses are formed to carry the gudgeon-pin bushes, and suitable flanges are provided to locate a sleeve which forms the guiding member of the piston.

This sleeve is made of cast iron and is secured to the lower piston flange by T-headed bolts. The sleeve works in a guide lined with anti-friction metal.

The advantages of this form of piston are that it gives a high mechanical efficiency and enables large diameter aluminium pistons to be employed with complete absence of piston slap.

The reasons underlying the increase in mechanical efficiency due to the use of these cross-head pistons are as follows. In practically all internal combustion engines the largest mechanical loss is that arising from the friction of the piston on the cylinder wall. In spite of the fact that the bearing pressure per unit area and the rubbing speed in the case of a piston are in general lower than the pressure and speeds found in ordinary bearings,

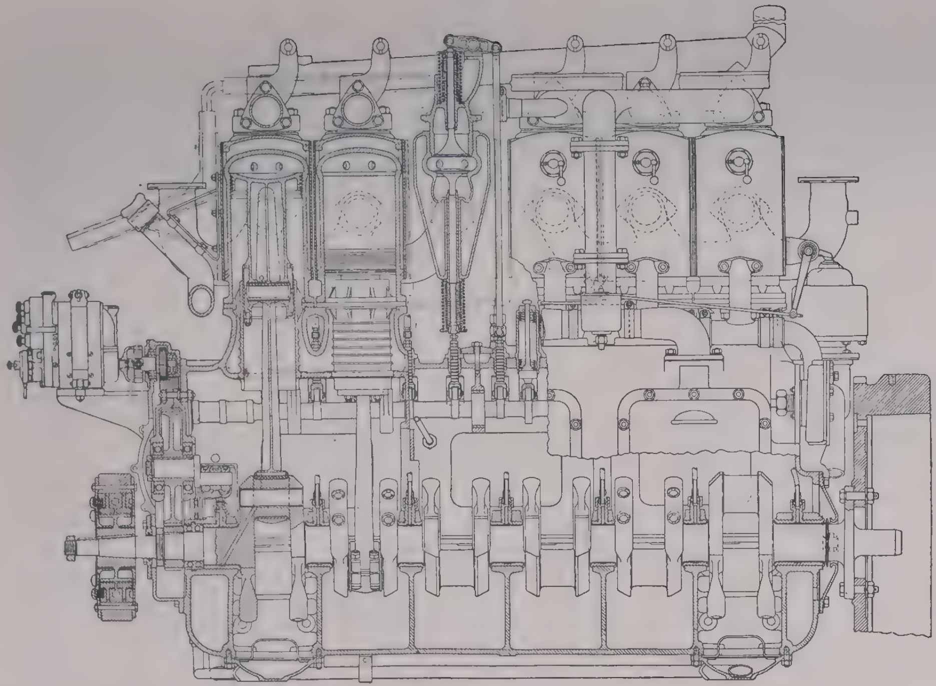


FIG. 8.

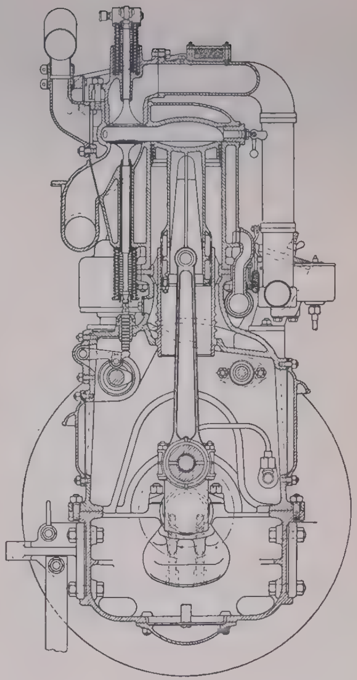


FIG. 9.

the friction arising from pistons is greater than in bearings. It appears probable that this is due to the fact that in the case of pistons as ordinarily arranged the rubbing surfaces are lubricated with oil fouled by particles of carbon and gummy residues.

In the cross-head piston the whole of the side thrust due to the obliquity of the connecting rod is taken by the cross-head guide, which is lubricated with clean, unfouled oil, and the head of the piston, carrying the rings, is relieved of all contact with the cylinder wall. Further, the reciprocating weight with the cross-head construction in aluminium is very considerably less than with a cast-iron trunk piston, and is comparable with the weight of a light aluminium trunk piston of the same size.

The lubrication of the cross-head guide can be copious without any risk of excess of oil being passed to the bore of the cylinder. The lubrication of the rings is effected by providing holes in the cross-head sleeve, which overrun the upper edge of the guide when the piston is near the top of its stroke. These holes communicate with the interior of the piston and allow oil to be drawn out and sprayed on to the cylinder wall by the partial vacuum caused by the upward movement of the piston. Any fouled oil draining from the cylinder bore can be led away and prevented from contaminating the oil in the crankcase.

The pistons are cooled by contact of the rings on the cylinder walls and by the circulation of air which is drawn into the chamber surrounding the cross-head guides. This air passing up under the pistons cools the piston heads, and the heat picked up by the air from the heads and guides warms the air before it passes to the carburettors which draw from the cross-head chamber. The heat thus picked up by the air is found to be just sufficient to supply the latent heat of evaporation of that portion of the fuel vaporised in the induction system on full load, and the extra heat picked up on reduced loads effectually prevents condensation of fuel in the induction system. The circulation of air over the top of the crankcase prevents undue heating of this part by conduction and by radiation from the piston crowns.

It will be observed that the gudgeon pin is of unusually short length, and it might be supposed that wear would in consequence be excessive. In practice it has been found that these pins are remarkably free from wear, for the following reasons. The ends of the pin are supported in bosses which are situated relatively closely together, so that the tendency of the pin to bend under load is small, and consequently one of the principal causes of trouble with gudgeon pin bearings is removed. Further, the pin is free to rotate both in the piston bosses and in the connecting rod small end, so that the rubbing speed is halved and local wear avoided.

The connecting rods are of mild steel 16 in. centre to centre length, giving a ratio of connecting rod length to crank throw of 4·26 : 1. The small end bush of bronze is secured in the rod by swaging a portion of the bush into a slot formed in the top of the rod. The big end bearing consists of a white metalled shell which is located in the connecting rod by means of a pin passing through the shank of the rod parallel with the crankpin axis, and engaging in slots in the flanges of the bearing shell. The big end cap is secured by four bolts.

The crankcase is of cast iron split on the crankshaft axis. The seven crankshaft bearings are white metalled shells supported in the lower half of the crankcase independently of the top half of the crankcase. The bearing caps are mild steel stampings to which the top half of the bearing shells are secured by means of a stem formed on the shell, passing through a hole in the cap. These stems are drilled up and are connected to the oil-distribution system, so that they serve the purposes of a bearing location and a connection for the introduction of oil.

The object of supporting the main bearings in the lower half was to enable the crankshaft, the main bearings, or the top half to be removed without disturbing the base which carries the engine feet. Any of the main bearings can be removed and replaced without disturbing the crankshaft, by removing the cap and top half of the bearing and rotating the lower shell round the shaft.

The crankshaft is a mild steel forging with solid journals and hollow crankpins. The outside diameter of the journals and crankpin is 2·875 in., and the bore of the pins 1·4375 in. The lengths of the journals are: flywheel end, 4·0 in.; forward end and centre bearing, 2·875 in.; intermediates, 2·125 in.

The flywheel is an iron casting 26 in. in diameter, bolted to a flange formed on the crankshaft. On the forward end of the shaft a Lanchester vibration damper is mounted.

Owing to the limitation imposed on the overall length of the engine, the length of the journal and crankpin bearings had to be restricted, and it was decided to favour the big end bearings and to rely on the use of balance weights to reduce the loading on the main bearings, should it be found necessary to reduce the load factor on these bearings owing to the poor wearing qualities of the mild steel crankshaft which had to be used.

The lubrication is arranged on the dry base system. Three oscillating valveless plunger pumps are provided at the gear end of the engine. The plungers are all operated by a crankpin formed on the end of the spindle carrying the idle wheel connecting the crankshaft pinion and camshaft wheel. One of the pumps supplies oil under pressure to the distribution system, and the other two pumps return the used oil to the external oil-tank. An oil sump is provided at each end of the base chamber. Each of the scavenger pumps is connected to one of the sumps, so that the crank chamber is kept free from accumulated oil even when the engine is tilted through large angles.

The distribution system consists of a pipe running the length of the engine, inside the crank chamber, with branch pipes leading to each main bearing. A relief valve is provided at the flywheel end of the main pipe, and a branch is led from an intermediate point in this pipe to an oil-pressure gauge. Oil passes from the main bearings to the crankpins by way of inclined holes drilled through the crank webs to connect the interior of the pins with a hole in the adjacent journal. The exit holes in the crankpins are drilled at the side of the pins at the point of minimum loading.

Owing to the restriction imposed on width, all the auxiliaries were grouped at the ends of the engine.

The auxiliaries consist of two magnetos, two governors, three oil-pumps, water-circulating pump, and an air-pressure pump for putting pressure on the petrol tank. One of the governors controls the maximum engine speed, and the other was intended to open the throttle when the speed dropped below 400 r.p.m. This latter governor was subsequently found to be unnecessary.

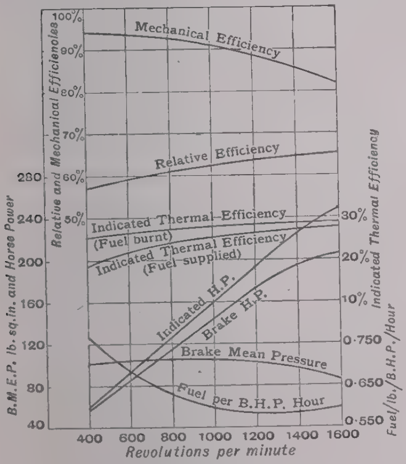
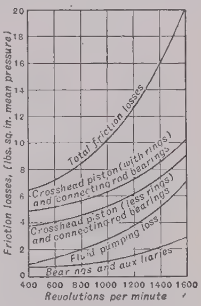


FIG. 10.

An interesting feature in connection with the valve driving gear is the method of providing for adjustment of mesh of the idle wheel, by mounting it in a spider bolted to the end of the crankcase so that its position may be adjusted.

Mixture is supplied to the cylinders by two vertical 55-mm. Zenith carburettors, which normally draw the whole of their air supply from the crosshead chamber. A hand-adjusted cold air supply is provided for use in very hot weather.



Mechanical loss curves

FIG. 11.

The performance of these engines is given in the accompanying graphs, which are self-explanatory. It may be pointed out, however, that as all the engines made were tested on swinging field dynamometers, the various frictional and pumping losses were easily ascertained under actual running conditions as to temperatures by motoring the engine with fuel and cooling water cut off. The motoring method of ascertaining the gross mechanical losses was checked by Morse's method of cutting out one cylinder at a time while running under full load, and noting the drop in torque equivalent to the indicated power of the cylinder cut out.

The following mean results were obtained in the last eight hours of a ten-hour test made on a standard engine to establish a heat balance sheet:

B.H.P.	162·9
Fuel (lb. per b.h.p. hour)	0·554
Brake thermal efficiency	24·7 p.c.
I.H.P.	187·0
Indicated thermal efficiency	28·4 p.c.
Indicated efficiency relative to the air standard	64·0 p.c.
Heat loss to jackets (B.Th.U.'s per hour)	418,000
Heat to indicated work.	28·4 p.c.
Heat to cooling water	24·9 p.c.
Heat to exhaust, radiation, etc. . .	46·7 p.c.

The fuel used was Shell spirit, sp.g. 0·725, lower heating value 18,600 B.Th.U.'s per lb. The air standard efficiency for the compression ratio of 4·34 : 1 is 44·4 per cent, and the mechanical efficiency 87 per cent.

The following are some of the principal data of these engines:

Number and arrangement of cylinders	Six, vertical, separate.
Bore	5·625 in.
Stroke	7·500 in.
Compression ratio	4·34 : 1
Normal b.h.p. and speed . . .	165 b.h.p. at 1200 r.p.m.
Brake mean pressure . . .	97·3 lbs. sq. in.

Mechanical efficiency	88 per cent.
Indicated mean pressure	110·6 lbs. sq. in.
Fuel consumption (0·730 s.g. spirit)	0·636 pint b.h.p. hr.
Brake thermal efficiency	23·6 per cent.
Indicated thermal efficiency	26·8 per cent.
Air standard efficiency	44·4 per cent.
Relative efficiency (fuel supplied)	60·4 per cent.

Gas Velocities (ft. per sec.) at 1200 r.p.m.

Choke tube	353·0
Carburettor body	168·0
Vertical induction pipe	165·2
Induction manifold	156·3
Inlet port	109·6
Inlet valve (period above mask)	168·3
,, ,, (total period)	130·8
Exhaust valve	148·25
Exhaust port	140·3
Exhaust branch pipes	140·3
Exhaust manifold	126·6

Weight of piston complete with rings and gudgeon	7·25 lbs.
Total reciprocating weight per cylinder	10·82 lbs.
Weight per sq. in. piston area	0·435 lb.
Mean inertia pressure at normal speed	33·3 lbs.
Weight of rotating mass of connecting rod	7·13 lbs.
Total centrifugal pressure at normal speed	1094 lbs.
Centrifugal pressure per sq. in. piston area	44·4 lbs. sq. in.
Mean average fluid pressure, including compression	43·0 lbs. sq. in.
Total loading from all sources per sq. in. piston	109·5 lbs. sq. in
Diameter of crankpin	2·875 in.
Rubbing velocity of big end (normal speed)	15·04 ft. sec.
Width of big end bearing (effective)	2·25 in.
Projection area of big end bearing (effective)	6·47 sq. in.
Ratio piston area/projected big end area	3·84 : 1
Mean average pressure on big end	421 lbs. sq. in.
Load factor on big end bearing	6330 lbs. ft. sec.

A. T. E.

PHASE (in Thermodynamics). See "Thermodynamics," §§ (28), (52); "Phase Rule," § (1).

PHASE RULE

I. INTRODUCTION

§ (1) INTRODUCTORY.—Liquid water may be in equilibrium with its saturated vapour throughout a wide range of temperature and pressure; but if one of these variables, let us say temperature, be fixed, there is only one pressure at which permanent equilibrium is possible. At 0° C. this pressure is 4·6 mm. of mercury, while at the standard boiling-point, 100° C., the equilibrium pressure is 760 mm. Again, at a certain temperature, a fraction of a degree above the normal freezing-point, ice, water, and vapour can coexist in stable equilibrium under the pressure of the vapour at that temperature. But, in this case, no variation is possible: at the triple point, with ice, water, and vapour in equilibrium, the temperature must be +0°·0068 C., and the pressure 4·6 mm. of mercury. If either temperature or pressure depart from these values, the equilibrium will be disturbed, and either ice, water, or vapour disappear.

In these systems we have but one *component*—water substance, the chemical compound of composition H_2O. There are three possible *phases*—solid ice, liquid water, and aqueous vapour. The thermodynamic investigation of the phenomena of equilibrium between the phases in systems of one or more components led Willard Gibbs[1] to formulate a simple law to which is given the name of the *Phase Rule.*

§ (2) DEFINITIONS.—In more complex systems, we may have equilibria such as that between calcium carbonate on the one hand and lime with carbon dioxide on the other. In this reaction

$$CaCO_3 \rightleftarrows CaO + CO_2.$$

When the velocity of change in opposite directions is equal, we get equilibrium analogous to that between water and its saturated vapour. The amounts of lime and carbon dioxide on the right-hand side of the equation are independent of each other. We can bring more lime into the system from outside without changing the total quantity of carbon dioxide free and combined. Lime and carbon dioxide are independent components of the system. But bringing in either lime or carbon dioxide or both of them will affect the equilibrium, and a change in the amount of calcium carbonate will be produced. Hence the quantity of calcium carbonate depends on the amounts of lime and carbon dioxide. Calcium carbonate is merely a phase in equilibrium with other phases. It is not an independent component of the system.

We are now in a position to appreciate the meaning of the following definitions:

A Phase is a mass chemically and physically homogeneous, or a mass of uniform concentration.

The Components of a phase or system are the substances contained in it which are of independently variable concentration.

II. THERMODYNAMIC THEORY OF THE PHASE RULE

§ (3) THE EQUILIBRIUM OF ISOTHERMAL SYSTEMS.—The Phase Rule can be deduced from the two classical laws of thermodynamics, which are themselves the expression of accumulated experience in the observed phenomena of the mutual transformations of heat and mechanical work.

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[1] *Trans. Connect. Acad.*, 1875–78, ii. and iii.; or reprint of Willard Gibbs' papers.

If heat and work are passed into a system, the first law or principle of conservation of energy tells us that the gain, $\delta\epsilon$, in the energy of the system is equal to the sum of the heat, δH, passed in (expressed in mechanical units), and the work, δW, done on the system. If the only work involved is that done against a uniform hydrostatic pressure, it may be written as $p\delta v$, but, if other forms of work are done, we may express them all in generalised co-ordinates as $\Sigma(\mathrm{X}\delta x)$, where X denotes an intensity factor, such as pressure or electromotive force, and δx the change in a quantity factor such as volume or quantity of electricity.

Hence the increase in energy of a system which takes in heat and work may be written as

$$\delta\epsilon=\delta\mathrm{H}+\Sigma(\mathrm{X}\delta x). \quad . \quad . \quad (\text{i.})$$

The second law of thermodynamics leads to the concept of entropy, which is such that the change $\delta\phi$ in entropy when heat passes isothermally and reversibly into a system is $\delta\mathrm{H}/\theta$, where θ is the temperature, measured on the absolute scale, at which the change occurs. Hence we get the relation $\delta\mathrm{H}=\theta\delta\phi$ for a reversible change.

If the change be non-reversible (*e.g.* if friction or conduction of heat through finite temperature ranges occur) the efficiency of the process is less, and the increase in entropy is greater. Thus for non-reversible processes $\delta\mathrm{H}<\theta\delta\phi$.

Now let us substitute these relations in equation (i.). We get for reversible operations

$$\delta\epsilon=\theta\delta\phi+\Sigma(\mathrm{X}\delta x), \quad . \quad . \quad (\text{ii.})$$

and for non-reversible operations

$$\delta\epsilon<\theta\delta\phi+\Sigma(\mathrm{X}\delta x). \quad . \quad . \quad (\text{iii.})$$

If we subtract from each side

$$\delta(\theta\phi)=\theta\delta\phi+\phi\delta\theta,$$

we obtain the equation and the inequality

$$\delta(\epsilon-\theta\phi)\overset{=}{<}-\phi\delta\theta+\Sigma(\mathrm{X}\delta x). \quad . \quad (\text{iv.})$$

Let us suppose that we keep the temperature constant, that is, that we deal with an isothermal system. Since there is no change in θ, $\delta\theta$ is zero, and we get

$$\delta\psi\overset{=}{<}\Sigma(\mathrm{X}\delta x), \quad . \quad . \quad . \quad (\text{v.})$$

where ψ is written for $\epsilon-\theta\phi$, a quantity which will often recur in our investigation.

We can now see the physical significance of this quantity $\delta\psi$. It denotes the total amount of work, of whatever kind, which is put into a system during a small reversible isothermal change in the system, and $-\delta\psi$, the decrease in the function ψ, is the work which can be obtained from the system during such a change. Hence ψ or its decrease is known as the free energy or the available energy, and is often written as A in chemical treatises.

If we restrict ourselves to operations carried out at constant volume, or, in the general case, where any kind of work is done, to operations where there is no change in x, δx vanishes, and we get

$$\delta\psi\overset{=}{<}0, \quad . \quad . \quad . \quad . \quad (\text{vi.})$$

for reversible and non-reversible operations respectively.

Now all real processes are more or less non-reversible. Reversibility, in the thermodynamic sense of the word, is a condition in which an infinitesimal change in one of the co-ordinates of the system, such as temperature or pressure, suffices to reverse the direction of change in the system. It is an ideal state, which we may approach more or less nearly, but cannot reach, since such influences as friction and conduction of heat cannot wholly be excluded. Hence for all real isothermal, constant-volume operations $\delta\psi$ must be less than zero, that is negative, and ψ must suffer a decrease.

Let us imagine that ψ has reached a minimum value. Then, for any further small change, it can decrease no more. If it alter at all, it must increase. But, since a real change must involve a decrease in ψ, no real change can occur when ψ is a minimum: the system must be in equilibrium.

Thus we reach the necessary and sufficient condition of equilibrium for an isothermal system in which the quantity factors x, such as volume, are kept constant. The condition is that the ψ function should be a minimum.

In mechanics we similarly get equilibrium when a potential is a minimum, and hence, by analogy, the ψ function is called a thermodynamic potential—the thermodynamic potential at constant volume.

In order to investigate the condition of equilibrium for a system at constant pressure, or, in the general case, when the intensity factors X in the expression $\Sigma(\mathrm{X}dx)$ for the work of all kinds are kept constant, we must return to relations (ii.) and (iii.),

$$\delta\epsilon\overset{=}{<}\theta\delta\phi+\Sigma(\mathrm{X}dx),$$

and subtract from each side

$$\delta\{\theta\phi+\Sigma(\mathrm{X}x)\}=\theta\delta\phi+\phi\delta\theta+\Sigma(\mathrm{X}\delta x)+\Sigma(x\delta\mathrm{X}),$$

thus obtaining the relations

$$\delta\{\epsilon-\theta\phi-\Sigma(\mathrm{X}x)\}\overset{=}{<}-\phi\delta\theta-\Sigma(x\delta\mathrm{X}). \quad (\text{vii.})$$

If we write ζ for $\epsilon-\theta\phi-\Sigma(\mathrm{X}x)$, and restrict ourselves to isothermal operations, we get

$$\delta\zeta\overset{=}{<}-\Sigma(x\delta\mathrm{X}), \quad . \quad . \quad (\text{viii.})$$

relations which indicate that $\delta\zeta$ has not such a definite physical significance as $\delta\psi$, the available energy. Nevertheless, ζ gives us the condition we seek.

If we work at constant pressure, or, in the general case, when all X's are constant, δX vanishes and

$$\delta\zeta \leqq 0, \quad . \quad . \quad . \quad . \quad \text{(ix.)}$$

where, as before, the equation characterises reversible and the inequality irreversible processes. Hence it follows that, when ζ is a minimum, no real, *i.e.* irreversible, process can occur, and the system must be in equilibrium.

We have now obtained a complete solution of the problem of the conditions of equilibrium of isothermal systems in the two cases which are at once the simplest and most important. When the x's (including the volume) are constant, ψ must be a minimum, and when the X's (including the pressure) are constant, ζ must be a minimum.

The functions ψ and ζ are thermodynamic potentials, and involve the internal energy, the temperature, the entropy, and the coordinates X and x of the system. ψ and ζ are therefore functions of the composition, the temperature, and the pressure of unit mass of the system.

§ (4) THE EQUILIBRIUM OF PHASES.—Let us once more return to the consideration of the system with which we began—a liquid, water, in contact with its saturated vapour. If a small mass δm of liquid evaporate at constant volume, there will be an increase, say, $\delta\psi_1$, in the value of ψ_1 for the vapour, and a decrease, $\delta\psi_2$, in that for the liquid, while, if the change occur at constant pressure, the corresponding quantities will be $\delta\zeta_1$ and $\delta\zeta_2$. The total rate of increase per unit mass evaporated is therefore

$$\frac{d\psi_1}{dm}-\frac{d\psi_2}{dm}, \text{ or } \frac{d\zeta_1}{dm}-\frac{d\zeta_2}{dm}.$$

In order that the system should be in equilibrium, ψ or ζ must be a minimum, and therefore the differential coefficient must vanish. Hence the necessary and sufficient condition of equilibrium for the component water substance in the two phases is

$$\left.\begin{aligned}&\text{at constant volume} \quad &\frac{d\psi_1}{dm}=\frac{d\psi_2}{dm},\\ &\text{and at constant pressure} \quad &\frac{d\zeta_1}{dm}=\frac{d\zeta_2}{dm}.\end{aligned}\right\} \quad . \quad . \quad \text{(x.)}$$

These quantities were named by Gibbs the chemical potentials, and, for brevity, the equations such as (x.) may be written as

$$\mu_1=\mu_2.$$

At the triple point, where ice, water, and vapour coexist in equilibrium, we have also

$$\mu_2=\mu_3,$$

and $$\mu_3=\mu_1.$$

But the last is not an independent equation, it is implicitly contained in the other two; for three phases, containing one component, we have two independent equations when the three phases are in equilibrium.

§ (5) THE PHASE RULE.—We are now ready to consider the general problem of equilibrium in a system containing r phases and n components.

For each component we can get a series of equations like those just obtained, giving the conditions of equilibrium for that component in pairs of coexisting phases. As before, the best possible equation will be implicitly contained in the others, and the number of independent relations is consequently one less than the number of phases, or $r-1$. In the whole system, containing n components, we shall have a series of these equations for each component, since, if the whole system is in equilibrium, each component must be in equilibrium in each pair of coexisting phases. For instance, in a mixture of water and alcohol in equilibrium with the mixed vapour, both for water substance and for alcohol substance must $\mu_1=\mu_2$. Hence, in the whole system, the number of independent relations is

$$n(r-1),$$

and these are relations between quantities which are functions of the composition, the temperature, and the pressure.

Let us now ask what is the number of independent variables in the system of r phases and n components.

In unit mass of each phase there may be n components. The composition of the phase is known if the masses of $n-1$ of these components are given. For instance, in one gram of a solution of a salt in alcohol and water, if we know that there is half a gram of water and a quarter of a gram of alcohol, we do not need to be told that there is a quarter of a gram of salt.

Altogether we have r phases of which this is true. Hence the number of independent variables defining the concentration or composition of the system is $r(n-1)$. But, besides the composition, we must know the temperature and pressure. The volume of unit mass will then of course be fixed, and is not an independent variable. Hence to the internal variables, defining the composition, we must add two, the temperature and the pressure. The whole number of independent variables is thus

$$r(n-1)+2,$$

and, to determine these variables, we have $n(r-1)$ independent equations, containing functions of those same variables.

If we have the same number of simultaneous equations as we have independent variables, the variables must each have one and only one determinate value. That is, the system can exist in equilibrium only at one temperature, one pressure, and one composition of each variable phase. Such a system is known as non-variant or invariant. It is defined by the relation

$$n(r-1)=r(n-1)+2$$

or

$$n-r+2=0. \quad . \quad . \quad . \quad \text{(xi.)}$$

On the other hand, if there are less independent equations than there are variables, the system will not be completely determined. It can exist in equilibrium in different conditions, and its amount of indeterminedness will increase with the excess of variables. For instance, if there is one more variable than there are equations, one unknown will remain unspecified. In the equilibrium between a liquid and its vapour, n is 1 and r is 2; hence

$$n-r+2=1. \quad . \quad . \quad . \quad \text{(xii.)}$$

This system is known as monovariant or univariant. It can exist in equilibrium throughout a wide range of temperatures and pressures. But, if one of these variables be fixed, the other is fixed also. At 50° C., for example, equilibrium is only possible between water and steam at a pressure of 92·3 mm. of mercury.

We now see that the excess of the number of variables over the number of independent equations between them may rightly be called the number of degrees of freedom of a system. If this number be written as F, we have

$$F=r(n-1)+2-n(r-1)=n-r+2 \quad \text{(xiii.)}$$

as the symbolic expression of Willard Gibbs' Phase Rule, which, put into words, is:

The number of degrees of freedom of a system in equilibrium is two more than the excess of the number of components over the number of coexisting phases.

III. Application of the Phase Rule

§ (6) One-component Systems.—As the simplest and most familiar application of the Phase Rule, let us deal with the phenomena of equilibrium between the different phases of water substance.

To obtain a non-variant system, we must have as in equation (xi.)

$$F=n-r+2=0.$$

Since there is only one component, n is unity, and we find

$$r=3.$$

That is, to get a system which is completely specified, three phases must coexist, or ice, water, and vapour can coexist in stable equilibrium at one temperature and one pressure only, viz. at the freezing-point +0°·0068 C., under the pressure of the vapour, which is 4·6 mm. of mercury.

The further relations of a one-component system, as illustrated by the Phase Rule, are best exemplified by means of a plane diagram, drawn between temperature and pressure as ordinates (*Fig.* 1). On this diagram the invariant system of three phases is represented by the point P, the so-called triple point. Both temperature and pressure are fixed and determined.

Let us imagine that, to this system of three phases, heat is added. Ice will gradually melt, but, as long as any ice remains, neither temperature nor pressure will change. The melting-point is constant and the vapour pressure is constant.

When the last particle of ice disappears only two phases are left. In the equation (xiii.) r is now 2, and F becomes unity. We have the system of liquid and vapour in equilibrium as explained above; we have a univariant system. On the diagram, this system must be represented by a simple line—that is, by a figure which, for one value of the abscissa, has one value of the ordinate only. In this case, the line is PA, the well-known vapour pressure curve of water. It ends at the critical point, where the distinction between liquid and vapour ceases and above which only one phase exists.

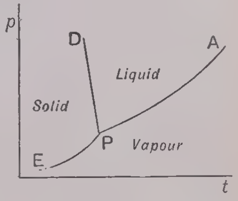

Fig. 1.

The slope from point to point of curves such as this is given by the so-called Latent Heat Equation,[1]

$$\frac{dp}{d\theta}=\frac{L}{\theta(v_2-v_1)}, \quad . \quad . \quad \text{(xiv.)}$$

where p is the pressure, θ the temperature measured on the absolute scale, v_2-v_1 the change in volume produced when unit mass of one phase passes into the other, and L the latent heat, *i.e.* the amount of heat, measured in mechanical units of energy, that must be added to cause this transformation. The Phase Rule enables us to predict the number and meaning of these curves of equilibrium, and the Latent Heat Equation gives their slope. Hence the Phase Rule and the Latent Heat Equation contain together the complete solution of the problem of equilibrium.

And now let us return once more to a mixture of three phases at the triple point. Instead of passing heat into this mixture, let us imagine it to be abstracted. Water will gradually freeze, but, so long as any liquid remains, the temperature and pressure are unaltered and we still have an invariant system.

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[1] See article "Thermodynamics."

But, when the last drop of water freezes, only two phases, ice and vapour, remain. Again, in the equation (xiii.) r is 2 and F is unity, and once more we must represent the system on our diagram by a line. This curve PE, showing the equilibrium between solid ice and aqueous vapour, is the vapour pressure curve of ice below the freezing-point. Since the latent heat of the change from solid to vapour near the freezing-point is the sum of those for the changes solid to liquid and liquid to vapour, the slope of this curve is greater than that of the vapour pressure curve of water, and the curve for ice lies below that for under-cooled water.

Finally, if instead of passing heat into or out of the invariant system, we attempt to increase the pressure, its rise will be only infinitesimal; vapour will condense, and the volume will diminish. When all vapour has gone, we get the univariant system liquid water and solid ice. The curve of equilibrium PD, since the change in volume from ice to water is small and negative, is shown by the latent heat equation to be very steep, and to slope upwards from right to left. It gives the depression of freezing-point as pressure increases, which is one degree centigrade for about 147 atmospheres.

§ (7) Two-component or Binary Systems.—As an example of a simple system containing two independent components, we will take water and an anhydrous salt such as potassium chloride. In the Phase Rule Equation (xiii.) n is 2, and to get a non-variant equilibrium we have

$$F = 0 = 2 - r + 2$$

or

$$r = 4.$$

The four possible coexisting phases are crystals of ice, crystals of salt, saturated solution, and the vapour, which, since the vapour pressure of the salt at ordinary temperatures is negligible, is the vapour of water only.

Our plane diagram, where the co-ordinates are temperature and pressure, is now inadequate, because a new independent variable appears—the concentration of the solution. For graphical representation, we shall want another axis, and therefore we must take a three-dimensional model, which can be indicated on a plane diagram by a perspective sketch (*Fig.* 2).

In this model, the invariant system of four coexisting phases is represented by a point P. The co-ordinates of this point are fixed by the temperature t corresponding to the freezing-point of the saturated solution under its own vapour pressure, the vapour pressure p, and the composition c of that solution at the freezing-point. For water and potassium chloride $t = -11°{\cdot}1$ C., $p = 2{\cdot}0$ mm. of mercury, and $c = 24{\cdot}6$ grams KCl to 100 grams H_2O.

This constancy of composition and of melting-point used to be thought characteristic of elements or definite chemical compounds only. And Guthrie, who first investigated the phenomena of these invariant systems, called the mixture of solids cryohydrates.

Let heat be passed into this non-variant mixture. Ice melts, and salt dissolves in the liquid so produced to form more saturated solution; the co-ordinates are unaltered. But eventually either salt or ice fails. If ice fails first, leaving excess of salt, we are left with salt, saturated solution, and vapour—three phases which, in accordance with the rule, form a univariant system represented by a line such as PA in our solid model, the vapour pressure curve of a solution saturated from point to point as the temperature rises.

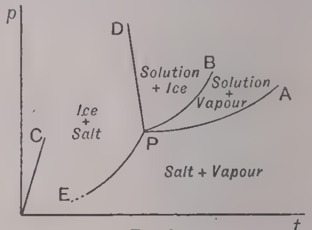

FIG. 2.

If, on the other hand, the supply of salt fails, leaving excess of ice, the solution grows more dilute as ice melts, and in the end, if the amount of ice is unlimited, the solution is infinitely dilute, *i.e.* this freezing-point curve ends at B, the freezing-point of pure water. Along its length the concentration varies fast—it lies quite out of the plane of the sketch diagram.

If heat be taken from the non-variant mixture, liquid vanishes, and we get PE the vapour pressure curve of mixed ice and salt, while, if the volume be diminished and finally pressure be increased, we obtain the last possible univariant system made up of ice, salt, and saturated solution, represented in *Fig.* 2 by the line PD.

The four curves so obtained mark out edges in the solid model, and these edges form lines of contact between sheet-surfaces. Such surfaces represent di-variant systems with two degrees of freedom, and, as the Phase Rule shows, two phases in equilibrium. For the example taken, these pairs of phases are written on the diagram (*Fig.* 2).

Finally, between the di-variant surfaces three-dimensional spaces exist, representing the three degrees of freedom characterising systems containing one phase only. The meaning of each space on the diagram can be seen by taking the phase common to the two surfaces which bound the space in question.

The complete significance of the diagram is now clear.

This diagram also represents the phenomena of equilibrium between water and a hydrated salt such as sodium sulphate, $Na_2SO_4 . 10H_2O$, "hydrate" being substituted for "salt" where it occurs in *Fig.* 2. In this case, the curve OA, giving the equilibrium between hydrate, solution, and vapour, ends at 32° C., where the hydrate melts and passes into anhydrous sodium sulphate and water. At this point, then, we get another non-variant system, the four necessary phases being solid hydrate, solid anhydrous salt, liquid solution, and the vapour. From this point start four univariant curves, repeating qualitatively the phenomena of *Fig.* 2.

When we pass from the qualitative to the quantitative study of these phenomena, the general sketch of the model must be replaced by a projection of the univariant curve in question on one or other of the three planes. Thus experiments on the connection between temperature and solubility are represented by projecting the curve OA on the C-*t* plane. In order to do this, the pressure at each point should be that of the vapour at that point, but, since the solubility of a solid does not change much with pressure, the simpler experiments made under constant atmospheric pressure give the theoretical univariant curve with enough accuracy.

Simple illustrations may be taken from solutions of salts in water, or, to open new ground, from alloys. *Fig.* 3 shows the equilibrium between silver and copper. Pure silver melts at 960°, and the addition of copper lowers the melting-point in a univariant manner. On the other hand, pure copper melts at 1081°·5, and the addition of silver similarly lowers the melting-point. The two curves cut each other at a point B, where there are 40 atomic per cents of copper and the temperature is 777°. Here we get four phases: crystals of the two pure metals, the liquid, consisting of the saturated solution of each in the other, and the vapour. By the Phase Rule this is a non-variant system; microscopic examination shows that its crystalline structure is more uniform than that of a mixture of any other proportions, and it is hence called a eutectic alloy. The crystals are those of pure silver, represented by E, mixed with those of pure copper, represented by F. Below 777° no liquid can exist, hence underneath the horizontal line EBF we have a region in the diagram which represents solid alloys made of mixtures of copper crystals and silver crystals in varying proportions. In the region AEB we have crystals of silver and a solution saturated with silver, and in CBF copper and a solution saturated with copper. Above the equilibrium curves AB and CB no solids exist, and the liquid is an unsaturated solution.

If a melted alloy, such as that represented by the point G in *Fig.* 3, of composition richer in copper than that of the eutectic mixture, be cooled, solid copper, represented by the point M, is deposited when the temperature falls to that corresponding to the point H. The residual liquid thus becomes richer in silver. It changes as indicated by the curve HB, till its composition is that of the eutectic. At B silver and copper are deposited together in eutectic proportions till the whole system is solid. Microscopic examination of the solid so obtained will show the larger crystals of pure copper first deposited, cemented together by a conglomerate made of the smaller, more regular crystals of both metals deposited eutectically.

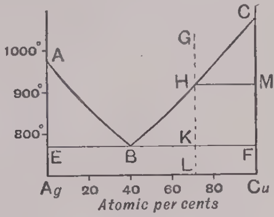

FIG. 3.

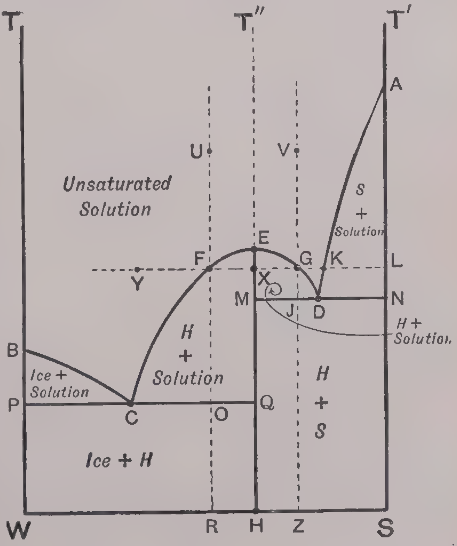

FIG. 4.

The copper-silver diagram of *Fig.* 3 will also represent, in a qualitative way, the equilibrium between water and an anhydrous salt. The point A represents the melting-point of ice and the point C that of the salt. For potassium chloride, for instance, this point is 730° C. Practically, the investigation of this case presents difficulties, since above 100° C., owing to the high vapour pressure of water, the experiments have to be carried out under rapidly increasing pressures, but the change thus produced in the state of equilibrium of the solid and liquid phases is very small, and, theoretically, the diagram brings out the fact that there is no real distinction between the so-called solute and solvent. Along the curve CB the solution is saturated with and in equilibrium with salt, and along AB it is saturated with ice. At the eutectic point B both ice and salt exist. *Fig.* 3, therefore, will represent the equilibrium between water and a salt, if ice be read for silver and salt for copper.

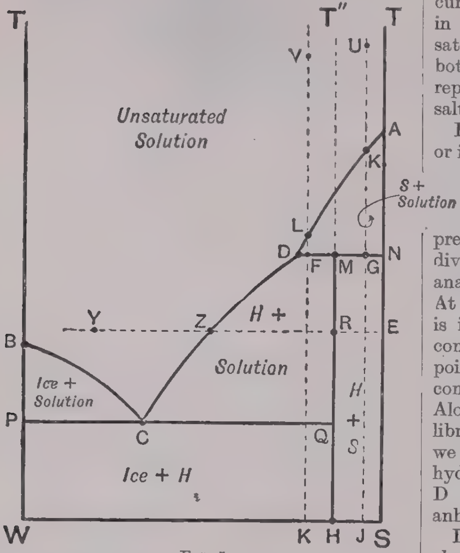

FIG. 5.

If water and salt combine to form a hydrate, or if two metals, such as antimony and copper, form a compound, we get as one possibility a diagram of the type shown in *Fig.* 4.[1] If the compound have the composition represented by H, the vertical line T″EH divides the figure into two, each part being analogous to the complete diagram in *Fig.* 3. At E the solid hydrate or other compound is in equilibrium with a liquid of the same composition, and we get a constant melting-point. Adding either component to this compound causes the melting-point to fall. Along the curve CED the hydrate is in equilibrium with the saturated solution. At C we have a non-variant eutectic point where hydrate and ice are the solid phases, and at D another eutectic point with hydrate and anhydrous salt as solids.

If a liquid represented by U be cooled along UR, solid hydrate, represented by X, appears at F. As more and more hydrate crystallises out, the composition of the liquid changes, the proportion of water increasing

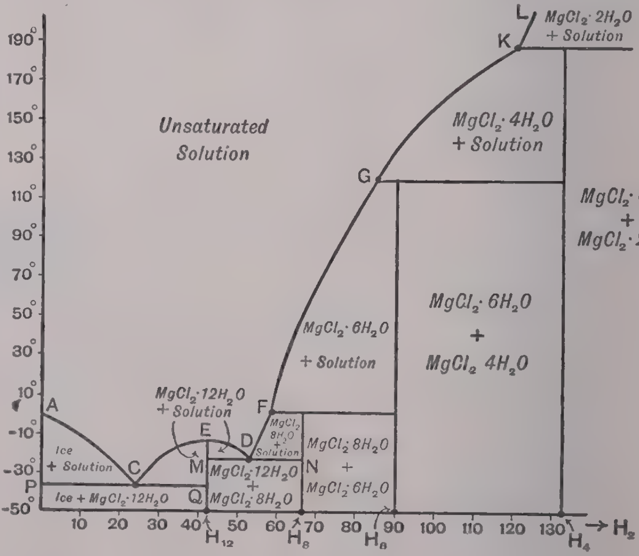

FIG. 6.

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[1] *Figs.* 4, 5, 6, and 12 to 18 are taken, by permission, from Dr. D. A. Clibbens' book, *The Principles of Phase Theory* (Macmillan & Co.).

with falling temperature as shown by the curve FC. At C ice appears.

On the other hand, if a liquid richer in salt than the hydrate be cooled along VZ, the hydrate X appears at G and the residual liquid becomes increasingly more saline till anhydrous salt crystals appear at D.

It is interesting to note that if water be evaporated at constant temperature from an unsaturated solution represented by Y, so that we pass along the horizontal line YL, the liquid will solidify at F, but the system will again liquefy at G, where the line once more enters the region of unsaturated solutions. It will remain wholly liquid till K, where anhydrous salt appears. When all water is evaporated, we are left with pure salt at L.

Another possibility is shown in *Fig.* 5. Here the non-variant point D has a liquid phase containing more water than does the hydrate—that is to say, is on the left of T″MH. Passing up the curve CD from left to right, anhydrous salt appears before the composition corresponding to the pure hydrate is reached. Liquids of composition V or U when cooled deposit anhydrous salt at L and K respectively, and hydrate when the residual liquid reaches the transition point D. Isothermal evaporation along YE produces hydrate at Z, which is converted into anhydrous salt when dehydration begins at R.

Both these possibilities are actually found in the phenomena of equilibrium of water and magnesium chloride, a salt which crystallises with 12, 8, 6, 4, and 2 molecules of water to form five different hydrates. The meaning of *Fig.* 6 will be clear without further explanation.

§ (8) MIXED CRYSTALS OR SOLID SOLUTIONS.—Hitherto we have dealt with systems where the only phase with continuous variation in composition was the liquid. The solids have been crystals, such as ice, salt, a hydrate, or a metal, of one definite chemical and physical constitution. But solids are found with continuous variation; they are called solid solutions or mixed crystals. For instance, it has long been known that in crystals of the alums, one metal such as sodium can replace another such as potassium by insensible gradations, causing no change in the form of the crystals.

The thermodynamic theory of these variable solid phases has been worked out by Roozeboom,[1] by a method originally due to van Rijn van Alkemade.[2]

We have already seen that the condition of equilibrium of each component in two coexisting phases is

$$\frac{d\psi_1}{dm}=\frac{d\psi_2}{dm}, \text{ or } \frac{d\zeta_1}{dm}=\frac{d\zeta_2}{dm},$$

according as the system is maintained at constant volume or at constant pressure. These differential coefficients give the slope of the curves in a diagram drawn between ψ and m or ζ and m as ordinates. When m and therefore the concentration of one component in the other is small, we get a characteristic dilute system, the energy relations of which correspond to those of any other dilute solution or of a gas. The work done during an isothermal compression of a gas is[1]

$$m\mathrm{RT}\log\frac{p_2}{p_1},$$

where m is the mass of the gas, R the gas constant, T the absolute temperature on the gas scale, and p_1 and p_2 the initial and final pressures.

In the problem which now faces us, it follows that the work done in introducing a further small quantity of substance is of the form $a+\log bm$, where a and b are independent of m. When the concentration is indefinitely small, m approaches zero, and its logarithm $-\infty$. The curve must therefore at first touch the axis of ψ or ζ. In the diagram of *Fig.* 7 the pure components A and B are represented by the two sides of the figure, while systems of mixed composition are denoted by intermediate points. In Divisions I., II., III., and IV. of the figure, the vertical axis gives the value of ψ or ζ; in Division V. it gives the temperature.

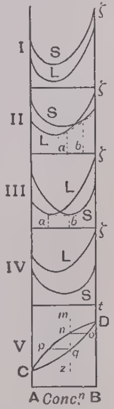

FIG. 7.

With two variable phases, liquid and solid, we have two curves, and we now see that each of these curves must start vertically downwards from each side of the diagram. Let us consider the possibilities of form in the rest of the curves.

The simplest possibility is that both curves should run from end to end with no changes in the sign of curvature, as in *Fig.* 7. Division I. in this figure shows the two phases above the melting-point of either component. The liquid being the stable phase, its potential must be less than that of the solid, and its curve lies below. At a melting-point, solid and liquid are in equilibrium, and the corresponding ends of the two curves coincide. At some lower temperature, for certain compositions, the solid is stable, and the two curves cut each other as shown in Division II.

Now the condition of equilibrium as given in equation (x.) is that $d\zeta_1/dm=d\zeta_2/dm$, *i.e.* that the curves should have a common tangent. This is satisfied by the points a and b, which therefore give the compositions of liquid and solid in equilibrium with each other at this temperature. Division III. represents similar equilibrium at a lower temperature, while IV. shows the curves below the melting-points, when the solid is the only stable phase.

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[1] *Zeits. phys. Chem.*, 1899, xxx. 385.

[2] *Ibid.*, 1893, xi. 289.

[1] See article "Thermodynamics."

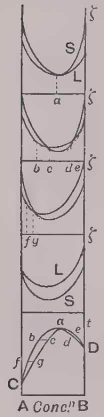

FIG. 8.

These four $m\zeta$ diagrams can now be used to construct an *mt* or freezing-point diagram, showing the relation between composition and melting-point as in Division V. At the temperatures of II. and III. a liquid of composition *a* is in equilibrium with a solid of composition *b*, and we thus get two curves in Division V., the upper corresponding to the liquids and the lower to the solids in equilibrium with each other. These curves are called the liquidus and solidus respectively.

When a liquid of composition *m* cools, solid of composition *o* appears when the liquid is at *n*. As the liquid varies in composition from *n* to *p*, the solid in equilibrium with it changes from *o* to *q*, at which latter point the whole system has become solid.

Instead of cutting each other as in *Fig.* 7, the $m\zeta$ curves may first come into contact at an intermediate point as in *Fig.* 8. This gives *mt* curves as shown in *Fig.* 8, Division V. If the $m\zeta$ curves leave each other at a point as in *Fig.* 9, the *mt* diagram is as shown therein, while, if one curve suffers a change in curvature as in *Fig.* 10, the thermodynamic potential between certain points is higher than it is at those points and the compositions between them are unstable, the crystals passing spontaneously into mixtures of the solid solutions represented by the points. The corresponding freezing-point diagram shows that at a temperature *c* a liquid of composition *c* is in equilibrium with solids of composition both *d* and *e*, no solid solutions of intermediate composition being stable. Between the two lines *dhl* and *ekm*, we get mixtures of two solid solutions of compositions represented by points on those lines.

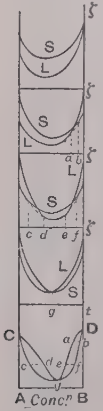

FIG. 9.

We are now ready to elucidate some of the complex phenomena of solid solutions which have been discovered, especially in the case of alloys. The experimental investigation consists chiefly, firstly, in observations on the rate of cooling, when, owing to the evolution of latent heat a slowing down in the rate of cooling marks a change of state whether from liquid to solid or from one solid to another; and secondly, especially in the case of alloys, in the microscopic examination of polished surfaces of the solids which have been suddenly cooled from a definite temperature by chilling in cold water. By this chilling, the crystalline structure at the given temperature is fixed permanently, and it can then be examined at leisure. It is found that each kind of element, compound, or solid solution has a characteristic crystalline structure by which its presence can be detected and its appearance and disappearance traced.

Let us take as an example the work of Heycock and Neville on the bronzes, that is, of alloys of copper and tin.[1] *Fig.* 11 shows the equilibrium curves from pure copper to an alloy containing 80 atomic percentages of tin. Above the "liquidus" curve, ABCDEFGH, the alloys consist of a homogeneous liquid, and below the "solidus" curve, $Ablmf\mathrm{E}_2\mathrm{E}_4\mathrm{H}''\mathrm{M}$, the whole mass is solid. But, even in this solid mass, changes of structure go on when the conditions are altered. For instance, the two curves *l*X and E′X, which cut each other at X, enclose a region which represents homogeneous solid solutions of varying composition. Along *l*X and E′X, these solutions are in equilibrium with other solids which crystallise out when the mass is cooled slowly (as down the dotted line) so as to cut one of these curves.

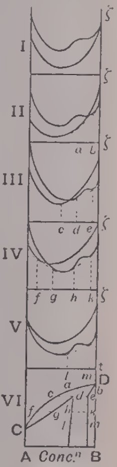

FIG. 10.

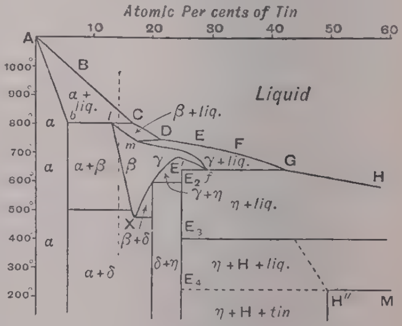

FIG. 11.

The meaning of the rest of the diagram is clear from what has already been said. The regions representing the different crystalline substances of definite composition *α*, *β*, *γ*, *δ*, *η* are marked with those letters, while the region in which β and γ join in varying proportions to form solid solutions is marked $\beta\gamma$.

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[1] *Trans. R.S.* A, 1904 ccii. 1.

§ (9) THREE-COMPONENT OR TERNARY SYSTEMS.—The Phase Rule Equation,

$$F=n-r+2,$$

shows that, for three components ($n=3$), to get a non-variant system, we must put $r=5$, that is, assemble five phases in equilibrium. If, as before, only one liquid and one vapour phase be possible, we shall want three distinct solids to complete our five phases.

When no mixed crystals are formed, the solids are invariable, and if also only one component be volatile, the liquid is the only variable phase. To express the composition of unit mass, we shall need to know the value of $n-1$ or 2 co-ordinates. Diagrammatic representation of the complete system, then, would need a four-dimensional model. By taking the pressure constant, as is legitimate when dealing only with solids and liquids, we can reduce our four dimensions to a possible three, while by working at constant temperature we can represent the equilibrium of three components on a plane diagram.

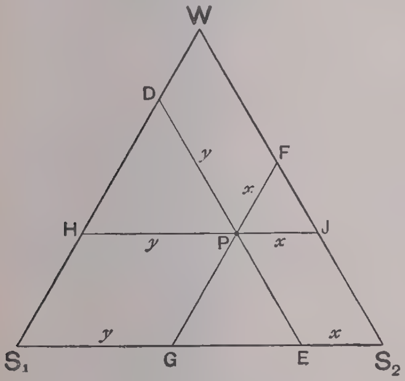

FIG. 12.

In order to do this, we will use a method introduced by Gibbs. Points at the angles of an equilateral triangle are taken to represent the three components in a pure state. A point in any side then represents a mixture of two components, and a point P within the triangle mixtures of three components, the amount of any one component being proportional to the distance of P from the side opposite to the angle representing that one component in a direction parallel to one of the other sides.

Thus, to take as an example a system of water W and two salts S_1 and S_2, the point P (*Fig.* 12) represents a solution of xS_1 and yS_2 in unit mass, the amount of water, $1-(xS_1+yS_2)$, being represented by PE or PG.

Let the composition of a saturated solution of S_1 in water be represented by A (*Fig.* 13). Similarly, let C and D represent saturated solutions of S_1 in water containing two different amounts of S_2. At F the amount of S_2 is enough to saturate the liquid with S_2 also, and here the solution is in equilibrium with both salts. For an isothermal system, F is an invariant point—the equilibrium is completely defined for the one temperature represented by the plane diagram. In the same way, the curve BEF represents liquids saturated with S_2 in solutions of S_1 in water of increasing concentration, and once more when we reach F the liquid is saturated with both salts.

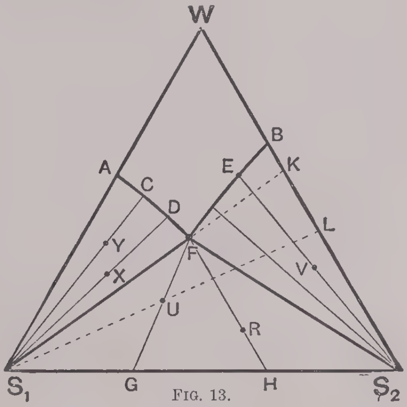

FIG. 13.

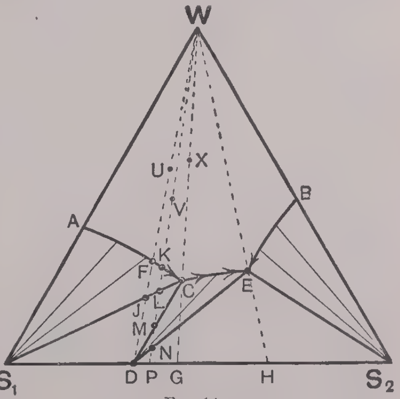

FIG. 14.

If the two salts form a compound double salt, its composition may be represented by a point D in the line S_1S_2 (*Fig.* 14). Its saturation curve in solvents of varying composition will be some such line as CE, the liquid being also saturated with S_1 at C, and with S_2 at E. These points, C and E, represent non-variant systems.

The phenomena of isothermal evaporation can be traced by following lines such as UFD, VLP, and XCG on the diagram. For instance, when we pass along UD, we start with an unsaturated solution prepared by dissolving the compound in water. At F it deposits crystals of pure S_1, and the composition of the residual liquid passes along the curve from F to C, where the liquid is also saturated with the compound. Hence, in such cases, the compound is said to be "decomposed by water," though theory now shows us that such reactions depend on the thermodynamic relations of equilibrium between solid and liquid phases. By whatever form of words we describe the phenomena, our theory enables us to foretell the conditions necessary for the separation of one constituent salt from a solution of two salts or a double salt.

Points between J and D represent complexes of both S_1 and the compound, in equilibrium with the invariant solution C, and, if we enter the space DCE, as by evaporation of a liquid represented by V or X, the deposited S_1 will redissolve, and we shall get the compound double salt as the only solid. This, in its turn, will become mixed with crystals of pure S_2 if we pass into the space DES_2.

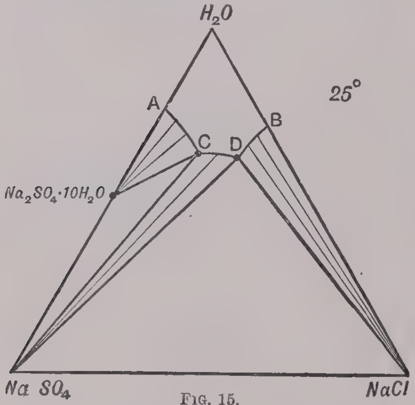

FIG. 15.

A somewhat similar diagram is obtained if either of the salts forms a solid hydrate with water. Its form is clear from what has been already said. The isotherm at 25° C. for water, sodium sulphate, and sodium chloride is given in *Fig.* 15.

The effect of changes in temperature may be represented by superposing these isothermal diagrams one on the other. We thus get a three-dimensional triangular prism in which the whole phenomena may be set forth.

§ (10) FOUR-COMPONENT OR QUATERNARY SYSTEMS.—When another or fourth component is present and $n=4$, even if we work at constant pressure and constant temperature, it will need a three-dimensional model to represent the composition of the liquid phase. This can best be done by using a regular tetrahedron, each of the faces being an equilateral triangle. Points on one face of the tetrahedron represent systems containing the components represented by the three corners of that face, just as in the case of the ternary systems already considered, and points inside the tetrahedron correspond to systems containing also the fourth component.

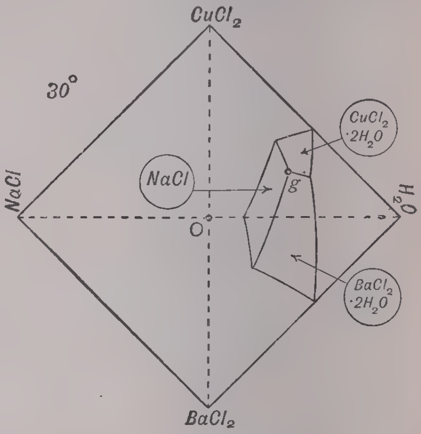

FIG. 16.

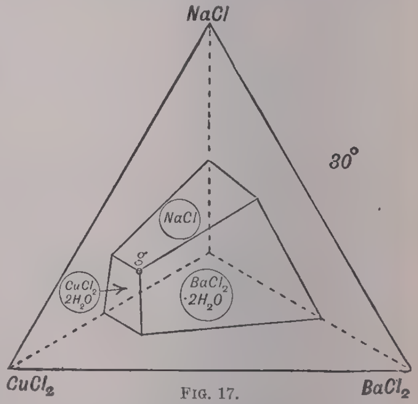

FIG. 17.

The tetrahedron may be drawn as a perspective sketch or projection on the base of the tetrahedron, or it may be imagined as standing on one edge, symmetrically about a vertical plane through that edge, and then be projected on a horizontal plane. We thus get a square, in which we can represent the system on a plane diagram. *Figs.* 16 and 17 show the equilibria of water with sodium, copper, and barium chlorides by these two methods for a constant temperature of 30°, where the stable solid phases are NaCl, $BaCl_2 . 2H_2O$, and $CuCl_2 . 2H_2O$. The point *g* represents the isothermal invariant solution which is saturated at 30° with these three solids, the composition being

NaCl	$BaCl_2$	$CuCl_2$	H_2O
10·49	1·97	36·12	51·42 per cent.

When double salts, anhydrous or hydrated, are formed, more complicated diagrams of course result.

Another type of quaternary system is given by mixtures of two salts with no ion in common, such as potassium nitrate and sodium chloride. Here a double reaction

$$KNO_3+NaCl \rightleftarrows NaNO_3+KCl$$

is possible, and, indeed, in solution the salts are ionised. Thus, from a liquid containing $\overset{+}{K}$, $\overset{+}{Na}$, $\overset{-}{NO_3}$, and $\overset{-}{Cl}$ any of the four possible salts may be deposited. Nevertheless, since the equation given above describes the reaction, the concentration of one salt can be expressed in terms of the other three, and, with water, we still have only four independent components. For instance, the amount of potassium nitrate is given by

$$KNO_3=NaNO_3+KCl-NaCl.$$

If the three salts KNO_3, $NaNO_3$, and KCl are represented by the three corners of the triangular base of a tetrahedron, the apex of which corresponds to water, solutions containing KNO_3 will be represented by points having a negative value of NaCl, that is, points such as *b* lying beyond the side $NaNO_3$, KCl of the triangle $NaNO_3$, NaCl, KCl of *Fig.* 18. Instead of drawing this as an equilateral, it is better to make it a right-angled isosceles triangle as in the figure. The point representing pure KNO_3 will then, if all compositions are expressed in molecular proportions, be at the fourth corner of the square.

The complete diagram of *Fig.* 18 represents the isotherms at four different temperatures. No hydrates or double salts appear, and the system is the simplest possible of its type.

Let us make a practical use of this diagram to investigate the conditions in which we can make crystals of potassium nitrate from sodium nitrate and potassium chloride. The solution containing equimolecular amounts of these two salts will be represented by the middle point of the line joining them, *i.e.* by the point *a*, where the diagonals of the square intersect. It appears from the diagram that at 100° this point *a* lies within the surface between the axes meeting at the point marked NaCl, and the lines meeting each other at P_1 and P_2. This surface represents solutions and solid NaCl, so that, if the liquid be evaporated at 100° from an unsaturated solution of sodium nitrate and potassium chloride in molecular proportions, it will first deposit sodium chloride. As the amount of this salt gets less, we recede more and more from the corner NaCl, that is, we pass along the diagonal through it. When we reach *b* the liquid becomes saturated also with potassium chloride. We must now stop the evaporation and filter off the sodium chloride. The liquid filtrate contains the salts in the proportions corresponding to the point *b*, and, at temperatures below 50°, this point lies within the surface between the axes meeting at the point marked KNO_3 and the lines meeting each other at P_1 and P_2. This surface represents solutions mixed with potassium nitrate. By cooling the liquid, therefore, we deposit crystals of potassium nitrate. As the liquid loses that salt, we pass away from the corner KNO_3 along the diagonal. Let us suppose the temperature has fallen to 0°. When we reach the point *c*, the solution becomes saturated with sodium chloride also, and the evaporation must be stopped, since we want pure potassium nitrate.

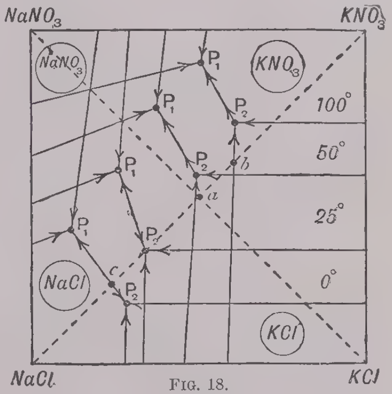

FIG. 18.

§ (11) MORE COMPLEX SYSTEMS.—The larger the number of independent components, the greater the difficulties both of experiment and also of graphical representations of results. With five components, however, much information is available.

At constant temperature and pressure, we still have four variables. But, if we leave out of our diagrams the constitution of the liquid phase as regards water, we can represent usefully the relations of equilibrium for four salts dissolved therein.

Instead of taking the salts themselves as components, we may take the ions instead.

A five-component system can either have four ions of one sign and one of the other, as, for instance, when four chlorides are present, or it may have three metals shared between two acid radicles.

For the theory of these complex systems, and its application to such practical problems as the separation of one salt from its mixture with several others, the reader is referred to Clibbens' *Principles of the Phase Theory* (from which some of the diagrams in this article are taken), to Roozeboom's *Die heterogenen Gleichgewichte vom Standpunkte der Phasenlehre*, and to the many papers on the subject which will be found in the *Zeitschrift für physikalische Chemie* and other chemical and physical journals.

W. C. D. W.

Pipe Line, Capacity of a. See "Hydraulics," § (55).

Pipe Lines, Losses in. See "Hydraulics," § (25).

Pitot Tube. An instrument used for determining the velocity of a fluid at a given point. See "Friction," § (11).

Planck's Constant and Theory of Temperature Radiation. See "Thermodynamics," § (58); "Quantum Theory," § (1), Vol. IV.

Planck's Formula for the law of spectral radiation,

$$E_\lambda = c_1\lambda^{-5}(e^{c_2/\lambda T} - 1)^{-1},$$

specifies the distribution of thermal emission intensities in the spectrum of the radiation emitted by a uniformly heated enclosure. See "Radiation, Determination of the Constants," etc., I. § (1), Vol. IV.

Planck's Radiation Formula: the formula which gives the distribution of energy of radiation along the spectrum. It has the form

$$E_\lambda = \frac{8\pi hc}{\lambda^5(e^{hc/k\tau\lambda} - 1)},$$

where T is the absolute temperature, λ the wave-length of radiation considered, c the velocity of light, E_λ the density of isotropic energy per unit wave-length, and k and h universal constants. See "Radiation Theory," § (6), Vol. IV.

Planetary Theory, Application of Dynamical Similarity to. See "Dynamical Similarity, The Principles of," § (9).

Plasters containing Gypsum: General Properties. See "Elastic Constants, Determination of," § (153).

Platinum, Influence of Purity of, on Resistance Thermometers. See "Resistance Thermometers," § (6) (ii.).

Platinum, Polished, Emissivity of, determined by optical pyrometer. See "Pyrometry, Optical," § (22).

Platinum, Specific Heat of, used as a secondary standard of temperature in the range above 500° C. See "Temperature, Realisation of Absolute Scale of," § (41) (i.).

Platinum-metal Thermocouples, used as secondary standard of temperature in range 100° to 500° and compared with gas-thermometers. See "Temperature, Realisation of Absolute Scale of," § (36).

Platinum Thermometer, Calibration of Box Coils and Bridge Wire of. See "Resistance Thermometers," § (13).

Platinum Thermometer Coil, Heating Effect of Current passing through. See "Resistance Thermometers," § (9).

Platinum Thermometers, Construction of. See "Resistance Thermometers," § (6).

Pneumatic Gauges. See "Meters: Liquid Level Indicators," § (15), Vol. III.

Poisson's Ratio:

- Definition of: The ratio of the lateral contraction to the elongation produced by a simple tension. See "Elasticity, Theory of," § (4).
- Method of Direct Experimental Determination. See "Elastic Constants, Determination of," § (67).
- Tabulated Values of. See *ibid.* § (67), Table 25.

Porous Plug Experiment, Joule and Thomson's. See "Thermodynamics," § (12).

Potential, Thermodynamic. See "Thermodynamics," § (51).

Potentiometer:

- Hausrath, White, Diesselhorst Type of, designed for the measurement of thermal E.M.F.'s so that thermoelectric effects at brush contacts are reduced to a minimum. See "Thermocouples," § (13).
- Portable Deflection Type of: a type of instrument for the measurement of thermal E.M.F.'s which occupies a position intermediate between the nul potentiometer and the moving coil deflection instrument. See *ibid.* § (15).
- Simple Circuit, a practical form for the measurement of thermal E.M.F.'s. See *ibid.* § (10).
- Suitable for the measurement of thermal E.M.F.'s. See *ibid.* § (9).
- For Thermoelectric Work, points needing care in the installation of: (1) Elimination of parasitic E.M.F.'s, (2) prevention of leakage into the potentiometer circuit from neighbouring lighting or furnace circuits. See *ibid.* § (26).
- Vernier Forms of, designed by Thomson and Varley for resistance and cell comparison work and modified for the measurement of thermal E.M.F.'s. See *ibid.* § (11).

Pouillet Instrument for the comparison of gas-thermometers with secondary standards of temperature in the range 500° to 1600°. See "Temperature, Realisation of Absolute Scale of," § (39) (ii.).

Pressure, Influence of, on Thermal Expansion of Solids and Liquids. See "Thermal Expansion," § (13).

PRESSURE, MEASUREMENT OF

§ (1) Introduction.—Pressure is defined as the force acting upon unit area, so that the problem of its measurement is resolved into the determination of the force acting upon a known area. In comparing any force with the gravitational force on a known mass, the value of the gravitational constant affects the result so that a pressure gauge which determines the weight supported upon unit area will vary as the gravitational constant varies. On the other hand, if the pressure gauge measures the force by the elastic deformation of some body, its calibration will be independent of the value of the gravitational constant.

Pressure may also be measured by the effect produced upon some physical characteristics of a body, and this may be termed an indirect determination of pressure. Such gauges may or may not be capable of use as absolute standards. For example, the effect of pressure upon the electrical resistance of a conductor may be used; but the data necessary for the calculation of the effect are in general insufficient to enable such a gauge to be calibrated except by comparison with another form of pressure gauge. A gauge in which the compression of a gas is utilised generally requires comparison with a standard gauge, not because the properties of gases have not been sufficiently examined, but because the elastic properties of the containing vessel are not sufficiently accurately known. The usual type of gauge utilising the elastic deformation of a body suffers from the same disability, since the elastic properties of metals are to a certain extent variable.

The particular type of gauge adopted depends upon the accuracy required, the rapidity with which observations are to be made, and upon the variability of the pressure. The following paragraphs will describe the usual forms of gauge used in the following order:

(i.) Primary static pressure gauges.
(ii.) Secondary static pressure gauges.
(iii.) Gauges for measuring varying pressures.
(iv.) Gauges for measuring very small differences of pressure.

All the types of pressure gauges described, with one exception, measure the difference between two pressures, and in most cases one of these is the pressure of the atmosphere. When this is the case it is necessary to observe the barometric height at the time the observations are made, and add the pressure of the atmosphere to the observed pressure in order to obtain the absolute pressure. The exception is the closed mercury column or compressed gas manometer (§ (6)). This manometer reads the ratio between the absolute pressure applied and the pressure at which the gas reservoir was originally filled.

The different types of pressure gauge are mentioned, but the descriptions are necessarily brief. References are, however, given which will supply fuller particulars when these are required. The relation between the more important pressure units is given in the following table:

Table

Conversion Factors for Pressure Scales

(Correct to 1 part in 10,000 at 0° C., taking $g=980\cdot617$)

	Lbs./in.^{2}.	In. of Mercury.	Atmospheres.	Mm. of Mercury.	Kg./cm.^{2}.	Dynes/cm.^{2}.
Lbs./in.^{2} . . .	1	2·0360	6·804 $\times 10^{-2}$	5·171 $\times 10$	7·031 $\times 10^{-2}$	6·894 $\times 10^{4}$
In. of Mercury .	4·912 $\times 10^{-1}$	1	3·342 $\times 10^{-2}$	2·540 $\times 10$	3·453 $\times 10^{-2}$	3·386 $\times 10^{4}$
Atmospheres . .	1·4696 $\times 10$	2·992 $\times 10$	1	7·600 $\times 10^{2}$	1·0333	1·0132 $\times 10^{6}$
Mm. of Mercury .	1·9337 $\times 10^{-2}$	3·937 $\times 10^{-2}$	1·3158 $\times 10^{-3}$	1	1·3596 $\times 10^{-3}$	1·3332 $\times 10^{3}$
Kg./cm.^{2} . . .	1·4223 $\times 10$	2·896 $\times 10$	9·678 $\times 10^{-1}$	7·355 $\times 10^{2}$	1	9·806 $\times 10^{3}$
Dynes/cm.^{2} . .	1·4504 $\times 10^{-5}$	2·953 $\times 10^{-5}$	9·869 $\times 10^{-7}$	7·501 $\times 10^{-4}$	1·0198 $\times 10^{-6}$	1

§ (2) Primary Manometers.—These manometers are such that they can be constructed and calibrated without reference to any other form of pressure gauge. The most common form is that in which a liquid column of known density is supported by the pressure to be measured. The calibration depends upon the determination of the density of the liquid and the provision of an accurate scale of length. The corrections that may be necessary for the height of the meniscus capillarity and change of density with pressure can be applied from the physical properties of the liquid used, which properties are generally known.

The second class of primary manometer is the loaded piston, in which the actual pressure upon a small piston is weighed. This gauge depends only upon the accurate measurement of piston and cylinder and upon the accuracy of the weights used. Some gauges mentioned in this section as primary gauges require comparison with another standard in order to obtain the highest accuracy. As an example the closed mercury column requires correction for the deformation of the vessel containing the gas, which deformation may cause errors in the volume calibration; and the loaded piston gauge, in which the load is applied by a spring, requires at least a separate calibration of the compression of the spring which cannot be accurately calculated from its dimensions. In both these cases it may be more convenient to calibrate by comparison with an open column or dead weight piston gauge than to determine the unknown quantities independently. They are treated, however, in this section owing to their similarity to the other types of gauge here described.

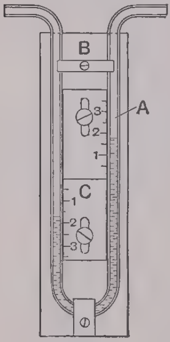

FIG. 1.

§ (3) OPEN LIQUID COLUMNS.—If the difference in height between the top of a column of liquid open to the atmosphere and the surface subjected to the pressure is observed, the pressure above the atmospheric pressure can be determined when the density of the column at every point is known. Whatever the liquid employed the temperature must be observed, in order to determine the mean density, and a correction must be made for the meniscus at the top of the column, and for the temperature of the scale used. For small pressures the liquid employed is generally water, and the simplest form is that used for measuring the pressure in gas mains or the pressure in the bellows of an organ. The gauge can be constructed in a few minutes, since it consists of a piece of glass tubing A (*Fig.* 1) fixed to a board B, with a scale C capable of small adjustment vertically. The pressure is represented by the difference in height between the two columns of liquid, and if the pressure is stationary the meniscus will be similar in each tube, so that no correction is required, provided that a similar part of the meniscus is observed in each case. The scale is usually marked in half-size units, so that when it is set to read zero for zero pressure, the pressure is determined by the reading of either column without reference to the other. It must be noted that if the zero is set to the top of the meniscus, the top of the meniscus must be observed in all other observations, and similarly for other settings of the zero.

In practice the unit of pressure used is the unit of height of the column; for example, an organ-builder speaks of the "weight" of the wind being 4 inches of water, meaning that a gauge of this type, filled with water, shows 4 inches difference in height between the two columns; and the Board of Trade regulations state that a gas company must supply gas to its customers at a pressure of at least $\frac{10}{10}$ inches of water. One inch of water corresponds to ·036 lb. per square inch approximately, the exact figure depending upon the temperature of the water.

This gauge must be in a truly vertical position when in use, otherwise the reading will be incorrect. If the gauge is set to zero correctly and afterwards becomes tilted at an angle α from the vertical, the true pressure height $h = \mathrm{H}\cos\alpha + d\tan\alpha$, where H is the difference of reading on the scales and d the distance between the tubes. If the scale lines are marked at right angles to the tubes the zero cannot be set unless the gauge is vertical, for any tilt shows the error $d\tan\alpha$, by the difference of head when the pressure is removed.

For higher pressures the only suitable liquid is mercury, and a column of mercury is usually employed by makers of gauges as their absolute standard of pressure up to 300 or 450 lbs. per square inch (20-30 atmospheres). An exceptionally high mercury column has been installed upon the Eiffel Tower at Paris.

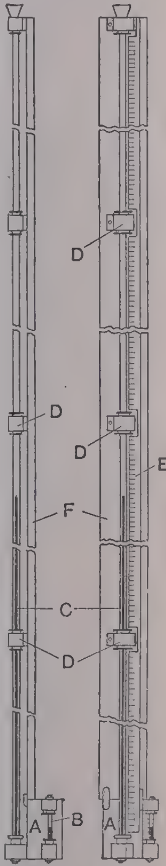

FIG. 2.

A similar construction can be employed to that described above, and for pressures up to one atmosphere this is often adopted. For larger pressures it is usual to employ one column and a large reservoir for the mercury at the bottom. *Fig.* 2 shows a mercury column of ordinary construction for measuring pressures to 200 lbs. per square inch, or

14 atmospheres. A is the reservoir for the mercury and B a small gauge glass for indicating the level in the reservoir. C is the column, D joints in the column, and E the scales, and F the board upon which the parts are fixed, and which itself must be firmly fixed to a vertical wall. Glass tubing suitable for the purpose can be purchased in 10-ft. lengths, but these have to be carefully selected, since the majority of tubes are not sufficiently straight. A joint such as D occurs at every 10 feet, and it is a great drawback to the usual construction that a certain piece of the column cannot be observed at each joint. This can be overcome at considerable expense by duplicating the tube, providing a second column from the same reservoir with the joints in positions differing from those in the first column. Another and better way is to construct the tube as shown in *Fig.* 3, where the main tube C is of steel, and at each joint a valve V is provided, allowing the mercury to flow into a glass tube G alongside, which is provided with a scale E. When one tube is full, the valve is closed and the readings for higher pressures obtained in the next tube above. Each tube overlaps the one above, so that the scale readings can be checked. It is more expensive than the usual form, and requires a double quantity of mercury, while provision must be made for catching the overflow from all the gauge glasses if the pressure is accidentally raised to too great an extent; but the convenience of being able to obtain continuous readings and the less cost of replacing a broken gauge glass compensate for the extra cost of construction.

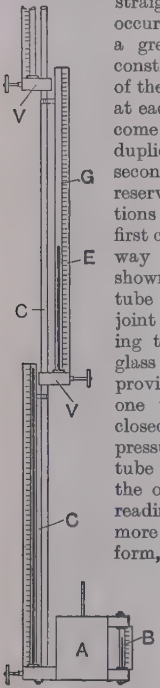

FIG. 3.

The best fluid for applying the pressure to the column is air, owing to its cleanliness, and also because a very small correction for the height of the air column above the liquid is required. The air should be contained in a gas bottle provided with a valve capable of very fine adjustment. However slightly the valve is opened, the pressure will gradually rise to the pressure in the bottle, so that when the desired pressure is reached the valve must be completely closed. When the column is used for calibrating other pressure gauges, it is advisable to provide a small leak valve, capable of fine adjustment, and to keep the pressure valve slightly open. When the column reaches the desired height, the leak valve is carefully opened to a small extent, and the pressure can thus be kept perfectly constant while observations are made. The inertia of a column of mercury is very considerable, so that it is not possible to obtain accurate readings while it is in motion.

An attachment, which is seldom fitted, is a means of lowering the mercury below the zero point when the apparatus is to be left out of use for some time. There is a definite chemical action of the mercury on the glass which causes the glass to become clouded at the surface of the mercury after a time, and this rapidly prevents the position of the zero being observed. It is always necessary to check the zero level in the column and in the reservoir, since this depends upon the quantity of mercury present. One method of preserving clearness at this position is to leave the column with a small pressure, say $\frac{1}{4}$ atmosphere in at all times, so that the clouded positions are clear of the zero points.

It is an advantage to have all the tubes of the column of the same size, and a reservoir of uniform section; for the depression of the zero is then proportional to the height, and an observation of the level in the reservoir is unnecessary at every reading. The reservoir being much larger than the tube, the meniscus in the reservoir is different from that in the tube. The reservoir is always so large that no correction is required for the height of the meniscus; but the gauge glass showing the level in the reservoir is invariably so small that the correction for the meniscus is greater than for the column tube. If, however, the column is always used for stationary pressure and the zero is set for definite positions on the meniscus in both tubes, no error occurs if all readings are taken at the same position of each meniscus. If the column is in motion, even to a small extent, an error is introduced, for one meniscus is flattened and the other raised by the motion. The amount of this error can be determined by taking observations with the pressure rising slowly and then with the pressure falling slowly. With the ordinary size of tube (10 mm.) and a movement of 1 cm. per minute, the difference will be found to be about 2 mm.

The height of the meniscus is determined by the size of the tube and by the condition of the surface of the walls. It is advisable to use as large a tube as possible, but the larger the tube the greater the inertia of the column, which in some cases is a disadvantage. In any case, whether the tube is large or small, time taken in the careful cleaning of the walls of the tubes is well spent.

In most cases a mercury column is erected passing through several floors of a building, and the temperature varies greatly as different parts. The temperature can be equalised to a certain extent by forcing the mercury to the top several times and allowing it to fall rapidly in order to mix the mercury up in the reservoir. The temperature of the mercury in the reservoir can then be taken as representing the mean temperature of the column. The scale is generally constructed of steel, and if the scale is correct at $\theta_1°$ C. the divisions will be larger as the temperature increases and will compensate in a small degree the change in density of the mercury. Thus the reading reduced to $\theta_1°$ C. will be $H(\rho_1/\rho_2 - \alpha(\theta_2 - \theta_1))$, where H is the observed height, ρ_2 the density of mercury at temperature $\theta_2°$, and ρ_1 the density at the standard temperature $\theta_1°$, and α the coefficient of expansion of the scale. At ordinary room temperatures the simple expression

$$H_1 = H_2\{1 - (\cdot 0001818 - \alpha)(\theta_2 - \theta_1)\}$$

may be used to give the corrected height H_1 at the standard temperature θ_1. In order to obtain great accuracy the whole column and scale should be enclosed in a water bath kept at constant temperature; but this is not convenient except in the case of very short columns.

The difficulties experienced in the use of columns are: (1) the correction for temperature; (2) the meniscus correction; (3) the inertia of the column when the pressure changes; and (4) the elimination of parallax errors in reading off the levels.

The first of these has been dealt with, and the meniscus correction when necessary can be made from the tables of capillarity depression. The error due to a moving column cannot be exactly allowed for; so that for great accuracy a steady pressure must be maintained. The parallax error makes it necessary that a lift be provided to carry the observer to the exact height required. Even then great care and experience is required to avoid small errors of this kind.

The error due to parallax can be completely avoided by dividing the scale on a piece of silvered glass and placing it behind the column. The surface of the mercury and its reflection together with the nearest scale division and its reflection can then be made coincident. The first impression of such a scale is a confused mass of lines, but a few minutes' practice enables an observer to place his eye in the position to obtain the necessary coincidences of lines and reflections. These scales are easily made with a dividing engine, either by ruling the lines with a diamond or by coating the glass with wax in which the lines are cut with steel tool and then etching the glass. When the lines are cut with a diamond it is unwise to cut up to the edges of the glass or fracture may easily occur. Etched lines can be carried to the edge since the etched line is not so sharp as the diamond scratch.

When a mercury column is required to give certain definite pressure readings only it can be made with a steel containing tube into the walls of which insulating plugs each carrying a platinum wire are inserted. These platinum wires then act as contacts which by completing an electric circuit indicate when that particular height is reached. This arrangement is very convenient for marking off the dials of pressure gauges in numbers, the current operating a magnet which marks the position of the index hands on the dials and at the same time operating a mechanical switch which cuts out the circuit and makes the circuit of the contact next above. In this case the pressure is generally kept slowly increasing, the error due to the inertia of the mercury in the column not being of importance in the marking of gauges for general use.

In calculating the total pressure due to a column of liquid it is clear that the total volume of mercury is not defined by the height to the top of the meniscus or the height to the bottom of the meniscus. That is to say, the volume of the meniscus should be allowed for. The meniscus height depends upon the condition of the surface of the glass and the bore of the tube.

A further correction is needed for the capillarity of the tube which occurs with the liquid employed. If the liquid wets the glass, as in the case of water or oil, the liquid stands too high and a negative correction is required for the capillarity. If, on the other hand, a liquid such as mercury is used, which does not wet the glass, a positive correction for the capillarity is required, the liquid standing at too low a level.

It is, in most cases, possible to avoid all corrections of this kind, for if the gauge glass is of uniform section and cleanliness, the correction will be constant, and if the same portion of the meniscus is observed at all times the correction is eliminated.

In any mercury column the joints between the lengths of glass tubes require careful making. The stuffing boxes for the tubes are generally packed with india-rubber rings, which prove quite successful. All metal fittings must, of course, be of iron or steel, and if joints are required between such pieces, care must be taken that the jointing material does not break away on the inside, all such pieces of dirt floating to the top of the column and spoiling the surface of the mercury. Plane-faced metal joints can be used, but require considerable mechanical skill in manufacture. A metal cone joint can be used without packing. If packed joints are used, a plain piece of paper, or paper soaked in gold size, is satisfactory. In the latter case, the gold size must be allowed to set hard before the mercury is introduced. A spigoted joint with a paper ring can be made perfectly satisfactory for any reasonable pressure (see *Fig.* 4). Valves must be designed so that no air-pockets can be formed in the column of liquid. Mercury column tubes should not be less than 1 cm. bore. Although the initial cost of the mercury is increased by increasing the bore, yet the saving due to the ease of cleaning and the greater convenience due to the smaller meniscus compensate for the greater initial cost.

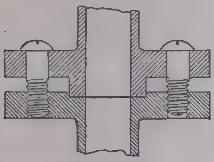

FIG. 4.

§ (4) MULTIPLE LIQUID COLUMNS.—In most laboratories it is not convenient to erect an open mercury manometer for more than about 20 atmospheres owing to the height required. For higher pressures three methods of reducing the height of the column can be used: (1) A series of mercury columns with compressed gas between them; (2) a compound mercury column with a series of U tubes filled partly with mercury and partly with a less dense liquid; (3) a closed mercury column or compressed gas manometer.

A Series of Mercury Columns.[1]—This form of gauge has been constructed by Professor Kammerlingh-Onnes at Leiden University, the maximum pressure available being 100 atmospheres. The principle of this gauge is shown in *Fig.* 5, where 1, 2, and 3 are mercury manometers of the usual type and which need not be of the same height. The pressure is applied to the left-hand tube of 1 and will force the mercury up in the right-hand tube which will compress the air between A and B. The total pressure at the left hand of 1 will be the sum of the differences of height of the mercury in the series of U tubes. The compression of the air in the intermediate parts of the tube would, however, be so great that a small part only of the total range of every syphon except the first could be utilised. This difficulty is obviated by admitting air at high pressure to the intermediate spaces until the maximum difference in height between the mercury columns in the various syphons is obtained. The pressure of the air in each column varies, but all the spaces can be filled to the required amount from a high-pressure gas bottle and a reducing valve. There is a correction to be applied for the weight of the air column in each tube, which correction can easily be estimated since the pressure in each space is known.

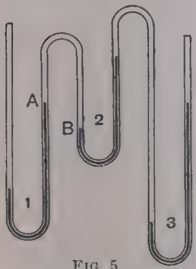

FIG. 5.

Fig. 6 shows the end manometer and three intermediate manometers of the Kammerlingh-Onnes gauge. Each intermediate manometer is used to read a pressure difference of four atmospheres while subdivisions of this pressure are read upon the left-hand manometer. The whole gauge contains 24 intermediate syphons like B, one end syphon A and also a small syphon for final adjustment reading ± 2 atmospheres. The whole is mounted upon the wall of the laboratory and the control valves placed at a convenient height. The large portions of tubes B are made of suitable thickness to withstand the maximum pressure reached in them and the bore varies from 6½ to 9 mm. according to the pressure. The observer is further protected from hurt if a tube bursts by a screen of plate glass. The capillary connecting tubes are small and strong enough to stand any reasonable pressure. To obtain an observation the level of the mercury in the various bulbs of tubes B is adjusted as nearly as possible to four atmospheres for each tube and the subdivision measured on a scale by the tube A. When filled with hydrogen, the correction for the head of the gas column amounts to 9 mm. for a pressure of 36,000 mm. of mercury and to 21 mm. for a head of

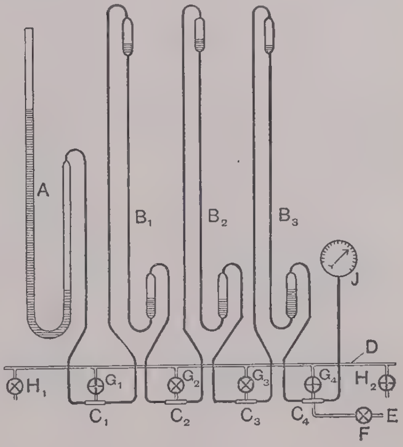

FIG. 6.

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[1] Communications from the Physical Laboratory of the University of Leiden, No. 44, 1898.

76,000 mm. The correction is thus small and the accuracy with which a large pressure can be read is very great. Care is required in the cleaning of the tubes and drying of the gas used and the adjustment to any desired pressure is a tedious process, but the accuracy of the readings is probably far greater than can be usefully employed.

§ (5) Compound Mercury Column.—In this gauge the intermediate spaces between the mercury columns are filled with a less dense liquid which compresses but slightly and the adjustment of the volume of liquid in the spaces is unnecessary. A compact form of this gauge is shown in *Fig.* 7, where the U tubes A are made up of straight glass tubes and metal joint boxes, arranged diagonally so that the upper column appears on the one side of the apparatus and the lower column of each tube on the other side. Each tube is 10 feet long so that each syphon gives a pressure of approximately four atmospheres. A number of valves V are provided, those at the top for filling the upper part of the columns with air-free oil or water and those at the bottom for adjusting all the mercury columns to the same height at zero pressure. The number of syphons in use is varied by opening the valve V at the top of the last column to be used. If n columns are in use the total pressure $P=np$ where p is the pressure read by each separate syphon. A correction must be applied for the compressibility of the oil or water used and the need for the correction is shown by the variation in the height differences in the syphons. Two measuring microscopes M are provided, one on each side of the apparatus. The adjustable frames carrying these are adjusted to the height required and the difference in the columns read off by means of an eyepiece micrometer.

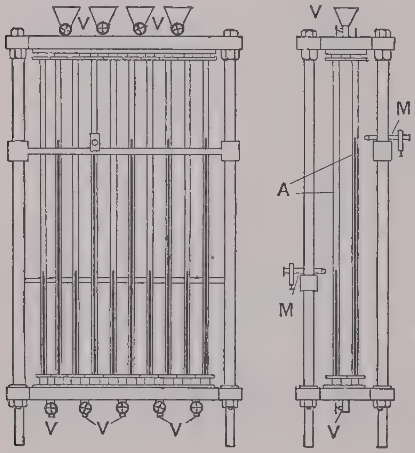

Fig. 7.

The value of the height readings of a compound mercury column in units of mercury can be represented by $H=h(1-\rho_2/\rho_1)$ where h is the observed height, ρ_1 the density of mercury, and ρ_2 the density of the liquid. The ratio ρ_2/ρ_1 will vary at different pressures because the compressibilities are not identical. The mean density for the whole column will be one half the maximum density and the compressibility of mercury cannot be neglected at high pressures if great accuracy is required.

An open mercury column of great height must be corrected for the mean density of the mercury. The compressibility of mercury is $3{\cdot}76\times10^{-6}$ per atmosphere, so that the observed height should be multiplied by the following constant at the pressures stated:

Pressure in Atmospheres.	Multiplier.
20	1·000038
50	1·000094
100	1·000188
150	1·000282

The correction amounts to 3 parts in 10,000 at 150 atmospheres, a quantity which is negligible for most work.

The compressibility of the second liquid used in a compound column is more than that of mercury and the true height in mercury of standard density is

$$H=h\left\{1-\frac{\rho_2}{\rho_1}(1+\tfrac{1}{2}A(C_2-C_1)\right\}$$

very nearly. Where A is the pressure in atmospheres and C_1, C_2 the compressibility of mercury and the liquid respectively.

The following table gives the value of H/h for compound columns filled with (1) mercury and water, and (2) mercury and olive oil.

Densities at 15° C – water 0·9991; olive oil 0·92; mercury 13·558. Compressibilities per atmosphere – water $49{\cdot}5\times10^{-6}$; olive oil $63{\cdot}3\times10^{-6}$; mercury $3{\cdot}76\times10^{-6}$.

Pressure.	$\frac{H}{h}$.	$\frac{H}{h}$.
0	·92632	·93229
50	·92623	·93219
100	·92614	·93209
150	·92605	·93199

From the table above it will be seen that for pressures up to 150 atmospheres the multiplier is constant to 1 part in 1000, and that it is only for much larger pressures or for a higher order of accuracy that the compressibility is of great importance.

It must be noticed that it is assumed in the above that all the mercury surfaces are covered with the second liquid. This need not be the case in the first and last tubes; for the last may be open to the atmosphere and the first to the pressure supply which may be a gas. When several tubes are in use, however, the error produced (which can be allowed for if necessary) is very small.

The following table gives the constant of a compound gauge of mercury and the other liquids mentioned. The values H/h are for atmospheric pressure, but the compressibility is given so that the constant at other pressures can be calculated as shown above.

Table of Densities and Compressibilities of Liquids

Liquid.	Density (15°).	$\frac{H}{h}$.*	Compressibility per Atmosphere.
Alcohol (Ethyl)	·792	·9416	77×10^{-6}
Chloroform . .	1·525 (0° C.)	·8876	$9{\cdot}5 \times 10^{-6}$
Glycerine . .	1·26	·9071	25×10^{-6}
Mercury . .	13·558	1	$3{\cdot}76 \times 10^{-6}$
Olive Oil . .	·92	·9323	$63{\cdot}3 \times 10^{-6}$
Paraffin . .	·80	·9410	$62{\cdot}7 \times 10^{-6}$
Water † . .	·9991	·9263	$49{\cdot}5 \times 10^{-6}$

* $\frac{H}{h} = \left(1 - \frac{\text{density of liquid}}{\text{density of mercury}}\right)$ at about 15° C and atmospheric pressure.

† The compressibility of water falls as the pressure rises. At 1000 atmospheres it is about 37×10^{-6}, and at 2500 atmospheres about $\cdot 26 \times 10^{-6}$. The figures given for all substances will be found sufficiently correct for pressures to 200 atmospheres.

§ (6) Closed Mercury Column or Compressed Gas Manometer.—In this gauge the compression of the gas is the measure of the pressure and the mercury column is used as an indication of the volume of the gas. In *Fig.* 8, A is a strong vessel of steel in which is fixed a glass vessel B, the upper end of which is a tube protruding from A through a stuffing box. At the lower end of B is a small curved tube filled with mercury and acting as a seal to prevent loss of the gas enclosed in B. The upper part of B is graduated and the volume from the top to each graduation and the whole volume must be determined. The vessel A is filled with sufficient mercury to reach above the small tube at the bottom of B and to fill the tube B. When pressure is applied to the mercury in A, it enters the tube B and compresses the gas into the upper graduated end. No reading can be taken until the gas is sufficiently compressed for the mercury to appear in the exposed part of the tube. The volumes of the lower part and tube must be arranged so that for the range of pressure desired the mercury can be seen in the tube. It is sometimes necessary to add a bulb at the top of the tube for this reason.

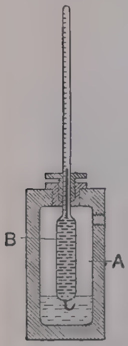

Fig. 8.

If the gas obeys Boyle's law and is compressed at constant temperature, the pressure in atmospheres is given by $P = V_1/V$, where V_1 is the volume of gas at a pressure of one atmosphere and V is the observed volume. The manometer is best filled with hydrogen for which gas the departure from Boyle's law can be represented by a simple expression, and $P = 1/\{(V/V_1) - \cdot 00066\}$ atmospheres.

The correction for temperature is somewhat difficult and should be eliminated by immersing the apparatus in a bath at constant temperature. The height of the column of mercury varies with the pressure and must be added to the pressure obtained above.

This gauge is not capable of very accurate observations over a large range since the scale length is limited, but the gauge is fairly compact, and if proper care is taken in the measurement of the volumes, in filling with pure and dry gas, and in keeping the temperature constant, it is a reliable primary gauge.

§ (7) Loaded Piston Pressure Gauges.—In these gauges a piston of known area is loaded with a measured weight from which the pressure acting upon unit area of the piston can be calculated. For this purpose it is necessary to construct a cylinder and closely fitting piston, free from friction and leakage. The load is sometimes stationary and applied by a lever to the top of the piston, a pivot or ball being used to reduce the friction when the piston is rotated. The rotation of the piston is necessary in order to reduce the friction. The friction is further reduced if the load is allowed to revolve with the piston, which eliminates the frictional loss behind the point of the piston This friction does not directly influence the pressure measurement but increases the power required to maintain the rotation of the piston. In order to obtain accurate measurements, it is advisable to have no rotating force beyond the inertia of the piston and load while making observations. The pressure is proportional to the load and inversely proportional to the area of the piston, so that for high pressures the load must be very large or else the area must be very small. If the area of the piston is small it may not be strong enough to support the load, but on the other hand, if the load is large the instrument becomes unwieldy and slow in operation.

This difficulty has led to the construction of differential pistons in which the pressure is applied between two pistons of slightly different area, and in one case at least the gauge has been constructed with the weight hanging on the piston, thus putting it in tension instead of compression. This arrangement gives two leakage areas instead of one, which is a disadvantage.

Several types of this gauge are shown in *Fig.* 9. (*a*) shows the simplest form in which the piston is in compression, and must be made stiff enough to withstand the load. A is the cylinder, B the piston, and C the load, and it is clear that the piston and load can be rotated without any difficulty. (*b*) is a similar gauge loaded by means of a lever E. A ball bearing should be interposed between the pivot and the piston so that the latter can be freely rotated. This form generally requires that the piston be mechanically rotated continuously, which slightly reduces the sensitivity. (*c*) shows a similar gauge loaded with a spring C_1 and has some disadvantages in that the spring must be accurately calibrated and, unless its motion is greatly magnified, gives a very short scale. It also requires a much longer piston than the other patterns, and this long piston must be of uniform area. (*d*) shows a differential piston in which the pressure is applied to the piston B_1 working in cylinder A_1 and transmitted to piston B_2 in cylinder A_2, the pressure on which may be measured by a mercury column or other means. This gauge has two leakage surfaces, and the two pistons must be perfectly in line. Some complication is needed to obtain the desired rotation of the pistons. (*e*) shows a differential gauge in which the piston is turned to two diameters, B_1 and B_2, so that the effective area is the difference between the areas of the two parts. This arrangement allows the piston to be put in tension instead of compression, and since the effective area may be very small, the load may be very small for a large pressure. It requires very accurate workmanship to keep the two parts of the piston and of the cylinder truly axial, it has two leakage surfaces, and requires extreme accuracy in the measurement of the piston diameters to keep the difference reasonably accurate. (*f*) is an inverted form of (*e*), which has been used with some success for pressures up to 3 tons per square inch, but since the pistons cannot be rotated, a small oscillation of less than 120° being possible, the gauge is not at all sensitive.

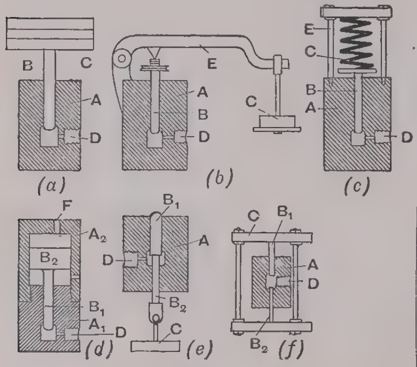

FIG. 9

Of all these types (*a*) is the best, being the easiest to construct and suitable for the highest pressures if care is taken in manufacture. It is also an extremely sensitive gauge, and in ordinary work will measure pressures more quickly than a mercury manometer and with equal accuracy. It is not possible to make this gauge very sensitive for pressures below 4 atmospheres, however, and for pressures up to this a mercury manometer is better.

The type of gauge to be described is that which has proved most accurate and sensitive, and is in use for measuring pressures from 30 lbs. to 24,000 lbs. per sq. in. It consists of a plain ground piston of hardened steel, fitting a lapped hole in a steel cylinder. The motion of the piston for the small sizes is restricted to $\frac{1}{8}$ in. in order to reduce the length, since the intensity of stress is very high under high pressure. The pistons vary in size from 0·2 sq. in. for measuring pressures up to 1200 lbs. sq. in. (80 atmos.) to 0·01 sq. in. for pressures up to 24,000 lbs. sq. in. (1600 atmos.). The largest one will measure pressures as low as 30 lbs. sq. in. (2 atmos. with fair sensitivity).

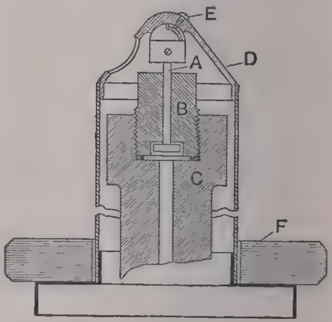

FIG. 10.

Fig. 10 shows the details of instrument. A is the piston provided with a button at the lower end to prevent it from leaving the cylinder, and a separate head fixed to the piston by a grub screw. B is the cylinder turned out of a piece of tool steel and mounted on column C of mild steel attached to the test apparatus. The spherical head of the piston is slotted on one side and carries a brass or aluminium tubular frame D with a ledge at the lower end. The head of the frame has a small pin E entering the slot in the spherical head of the piston, and thus ensuring that frame and piston rotate together. The load consists of annular weights of cast iron passing over the frame and resting on the ledge. Openings are made in the upper part of the frame, so that the position of the piston can be observed.

The instrument is very simple in construction, and the only difficulty in manufacture is the grinding of the piston and lapping out of the cylinder. The part of the piston working within the cylinder must be parallel, and it is advisable to have it of exact size, so that the area may be a simple fraction of an inch. The weights are then carefully calibrated weights of standard values, and no correction factors are needed. The smallest piston referred to above is ·11286 of an inch in diameter, and for all the working part of the piston this piston is circular and parallel, but it is not easy to make the piston to this accuracy, since the last figure is not measurable by any but measuring machines of the highest accuracy. The piston in question was one of two which were made together and gradually reduced to size by cautious lapping until the figure required was obtained. The second one, which was rejected, varied but little from ·11286 and would have been accurate enough for any ordinary work, but not for a standard of pressure. The cylinder was lapped out until the piston could just slide in by pushing, but without driving. The cylinder cannot be measured directly, and would be assumed to be the same size as the piston, which fitted it thus tightly. An attempt was made to measure it by driving a very soft copper plug into it and then measuring the plug. The plug was found to measure ·11286 of an inch, or the same measurement as the piston. The leakage from these pistons is found to be exceedingly small, and the friction is also very small. If the piston is rotated with, say, 100 lbs. upon it, it will continue in motion for some ten minutes, during which time the pressure remains constant. This has been examined by means of an ordinary pressure gauge, with a microscope adjusted to show any movement of the hand. With a pressure of 20,000 lbs. per sq. in. it was found that the hand of the gauge remained stationary while the piston was rotating, but sharply took a new and definite position on the addition of 0·01 lb. to the load, equivalent to 1 lb. per sq. in.

A gauge of this kind must work with oil, as the lubricant for the piston, and if the oil is at all acid, corrosion of piston and cylinder will take place, thus altering their dimensions. The best oil has been found to be the tasteless castor oil sold for medicinal purposes. The oil must be freed from air, otherwise the sensitivity of the gauge is reduced very considerably and it is very difficult to get the minute oil bubbles out of the castor oil used. In time, however, the air bubbles gradually leak out through the piston, and although the sensitivity may be small when the gauge is freshly filled, if a high pressure is kept on for a few hours and the piston kept rotating, the air bubbles will be removed and the sensitivity increased.

For use on pressure gauge testers which are filled with water a syphon arrangement must be used in order to keep the water away from the piston.

The effective area of the piston may be altered by the strain of the piston and cylinder under very high pressure. The piston is subject to constant compression axially, and to varying compression radially, while the cylinder is enlarged by the internal pressure varying from the full pressure at the bottom to zero at the top, and also to axial stresses varying according to the manner of fixing. The effective area of the small piston referred to above is calculated to be one part in 10,000 more at 20,000 lbs. per sq. in. than at small pressures.

Gauges of this type have been constructed by Mr. Bridgman of still smaller diameter and for measuring higher pressures.[1] The pistons of these gauges were very small and were not specially ground to fit, suitable pieces of hard steel wire being selected. To reduce the leakage, the fluid used was a mixture of molasses and oil.

§ (8) Secondary Manometers.—These gauges must be calibrated by comparison with a primary gauge, since the properties upon which they depend cannot be determined with sufficient accuracy to enable their true calibration to be calculated. Most gauges of this type depend upon the elastic deformation of some metal part, and it is often possible to copy the dimensions adopted for a calibrated gauge in order to reproduce similar gauges having the same calibration.

The types of gauge described later for determining the maximum pressure to which they have been subjected are generally calibrated by comparison with a primary gauge or a calibrated secondary gauge, although those depending upon the reduction in volume of a confined volume of air are capable of construction as primary standards. The difficulty of determining the volumes and the deformation of the envelopes makes it a more practical matter to obtain the calibration by comparison with another type of gauge. The methods of measurement mentioned in this chapter are those chiefly used, but there are many other properties of materials which might be utilised for the purpose under special circumstances.

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[1] *Am. Acad. Proc.*, 1909–10, xliv. 8.

§ (9) Elastic Gauges.—The majority of pressure gauges used belong to this type and depend upon the elastic properties of some metal part. In all cases it is most important that the elastic limit of the material is not approached, since this will lead to a continual change in the zero of the gauge. They all show a certain "hysteresis" effect in that the calibration as the pressure increases does not agree with the calibration as the pressure decreases. This effect is small if the maximum stress in the material is small. A small stress necessarily entails a small strain and consequently the actual motion produced has to be magnified by mechanical or other means in order to obtain a readable scale. The mechanism employed for the magnification of the motion introduces errors in the readings due to friction and general slackness of the parts. The errors arising from this cause are often greater than errors produced by imperfections in the elasticity of the deformed member. For special purposes it is always possible to obtain a more accurate gauge by dispensing with the mechanism and using an optical lever system. The great advantages of elastic gauges, however, are their compactness, portability, and general handiness, all of which are lost when the optical system is introduced.

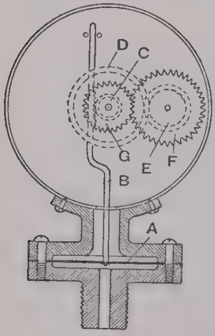

Fig. 11.

§ (10) Schaffer Diaphragm Gauge.—The simplest form of this manometer is the Schaffer diaphragm gauge (*Fig.* 11). The diaphragm A is subjected to pressure on its lower side and the motion of the diaphragm communicated to the rod B which carries a rack at its upper end gearing with the pinion C. This pinion is fixed to wheel D which gears with the small wheel E attached to wheel F which in turn gears with the small wheel to which the index hand is attached. This somewhat complicated gear is necessary owing to the small motion of the diaphragm permissible, and the great advantage of the Bourdon tube gauge to be described later is the more simple mechanism required. This gauge, however, is still occasionally used on traction engines and machinery subject to vibration, since the natural period of vibration of the diaphragm is much less than that of a Bourdon tube suitable for the same pressure. For laboratory work this type of gauge has some advantages in that it can be constructed in any ordinary workshop, and by the substitution of an optical lever for the gearing and index an open scale of pressure can be obtained.

§ (11) Bourdon Tube Gauge.—The most common form is the Bourdon tube gauge, which is used universally for measuring large or small pressures. In this gauge the pressure is applied to the inside of a tube of bronze or steel, generally oval in section and bent into the arc of a circle. The effect of the internal pressure is to increase the radius of curvature of the tube, and one end being fixed to the case, the motion of the other end is utilised to operate an index hand by means of a rack and pinion. The limit of elastic deformation must not be approached, otherwise the tube will take a gradual permanent set and the gauge readings will increase. The gauge is shown in *Fig.* 12, where B is the tube fixed to the case at A. The free end of the tube is connected to a quadrant rack D by a link C. The rack D gears with a pinion E, on the shaft of which the index hand is fixed. A small restraint is added in the form of a hair-spring F.

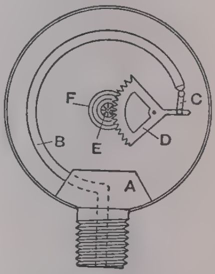

Fig. 12.

If the tube is made very stiff, the magnification must be large, and unavoidable slackness in the pin joints will produce uncertainty in the reading. In order to reduce the magnification makers generally allow the tube to deform more than is advisable, and most gauges are much more constant if worked to about 75 per cent of their designed load. A gauge of the cheapest type, calibrated to 75 per cent of its intended maximum, has been found to give exact repetitions of its reading for several years. The scale division of this gauge, however, was smaller than that of a standard gauge for the same pressure.

The Bourdon tube gauge is the most convenient pressure indicator for a large scope of work, because the gauge quickly records any change of pressure, which can be read off directly on the dial. These gauges can be purchased for any range of pressure, such as vacuum up to zero, vacuum and pressure from 0 to 10 lbs., and any range of pressure from zero to 5 lbs. per sq. in., or zero up to 12 tons per sq. in. The usual sizes made have

6″ and 10″ dials, but smaller and larger dials can also be obtained. The mechanism is not always increased in size for the larger gauges, the dials being enlarged and the index hand increased in length and breadth simply to enable them to be read from a considerable distance.

(i.) *The dials of gauges* of this kind must be carefully chosen. The index hand should move as close as possible to the dial, but must not touch it at any point of its motion. Considerable parallax errors can be obtained if the index hand is a small distance from the dial, and practice is required before an observer instinctively places his eye in the true position to avoid this. The index hand itself is made of considerable width for ordinary gauges for engineering purposes because it is more important that it should be easily seen than that the pressure should be observed with great accuracy. For laboratory work it is convenient to reduce the width of the pointer almost to a point and, generally speaking, the dial is better marked in dots than in lines. A very general fault is for the index hand to overlap the divisions, which renders accurate reading impossible. When greater accuracy is required, the pointer should be flattened to a knife edge, turned edgewise and a mirror let into the dial behind the flattened part of the hand. This enables very accurate observations to be made, and eliminates the error due to parallax. This type of dial can be obtained to special order.

Examples of the dials referred to are shown in *Fig.* 13, where (*a*) is the usual form of pointer, which is shown overlapping the divisions—a very common fault; (*b*) shows the fine point with the scale marked in dots, the best method of marking; and (*c*) shows the reflector scale type, which is recommended for special work.

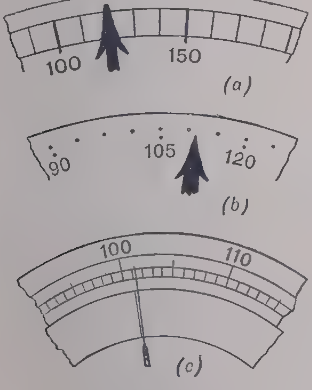

FIG. 13.

(ii.) *Overhauling a Gauge.*—If a new gauge is obtained, it may be found that its motion is irregular, the pointer setting at different places each time it is tapped. An examination of the mechanism may show that small particles of metal dust have been left in the case and have fallen between the pinion and the rack, and a thorough cleaning with a camel's-hair brush loaded with petrol may put the gauge right. If the fault still remains it may be that the rack and pinion do not mesh sufficiently closely in gear, a point which can sometimes be rectified. Or it may be found that the pinion shaft is not circular, which must be put right before the gauge will be satisfactory. It should be noted that the mechanism is somewhat delicate, and the index hand should only be removed by means of a special little drawing tool, as shown in *Fig.* 14. The jaws A press on the back of the boss of the hand, and the screw point B presses against the spindle. A turn of the screw will then remove the hand without bending the pivot. To replace the hand it must be pushed straight on in the correct position and then tightened by a tap with a very light instrument-maker's hammer; it is not possible to put the hand on loosely and then twist it into place without risk of damage to the mechanism.

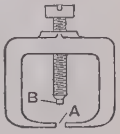

FIG. 14.

If an old gauge is suddenly found to behave erratically, it will generally be found that the tube is punctured, although the hole may be so small as not to be found easily. In such a case the gauge must be returned to the makers for a new tube to be fitted.

(iii.) *Correction for Temperature.*—If a pressure gauge is required to indicate small differences of pressure, it can be calibrated at different temperatures and the readings corrected accordingly. It is necessary to choose a gauge of good quality which repeats its readings nearly exactly whether the pressure is rising or falling, and to have the knife-edge index with mirror shown in *Fig.* 13 (*c*). The curve of corrections at any constant temperature cannot be represented as a mathematical function of the pressure, since such corrections are largely due to small errors in marking the scale of the dial. It has been found, however, that the alteration to the correction at any particular pressure due to change of temperature is a simple function of the temperature and pressure. As an example a 10″, 250-lb. gauge was calibrated at 65° F. as a standard temperature and its curve of corrections obtained. It was then calibrated at several temperatures between 50° and 100° F., and the change in correction Δc at temperature $\theta°$ F. was found to be $\Delta c = -\cdot 00018\ \mathrm{P}\ (\theta - 65)$, where P is the pressure in lbs. per sq. in. The following table shows the actual corrections at three temperatures:

TABLE OF CORRECTIONS

Temp. °F.	Pressure—lbs. per sq. in.				
	50.	100.	150.	200.	250.
50	−0·38	+0·04	+0·25	+0·31	+0·16
65	−0·51	−0·23	−0·15	−0·23	−0·52
90	−0·73	−0·68	−0·82	−1·13	−1·64

In practice it is often found that a pressure gauge is fixed a few inches from the hot boiler, and although a water syphon is always fitted to prevent the heating of the Bourdon tube by steam, the temperature of the whole gauge will often exceed 90° F. There will then be 1 lb. per sq. in. difference in the correction of such a gauge at 250 lbs. per sq. in. if the gauge had been calibrated at the ordinary temperature of a laboratory. The error thus caused is appreciable in efficiency tests, although its effect is not very large.

Bourdon tube gauges are always liable to change of zero, especially if they are subject to slight vibration such as occurs while they are carried about. When the zero can be observed any change in this will apply to all the corrections at other parts of the scale. If, as is generally the case, a peg is provided to arrest the index hand just above the zero point, the calibration will be doubtful after the gauge has been despatched by train or carrier. It is better to have the zero free by removing the peg and to place a piece of cork between the end of the tube and the case as a temporary stop during transport.

§ (12) SPURGE HIGH-PRESSURE MANOMETER.—The scale length of a Bourdon tube gauge is limited, being about 13 inches on a 6-inch dial and 22 inches on a 10-inch dial. The value of one-inch motion of the index becomes very great when the maximum pressure registered by the gauge increases. It is often desirable to read high pressures with much greater accuracy, or, in other words, to provide a much more open scale. This can be done by substituting a vessel in the shape of a tube or a sphere for the Bourdon tube, and measuring the cubical expansion or compression of the vessel. The measurement of the expansion can be best made by observing the displacement of a liquid either from the inside or the outside of the tube. *Fig.* 15 shows a manometer of this type constructed by Mr. Jas. Spurge. It consists of an outer cylinder A filled with water and a smaller cylinder B which communicates with the outside of the apparatus by a capillary tube C connected to the pressure supply. The cylinder A is filled with water, and on the pressure in chamber B increasing, some of the water in A is forced into the measuring vessel D. The vessel D is provided with a fiducial mark E, at which level the water is kept by operating a displacement piston F by means of a micrometer screw G. The amount of water flowing from A is thus measured by the rotation given to the micrometer head G. The water in A being at all times open to the atmosphere is not compressed, and as long as the whole apparatus remains at one temperature the ratio of the displaced volume to the total volume remains constant, except for the small error produced by temperature expansion of the metal cylinder. An additional plunger H is provided for adjusting the zero so that the micrometer head reads zero at zero pressure. The vessel B is subjected to axial and longitudinal compression so that the volume of water displaced is not exactly a linear function of the pressure. The calibration of gauges made from one quality of steel is, however, very constant. The gauge is very sensitive and can be made to read a pressure of 300 atmospheres to one part in 2000. The sensitivity depends upon the ratio of the area of the displacement piston to the volume of liquid in A and upon the use of a small tube for the fiducial mark E.

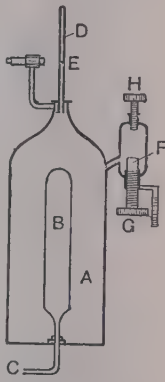

FIG. 15.

§ (13) ELECTRICAL RESISTANCE MANOMETERS.—These manometers depend upon the change in the electrical resistance of a metal when subjected to pressure. The metals used for this purpose are mercury, platinum, and manganin. The first must be contained in a capillary tube of non-conducting material, such as glass, and the solid metals are used in the form of wires. In the case of mercury the elasticity of the glass-containing tube affects the resistance of the mercury, the compression of the bore altering the length of the thread of mercury. Such a gauge must, for this reason, be calibrated by comparison with a primary manometer. Manganin wire is not very constant, its variation in resistance depending upon its previous history.[1] This must also be calibrated against a standard gauge. It is possible that a pure platinum wire may be sufficiently constant in its resistance variation to enable such a gauge to be used as a primary standard for very high pressures.

[1] *Amer. Acad. Proc.*, 1911, xlvii.

M. A. Lafay gives the resistance variation as follows :[1]

Mercury :

$$\frac{r-r_0}{r_0}=-32\cdot7\times10^{-6}p+1\cdot1\times10^{-9}p^2,$$

where p is in kg./cm.^{2}.

Platinum :

$$\frac{r-r_0}{r_0}=-1\cdot86\times10^{-6}p.$$

Manganin :

$$\frac{r-r_0}{r_0}=+2\cdot23\times10^{-6}p,$$

where p is in atmospheres, r_0 is the resistance at atmospheric pressure, and r the resistance at any other pressure. The positive sign for manganin is a peculiarity of this material.

Mr. Bridgman finds that a gauge of this kind repeats itself extremely well for increasing and decreasing pressures, as is shown by the following table :[2]

TABLE

Pressure in Kgm./Cm.^{2}.	Slider Displacement (Cm.).	
	Increasing.	Decreasing.
917	4·89	4·89
2018	10·17	10·19
3719	17·75	17·74
5348	25·70	25·65
6452	27·45	27·43

All that is necessary for the construction of this gauge is a vessel sufficiently strong to hold the pressure, filled with a non-conducting fluid and provided with insulated pressure tight terminals for taking the leads to the resistance - measuring apparatus. Provision must also be made for keeping the apparatus at constant temperature. The resistance wire or glass tube containing the mercury is attached to the inside ends of the insulated terminals and suspended in the fluid to which the pressure is applied. The change of resistance is not very great, so that a sensitive resistance bridge is required for measuring the change with accuracy.

§ (14) CRUSHER GAUGES.—When a single high - pressure measurement is required, a crusher gauge of the kind devised for the determination of the pressure obtained in the breech chamber of a gun can be used. The measurement depends upon the permanent compression of a small cylinder of copper. A number of these copper cylinders must be made from one sample of material, and a few of them chosen at random compressed at known loads in a testing machine. A curve can then be drawn showing the compression of the cylinder for any load.

Fig. 16 shows the form of the gauge. A is a steel cylinder in which slides a piston B, the upper end of which is subjected to the pressure to be measured. Leakage is prevented by a cup-shaped disc of soft copper C. The cylinder A screws into the body D in which the cylinder of copper E is placed. The cylinder E is kept in a central position by means of a light spring, not shown, which does not prevent the radial deformation of the cylinder. After the test the cylinder is removed, its compression measured, and the pressure read off from the chart. The actual load on the copper depends upon the diameter of the piston, and as the whole apparatus is self-contained, no external connections are needed, and it can therefore be used for the highest pressures attainable.

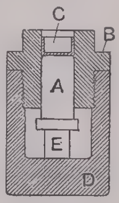

FIG. 16.

§ (15) SOUNDING APPARATUS.—Two forms of recording pressure gauges are in common use for determining the depth of water under a ship. One of these is a compressed-air gauge and consists of a metal vessel with a small opening for the admission of water provided with a non-return valve. This opening must be kept at the bottom of the apparatus as it sinks in the water. The vessel is initially full of air, but as the pressure of the water increases some passes the non-return valve, compressing the air inside until equilibrium is reached. When the vessel is hauled back to the ship the non-return valve prevents the escape of the water, and the volume of the water found in the gauge is a measure of the maximum pressure reached. This water is measured in a calibrated vessel, or by means of a divided scale dropped in the vessel, and the depth of water read off. The true depth varies with the amount of salt in the water and a correction must be made when the apparatus is used in river mouths or land-locked waters. After emptying, the sounding apparatus is ready for use again. The depth recorded is independent of the direction of the heaving line, but the inlet must remain in a downward position otherwise air will escape as the water enters and a false reading will be obtained.

The second form of sounding apparatus of this kind consists of a narrow glass tube closed at one end and coated on the inside with a chemical which will be coloured by the action of sea water. This is dropped in a suitable sinker with the open end of the tube downwards when the water enters the tube, compressing the air contained therein a certain distance depending upon the pressure. Chemical action then takes place, and when the tube is hauled back a measure of the coloured portion of the bore on a special scale will give the maximum pressure reached. These tubes are as a rule used only once, since the cleaning off of the coating and refilling is a difficult matter. The tubes are, however, very cheap to make and a large stock of them can be carried in a small space. The original length of the tube must be made exact, otherwise the scale will be incorrect.

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[1] *Comptes rendus*, 1909, cxlix. 566.
[2] *Amer. Acad. Proc.*, 1909–10, xliv. 8.

§ (16) SOUNDING TUBE BY M. BERGET.[1]—In this apparatus the compression of water instead of air is utilised. It consists of a large reservoir filled with water and closed by a capillary tube silvered on the inside surface, which contains a thread of mercury and is open to the outside. As the gauge descends the water is compressed and the thread of mercury moves along the capillary, dissolving the silver lining and thus giving a permanent record of the maximum depth obtained. The sensitivity can be increased by enlarging the reservoir and providing a smaller capillary. The temperature of the reservoir must be known and can be determined by a recording thermometer affixed to the apparatus.

§ (17) MEASUREMENT OF CYCLES OF VARYING PRESSURE.—For observations of a continual change of pressure the elastic gauge in some form is generally used. For slow changes a continuous photograph of a mercury column can be taken upon a clockwork drum covered with sensitive paper, but such an arrangement can only be used to record very slow changes. For such work it is better to use an ordinary gauge, in which the index hand is replaced by a marking-point moving upon a paper disc or drum, rotated by clockwork. This apparatus can be readily obtained from pressure-gauge makers.

When the changes of pressure are more rapid, such as those occurring in the cylinder of a steam or gas engine, a different method is necessary.

§ (18) STEAM-ENGINE INDICATORS.—The ordinary steam-engine indicator consists of a small cylinder, containing a freely moving piston, the motion of which is restrained by a calibrated spring. A rod is attached to the piston, and the end of this, either directly or through a magnifying motion, records the pressure upon a paper drum. The drum can be driven by clockwork but is generally given a reciprocating motion from the crosshead of the engine, so that the pressure is recorded with reference to the position of the piston, and the area obtained is a measure of the work done.

The choice of the size of the piston and the amount of motion allowed to the spring depends upon the speed of the engine. The inertia of the piston is more with a large piston, and with the larger motion, but the accuracy with which the record can be measured is proportional to its height.

(i.) *Richards Indicator.*—For slow speed steam engines this indicator (*Fig.* 17) is perfectly satisfactory. It consists of a cylinder A in which the piston B, restrained by a spring C, moves. The motion is transmitted through a simple Watt parallel motion to the point O, which records on the drum E. The marking-point may be of gun-metal, when special metallic paper must be used upon the drum, or a hard lead pencil marking upon ordinary paper can be used.

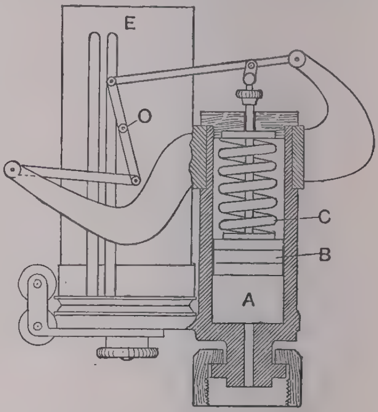

FIG. 17.

(ii.) *Crosby Indicator.*—As the speeds of engines increased, error, due to inertia of the parts, increased, and other indicators, of which the Crosby is a type, were introduced. In these the motion of the piston is reduced and greater magnification is provided by specially designed link motion. The Crosby indicator is shown in *Fig.* 18, in which the

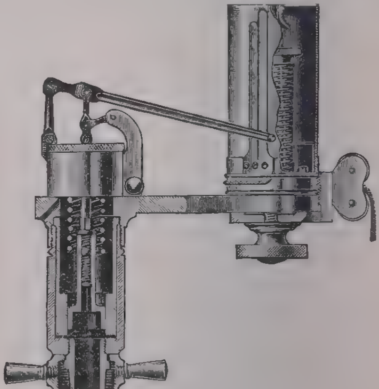

FIG. 18.

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[1] *Comptes rendus*, 1914, clviii. 1465.

parts are similar to those of the Richards indicator, except in the link motion, which gives a magnification of 5 to 1 instead of 2 to 1.

When high steam pressures and superheated steam came into use, the position of the spring under the cylinder rendered it liable to change, owing to the temperature which it attained. This became of greater importance still when the use of the explosion engine became general. Several indicators have been devised in which the spring is placed outside the hot cylinder.

(iii.) *Dobbie-McInnes Indicator.*—This example of an external spring indicator is shown in *Fig.* 19. The spring is outside the cover of the cylinder, which prevents it from being overheated, and the spring diameter is not limited by the diameter of the piston. It is possible to use very stiff springs and very high pressures can be recorded. These indicators are satisfactory for speeds up to about 400 or 500 cycles per minute. For higher speeds a smaller piston, only one quarter of a square inch in area, is used, and the height of the diagram is reduced to $1\frac{1}{2}$ inch. For small explosion engines the piston is reduced in area still more and the height of diagram limited to 1 inch. These small indicators can be used with fair success upon petrol engines running at 1000 or 1200 cycles per minute; but for this speed the optical indicator is a better instrument, although more cumbrous.

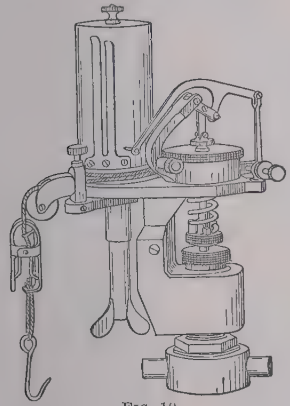
FIG. 19.

The faults of records obtained from indicators of this type at high speeds are due to two causes: the first is the inaccuracy of the height or pressure ordinate of the diagram, due to the inertia of the piston and link motion of the indicator; and the second is the inaccurate positions of the pressure ordinates due to the inertia of the drum and driving mechanism, and the stretch of the cord due to these inertia forces. In steam-engine diagrams, where the variation of pressure is not great during a working stroke, the error produced is not very serious, but in the case of an explosion engine, in which the record is asymmetrical, the high-pressure peak being at one end of the diagram, the errors produced are very large. These records are generally used to calculate the mean effective pressure on the piston, *i.e.* the mean difference between the pressures on the forward and the return strokes. To obtain this, the area of the diagram is divided by the actual length of the base line of the recorded curve, and not the length which is given at slow speeds, when inertia forces and stretch of cord are nearly absent. Thus the errors are, to a certain extent, eliminated. If, however, the record is used for determining the exact position of valve settings and such observations, the errors produced may be very great.

The motion to the drum may be derived directly from the crosshead of the engine by means of a simple lever system or a differential drum, or it may be derived from a small copy of the crank and slider mechanism, driven from the engine shaft by a chain, or other means free from slip. In any case it is necessary that the circumferential motion given to the indicator drum shall be an exact copy of the motion of the piston, and in phase with this motion.

The piston of the indicator must be lubricated with oil, and this oil is soon removed when wet steam or hot gases are present. For this reason it is best to remove the piston and link motion when the indicator is not in use, and replace them cleaned and oiled when a record is desired. This also prevents the spring from becoming overheated, and thus preserves its accuracy. After use it is most important that the whole indicator be thoroughly cleaned and oiled and put away in its proper box. The very faulty diagrams so often obtained on engine tests are largely due to neglect of these precautions.

§ (19) OPTICAL INDICATORS.—For high-speed engine tests, and for recording explosion pressures, optical indicators are the only instruments which can be used with success.[1] In these indicators the motion of the piston is reduced to a minimum, or a diaphragm is used and the magnifying gear is replaced by a beam of light, friction and inertia being eliminated in this part of the mechanism. The instruments consist of a spring-loaded piston or a diaphragm (which requires no additional spring) subjected to the pressure, and the motion of which is transmitted to a very light pivoted mirror. A ray of light is projected upon this mirror and its reflection thus strikes out a straight line, the length of which is proportional to the pressure. In order to obtain a diagram, motion must be given to this reflected ray in a direction at right angles, such motion being a copy of the motion of the engine piston. This may be done by rotating the frame containing the pressure mirror, or by deflecting the light ray by means of an independent mirror. The second motion of the light ray may be avoided by moving the ground-glass screen or photographic film upon which the light spot is received. This, however, introduces inertia forces which can easily be avoided by the above methods. For records of explosions in closed cylinders the diagram is required upon a time base, and the second reflection is then unnecessary, a revolving drum, holding the sensitive paper, alone being required. These records usually cover a very short interval of time and clockwork is unsuitable. The drum is then driven by an electric motor at approximately constant speed and an independent time record made by a ray of light reflected from a mirror attached to a tuning-fork. Another method is to arrange a tuning-fork so that the light from the diaphragm mirror is occulted at each vibration of the fork, and a dotted line thus obtained on the recorded diagram. In *Fig.* 20 (*a*), (*b*), (*c*), and (*d*) are shown four different types of these optical indicators. (*a*), due to Professor Hopkinson, shows a piston instrument in which the piston P is restrained by a straight bar spring S. The mirror is pivoted between spring supports L and the second motion obtained by oscillating the whole frame F by means of a convenient system of levers attached to the engine crosshead. (*b*) shows the instrument used by Professor Watson, in which a corrugated diaphragm D is used and kept cool by a water-jacket J. The mirror M is supported on three pivots, two of which are fixed and the third receives motion from the diaphragm. The mirror K gives the second deflection to the light ray, being oscillated by a crank and connecting-rod mechanism C in which the ratio of the connecting-rod length to the crank radius is made similar to that of the engine upon which it is used. The crank is rotated by means of a chain from the engine shaft, and an adjustment is provided for securing correct phase position relatively to the engine piston. (*c*) is the Carpentier-Hospitalier indicator, made by Messrs. Van Raden & Co. of Coventry. In this indicator the flat diaphragm D is connected to the pressure source by a small tube of some length, in order to keep the diaphragm cool. The mirror M is supported upon three pivots, one of which is fixed, one receives motion from the diaphragm, and the third receives motion from a crank mechanism which copies the motion of the engine. The two motions are thus communicated to the single mirror. The copying motion is driven from the engine by a flexible shaft to wheel J, the phase position

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[1] See "Cathode Ray Manometer."

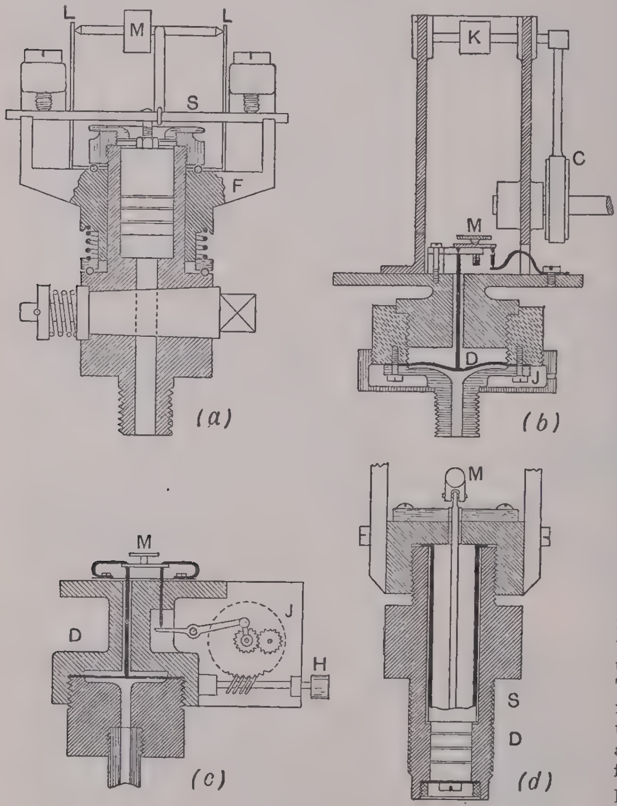

FIG. 20.

being adjusted by the knurled head H and the ratio of connecting-rod to crank being adjusted to agree with that of the engine. (*d*) shows the indicator used by Professor Petavel for explosions in closed cylinders, in which a piston D with extremely restricted motion is used and the spring consists of a thin tube S. In this case time records were required, so that but one moving mirror M was required.

These four illustrations show the chief methods of construction of optical indicators for different purposes. The essential points to be observed in the design of such instruments are the cooling of the diaphragm and the lightness of the moving parts, together with their rigidity. However light the parts are made, they will not follow an extremely abrupt change of pressure such as is obtained on detonation of an explosive charge, and unless great care is taken the mirror mechanism will be damaged under these circumstances. The rigidity of the pivot supports of the mirror can be made satisfactorily by a skilful mechanic, but a fault which often occurs is caused by the straining of the mirror under the shock of explosion. The mirror is generally of glass, fixed in a steel frame, but it would appear to be possible to shape the steel holder to the curvature required and coat it with silver, thus avoiding the attachment of glass to metal.

With spring-loaded pistons it is possible to make the springs to dimension after trial, so that the scale of pressure can be made an exact number of units per unit of motion, but with diaphragms the scale must be determined by trial under static pressure. The actual scale on the diaphragm can be altered by altering the length of the light-path, but this is not, in general, convenient, since in most cases it requires alteration in the curvature of the mirror, and always requires alteration in the light-tight enclosure. For this cause it is generally more convenient to make a special scale for measuring up the diagrams.

One other type of optical indicator may be mentioned, namely, that used by J. Kirner[1] for recording the pressure in a gun-barrel during the explosion of the charge. The ordinary type of optical indicator requires some modification to enable it to record with accuracy the rate of rise of pressure under these circumstances. The gauge referred to has two glass lenses of great strength, which are pressed together by the pressure to be measured. The pressure is indicated by an interference method, the spreading of the Newton rings being observed. An arc lamp was used for producing the ray of light and a photographic record taken of the shift of the Newton rings as the explosion took place. The inertia of this indicator is practically negligible, but considerable complication is introduced both in obtaining the record and in interpreting it.

§ (20) Micromanometers.—The gauges to be described are designed for the measurement of very small differences of pressure when both pressures are nearly atmospheric. A distinct class of gauge for measuring very small absolute pressures, called vacuumometers, is discussed elsewhere. For the latter purpose many other physical properties of attenuated gases are available, but for measurements in the neighbourhood of the atmospheric pressure two means only are available—liquid columns and diaphragm manometers. In both cases the principle of construction lies in the magnification of the very small difference in the level of the liquid surfaces or of the minute motion of the diaphragm. In the case of liquid columns it is important to arrange that the temperature of both columns is identical, since the difference to be measured may be comparable with the difference produced by temperature even in a short column of liquid.

§ (21) Diaphragm Gauge and Optical Lever.—This apparatus is shown in *Fig.* 21.

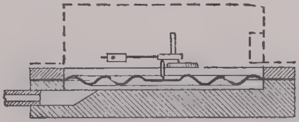

Fig. 21.

A very thin corrugated diaphragm is fixed above the pressure space. The motion of the diaphragm is communicated to a mirror mounted on three pins forming an optical lever. The reflected ray of light can be received on a screen at a distance. The sensitivity depends upon the distance between the fixed and moving supports of the mirror, which may be reduced to about $\frac{1}{4}$ mm. if necessary, and the distance of the scale. The greater the distance of the scale the more powerful must be the light. It is possible to measure a deflection of the diaphragm of about ·02 μ. The gauge must be calibrated at its highest readings by comparison with a liquid column and the pressures will be proportional to the deflection.

If differences of pressure are required the whole apparatus must be enclosed in a cover, light rays passing through a glass window, as shown dotted in *Fig.* 21, the space enclosed being connected to one source of pressure. This gauge is easily disturbed by external vibration and can only be used when such disturbances are absent.

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[1] J. Kirner, *Zeitschrift Ver. Deuts. Ing.*, 1909, liii. 41.

§ (22) Diaphragm Gauge of Scheel and Heuse.[1]—This gauge is somewhat similar to the last, but has a light copper diaphragm, and the deflection is measured by the Fizeau interference method instead of by the optical lever. The gauge is thus more compact and not so sensitive to external vibration. Differences of pressure of the order of 0·1 mm. of mercury can be measured to 0·00001 mm. and a change of pressure of 0·001 mm. corresponds to four interference bands of yellow helium light.

§ (23) Collodion Diaphragm Gauge.[2]—M. Lafay has constructed a very sensitive diaphragm of silvered collodion in which the Fizeau interference method is utilised to indicate when the diaphragm is in its normal position. The deformed diaphragm is brought back to its initial position by means of an electrostatic charge, the quantity of which is measured and the pressure determined therefrom. This has the advantage that the actual pressure measurement is simpler than the counting of the interference bands, the light system being used simply to indicate the null position.

§ (24) Liquid Columns. — The simplest method of magnifying the difference in height is to make one of the tubes at a small angle to the horizontal instead of vertical. If the position of the meniscus is measured along the tube the height $h=l \sin \alpha$ where α is the angle to the horizontal and l the motion along the tube. If α is made 5°, the magnification is 12 times. A disadvantage of this method is that the smaller the slope the greater difficulty is experienced in determining the position of the liquid in the tube, and in any case a small tube must be used.

§ (25) Roberts Compensated Manometer.[3]—This gauge, shown in *Fig.* 22, consists of two tubes A and B connected by a capillary C, the whole being filled with liquid except for a bubble of air D left in the horizontal tube C. A very small change in head in either A or B thus causes a large motion of the bubble in C, and the tubes A and B being close together they can be maintained at the same temperature. If A and B are 10 mm. in diameter and C is ½ mm. bore, the magnification is 400 to 1, a reading of 0·001 mm. being easily made.

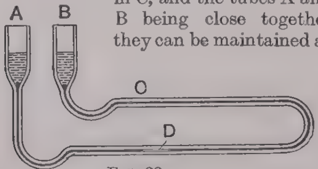

Fig. 22.

§ (26) Micrometer Water-gauge. — This gauge, shown in *Fig.* 23, is constructed of two water-vessels A and B connected by a small tube C. The level of water is measured by micrometer heads D, the reading being taken when the point attached to the micrometer screw just touches the surface of the water. As the point is brought down very slowly to the water surface, the latter will suddenly appear to jump up to meet the point and will adhere to the point. If this contact is always observed it is possible to read to ·025 mm. on each head corresponding to ·05 mm. difference in head. This corresponds to 0·004 mm. of mercury. The water surface and the points must be kept clean, and the points must be withdrawn from the surface before each reading. The apparatus must be firmly fixed, since vibrations on the surface of the liquid will prevent accurate observations of the position of contact.

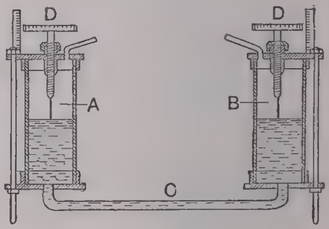

Fig. 23.

§ (27) Chattock Gauge.—*Fig.* 24 shows this gauge, which is simple to construct and only requires reasonable care in operation to give very reliable observations of small differences of pressure. The gauge is constructed in glass, A and B being two water-vessels attached to the pressure sources. These vessels communicate with a central vessel C, one to the body of the vessel and the other to an internal tube D. The central vessel C is filled with any moderately transparent liquid lighter than, and non-mixable with, water. The whole gauge is mounted upon an upper frame F, which in turn is supported at one end from a stand G and carried at the other end by

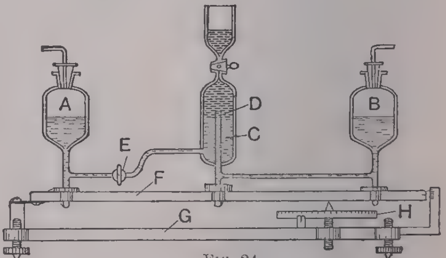

Fig. 24.

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[1] *Deutsch. Phys. Gesell. Verh.*, 1909, xi. 1.

[2] *Comptes rendus*, 1909, cxlix. 1115.

[3] *Proc. Royal Society*, 1906, A, lxxviii.

an adjustable screwed support with a micrometer head H. The excess of pressure in vessel B will force a bubble of water up the tube D into the oil surrounding its end A lamp is fixed at the back of the gauge to illuminate this bubble, and a microscope with an objective of about 25 mm. focus is employed to observe the bubble. The light is then adjusted until a bright line either golden or red in colour is observed in the microscope, and this line is brought into coincidence with the fiducial line in the eyepiece of the microscope by tilting the gauge by the micrometer head H. When the gauge is properly filled with clean liquids this bright line will be found to be quite definite in position for any position of the gauge. The line is adjusted for equal pressures in the two tubes, and when pressure is applied is brought back to position by tilting the gauge. The micrometer readings are thus proportional to the pressure differences in the water vessels. A common size is to make l the distance between the micrometer and support 25 cm., L the distance between the tubes $= 35$ cm. With 1 mm. pitch of screw and 100 divisions on the micrometer head one unit $= \frac{35}{25} \times \cdot 01 = \cdot 014$ mm. of water, and readings can be made to one-tenth of a division, corresponding to about ·0001 mm. of mercury. The ground joints at the top of the water vessels are required so that the gauge can be properly cleaned, and these and all valves must be kept well greased to prevent leakage. If a sudden rise of pressure takes place the bubble of water may become detached and fall to the bottom of vessel C, thus altering the adjustment of the gauge. A tap E is provided so that the gauge can be shut off if this seems likely to occur. It is well to provide two branch taps in the pressure pipes, which can be opened to the atmosphere in order to check the zero without disconnecting the gauge from the test apparatus.

The sensitivity of the gauge depends upon the definition of the bright line seen in the microscope, and this depends chiefly upon the liquid used. Otherwise the sensitivity could be increased by bringing the vessels A and B together, thus making the ratio l/L large. The sharpness of definition of the bright line depends chiefly upon the two liquids employed. The best result is obtained by the use of salt water and pure castor oil. The salt is used in order to prevent a fungus-like growth which occurs at the surface of pure water in contact with the oil. The readings of the gauge must be corrected for the density of the water used, the salt being added until the density is about 1·07. With this solution the gauge will remain in working order for many months. The use of the microscope is somewhat tiring during prolonged series of observations, and it can be replaced by a projection lens which throws an image of the bubble on a ground-glass screen. If the surrounding light is dim the position of the bright line can be adjusted accurately and with less strain to the eye.

The range of pressure measurable by these gauges may be increased by increasing the distance between the water vessels A and B. The new piece of glass work can be attached to the standard frame, avoiding the cost of the more expensive part of the apparatus. If a still greater range of pressure is to be measured, the glass part can be filled with mercury instead of water. It is evident that the larger the range of pressure available the smaller the sensitivity of the gauge. It is, however, a great convenience to be able to alter the range without great expense, and this renders the gauge particularly suitable for general laboratory work.

§ (28) Conclusion.—The manometers in common use are of three kinds—the liquid column, the loaded piston, and the elastic gauge. Gauges of each of these kinds have been briefly described. The range of pressure measured by the different types may be given here:

Liquid Columns:		
Open column . .	0 to 50	atmospheres.
Multiple and compound columns .	0 to 200	,,
Closed column . .	up to 3000	,,
Loaded Piston . . .	3 to 6000	,,
Elastic Gauges:		
Bourdon tube . .	0 to 1800	,,
Diaphragm gauges and optical indicators . . .	0 to 100	,,
Spurge manometer .	0 to 500	,,

The sensitivity of the different micromanometers is as follows:

Metal diaphragm . .	1×10^{-2}	mm. of mercury.
Very thin metal diaphragm	1×10^{-5}	,, ,,
Collodion diaphragm .	1×10^{-4}	,, ,,
Inclined liquid column .	1×10^{-3}	,, ,,
Roberts' micromanometer	1×10^{-4}	,, ,,
Micrometer water gauge .	1×10^{-2}	,, ,,
Chattock gauge . .	1×10^{-4}	,, ,,

When an absolute pressure of very small amount is to be measured other and quite novel methods of measurement can be adopted. These generally depend upon the molecular motions of gases and are only available when by rarefaction the molecular path is comparatively large. Examples of these are the repulsion of two plates suspended in a rarefied gas, the measurement of the electric current flowing to a collector, or the radiation from a hot wire. Manometers of this kind will measure pressures of the order 10^{-6} mm. of mercury, but the methods are not available when the absolute pressure is much greater than 10^{-3} mm. of mercury. Such gauges will be found described in another portion of this Dictionary. C. J.

Pressure-coefficient of Expansion of a Gas, Experimental Determination of, with constant volume. See "Thermal Expansion," § (16).

Pressure-coefficients of Various Thermometric Gases, tabulated. See "Temperature, Realisation of Absolute Scale of," § (18), Table 2.

Pressure Corrections, to be applied to the readings of a thermometer to allow for the effect of changes of pressure either inside the thermometer or exterior to it. See "Thermometry," § (3) (*b*) and (*c*).

Pressure-difference between the two Surfaces of a Soap Film,

$$p = 2T\left(\frac{1}{R}+\frac{1}{R_1}\right),$$

T being the tension, and R and R_1 the principal radii of curvature of the film.

Pressure Distribution for Streamline Flow. See "Ship Resistance and Propulsion," § (20).

Pressure Engines (Hydraulic). See "Hydraulics," § (54).

Pressure of a Gas, due to Molecular Impacts on the Containing Walls. See "Thermodynamics," § (66).

Pressure of Radiation: the pressure which, by the second law of thermodynamics, radiation must exert. From the electromagnetic theory, Maxwell showed that this pressure, for isotropic radiation, is numerically equal to one-third of the total energy of radiation of all frequencies in unit volume. See "Radiation Theory," § (5) (i.).

Pressure Turbines (Hydraulic). See "Hydraulics," § (52).

Pressures, maximum and mean in internal combustion engines. See "Engines, Thermodynamics of Internal Combustion," §§ (48)-(54).

Prony Brake. See "Dynamometers," § (2) (i.).

Propeller Dynamometer for testing Airscrews. See "Dynamometers," §§ (7), (9).

Propellers, Air and Water. See "Ship Resistance and Propulsion," § (40) *et seq.*

Properties of a Fluid as exhibited by Charts. See "Thermodynamics," § (42).

Propulsion of Ships. See "Ship Resistance and Propulsion," V.

Pulleys. See "Mechanical Powers," § (2).

Pulsometer Pump. See "Hydraulics," § (41).

Pumps. See "Hydraulics," Part II.

Pyrometer:

Effect upon Reading of, when the Focussing Distance is increased, tabulated. See "Pyrometry, Total Radiation," § (14) (iii.), Table IV.

Extension of Scale of, above 1400° C. See "Pyrometry, Optical," § (3).

Féry's Mirror. See "Pyrometry, Total Radiation," § (7).

Féry "Spiral." See *ibid.* § (8).

Féry's Telescope. See *ibid.* § (6).

Foster Fixed-focus. See *ibid.* § (10).

Optical, Calibration of, by comparison with a standard instrument. See "Pyrometry, Optical," § (9).

Optical; Disappearing Filament Type: a type of pyrometer depending on the matching of the brightness of a lamp filament against that of the hot object. See *ibid.* § (3).

Polarising Type of. See *ibid.* § (6).

Radiation, Calibration of. See "Pyrometry, Total Radiation," § (16).

Radiation, Sources of Error in Practical Forms of. See *ibid.* § (14).

Recording: used in manufacturing processes where it is necessary to keep a continuous record of the temperature of the furnace. See "Thermocouples," § (16).

Recording Deflection: instruments for the measurement and continuous record of temperature, the instruments being of the millivoltmeter type and the record being made by periodically depressing a pointer into contact with a chart. See *ibid.* § (17).

Recording Resistance, used in industrial work to give a continuous record of the temperature of a furnace or kiln. See "Resistance Thermometers," § (20).

Spectroscopic Eyepiece of: a red filter glass for producing approximately monochromatic radiation. See "Pyrometry, Optical," § (14).

Thermoelectric: the most generally used of all appliances for the measurement of high temperatures, and now developed into an instrument of precision. See "Thermocouples," § (1).

Thermoelectric: Cold Junction Correction for. See *ibid.* § (19).

Thwing Radiation. See "Pyrometry, Total Radiation," § (11).

Total Radiation, compared with the Optical Type. See *ibid.* § (15).

Total Radiation: instruments based on the fourth-power law, for the measurement of high temperatures, and really specially designed thermopiles. See *ibid.* § (5).

Use of a Radiation, with a Source of Insufficient Size. See *ibid.* § (13).

Use of Rotating Sector to reduce the intensity by a definite amount. See "Pyrometry, Optical," § (15).

Whipple Closed Tube. See "Pyrometry, Total Radiation," § (12).

PYROMETRY, OPTICAL

The measurement of high temperatures by means of optical pyrometers is based on the well-known fact that the intensity of the light emitted by a hot object increases as its temperature is raised.

The mathematical relationship between the intensity of the light of any particular wave-length and the temperature can be deduced from theoretical considerations for a surface which is a "full radiator," [1] and the fact that the radiation issuing from a uniformly heated furnace approximates closely to "full radiation" has greatly simplified optical pyrometry.

§ (1) Theory of Heat Radiation.—Boltzmann [2] in 1884 deduced the relationship between the total radiation from a black body and its temperature, according to which the total radiation varies as T^4. This is generally known as the Stefan-Boltzmann or the fourth-power law.

In 1896 Wien further developed the theory of radiation. His first deduction is known as Wien's displacement law:

$$\lambda_m = bT^{-1} \text{ or } E_m = kT^5$$

where λ_m is the wave-length of maximum energy, E_m is the maximum energy ordinate, and T the absolute temperature.

For the distribution of the energy among the various wave-lengths in the spectrum Wien deduced the expression

$$E_\lambda = c\lambda^{-5} f(\lambda T),$$

in which $f(\lambda T)$ could not be determined by purely thermodynamical reasoning. E_λ is the energy emitted of wave-length λ.

By making certain arbitrary assumptions concerning the radiation emitted by vibrating gaseous molecules, he succeeded in resolving the function of λT, and obtained the relationship

$$E_\lambda = c_1 \lambda^{-5} e^{-\frac{c_2}{\lambda T}}$$

for the distribution of energy among the various wave-lengths in the spectrum.

Experimental work, which will be referred to later, showed that the above expression only represented the facts for a limited range of λ and T. Since that time various attempts, based on plausible assumptions, have been made to discover the correct expression, but without success.

The one formula which does represent the experimental data closely under all conditions is that of Planck:

$$E_\lambda = c_1 \lambda^{-5} \frac{1}{e^{c_2/\lambda T} - 1}.$$

The subject has aroused considerable interest during recent years, and reference must be made to the literature of the subject for fuller information.[3]

It will be observed that for small values of λ and T the term $e^{c_2/\lambda T}$ is large compared with unity and Planck's equation approximates to that of Wien; the concordance is sufficiently close for wave-lengths in the visible spectrum to permit of the use of the simple form of Wien's equation for the range of temperature covered in practical optical pyrometry.

§ (2) Experimental Study of the Distribution of Energy in the Spectrum of a "Full Radiator" at various Temperatures.—In 1899 Lummer and Pringsheim, who had already proved by experiment the validity of the Stefan-Boltzmann or fourth-power law, published a further contribution to the subject. Their experiments on this occasion were directed to the determination of the distribution of energy in the spectrum of a "full radiator." At first they employed the various types of uniformly heated enclosures, which they had constructed for their experiments to test the Stefan-Boltzmann law. Later they introduced many practical improvements in the apparatus, and, by basing their method of measuring temperature on the fourth-power law, they were able to continue their experiments to very high temperatures. This procedure also led to a great simplification in the form of their "full radiator," which could then be reduced to an electrically heated carbon tube.

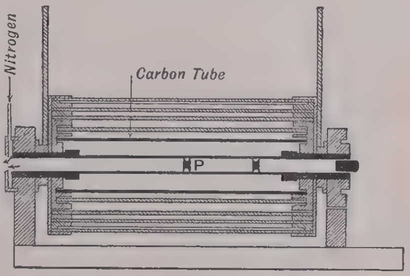

Fig. 1.

(i.) *Description of the Experimental Arrangements.*—The carbon tube furnace construction is shown in *Fig.* 1.

The tube was of uniform wall thickness (1·2 millimetre), 34 cm. long, and 1 cm. internal diameter. The ends were inserted into heavy carbon blocks, copper-plated and fixed into metallic clamps.

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[1] In radiation pyrometry generally the term "full radiator" or "black body" denotes one that will absorb all the radiation that it receives—that is to say, it will neither reflect nor transmit any of the incident radiation. There is no known substance that has strictly this property, the nearest approach being probably untreated carbon. Kirchhoff demonstrated that a hollow cavity with walls at a uniform temperature possesses the properties of a "black body." If a small opening is made in the wall of the uniformly heated cavity, the radiation issuing from the hole will obey the laws of black body radiation.

[2] See "Radiation Theory," § 5, Vol. IV.

[3] See list at end of article.

A stream of nitrogen was passed into the cap at the mouth of the furnace to diminish oxidation.

The plug P (in the centre of the tube) and the left half of the furnace was equivalent to a " black body " in respect of radiation.

The spectrum was produced by a fluor-spar prism, and corrections applied to convert the prismatic to normal energy curves by means of the known dispersion curve of fluor-spar.

The distribution of energy was measured by means of a linear bolometer, which was enclosed in an air-tight case to diminish the absorption effects of carbon dioxide and water vapour. The bolometer consisted of a single strip of platinum foil 0·6 mm. wide. The distribution of energy in the spectrum for various temperatures between 700° and 1600° C. was obtained. From these curves the values of λ_m and the energy corresponding to E_m could be read. The experimental results were in agreement with the two laws:

$$\lambda_m T = \text{constant},$$

$$E_m T^{-5} = \text{another constant}.$$

Wien's distribution formula,

$$E_\lambda = c_1 \lambda^{-5} e^{-\frac{c_2}{\lambda T}},$$

was found to hold for values of λT less than 3000 μ degrees. Since the longest wave-length used in optical pyrometry is less than 0·7 μ, the equation is valid for the entire range of temperature that has to be measured in practical work.

(ii.) *Variation with Temperature of the " Maximum Energy Ordinate E_m."*—To verify the relationship

$$E_m - kT^5$$

Lummer and Pringsheim employed a Lummer-Brodhun spectro-photometer, and worked with different parts of the luminous spectrum from red to violet. Owing to the rapid increase in the intensity of the luminous radiation with temperature, it was necessary to employ a number of absorption plates; at the highest temperature the intensity was reduced to $\frac{1}{8000}$ part to bring it within measurable limits.

Temperature by 4th-Power Law (Various Distances), ° C.	Temperature by $E_m = kT^5$, ° C.
2345	2325
2348	2327
2339	. .

They were able to obtain an accuracy of about ±20° C. with the total radiation pyrometer, and the calculated temperatures were in agreement within these limits of accuracy.

The relationship $\lambda_m T = \text{constant}$ was also checked and the value 2930 obtained for the constant of a " full radiator." When the radiation from polished platinum was studied the constant was found to be 2620.[1] Hence it is possible to estimate roughly the temperature of any object whose radiation is intermediate in character between that from a perfectly black body and polished platinum by determining λ_m by means of a bolometer and a dispersion apparatus.

This has been done for a number of radiators by Lummer and Pringsheim. The maximum value of T is given by $2930/\lambda_m$ and the minimum by $2620/\lambda_m$.

Hot Object.	λ_m.	$T_{\text{Max.}}$, ° C.	$T_{\text{Min.}}$, ° C.
	μ	abs.	abs.
Arc light . . .	0·7	4200	3750
Nernst lamp . .	1·2	2450	2200
Welsbach mantle .	1·2	2450	2200
Incandescent lamp .	1·4	2100	1875
Candle . . .	1·5	1960	1750
Argand burner .	1·55	1900	1700

(iii.) *Comparison of Wien's Law and Stefan-Boltzmann's Law to* 2800° C.—The work of Lummer and Pringsheim and their contemporaries was carried out before the researches of Holborn and Valentiner, of Jacquerod and Perot, and of Day and Sosman had established the high temperature scale in terms of the gas thermometer, consequently it is difficult to form any precise estimates of the limits of accuracy to which the radiation laws may be regarded as proven at high temperatures. Recently Mendenhall and Forsythe have made a comparison up to 2800° C. between two pyrometers, one based on " fourth-power " law and the other on Wien's law. The pyrometers were calibrated by observations of the melting-points of gold and palladium, the values for which, on the scale of the nitrogen gas thermometer, had been determined by Day and Sosman.

(iv.) *The Optical Pyrometer.*—This, as used by Mendenhall and Forsythe, was of the disappearing filament type described in § (3). The principle of the instrument is that of a telescope. An image of the hot object is superposed on the filament of a small electric lamp. Matching is effected by making the apparent brightness of the image identical with that of the filament by varying the current through the latter.

Since Wien's law is applicable to monochromatic radiation and not to the entire visible spectrum, it is necessary to isolate as narrow a spectral range as possible. Generally this is effected by the use of a piece of good red glass; in the present case, however, they employed a spectroscopic eyepiece. The latter has the theoretical advantage of giving a narrower band and consequently a nearer approach to the ideal conditions contemplated by Wien's law. The width of the band transmitted was determined and found to be 200 A.U. (0·02 μ) with a centre at $\lambda = 0{\cdot}658\ \mu$.

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[1] There is no theoretical basis for the application of the law to the radiation from platinum, and subsequent investigations have shown that $\lambda_m T$ is not a constant for polished metallic surfaces, but increases with temperature. The constant value obtained by Lummer and Pringsheim is due possibly to the small range of temperature employed, or to lack of polish on the radiating surface.

The calibration of the instrument, *i.e.* the relationship between intensity of radiation and the current necessary to match it, was effected by the use of a system of rotating discs of measured aperture.

Taking the logarithmic form of Wien's law,

$$\log E_\lambda = K_1 - \frac{c_2}{\lambda} \cdot \frac{1}{T}.$$

If balance was obtained with clear aperture on a "black body" at temperature T_1, and an apparent temperature T_2 was obtained through a sector of transmission ratio k, then

$$\log_{10} \frac{1}{k} = \frac{c_2}{\lambda}\left(\frac{1}{T_1} - \frac{1}{T_2}\right) \times \log_{10} e,$$

so that, by a series of observations on a furnace maintained at a constant known temperature, it was possible, by employing a series of discs with various values of k, to calibrate the pyrometer over a range of temperature. The apertures of the discs were measured by means of a dividing engine. Two steady temperatures were used as checks on the calibration.

The minimum aperture employed was about "$\frac{1}{180}$." This proved difficult to make accurately, and on measurement was found to be 2° 1′ 50″ instead of 2°. This error, however, would only produce a divergence of 5° C. in the computed scale if the nominal value of 2° angle had been taken.

(v.) *The Total Radiation Pyrometer.*—It will be observed from a study of *Fig.* 4, § (4)—article on "Total Radiation Pyrometry"—that the "total radiation" pyrometer was enclosed in the evacuated chamber containing the furnace, while the optical pyrometer observations were taken through a glass window. A small correction was necessary for the absorption of this window in the visible radiation.

The results of the intercomparison are summarised below:

No. of Comparisons.	Temp. °C.	$T_{\text{Optical}} - T_{\text{Total Radiation}}$.	Range of Observations
9	1750	Less than ±0·5°	2°
7	2200	„ „ +2°	4
3	2500	About +2°	4
4	2800	„ +4°	7

The difference is systematic but not greater than the possible error of experiment. By an alteration in the assumed value of either c_2 or λ the systematic difference could be eliminated. For example, Mendenhall and Forsythe state that if, instead of 0·658 μ, the value 0·657 μ is taken the differences disappear.

I. Practical Types of Optical Pyrometers

§ (3) The Disappearing Filament Type.—This type of pyrometer was introduced about twenty years ago by Morse in America, but the principle involved—the matching of the brightness of a lamp filament against that of the hot object—was in use as far back as 1888.

In its earliest form the Morse Pyrometer consisted of a metal tube about 3 inches in diameter and 8 inches long, open at both ends, and provided on one side with a projection serving as a means for holding an incandescent lamp.

At the centre of the tube was mounted the lamp, which was connected in series with a battery, rheostat, and milliammeter.

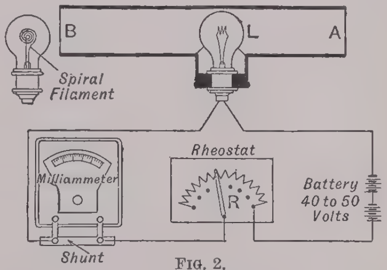

Fig. 2.

The instrument is shown diagrammatically in *Fig.* 2.

In making a temperature measurement the operator holds the pyrometer in front of his eye and, looking through, observes the lamp filament superposed on the furnace or hot object as background. Owing to the different distances of lamp and furnace from the observer it is necessary to vary the accommodation of the eye when looking at one object and then the other.

By adjustment of the rheostat the current in the lamp is varied progressively until the lamp filament and furnace appear equally bright. When the filament disappears against the furnace as background, the current through the filament is a measure of the temperature.

Holborn and Kurlbaum modified the instrument by adding an objective and eyepiece. The objective projects an image of the furnace upon the plane of the lamp filament, and the fatigue of the eye due to constantly varying the accommodation is avoided.

Theoretically, the image of any source as observed through a particular telescope will not vary in brightness with a change in distance from the source (except, of course, differences due to air absorption, etc.), providing a certain solid angle is always filled with radiation from the source and this angle is of such size that the cone of rays entering the eye is constant. This angle is generally determined by having the eyepiece at a fixed distance from the pyrometer lamp and having before the eyepiece a limiting diaphragm of such size that it is always filled with light from the objective lens. It is also necessary to have a fixed diaphragm between the objective lens and the pyrometer lamp (see *Fig.* 3).

In all modern instruments of this type a red filter glass[1] is fixed in the eyepiece.

This serves two purposes:

(*a*) The matching of the intensities is facilitated, as practically monochromatic radiation is obtained, so there are no colour differences at high temperatures. This is of particular value in dealing with surfaces which do not radiate light of the same composition as that emitted by a black body, since the intensity of radiation of any one colour from such surfaces increases progressively in a definite manner as the temperature rises.

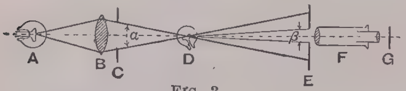

FIG. 3.

A, hot object; B, objective lens; C, entrance cone diaphragm; D, pyrometer filament; E, eyepiece diaphragm; F, eyepiece; G, red glass.

(*b*) The scale of the instrument can be extrapolated on the basis of Wien's law, employing a rotating sector or absorption glass for cutting down the intensity of the source.

In recent years Forsythe and his collaborators at the Nela Research Laboratory have made a thorough study of this form of pyrometer, and shown how many of the errors in temperature measurements with the instrument may be avoided by attention to the details of the design.

A modern form of disappearing filament optical pyrometer made by the Leeds & Northrup Co., which is largely used in the industries, is shown in *Fig.* 4. The tube is of aluminium and the instrument constructed as lightly as possible for convenience in using.

In the Standard Pyrometer of the National Physical Laboratory, used for precision work, two lamps are fitted, and they can be readily interchanged so that a check is obtained on the permanency of the calibration. The rotating sector is arranged just in front of the lamps, as the experiments of Mendenhall and Forsythe have proved that in this position the definition is practically independent of the position of the opening of the sector relative to the filament when crossing the field. The difficulty only occurs when taking the temperature of small objects such as incandescent lamp filaments.

Extension of the Scale above 1400° C.—The range of temperature that can be measured in the ordinary way by an optical pyrometer without some device for cutting down the intensity of the radiation from the hot object is from 700° to 1400° C. The comparison source cannot be run at much higher temperatures than 1400° C. without rapid deterioration. The scales are extended by the use of absorption devices or rotating sectors. With the early form of disappearing filament pyrometer, the absorbing device generally used consisted of two black glass mirrors inclined at an angle of 45°. In this arrangement the beam of light from a hot object is reflected twice at an angle of 45° incidence, and thereby weakened to about 1/200 of its original intensity.

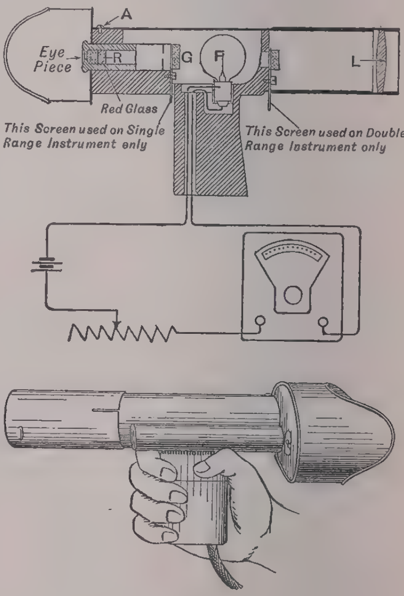
A
Eye Piece
R
G
F
L
Red Glass
This Screen used on Single Range Instrument only
This Screen used on Double Range Instrument only

FIG. 4.

To extend the scale of the pyrometer up to about 2700° C., the usual practice at the present day is to insert a piece of neutral-tinted glass in the path of the beam from the furnace or to employ rotating sectors of definite aperture.

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[1] A detailed discussion of the red filter glass and absorption screens is given later, as they perform the same functions in all types of optical pyrometers.

A discussion of the methods of computing the transmission factor is given later.

§ (4) MICROPYROMETER.—Burgess has devised a valuable instrument of the disappearing filament type, which he terms a *micropyrometer*, for the determination of the melting-points of minute specimens of materials.

The apparatus (*Fig.* 5) is somewhat similar to a Joly meldometer, with a microscope pyrometer, which is focussed on the specimen resting on a strip of platinum heated by a current.

Within the Huyghens eyepiece of an ordinary microscope a small incandescent lamp is mounted. In series with the lamp there is an ammeter and rheostat operated in precisely the same manner as in the case of a disappearing filament type of optical pyrometer. The eyepiece of the microscope is fitted with a piece of good red filter glass.

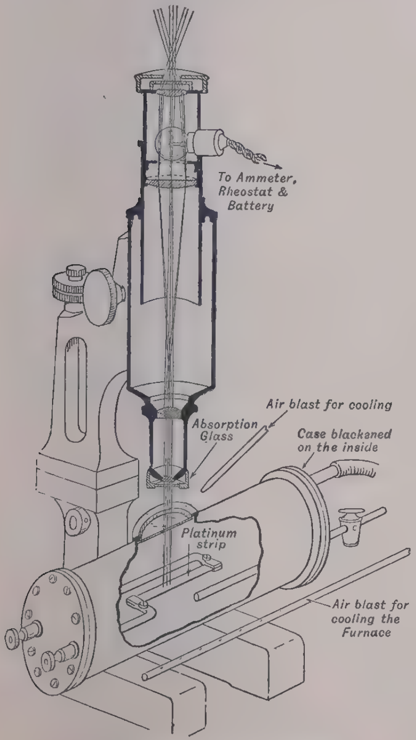

FIG. 5.—Burgess' Micropyrometer.

For temperatures exceeding 1400° C. an absorption glass is placed between the microscope objective and the furnace window.

The tip of the filament of the lamp is set to the same brightness as the platinum strip viewed from above at the instant of melting of the metallic or other specimen on which the microscope is focussed. The eye of the observer sees the specimen, the platinum strip, and the lamp filament all in focus at once.

§ (5) CALIBRATION.—The methods generally employed for the calibration of optical pyrometers cannot be adopted for this particular form of disappearing filament pyrometer. The most convenient method is that originally used by Joly, which is to observe the lamp currents at the known melting-points of two or more pure substances, such as gold, nickel, and palladium, and from the equation expressing the relation between temperature and current (in lamp), the temperature of melting of any specimen may be computed. For moderate temperature intervals the equation $\log C = a + b \log T$ may be used, where C is the current, T is the absolute temperature, b a constant very nearly unity.

The melting-points of nickel and palladium were taken for calibration purposes.

The accuracy of the equation was checked by extrapolating to the melting-point of platinum, which could be directly observed by matching the lamp and strip at the instant the latter burnt out. The value 1755° C. ± 5 was obtained from six observations.

An atmosphere of pure hydrogen was maintained in the enclosure, to prevent oxidation of the metal under test.

For those metals for which the melting is sharp, such as nickel, cobalt, and iron, a precision of 1° to 2° was possible with only a few thousandths of a milligram of material.

§ (6) THE POLARISING TYPE OF OPTICAL PYROMETER.—König in 1894 described a new type of spectrophotometer. In this instrument the two beams of light were resolved into two spectra and comparison effected between beams of identical colour throughout the spectrum. In 1901 Wanner applied the essential principles of this instrument to the design of an optical pyrometer. In this case the radiation from the hot object is the one beam, while the light of constant intensity from an electric lamp supplies the comparison beam.[1]

(i.) *Description of the Instrument.*—The essential features of the instrument will be understood from *Fig.* 6.

The radiation from the hot object is received through a small circular hole while the electric lamp illuminates the matt surface of a right-angled prism, which in turn directs the light on to a second circular hole symmetrically disposed with the first about the optical axis of the system.

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[1] The current through the lamp is maintained at a predetermined value by means of a rheostat and ammeter. From time to time the intensity of the beam given by the electric lamp is matched against that from an amyl acetate lamp by adjustment of the current. This renders the scale of the pyrometer independent of the permanency of the electric lamp.

A lens renders the two beams parallel, and a Rochon prism splits each beam up into components polarised at right angles.

The function of the biprism lens is to produce deviation in the beams of such amount that an image from each of the two sources is brought into juxtaposition. On consideration it will be seen that the biprism splits each image up into two, thus bringing the total up to eight.

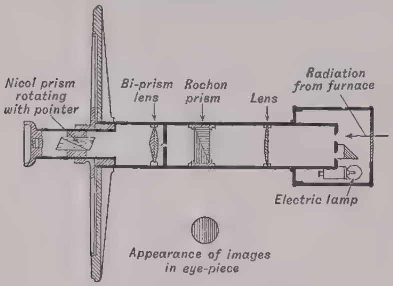
Nicol prism rotating with pointer
Bi-prism lens
Rochon prism
Lens
Radiation from furnace
Electric lamp
Appearance of images in eye-piece

FIG. 6.—Diagram illustrating Wanner's Modification of König's Spectrophotometer.

These images are semicircular patches uniformly illuminated. The two in juxtaposition are polarised at right angles, and are viewed through the eyepiece, the other images being screened out.

The Nicol prism N can be rotated around the optical axis, its position being indicated by a pointer attached.

To understand the precise functions of the various optical parts it is advisable to consider the effect of each individually.

In *Fig.* 7 the contribution of each component is shown. The circular holes are at the focus of the lens, so the images produced will be uniformly illuminated circular discs, *i.e.* of the face of the lens, which the biprism splits up into semicircles.

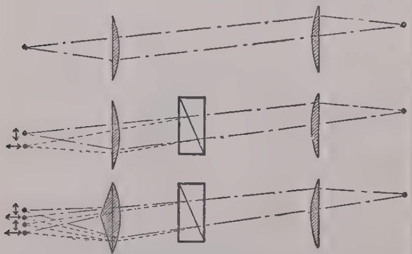

FIG. 7.—Diagram showing Junctions of various Optical Parts of the Spectrophotometer.

The arrows ⟷ and ↕ indicate plane of polarisation. Eight images are formed in all, of which six are stopped out.

To understand the function of the Nicol prism suppose for the moment that the two beams are of equal intensity, then, with the plane of polarisation of the Nicol prism making an angle of 45° with the direction of polarisation of either beam, a uniformly illuminated circle would be observed having a diametrical line across where the two fields come into contact.

Rotation of the Nicol prism in either direction will cut down the intensity of one of the beams and increase that of the other.

Hence, if the beams are initially of unequal intensity, matching of the intensities, as viewed through the eyepiece, is possible for a certain position of the prism between the extinction positions 0° and 90°.

(ii.) *Theory of the Polarising Type of Pyrometer.*—It is proved in textbooks of Light that if I_1 and I_2 are the intensities of two plane polarised beams of radiation matching at angles ϕ_1 and ϕ_2, a beam of constant intensity, such as that from an electric lamp, when viewed through a Nicol prism, then:

$$\frac{I_1}{I_2}=\frac{\tan^2\phi_1}{\tan^2\phi_2}.$$

In optical pyrometry, by the insertion of a direct-vision prism or a piece of suitable red glass in the path of the two beams it is possible to work with narrow spectral bands and consequently apply Wien's law.

According to this law the intensity of light of wave-length λ emitted by a "full radiator" is given by the expression

$$I=\frac{c_1}{\lambda^5}e^{-\frac{c_2}{\lambda T}}.$$

Suppose I_1 is the intensity of wave-length λ at temperature T_1; I_2 is the intensity of wave-length λ at temperature T_2.

By Wien's law

$$\frac{I_1}{I_2}=e^{\frac{c_2}{\lambda}\left(\frac{1}{T_2}-\frac{1}{T_1}\right)}.$$

Hence $$\frac{I_1}{I_2}=\frac{\tan^2\phi_1}{\tan^2\phi_2}=e^{\frac{c_2}{\lambda}\left(\frac{1}{T_2}-\frac{1}{T_1}\right)}$$

Taking logarithms to the base e,

$$2(\log\tan\phi_1-\log\tan\phi_2)=\frac{c_2}{\lambda}\left(\frac{1}{T_2}-\frac{1}{T_1}\right),$$

so that the relation between ϕ and T is of the form

$$\log_e\tan\phi=a+\frac{b}{T}.$$

Hence, if a series of values of ϕ and T are obtained, when log tan ϕ is plotted against 1/T, the points should fall on a straight line.

Theoretically, a single determination of the angle ϕ corresponding to a known temperature T, together with a knowledge of c_2 and λ, should suffice to give a complete calibration of the instrument. In practice, however, it is very difficult to obtain perfection in the optical parts and freedom from strain in the lenses, and this affects the polarised beam. So it is generally advisable to calibrate the instrument against a "black body" over the entire range.

Experience with commercial forms of the pyrometer indicates that the divergence from the theoretical line is greatest near the extinction positions owing presumably to defects in the Nicol prisms and scattering of light.

§ (7) Calibration of the Instruments Empirically.—The calibration of optical pyrometers can be readily effected by reference to a standardised thermocouple in an electric furnace arranged to give approximately full radiation by a series of diaphragms suitably disposed.

Fig. 8 shows a furnace arranged for optical pyrometer calibration up to 1370° C.

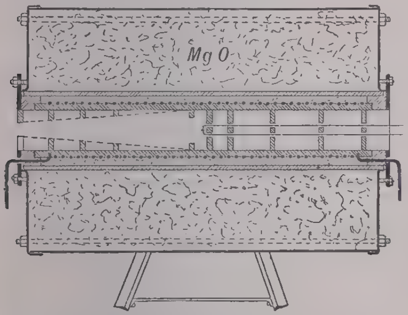

Fig. 8.—Diagram illustrating the Construction of a Furnace for Optical Pyrometer Calibration.

The pyrometer is sighted upon the central diaphragm, which has the junction of a thermojunction on its surface to give the temperature of the enclosure.

For representing the relationship between current and temperature in the case of the disappearing filament type of pyrometer a parabolic formula

$$C = a + bT + cT^2$$

is sufficiently accurate. Whilst for the polarising type the formula

$$\log \tan \phi = a + \frac{b}{T}$$

should hold for the major portion of the scale if the optical parts are in correct adjustment.

§ (8) Standardisation by Observation of Transition Points.—It is possible to calibrate optical pyrometers by direct observations of freezing- or melting-points, without the use of a thermocouple as intermediary.

In the case of materials which require a reducing atmosphere and do not react at high temperatures with graphite, Kanolt employed the following method:

The substance was contained in a graphite crucible with re-entrant tube carried from the lid, as shown in *Fig.* 9.

The crucible was heated in a graphite spiral furnace and the pyrometer sighted on the bottom of the tube; this ensured that approximately black body conditions were obtained.

On plotting the heating or cooling curves a well-defined halt was observed at the transition point.

The following metals and salts were employed: Antimony, 630° C.; copper-silver eutectic, 779° C.; silver, 960·5° C.; copper, 1083° C.; diopside (melting), 1391° C. Prolonged heating of diopside in contact with graphite had no apparent effect on the value obtained for the melting-point.

Attempts have been made by Hoffman and Meissner to employ a similar method in the case of the palladium freezing-point.

A hard porcelain crucible and tube were used with an oxidising or neutral atmosphere around the metal. It was found that the molten palladium attacked the porcelain with the formation of a brownish substance.

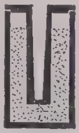

Fig. 9.—Graphite Crucible with Re-entrant Tube supported from the Lid.

An additional difficulty was the "spitting" of the fused metal, accompanied by considerable temperature fluctuations; this could not be prevented even by the passage of a stream of pure nitrogen into the metal.

For the direct calibration of a pyrometer in terms of the melting-point of palladium the simplest procedure is to make the palladium wire a part of an electrical circuit and heat it up in a furnace under "black body" conditions.

The melting-point can be detected by the break of the circuit, and the temperature at this instant should be noted.

The melting-point of platinum can also be employed as a fixed point in the same way.

§ (9) Calibration of Optical Pyrometers by Comparison with a Standard Instrument.—When an optical pyrometer has been standardised by reference to high temperature melting-points and its scale calculated, it is not a difficult matter to calibrate other instruments by comparison.

A carbon tube furnace is suitable for temperatures up to about 2500° C. This type of furnace is very simple in construction. The carbon tube is clamped in water-cooled electrodes and heated by a current of several hundred amperes.

The incandescent tube is protected from oxidation by filling the furnace shell with finely divided lamp-black and the ends are closed by thin glass windows. A stream of nitrogen is passed through the tube to clear away any smoke produced.

When the furnace is at a steady temperature the pyrometers are sighted in turn upon a plug of carbon fixed about midway in the tube.

If instruments of identical construction have to be compared it is not essential to have a "black body," and the plate of a tungsten arc lamp[1] constitutes a convenient hot object to sight upon. By arranging the ionising filament so that it can be heated independently of the current through the arc, it is possible to obtain a range of temperatures for the plate.

§ (10) Relative Merits of the Disappearing Filament and Polarising Types of Optical Pyrometers.—The filament type is essentially a telescope, and consequently it is easy to select out the object whose temperature is desired; the polarising type does not permit of a sharply defined image; in fact, the field should be a uniformly illuminated semicircular patch. A blurred image of the hot object is, however, distinguishable by moving the eye about a little.

With the polarising type it is necessary, when taking the temperature of a metallic surface, to sight normally since the light given out at oblique incidence is largely polarised.

The polarising type has the following advantages:

1. Extrapolation of the scale on the basis of Wien's or any other radiation law is readily effected, as the instrument is essentially a photometer.

2. The temperature scale is independent of the permanency of the electric lamp, which is set from time to time against an amyl acetate lamp.

It should be observed, however, that accuracy in this setting on the amyl acetate is of fundamental importance.

§ (11) Wave-length of the Radiation transmitted by the Red Filter Glass.—Over the range 700° to 1400° C., through which it is possible to calibrate an optical pyrometer against a thermo-element under "black body" conditions, it is immaterial what the wave-length transmitted by the filter glass may be. But when it becomes necessary to extend the scale to high temperatures, the use of a rotating sector or an absorption glass is necessary to cut down the intensity. The higher range is calculated on the basis of Wien's law as follows:

Calculation of the Constant of an Absorption Glass or Rotating Sector.—It is assumed that the absorption glass or sector cuts down the intensity of the light from the hot object in the ratio of k *to* 1. If T_1 is the observed temperature without absorption glass, and T_2 the apparent temperature of the same object with absorption glass, then by Wien's law

$$E=c_1\lambda^{-5}e^{-\frac{c_2}{\lambda T_1}},$$

$$kE=c_1\lambda^{-5}e^{-\frac{c_2}{\lambda T_2}},$$

$$\log_e k=\frac{c_2}{\lambda}\left(\frac{1}{T_1}-\frac{1}{T_2}\right),$$

where c_2 is a numerical constant[2] whose value is 14,350 micron degrees, and λ is the wave-length; so that T_1 can be calculated from the observed T_2 if k and λ are known from independent measurements.[3]

If the red filter glass was strictly monochromatic the wave-length transmitted λ would be a constant. Thus it would be the same for any energy distribution and temperature of source.

As it is impossible in practice to obtain absolutely monochromatic filter glass, the question arises which particular wave-length in the spectral band transmitted by the glass is to be employed in the calculations?

In order to decide upon the appropriate wave-length it is necessary to consider the function of the filter glass.

Now the quantities which are actually compared in pyrometry are the integral luminosities as observed through the filter glass, so it is obvious that the wave-length to be employed in calculation (or the "effective" wave-length as it is termed) must be such that for any definite temperature interval the ratio of the radiation intensities for this wave-length according to Wien's law shall equal the ratio of the integral luminosities as observed through the glass.

§ (12) Determination of the "Effective" Wave-lengths for various Temperature Ranges.—The following method for the determination of the "effective" wave-length for the interval between two definite temperatures of a full radiator has been described by Hyde, Cady, and Forsythe. Other methods, depending on a knowledge of the sensibility curve of the eye, have been described by the same authors, by Pirani, and by Foote, who gives a mathematical treatment of the question.

(i.) *Direct Determination.*—The ratios of the intensities of emission of the source for a number of wave-lengths are measured, and these ratios compared with the ratio of the integral luminosities of the radiation from the source, when observed through the red glass under test.

These measurements can be made in two ways:

In one set of measurements the ratios of the intensities of radiation are measured with a spectrophotometer, and the ratio of the integral luminosities with a Lummer-Brodhun photometer having the red glass over the eyepiece. In the other set the ratio of the intensities are measured with a spectral photometer (Henning type), and the ratio of the integral luminosities with an optical pyrometer, either disappearing filament or polarising type, with the red glass over the eyepiece.

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[1] These lamps are manufactured by the Ediswan Co. for optical projection purposes.

[2] For a discussion of the value of this constant see "Radiation, Determination of Laws of," §§ (9), (11), (12).

[3] This expression is also applicable for calculating the small absorption effect of a glass window closing a furnace.

(ii.) *Indirect Determination.*—The results obtained by calculation from the transmission curve of the red glass (from which the integral luminosities are calculated on the basis of Wien's law) and the sensibility curve for the eye are in close agreement with those obtained by direct experiment.

The procedure for obtaining the values of λ is laborious, and is resorted to only in the case of standard instruments for use at temperatures exceeding 1500° C.

Foote adopts the indirect method, but develops the method of computation, so that the effective wave-length for a definite temperature is obtained instead of over an interval of temperature.

His method of calculation is as follows:

Let the transmission coefficient of the glass, *i.e.* the ratio of energy transmitted to the incident energy, be denoted by k. Then $k=f(\lambda)$, where $f(\lambda)$ is an unknown function to be determined experimentally. And let the visibility curve for the average eye be represented by

$$\mathrm{V}=\mathrm{F}(\lambda).$$

It is not necessary to know the mathematical equations for these as the graphic forms-of the functions are quite convenient.

Now according to Wien's law the intensity at temperature T_1 is

$$\mathrm{E}_{\mathrm{T}_1}=c_1\lambda^{-5}e^{-\frac{c_2}{\lambda\mathrm{T}_1}},$$

and at T_2

$$\mathrm{E}_{\mathrm{T}_2}=c_1\lambda^{-5}e^{-\frac{c_2}{\lambda\mathrm{T}_2}},$$

so that

$$\frac{\mathrm{E}_{\mathrm{T}_1}}{\mathrm{E}_{\mathrm{T}_2}}=e^{\frac{c_2}{\lambda}\left(\frac{1}{\mathrm{T}_2}-\frac{1}{\mathrm{T}_1}\right)}.$$

Since λ may have any arbitrary value, it can be chosen so as to give the ratio of $\mathrm{E}_{\mathrm{T}_1}$ to $\mathrm{E}_{\mathrm{T}_2}$ the same value as the ratio of the integral luminosities seen through the filter glass.

Let L_1 = luminosity at temperature T_1,
L_2 = luminosity at temperature T_2.

Then

$$\mathrm{L}_1=\int_0^\infty \mathrm{E}k\mathrm{V}d\lambda=\int_0^\infty \mathrm{E}(\lambda,\mathrm{T}_1)k(\lambda)\mathrm{V}(\lambda)d\lambda,$$

$$\mathrm{L}_2=\int_0^\infty \mathrm{E}(\lambda,\mathrm{T}_2)k(\lambda)\mathrm{V}(\lambda)d\lambda,$$

$$\frac{\mathrm{L}_1}{\mathrm{L}_2}=\frac{\int_0^\infty \mathrm{E}(\lambda,\mathrm{T}_1)k\mathrm{V}d\lambda}{\int_0^\infty \mathrm{E}(\lambda,\mathrm{T}_2)k\mathrm{V}d\lambda}.$$

L_1 and L_2 can be determined by graphical integration. Let the ratio be any definite number. It is possible to so choose λ in the relation $\mathrm{E}_{\mathrm{T}_1}/\mathrm{E}_{\mathrm{T}_2}$ that $\mathrm{E}_{\mathrm{T}_1}/\mathrm{E}_{\mathrm{T}_2}=\mathrm{L}_1/\mathrm{L}_2$. Call this value of λ, λ_m. By substituting in the above expressions we have

$$\text{Mean } \lambda_m=\frac{c_2\left(\frac{1}{\mathrm{T}_2}-\frac{1}{\mathrm{T}_1}\right)}{\log_e \mathrm{L}_1-\log_e \mathrm{L}_2}.$$

This is merely the mathematical form of the definition of effective wave-length proposed by Hyde, Cady, and Forsythe

Instead, however, of referring the effective wave-length to a given temperature interval T_1 to T_2 it may be referred to a definite temperature by letting the two temperatures approach one another.

Assuming T_1 and L_1 constant and dropping the suffixes, let T_2 and L_2 approach these values as their respective limits. Then

$$\lambda_m=\text{limit of }\frac{c_2\left(\frac{1}{\mathrm{T}_2}-\frac{1}{\mathrm{T}}\right)}{\log \mathrm{L}-\log \mathrm{L}_2}$$

becomes, when numerator and denominator have been separately differentiated with respect to T,

$$\lambda_m=\frac{-c_2\frac{1}{\mathrm{T}^2}}{-\frac{1}{\mathrm{L}}\frac{d\mathrm{T}}{d\mathrm{L}}}=\frac{c_2\mathrm{L}}{\mathrm{T}^2\frac{d\mathrm{L}}{d\mathrm{T}}},$$

but

$$\mathrm{L}=\int_0^\infty \mathrm{E}k\mathrm{V}d\lambda,$$

$$\frac{d\mathrm{L}}{d\mathrm{T}}=\frac{d}{d\mathrm{T}}\int_0^\infty \mathrm{E}k\mathrm{V}d\lambda=\int_0^\infty k\mathrm{V}d\lambda\frac{d\mathrm{E}}{d\mathrm{T}},$$

since only E is a function of T.

Again,

$$\mathrm{E}=c_1\lambda^{-5}e^{-\frac{c_2}{\lambda\mathrm{T}}},$$

$$\frac{d\mathrm{E}}{d\mathrm{T}}=c_1\lambda^{-5}e^{-\frac{c_2}{\lambda\mathrm{T}}}\left[\frac{c_2}{\lambda}\frac{1}{\mathrm{T}^2}\right],$$

$$\frac{d\mathrm{L}}{d\mathrm{T}}=\int_0^\infty k\mathrm{V}d\lambda c_1\lambda^{-5}e^{-\frac{c_2}{\lambda\mathrm{T}}}\left(\frac{c_2}{\lambda}\frac{1}{\mathrm{T}^2}\right)$$

$$=\frac{c_2}{\mathrm{T}^2}\int_0^\infty \frac{k\mathrm{VE}d\lambda}{\lambda},\text{ since } \mathrm{E}=c_1\lambda^{-5}e^{-\frac{c_2}{\lambda\mathrm{T}}};$$

$$\therefore\ \lambda_m=\frac{c_2\mathrm{L}}{\mathrm{T}^2\frac{c_2}{\mathrm{T}^2}\int_0^\infty \frac{k\mathrm{VE}}{\lambda}d\lambda}\quad\frac{\mathrm{L}}{\int_0^\infty \frac{k\mathrm{VE}}{\lambda}d\lambda}$$

$$\lambda_m=\frac{\int_0^\infty k\mathrm{VE}d\lambda}{\int_0^\infty \frac{k\mathrm{VE}}{\lambda}d\lambda}.$$

This is the effective wave-length for a glass of transmission $k=f(\lambda)$ for temperature T.

To find λ_m, therefore, it is necessary to plot the transmission curve, k, for the glass, the visibility curve, V, for the eye, and the energy curve, E, of the source.

The product of corresponding ordinates would give a new curve, the area of which is the numerator of the fraction representing λ_m.

Similarly a curve obtained by dividing each ordinate of the previous curve by its λ would give a curve the area of which is the denominator of the fraction

A typical series of curves is shown in *Fig.* 10.

But Foote points out that instead of plotting λ in terms of EkV as ordinates, the ordinary luminosity curve, we may plot VE$k/\lambda$ as ordinates. Then the value of λ_m is that value which corresponds to the centre of gravity of the curve with respect to the EkV/λ axis.

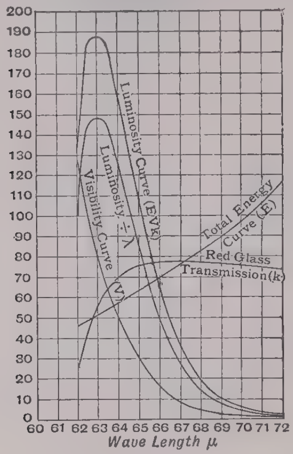

FIG. 10.

Hence the true effective wave-length of a pyrometer glass is the wave-length corresponding to the centre of gravity of the curve $\phi(\lambda)=\mathrm{EV}k/\lambda$ plotted in terms of λ, that is, the luminosity at any wave-length divided by the wave-length and expressed in terms of the wave-length.

The value of L can be determined for various values of T, and hence a curve found for λ_m as a function of T.

In *Fig.* 11 is shown the results obtained for typical glasses employed for filter screens.

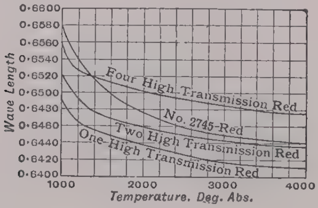

FIG. 11.

§ (13) EFFECT OF CHANGE OF TEMPERATURE OF THE FILTER GLASS ON ITS TRANSMISSION COEFFICIENT.—Another factor which influences the transmission coefficient of the red glass is its actual temperature.

Some observations have been made by Forsythe on the transmission curves for a specimen of red glass when maintained at two different temperatures 20° and 80° C. The results are shown in *Fig.* 12. Curve A gives the transmission of the glass at 20°, while curve B is the corresponding one when its temperature is 80°. The transmission is shown to decrease with increasing temperature; the coefficient of change being greatest in the shorter wave-length. The change is such as to make the transmission band appear to shift to longer wave-lengths as the temperature is increased.

A test was made of the effect of this temperature shift of the transmission band on temperature measurements when the red glass was used as a filter screen before the eyepiece of a pyrometer. The temperature of a carbon filament lamp operated at a temperature of about 1900° K[1] was measured with the red glass temperature at 20° and 80° C., using a sector disc with a 2° opening, as this gives a larger effect than a sector disc of greater transmission. It was found that there was a decrease of about 5° C. in the temperature obtained when the glass was heated to 80° over that obtained with a glass at room temperature. Hence it may be inferred that for all ordinary room temperature changes the effect is negligible.

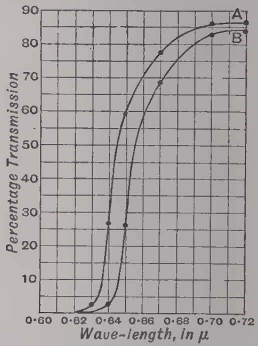

FIG. 12.—Spectral Transmission of a Single Thickness of Glass F-4512.
A at 20° C., *B* at 80° C.

§ (14) SPECTROSCOPIC EYEPIECE.—Practically all modern optical pyrometers have red filter glasses for producing approximate monochromatic radiation, and as shown above it is possible to obtain the effective wave-length of such a glass, which is equivalent for the purposes of calculation to the case of perfect monochromatism.

The early forms of optical pyrometer of the polarising type had direct vision prisms in place of filter glasses, but practical requirements demanded that the slit opening should be so wide that the spectral band transmitted was no better than that transmitted by a filter glass. If, however, the conditions of the experiment are such that narrower slits are possible the system offers certain advantages. Mendenhall, in some of his work on the disappearing type of optical pyrometer, used a spectroscopic eyepiece. The fitment consists of a small auxiliary eyepiece, and a totally reflecting prism which slides in a side tube just beyond the lamp. The images of the comparison filament and of the hot surface are thrown in sharp focus across the middle of the spectroscope slit by means of an intermediate achromatic lens, the primary image of the hot surface having previously been brought into the plane of the comparison filament by focussing in the usual way.

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[1] The symbol K denotes the absolute thermodynamic scale of temperature.

The eyepiece and ocular slit of variable width are movable with a micrometer screw, giving about 500 divisions for the visible spectrum, and it is easy to work with an ocular slit covering not more than 25 A.U. ($=0{\cdot}0025\ \mu$). The field then shows a central band due to the filament bordered by light from the hot object.

A comparative series of tests on the same object under the same conditions, employing in the one case a spectroscopic eyepiece and in the other a piece of red Jena glass having a maximum ordinate at $\lambda=0{\cdot}652\ \mu$, is given below. The distribution of light in the transmission spectrum of the red glass was determined by spectrophotometric observations:

Temperature, ° C. Red Glass.	Temperature, ° C. Spectroscopic Eyepiece.
1063	1062
1858	1861
1990	1990
2000	1995
2370	2380

§ (15) ROTATING SECTOR METHOD OF REDUCING THE INTENSITY BY A DEFINITE AMOUNT.

—Rotating sectors have been advocated by some observers for cutting down the intensity, since it is then possible to obtain the coefficient of transmission by actual measurement of the sector.

(i.) *The Sectors.*—Mendenhall employed sectors 13·5 cm. in diameter. With discs of this diameter it was found possible to cut radial openings as small as 3°, giving a factor of 1/120.

The absence of any error due to diffraction was proved by comparing the effect of ten equal and separate openings with that due to a single opening of the same total area.

The range of temperature available with various-sized sectors is illustrated by the following example:

A pyrometer calibrated with full aperture had a scale covering the range up to 1549° C. (melting-point of palladium): with a 1/60 sector the range was from 1755° to 2482° C.; and with a 1/180 sector the scale was extended to include the melting-point of tungsten (3300° C. approx.).

For the very small apertures it is advisable to use a large disc: Mendenhall employed one of 27 cm. diameter with an opening of 1·7 mm. at its narrowest part, whilst sectors 35 cm. n diameter or bigger are desirable in precision work.

The table below gives the transmission value of a number of sectors and the apparent temperatures T_2.

TEMPERATURE CORRESPONDING TO DIFFERENT PERCENTAGES OF THE RADIATION FROM A BLACK BODY HELD AT THE TEMPERATURE OF MELTING PALLADIUM (1828 DEG. K), USING RED GLASS WITH AN EFFECTIVE WAVE-LENGTH WHICH VARIES AS IS SHOWN IN COLUMN 2.

$c_2=14{,}350\ \mu$ degrees.

Transmission of Sector.*	λ_m.*	T_2 Deg. K.
0·749	0·6652	1785
·499	·6653	1727
·2443	·6655	1632
·0830	·6657	1509
·0336	·6658	1426
·01668	·6659	1356
·00542	·6662	1267

* Last digit in this column approximated. Sectors about 35 cm. in diameter.

(ii.) *Speed of Rotation of the Sectors.*—Forsythe has investigated the speed necessary for rotating sectors and his conclusions are:

The sector must rotate at such a speed that no flicker is noticeable. To accomplish this the alternations must be at least 30 to 40 per second. This is for the condition where the open and closed spaces of the sector are about equal in size. If there is a very great difference between the open and closed parts of the sector, as for instance in the case of the 2-deg. sector with two 1-deg. openings, the speed must be higher. If the motor available will not rotate the sectors fast enough when there is but one opening and one closed part, it is often a great help to make more open spaces. A good plan is to have six openings, which will reduce the necessary speed considerably. For a small sector having a 1-deg. opening this is impossible without making the sector too large, because if the sector opening is too small there is danger of an error due to diffraction. In this case, with a sector 35 cm. in diameter, it is necessary to have a motor that will rotate it something like 3500 r.p.m.

In *Fig.* 13 are shown the values found for the transmission of a 180-deg. sector as a function of the speed in alternations per second. This transmission was measured with the optical pyrometer. It can be seen, as shown in the curve, that the value found is constant and equal to the transmission of the sector for alternations above about 30 per second, but below that the transmission at first becomes smaller and then larger, and an apparent transmission of over 100 per cent is obtained for a speed of alternations of about 1 per second. The transmission, of course, approximates 100 per cent for zero speed.

The values found for the transmission for speeds corresponding to alternations between 10 and 30 per second are due to a flicker which bothers the observer. It might be that another observer would get slightly different values for the transmission of such speeds. For speeds slower than 10 alternations per second the observer attempts to make settings when the open part of the sector is passing before the opening. There is a very great flicker and settings are attempted, using only the bright part. The value found for extremely low speeds is due to the apparent over-shooting of the brightness of the source when viewed only for the short time as compared with the pyrometer filament, that always has about the same brightness.

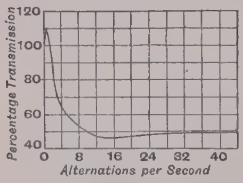

Fig. 13.

If the sector has a long opening measured in the direction of the radius, care must be taken to always use about the same part of this opening. This is to avoid error due to the fact that the radial sides of the openings may not be straight or may not be in a radial direction.

§ (16) Absorption Glass for reducing the Intensity.—For most work the use of a neutral-tinted glass is to be preferred on account of its simplicity, but it has the disadvantage that the effective wave-length shifts towards the green with increasing temperature. The best neutral glasses are far from being neutral. The transmission coefficient of the glasses obtainable varies considerably through the visible spectrum.

In *Fig.* 14 are shown some typical transmission curves for various absorbing glasses. The degree to which it is necessary for the absorbing screen to have a spectral transmission independent of the wave-length depends on the monochromatism of the filter glass. It is evident that if the red filter glass is absolutely monochromatic any absorbing glass will be satisfactory. If it transmits a band of appreciable width, then colour differences are encountered which renders the matching of the brightness extremely difficult.

The total transmission of the absorbing screen when used in conjunction with a red glass in the eyepiece in the usual manner can be calculated for any black body distribution by the following formula:

$$\text{Total transmission coefficient} = \frac{\int_0^\infty EVk_Rk_Bd\lambda}{\int_0^\infty EVk_Rd\lambda},$$

where E = black body energy for interval λ to $\lambda + d\lambda_1$ and V = visibility. k_R and k_B = spectral transmission of red and absorbing glasses respectively.

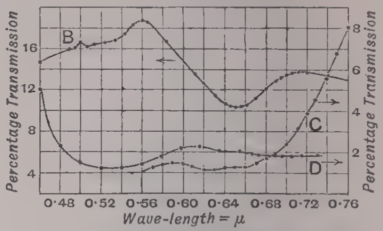

Fig. 14.—Spectral Transmission of Various Absorbing Glasses.

Curve *B*, Jena absorbing glass 1·5 mm. thick; curve *C*, Noviweld, obtained from Corning Glass Works, shade about 6; curve *D*, Leeds & Northrup absorbing glass made of purple and green glass.

It is evident that if the spectral transmission of the absorbing glass is different for different wave-lengths, the total transmission will be a function of the temperature of the source under investigation.

In *Fig.* 15 is shown, as a function of the temperature of the source, the total transmission for red light of the absorbing glasses having the spectral transmission given by curves B and C, *Fig.* 15.

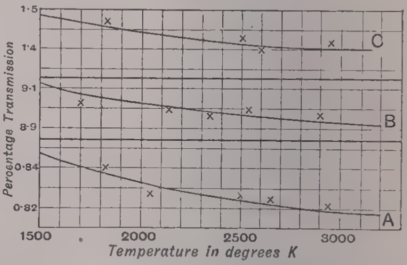

Fig. 15.—Total Transmission of Absorbing Glasses, as a Function of Temperature when used with Red Glass No. 4512—5·8 mm. thick.

Curve *A*, two pieces Jena absorbing glass; *B*, one piece Jena absorbing glass: *C*, Noviweld glass from Corning Glass Works. Curves drawn through points calculated from the above equation. Crosses represent values of transmission obtained with optical pyrometer.

The data were obtained by Forsythe, who employed a double thickness of the red filter glass whose spectral transmission is shown in *Fig.* 13.

§ (17) Distribution of Energy in the Heat Emission Spectrum of the Metals and Correction to Optical Pyrometer Readings.—While Planck's formula is capable of representing with considerable accuracy the distribution of energy among the wave-lengths of the spectrum of a "full radiator," the corresponding problem for a metal has not yet been solved. Attempts have been made to apply a modification of Wien's form of equation to represent the distribution of energy in the heat spectra of the metals, such as

$$E_\lambda = c_1'\lambda^{-a}e^{\frac{-c'_2}{\lambda T}}.$$

For platinum Paschen obtained the value 6·4 for a; Lummer and Pringsheim, 6·0; while M'Cauley, in a detailed investigation, was unable to find any constant value of a to satisfy either the equation or a modified form of Planck's.

A considerable amount of experimental data has been accumulated, which is of practical value, insomuch that it permits of the calculation of the corrections to temperature observations on such surfaces when taken with an optical pyrometer calibrated under "black body" conditions.

Unfortunately, however, such corrections can only be regarded as approximations, since it has been shown that the slightest oxide film seriously affects the emissivity of a metallic surface.

The work done on this subject may be roughly grouped under two headings: the one in which experiments have been made with spectral bands of appreciable width, such as those obtained in practical forms of optical pyrometers, and over extended ranges of temperature; the other in which experiments have been made with almost monochromatic illumination for various wave-lengths in the spectrum, the investigation being usually confined to a few steady temperatures.

§ (18) Emissivity of Iron Oxide.—When iron is heated in air to a temperature of about 800° C. its surface becomes covered with a brittle coating of oxide, apparently consisting of FeO and Fe_2O_3 in various proportions. In appearance the surface is silky and pitted with minute depressions, which produce the same effect as a collection of minute "black bodies," and consequently the surface has a high emissivity. Burgess and Foote have measured the emissivity of this oxide surface.

A small sample of the oxide was heated up on a platinum strip and its apparent temperature observed with a disappearing filament type of optical pyrometer, having a red glass with transmission band at $\lambda = 0{\cdot}65\ \mu$.

The true temperature was obtained by placing on the platinum strip minute specimens of NaCl, Na_2SO_4, and Au.

The table below gives the emissivity of the oxide surface at various temperatures:

Melting-point of	True Temperature, ° C.	Apparent Temperature, ° C.	Average Deviation from Mean.	Emissivity.
NaCl	801±1	801	±1·5	1·00
Na_2SO_4	884±1	882	±3	0·97
Au	1063±2	1058	±2	0·94

The emissivity is (§ (12) (ii.)) given by the equation

$$\frac{1}{T} - \frac{1}{S_\lambda} = \frac{\lambda}{{\cdot}4343 c_2} \log_{10} E_\lambda,$$

where T is the true temperature,
S_λ is the observed temperature,
λ is the wave-length of light considered measured in μ units,
c_2 is Wien's constant (it was assumed to be 14,450).[1]

The emissivity appears to decrease slightly with increasing temperature, which is the case also with nickel and other oxides.

The authors ignore the observation at 801° C., and by extrapolation—assuming a straight line—obtain the following corrections to reduce apparent to true temperatures:

Observed Temperature, ° C.	Correction, ° C.
600	0
700	0
800	+ 1
900	+ 2
1000	+ 4
1100	+ 6
1200	+10

§ (19) Emissivity of Copper and Cuprous Oxide.—Burgess investigated the emissivity of a clear copper surface and one of the oxides, employing the disappearing filament type of pyrometer with red ($\lambda = 0{\cdot}65\ \mu$) and green ($\lambda = 0{\cdot}55\ \mu$) glasses.

The copper was contained in shallow crucibles of magnesia and graphite, about 8 cm. inside diameter. These were heated in a gas furnace, and by regulating the gas and air supply it was possible to obtain either the clear metal surface or the oxide surface. The crucible was set in the furnace so that the flames could not play on the surface, and also in such a position as to avoid direct radiation from the walls of the crucible into the pyrometer.

In the case of the rough oxide surface diffuse reflection from the surface might vitiate the observations. But when sighting on the mirror-like surface of the molten metal only specular reflection was possible, and there could be no error due to this cause, unless the image of some hot object was actually visible in the telescope.

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[1] The value generally accepted at the present date is 14,350. The factor $\cdot 4343 = \log_{10} e$.

Temperatures were measured by means of a thermo-element bent into the form of a crook.

From the observations the following equations were deduced, in which t is the true temperature centigrade and s the apparent.

(i.) *Clear Molten Copper Surface.*

Red light $(\lambda = 0{\cdot}65\ \mu)$,

$$t = 1{\cdot}515\, s - 359.$$

Green light $(\lambda = 0{\cdot}55\ \mu)$,

$$t = 1{\cdot}515\, s - 477.$$

Hence at the melting-point the apparent temperature, with the red glass, is too low by 130°.

The green glass gives a temperature 78° higher than the red glass throughout the range.

Burgess states that in deducing the equations a slight allowance was made for the fact that the optical readings would tend to be too high on account of the slightest traces of impurity on the copper surface.

The maximum difference between the observed and computed values of the temperature over the range 1073° to 1200° C. was about 13°.

(ii.) *Cuprous Oxide Surface.*—The formation of an oxide film caused an apparent increase of 100° in the temperature of the molten copper when observed with the red glass, and 35° in the case of the green glass.

The smaller increase with the green glass is due to the fact that molten copper radiates strongly in the green. This greenish appearance persists in the case of incandescent solid copper, as may be observed by adjusting the gas feed to remove surface oxidation.

The relation between the apparent and true temperatures, in the case of the oxide, is not quite linear. This can be seen by a consideration of the data in the Table below, which refer to the red light of wave-length $\lambda = 0{\cdot}65\ \mu$.

Pyrometer Reading, ° C.	True Temperature, ° C.
Molten Copper	
950	1082
975	1118
1000	1156
1025	1193
1050	1231
Cuprous Oxide	
900	903
950	958
1000	1020
1050	1087
1100	1159
1150	1233

EMISSIVITIES

Temperature, ° C.	$\lambda = 0{\cdot}65\ \mu$.	$\lambda = 0{\cdot}55\ \mu$.
	Molten Copper	
1075	0·17	0·47
1125	0·15	0·38
1175	0·14	0·32
1225	0·13	0·28
	Cuprous Oxide	
1000	0·80	. .
1100	0·60	0·68
1200	(0·49)	(0·49)

The corrections necessary to temperature observations with an optical or total radiation pyrometer when sighted on copper and copper oxide surfaces are shown graphically in *Fig.* 16

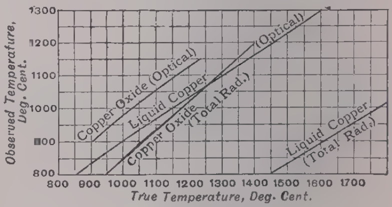

FIG. 16.—Black Body Temperatures for Liquid and Oxidised Copper $(\lambda = 0{\cdot}65\mu)$.

§ (20) EMISSIVITY OF SOLID AND LIQUID GOLD.—Gold is one of the metals which does not oxidise appreciably when heated.

Both copper and gold emit greenish or bluish light at high temperatures. Stubbs and Prideaux made a study of the emissivity of gold. The radiation of the various wave-lengths was measured by means of a König spectrophotometer and direct comparison made with the radiation from a "black body" at the same temperature.

The metal was contained in a silica capsule of 4·5 cm. diameter and 6 mm. deep. The "black body" was placed in exactly the same position for the second experiment. It consisted of a cylindrical graphite block, 11 cm. long, 5 cm. in diameter, in the centre of which a hole 12 mm. in diameter by 9 cm. deep was bored.

True temperatures were obtained by means of thermocouples.

In the case of the gold the two couples dipped into the metal, one on either side of the field of view. Owing to the shallowness of the depth of immersion an error of the order of 6° at 1000° C. was introduced in the observed readings. A correction was applied, this being determined by observing the apparent freezing-point. Two couples were also embedded in the graphite. The black body conditions were sufficiently perfect to make it impossible to distinguish the white thermocouple when it projected into the central hole.

(i.) *Impurities or Inequalities in the Gold Surface.*—Any slight surface film present was found to produce an increase in the red radiation in which the gold spectrum was weak.

The impurities were got rid of in the case of the molten surface by the application of borax.

The solid gold surface proved to be more difficult to obtain free from oxide.

It was found that a surface turned flat, treated with four grades of fine emery powder and then with jeweller's rouge, gave an apparently perfect mirror. On heating, however, a conspicuous red film was produced. This film could be removed by repeatedly treating the surface with borax at a temperature near the melting-point. Although a clear, solid gold surface was obtained when the liquid solidified, it was generally uneven owing to contraction, crystallisation, etc., and consequently reflected heat from the furnace walls into the spectrophotometer.

Occasionally by very slow cooling an area could be obtained which was free from unevenness.

EMISSIVITY OF GOLD IN THE SOLID STATE

Wave-length.	Temperature, °C.				Mean Emissivity.
	949°.	1040°.	1061°.	1046°.*	
0·7014	[0·164]	[0·134]	0·103	0·103	0·103
0·6712	[0·146]	0·114	0·114	0·116	0·114
0·6409	[0·169]	0·144	0·143	..	0·143
0·6149	[0·198]	0·172	0·171	0·178	0·175
0·5895	0·241	0·229	0·221	..	0·229
0·5649	0·315	0·291	0·289	0·361	0·301
0·5418	0·379	0·371	0·366	..	0·371
0·5186	0·520	0·465	..	0·516	0·494
0·4961	0·595	0·492	..	0·541	0·531

* Data given in this column refer to a surface naturally crystallised from the molten state.
Values enclosed in brackets [ ] were obtained with a surface having a slight film over it.

The following values were obtained for the emissivity of molten gold at the temperature of its freezing-point :

EMISSIVITY OF GOLD IN THE MOLTEN STATE

Wave-length.	Emissivity.	Wave-length.	Emissivity.
0·7014	0·184	0·5649	0·347
0·6712	0·203	0·5418	0·390
0·6409	0·232	0·5186	0·434
0·6149	0·263	0·4961	0·473
0·5895	0·304	0·4750	0·503

For solid gold at various temperatures the values tabulated below were obtained. The data are shown graphically in *Fig.* 17.

Stubbs and Prideaux observed that in passing from the solid to the molten state a sharp discontinuity occurred in the emissivity wavelength relationship, the relative emissivity increasing in the red and decreasing in the violet. Roughly the emissivity for red light of the solid gold is only about three-fifths that of molten gold.

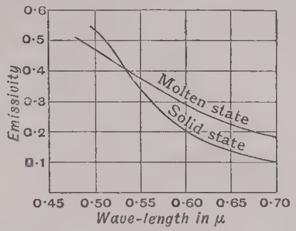

FIG. 17.—Diagram to illustrate Emissivity of Gold.

For the limited range of temperature investigated (110° C. in the case of the molten state) no change of the relative emissivity of either molten or solid gold with temperature was observed.

From the above values of the emissivities the "black body" or true temperatures corresponding to various values of the apparent temperatures can readily be calculated from the relationship :

$$\frac{1}{T}-\frac{1}{S_\lambda}=\frac{\lambda}{c_2}\log_e\frac{E_\lambda'}{E_\lambda},$$

where E_λ'/E_λ is the emissivity for wavelength λ.

TRUE TEMPERATURE, 1063·2° C.

Wave-length in μ.	Apparent Temperature, °C.	
	Solid State.	Molten State.
0·7014	891·5	931
0·6490	925·5	956·5
0·5895	964	982
0·5418	1000	1003
0·4961	1025·5	1019

The data in preceding table give the apparent temperatures by Wien's equation for various wave-lengths corresponding to a true temperature of 1063·2° C.

§ (21) SOLID AND MOLTEN COPPER AND SILVER.—An investigation on the same lines as the above described in the case of gold was carried out by Stubbs for the metals silver and copper.

(i.) *Copper.*—The metal was contained in a silica pot and heated in a closed furnace with a hydrogen atmosphere, the observations being taken through a glass window (microscope cover glass).

The block of copper (4·2 cm. diameter) was turned up to a flat surface, rubbed smooth with emery paper, and polished with metal polish. The use of rouge was avoided as it was found to tarnish on heating. No trace of filming was observed, and the surface showed no deterioration until a temperature of about 10° from the melting-point had been attained, when recrystallisation rapidly set in, and this produced stray reflection of light from the furnace wall into the photometer, owing to the roughening of the surface.

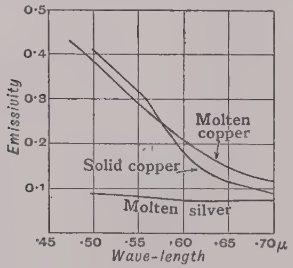

FIG. 18.—Diagram showing Emissivities of Solid and Molten Copper.

A perfect mirror of molten copper, free from film, was obtained without difficulty.

The emissivities for the solid and molten copper for various wave-lengths are shown in *Fig.* 18. The values are for a temperature of 1010° C. for solid copper and 1130° C. for molten copper. As in the case of gold, there is a discontinuity at the melting-point, but of smaller magnitude.

The values differ markedly from those obtained by Burgess described above.

This may be due to the fact that the experimental conditions in Burgess' work were less favourable, and that the red glass of his pyrometer transmitted a comparatively wide spectral band (100 $\mu\mu$), while the width of the band in the photometer employed by Stubbs and Prideaux was 8 $\mu\mu$.

(ii.) *Silver.*—It was unnecessary to take the extreme precautions for the exclusion of oxygen in the case of silver as for copper. A satisfactory reducing atmosphere was produced by placing a small quantity of powdered graphite in the furnace below the crucible containing the silver. The solid metal surface lost its polish on heating, so that no measurements could be made of its true emissivity.

The emissivity of the molten surface for various wave-lengths is shown in *Fig.* 19.

There appeared to be a slight increase in the relative emissivity with temperature, but owing to the small magnitude of the coefficient the authors are in some doubt as to its real nature.

The apparent temperatures corresponding to the various wave-lengths at the melting-points of the metals are given below:

Wave-length in $\mu\mu$.	Copper, m.p. 1083° C. Apparent Temperature.		Silver, m.p. 961° C. (Liquid.) Apparent Temperature.
	Solid.	Liquid.	
700	896	917	792
650	924	942	804
600	966	973	816
550	1007	1003	831
500	1028	1026	845

§ (22) EMISSIVITY OF POLISHED PLATINUM.—The permanency of a polished platinum surface greatly simplifies the study of the emissivity of the metal at high temperatures.

Holborn and Kurlbaum, and Waidner and Burgess have independently investigated the difference between the apparent and true temperatures for approximately monochromatic radiation of a platinum surface. The former employed a small box of platinum, with a thermocouple in the interior, to give the true temperature, the apparent temperature being obtained by means of an optical pyrometer.

The latter worked with a Joly meldometer, which consists of an electrically heated platinum strip the expansion of which is measured by a micrometer screw.

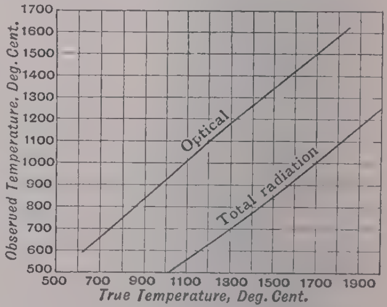

FIG. 19.—Relation between True and Apparent Temperatures for Platinum ($\lambda = 0·651\ \mu$).

The apparent temperature of the strip was determined with a "disappearing filament" type of optical pyrometer.

True temperatures were obtained by calibrating the strip by melting-points of pure metals and salts.

The electric contact micrometer permitted of accurate determinations of lengths corresponding to the melting-temperatures of minute specimens heated on the strip.

The relation between apparent and true temperatures is shown in *Fig.* 19.

§ (23) Distribution of Energy in the Spectra of Platinum, Palladium, and Tantalum.—McCauley has investigated the distribution of energy in the heat spectra of platinum, palladium, and tantalum over a wide range of temperature.

The experiments were made on electrically heated strips 3·5 cm. long, 0·05 mm. thick, by 7 mm. wide, folded into wedge-shaped filaments with 12° angular opening.

The "true temperature" was deduced from observations of the radiation from the interior of the wedge, using a disappearing filament type of optical pyrometer.[1]

The distribution of energy was obtained with the usual spectro-bolometric arrangement.

With tantalum great precautions had to be taken to obtain the highest vacuum in the enclosure containing the filament. The slightest trace of residual gas would cause the resistance to increase rapidly, the metal becoming brittle and losing its metallic appearance.

If the vacuum was satisfactory the metal after heating remained bright and ductile except for a slight flaking off at the highest temperatures.

The spectral energy curves for platinum, palladium, and tantalum were perfectly continuous and showed no bands of selective emission. They resembled generally those of a "black body," but no modifications of Wien's or Planck's formulae were found capable of resembling the curves over the range. Taking the general form of Planck's equation,

$$E_\lambda = c_1\lambda^{-a}\frac{1}{e^{c_2/\lambda T}-1},$$

and determining a mean value of a from the data of a given isothermal curve, McCauley found that the computed emission for palladium was in general from 4 to 7 times smaller than the observed value at 6 μ. The agreement was better for shorter wave-lengths and correspondingly worse for longer ones.

For all three metals the emission diminishes more rapidly than that of a "black body" at the same temperature in the infra red.

The wave-length of maximum emission shifts much more slowly towards the shorter wave-lengths for increasing temperature than for a "black body," especially at the higher temperatures. The product $\lambda_m T$ increases with temperature, and the constant value for platinum found by Lummer and Pringsheim is probably in error. Their value of $\lambda_m T = 2620$ is correct only in the neighbourhood of 2000° abs.

§ (24) Melting-points of Refractory Elements and Compounds.—For melting-point determinations at temperatures exceeding 1500° C. it is advisable to employ some type of pyrometer based on the laws of radiation, preferably of the optical type.

The measurement of the apparent temperature usually presents no difficulty but the determination of the emissive power, and hence the correction of "apparent" to true temperatures is a troublesome operation.

If the conditions permit of the material being heated under black body conditions, then the true melting-point can be obtained by direct observation.

Kanolt, for example, determined the melting-point of the refractory oxides—lime, magnesia, alumina, and chromium oxide—by heating in a graphite spiral furnace. The material was contained in a crucible with re-entrant tube, and a heating up curve taken with a pyrometer sighted on the bottom of the re-entrant tube (see *Fig.* 10). At the melting-point a halt in the time-temperature curve is observed.

The material of the crucibles had to be chosen so as not to react with the charge. Graphite and tungsten crucibles were found satisfactory for the above oxides.

Sometimes the material is so costly that sufficient quantity to fill a crucible cannot be obtained, so the direct method cannot be employed.

Mendenhall and Ingersoll adapted the Nernst glower as a source of high temperature heat supply for the determination of the melting-points of rhodium and iridium. A polarising type of pyrometer was used for taking the temperature of the Nernst filament (see *Fig.* 20). The apparatus was calibrated from the known values of the melting-points of gold, palladium, and platinum. The metal under test was in the form of a minute globule, diameter of the order of 0·05 mm., which was observed by means of a microscope. By careful manipulation of the current through the glower it was possible to maintain one part

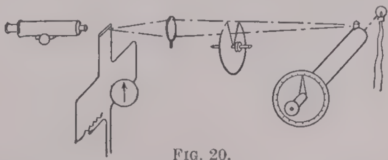

Fig. 20.

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[1] This device for obtaining "full radiation" is discussed in detail in § (24).

melted and the other part solid in the case of most of the metals. The method, of course, is only applicable to metals which draw into clear metallic beads and which give consistent readings for successive melts.

The metals tungsten, tantalum, and molybdenum being readily oxidisable cannot be studied by any of the previously described methods. Hence the experimental arrangements must be such that the electrically heated sample gives approximately "full radiation."

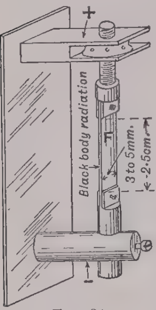

FIG. 21.

Mendenhall accomplished this by means of a narrow wedge opening formed by folding on itself a sheet of the material being studied.

The arrangement of the apparatus is shown in *Fig.* 21, where F is a flat conducting ribbon, heated by a longitudinal electric current, as shown, and folded on a line parallel to the length so that the resulting cross-section perpendicular to the current flow is a very narrow V—say with about 10° angular opening. If the ribbon is of uniform thickness and width, it will be raised to a uniform temperature by a given current, except near the ends. The inside of the V might be then expected to be a close approximation to a black body, since it has but a small opening and uniformly heated walls, and if this were so, observations on it with an optical pyrometer would give the true temperature of the inside walls. The outside of the V will give radiation characteristics of the material of the ribbon.

The questions arise :

(1) How closely does the radiation from the inside of the V approximate to that of a black body at the temperature of the inside walls ?

(2) How much real temperature difference is there between the inside and outside surfaces of the wall of the V ?

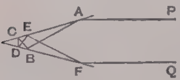

FIG. 22.

The first of these two questions is answered by considering the building up of radiation within the V opening by multiple reflection. In *Fig.* 22 a V opening is formed by bending a specular reflecting sheet. Points A, B, C, D, E, and F are points of reflections for a ray which may be imagined as entering at P. If the material of the V is radiating, in consequence of its temperature, for any range of wave-lengths, the brightness of the point F, as viewed from Q, may be considered as made up of various components ; first, that due to the natural radiation from F ; second, that due to the natural radiation from E reflected at F ; third, that due to the natural radiation from D which is twice reflected at E and at F, etc. Limiting ourselves to a small wave-length interval, remembering according to Kirchhoff's law that the reflection factor r is equal to $1-e$, and representing by b' the spectral brightness of a black body at the temperature of the material of the V, and by b'' the corresponding spectral brightness of the point F as viewed from Q, we have

$$\begin{aligned} b'' &= eb' + reb' + r^2eb' + \ \dots\ r^neb' \\ &= eb'(1-r^n). \end{aligned}$$

With a V opening of 10°, as suggested by Mendenhall, n will be equal to 18. Thus with r equal to 0·7 (which is roughly the value for the material used originally by Mendenhall) b''/b is found to be 99·8 per cent, that is, the radiation from the V cavity may be said to be 99·8 per cent black, a satisfactory approach to black-body radiation.

The second question relating to the temperature difference between the inside and the outside of the V opening was settled by computing the difference in temperature from the known dimensions, the electrical input, and the thermal conductivity of the material. For the platinum wedges used, Mendenhall found a difference of the order of a few tenths of a degree. His results on platinum agreed quite well with the previously mentioned results by Holborn and Kurlbaum and by Waidner and Burgess.

Later Mendenhall and Forsythe applied this method with considerable success to tungsten, tantalum, molybdenum, and carbon.

While the V method of obtaining the true temperature of the material being investigated was theoretically a considerable advance, it left some uncertainties. The method demanded a uniform temperature over relatively large plane surfaces. Moreover, in certain cases, particularly in connection with tungsten, trouble was experienced due to the two separate sheets, found necessary at that time in making up the V, separating so as to leave a gap between the two parts.

Worthing devised another method of effecting the same object. He employed a hollow cylindrical tungsten filament perforated with small holes. This was mounted in a large lamp bulb.

Determinations of the brightness were made by sighting on a hole and on the adjacent surface. The ratio of the latter brightness to the former when corrected for (1) the difference in temperature between the interior and the surface, (2) for the departure from "full radiation" of that from the interior due to the presence of the hole, (3) for the lack of symmetry in the temperature distribution over the filament, gives the emissive power for the metal at that temperature.

A linear relation was obtained between the emissive power and the true temperature over the range 900° to 2900° C.; the values at these two temperatures being 0·467 and 0·406 respectively for $\lambda = 0{\cdot}666\ \mu$. The corresponding corrections to convert apparent to true temperatures are 33° and 375° C.

Assuming that the linear relationship between emissive power and temperature for solid tungsten continues to hold up to the melting-point, this occurs at 3360° C.

The apparent temperature (for red radiation) of the melting-point is 2870° C., and the emissivity 0·390.

It is advisable to make the intensities of the light from the lamps and the standard approximately equal to facilitate comparisons of the colours. In the table below are given some data by Hyde for commercial tungsten lamps in which the true temperature, the colour temperature, and the apparent or brightness temperature are compared with the lumens per watt. It might be remarked that the temperature scale is based upon the melting-point of gold at 1063° C., and the value 14350 for c_2 in Wien's equation. This gives the value 1555 for the melting-point of palladium, whereas the value obtained by Day and Sosman was 1550° C. The difference, however, is practically within the limit allowed for experimental error.

Summary of Determination of Melting-points of Tungsten and Tantalum

Observer.	Date.	Method.	Value, ° C.	Average Deviation from Mean, ° C.
		Melting-point of Tungsten		
Waidner and Burgess	1907	Extrapolation	3080	28
Pirani	1910	”	3250	. .
Ruff	1907	Carbon tube furnace	2650	33
Wartenberg . .	1910	Apparent temperature and reflecting power	2930	30
Forsythe . . .	1911	Vacuum furnace	2970	27
		Wedge method	2970	. .
		Apparent temperature and wedge	3030	. .
Langmuir . . .	1915	Arc method	3270	30
Worthing . . .	1916	Hollow filament method	3360	. .
		Melting-point of Tantalum		
Waidner and Burgess	1907	Extrapolation	2900	. .
Pirani . . .	1910	”	3000	. .
	1911	Apparent temperature and reflecting power	2700	. .
Forsythe . . .	1911	Wedge method	2800	. .

§ (25) Colour Match Method of determining Filament Temperatures.—A method of estimating the temperature of lamp filaments which has come into extensive use in recent years is that based on colour matching. By the colour temperature is meant the temperature of a full radiator when its radiation matches in colour the radiation from the incandescent metal. The relation between colour temperature and true temperature for tungsten has been determined by Hyde, Cady, and Forsythe, so that it is now possible to estimate the temperature of the filament of a tungsten lamp by matching its colour against that of a standard whose temperature is known.

In practice the comparison standard is not a "black body" but a combination of a vacuum lamp with a filter. This lamp is run at different voltages to give the desired colour for matching. The lamp filaments to be tested are compared by means of Lummer-Brodhun photometer head with the standard.

Melting-point of Tungsten = 3675° K.

Brightness Temperature. ° K.	True Temperature. ° K.	Corrected for End Effects and Bulb Absorptions.	
		Colour Temperature. ° K.	Lumens per Watt.
1600	1701	1714	0·78
1650	1758	1775	1·04
1700	1816	1837	1·40
1750	1874	1898	1·81
1800	1932	1960	2·31
1850	1990	2021	2·90
1900	2049	2082	3·56
1950	2108	2144	4·35
2000	2167	2205	5·26
2050	2227	2266	6·30
2100	2287	2327	7·45
2150	2347	2389	8·69

Above table based on the following constants:
Melting-point of gold, 1336° K.
Melting-point of palladium, 1828° K.
These result in Wien's constant c_2 being equal to $14350\ \mu \times$ deg.

§ (26) Temperature Measurement in Terms of Total Intrinsic Brilliancy.

—An empirical method of estimating temperatures which is occasionally used in connection with light sources, such as incandescent lamp filaments, is one developed by Rasch.

He proposed the relationship

$$\log \mathrm{H} = -\frac{\mathrm{A}}{\mathrm{T}} + \mathrm{B}$$

for connecting the temperature T (abs.) of a "black body" and its total intrinsic brilliancy H.

It will be observed that this equation is merely Wien's law, in which monochromatic light is replaced by the total radiation visible to the normal eye.

This, of course, is a somewhat questionable assumption, but some support is given to it by the fact that Crova has shown that, for all ordinary light sources giving a continuous spectrum, there is one wave-length for which the monochromatic intensity is proportional to the total intensity. This wave-length is about 0·58 μ.

In order to test Rasch's equation it is necessary to measure the intrinsic brilliancy of a "black body," *i.e.* the candle-power per sq. mm. It is a difficult matter to do this with sufficient accuracy on the usual type of uniform temperature enclosure form of "black body," and the following indirect method was adopted by Nernst.

Nernst lamp filaments were measured, and the relation between the watts expended and the candle-power obtained was observed in the usual manner by photometric measurements.

A filament was then set up in front of a "black body" furnace, and at a series of steady temperatures the filament was brought to a temperature when it disappeared against the background, *i.e.* emitted the same intensity of light.

From the watts expended in the filament the intrinsic brilliancy of the filament, and hence that of the furnace, was obtained.

The actual temperatures of the furnace were obtained by means of a Wanner pyrometer standardised by the melting-point of gold (1064° C.), and assuming c_2 in Wien's equation to be 14600.

Rasch's equation was verified within 10° over the temperature range from 1400° to 2300° C., and from the experiments the constants of the equation were obtained:

$$\mathrm{T} = \frac{11230}{5{\cdot}367 - \log_{10} \mathrm{K}},$$

where K is the intrinsic brilliancy in Hefner candles per sq. mm.

This equation requires a correction factor if employed for taking the temperature of surfaces which are not full radiators.

For example, tungsten has an emissivity of about 0·51 for light of $\lambda = 0{\cdot}56\ \mu$.

Hence, if H is the intrinsic brilliancy of a tungsten filament, and K that of a "black body" at the same true temperature, then $\mathrm{H} = 0{\cdot}51\mathrm{K}$.

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E. G.

PYROMETRY, TOTAL RADIATION

§ (1) THE "FOURTH-POWER" LAW OF RADIATION.—At temperatures exceeding 1300° C. the practical difficulties encountered in the use of thermoelements, resistance thermometers, and gas thermometers are very considerable, even under the favourable conditions prevailing in the laboratory. For industrial work the difficulties are vastly greater and other methods have to be resorted to, such as those based on the laws of radiation. With pyrometers of the radiation type it is not necessary to subject any portion of the instrument to the temperature of the furnace, and there is no upper limit to the temperature which can be measured.

§ (2) THE STEFAN-BOLTZMANN LAW.—The earliest suggestion of a simple relation connecting the radiation from a surface and its temperature was that of Stefan in 1879, who obtained empirically the relationship between total radiation and temperature now generally known as the "fourth-power law."

In 1884 Boltzmann gave a theoretical proof of the fourth-power law based on thermodynamic principles and Maxwell's electromagnetic theory of light. He pointed out that the law was only valid for an "ideal black body."

The radiation emitted by such a body would possess a character independent of the properties of any particular substance and would be identical with the radiation within a uniformly heated enclosure. This conception of a perfect black surface and its practical realisation, by means of an enclosure at a uniform temperature, is due to Kirchhoff.[1]

He demonstrated conclusively that the radiation issuing from a small hole in a uniformly heated enclosure would be "full radiation" for that temperature, such as would be emitted by an ideal black body.

The simplicity of the law and the fact that most industrial furnaces, etc., are fair approximations to uniform temperature enclosures have been factors of immense service in the development of high-temperature pyrometry.

The formal statement of the Stefan-Boltzmann law is

$$S = \sigma(\theta^4 - \theta_0^4),$$

where S is the energy per sq. cm. per sec.,

σ a numerical constant the experimental value of which is $1{\cdot}36 \times 10^{-12}$ gm. cals. per sec. per cm.^{2},

θ the absolute temperature of the surface,

θ_0 the absolute temperature of the surroundings receiving the radiation.

It will be observed that as a method of defining absolute temperature the law is independent of the specific properties of any particular substance; the ideal black body in this respect playing the same rôle as that of a perfect gas in the definition of the gas scale.

A comparison at one temperature above zero with the gas scale would suffice to determine σ and hence connect the two scales.

Since, however, Boltzmann's demonstration involves an imaginary thermodynamic cycle with radiation as working fluid, it is necessary to confirm the theoretical deduction by experimental observations over an extended temperature range. Shortly after its formulation the law was submitted to test by various investigators; the most comprehensive series of experiments were carried out by Lummer and Pringsheim, who investigated the radiation from a "black body" enclosure over the temperature range 100° to 1300° C.

§ (3) LUMMER AND PRINGSHEIM'S EXPERIMENTS.—For the measurement of the radiation the authors employed a modified form of Langley bolometer—an instrument depending on the change of electrical resistance of platinum with temperature. Details of the instrument are given later.

(i.) *The Apparatus.*—The general disposition of the apparatus is shown in *Fig.* 1.

A was a hollow vessel containing boiling water. This source of radiation was used as a standard of reference for calibrating the bolometer from time to time, since only by this means could the variations produced by changes in the battery current and galvanometer sensibility be eliminated.

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[1] Kirchhoff defines a "black body" as one which has the property of allowing all incident rays to enter without surface reflection and not allowing them to leave again. See "Theory of Heat Radiation" (Planck; trans. by Masius).

The radiation could be cut off from the bolometer by means of a water-cooled shutter.

The "black body" C was employed for the range of temperature from 200° to 600° C. It consisted of a hollow sphere of copper blackened inside with platinum black and contained in a bath of well-stirred molten salt. This salt bath could be maintained at any desired temperature by regulating the flame. The temperature was measured by means of a high-range mercury thermometer and a thermoelement.

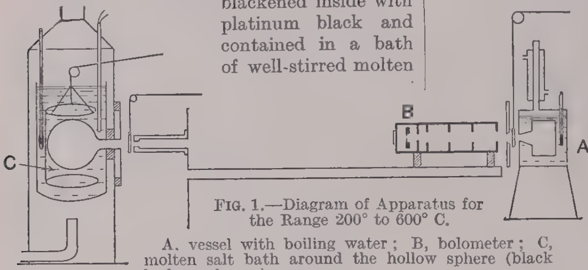


Fig. 1.—Diagram of Apparatus for the Range 200° to 600° C.

A, vessel with boiling water; B, bolometer; C, molten salt bath around the hollow sphere (black body enclosure).

The procedure in carrying out the observations was as follows:

The bath was heated up to the desired temperature and maintained steady; then the water-cooled shutter was raised to allow radiation to fall on the receiving face of the bolometer. When the galvanometer deflection had attained its maximum value the shutter was lowered and galvanometer zero redetermined; if it differed slightly from the previous value the mean was taken.

For higher temperatures, from 600° to 1300° C., the construction of the "black body" is shown in *Fig.* 2. D was an iron cylinder (coated inside with platinum black) enclosed in a double-walled gas muffle. The temperature of the interior of the iron cylinder was obtained by a thermoelement enclosed in a porcelain tube T passing through the furnace.

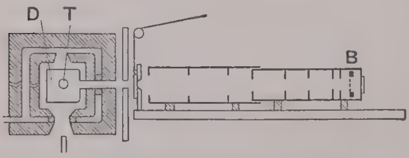


Fig. 2.—Diagram showing Construction of "Black Body," for the Range 6000° to 1300° C.

D, iron cylinder, "black body" enclosure; T, porcelain tube carrying thermoelement; B, bolometer.

(ii.) *The Bolometer.*—Essentially this is a Wheatstone's bridge, the four arms of which consist of grids of thin platinum foil similar in all respects. The method of connecting up the grids is shown in *Fig.* 3. Grids 1 and 3 are in the opposite arms of the bridge, and the strips of 3 are set to receive the radiation passing through the gaps of 1. The other two grids, 2 and 4, are similarly disposed, but shielded from radiation by a box.

To prevent wandering of the galvanometer zero, the disposition must be as symmetrical as possible, and the whole instrument enclosed in a well-lagged box provided with diaphragms to cut down the radiation falling on the absorbing surface to a parallel beam of about 16 mm. in diameter.

The grids are of foil from one to two thousandths of a millimetre thick and with a resistance of about 60 ohms each.

Theoretically the quantity of radiation received per unit time, for a given difference of temperature by unit area, at a distance r from a point source, varies inversely as r^2. Lummer and Pringsheim ascertained that their experimental arrangements complied with this theoretical condition, by taking observations at varying distances between the "black body" and bolometer. It was found that the galvanometer deflections varied inversely as the square of the distance. Since the quantity of radiation received varied as the difference of the fourth powers of the absolute temperatures of the radiator and receiver, it was necessary to vary the sensitivity of the bolometer in order to keep the galvanometer deflections within measurable limits. Two means of effecting this were employed: (1) variation of the sensitivity of the Wheatstone bridge by changing the battery current; (2) alteration of the distance between the "black body" and the bolometer.

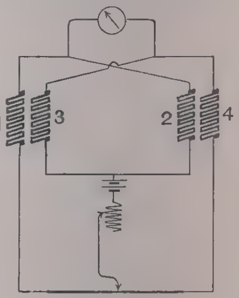


Fig. 3.

(iii.) *The Observations.*—The observations were all reduced to a common unit (arbitrary), based on the radiation from the "black body" at 100° C. at a standard distance of 633 mm.

If d was the deviation of the needle for "black body" at absolute temperature θ, and k a constant, the mean value of which for their instrument was $123{\cdot}8 \times 10^{-10}$, then

$$d = k(\theta^4 - 290^4),$$

where 290° was the absolute temperature of the water-cooled shutter (*i.e.* 17° C.).

In Table I. the experimental observations are compared with values calculated from the fourth-power law. The calculated temperatures are obtained by taking the mean value of k.

Table I

Temperature (° abs. C.) of Black Body.	Deflection (Reduced).	$k\times10^{10}$.	$\theta_{\text{calc.}}$	$\theta_{\text{obs.}}-\theta_{\text{calc.}}$
373·1	153	127	374·6	− 1·5
492·5	638	124	492·0	+ 0·5
723	3,320	124·8	724·3	− 1·3
745	3,810	126·6	749·1	− 4·1
*789	4,440	(116·7)	778·0	+11·0
810	5,150	121·6	806·5	+ 3·5
868	6,910	123·3	867·1	+ 0·9
*1092	16,400	(115·9)	1074	+18
*1112	17,700	(116·3)	1095	+17
1378	44,700	124·2	1379	− 1
1470	57,400	123·1	1468	+ 2
1497	60,600	120·9	1488	+ 9
1535	67,800	122·3	1531	+ 4

* These experiments were carried out with the gas furnace at temperatures overlapping those obtained with the salt bath. The large discrepancies are due to lack of uniformity of temperatures with a small flame. It will be seen that the observation at 789° C. is eliminated by observations 4, 6, and 7 with the nitrate bath.

Between 100° C. and 1000° C. the deviations of the calculated from the observed temperatures are small, of the order of 3°, and exhibit no systematic variation. Between 1000° C. and 1360° C. the discrepancies are greater, and the observed values are systematically larger than the calculated. It should be remembered, however, that their temperature scale is based on the gas thermometer work of Holborn and Day, and this extended to 1150° C. only. Beyond this point the values are based on extrapolation of the E.M.F. temperature curves of thermocouples, a procedure which has since been shown to lead to erroneous results. Up to 1100° C. it is probable that the temperature scale of Day and Holborn is reliable to about 3°. Consequently Lummer and Pringsheim's experiments establish the validity of the fourth-power law to 1150° C. to the same order of accuracy as the gas scale was known at that time.

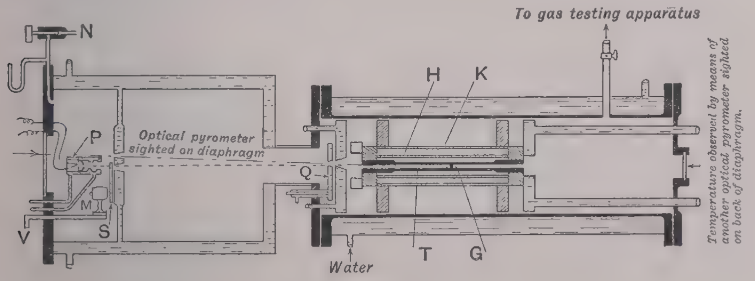

To gas testing apparatus
N
H
K
P
Optical pyrometer sighted on diaphragm
Q
M
V
S
Water
T
G
Temperature observed by means of another optical pyrometer sighted on back of diaphragm.

Fig. 4.

§ (4) Fourth-power Law between 1063° C. and 1549° C.—In an investigation, whose primary object was the comparison of the "optical scale," based on Wien's distribution law, and the "total radiation" scale, based on the fourth-power law, Mendenhall and Forsythe checked the Stefan-Boltzmann law at the two temperatures 1063° C. and 1549° C. These temperatures are the melting-points of gold and palladium respectively as determined by Day and Sosman.

The comparison was effected in an indirect manner. The melting-points of gold and palladium were observed in terms of the scale of a certain optical pyrometer under "black body" conditions. The temperature of a carbon tube furnace could then be maintained at these two temperatures by observations with the optical pyrometer, which merely served as a transfer instrument. The apparatus for verifying the fourth-power law is shown in *Fig.* 4.

The "black body" is the graphite tube T, 30 cm. long, 14 mm. inside diam., and 3 mm. wall thickness. A graphite diaphragm G is placed 1 cm. from the centre, the left-hand segment being used as "black body." The coaxial tubes H and K are merely to reduce the heat loss by radiation, etc.

The apparatus was water-cooled; the total radiation thermopile P being protected from stray radiation by water-cooled diaphragms, while a movable water-cooled shutter Q contained the limiting aperture. The geometry of the apparatus was so arranged that radiation from the graphite diaphragm G alone entered the thermopile. A motor M, movable by means of a rod V, carried a sector S of definite aperture on its shaft. By a movement of the rod the motor could be swung up and the rotating sector placed in front of the thermopile so that the radiation from the furnace was reduced in a known ratio, depending on the clear aperture of the sector.

The thermopile consisted of a single Bi-Sb and Sb-Cd alloys couple, the "hot" junction being soldered to a very light receiving disc of silver foil 3 mm. in diameter, a similar disc being attached to each cold junction, where the alloys joined copper leads. The silver disc "hot" junction was blackened with acetylene smoke and mounted at the centre of a hemispherical concave mirror, so as to make the absorption as perfect as possible. The thermocouple was directly connected with a low-resistance galvanometer, a resistance box being connected in series to control the sensibility.

Special attention was given to the elimination of any possible error due to absorption of radiation by the gases inside the furnace. With a hot object, such as graphite, it is impossible to obtain a perfect vacuum, and fluctuations of pressure would have had a serious influence on the sensibility of the thermopile.

During the observations a steady pressure (from 5 to 15 mm. of mercury) was maintained in the apparatus by controlling a valve governing the nitrogen supply; the pressure could be maintained constant within 0·2 mm. by careful regulation; a Fluess and Gaede pump steadily exhausting at the other end.

To ascertain whether the residual gas exerted appreciable absorption on the radiation, samples were drawn off from time to time to a side tube and the deflections of an auxiliary thermopile read with and without the gas, employing a Nernst filament as radiator; the authors claim that they could detect such effect if it amounted to $\frac{1}{10}$ per cent.

§ (5) Theory of the Sectored Disc.—The object of a sectored disc is to cut down the radiation by a definite fraction so that the same galvanometer deflection is obtained for two different temperatures of the radiator or "black body."

Suppose the deflection x is obtained when radiation from an object at absolute temperature T_1 is received through a clear aperture sector, and the same deflection obtained for temperature T_2 with a rotating sector having transmission ratio R (where $R < 1$). Then, if A denote the area of the disc,

$x = kAT_1^4$ for the first condition,

and $x = kRAT_2^4$ for the second condition.

Hence $T_1^4 = RT_2^4$ or $T_2 = \frac{T_1}{\sqrt[4]{R}}$.

In practice it was found impossible to bring the two deflections to absolutely the same value, consequently it was assumed that the deflections were proportional to the total energy in the two cases.

The aperture in the sector was cut in the ratio $[1336/1822]^4$, where 1336 is the melting-point of gold and 1822 the melting-point of palladium, in absolute temperatures.

Consequently, the ratio of the galvanometer deflections at these two temperatures, if the fourth-power law was obeyed, should be unity.

From twelve comparisons the observed mean value was 1·001. The maximum value of the ratio found was 1·007 and the minimum 0·998. A few typical observations are shown in Table II.

Table II

Deflection at 1063° C.	Deflection at 1549° C., with Sectored Aperture.	Ratio Column 2 to Column 1.
29·28	29·33	1·002
29·08	29·17	1·003
29·33	29·33	1·000
28·29	28·27	0·999
28·27	28·25	0·999
	Mean . . .	1·001

This indicates an agreement well within the possible limits of experimental error of ±0·5 per cent at each of the two temperatures.

All the experimental evidence available supports the conclusion that the Stefan-Boltzmann law is valid over the entire temperature range covered by the gas thermometer. It may therefore be employed with confidence, in view of its plausible theoretical foundation, as the basis of methods for the evaluation of high temperatures.

I. Total Radiation Pyrometers

Pyrometers based on the fourth-power law for the measurement of high temperatures are merely thermopiles so arranged that the instruments are (1) direct reading, (2) robust, (3) quick in action, and (4) designed to render the readings independent of the distance between pyrometer and hot body, within certain limits.

§ (6) Féry's Telescope Pyrometer.—Féry appears to have been the first to evolve a practical form of pyrometer based on the Stefan-Boltzmann law, and capable of measuring temperatures between 500° C. and 1500° C.

In the early types the instrument consisted of a telescope having a minute thermocouple connected to a sensitive portable galvanometer. The hot junction and the source of radiation were brought to the conjugate foci of the lens by focussing in the usual manner.

The difficulty with this type was the lens, which had to be transparent for both the visible and the infra-red.

Fluorite was found satisfactory for temperatures above 900° C., since it possesses a nearly constant coefficient of absorption throughout the spectrum. The indications of the instrument, however, did not follow the fourth-power law owing to the fact that fluorite has an absorption in the infra-red at about 6 μ, and is not transparent for wave-lengths greater than 10 μ.

For industrial work fluorite was too costly, so glass was employed and the instrument calibrated empirically over the working range. The ordinary varieties of glass are opaque for wave-lengths greater than 2 μ.

§ (7) FÉRY'S MIRROR PYROMETER. — These difficulties were avoided by the use of a concave mirror to collect the radiation.

Fig. 5 represents a modern type of instrument. The mirror C is capable of being racked backwards and forwards to focus the radiation on the thermocouple receiver at N, in front of which is a limiting diaphragm. The cold junctions of the couple are shielded from radiation by a tongue and a box M surrounding both the couple and the inclined mirrors immediately in front of the thermocouple receiver.

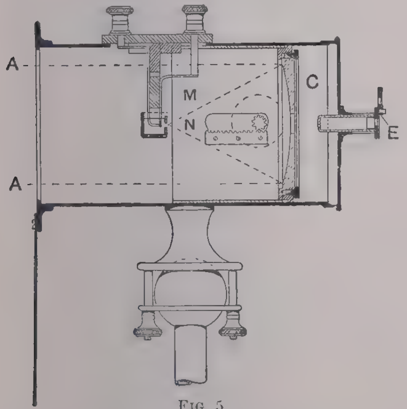
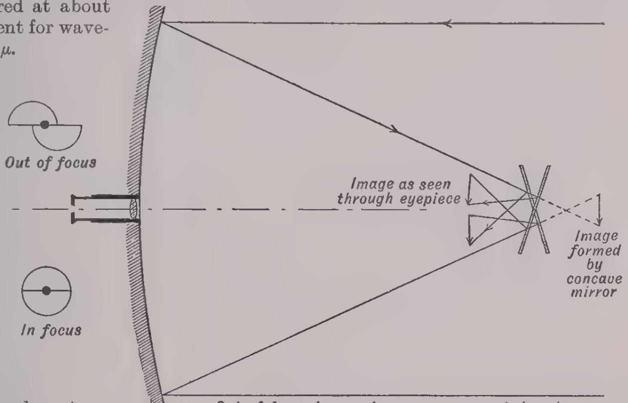


FIG. 5.

To enable the observer to focus the radiation accurately on to the hot junction, Féry employs an ingenious device. Two semicircular mirrors, inclined to one another at an angle of 5° to 10°, are mounted in the thermocouple box, an opening of about 1·5 mm. at the centre of the mirrors forming the limiting diaphragm immediately in front of the couple. Then, unless the image of a straight line viewed through E is exactly in the same plane as the two inclined mirrors, it will appear broken at the plane of intersection of these mirrors. *Fig.* 6 illustrates the paths of the rays producing the distorted images. The observer moves the concave mirror until the relative displacement of the two halves of the image disappears; an operation within the capacity of a workman.


Out of focus
In focus
Image as seen through eyepiece
Image formed by concave mirror

Out of focus image shown on exaggerated scale

FIG. 6.

In the earlier forms of this instrument the concave mirror C was of glass silvered on the back. Since glass is a very good reflector of the infra-red, the heat rays were reflected in part from the front air-glass surface and in part from the back glass-silver surface. The two groups of rays were brought to the same focal point by making the radii of curvature of the two surfaces slightly different. If, however, the thickness of the glass is small, 1 to 2 mm., the same radius of curvature can be used for the two surfaces without appreciable error.

Later instruments have a glass mirror with a gold deposit on the front surface, others gold or nickel on copper.

(i.) *Independence of Distance.*—So long as the hot object formed by the concave mirror is sufficiently large to overlap the limiting diaphragm immediately in front of the sensitive thermoelement, then it is the intensity of the heat image and not the total heat reflected that is measured by the instrument. Now it can be easily shown that theoretically at least this intensity is independent of the distance from the hot object. If, for example, the distance between the instrument and the hot object is doubled, then the total amount of heat received by the concave mirror is reduced to one-fourth, but the area which the image covers is simultaneously reduced to one-fourth, so that the actual heat intensity of the image remains constant.

(ii.) *Relation between Size of Object and the Distance for the Féry Type of Pyrometer.*—It is a simple matter to calculate the minimum size of object required by the geometry of the Féry optical system.

The relation between the size of object and image formed by a concave mirror is $O = I(u/f - 1)$, where O is the diameter of the object, I that of the image, u the distance from the object to the mirror, f the focal length.

For the ordinary type of Féry pyrometer the aperture in the diaphragm in front of the receiving disc is about 1·5 mm. diameter, f the focal length about 7·6 cm.

The table below, due to Burgess and Foote, gives the size of source for various distances, assuming the above data.

TABLE III

u (Cm.).	Diameter Source (Cm.).
70	1·2
80	1·4
100	1·8
150	3·1
200	4·2
300	6·3
500	10·7

§ (8) FÉRY "SPIRAL" PYROMETER.—The construction of this instrument resembles that of the thermoelectric type, except that the couple is replaced by a bimetallic spring spiral (*Fig.* 7) carrying an aluminium pointer P, which is moved over a dial D, graduated in degrees centigrade, under the influence of the differential expansion and uncoiling of the spring when radiation is concentrated upon it. In some of the instruments the spring is trimetallic, the thermal expansion coefficient of the intermediate metal being itself intermediate to that of the two outer metals. Thus, gold, platinum, and invar have been employed. The spiral is similar to that used in the metallic thermometer of Breguet. A strip 0·02 mm. thick and 2 mm. wide is coiled by several turns into a spiral 2 mm. in diameter. The centre of the spiral is connected by a shank to a small disc, and on this disc is usually mounted the pointer. (In *Fig.* 7 a slightly different mounting is shown. The shank is fixed and the pointer is mounted at the other end of the spiral.) Usually a mirror is placed behind the spiral so that the radiation which passes through and between the turns of the spiral is reflected back upon it.

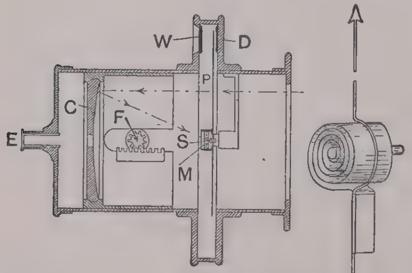


FIG. 7.

Enlarged view of bimetallic receiver

It is of interest to consider the method of spacing of the temperature scale engraved on the instrument. Suppose that the scale is first spaced linearly or in terms of angular deflection of the pointer:

Let d = angular deflection,
T_0 = absolute temperature of spiral,
T = absolute temperature of furnace,
E = energy falling upon spiral.

The angular deflection of the pointer is approximately proportional to the temperature of the spiral; the temperature of the spiral is approximately proportional to the energy absorbed by it; this energy is approximately proportional to the fourth power of the absolute temperature of the furnace; or

$$d \propto T_0 \propto E \propto T^4.$$

Hence $$d = \text{const. } T^4.$$

Hence, determining the deflection corresponding to any one furnace temperature fixes the constant in the above relation and permits the computation of the temperature corresponding to all other deflections.

Actually, the pyrometer does not exactly follow the fourth-power law but rather the relation

$$d = cT^b,$$

where b is an empirical constant slightly different from 4. If a calibration is made at a number of different temperatures, the exponent b may be determined from the slope of the best straight line drawn through the observations, plotting log d against log T. The spiral pyrometer has an especial advantage in being self-contained, requiring no accessories such as lead wires, galvanometer, etc., but its accuracy is not equal to that of the thermoelectric instruments. The readings depend somewhat upon the position in which the pyrometer is held and upon the previous condition of the instrument. For example, tilting the case to the right or left alters the reading, and slightly different readings may be expected when (1) the pyrometer has been sighted upon a source at a higher temperature immediately before taking a certain reading, and (2) when the initial source sighted upon was at a lower temperature.

§ (9) Extension of the Temperature Range by the Use of Diaphragms.—Féry instruments are often provided with a second scale extending over a higher range of temperature. This is effected by the addition of a sector diaphragm over the front of the pyrometer, which cuts down the radiation by a definite fraction. Whilst theoretically the "law" of the instrument should be unaffected by the addition of the diaphragm, it is generally found that the index is not exactly the same with and without the diaphragm. The change is probably due to secondary radiation from the heated diaphragm, and also to change in the distribution of the air currents within the case, caused by the presence of the diaphragm. The extrapolation of the scale temperature is in practice effected by assuming the validity of the "law" of the instrument as determined by the experiments over the range of temperature measurable with thermoelements.

§ (10) The Foster Fixed-focus Pyrometer.—The construction of this pyrometer will be understood from *Fig.* 8.

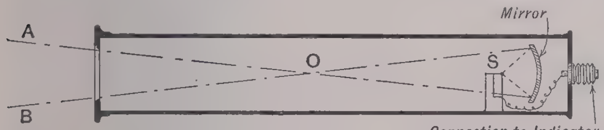
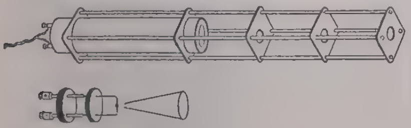


Fig. 8

The receiving disc on the couple and the front diaphragm of the pyrometer are located at the conjugate foci of the mirror. Then, so long as the cone AOB is filled by the radiation from the hot object, the readings are independent of the distance. The position of the point O is marked by the wing nut on the telescope tube. The angle is made such that the diameter of the source sighted upon must be at least one-tenth (or in some instruments one-eighth) the distance from the source to the wing nut.

§ (11) Thwing Radiation Pyrometer.—The Thwing pyrometer (*Fig.* 9) is somewhat similar to the Foster, but has a cone instead of a concave mirror. The receiving disc of the couple is situated at the apex. Radiation from the furnace enters the diaphragm and falls upon the hollow conical mirror. The hot junction of a minute thermocouple is located at the apex of the cone, and the cold junctions are outside. By multiple reflection along the sides of the conical mirror the radiation is finally concentrated upon the hot junction of the couple. The Thwing instrument is so constructed that the source must have a diameter at least one-eighth of the distance from the source to the receiving tube; thus, at 8 feet (2·4 m.) from a furnace the source must be 1 foot (0·3 m.) in diameter.



Arrangement of thermoelement and cone

Fig. 9.

For permanent installations, the tube is ventilated and has several extra diaphragms to prevent the local heating of the instrument and stray radiation reaching the couple.

§ (12) Whipple Closed Tube Pyrometer.—Whipple has introduced a modification of the Féry pyrometer. In this type a closed tube of salamander or fireclay is inserted into the furnace or molten metal, and the radiation from the hot end focussed on a minute thermocouple; the instrument being of the fixed focus type.

For taking temperatures of molten metals, the radiation from which departs considerably from full radiation, and for furnaces with a smoky atmosphere, this form of pyrometer has proved to be of service. The drawback of this type for very high temperature work is the impossibility of obtaining an impermeable tube, and should oil, vapour, or fumes pass into the interior it would seriously vitiate the results.

§ (13) Use of a Radiation Pyrometer with a Source of Insufficient Size.—It is sometimes necessary to use a radiation pyrometer at such a distance from a small source that the aperture of the instrument is not completely filled.

Thus, with the Féry pyrometer, the image of the source formed at the receiver may be smaller than the limiting diaphragm immediately before the couple. The most satisfactory method of using the radiation pyrometer under such conditions is to construct a new limiting diaphragm of the proper size and recalibrate the pyrometer, sighting upon a black body. Another method which may be employed with small sources is to compute[1] the actual size of the image found at the receiver and correct the observed deflection, making use of the assumption that the galvanometer deflection is proportional to the area of the image as long as the image is smaller than the limiting diaphragm. Thus, if the area of the opening to the receiver of the Féry pyrometer were 1 mm.^{2}, and the area of the image of the source formed by the gold mirror were 0·5 mm.², the correct temperature would be given by the value corresponding to a deflection twice that of the actually observed deflection. Errors due to aberrations of the gold mirror will affect the measurements to some extent.

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[1] $\frac{O}{I} = \left(\frac{u}{f} - 1\right)$,

where O is the diameter of the source,
I is the diameter of the image,
f is the focal length of the concave mirror,
u is the distance from source to mirror.

As a rough check upon this method of using a Féry pyrometer Burgess and Foote made the following measurements: The area of image required by the pyrometer was 1·77 mm. The source remained at approximately a constant temperature 1260° C., and its size was altered by means of water-cooled diaphragms.

Area of Image.	Observed E.M.F.	Area of Receiver / Area of Image.	Observed E.M.F. × Ratio of Areas.	Temperature computed.
Mm.²	Millivolts.			° C.
0·302	1·04	5·86	6·09	1290
0·695	2·33	2·55	5·94	1280
0·807	2·78	2·19	6·09	1290
1·564	4·32	1·13	4·88	1200
1·77	5·64	1·00	5·64	1260
			Mean .	1265

The above-computed temperatures have a wide range, but without doubt, if sufficient care were taken, the accuracy could be increased, possibly to ±20° C. This method will not give as satisfactory results as may be obtained by replacing the limiting diaphragm with one of smaller opening and then recalibrating the instrument.

§ (14) SOURCES OF ERROR IN PRACTICAL FORMS OF RADIATION PYROMETERS.—Since the ideal radiation pyrometer would give galvanometer deflections proportional to the intensity of the radiation emitted by the hot object, and hence the difference in the fourth powers of the absolute temperatures, a calibration at one temperature would be sufficient to supply all the data necessary for the computation of the temperature scale. When T is large compared with T_0, the deflections should be proportional to T^4 (T_0^4 being negligible).

It is generally found, however, that the index is not 4, but varies between the limits 3·8 to 4·2 for various instruments.

In any particular case the value of the index may be obtained by plotting the logarithms of the deflections and temperatures. The experimental points will in general be found to lie on a straight line.

Many factors contribute to produce variations in the value of the index from 4.

(*a*) The electromotive force generated by the thermocouple is not strictly proportional to the temperature difference between the hot and cold junctions. When the rise in temperature of the receiving disc above the surroundings is of the order of 80° this factor has an appreciable influence.

(*b*) The thermocouple and inclined mirrors are enclosed in a small cell. Stray reflections from the walls falling on the receiving disc produce disturbances.

(*c*) The rate of heat loss from the junction is not strictly proportional to its temperature excess.

(*d*) Conduction of heat along the couple wires produces a slight temperature rise in the cold junction.

(*e*) In addition there are various errors to which the millivoltmeter readings are liable.

Occasionally one finds that an instrument will follow the fourth-power law with considerable exactitude. This is to be ascribed to the fact that the small residual effects accidentally neutralise each other's influence, rather than to theoretical perfection of design.

The other characteristics of practical types of radiation pyrometers which require study in the case of each individual instrument are:

(i.) *Absence of "Lag."*—So that the final reading is quickly attained. While theoretically an infinite time is required to reach the equilibrium state (*i.e.* when the receiving disc emits as much heat as it receives), most practical types reach the steady state in a minute or so. The time interval required depends, of course, on the individual pyrometer.

Occasionally a maximum reading will be quickly reached and then it begins to decrease, the final value only being reached after fifteen to twenty minutes. This anomalous behaviour is generally due to conduction along the wires of the couple and to secondary radiation from the sides of the cell and the diaphragms.

Such an instrument must be calibrated under the same conditions as it is to be used in practice.

(ii.) *Effect of Size of Image.*—It is necessary to ascertain how far the indications are independent of the distance from the source to pyrometer and independent of the size of the source when this is above the minimum size required by the geometry of the instrument.

The dependence of the indications on the size of the image is probably the most serious source of error inherent to practical types of radiation pyrometers, and can only be overcome by careful construction and arrangement of the mechanical parts of the instrument. Burgess and Foote studied the effect of variation in the size of the hot object and the focussing distance on commercial types of total radiation pyrometers.

In the geometrical theory of the Féry pyrometer it is assumed that the source is of a size sufficient for its image to cover the thermocouple receiver or the limiting diaphragm immediately in front of the receiver. Usually this limiting diaphragm is the hole in the focussing mirrors in front of the thermocouple. As long as this opening is covered by the image of the source, one might expect that the reading of the pyrometer would be independent of distance or size of source. Their experiments, however, show that, in general, the reading of the pyrometer decreases with increasing focussing distance and with decreasing size of source, even though the image of the source always covers the receiver; some instruments show this effect much more markedly than others. It is even possible to obtain a positive reading, as shown by Kanolt, when the pyrometer is sighted on a hole in a heated surface, although the image of the hole covers the opening to the thermocouple receiver. The cause of this effect is best determined from a consideration of the size of the image of the source.

Errors which are surprisingly big in magnitude, and which completely outclass those resulting from other causes, may arise in the variation of the size of the image produced by the pyrometer. The size of the image may be altered by (*a*) varying the focussing distance, the size of the source remaining constant, and (*b*) by varying the size of the source, the focussing distance remaining constant.

In the study of these effects they employed a large nickel strip as the radiator, in front of which, at a distance of 1 or 2 cm., was placed diaphragms of various openings. By water cooling, the temperature of the diaphragms was maintained at about that of the room, so that there was no effect of radiation from them, and the thoroughly blackened surface of the diaphragms absorbed practically all the heat falling upon it, hence the radiation loss from the surface of the heated strip behind the diaphragm was the same as that from the exposed surface of the strip. Consequently there was no variation in either the apparent or true temperature of the strip when the size of the diaphragm opening was changed. Except for very close distances from the pyrometer to the strip, the diaphragm openings acted as the real source of the radiation. In this manner five sizes of the radiating source were obtained, circular areas having diameters of 1·95, 3·5, 5·6, 8·5, and 12 cm.

The pyrometer was mounted upon a carriage which rolled on parallel tracks similar to an ordinary photometer bench. The strip and water-cooled diaphragms were properly adjusted at one end of the bench, and the apparatus aligned so that the image of the diaphragm opening employed was always centred upon the pyrometer receiver for all focussing distances (usually 80 to 300 cm.). Curve A, *Fig.* 10, represents the variation in E.M.F., with the diameter of the image of a uniform temperature source for a Féry pyrometer which showed the effect of these errors in a marked degree.

This particular pyrometer had focussing mirrors of thin glass silvered on the back surface; the opening in these mirrors, which formed the limiting diaphragm, was 1·5 mm. in diameter. A 0·6 cm. diaphragm was located immediately in front of the focussing mirrors, and the inside diameter of the thermocouple box was 1·1 cm. In the present case the diameter of the image was varied from 0·05 to 0·9 cm. by employing sources of diameters 2 to 11 cm. at focal distances of 70 to 250 cm. In the section *ab* of the curve the image was not large enough to cover the 0·15 opening in the focussing mirrors. For this range the E.M.F. is approximately proportional to the area of the image or to the (diameter)^2, so that AB is a parabola. If the focussing mirrors formed a perfect diaphragm, completely shutting out all radiation except that passing through the opening, the point *b* would represent a maximum reading, and the curve would continue horizontally along the line *bc*. Actually the E.M.F. increases up to the point *e*, where the image has a diameter of about 1·1 cm., the inside diameter of the thermocouple box,

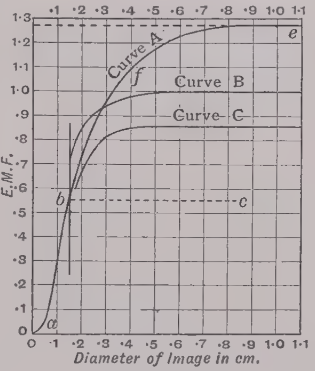

FIG. 10.

although for all points from b to e the size of the image was sufficient to cover the receiver. The large increase along be is due to the heating of the silvered glass focussing mirrors, and the amount of this heating increases with the size of image, until the image completely fills the inside of the thermocouple box. This heat is communicated to the thermocouple by radiation and by convection currents set up within the receiver. The errors in measurement resulting from variations in size of image are readily apparent. For example, suppose the instrument were calibrated by sighting at 150 cm. distance upon a black body having an opening of 3 cm., and were used for the measurement of the temperature of a source 11 cm. in diameter at a distance of 100 cm. In the first case the image diameter is about 0·15 cm. and in the latter 0·9 cm., corresponding on the curve to the points b and e respectively. The E.M.F. at e is 227 per cent of that of b, so that as used the instrument would indicate E.M.F.'s in error by 127 per cent. Suppose this instrument obeys the law $E = aT^4$, then by differentiation

$$\frac{\delta E}{E} - 4\frac{\delta T}{T},$$

which states that the fractional error in the absolute temperature is one-fourth the fractional error in E.M.F. Hence an error of 127 per cent in E.M.F. is equivalent to an error 32 per cent in the absolute temperature. Thus, for a source of 11 cm. diameter, 100 cm. distance, and at a temperature of 1500° C., this pyrometer would read about 2070° C., or a temperature too high by 570° C.

The example cited is an extreme case, and the variation of E.M.F. with diameter of the image was greater for this instrument than for any other examined by Burgess and Foote. Actually, a pyrometer would not be calibrated with the minimum-sized image required by the optics of the instrument. It is rather difficult to centre such an image, so that usually the image is made large enough to overlap the opening of the receiver. Probably an image diameter of approximately 0·4 cm. is more often employed in calibration, and, in general use, the variation in image diameter may be of from 0·2 to 1·1 cm. Thus calibrated, this pyrometer would indicate E.M.F.'s too small by 32 per cent when the smaller image is used, and too great by 17 per cent with the larger image.

Curves B and C illustrate the variations of E.M.F. with image diameter for two other Féry pyrometers. The size of image was varied by both altering the size of the source and of the focussing distance. The fact that the points obtained by either method coincide equally well with the curve precludes the possibility of any experimental errors, such as temperature gradient across the nickel strip used as a source, or the change in temperature of the strip due to the use of various sized diaphragms. It also indicates that the error due to the atmospheric absorption is small, at least in comparison with errors involved in the heating of the focussing mirrors. These curves represent the behaviour of the two best acting pyrometers examined. In the case of B, the E.M.F. for 1·1 cm. image is 133 per cent of that obtained with an image of 0·15 cm., the diameter of the limiting diaphragm in front of the receiver, and for C this relation is 162 per cent. If these instruments were calibrated with an image diameter of 0·4 cm., the E.M.F. developed by B would be too great by 2 per cent for a 1·1 cm. image and too small by 11 per cent for a 0·2 cm. image, and by C, too great by 2 per cent for the larger image and too small by 18 per cent for the smaller image, with errors of one-fourth these percentages when referred to absolute temperatures.

Pyrometer B had a metal diaphragm located between the silver glass focussing mirrors and the receiver, while in C this diaphragm was absent, but the focussing mirrors were of gold-plated sheet-copper. These two instruments show a smaller effect of variation in size of image than does pyrometer A, for the reason that the metal diaphragm in B and the copper mirrors in C were good heat conductors, in contrast with the glass mirror diaphragm of A, and allowed part of the heat to be carried to the walls of the thermocouple box, where it was dissipated by radiation and convection on the outside, away from the thermocouple.

The fact that all three curves are asymptotic to constant values proves that this type of pyrometer should be calibrated and used on objects giving images much larger than the minimum specified by the theory of the instrument.

(iii.) *Summary of Focussing Errors for a Féry Pyrometer.*—The principal error results from a variation in the size of the image of the source formed by the large condensing mirror, and is due to the heating of the limiting diaphragm or the focussing mirrors immediately in front of the thermocouple receiver. The amount of this heating increases with increasing size of image. On account of this fact, the pyrometer readings for a source of constant size decrease with increasing focussing distance; and for a constant focussing distance the readings increase with increasing size of source. With ordinary use of this pyrometer, errors of this type may amount to several hundred degrees in extreme cases, and, in general, to 50° or more, unless certain specified methods of procedure are employed; for example, the use of an image of a definite size for every measurement.

The following table presents a summary of the various errors, the magnitudes of which depend upon the focussing distance:

TABLE IV

EFFECT UPON THE PYROMETER READING OF INCREASING THE FOCUSSING DISTANCE

Reading increases on Account of	Reading decreases on Account of
1. Variable aperture. 2. Shading of concave mirror by thermocouple box.	1. Atmospheric absorption. 2. Convection currents from source to receiver. 3. Stray reflection in receiver and telescope tube. 4. Reradiation to couple from side walls of pyrometer. 5. Image of source becoming smaller.

With the Foster fixed focus the principal source of the error is secondary radiation from the side walls and diaphragms of the front part of the pyrometer tube to the thermocouple receiver, and stray reflection from the side walls.

(iv.) *Effect of Dirt and Oxidation upon the Condensing Device.*—It has been found that a slight film of oxide on the surface of the concave mirror of the Féry or Foster radiation pyrometers does not seriously alter the amount of radiation reflected, which amounts to about 96 per cent of the incident energy, as the greater part of the radiation exists in the form of long wave-lengths, which the tarnished mirrors reflect without difficulty. This experimental fact has frequently been misunderstood and the impression obtained that in spite of dirt accumulation, stains, and scratches the gold surface remains unchanged in its reflecting power; such is not the case. Pyrometers subjected to severe use in the industries soon become coated with dust and dirt, and errors of 100° C. are not unusual when the mirrors become dirty. The instruments will, of course, read low.

In exceptional cases the front of the pyrometer is covered by a sheet of glass so as to prevent the access of dust and fumes into the interior of the pyrometer. The glass reduces the sensitivity of the instrument very considerably owing to absorption, and experiments show that the variation of the E.M.F. is generally nearly proportional to the fifth power of the absolute temperature.

§ (15) ADVANTAGES AND DISADVANTAGES OF THE TOTAL RADIATION PYROMETER AS COMPARED WITH THE OPTICAL TYPE.—The total radiation pyrometer has the advantage over the optical type, insomuch that it is direct reading. There is no necessity for the observer to judge equal intensities as is the case with optical pyrometers. A total radiation pyrometer can be coupled up to a recorder of the type employed with thermoelements and a continuous record of temperatures obtained.

It is also applicable to lower temperatures than can be measured with the optical type. On the other hand, departure from black body conditions or the presence of CO_2 or water vapour causes greater errors in the reading of the total radiation type than in the case of the optical.

§ (16) CALIBRATION OF RADIATION PYROMETERS.—Instruments are generally calibrated by comparison with a standard instrument over the range 500° to 1400° C. A uniformly heated muffle forms a convenient source of radiation closely approximating to a full radiator.

The standard instrument requires more elaborate study. For this purpose a large platinum foil wound electric furnace is convenient; this should be provided with suitably disposed diaphragms, and the pyrometer focussed on a plug of refractory material fixed in the centre of the furnace. Across the face of this plug one or more platinum-rhodium couples should be stretched, so that the mean temperature of the surface is obtained with accuracy.

To obtain a cone of radiation of sufficient size to fill the field of the pyrometer it is advisable to employ a furnace with an aperture of over three inches in diameter.

At high temperatures a considerable amount of cooling takes place by convection from the open mouth of a horizontal furnace. Some improvement can be effected by inclining the mouth downwards at an angle of 20° to 30°, but for the steadiness of temperature and for economy a vertical arrangement of the furnace, opening downwards, is the most satisfactory.

For temperatures up to 1100° C. the furnace can be wound with "nichrome" or similar alloy tape about 1 cm. wide by 1 mm. thick.

For higher temperatures up to 1400° C. it is necessary to employ platinum foil, since the life of a nickel-chromium alloy winding is only a few hours at a temperature of 1300° C.

(i.) *Computation of Calibration Data.*—The thermoelectric type of radiation pyrometer obeys the relation $E=a(T^b-T_0{}^b)$, where E is the E.M.F. developed when the pyrometer is sighted on a black body at an absolute temperature T, the temperature of the receiver being T_0, and a and b are empirical constants. In general, $T_0{}^b$ is negligible in comparison with T^b, so that one may write $E=aT^b$. The constants a and b must be determined for each instrument.

Although two calibration points serve to determine a and b, observations are usually made at five or more different temperatures, and the best curve is drawn through all the points. Since an exponential curve of the correct form is difficult to adjust graphically, the curve is rectified into a straight line by plotting log E in terms of log T.

Thus, expressed in logarithmic form, the equation for the pyrometer becomes

$$\log E = \log a + b \log T,$$

which is a linear relation between log E and log T, the slope for the straight line determining the constant b.

(ii.) *Calibration by Sighting on a Heated Strip.*—The standardisation of a radiation pyrometer under black body conditions against a thermoelement is a somewhat tedious operation owing to the time required by the furnace in settling to equilibrium.

When a calibrated instrument is available this can be used as working standard and other instruments tested by comparison with it. For this purpose it is not essential to have black body conditions, and any furnace will meet the requirements provided the surface sighted upon is uniform in temperature.

The Bureau of Standards has discarded the use of a furnace in favour of an electrically heated strip. Of the metals available for the purpose of making the strip, nickel appears to be most satisfactory. When heated in air a firm and uniform coat of black nickel oxide (NiO) forms on the surface. Such a strip can be used almost indefinitely up to 1300° C. and rapid changes of temperature can be made from 500° C. to 1300° C. The cooling from 500° to room temperature must be done slowly, or flaking of the oxide will occur.

In the apparatus employed a strip 17 cm. long (exposed section), 13 cm. wide, and 0·015 cm. thick is mounted vertically between water-cooled brass-clamp terminals. This is heated by an adjustable current (maximum 1500 amperes) supplied by a low-tension transformer. A strip of this size furnishes a source of circular area and diameter of 12 cm. which is uniform to within 2° at 1200° C. over its entire surface.

The advantage gained in using a strip several centimetres longer than its width is marked. The temperature variation across the width of the strip is practically nil, the main variation occurring along the lower edge. Thus, the temperature gradient along a vertical section of the strip is not symmetrical, the bottom of the strip being cooler for several cm. than the top. Using a strip 17 by 13 cm. the centre of the 12 cm. uniform temperature area is located 6·5 cm. from either side, about 7·5 cm. from the top, and 9·5 cm. from the bottom.

In the use of this nickel-oxide source for the calibration of pyrometers it is essential that the instruments compared be of similar type, so that the departure from "blackness" of the strip will affect each pyrometer in the same manner. Large errors would be involved in the comparison of an optical and radiation pyrometer by this method unless the observations were corrected both for the monochromatic and for the total emissivity of nickel oxide.

§ (17) Total Radiation from Oxide and Metallic Surfaces.—While muffle furnaces and heating chambers employed in the industries closely approximate to "full radiators," the surfaces of metallic objects depart considerably from the ideal contemplated by the Stefan-Boltzmann law. Consequently, a radiation pyrometer calibrated on a "full radiator," if employed to take the temperatures of such surfaces, will give readings which are too low.

When the surface is oxidised, the difference between the apparent and the real temperature will be a function of the condition of the surface, and it is difficult to apply a correction with any degree of certainty.

The radiation from some of the commoner metals has been investigated with the conclusions summarised below.

§ (18) Definition of the Emissivity of a Surface.—At the present time the term "emissivity" is used to denote the ratio of the heat emitted by unit area of the surface to that emitted by an equal area of a "full radiator" at the same temperature, and not in the older sense of the term, when it denoted the heat emitted per unit time divided by the temperature excess of the surface above the surroundings. Hence if Q is the total radiation emitted by the unit area of the surface at absolute temperature T to surroundings at temperature T_1, and σ is the "black body" constant, then

$$Q = E\sigma(T^4 - T_1{}^4),$$

where E is defined as the emissivity constant for the surface at T.

In the following brief review of experimental work on the determination of emissivities attention will be confined to those investigations which have been made primarily with a view to the evaluation of the corrections to the total radiation pyrometer readings when used for taking the temperatures of such materials in the open.

(i.) *Emissivity of Nickel Oxide Surface.*—Burgess and Foote employed an ordinary Féry radiation pyrometer for the measurement of the emissivity coefficient. This pyrometer was calibrated to give true temperatures under black body conditions.

Observations were made of the apparent temperatures, the corresponding true temperatures being obtained by methods described below.

If E denotes the total emissivity of an approximately non-selective, radiating surface at absolute temperature T_1, and S the apparent temperature observed with the radiation pyrometer, and Q the radiation, then

$$Q = \sigma(S^4 - T_0^4),$$

$$Q = E\sigma(T_1^4 - T_0^4),$$

hence $$\sigma(S^4 - T_0^4) = E\sigma(T_1 - T_0^4),$$

so that $$E = \frac{S^4 - T_0^4}{T_1^4 - T_0^4}.$$

Above 600° C. the term T_0^4 may be neglected, as a simple calculation will show:

If, for example, E is about 0·5, the error in its value by neglecting T_0 would be only 0·007 at 600° C. and 0·0002 at 1300° C.

Hence the emissivity may be calculated by the simple expression

$$E = \left(\frac{S}{T_1}\right)^4.$$

Since in practical types of radiation pyrometers the index is rarely exactly 4, the above expression requires slight modification in such cases.

Taking the characteristic exponent of the empirical relationship between e the E.M.F. and temperature as b so that

$$e = aT^b,$$

then, if e' is the E.M.F. generated when sighted on a radiating surface of apparent temperature S° (absolute) and true temperature T,

$$E = \left(\frac{e'}{e}\right)^{\frac{4}{b}}.$$

To obtain the true temperatures corresponding to the apparent temperatures two methods were available which gave results in close accord.

(1) The application of the idea embodied in the Joly meldometer, in which the melting-points of microscopic specimens of various substances were observed, such as NaCl (800° C.); Na_2SO_4 (884° C.); Au (1063° C.).

Experiments were made to ascertain that the temperature of the strip did not differ appreciably from that of the specimens.

(2) By the use of an optical pyrometer calibrated to read true temperatures when sighted on such a surface. This pyrometer is based on the principle of matching the intensity of the light from an electric lamp filament with the light from the hot object.

The pyrometer was calibrated initially for "full radiator" conditions, and then the departure of the nickel oxide surface was obtained by sighting on the surface of a nickel tube, electrically heated, and into a small diaphragmed enclosure in the centre of the tube. By properly locating the diaphragms in the interior of the tube "black body" conditions could be realised quite satisfactorily.

Theoretically the experiment might be simplified by using the total radiation pyrometer directly to sight on the outside of the oxidised tube and on the interior: the first observation giving the apparent temperature while the second would give the true temperature. But, owing to the large aperture that would be necessary in comparison with that required for an optical pyrometer, this method of obtaining the true temperature would present practical difficulties.

It should be remarked, however, that the principle employed (of measuring the radiation by concentrating it on the thermocouple by means of a metallic mirror) is only valid so long as coefficient of reflection of the surface does not vary with the wave-length.

This appears to be the case for gold over the spectrum range from μ to 14 μ.

The variation of emissivity with temperature of NiO is given in Table V. while corrections to "apparent" temperatures are given in Table VI.

TABLE V

Temperature, ° C.	Emissivity.	Temperature, ° C.	Emissivity.
600	0·54	1000	0·75
650	0·59	1050	0·78
700	0·62	1100	0·81
750	0·65	1150	0·84
800	0·68	1200	0·86
850	0·70	1250	$0{\cdot}86_5$
900	0·72	1300	$0{\cdot}86_7$
950	0·73	..	..

TABLE VI

CORRECTIONS WHICH MUST BE ADDED TO THE APPARENT READINGS OF RADIATION PYROMETERS TO GIVE TRUE TEMPERATURES WHEN SIGHTED ON AN OXIDISED NICKEL SURFACE

Apparent Temperature, ° C.	Correction.	Apparent Temperature, ° C.	Correction.
500	120°	1000	75°
600	110	1100	65
700	100	1200	55
800	95	1250	50
900	85	..	..

(ii.) *Iron Oxide.* — A knowledge of the emissivity of iron oxide is of considerable importance technically, since it permits of correction to the readings of radiation pyrometers when taking the temperatures of billets, rails, etc.

Burgess and Foote made observations on

the same lines as those described above in the case of nickel; in this case, however, electrically heated iron tubes of various sizes were employed as radiators.

The results are given in Table VII.

TABLE VII

True Temperature, ° C.	Emissivity.	Correction to Apparent Temperature.
500	0·85	30°
600	0·85	30
700	0·86	35
800	0·87	35
900_1	0·87	40
1000_1	0·88	40
1100_1	0·88	45
1200	0·89	..

It is possible to calibrate a total radiation pyrometer to give approximately true temperatures when sighted on an oxide surface by inserting a resistance coil in series with the indicator when standardising on the customary "black body" furnace. The value of the resistance can be calculated from the constants of the instrument.

(iii.) *Temperature Gradient through the Oxide Layer.*—Iron oxide is a comparatively poor heat conductor, consequently it might be expected that the true surface temperature would be appreciably below that of the body of the metal.

Experiments with a thermocouple inside the tube to give the true temperature showed that the gradient through the oxide was considerable and apparently independent of the size of the tube. It would appear that the thickness of the oxide layer is automatically rendered of the same order of magnitude for different times of heating by the flaking off which occurs.

Data obtained by Burgess and Foote concerning this gradient are in good agreement with those obtained by Burgess, Crowe, Rawdon, and Waltenberg on rail sections, the couple being inserted in a small hole drilled parallel to the length of the rail and as near the oxide as possible.

Temperature, Inside Layer, ° C.	Temperature, Outside Layer, ° C.
610	600
715	700
820	800
930	900
1080	1000

These results show that any method of obtaining the temperature of rails, ingots, etc., by observations of the surface temperature, is liable to serious error unless account is taken of the gradient through the oxide film.

(iv.) *Emissivity of Molten Metals.*—The total radiation pyrometer is of very limited use with molten metal surfaces. Such surfaces can never be freed from haze or fog, and the radiation from the walls of the furnace or crucible reflected at the molten surface is apt to produce serious errors.

Burgess has made some observations on the difference between the apparent and real temperatures in the case of metallic copper and cuprous oxide surfaces, using a Féry pyrometer.

The following relationships were found to be approximately true. In these equations t is the true temperature centigrade and s the apparent.

Molten copper, clear surface—

$$t = 3{\cdot}55s - 1018.$$

Surface covered by cuprous oxide—

$$t = 1{\cdot}41s - 169.$$

The difference between the apparent temperatures—when the pyrometer was first sighted on the clear copper surface and then on the oxide surface, both being at the same temperature—amounted to as much as 300° C.

The apparent freezing-point of copper (clear surface) was found to be 600° C. compared with the true value of 1083° C.

The emissivities of the two surfaces at various temperatures are given in Table VIII.

TABLE VIII

Temperature, ° C.	Emissivity (Molten Copper).	Temperature, ° C.	Emissivity (Cuprous Oxide).
1075	0·16	800	0·66
1125	0·15	900	0·60
1175	0·15	1000	0·56
1225	0·14	1100	0·54
1275	0·13	..	..

Thwing has made some observations on the emissivities of both molten iron and molten copper relative to that of iron in the solid state (presumably oxidised). No details are given concerning the experiments. He states that molten cast iron at 1300° C. to 1400° C. has an emissivity of 0·29 that of the solid metal. Mild steel (molten) at 1600° C. has a relative coefficient of 0·28, which coefficient appears to hold up to 1800° C. Molten copper has an emissivity of 0·14 that of solid iron.

Some of the experiments appear to have been made on the streams of molten metals issuing from the furnaces.

Table IX. summarises existing knowledge as regards the corrections required to apparent temperatures given by total radiation pyrometers to convert to true temperatures.

TABLE IX

TRUE TEMPERATURES AND APPARENT TEMPERATURES MEASURED BY RADIATION PYROMETERS WHEN SIGHTED UPON VARIOUS MATERIALS

Observed Temperature, ° C.	True Temperature, ° C.				
	Molten Iron.	Molten Copper.	Copper Oxide.	Iron Oxide.	Nickel Oxide.
600	..	1130	720	630	710
650	..	1210	775	..	755
700	..	1290	830	735	800
750	..	..	890	..	845
800	1200	..	945	840	895
850	1270	..	1000	..	940
900	1340	..	1060	945	985
950	1410	..	1115	..	1030
1000	1475	..	1170	1050	1075
1050	1550	..	..	..	1120
1100	1610	..	..	1155	1165
1150	1680	..	..	..	1210
1200	1750	..	..	1260	1255

§ (19) **Absorption in the Medium through which the Radiation passes.**—The radiation has generally to pass through a gaseous medium before reaching the pyrometer. At present the information available concerning the effect of any absorption by the medium on pyrometric observations is very scanty.

The subject is complicated by the fact that most gases and vapours have fairly sharply defined absorption bands, and that the distribution of energy among the wave-lengths of the continuous spectrum emitted by a "black body" varies with the temperature. Should one of the absorption bands coincide with the maximum energy wave-length of the spectrum its influence would be very marked, whereas at another temperature that particular wave-length might contain but a very small fraction of the total energy in the spectrum, consequently the loss by absorption would be insignificant.

It is known that CO_2 has absorption bands of wave-lengths 4·4, 2·7, 14, and 15 μ in the infra-red, the band at 4·4 μ being a strong one. Water vapour has a number of absorption bands in the neighbourhood of 6 μ.

Hence the presence of cooler strata of CO_2 and other gaseous products in the furnace will lower the readings of the pyrometer. The writer on one occasion observed an error of 40° C. in the readings of a pyrometer when taking the temperature of a furnace near the mouth of which water vapour was present due to the drying-out of the furnace.

Tyndall, about 1859, made a thorough investigation of the diathermancy of gases and vapours. The apparatus employed consisted of a brass tube closed at the ends by plates of rock salt. Facing one end of the tube was a source of radiation, such as a cube containing boiling water or a glowing spiral of platinum. At the other end was placed a thermopile. When the interior of the tube was exhausted the deflection of the galvanometer connected with the thermopile was reduced to zero by bringing up a compensating cube to the other face of the thermopile. The gas under test was then introduced into the tube; if it exerted any absorption effect the galvanometer needle would be deflected. The fraction of radiation absorbed could be obtained by observing the full deflection produced when a screen was interposed between the thermopile and the tube. The compensating cube then produced the same effect as the radiation which previously traversed the exhausted tube.

Tyndall found that air, oxygen, hydrogen, and nitrogen, if carefully purified, exerted no sensible absorption, while water vapour and carbon dioxide had a marked absorption. Vapour of organic compounds also had a considerable absorbing effect.

E. G.

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— Q —

QUANTUM THEORY:

- Application of, to the behaviour of gases under various conditions. See "Thermal Expansion," § (26).
- Explanation of Variation of Atomic Heat with Temperature. See "Calorimetry, the Quantum Theory," § (44).
- Formulae for Specific Heat, Experimental Test of, by E. H. Griffiths and Ezer Griffiths. See *ibid.* § (45).

See also Vol. IV.

— R —

RADIATION:

- Confirmatory evidence of the laws of, from a consideration of the inter-related phenomena of atomic structure, of X-rays, of ionisation and resonance potential, and of photoelectrical action. See "Radiation, Determination of the Constants of," etc., IV. § (11), Vol. IV.
- Constant of Spectral, determinations of. See *ibid.* III. § (7), Vol. IV.
- Formula and Coefficient of Total: verification by experiment of Stefan-Boltzmann law. See *ibid.* IV. § (9), Vol. IV.
- Formula and Constant of Spectral: experimental evidence shows that, throughout the spectrum, from 0·5 μ to 50 μ, Planck's formula fits the observed spectral energy distribution more closely than any other equation yet proposed. See *ibid.* IV. § (11), Vol. IV.
- "Fourth-power" Law of, used to measure high temperatures. See "Pyrometry, Total Radiation," § (1).
- From a Black Body, discussion of. See "Radiation, Determination of the Constants of," etc., I. § (2), Vol. IV.
- From Flames. See "Engines, Thermodynamics of Internal Combustion," § (57).
- Losses in internal combustion engines. See *ibid.* § (61).
- Measurements of Solar and Stellar. See Vol. III.
- Theory of Heat: laws of Boltzmann, Wien, and Planck. See "Pyrometry, Optical," § (1).
- Total, of a Black Body, used as a secondary standard of temperature in the range above 500° C. See "Temperature, Realisation of Absolute Scale of," § (41) (iii.).
- Transmitted by Red Filter Glass of Optical Pyrometer, Wave-length of. See "Pyrometry, Optical," § (11).

RADIATION, COEFFICIENT OF TOTAL:

- Indirect and substitution methods of experimental evaluation of. See "Radiation, Determination of the Constants of," etc., II. § (5), Vol. IV.
- Modern methods of experimental evaluation of. See *ibid.* II. Vol. IV.
- Thermometric methods of evaluation of, with "black" receivers. See *ibid.* § (4), Vol. IV.

RADIATION, DETERMINATION OF THE CONSTANTS OF. See Vol. IV.

RADIATION THEORY. See Vol. IV.

RADIUS OF GYRATION. The square root of the ratio of the moment of inertia of a body about a line to the mass of the body,

$$k=\sqrt{\left\{\frac{\Sigma(mr^2)}{M}\right\}}.$$

RAILWAY DYNAMOMETER CARS—FOR THE MEASUREMENT OF TRACTIVE EFFORT AND RESISTANCE. See "Dynamometers," § (5) (iv.).

RAIN GAUGES. See "Hydraulics," § (3). See also Vol. III.

RAINFALL, DISTRIBUTION AND ANNUAL VARIATION OF. See "Hydraulics," §§ (1) and (2).

RAKE OF A PROPELLER. A blade is said to be raked forward or aft according as the centre line of the blade at the tip is forward or aft of the centre line at the root. See "Ship Resistance and Propulsion," § (41).

RANKINE CYCLE. See "Steam-engine, Theory of," § (3).

- Reversibility of. § (7).
- For Wet Steam. § (5).

RAYLEIGH'S FORMULA FOR RADIANT ENERGY: a formula, due to Lord Rayleigh, giving an approximation to the distribution of radiant energy along the spectrum, on the side of the long wave-lengths. It is the expression which should be found if the classical system of mechanics were valid, and has the form

$$E_\lambda=\frac{8\pi kT}{\lambda^4}.$$

See "Radiation Theory," § (6), Vol. IV.

REACTION TURBINES. See "Turbine, Development of the Steam," § (2); "Steam-turbines, Physics of," §§ (11), (16).

RÉAUMUR, introducer of a scale of temperature, still used in parts of Central Europe and Russia, in which zero and 80 correspond to the freezing- and boiling-point of water respectively. See "Thermometry," § (2).

Rectifying Column used in Fractional Distillation of Air. See "Gases, Liquefaction of," § (2).

Redundant Members in a Frame. See "Structures, Strength of," § (25).

Refractory Materials, Thermal Conductivity of. See "Heat, Conduction of," § (4) and Table II.

REFRIGERATION

§ (1) Introductory. General Theory.—The process of refrigeration consists in removing heat from a body that may be (and generally is) colder than its surroundings. In cold storage, for example, the contents of a chamber are kept at a temperature lower than that of the air outside, by extracting the heat which continuously leaks in through the imperfectly insulating walls. In ice-making, heat has first to be removed from water in a can or tank to bring its temperature down to the freezing-point, and then the "latent heat" has to be removed as the water freezes. A fundamental principle of Thermodynamics (*q.v.*), known as the Second Law,[1] is that heat does not pass spontaneously from a cold body to a hot body, and it is impossible to have a purely self-acting machine that will go on conveying heat from a cold body to a hot body. To maintain a refrigerating process accordingly requires some expenditure of energy. It is generally done by means of a mechanically driven heat-pump, working on what is essentially a reversed heat-engine cycle. It may also be done by the direct use of high-temperature heat without intermediate conversion of that heat into work, by means of devices which will be mentioned later.

Any process of refrigeration involves the use of a working substance which can be made to take in heat at a low temperature and discharge heat at a higher temperature. The heat is discharged by being given up to the air outside or to any water that is available to receive it. The process is a pumping up of heat from the level of temperature of the cold body, at which heat must be taken in, to the level at which heat may be discharged. These temperature levels should be as near together as is practicable, in order that no unnecessary work may be done: in other words the action of the working substance should be confined to the narrowest possible range of temperature. The temperature of discharge should be no higher than is necessary to get rid of the heat, and the lower limit should be no lower than will ensure transfer of heat into the working substance, from the cold body from which heat is to be extracted.

Let T_1 be the temperature at which heat is discharged and T_2 the temperature at which it is taken in from the cold body. Consider a complete cycle in the action of the working substance. Let Q_1 be the quantity of heat which is discharged and Q_2 the quantity which is taken in from the cold body; and let W be the thermal equivalent of the work spent in driving the refrigerating machine. Then, by the conservation of energy,

$$Q_1 = Q_2 + W.$$

The useful refrigerating effect is measured by Q_2, and the "coefficient of performance," which is the ratio of that effect to the work spent in accomplishing it, is Q_2/W.

We have first to inquire what is the highest possible coefficient of performance when the limits of temperature T_1 and T_2 are assigned. We know by the principle of Carnot ("Thermodynamics," §§ (18) to (20)) that when heat passes down from T_1 to T_2 through a heat-engine, the ideally greatest efficiency in the conversion of heat into work is obtained when the engine is thermodynamically reversible. In that case

$$\frac{Q_1}{T_1} = \frac{Q_2}{T_2}.$$

The output of work W is $Q_1 - Q_2$. Hence the ideally greatest output of work is related to Q_2, the heat rejected at the lower limit of temperature, by the equation

$$W = \frac{Q_2(T_1 - T_2)}{T_2}.$$

A corresponding proposition in the theory of refrigeration (which will be proved immediately) is that the ideally greatest coefficient of performance of a refrigerating machine, working to pump up heat from T_2 to T_1, is obtained when the machine is thermodynamically reversible. In that case the same relation holds, namely

$$\frac{Q_1}{T_1} = \frac{Q_2}{T_2},$$

and the amount of work W which is spent in driving the machine (and is equal to $Q_1 - Q_2$) is related to Q_2 by the equation

$$W = \frac{Q_2(T_1 - T_2)}{T_2}.$$

In other words, the *greatest* amount of work that is theoretically obtainable in letting heat pass down through a given range of temperature is the *least* amount of work that will suffice to pump up the same quantity of heat through the same range.

To show that no refrigerating machine can be more efficient than one that is reversible, we shall use an argument of a quite general character, based on the Second Law of Thermodynamics. Let E, *Fig.* 1, be a reversible refrigerating machine, reversed and therefore serving as a heat-engine. It takes a quantity of heat, say Q_1, from the hot body, and delivers a quantity Q_2 to the cold body, converting the difference into work. Let all the work W which it develops be employed to drive a refrigerating machine R; and assume that there is no loss of power in the connecting mechanism. Accordingly the two machines, thus coupled, form a self-acting combination.

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[1] See "Thermodynamics," § (17).

If the machine R could have a greater coefficient of performance than the reversible machine E, that would mean that the ratio of Q_2 to W would be greater in R than in E. Hence (W being the same for both) R would take more heat from the cold body than E gives to it, and R would also give more heat to the hot body than E takes from it. The result would be a continuous transfer of heat from the cold body to the hot body by means of a purely self-acting agency. This would be contrary to the Second Law of Thermodynamics: we conclude, therefore, that no refrigerating machine can have a higher coefficient of performance than a reversible machine working between the same limits of temperature.

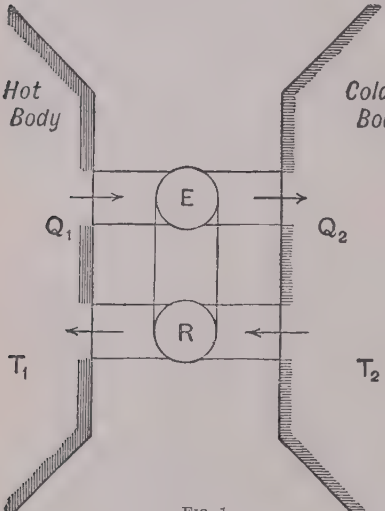

FIG. 1.

It follows that all reversible refrigerating machines working between the same limits of temperature have the same coefficient of performance. It also follows that the value of this coefficient is that which would be found in a reversed Carnot cycle, namely

$$\frac{Q_2}{W}=\frac{T_2}{T_1-T_2}.$$

This is the ideally highest coefficient; it measures the performance of what may be called a perfect refrigerating machine. The coefficient of performance in any real machine is necessarily less, for the cycle of a real machine falls short of reversibility.

The following are numerical values of this expression, namely values of the coefficient of performance in a perfect or reversible refrigerating process, for various ranges of temperature. Though these are ideal figures, representing a theoretical limit which cannot be reached in practice, and is in fact not nearly reached, they illustrate the importance of making the range of temperature as small as possible by taking in the heat which has to be extracted from the cold body at a temperature no lower than can be helped, and by discharging it after the least practicable rise.

COEFFICIENTS OF PERFORMANCE OF A PERFECT REFRIGERATING MACHINE

Lower Limit of Temperature (Centigrade).	Upper Limit of Temperature (Centigrade).				
	10°.	20°.	30°.	40°.	50°.
−20°	8·4	6·3	5·1	4·2	3·6
−15°	10·3	7·4	5·7	4·7	4·0
−10°	13·1	8·8	6·6	5·3	4·4
− 5°	17·9	10·7	7·7	6·0	4·9
0°	27·3	13·6	9·1	6·8	5·5
5°	55·6	18·5	11·1	7·9	6·2
10°	..	28·3	14·1	9·4	7·1

§ (2) THE VAPOUR-COMPRESSION PROCESS.—The working substance in a refrigerating cycle may be a gas which remains gaseous throughout, such as air. More commonly it is a fluid which is alternately condensed and evaporated. During evaporation at a low pressure the fluid takes in heat from the cold body; it is then compressed and gives out heat in becoming condensed at a relatively high pressure. A machine for carrying out this process is called a vapour-compression refrigerating machine. The selection of the fluid is governed by practical considerations. Water is used in some cases, but a serious drawback to its use is the very large volume and low pressure of the vapour at low temperatures. There are obvious advantages in using a fluid whose vapour-pressure is neither inconveniently small at the lower limit of temperature nor inconveniently large at the upper limit. The fluids most commonly used are ammonia and carbonic acid. Ammonia has a convenient range of vapour-pressure throughout the range of temperature with which we are concerned in practical refrigeration. It has the drawback that it acts chemically on copper and brass: accordingly none of the parts of an ammonia plant that are exposed to contact with the working substance may be made of these metals. From the thermodynamic point of view ammonia admits of very efficient working; it is, in fact, the favourite substance when economy of power is the chief factor in determining the choice. With carbonic acid the vapour-pressure is considerably higher, the critical point is reached at a temperature that may come within the range of operation, and the thermodynamic efficiency is somewhat less. Notwithstanding these objections, carbonic acid is frequently preferred, especially on board ship, where it is more harmless should any of the fluid escape by leakage into the room containing the machine. For use on land, especially when the vapour-compression process is carried out on a large scale, as in ice-making or in the cooling of large stores, and the highest thermodynamic efficiency is aimed at, ammonia is usually chosen. Other fluids with lower vapour-pressures are occasionally preferred, especially in small plants, such as sulphurous acid, ethyl chloride, or methyl chloride.

If the reversed Carnot cycle were actually followed, the choice of working fluid would make no difference to the efficiency: the coefficient of performance for any fluid would have the value $T_2/(T_1-T_2)$. But a part of the reversed Carnot cycle is omitted in practice, with the result that the coefficient is reduced, and the extent of the reduction depends on the nature of the fluid; it is greater in carbonic acid than in ammonia.

To carry out a reversed Carnot cycle completely, with separate organs for the successive events which make up the cycle, would require:

(1) A compression cylinder in which the vapour is compressed, from the pressure corresponding to T_2, to the pressure corresponding to T_1.

(2) A condenser in which it is condensed at T_1. A typical form of this organ would be a surface condenser in which the working fluid gives up its heat to circulating water.

(3) An expansion cylinder in which it expands from T_1 to T_2.

(4) An evaporator in which it takes up heat at T_2 from the cold body from which heat is to be extracted. This vessel is sometimes called the "refrigerator."

In nearly all refrigerating machines the expansion cylinder is omitted for reasons of practical convenience, and the fluid streams from (2) to (4) through a throttle valve with an adjustable opening, called the "regulator" or "expansion-valve." In passing the expansion-valve the pressure of the working fluid falls to that of the evaporator; its temperature falls to T_2 and part of it becomes evaporated before it begins to take in heat from the cold body.

The omission of an expansion cylinder, with the substitution for it of an expansion-valve, simplifies the machine, but it introduces into the cycle a definitely irreversible step. It thereby reduces the coefficient of performance for two reasons. The work which would be recovered in the expansion cylinder is lost; and also the refrigerating effect in the evaporator is reduced, for more of the liquid is vaporised in the act of streaming through the expansion-valve than would be vaporised in adiabatic expansion, consequently less is left to be evaporated by subsequently taking in heat from the cold body. The loss of efficiency from these two causes is not, however, very important under ordinary conditions. If the expansion cylinder were retained as part of the machine its effective volume would need adjustment relatively to that of the compression cylinder, in order to secure the best results under varying conditions as to the limits of temperature. Rather than introduce this complication, it is worth while to make the slight sacrifice of thermodynamic efficiency which is involved in letting the compressed fluid pass back to the low-pressure side of the apparatus through an expansion-valve instead of through a cylinder in which it would do work in expanding to the lower pressure.

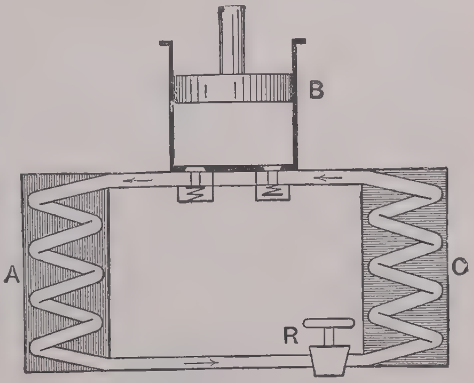

FIG. 2.—Organs of a Vapour-compression Refrigerating Machine.

In the usual type of vapour-compression refrigerating machine, accordingly, the expansion cylinder is omitted, and the organs are those shown diagrammatically in *Fig.* 2. They are (1) the compression cylinder B; (2) a condenser A such as a coil of pipe cooled by circulating water, in which the working substance is condensed under a relatively high pressure and at the upper limit of temperature T_1; (3) an expansion-valve or regulator R through which it streams from A to C; (4) the evaporator C, in which it is vaporised at a low pressure by taking in heat from the cold body at the lower limit of temperature. The evaporator may for instance be a coil of pipe fixed in the cold chamber (generally near the ceiling) and taking in heat from the surrounding atmosphere of that chamber; often it is a coil placed in a tank and surrounded by cold circulating brine which serves as a vehicle for conveying heat to the working substance from a cold chamber or from cans for ice-making or other objects that are to be refrigerated.

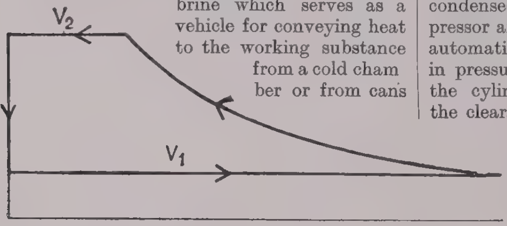

FIG. 3.—Indicator Diagram of Compression Cylinder.

The action of the compression cylinder is shown by the indicator diagram, *Fig.* 3. During the forward stroke of the compressor the valve leading to A is shut and that leading from C is open. A volume V_1 of the working vapour is taken in from C at a uniform pressure corresponding to the lower limit T_2. In most actual cases what is taken in is not dry-saturated vapour but a wet mixture, the wetness of which is regulated by adjusting the expansion-valve R. This is in order that the subsequent compression may not produce much, if any, superheating. It is possible to make the compression wholly "wet" by taking in a sufficiently wet mixture: more generally the expansion-valve is adjusted so that the vapour is moderately wet to begin with, and becomes slightly superheated by compression. At the end of the forward stroke the valve leading from C closes, and the piston is forced to move back, compressing the vapour or wet mixture in the cylinder until its pressure becomes equal to that in A. This compression reduces the volume of the fluid in the cylinder to V_2. The valve leading to A then opens, and the back stroke is completed under a uniform pressure while the working substance is discharged into A and condensed there. The valves of the compressor are spring valves which open and close automatically in consequence of the changes in pressure, and are situated in the cover of the cylinder in such a manner as to make the clearance negligibly small. For the same reason the ends of the piston are often curved. These features are illustrated in *Fig.* 4, which is a sectional drawing of an ammonia compression cylinder showing the form of the piston and the arrangement of the admission and delivery valves at each end. To complete the cycle, the same quantity of working substance is allowed to pass directly from A to C through the expansion-valve R.

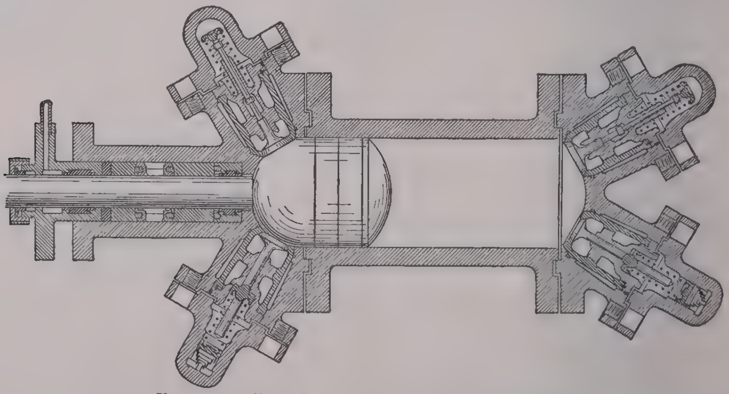

FIG. 4.—Section of Ammonia Compression Cylinder.

The temperature T_1 at which condensation takes place is in practice necessarily a good deal higher than that of the circulating water by which the condenser is kept cool, for a large amount of heat has to be discharged from the condensing vapour in a limited time. But it is important that the condensed liquid should be brought as nearly as possible to the lowest temperature of the available water-supply before it passes the expansion-valve, though it may have been condensed at a considerably higher temperature, and sometimes a supplementary vessel called a "cooler" is added for this purpose.

The complete vapour-compression cycle is exhibited in the entropy-temperature diagram of *Fig.* 5, which is drawn for ammonia as working substance, and *Fig.* 6, which is drawn for carbonic acid. There *dg* and *ch* are portions of the boundary curves of the liquid and the vapour in the saturated state. The point *a* represents the condition of the mixture which is drawn into the compression cylinder when compression is about to begin; its wetness is measured by the ratio *ah*/*gh*. The line *ab* represents adiabatic compression to the pressure of the condenser. The next process consists of cooling and condensing at this constant pressure; it is made up of three stages, *bc*, *cd*, and *de*. In the first stage, *bc*, the superheated vapour is cooled to the temperature at which condensation begins; in the next stage, *cd*, the vapour is completely condensed; in the third stage, *de*, the condensed liquid is cooled to the lowest available temperature before it passes the expansion-valve. The lines *bc*, *cd*, and *de* form parts of one line of constant pressure. In the diagram for ammonia, *Fig.* 5, *de* is practically indistinguishable from the boundary line, but in the diagram for carbonic acid, *Fig.* 6, the distinction is very apparent because we are there dealing with a liquid that is highly compessible in consequence of its nearness to the critical state. The line *ef* represents the process of passing through the expansion-valve, in which the pressure falls to that of the evaporator. This process takes place too quickly for any substantial amount of heat to enter the fluid from outside; consequently *ef* is a line of constant total heat, for in a throttling process the total heat I does not change ("Thermodynamics," § (32)). As a result of passing the expansion-valve the working substance comes into the condition shown by the point *f*. The proportion which is converted into vapour by the act of passing the valve is shown by the ratio *gf*/*gh*. Lastly, we have the process of effective evaporation when the substance is usefully extracting heat from the brine or other cold body, by evaporating in the refrigerator. This is represented by the line *fa*, during which the dryness changes from *gf*/*gh* to *ga*/*gh*.

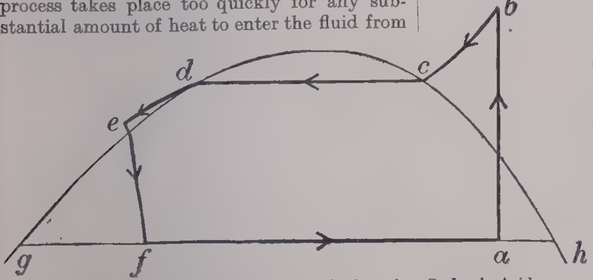

FIG. 6.—The Vapour-compression Cycle, using Carbonic Acid.

The refrigerating effect, that is to say, the amount of heat taken in from the cold body, is represented by the area under the line *fa*, measured down to a base-line corresponding to the absolute zero of temperature (see "Thermodynamics" §§ (24) and (42)).

The amount of heat rejected during cooling and condensation of the vapour and subsequent cooling of the condensed liquid is the area under the lines *bc*, *cd*, and *de*.

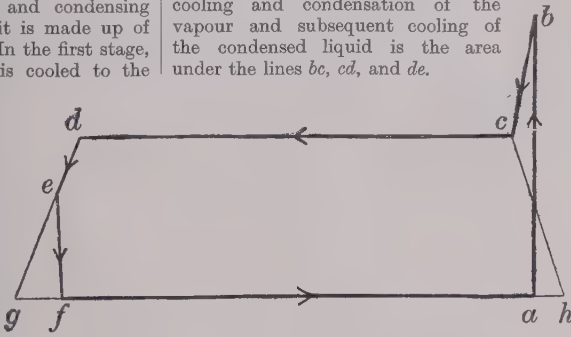

FIG. 5.—The Vapour-compression Cycle, using Ammonia.

The thermal equivalent of the work spent in carrying the working substance through the complete cycle—which is simply the work spent on it in the compressor—is the difference between those two quantities. It should be noted that the work spent is not measured by the area *abcdefa*, enclosed by the lines which represent the complete cycle, because the cycle includes the irreversible step *ef*. In consequence of that the work spent is greater than the enclosed area by the area under the line *ef*.

When carbonic acid is used as the working substance, the temperature of the cooling water may be so high that the pressure during cooling is above the critical pressure. This case is illustrated in *Fig.* 7. The line *be* then becomes a continuous curve lying entirely outside of the boundary curve. The working substance passes from the state of a superheated vapour at *b* to the state at *e* without any stage corresponding to *cd* in *Fig.* 6, in which it is a mixture of liquid and vapour. As before, the refrigerating effect is measured by the area under *fa*; the heat rejected to the cooling water is measured by the area under *be*; the difference between these two quantities measures the work spent, and is greater than the area of the closed figure *abefa* by the area under the irreversible step *ef*. Even when the temperature of the cooling water is above the critical temperature, a substantial amount of refrigerating effect is obtained, though the thermodynamic efficiency of the cycle is less than when the upper limit is low enough to allow the compressed gas to become liquid.

In all these vapour-compression cycles the conditions are to some extent ideal, for it is assumed that the compression *ab* is adiabatic, and that in passing the expansion-valve the substance takes in no heat by conduction. Under these conditions the refrigerating effect, the work of compression, and the heat rejected may very usefully be expressed as follows in terms of the total heat of the substance at the various stages of the operation.

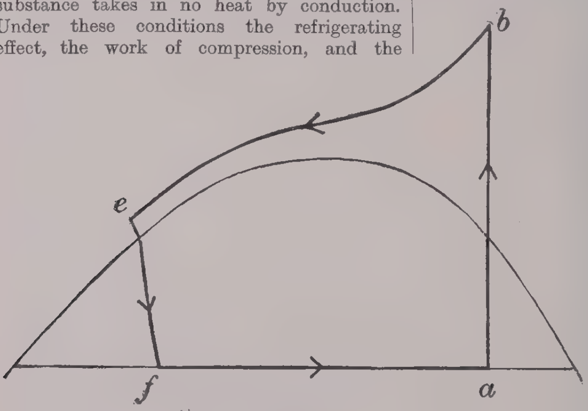

FIG. 7.—Cycle for Carbonic Acid, with Compression above the Critical Pressure.

The refrigerating effect, that is to say the amount of heat taken in from the cold body, is $I_a - I_f$, where I_a is the total heat at *a* and I_f is the total heat at *f*. This is because the (reversible) operation *fa* is effected at constant pressure. For the same reason the amount of heat rejected to the condenser and cooler is $I_b - I_e$, where those quantities designate the total heat at *b* and at *e* respectively. Further, since in the process *ef* of passing the expansion-valve there is no change of total heat, $I_f = I_e$. We may therefore state the amount of heat rejected as $I_b - I_f$.

Again, the work spent in the compressor is (in thermal units) $I_b - I_a$. It is the thermal equivalent of the area of the indicator diagram in *Fig.* 3, namely $A\int_a^b V dP$, which is equal to $I_b - I_a$ by the general principle proved in "Thermodynamics," § (38).

That these results are in agreement with one another is seen by considering the heat-account of the cycle as a whole:

Work spent = Heat rejected − Heat taken in.

$$I_b - I_a \quad = \quad (I_b - I_f) \quad - \quad (I_a - I_f).$$

§ (3) PERFORMANCE OF A MACHINE.—The *coefficient of performance*, which is the ratio of the heat taken in from the cold body to the work spent in the compressor, is

$$\frac{I_a - I_f}{I_b - I_a}.$$

Hence estimates of performance are easy when we can find the total heat of the liquid just before the expansion-valve, and that of the vapour before and after compression. These quantities are most readily found by representing the cyclic process on a Mollier chart of entropy ϕ and total heat I for the given working substance. Fairly complete data are available for ammonia, carbonic acid, and sulphurous acid, and $I\phi$ charts for these substances will be found in a Report of the Refrigeration Research Committee of the Institution of Mechanical Engineers (*Min. Proc. Inst. Mech. Eng.*, Oct. 1914).

In drawing such charts a geometrical device is resorted to for the purpose of making the diagrams at once open and compact, with the effect that measurements may be made with sufficient accuracy on a chart of reasonable size. This device, which Mollier originally adopted in drawing his $I\phi$ chart for carbonic acid, is to use oblique co-ordinates. The lines of constant I are horizontal; the lines of constant ϕ, instead of being perpendicular to them, are inclined at a small angle. The result is that when the chart is drawn the curves on it are sheared over, as compared with the form they would have on a chart with rectangular axes, and there is a gain in clearness and in the precision with which one may measure the changes of total heat that occur in the several stages of the vapour-compression process. *Fig.* 8 shows, on a small scale, an $I\phi$ chart, with oblique co-ordinates, for ammonia, and *Fig.* 9 shows one for carbonic acid. Lines of constant pressure and lines of constant temperature are drawn, and, in the wet region, lines of constant dryness. (Cf. "Thermodynamics," § (42)). In each case the region useful in refrigeration is included, and in *Fig.* 9 the region extends both above and below the critical point. On such charts it is easy to draw the ideal diagram for any assigned temperatures of evaporation, condensation, and subsequent cooling, and for any assigned wetness at the beginning of compression, and in that way to find graphically those changes in the total heat which determine the amount of the refrigerating effect and the coefficient of performance.

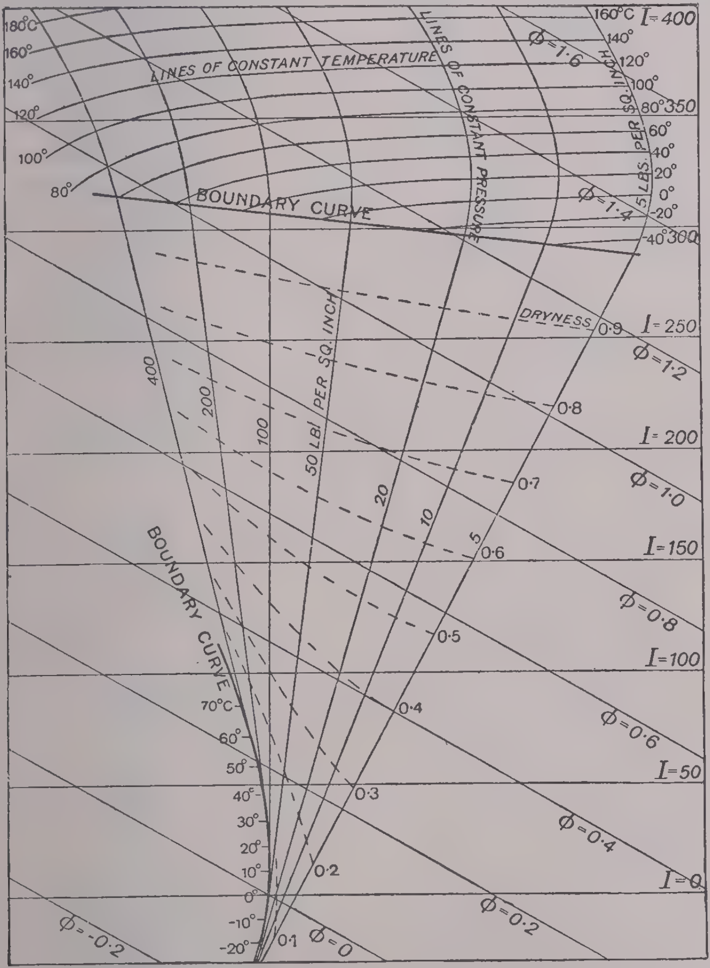

FIG. 8.—Mollier Iφ Chart for Ammonia (oblique co-ordinates).

Fig. 10 illustrates how the ideal action of a vapour-compression refrigerating machine is represented on the Iφ chart. In this example the working substance is carbonic acid, the lower limit of temperature at which evaporation occurs is supposed to be – 10° C., the temperature of condensation is 25° C., and the condensed liquid is cooled to 15° C. before it streams through the expansion-valve. The cycle begins at the point *a*, which represents the state of the substance when it is about to enter the compressor. This point is on the constant-pressure line corresponding to the process of evaporation in the evaporator, and its distance from the two boundary curves corresponds to the proportion of vapour to liquid in the mixture. If the compression is to be completely "dry," the starting-point will be at a_1: more generally the substance is slightly wet when compression begins. The straight line *ab*, drawn parallel to the lines of constant entropy on the chart, is the process of adiabatic compression. The position of *b* is determined by the intersection of this line with a line of constant pressure corresponding to the known upper limit of pressure at which condensation is to occur. The temperature reached in the process of compression is seen by the position of *b* among the lines of constant temperature. When compression begins at a point such as *a* it involves some superheating. But if the mixture is so wet to begin with that the adiabatic compression line through *a* does not cross the boundary curve before the upper limit of pressure is reached, there is no superheating, and in that case the process is spoken of as "wet" compression. This would require the compression to have begun at a_c instead of *a*. By beginning at *a* it carries the substance into the region of superheat before compression is completed at *b*. Next we have the constant-pressure process of cooling and condensation and further cooling, represented in its three stages by the lines *bc*, *cd*, and *de*, the position of *e* being fixed by the temperature to which the liquid is known to be cooled before it reaches the expansion-valve. Then a horizontal straight line through *e* (a line of constant total heat) represents the process of streaming through the expansion-valve, and determines a point *f*, on the evaporation line, which exhibits the condition in which the substance enters the evaporator. The process of evaporation *fa*, which is the effective refrigerating process, completes the cycle. The values of I_a, I_b, I_e, and I_f (which is the same as I_e) are read directly by measurement from the chart, and from them the work spent in compressing the substance, which is $I_b - I_a$, and the refrigerating effect, which is $I_a - I_f$, are determined. The position of the starting-point *a*, between a_c and a_1, which determines how far the compression will be wet or dry, does not greatly affect the thermodynamic efficiency of the process. Between the two extremes there is a certain degree of initial dryness which gives a slightly higher coefficient of performance than is obtained either

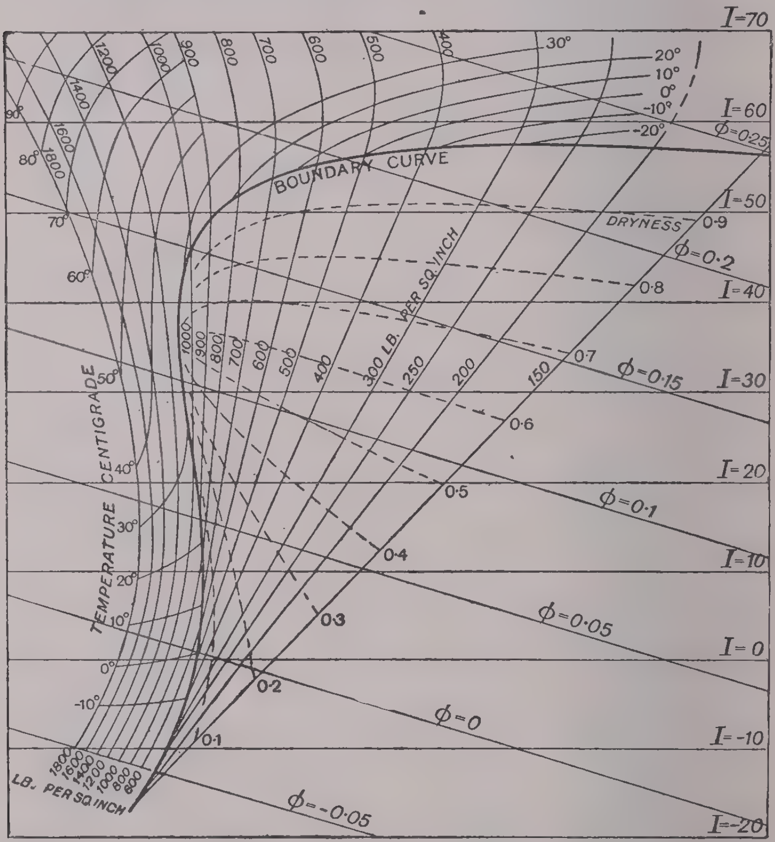

FIG. 9.—Mollier Iϕ Chart for Carbonic Acid (oblique co-ordinates).

by starting as at a_1 with dry vapour or as at a_c with a mixture so wet that compression does no more than vaporise the liquid it contains. The position of a for maximum efficiency may be found thus: The refrigerating effect for any position of the compression starting-point a is proportional on some scale to the length fa. The work spent in compressing the fluid is proportional, on another scale, to the length ab. Hence the position of the compression line ab which will give the highest coefficient of performance is that which gives the smallest ratio of ab to fa. This position is found by drawing a tangent from f to the curve of constant pressure dcb_1. The compression line ab is then drawn through the point of contact b, and this fixes a as the starting-point for maximum efficiency in the ideal cycle with adiabatic compression.

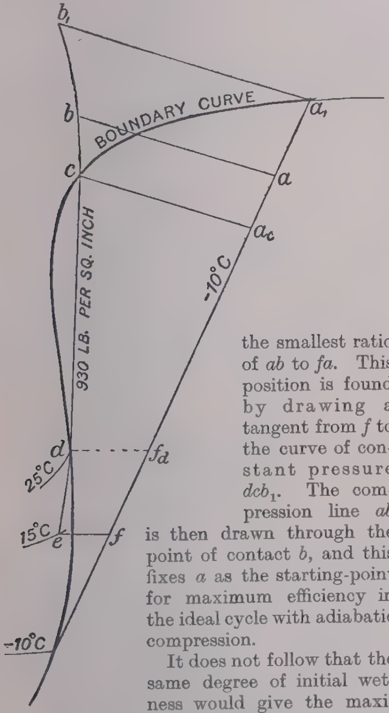

FIG. 10.

It does not follow that the same degree of initial wetness would give the maximum coefficient in a real compressor, for the performance of a real machine is complicated by transfers of heat between the working substance and the metal. In general such transfers will be less when the working substance is dry. On the other hand, with a wet mixture, what is called the volumetric efficiency of the apparatus is greater, since a larger quantity of the working substance passes through the machine for every cubic foot swept through by the piston, and this tends to reduce the proportion of those losses that arise from mechanical friction, and from radiation and conduction between the apparatus and its environment.

Incidentally, *Fig.* 10 illustrates the loss of refrigerating effect that would result from omitting to cool the condensed working fluid down to the lowest available temperature before it passes through the valve. If instead of being cooled to 15° C. it were allowed to pass through the valve when its temperature is still 25° C. (the temperature of condensation), the operation of passing the valve would be shown by the line df_d, and the process of effective evaporation would begin from the state f_d instead of the state f.

Whatever be the working substance, an essential feature of any vapour-compression refrigerating machine is that the vapour must be pumped up from the low-pressure region in which it has been evaporated to the high-pressure region in which it is to be condensed. But this pumping up may be effected in more than one way. The usual way is by means of a cylinder and piston, and so long as the vapour-pressure is moderately high the use of a compressing piston is quite satisfactory. But when the vapour-pressure is very low, as it would be if water were used for the working substance, the volume to be swept through by a compressing piston would be so large as to be very inconvenient, and the amount of work which would be wasted through friction between the piston and cylinder would be an excessive addition to the legitimate work of compression. Not only would the machine be exceedingly bulky but its practical efficiency would be exceedingly low. At 0° C., for example, the density of water-vapour is so small that about 365 cubic feet of it are required to absorb as much latent heat as one cubic foot of ammonia-vapour. Hence to use water-vapour as a refrigerating agent some appliance must be resorted to which will avoid the bulk and frictional waste of an ordinary compression pump. One such appliance is an ejector or jet pump, in which an auxiliary stream of vapour, supplied at a comparatively high pressure, forms a motive jet which drags with it the vapour to be "aspirated," namely the vapour which has been formed by evaporation at low pressure, so that both pass on together to be condensed. An independent supply of steam at a higher pressure forms the motive jet. It acquires a high velocity in a discharge nozzle of the type which first converges and then diverges. The low-pressure vapour to be aspirated is allowed to enter the nozzle, from the side, at the reduced section, where the velocity is greatest and the pressure is least. The jet communicates some of its momentum to that vapour, and the mixed stream passes on to the condenser through the divergent channel, losing velocity and gaining pressure as it goes. This enables the pressure of the working substance to rise from the lower limit at which the aspirated vapour is formed to the higher limit at which it is condensed. In refrigerating machines constructed to act in this way the quantity of vapour in the motive jet is as much as three or even five times the quantity that is aspirated. The thermodynamic efficiency of the method is found on trial to be only moderate, but the apparatus has advantages in point of simplicity and in the absence of any working substance other than water. It has been applied not only to cool water, but also to maintain a temperature considerably below 0° C., in which case brine is substituted for fresh water as the working substance whose vapour is aspirated, and the cooled brine is prevented from becoming too dense by systematically returning to it enough water to compensate for the evaporation.

§ (4) Refrigeration by Compression and Expansion of Air.—So long as the working substance in any refrigerating machine is a vapour which becomes liquefied during the operation, it is practicable, as we have seen, to dispense with the expansion cylinder. The step-down in temperature, which occurs while the substance passes the expansion-valve, is an example of the Joule-Thomson cooling effect of throttling (see "Thermodynamics," § (50)). This effect is large when the substance is a mixture of liquid and vapour. It is also large in a gas near its critical point, and hence a machine using carbonic acid under tropical conditions can be effective without an expansion cylinder although the substance may not have been liquefied under compression. A gas near its critical point is very far from perfect and does not even approximately conform to Joule's Law. A gas which conforms to that law would suffer no step-down of temperature in passing an expansion-valve. With a gas such as air, which is nearly perfect at the temperatures and pressures that occur in ordinary refrigeration, the step-down would be too small to serve the desired purpose. Hence with air for working substance an expansion cylinder becomes an essential part of the machine. Refrigerating machines which use air, and cool it by means of expansion in a cylinder in which it does work against a piston, are amongst the oldest effective means of producing cold by mechanical agency. They are still used for the direct cooling of the atmosphere of cold stores, but their use is now less common, because machines in which the working substance is a condensible vapour are not only more compact but give a better thermodynamic return for the work spent in driving them. Historically, refrigerating machines which use air are important, for it was by their successful use that the cold-storage industry was created and the business was established of conveying refrigerated cargoes overseas.

The air-machines which are in actual use operate by taking in a portion of air from the chamber that is to be kept cold, compressing it more or less adiabatically with the result that its temperature rises considerably above that of the available water supply, then extracting heat from it in the compressed state by means of circulating water, then expanding it in a cylinder in which it does work, with the result that its initial pressure is restored and its temperature falls greatly below the initial temperature. It is then returned into the atmosphere of the cold chamber, with which it mixes; the object being either to lower the temperature in the chamber or to keep it from rising through leakage of heat from outside. This type is known as the Bell-Coleman air-machine.

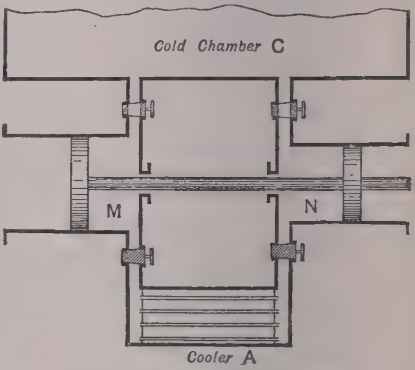

Fig. 11.—Organs of an Air-machine.

As applied to the cooling of a chamber such as a cold store or the hold of a ship, the apparatus takes the form shown diagrammatically in *Fig.* 11. In the phase of action shown there the pistons are moving towards the left. Air from the cold chamber C is being drawn into the compression cylinder M. In the return stroke it will be compressed from one atmosphere to about four, with the result that its temperature may be raised to 130° C. or higher It is delivered under this pressure to the cooler A, where it gives up heat to the circulating water and comes down to near atmospheric temperature. It then passes, still at high pressure, to the expansion cylinder N, where it does work in expanding down to the original pressure of about one atmosphere and thereby becomes very cold, reaching a temperature of perhaps – 60° C. or – 70° C., in which condition it is returned to the cold chamber. An ideal indicator diagram for the whole cycle is given in *Fig.* 12, where *fcbe* shows the action of the compression cylinder and *eadf* shows that of the expansion cylinder. The area *abcd* measures the net amount of work that is expended. In the diagram the compression and expansion are both treated as adiabatic, and the volume of A as well as that of C is assumed to be so large that during delivery of the air its pressure does not sensibly change. Writing K_p for the specific heat of air at constant pressure, and T_a, T_b, T_c, and T_d for the temperature of the working air, at the points *a*, *b*, *c*, and *d* of the diagram, we have $Q_A = K_p(T_b - T_a)$ for the heat rejected to the cooling water, and $Q_C = K_p(T_c - T_d)$ for the heat usefully extracted from the cold chamber. The net amount of work expended is equal to $Q_A - Q_C$. The coefficient of performance is $Q_C/(Q_A - Q_C)$. Since the ratio of temperatures in the adiabatic expansion of a gas depends on the ratio of volumes,

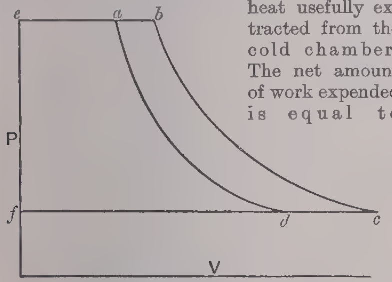

Fig. 12.—Indicator Diagram of Air-machine.

$$\frac{T_b}{T_c}=\frac{T_a}{T_d};\ \text{from which}\ \frac{T_c-T_d}{T_b-T_a}=\frac{T_c}{T_b}.$$

Hence $\dfrac{Q_C}{Q_A}=\dfrac{T_c}{T_b}$ and $\dfrac{Q_C}{Q_A-Q_C}=\dfrac{T_c}{T_b-T_c}$.

This coefficient of performance is low because of the very large range of temperature through which the working air is carried. For this reason, and also because of greater frictional losses, an actual air-machine gives results that compare unfavourably with those obtained by using vapour compression.

In the working of air-machines the presence of moisture has to be reckoned with. The air coming from the cold chamber is more or less saturated: during expansion it becomes supersaturated, and the water from it would be deposited as snow in the expansion cylinder, and might interfere with the action of the mechanism if preventive devices were not introduced. One such device is to divide the whole expansion into two stages by making it compound. In the first stage the expansion is carried only far enough to cool the air to a temperature just above the freezing-point. In that way nearly all the moisture is deposited in the form of water, and is easily drained away before the final stage, which would freeze it, begins. Another device is to condense out most of the moisture before expansion, by passing the compressed air through pipes, called drying pipes, which bring its temperature down to near the freezing-point before it enters the expansion cylinder.

§ (5) Direct Application of Heat to produce Cold. *Absorption Machines.*—As was mentioned at the beginning of this article, in some refrigerating appliances there is no application of mechanical power: the agent is heat, which is supplied from a high-temperature source, and is employed in such a way as to cause another quantity of heat to pass from a cold body and to be discharged at a temperature intermediate between that of the cold body and the hot source. Such machines act by the absorption of one substance by another, to form a solution or compound, and the subsequent separation of the constituents by the agency of heat. In such machines the efficiency of the action from the thermodynamic point of view is measured by the heat ratio Q_2/Q, where Q_2 is the heat extracted from the cold body, and Q is the high-temperature heat which is supplied to carry out the operation.

A typical example is the ammonia-absorption refrigerating machine, in which the vapour of ammonia is alternately dissolved by cold water under a relatively low pressure, and distilled from solution in water under a relatively high pressure by the action of heat. The ammonia vapour, driven off by applying heat to a solution, is condensed in a vessel which is kept cool by means of circulating water. In this way anhydrous liquid ammonia is obtained at high pressure, which (just as in a compression machine) is then allowed to pass through an expansion-valve into a coil or vessel forming the evaporator. A low pressure is maintained in the evaporator by causing the evaporated vapour to pass into another vessel, called the absorber, where it comes into contact with cold water in which it becomes dissolved. When the water in the absorber has taken up a sufficient proportion of ammonia, it in turn is heated to give off the vapour again under high pressure. In the simplest form of the apparatus the same vessel serves alternately as absorber and as generator or distiller. For continuous working there are separate vessels, and the rich solution is transferred from the absorber to the generator by a small pump, while the water from which ammonia has been expelled flows back to the absorber to dissolve more ammonia. The scheme of such an apparatus is shown diagrammatically in *Fig.* 13. Heat is applied to the solution in the generator by means of a steam-coil. The gas passes off at top to the condenser, then through the expansion or regulating valve to the evaporator, and then on to the absorber, where it meets a current of water or very weak solution that has come over from the bottom of the generator. Between the generator and absorber is the interchanger, a device for economising heat by taking it from the water that is returning to the absorber, and giving it to the rich solution that is being pumped into the generator. This rich solution is delivered at the top of the column in the generator; as the liquid parts with the ammonia it becomes denser and falls to the bottom, where it escapes to the absorber through an adjustable valve. When water absorbs ammonia a large amount of heat is given out. Hence the absorber as well as the condenser has to be kept cool by means of circulating water or otherwise. Under the most favourable conditions the quantity of heat which such a machine takes in from the cold body is considerably less than the quantity of high-temperature heat that has to be supplied, for it needs a greater number of thermal units to separate ammonia gas from solution in water than simply to evaporate the same amount of liquid ammonia.

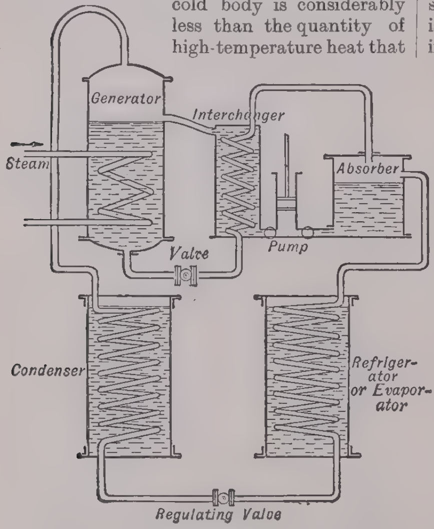

Fig. 13.—Organs of an Ammonia Absorption Machine.

In another type of absorption machine water-vapour is the substance which is absorbed: it is taken up by sulphuric acid, from which it may again be separated by the agency of heat. Such a machine has been used for ice-making, the evaporation of part of the water serving to freeze the rest. In this case also the heat ratio, namely, the ratio of heat usefully extracted to heat supplied, is less than unity, for it takes more heat to separate the vapour of water from a sulphuric-acid solution than from pure water. It is a familiar fact that when water is mixed with sulphuric acid much heat is given out.

It is obvious that a better thermodynamic result would be attainable if the process of absorption of the vapour were attended by the giving out of less heat than is equivalent to the latent heat of the vapour itself. This is the case in a process patented by Mr. W. W. Seay, in which ammonia vapour unites with certain anhydrous salts, for which it has much affinity, such as the sulphocyanide of ammonium (NH_4CNS), or the nitrate, bromide, or iodide. Any one of these salts forms a suitable absorbent. The ammonia vapour unites with the dry salt to form a liquid solution, from which the ammonia vapour can again be driven off by the application of heat, leaving the salt dry and ready to serve again as the absorbent. The vapour is strictly anhydrous, for no water is present in the working substance at any stage. The heat given out during absorption of the ammonia vapour by the salt is substantially less than the latent heat of the vapour itself at the same pressure, for part is taken up in liquefying the salt. Similarly, the heat required to effect a separation of ammonia vapour from the salt is substantially less than the latent heat of the vapour, for part is supplied by the solidification of the salt. Consequently, when this process is made use of for the purpose of refrigeration, the ratio of the heat which is extracted from the cold body to the high-temperature heat, which is supplied to the generator, would be greater than unity, if it were not for such losses as occur through imperfection in the working. Practical difficulties in the use of such salts arise from the fact of their turning solid during the operation, and from their tendency to act chemically on the metal of containing vessels.

Any appliance for the production of cold by the agency of heat requires a supply of heat at a temperature higher than that of the surroundings. There are necessarily three temperatures to be considered: (1) the low temperature T_2 of the cold body from which heat is being extracted; (2) the intermediate temperature T_1 of the available condensing water or other "sink" into which heat can be rejected; and (3) the high temperature T of the source from which heat is supplied to perform the operation. Any such appliance may be regarded as equivalent to the combination of a motor or heat-engine driving a refrigerator or heat-pump (*Fig.* 14). A quantity Q of high-temperature heat goes in at one place, and thereby causes a quantity Q_2 of low-temperature heat to go in at another place. Heat is rejected at the intermediate temperature T_1, and the heat so rejected is equal to the sum of Q and Q_2, for no work is done by the appliance or spent upon it as a whole. This description applies whether the appliance is actually a mechanical combination of a heat-engine with a heat-pump, or is an absorption machine with no conversion of heat into work and work into heat. In either case we have to consider what is the ideally greatest ratio of the low-temperature heat Q_2, which is extracted from the cold body, to the high-temperature or driving heat Q, when the three temperatures T_2, T_1, and T are assigned.

Suppose, first, that the machine consists of a perfect (reversible) heat-engine driving a perfect (reversible) heat-pump. Then it is easy to calculate the ratio of the heat extracted Q_2 to the heat supplied Q. Writing W for the heat-equivalent of the work developed in the heat-engine and employed to drive the heat-pump, we have

$$W=\frac{Q(T-T_1)}{T},$$

since the heat-engine is reversible. Again, since the heat-pump is also reversible,

$$W=\frac{Q_2(T_1-T_2)}{T_2}.$$

Hence $$\frac{Q_2}{Q}=\frac{T_2(T-T_1)}{T(T_1-T_2)},$$

which gives the required ratio of heats.

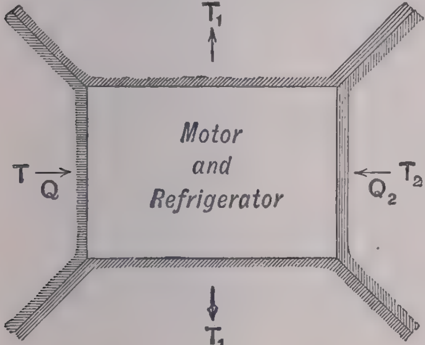

FIG. 14.

§ (6) EFFICIENCY OF ABSORPTION PROCESSES. —The importance of this result lies in the fact that no other method of applying heat to produce cold can give a higher ratio of T_2 to Q, the three temperatures T, T_1, and Q_2 being assigned. To prove this, imagine the combination of reversible heat-engine and reversible heat-pump to be reversed; it will then give out an amount of heat equal to Q to the hot body and an amount equal to Q_2 to the cold body, and it will take in an amount equal to $Q+Q_2$ from the intermediate body at T_1. It will still develop no work as a whole, nor require work to be spent in driving it. Imagine further that between the hot body and the cold one there are two appliances working—both using the same intermediate temperature—one of which is this reversed combination, and the other is a refrigerating machine (such as an absorption machine) whose efficiency we wish to compare with that of the combination. Then if it were possible for that machine to have a higher efficiency than the combination, it would extract more heat than Q_2 from the cold body for the same expenditure of high-temperature heat Q. Hence, when both work together, namely, the combination working reversed and the other machine working direct, the cold body would lose heat while on the whole the hot body would lose none. In other words, we should then have an impossible result, namely, a simple transfer of heat, by a purely self-acting agency, from a cold body at T_2 to a warmer body at T_1, the intermediate temperature. The agency would be self-acting in the sense of being actuated by no form of energy, mechanical, thermal, or other. Such a result would be a violation of the Second Law of Thermodynamics. The conclusion is that no means of employing heat to produce cold, whether directly as in an absorption machine, or indirectly as in a compression machine driven by an engine, can be more efficient (for the same three temperatures) than the combination of a reversible heat-engine driving a reversible heat-pump. Hence the expression

$$\frac{Q_2}{Q}=\frac{T_2(T-T_1)}{T(T_1-T_2)}$$

measures the ideally greatest ratio of heat extracted to heat supplied. Any real appliance will show a smaller heat-ratio in consequence of irreversible features in its action.

It is instructive to consider this action in relation to the entropy of the system as a whole. So long as the action is completely reversible, the entropy of the system does not change. The above expression may be written in the form

$$\frac{Q_2}{T_2}(T_1-T_2)=\frac{Q}{T}(T-T_1),$$

from which $$\frac{Q+Q_2}{T_1}=\frac{Q}{T}+\frac{Q_2}{T_2}.$$

This expresses the conservation of entropy for the complete reversible operation. The entropy of the system as a whole does not change, for the term on the left is the gain of entropy by the body at T_1 to which heat is rejected; the two terms on the right are the losses of entropy by the hot body and cold body respectively. The whole action may be regarded as a transfer of entropy from two sources at T and T_2, to an intermediate sink at T_1. So long as the action is reversible this transfer occurs without affecting the aggregate entropy, but if it is not completely reversible the aggregate entropy will increase; in that case the term on the left becomes greater than the sum of the terms on the right.

Again, the equation shows that, under reversible conditions, the product of the entropy lost by the hot source (through the removal of the heat Q) into the drop in temperature which that heat undergoes, namely, from T to T_1, is equal to the product of the entropy lost by the cold body into the rise of temperature of the abstracted heat Q_2. Each of these products is in fact a measure of W, the work which the heat-engine produces and the heat-pump consumes in the ideal combination of reversible engine with reversible pump.

A mechanical analogue to this thermal operation is obtained if one thinks of a machine for lifting water in a bucket from a low level to an intermediate level, by letting water come down in another bucket from a high level to the same intermediate level. If the buckets are connected by frictionless pulleys the operation is mechanically reversible. Let H, H_1, H_2 represent the height of the high level, the intermediate level, and the low level respectively, above any convenient datum level. Then the energy equation is

$$M(H - H_1) = M_2(H_1 - H_2),$$

where M is the weight of water that comes down from H to H_1, and M_2 is the weight of water that is lifted from H_2 to H_1. On comparing this with the above equation for a corresponding reversible thermal process, it will be seen that tho analogue of weight of water is not quantity of heat, but entropy, namely, the quantity of heat divided by the temperature of supply.

The reversible thermal operation may be represented by means of an entropy temperature diagram (*Fig.* 15). There the area *abon* represents the heat which is supplied at the high temperature T; and the area *abcd* represents the work which would be done in a perfect heat-engine by letting down that quantity of heat from T to the lower level T_1. Between the given levels of temperature T_1 and T_2 draw a rectangle *defg* whose area is equal to the area *abcd*, and produce *ef* to meet the base line for zero temperature in *m*. Then the area *fgnm* represents the refrigerating effect, namely the heat extracted from the cold body at T_2. The amount of heat discharged at the intermediate level T_1 is equal to the area *ecom*, which is equal to the sum of the areas *abon* and *fgnm*.

§ (7) THE REFRIGERATING MACHINE AS A MEANS OF WARMING.—In any such appliance, whether reversible or not, the quantity of heat delivered at the intermediate temperature T_1 is greater than the quantity supplied at the higher T by the amount of the heat raised from the low temperature T_2, and may, as we have seen, be much greater. This fact is the basis of an interesting suggestion made by Kelvin in 1852, that in the warming of rooms it would be thermally more economical to apply the heat got from burning coal in this indirect way than to discharge it into the room to be warmed. The thermodynamic value of high-temperature heat is wasted if we allow it directly to enter a comparatively cold substance. That value might be better utilised by employing the high-temperature heat to pump up more heat, taken in from, say, the outside atmosphere, to the level to which the room is to be warmed. By using, for example, an efficient steam-engine to drive an efficient heat-pump, a small quantity of heat supplied at a high temperature will suffice to raise a much greater quantity of heat through the small range that is required, and consequently to produce a much greater warming effect. Similarly, if a supply of power from any source is available as a means of warming to a moderate temperature, it will be turned to better account for that purpose if we set it to drive a heat-pump than if we simply convert it into heat.

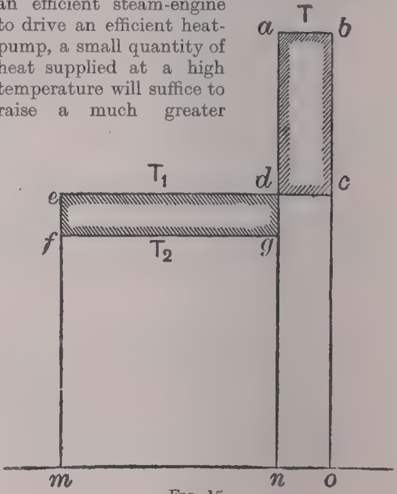

Fig. 15.

For methods of producing extreme cold in a gas, by the cumulative use of the Joule-Thomson cooling effect in passing a throttle-valve, in conjunction with a thermal interchanger which enables the stream of gas that has passed the valve to extract heat from the stream that is still on its way to the valve, see the article on "Liquefaction of Gases." In that article, as in this one, the writer has followed the lines of treatment developed in his books on *Thermodynamics for Engineers*, and *The Mechanical Production of Cold* (Camb. Univ. Press), to which reference should be made for further particulars.

J. A. E.

Refrigerators, Specific Heat of Liquids used for, determined by the electrical method. See "Calorimetry, Electrical Methods of," § (7).

Regenerative Cooling. See "Gases, Liquefaction of," § (1).

Regenerator, Stirling's and Ericsson's. See "Thermodynamics," § (27).

Regnault, 1861, compared gas thermometers with secondary standards of temperature in range 500° to 1600°. See "Temperature, Realisation of Absolute Scale of," § (39) (v.).

Experiments of, in determination of Latent Heat of Steam. See "Latent Heat," § (1) (i.)

Regnault's Law. See "Engines, Thermodynamics of Internal Combustion," § (15).

Reheat Factor. See "Turbine, Development of the Steam," § (16); "Steam Turbine, Physics of," § (8).

Reservoirs for Storage of Rainfall. See "Hydraulics," § (19).

Resistance, Varying Circuit, Compensation for, in temperature indicators of the millivoltmeter type. See "Thermocouples," § (8).

Resistance Bridges for Thermometric Work. See "Resistance Thermometers," § (7).

Resistance Coefficient, Non-Dimensional, for the Motion of a Body through a Viscous Fluid. See "Dynamical Similarity, The Principles of," § (19).

Resistance due to Fluid Motion over a Solid Boundary. See "Friction," § (13).

Resistance Manometers, Electrical. See "Pressure, Measurement of," § (13).

Resistance of Solid Bodies when towed in Fluids. See "Friction," § (14).

Resistance Thermometer:

Standardised at 0°, 100°, and 444·5° C., gives temperatures identical with the gas scale up to 1080° C. See "Resistance Thermometers," § (18).

Standardisation of, for temperatures up to 600°. See *ibid.* § (14).

Used to determine Low Temperatures, by Holborn and Wien in 1901. See *ibid.* § (17).

RESISTANCE THERMOMETERS

§ (1) Historical.—The foundation of a method for the measurement of temperature based on the change of resistance of platinum is due to Sir William Siemens, who in 1871 constructed a practical form of pyrometer on this principle. He also devised an ingenious form of resistance bridge, with the object of eliminating uncertainties due to change in the resistance of the leads, consequent on the variations in the depth of immersion of the pyrometer stem in the hot region.

The constructional details of the early Siemen pyrometers were unsatisfactory. The platinum wire was wound on a pipeclay cylinder and enclosed in a tube of wrought iron.

A committee of the British Association, appointed in 1872–73 to test these pyrometers, made an unfavourable report on their permanency, and for some years this method of temperature measurement fell into disuse. Professor A. W. Williamson, Chairman of the British Association Committee, suggested that the changes in the resistance of the platinum were due to the reducing atmosphere produced by the highly heated iron casing, which would cause the platinum to combine with a trace of the reduced silicon taken from the pipeclay cylinder. Analysis proved the truth of this theory, and pointed to the desirability of an oxidising or neutral atmosphere around the platinum wire.

Siemens showed later that a sheath of platinum eliminated this trouble, and *Fig.* 1 illustrates his improved type of pyrometer.

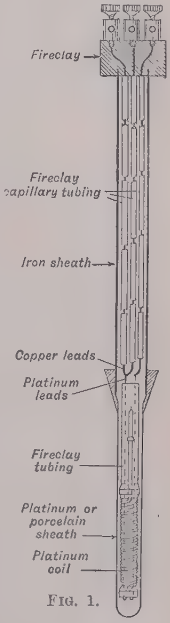

Fig. 1.

About this time the thermoelectric method of measuring temperature was being developed with conspicuous success by Le Chatelier in France and Barus in America. The simplicity of the thermoelement and its direct reading indicator as compared with the resistance thermometer outfits of those days, led to its general adoption in the industries in preference to the Siemens pyrometer.

Between 1887 and 1896 the resistance thermometer as a scientific instrument received thorough study in the hands of Callendar, E. H. Griffiths, Heycock, and Neville, who proved beyond question its reliability and extreme precision when used with due precautions and with appropriate electrical appliances.

§ (2) Callendar's Resistance Thermometer.—Callendar[1] in 1886 made a direct determination of the resistance of a particular specimen of platinum wire at various temperatures up to 600° C. The platinum spiral was sealed into a bulb of the air thermometer by means of which the temperature was obtained.

By this device Callendar avoided one of the greatest experimental difficulties of that time in gas thermometry, viz. the maintenance by gas heating of a large enclosure at a constant and uniform temperature.

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[1] *Phil. Trans. Roy. Soc.* A, 1887, clxxviii.

(i.) *The Gas Thermometer.*—His apparatus is shown diagrammatically in *Fig.* 2. The gas thermometer was designed for both constant volume and constant pressure work.

Instead of the customary practice of confining the gas by a column of mercury, a sulphuric acid gauge was employed, which increased the sensitivity sevenfold, and also eliminated capillary errors entailed by the use of a small bore tube.

The thermometer bulb was made from a piece of hard glass tubing, the coefficient of cubical expansion being deduced from measurements on a length of the same tubing as that from which the bulb was made. It was observed that the expansion was irregular, on account of the structural and hysteresis changes in the glass.

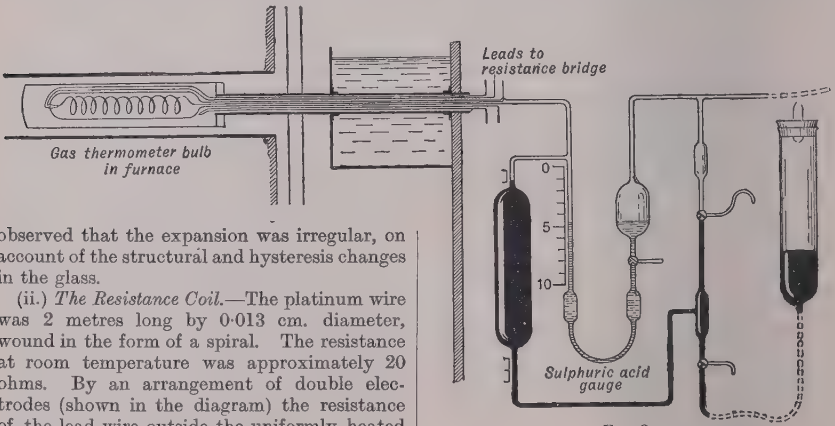

FIG. 2.

(ii.) *The Resistance Coil.*—The platinum wire was 2 metres long by 0·013 cm. diameter, wound in the form of a spiral. The resistance at room temperature was approximately 20 ohms. By an arrangement of double electrodes (shown in the diagram) the resistance of the lead wire outside the uniformly heated region could be allowed for.

One end of the coil was led out through the capillary connecting the bulb to the manometer (*Fig.* 3). A double lead of the same wire, and which was coated with the same hard glass to imitate it exactly, was laid alongside. The other double lead consisted of 10 cm. of fine platinum wire, to the mid-point of which the other end of the spiral was fused. The fine wires were bent double and fused through the glass, while the projecting ends were fused on to thick (0·096 cm.) platinum wires coated with hard glass and laid alongside the others.

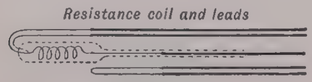

FIG. 3.

The resistance of the leads was determined on each occasion, and the correction amounted to about 3 per cent of the total resistance.

The experiments showed that to the degree of accuracy attainable in the observations (about 1°), the variation of resistance of platinum with temperature could be represented by a parabolic formula.

Callendar introduced nomenclature which has since come into general use.

The platinum temperature *pt* is defined as

$$pt=\frac{R-R_0}{R_1-R_0}\times 100,$$

where R_0 is the resistance at 0° C.,
R_1 is the resistance at 100° C.,
R is the resistance at t° C.

The quantity R_1-R_0 is generally referred to as the "Fundamental Interval" (F.I.) of the thermometer.

He showed that the difference between the true temperature t, as measured by the air thermometer, and the platinum temperature *pt* was represented by the parabolic formula

$$t-pt=\delta\left\{\left(\frac{t}{100}\right)^2-\left(\frac{t}{100}\right)\right\},$$

where δ is the coefficient for that particular sample of wire and whose numerical value is about 1·5.

The subsequent investigations of Callendar and Griffiths showed that this was generally true for wires of varying degree of purity, when the appropriate values of the coefficients were inserted in the parabolic formula.

In a direct determination of the boiling-point of sulphur with the gas thermometer, they obtained the value 444·53° C., which was about 4° lower than that previously obtained by Regnault.

In order to determine δ they concluded that the boiling-point of sulphur was the most suitable fixed point in conjunction with ice, and steam, for calibration purposes.

§ (3) Determinations of the Boiling-point of Sulphur.[1]—Subsequent to the work of Callendar and Griffiths, numerous determinations of the boiling-point of sulphur have been made, all of which have closely confirmed the value obtained by these investigators.

In the course of their comparison of the resistance thermometer with the gas thermometer, Harker and Chappuis, and Holborn and Henning, made some determinations of the boiling-point of sulphur on the gas scale, employing the resistance thermometer as intermediary. Their values are indirect, insomuch that the gas thermometer was not directly employed to determine the temperature of the sulphur vapour, but are entitled to full weight, since the resistance thermometer was directly compared with the gas thermometer in salt baths, at temperatures in the vicinity of the sulphur boiling-point.

Day and Sosman (1912) made a direct determination of the sulphur point, using a nitrogen-filled thermometer, whose bulb was of platinum-rhodium alloy. It was of 205 c.c. capacity, and the nitrogen was under an initial pressure of 500 mm. of mercury.

Gas Thermometer Determinations of the Boiling-point of Sulphur since 1890

Date.	Author.	Thermometer.	Gas.	Pressure, mm.	Original Figure, ° C.	Thermodynamic Scale, ° C.
1890	Callendar and Griffiths	Constant pressure	Air	760	444·53	444·91
1902	Chappuis and Harker	Constant volume	Nitrogen	530	444·70	444·80
1908	Eumorfopoulos	Constant pressure	Air	760	444·55	444·93
1911	Holborn and Henning	Constant volume	Hydrogen	623	444·51	444·51
			Helium	612	444·39	..
			Nitrogen	625	..	..
1912	Day and Sosman	Constant volume	Nitrogen	502	444·45	444·55
1912	Dickinson and Mueller	Constant volume	Nitrogen	502	444·28	444·38
1914	Eumorfopoulos	Constant pressure	Nitrogen	415	444·36	444·57
				792	444·13	444·53
1917	Chappuis	Constant volume	Nitrogen	500-564	444·48	444·59

Great precautions were taken to eliminate systematic errors. An aluminium shield surrounded the bulb in the sulphur tube, to prevent the condensed sulphur from lowering its temperature below the true boiling-point.

To ascertain whether the temperature of the walls of the tube had any influence, in some of the experiments the external jacket was heated until the temperature of the air gap was as high as that of the sulphur vapour within the tube. Provided the tube was full of vapour, this produced no apparent change in the value obtained.

A direct comparison of the temperature given by this form of apparatus with the Meyer tube form of sulphur boiling apparatus (*Fig.* 17), devised by Callendar and Griffiths, showed a systematic difference of but 0·04° C.

The probable source of the small divergences in the results obtained by the various observers since the time of Regnault is uncertainty in the coefficient of cubical expansion of the bulb material of the air thermometer.

Fused quartz has the smallest coefficient of the materials available for the construction of the bulb of gas thermometers, and a thermometer with a bulb of this material was used by Eumorfopoulos.

The value $444{\cdot}5_3$° C. is, at the present time, the generally accepted value for the boiling-point of sulphur on the thermodynamic scale in this country, although the value $444{\cdot}6_0$° C. is used in America and $444{\cdot}5_5$° in Germany.

§ (4) Purity of the Sulphur.—Ordinary commercial stick-sulphur manufactured by the "Chance process" is satisfactory, since no difference has been observed between its boiling-point and that of highly purified sulphur.

Usually when sulphur is boiled for the first time volatile impurities distil off. A black residue (FeS) is generally found, but in the quantity in which it is generally present this appears to have no influence.

§ (5) Variation with Pressure of the Boiling-point of Sulphur.—The influence of pressure on the boiling-point of sulphur is quite considerable in the vicinity of 760 mm., an increase of 1 mm. in the barometric height raises the boiling-point by approximately 0·09°.

The relation between temperature and pressure over the range of importance in practical work has been investigated by Holborn and Henning, by Harker and Sexton (1908), and by Mueller and Burgess (1919). The results of these investigators are in substantial agreement. Over the range 700 to 800 mm. pressure of mercury, the relation between temperature and boiling-point may be represented by the formula

$$t=t_s+0{\cdot}0910(p-760)-0{\cdot}000049(p-760)^2,$$

where t is the boiling-point at pressure p, t_s the boiling-point at 760.[1]

The freezing-point of zinc (419·4° C.) is sometimes recommended as a third fixed point instead of sulphur.

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[1] See also "Temperature, Realisation of Absolute Scale of," § (35) (iv.).

[1] The formula accepted by the Reichsanstalt is $444^{\circ}{\cdot}55+0{\cdot}0908(p-760)-0{\cdot}000047(p-760)^2$.

As the sulphur-point is so well established and convenient to use, there do not appear to be very valid reasons for discarding it in favour of a freezing-point determination.

§ (6) CONSTRUCTION OF PLATINUM THERMOMETERS.—*Fig.* 1 represents the original type of high-temperature resistance thermometer. Its chief defects are the considerable lag in its indications and the liability to contamination of the platinum wire due to the fireclay former on which the coil is wound.

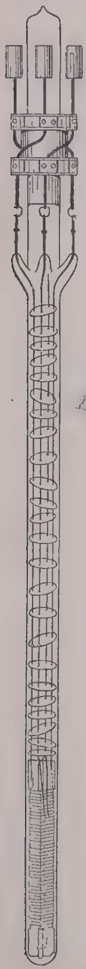

FIG. 4.

Callendar and Griffiths devised the form on which the platinum coil was wound on a mica rack with leads of heavy platinum wire. A similar pair of wires to the leads, in the form of a loop, were laid alongside. This loop was connected in the opposite arm of the bridge so as to compensate for the resistance of the leads to the coil at all temperatures.

Fig. 4 illustrates a hermetically sealed type of thermometer designed by Professor E. H. Griffiths for laboratory work of high precision. By sealing the sheath it is possible to eliminate any possibility of moisture condensing on the mica and impairing the insulation. Insulation troubles are frequent in high-resistance thermometers unless precautions are taken to prevent access of moist air.

Of the various insulating materials hitherto investigated for high-temperature work, good quality mica has proved to be the most satisfactory for thermometer construction.

Exposure to a temperature of about 1000° C. causes dehydration of the mica, which in consequence becomes silvery white and brittle. If reasonable care is taken it is quite satisfactory in this condition.

(i.) *Resistance Thermometers for Calorimetric Work.*—For calorimetric work it is desirable to have a sensitive thermometer of small lag, so that its indication at any instant is a true measure of the temperature of the liquid.

Thermometers of the type illustrated in *Fig.* 4 are satisfactory provided the coil is made as light as possible and the sheath of very thin glass.

With the object of reducing the thermometer lag to a minimum, Dickinson and Mueller have developed the type of thermometer illustrated in *Fig.* 5. The coil is wound on a flat strip of mica and enclosed in a sheath of silver tubing flattened down to fit it closely.

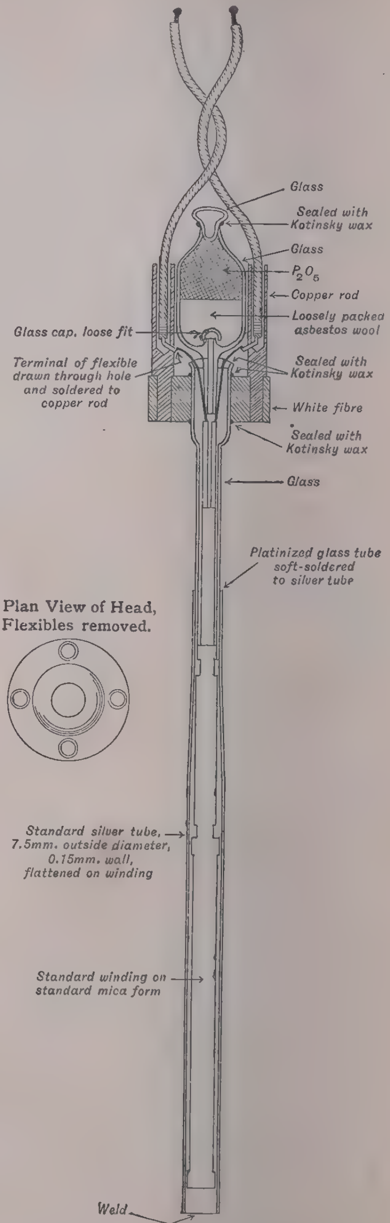
Glass
Sealed with Kotinsky wax
Glass
P_2O_5
Copper rod
Loosely packed asbestos wool
Glass cap, loose fit
Terminal of flexible drawn through hole and soldered to copper rod
Sealed with Kotinsky wax
White fibre
Sealed with Kotinsky wax
Glass
Platinized glass tube soft-soldered to silver tube
Plan View of Head, Flexibles removed.
Standard silver tube, 7.5mm. outside diameter, 0.15mm. wall, flattened on winding
Standard winding on standard mica form
Weld

FIG. 5.

Electrical insulation is effected by the use of strips of mica somewhat wider than that on which the coil is wound.

The leads are made of thin strips of copper about one-tenth of a millimetre thick and two or three millimetres wide.

If the thermometer is of the compensating lead type the compensating loop is closed by a piece of the same platinum wire as that of which the coil is wound.

All platinum joints are made by fusing with an arc, using a pure graphite electrode, while the copper platinum joints are made either by welding or with silver, using borax as flux.

In the head of the thermometer a drying capsule is arranged containing phosphorus pentoxide.

(ii.) *Influence of the Purity of the Platinum.*—A fairly accurate idea of the purity of the platinum wire may be formed from a consideration of its coefficients of resistance. The purest specimens give values of a (the mean coefficient between 0° and 100° C.) as high as 0·00389 and δ about 1·50 or slightly less. Impure wire, on the other hand, may give values of a only 60 per cent of the above and of δ up to 1·6.

The parabolic formula has been found to hold rigorously when the purity does not vary very far from the first-named values. When a thermometer is constructed of impure wire it is generally found that the zero will not remain constant after exposure to high temperatures, and consequently the reliability of the instrument is impaired.

Since only a small quantity of the material is required, it is advisable to wind the coil of the purest platinum obtainable. The heavy wire leads may be of commercially pure wire.

For work up to temperatures of 500° C. silver leads are quite satisfactory, provided an intermediate piece of platinum is introduced between the fine wires and the silver, so as to avoid the risk of contamination in the autogenous welding.

Above 600° C. the volatilisation of the silver causes contamination of the platinum coil and also deterioration of the insulation of the mica rack.

(iii.) *Test of Insulation Resistance.*—The presence of moisture in the thermometer, due to a leak or to exhaustion of the drying material, may occur. The resulting phenomena are very characteristic and easily recognised. If the bridge, with the thermometer in circuit and galvanometer circuit closed and a key in the battery circuit, is balanced by adjusting resistance with the battery key closed, then on opening the battery circuit there will be a large deflection of the galvanometer, which gradually diminishes, and on closing it again another large deflection in the opposite direction. The latter slowly diminishes the circuit is kept closed. This phenomenon is readily distinguished from that due to the use of an excessive measuring current, by the absence of the galvanometer deflection in the latter case when the battery circuit is opened.

The presence of moisture also reduces the insulation resistance between the coil and sheath. This insulation resistance is easily tested and should exceed 200 megohms.

§ (7) Resistance Bridges adapted for Thermometric Work.—The requirements of platinum thermometry differ in many respects from those of ordinary resistance comparison work.

Compensation for the resistance of the leads necessitates the use of a bridge with equal ratio arms. Another requirement is that it should be capable of measuring changes in resistance to a high order of accuracy. For example, a thermometer constructed with a resistance coil of 2·56 ohms at 0° will have an increase in resistance of about 1 ohm when heated to 100° C. Hence to measure temperatures to $\frac{1}{100}$° demands resistance measurements $\frac{1}{10,000}$th of an ohm.

In practice this presents little difficulty, since balance to the nearest 0·05 is obtained by the set of coils, and final balance obtained by means of a bridge wire or set of shunted coils.

In resistance thermometry we are only concerned with changes of resistance; the absolute value of the unit employed is of little consequence, provided the relative values of the coils are accurately known.

Methods of calibrating the bridge coils and wire are described later.

Types of Resistance Bridges—(i.) *Siemens' Three-lead Bridge.*—This bridge is primarily of historical interest as representing the first attempt to eliminate lead resistance in platinum thermometry work. *Fig.* 6 shows the connections.

The coils Q and S are the equal ratio arms. The thermometer coil P has three leads connected in the manner shown in diagram.

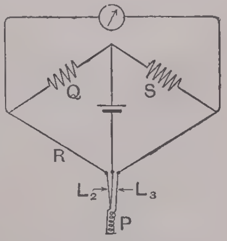

Fig. 6.—Siemens' Three-lead Bridge.

Q and S, ratio arms; P, thermometer coil; L_2 and L_3, equal leads.

The lead L_2 is adjusted in the construction of the thermometer equal to L_3. When R is adjusted equal to P the bridge is balanced and the lead resistance completely eliminated.

The defect of this form of bridge is that a ide wire cannot be used in connection with R to measure minute changes of resistance.

This difficulty can, however, be surmounted by the use of a set of shunt coils of the type described later in connection with shunted coil bridges. One setting would then determine the resistance and hence the temperature.

Siemens' procedure was to shunt one of the ratio arms and calibrate empirically.

(ii.) *The Callendar and Griffiths Bridge.*—The connections on this bridge are shown in *Fig.* 7. It was evolved for use with the compensating leads type of thermometer.

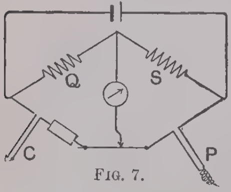

FIG. 7.

In the latest type of bridge manufactured by the Cambridge Scientific Instrument Co., mercury contacts of the form shown in *Fig.* 8 replace the usual plugs.

The ends of the coil are soldered to brass posts terminating in mercury cups. A ∩-shaped bar is carried by a light spring, which is depressed when the weighted plug D is inserted into the hole G.

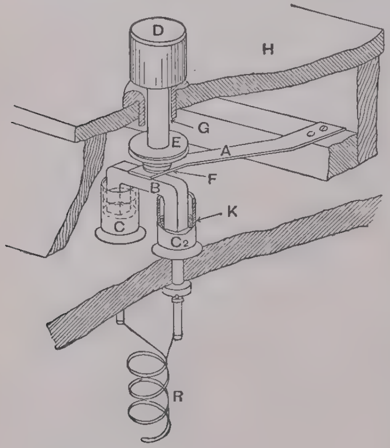

FIG. 8.

A, spring; B, ∩-shaped bar (copper link); C and C_2, mercury cups; D, weighted plug; E, spring-plate; F, helical spring; G, opening; H, bridge top; K, mercury; R, coil.

In modern resistance bridges the coils are of manganin and immersed in well-stirred oil—a good grade of paraffin oil is quite satisfactory for this purpose.

Manganin has an extremely small temperature coefficient of resistance, and also a very small thermal E.M.F. against copper, in this respect affording a marked contrast to constantan or eureka, which has an E.M.F. of about 40 microvolts per degree against copper.

Manganin is, however, subject to gradual changes of resistance and consequently the coils require calibration from time to time.

Recent experiments by Rosa and by Smith have shown that the shellac varnish coating of resistance coils absorbs moisture and in the accompanying change of volume strains are set up in the wire. To eliminate this effect of humidity, present-day standard coils are hermetically sealed, using moisture-free paraffin. This mode of construction might well be applied to the coils of resistance bridges.

The bridge wire is usually of manganin, and both it and the contact maker are immersed in the oil.

For industrial use the sensitive type of resistance thermometer, bridge, and galvanometer are out of the question, and several modifications have been evolved with a view to obtaining robustness combined with ample, if moderate, sensitivity.

In the Whipple indicator the resistance box is replaced by a long bridge wire wound spirally on a drum. A sensitive pivoted galvanometer is fixed in the top of the case and balance obtained by rotating the drum. The instrument has a scale graduated directly in ° C., and readings may easily be taken to about ⅓° in the range 0° to 1100° C.

(iii.) *Calorimetric Bridge.*—Messrs. Dickinson and Mueller have arranged the equal arm bridge[1] in a convenient form for calorimetric work. The wiring diagram is shown in *Fig.* 9; it will be observed that, while the balance of such a bridge is adjustable at three points, the contact resistance at these three points are so placed as to have a minimum effect upon the accuracy of the bridge.

The slide wire contact is in series with the battery. The contacts of the rheostats R and R_1 are each in series with a ratio coil of 200 ohms or over, where, even though considerable contact resistance were present, the percentage effect would be small. This arrangement of the Wheatstone Bridge also has the advantage of maintaining an almost constant resistance at the terminals of the galvanometer for all bridge settings, thus maintaining both constant damping conditions and deflections proportional to the want of balance for all bridge settings.

The rheostat R provides for the adjustment of the bridge by coarse steps, and the rheostat R_1 by fine steps, while the total continuous slide wire is properly proportioned to be equivalent in its total to one step on R_1 with suitable overlap. The slide wire consists of eleven turns of manganin wire, wound spirally on a marble cylinder, providing in effect a continuous scale 240 inches long. Ten turns of the slide wire are equivalent to one step of the rheostat R_1. One half of the additional turn is located at the high end and the other at the low end of the scale, thus providing the overlap for steps on R_1. One turn of the slide wire is equivalent to ·01 ohm or approximately ·1° C. in a thermometer whose R_0 is approximately 25·3 ohms. As there are 200 divisions in each turn, $\frac{1}{5}$ of one division (equal to a distance of $\frac{1}{50}$ inch) is approximately equivalent to ·0001° C. The rheostat R_1 is composed of ten ·1 ohm coils and hence covers a range of approximately 10° C. by steps of 1° C. The rheostat R is composed of ten 1 ohm coils, thus covering a range of 100° C. An additional ·5 ohm resistance, whose principal function is described below, extends the range an additional 5° C., thus making the over-all range 115° C. The coils of the rheostat R_1 must be adjusted to an accuracy of ·00001 ohm, this being equivalent to ·0001° C. in the resistance thermometer, hence the plug of this decade may be shifted during a test. On the other hand, the coils of the rheostat R are each 1 ohm. To adjust these to an accuracy of ·00001 ohm would be useless, since the manganin will not remain constant to $\frac{1}{1000}$ per cent, which would be the accuracy of such an adjustment. Hence, the plug of the rheostat R should not be distributed during any one test. To guard against the necessity of disturbing R, a ·5 ohm resistance, which may be included in the circuit at will, is connected in series with the rheostat R_1, between the posts T_0 and T_5. If the initial temperature of a test requires a setting in the upper half of the rheostat R_1 with the thermometer lead connected, a rise of temperature during the test might readily require the plug R to be moved in order to maintain balance. To avoid this, the thermometer lead may be connected at T_5 ohm at the beginning of the test so that the balance point will be in the lower half of the rheostat R_1, and a subsequent rise of 5° C. may be measured without touching R.

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[1] Manufactured by the Leeds and Northrup Company.

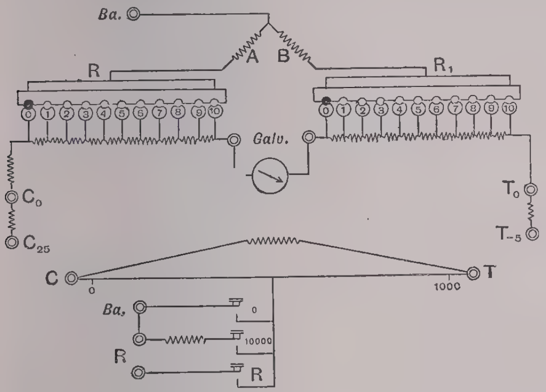

Fig. 9.

(iv.) *Potentiometer Method of measuring Resistance.*—The well-known potentiometer method for determining resistances, by comparison with a standard, has been applied to resistance thermometry.

The thermometer coil has four leads: two current and two potential; the current from a steady battery is passed through a standard oil immersed resistance coil and the thermometer coil in series. By measuring the fall of potential across the two coils separately, the resistances may be calculated in the usual manner.

(v.) *Smith's Difference Bridge.*—In this form of bridge the connections are so arranged that by two observations and a reversal of connections the resistance of the leads is eliminated without requiring absolute equality of lead resistance. *Fig.* 10 shows the disposition of the bridge connections for the first balance position. P is the thermometer coil with current loads L_1 and L_4 and potential leads L_2 and L_3.

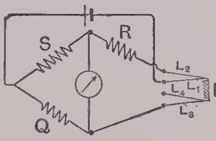

Fig. 10.—Smith's Difference Bridge.

P, thermometer coil; L_1, current lead; L_2, potential lead; L_3, potential lead; L_4, current lead: R, adjustable arm of bridge; S and Q, equal ratio arms; •, mercury cup contacts.

Q and S are equal or nearly equal ratio arms, and R is the adjustable arm of the bridge.

When the balance is obtained, then

$$P + L_3 = \frac{Q}{S}(R + L_2).$$

The connections are then transposed by a mercury switch, so that the potential lead L_2 is disconnected from R and joined to S; L_3 to R; the battery lead from L_1 to L_4; and P and R are interchanged.

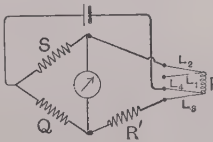

Fig. 11.

Fig. 11 represents the connections.

On rebalancing

$$P+L_2=\frac{S}{Q}(R'+L_3).$$

Now Q has been adjusted during the construction of the bridge to be very nearly equal to S, so that we can put

$$\frac{Q}{S}=(1+a),$$

where a is a small quantity. Then

$$P=\frac{R+R'}{2}+\frac{a}{2}\{R-R'+L_2-L_3+a(R'+L_3)\}.$$

If Q=S within 2 parts in 10,000, then $a=0{\cdot}0002$. So that P is equal to $\frac{1}{2}(R+R')$ within 2 parts in 100 millions, assuming $L_2=L_3$. If L_2 and L_3 each have a resistance of 0·1 ohm, but differ by 10 per cent, then the error introduced by neglecting a and taking the equation $P=\frac{1}{2}(R+R')$ as exact is equivalent to about 0·0001° C. on a thermometer with F.I. of 1 ohm.

The reversals which have been indicated above are conveniently made by means of a six-pole mercury contact-switch with connections as shown in *Fig.* 12. Thick copper links, *a*, *b*, and *c*, are attached to an ebonite disc movable on a vertical axis. The change over is effected by lifting clear of the mercury cups, rotating through 90° and then lowering. The second position of the links is shown dotted in *Fig.* 12. It is, of course, easy to arrange for the change in position of the battery lead to be made simultaneously by contacts attached to the same disc.

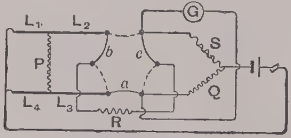

FIG. 12.—Connections to Six-pole Mercury Contact Switch.

L_1, L_2, L_3, L_4 leads to the thermometer coil; Q and S, ratio arms; R, variable arm of bridge; P, thermometer coil; *a*, *b*, *c*, heavy copper links dipping into mercury cups.

This bridge method possesses one unique advantage, insomuch that it eliminates lead resistance without assuming absolute equality between the leads. On the other hand, it suffers under the disadvantage of requiring two settings for each resistance reading, and that six contacts have to be broken and made; the contact resistances being assumed unchanged.

If the leads are made very nearly equal, R and R′ will only differ by an extremely small amount. This increases both the speed of working and the accuracy, since the above equation assumes reasonable identity in L_2 and L_3.

Hence the bridge is primarily of value when working at steady temperatures.

It will be observed that a bridge wire cannot be employed. Small changes of resistance can, however, be obtained by means of shunted coils. The principle of this method is illustrated by *Fig.* 13.

The ten coils CD each of 0·1 ohm are in series; any number of the coils EF can be placed as a shunt across the corresponding number of coils in CD by moving the bar AB.

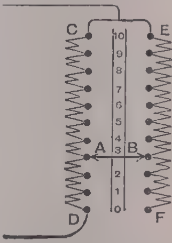

FIG. 13.—Diagram of Shunted Coils (Smith System).

AB, movable cross-bar; CD, coils of 0·1 ohm each: EF, shunt coils (9·9 ohms each).

Consider the bar in the position marked 0. We have then ten 0·1 ohm coils shunted by ten coils of 9·9 ohms each.

So the effective resistance is

$$\frac{1}{X}=\frac{1}{1}+\frac{1}{99}.$$

Hence

$$X=0{\cdot}990.$$

Suppose now the bar AB is moved to position marked 1.

We have now one coil of 0·1 ohm in series with nine coils of 0·1 ohm shunted by nine coils of 9·9 ohms.

Hence, if Y is the effective resistance of the hunted portion,

$$\frac{1}{Y}=\frac{1}{0{\cdot}9}+\frac{1}{89{\cdot}1}.$$
$$Y=0{\cdot}891.$$

So that the total resistance is given by $0{\cdot}1+0{\cdot}891=0{\cdot}991$. Hence moving the bar one step has increased the resistance by 0·001 ohm.

Similarly it can be shown that each step has a corresponding effect, so that the arrangement is capable of giving a total increase of 0·01 ohm distributed over ten steps.

It is obvious that the studs could be distributed on a circle, so the movement of AB would be one of rotation.

Fig. 14 illustrates the R arm of a bridge constructed on this principle. Coils of 0·01 ohm and upwards (apart from the dials) have mercury contacts bridged by ∩-shaped pieces of copper.

The three dials are employed to produce the small changes of resistance.

The contact-brushes enable one or more of the lower resistance coils to be shunted by the higher resistance ones, the shunting producing a diminution of the total resistance.[1]

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[1] As will be explained later, in constant-current bridges the arm R is decreased in resistance with increase in resistance of the platinum thermometer.

This device of shunted coils does not involve any great accuracy in the adjustment of the coil values.

For instance, consider the 0·001 ohm dial. Nominally the coils composing this dial are exactly 0·1 ohm and 9·9 ohms. If, however, the coils are badly adjusted so that, instead of a resistance coil being 9·9 ohms, it is 9·95 ohms, the change when this coil is used as a shunt resistance would be five millionths of an ohm less than 0·001.

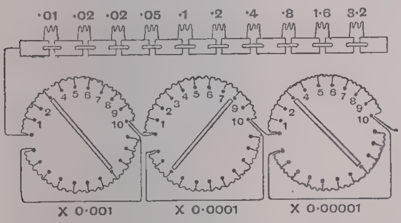

FIG. 14.—Diagram of R Arm of Bridge.

The resistance coils in the three dials are as follows:

For the 0·001 dial ten resistances of 0·1 ohm each and ten of 9·9 ohms each.

For the 0·0001 dial ten resistances of 0·05 ohm each and ten of 24·95 ohms each.

For the 0·00001 dial ten resistances of 0·01 ohm each and ten of 9·99 ohms each.

§ (8) TEMPERATURE CONTROL.—While, in work of the highest precision, it is necessary to control the temperature of the bridge coils within narrow limits, a fair degree of accuracy may be obtained by applying temperature corrections to the coil values. With manganin coils of good quality measurements can be made to about 1 part in 25,000, if the coil temperatures are known within 1° or 2°, and the indications of a mercury thermometer with its bulb near the coils should give the coil temperatures within this limit.

For great accuracy the bridge must be mounted in an oil bath and thermostatic control employed. A convenient arrangement is to have the motor for circulating the oil mounted on the bridge top with its axis vertical, and coupled a screw propeller working in a vertical tube, which also contains a heating coil. The oil is circulated through the tube, along the bottom of the box under a false bottom, thence upward and past the coils, and through the tube again. A liquid-in-glass thermo-regulator is mounted on the lower side of the false bottom.

Resistance measurements on a copper coil mounted in the bridge similar to the sealed coils used for the 10-ohm and 1-ohm decades, and arranged so that its resistance could be measured with the bridge, have shown that in such coils the fluctuations in the temperature, as the regulator operated, are almost completely damped out.

§ (9) HEATING EFFECT OF THE CURRENT PASSING THROUGH THE THERMOMETER COIL.—In order that the thermometer bulb may be of small dimensions and the thermometric lag reduced to a minimum, the coil must be made of wire of about 6 mils in diameter, consequently the heating effect of the measuring current on the value of the resistance is quite appreciable. The bridge coils are sufficiently heavy and well cooled to make the effect on them negligible, hence the limiting value of the current is determined solely by considerations of the thermometer coil.

At any given temperature the increase in resistance is proportional to the square of the current.

The same expenditure of watts at different temperatures does not, however, produce the same heating effect, since the rate of cooling of a surface by convection and radiation is a function of its absolute temperature.

The precise laws governing the phenomenon have not yet been investigated, and it is only possible to approximate to a constant heating effect for all temperatures by keeping the *current* through the thermometer constant.

Callendar states: "The cooling effect of conduction and convection currents in air in the thermometer tube increases nearly in proportion to the absolute temperature. The effect of radiation also becomes important at high temperatures, and the cooling is then more rapid. If, therefore, the watts are kept constant, the heating effect will diminish as the temperature rises, and a small systematic error will be produced. Assuming that the rate of cooling increases as the absolute temperature θ, and that the watts are kept constant, the heating effect at any temperature θ is $273h/\theta$, where h is the heating effect in degrees of temperature at 0° C."

This train of reasoning led Callendar to conclude that a better rule is to keep the *current* through the thermometer the same at all temperatures, as in that case the heating effect also is nearly constant, if the current flows sufficiently long for the steady state to be attained.

The table below shows the heating effect of the measuring current on two thermometers, using a current of 0·1 amp.—a current ten times larger than that customary in precision work.

Temperature, ° C.	Increment of Temperature above Surroundings (Diameter of Wire ·15 mm.).
0	1·62°
100	1·68
444·5	1·85

Hence, for the usual value of current ·01 amp. the rise would be ·016°, ·017° at 0° and 100° respectively.

The above values are of course only strictly applicable to the particular thermometers investigated.

To eliminate the effect, the following rule has been proposed by Callendar: "Take away one-third the difference in reading when the battery consists of two secondary cells in series and in parallel from the reading when in parallel."

§ (10) Modification of a Bridge to obtain Constant Current through the Thermometer. — To satisfy the condition of constant current through the thermometer at all temperatures it is necessary either to vary the resistance in series with the battery according to a calculated table or arrange the bridge so that the resistance of each arm remains constant.

To effect this it is only necessary to make the value of the arm R a fixed value greater than the maximum ever attained by the thermometer coil, and then insert a variable resistance (plugs and dials) in series with the thermometer to form the other arm of the bridge. Under these conditions an increase in resistance of the thermometer coil is counterbalanced by a decrease in the variable resistance. In such a case the arm R may be composed of simply one coil of the required value.

The system of coils shown in *Fig.* 14 is arranged for this purpose.

§ (11) Determination of the Bridge Centre.—In thermometers of the compensated lead type it is necessary to determine the bridge centre from time to time, as this is the base point from which the resistance is measured. For this purpose the P_1, P_2 and C_1, C_2 ends of the leads should be short-circuited at the thermometer head. Any change with time in the resistance of the flexible leads can thus be detected. It is scarcely necessary to point out that the leads from the bridge to the thermometer should be approximately equal in resistance and the junctions well made.

§ (12) Elimination of Thermoelectric Effects. — One of the troubles of precision resistance measurements is the thermoelectric effect in the circuits, particularly under conditions where there are big temperature gradients in the thermometer head. The magnitude of the effect is readily seen by closing the galvanometer key with the battery circuit left open. The galvanometer spot under the circumstances will generally take up a new position, and the movement is a measure of the thermoelectric effects in the system.

It is the practice, therefore, to work with the galvanometer circuit always completed and observe the deflection when the battery circuit is made or reversed.

Reversal of the battery is preferable, since this procedure permits the heating effect of the current on the thermometer to become settled, and thus eliminate the initial drift when the battery is first made, owing to the heating effect of the current on the resistance.

To eliminate induction effect Professor E. H. Griffiths devised a thermoelectric key. In this key there is a series of spring tongues so arranged that the galvanometer circuit is always made. When the key is depressed, the galvanometer circuit is broken momentarily, the battery circuit completed, and then the galvanometer circuit remade. By this sequence the galvanometer circuit is open during the period the current is growing in the circuit, and consequently there is no inductive kick of the light spot. It is easy to arrange a battery reversal key on the same principle.

The various junctions and connections in the keys are a frequent source of thermal E.M.F., so it is advisable to thoroughly box in the entire key including the terminals; some observers have even found it desirable to immerse the key in oil with only the handle projecting.

With the non-inductive windings of the resistance coils now used the induction effect is usually negligibly small, so it is sufficient to have a plain battery reversal key with an "off" position.

§ (13) Calibration of Box Coils and Bridge Wire.—For platinum thermometry work the relative values only of the coils and bridge wire are of importance. The method of calibration is closely analogous to that employed for the standardisation of a set of weights.

Instead of a thermometer a variable rheostat is connected to the P_1, P_2 terminals of the bridge. This resistance must be capable of fine adjustment; a convenient type which can readily be constructed is shown in *Fig.* 15. It consists of four dials of 10 coils each, the coil values being 0·1 ohm, 1 ohm, 10 ohms, and 100 ohms respectively. A trough of mercury with a ∩-shaped piece of copper permits of fine adjustments.

The method of construction will be readily understood from the diagram. Each coil terminates in mercury cups, so that a movement of the bar cuts out any number. The coils of course need not be accurately known.

As an alternative, ordinary P.O. resistance boxes may be used, one box forming a shunt on the other. This method of successive shunts is, however, rather laborious.

(*a*) *Calibration of the Bridge Wire.* — To the terminals C, C of the bridge is connected a short length of resistance wire terminating in two massive pieces of copper, which are drilled to contain mercury cups. Then by the insertion of a ∩-shaped piece of copper the resistance wire can be short-circuited without interfering with its connections to the bridge.

The resistance of the wire should be about 1 unit of the bridge wire.

The variable rheostat is adjusted to bring the contact maker to one end of the bridge wire; to effect this it will generally be necessary to withdraw a plug from the box.

The bridge is balanced in the usual manner. The calibrator coil (connected to C, C) is then short-circuited and balance again obtained at a distance about 1 unit away on the bridge wire. The operation should be repeated a few times.

The rheostat is readjusted so that a new position is taken up on the bridge wire adjacent to the second balancing point and the operation repeated. By a series of such steps the entire length of the bridge wire may be covered.

If we suppose r to be the resistance of the calibrator coil and l_1, l_2 the lengths of wire corresponding to it at various points, then

$$\frac{1}{l_1} \propto r, \text{ etc.}$$

Hence. by plotting graphically the reciprocals of l_1, l_2, etc., as ordinates with the mean bridge wire reading as abscissae, a curve can be obtained representing the variation in resistance per unit length along the bridge wire.

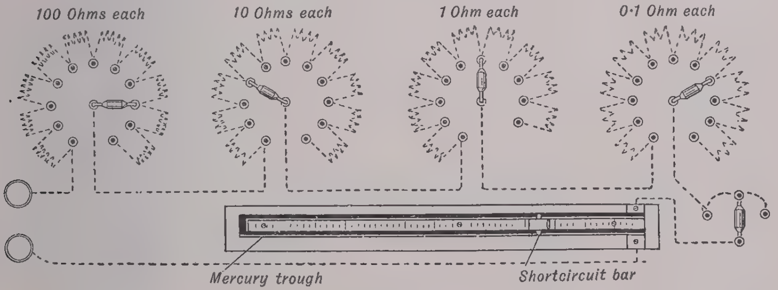

FIG. 15.

The values can be converted into those of the "mean box unit" referred to below, by obtaining the resistance of a length of the bridge wire in terms of one of the box coils by the usual substitution method.

(*b*) *Calibration of the Box Coils.*—For convenience it is assumed that the coils are arranged on the binary system, and that the nominal values are 5, 10, 20, 40, 80, etc.

Balance is obtained at any convenient place on the bridge wire by adjustment of the variable rheostat; plug 5 is then withdrawn and the change of bridge wire reading to restore balance observed. The rheostat is readjusted to bring the balance point back to approximately the same position as when plug 5 was in; plug 10 is then withdrawn, plug 5 inserted, and the change in bridge wire reading observed as before.

The same procedure is followed until the difference between the highest coil in the bridge and the sum of the series below is obtained in terms of a length of the bridge wire.

Let the successive differences in bridge wire readings be $x_1, x_2, \ldots x_9$.

Assuming coil 1280 to be the highest in the set. Then we obtain the following series of equations:

$$\text{Coil } 1280 - (\text{coils } 640 \text{ to } 5) = x_9$$
$$\text{Coil } 640 - (\text{coils } 320 \text{ to } 5) = x_8$$
$$\text{Coil } 320 - (\text{coils } 160 \text{ to } 5) = x_7, \text{ etc.}$$

By subtraction—

$$\text{Coil } 1280 - 2 \times \text{coil } 640 = x_9 - x_8$$
$$\text{Coil } 640 - 2 \times \text{coil } 320 = x_8 - x_7$$
$$\cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot$$

to $\quad \text{Coil } 10 - 2 \times \text{coil } 5 = x_2 - x_1.$

Now the values of $x_1, x_2, \ldots x_9$ in terms of coil 5 are already known from the previous operations in connection with the bridge wire calibration.

Hence the values of coils 640 to 10 in terms of coil 5 may be found.

Knowing the values of all the coils in terms of coil 5, it is then easy to express them all in terms of the mean coil, and hence in terms of the mean box unit, a corresponding correction being made in the integrations of the bridge wire.

It is preferable, however, to express the coils in terms of the international ohm, and this, of course, can be done by ascertaining the resistance of a standard coil, say 10 ohms, on the bridge.

By expressing the coils in terms of an absolute standard it is possible to keep note of the variations with time in the coils.

The same procedure is followed in the calibration of a bridge fitted with a set of shunted coils instead of a bridge wire. For the shunted coil dials the changes of the shunted coils are of far less importance, and there is little difficulty in adjusting them to the required degree of accuracy.

They possess the advantage of not being subjected to wear, as is the case with a bridge wire.

§ (14) STANDARDISATION OF A RESISTANCE THERMOMETER.—For temperatures up to 600° a platinum resistance thermometer is generally standardised at the temperature of melting ice (0° C.), of the vapour of water boiling under normal pressure (100° C.), and of the vapour of sulphur boiling under normal pressure (444·5°).

For the ice-point the thermometer should be well immersed in finely-crushed ice moistened with water. Unless the thermometer has been carefully sealed or provided with a drying tube to prevent access of moist air, prolonged exposure to the low temperature will cause electrical leakage owing to the deposition of moisture on the mica.

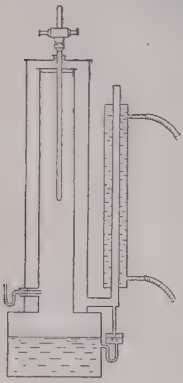

FIG. 16.

The steam-point is determined in a standard form of hypsometer such as that shown in *Fig.* 16. It is advisable to take precautions to prevent escape of steam rising around the head of the thermometer, as the thermal effects produced are apt to be troublesome. Alternate readings of the barometer and bridge should be taken, and correction made if necessary for the difference in level of the mercury cistern of the barometer and hypsometer, if the difference in level is considerable.

§ (15) THE SULPHUR BOILING-POINT.—Experiments with platinum resistance thermometers have shown that it is possible to measure the temperature of the sulphur boiling-point with a precision of a few 100ths of a degree without difficulty. Consequently it is advantageous to specify the conditions under which the sulphur boiling-point is taken, so that the point is reproducible to this degree of accuracy, even although its absolute value may not, at present, be known to a better than $\frac{1}{5}$ of a degree. During the past twenty-five years an immense amount of study has been given to the sulphur boiling-point apparatus so as to ascertain the magnitude and effect of any variations in conditions. The same precautions that have been found to be necessary in taking the sulphur boiling-point are applicable in a lesser degree to any other boiling-point determination, but the sulphur boiling-point, on account of the fact that it serves as the third fixed point for defining the temperature scale between −40° and +500° C., has received the most exhaustive study.

Callendar and Griffiths, in the course of their work on the development of the resistance thermometer method, found that the tube of a Meyer apparatus was well adapted for boiling the sulphur. See *Fig.* 17.

They also found it necessary to fit the thermometer with an asbestos or aluminium cone as shown in *Fig.* 17. This cone serves two purposes: (1) It prevents the condensed sulphur from running down over the bulb and cooling it below the temperature of the surrounding vapour; and (2) it eliminates direct radiation from the bulb to the colder walls of the large tube.

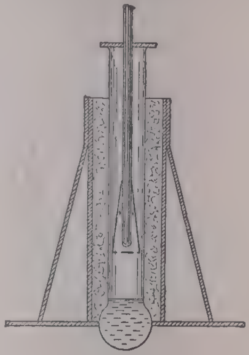

FIG. 17.

They made a careful investigation of these effects; the error due to the first cause was found to be about 0·28°, while that due to the second cause amounted to 0·49°; consequently an unprotected thermometer would read nearly a degree low in sulphur.

Recently a special study of the type of radiation shield to be employed around the thermometer has been made by Mueller and Burgess.

The various forms of shields investigated are shown in *Fig.* 18. It was noted that the iron shields, either with or without the lower disc, gave practically the same value for the temperature. It was, however, found that when a polished shield of the simple cone type was used the readings were ·2° low with a glass thermometer, and ·02° low with a porcelain enclosed thermometer. This effect was first observed by Meissner, and shown to be dependent upon the reflecting power of the interior of the shield. When the aluminium cylinder type was employed with the walls sharply corrugated to form a series of wedges, which is therefore a good radiator, it was found to be as effective as the other shields. Inadequate shielding is also usually accompanied by considerable variations in temperature, sometimes amounting to 0·1°, when the thermometer is displaced vertically, but the absence of such variation is not necessarily proof of adequate shielding. Nor does it prove that there is no superheating of the vapour, as in one instance constant temperatures were observed with a displacement of 4 cm. where, owing to insufficient depth of liquid sulphur in the tube, the vapour was superheated about 0·5°.

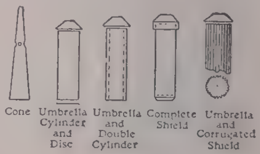
Cone
Umbrella Cylinder and Disc
Umbrella and Double Cylinder
Complete Shield
Umbrella and Corrugated Shield

FIG. 18.

From their investigation Mueller and Burgess came to the conclusion that a simple sheet-iron cylinder from 1½ to 2½ cm. larger in diameter than the thermometer tube and about 4 cm. or more longer than the coil, open below and with an umbrella above, was the most satisfactory form of shield, which is practically the same as that originally devised by Callendar and Griffiths. The umbrella should fit the thermometer tube closely and extend beyond the end of the cylinder, leaving a space 5 mm. to 1 cm. high between the umbrella and cylinder for circulation of the vapour.

§ (16) SPECIFICATIONS OF SULPHUR BOILING-POINT APPARATUS. — The following specifications proposed by Mueller and Burgess as the result of their experiments are primarily directed to be of assistance in obtaining standard practice in resistance thermometer calibration :

"1. *Boiling Apparatus.*—The boiling tube is of glass, fused silica, or similar material, and has an internal diameter of not less than 4 nor more than 6 cm. The length must be such that the length of the vapour column, measured from the surface of the liquid sulphur to the level of the top of the insulating material surrounding the tube, shall exceed the length of the thermometer coil by at least 20 cm.[1] Heating is by any suitable heater at the bottom of the tube, and the arrangement must be such that the heating element, and all conducting material in contact with it, terminate at least 4 cm. below the level of the liquid sulphur. If a flame is allowed to impinge directly on the tube the heat insulation must extend at least 4 cm. below the level of the liquid sulphur. There should be a ring of insulating material above the heater, fitting the tube closely, to prevent superheating of the vapour by convection currents outside the tube. Above the heater the tube is surrounded with insulating material, not necessarily in contact with it, and of such character as to provide heat insulation equivalent to a thickness of not less than 1 cm. of asbestos. The length of this insulated part has already been specified. Any device used to close the top of the boiling tube must allow a free opening for equilisation of pressure.

"2. *Purity of Sulphur.*—The sulphur should contain not over 0·02 per cent of impurities. It should be tested to determine whether selenium is present.

"3. *Radiation Shield.*—The radiation shield consists of a cylinder open at both ends, and provided with a conical umbrella above. The cylindrical part is to be 1·5 to 2·5 cm. larger in diameter than the protecting tube of the thermometer, and at least 1 cm. smaller than the inside diameter of the boiling tube. The cylinder should extend 1·5 cm. or more beyond the coil at each end. The umbrella should fit the thermometer tube closely, should overhang the cylinder, and be separated from the latter by a space 0·5 to 1·0 cm. high. The inner surface of the cylinder must be a poor reflector, such as sheet-iron, blackened aluminium, asbestos, or a deeply corrugated surface.

"4. *Procedure.*—The sulphur is brought to boiling,[2] and the heating is so regulated that the condensation line is sharply defined and is 1 cm. or more above the level of the top of the insulating material. The thermometer, enclosed in its shield, is inserted into the vapour, taking care to have the thermometer coil properly located with respect to the shield, and the thermometer and shield centred in the boiling tube. After putting the thermometer into the vapour, time must be allowed for the line of condensation again to reach its proper level. Simultaneous readings of the temperature and barometric pressure are then made. In all cases care should be taken to prove that the temperature is not affected by displacing the thermometer 2 or 3 cm. up or down from its usual position.

"5. *Computations.*—Temperatures[3] are calculated from the pressure by use of the formula

$$t = 444{\cdot}60^\circ + 0{\cdot}0910\,(p-760) - 0{\cdot}000049\,(p-760)^2.$$

If necessary, account should be taken of any difference in pressure between the levels at which the thermometer bulb and the open end of the barometer respectively are located. Pressures are to be expressed in the equivalent millimetres of mercury at 0° and under standard gravity ($g = 980{\cdot}665$)."

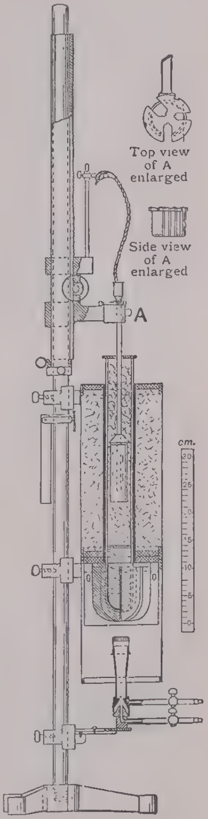

FIG. 19.

Example of Method of Reduction of Observations taken in the Standardisation of a Platinum Thermometer.—The resistance observations were corrected for the errors of the coil values, the temperature, and the value of the bridge centre deduced.

The resistance at the temperature of melting ice was 257·916 units.

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[1] This length was arrived at as follows: The minimum distance from the liquid surface to the bottom of the shield was taken as 6 cm.; excess length of shield over length of thermometer coil, 6 cm.; distance available for displacing thermometer, 6 cm.; minimum distance from top of shield to level of top of insulation, 2 cm.

[2] If the sulphur has been allowed to solidify in the bottom of the tube, it must be melted from the top downwards, to avoid breaking the tube. A better procedure is that recommended by Rothe, namely, on completing work with the apparatus, it is turned so that the tube makes an angle of 30° or less with the horizontal, so that the sulphur on solidifying extends along the sides of the tube, in which position it may be melted down with less danger of breaking the tube. Even when the procedure recommended is followed, breakage of tubes may be reduced by carefully melting the sulphur from the top downwards over a Bunsen burner before applying heat to it in the apparatus.

[3] International agreement has not yet been arrived at as to the value to be ascribed to the sulphur boiling-point.

In steam the resistance was 357·953.

The barometric height during the steam-point observations was 766·26, and the temperature of the mercury column and scale 17·55° C.

The barometric height has to be reduced to that corresponding to a column at 0° in latitude 45°.

Tables are available for this reduction in the case of standard types of barometers possessing a brass scale, such as the Fortin type.

The correction for temperature in this case is $-2{\cdot}19$ mm., while the latitude correction is approximately $+0{\cdot}45$ for places on the parallel through London. Hence the corrected value of the pressure is 764·52 at 0°, latitude 45°.

The boiling-point of water under a pressure of 764·52 mm. is 100·1654° C. according to Broch's recalculation of Regnault's observations.

Hence increase in resistance for a temperature change from 0° to 100·1654° C. = 100·037 units.

To obtain the value of the F.I. the resistance at 100° C. is required.

$$\delta\mathrm{R} \text{ for } 0{\cdot}1654 = \frac{100{\cdot}037}{100{\cdot}1654} \times 0{\cdot}1654 \times 0{\cdot}985 = 0{\cdot}163.$$

[The coefficient 0·985 is the value of $(\Delta pt)/\Delta(t)$ near 100° C. for a thermometer of $\delta = 1{\cdot}50$. The general formula is

$$\frac{(\Delta pt)}{\Delta(t)} = \left(1 - \delta\frac{2t-100}{10{,}000}\right)\Big].$$

Hence $$\mathrm{R}_{100} = 357{\cdot}790,$$

so that $$\mathrm{R}_{100} - \mathrm{R}_0 \equiv \mathrm{F.I.} = 99{\cdot}874.$$

The resistance in sulphur was 679·165, and the barometric height 766·23 at 17·5°. Correcting for temperature ($-2{\cdot}19$ mm.), and latitude $+0{\cdot}45°$, barometric height at 0° and latitude 45° = 764·49 mm. For this pressure the boiling-point of sulphur at this temperature is 444·94° C.

The platinum temperature (pt) corresponding to the above value of the resistance is

$$\frac{679{\cdot}165 - 257{\cdot}916}{99{\cdot}874} = 421{\cdot}78^\circ \text{ C.},$$

so that $$t - pt = 444{\cdot}94 - 421{\cdot}78 = 23{\cdot}16.$$

Now

$$\left[\left(\frac{t}{100}\right)^2 - \frac{t}{100}\right] = 15{\cdot}347 \text{ for } t = 444{\cdot}94.$$

Hence $$\delta = \frac{23{\cdot}16}{15{\cdot}347} = 1{\cdot}51.$$

§ (17) DETERMINATION OF LOW TEMPERATURES BY MEANS OF RESISTANCE THERMOMETERS.—Holborn and Wien in 1901 extended their previous comparison between the gas and platinum thermometers between 0° and 500° C., by making comparisons at $-78°$ C. and $-190°$ C., temperatures obtainable by the use of solid CO_2 and liquid air. The coil of the platinum thermometer was enclosed within the bulb of the gas thermometer, so as to minimise the error due to the slowly varying temperature. They found that the parabolic formula obtained by standardising in ice, steam, and sulphur vapour represented the gas scale between $-78°$ and 500° C., but below $-78°$ C., deviated; the divergence amounting to 2·3° at $-190°$ C., the platinum thermometer reading too low.

Travers and Gwyer, in 1905, made comparisons at the same two temperatures with improved apparatus of greater sensitivity. They found the departure of the platinum thermometer at $-190°$ C. to be 2·23°, a value in close agreement with that of Holborn and Wien.

Henning, in 1913, carried out a detailed investigation over the range 0° to $-200°$ C. The comparison was effected between the hydrogen gas thermometer and a number of platinum thermometers constructed of wire of varying degrees of purity. The lowest temperatures were obtained by means of liquid air baths, while the intermediate points were obtained in a bath of alcohol or petroleum ether cooled by liquid air.

His experiments showed that the parabolic formula was not valid below $-40°$ C., and, moreover, that platinum thermometers constructed of wires of varying purity were not consistent, but gave results differing by as much as half a degree when immersed in the same bath, if the observations were reduced by the parabolic formula obtained from the ice, steam, and sulphur points.

He found, however, that the scales of two different thermometers could be connected over this range by a formula involving only one constant, and that the constant could be determined by a comparison of the thermometers at a single temperature.

Thus, if platinum temperatures pt' and pt are deduced from observations with two thermometers compared at the same temperature, the following empirical relationship holds good:

$$pt' - pt = cpt(pt - 100).$$

The constant c may be obtained by a comparison of the two thermometers at one low temperature, as, for example, in a liquid air bath.

Although the above formula would appear to require for its evaluation the use of a standard platinum thermometer whose scale has been directly compared with the gas thermometer, it is possible to utilise Henning's data by making the assumption that his gas thermometer would give the accepted value $-182{\cdot}9_5^\circ$ C. for the boiling-point of oxygen.

The curve in *Fig.* 20 is plotted from data given by Henning for his standard thermometer, whose a was $a = 0{\cdot}0039150$ and $\delta = 1{\cdot}484$.

Hence, if a platinum thermometer has to be calibrated down to low temperatures, it is only necessary to determine the resistance in ice, steam, sulphur vapour, and boiling oxygen.

The departure of the scale at the oxygen-point can then be compared with Henning's value at $t = -182 \cdot 9_5$° C.

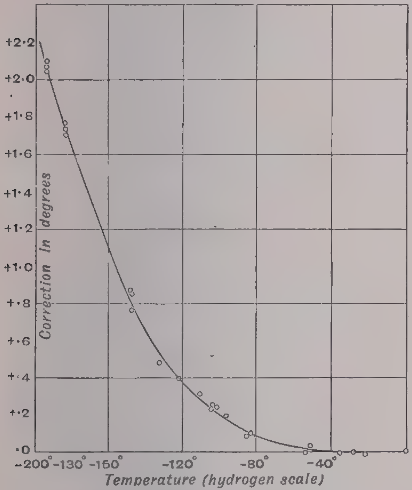

FIG. 20.—Curve plotted from Henning's Data for his Standard Thermometer.

In the table below the values of *a*, δ, and *c* are given for the various platinum thermometers tested, and it will be observed that there is no obvious connection between *c* and the δ of the parabolic formula.

100*a*.	δ.	$c \times 10^5$.
0·38624	1·510	−0·68
0·38874	1·492	+0·09
0·39131	1·491	+0·06
0·39132	1·491	+0·11
0·39150	1·484	0
0·39134	1·486	−0·10
0·39143	1·482	+0·10

§ (18) EXTRAPOLATION OF THE SCALE OF THE PLATINUM THERMOMETER FOR THE DETERMINATION OF HIGH-TEMPERATURE MELTING-POINTS.—The work of Heycock and Neville immediately followed that of their colleagues Callendar and Griffiths, and was primarily directed towards the determination of the freezing-points of metals and their alloys.

To measure temperatures in the vicinity of 1000° C., it was necessary to extrapolate the parabolic formula over a range of 400° C., since at that time there were no reliable determinations of the melting-points of metals in terms of the gas scale.

For example, gold, a metal obtainable in a state of high purity, had, according to Barus, a freezing-point value of 1093° C. This was determined by means of a platinum *vs.* platinum-iridium couple calibrated in terms of a gas thermometer. Holborn and Wien, at the same time (1892) and employing an almost identical method, obtained the value 1072° C., a discrepancy of 21°.

Heycock and Neville investigated the freezing-point of gold among those of other metals, and their work showed that it gave a sharp, well-defined transition point which renders it an excellent "fixed point" for calibration purposes.

Employing thermometers constructed of wire of various degrees of purity and reducing the observations by the parabolic formula, they obtained the values given below:

FREEZING-POINT OF GOLD. PURITY 99·95 PER CENT

Thermometer Number.	Platinum, Temperature ° C.	δ.	$d = t - pt$ ° C.	t ° C.
13	908·7	1·500	153·2	1061·9
15	852·9	2·040	208·3	1061·2
18	900·7	1·574	180·7	1061·4
13A	903·3	1·553	158·6	1061·9
14	907·7	1·511	154·3	1062·0
	Weighted mean . .			1061·7°

This value for the freezing-point of gold is in close agreement with the recent determination of Day and Sosman, 1062·4° C.

A comparison of the most reliable recent determinations of the freezing-points of the metals, expressed on the gas thermometer scale, with those obtained previously by Heycock and Neville proves conclusively that the resistance thermometer standardised at 0°, 100°, and 444·5° C. will give temperatures identical with the gas scale up to 1080° C., within the limits of experimental error to which the gas thermometer scale is known.

This is further confirmed by several direct comparisons between the resistance thermometer and the gas thermometer over the same range.

The application of resistance thermometers to the determination of high-temperature freeze-points is an operation which requires considerable care if results of the highest order of accuracy are desired.

The thermometer usually has a bulb of 3 to 4 cm. in length and enclosed in a heavy porcelain sheath. Consequently it is necessary to allow for a depth of immersion of from 6 to 8 cm. in the metal. Further, the rate of cooling should be slow, to diminish possible error due to lag. Heycock and Neville state that the freezing-point of gold could be read to $\frac{1}{100}$° without difficulty. In a study of the effect of high temperatures on the constants of platinum thermometers they found that the first few heatings to 1000° C. increased both the R_0 and the F.I. This effect they ascribed to the thickening of the mica plates after exposure to the high temperature and the consequent straining of the fine wire when the coil cools and the wire contracts on to the larger frame.

They concluded that the constants should be determined *before* each temperature measurement of importance, and, provided the leads had been well annealed originally, that these constants should be used in calculating the temperature, regardless of what the values of the constants might be after the experiments. A thermometer should of course be thoroughly annealed before standardisation.

§ (19) Kelvin Double Bridge Method.—In order to overcome the difficulties associated with the use of fine-wire thermometers at high temperatures Northrup has proposed the use of low-resistance, heavy-wire thermometers with the Kelvin double bridge for measuring the resistance.

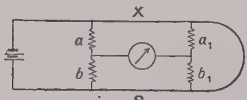

Fig. 21.

As is well known, the bridge connections, due to Lord Kelvin, shown in *Fig.* 21, are the best arrangement yet devised for measuring a very low resistance The bridge is balanced, when

$$\frac{a}{b}=\frac{a_1}{b_1}=\frac{X}{S}.$$

The first two terms being made equal by construction,

$$X=\frac{a}{b}S.$$

With these bridge connections it is stated that 0·01 ohm can be measured to the same precision as 100 ohms by the ordinary bridge arrangements. By taking advantage of this bridge as a reading device a high-temperature thermometer of robust design can be constructed.

The resistance coil is in the form of a small spiral supported by two mica washers. The current and potential leads are of a cheaper grade of platinum. In fact, it is a positive advantage to have the potential leads of an impure platinum, because of its low coefficient, which may be about 0·6 that of pure platinum. The connections, as arranged for measuring a number of thermometers, are shown in *Fig.* 22.

To measure a temperature with this arrangement, the terminals p, p' are moved by a switch to the potential terminals of the thermometers to be measured, while the thermometers to the right of the one being measured are cut out of circuit by y, which keeps the resistance of the "yoke" low, as required by theory. A balance on the galvanometer is obtained by moving the plug N and the slider S. The slide wire on which S moves would consist of a substantial manganin wire lying over a scale, marked off in degrees centigrade, if it is desired to make the bridge direct-reading. The only uncertain element in the method is the possibility of the ratio a/b and a_1/b_1, *Fig.* 21, becoming variable in an unknown way through a change in the resistance of that portion of the potential leads which lie in the thermometer tube. This uncertainty, however, is practically avoided if the resistance a is made sufficiently high. Calculation shows that, if a is chosen as high as 250 ohms, the maximum error from this cause will not exceed 0·1° C. The resistance a may, however, be as high as 1000 or even 5000 ohms, thus practically reducing the error to zero.

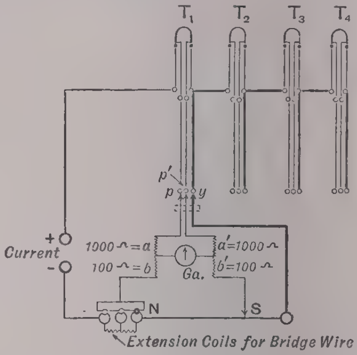

Fig. 22.

The necessity of having a high resistance in the ratio coils requires that the galvanometer used shall have a higher sensibility than can be obtained in a portable pointer instrument. There are, however, available several very convenient forms of semi-portable suspended-coil types of galvanometers, having an attached telescope and scale which are amply sensitive for the purpose.

§ (20) Recording Resistance Pyrometer.—In industrial work it is frequently necessary to have a continuous record of the temperature of a furnace or kiln during the course of a complete run. To meet these requirements Callendar devised his automatic resistance bridge which gave a graphical record of the position of the bridge-wire contact maker on a clockwork driven drum.

In the design of the instrument innumerable difficulties had to be overcome, particularly in connection with the relay action operated by the galvanometer pointer. It is evident that the motion of the galvanometer pointer must close or open an electric circuit, and the practical problem was to make this contact a reliable one with the light pressures that are available.

Fig. 23 is a general view of the recorder,

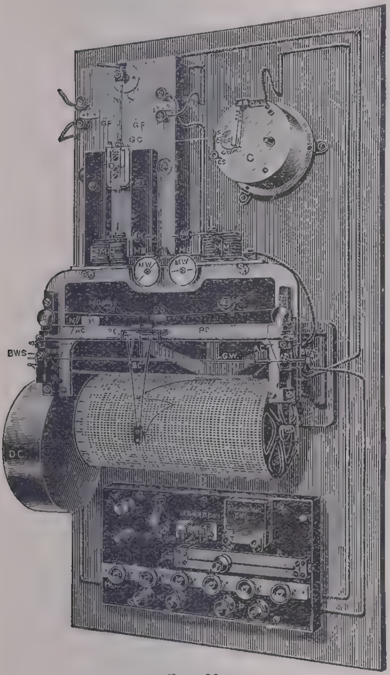

FIG. 23.

whilst *Fig.* 24 is a diagram of the connections. The movement of the slide-wire contact is effected by clockwork pulling a string, the operation of the clock being controlled by two relays.

According as the moving coil of this galvanometer GC deflects in one direction or the other, a relay circuit is completed through one or other of two electromagnets MM. Each of these magnets is mounted on a clock, the movement of which is prevented by a brake. When a current passes through the magnet this brake is lifted, allowing the clockwork to revolve. These clocks are connected by differential gearing with a recording pen carriage PC, which is hauled in one direction or the other, according as the brake is lifted from the corresponding block. The bridge slider moves with this pen and tends to restore balance. As soon as this is done the galvanometer coil returns to its normal position, the relay is cut out, the brake springs back, stopping the clock, and the recording pen P comes to rest, until the equilibrium of the circuits is again disturbed. The main difficulty in devising a satisfactory instrument on this general plan has been that of obtaining a delicate and reliable relay. The total current available for operating this is necessarily small, and in such cases the contacts are very liable to stick. This difficulty Callendar has overcome by mounting the contact on one of the arbors of a clock movement C. Metallic springs, CF, press on the contact surfaces, polishing them as they are rotated by the clock. With this arrangement the make-and-break is effected sharply and certainly, in spite of the very small force which is available for pressing the two contacts together. The contact piece consists of a ring of platinum CW, forming the tyre of a wheel mounted on one of the shafts of the clock. A spring fork connected electrically with one terminal of a voltaic cell, or secondary battery, grips this metallic tyre on either side, and polishes the contact surfaces as they move round. Contact is made by one or other of two pieces of stout platinum or gold foil fixed at the end of the long horizontal rod, which, as shown in *Fig.* 23, is carried by, and moves with, the coil of the D'Arsonval galvanometer GC. This rod carries with it two insulated-copper wires GF, which are connected at the contact-making end with one or other of the two platinum wires above mentioned. At the other end the wires connect with one or the other of the two magnets MM, controlling the clock brakes. These magnets are clearly shown in the

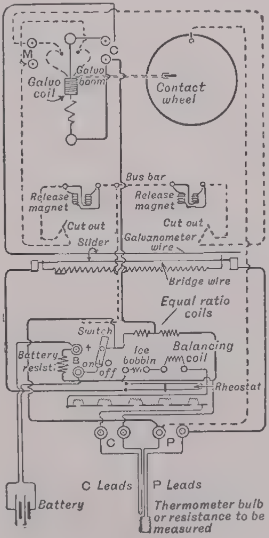

FIG. 24.

figures, mounted above the cases containing the clockwork.

The clockwork consists of two clocks connected with a simple differential gear, so that the screw pulley PS, which drives the silk cord PD connected to the pen slide, is turned in one direction or the other according to the deflection of the relay.

The carriage carrying both the recording pen and the Wheatstone bridge slider, is coupled at either end with a cord making two complete turns round the hauling spindle, as shown. A spring fastening at each end of the cord keeps the tension properly adjusted. Just below the guide-bar, on which this carriage moves, are the bridge and galvanometer wires over which the slider passes. The two lie in the same horizontal plane, and the slider consists of a platinum silver fork bridging the space between them. The front wire is connected at either end with the battery, whilst at the back is connected to the D'Arsonval galvanometer. The potential along this battery wire of course falls from end to end, and as the slider moves along the potential of the galvanometer wire is raised or lowered accordingly. A cut-off is arranged at either end of the travel of the pen carriage, which breaks the magnet circuits, and thus prevents the pen overrunning its cylinder. This latter consists of a light drum of very thin brass, to which squared paper can be fixed in the usual way. The spindle carrying this drum is connected by means of toothed gearing to a clock DC fixed to the frame of the instrument.

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- In open water. See *ibid.* § (42).
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- Design of. See "Structures, Strength of," §§ (29) and (30).
- Whirling of, considered by the method of dimensions. See "Dynamical Similarity, The Principles of," § (47).

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SHIP RESISTANCE AND PROPULSION

I. Ship Resistance

§ (1) Historical.—For all practical purposes it may be said that the existing knowledge of ship resistance has been acquired during the last century. It is true that about 3000 B.C. the Egyptians had some knowledge of this sort, as even then, their vessels were rounded in section, fined at the ends, and given considerable sheer, *i.e.* upward curvature of the deck and form at the ends. In England and Northern Europe the advantages of rounding off the fore end were known some 700 to 1100 B.C., as all the large "dug-outs" of this period, which have been unearthed, have this feature. At a later period (A.D. 300 to 1000), the Vikings and the English gave their vessels a form eminently suited for speed, indicating a good general knowledge of ship resistance. But such a knowledge as existed then, both in Northern waters and on the Mediterranean, was swamped and slowly lost in the effort to build heavy sailing-ships of large burthen. Centuries passed, and despite improvements in construction and in arrangements of sails, little improvement of form took place until about 1800 A.D.

§ (2) Early Experiments.—Amongst those who have worked at the problem of ship resistance must be included Newton, but his work has not stood the test of experiment.[1] He was followed by Euler, D'Alembert, the Marquis de Condorcet, the Abbé Bossut, and M. Romme (1778). Their experiments confirmed the belief that resistance in general varied with the area of the surface exposed, and with the square of the velocity of movement, but the general theory of ship resistance in other respects advanced but very little. The experiments of Bird in 1775, Gore in 1792, and those carried out in Scandinavia concurrently with Beaufoy's, contributed nothing of any importance on the subject. Up to about 1800 A.D. the accepted theory of ship resistance was based on the idea that the vessel had to overcome the inertia of the water thrust away in front and drawn in behind, and experienced a resistance due to the inertia of the disturbed water.

§ (3) Beaufoy.—The first successful effort to separate the resistance of a ship into component parts, and to use model experiments for this purpose, was made by Beaufoy. He clearly discriminated between skin friction and resistance due to "dynamic pressures," and he was obviously aware of the added resistance experienced by a body on the surface of the water compared with its resistance when submerged. He attempted to separate out the skin resistance of certain surfaces,[2] and showed that, assuming such resistance varied with the nth power of the speed, n varied from 1·71 to 1·82. But Beaufoy, like his predecessors, was handicapped by a want of knowledge of how to apply his experiment results to the estimation of resistance of the full-size ship, and it was left for the late William Froude to propound a method for doing this, and to demonstrate its accuracy. In all essentials the modern treatment of ship resistance is based upon the work of W. Froude. His theories and methods, which are summarised in the following paragraph, have been tested by comparing results obtained with a model and those obtained by towing a ship at sea, and by sixty years of steady application in ship design.

§ (4) W. Froude's Investigations.—Apart from the air-resistance of deck erections and of the hull above water, the resistance experienced by any ship may be regarded as consisting of that due to skin friction, eddy-making, and the formation of waves. From a number of experiments, W. Froude concluded that the frictional resistance of a ship and its model were practically the same as those of plane surfaces of the same respective lengths, areas, and smoothness, moving at the same velocities. To deal with the remaining portion of the resistance of ship and model, he proposed what is generally known as the "law of corresponding speeds." This states that the resistances of similar ships are in the ratio of the cube of their dimensions, when their speeds are in the ratio of the square root of their dimensions. Speeds connected by this relation are generally known as "corresponding speeds."[3] Or to express the law in symbols,[4] it can be shown that the resistance R can be expressed by the formula

$$R = \rho v^2 l^2 F\left(\frac{gl}{v^2}\right),$$

where ρ is the density of the fluid, v the speed, and l a linear quantity, defining the scale usually taken as the length of the ship. Thus if the resistance of the ship is to be accurately represented by that of the model, we must have

$$\frac{l}{v^2} = \frac{L}{V^2},$$

where lv refer to the model, LV to the ship. Then

$$F\left(\frac{gl}{v^2}\right) = F\left(\frac{gL}{V^2}\right),$$

while, since v^2 is proportional to l, R, which is proportional to l^2v^2, varies as l^3.

This law is true for resistances arising from dynamic conditions which are similar, irrespective of size, and is generally applied to that due to the formation of waves and eddies, which is assumed to constitute all that resistance experienced by ship and model over and above the friction resistance. To this resistance Froude gave the name "residuary." If, therefore, the total resistance of a model be ascertained by experiment, and its friction resistance calculated as above be deducted from it, the residuary resistance of the ship at the corresponding speed can be inferred from the result, and when added to the calculated friction resistance gives the total for the ship. This procedure has been tested by comparing estimates based on model experiments with the measured resistance of a similar full-size ship towed under the same conditions. The first tests of this kind were made by W. Froude on the *Greyhound* in

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[1] Beaufoy's *Hydraulic Experiments*, p. xxi.

[2] *Ibid.* Table II.

[3] See "Dynamical Similarity, Principles of," §§ (10), (39).

[4] See "Friction," § (24) (ii.).

1873,[1] and by Mr. Yarrow on a first-class torpedo boat in 1882.[2] In both cases the measured resistance exceeded that deduced from the model tests, but the agreement was sufficiently close to justify the use of this method of estimating powers for ships, and with some modifications in detail it is still in use.

§ (5).—Of these modifications the most important is in the method of calculating frictional resistance. Froude assumed: (i.) That for all lengths greater than 50 ft. the resistance could be obtained, for the first 50 ft. by using the constants obtained for a 50-ft. plank, and that for all the surface beyond this length the resistance per square foot would be the same as that of the last foot of the 50-ft. plank, this being determined by an analysis of his results; (ii.) that the resistance of a clean plated ship could be calculated from that of his smooth planks without any correction for roughness or form effect. These assumptions, although still in considerable use, are not now necessary, and are not made in the data and methods of calculation here given.

For the sake of clearness, each of the three main factors in resistance is treated separately. Although this is the usual practice it is not strictly correct, as all three are to a small and usually not important extent interdependent. These cross effects are dealt with under the separate headings.

The section on stream-lines is intended mainly as an introduction to the subject of ship wave formation, and its close ally, ship form design. The data given, however, serve to illustrate points raised in discussing frictional and eddy resistance.

II. Skin Friction Resistance

§ (6) Skin Friction.—This is one of the most prolific causes of resistance to the movement of ships through water. In all well-designed vessels it amounts to about 80 per cent of the whole resistance at low speeds, and even in a torpedo-boat destroyer at its maximum speed of about 40 knots, 40 per cent of the whole resistance is due to skin friction. In but one type of vessel, viz. the hydroplane (§ (35)), is it a comparatively unimportant factor, and then only at high speeds.

It may be defined as the tangential force between the surface of a moving body and the layer of water with which it is in contact. The nature of this tangential force is little known. Its effect in the case of a smooth surface is to set in motion a comparatively thin layer of water in the immediate neighbourhood of the body, and this layer is usually called *the frictional belt*. Terzaghi's experiments[3] with thin films between glass plates suggest that the innermost skin or film of the water in contact with the body has shearing and tensile strength as well as viscosity, and that the properties of this film are different from those of larger bodies of water. To what extent this may hold good for the boundary of a ship form is, however, not known. Hele Shaw's[4] experiments suggest that there is a thin film (much thicker than Terzaghi's film) in contact with the body in which the movement is purely straight line or viscous, and that sinuous movement exists in the remainder of the frictional belt. Stanton's experiments have confirmed this. The layer is approximately one-tenth millimetre in thickness, but gradually merges into the surrounding sinuous flow streams. The fluid in contact with the moving surface moves with the velocity of the surface at the point of contact.

§ (7).—Calvert's[5] experiments with long planks show that the forward motion of this belt at the after end of a plank increases with the length of the plank, and both Calvert and Ahlborn[6] have found that, for a perfectly plane surface, increase of velocity of the surface through the fluid did not materially increase the thickness of the frictional belt, but only increased the accelerations of the particles inside it, a result in general accord with the equations of motion, when viscosity is the only external force acting. Ahlborn also found that the thickness of the belt at the after end of his planks increased but very slowly with increase of length of surface, particularly when this was smooth.

§ (8) Resistance of Smooth Planks.—The amount of energy dissipated in this way, by smooth bodies moving in water, has been investigated by Beaufoy, Froude, Gebers,[7] and Baker,[8] etc. The results obtained for smooth planks, by these experimenters, are given in *Fig.* 1. The ordinates are values of $R/\rho V^2$, and are plotted to a base of VL/ν, where

R is the resistance in lbs. per square foot,
V is the speed in ft. per second,
L is the length of surface in feet,
ν is the kinematic viscosity of the water,
ρ is the density in lbs. per cubic foot,

for in this case we can show that

$$R=\rho V^2L^2F\left(\frac{VL}{\nu}\right).$$

Provided that one is dealing with bodies similar in form and nature of surface, the value of $R/\rho V^2$ at any value of VL/ν should be the same, whatever the dimensions of the bodies.[1] Although none of the planks whose results are given were made "similar" in all their dimensions, the figure shows that except at low VL/ν this difference had little effect, and the mean curve may be taken as defining the resistance of a long smooth plank in water. This curve passes well through the spots obtained by Froude and Baker. Geber's results lie a little below these. His experiments were made with the top of his planks out of water, and the difference in result may be due either to this or to the very great care taken by Gebers to obtain a perfectly smooth surface.

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[1] *Inst. Naval Architects Trans.* xv.

[2] *Ibid.* xxiv.

[3] *Phys. Rev.*, 1920.

[4] *Inst. Naval Architects Trans.* xl.

[5] *Ibid.* xxxiv.

[6] *Schiffbautechnischen Gesellschaft*, x.

[7] *Schiffbau*, ninth year, Nos. 12 and 13.

[8] *N.E.C. Inst. Eng. and Ship. Trans.* xxxii.

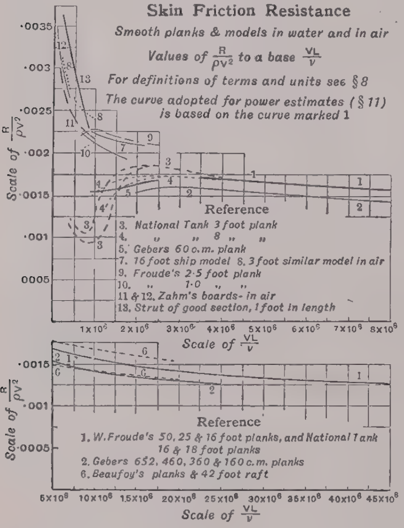

FIG. 1.

Beaufoy's experiments were made from 1786 to 1798, his object being to determine the friction of a surface following behind a tapered fore end, whose resistance was separately measured. This procedure, and other conditions of the experiments, lead to some small inaccuracies, and the difference between his results and the mean curve are attributed to these.

The results for planks of short length differ considerably, not only between the work of different experimenters, but between different planks tested by the same individual. Careful experiments made in the National Tank have shown that in this region of VL/ν a great deal depends upon the accurate setting of the experiment, and that with a perfectly uniform speed of advance a *much* lower resistance is obtained than is the case even with quite small irregularities in speed. It is believed that in this region, when the speed is steady, the flow is more nearly viscous in character, but is easily changed to sinuous with any irregularity. This probably also explains the large difference (allowing for density) between the friction of planks obtained in air and in water at the same VL/ν. In an air channel the fluid is everywhere in sinuous and disturbed motion, but with water there is no such motion to start with, and its development may sometimes be avoided; the frictional resistance will then be smaller.

§ (9) CHARACTER OF SURFACE.—Experiments made by W. Froude, Baker, and Taylor, with ship models varying from 10 to 20 ft. in length, show that no appreciable variation of resistance is experienced with surfaces having a coating of shellac varnish, red-lead paint, black-lead varnish, paraffin wax, and a number of ships' "compositions," and it can be concluded that the data for smooth planks hold good for any reasonably hard smooth surface. It has sometimes been suggested that skin friction might be reduced by blowing air into the water at the surface of the body. Very complete experiments made on a large paddle steamer showed that the presence of the air had very little effect on the resistance —and such effect as was detectable was not good. Experiments with planks lubricated with oil have been made in the Washington Tank. The presence of the oil increased the resistance by 5 per cent, but this diminished as the oil was washed off.

A ship's surface is made up of a number of strips of plates worked longitudinally so that the edges of the plates are exposed. The area of wetted surface is increased slightly by this, and in the case of a moderately fine form the resistance was also increased 3·7 per cent. If these longitudinal strakes of plating are made up of a number of short lengths of plate, the after end of one lapping over the fore end of the next, there is a considerable increase in resistance. This is the condition which exists in all ordinary ships, and experiments with models having similarly arranged surfaces, with the plating thickness correct to scale, indicated a 10 per cent increase in resistance. The increase varied with the thickness of plates, the above figure being for a 400-ft. ship with strakes of $\frac{3}{4}$-inch plating 4 ft. wide, the plates being 20 to 24 ft. long. Other experiments with these ship models were made with the foremost eighth of the length perfectly smooth, the rest of the surface being coated with plates as above. These showed that 40 per cent of the whole *increase* in resistance caused by the plate edges and ends is due to these in the foremost eighth of the length. Experiments with a model having a "calico" surface support this result. The extra resistance due to 1 sq. ft. of calico, placed anywhere about amidships, on an otherwise smooth model of about 16 ft. in length, varied from one-half to one-third of that for the same surface placed within 2 ft. of the bow.

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[1] "Dynamical Similarity, Principles of," § (14).

The effect of a uniform roughening of the surface is given in the accompanying table, derived from W. Froude's experiments with planks. There is a marked falling off in effect of a given state of roughness as length is increased, that is, as the dimensions of the surface increased relative to the size of the particles forming the roughness. In applying such results to a ship it is essential that the roughness of the model shall be "similar" to that of the ship. Thus a growth of weed about 3 in. in length on the ship would be represented on an average model by a very rough calico surface, and not by weed of the same length.

TABLE I

RELATIVE RESISTANCE OF UNIFORMLY ROUGH AND SMOOTH SURFACE

Nature of Surface.	Length of Surface in Feet.			
	2.	8.	20.	50.
	$\frac{\text{Resistance per Sq. Ft. Rough}}{\text{Resistance per Sq. Ft. Smooth}}$.			
Calico	2·2	1·95	1·93	1·89
Fine sand . . .	2·0	1·87	1·95	1·62
Medium sand . .	2·25	1·95	1·93	1·95
Coarse sand . .	2·75	2·24	2·14	. .

§ (10) SOLID BODIES.—There are no direct experimental data on the friction of solid bodies in water, and very little guidance is to be obtained from theory. For very low velocities—*i.e.* with viscous flow—Lee[1] has shown that the diameter of the circular section of equal resistance to that of any elliptical section is equal to the sum of the semi-axes of the elliptical section, and for a thin plank will be one-half the width of the plank. There is no authority, however, for assuming the same rule to apply in ordinary turbulent flow.

If the velocities in the theoretical stream-lines for a non-viscous fluid around any solid body are obtained by calculation it will be found that the mean value of the rubbing velocity of the streams, taken over the whole form, generally exceeds the velocity of the form itself, the excess varying with the fulness of the body. For flow in two dimensions only this excess varies from 11 per cent for a full form to 6 per cent for a fine one. For a three-dimension form of prismatic coefficient ·53 the mean velocity of the streams in contact with the form was 1·03 times the velocity of the form. Such an increase in velocity must carry with it a corresponding increase in resistance. Experiments in a tank with many ship models at low speed when the wave-making was negligible showed that the resistance exceeded that calculated for a plank of the same wetted area and length. The excess varied from 4 per cent for a long fine form such as a torpedo-boat destroyer to 10 per cent for a battleship or liner form, and 10 to 14 per cent for a full type of cargo vessel.

§ (11) POWER ABSORBED BY FRICTION RESISTANCE.—This can be estimated from the data already given or from the following formula:

$$\text{Effective horse-power} = \cdot 000024 SV^{2\cdot 86},$$

where S is the wetted area in square feet, V is the ship speed in knots.

This contains no allowance for either the roughness due to the plate edges and ends, which amounts to about 6 per cent on an average, or for the effect of form as given above. An addition, amounting to 10 per cent in long fine vessels and 16 per cent in short full-cargo vessels, is required to allow for these effects. The index is based on a logarithmic plotting of the data of *Fig.* 1, and represents the experiment results for planks at all high speeds.

§ (12) DIMENSIONS AND WETTED SURFACE.—The smaller the wetted surface of any ship can be kept, consistent with non-wave-making, the better will be the result. Calculations have shown that with fixed length the surface is remarkably non-sensitive to such factors as fulness of form, but depends mainly upon the principal cross-dimensions. The wetted surface per ton of displacement will decrease with either increased draft, the beam being fixed, or with increased beam, the draft being fixed, and the best ratio of beam to draft is that which gives the same reduction of skin per ton displacement for either beam or draft variation. If a simplified form of ship having all horizontal sections of the shape shown in *Fig.* 2 be considered, the best ratio of beam (2*b*) to draft (*a*) is given by

$$\frac{2b}{a} = 2\frac{2c^2 + l\sqrt{b^2 + c^2}}{(l + c)\sqrt{b^2 + c^2}}.$$

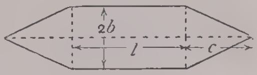

FIG. 2.

For a ship form with rounded sections the ratio is about 6 per cent less than the formula indicates, and is about 2·8 for a form having parallel body for about 30 per cent of its length amidships, and somewhat less than this for forms with 50 per cent of parallel body.

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[1] *Inst. Naval Architects Trans.* lviii.

When the displacement and length are kept constant, if the ratio of beam to draft is near that given above it may be changed to a quite fair extent without any noticeable effect upon the wetted surface of the ship.

III. Eddy Resistance

§ (13) Eddy-making.—In the strict sense of the word eddy, all frictional resistance might be classed under this heading, as the frictional belt consists largely of eddies. But by long usage it has come to mean that resistance due to eddy formation or broken stream-line flow, produced by some cause apart from skin friction or only indirectly due to skin friction.

When water is made to flow along a pipe, from a large to a small diameter, no matter how rapid the change in diameter is effected, no material break up of the flow takes place, but if the flow be reversed it is found that, unless the increase in sectional area of the pipe is very gradual, the steady flow will be broken up, and there will be a tendency to form a central stream surrounded by an eddying annulus. This same phenomena is always liable to occur around any solid body such as a ship, at such parts where the flow may be divergent in character. For the maintenance of the stream-line flow around such forms, areas of high pressure are required at the two ends, with regions of variable but always low pressure between, and the particles of water as they meet the after body have to pass from low- to high-pressure areas, just as did the water in the expanding pipe; and if the rate of expansion of the stream-tubes necessary to keep contact with the form exceeds a certain amount they will not follow the form. Moreover, the water particles moving into the high-pressure area automatically render up what speed they may have, but if this is not sufficient to maintain the necessary pressure they tend to stop still. What will then happen is not very clear, but there will be a break-down in the general stream-line arrangements at this part. In a perfect fluid of infinite extent such break-downs will be possible when the total change of velocity head is equal to that corresponding to discharge into *vacuum*, and at a free water surface, where the atmospheric pressure must in any case be maintained, the requisite velocity change will be very much smaller. With a viscous fluid flowing past a ship the streams quite close to the form will have lost a good deal of their speed before reaching the after body, and the liability to form eddies will therefore be increased. When there is a free surface to the fluid—as is the case with all ships except submarines—the chance of eddy formation is still further increased as indicated above.

The pressures which would exist without any free surface are different from those actually existing, owing to the formation of waves. The energy dissipated in these waves is extracted from the stream-lines, and there is less aggregate energy in these close to the form at the after end than at the fore end. In addition to this it must be remembered that at the after end there is a considerable upward flow of the particles closing in around the stern, and this demand for greater potential energy is sometimes more than the particle can satisfy, and a break-down occurs.

Eddy-making of this kind, therefore, is to some extent dependent upon the bulkiness and abruptness of feature of the ship, and to a smaller extent upon the wave-making (which will vary with the speed) and the skin friction. The energy lost in the eddies thrown off will vary with the square of the whirling velocity, the size of the eddies formed, and the rate at which they are shed. There is no experimental information on the latter point, but provided that this factor can be assumed to be relatively the same in model and ship, and in so far as the break-down in flow producing the eddies is dependent on form and flow in general, the resistance due to it will vary in accordance with Froude's law of comparison. But since the frictional resistance is known not to vary as the square of the velocity, from what has been said it is evident that the accuracy of the eddy-resistance estimates, based upon the use of Froude's law, will be affected to the small extent to which any break-down in flow depends upon this factor.

§ (14) Eddy Resistance.—Experiments with ship models have shown:

(i.) When eddy-making is produced by abruptness or bluntness of form the consequent resistance varies with the square of the velocity within the limits of the experiments.

(ii.) With increase of speed the extent of the eddy formation increases slightly, *i.e.* the break up of the stream occurs at a position which moves forward slightly with high speed.

(iii.) To avoid eddy-making the after end of the form must be so designed that the stream-lines if they are to exist shall in no case be inclined at an angle in excess of 16° to 20° from the line of motion of the form. The lower figure should be used with high velocities (for example, for airships and torpedoes 16° is the maximum slant of a longitudinal section of the tail) and the upper figure for low velocities (as in cargo vessels).

(iv.) For ordinary mercantile ships of low speed, having reasonably good lines, the minimum length in feet of actual tapered after end, measured from the section at which the reducton of sectional area begins, to the stern post, is given by

$$L=4\cdot1\sqrt{\text{immersed midship section area in sq. ft.}}$$

Wave-making considerations may, of course, demand a greater length than is given by the above formula.

§ (15) Head Resistance.—A second form of eddy resistance developed by a ship, sometimes called "head resistance," is due to such features as condenser scoops, bracket arms, web supports to the propeller shafts, thick stems, and stern posts. This resistance, however, constitutes only a small portion of the total. Thick stems, stern posts, and condenser scoops present a flat face to the streams they meet. Their resistance is given by the formula

$$R = 1{\cdot}12V^2,$$

where R is resistance in lbs. per sq. ft. of area of projected surface, V is the velocity in ft. per sec.

If the propeller shafts are carried in a case supported by a deep web or "bossing" from the ship's side, the resistance of this bossing can be reduced to almost its skin friction, provided the plane in which the web is worked is along the natural line of flow of the streams in this part. If the propeller shaft is supported by large bracket arms near the propeller the resistance of these struts or arms is given by the following formulae for two types covering a rather bad and a normal section.

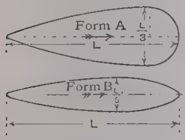

Fig. 3.

Strut arms of section A, *Fig.* 3,

$$\frac{R}{\rho V^2} = {\cdot}014 \text{ at and above } \frac{VL}{\nu} = 6 \times 10^5.$$

Strut arms of section B,

$$\frac{R}{\rho V^2} = {\cdot}004 \text{ at and above } \frac{VL}{\nu} = 8 \times 10^5.$$

R is the resistance in lbs. per sq. ft. of the product length L of section and length of arm or strut.

ρ is the density of the fluid $= 1{\cdot}99$ for salt water.

ν is the kinematic viscosity of the fluid, which for salt water varies from $1{\cdot}98 \times 10^{-5}$ at freezing-point to $1{\cdot}29 \times 10^{-5}$ at 15° C.

In both cases it is assumed that the plane of the strut arms is in the line of flow. At lower VL/ν values the $R/\rho V^2$ values steadily increase, and the rate of increase becomes abnormal below $VL/\nu = 1{\cdot}0 \times 10^5$.

The velocities to be used in these calculations are those of the streams passing the scoop, struts, etc., which, owing to the stream-line effect and the frictional belt, will vary from ·6 to ·5 the velocity of the ship. The higher figure holds for fine ships, the lower for full-bodied ocean tramps.

IV. Stream-lines of Ship Forms

§ (16) Stream-line Motion.—The forces and velocities produced in a fluid by the motion of the body through it are usually calculated on the assumption that the fluid is non-viscous, perfectly homogeneous, and incompressible. The motion of the particles in such cases is fully determined by the well-known equations of motion. The justification for applying the deductions from such calculations to the flow around ship forms lies in the fact that it is found experimentally that the main body of water does flow in much the same manner as the mathematical solutions would lead one to expect.

In ship design a knowledge is required of the relative velocities of ship and fluid particles at various positions along the ship, when it is moving at a uniform speed. The problem is more easily solved mathematically, and pictured mentally, by adopting the standpoint of an observer on the vessel, and considering it as fixed and the fluid moving past it. When at every point in the fluid the velocity and direction of flow remain constant in time, the motion is said to be "steady." A number of fluid particles, originally moving with uniform velocity in a straight line, will then continue to follow one another, but along a deflected path, as they approach and recede from the body, and will constitute what is called a "stream-tube." The extent and nature of these displacements can sometimes be determined if the shape of the body can be expressed by certain algebraic formulae.[1]

§ (17) Theory.—The motion, when expressed as an equation, is not always easily pictured, and a graphical interpretation of the result is generally more convincing. As is explained in the article on "Stream-line Motion," the motion of the particles can be represented by tubes through which they move; the cross-section being constricted where the particles flow faster and expanded where they are retarded, the actual velocity at any point varying inversely as the cross-section of the tube. The calculations are considerably simplified by assuming that the flow is "two dimensional" in character, *i.e.* takes place in one plane only.

For example, the flow around an oval of length $2\sqrt{(a^2+2ab)}$ and breadth equal to $2b$ is given by the equation

$$\psi = Uy - \frac{Ub}{2\cot^{-1}(b/a)} \tan^{-1}\frac{2ay}{x^2+y^2-a^2},$$

ψ being the stream function, and U the velocity in the uniform portion of the stream.

This equation is obtained by combining the motion of a single source and sink distant $2a$ apart with a uniform flow of velocity U in the direction from source to sink. The blunt end of the body for which the above determines the stream-lines bears little resemblance to ship lines, which are usually given a sharp angular termination.

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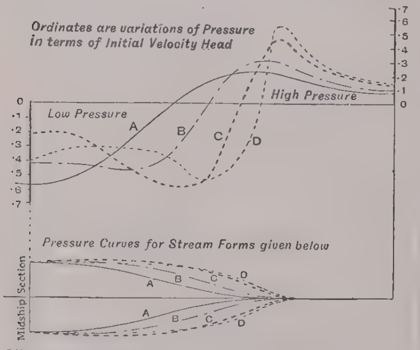
[1] See "Stream-line Motion."

§ (18).—Taylor[1] has developed a method of obtaining ship-shape stream forms and their stream-lines by combining an infinite number of sources and sinks of varying strengths lying on a straight line, with a uniform flow parallel to the line of the sources and sinks. The values of ψ due to the sources and sinks are calculated at a number of points for a series of abscissa values. For each abscissa value a curve is plotted so that its horizontal ordinate at any y value is the ψ value for that y and abscissa. Points on a stream-line are given by the equation $\psi = \mathrm{U}y + \text{constant}$. Such points can be picked out and the stream-line drawn by joining them. An infinite variety of forms can be obtained, by varying the strengths of the sources and sinks at different points, and the stream form will be a closed one, provided the total source and sink strengths are equal. The length of the stream form will be the length of the source-sink line; its breadth will depend upon the relative strengths of the sources and the uniform flow. A number of forms obtained by this method are given in *Fig.* 4, together with the distribution of pressure along them.



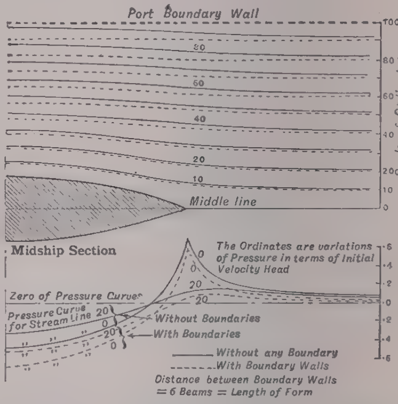
The stream forms are symmetrical about amidships.

FIG. 4.

This method has been extended[2] to determine the stream pressures around two-dimensional forms with pointed ends, symmetrically placed midway between boundaries parallel to the direction of motion. For this the stream function $\psi = \mathrm{A} \tan^{-1} y/x$, for a source or sink in an infinite fluid, is replaced by

$$\psi = \mathrm{A}' \tan^{-1} \frac{\tan (\pi x/2d)}{\tan (\pi y/2d)},$$

$2d$ being the distance between the boundaries. An infinite number of such sources and sinks in a straight line, of varying strength determined by the constant A′, are combined with a uniform flow. The procedure is much the same as for the unrestricted fluid. One example in which the source-sink strengths have been so adjusted as to give the same form as in open water is given in *Fig.* 5 to illustrate the effect of boundary walls (for effect upon resistance see § (36)).



Stream Lines and Pressure Curves for angular form with and without boundaries.

FIG. 5.

It is possible by the source-and-sink method to calculate the flow around solids of revolution, moving along their axes of revolution, and the case of such a solid in a concentric cylindrical boundary can also be treated in this way.

§ (19) EXPERIMENTAL METHODS.—Such investigations neglect many factors which may be important, especially in the case of the flow around ships, and for such purposes experimental means of determining the stream-lines have been developed. Taylor's method[3] consists of coating the surface of a wooden model with glue, which is painted over with a strong solution of sesqui-chloride of iron ($\mathrm{Fe_2Cl_6}$). The model is towed in fresh water at a speed corresponding to the normal speed of the ship it represents, and a strong solution of pyrogallic acid is ejected through small holes bored through the bottom. The acid is washed aft by the water, and coming into contact with the $\mathrm{Fe_2Cl_6}$ leaves a dark divergent mark or pencil on the surface of the model. Such lines show that, broadly speaking, in ships with cross-sections approximating to a rectangle amidships, the flow is divided into two parts. The first, near the surface, approximates to two-dimensional flow in an almost horizontal plane, and the second passes down in steep diagonal planes under the bottom at the fore end, and rises again along similar planes at the stern. The line of division of these two main flows is approximately along a diagonal plane at the bilge, and recent work at the National Tank has led to the conclusion that, where these two types of flow meet, the actual flow is frequently unstable, and there is probably some difference between the velocity of flow along the sides and bottom. This method only defines the flow close to the surface, and the stream-lines so obtained are to a certain extent affected by the frictional belt. They are, therefore, not absolutely reliable as a guide for the direction to be given bilge keels, shaft webs, and other necessary under-water projections from the ship's surface. For this purpose a number of small flags are used, each of depth equal to that of the proposed projection, each carefully balanced in water, and secured to a spindle passing through a gland in the surface of the model, at the point where the flow is to be determined. The inner end is fitted with a pointer moving over a graduated scale. The positions taken up by the pointers are recorded during each test, and the flag positions are afterwards transferred to the model surface. These flags have been used successfully on several occasions to determine the flow at some distance from the hull, as well as close up to the form.

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[1] *Inst. Naval Architects Trans.* xxxv.

[2] *Ibid.* lv.

[3] *American Soc. Naval Architects Trans.* xv.

§ (20) Pressure Distribution. — The general characteristics of the pressure curves for ship-shape stream forms are as follows: From an infinite distance in both directions the pressure slowly rises from zero until it reaches the neighbourhood of the ends of the form. It then rises rapidly to a maximum positive pressure, falls sharply to a maximum negative pressure, and the negative pressure persists along the centre portion of the form.

A close study of such pressure curves gives a clue to the particular features of the form which produce large pressure differences, and is a valuable aid to the ship designer in determining the shape of the water-lines near the surface. The value of the maximum pressure is greater the fuller the form, and occurs nearer the end when the slope of the form at that end is increased. Conversely, the finer the form the lower the maximum positive pressure, but the greater becomes the length of the form over which the positive pressure occurs. The governing feature producing the maximum negative pressure between the ends and the centre of the form is the curvature, or rate of change of shape from the entrance to the section of maximum beam. The quicker this curvature, the greater is the maximum negative pressure.

Rankine has shown that for two-dimensional flow round an oval whose length/breadth equals $\sqrt{3}$ (called a lissoneoid) the pressure changes are more gradual than for any other oval. If the ratio is less than $\sqrt{3}$ the pressure curve has its maximum negative pressure between each end and the centre of the oval, while if it is greater than $\sqrt{3}$ the greatest negative pressure is at the middle of the oval. This suggests that, ignoring wave-making, the best proportion of length of entrance/half-beam in a ship would be $\sqrt{3}$, but many experiments with ship models show that this ratio can be exceeded with good results.

The maximum pressures of the various forms of *Fig.* 4 occur at different positions along their length. If the ship designer can so arrange his form in the immediate vicinity of the surface in such a way that the maximum negative pressure of one water-line is partially over the positive pressure of a water-line some small distance below it, the net resultant pressure will be small, and the surface movement will also be small. This has been achieved in certain low-speed cargo boats. A full surface water-line is essential for commercial reasons, and can be associated with finer lower water-lines stepped back relative to the upper ones, so that, due to both their finer angle and the actual set-back, they tend to create their positive pressures in the same section as the fuller load water-line is trying to create negative pressures. (See § (31).)

§ (21).—A word of caution is necessary in drawing deductions from theoretical stream-lines, and applying them to practice without experimental confirmation. It must not be forgotten that in the theory a "perfect" fluid is assumed. With this there is ordinarily no possibility of a break-down in the flow, as explained in the section on eddy-making, and the effect of the free surface and its movement under any change of pressure is ignored. Also flow in three dimensions is at present calculable only on the assumption of the flow being symmetrical about an axis, or, in other words, by ignoring the effect of gravity upon the system as a whole.

V. Wave Resistance

§ (22) Waves.—The nature of the stream-line pressures set up around a moving form has already been described. If the fluid were a perfect one, and the body well below the surface, the total fore-and-aft force on the form would be zero. This would also be the case at the surface if the surface is supposed rigid, despite the fact that the presence of this boundary surface had caused considerable differences in the fluid pressures. In such a case the surface would be subjected to pressures varying in the systematic fashion already described, and if its hypothetical rigidity is abandoned it will form into humps and hollows under the influence of these pressures. This alteration in pattern of surface effects another modification in the stream pressures, and these will therefore vary to some extent with the waves formed. In the case of a ship travelling on the water, the total weight is constant, and, neglecting any vertical accelerations, the immersion of the ship when in motion must always automatically vary so that the vertical resultant pressure on it is equal to its weight, and this alteration affects the whole system to a small extent. The final pressure system created is therefore the balance achieved under these varying influences, consistent with the free surface being one of constant pressure, and the upward force on the hull remaining constant. Its general characteristics remain the same as when the form is submerged, but their relative magnitude and importance change with form and speed.

§ (23) THEORY.—The disturbance of the surface produced by a point of pressure travelling in a straight line over the surface of the water has been considered by Kelvin [1] and Havelock.[2] This consists of a system of transverse waves travelling with the originating point, associated with divergent waves all radiating from the point, the whole pattern being contained within two straight lines radiating from the point at an angle of 19·46° to the line of motion. The height of successive transverse wave crests at the middle line diminish in the inverse ratio of the square root of their distance from the point. Near the outer boundaries of the wave system, the crest of each transverse wave is bent back and joins a divergent wave in a cusp, and these three waves form a sort of triangle with curved sides, of which the cusps at the corners are the highest points. The heights of these cusps diminish with distance from the point at a slower rate than do the transverse waves, so that the divergent waves become relatively more marked towards the rear.

There are, however, very serious differences between such a point of pressure and the pressure system of a ship, in fact the whole science of ship form design has arisen largely from the study of these differences. Havelock [3] has traced the effect of a travelling pressure disturbance similar in character to those of *Fig.* 4, the work being confined to two dimensions, and transverse waves only being formed. If this pressure is independent of the speed, there is a certain speed above which the wave formation continually diminishes. If the pressure varies as some power of the speed less than two, the resistance tends to a limiting finite value as speed increases indefinitely, corresponding to diminished altitudes of waves at the higher speeds. The interference between the waves set up by the two ends, and other features, are similar to those described later, but the supposition of two-dimensional motion rules out any divergent waves.

§ (24) WAVE TRAINS.—The deformation of the water surface resulting from the motion of all ships, except hydroplanes or skimming boats, has always the same general characteristics. Most noticeable are two trains of waves of short length along each crest line, commencing with a heaped-up wave at the bow, and trailing away on each side along a diagonal line, in such a way that each wave is stepped behind its predecessor. These are called the bow divergent waves. Between the divergent waves on either side of the ship other waves are formed, having their crest lines near the ship at right angles to the direction of motion, but bent backwards slightly as they approach and coalesce with the divergent waves. These waves have a definite length between consecutive crest lines, given in feet by $2\pi V^2/g$, where V is the ship's speed in feet per second, and successive waves vary in height in the inverse ratio of the square root of their distance from some point in the bow. These waves are known as the bow transverse waves, and are most readily seen in profile along the side of full-ended ships, such as ocean cargo vessels having long lengths of parallel body, when forced above their natural or economical speed. The outer end of each transverse wave is associated with a divergent wave, and remains associated with it at all speeds, *i.e.* as the divergent wave moves out along the diagonal line, the transverse wave drops aft and increases in length of crest line. These two sets of waves together constitute the principal wave features of the bow, and are known as the bow wave system. The middle portion of a ship, throughout which the section remains the same, does not give rise to any additional waves, but at the after end another wave system, consisting of divergent and transverse waves, is formed. The transverse waves, in this case, commence with a trough, which tends to form where the sections of the ship begin to reduce in area, and is followed by a crest at the stern post. The divergent system commences at a varying distance abaft the first trough, but is hardly visible in a well-designed low-speed ship.

§ (25) CHARACTERISTICS OF DIVERGENT WAVES.—The first wave, at the bow of a low-speed ship, is generally called a "bow breaker" when it extends across the bows nearly at right angles to the middle line, with the outboard ends curved back, frequently foaming along the front slope of the crest.

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[1] *Math. and Phys. Papers*, ii. and iv.

[2] *Roy. Soc. Proc.* A, lxxxi.

[3] *Ibid.* lxxxix.

The rest of the waves following the bow breaker are different in character, their crests being concave outwards, all of them making a much smaller angle with the direction of motion. Each succeeding wave is longer and smaller in height than the one in front of it. The perpendicular distance between the crest lines of consecutive waves when clear of the ship, is given by $(2\pi V^2/g) \sin^2 \beta$, their velocity measured in the same way being $V \sin \beta$, where V is the ship speed in feet per second, and β the angle between the crest line and the direction of motion.

A straight line drawn from the bow through the points of maximum height of all the divergent waves on one side of the bow, makes an angle with the middle line approximately one-half that of the individual crest lines, and this relation holds good at all speeds. Increase in speed decreases all the crest-line angles, and in vessels of moderate speed the bow breaker becomes similar in character to the other echelon waves.

At low speeds all the particles forming these waves maintain their general position relative to each other, but with increase in speed the particles of the surface levels near the stem move out and in very rapidly, as well as up and down, as the divergent wave crest passes. At still higher speeds, only the outward and upward movement remains, and in such vessels as destroyers and hydroplanes, the surface levels at the crest of the primary bow divergent wave consist of water thrown out in a curling broken stream from the ship's bow, and the energy in this water is irrecoverably lost.

The angles between the crest lines and the middle line of the ship vary from 10° in a destroyer travelling at 35 knots, to 26° for a liner at normal speed, and 33° for a full-ended ocean cargo vessel at about 10 knots. Sharper angles to all the level lines of the fore end of the ship usually produce smaller angles of divergence, and diminish the height of the bow breaker and the amount of broken water along its forward edge.

The divergent waves at the stern in low-speed ships are hardly noticeable, but when the form has rapid change in curvature as the lines approach the stern, a set of divergent waves becomes apparent, commencing with its primary crests a little abaft the first hollow of the stern transverse system. The angles of the crest lines and other characteristics are practically the same as those of the bow system. In high-speed ships the primary crest of the divergent system starts from a point near the after end of the ship, and is noticeable at all speeds, but its importance diminishes with speed.

§ (26) CHARACTERISTICS OF TRANSVERSE WAVES.—Each wave of this system is of finite length measured along its crest. In contour these waves depend to some extent upon the ship's form, but they are usually much steeper on the forward than on the after slope. They are not so well marked as the divergent waves at ordinary speeds, but owing to the larger amount of water involved, the energy they absorb is considerable.

If the bow is sufficiently removed from the stern for the motion set up by it to be negligible there, then two complete wave systems are formed. As a rule this is not the case, and the waves seen at the stern are the result of the two systems coalescing. They naturally vary in height and position with the ship's speed, length, etc. But for all moderate speeds the positions of the bow and stern pressure systems accompanying the ship do not change materially as the speed varies, and the primary bow crest and primary stern hollow always tend to form at the same positions along the ship's length. Observation and analysis of many experiments have shown that the distance between a point half-way between the primary bow crest and hollow, and the trough of the primary stern wave, can be expressed in terms of the ship's length and fulness, and is given by $d = pL$, where

L is the length of the ship;

p is the prismatic coefficient, or the ratio of the immersed volume to the product of the length and largest section area;

so that d is the displacement divided by the largest section area.

The height of the resultant crests will depend upon the phase difference between the two systems. If kh_1 is the depth of the bow system wave hollow at the stern, and h_2 the stern system hollow, assuming the waves are trochoidal in contour, the depth of the resultant wave formed will be given by

$$h^2 = k^2 h_1^2 + h_2^2 + 2kh_1 h_2 \cos \frac{gD}{V^2}, \quad . \quad (1)$$

where $D = d - \frac{1}{4}(2\pi V^2)/g$, or is a measure of the phase difference of the two systems.

The fluctuations produced in the height of the following waves when the phase difference of the bow and stern systems is varied, can be seen from the wave profiles given in *Fig.* 6. These results were obtained with a series of ship models having identically the same bow and stern, between which varying lengths of perfectly parallel body were introduced. The dimensions of the form are given in the figure; the speed at which the measurements were taken was 340 ft. per minute. The positions of the primary bow and stern waves were unaffected by the amount of parallel body between them. The actual stern system is hidden in the model, but careful analysis of the different combinations shows that the primary stern waves are as dotted in the figure, and the combination of these with the actual bow system, gives the final set of waves at the after part of the ship.

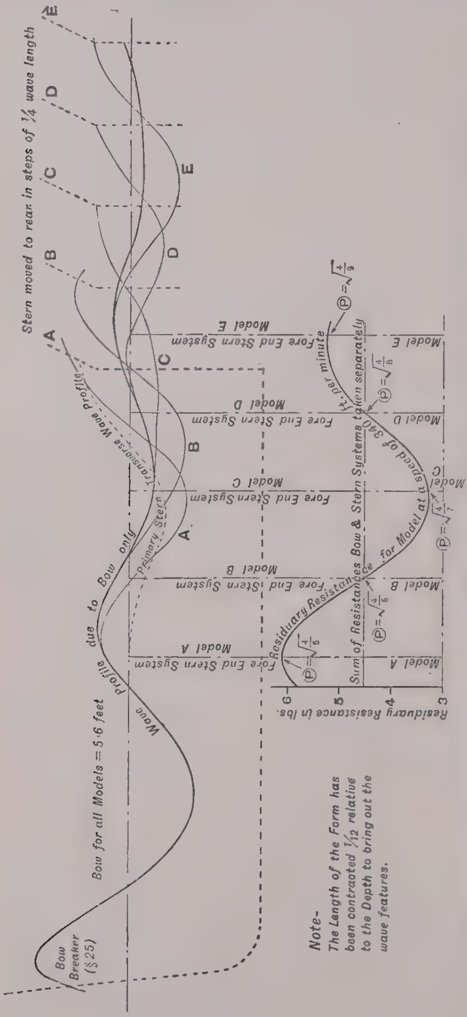
Stern moved to rear in steps of ¼ wave length
A
B
C
D
E
Bow for all Models = 5·6 feet
Bow Breaker (§ 25)
Wave Profile due to Bow only
Transverse Wave profile
Primary Stern
Model A
Model B
Model C
Model D
Model E
Fore End Stern System
Residuary Resistance in lbs.
6
5
4
3
Sum of Resistances Bow & Stern Systems taken separately
Residuary Resistance for Model at a speed of 340 ft. per minute
Ⓟ=√4/6
Ⓟ=√4/6
Ⓟ=√4/7
Ⓟ=√4/8
Ⓟ=√4/9
Note- The Length of the Form has been contracted 1/12 relative to the Depth to bring out the wave features.

FIG. 6.—Wave-profiles and Residuary Resistance for Ship Model, varying Phase between Bow and Stern Systems (§ (28)).
Beam of all Models = 2·09 feet; Draught = 0·93 feet; Speed of Experiments = 340 feet per minute.

§ (27) ENERGY ABSORBED IN WAVES.—Every wave created by any disturbance of the water surface involves the expenditure of an amount of energy, and if the wave contour is trochoidal, this is given in foot-pounds by $8Lh^2(1-\pi^2h^2/2L^2)$ per foot run of wave created, where L is its length and h its height in feet.

Approximately one half of this is absorbed in potential energy, the half height of the wave from crest to hollow being $\pi h^2/4L$ ft. above the still-water level, and the other half in kinetic energy involved in the rotary motion set up in the water. Amongst a group of uniform waves, each particle retains its energy of motion, but its potential energy depends upon its position. When a particle is to the *rear* of its mean position, it is always *acquiring* potential energy as it rises to the crest, and it gives this up when it is *forward* of its mean position as the back slope of the wave passes. In this way part of its energy is transmitted through the water, and the speed at which the whole group can travel without a supply of energy from external sources will depend upon the relation between the energy so transmitted and the whole energy in the waves. In trochoidal waves this is one-half, and a group of such waves if left to itself will advance at one-half the velocity of the individual waves. At each end of such a group there will be two or three waves of diminishing height, but otherwise the whole group will maintain its uniform height and character for considerable distances.

If a travelling disturbance is creating such a group of waves, the fore end of the group must move with the velocity of the disturbance, *i.e.* the velocity of the individual waves, but well clear of the disturbance the group will move as before with one-half this velocity, and the length of the group will increase at a rate equal to one-half the velocity of the disturbance. The energy required in unit time to enlarge the group in this way, divided by the velocity of the disturbance, is the wave resistance encountered by it. In a ship the waves formed are not necessarily trochoidal or uniform, but their general characteristics are the same, and the ship-wave resistance is governed by the same considerations as detailed above for a regular group, and the fluctuations in wave height of the final group at the rear of the ship should be reflected in the ship resistance.

Havelock has shown that a travelling pressure disturbance similar to those of *Fig.* 4

will create waves and experience resistance which will vary with

$$\frac{1}{a^4}\frac{A^2}{V^4}\epsilon^{-\frac{2ag}{V^2}}\sin^2\left(\frac{ag}{V^2}\right),$$

where a is nearly but not quite the longitudinal distance between the maximum and minimum values of the pressure, and A is a measure of the magnitude of the pressure. If A be taken to vary with the square of the velocity, as would be the case for a submerged body, the resistance consists of an exponential term and an oscillating factor. The combination of two such pressure disturbances, the rear one of less magnitude than the forward one, gives a number of terms containing the same exponential function, added to terms with oscillating factors in them. These oscillations reach a maximum when waves are formed of length corresponding to the distance between the two pressure humps, or a pressure hump and hollow at one end—particularly the bow.

§ (28).—The effect of the fluctuations in wave height upon the resistance can be seen from the lower part of *Fig.* 6. This shows the residuary resistance of the models at the same speed at which the waves in the upper part of the figure were measured. The wave contours marked A, B, C, etc., correspond to abscissae marked A, B, C, etc., in the lower part of the figure. When a trough of the bow system combines with a primary stern trough, it produces a large wave and a maximum resistance. The aim of the designer is therefore to choose such dimensions and form that the resultant waves formed at the usual or service speed are of minimum height, *i.e.* that D of equation (1), § (27), shall be equal to $2n+1/2$ (wave-length corresponding to the speed) or $(2n+1/2)(\pi V^2/g)$. This can be put in a somewhat different and practical form by writing

$$\textcircled{P}=\sqrt{\frac{\text{wave-length at speed V}}{\text{prismatic coefficient}\times\text{length}}}=\cdot 746\frac{V}{\sqrt{pL}},$$

where V is the speed in knots and L the ship length in feet.

For minimum wave resistance

$$\textcircled{P}=\frac{1}{\sqrt{n+\frac{3}{4}}}\doteq\sqrt{\tfrac{4}{3}},\quad\sqrt{\tfrac{4}{7}},\quad\sqrt{\tfrac{4}{11}},\text{ etc.,}$$

and for maximum wave resistance

$$\textcircled{P}=\frac{1}{\sqrt{n+\frac{1}{4}}}=\sqrt{\tfrac{4}{5}},\quad\sqrt{\tfrac{4}{9}},\quad\sqrt{\tfrac{4}{11}},\text{ etc.}$$

The highest speed for a maximum resistance is given by Ⓟ=1·5 to 1·6. This holds good for all destroyers, motor boats, etc., and any growth of wave resistance beyond this speed will be continuous. When the cosine term in equation (1), § (27), is zero, the resultant wave resistance is that due to the bow and stern added together, and is equal to that which would be obtained if the middle parallel portion was long enough to enable the waves of the bow system to spread, so as not to interfere with the stern system. The speed-length values at which this occurs are given by

$$\textcircled{P}=\sqrt{\tfrac{4}{4}},\quad\sqrt{\tfrac{4}{6}},\quad\sqrt{\tfrac{4}{8}},\text{ etc.}$$

§ (29).—A second form of interference amongst the waves formed by a ship is due to rapid change in sectional area of either end, producing a maximum negative pressure close to the maximum positive pressure of the end. When the speed is sufficient to form a wave with crest and hollow approximating to these maximums, the resistance will increase at a greater rate. If this form is associated with double curvature of the load water-line, two sets of divergent waves may be formed, one as usual near the stem and the second near the point of inflection of the water-line. The first set will have the smaller obliquity to the direction of motion because of the finer angle at the stem, and these may cross the second set, giving rise to an interference effect similar to that of the transverse waves. The usual result of such interference is to produce an increase above the normal resistance at some particular speed. It is difficult to give any accurate formula for the speed when these effects occur, but an increase in resistance at a speed given in knots by

$$V=1{\cdot}08\sqrt{\text{length of tapering fore part of ship}}$$

is usually found to be due to the form of the bow being unsatisfactory in this respect.

§ (30) Effect of Type and Form on Resistance.—To form a correct estimate of the effect of any form characteristic, it is best to reduce results to some standard form. Ship-resistance values are therefore usually plotted as Ⓒ ordinates to an abscissa of either Ⓟ, $V/\sqrt{L}$, or Ⓚ, where

Ⓚ is the ratio of the ship's speed to the speed of a wave whose length is one-half the side of a cube having contents equal to the ship's displacement,

or
$$\textcircled{K}=\cdot 5834\frac{V}{\Delta^{\frac{1}{6}}},$$

and

$$\textcircled{C}\text{ is }\frac{\text{resistance}}{\text{displacement}\times\textcircled{K}^2}=427{\cdot}1\frac{\text{horse-power}}{\Delta^{\frac{2}{3}}\times V^3};$$

Ⓟ is as defined in § (28),
V being speed in knots,
Δ displacement in tons,
L the length of the ship in feet.

Typical curves of Ⓒ are given in *Fig.* 7.

A constant value of the Ⓒ ordinate means that the resistance varies as the square of the speed, and the absence of any serious wave-making. If in such a case the Ⓒ value is high, this usually means that the virtual size of the ship is greater than the actual, owing to dead-water or eddy formation, either at the upper levels at the stern, or due to a too abrupt turn of the bilge lines.

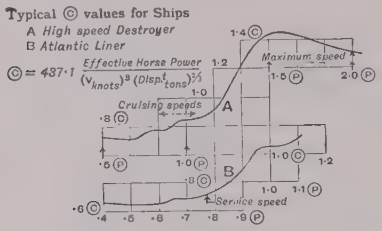

FIG. 7.

At low speed the skin friction should account for some 84 to 94 per cent of the whole resistance, the former figure holding good for cargo vessels and the latter for narrow and fine-lined vessels. Since frictional resistance varies with a power slightly less than the square of the speed, the ordinate of the Ⓒ curve should diminish slightly as the speed is increased, until wave-making makes itself felt. Divergent waves show themselves by a steady rise in the Ⓒ value as speed is increased, the rise commencing at quite low speeds.

§ (31) CARGO SHIPS.—These are not intended for more than 13 knots on a ship length of 400 ft. A single hump in the Ⓒ curve usually denotes the presence of a marked entrance wave, due to the bow lines being too fine or hollow. Such vessels require full (and sometimes convex) bow level lines near the surface, associated with finer lines at the lower part of the ship. This produces V-shaped bow sections with considerable flare, a result which can also be obtained by raking the stem forward and upward. In this way the positive pressure produced by the lower levels is underneath and tends to counterbalance the negative pressure of the upper ones, and the net result is to reduce the bow breaker (see § (25)) and its succeeding hollow, and at the same time ensure that the second bow divergent crest is well away from the form. In vessels of this type, for simplicity of construction, a considerable portion of the length amidships has the same cross-section. This portion requires to be shaped into the tapering entrance by very gentle curvature, particularly at the bottom, so that there shall be no locality of high-pressure change.

The stern lines are usually settled by the considerations given in the section on eddy resistance. A length somewhat greater than that of the entrance is usually required, and it follows that the bow waves are always of more importance than the stern ones.

§ (32) MODERATE SPEED VESSELS.—This class includes the intermediate passenger steamers, fruit and meat steamers, and others working at speeds from 13 to 17 knots for a 400-ft. ship, or at Ⓟ values of ·55 to ·7. There is a certain length of both entrance and run necessary to avoid abnormal wave-making. This should be a little greater than that given by the formula $V=1{\cdot}08\sqrt{L_e}$, L_e being the length of entrance or run. The transverse waves depend largely upon the longitudinal distribution of the displacement, and the shape given to the load water-line. The former is usually shown by a curve whose abscissa represents the length of the ship, and ordinate at any point, the area of the immersed cross-section at that point. This curve requires a slight hollow in it at both ends, the points of inflection being drawn as near amidships as possible so as to secure long ends. The stern water-lines should be as straight as possible. The bow lines should be either quite straight at the ends or slightly hollow, according to the features of the form and the speed.

§ (33) LINERS, CRUISERS, AND HIGH-SPEED MERCANTILE VESSELS AND STEAM YACHTS.—These have speeds in the neighbourhood of Ⓟ values of ·8 to 1·1, or, roughly, 17 to 23 knots for a 400-ft. ship. It is essential in these that the product (pL) shall be so arranged that the service speed gives a Ⓟ value of the right order for minimum wave-making (see § (28)), which for these vessels means placing the second crest of the bow transverse waves as near as possible along the beginning of the run, or somewhat aft of amidships, where it frustrates the formation of the stern system. Fine forms are necessary, and the curve of areas should be markedly hollow at both ends, the stern being somewhat fuller than the bow, and the point of inflection in the curve should be kept well towards amidships, there being no parallel middle body. The load water-line at the fore end should be hollow to keep down the divergent waves, and this with the long entrance keeps the bow wave system small. Filling out the ends, particularly the bow, is always bad, and it is better to use fine ends associated with a large midship section (which can be made almost rectangular) than to cut away the midship section and fill out the ends.

§ (34) CHANNEL AND MAIL STEAMERS.—When the speed is materially in excess of unity Ⓟ value (which for most vessels of this type means a speed in knots about equal to the square root of the length over all) the hollowness of the ends of the area curve must be eliminated. The fine angle of the entrance is still required, but the load line should be practically straight for some distance from the bow, and its maximum ordinate should be well aft of amidships. A cruiser stern, *i.e.* a stern of which the surface water-lines extend some distance aft of the rudder and the lower levels, is of great advantage to these vessels, as it increases the virtual wave-making length, and enables the curvature of the stern lines to be kept much easier than is possible with the vertical sternpost and the usual overhanging mercantile stern well out of water.

§ (35) T.B. DESTROYERS AND HYDROPLANES.—At very high speeds, two things may happen with any vessel. The hydrodynamic pressures may be sufficient, and the form may be adapted to make use of them, so that the ship is lifted to and supported at the surface, and "planes" at the top without serious wave-making. Alternatively it may continue to cleave through the water at all speeds. In this case the divergent waves at the bow continually increase, but the primary hollow of the bow system is now at or abaft the vessel's stern, and bow and stern transverse waves tend to cancel one another. A typical Ⓒ curve (A) is shown in *Fig.* 7. The drop in Ⓒ value at top speeds will continue so long as the cancelment of the transverse waves and the general diminution of their height, due to their high speed, exceeds the growth of the divergent waves. Length of water-line entrance and fine angles forward are essential if the bow divergent waves are to be reasonable. Aft, the water-lines may be quite full with an abrupt finish. All the stream-lines near the stern now tend to form in planes parallel to the central plane of the ship, and to avoid rapid pressure changes at the stern, vertical fore and aft sections of the form (called buttock lines) should be fairly *flat*, the slope from the water-line at the stern to the keel being as small as possible. The essential factor in the attainment of high speed is the ratio $(\text{displacement})^{\frac{1}{3}}/\text{length}$. The more this is reduced, the less is the horse-power per ton required to propel the vessel at these high speeds.

If the vessel lifts to the surface it is called a *hydroplane*. Such a vessel skimming on the surface of the water represents a single-pressure disturbance, and the energy involved in whatever waves or stream-line motion are set up is not recoverable in any shape or form. The lines along which the water will flow must be given the smallest possible curvature and small angles to the horizontal. Flat cross-sections, meeting the sides at a sharp angle, give the best result, but are not suited for meeting rough water, and in practice a compromise between this and the ordinary flaring ship bow must be adopted. The sharp corners where the bottom meets the sides—called the "knuckle" at the stern and the "chine line" forward—is necessary to effectively beat down the water. Without them water would cling to the sides of the boat, and as speed is increased would mount up and ultimately swamp it. The chine line should be quite full in plan, lifted above the water at the fore end, and should be arranged so that when skimming on the water it makes about 3° to $4\frac{1}{2}°$ to the horizontal. Below the chine the bottom must be arranged to prevent the formation of divergent waves, *i.e.* the sections must be given a flat form just under the chine. If for the sake of longitudinal stability, or to attain greater lifting power and efficiency, the planing bottom is broken into two parts by working a step across it, the vessel will travel supported on this step and the tip of the stern. The step should be placed about amidships, and should be such that the longitudinal tangent to the planing bottom at the fore side of the step makes only a small angle with the line joining the step and the tip of the stern. Air must be admitted to the bottom abaft the step, otherwise considerable suction will be developed in this region, and the greater the supply of air, so long as there is no interference with the stream-line flow, the better is the performance.

§ (36) SHOAL WATER, CANAL WALLS, AND RESISTANCE.—When the water in which a ship works is of finite breadth or of small depth, the shape of the stream-lines is necessarily different from those in an infinite fluid. The general character of this alteration is shown by *Fig.* 5. Its effect is to produce a longer belt of reduced pressure along the middle portion of the form, and a greater rate of distortion of the streams at the ends. These changes have a threefold effect upon the resistance of the ship. The former involves greater mean relative velocity of the water past the ship, and, therefore, an increase in the frictional resistance of about the same percentage at all speeds, since the effect depends only on the boundary conditions. The increased rate of distortion at the ends involves a less perfect replacement of the water behind the ship, and this also means a constant percentage increase in resistance at all speeds—as long as eddy resistance varies as the speed squared. Moreover, any great distortion of the streams at the stern renders them more or less unstable, and is liable to produce bad manoeuvring of the ship. Experiments have shown that to avoid *all* such boundary effects a width of channel equal to eight beams is required in deep water, and a depth of about six drafts for a cargo boat, and 7·5 drafts for a liner, with plenty of breadth in the channel.

Both the above causes of resistance are fully developed at low speeds. To them must be added the larger waves formed due to the greater disturbance and rate of change of pressure. This effect is evident at speeds given by

$$V^2_{\text{knots}} = 5{\cdot}0 \text{ (depth of water in feet)}$$

and increases continuously up to a critical speed given by

$$V^2_{\text{knots}} = 11{\cdot}5 \text{ (depth of water in feet).}$$

The constant is slightly lower when V is high compared with the natural wave-making speed of the ship, *i.e.* when it is a little above that giving a Ⓟ for maximum wave-making. At the critical speed the resistance is abnormally large, exceeding the deep-water value in some cases by 60 to 100 per cent. At still higher speeds this increase rapidly dies out, as the transverse waves are greatly diminished in height, and ultimately becomes somewhat less than in deep water.

The general effect of breadth of channel, apart from that already mentioned, is to augment whatever wave-making would normally be present, and to produce these waves at somewhat lower speeds than in open water. If both depth and breadth are restricted, an area of channel 200 times the largest immersed section of the ship is required to entirely avoid increase in resistance at low speeds. With a channel of one-fourth this section the increase at low speeds was about 8 per cent in two cases tried, but increased very much at speeds producing shallow-water waves.

VI. PROPULSION OF SHIPS

§ (37) THEORETICAL CONSIDERATIONS.—This can be effected either by such mechanical means as a tow rope, an endless chain, or by poling—methods only feasible in inland waters—or by means of either of the two fluids in contact with the ship, *i.e.* the air and the water. With either of these, a forward thrust can only be obtained by the production of a change of momentum having a sternward component. This sternward momentum may be produced in the air by the action of sails, or in the water by oars, paddles fixed to rotating wheels, a screw propeller, or the ejection of water from piping; but whichever propulsive agent is used, for efficiency the change of momentum must be as nearly as possible in the direction opposite to the line of motion, and must be effected with the least possible shock.

If U is the sternward velocity imparted to the fluid,
Q is the quantity of fluid acted upon per second,
V is the relative velocity of the impeller and the undisturbed fluid,
w is the weight of unit volume of the fluid,

the thrust T of the impeller will be given by

$$T = \frac{w}{g} UQ.$$

The useful work done equals

$$TV = \frac{w}{g} QUV.$$

$$\left.\begin{array}{l}\text{With no loss due to shock, the}\\ \text{energy in the race due to the}\\ \text{velocity U imparted to it is}\\ \text{the only and inevitable loss}\end{array}\right\} = \frac{w}{2g} QU^2,$$

and the efficiency therefore equals

$$\frac{V}{V + (U/2)}.$$

This implies that one-half the velocity U is imparted to the water by the time it has reached the propeller, and the other half after passing through it. If the race is rotational, and the angular and translational velocities are assumed to be the same throughout it, the equations for thrust and efficiency become

$$T = \frac{w}{g} Q\left(U - \frac{r^2\omega^2}{4(V + U)}\right),$$

$$\text{efficiency} = \left(\frac{1 - \bar{s}}{1 - (\bar{s}/2)}\right)\left(1 - \frac{\bar{s}r^2\omega^2}{4U^2}\right),$$

where $\bar{s} = U/(U + V)$, r is the external radius of the race column, and ω its angular velocity. It should be noticed that the loss due to rotation varies with $r\omega/U$, which is the tangent of the inclination of the spiral path of the race particles with the line of motion.

The quantity $\bar{s}$ is the ratio of the sternward velocity imparted to the fluid by the impeller to the resultant velocity relative to the impeller of the sternward flowing stream, and is clearly connected with the slip ratio, as defined in § (41).

For maximum efficiency two conditions must be satisfied:

(i.) The momentum must be imparted only in a sternward direction, and without any loss of energy in shock during the acceleration of the water set in motion.

(ii.) The sternward velocity imparted to the water must be a minimum, or for constant thrust the propeller must act upon as large a body of water as possible.

If the momentum is imparted *suddenly* to the water, the mean velocity of the water past the propeller becomes (U + V). The useful work remains the same as before, but the work put in is now T(U+V), and the best efficiency becomes V/(V+U). In this case the whole velocity U of the race is imparted to the water at the propeller.

There are other losses in propulsion, differing according to the mechanism used. Chief among these are those due to friction of the propeller surface and turbulent flow produced in impelling the water, but these equations serve to show broadly on what factors efficiency depends.

§ (38) Screw Propeller.—These equations hold good irrespective of the form of propeller producing the momentum, and may be applied to propulsion by oars, by paddle wheels, jet or screw propellers. Of these different mechanisms for propulsion by far the most important in actual practice is the screw propeller. This consists of a series, usually three or four, of fan-shaped blades which rotate at the stern of a ship about an axis parallel to the keel, and are so shaped as to produce by their action on the water a thrust which propels the ship. Owing to its large diameter the propeller acts upon a large quantity of water which it impels to the rear with a steady thrust, this sternward moving water forming a fairly well-defined column at the rear of the propeller called the "race," or slip stream. It is this steadiness of its thrust which separates the screw from the paddle and still more so from the oar (see § (53)).

A somewhat better method of considering the efficiency of a *screw* propeller than the general one already given is due to Mallock.[1] A propeller is made up of a number of blades, and each blade may be considered as made up of a number of annular strips whose sections are known. If such an elementary annular strip is moving along the axis OY, and is rotating at the same time about an axis parallel to OY, its path then will be along a line OX, and we assume this is inclined at a small angle to the chord of the element.

Its motion will be resisted by a force R acting in the direction XO; let DE represent this force. It will also be subject to a force L due to the pressure of the water at right angles to OX represented in *Fig.* 8 by AE; A being a point on the line of motion OY. Now the force AE is equivalent to EO and OA, thus the forces are DO and OA, and of these DO is equivalent to BO and DB. Thus, resolving the forces parallel and perpendicular to OY, we have for the components in these two directions BA and DB; of these BA represents the thrust due to the element and DB a force at right angles to the direction of rotation required to keep the element in rotation.

The efficiency of the element as contributing to the thrust is given by the ratio of the components of these forces normal to OX, for the components in this direction do no work on the element. Thus the efficiency is FE/FD.

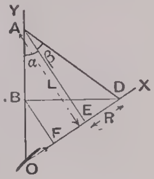

Fig. 8.

Now let α be the angle which the direction of motion OX makes with the normal to OY, and β the angle given by the relation $\tan\beta = R/L$. Then BAE $=\alpha$, and FAD $=\beta$. Also BDO $=\alpha$.

Again FD $=$ BD $\cos\alpha =$ AB $\tan(\alpha+\beta)\cos\alpha$ and FE $=$ perpendicular from B on AE $=$ AB $\sin\alpha$. Hence

$$\text{Efficiency} = \frac{FE}{FD} = \frac{AB\sin\alpha}{AB\tan(\alpha+\beta)\cos\alpha} = \frac{\tan\alpha}{\tan(\alpha+\beta)}.$$

Provided that R and L are known, the efficiency and thrust of every element of a blade can be determined, and by integration that of the propeller is found. There is a large amount of data of this kind for aerofoil sections, but when applied in this way to propeller calculations great caution is required. Due allowance must be made in the values of R and L for the interference of one blade with another, for the effect of shape of blade and its "aspect" ratio, and the large variation of velocity and pressure in the fluid at different radii. As a rule the method is fairly good for efficiency calculations, but is not accurate for quantitative thrust data, for which recourse must be made to experiments.

§ (39) Screw Propeller Experiments.—These are necessarily made on a small scale, but experiments by Taylor[2] and Gebers[3] have shown that the scale effect for water propellers in passing from 3 to 12 in. diameter and from 8 to 24 in. diameter is very small—less than 2 per cent. Tests with air propellers of (*a*) 2 and 15-ft. diameters, and (*b*) 6·2 and 14-ft. diameters have agreed to within 3 to 4 per cent. It is generally accepted therefore that model experiments can be used for estimating both thrust and efficiency of full-sized ship screws.

§ (40) Water and Air Propeller Differences.—Although the underlying theory for both types of propeller is precisely the same, the propellers themselves differ in several important respects. These differences are brought about partly by the conditions under which they have to work, partly by considerations of material and construction of the propellers themselves. In an air propeller compared with a water propeller—

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[1] *Inst. Naval Architects Trans.* l. 196. For other theories of screw propeller action reference should be made to the following: Theory of Cotterill, *Inst. Naval Architects Trans.* xx.; R. E. Froude, *ibid.* xix., xxx., xxxiii.; Greenhill, *ibid.* xxix.; Henderson, *ibid.* lii. lv.; Rankine, *ibid.* vi.

[2] *Am. Soc. Naval Architects and Mar. Eng. Trans.* xiv.

[3] *Schiffbau-Gesellschaft Jahrbuch*, 1910.

(i.) The total thrust for a given diameter is relatively small.

(ii.) The volume of air dealt with in unit time is large, but owing to its low specific gravity its mass is relatively small, hence

(iii.) The attainment of high thrust per unit area of blade, with reasonable efficiency, requires high axial and rotational velocities compared with those of a water propeller.

(iv.) Cavitation can only take place in the air at excessively high velocities, whereas in the water there is a limiting change of pressure, or thrust per unit area of blade, which can be produced at any part of the screw disc without a breakdown in the flow. To avoid this in high-speed ships the propeller blades are given wide tips with thick roots, and these represent a considerable departure from an air screw, although reasonably good efficiencies are obtained with them.

(v.) The air propeller is much more flexible. At and above normal speed of advance, at thrusts involving a slip of about 30 per cent, the blade bends and the pitch may change. With a water propeller there is very little movement, owing to the rigidity of the material from which it is made.

§ (41) Elements of Propulsion.—Definitions of terms used:

The *driving face* or *front* of a screw blade is the surface seen when looking from aft to forward.

The driving face is usually helicoidal in form, and a blade of uniform pitch is one whose face is a portion of a true helicoidal surface.[1]

The *face pitch of a propeller* is the distance parallel to the axis of rotation through which a point on the face would advance in one complete rotation of the generating line of the helicoid.

If a propeller, considered as a thin sheet, be made to move through the water so that its advance for each complete rotation is equal to its pitch, every point on its surface will move parallel to the surface. At no point will any blade have a component of velocity normal to its surface, and the propeller will exert no thrust on the water. We might define the pitch of the face as the advance per complete rotation at which the blade exerts no thrust.

In practice, even if the face of the blade be of constant pitch, the back and edges of the propeller will have some effect on the water, but in all cases a speed of advance can be found for which the thrust vanishes.

The *effective mean pitch* of a propeller is the distance through which the propeller advances in one complete rotation when producing no thrust.

The *pitch ratio* or *pitch diameter ratio* is the ratio of the pitch to the diameter of the propeller; if effective mean pitch be used in the numerator of the ratio, it becomes the *effective pitch ratio.*

Slip and Slip Ratio.—If P be the pitch of the propeller as just defined, and N the number of revolutions per second, then PN will be the distance traversed per second when producing no thrust; now V is the distance actually traversed, and it is found both by observation and experiment that the thrust depends on $PN - V$. This quantity is known as the slip, and its ratio to PN or the expression $1 - V/PN$ is called the slip ratio and denoted by s.

The *developed area* is the sum of the actual areas of the blades irrespective of shape.

Disc area ratio is the ratio of the developed area to the area swept out by the tips of the blades.

Blade-width ratio is the ratio of the maximum width of blade along its surface, to the radius of propeller.

Rake.—A blade is said to be raked forward or aft according as the centre line of the blade at the tip is forward or aft of the centre line at the root.

Skew back is the displacement of the centre line of a blade from the normal to the axis when viewed from aft. It usually increases towards the tip, and is measured by the movement of the tip, circumferentially from the normal.

When a ship is propelled by a screw the velocity and pressure changes which the screw produces in the water in front of itself, affects the ship resistance. Also the forward motion produced in the water by the passage of the ship, gives rise to a following current of water called the ship's "wake," in which the propellers have to work. The net efficiency of a screw as a propelling agent will therefore depend upon these effects of screw upon ship, and ship on screw. Let

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[1] A helicoid is the surface traced out by a straight line one end of which moves uniformly along a straight line—the axis of the screw or helicoid—while the line itself rotates uniformly about that axis.

If we imagine a circular cylinder described about the axis of rotation, the rotating line will cut the cylinder in a helix or spiral curve which is inclined everywhere at the same angle to a plane at right angles to the axis, so that, if the cylinder were cut open by a line parallel to the axis and developed into a plane, the trace would develop into a straight line inclined at a constant angle to the base of the cylinder; let this angle be α and let the diameter of the cylinder be d.

The distance, measured parallel to the axis of rotation, through which the rotating line moves during one complete rotation, is known as the pitch of the helix or screw; if p be the pitch then clearly from a consideration of the developed cylinder we have $p = \pi d \tan \alpha$. The ratio of p to d which is equal to $\pi \tan \alpha$ is known as the pitch ratio or pitch diameter ratio of the helix.

V be the speed of the ship in knots,
N the revolutions of the screw,
T the thrust of the screw behind the ship, which equals the resistance of the ship with screw working,
R the resistance of the ship with no screw,
S the shaft h.p. delivered to the screw.

Let the screw be set working without the ship being present, the same revolutions being maintained, with the speed of advance adjusted to V_1 so that the thrust is still T, but the shaft h.p. absorbed by it is now S_1. If the conditions behind the ship were uniform over the whole disc area of the screw, this speed, V_1, would be equal to the mean speed of the propeller through the water in the wake of the ship, and generally this is assumed to be the case. The propulsive efficiency of the screw behind the ship can now be written

$$\eta=\frac{RV}{S}=\left(\frac{R}{T}\right)\left(\frac{V}{V_1}\right)\left(\frac{S_1}{S}\right)\left(\frac{TV_1}{S_1}\right).$$

The last term is the screw efficiency in the open water. The other three terms express the effect of the ship and screw upon each other.

Writing $R/T=(1-t)$, t is a measure of the fractional excess of the screw thrust over the tow-rope resistance at the same speed, V, and is called the "thrust deduction fraction."

Writing $V/V_1=(1+w)$, w is a measure of the fractional excess of the ship speed over the velocity, V_1, which represents the mean speed of the water at the screw, and is called the "wake fraction."

The term S_1/S is known as the "relative rotative efficiency," and is a measure of the relative powers required for the development of a given thrust at given revolutions in open undisturbed water and behind the ship.

The propulsive efficiency can therefore be written

$$\eta=\begin{pmatrix}\text{screw efficiency}\\ \text{for undisturbed}\\ \text{water conditions}\end{pmatrix}(1-t)(1+w)\begin{pmatrix}\text{relative}\\ \text{rotative}\\ \text{efficiency}\end{pmatrix}.$$

The product $(1-t)$ $(1+w)$ is called the "hull efficiency," since it represents the ratio of RV, the tow-rope h.p. of the ship, to TV_1, the thrust h.p. of the screw, when developing in open water at the correct revolutions, the thrust necessary for the propulsion of the ship.

The net propulsive efficiency of *the ship* will be that of the screw, multiplied by the mechanical efficiency of the engine and the transmission gear between it and the screw.

§ (42) SCREW PROPELLERS IN OPEN WATER.—The general conclusions to be drawn from both theory and experiment are:

(i.) That the thrust of a given screw at a given slip varies as the square of the speed of advance through the water, and at a given speed and slip ratio will vary as the square of the diameter, D.

(ii.) The thrust and efficiency of any given screw will depend upon the slip ratio (s) corresponding to its revolutions and speed of advance at any given moment. Of the experiment results published, Froude's,[1] Taylor's,[2] Durand's,[3] and Geber's[4] are the most important. To a large extent they cover the same ground and generally corroborate each other in defining the effect of pitch, width of blade, etc., upon the result obtained. For all practical purposes Froude's data may be used, and his method of presenting results has been followed. It should be noted that whereas both Taylor and Geber use the face pitch of the screw in defining slip and true pitch, Froude uses a pitch calculated from the revolutions and speed of advance, so that slip shall be zero when thrust is zero. This is more in accord with sound theory, but the pitches obtained in this way require to be multiplied by a factor to get the face pitch of the ship's screw. Froude gives the face pitch as 1/1·02 times the nominal pitch for ordinary screws, but it has been found that for small pitch ratios and large disc area ratios 1/1·04 gives better agreement between estimate and ship-trial result.

Froude's experiments were made with screws of uniform pitch, having a diameter of ·8 foot, mounted on the fore end of a horizontal driving shaft clear of all obstructions, the centre of the screw being immersed ·8 of its diameter. All the screws were given a small boss ·91 inch in diameter. The thickness of each blade varied from a quite small amount at the tip to ·27 inch at the root where it joined the boss. The thickest section was always at the centre of the blade, and at right angles to the axis of the screw. Propellers with three and four blades were tried. The outlines of the blades were generally ellipses, but for some of the three-bladed propellers the tips were made specially wide. The width of blade was varied so as to cover a disc area ratio (§ (41)) from ·3 to ·75, and the pitch ratios covered a range from ·8 to 1·4. The thrusts obtained were expressed in a formula:

$$\frac{T}{D^2V^2}=B\frac{p+21}{p}\,\frac{1{\cdot}02s(1-{\cdot}08s)}{(1-s)^2},$$

where T is the thrust in lbs.,
V is the speed of advance in units of 100 ft. per min.,
P is the effective or analysis pitch,
p is the effective or analysis pitch ratio, which is equal to P/D,
B is a blade factor depending upon the number and type of blades, and the disc area ratio,
s is the slip ratio.

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[1] *Inst. Naval Architects Trans.* l.

[2] *Am. Soc. Naval Architects and Mar. Eng. Trans.* xii.

[3] *Researches on the Screw Propeller.*

[4] *Schiffbau-Gesellschaft Jahrbuch*, 1910.

This formula is based in part upon theoretical considerations. If it be assumed that T for unit diameter varies as the slip and as the square of the speed of rotation of the propeller, then

$$T = aN^2s.$$

The area, and for given revolutions the velocity of the blades, both vary as the square of the diameter, hence

$$T = aD^4N^2s, \quad \text{or} \quad \frac{T}{D^2V^2} = \frac{a}{p^2}\,\frac{s}{(1-s)^2}.$$

Analysis of the experiments showed that a varied as $p(p+21)$, and with constant pitch ratio a small correction for slip was required. B denotes the thrust capacity of the propeller as dependent on type, *i.e.* number, area, and shape of blades.

The efficiencies for screws having three elliptical blades and a disc area ratio of ·45 are given in *Fig.* 9, each curve being for a particular pitch ratio. All the curves possess the same general characteristics. The maximum efficiency is reached between 20 and 25 per cent slip, and falls off slowly at higher slips. High pitch ratios give best efficiency at normal slip ratios, but their advantage decreases as slip increases and is gone at 40 per cent slip. For any other disc area ratio, a small correction is required, the efficiencies decreasing as blade area increases. For wide-tipped blades a constant deduction of ·024 is to be made, and for four-bladed screws a deduction of ·0125 in addition to the above disc area ratio correction.

Although a four-bladed, or three-bladed wide-tipped propeller, working at the same slip as a three-bladed elliptical propeller, has slightly smaller efficiency, the more effective surface of both the former gives a higher thrust value B, and in some cases this can be utilised to improve the efficiency, as, all other conditions being the same, a smaller slip is required for a given thrust, and for slips above about 24 per cent this means better efficiency. In practice, therefore, there is very little difference to be obtained on this score.

To facilitate the use of the data for ship calculation, the results are better expressed in terms of horse-power, etc.

If H = the thrust horse-power of the screw, *i.e.* the power it delivers in thrust,
N = revolutions in units of 100 per minute,
V_1 = speed of screw through wake water in knots,

$$\frac{T}{D^2V^2} = \frac{H}{D^2V_1^3} \times \frac{33000}{100(1{\cdot}013)^3},$$

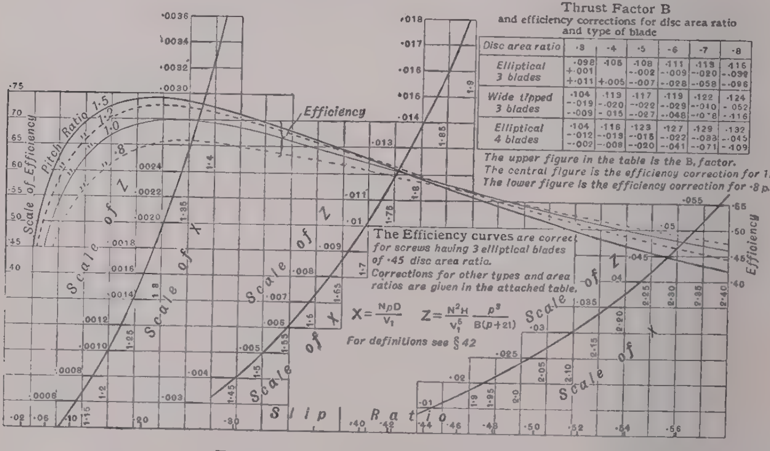
or

$$\frac{H}{D^2V_1^3} \times \frac{p}{B(p+21)} = {\cdot}00322\frac{s(1-{\cdot}08s)}{(1-s)^2},$$

and since

$$\frac{NpD}{V_1} = \frac{1{\cdot}013}{1-s} = X,$$

$$\frac{N^2H}{V_1^5} \times \frac{p^3}{B(p+21)} = {\cdot}00322(1{\cdot}013)^2\frac{s(1-{\cdot}08s)}{(1-s)^4} = Z.$$



Thrust Factor B
and efficiency corrections for disc area ratio
and type of blade

Disc area ratio	·3	·4	·5	·6	·7	·8
Elliptical 3 blades	·098 +·001 +·011	·105 +·005	·108 −·002 −·007	·111 −·009 −·028	·119 −·020 −·058	·116 −·039 −·096
Wide tipped 3 blades	·104 −·019 −·009	·113 −·020 −·015	·117 −·022 −·027	·119 −·029 −·048	·122 −·040 −·078	·124 −·052 −·116
Elliptical 4 blades	·104 −·012 −·002	·116 −·013 −·008	·123 −·015 −·020	·127 −·022 −·041	·129 −·033 −·071	·132 −·045 −·109

The upper figure in the table is the B. factor.
The central figure is the efficiency correction for 1.2 p.r.
The lower figure is the efficiency correction for ·8 p.r.

The Efficiency curves *are correct for screws having 3 elliptical blades of ·45 disc area ratio. Corrections for other types and area ratios are given in the attached table.*

$X = \frac{NpD}{V_1}$ $\quad Z = \frac{N^2H}{V_1^5}\,\frac{p^3}{B(p+21)}$

For definitions see § 42

FIG. 9.—Curves of Screw Efficiency and X-Z.

Both X and Z are functions of slip, and a curve of Z to a base of X is given in *Fig.* 9. The term V_1 is given by $V/(1+w)$, where V is the ship's speed (see Table II. for w values), and in estimating ship powers H is usually taken some 7 per cent more than the tow rope power, to allow for air resistance of upper works, rudder resistance, etc.

It should be understood that these results are for screws having clean smooth surfaces; the effect of a rough surface is dealt with in § (46).

§ (43) VARIATIONS OF PROPELLER BLADES. (i.) *Rake and Skew Back.*—Model experiments show that raking the blades forward or aft up to 15° has no effect upon either efficiency or thrust in open water. But rake aft is of considerable advantage in both single and twin screw ships, in keeping the blade tips away from the hull surface, where, owing to the frictional drag, the feed of water to the blade tip is very poor. Skew back or bending the blades in a transverse plane carries no propulsive advantages, but where there is any considerable amount of weed in the water, it helps the blades to clear themselves.

(ii.) *Disc Area Ratio.*—For all practical purposes the efficiency of a propeller is unaffected by its disc area ratio, provided that it is less than ·5, *i.e.* provided that the total area of the blades is less than one-half the area swept out by the tips. With large disc area ratios the efficiency drops, particularly with small pitch ratios. Froude found that this loss was practically independent of slip, and gave correction curves for several pitch ratios, to be applied to the standard efficiency curves of *Fig.* 9. This loss is chiefly due to the reduction of the ratio (blade width)/(gap between blades) with increase of area. It is well known from tests with flat blades in water and in air, that increase in this ratio reduces the thrust per unit area and the efficiency. The ratio can be written in the form $nb/(2\pi r \sin \alpha)$, where b is the width of the blades at radius r, n is the number of blades, and α is the pitch angle. Increase of n or b, and decrease in α (or pitch), will therefore have an adverse effect.

The thrust of the screw drops with area as shown by the B curve of *Fig.* 9, but the thrust per unit area increases, as the disc area ratio decreases. Excessively wide blades do not increase the thrust value of a screw materially. This can be improved somewhat by moving the area out towards the tips of the blades, but such wide blades are only used for the avoidance of cavitation.

(iii.) *Variable Pitch Ratio.*—Froude's results are for screws of uniform pitch ratio. The general effect of this factor can be seen from a study of *Fig.* 9. It is not uncommon for the pitch of screws to increase from the leading to the trailing edge—a change which produces slight hollow of the driving face. Thornycroft tested this on some model screws, and his results showed a slight improvement in efficiency. Taylor has tried the reverse, viz. a slight decrease of pitch at the leading edge. With a small pitch ratio (·8) a little improvement is effected at moderate slips, but, with a pitch ratio of 1·2 on the face, there was a general loss of about 7 per cent at all high slips. Taylor's experiments with blades having *rounded* faces showed that to produce the same thrust as with flat faces, the revolutions had to be increased—in some cases 10 to 15 per cent, but there is not sufficient data to clearly define the effect of gaining or decreasing pitch (*i.e.* hollow or rounded face) on thrust.

§ (44) ROUGHNESS OF SURFACE OF BLADES.—All the results given in § (42) are for screws with smooth surfaces. If these are made rough the efficiency drops. Thus, with a propeller of 6 feet diameter fitted to a pinnace of 18·5 tons weight, giving the propeller a surface equivalent to that of coarse sand, involved an increase of 8 per cent in the revolutions for the same speed as before, and an increase of the power at all normal speeds from 12 to 20 per cent. With small propellers[1] 1·3 feet in diameter, the same kind of roughness reduced the maximum efficiency from 72 to 36 per cent. The relatively greater effect produced here was due to the size of the particles on the surface being much greater compared with the dimensions of the propeller than in the former case. The same small propeller with a surface as cast showed 9 per cent less efficiency at small slips and 4 per cent at high slips, than when polished. Allowing for increase in size, the loss on a 10-ft. diameter propeller with such a surface would be of the order of 2 per cent.

§ (45) SCREW PROPELLERS BEHIND SHIPS.—The propulsive efficiency has been shown to be equal to the product of the screw efficiency in the open water and three other terms, viz. $(1+w)$ $(1-t)$ (rotative efficiency), which vary as the wake velocity and the interaction between the screw and ship. It has been found by many experiments that these terms are not independent of each other. If the wake fraction w is high, t is also usually high. Provided there is streaming water at the stern (or no actual break-down of the stream-line flow), experience shows that these three terms together approach unity. For twin-screw vessels of moderate fineness, their total value is about ·96 to ·98 at low speeds, and for single-screw vessels the value rises to 1·0 and occasionally 1·06. With full sterns carrying any dead-water (*i.e.* eddy water at those parts where, owing to the fulness, there is no steady flow), particularly with single-screw vessels, their total value falls off, owing to the poor conditions under which the propeller has to work. There is little data on this at present, but values down to ·85 have been obtained in practice.

But although these factors tend to cancel each other as regards total efficiency apart from the screw itself, a proper valuation of the wake fraction is necessary, as the velocity of the screw through the water, and therefore its slip and efficiency, depend upon it.

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[1] *Am. Soc. Naval Architects and Mar. Eng. Trans.* xxiv.

Wake is the water at the stern of a ship to which the latter has imparted forward motion. The absolute velocity of this water is called the "wake velocity," but it is usually only measured for that portion of the wake in which the propellers work. In the nomenclature already used this velocity is $(wV/1+w)$. This forward velocity is due to three causes, the relative effect of which varies in different types of ship :

(i.) The frictional belt increases in thickness continuously towards the stern, and, where the lines are closing in around the stern, tends to intermingle with the surrounding water — an effect which increases with the fulness of the after-body.

(ii.) When the stern is too full and rounded for proper stream-line flow, eddies form and constitute a mass of water called "dead-water," which is dragged along at the same velocity as the ship. This occurs at the surface level lines of all ocean cargo boats.

(iii.) If the ship is moving fast enough to form marked waves, the orbital velocity of the particles forming the waves will add to the forward movement of the water where a wave crest is formed, and tend to cancel it where there is a wave hollow. In torpedo-boat destroyers this effect is very marked at all high speeds when the stern is riding in a marked wave hollow, and is sufficiently great in some cases to more than cancel any forward wake due to friction (see Table II.).

The intensity of the wake velocity varies at different points, decreasing both towards the keel and outwards from the middle line. Quite close to the form, *i.e.* within 2 or 3 feet of the surface in a ship of 400 feet length, it is very high—in some cases as much as one-half the ship's velocity—and since the action of the screw is improved by working it in a stream of which the velocity is the same at all parts, the screws are so placed on a ship that the blades are well clear of the hull. Experience and experiment show that the wake fraction is practically independent of the form and fulness of the fore part of a ship. Fulness of the after end increases the wake of single-screw ships, but in twin-screw ships its effect is very small for good forms, provided that the clearance between the blade tips and the hull remains about the same. Small clearance gives high wake fractions, but somewhat lower hull efficiency.

TABLE II

SHIP WAKE AND HULL EFFICIENCY VALUES

Ship.	Length. Feet.	Breadth. Feet.	Draft. Feet.	Displacement. Tons.	Prismatic. Coefficient.	Speed. Knots.	Wake. Fraction.	Hull. Efficiency.
Single-Screw Ships								
Flavio Gioja . .	247	41·6	16	2500	·62	15	·24	—
Vessel L2 . .	400	60	18	7450	·62	—	·22	1·02
Vessel X . .	400	58·0	21·5	13600	·68	14·5	·24	·97
Vessel L . .	400	58·8	17·6	7720	·68		·32	1·08
Monarch . .	330	57·5	23·7	8100	·73	15	·37	1·0
Vessel M . .	400	53·3	24	10528	·73	10	·37	1·09
Vessel M2 . .	400	57·3	26	13140	·79	10	·45	1·07
Twin-Screw Ships								
Iris	300	46	18·1	3290	·55	16·5	·06	·96
Cruiser I. . .	400	71·2	26·2	11080	·57	20	·10	·98
T.B. Destroyer .	400	39·5	10·3	2050	·59	40	– ·006	·97
Cruiser 2 . .	400	52·5	22	6500	·57	—	·07	·97
Liner A . . .	400	52	18·5	6500	·63	—	·09	·98
Italia . . .	400	72·8	28·3	13850	·65	14·5	·14	1·0
Liner B . . .	400	50	19·5	7200	·67	—	·15	·95
Majestic . . .	390	75	27·5	14850	·69	17	·16	·94
Liner C . . .	400	54	19·3	8400	·73	15	·20	·99

Since the passage of a ship through the water inevitably sets up a forward moving wake, and it is equally inevitable that for the production of thrust the screw must set up a rearward moving column, in so far as these two can be made to cancel one another, there will be less velocity in the water left behind by the ship, and the energy required for propulsion will be decreased. By placing the propeller well aft this is partially achieved, and the energy thus saved is sometimes called the wake again. This constitutes a distinct advantage of screws placed at the stern, and gives a single screw a slight advantage over any other, provided that no portion of the screw is required to work with water in violent eddy formation ; the extent of this advantage is shown by the hull efficiencies of Table II.

§ (46) CAVITATION.—This is the name given to the formation of cavities in the water on the blade surface of a screw propeller—usually near its leading edge. It was first noticed in 1894, on the trials of the *Daring*. The formation of such cavities shows itself by absence of proper increase in thrust with increase in revolutions of the propeller shaft. When it is present it is believed that coreless vortices are continually formed, close up, and collapse on the surface, and the concentration of the energy of collapse on a small portion of the surface produces a hammering action which erodes the blade and sets up vibration in the propeller.

Cavitation may be produced by several different causes. A propeller produces its thrust partially by increased pressure on its rear face, and partly by suction on the leading face. When the suction at any point has reached the still water pressure, increase of speed of rotation cannot produce more suction, and the water supply to the screw will therefore remain the same although the revolutions have been increased, and will not be equal to the demands of the screw. This results in a break up of the flow, and the momentary formation of cavities. This break up will first show itself where the suction and thrust are greatest, which occurs at about one-seventh of the radius from the blade tips, towards the leading edge. The suction is always on the back of the blade, but if this is very full near the leading edge, it tends to produce at small slips suction on the driving face close up to this edge. The avoidance of cavitation therefore requires that good suction must be obtained without any high *local* value. For this the leading edge must be sharp, the back have no sudden change of shape, and the contour of the blade tips must be well rounded.

If the propeller does not rotate uniformly, the maximum thrust (on which cavitation depends) exceeds the mean (on which propulsion depends) by an amount depending on the variation in rate of rotation during a revolution. With turbine drive the rate of revolution is fairly constant, but with reciprocating engines[1] the departure from mean rate varies from some 4·6 per cent with a four-crank balanced engine in a high-speed passenger ship to 12 per cent with a two-crank compound engine in a cargo boat. The limiting thrust at any point, above which cavitation will occur, will diminish in sympathy with such variation in the rate of revolution.

A third cause of cavitation is the difference in the water supply to the tips of the blades when these are close to the hull surface. In this region the water is being drawn forward with the ship, and the slip angle of the screw blade is thereby greatly increased, and the thrust becomes correspondingly great. The higher the tip velocity, the greater should be the clearance between the hull and the screw tip. Cavitation due to this cause usually produces emphatic local vibration of the hull plating in the immediate neighbourhood of the screw tip.

The maximum pressure per square inch of the blade surface which a propeller can exert will depend upon many practical details. The maximum suction at any point which can be produced is that equivalent to the atmospheric pressure (14·6 lbs. per sq. in.) plus the depth of water. But taken over the whole blade surface the pressure will be considerably lower than this, as the actual thrust is always much greater towards the tip than at the root. With deep immersion, directly driven turbine screws having good clearance from the hull, on a fine lined ship, have given pressures up to 13·5 lbs. per sq. in. of blade surface. On the other hand, Barnaby[2] gives pressures of only 8 to 9 lbs. per sq. in. for propellers driven by four-cycle internal combustion engines, in which the turning moment varies considerably during a revolution.

§ (47) MULTIPLE SCREWS.—When a ship is propelled by a number of screws, each on a separate shaft, each may be considered by itself, and its thrust and efficiency be obtained as already detailed. The screw discs, when projected on a transverse plane, should if possible clear each other, as the race column of the forward one does not have a good effect on the after one. Luke's[3] experiments show that for a reasonably fine vessel, having a block coefficient of ·6, the after screws have no effect on the forward ones, but the forward screws decrease the wake fraction of the after ones (in this particular case from ·2 to ·14) so that their hull efficiency was 5·5 per cent less than it was with no forward screws. Generally it can be said that the best result is obtained with the smallest number of screws consistent with the development of the required thrust for propulsion at a reasonable slip ratio.

(i.) *Tandem Screws.*—These consist of two screws placed on the same shaft, necessarily turning in the same direction with the same revolutions. They are inefficient in working, and the thrust developed by them is little more than that developed with a single screw of the same diameter. This result appears from Luke's and Durand's experiments to be independent, broadly speaking, of the relative pitches of the forward and after screws. A fast passenger steamer, the *King Edward*, originally fitted with five screws, two each on two wing shafts and one at the middle line, did better when the outer ones were removed and only one screw working at lower revolutions was fitted on each shaft.

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[1] *Inst. Naval Architects Trans.* xlvii.

[2] *Marine Propellers* (1920).

[3] *Inst. Naval Architects Trans.* lvi.

(ii.) *Contrary Turning Screws.*—In small ships, two propellers can be placed on the same shaft line and be turned in opposite directions. Rota has tried this on a pinnace and in models, and the results give a reputed gain of 20 to 25 per cent in efficiency. Such propellers are used for the propulsion of torpedoes, the diameter of the after one being reduced so that the opposed torques on their shafts shall balance each other, and a good efficiency is obtained.

Luke [1] has tested the efficiency of such a combination in open water. His screws were all three-bladed, 6 in. in diameter, placed close together, the leading one being of 1·2 pitch ratio. The best efficiency was obtained with an after screw of 1·6 pitch ratio, and was ·86 that of a single screw. The thrust was roughly twice the thrust of a single screw of the same diameter. When placed behind a model, the contrary turning screws showed a larger wake and a consequently greater hull efficiency, representing a total improvement of 15 per cent on a full model, and just sufficient to cancel its lower "open" efficiency on a fine one—comparisons being made with twin screws in each case. This arrangement has not been tried on any large ship, and since it is not clear to what extent the rotative efficiency of the screws were affected behind the ship, the total effect may be less advantageous than the above result suggests.

§ (48) PROPELLERS WITH GUIDE BLADES.—The action in contrary turning screws is somewhat allied to that between a screw propeller with fixed guide blades placed immediately at its rear so that the water in the race leaves the guide blades without whirl of any kind. This arrangement was first tried by Thornycroft in his "turbine" propeller. This consisted of a screw propeller worked inside a cylindrical casing, the blades being secured to a boss which increased in sectional area from the leading to the trailing edge of the screw, the section of the channel between the containing cylinder and the boss being proportioned to suit the acceleration of the water produced by the blades. The pitch of the blades increased towards the after edge, aft of which were numerous guide blades fixed to the containing cylinder and to the long tapering after part of the boss. The rotation of the water set up by the blades is converted into fore-and-aft motion by the guide blades, and utilised except for loss in friction of guides. Barnaby states that the thrust delivered by the blades amounts to about one-third of the whole. Wagner [2] has adopted similar guide blades, but without any containing cylinder or tapered boss, and obtained an increase in propulsive efficiency of about 8 per cent on a cargo vessel at 12·5 knots, and 11 per cent on a first-class torpedo boat at 32 knots.

§ (49) HYDRAULIC OR JET PROPELLER.—In this propeller, water from the fore end or bottom of the ship is drawn into a centrifugal pump, and discharged through pipes in a sternward direction. Compared with a screw propeller the quantity of water acted on by the pump is necessarily small, unless abnormally heavy machinery is used, and to obtain the necessary reaction the velocity imparted to the water must, therefore, be high, and this militates seriously against efficiency. In addition to this defect there is considerable loss of energy due to friction of the piping and its bends, and some loss by shock at the water inlet unless this is shaped as a scoop, so that the water retains its velocity relative to the ship on entering it. The efficiency of the jet alone, with an efficient scoop at the entrance, is approximately ·7, and that of the pump action is ·5 to ·6, giving a total efficiency of ·35 to ·42 against ·6 to ·75 in a screw propeller. Its only sphere of usefulness is in cases where an external water propeller would be dangerous, and a modern air propeller cannot be used.

§ (50) PADDLE WHEELS.—A paddle wheel consists of a wheel rotating about a transverse horizontal axis, having paddle blades (or "floats" as they are termed), also in transverse planes at its periphery. The wheel is placed either over the stern (when the vessel is called a stern-wheeler), or on each side at about the middle of the ship, so that the blades send a stream of water aft when the wheel is rotated. The wheel requires to be so arranged that at the working speed of the ship, the apparent slip, measured at the outer edge of the blades, shall be of the order of 20 to 25 per cent, and the paddles shall be just immersed, due regard being paid to the change of water level near the wheels when under way. In small fast passenger vessels, at service speed, the water level usually drops relative to the ship at the wheel position.

The maximum theoretical efficiency is determined by the equations already given (§ (37)). To avoid loss by shock, the paddles of modern wheels are pivoted on a transverse axis at their centre, and rocked about this axis by an eccentric, so arranged that on entering and leaving the water the motion of the blades relative to the water shall be parallel to their surface, but when in the water they shall face as near sternward as possible. The breadth of a wheel varies from one-third to one-half the breadth of the ship, so that a large quantity of water can be dealt with, and only small velocities need be imparted to it, to obtain the necessary thrust. This propeller is only of use in vessels of which the draft is fairly constant. It is sometimes preferred to screws in shallow water, as the banks and bottom are not eroded so much by its wash.

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[1] *Inst. Naval Architects Trans.*, lvi.

[2] *Schiffbau-Gesellschaft Jahrbuch*, 1911.

§ (51) OARS.—The action of an oar is very similar to that of a paddle-wheel blade, and its efficiency is determined by the same factors. In a rowing eight the average speed may be taken as ten miles per hour, and the speed of slip of oar blade through the water as about 5 ft. per second (corresponding to a pull on the end of the oar of about 50 lbs.). Neglecting all motion other than sternward in the slip stream, the efficiency of a blade will be 74 per cent. The lateral and eddy motion set up is considerable, but even allowing for this, the blade efficiency is greater than that of a jet propeller.

§ (52) SAILS.—The general principles of propulsion by sails are the same for all ships, whether they have square or fore-and-aft rig. The propulsive effect will depend upon the change in momentum in a fore-and-aft direction of the air impinging on the sails, which varies with the speed and direction of the wind relative to the sails, and the inclination of the latter to the desired course. When a sailing ship has attained uniform speed under certain conditions of wind and sail area, its actual course will be inclined at a small angle (known as the leeway angle) to her keel line. This angle is generally small, seldom exceeding 10°, and its tangent gives the ratio of the lateral to the ahead speed. The angle must be so adjusted by the rudder or by shifting sail, until balance between air and water forces has been obtained. In reckoning wind pressure, due account must be taken of the motion of the ship relative to the wind. If the ship's course when sailing on a wind is along the line AB, and its keel line is AC, and YY is the position of the sails, let WA be the wind force in magnitude and direction. If WW_1 is the uniform speed of the vessel, W_1A is the true wind. The resultant force, AR, on any plain area such as YY, will act at a small angle δ aft of the normal to YY, varying from nil when running before the wind with YY square to AC, to about 10° to 15° in square rig and somewhat less than this in yachts, when sailing as close to the wind as possible. It should be observed that to make headway at all, the angle between the yards and the course must never be less than this angle.

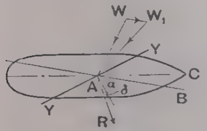


FIG. 10.

The sail force, AR, must be balanced by the water forces. When a ship form is towed through the water at small angles of yaw, the angle between the tow line and the keel increases very rapidly with yaw, attains a maximum of about 70° for ordinary ships at about 10° yaw, and remains there for a considerable further increase in leeway angle. The water force along the keel line of the ship does not vary much for small angles of yaw, but it grows rapidly with angles beyond 10°. For this reason it is never efficient to sail a ship at large leeway angles. Since the angle between the water force AR and the ship's course cannot exceed about 70°, and taking the angle δ to be 10° to 15° when close hauled to the wind, there is no purpose served in bracing the plane of the sail nearer than 30° or 35° from the ship's course, or with an angle of yaw of 10°, 20° to 25° to the keel line. If it be assumed that the sails will "draw" with the yards braced to within 15° of the apparent wind, the vessel can maintain a course not nearer than 45 to 50 degrees to this wind. The above figures are approximately correct for large square-rigged ships, and are independent of speed; fine-lined fore-and-aft-rigged vessels, with large central fins or dropped keels, would have larger a values and sail somewhat closer to the wind. The closer the plane of the sails is brought to the keel line of the ship, for a given wind, the greater is the angle W_1AY between the relative wind and the sails. This increases the magnitude of the force AR, and is an advantage so long as the increase in leeway angle resulting from the greater lateral wind force does not increase the water resistance to ahead motion more than the propulsive air force has increased from the greater value of AR—a matter which depends upon the area of sails used and many other seamanship items. G. S. B.

SHOAL-WATER AND SHIP RESISTANCE. See "Ship Resistance and Propulsion," § (36).

SIEMENS, SIR WILLIAM, maker in 1871 of a practical form of pyrometer based on the change of resistance of platinum with change of temperature. See "Resistance Thermometers," § (1).

SILICA:
  Coefficient of Apparent Expansion of Mercury in, determined by Harlow. See "Thermal Expansion," § (11) (ii.).
  Fused, used in tube form as protection for thermoelements up to 1000° C. in an oxidising atmosphere free from alkalis. See "Thermocouples," § (4) (ii.).

SILICATES:
  "Interval" and Instantaneous Mean Atomic Heats of, White's tabulated values. See "Calorimetry, Method of Mixtures," § (10), Tables I., II.
  Specific Heats of, at High Temperatures. See "Calorimetry, Method of Mixtures," § (10).

SILVER:

Atomic Heat of, at low temperatures, Nernst's values for, tabulated. See "Calorimetry, Electrical Methods of," § (11), Table VI.

Solid and Molten, Emissivity of, determined by optical pyrometer. See "Pyrometry, Optical," § (21).

Specific Heat of, at various temperatures, tabulated, with the Atomic Heat. See "Calorimetry, Electrical Methods of," § (10), Table V.

SIMILITUDE, APPLICATION OF PRINCIPLE TO CONVECTION CURRENTS. See "Heat, Convection of," § (2) (iii.) (iv.) and § (4) (iv.).

SIMPLE HARMONIC MOTION

THIS may be defined as the orthogonal projection of uniform circular motion. Thus if a point Q (*Fig.* 1) be supposed to describe a circle with constant velocity, and if its position at each instant be projected orthogonally on a fixed diameter AOA′, the particular type of rectilinear oscillation which the projection P executes between its extreme positions A, A′ is called a "simple harmonic," or sometimes merely a "simple" vibration.

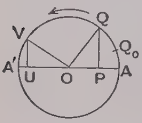


FIG. 1.

If ω be the angular velocity of Q in the circle the interval between two successive transits of P in the same direction through any given position will be $2\pi/\omega$. This is called the "period" of the vibration; its reciprocal $\omega/2\pi$ which gives the number of complete vibrations per unit time is called the "frequency." The distance (a say) of the extreme positions A, A′ from the mean position O is called the "amplitude." The angle AOQ is called the "phase."

The velocity of Q is at right angles to OQ and equal to ωa. It is therefore represented by the vector $\omega \, . \, \overrightarrow{\mathrm{OV}}$, where OV is the radius drawn 90° ahead of OQ. The component in AA′ is represented by $\omega \, . \, \overrightarrow{\mathrm{OU}}$, which is therefore the velocity of P. Again, the acceleration of Q is represented by $\omega^2 \, . \, \overrightarrow{\mathrm{QO}}$, and the component of this in AA′ is $\omega^2 \, . \, \overrightarrow{\mathrm{PO}}$, which is accordingly the acceleration of P.

It is this property, that the acceleration is directed always towards a fixed point, and is proportional to the distance from that point, which gives simple harmonic motion its special importance in Mechanics. A body, or (more generally) any system having one degree of freedom, which is *slightly* displaced from a position of stable equilibrium is urged back towards this by a force approximately proportional to the displacement. This is the case, for instance, with a pendulum, or a galvanometer needle. If the body be left to itself its motion will (so far as the approximation holds) be simple harmonic; for we can always construct a type of simple harmonic vibration which obeys the required law of acceleration, and also satisfies prescribed initial conditions of displacement and velocity.

If Q_0 be the initial position of Q in *Fig.* 1, the angle AOQ_0 is the initial phase. Denoting this by ϵ, we have $AOQ = \omega t + \epsilon$. Hence if x denotes the displacement OP, with the usual convention as to sign, we have

$$x = a \cos (\omega t + \epsilon). \quad . \quad . \quad . \quad (1)$$

If we represent this function graphically, with t as abscissa, and x as ordinate, we get a curve of sines, as in *Fig.* 2.

For this reason simple harmonic vibrations are sometimes described as "sinusoidal."

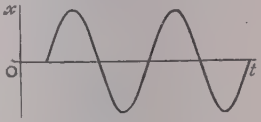


FIG. 2.

The preceding statement is equivalent to this, that (1) constitutes the *general* solution of the typical equation of the small motion of a body about a position of stable equilibrium, viz.

$$\mathrm{M}\frac{d^2x}{dt^2} = -\mathrm{K}x, \quad . \quad . \quad . \quad (2)$$

provided the value of ω be suitably chosen. We find, in fact, on substitution, that (2) is satisfied provided $\omega^2 = \mathrm{K}/\mathrm{M}$, and since the constants a and ϵ are at our disposal they can be adjusted so as to fulfil prescribed initial conditions of displacement (x) and velocity (dx/dt). A form of solution which is equivalent to (1) is

$$x = \mathrm{A} \cos \omega t + \mathrm{B} \sin \omega t, \quad . \quad . \quad (3)$$

the arbitrary constants being now A and B. Since the values of x and dx/dt recur whenever ωt increases by 2π, the period is $2\pi/\omega$, or $2\pi\sqrt{(\mathrm{M}/\mathrm{K})}$. It is to be noted that this depends only on the nature of the dynamical system considered, and is independent of the initial conditions, and therefore of the amplitude. The oscillations are accordingly said to be "isochronous"; but it must be remembered that the equation (2) is usually obtained as an approximation, in which powers of x higher than the first are neglected, and that it therefore ceases to be practically valid when the amplitude exceeds a certain limit. The structure of the formula $2\pi\sqrt{(\mathrm{M}/\mathrm{K})}$ for the period should be noticed on account of its wide applications and still wider analogies. The period varies as the square root of the ratio of two quantities, one of which (M) represents the inertia, and the other (K) the elasticity or "spring" of the system. For instance, in the case of a body vibrating under the torsion of a suspending wire, M is the moment of inertia of the body, and K the torsional rigidity of the wire.

In all practical cases vibrations are affected more or less by friction. This may be allowed for in many cases by introducing into the equation a retarding force proportional to the velocity, thus

$$\mathrm{M}\frac{d^2x}{dt^2} = -\mathrm{K}x - \mathrm{R}\frac{dx}{dt}. \quad . \quad . \quad . \quad (4)$$

Writing $\frac{\mathrm{K}}{\mathrm{M}} = \omega^2, \quad \frac{\mathrm{R}}{\mathrm{M}} = 2k, \quad . \quad . \quad . \quad . \quad (5)$

we obtain $\frac{d^2x}{dt^2} + 2k\frac{dx}{dt} + \omega^2 x = 0, \quad . \quad . \quad . \quad (6)$

which is the typical equation of "damped" oscillations. This is satisfied by $x = \mathrm{A}e^{\lambda t}$ provided

$$\lambda^2 + 2k\lambda + \omega^2 = 0. \quad . \quad . \quad . \quad . \quad (7)$$

When the friction is sufficiently small, more precisely when $k < \omega$, we have

$$\lambda = -k \pm i\omega', \quad . \quad . \quad . \quad . \quad (8)$$

where $\omega' = \sqrt{(\omega^2 - k^2)}. \quad . \quad . \quad . \quad . \quad (9)$

Hence $x = e^{-kt}(\mathrm{A}e^{i\omega' t} + \mathrm{B}e^{-i\omega' t}), \quad . \quad . \quad (10)$

or, in real form,

$$x = e^{-kt}(\mathrm{F}\cos\omega' t + \mathrm{G}\sin\omega' t). \quad . \quad (11)$$

This may be described as a simple harmonic vibration whose amplitude diminishes exponentially according to the law e^{-kt}. The ratio of one elongation to the next (on the opposite side) is $e^{\pi k/\omega'}$; the logarithm of this to base 10, viz. $\pi k/\omega' \times \log_{10} e$, is called the "logarithmic decrement." The formula (9) shows that the period $2\pi/\omega'$ is lengthened by the friction, but if the ratio k/ω is small the effect is only of the second order and may often be neglected.

When $k > \omega$ the roots of the auxiliary equation (7) are real and negative. Denoting them by $-\alpha$, $-\beta$, we have

$$x = \mathrm{A}e^{-\alpha t} + \mathrm{B}e^{-\beta t}. \quad . \quad . \quad . \quad (12)$$

There is now no true oscillation; the body passes once (at most) through its mean position, towards which it finally creeps asymptotically. This type of motion is described as "aperiodic," or "dead-beat." In the intermediate case where $k = \omega$ the solution is

$$x = (\mathrm{A} + \mathrm{B}t)e^{-kt}, \quad . \quad . \quad . \quad (13)$$

and the same remarks apply.

When in addition to the restoring force represented by $-\omega^2 x$ there is an extraneous disturbing force whose accelerative effect is X we have, if friction be neglected,

$$\frac{d^2x}{dt^2} + \omega^2 x = \mathrm{X}. \quad . \quad . \quad . \quad (14)$$

The most important case is where X is a simple-harmonic function of t, say

$$\mathrm{X} = f\cos pt. \quad . \quad . \quad . \quad (15)$$

The solution then is

$$x = \frac{f}{\omega^2 - p^2}\cos pt + \mathrm{A}\cos\omega t + \mathrm{B}\sin\omega t. \quad (16)$$

The first term represents the "forced oscillation" due to the disturbing force; it has the same period $2\pi/p$ as the latter, and its phase is the same or the opposite, according as $p \lessgtr \omega$, *i.e.* according as the imposed period is longer or shorter than the natural period $2\pi/\omega$. The remaining terms represent a "free" vibration superposed on the former; the constants A, B depend as before on the initial conditions. The amplitude of the forced oscillation becomes very great when the forced and natural periods are nearly coincident. This is exemplified by the phenomenon of "resonance" in Acoustics, but for a complete discussion it is necessary to take into consideration the effect of dissipation forces.

When friction is taken into account the equation to be solved is

$$\frac{d^2x}{dt^2} + 2k\frac{d\omega}{dt} + \omega^2 x = \mathrm{X}. \quad . \quad . \quad (17)$$

One method is to examine what extraneous force would be required to maintain a prescribed oscillation

$$x = \mathrm{C}\cos pt. \quad . \quad . \quad . \quad (18)$$

We find

$$\begin{aligned}\mathrm{X} &= \mathrm{C}\{(\omega^2 - p^2)\cos pt - 2kp\sin pt\} \\ &= \mathrm{CH}\cos(pt + \alpha), \quad . \quad . \quad . \quad . \quad (19)\end{aligned}$$

where H and α have been chosen so as to make

$$\mathrm{H}\cos\alpha = \omega^2 - p^2, \quad \mathrm{H}\sin\alpha = 2kp. \quad (20)$$

Changing the origin of t, it appears that a disturbing force

$$\mathrm{X} = f\cos pt \quad . \quad . \quad . \quad (21)$$

would give rise to the forced oscillation

$$x = \frac{f}{\mathrm{H}}\cos(pt - \alpha). \quad . \quad . \quad (22)$$

On this may be superposed a free oscillation of the type (11) or (12) or (13) as the case may be. The free oscillation, however, and therefore the influence of the initial conditions, gradually decays until the forced oscillation is alone sensible.

An alternative way of obtaining the above result is to put, in (17), $\mathrm{X} = fe^{ipt}$, and to discard in the end the imaginary part of the result. The solution then is

$$x=\frac{fe^{ipt}}{\omega^2-p^2+2ikp}=\frac{fe^{ipt}}{\mathrm{H}e^{i\alpha}}=\frac{f}{\mathrm{H}}e^{i(pt-\alpha)}, \quad (23)$$

the real part of which is as in (22).

It appears from (22) and (20) that the phase of the displacement lags behind that of the force by an angle α determined by

$$\tan\alpha=\frac{2kp}{\omega^2-p^2}. \quad . \quad . \quad . \quad (24)$$

This angle lies in the first or second quadrant according as $p \lessgtr \omega$. If the ratio k/ω be small the angle approximates as a rule to 0 or 180° respectively, which are the values when there is no friction, but if p is sufficiently nearly equal to ω the angle approximates to 90° on one side or the other. Since

$$\mathrm{H}^2=(\omega^2-p^2)^2+4k^2p^2, \quad . \quad . \quad (25)$$

the amplitude of the forced oscillation is greatest when p is in the neighbourhood of ω, provided k/ω be small. In any case, the maximum amplitude is $f/2k\omega'$, and is therefore greater the smaller the frictional coefficient $2k$, as was to be expected. A more important matter is the influence of the period on the absorption of energy. The rate at which the extraneous force does work is

$$\mathrm{X}\frac{dx}{dt}=-\frac{pf^2}{\mathrm{H}}\cos pt\sin(pt-\alpha)$$
$$=\frac{pf^2}{2\mathrm{H}}\{\sin\alpha-\sin(2pt-\alpha)\}, \quad (26)$$

the mean value of which is

$$\frac{pf^2}{2\mathrm{H}}\sin\alpha \quad \text{or} \quad \frac{f^2}{4k}\sin^2\alpha. \quad . \quad . \quad (27)$$

This attains its maximum value $f^2/4k$ when $\alpha=90°$, or $p=\omega$ exactly. It is to be noticed that although the resonance is more intense the smaller the value of k it is concentrated on a narrower range of frequency. To show the effect of a slight deviation from the critical value of p, put $p/\omega=1+z$, where z is small, and $k/\omega=\beta$. We find $\tan\alpha=-\beta/z$, or $\alpha=\frac{1}{2}\pi+z/\beta$, approximately, in circular measure. The formula (27) becomes

$$\frac{f^2}{4\omega}\cdot\frac{\beta}{\beta^2+z^2}. \quad . \quad . \quad . \quad (28)$$

The graph of the second factor is shown in *Fig.* 3 for various values of β. The principle here established has many applications in Acoustics. For instance, the vibrations of a piano wire are only slowly given up to the air, and a close coincidence of pitch is therefore necessary in order that it may respond audibly to a note sounded in its neighbourhood. On the other hand the column of air in an organ pipe, which has little inertia and readily gives up its vibrations to the outer air, will respond with only a slight variation of intensity to a much wider range of frequencies.

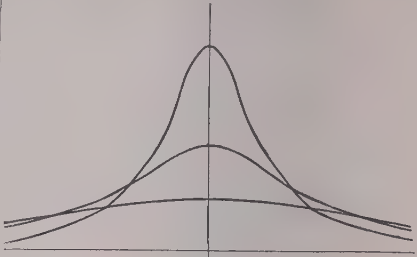


FIG. 3.

H. L.

Skew-back in a Screw Propeller is defined as the displacement of the centre line of a blade from the normal to the axis when viewed from aft. See "Ship Resistance and Propulsion," § (41).

Skin Friction. See "Ship Resistance and Propulsion," § (6).

Slide-valve for Steam Engine. See "Steam Engine, Reciprocating," § (2) (ii.).

Slipping of Belts—Elastic stretching of leather belts passing over pulleys. See "Friction," § (36).

Slipping between Surfaces in Rolling Contact. See "Friction," § (37).

Smith, A. W.; measurement of latent heat of water. See "Latent Heat," § (4).

Sodium, Specific Heat of, in the Annealed and Molten States, at various temperatures, tabulated, with the Atomic Heat. See "Calorimetry, Electrical Methods of," § (10), Table V.

Solid Angle subtended by a Surface at a Point. Describe a cone with its vertex at the point by radii drawn from the point to all points of the boundary of the surface, and let A be the area of a sphere of radius R which is intercepted by this cone. Then the ratio A/R^2 is known as the solid angle subtended at the point by the surface; it is clearly equal to the area intercepted by the cone on a sphere of unit radius.

Solid Piston Air-pumps. See "Air-pumps," § (11).

Solid State, Change of Specific Volume in passing to. See "Thermal Expansion," § (31).

Solids:

- Methods of measuring Thermal Conductivity of. See "Heat, Conduction of," §§ (3)-(6).
- Specific Heat of, by Electrical Methods. See "Calorimetry, Electrical Methods of," § (8).

Solutions, Thermodynamics of. See "Thermodynamics," § (63).

Sound, Velocity of, used to determine γ. See "Thermodynamics," § (58). See also Vol. IV.

Sounding Apparatus. One type consists of a compressed air gauge fitted with a non-return valve. The pressure attained is shown by the quantity of water which enters the tube. The second type consists of a tube closed at one end and coated inside with a chemical which will be coloured by the action of the sea water. This is dropped in a sinker with the open end downwards, and again the pressure can be calculated from the amount of water which has entered, shown by the discoloration produced. See "Pressure, Measurement of," § (15).

Sounding Tube of M. Berget. See "Pressure, Measurement of," § (16).

Specific Heat:

- The ratio of the amount of heat required to raise the temperature of a substance one degree to that required to raise the temperature of an equal mass of water one degree—usually from 14°·5 C. to 15°·5 C.—is called its specific heat. In some cases the range 17° C. to 18° C. is selected. The conditions under which the heating is to occur may be limited in various ways, the two most important being (*a*) constant volume and (*b*) constant pressure. See "Thermodynamics," § (13). See also "Specific Heat of Saturated Vapour," § (53); "Specific Heat, Ratio of, in Gases," § (58); "Specific Heat, Various Expressions for," § (48).
- Application to the Non-Metals of Debye's and Einstein's Formulae for, deduced from the Quantum Theory. See "Calorimetry, the Quantum Theory," § (46).
- Formulae of Nernst and Lindemann and of Debye, tested by Nernst on the data for diamond, and results summarised in tabular form. See *ibid.* § (46), Table IV.
- Tabulated Comparison of Experimental Values for, with Formulae deduced from the Quantum Theory. See *ibid.* § (45), Table I.

Specific Heats:

- Of Elements at about 50° abs., measured by the liquid hydrogen calorimeter and tabulated. See "Calorimetric Methods based on the Change of State," § (8), Table V.
- Of Gases, tables of. See "Engines, Thermodynamics of Internal Combustion," § (70), Tables VI.-XI.a; "Specific Heat of Gases at High Temperatures."
- At Low Temperatures, Appliances for the Measurement of, by the Method of Mixtures. See "Calorimetry, Method of Mixtures," § (11).
- Variable. See "Engines, Thermodynamics of Internal Combustion," § (78); "Specific Heat of Gases at High Temperatures."

Specific Weight and Volume of Gases, table of. See "Engines, Thermodynamics of Internal Combustion," § (68), Table I.

Sprengel Pump. See "Air-pumps," § (18).

Spur Gear and Driving Chain Testing Machine (National Physical Laboratory). See "Dynamometers," § (6) (ii.).

Spurge High-pressure Manometer. See "Pressure, Measurement of," § (12).

Squirrel Cage Speed Indicator. See "Meters," § (9), Vol. III.

States of Aggregation. See "Thermodynamics," § (28).

Steam:

- Latent Heat of, Comparison of Data for, by Callendar. See "Latent Heat," § (5).
- Latent Heat of, Formulae for Variation of, with Temperature. See *ibid.* § (6).
- Specific Heat of, at atmospheric pressure, investigated by Holborn and Henning. See "Calorimetry, Method of Mixtures," § (17).
- Specific Heat of, determined by Brinkworth by the continuous flow electrical method at atmospheric pressure between 104° C. and 115° C. See "Calorimetry, Electrical Methods of," § (14).
- Specific Heat of, determined by Callendar's continuous electrical method, the variation of the specific heat with pressure being found by subsidiary experiments, using the throttling calorimeter method, by Callendar and Professor Nicholson. See "Calorimetry, Method of Mixtures," § (17).
- Specific Heat of, Regnault's Value for. See *ibid.* § (17).

Steam Charts and Tables. See "Thermodynamics," §§ (42) and (61).

STEAM ENGINE, RECIPROCATING

§ (1) Description.—The reciprocating steam engine is so named because of the working substance used and the motion of one of its fundamental parts. The cylinder, piston, and some means of controlling the working substance are essentials. The motion of the piston within the cylinder is one of reciprocation, hence the term "reciprocating." The piston is a movable division plate constrained to move axially within the bore of the cylinder. The piston must be of adequate strength to withstand without appreciable deformation the steam loads to which it is subjected, and its circumference must be such a good fit in the bore of the cylinder that steam cannot readily pass from the one to the other side of the piston. The friction between the piston and the cylinder must be a minimum. The limits of reciprocation of the piston are usually definitely defined, and the distance between the limits is called the "stroke of the piston." Difference in magnitude of the loads acting on the two sides of the piston causes motion of the piston.

Difference of purpose in view has led to variation in detail and general appearance, and some unusual feature, or perhaps special service for which the engine is suited, has provided a class name for engines of similar outline or special service. A list of the classes to which engines are assigned includes—marine, locomotive, stationary, portable, vertical, horizontal, diagonal, oscillating, drop-valve, Corliss, winding, pumping, high-revolution. All, however, are alike in one fundamental respect—all have a cylinder within which a piston reciprocates, and all have some means of controlling the steam entering and leaving the engine cylinder.

In this article a very commonplace simple engine is considered first, and afterwards various departures from this simple engine, which have resulted in establishing special classes of engines, are noted.

Fig. 1 shows diagrammatically the cylinder and piston of a "double-acting" engine. In this figure the ends or "covers" of the cylinder are shown containing the necessary control valves for regulating the steam entering or leaving the cylinder. When the piston is moving towards the right the left steam valve S is open, allowing live steam to flow into the gradually increasing space due to the piston moving away from the left cylinder cover; and the right exhaust valve E is open to allow the steam used during the previous stroke to escape (or exhaust) from the cylinder. The return stroke of the piston is caused by closing these two valves and opening the alternative pair of valves. The engine is called "double-acting" to differentiate it from the "single-acting" engine in which but one stroke of each two strokes is a power stroke. The single-acting engine is rarely used, and then only for some very specialised duty. The double-acting engine has the advantage in that each stroke is a power stroke, and, therefore, for a given size and correspondingly almost equal weight of engine, the double-acting engine has an output approximately double that of the single-acting engine.

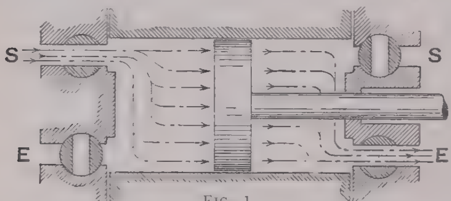
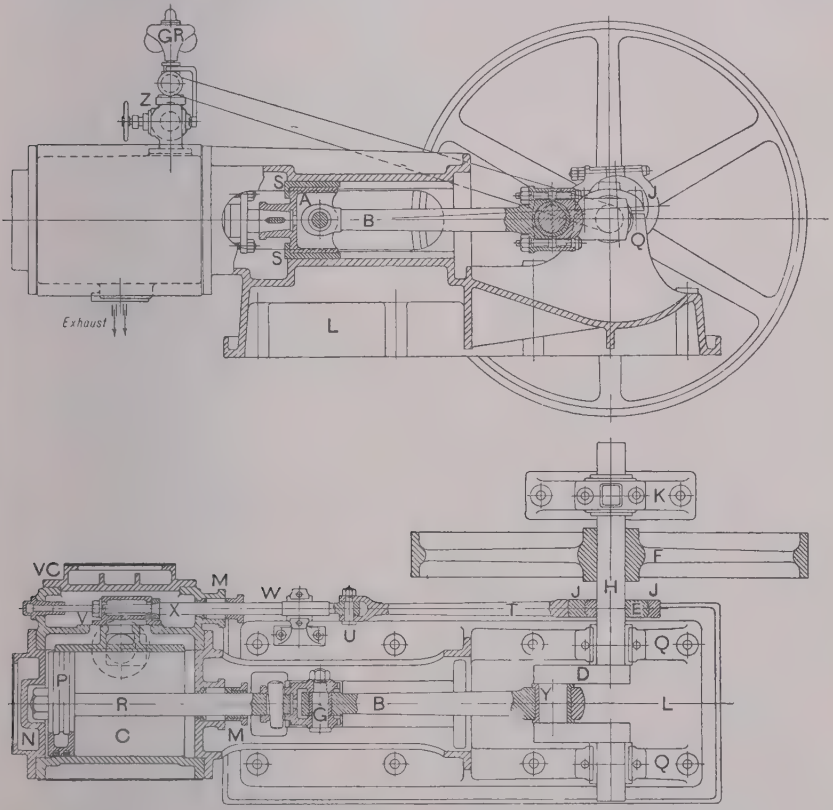


FIG. 1.

Whilst the motion of the piston is one of reciprocation, the motion usually desired, for ease in transmission, is rotary. Several mechanisms have been devised and used to transform from the one to the other motion, and out of these the crank and connecting-rod mechanism holds the premier position because of its compactness, small number of wearing parts, and reliability.

§ (2) SIMPLE DOUBLE-ACTING ENGINE. (i.) *Description.*—In *Fig.* 2 is shown in sectional plan view and elevation a simple double-acting horizontal steam engine. One end of the "piston rod" R is attached to the "piston" P. The piston rod passes through a suitable steam-tight "stuffing-box with gland" M, and terminates in a "cross-head" A. A "connecting rod" B couples together the cross-head and the "crank" D of the "crank shaft" H. The cross-head end of the connecting rod moves in a straight line; the crank end of the rod moves in a circle. To allow for the resulting angular displacement of the connecting rod, the connections at the cross-head and crank ends are not rigidly fixed as are the connections between the piston, or cross-head, and piston rod, but are pin connections. The pin G at the cross-head end is called the "gudgeon," or "cross-head pin," and that in the crank the "crank pin" Y. The crank shaft is constrained in its motion by "crank-shaft bearings" Q, integral with the "engine frame," or "bedplate," L. The bedplate is anchored to a suitable heavy foundation. At any instant the load causing reciprocation is transmitted along the piston rod and through the connecting rod to the crank pin, and produces rotation of the crank shaft. Twice in each revolution the piston rod, connecting rod, and crank are in line. The engine in such position is said to be on the "dead centre." At all other times the lines of action of the forces along the piston rod and connecting rod are not coincident, and hence bending of both rods would occur if a support attached to the cross-head were not provided. The support consists of the "shoes" S, secured to and reciprocating with the cross-head whilst bearing against the "guide surfaces."

As the piston rod, connecting rod, and crank are in line twice in each revolution of the crank shaft, the turning moment twice in each revolution is correspondingly zero. If a diagram be drawn as in *Fig.* 3 (*a*), the base line representing the circumference of the circle described in one revolution by the crank pin and the ordinates representing the resolved values of the forces transmitted along the connecting rod acting normally to the crank D and at the crank pin Y, it will be noticed that there is a cyclical variation of considerable range. The resistance offered by whatsoever the engine may be driving is usually nearly constant. To absorb the excess of energy supplied during one part of each half revolution and to make up the deficiency during the remainder of the half revolution a "flywheel" F is fitted to the crank shaft. The range of the cyclical variation in crank effort may be considerably reduced, as at (*b*), by coupling together the crank shafts of two engines so that in end view the cranks are 90° apart; the addition of a third engine would further reduce the range, as at (*c*), provided that the cranks in end view are about 120° apart. In (*b*) and (*c*) the heavy solid line marks the sum of the energy supplied by the two or three engines respectively.



A. Cross-head.
B. Connecting Rod.
C. Cylinder.
D. Crank Web.
E. Eccentric Sheave.
F. Flywheel.
G. Gudgeon Pin.
H. Crank Shaft.
J. Eccentric Strap.
K. Bearing.
L. Bedplate.
M. Stuffing-box and Gland.
N. Cylinder Cover.
P. Piston.
Q. Crank-shaft Bearing.
R. Piston Rod.
S. Cross-head Shoe.
T. Eccentric Rod.
U. Eccentric Rod Pin.
V. Slide Valve.
W. Valve Spindle Guide.
X. Valve Spindle.
Y. Crank Pin.
Z. Combined Stop Valve and Throttle Valve.
GR. Governor.
VC. Valve Chest.

FIG. 2.

(ii.) *Valves.*—The means used for controlling the steam to and from the engine cylinder are numerous. Should either rocking or drop valves be used, the valves may be arranged in the cylinder ends as shown diagrammatically in *Fig.* 1. Considerable modification in the design of the cylinder is necessary should the more common sliding valve be used. As commonly arranged, one sliding valve controls both the supply of steam to and the removal of the steam from both ends of the cylinder of a double-acting engine.

The simplest sliding valve, called the "D slide valve" because of its resemblance to the letter D in its longitudinal section, consists in its elementary form of an open-top metal box.

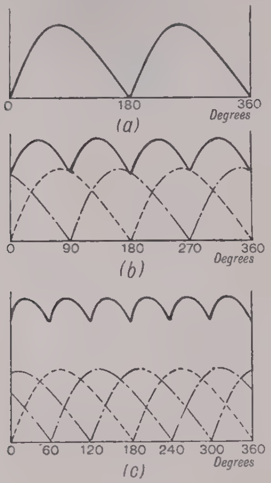
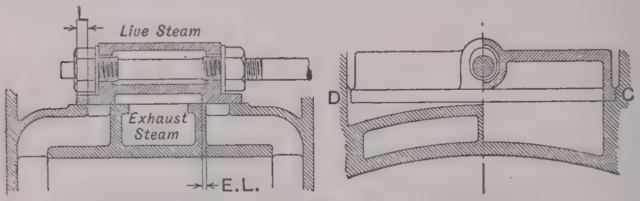


FIG. 3.



Longitudinal Section. FIG. 4. Transverse Section.

If the open top of the metal box be placed on a suitable surface and the box be caused to reciprocate, the end walls of the box would cover and uncover suitable openings provided in the surface upon which reciprocation takes place. In *Fig.* 4 a common D slide valve is shown at the middle of its travel and in its relation to the cylinder passages. The live-steam space is outside the slide valve, and the exhaust space is within the slide valve. Even though relieving devices are used (there is no provision for such a device in *Fig.* 4) the load acting on the back of the valve and pressing the valve on to the cylinder face is considerable. It is therefore desirable to make the area subject to steam load as small as possible and to have the amount of reciprocation a minimum. The length of the steam ports or passages cannot exceed the diameter of the cylinder, and the area of each of the ports must allow the steam to pass through the port at a reasonable velocity. In *Fig.* 4 the valve of the simple engine of *Fig.* 2 is shown to a larger scale.

The usual method of securing the slide valve to its "spindle" is by means of nuts, so that a slight adjustment of the valve upon its spindle is an easy matter. The valve spindle passes through the end of the "valve chest" VC as shown in *Fig.* 2, a stuffing-box and gland M being used to prevent the escape of steam from the valve chest to the atmosphere, and ends with a pin connection to which the "eccentric rod" T is attached. A "valve spindle guide" W is necessary to give support to the valve spindle because of the angular displacement of the eccentric rod relative to the valve spindle during each stroke of the piston. The valve spindle is enlarged where the guide is provided, the enlargement reciprocating within the guide secured to the engine frame. In comparison with the engine piston the amount of reciprocation of the slide valve is small, and the load taken by the valve spindle guide is not great.

A pictorial view of the slide valve, with a portion of its spindle, raised bodily above the valve face of the cylinder upon which it reciprocates, is shown in *Fig.* 5. One corner of the slide valve is broken to show the general distribution of metal in the valve. In the actual valve there is of course no such break. The valve is guided in its reciprocation by the machined sliding surfaces A and B moving upon similar surfaces machined in and forming part of the valve chest; these are marked C and D in *Fig.* 4.

(iii. *The Eccentric.*—The motion of the slide valve is provided by a virtual crank and connecting rod called an "eccentric sheave" and

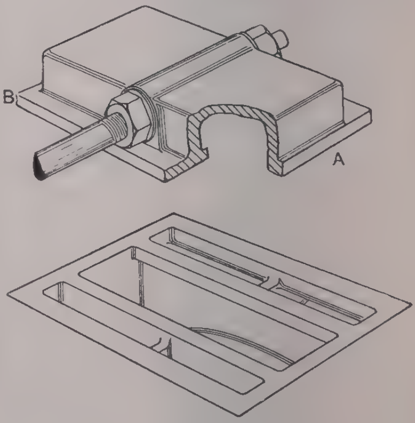


FIG. 5.

"eccentric strap and rod" respectively. The eccentric sheave is a crank wherein the crank pin is sufficiently large to embrace the crank shaft, although for manufacturing or other convenience it is usually separate from the shaft. In the three sketches of *Fig.* 6 it will be noticed that the same eccentricity has been maintained, and that at (*a*) the strength of the shaft is much impaired by forming the valve crank, whilst at (*c*) the full strength of the shaft is maintained. To prevent longitudinal motion of the eccentric strap, flanges or their equivalent are provided on the eccentric sheave. *Fig.* 7 shows an eccentric sheave with its strap and eccentric rod suitable for a high-power marine engine. The eccentric sheave is driven by a key fitted partly into the crank shaft and partly into the sheave. The sheave is made of cast iron and revolves within a white-metal bearing secured to the eccentric strap. The mild-steel eccentric rod is provided with bronze bearings for the pin connection.

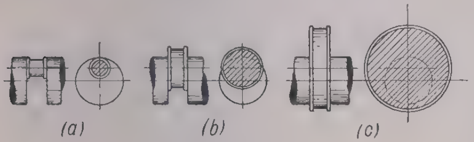

(a) (b) (c)

FIG. 6.

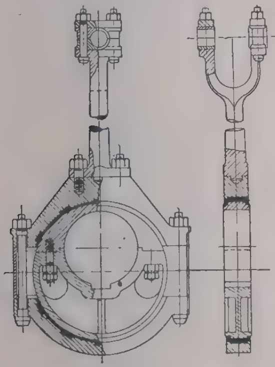


FIG. 7.

Whilst the throw of the eccentric must be such as to give the necessary amount of reciprocation to the slide valve which it drives, the angular position of the sheave relative to the engine crank must be such that the valve functions at the correct times. In most engines it is highly desirable to admit steam only during a portion of the stroke of the piston rather than during the whole of the stroke. This can readily be effected by increasing the thickness of the end walls of the elementary metal box or slide valve, *i.e.* by adding "steam lap" to the valve, and altering correspondingly the angular position of the eccentric sheave. The lap of a valve is defined as the amount by which the edge of the valve overlaps the corresponding edge of the corresponding port when the valve is in its middle position with regard to its travel. Should the lap be over the steam edge of the port, L of *Fig.* 4, the lap is "steam lap"; should the lap be over the exhaust edge of the port, EL of *Fig.* 4, the lap is "exhaust lap." The steam laps at both ends of the valve are not necessarily equal; the exhaust laps also are not necessarily equal, and sometimes one or both are negative.

(iv.) *Cut-off.*—The admission of steam during a portion of the stroke of the piston instead of during the complete stroke is known as "cutting off" the steam supply, and the point of the stroke at which the steam supply actually ceases is called the "point of cut-off." It may be of interest to note that if the cut-off is at one-half of the stroke and the engine is running at 300 revolutions per minute, the period during which the steam may enter the engine cylinder is but one-twentieth of a second. During this short interval of time the slide valve from the closed position opens the steam port to the full and closes it again. Throttling, and consequent reduction in pressure of the steam entering the cylinder, occurs when the port has just been opened by the slide valve and also when the valve has almost closed the port. In the cycle of events the opening of the port to admit steam to the cylinder is termed "admission"; the closing of the port is termed "cut-off." The opening of the port to allow the used steam to leave the engine cylinder is termed "release"; the closing of the port against the passage of the exhaust steam is called "compression," because a small amount of steam is trapped in the engine cylinder between the moving piston and the fixed cylinder cover. The admission of steam when, say, about 90 to 95 per cent of the previous stroke has been accomplished aids in bringing the moving parts of the engine to rest ready for the new stroke. The amount by which the slide valve is open to steam when the piston is at the commencement of a stroke is called the "lead" of the valve. The leads for both ends of the valve are not necessarily equal. In vertical engines the lead for the stroke in which the reciprocating mass is lifted against gravity is usually greater than for the reverse stroke. The lead of a slide valve is dependent upon the angular position of the eccentric sheave in relation to the engine crank. Lead is necessary in order that admission may take place before the commencement of a new power stroke.

(v.) *Cylinder.*—The cylinder, including the valve chest with steam passages, and often one of the cylinder covers, forms one casting. A high-quality cast iron is used to withstand the wearing action of the reciprocating piston and valve. The piston packing usually consists of spring rings, *i.e.* rings turned a little larger than the cylinder bore and sufficiently cut out to allow the rings to close to a little less than the cylinder bore. The tendency to open out to their original diameter produces pressure on the bore of the cylinder and forms a moving steam-tight joint. To lubricate the piston a sight-feed displacement lubricator is provided to pass a small amount of mineral oil per revolution into the engine cylinder. Necessary bosses for drain and other connections, and flanges to which the steam and exhaust connections may be made, are integral with the cylinder casting. A steam "stop valve" must be provided adjoining the engine. If the steam be not superheated it is well to fit a "separator" to the steam main close to the engine. The separator, either by setting the steam in rotation or by means of baffle plates, removes a large percentage of the entrained water due to condensation in the steam mains or to other causes. The drainage of the valve chest and the engine cylinder should be accomplished by leading pipes to a "steam trap," *i.e.* an automatic device which whilst allowing water to escape will not permit steam to pass. On small engines in particular the drainage provision is frequently very crude and wasteful.

Every precaution should be taken to prevent radiation loss by adequately covering with asbestos or other non-conducting material all parts subjected to high temperature. For appearance the non-conducting material used on engine cylinders is often covered with thin planished steel; on steam and exhaust mains canvas is frequently used to give a substantial yet neat finish. The monetary loss due to radiation from inadequately clothed hot surfaces is considerable and continues so long as the plant is running.

The steam load acting on the moving engine piston at any instant also acts upon the fixed cylinder cover. These two loads, being action and reaction, are of equal magnitude. If the cylinder be not integral with the engine frame it is secured by bolts thereto. The frame also carries the guide surfaces for the cross-head shoe, and the crank-shaft bearings. Considerable stiffness of the frame is necessary to contend with the rapidly altering loads. Even at the comparatively slow speed of 60 revolutions per minute the loads change twice per second. Heavier loads than those due to steam thrust, however, are imposed upon the frame. In the single-cylinder engine of *Fig.* 2 it is not possible to balance[1] the reciprocating masses even if the revolving weights be balanced, which in many engines is not done, so that vibratory forces are set up as soon as the engine runs. These forces are quite independent of the work being done by the engine. If the engine does no work, but is motored round by some external agent, the forces still act. The forces are dependent upon the weights and velocities of the moving parts. Vibration ensues even though the engine be anchored to a heavy brickwork or concrete foundation. The desirability of designing the moving parts so that they shall be of a minimum weight consistent with necessary strength is apparent. Multi-crank engines can be designed so that the collective effect of the separate moving parts is to balance.

(vi.) *The Governor.*—If the load against which the engine is working be suddenly removed, the speed of the engine will increase. To prevent undue increase in speed a revolution regulator or "governor" is provided. The action of the governor is due to centrifugal force. When the speed increases, certain weights driven by the engine itself change position and, by reducing or modifying the steam supply, bring about a reduction of the speed of the engine to the normal. The function of the flywheel is to act as an energy store and regulator; the function of the governor is to regulate the number of revolutions made per minute by the engine.

Fig. 8 shows a typical form of governor. The vertical spindle S is driven by gearing from the engine shaft. Two arms terminating in balls B are pivoted to the upper end of the vertical spindle. To each arm is pin-jointed a link L. The lower end of each of the two links is pin-jointed to a sleeve A which is free to slide on the vertical spindle. The vertical position of the sleeve depends upon the position of the governor balls, which in turn depends upon the speed of rotation of the vertical spindle and therefore upon that of the engine shaft. The position of the

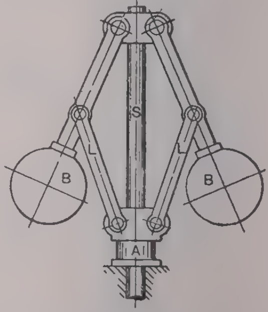


FIG. 8.

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[1] See "Engines and Prime Movers, Balancing of," § (5).

sleeve determines the quality or the quantity of the steam supplied to the engine cylinder.

There are two distinct methods by which the governor may control the engine revolutions. Through such levers and links as are necessary the governor may either operate a throttle valve or it may alter the point of cut-off in the engine cylinder. The position of the valve of the throttle valve determines the quantity of steam passing to the engine valve chest. If the quantity passing through the throttle valve be curtailed, expansion occurs (without any work being done) immediately the steam passes through the valve and before entry into the cylinder. By throttling, therefore, the initial pressure of the steam entering the cylinder depends upon the engine speed. In the alternative method the governor may be arranged to alter either or both the valve travel and the angular position of the eccentric sheave whilst the engine is running. The initial pressure of the steam entering the engine cylinder is constant but the quantity admitted per stroke is variable and depends upon the engine speed.

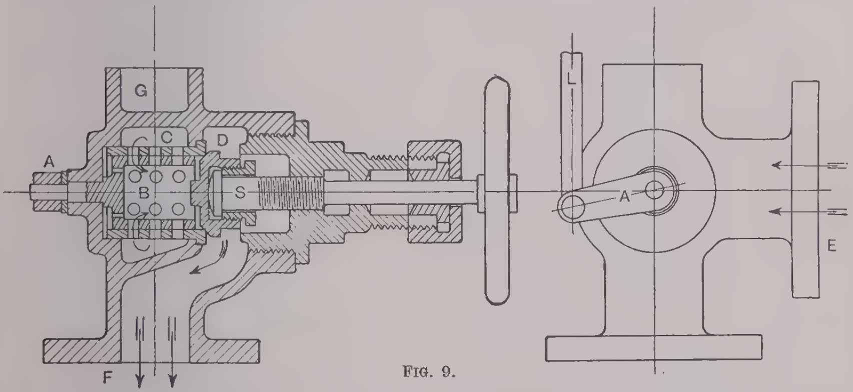

FIG. 9.

The throttling method of control is common for engines developing no great power. A simple throttle valve combined with a steam stop valve arranged within the one casing is shown by *Fig.* 9. One end of the link L is connected to the sleeve of a centrifugal governor, and the other end to the lever A for rotating a cylindrical throttle valve. The position of the governor balls and sleeve as before depends upon the speed of the engine. Any vertical displacement of the sleeve is communicated by the link to the lever and produces slight rotary movement of the throttle valve B within its casing C. As the perforations of the throttle valve and the throttle valve casing are coincident at normal speed engine, slight rotation of the throttle valve means reduction in available area through the perforations and therefore throttling of the steam. The governors for small throttle valves are usually belt driven, but as the belt drive is not a positive drive it is not suited for a close control of the engine speed. A safety device is sometines provided to prevent excessive engine speed in case of breakage of the governor belt. The steam main is connected to the flange E; flange F is connected to the cylinder valve chest. The arrows indicate the direction of flow of the steam when the stop valve is open. The governor base piece fits into the machined portion marked G. The combined stop and throttle valve complete with a pendulum governor are shown in position on the engine, *Fig.* 2, at Z and GR respectively.

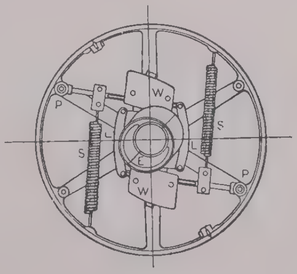

FIG. 10.

Fig. 10 shows a shaft governor arranged to alter the angular setting of the eccentric sheave and so control the cut-off. The gear is symmetrical in outline. Pivoted at P to a wheel mounted on the engine crank shaft is a pendulum weight and arm W. Between the pivot and the weight is attached a spring S. The other end of the spring is secured to the wheel. The eccentric sheave E is integral with a plate bored an easy fit for the crank shaft so that rotation of the sheave may readily be accomplished. A connecting link L is secured to the plate and the pendulum weight arm by pins. On the engine revolutions increasing beyond the normal, the weights under centrifugal action move outwards despite the resistance offered by the springs, and by means of the links slight rotation of the eccentric sheave ensues. An adaptation of this governor permits of simultaneous variation of the angular position and of the throw of the eccentric sheave.

Governing by altering the cut-off is sometimes performed by the gear shown in *Fig.* 11. Pivoted at P to the governor stand is a slotted link SL, to which is attached by a pin the eccentric rod ER. The angular position and throw of the eccentric is invariable. The end of the valve rod VR terminates in a pivoted block B capable of sliding within the slot of the link. The lever L is fulcrumed at A to the governor stand. Being attached to the governor sleeve at C, its angular displacement depends upon the speed of the governor. At D a supporting rod R is jointed to the other end of the lever L. The lower end of R is attached to the valve rod by a pin and carries the weight of the valve rod end. The angular displacement of the slotted link does not depend upon the speed of the engine; the longitudinal travel of the slide valve depends upon the position of the block within the slotted link, and as the position of the block is determined by that of the governor sleeve, the governor control is by varying the cut-off.

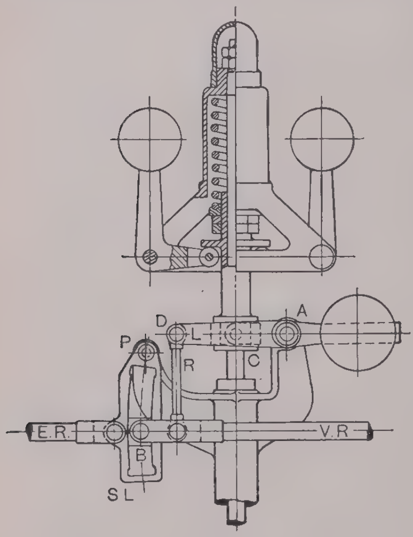

FIG. 11.

(vii.) *Reversing Gear.*—The most common form of gear for reversing the direction of rotation of an engine is the Stephenson Link Motion Reverse Gear. *Fig.* 12 illustrates this gear. Two eccentric sheaves keyed to the engine crank shaft are used, one being for ahead running and the other for running in the reverse direction. The eccentric rods ER are coupled to opposite ends of a slotted link S. A block B capable of sliding within the slot is pin-jointed to the valve spindle end VS. The slotted link is placed in any desired position with respect to the block by means of the pull rods R and the reversing shaft and lever L. The requisite motion for the reversing shaft may be provided by a screw or other convenient form of control. When the block is immediately in front of the one eccentric rod connection, the valve travel is due to that one eccentric sheave; and when placed immediately in front of the other eccentric rod connection the valve travel is due to the reverse direction eccentric sheave, and the engine runs in the reverse direction. At any intermediate position the valve travel is due to the sum of the effective motions of the two eccentrics. In addition to permitting of reversing, this gear allows of "linking up," *i.e.* altering the cut-off (whilst the engine is running) to cope with the conditions under which the engine is working. The Allan and Gooch Link Motions, modifications of the Stephenson, are not very frequently used; other gears, requiring but one eccentric sheave, or perhaps none, are sometimes employed.

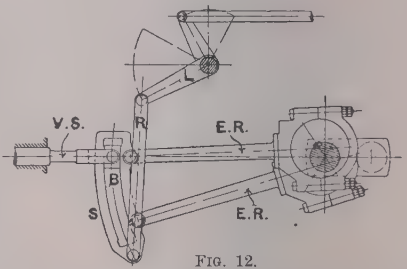

FIG. 12.

(viii.) *Clearance.*—The limit of the piston's stroke is some small distance from the cylinder cover nearest the piston. This small dimension, ranging from about one-quarter inch for small pistons to one inch for pistons one hundred or more inches in diameter, is called the "clearance," or the "mechanical clearance," of the piston. Clearance allows for slight irregularities in manufacture of the various parts of the engine as well as for subsequent slight variations in the relative positions of the various parts due to wear and readjustment, and temperature changes. The "clearance volume" is the volume due to the mechanical clearance plus the volume of the steam passage or passages in association with that particular end of the engine cylinder. The working volume of the cylinder is the volume swept per stroke by the piston, *i.e.* the cross-sectional area of the cylinder multiplied by the stroke of the piston. The clearance volume is usually somewhere between 7 and 15 per cent of the working volume.

(ix.) *Expansive Working.*—If steam be used expansively, *i.e.* if the cut-off be at some fraction of the stroke instead of at the completion of the stroke, the terminal volume of the steam is larger than the volume of the steam admitted, and correspondingly the terminal pressure of the steam is lower than the initial admission pressure. The actual volume of steam expanding in the engine cylinder is the volume admitted plus the volume of the residual steam in the clearance space, and the "actual ratio of expansion" is the volume swept by the piston — the working volume—plus the clearance volume, divided by the expanding volume at the point of cut-off. From the point of cut-off to the completion of the stroke the volume of the expanding steam gradually increases whilst correspondingly the pressure falls. The pressure of the steam leaving the cylinder is usually at a few pounds per square inch below the terminal pressure due to expansion.

By using steam expansively a greater amount of work is performed per pound of steam used than if the steam were admitted during the whole of the stroke. If a diagram be drawn giving the pressure of the steam at any instant during the stroke of the piston, the area of the diagram represents to some scale the work done per stroke of the piston. Such a diagram may be obtained from the actual engine cylinder by means of an "indicator," and the diagram is called an "indicator diagram." If the indicator and the gear for driving the indicator be in good condition and well designed, the resulting indicator diagrams faithfully record the conditions existing within the engine cylinder. Finality in steam-engine design has not been reached, and the constant endeavour of designers is to increase the output per unit of working volume of the cylinder. Assuming the length of the engine stroke and the diameter of the engine cylinder to be fixed, and saturated steam to be used, any increase in output must be due to some change resulting in an increase of the area of the indicator diagram or an increase of the number of revolutions per minute. Should increase of the latter be not allowable, for increased output the area of the indicator diagram must be increased, and this can be accomplished (if the cut-off remain unaltered) by increasing the initial pressure and lowering the back pressure. The lowering of the exhaust pressure is accomplished by causing the engine to exhaust into a vacuum. It must be remembered that to provide plant to create a vacuum is initially costly and also expensive in upkeep. Increasing the initial pressure introduces operating troubles inseparable from the use of high-pressure steam.

§ (3) COMPOUND ENGINES.—To use steam as efficiently as possible the heat content of the steam at entry to the engine cylinder must be high, and at the exit from the cylinder it must be low. The initial pressure therefore being high, since it is desired that the exhaust and terminal pressures should be nearly equal, the ratio of expansion is high. In a single-cylinder engine of usual design the early cut-off thus necessitated is very undesirable, but if the same ratio of expansion be performed by steps or stages in two or more cylinders many of the objections disappear. Such step or stage expansion is termed "compounding." The first step is performed in a "high-pressure cylinder," and the last step in a "low-pressure cylinder." If the engine be a two-stage engine it is called a "compound engine"; if three stages are used the engine is called a "triple expansion engine" and the intermediate stage is performed in an "intermediate pressure cylinder"; if four stages are used the engine is called a "quadruple expansion engine" and the intermediate stages are performed in "first intermediate pressure" and "second intermediate pressure" cylinders respectively. Sometimes it is considered advantageous to perform the work of the high-pressure cylinder (or perhaps the low-pressure cylinder) in two cylinders. In marine service four-cylinder triple expansion engines are not uncommon. In such engines there are two low-pressure cylinders collectively doing the work which ordinarily one low-pressure cylinder would do in the more common three-cylinder engine. Compound locomotives sometimes have two high-pressure cylinders exhausting into one low-pressure cylinder, and although there are three cylinders the engine is not a triple expansion but a three-cylinder compound locomotive. Sometimes the cylinders are arranged "tandem," *i.e.* the piston rod of one cylinder is directly connected to the piston of the adjoining cylinder, so that only one crank and connecting rod is required for the two pistons and piston rods. The ordinary single-cylinder engine is called a "simple" engine, and if two such engines are arranged on the same frame and drive a common shaft the arrangement is said to be "twin simple engines" or a "twin engine."

§ (4) THE CONDENSER. — In compound, triple, and quadruple expansion engines the terminal pressure in the low-pressure cylinder is usually between six and four pounds per square inch absolute. A lower pressure than this is not usually desired because of the increase in size of the low-pressure cylinder due to the rapid increase in volume of the steam at the higher vacuum. The steam is condensed in a "condenser," and the pressure in the condenser is slightly less than in the engine cylinder. The type of condenser used is determined largely by the amount of water available for condensing the steam. In marine service, where there is unlimited water to be had, the surface condenser naturally is used. In essence, the surface condenser consists of a battery of small-bore tubes, usually about three-quarters of an inch in bore, arranged within a casing so that on one side of each tube the exhaust steam circulates whilst the cooling water passes on the other side. The condensing water and the condensed steam are thus kept separate and distinct from one another. For land service where the amount of water for condensation of the exhaust steam is strictly limited, an evaporative form of surface condenser is sometimes used. In this condenser the steam is condensed within the tubes by a small quantity of water flowing across their exterior surfaces. In the class of condenser commonly used for land purposes the steam and condensing water mingle, and the condenser is called a "jet condenser." There are two distinct varieties of jet condensers — the "parallel flow" and the "counter flow." In the parallel flow the steam and the condensing water both enter at the same end of the condenser, whilst in the counter flow the steam enters at the bottom and the condensing water at the top of the condenser. The counter flow is the more efficient of the two arrangements.

The air leakage past the gland where the low-pressure piston rod enters its cylinder is considerable, and, together with the air passed over from the boiler with the steam, necessitates the continual use of an "air-pump" in order that a vacuum may be obtained and maintained. Generally, in addition to removing the air from the condenser the air-pump removes the water of condensation from the surface condenser, or the water of condensation plus the injection water from the jet condenser. Usually the air-pump is driven through the medium of levers and links from the engine cross-head or piston rod. In horizontal engines the condenser (with its pumps) is generally arranged under the engine in order that the exhaust pipe may be of minimum length. Long exhaust pipes are inefficient and undesirable. For the same reason the condenser in vertical engines (so common in marine service) is mounted on the engine framework as close as possible to the low-pressure cylinder. The condensing water is circulated by a "circulating pump" sometimes driven by the engine or sometimes independently.

§ (5) VALVES.—With the simple D slide valve of *Figs.* 4 and 5, independent control of the cycle of events is not possible. Should the valve be set to give an early cut-off, an early release and compression also occur. To permit the timing of the admission portion of the cycle to be altered without affecting the timing of the exhaust has been the object of many schemes. The arrangement shown in *Fig.* 13, called the "Meyer expansion valve," has been much used to this end. Two complete sets of eccentrics with rods and valve spindles are necessary. The main valve is driven by its spindle D and reciprocates in the usual manner over the three passages of the cylinder valve face. The main valve is in essence a simple slide valve with the addition at each end of a steam port S. As in the simple slide valve the steam lap is the amount L. The lap L controls the passage leading to the left end of the engine cylinder, but the separately reciprocated expansion valve A controls the steam supply to the corresponding main valve port. Hence the steam supply to the engine cylinder is determined by A and B, and the exhaust from the cylinder is determined by the main valve exhaust edges F and G. Varying the cut-off whilst the engine is running is readily achieved by providing right- and left-hand screw threads on the valve spindle C, and arranging that C may be rotated slightly without affecting its reciprocation. Partial rotation of C either closes or separates somewhat the parts A and B, and therefore alters the period of opening of the ports S to steam. The screw threads are of the same pitch, *i.e.* rotation of C produces equal longitudinal movement of A and B, but in opposite directions.

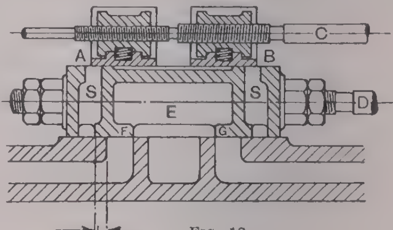

FIG. 13.

It is common practice in large stationary engine design to discard the slide valve and substitute in its place four valves, two valves for controlling the steam supply to the cylinder and two valves for regulating the exhaust from the cylinder. A great advantage of such an arrangement lies in the curtailing of the waste clearance volume due to long steam passages. Whether the four valves are arranged in the cylinder covers or in the cylinder barrel itself, the length of the passage from the valves to the cylinder barrel is short. In horizontal engines it is usual to place the steam valves on the top and the exhaust valves underneath the cylinder barrel. If reasonable care is taken in the design of these valves, very efficient drainage for any steam condensed within the cylinder and its passages is possible. It is preferable to use separate valves for the high- and low-temperature steam rather than one valve for alternating temperatures. For the four-valve scheme two distinct forms of valves and gears have been devised, and each has given its name to reciprocating engines using that special form. Engines fitted with rocking valves are named after the originator of such valves in their application to steam engines, and are called "Corliss engines." Engines employing the other form are known as "Drop-valve engines" because a drop valve is used.

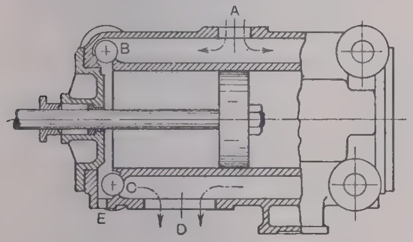

FIG. 14.

(i.) *Corliss Valves.* — A typical cylinder barrel with covers for a Corliss engine is shown in *Fig.* 14. Part of this figure is in longitudinal section and part in outside view. The closeness of the valves to the working volume of the cylinder is very marked. At A the steam enters a steam belt running longitudinally above the cylinder barrel. This belt terminates at each end in a casing B containing the steam valve for that end of the cylinder. The exhaust valves are arranged to work within similar-shaped casings C, and the two exhaust casings are connected by a longitudinal belt underneath the cylinder. The exhaust pipe is connected to a suitable facing D on the exhaust belt. At E drainage provision is made. Cross-sections of the valves are shown in *Fig.* 15. The steam valve at the point of admission is shown at (*a*), and full open to steam at (*b*). The exhaust valve is shown at point of release at (*c*), and full open to exhaust at (*d*). The valve spindle attached to each of the valves protrudes through a steam-tight stuffing-box with gland, and is fitted with a lever, as shown dotted in the four sketches.

Each of the two sets of rocking valves is worked by an eccentric. This permits of ready variation in both steam and exhaust cycles. In one arrangement of the valve gear the exhaust valve levers are connected by links to a circular plate mounted on a spindle fixed to the cylinder about midway between the valves. The exhaust eccentric rod is also coupled to this circular plate and gives to it a vibratory motion which is transmitted through the connecting links to the rocking exhaust valves. For the steam valves a similar plate is used, and the eccentric for the steam valves is coupled to the plate to provide the desired motion. The steam valves work under the joint influence of the steam valve eccentric and a governor. The mechanism is complicated, and is such that the cut-off is always rapid. Governing is by variation in cut-off. Large vertical engines have been constructed in addition to the more common horizontal pattern for driving mills, factories, electric-light plant, etc., and have proved successful in normal working. Given that the design is satisfactory, sweet and efficient running of the Corliss engine depends upon the care with which the numerous adjustments are made and maintained. If not given adequate attention the working is noisy.

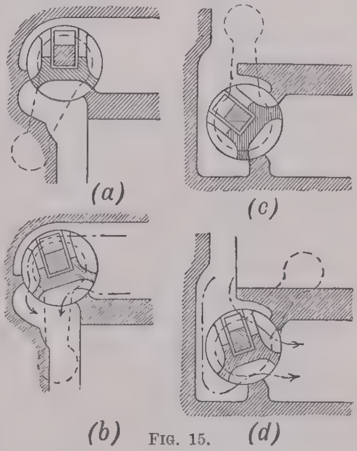

FIG. 15.

(ii.) *Drop Valves.* — The valves and valve operating mechanism of the drop-valve form of engine differ in principle from those already described. To allow steam to pass, the valves are raised bodily from their seats, closing being effected by dropping the valves on to their seats. Four valves are required per cylinder—two steam and two exhaust valves. The operation of these four valves is through the agency of a "lay shaft." The axes of the lay shaft and the engine cylinder are parallel, and the lay shaft extends almost the whole length of the engine. The motion of the lay shaft is one of rotation, and the speed of rotation is usually the same as that of the engine crank shaft. As the lay shaft is arranged at right angles to the crank shaft, rotation is transmitted either by bevel wheels or skew gearing.

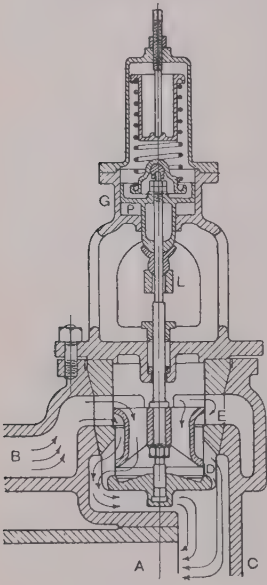

Fig. 16.

The commonest form of lift valve is that shown at D, *Fig.* 9. The surface of the valve actually in contact with the valve seat when the valve is closed is small, in order to obtain steam-tightness. The angle of the valve seat is 45°. If the pressures above and below the valve when the valve is closed are not equal, some form of gear is required either to keep the valve in position or to open the valve when desired. In *Fig.* 9 a screwed spindle S is provided for both purposes. The unbalanced steam lead on the valve may be, and in fact usually is, considerable. For some purposes the lack of balance of such steam leads is not of great moment, but in valves controlling the steam distribution of an engine making perhaps 200 revolutions per minute any considerable lack of balance is prohibitive, since the operating gear would be subjected to unnecessarily large loads. The balanced double-seated valve of *Fig.* 16 has been designed to overcome these operating difficulties. The seats are at D and E, and the valve is shown full open. The cover securing in position the valve cage also carries the gear for imparting the requisite motions to the valve at the correct intervals. The nearness of the valve to the cylinder bore A and the cylinder cover C is noticeable, and the waste clearance space is correspondingly small. Difficulty has been experienced in keeping steam-tight the valve faces of double-beat valves, and as a result modifications to the design shown have been introduced, but the balancing of the steam leads is always kept in view. As in the Corliss engine, steam to the valve comes by way of a longitudinal belt B. In an engine running at 180 revolutions per minute the period for steam admission, if the cut-off be at one-quarter stroke, is one twenty-fourth of a second. During this short interval the valve must both open and close. A positive mechanism is imperative, therefore, for operating the valves. Usually the valves are pressed open by some form of cam gear and forcibly closed by a strong spring. The operating forces are clearly in the nature of a hammer blow and the gear might readily be described as percussive. The necessity for the elimination as far as possible of all unbalanced loads is clear. If the design of the gear is not theoretically sound no amount of care and attention by engine attendants can prevent noisy working, severe wear and tear, and unreliability in service. The hammer blow of the closing valve is softened considerably by trapping air under the piston P working within the spring cylinder G. Sometimes oil is used instead of air. The amount of cushioning effect may readily be controlled by a small valve not shown in the illustration. The end of the lever for pressing open the valve is lettered L. The valve gear is not shown. An eccentric complete with its strap and rod is necessary for operating each valve. The four eccentric sheaves are mounted on the lay shaft. Governing is accomplished by varying the point of cut-off. Either a shaft governor or the more common pendulum governor may be used for controlling the admission period. The exhaust valves are not governor-controlled. The remainder of the engine presents no unusual features other than that the crank and connecting rod are enclosed and that lubrication of the bearings is forced instead of the customary oil-box and cotton-wick method being used. The oil in the simple common method is transferred from the oil-box to the bearing by capillary action. Grooves cut in the bearing allow the oil to come into contact with the journal within the bearing.

Some manufacturers prefer to substitute gridiron slide valves for the exhaust drop valves. Essentially a gridiron slide valve is a metal plate perforated with perhaps three slots. This plate slides upon a valve face perforated with similar slots, thus allowing communication with the exhaust to be established when desired. By the use of several slots a considerable opening to exhaust is possible with but a short valve travel. The gridiron valves are driven by eccentrics from the lay shaft, one valve being provided at each end of the engine cylinder. With this arrangement also the waste clearance volume is small and the cylinder drainage is automatic.

§ (6) UNA-FLOW ENGINE.—The general appearance of the drop-valve engine is not very dissimilar to that of the una-flow engine illustrated by *Fig.* 17. The cylinder of the una-flow engine is almost twice the length of the cylinder of the corresponding drop-valve engine.

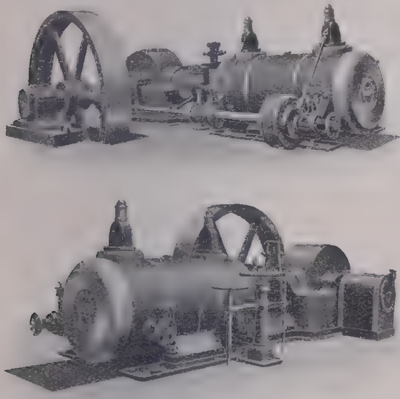

FIG. 17.—Una-flow Engine. Designed and built by Messrs. Robey & Co., Ltd., Lincoln, England.

In the engines so far described, each end of the cylinder is alternately heated by incoming steam and cooled by outgoing steam. The four-valve scheme is preferable to the D slide valve in this respect. A further modification of the four-valve arrangement is to eliminate the mechanically operated exhaust valve and so make the cylinder that the steam flow is always in one direction. Because of the steam flow being always in the one direction such engines are called "una-flow" (sometimes "uni-flow") engines. The general appearance of a una-flow engine is illustrated in *Fig.* 17. The engine is a single-cylinder engine of about 100 horse-power at about 180 revolutions per minute. Nearly all the working parts are completely enclosed. Steam admission is controlled by drop valves worked by eccentrics from a lay shaft, as in the drop-valve engine. The drop valves are arranged in the cylinder covers, as may be seen from *Fig.* 18. The waste clearance volume is very small, and the total clearance volume is about 2 per cent of the working volume. There are no exhaust valves in the usual sense of the expression. The cylinder barrel and piston are unusually long. The length of the piston is about 90 per cent of the length of the piston stroke. At the centre of length of the cylinder barrel is an exhaust belt. Access to this belt from the cylinder barrel is obtained through a number of slots cut in the circumference of the cylinder barrel. The length of each of these slots is about 10 per cent of the length of the piston stroke.

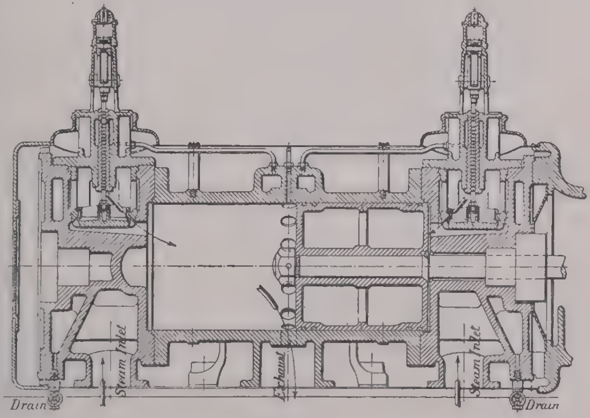

FIG. 18.

In a single-cylinder una-flow engine the ratio of expansion is as high as in a quadruple expansion engine. Consequently the cut-off is very early, being rarely later than one-tenth of the stroke. When the piston has nearly reached the end of its stroke it uncovers the exhaust slots and allows an almost unrestricted escape for the used steam. On completing about 10 per cent of the reverse stroke the piston closes the exhaust slots and such steam as is trapped is compressed. Compression continues until the end of the stroke is nearly reached and the admission valve is ready to open again. The compression period is very long and the maximum compression pressure correspondingly very high. The indicator diagram obtained from the una-flow engine cylinder is strikingly different from that usually associated with steam-engine practice. In the normal design of una-flow engine a high vacuum is necessary to prevent excessive compression pressure. A jet condenser is usually provided and is placed immediately under the engine cylinder. The exhaust slots are so large in area as to offer practically a negligible resistance to the exhaust steam, and the vacuum in the engine cylinder is practically that in the condenser. To provide against damage due to excessive rise in compression pressure, should there be leakage past the admission valve or a fall in vacuum, spring-loaded relief valves are provided. Provision is made for running the engine temporarily non-condensing by throwing into communication with the bore of the cylinder an additional clearance space in each cylinder cover. These additional clearance spaces are controlled by valves.

Superheated steam is generally used for una-flow engines. The piston under the action of high temperature needs lubrication. A mechanical sight-feed lubricator is provided for the purpose. Lubrication of the main bearings, connecting rod, and cross-head is forced, the oil being circulated by an oil pump driven by the engine itself.

In *Fig.* 17 a large cylindrical casing around a portion of the lay shaft and almost touching the front eccentric will be noticed. This casing encloses a shaft governor capable of fine speed regulation. It moves the eccentrics and so varies the cut-off to suit the load on the engine. The handwheel at the extreme end of the lay shaft is for speed adjustment by altering the governor conditions whilst the engine is in motion.

Uniformity of turning moment is usually a requisite in engineering practice. As has been shown by *Fig.* 3, a single-cylinder engine gives a maximum cyclical variation and range. The flywheel therefore must be much heavier than would be necessary for a multicrank engine developing in the aggregate the same power as the single-cylinder engine. Although the flywheel of the una-flow single-cylinder engine is extraordinarily heavy, the una-flow engine is cheaper to build and operate than the multicylinder engine. Una-flow single-cylinder engines have been used for driving alternators direct-coupled to the engine crank shaft to run in parallel where the permissible cyclical variation of turning is extremely small.

§ (7) HIGH-SPEED ENGINES.—The so-called "high-speed engine" is due to the demand of electrical engineers for an engine to drive when coupled direct to electric generators. Its correct designation is "quick revolution," as the mean piston speed due to the short stroke employed is comparatively slow. Its great success for this exacting service led to its application elsewhere, and although the steam turbine has practically superseded the quick-revolution engine for electrical purposes, the engine, nevertheless, is in demand as a convenient, reliable, and moderately economical prime mover. It is usually vertical, and therefore possesses the advantage of requiring but little floor-space. It is very compact, of no great weight per unit of power developed, and is made either as a simple, compound, or triple expansion engine, using either saturated or superheated steam. This form of engine is conducive to high efficiency, but if high efficiency is to be maintained the engine must be kept in good running condition. Quick-revolution engines will not work satisfactorily under conditions of neglect such as are associated too frequently with the running of the slow-revolution engine.

The difficulties of adequately lubricating the moving parts of the high-revolution engine were not satisfactorily solved until the enclosed form of engine was designed. This type of engine is illustrated by *Fig.* 19. All the working parts are enclosed in an oil-tight case, and usually all that is in sight outside the case is a small piece of the piston rod and of the valve rod. Lubrication of the various bearings is effected under pressure. The maximum oil pressure used is about thirty pounds per square inch. A small plunger-pattern oil pump driven from the end of the crank shaft circulates the oil through passages and pipes. The bearings are flooded with oil. The oil escapes at the ends of the bearings and, dropping into the well formed by the crank case, passes through some form of cooler and is strained before entering the oil pump to continue its journey once more to the bearings. If each bearing is to receive its fair share of lubricant, all the bearings must be finely and uniformly adjusted. When the engine is running, nothing of the oiling provision is visible except an oil-pressure gauge. The quantity of oil in the oiling system is usually such that the crank and the lower end of the connecting rod pass through the store of oil in about one-sixth of a revolution. This materially aids the lubrication of the crank pin. There is an inevitable leakage of condensate where the piston and valve rods enter the casing, and the water finds its way into the crank case. Provision should be made (if it has not been done by the engine builders) for periodically removing this water and also the water of separation from the lubricating oil placed in the crank case. If such be not made, the crank churns the oil and water into a viscous mixture of about the consistency of cream and of little value as a lubricant, and the pressure in the oiling system drops to an unsafe figure, about five pounds per square inch. If the water be removed periodically, the same oil may be used in the crank case for many weeks running. A valveless oil pump is preferable.

If a quick-revolution engine runs at, say, seven times the speed of an ordinary engine of equal power—and this is not unusual—the time for heat exchange between the steam and the cylinder walls is one-seventh that of the ordinary engine. When running at 420 revolutions per minute the period of exhaust occupies less than one-fourteenth of a second. As the time for heat flow is so short the cylinder condensation due to heat flow is correspondingly small.

Because of the high number of revolutions the size of the engine cylinder is relatively small. This contributes to a minimum heat loss due to radiation per pound of steam entering the engine, and therefore the high-revolution engine is again preferable. Additionally, the amount of covering provided on the hot surfaces in high-revolution engines is in marked contrast to the almost unclothed cylinders so common in slow-revolution engine practice. The cylinders, pistons, and valves are arranged to be self-draining and water pockets are eliminated. When arranged for using saturated steam a "separator" is provided to drain as much as possible of the moisture held in suspension in the steam before the steam enters the valve chest.

Steam distribution is usually controlled by a "piston valve." A piston valve is a D slide valve in which the flat valve face is bent into a cylinder. Piston valves are free from friction due to steam load and occupy small space. The valve gear is of an unusually substantial construction because of the rapidly alternating loads to which it is subjected. Enclosed engines are not arranged for reversing when running, but often, especially in single-cylinder engines, the eccentric sheave is so arranged that only a few minutes' work is required to set it for running in the reverse direction.

Usually a governor is arranged with its axis horizontal, and is placed at the free end of the crank shaft. The governor presents no special features other than that it works in a horizontal position. The hinged weights under the control of the powerful springs occupy definite positions at definite engine speeds. The position of the weights is communicated to the sleeve and through the medium of a lever and rod to a double-beat governor control valve placed at the steam entrance to the valve chest. The position of the valve determines the quantity of steam passing the valve, and control of the engine speed is therefore by throttling. With this form of governor control it is possible to work with a maximum momentary variation in speed of but 2 per cent and a permanent variation of less than 1 per cent when full load is suddenly removed from the engine. With the exception of the weights and springs the moving parts of the governor gear are very light and easily operated. A heavy flywheel is essential, particularly for the single-cylinder engines. A tachometer or revolution indicator is provided, driven from the engine shaft by a belt.

§ (8) THE LOCOMOTIVE.—The present-day locomotive engine is a compact self-contained steam plant remarkably powerful for the space occupied, whilst possessing exceptional flexi-

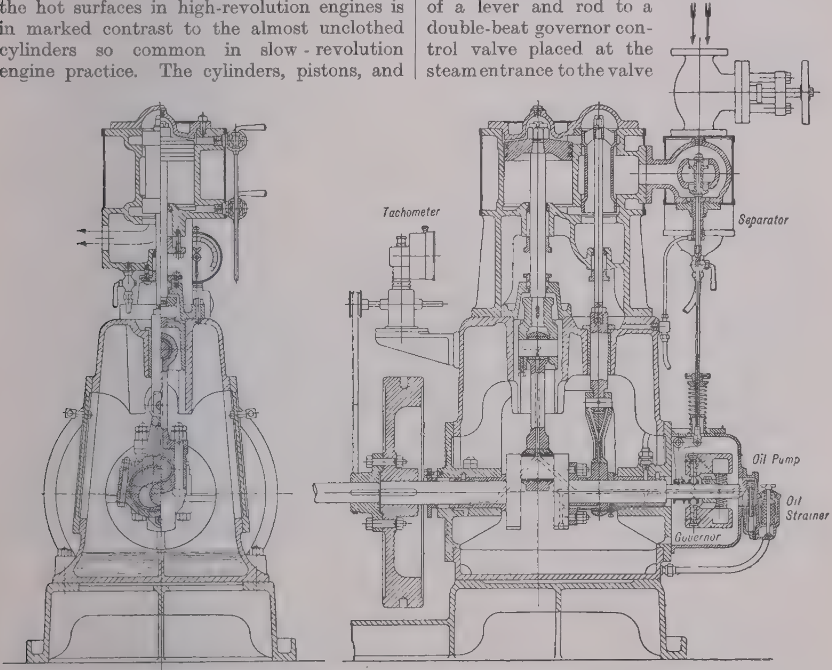

FIG. 19.—Enclosed Steam Engine. Built by Messrs. Alley & MacLellan, Ltd., Glasgow, Scotland.

bility in operation. The form of steam generator used and the general disposition of the engine parts still bears the stamp of Stephenson's genius. Whilst in other branches of steam engineering the compound engine has proved its right to the first place, in locomotive practice generally the simple engine prevails. The conditions under which locomotives must work are unusual and severe. Minute by minute almost, the conditions change as different gradients are encountered, and often extreme acceleration or retardation occurs at short intervals. The conditions of service debar the use of a condensing provision. Therefore for locomotive service the simple engine has survived.

The steam boiler is secured at one end to the frame plates; at the other end it is supported, so allowing freedom for the change of length due to temperature variation. The engine cylinders are secured to the same frame plates at the anchored end of the boiler. This avoids racking the steam pipe between the boiler and the engine cylinders. If the cylinders be placed within the frame plates the locomotive is called an "inside cylinder engine"; if placed outside the frame plates the locomotive is called an "outside cylinder engine." The exhaust from the cylinders is used to augment the draught and therefore promotes rapid steam generation. If the exhaust were not used for draught acceleration, a much larger and heavier boiler would be required. The pistons, piston rods, cross-heads, and connecting rods are similar to those used in simple land engines, although, owing to restricted space, the detail is different. The locomotive engine is not provided with a governor. By an intelligent use of the reversing gear and the steam admission valve or "regulator" the engine power can be easily adjusted by either or both throttling and varying the cut-off to suit the constantly altering running conditions. The double-eccentric reversing gear, as illustrated in *Fig.* 12, together with a simple D-slide valve is commonly used, but many variations in valve gear are to be found.

Keyed to the crank shaft, or "crank axle," are the two "driving wheels." The driving wheels take the place of the flywheel or flywheels of the stationary engine. The rim of each wheel is provided with a tyre having a cross-section suitable for the rails on which the wheels roll. Adhesion between a tyre and a rail depends upon, amongst other factors, the weight pressing the two together. In order to take advantage of the weight of the locomotive, and in view of the weight upon any one axle being restricted to some certain amount depending upon the construction of the track, etc., the wheels of two or more axles are usually coupled together. The coupled wheels act as supplementary driver wheels. To enable the wheels to be coupled, they are provided with crank pins, and the coupling rods connect these crank pins and cause all the wheels to revolve together. In the common two-cylinder locomotive the engine cranks are arranged at right angles to one another. This is beneficial, not only because it reduces the fluctuation of the twisting moment, but because it enables the engine to start from almost any position in which it may have come to rest. The axles of the coupled wheels other than the crank axle are plain axles.

A marked difference between the stationary and the locomotive engine is that in the locomotive the crank-shaft bearings — "axle boxes"—and the bearings for the other axles are not rigidly fixed to the engine frame. Each bearing is spring supported. Whilst constrained horizontally the spring support allows the axle box a certain limited movement vertically. A typical axle box with its spring support is shown in *Fig.* 20. The axle box slides vertically within the guide bolted to the engine frame. The spring absorbs shocks due to inequalities in the track.

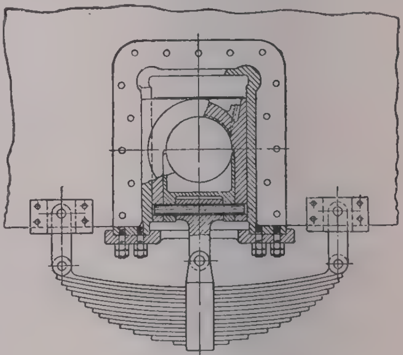

FIG. 20.

Although an engine speed of about 260 revolutions per minute is quite common, and although the conditions respecting dust and grit are anything but ideal, the simple means used for lubricating the axle within the axle box is effective. In marked contrast is the care and elaboration of detail found necessary for stationary engines running at similar speeds. Only the upper portion of the axle box comes in contact with the axle. The lower portion of the axle box is provided with a hollow "keep" which acts as an oil container. A felt lubricating pad is lightly pressed against the axle by slender springs, and as the pad is partly immersed in the oil in the keep the surface in contact with the axle is well supplied with lubricant. Additionally, oil is conveyed from an oil box formed in the top part of the axle box to suitable oiling grooves cut in the bearing surface in contact with the axle.

A six-coupled, simple, inside-cylinder, side-tank locomotive is shown in *Fig.* 21. An unusual degree of flexibility is given to the axle boxes for the trailing axle (under the coal bunker) to permit the locomotive to negotiate curves readily. Whilst but a moderate sized engine, this locomotive is typical of a class in great demand, under certain conditions, for home railway service.

§ (9) Marine Engines.—The modern marine reciprocating steam engine at first view does not appear to have much in common with the simple engine already described. Its difference in form and detail is solely due to the endeavour to satisfy the requirements peculiar to the use of reciprocating engines for the propulsion of ships. The form of cross-section of the vessel, the necessity of having the crank shaft relatively close to the keel of the vessel in order to secure immersion of the propeller when the vessel is running without cargo, the desirability of being able to run continuously for many days without impairing the ability of the engine to reverse instantaneously on demand, the ability to obtain unlimited supplies of cooling water for use in condensers, and the inability to obtain, other than to a very limited extent, fresh water for use in steam boilers—all these, as well as other factors, have influenced the moulding of the simple form of reciprocating engine into the marine engine of to-day. The largest reciprocating steam engines ever built have been for marine service, and the average size of engine in use at present on board ship is much larger than used elsewhere. A "tramp" steamer equipped with a two thousand horse-power engine is very common; an engine of this size for land service is rare. Whilst for certain marine services the reciprocating steam engine has given place to the geared steam turbine, this is not because of mechanical defect or inability to perform exacting duty. The average engine is remarkably reliable and frequently works under the most adverse conditions. For "tramp" steamers the reciprocating steam engine holds the premier position.

Triple expansion engines are customarily used, although quadruple expansion engines with higher steam pressures and superheated steam are not uncommon. The engines are vertical with the cylinders arranged immediately above the crank shaft. A typical engine is shown in *Fig.* 22. The rear columns for supporting the cylinders are of cast iron and the front columns are of wrought steel. The engine is therefore more open for inspection when running, and is less massive in appearance than when the front columns are of cast iron also. The condenser is at the back of the engine, and is carried upon brackets forming part of the rear columns. The sea water used for condensing the exhaust steam passes through the nests of small-bore tubes forming the condensing surface, and the steam is condensed on the exterior surfaces of these tubes. For circulating the condensing water a plunger pump is usually provided. This pump, together with the air pump and sometimes also feed and bilge pumps, is driven from one of the cross-heads by means of a lever and links. The pumps are arranged under and at one side of the condenser. The duty of the air pump is to remove from the condenser such air as inevitably finds its way in amongst the exhaust steam through the piston rod and perhaps valve rod stuffing-boxes, and also the condensed steam.

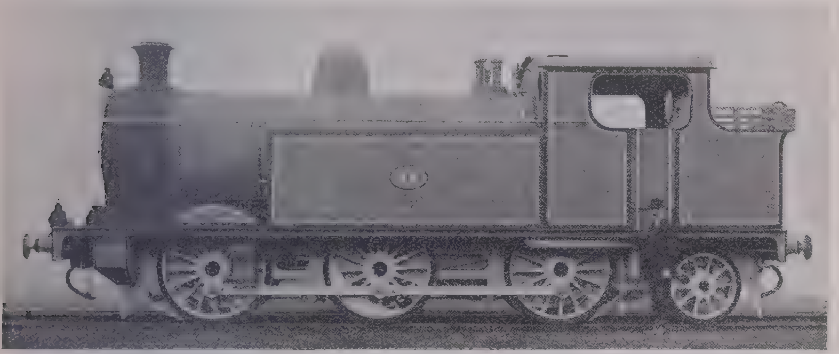

Fig. 21.—Six-coupled Tank Engine for the Brecon and Merthyr Railway. Built by Messrs. Robert Stephenson & Co., Ltd., Darlington, England.

The left end of the crank shaft of *Fig.* 22 is coupled to the line shafting terminating in the propeller shaft. The propeller screws its way through the water and, in doing so, transmits a considerable thrust to the ship. Adjoining the engine is a thrust-block bearing in which a number of collars formed on the thrust shaft, the shaft directly coupled to the crank shaft, bear against an equal number of fixed bearing surfaces, so transmitting the thrust of the collars to the thrust block and thence to the ship itself. The crank shaft is in three sections. To facilitate overhauling of the engines when in port, a small simple engine is provided at the left end of the main engine. The small engine drives a worm which, by gearing into a suitable wheel secured to the crank-shaft coupling, is able to slowly rotate the main engines to any desired position for inspection.

The ordinary forms of governor used for land engines are unsuited for marine service. A special inertia governor is sometimes employed to check the "racing" of the engines, which occurs if the propeller be momentarily lifted out of the water by the pitching of the vessel during rough weather. This governor operates a disc throttle valve placed in the main steam pipe close to the engine. Many engineers prefer hand control of the throttle valve and do not fit a governor. In the illustration the vertical rod to the right of the right column is connected at the upper end to the small lever of the throttle valve. At the lower end of the vertical rod is a hand lever. No governor is provided.

The Stephenson pattern link motion is fitted for reversing. The detail is slightly different from that of *Fig.* 12. The reversing shaft is at the back of the engine. It receives vibratory motion from a small steam engine placed on the main engine bed-plate and midway between the middle and right front columns. The handwheel of the engine is specially

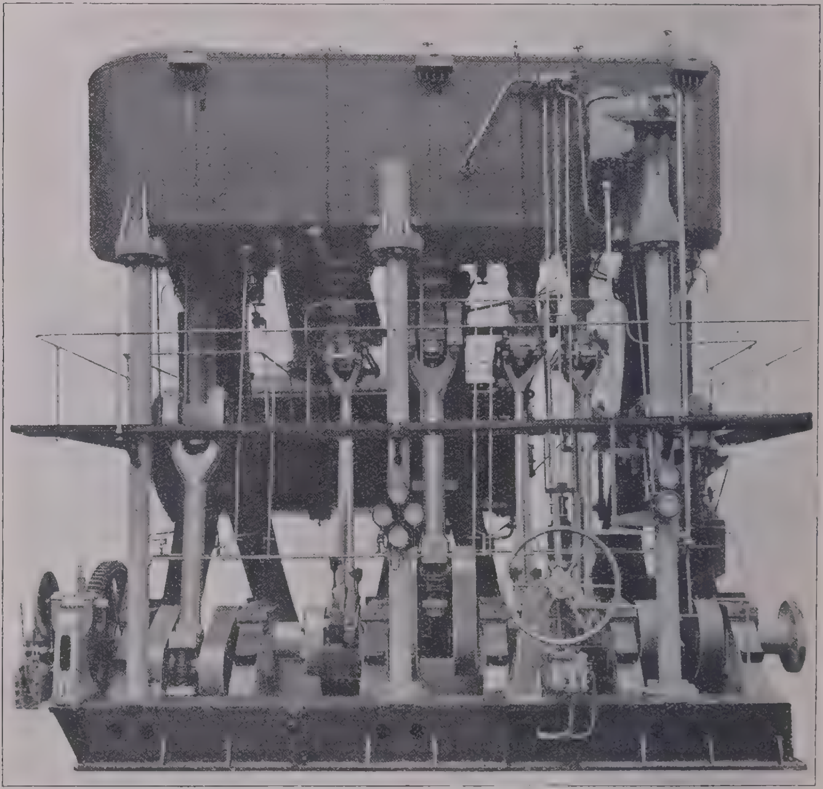

FIG. 22.—Triple Expansion Marine Engine. Designed and built by Messrs. The North-Eastern Marine Engineering Co., Ltd., Wallsend-on-Tyne, England.

prominent. This simple engine, through a worm, drives a wheel to which is fixed a crank pin. A rod connects this crank pin with the arm fixed to the reversing shaft. The arrangement is such that whilst the crank pin describes a circle the end of the lever of the reversing shaft vibrates through about 90°. As there are no stops for the reversing links to strike against, this gear is very convenient. It is called the "all round" reversing gear. The engine as a whole can be linked up by the reversing gear, as in the locomotive. Sometimes it is considered advisable to alter independently the cut-offs in the three cylinders, and a simple provision is made for this to be done without interfering with the possibility of immediately reversing the engine as a whole.

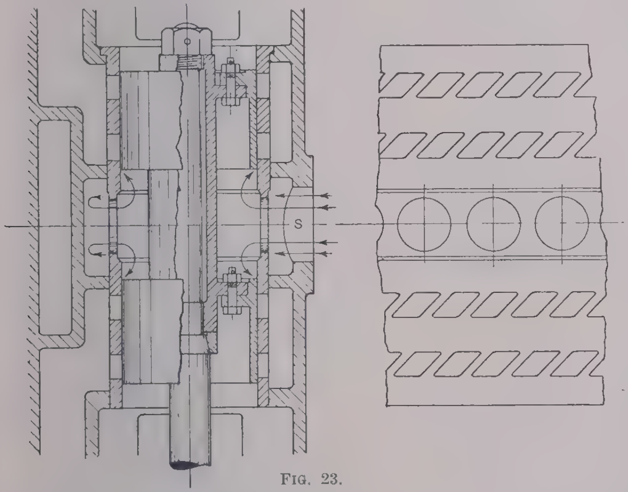

FIG. 23.

The high-pressure cylinder is fitted with a piston valve. A typical piston valve is illustrated in *Fig.* 23. The steam supply enters at S. The exhaust edges of the valve are the outside edges in this instance. The arrangement shown keeps the high-temperature steam away from the valve rod stuffing-box. The valve works within a liner which is fitted for ease of renewal in case of wear. A partial development of the liner is shown, from which the slope of the connecting bars between the port slots will be noticed. The angle is arranged so that ridges will not be formed on the surface of the reciprocating valve. The separate steam and exhaust openings cut in each end of the valve liner allow for a long guiding surface to be given to the valve. In principle the valve is a simple D-slide valve. The valves for the intermediate and low-pressure cylinders are double-ported, flat-slide valves, *i.e.* valves arranged to give a large opening for a small travel.

§ (10) NEWCOMEN'S ENGINE.—The first successful and practical reciprocating steam engine was invented and constructed early in the eighteenth century by Thomas Newcomen. The steam pressure was low—rarely, in fact, did it exceed that of the atmosphere. Rapid development was delayed because of inability to obtain steam generators capable of working at pressures above that of the atmosphere. In the early days of the Newcomen engine an attendant was required to operate the control valves, but eventually the engine was made self-acting by introducing valve-operating rods attached to the overhead beam. About the year 1711 the Newcomen engine began to be used for the pumping of water from mines, and for about three-quarters of a century it was the best prime mover in this country.

Fig. 24 shows schematically the Newcomen engine. The piston is shown at the top of its stroke. Steam is generated in the boiler A, whence it passes through the control valve V, when required, into the cylinder C. When the cylinder is full, the control valve V is closed and a jet of cold water is sprayed into the cylinder through the spray pipe S. The steam in consequence is condensed and a slight vacuum (dependent upon the fit of the piston in the cylinder) is formed under the piston P. The pressure of the atmosphere, acting directly on the exposed surface of the piston, forces the piston to the bottom of the cylinder. The condensed steam and the injection water escape from the cylinder by the escape valve E and pass into the feed-water tank F. At opposite ends of the oscillating beam B are attached by chains the weighted mine pump rods M and the piston. The weights of the parts attached to the beam are so arranged that on completion of the down stroke, when steam is admitted again to the cylinder, the piston is readily taken to the top of its stroke.

No attempt is made in the Newcomen engine to use the expansive properties of steam, the function of the steam used being merely to facilitate the formation of a vacuum. Leakage of air into the cylinder is prevented by sealing the upper surface of the piston with a layer of water. The heat wastage due to the alternate use of the cylinder as a steam receptacle and as a condenser is enormous, and the thermal efficiency of the engine is correspondingly small. The Newcomen engine is a single-acting atmospheric engine.

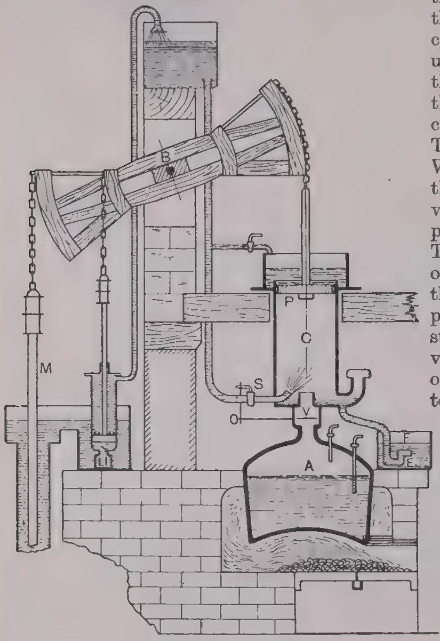

FIG. 24.

§ (11) WATT'S ENGINE.—James Watt discerned its inherent defect when repairing a model of the Newcomen engine for Glasgow University. In 1769 he patented his improvements, and in his specification laid down basic principles which to the present time have determined the development of the steam engine. Although his patent was of far-reaching importance, for a few years it resulted in nothing more than an improvement of the Newcomen type of engine—still single acting, with steam carried the full stroke, only suitable for pumping, but able to make an increased number of strokes per minute and less wasteful of heat than formerly. His separate condenser was generally worked by injection, although he saw the potentialities of the surface condenser and even made a model condenser similar in essentials to those used with modern marine engines.

Fig. 25 shows the Newcomen engine as improved by Watt in 1769. The engine is single acting, but the top of the cylinder is closed. The closing of the cylinder top and the providing of a steam jacket surrounding the cylinder body keeps both piston and cylinder warm. The steam acting on the upper surface of the piston takes the place of the atmosphere in the Newcomen engine. Only the lower end of the cylinder is allowed to come into communication with the condenser. Three valves are used to control the steam; V_1, V_2 and V_3. V_1 is the steam valve, V_2 is the equilibrium valve, and V_3 is the exhaust valve. All three valves are operated by a plug rod P and tappet levers (not shown). The plug rod receives its motion from the overhead beam B. The method of operating the engine is as follows, commencing with the piston at the top of its stroke and ready to start on the down stroke: the equilibrium valve V_2 is closed, the exhaust valve V_3 is opened to allow the under side of the piston to be subjected to the vacuum of the condenser C into which the escaping steam from the under side of the piston flows. The steam valve V_1 is opened, and the steam passing on to the upper side of the piston forces the piston down to the bottom of the stroke. Here the two open valves are closed and the equilibrium valve V_2 is opened, so allowing gravity acting through the weighted pump rods M to bring the piston to the top of its stroke. For removing the condensed water and condensed steam, and any air that may have entered into the cylinder and condenser, an air pump A is provided. The discharge from this pump is into the hot well H, from

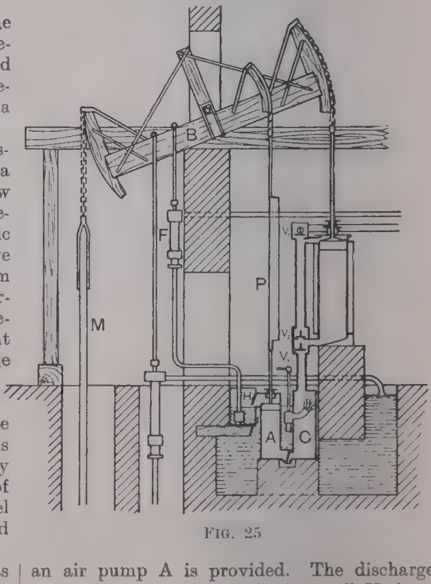

FIG. 25

which the feed pump F draws its supply of hot water for feeding the boiler.

Watt saw a multitude of uses for his engines if the reciprocating motion of his piston rod could be changed to the rotary motion of a shaft. To achieve this end he devised the crank and connecting-rod mechanism, although retaining the overhead beam, but on some one patenting Watt's device he produced the sun and planet wheel mechanism and patented the detail in 1781. On the expiry of the patent relating to the crank and connecting rod, the sun and planet mechanism was discarded and the crank and connecting rod came into practically universal use as a reciprocating engine mechanism.

Watt's next object was to eliminate the wasted stroke of his single-acting engines of 1769, and in 1782 he patented his double-acting engine. At this time he also patented the idea of using steam expansively, *i.e.* of admitting live steam only for a portion of the stroke of the piston and allowing the quantity admitted to continue doing work by expansion until the piston reaches the end of its stroke. In view of each stroke now being a power stroke the original chain form of connection between the piston rod and the oscillating beam was no longer feasible. To take its place he devised the so-called parallel motion. The function of the parallel motion is to guide the upper end of the piston rod so that, whatever the angle the beam may make (within the designed limits) the end of the piston rod is not deflected from the vertical. The pressures at which his engines worked rarely exceeded seven pounds per square inch above the atmosphere, although he was well aware of the advantages of using steam expansively. Further patents were the throttle valve for regulating the admission of steam to the engine cylinder, and the centrifugal double pendulum governor for controlling the engine speed by operating the throttle valve. He also devised the first indicator for recording graphically the state of affairs existing within the engine cylinder at any part of the stroke. To Watt is due the present method of rating the duty of engines — the horse-power. He defined one horse power to be the raising of 33,000 pounds through one foot in one minute. In partnership with Matthew Boulton he carried on as a commercial venture the manufacture and sale of his engines at works in Birmingham.

Watt's double-acting engine of 1782 is shown diagrammatically in *Fig.* 26. In addition to the then customary arrangements for pumping, a continuous rotary motion of a shaft is provided through the medium of his sun and planet wheel mechanism. Valves V$_1$ and V$_3$ are steam valves; V$_2$ and V$_4$ are exhaust valves. Commencing with the piston at the top of its stroke the cycle of events is as follows—exhaust valve V$_2$ and steam valve V$_3$ are closed, and steam valve V$_1$ and exhaust valve V$_4$ are opened. Steam may now enter the cylinder above the piston and press down the piston, the steam from the previous stroke meanwhile exhausting into the condenser C through the exhaust valve V$_4$. The purpose of the injection jet J is to condense the steam. The steam valve V$_1$ may be closed before the completion of the piston stroke and the expansive properties of the steam used to complete

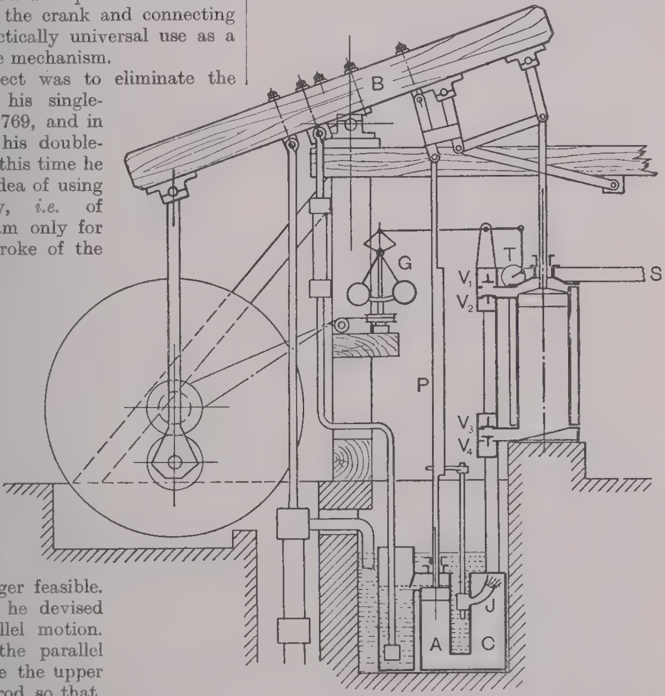

FIG. 26.

the stroke. For the return stroke the steam valve V_3 and the exhaust valve V_2 are opened and valves V_1 and V_4 are closed. When the engine has been thoroughly warmed up and is under way, the opening and closing of the valves is performed automatically by a plug rod P driven by the oscillating beam B. The plug rod moves levers connected to the valves and so operates the valves. The engine speed is controlled by a centrifugal pendulum governor G driven from the engine shaft by a belt. The governor actuates a throttle valve T placed in the steam pipe S, near the cylinder. The throttle valve consists of a disc mounted on a spindle working within the steam pipe. The angular displacement of the disc from the closed to the open position is about ninety degrees.

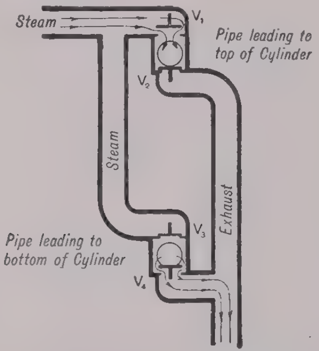
Steam
V_1
Pipe leading to top of Cylinder
V_2
Steam
Exhaust
V_3
Pipe leading to bottom of Cylinder
V_4

FIG. 27.—Cross-section of Steam and Exhaust Passages of Watt's Double-acting Engine.

§ (12) HORNBLOWER'S ENGINE. — To Jonathan C. Hornblower belongs the honour of making the first compound engine. His engine, patented in 1781, consisted of two cylinders of different diameters placed side by side, the steam doing work successively on the two pistons. Both pistons were coupled to the same end of an overhead oscillating beam. Owing to an infringement of Watt's patents the expansive condensing engine introduced by Hornblower was abandoned for many years. About the commencement of the nineteenth century the idea was revived. To what extent Hornblower appreciated the advantages of the compound engine is uncertain, but nevertheless his work marks the greatest advance made in reciprocating steam-engine construction since Watt applied his genius to improving the Newcomen engine.

The revival of the compound engine did not pass unchallenged, and its competitor proved itself to be so satisfactory under the prevailing conditions that compounding again fell into disuse. Its competitor was the single-cylinder high-pressure engine, which, at this time, had the advantage of simplicity coupled with unusual economy. Introduced originally by Richard Trevithick for propelling vehicles along roads, it eventually, when modified somewhat, took the form of a pumping engine and for many years held sway under the name of the Cornish pumping engine.

§ (13) THE CORNISH ENGINE. — In the Cornish pumping engine, as was customary at that time, the cylinder was placed to work on to one end of an overhead beam. At the other end of the beam was attached the usual heavy pump rod working a pump placed at the foot of the mine shaft. The down stroke of the piston was due to steam being admitted above the piston; on the stroke being completed the two sides of the piston were placed in equilibrium by opening an equilibrium valve similar to that used in Watt's single-acting engine. The heavy pump rods, under the action of gravity, descended, doing work in their descent and bringing the piston again to the top of the cylinder ready for another power stroke. The valuable expansive properties of the steam were used by cutting off the supply of steam early in the stroke. This, combined with the relatively high initial pressure used, and the beneficent effects in the present engine of the inertia of the large moving masses employed, resulted in unusual economy. A small plunger pump was used for controlling the frequency of the engine strokes. The plunger of the control pump was raised by the engine beam; the descent of the plunger was controlled by the rapidity with which the fluid under the plunger was allowed to escape through an adjustable orifice. The steam and exhaust valves were controlled by levers and catches which the plunger in descending operated. The plunger control pump was called a cataract. A similar pump was used for working the equilibrium valve, and was set so that a decided pause occurred when the pump rods were at the top of their stroke, thus allowing time for the pump cylinder to fill with water. Whilst the efficiencies of these engines were undoubtedly very good, some of the claimed results of tests are of so startling a character as to cause the trial data to be regarded with suspicion. These engines played a prominent part in the development of the steam engine as a reliable prime mover, and especially drew attention to the advantages accruing by the use of high-pressure steam expansively.

§ (14) M'NAUGHT'S ENGINE.—About the middle of the nineteenth century the second revival of the compound engine took place. Facilities for generating steam at higher pressures than had been common hitherto, the need for augmenting the power developed by existing machinery, and the cry for greater economy in steam consumption —

these, and other factors, led M'Naught in 1845 to improve the existing Watt beam engine by adding a high-pressure cylinder. The piston rod of this cylinder was connected to the engine beam at some convenient position, the stroke of the new cylinder being made to suit the position chosen. After working expansively in the high-pressure cylinder the steam passed into the original cylinder, now the low-pressure cylinder, to do further work before exhausting into a condenser. As might be expected it was found that the many engines so treated had their total power greatly increased and showed a material gain in thermal efficiency. Compound engines now came into general use, and their advantages became more pronounced with the advent of still higher boiler pressures. Generally, with the exception of locomotives, the majority of large engines are now stage expansion engines. In the locomotive factors exist which render the stage expansion idea difficult to apply. The largest development of the reciprocating compound engine has been in the marine service. The oscillating beam is no longer used; through the medium of Watt's crank and connecting-rod mechanism rotary motion is produced without the intervention of the beam. Modern engines are very different in appearance from those of the early days, occupy less space, weigh less per unit of power developed, and are much more economical in steam consumption, but let it be remembered that the debt we owe to those idealists and engineers of the past two centuries is great.

The development since the middle of the nineteenth century has been more in the direction of size of unit rather than in the application of hitherto unconsidered principles. With increase in skill in boiler-making and the ability to obtain boiler material of a quality and reliability unknown to the earlier boiler-makers, steam pressures gradually increased until now pressures of 200 pounds per square inch are common. The two-stage expansion engine has given place to the three- and sometimes the four-stage expansion engine, with, naturally, increased heat efficiency. The advantages of superheating the steam, *i.e.* heating the steam above the temperature due to formation, have been investigated, and despite much discouragement the designing and operating of superheated steam plants has received considerable attention. The results as regards heat efficiency have been encouraging, and rapid development is assured.

The bulk of steam-engine development has been due to the activities of the British. In relatively recent years others, however, have entered the field to aid in the advancement of applied science. In 1885 L. J. Todd, realising the almost thermal instability existing in the ordinary form of reciprocating engine cylinder, attempted to overcome the inherent defect by inventing and making the one-directional-flow engine, usually called the una-flow (or sometimes the uni-flow) engine. In the ordinary form of engine there is frequent and continuous reversal of steam flow so long as the engine is running. This materially affects the heat efficiency of the engine. In his engine he has arranged the steam flow so that it is always in the same direction. The results of his researches during the few succeeding years were unheeded, and it was left to German engineers to continue his labours and by careful and precise effort to bring his ideas to practical achievement. Continental engineers have shown remarkable perseverance and ingenuity in perfecting this important form of engine. The unusual features are that the steam ends of the cylinder are always hot and the exhaust portion of the cylinder always cold, and that the ratio of expansion in a single cylinder is as great as that usually found over the cylinders of a triple or quadruple expansion engine of the ordinary alternating-flow pattern. Only recently have British engineers turned their attention to the design and manufacture of the una-flow engine, and in 1920 there was in process of manufacture a larger una-flow engine than had hitherto been constructed. It is said that up to the end of 1911 una-flow engines aggregating over one half-million horse-power had been made or were in process of manufacture.

§ (15) STEPHENSON'S "ROCKET."—As has already been mentioned, Trevithick applied the reciprocating steam engine to the propulsion of road vehicles. It was not, however, until 1829, twenty-five years after Trevithick had shown the possibility of steam-driven vehicles, that the question of horse versus steam traction for railways was definitely settled. In this year locomotive trials were conducted, and the engine, "Rocket," made by George Stephenson, proved itself vastly superior to all competitors and gave a general outline to the locomotive which persists even yet. The cylinders of the early locomotives were placed vertically; in the Rocket they were inclined and afterwards placed almost horizontally. The exhaust from the engine cylinders was used to accelerate the draught, so giving a ready combustion of the fuel, even with a small boiler. Stephenson's boiler was a multi-tubular one. The products of combustion passed through these tubes on the way to the chimney. The tubes provided a large heating surface, and in this respect his boiler was in advance of contemporary boilers. A crude arrangement of gabs was provided to facilitate reversing the direction of motion of the engine. This arrangement eventually developed into the now common link-motion reversing gear. The link motion also has the advantage of allowing variation in the ratio of expansion to be made without stopping the engine. The manufacture of locomotives was taken up by his son Robert Stephenson at Newcastle-on-Tyne, and had a far-reaching effect upon the commercial expansion of Great Britain. Restrictions in loading gauge have cramped the development of the locomotive in Great Britain, but where the restrictions have not been so severe the steam locomotive has become extraordinarily powerful and of considerable bulk and weight.

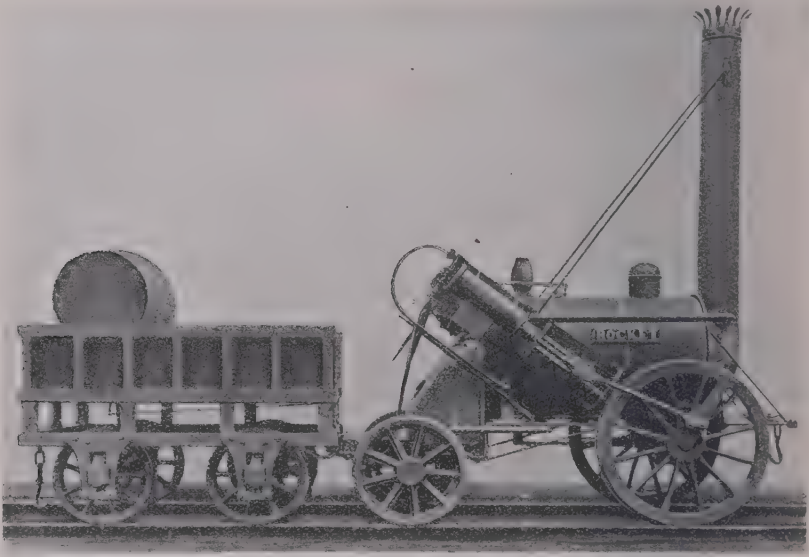

FIG. 28.—George Stephenson's "Rocket."

§ (16) THE FIRST MARINE ENGINE.—Another natural application of the steam engine was to the propulsion of ships. The first practical application was to a tug, *Charlotte Dundas*, which was tried in the Forth and Clyde Canal in 1802. A Watt double-acting condensing engine was installed. It was placed horizontally, and by means of a connecting rod drove the crank of a shaft carrying a paddle wheel placed at the stern of the vessel. The trial was successful, but outside interests prevailed, and steam towing was abandoned. In 1807 an American, Robert Fulton, prepared a vessel on the Hudson River for engines made to his design by Boulton and Watt. His venture was a success, and for the first time steam navigation was commercially feasible. Since then improvements have been in design rather than in principle. The modern marine reciprocating steam engine is wonderfully reliable, and compares very favourably in economy with the best of land engines, yet for certain marine services it has been superseded by the steam turbine and the internal combustion engine. Its days are, however, by no means over, and for several classes of work it will not readily be replaced.

Although locomotive and marine engines may justly claim the bulk of the power of the reciprocating engines manufactured, the other services to which this form of engine has been applied are legion, and in the aggregate the power developed is enormous. Each well-defined branch of industry has its own peculiar requirements to be satisfied, and hence it is that there is so much variation in the detail of land engines. Unquestionably, many of the smaller size engines are inherently wasteful, and the wastage is frequently amplified by lack of knowledge and sheer indifference on the part of owners and attendants, yet they retain their position as prime movers because of their freedom from break-down and their flexibility in operation.

A. C.

STEAM ENGINE, THEORY OF

§ (1) INTRODUCTION.—The theory of the steam engine considered as a heat engine, that is to say, considered as an appliance by which work is done through the agency of heat, is a development of "Thermodynamics" (*q.v.*). The engine may be of the piston and cylinder type, where work is done during the expansion of the working substance and as a direct result of that expansion, in consequence of the pressure which the substance exerts on its containing envelope, part of which moves so as to increase the volume of the contents. Or it may be of the turbine type, where a stream of the working substance acquires kinetic energy by passing from a region of comparatively high pressure to a region of lower pressure, and work is done by the impulse or reaction of the stream on moving vanes. In either case the action is governed by the First and Second Laws of Thermodynamics. A fraction of the heat which is supplied to the working substance disappears as heat, by being converted into the mechanical form of energy which it is the function of the engine to produce; and the principles which determine how large that fraction may be are those laid down by Carnot in his discussion of the conditions under which an engine will most efficiently perform work by the agency of heat. In either case it has to be recognised that the fraction convertible into work is limited by the temperature at which the working substance takes in heat from the source, and the lower temperature at which it rejects the unconverted remainder, and also that the ideal limit of efficiency which these temperatures impose will be more and more closely approached the more nearly the actions that occur within the engine are reversible. If all the heat which the working substance takes in were taken in at an absolute temperature T_1 and all the heat which it rejects were rejected at an absolute temperature T_2 the ideal limit of efficiency would be

$$\frac{T_1 - T_2}{T_1},$$

as is shown in "Thermodynamics," § (20). In § (40) of that article an imaginary reversible steam engine is described, in which heat is taken in by evaporation of the working substance at a temperature T_1 and is rejected by condensation of the working substance at a temperature T_2. Within that imaginary engine there is no irreversible feature; the substance changes its temperature from T_1 to T_2 by adiabatic expansion, and from T_2 back to T_1 by adiabatic compression.

No real engine works in a strictly reversible manner, and the ideal efficiency which corresponds to reversible working is to be regarded as a standard by comparison with which the actual performance should be judged. The actual efficiency, namely the actual ratio of work done to heat supplied, may be measured by observing the indicated horse-power, the quantity of steam supplied, and the conditions as to temperature and pressure of the supply. The actual efficiency will necessarily be less than the ideal efficiency which corresponds to reversible working under the assigned conditions as to supply and rejection of heat. Its ratio to the ideal efficiency is called the "Efficiency Ratio." The efficiency ratio is an important criterion of performance, but it must be borne in mind that when one engine is compared with another, we are concerned not only with the efficiency ratio of each, but also with the ideal standards, which may differ widely owing to differences in the conditions under which the engines receive heat or reject it.

§ (2) Cycle of Operations (Carnot).—If the heat which the working substance of a steam engine takes in were exclusively the heat of evaporation of the steam, which is received at the temperature of the boiler, it would be proper to take the efficiency of the Carnot Cycle, namely $(T_1 - T_2)/T_1$, as the ideal standard with which the actual efficiency should be compared. But this would require the feed-water to be already at the temperature T_1 before it begins to receive heat, a condition which requires all the operations to occur in a single vessel, as in the imaginary engine of Carnot. When the organs are separated into boiler, cylinder, condenser, and feed-pump, adiabatic compression from T_2 to T_1 becomes impracticable, and in lieu of it we have, as an essential part of the cyclic process, the heating of the feed-water from T_2 to T_1 by direct application of heat. Thus the cycle to be considered is one in which part of the heat is necessarily received at temperatures lower than T_1, namely that part of the heat which serves to warm the feed-water up to the temperature of the boiler. It is easy to imagine an ideal cycle of operations in which the working substance takes in heat in this manner but in which the internal actions are completely reversible.

Taking the Carnot cycle with its four operations, as described in "Thermodynamics," § (40), let it be modified as follows. Let the first and second operations occur as they do there, namely the vaporisation of the water at T_1, and the expansion of the steam from T_1 to T_2; but let the third operation, namely the condensation at T_2, be continued until the steam is wholly condensed. The substance then consists of water at T_2, and the cycle is completed by heating it, in the condition of water, from T_2 to T_1. In an engine where all the operations occur in a single vessel this could be done by increasing the pressure exerted by the piston from P_2 to P_1 before applying the hot body; this prevents steam from forming during the heating of the water.

The indicator diagram of the cycle modified in this manner is shown in *Fig.* 1. There *ab* is the operation of forming steam, from water, at T_1 and P_1; *bc* is adiabatic expansion from T_1 and P_1 to T_2 and P_2. During this operation part of the steam becomes condensed. Then *ce* completes the condensation at T_2 and P_2. In the fourth operation the pressure of the condensed water is raised from P_2 to P_1 and its temperature from T_2 to T_1. During that operation the change of volume is negligible in comparison with that which takes place in the other operations.

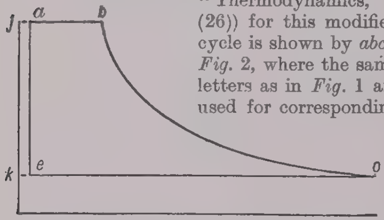

FIG. 1.—Indicator Diagram of Ideal Steam Engine with complete Adiabatic Expansion.

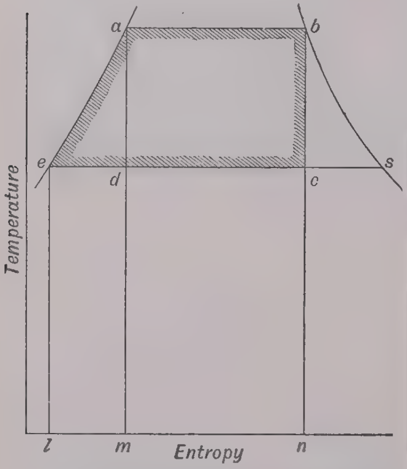

FIG. 2.—Entropy-temperature Diagram for Ideal Steam Engine with Complete Adiabatic Expansion.

The entropy-temperature diagram (see "Thermodynamics," § (26)) for this modified cycle is shown by *abce*, *Fig.* 2, where the same letters as in *Fig.* 1 are used for corresponding operations. As in the Carnot cycle, *ab* represents the conversion of a pound of water at T_1 into dry saturated steam at T_1, and *bc* represents its adiabatic expansion to T_2, resulting in a wet mixture at *c*, the dryness of which (that is to say, the fraction that is present as vapour) is measured by the ratio *ec*/*es*. Then *ce* represents the complete condensation at T_2 of the steam in this wet mixture, and *ea*, which practically coincides with the boundary curve, represents the reheating of the condensed water from T_2 to T_1.

The working substance behaves reversibly throughout all these operations, and therefore the work done in the cycle is represented by the closed area *abce* in the entropy-temperature diagram of *Fig.* 2. The diagram further exhibits the heat taken in and the heat rejected. The whole heat taken in is measured by the area *leabn*, the base line *ln* being drawn at the absolute zero of temperature, and of this the area *leam* measures the heat taken in during the last operation, while the water is being reheated, and the area *mabn* measures the heat taken in during the first operation, while the water is turning into steam. The area *ncel* measures the heat rejected, namely during the condensing process *ce*.

An important algebraical expression for the work done in the cycle is obtained by making use of the "total-heat" function I, explained in "Thermodynamics," §§ (31), (38). In the indicator diagram of *Fig.* 1, let the lines *ba* and *ce* be produced to meet the line of no volume in *j* and *k*. Then by § (38) the area *jbck* is an amount of work equivalent to the difference of total heats of the working fluid at *b* and *c*, $I_b - I_c$, namely the "heat-drop" of a pound of steam in expanding adiabatically from the condition at *b* to the condition at *c*. The small area *jaek* (the size of which is exaggerated in the sketch) is $(P_1 - P_2)V_{w2}$, where V_{w2} is the volume of a pound of water at T_2. (We may take the volume of water to be practically constant for the purposes of this calculation.) Hence the thermal equivalent of the work done in the cycle, per pound of steam, is

$$I_b - I_c - A(P_1 - P_2)V_{w2},$$

A being the factor for converting from units of work to units of heat. The same quantity of heat is represented in the entropy-temperature diagram, *Fig.* 2, by the area *eabca*.

§ (3) RANKINE CYCLE.—This modified cycle is practically important because it is the nearest approach to a Carnot cycle that can be aimed at when the operations of boiling, expanding, and condensing are conducted in separate vessels. The ideal engine already considered had one organ only—a cylinder which also served as boiler and as condenser. We come nearer to the conditions that hold in practice if we think of an engine with separate organs, shown diagrammatically in *Fig.* 3, namely a boiler A kept at T_1, a non-conducting cylinder and piston B, and a surface condenser C kept at T_2. To these must be added a feed-pump D, which returns the condensed water to the boiler. Provision is made by which the cylinder can be put into connection with the boiler or condenser at will.

With this engine the cycle of *Fig.* 2 can be performed. An indicator diagram for the cylinder B is sketched in *Fig.* 4.

Steam is admitted from the boiler, giving the line *jb*. At *b* "cut-off" occurs, that is to say the valve which admits steam from the boiler to the cylinder is closed. The steam in the cylinder is then expanded adiabatically to the pressure of the condenser, giving the line *bc*. At *c* the "exhaust" valve is opened which connects the cylinder with the condenser. The piston then returns, discharging the steam to the condenser and giving the line *ck*. The area *jbck* represents the work done in the cylinder B. The condensed water is then returned to the boiler by the feed-pump, and the indicator diagram showing the work expended upon the pump during this operation is sketched in *Fig.* 5. It is the rectangle *keaj*, where *ke* represents the up-stroke in which the pump fills with water at the pressure P_2, and *aj* represents the down-stroke in which it discharges water to the boiler against the pressure P_1. If we superpose the diagram of the pump on that of the cylinder we get their difference, namely *abce* (*Fig.* 1), to represent the net amount of work done by the fluid in the cycle. It is the excess of the work done by the fluid in the cylinder over that spent upon the fluid in the pump.

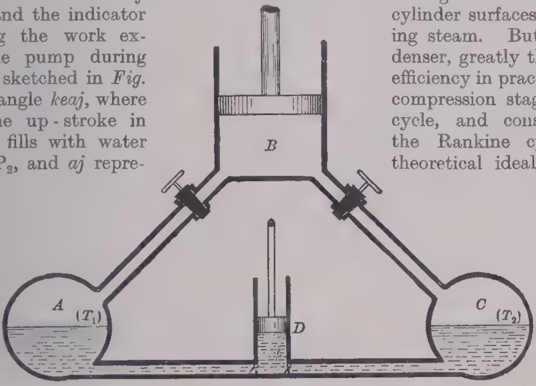

FIG. 3.

This cycle is commonly called the Rankine Cycle. Like the Carnot cycle it represents an ideal that is not practically attainable, for it postulates a complete absence of any loss through transfer of heat between the steam and the surfaces of the cylinder and piston or through irreversibility in the motion of the working fluid. But it affords a very valuable criterion of performance by furnishing a standard with which the efficiency of any real engine may be compared—a standard which is less exacting than the cycle of Carnot, but fairer for comparison, inasmuch as the fourth stage of the Carnot cycle is necessarily omitted when the steam is removed from the cylinder before condensation. A separate condenser is indispensable in any real engine that pretends to efficiency.

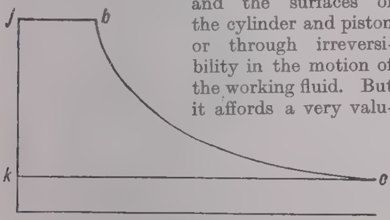

FIG. 4.

The use of a separate condenser was in fact one of the great improvements which distinguished the steam engine of Watt from the earlier engine of Newcomen, where the steam was condensed in the working cylinder itself. The introduction of a separate condenser enabled the cylinder to be kept comparatively hot, and thereby reduced immensely the loss that had occurred in earlier engines through the action of chilled cylinder surfaces upon the entering steam. But a separate condenser, greatly though it adds to efficiency in practice, excludes the compression stage of the Carnot cycle, and consequently makes the Rankine cycle the proper theoretical ideal with which the performance of a real engine should be compared.

The efficiency of the Rankine cycle is less than that of a Carnot cycle with the same limits of temperature. This is because, in the Rankine cycle, the heat is not all taken in at the top of the range. In the Rankine cycle, as in Carnot's, all the internal actions of the working substances are, by assumption, reversible, and consequently each element of the whole heat-supply produces the greatest possible mechanical effect when regard is had to the temperature at which that element is taken in. But part of the heat is taken in at temperatures lower than T_1, namely that quantity of heat which is required to warm the water up to the temperature of the boiler. Hence the average efficiency is lower than if all had been taken in at T_1, as it would be in the cycle of Carnot.

Each pound of steam that passes through the engine does a larger amount of work in the Rankine cycle than it does in the Carnot cycle. This will be apparent when the areas are compared which represent the work in the corresponding entropy diagrams—the area *abce* with the area *abcd* in *Fig.* 2. But the quantity of heat that has to be supplied for each pound in the Rankine cycle is also greater, and in a greater ratio: it is measured by the area *leabn* in *Fig.* 2, as against *mabn*. Hence the efficiency is less in the Rankine cycle. In the Rankine cycle, of the whole heat-supply the part *leam* does only the comparatively small amount of work *ead*, and the remainder of the heat-supply, namely *mabn*,

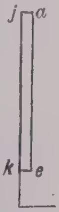

FIG. 5.

does the same amount of work as it would do in a Carnot cycle.

To express the efficiency of a Rankine cycle we may take in the first instance one in which the steam supplied to the cylinder is dry and saturated. The whole amount of heat taken in is the quantity required to convert water at P_1 and T_2 into saturated steam at P_1. This quantity is $I_{s1}-[I_{w2}+A(P_1-P_2)V_{w2}]$, for the total heat of the water at P_1 and T_2 is greater than I_{w2} by the quantity $A(P_1-P_2)V_{w2}$. I_{s1} is the total heat of the steam as supplied from the boiler.

The work done is, as we have seen, equal to the heat-drop *minus* the work spent in the feed-pump, or $I_{s1}-I_c-A(P_1-P_2)V_{w2}$, where I_c is the total heat of the wet mixture after adiabatic expansion.

The efficiency in the cycle as a whole is therefore

$$\frac{I_{s1}-I_c-A(P_1-P_2)V_{w2}}{I_{s1}-I_{w2}-A(P_1-P_2)V_{w2}},$$

which is approximately equal to

$$\frac{I_{s1}-I_c}{I_{s1}-I_{w2}}.$$

The feed-pump term $A(P_1-P_2)V_{w2}$, which occurs in both the numerator and the denominator of this expression, is relatively so small that it is often omitted in calculations relating to ideal efficiency, just as it is omitted in stating the results of tests of the performance of real engines. In such tests it is customary to speak of the work done per pound of steam, without making any deduction for the work that has to be spent per pound in returning the feed-water to the boiler. But to make the analysis of a Rankine cycle complete, the feed-pump term has to be taken into account, and it is only then that the area of the entropy-temperature diagram gives a true measure of the work done. The heat-drop, by itself, is not an accurate measure of the work done in the Rankine cycle as a whole, nor is the heat-drop equal to the enclosed area of the entropy-temperature diagram, but to that area minus the thermal equivalent of the work spent in the feed-pump.

If, however, we are concerned only with the work done *in the cylinder* of the ideal engine, then the heat-drop alone has to be reckoned. It is the exact measure of that work. The ratio of the heat-drop to the heat supplied shows what proportion of the supply is converted into work in the cylinder, under the ideal conditions of adiabatic action: it is a ratio nearly identical with the efficiency of the Rankine cycle, and even more useful as a standard with which to compare the performance of a real engine. In the actual performance of any real engine the amount of work done in the cylinder necessarily falls short of the adiabatic heat-drop, because the working substance loses some heat to the cylinder walls. The extent to which it falls short is a matter for trial, and once that has been ascertained by trials of engines of given types, estimates may be made of the performance of an engine under design, using the adiabatic heat-drop as the basis of the calculation, with a suitable allowance for probable waste.

§ (4) TOTAL HEAT: WET STEAM. — To determine the ideal performance it is essential to calculate the adiabatic heat-drop under any assigned initial and final conditions. For this purpose we have to find I_c, the total heat of wet steam after adiabatic expansion. To bring the mixture at c (*Fig.* 2) into the condition of water at e would require the removal of a quantity of heat equal to the area under ec, namely $T(\phi-\phi_w)$, where ϕ is the entropy at c, and ϕ_w is the entropy of water (at e). On the other hand, to bring it to the condition of saturated steam would require the addition of a quantity of heat equal to the area under cs, namely $T(\phi_s-\phi)$. Hence the total heat of the mixture at c is

$$I_c=I_w+T(\phi-\phi_w)$$

or

$$I_c=I_s-T(\phi_s-\phi).$$

Of these two expressions the second is the more convenient, because steam tables generally give more complete sets of values of ϕ_s than of ϕ_w.

The entropy ϕ of the wet mixture is the constant entropy under which adiabatic expansion has taken place: it is to be calculated from the initial conditions. This method of finding the total heat, after adiabatic expansion, makes no assumption as to what the state of the steam was before expansion: it is equally valid whether the steam was dry, wet, or superheated to begin with. What is assumed is that after expansion the steam is wet, and that will in general be true even if there be a large amount of initial superheat. It is also assumed that the vapour and liquid in the wet mixture are in thermal equilibrium.

If the steam is dry and saturated at the beginning of the adiabatic expansion, its initial total heat is I_s, and the entropy throughout expansion is equal to ϕ_{s1}. Under these conditions the total heat after adiabatic expansion is

$$I_c=I_{s2}-T_2(\phi_{s2}-\phi_{s1}),$$

and the heat-drop is

$$I_{s1}-I_c=I_{s1}-I_{s2}+T_2(\phi_{s2}-\phi_{s1}).$$

To take a numerical example, let the steam be supplied in a dry-saturated state at a pressure P_1 of 180 pounds per square inch (absolute), and let it expand adiabatically to a pressure P_2 of 1 pound per square inch, at which it is condensed. With these data we find from Callendar's Steam Tables that $T_1=462^\circ{\cdot}58$, $T_2=311^\circ{\cdot}84$, $\phi_{s1}=1{\cdot}5620$, $\phi_{s2}=1{\cdot}9724$, $I_{s1}=668{\cdot}53$, $I_{s2}=612{\cdot}46$.

Hence the total heat after adiabatic expansion to the assumed pressure of condensation is

$$I_c=612{\cdot}5-311{\cdot}8(1{\cdot}9724-1{\cdot}5620)=484{\cdot}5,$$

and the heat-drop

$$I_{s1}-I_c=668{\cdot}5-484{\cdot}5=184{\cdot}0.$$

If we consider the Rankine cycle as a whole the feed-pump term $A(P_1-P_2)V_w$ is

$$\frac{(180-1)144\times 0{\cdot}0161}{1400}=0{\cdot}30.$$

Deducting this from the heat-drop we have 183·7 pound-calories as the thermal equivalent of the net amount of work done in the Rankine cycle.

The heat supplied is

$$I_{s1}-I_{w2}-A(P_1-P_2)V_{w2}$$
$$=668{\cdot}5-38{\cdot}6-0{\cdot}30=629{\cdot}6,$$

and therefore the efficiency of the Rankine cycle is

$$\frac{183{\cdot}7}{629{\cdot}6}=0{\cdot}292.$$

This figure would be scarcely altered if the feed-pump term were left out of account. A Carnot cycle with the same limits of temperature would have the efficiency 0·326. The difference between this and 0·292 shows the loss which results in the Rankine cycle from not supplying all the heat to the best possible thermodynamic advantage, namely at the top of the temperature range.

§ (5) RANKINE CYCLE: WITH WET STEAM.—In the Rankine cycle described above the steam was supplied to the cylinder in the dry-saturated state. But the term "Rankine cycle" is equally applicable whatever be the condition of the working substance on admission, whether wet, dry-saturated, or superheated. As regards the action in the cylinder, all that is assumed is that the substance is admitted at a constant pressure P_1, is expanded adiabatically to a pressure P_2 and is discharged at that pressure, and that in the process there is no transfer of heat to or from the metal, nor any other irreversible action. In these conditions the heat-drop in adiabatic expansion from P_1 to P_2 is the thermal equivalent of the area *jbck* in *Fig.* 1, and therefore measures the work done in the cylinder, no matter what the condition of the substance on admission may be.

A Rankine cycle for steam that is wet on admission to the cylinder is shown on the entropy-temperature diagram by the figure *ab′ce* in *Fig.* 6. The point *b′* is placed so that the ratio of the length *ab′* to *ab* is equal to q_1, the assumed dryness on admission. The line *b′c* represents adiabatic expansion from P_1 to P_2, *ce* represents condensation at P_2, and *ea* represents, as before, the heating of the condensed water. The area with the shaded boundary is the thermal equivalent of the work done in the cycle.

The total heat before adiabatic expansion is $I_{w1}+q_1L_1$ or $I_{s1}-(1-q_1)L_1$, and the heat supplied is the excess of this quantity above

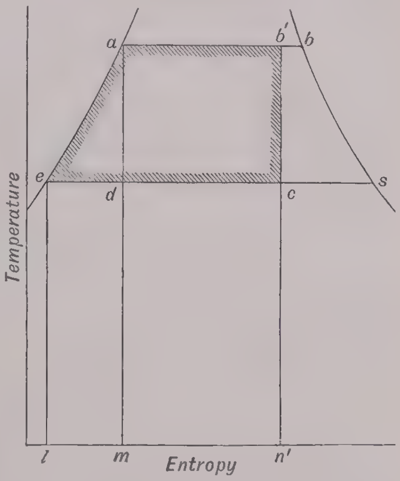

FIG. 6.—Rankine Cycle with Steam initially wet.

$$I_{w2}+A(P_1-P_2)V_{w2}.$$

The entropy ϕ during adiabatic expansion is

$$\phi_{w1}+q_1L_1/T_1 \text{ or } \phi_{s1}-(1-q_1)L_1/T_1,$$

and the total heat after adiabatic expansion is

$$I_{s2}-T_2(\phi_{s2}-\phi).$$

The heat-drop is got by subtracting this from the total heat before adiabatic expansion.

The efficiency (which, as before, is practically equal to the heat-drop divided by the heat supplied) is slightly less than when the steam is saturated before expansion; the reason being that the proportion of heat supplied at the upper limit of temperature is now rather less, because part of the water remains unconverted into steam.

As a numerical example let the steam have the same limits of pressure as before, but contain 10 per cent of water on admission. Then q_1 is 0·9 and the total heat per pound of the mixture before expansion, which is $I_{s1}-0{\cdot}1L_1$, is 620·9. The heat supplied is $620{\cdot}9-38{\cdot}9=582{\cdot}0$. The entropy is

$$\phi_{s1}-qL_1/T_1=1{\cdot}4591.$$

The total heat after expansion $I_{s2} - T_2(\phi_{s2} - \phi)$ is 452·4; the heat-drop is therefore 168·5. Allowing for the feed-pump term, the efficiency in the complete Rankine cycle is 0·289, as against 0·292 when there was no initial wetness.

In practice the steam supplied to an engine would be wet only if there were condensation in the steam-pipe, such as would occur if the pipe were long or insufficiently covered with non-conducting material, or if the boiler "primed." Priming is a defective boiler action which causes unevaporated water to pass into the steam-pipe along with the vapour. The above example will show that a moderate amount of wetness has no more than a small effect on the efficiency of the Rankine cycle. But its practical effect in reducing the efficiency of an actual engine is much greater, because the presence of water in steam increases the exchanges of heat between it and the metal of the cylinder, and consequently makes the real action depart more widely from the adiabatic conditions which are assumed in the ideal operations of the Rankine cycle.

§ (6) RANKINE CYCLE: SUPERHEATED STEAM.—On the other hand, if the steam be superheated before it enters the engine, the exchanges of heat between it and the metal are reduced; the action becomes more nearly adiabatic, and the performance of the real engine approaches more closely the ideal of the Rankine cycle. This is the chief reason why superheating improves the efficiency of a real engine of the cylinder and piston type. In steam turbines also it is beneficial, partly for the same reason, and partly because it reduces internal friction in the working fluid by keeping the fluid drier than it would otherwise be during its expansion through the successive rings of blades. Superheating is now very generally employed in steam engineering. It is therefore important to consider the Rankine cycle for steam that is initially superheated.

In the entropy-temperature diagram, *Fig.* 7, the line *bb′* represents the process of superheating steam that was dry-saturated at *b*. During this process its entropy and its temperature both increase, and when the pressure and temperature at any stage in the superheating are known the corresponding entropy is found from the tables relating to superheated steam. If we assume that the pressure during superheating is constant, and equal to the boiler pressure, the line *bb′* is an extension, into the region of superheat, of the constant-pressure line *ab*. During the process of superheating the steam takes in a supplementary quantity of heat equal to the area under the curve *bb′*, measured down to the base line, namely *nbb′n′*. This quantity of heat may also be found from the tables, being equal to the excess of the total heat $I_{b'}$ over that of saturated steam of the same pressure. Callendar's Tables give values of the total heat of superheated steam, as well as its entropy, for a wide range of pressures and temperatures. During the subsequent process of adiabatic expansion *b′c′* the steam loses superheat, and if the process is carried so far that the adiabatic line through *b′* crosses the boundary curve, it becomes saturated and then wet, and the final condition is that of a wet mixture at *c′*. The total heat of this wet mixture may be found by the method already described.

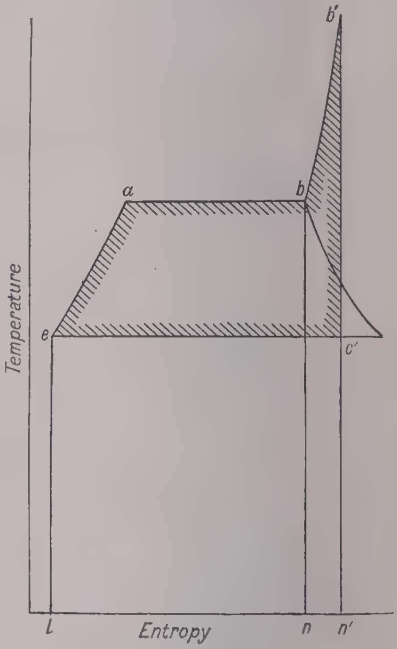

FIG. 7.—Rankine Cycle with Steam initially superheated.

The work done in the Rankine cycle as a whole is the area *eabb′c′*, and the heat taken in is the area *leabb′n′*. Both these quantities are readily calculated without the help of the diagram. To find the work done in the cycle we have only to calculate the heat-drop during adiabatic expansion, namely $I_{b'} - I_{c'}$, and subtract from that the small term which is the thermal equivalent of the work done in the feed-pump, namely $A(P_1 - P_2)V_{w2}$. The heat supplied is

$$I_{b'} - I_{w2} - A(P_1 - P_2)V_{w2}.$$

As a numerical example we may again take $P_1 = 180$ and $P_2 = 1$, and assume that superheating is carried so far as to raise the temperature of the steam to 400° C., which is a limit very rarely exceeded in practice. As a rule the temperature after superheating is considerably lower than this. With these data the steam tables show that the total heat of the superheated steam is 780·8 and its entropy is 1·7633. The heat supply is $780{\cdot}8 - 38{\cdot}9 = 741{\cdot}9$. After adiabatic expansion the steam is wet, and its total heat, which is $I_{s2} - T_2(\phi_{s2} - \phi)$, is 547·2. The adiabatic heat-drop is therefore 233·6.

If we deduct the small feed-pump term (0·3) the efficiency of the cycle as a whole is $233{\cdot}3/741{\cdot}9 = 0{\cdot}314$. This is rather better than the figure for saturated steam (0·292), because a portion of the heat is now supplied at a higher temperature. Even with the extreme amount of superheating, however, which is assumed in this example, the main part of the heat is still supplied at the temperature of saturation, and therefore there is no great gain in theoretical efficiency as expressed by the ideal figure for the Rankine cycle. The practical advantage of superheating is much more considerable, for reasons which have already been indicated, than might be expected from this comparison of the two ideal cycles.

Whatever the initial state be, whether dry-saturated, wet, or superheated, the internal action of the working substance in the Rankine cycle is reversible.

§ (7) Rankine Cycle Reversible.—An ideal engine performing a Rankine cycle may be regarded as a strictly reversible engine taking in heat at various temperatures and consequently extracting the greatest possible amount of work out of each element of the heat supplied, having regard to the temperature at which the element of heat was supplied. In the heating of the feed-water a part of the heat supply is taken in at temperatures ranging from T_2 to T_1. But any element of heat, taken in at a temperature T, acts as efficiently as it would do in a Carnot cycle: the efficiency of conversion of that element is equal to

$$\frac{T-T_2}{T}.$$

Consequently the general efficiency of an ideal engine working on the Rankine cycle is the highest possible efficiency that is compatible with the condition that the substance is to be completely condensed at the lower limit of temperature and returned to the boiler by a separate pump, instead of having its cycle completed by adiabatic compression as in the engine of Carnot. In other words, the work which the steam does in the cylinder of an ideal Rankine engine is the greatest amount of work that can conceivably be done by the steam in passing through any engine, of whatever type, having regard to the temperature at which the working substance has taken in its heat, and to the temperature at which it rejects heat during its complete condensation before being returned to the boiler. But we know that this work is measured by the adiabatic heat-drop. Consequently the adiabatic heat-drop measures the greatest conceivable performance of the steam in passing through any engine (including any engine of the turbine type) when the conditions of supply and of condensation are assigned.

Whatever, therefore, be the nature of the engine, the adiabatic heat-drop serves as an ideal standard with which to compare the actual performance. Thus a steam turbine, equally with an engine of the cylinder and piston type, cannot exceed and necessarily falls short of the ideal performance as measured by that heat-drop. In the design of steam turbines the calculated value of the adiabatic heat-drop, after making a deduction which is determined by experience with similar machines, accordingly forms the basis on which the designer estimates the performance to be expected. Tests of good engines show that in favourable cases about 70 per cent of the adiabatic heat-drop is actually converted into work.

To secure high efficiency there are obviously two separate conditions to be aimed at: (1) that there shall be a large heat-drop in comparison with the heat of formation of the steam; in other words, a high *ideal* efficiency; (2) that there shall be a large Efficiency Ratio. As regards the ideal efficiency, it is important to notice that while some advantage is obtained by increasing the admission pressure, a far greater advantage is obtained by lowering the exhaust pressure.

That this is so will be clear from the following tabulated results which relate to saturated steam. The first table shows how the adiabatic heat-drop and the efficiency of the Rankine cycle are affected by taking different initial pressures, ranging from 100 to 300 pounds per square inch, but with the same pressure of exhaust throughout.

Rankine Cycle for Saturated Steam.
Effect of varying the Initial Pressure

Initial Pressure (pounds per square inch, absolute).	Heat-drop to a Final Pressure of 1 lb. per square inch.	Work done per lb. of Steam, allowing for Work spent in Feed-pump.	Heat supplied per lb. of Steam.	Efficiency of the Rankine Cycle.
		lb. calories.	lb. calories.	
300	202·0	201·5	634·8	0·317
260	197·0	196·6	633·4	0·310
220	191·1	190·8	631·7	0·302
180	184·0	183·7	629·6	0·292
140	175·0	174·8	626·8	0·279
100	162·9	162·8	623·0	0·261

From these results it will be apparent that when the admission pressure is high very little improvement in the efficiency is brought about by even a large increase of pressure.

§ (8) High Vacuum.—On the other hand, it is of great advantage to have what engineers call a "high vacuum"—that is to say, to make the pressure of condensation as low as possible. If a high vacuum can be effectively utilised we obtain from the steam the work which it is capable of doing, during the last stages of the expansion, when its pressure is low and its volume is very large. The following table illustrates the gain in heat-drop and in efficiency that results, in the Rankine cycle, from reducing the lower limit of pressure. In this example the admission pressure P_1 is assumed to be 180 pounds per square inch, and only P_2 is altered.

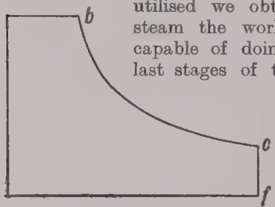

Fig. 8.—Indicator Diagram for Ideal Engine with Incomplete Expansion.

Rankine Cycle for Saturated Steam. Effect of varying the Final Pressure

Final Pressure (pounds per square inch, absolute).	Heat-drop from an Initial Pressure of 180 pounds per square inch.	Work done per lb. of Steam, allowing for Work spent in Feed-pump.	Heat supplied per lb. of Steam.	Efficiency of the Rankine Cycle.
	lb. calories.	lb. calories.	lb. calories.	
4	144·5	144·2	601·1	0·240
3	153·1	152·8	607·5	0·251
2	164·9	164·6	616·1	0·267
1·5	173·0	172·7	621·8	0·278
1	184·0	183·7	629·6	0·292
0·5	201·8	201·5	642·0	0·314

The last figure corresponds to a vacuum of nearly 29 inches of mercury with the barometer at 30 inches.

To secure in a real engine the full benefit of a high vacuum the steam must continue to do useful work in expanding down to the pressure at which condensation is to take place. In engines of the cylinder and piston type this is impracticable for two reasons: the volume of the steam becomes excessive, and the mechanical friction of the piston against the cylinder becomes relatively so great as to absorb all the work done in the final stages. But with the steam turbine these considerations do not apply; there is then nothing to prevent the steam from continuing to do useful work as it expands right down to the pressure of the condenser, and special pains are accordingly taken to maintain a good vacuum in the condenser of a steam turbine. It is largely for this reason that good steam turbines achieve in practice a greater efficiency than even the best engines of the cylinder and piston type.

When steam is released from the cylinder at a pressure substantially higher than the pressure in the condenser its expansion is said to be incomplete. The effect is to lose available work represented by the toe that is cut off the pressure-volume diagram, as in *Fig.* 8, and to make a corresponding reduction in the efficiency. Release takes place at *c* and the pressure falls to *f* while the piston is stationary.

To exhibit incomplete expansion on the entropy-temperature diagram, imagine that instead of letting part of the steam escape from the cylinder by opening the exhaust valve, we produce the same effect within the cylinder itself by applying a receiver of heat which will bring the pressure down to the lower limit P_2, by causing part of the contents to condense before the piston begins its return stroke. The piston being stationary, the volume of the working substance does not alter during this process. If we imagine the receiver of heat to have a temperature which falls progressively from that of the steam at *c* to the final temperature (T_2) at *f*, this removal of heat takes place reversibly. The work done by the steam is not affected by substituting this reversible process for the action of the condenser, because the pressure in the cylinder is in no way altered by the substitution, but we are now able to draw a curve that will represent the process on the entropy-temperature diagram.

This is done in *Fig.* 9, where the curve *cf* represents the condensation of part of the steam at constant volume, while the piston is at rest before beginning its return stroke. The constant volume in this process is to be reckoned per lb. of steam: it is the volume of the cylinder divided by the quantity of the working substance in it: in other words, it is the volume per lb. of the wet steam at *c*. Call that volume V_c. Than at any level of temperature such as *gih*, a point *i* on the

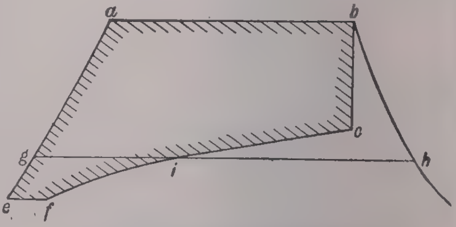

Fig. 9.—Entropy-temperature Diagram for Ideal Engine with Incomplete Expansion.

constant-volume curve which represents the process is found by taking

$$\frac{gi}{gh}=\frac{V_c}{V_h},$$

where V_h is the volume of 1 lb. of saturated steam at that temperature. The area of the figure within the shaded lines represents the thermal equivalent of the work done in the complete cycle. The corner cut off by the curve *cf* shows what is lost by incomplete expansion as compared with the work done in a Rankine cycle.

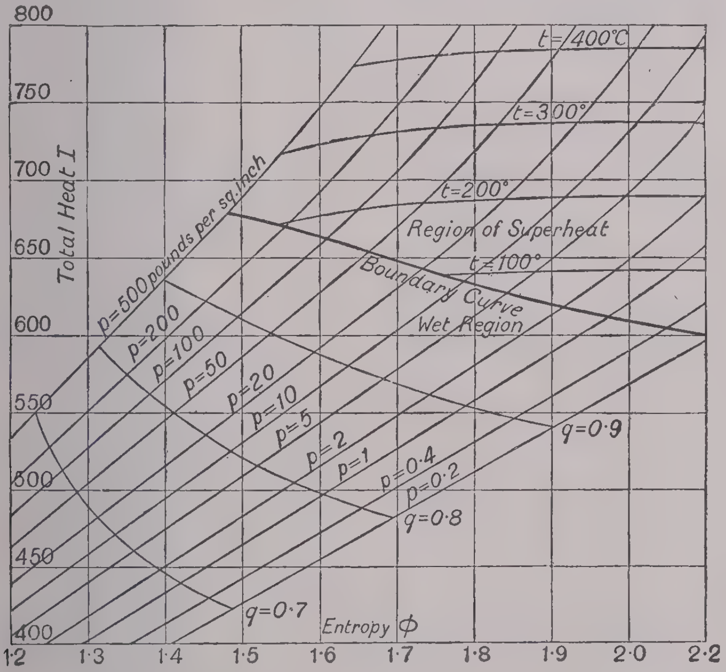

FIG. 10.—Mollier's Chart of Total Heat and Entropy.

§ (9) TOTAL HEAT. *Entropy Chart.*—It has been pointed out above that the adiabatic heat-drop, that is to say, the loss of total heat which the steam would undergo if it were to expand adiabatically from the condition at which it enters the engine to the temperature at which it leaves the engine, measures the greatest amount of work that is ideally obtainable from it in passing through the engine when these conditions as to entry and exit are assigned. This is true of an engine of any type: it applies to the steam turbine no less than to the reciprocating engine. To determine the adiabatic heat-drop I_1-I_2, for any assigned conditions of entry and exit, is accordingly a matter of fundamental importance not only in judging how far the ideal standard of efficiency of an engine under design will be affected by changes in the boiler pressure, in the temperature of superheat, or in the condenser pressure, but also in comparing the actual with the ideal performance of an engine under test. When steam tables such as Callendar's are available the adiabatic heat-drop between given initial and terminal conditions is readily calculated; but it may also be very conveniently found by direct measurement from the Mollier chart of Total Heat and Entropy, briefly described in "Thermodynamics," § (42). The useful part of that chart for steam is given (on a small scale and in skeleton form only) in *Fig.* 10. There the adiabatic heat-drop is simply measured by the length of the vertical line which represents a process of adiabatic expansion (ϕ = constant) from the condition of the steam as it enters the engine to the condition under which it is condensed. The advantage of a high vacuum is apparent from the effect which a low final pressure has on the length of such a line. Any process of throttling, such as steam undergoes when it passes to a region of lower pressure through a reducing valve or other constricted orifice, by which eddying motions are set up, the energy of which takes the form of heat as the eddies subside, may be shown on the Mollier chart by a horizontal line (I = constant). The effect of such a process is slightly to dry steam that is wet, and to superheat steam that is initially dry. At the same time the steam, if initially dry-saturated or superheated, becomes slightly cooled. These effects of throttling will be apparent when a horizontal line is drawn in the figure, from a point representing any condition of the steam, wet or dry-saturated or superheated, in the direction to the right, which corresponds to increase of entropy. The effects of a throttling process or of any dissipation of energy through friction in the movement of a fluid are to increase the entropy. They are thermodynamically disadvantageous because they involve irreversible changes of state.

Such effects occur to a greater or less degree in the action of any real engine or turbine and constitute one reason why the actual performance falls short of the ideal. In the steam turbine they form a chief source of thermodynamic loss: in the reciprocating engine they are less important. But even in the simplest form of reciprocating engine there is some frictional loss in the steam-pipes and passages (including leakage through imperfectly working valves), and in the compound engine, where the expansion of the steam takes place in two or more stages, the transfer of the partially expanded steam from one cylinder to the next of the series involves a further loss of the same kind.

§ (10) MULTIPLE EXPANSION.—There are, however, good reasons why, notwithstanding this fact, engines which divide the whole expansion into three or four stages (called triple-expansion and quadruple-expansion engines) are practically more efficient in the sense of coming nearer to the adiabatic ideal. The main reason is that a division into stages reduces another source of loss, namely the loss that arises from exchange of heat between the working steam and the metal surfaces with which it comes in contact. In any reciprocating engine the losses due to this cause are substantial. Steam entering the cylinder comes in contact with surfaces which have been chilled during the later stages of the previous stroke when the cylinder was full of wet steam at a lower pressure. Some of the entering steam accordingly condenses on the exposed surfaces, forming a layer of water which is at least partially evaporated later when the pressure has been reduced by expansion. This action, which goes on in every stroke after a uniform régime has been attained, makes the exchange of heat between the working fluid and the metal surfaces greater than if the fluid were a dry gas. Thus the action differs largely from the ideal action, which is *adiathermal*, and as the exchanges of heat take place irreversibly they involve thermodynamic loss. Initial superheating does a good deal to reduce this loss and consequently improves the efficiency ratio. By limiting the amount of the expansion in any one cylinder and so avoiding extreme variation of temperature on the part of each metal surface, the loss can be kept within bounds: hence the practical advantage of dividing the expansion into stages. The number of such stages will depend on the boiler pressure: with the highest pressures that are used in marine practice quadruple expansion is more economical than triple expansion, and triple expansion greatly more economical than expansion in two stages. In this connection reference should be made to an elaborate investigation by Professors Callendar and Nicolson of the exchanges of heat between the steam and the cylinder wall (*Min. Proc. Inst. C.E.*, 1897, cxxxi.).

In the steam turbine losses from this source are comparatively small, although the expansion is continued to lower temperatures, for no part of the internal surface is subject to periodic fluctuations of temperature such as occur in the working of a cylinder and piston. There is, of course, some loss by conduction to the outside; but the main loss is that which may be comprehensively described as due to fluid friction. Omitting the (small) loss of heat by conduction, at any stage in the passage of the fluid through the turbine, when expansion has occurred down to any given pressure, the fluid has suffered less drop of total heat than it would have suffered had there been no friction. There is less useful mechanical effect; but there is more internal energy left in the fluid, and the volume is greater, than if there had been no friction, for the heat developed by friction goes to increase the stock of internal energy. In a turbine such as Parsons', where the stages are very numerous, the whole process is made up of a series of small steps which may be approximately represented by a continuous curve on the pressure-volume diagram or on other diagrams. A diagram such as *Fig.* 11 then represents the complete action. The outer curve BC′ is a continuous line drawn through points which represent the volume of the steam at the beginning of each stage. The difference between it and the adiabatic curve BC shows how the volume is increased

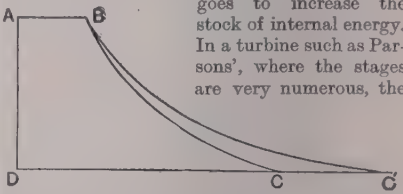

FIG. 11.

in consequence of the internal losses. The area ABCD represents the ideal output of work which would be obtained if the adiabatic heat-drop $I_1 - I_2$ were fully utilised. The greater area ABC′D does not measure an actual output of work, but an artificial quantity which may be called the "gross apparent work." Of this gross apparent work a part is being continuously reconverted into heat as the expansion proceeds, namely a quantity sufficient to supply enough heat to bring the expansion curve out from BC to BC′. At the end of the operation the net amount of work done by the steam is less than the adiabatic area ABCD by the equivalent of $I'_2 - I_2$, where I'_2 is the total heat at C′ and I_2 is the total heat at C, if we assume that there is no escape of heat by conduction, and also no appreciable kinetic energy in the stream of steam which is finally discharged from the turbine. In that case the whole actual heat-drop is converted into work, and the fraction

$$\eta_t = \frac{I_1 - I'_2}{I_1 - I_2}$$

expresses the theoretical efficiency ratio of the turbine as a whole, namely the ratio of the work done on the rotor to the work ideally obtainable by adiabatic expansion through the same range.

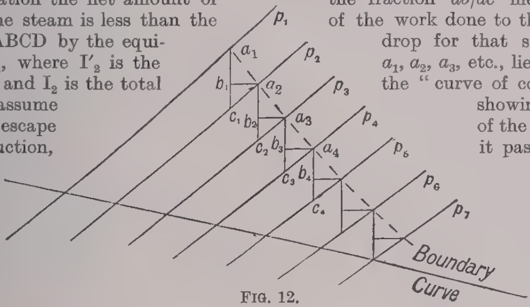

FIG. 12.

§ (11) TURBINES.[1]—The successive steps in the action of any compound turbine are most clearly shown by using the Mollier diagram of entropy and total heat, in the manner illustrated above in *Fig.* 12, where some of the early stages are shown for a turbine supplied with superheated steam. In the first stage the pressure drops from p_1 to p_2, in the second stage from p_2 to p_3, and so on. In the first stage, adiabatic expansion from p_1 to p_2 would be represented by a_1c_1, and the length of that line would be a measure of the adiabatic heat-drop, but the actual heat-drop is a smaller quantity equal to the length a_1b_1. Assuming no loss of heat by conduction, the length a_1b_1 measures the heat converted into work while the steam passes through the first stage. The condition of the steam at the end of the first stage and beginning of the second is represented by the point a_2, which is found by drawing a line of constant total heat through b_1 to meet the constant-pressure curve p_2. In the second stage, adiabatic expansion would give the line a_2c_2. The actual heat-drop, which is a measure of the work done, is a_2b_2, and the condition of the steam as it passes on to the third stage is represented by a_3. Similarly in the third stage the work done is a_3b_3, the steam passes to the fourth stage in the condition a_4, and so on. The diagram shows the process of expansion by stages down to the boundary curve; it is readily extended into the wet region. In each stage the fraction ab/ac measures the ratio of the work done to the adiabatic heat-drop for that stage. The points a_1, a_2, a_3, etc., lie on what is called the "curve of condition," a curve showing what the state of the steam would be as it passes from stage to stage on the assumption that no heat is lost to the outside. The curve of condition consequently corresponds to the outer curve BC′ of *Fig.* 11. The total work done on the rotor is the sum of the amounts of work done in the successive stages, namely Σab.

Taking any stage of a compound turbine, the ratio of the work done, to the adiabatic heat-drop in that stage, may be called the *stage efficiency* and denoted by η_s; thus

$$\eta_s = \frac{ab}{ac}.$$

The total work done on the rotor

$$\Sigma ab = \Sigma\eta_s(ac),$$

and if η_s can be treated as constant from stage to stage,

$$\Sigma ab = \eta_s \Sigma ac.$$

In a compound turbine the quantity Σac is greater than the whole adiabatic heat-drop between the initial and final pressures $I_1 - I_2$ to an extent that depends upon the stage efficiency. The ratio

$$R = \frac{\Sigma ac}{I_1 - I_2}$$

is called the *Reheat Factor*. The reheat factor is relatively high when the stage-efficiency is low, or, in other words, when there is much loss through irreversible action within each stage. Referring to *Fig.* 11, the reheat factor is equal to the ratio of the area ABC′D to the area ABCD.

If we may treat η_s as constant, we have

$$\eta_s R = \frac{\eta_s \Sigma ac}{I_1 - I_2} = \frac{\text{work done on rotor}}{\text{adiabatic heat-drop}} = \eta_t$$

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[1] See also articles "Steam Turbine, The Physics of the"; "Turbine, Development of the Steam."

under the conditions postulated, which make the actual heat-drop a measure of the work done on the rotor.

From the equation $\eta_t = \eta_s R$ it will be seen that in a compound turbine η_t is greater than the stage-efficiency η_s, since R is greater than unity.

The foregoing expressions involve the proviso that there is no leakage of heat. But when there is leakage of heat, or appreciable kinetic energy in the steam at its exit from the turbine, the actual heat-drop $I_1 - I'_2$ includes a quantity representing the loss due to these causes, in addition to the work done on the rotor. Let that loss be expressed as a fraction of the adiabatic heat-drop, namely

$$x(I_1 - I_2).$$

Then $$I_1 - I'_2 - x(I_1 - I_2)$$

is that part of the actual heat-drop which is converted into work on the rotor.

Hence, allowing for this loss, the net or real efficiency-ratio of the turbine becomes

$$\frac{I_1 - I'_2 - x(I - I_2)}{I_1 - I_2} = \eta_t - x.$$

The amount of work obtained from the steam is therefore

$$(\eta_t - x)(I_1 - I_2).$$

Writing η_r for the real efficiency-ratio, its relation to the other quantities is given by the equation

$$\eta_r = \eta_t - x = \eta_s R - x.$$

In the process of designing a turbine a value is estimated for the stage-efficiency η_s; then the curve of condition is deduced, which allows the reheat factor to be found, and also the probable volume and velocity of the steam at each stage.

These considerations apply to the complete action of the steam in passing through the turbine, from the point of admission to the point of exhaust. Considered in detail, the action takes place in a series of steps, in each of which the steam expands in such a manner as to form a jet, or more usually a group of many jets, in which the stream acquires kinetic energy as a consequence of its expansion, and this kinetic energy is subsequently given up to rapidly moving blades which are carried by the rotor. An account of the action will be found in the article on Steam Turbines; all that need be added here on this point is to sketch the thermodynamic theory of the formation of a jet when steam or any gaseous fluid expands through a nozzle, or a channel between guide-blades, forming a passage which is completely filled by the gaseous stream.

§ (12) THEORY OF JETS.[1]—Suppose steam or any other gas to be flowing through a nozzle from a region where the pressure is P_1 to a region where it has the lower value P_2. To simplify matters we shall assume that the process is adiathermal, that is to say, that no heat is taken in or given out by conduction during the flow. In consequence of the fall of pressure each element of the stream expands while it advances, and the work which it does in expanding gives energy of motion to the element in front of it. The steam therefore acquires velocity as it proceeds through the channel and also increases in volume. Imagine two partitions A and B across the channel, taken perpendicular to the direction of the stream-lines, A being in the region of higher pressure. Let P_a, v_a, V_a, and E_a represent respectively the pressure, the velocity, the volume of unit mass, and the internal energy of the fluid, at A, and let P_b, v_b, V_b and E_b represent the corresponding quantities at B. In flowing from A to B each unit of mass gains kinetic energy by the amount

$$\frac{v_b^2 - v_a^2}{2g}.$$

It loses internal energy by the amount $E_a - E_b$. As it enters the space between A and B it has work done upon it by the fluid behind, equal to P_aV_a. In passing out at B it does work on the fluid in front, equal to P_bV_b. Hence by the conservation of energy

$$\frac{v_b^2 - v_a^2}{2g} = E_a - E_b + P_aV_a - P_bV_b.$$

But $E_a + P_aV_a$ is I_a, the total heat of the fluid at A, and $E_b + P_bV_b$ is I_b, the total heat at B; consequently

$$\frac{v_b^2 - v_a^2}{2g} = I_a - I_b,$$

or the gain of kinetic energy, in adiathermal flow through a nozzle, is measured by the heat-drop. This applies as between any two points in the flow; and for the whole passage from the region of pressure P_1 to the region of pressure P_2 we have

$$\frac{v_2^2 - v_1^2}{2g} = I_1 - I_2.$$

When, as frequently happens, the initial velocity is sensibly zero, we accordingly have

$$\frac{v^2}{2g} = I_1 - I_2$$

as the fundamental equation for finding the velocity v which an expanding fluid acquires in a jet, starting from rest.

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[1] See also articles "Steam Turbine, The Physics of the"; "Turbine, Development of the Steam."

So far there has been no assumption as to absence of losses through friction or eddy currents. If we now assume that there is no such loss, the heat-drop $I_1 - I_2$ is to be interpreted as the adiabatic heat-drop which the fluid undergoes in expanding without change of entropy from P_1 to P_2. For steam this heat-drop may be determined from the tables or measured from the Mollier diagram. It has been shown ("Thermodynamics," § (38)) that the adiabatic heat-drop is equal to the area of the ideal indicator diagram ABCD (*Fig.* 13), or $\int_{P_2}^{P_1} V\,dP$, which represents the largest amount of work obtainable in expansion between these limits. In the assumed case the whole of that work takes the form of kinetic energy on the part of the stream as it issues under the final pressure P_2. We therefore have, in the ideal case,

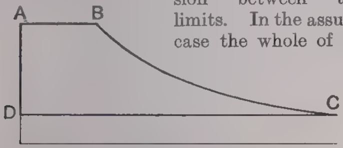

FIG. 13.

$$\frac{v_2^2 - v_1^2}{2g} = \int_{P_2}^{P_1} V\,dP.$$

Expansion along the adiabatic curve BC may generally be expressed by a formula of the type $PV^{\lambda} = \text{constant}$, where λ is a constant index. Then

$$\int_{P_2}^{P_1} V\,dP = \frac{\lambda}{\lambda - 1}(P_1V_1 - P_2V_2)$$

$$= \frac{\lambda}{\lambda - 1}\left[1 - \left(\frac{P_2}{P_1}\right)^{\frac{\lambda-1}{\lambda}}\right]P_1V_1.$$

Hence when an expanding fluid starts from rest, at the initial pressure P_1, to form a jet, we obtain

$$\frac{v^2}{2g} = \frac{\lambda}{\lambda - 1}\left[1 - \left(\frac{P}{P_1}\right)^{\frac{\lambda-1}{\lambda}}\right]P_1V_1$$

as an equation from which to find the velocity v when the pressure has fallen from P_1 to any lower pressure P, under the assumed condition of frictionless flow and no conduction of heat. It is convenient to write Z for the ratio in which the pressure has fallen, P/P_1. This gives, for the velocity at any stage,

$$v = \sqrt{\frac{2g\lambda}{\lambda - 1}\left(1 - Z^{\frac{\lambda-1}{\lambda}}\right)P_1V_1},$$

and for the volume of the steam (per unit of mass) at any stage

$$V = \frac{V_1}{Z^{1/\lambda}}.$$

As the flow along the nozzle proceeds Z becomes less, and v and V both increase. The proper form for the nozzle depends on the relation of V to v. Writing A for the cross-section of the nozzle at any point, the volume passing per second is vA and the mass Q passing per second is vA/V. This is the same at all sections: hence A is equal to QV/v, and its value may accordingly be calculated for any assigned value of Z, when the index λ is known. For air the value of λ is approximately 1·4, for dry steam 1·3.

On making this calculation with values of Z gradually decreasing from unity, to correspond with successive stages in the expansion of steam or air through a nozzle, starting from rest, it will be found that A, which is proportional to V/v, at first diminishes, then passes a minimum at a certain value of Z, and then increases as the pressure continues to fall. Hence when the total drop in pressure is large, the proper form for a channel for adiabatic flow is at first convergent and afterwards divergent: in other words, the form that will allow the heat-drop to be applied to the best advantage in giving kinetic energy to the stream is one in which the nozzle at first contracts to a narrowest section or "throat" and afterwards expands to an extent that depends on the ratio of the initial pressure to the pressure against which the stream is to be discharged.

If that ratio is not large the convergent portion of the nozzle only is required. When the whole drop of pressure is divided into many successive stages, this is the case: each of the blade-channels in a Parsons' turbine, for example, is convergent throughout its course. But in De Laval's turbine, where the whole available drop of pressure may be utilised in a single step, the nozzle takes the convergent-divergent form. The throat, where A is a minimum, is approached by a rounded entrance which allows the stream-lines to converge, and from the throat to the discharge-end the nozzle gradually expands, generally as a simple cone, until an area of section is reached which is appropriate to the final value of the pressure-ratio Z. The divergent taper from the throat onwards is made sufficiently gradual to preserve stream-line motion as completely as is practicable.

From the foregoing equations we have

$$\frac{Q}{A} = \frac{Z^{\frac{1}{\lambda}}}{V_1}\sqrt{\frac{2g\lambda}{\lambda - 1}\left(1 - Z^{\frac{\lambda-1}{\lambda}}\right)P_1V},$$

which may be applied to calculate the proper section A for a given discharge Q, at any place where the pressure has fallen from the initial value P_1 to the value ZP_1. For the purpose of designing a nozzle there are two places where this calculation has to be made, namely the throat and the end where discharge occurs. At the discharge-end Z is known, being the ratio of the back-pressure, against which the stream is to be discharged, to the initial pressure. We have to find Z_t, namely the ratio at the throat, and this is determined from the consideration that at the throat Q/A is a maximum. The above equation may be written

$$\frac{Q}{A}=\sqrt{\frac{2g\lambda}{\lambda-1}\,\frac{P_1}{V_1}}\sqrt{\left(Z^{\frac{2}{\lambda}}-Z^{\frac{\lambda+1}{\lambda}}\right)}.$$

Hence (by differentiating) the condition for a maximum is found to be that

$$\frac{2}{\lambda}Z^{\frac{2-\lambda}{\lambda}}-\left(\frac{\lambda+1}{\lambda}\right)Z^{\frac{1}{\lambda}}=0,$$

which gives

$$Z_t=\left(\frac{2}{\lambda+1}\right)^{\frac{\lambda}{\lambda-1}}.$$

This is sometimes called the critical ratio of pressures in the expansion of a jet.

When λ is 1·4, as it is in air and other nearly perfect gases, we have $Z_t=0{\cdot}528$. When λ is 1·3, as it is in dry steam, Z is 0·545.

It follows that for a dry steam jet the channel should be entirely convergent so long as the back-pressure is not less than 0·545 P_1, but when the back-pressure is less than this there should be a throat and a divergent extension beyond the throat.

Substituting the above expression for Z_t in the expressions already given, we obtain, for the pressure, volume, velocity, and section at the throat,

$$P_t=Z_tP_1=\left(\frac{2}{\lambda+1}\right)^{\frac{\lambda}{\lambda-1}}P_1,$$

$$V_t=\frac{V_1}{Z_t^{1/\lambda}}=V_1\left(\frac{\lambda+1}{2}\right)^{\frac{1}{\lambda-1}},$$

$$v_t=\sqrt{\frac{2g\lambda P_1V_1}{\lambda+1}},$$

$$\frac{Q}{A_t}=\frac{v_t}{V_t}=\left(\frac{2}{\lambda+1}\right)^{\frac{1}{\lambda-1}}\sqrt{\frac{2g\lambda P_1}{(\lambda+1)V_1}}.$$

The proper area A at any place where the pressure-ratio is Z is related to the throat-area A_t and the throat pressure-ratio Z_t by the following expression, which is readily found from the foregoing:

$$\frac{A^2}{A_t^2}=\frac{Z_t^{\frac{2}{\lambda}}-Z_t^{\frac{\lambda+1}{\lambda}}}{Z^{\frac{2}{\lambda}}-Z^{\frac{\lambda+1}{\lambda}}}=\left(\frac{Z_t}{Z}\right)^{\frac{2}{\lambda}}\left(\frac{1-Z_t^{\frac{1-\lambda}{\lambda}}}{1-Z^{\frac{1-\lambda}{\lambda}}}\right).$$

This is convenient in determining the final area of section that should be provided when the back-pressure is assigned.

It follows from these equations that the discharge through a given orifice under a given initial pressure P_1 depends only on the cross-section at the narrowest part of the orifice, and is independent of the back-pressure, provided the back-pressure is not greater than Z_tP_1. By continuing the expansion in a divergent nozzle after the throat is passed, the amount of the discharge is not increased, but the fluid acquires a greater velocity before it leaves the nozzle, because the range of pressure which is effective for producing velocity is increased. The heat-drop down to the pressure at the throat determines the amount of the discharge, and the remainder of the heat-drop, which would be wasted if there were no divergent extension of the nozzle, is utilised in the divergent portion to give additional velocity to the escaping stream. This velocity is given in a definite and useful direction, whereas if there were no divergent extension of the nozzle the fluid, after leaving the nozzle, would expand laterally, and its parts would acquire velocity in directions such that no use could be made of the kinetic energy so acquired.

Imagine steam or any gas to be expanding through a nozzle which has no divergent extension, into a chamber where the back-pressure P_2 is less than the value of P_t as calculated above. In that case the pressure in the jet, where it leaves the nozzle, will be P_t, and the further drop of pressure to P_2 will occur through scattering of the stream. The discharge in that case is not increased by any lowering of the back-pressure P_2, because any lowering of P_2 does not affect the final pressure in the nozzle, which remains equal to P_t. Osborne Reynolds explained the apparent anomaly by pointing out that the stream is then leaving the nozzle with a velocity equal to that with which sound (or any wave of expansion and compression) is propagated in the fluid, and consequently any reduction of the pressure P_2 cannot be communicated back against the stream: its effects are not felt at any point within the nozzle. The pressure in the stream at the orifice therefore cannot become less, however low the back-pressure P_2 may be. But if P_2 is increased so as to exceed P_t, the lateral scattering close to the orifice ceases, the velocity is reduced, the pressure at the orifice then becomes equal to P_2, and the discharge is reduced.

In applying these results to a nozzle of any form, the least section is to be regarded as the throat: if there is a divergent extension beyond the least section, the amount of the discharge is not affected, though the final velocity of the stream is increased. Taking a nozzle of any form, and a constant initial pressure P_1, if we reduce the back-pressure P_2 from a value which, to begin with, is just less than P_1, the discharge increases until P_2 reaches Z_tP_1, where Z_t has the value given above. After that, any further reduction of P_2 does not increase the discharge. But the velocity which the fluid acquires before it leaves the nozzle may then be augmented by lowering P_2 and adding to the divergent portion of the nozzle. The nozzle will be rightly designed when it provides for just enough expansion to make the final pressure equal to the back-pressure; the jet then escapes as a smooth stream, and the energy of expansion is utilised to the full. If the nozzle does not carry expansion far enough—if, in other words, the final pressure in the nozzle exceeds the back-pressure — energy will be wasted by scattering. If, on the other hand, the back-pressure is too high for the nozzle, so that the nozzle provides for more expansion than can properly take place, vibrations are set up in the fluid stream which cause some waste.

It has been pointed out by Callendar that in the formation of a steam-jet expansion occurs too fast to allow a condition of equilibrium to be attained as regards condensation. Steam that is superheated or dry at entry remains more or less completely dry during the first stages of its passage through the nozzle, even when it has expanded so considerably that its equilibrium condition would be that of a wet mixture. If the steam is superheated to begin with, it behaves like a gas in the initial stage of the expansion, and its equilibrium is stable until its condition crosses the boundary or saturation line, that is to say until its temperature falls to the value corresponding to saturation at the pressure then reached. As expansion proceeds beyond that stage a metastable state is temporarily set up in which the steam is supersaturated. If the steam is saturated to begin with, a metastable state is produced as soon as expansion begins. It is because of this supersaturation that the index λ for a steam-jet is to be taken as 1·3 instead of a value such as 1·135, which would approximately correspond to adiabatic expansion under conditions of equilibrium. When the latter index was used, as was generally done before Callendar drew attention to the true character of the action, the calculated discharge for a given size of throat was too small, with the result that the calculated discharge under ideal frictionless conditions fell short of the discharge which Rateau and other observers observed with actual nozzles, although the effect of friction is to reduce the real discharge below its ideal value. When account is taken of supersaturation, by using 1·3 as the index, the calculated discharge becomes, as it should be, slightly greater than the actual discharge. The experimentally measured discharge is about 3 or 4 per cent less than the calculated discharge under the ideal conditions which the calculation assumes. If, as is probable, the steam remains almost wholly dry up to the throat, this difference is to be ascribed mainly to friction.

§ (13) TESTS OF THERMODYNAMIC PERFORMANCE.—In testing the efficiency of an engine considered as a heat-engine we must measure the work done during a given time and compare that with the heat supplied. The work done may be determined, in an engine of the piston and cylinder type, by taking indicator diagrams from which the mean effective pressure of the steam is found by measurement, so that the horse-power developed within the cylinder or cylinders is readily calculated. The indicated horse-power, important as it is thermodynamically, is not a fair criterion of useful performance. That is found by measuring the brake horse-power, or its equivalent, which is less than the indicated power by the amount that the engine expends in overcoming the friction of its parts and in driving any necessary auxiliary mechanism essential to its own running. In turbine and other engines which are employed to generate electricity, it is usual to measure the electrical horse-power, thereby determining a net effective output which is less than the (hypothetical) brake horse-power by electrical and mechanical losses which occur in the dynamo. It is obvious that any figures for efficiency must be interpreted with reference to the stage at which the power developed is measured, whether in the cylinder (by means of an indicator), or on the shaft (by means of a dynamometer), or after conversion into some other form, as by a dynamo or by a pump.

The heat supplied to the engine may be found by direct measurement, or it may be inferred by measuring the rejected heat. The latter is a less usual procedure in engine tests. In any case, however, a measurement of the rejected heat furnishes a useful check on the accuracy of the other method: the most satisfactory trials are made by measuring the heat rejected as well as the heat supplied and the work done. A balance-sheet can then be drawn up in which the heat given to the engine may be more or less completely accounted for and the margin of error can be seen.

To find the heat supplied, we must observe the quantity of steam which passes through the engine, and its conditions of supply, as to pressure and (if there is superheating) as to temperature. The quantity passing through may be found by observing either (1) the boiler feed, or (2) the air-pump discharge, provided the engine uses a surface condenser.

To infer the steam supply from the boiler feed requires a prolonged run, for the level of water in the boiler cannot be read very accurately, and the whole consumption of feed-water should be great in order that possible errors due to that may become negligible. A single trial by this method may require to be continued for some hours, during which the conditions of working of the engine must be kept as nearly uniform as possible. On the other hand, a very short time suffices for an accurate measurement of the rate at which condensed water is discharged by the air-pump, once uniform working is established, and it is easy to get satisfactory trials under various conditions in comparatively quick succession, by varying the amount of the external "load," from zero up to the largest load that the engine can properly undertake.

The results of a series of trials are often exhibited by drawing a curve of total steam consumption per hour, or per minute, in relation to load, the load being expressed in indicated horse-power, or in brake horse-power, or in kilowatts if the net output is measured as electrical energy. Such a curve is called a Willans Line. It is in general nearly straight: an example is shown in *Fig.* 14, which relates to tests of a Parsons Turbine at various loads, made under nearly uniform conditions as to the pressure and temperature of the steam throughout the series. The (nearly) straight line shows the consumption of steam per hour in relation to the output of power in kilowatts. On the same diagram is a curve, showing the number of pounds of steam used per kilowatt-hour, which tends towards a minimum. When the engine is overloaded the Willans Line tends to bend upwards, with the result that the other curve passes a minimum, and the steam consumption per horse-power increases slightly at the highest loads. When the Willans Line is straight the whole consumption of steam at any load may be regarded as made up of two parts —the constant unproductive consumption that takes place without any effective output, and a further consumption that is simply proportional to the output. Even when the Willans Line is drawn to exhibit the relation of indicated work to steam consumption, this remark holds good. When extended backwards it does not pass through the origin, but above it. The consumption is approximately $a(n+b)$, where n is the number of horse-power, and a is the quantity of steam which is required per horse-power after the unproductive supply ab has been furnished.

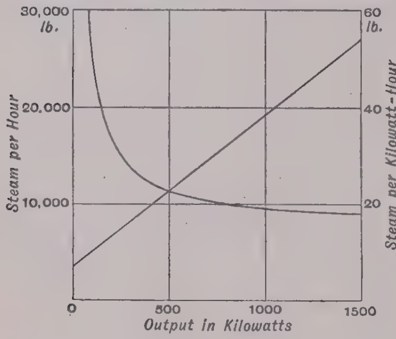

FIG. 14.

J. A. E.

REFERENCES

Rankine's *Steam Engine* was for long the most authoritative work on the subject, but its interest is now mainly historical. Perry, *The Steam Engine*; Dalby, *Steam Power*; Stodola, *The Steam Turbine*; Callendar, *Steam Tables*; Ewing, *The Steam Engine and other Heat Engines*; also *Thermodynamics for Engineers*. The writer has used the two last-named books (with the permission of the publishers, the Cambridge University Press) as the foundation of this article. See also the article "Thermodynamics."

STEAM TABLES, USE OF. See "Steam Engine, Theory of," § (9).

STEAM TURBINE, PHYSICS OF THE

THE following symbols, constants, and tables are used throughout this article, except where otherwise stated:

Units; lbs., ° F., ft., in.; hours, minutes, seconds.

A = area in square feet.
a = area in square inches.
α = velocity ratio = u/v.
d = mean diameter of blade ring in inches.
η = efficiency.
g = acceleration due to gravity = 32·2 feet per second.
γ = ratio of the specific heats of steam,
= 1·135 for adiabatic expansion of dry saturated steam,
= 1·30 for adiabatic expansion of superheated steam.
H = homogeneous head = $144pV$.
h = blade height in inches.
I = British thermal units per pound of steam.
$K = \Sigma N d^2 R^2 \times 10^{-9}$.
k = blade opening = $\sin\theta$ in *Fig.* 14.
λ = index in law of expansion pV^{λ} = constant.
N = number of rows of blades on cylinder or spindle.
n = revolutions per second.
$\omega = 2\pi n$ = angular velocity.
p = pressure in lbs. per sq. in. (absolute).
$p_1, p_2 \ldots p_o$ = pressures along turbines.
ϕ = entropy.
p_o = pressure at the exhaust end of turbines.
Q = pounds of steam per hour.
q = pounds of steam per second.
R = revolutions per minute.
T = temperature ° F. absolute.
t = temperature ° F.
u = blade velocity in feet per second.
V = specific volume of steam in cubic feet per pound.

v = steam velocity in feet per second.
w = work done in foot-pounds per pound of steam.
X = expansions by pressure $= p_1/p_2$.
$Z = p_2/p_1 = 1/X$.

(*Callendar's Steam Tables*, 1915, and *Peabody's Steam and Temperature-Entropy Tables*, 8th Edition, 1914.)

I. The Application of Thermodynamics to the Steam Turbine

§ (1) Homogeneous Head.—The quantity pV is not so variable in the case of saturated steam as either p or V, and its use simplifies many of the equations which follow. The value of $144pV$ is defined as the homogeneous head and denoted by H. Thus $H = 144pV$.

For saturated steam the following empirical formula may be used:

$$H = 8000\ (6 + \log p).$$

For superheated steam, which has nearly the properties of a perfect gas, the homogeneous head may be taken as proportional to the absolute temperature; so that if T is the temperature of saturated steam and T_s the temperature of superheated steam,

$$H_s = H\frac{T_s}{T}.$$

§ (2) The Quantity of Steam flowing through Nozzles.—The theory of the steam turbine is based on adiabatic expansion, although in practice radiation and other factors affect the result. If one pound of steam enters a nozzle at a pressure of p_1 and with a velocity v_1 and leaves with a velocity v at a lower pressure p, we have

$$v^2 - v_1^2 = 2gw. \quad . \quad . \quad . \quad (1)$$

If the initial velocity v_1 is negligible

$$v^2 = 2gw \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$
$$= 2 \times 32{\cdot}2 \times 778 \times I$$

and

$$v = 224\sqrt{I}\ (\text{more correctly } v = 223{\cdot}8\sqrt{I}). \quad (3)$$

In the case of a perfect gas working adiabatically it is proved[1] that the relation between pressure and volume is given by the equation

$$pV^{\gamma} = \text{constant},$$

where γ is the ratio of the specific heats and has the value 1·408 for air.

If the gas be not perfect an equation of similar form still holds, but the exponent will no longer be equal to γ. We may, however, write for steam expanding adiabatically

$$pV^{\lambda} = \text{constant}.$$

The value of λ will depend on the condition of the steam.

It is shown in the article on the "Steam Engine, Theory of,"[2] that, as the amount of expansion in a nozzle is increased beyond a certain point, the flow of the steam does not increase but reaches a maximum; this is called the critical expansion for the nozzle. The point is attained when the expression

$$Z^{\frac{2}{\lambda}} - Z^{\frac{\lambda+1}{\lambda}}$$

has such a value of Z as to make it a maximum; this is found by differentiating the expression and equating the result to zero. We then get for the critical value

$$Z_t = \left(\frac{2}{\lambda+1}\right)^{\frac{\lambda}{\lambda-1}}, \quad . \quad . \quad . \quad (4)$$

and for this the flow has a maximum value. Z_t is the ratio of expansion p_t/p_1 at the throat of a nozzle. It has the following values for different values of λ.

λ.	Z_t.
1·1	0·585
1·135	0·577
1·2	0·564
1·3	0·546
1·4	0·528

The formula for the maximum discharge of steam, when the drop in pressure is more than the critical, may be written

$$q_t^2 = 2g144A^2\frac{p_1}{V_1}\frac{\lambda}{\lambda+1}\left(\frac{2}{\lambda+1}\right)^{\frac{2}{\lambda-1}} \quad . \quad (5)$$

or

$$q_t^2 = m_1A^2\frac{p_1}{V_1},$$

or

$$q_t^2 = m_2\frac{A^2p_1^2}{H_1},$$

where m_1 and m_2 are constants.

Hence

$$q_t = \sqrt{\frac{m_2}{H_1}}Ap_1.$$

Let

$$m = \sqrt{\frac{m_2}{H_1}}.$$

Then $q_t = mAp_1$ lb. of steam per second

or $\quad Q_t = Map_1$ lb. of steam per hour, $\quad . \quad (5a)$

where a is in square inches and p_1 is in lbs. per sq. in. abs. and M is another constant.

Rateau's empirical formula for the discharge of saturated steam beyond the critical expansion is based on this formula, and may be written,[3] employing English units,

$$Q_t = 3{\cdot}6(16{\cdot}57 - \log p_1)ap_1. \quad . \quad (6)$$

In this expression M has been made equal to $3{\cdot}6\ (16{\cdot}57 - \log p_1)$.

For a long time a difficulty arose in reconciling experimental results for saturated steam with the above theory, the actual discharge of a nozzle proving greater than the theoretical discharge as calculated by a formula similar to (5). The value for λ was taken as 1·135, on the assumption that the steam was in stable equilibrium throughout; but H. L. Callendar[1] shows that the rapidity of flow through a nozzle is such that condensation is not instantaneous when the pressure drops, and that consequently the steam is behaving as a gas, and a more correct value of λ to take is 1·3. The steam is in a metastable condition, and is said to be supersaturated.

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[1] See "Thermodynamics," § (15).

[2] See "Steam Engine, Theory of," § (12).

[3] See H. M. Martin, *Steam Turbines*, 1913, p. 17.

The following table shows values of the constant M for different pressures of saturated steam:

p_1 lb. sq. in. abs.	$H=144p_1V_1$ (Callendar's Tables).	M in Equation (5a) when		M in Equation (6) (Rateau).
		$\lambda=1·3$.	$\lambda=1·135$.	
300	68,386	52·12	49·64	50·72
250	67,680	52·39	49·91	51·01
200	66,816	52·73	50·22	51·37
150	65,686	53·16	50·63	51·80
100	64,094	53·83	51·23	52·45
50	61,344	55·02	52·41	53·53
14·689	56,668	57·25	54·54	55·44
10	55,282	57·93	55·22	56·05
5	52,877	59·27	56·64	57·17
1·0	47,966	62·24	59·33	59·65

It is thus seen that for pressures above atmospheric pressure Rateau's experimental results are about 2¼ per cent above that expected for saturated steam with λ=1·135, but are about 3 per cent below that which would be given if there were no condensation and λ=1·3. Part of this difference is due to the friction of the steam in the nozzle and part may be due to partial condensation taking place.

Superheated steam behaves so nearly as a perfect gas that the relation H/T=constant may be used.

Since $$M \propto \frac{1}{\sqrt{H}} = \frac{\sqrt{T \text{ saturated}}}{\sqrt{T \text{ superheated}}},$$

it may therefore be corrected for superheated steam in the proportion of

$$\frac{\sqrt{T \text{ saturated}}}{\sqrt{T \text{ superheated}}},$$

e.g. at 250 lb. per sq. in. (abs.), with 200° F. superheat,

$$M_s = 51·01\sqrt{\frac{862}{1062}} = 45·96.$$

It should be noted that if the critical expansion has not been reached, the equation of continuity may be used to determine the quantity of steam flowing through a nozzle. That is,

$$Av = qV,$$

from which $$Q = \frac{25av}{V} \text{ lb. per hour.} \quad . \quad . \quad (7)$$

The number of heat units corresponding to one brake horse-power per hour would be

$$\frac{33,000}{778} \times 60 = 2545·6 \text{ B.Th.U.};$$

or to one kilowatt-hour

$$\frac{2545·6}{0·746} = 3411·9 \text{ B.Th.U.}$$

These values, divided by the adiabatic heat drop (I) give the number of pounds of steam required for an ideal turbine working on the Rankine cycle. The ratio between such an ideal consumption and the actual consumption of any particular turbine is known as *the coefficient of performance* or *efficiency ratio* of that turbine. This factor may, of course, be equally well calculated on efficiencies or heat units.

If the initial velocity is negligible,

$$v^2 = 2gw,$$

or, substituting[2] for w,

$$v^2 = 2g144\frac{\lambda}{\lambda-1}p_1V_1\left(1 - Z^{\frac{\lambda-1}{\lambda}}\right)$$

$$= 2gH_1\frac{\lambda}{\lambda-1}\left(1 - Z^{\frac{\lambda-1}{\lambda}}\right). \quad . \quad . \quad (8)$$

When the discharge is a maximum, the value for Z_t from equation (4) may be substituted in (8). Then

$$v_t^2 = 2gH_1\frac{\lambda}{\lambda-1}\left(1 - \frac{2}{\lambda+1}\right),$$

or $$v_t^2 = 2gH_1\frac{\lambda}{\lambda+1}. \quad . \quad . \quad . \quad (9)$$

By definition,

$$H_t = H_1\frac{p_tV_t}{p_1V_1} = H_1p_t\left(\frac{p_1}{p_r}\right)^{\frac{1}{\lambda}}$$

$$= H_1Z^{\frac{\lambda-1}{\lambda}}.$$

But $$Z_t = \left(\frac{2}{\lambda+1}\right)^{\frac{\lambda}{\lambda-1}}$$

at the throat for maximum discharge; therefore

$$H_t = \frac{2}{\lambda+1}H_1,$$

substituting this value in equation (8), gives

$$v_t^2 = \lambda gH_t, \quad . \quad . \quad . \quad (9a)$$

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[1] "On the Steady Flow of Steam through a Nozzle or Throttle," *Proc. Inst. Mech. Eng.*, Jan. 1915, pp. 53 *et seq.*

[2] See "Steam Engine, Theory of," § (12).

which is the velocity of sound in the steam[1] considered as a perfect gas.

This result can be explained by considering that the greatest velocity at which any reduction of pressure on the exit side of the throat can be carried back through the throat is the velocity of sound; and therefore any drop of pressure beyond the critical will not affect the flow of steam at the throat.[2]

II. Nozzles and Blading

§ (3) The Divergence and Efficiency of Steam Nozzles.—Denoting by the suffixes $_t$ and $_0$ the conditions in the throat and at the exit of a steam nozzle, the equation for continuity gives

$$q=\frac{\mathrm{A}_t v_t}{\mathrm{V}_t}=\frac{\mathrm{A}_0 v_0}{\mathrm{V}_0},$$

so that

$$\frac{\mathrm{A}_0{}^2}{\mathrm{A}_t{}^2}=\frac{v_t{}^2\mathrm{V}_0{}^2}{v_0{}^2\mathrm{V}_t{}^2}. \quad . \quad . \quad . \quad (10)$$

$$\frac{\mathrm{V}_0}{\mathrm{V}_t}=\left(\frac{p_t}{p_0}\right)^{\frac{1}{\lambda}} \text{ and } p_t=\mathrm{Z}_t p_1 \text{ or } p_1\left(\frac{2}{\lambda+1}\right)^{\frac{\lambda}{\lambda-1}},$$

so that

$$\frac{\mathrm{V}_0{}^2}{\mathrm{V}_t{}^2}=\left[\frac{p_1\left(\frac{2}{\lambda+1}\right)^{\frac{\lambda}{\lambda-1}}}{p_0}\right]^{\frac{2}{\lambda}}=\left(\frac{p_1}{p_0}\right)^{\frac{2}{\lambda}}\left(\frac{2}{\lambda+1}\right)^{\frac{2}{\lambda-1}}$$

$$=\mathrm{Z}^{-\frac{2}{\lambda}}\left(\frac{2}{\lambda+1}\right)^{\frac{2}{\lambda-1}}.$$

From equations (8) and (9)

$$\frac{v_t{}^2}{v_0{}^2}=\frac{2g\mathrm{H}_1\frac{\lambda}{\lambda+1}}{2g\mathrm{H}_1\frac{\lambda}{\lambda-1}\left(1-\mathrm{Z}^{\frac{\lambda-1}{\lambda}}\right)}$$

$$=\frac{\lambda-1}{(\lambda+1)\left(1-\mathrm{Z}^{\frac{\lambda-1}{\lambda}}\right)}.$$

Substituting these values in equation (10),

$$\frac{\mathrm{A}_0{}^2}{\mathrm{A}_t{}^2}=\frac{\lambda-1}{\lambda+1}\left(\frac{2}{\lambda+1}\right)^{\frac{2}{\lambda-1}}\frac{1}{\mathrm{Z}^{\frac{2}{\lambda}}\left(1-\mathrm{Z}^{\frac{\lambda-1}{\lambda}}\right)}$$

$$=\frac{\lambda-1}{2}\left(\frac{2}{\lambda+1}\right)^{\frac{\lambda+1}{\lambda-1}}\frac{1}{\mathrm{Z}^{\frac{2}{\lambda}}-\mathrm{Z}^{\frac{\lambda+1}{\lambda}}}, \qquad (10a)$$

giving an equation for the flare of a nozzle that contains only λ and Z. As Z is fractional, a more convenient form for calculation is obtained by writing $\mathrm{X}=1/\mathrm{Z}=p_1/p_0$, or the ratio of expansion by pressure. Then equation (10a) becomes

$$\frac{\mathrm{A}_0{}^2}{\mathrm{A}_t{}^2}=\frac{\lambda-1}{2}\left(\frac{2}{\lambda+1}\right)^{\frac{\lambda+1}{\lambda-1}}\frac{\mathrm{X}^{\frac{\lambda+1}{\lambda}}}{\mathrm{X}^{\frac{\lambda-1}{\lambda}}-1}. \qquad (10b)$$

Fig. 1 shows the theoretical divergence of nozzles ($\mathrm{A}_0/\mathrm{A}_t$) for various ratios of expansion (X) as obtained from equation (10b). It will be noted that for steam the curves are practically straight lines.

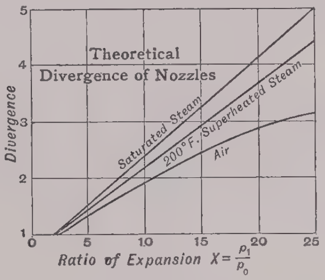

FIG. 1.

There is not as yet sufficient experimental evidence available to establish the best angle for the cone of the divergence. In practice it is found to vary between 4° and 30°. Experience shows that 12° gives very satisfactory results. In a similar way, there is a diversity of opinion as to the best angle between the axis of a nozzle and the direction of rotation of the moving blades. For a diverging nozzle this angle is usually made between 18° and 20°, but for non-diverging nozzles it is found to vary between 12° and 20°.

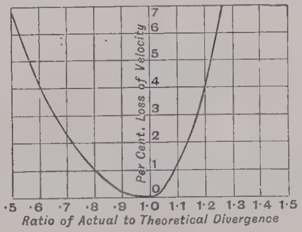

FIG. 2.

Fig. 2 is a combination of two curves obtained by W. J. Goudie[3] from an analysis of data published by Dr. Steinmetz, and shows the percentage loss of velocity for various ratios of actual to theoretical divergence. The advantages of under-expansion compared with over-expansion are clearly seen, a reduction of 20 per cent from the theoretical flare showing a loss of only 1 per cent in the exit velocity, where an excess of 20 per cent gives a corresponding loss of 4 per cent. In practice, therefore, nozzles are generally given about 10 per cent less divergence than theory would indicate. The effect of friction in the nozzle tends to increase the flare slightly, but in practice this correction is negligible. If $pV^{\lambda}=K$ for a frictionless nozzle, and $pV^{\lambda'}=K'$, if friction is accounted for, H. M. Martin points out [1] that

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[1] Newton's *Principia*, Book ii. section 8, referred to in Deschanel's *Natural Philosophy*, Everett's translation, 1875, p. 803.

[2] See Osborne Reynolds, "On the Flow of Gases," *Phil. Mag.*, March 1886, reprinted in his *Scientific Papers*, ii. 318.

[3] *Steam Turbines*, 1917, pp. 133-134.

$$\frac{\lambda'}{\lambda'-1}=\frac{1}{\eta}\,\frac{\lambda}{\lambda-1} \quad \text{or } 1-\frac{1}{\lambda'}=\eta\left(1-\frac{1}{\lambda}\right)$$

holds good, where η = the efficiency, which in practice is always above 90 per cent. Hence λ' is very nearly equal to λ.

In considering the efficiency of diverging nozzles it is necessary to distinguish between the loss of velocity of the steam and that of the kinetic energy. Equation (2) (*ante*) gives

$$v^2=2gw, \quad . \quad . \quad . \quad . \quad (2)$$

and from this we obtain by differentiation the result (dv and dw both being small),

$$2\frac{dv}{v}=\frac{dw}{w}, \quad . \quad . \quad . \quad . \quad (2a)$$

which shows that the percentage velocity loss is one-half the percentage energy loss.

For diverging nozzles, in which the heat drop (I) is well beyond the critical, H. M. Martin [2] gives the following empirical formula for the percentage loss due to kinetic energy:

$$\text{Percentage loss}=0{\cdot}06(\text{I}-45). \quad . \quad (11)$$

The corresponding efficiency is then given by the expression

$$\eta=102{\cdot}7-0{\cdot}06\ \text{I}. \quad . \quad . \quad (11a)$$

For example, with a heat drop of 125 B.Th.U. the loss of kinetic energy would be 0·06(125 – 45) or 4·8 per cent, and by equation (2*a*) the loss due to velocity would be half this amount or 2·4 per cent.

§ (4) BLADE FORMS AND THE EFFECT OF BACK RADIUS.—In reaction blading, expansion of the steam takes place in both the fixed blades or nozzles and the moving blades. A combination of one row of fixed blades and one row of moving blades forms one stage of a reaction turbine, and the heat drop per stage is generally less than the critical. In the case of impulse blading, all or nearly all the expansion takes place in the nozzles before the steam reaches the blades, and each set of expanding nozzles, together with their wheel, which may have one or more rows of revolving blades, is known as one stage of an impulse turbine. The heat drop through the nozzle of a multiple row wheel often exceeds the critical.

In drawing the cross-section of blading, the standard taken is the axial width of the blade. If α and β are the inlet and exit angles of the steam, the angle of the blade $\theta=\alpha+\beta$. This blade angle may vary very considerably in different types of blades. The curves of reaction blades do not lend themselves to any geometrical construction, but are based on practical experience. An example of reaction blading is shown in *Fig.* 3, from which it will be noted that the curves are made to sweep round gradually and evenly. It is safe to remember that an artistic blade is a good blade.

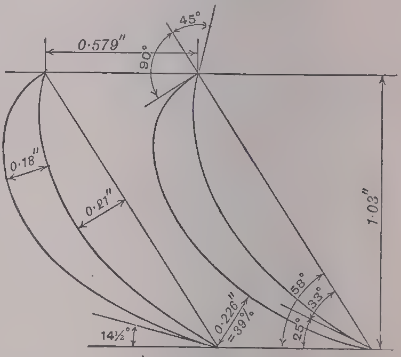

FIG. 3.

The shape of impulse blades is controlled to a large extent by the back radius (r). A large back radius gives a thin blade and a close pitch. A small radius gives a thick and stronger blade with a wide pitch. An example of impulse blading is shown in *Fig.* 4. The steam passage should converge without any irregularity. The small diagram shows a method of finding the slope of the chord YZ if the entrance and exit angles, α and β, are known. From this figure $\phi+\psi=90°$, $2\phi+(\alpha+\beta+1°)=180°$ and $\psi=\frac{1}{2}(\alpha+\beta+1)$.

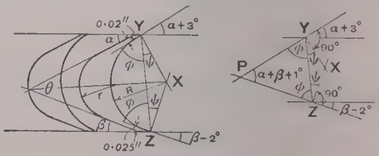

FIG. 4.

The point X may be obtained by making two right angles PYX and PZX, and the inside of the blade drawn with radius R(=XZ) from X. The back radius may then be put in from a centre lying on the line joining the point X to the apex of the blade.

§ (5) CARRY OVER.—In the foregoing theory it was assumed that the steam enters the nozzle at zero velocity; but in an actual turbine, in most cases, it enters at a velocity that is some fraction of the leaving velocity of the previous stage. The amount of energy that is due to this fraction of the leaving velocity is known as the "carry over." The frictional losses as the steam passes through the nozzle must be set against the extra energy due to this carry over, but the net result may be an alteration in the exit velocity of the steam as it leaves the nozzle. Let the proportion of the energy of the carry over that passes through the nozzle be denoted by m, and the proportion of the adiabatic heat drop utilised in the nozzle be denoted by M; then if v_1 is the velocity of carry over, v the velocity due to the heat drop, and v_0 the exit velocity,

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[1] *Engineering*, 1918, cvi. 53.
[2] W. J. Goudie, *Steam Turbines*, 1917, p. 128

$$v_0{}^2 = mv_1{}^2 + \mathrm{M}v^2. \quad . \quad . \quad (12)$$

Writing

$$z = v_1/v_0,$$

$$v_0{}^2 = \frac{\mathrm{M}}{1 - mz^2}v^2$$

$$= \delta v^2,$$

where

$$\delta = \frac{\mathrm{M}}{1 - mz^2}.$$ [1]

The value of z can be obtained from the velocity diagram. The values of m and M are very difficult to determine; but Goudie suggests [2] for impulse turbines that $m = 0{\cdot}8$ and $\mathrm{M} = 0{\cdot}90$ to $0{\cdot}93$, the higher figure being taken when there is superheat. As z is generally of the order of 0·3, this gives values of δ from 0·97 to 1·00, showing that the velocity of exit v_0 is practically equal to that due to adiabatic drop or v, as far as impulse turbines are concerned.

In the case of reaction turbines where the fixed and moving blades are the same, and there is an approximately equal heat drop, H. M. Martin [3] from an analysis of a marine turbine derived values of $m = 0{\cdot}52$ and $\mathrm{M} = 0{\cdot}90$ for this type of turbine; and Goudie publishes a curve [4] which shows that the coefficient δ varies from 1·02 to 0·97 as the velocity ratio (α) of the reaction turbine increases from 0·6 to 0·9. As these values of α may be considered the extreme limits in general use, the value of δ is so nearly unity that for all practical purposes of design the velocity of discharge may be taken as that due to the adiabatic heat drop through the stage of the reaction turbine.

This means that in the light of our present knowledge the friction and losses in the nozzle appear to be approximately equal to the energy supplied by the carry over, so that the steam enters the next stage with zero velocity; but the point cannot be said to be fully established, partly because of the doubt as to the exact values of m and M and partly because of another factor, namely the exact angle of the steam issuing from a nozzle. It has always hitherto been assumed that this angle corresponds with the exit angle of the nozzle or blade, but the point has never been properly investigated, and should it prove otherwise it will affect the value of z and therefore of the coefficient δ.

§ (6) VELOCITY DIAGRAMS AND EFFICIENCY CURVES.—The inlet angle α of a blade can be determined by vector diagrams, which also form a graphical method of obtaining the various velocity components of the steam at different stages of the turbine. The work done per pound of steam, and hence the theoretical efficiency of the blading, can also be determined. The accompanying figure shows such a diagram for a one-row impulse wheel. Steam leaves a nozzle with a velocity v and enters the moving blade in the direction of the axis of the nozzle inclined at an angle ϵ to the direction of rotation of the wheel. The mean peripheral velocity of the blading at a radius r is represented by u. The line v_1 completing the triangle gives both the magnitude and the direction of the inlet steam velocity relative to the blade and the inlet angle α. The exit steam velocity v_2 relative to the blade will be less than v_1 owing to frictional losses, which may cause from 10 to 15 per cent loss of velocity. The exit angle β is either assumed or obtained from the blade angle θ, and the closing line v_0 gives the magnitude and direction of the velocity of exit from the blade. In practice it appears that the energy the steam should possess in consequence of this velocity is utilised in overcoming the friction and in eddy-making.

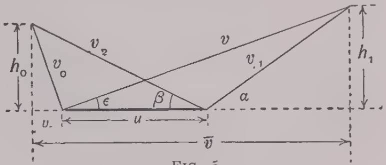

FIG. 5.

The ratio of the components of the relative velocities at right angles to the direction of motion of the blade, namely h_1/h_0, forms a measure of the ratio of the exit blade height to the nozzle height if the possible effects of reheat and drop of pressure in the blade are neglected. The change of the moment of momentum between inlet and exit gives the torque (T) on the shaft.

$$\mathrm{T} = \frac{\bar{v}r}{g} \text{ ft.-lbs.},$$

where r is the mean radius of the blades and $\bar{v}$ is the vector sum of the transverse components (v_1 and v_2) of the relative velocities. The work done per pound of steam

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[1] Cf. W. J. Goudie, *Steam Turbines*, 1917, p. 417.

[2] *Ibid.* p. 127.

[3] H. M. Martin, *The Design and Construction of Steam Turbines*, 1913, p. 110.

[4] W. J. Goudie, *Steam Turbines*, 1917, p. 418.

$$w = \omega T = \frac{\omega \bar{v} r}{g},$$

but $\omega = u/r$, so that

$$w = \frac{u \bar{v} r}{rg} = \frac{u \bar{v}}{g} \text{ ft.-lbs. per pound of steam.} \quad (13)$$

Again, from equation (2) the velocity energy available in the steam as it enters the blade is $v^2/2g$. Hence the theoretical thermal efficiency of the blade

$$\eta = \frac{u\bar{v}/g}{v^2/2g} = \frac{2u\bar{v}}{v^2}. \quad . \quad . \quad . \quad (14)$$

If there is more than one row of moving blades in one stage, the values of $\bar{v}$ must be obtained for each row and added together;

then $$w = \frac{u\Sigma\bar{v}}{g} \text{ and } \eta = \frac{2u\Sigma\bar{v}}{v^2}.$$

The relation between the blade speed and the steam speed, or the value u/v, is known as the velocity ratio of the turbine and plays an important part in the design of any particular machine. The mean peripheral velocity of the blades u may be calculated from the dimensions of the turbine:

$$u = \frac{\pi R d}{60 \times 12} = \frac{Rd}{230} \text{ ft. per sec.} \quad . \quad (15)$$

The velocity of the steam v in this relation is usually calculated on the adiabatic heat drop obtained from formula (3) (*ante*): $v = 224\sqrt{I}$, where I is the adiabatic heat drop between the steam pressures in lbs. per sq. in. absolute at the inlet and the exit of the stage.

Fig. 6 shows the relation usually obtained in practice between the actual thermal efficiency of the blading and the velocity ratio, for different types of turbine stages. It should be noted that the efficiencies upon which these curves are based, whilst not including mechanical friction, windage, or leakage losses in the turbine itself, do allow for the frictional effect and the spilling of the steam whilst passing through the nozzles and blading. In Parsons blading the carry over is included, but not in the case of impulse blading. It is advisable in practice to keep the velocity ratio of any particular type below the maximum efficiency shown by these curves, partly because the higher velocity ratio generally means a larger and more costly turbine, and partly because high velocity ratio and consequently increased size mean larger losses, due (for example) to the skin friction of the discs and other revolving parts of the turbine. As a result, the best efficiency is actually obtained with a velocity ratio somewhat below that shown by the maximum efficiency in the curves given in *Fig.* 6. The following range of values is found in modern practice for large turbines:

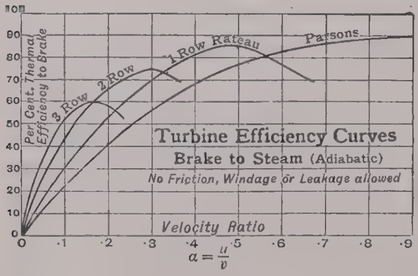

FIG. 6.

	$a = \frac{u}{v}$.
Reaction turbines	0·75-0·90
One-row impulse wheels . . .	0·45-0·52
Two-row impulse wheels . . .	0·22-0·28
Three-row impulse wheels . . .	0·13-0·15

§ (7) THE PARSONS DESIGN COEFFICIENT K.—From equation (3) the average steam velocity per stage (N) in the case of an impulse turbine is given by

$$v = 224\sqrt{\frac{I}{N}},$$

while in the case of a reaction turbine, where there are two rows of blades with a pressure drop in each stage, $v = 224\sqrt{2I/N}$, but

$$v = \frac{u}{a} \text{ and } u = \frac{Rd}{230} \text{ (from equation (15)).}$$

Therefore for impulse turbines

$$\frac{Rd}{230a} = 224\sqrt{\frac{I}{N}}$$

or $$\frac{NR^2d^2 \times 10^{-9}}{a^2} = 2{\cdot}65\,I.$$

Let $NR^2d^2 \times 10^{-9} = K$, then

$$a = \frac{u}{v} = \sqrt{\frac{K}{2{\cdot}65\,I}} \quad . \quad . \quad (16)$$

and $$a = \sqrt{\frac{K}{1{\cdot}32\,I}} \text{ for reaction turbines.} \quad (16a)$$

The quantity K, first employed by Parsons, forms a convenient coefficient for use in turbine design. For turbines in which the blade rings are not all the same diameter, K is computed separately for each diameter and added together to obtain the design coefficient

$$K = (N_1d_1{}^2 + N_2d_2{}^2 + \text{etc.})R^2 \times 10^{-9},$$

and from this the mean velocity ratio can be obtained. N is equal to the number of stages. In the reaction turbine this is taken to mean either the rows of blades on the spindle or in the casing, but not both.

Impulse turbines often consist of a multiple-row wheel followed by single-row wheels.

In that case, since N varies as the square of the velocity ratio, a two-row wheel is approximately equivalent to $(0{\cdot}45/0{\cdot}26)^2=3$ one-row wheels. Similarly, a three-row wheel corresponds approximately to $(0{\cdot}45/0{\cdot}15)^2=9$ one-row wheels.

Similar turbines may be said to be those of different sizes and output but of the same design; that is to say, with the same velocity ratio, stresses, and design coefficient and the same number of stages.

In such a case, from the definition of the design coefficient K, the diameter varies inversely as the speed, and the area of the steam passages through the blades as the square of the diameter. The total quantity of steam passing through, and therefore approximately the output, varies directly as the square of the diameter or inversely as the square of the speed. For example, taking as a normal machine 3000 kw. at 3000 r.p.m., then a similar machine running at half the speed would give $2^2=4$ times the output, or 12,000 kw., and one running at 1000 r.p.m. 27,000 kw.

§ (8) REHEAT FACTOR.—As the steam passes through a turbine, there is always a certain amount of leakage over the tips of the blades in a reaction type, and through the diaphragm glands in an impulse type. In both cases, also, heat is generated by the friction and eddying of the moving steam, some of which undoubtedly becomes reabsorbed, with the result that the steam in the exhaust tends to be drier than would be accounted for by pure adiabatic expansion. For both these reasons the specific volume of the steam may be larger, and the total heat drop available may be greater than that calculated by the use of adiabatic heat tables. The steam is said to be reheated, and the ratio of the heat drop available to the adiabatic heat drop is known as the reheat factor.

The effect of reheating may be seen on the accompanying Mollier diagram (*Fig.* 7). If the initial state of the steam is represented by the point a_1 on the line of constant pressure p_1, the adiabatic heat drop available in the first stage of a turbine in which the steam expands to a pressure p_2 is shown by the vertical line a_1a_2. If frictional and spilling losses are taken into account, the adiabatic heat drop will be

$$\eta_s \times a_1a_2 = a_1c_2,$$

where η_s is the stage efficiency based on such losses. The reheating effect causes the point c_2 to move at constant pressure to b_2 on the pressure line p_2, and this represents the condition of the steam at the end of the first stage. If the process is repeated for a multi-stage turbine working between p_1 and p_0 a series of steps is obtained; and since the lines of constant pressure diverge on a Mollier diagram it is evident that the available heat drop, $a_1a_2+b_2a_3+b_3a_4+\ .\ .\ .\ b_4a_0$, is slightly more than the adiabatic heat drop as represented by the line a_1d_0. As already stated, the ratio between these heat drops gives the reheat factor. Provided the stage efficiencies are known or assumed, this method can be applied whether the steam is originally superheated or whether the steps cross the saturation line or not. In the latter case, it is probable that a more accurate forecast of the reheat factor would be obtained if the pressure curves below the saturation line represented a supersaturated condition of the steam rather than a state of thermal equilibrium. Such an $I-\Phi$ chart, based on Callendar's tables, has been computed and drawn by H. W. Martin.[1] This effect of supersaturation is indicated on *Fig.* 7 by the dotted line.

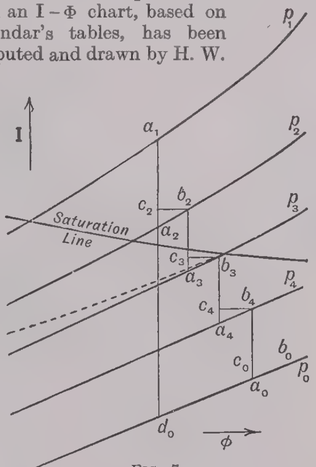

FIG. 7.

III. TYPES OF STEAM TURBINES

§ (9) All steam turbines receive their steam at a higher pressure, extract work from it by passing the steam through various arrangements of nozzles (or fixed blading) and moving blades, and reject the steam at a lower pressure and consequently at a larger volume.

If all, or nearly all, this drop in steam pressure takes place in the stationary nozzles, the design is known as the *Impulse Turbine.* When, on the other hand, there is nearly equal drop of pressure in both the fixed and the moving blades as the steam is passing through them, the design is generally referred to as *Reaction Turbine.*[2]

These names, impulse and reaction, have

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[1] "A New Theory of the Steam Turbine," *Engineering*, cvi. 1.

[2] See also "Turbine, Development of the Steam," § (2).

been adopted from the analogy of the water turbine, though, like a number of other terms current in engineering practice, they cannot be said to be a logical definition of what actually takes place in either type. To a less extent, the same objection holds good to the alternative definitions of velocity type and pressure type that occur in a number of text-books, but which are not commonly used in practice.

§ (10) THE IMPULSE TYPE.—In the impulse turbine, by suitably proportioning the cross-sectional areas of the nozzles the increase in volume or expansion due to the pressure drop causes the steam to issue with a comparatively high velocity. The moving blades are so curved that as much as is practicable of this high velocity is absorbed in causing the wheel to revolve, and the steam leaves with considerably less velocity, but the pressure on both sides of the moving blades remains practically unaltered. Each set of expanding nozzles, together with their wheel, which may have one or more rows of revolving blades, is known as *one stage* of the turbine. If the impulse turbine has only one stage, as in the case of the single wheel with multiple rows or of the De Laval type with one row, the pressure drops in the expanding nozzles to that of the exhaust, or nearly so. The moving blade passages are so shaped that the exit velocity of the steam is just sufficient to clear the wheel. If there are two or more stages, such as are found in the Rateau, Zoelly, and Curtis types, the pressure drops in steps through each stage, but remains the same or nearly the same on each side of the moving blades.

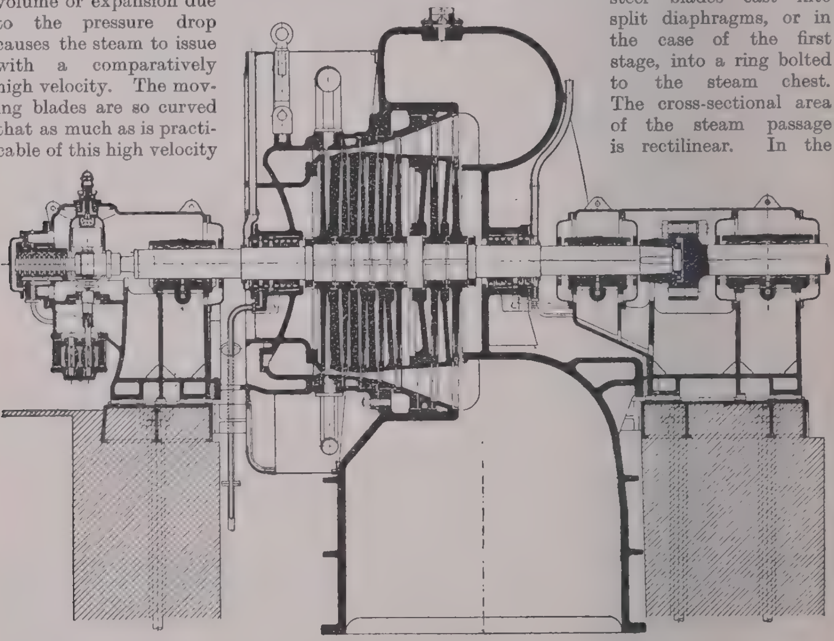

FIG. 8.

(i.) *Zoelly Turbine.*—A longitudinal section through a modern type of impulse turbine is shown in *Fig.* 8. It represents a Zoelly turbine as made by the firm of Escher Wyss & Company at Zurich. The size illustrated is designed to develop 15,000 horse-power when running at 3000 r.p.m. There are seven stages, each consisting of a row of nozzles followed by a single row of moving blades. The nozzles are formed by a number of nickel steel blades cast into split diaphragms, or in the case of the first stage, into a ring bolted to the steam chest. The cross-sectional area of the steam passage is rectilinear. In the earliest stages the nozzle arc does not extend completely round the circumference, but is arranged symmetrically in segments. This is known as partial admission. After full peripheral admission is reached, the blade heights are increased to correspond with the increasing volume of the steam. The revolving blades are also frequently made of nickel steel or phosphor bronze, and are fixed to the periphery of steel discs or wheels. The wheels are mounted on expanding rings, which are keyed on to the shaft and kept in position by nuts at each end. The nuts and sleeves tighten up against the collar, which can be seen on the shaft. For turbines running at 1500 r.p.m. or under this shaft is made rigid and is run considerably below the first critical or whirling speed; but in this case, at 3000 r.p.m., a flexible shaft is used, which runs through the critical speed under no-load during the process of warming up the turbine. Experience shows that this procedure can be followed, providing that the working speed is kept at not less than 30 per cent above or below the critical speed of the rotor. The flexible shaft has not entailed the use of spherical seatings for the shaft bearings. These are two in number, and are lined with white metal. They are lubricated by oil under pressure. As there is little or no change in pressure whilst the steam is passing through the moving blades, there is, theoretically, no axial thrust along the shaft of an impulse turbine. It is, however, customary to fit a thrust block, shown as an extension of the shaft at the high-pressure end; the thrust block is usually made adjustable so as to register and keep the correct position of the rotor relative to the casing. Such blocks can then take up any slight axial thrust in either direction. It will be noted that the casing as a whole is not rigidly fixed to the two main bearings, but that these latter are mounted in separate housings on two cross girders embedded in the foundation. The casing is carried on sliding supports, which allow for any expansion due to heating effects. The bearing housings can also move axially, so that provision is made for the turbine to adjust itself relative to the centre line of the exhaust, which is fixed by the condenser.

The main inlet of the steam is not shown. The steam is admitted through a throttle governor operated by an oil relay into one side of the annular space to which the first row of nozzles is bolted. The steam passes through strainers to the nozzle segments arranged symmetrically in front of the first row of moving blades. A supplementary steam inlet is provided for overloads, in this case between the first and the second stage. The steam then passes through the remaining stages into the exhaust branch that surrounds the last row of blades, and from there to the condenser underneath the turbine. The shaft as it enters and leaves the turbine casing passes through carbon packed glands, which effectively prevent any steam from escaping on the live side and maintain the required vacuum at the exhaust end. In order to prevent leakage from stage to stage at the shaft, it is necessary to keep down the clearance between the fixed diaphragms and the revolving wheel hubs as much as possible. At the same time a certain amount of latitude must be allowed for any slight deflection of the shaft. If the pressure drop is considerable some form of flexible gland is occasionally used, such as carbon segments held in position by springs; but in most cases the difficulty is surmounted by leaving a space between the diaphragm and the hub and inserting a number of wedge-shaped soft metal rings on the inner circumference of the diaphragm, with their apex towards the shaft, so that should touching occur no material damage will be done.

(ii.) *Metropolitan-Vickers Turbine.*—Another example of a modern impulse turbine is shown in *Fig.* 9. It is designed by the Metropolitan-Vickers Company of Manchester, and shows one two-row wheel followed by thirteen single-row wheels. The normal output is 12,500 kilowatts, and the speed is 3000 revolutions per minute. Instead of allowing the steam to fill the annular space in front of the first stage, it is admitted into one, two, or three separate nozzle boxes, according to whether half-load, full load, or overload is required. These boxes, which are made of cast steel, are so shaped as to be free to expand without affecting the alignment of the nozzles that form part of the first stage. As in the Zoelly turbine, provision is made to allow for any expansion or distortion of the materials of the turbine under the heating effects of the steam. An arrangement of multi-exhaust blading is shown, which is embodied in most of the large impulse turbines made by this Company. It is designed to obviate the difficulty of dealing with large quantities of steam at very low pressures, which would otherwise entail a large diameter of blade ring or the use of excessive lengths for the exhaust blades. The moving blade in the last stage but two is divided into two portions. The outer half is shaped to allow the steam passing through it to expand to the pressure of the exhaust. The remainder of the steam is by-passed through the inner half without expansion, and is again divided up by means of a specially shaped fixed nozzle to repeat the process through the next row of moving blades. In the last stage all the remaining steam expands to the last row of moving blades and passes through to the exhaust. The effect produced may be taken as the equivalent of a blade height equal to the sum of the two outer portions of the divided blades plus the length of the moving blade in the final stages, and thus the loss due to the velocity of the steam leaving the last row of blades can be reduced.

§ (11) The Axial-flow Reaction Type, Parsons Turbines.—In axial-flow reaction turbines, introduced by Parsons, the high-pressure steam first enters a row of fixed blades, in which it is caused to expand slightly and at the same time to increase in velocity. The steam is then passed through a row of moving blades so proportioned that whilst the increase of velocity is absorbed the pressure also diminishes as much as it did in the

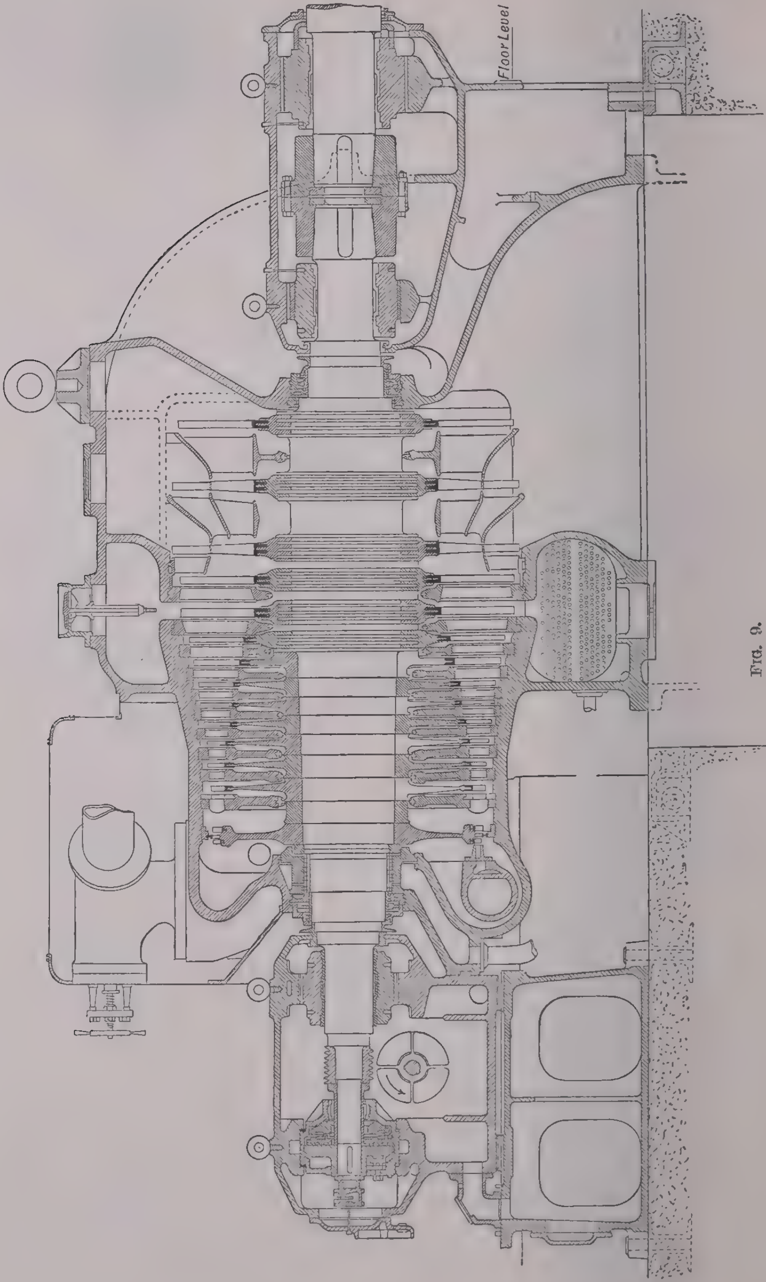

FIG. 9.

previous row of fixed blades. This combination of one row of fixed blading and one row of moving blades forms *one stage* of a reaction turbine, and there are always a large number of such stages as compared with the impulse type. The pressure falls gradually throughout the length of the turbine; and as the specific volume of the steam is correspondingly increasing, the diameter of the drum carrying the blades is stepped up to prevent the blade heights from becoming excessive. Each diameter is referred to as a *drum*, so that a reaction turbine often consists of *a high-pressure drum, an intermediate-pressure drum, and a low-pressure drum,* on each of which there is an appropriate number of stages.

In very large turbines the low-pressure drum is sometimes made as a separate machine, in which the steam generally enters at the middle and divides right and left to exhaust at each end. In this way the blade heights are halved. Such a machine is known as a *double-flow* turbine. In a few cases the double-flow low-pressure drum is included in one casing or cylinder.

For mechanical reasons of construction the heights of the blades are kept the same for a number of stages; but they may be stepped up on each drum to approximate more closely to the conical reaction turbine, which is the ideal shape for this type not yet completely achieved in practice.

(i.) *Parsons Turbine.*[1]—*Fig.* 10 shows a modern example of a 10,000-kilowatt high-pressure tandem turbine, as made by the firm of C. A. Parsons & Company at Newcastle-upon-Tyne. It is designed to run at 2400 revolutions per minute. The live steam, after passing through the main inlet valve (not shown) enters the annular space surrounding the first stage from underneath. Overloads are automatically taken up by the action of the by-pass valve shown above this space, which admits the full-pressure steam to a second annular space surrounding the second section of stages. The steam then passes through the remaining rows of blades on the high-pressure drum, and is carried by a steampipe connected to the large flange shown below the second main bearing to the centre of the second turbine, where it divides to pass through more stages to exhaust at both ends into condensers connected to the two exhaust branches. The low-pressure drum is therefore balanced, but the first turbine requires *balance pistons* or *dummies* to counteract the axial thrust on the moving blades. These dummies are shown to the left of the first stage. They are three in number, one for each diameter of the turbine drum; connecting pipes (not shown) maintain an equal pressure between the dummy and the corresponding section of the

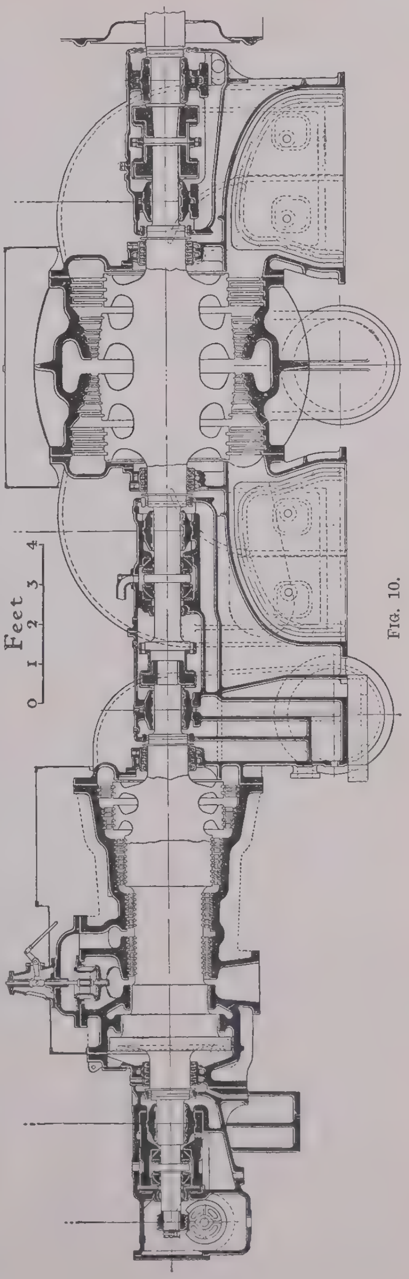

FIG. 10.

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[1] See also "Turbine, Development of the Steam."

turbine. It will be noted that each turbine has an end-thrust bearing fitted in front of the left-hand main bearing in each case. This ensures a correct register of the fixed to moving blades, as in the impulse type.

§ (12) Combination Reaction and Impulse Turbine.—In the impulse turbine the wheels carrying the moving blades are frequently referred to as *discs*, whilst the cylindrical spindle on which the moving blades of the reaction turbine are mounted is called the *drum*. Many modern turbines that form a combination of the two are therefore known as the *disc and drum type*. In such a case a number of stages of reaction blading at the high-pressure end of the turbine are replaced by one or more impulse stages, with a considerable saving in the over-all length of the turbine.

Richardson Westgarth's Turbine.—A disc and drum turbine as made by the firm of Richardson, Westgarth, & Company of Hartlepool is illustrated in *Fig.* 11. It has an output of 15,000 kilowatts when running at 1500 revolutions per minute. It consists of a two-row impulse wheel followed by two sections of reaction blading. The reaction stages approximate more closely to the conical type. The axial thrust is balanced by one dummy piston, seen just to the left of the impulse wheel; its diameter is approximately the mean of that of the two reaction sections. Other features, already referred to under the impulse and reaction types, are embodied in this design.

§ (13) The Radial-flow Reaction Turbine.—Nearly all successful turbines of the present day work with the steam flowing axially—that is to say, parallel with the shaft or spindle. There is, however, one notable exception, the Ljungström turbine, in which the steam enters the first stage near the spindle and flows radially outwards at right angles to the main axis of the turbine. In this turbine there are two discs, each carrying reaction blades, which project from them axially and which are connected to them by specially shaped expansion rings. The two discs revolve in opposite ways, so that the speed of the rings of blades relatively to one another is doubled, which enables the whole of the expansion to be carried out in a single pair of discs.

(i.) *Ljungström Turbine.*—A diagram of a 1500-kilowatt Ljungström turbine designed to run at 3000 revolutions per minute is shown in *Fig.* 12. It is made in England by the Brush Electrical Engineering Company of Loughborough. High-pressure steam enters through a pipe that is brought into the exhaust and passed by two expansion joints to annular steam chests surrounding the labyrinth packing gland on the end of each shaft. From these annular spaces the steam has direct access to the centre of the turbine, where it enters the first stage and flows radially to the periphery of the turbine. Arrangements are made for by-passing live steam into the turbine after the first few stages to cope with overloads. In order to avoid excessive blade length, the last few stages are arranged for parallel flow. In the larger machines the last stage may be fitted with the Parsons axial-flow reaction blading. Such an arrangement can be seen in *Fig.* 13, which is a compounded section of half the turbine as far as the shaft. The right half of this drawing shows the upper part of the turbine and details of the by-pass for overloads, the left half showing the underneath part with the main steam inlet.

Elaborate precautions are taken to allow for an expansion of all parts subject to high temperature without affecting their relative positions. This is satisfactorily achieved by the use of a number of expansion rings, whose cross-section somewhat resembles that of a dumb-bell. The metal in the disc or blade ring, as the case may be, is closed round the head of this dumb-bell by rolling to form a circular socket joint that is firm but flexible. It will be noted that all parts exposed to high-temperature steam, including the radial labyrinth packing and the steam chest itself, are linked up to the outer casing by means of these expansion rings. The radial labyrinth glands are so proportioned that they balance any axial thrust of the steam tending to force the blade discs apart.

Each shaft is direct-coupled to an alternator, and the stator windings of the two alternators are permanently connected in parallel, so that electrically the two machines form a single unit, though mechanically they are revolving in opposite directions at half the speed of the relative velocity of the blades to one another.

The Ljungström turbine belongs to the reaction type, since there is a drop in pressure in each of the two rings of blades forming one stage. The heat drop through each stage will be less than the critical value, and the blade areas may therefore be determined from the equation of continuity. Equation (7) may be written

$$a=\frac{QV}{25v}=\pi khd,$$

where k is the blade opening.

But $$v=\frac{u}{\alpha}=\frac{Rd}{230\alpha},$$

hence $$h=\frac{230\alpha QV}{25\pi Rkd^2},$$

or $h \propto V/d^2$ as long as the velocity ratio (α) remains constant from stage to stage. This is approximately true in the larger sizes of Ljungström turbines. The solution of this

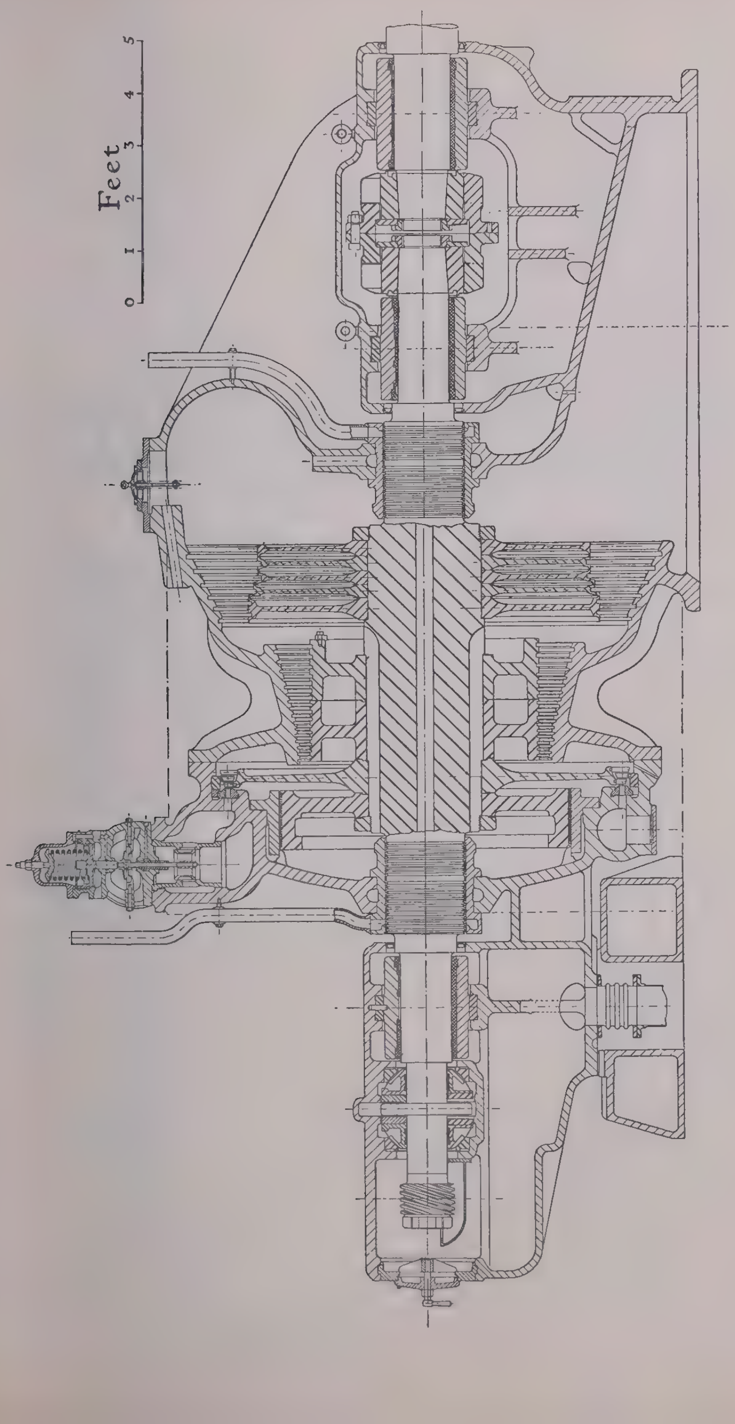

FIG. 11.

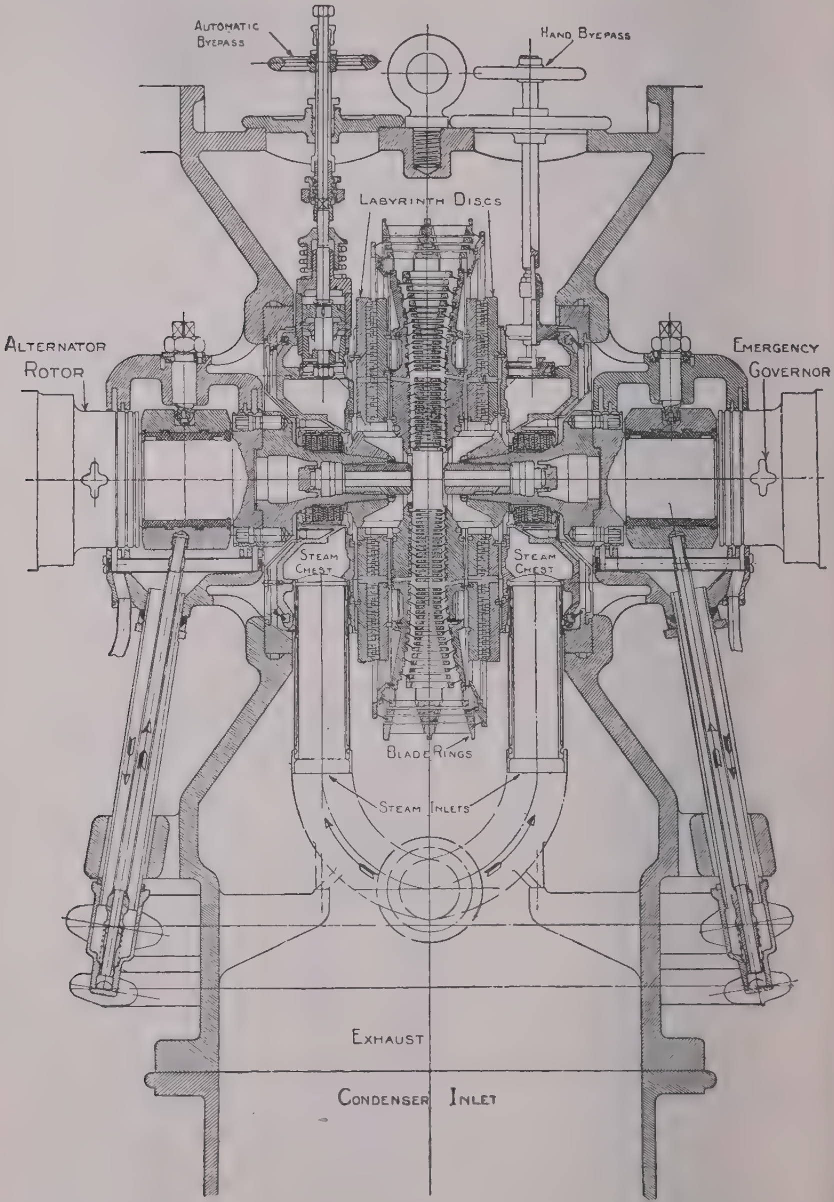
Automatic Byepass
Hand Byepass
Labyrinth Discs
Alternator Rotor
Emergency Governor
Steam Chest
Steam Chest
Blade Rings
Steam Inlets
Exhaust
Condenser Inlet

FIG. 12.

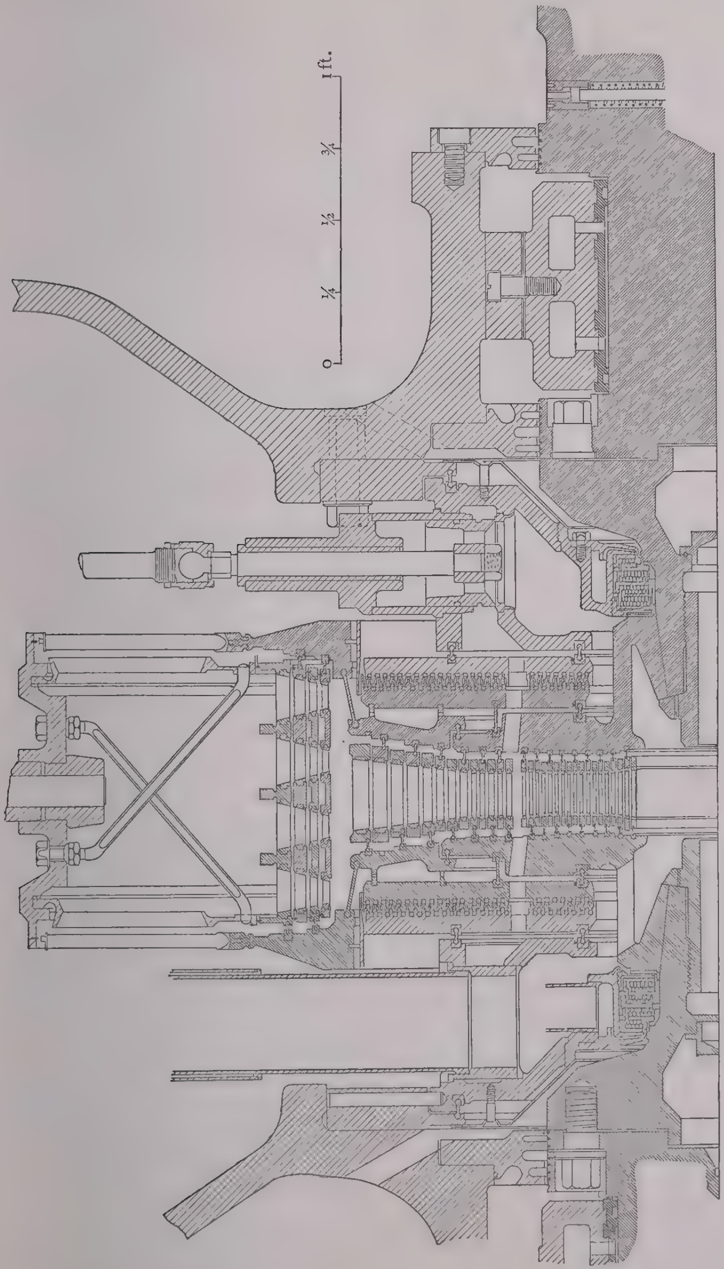

FIG. 13.

equation accounts for the convergence and divergence of the steam passage in this type of turbine.

§ (14) TURBINES FOR SPECIAL PURPOSES.—The requirements of a number of industries for steam other than for power generation have resulted in the design of modified types of steam turbines, both impulse and reaction, which have gradually come to be known by distinguishing names. For instance, a turbine may be made to work non-condensing to exhaust either at atmospheric pressure or above it. This enables a constant supply of low-pressure steam to be available for manufacturing purposes. Such a turbine is often called a *back-pressure turbine.* More commonly, the demand for low-pressure steam is not regular, but varies from time to time, as in heating the factory, which is a winter demand, or in a number of manufacturing processes that are intermittent. In that case the turbine is so arranged that steam may be drawn off as required from a point just above atmospheric pressure, whilst the remainder of the steam passes through the low-pressure end of the turbine to the condenser in the usual way. The name *reducing turbine* is applied to such a design, though in the United States of America they are often called *bleeder* or *extraction turbines.*

In a reciprocating engine, on account of fluid friction and the size of low-pressure cylinder that would be required, it is rarely possible to carry the expansion down to the vacuum in the condenser, and release generally takes place at 2 or 3 lbs. per square inch above the pressure in the condenser. Thus the whole of the energy in the last part of the expansion is lost in such an engine, and the extra heat drop due to high vacua is not available. As a result, vacua for reciprocating engines are generally between 25″ and 27″, and are rarely higher. In the case of a steam turbine, however, by suitably proportioning the exhaust blades the very highest vacuum possible in practice can as a rule be made use of, and therefore vacua from 28½″ or 29″, or even more, are common. The great advantage that a steam turbine has in being able to utilise the highest vacuum possible is at once seen if the heat drop available by Peabody's tables from (say) saturated steam at 215 lbs. absolute to various vacua is considered.

To 25″	vacuum	294	B.Th.U. available.
26″	„	306	„
27″	„	320	„
28″	„	343	„
29″	„	370	„

Here it is seen that between 27″ and 29″ there is a difference of 50 B.Th.U., or, say, 15 per cent.

This capacity of the steam turbine for utilising high vacua has led to the introduction of *Exhaust Turbines,* where the steam from a reciprocating engine exhausting at about atmospheric pressure is further utilised; and as a rule the power derived from such a turbine is about the same as from the reciprocating engine. Thus the power obtained with a given amount of steam is doubled. In many cases the reciprocating engine and the steam turbine form one unit; but if the reciprocating engine works intermittently, as in the case of a rolling-mill engine or a winding engine, a thermal accumulator is fitted between the two to give an approximately constant supply of steam to the steam turbine. In some cases, however, the supply of exhaust steam is at certain times liable to fail or to be insufficient to supply the turbine, and then a high-pressure stage is often fitted, to which high-pressure steam is automatically turned on when the low-pressure supply fails or is not sufficient. The turbine is then called a *Mixed-pressure Turbine.*

IV. PRINCIPLES OF STEAM TURBINE DESIGN

§ (15) THE IMPULSE TYPE. (i.) *Conditions of Service.*—In designing any particular turbine the following conditions would be known, or should be assumed: type, output, speed, initial steam pressure on the boiler side of the stop valve, superheat, and vacuum.

The mean blade speed (u) is limited by rotational stresses, a common European practice being 600 feet per second, though in America speeds of 800 feet per second have been used.

The mean diameter of the blade ring (d) follows directly from equation (15),

$$d=\frac{230u}{R}=\frac{138,000}{R}, \quad . \quad . \quad (15a)$$

when the blade speed is 600 feet per second. To estimate the number of stages, the available heat drop per pound of steam through the turbine is required. This can be considered in a variety of ways (see § (8)); but, as a rule, it is sufficiently accurate for this particular purpose to assume simple adiabatic expansion from the initial pressure on the boiler side of the stop valve. Unless a very large scale Mollier diagram is available, heat-drop or temperature-entropy tables should be used. In this connection it is worth noticing that Peabody's temperature-entropy tables only go down to 0·594 lb. per square inch. If a lower pressure is required, it is necessary to extrapolate; but the ratio of the heat drop to the temperature curve may be taken as a straight line in this region, which makes it possible for the required figures to be easily obtained.

Velocity ratios (α) should be chosen with the aid of the curves or table in § (6). Then

$$v_1=\frac{u}{\alpha_1} \text{ feet per second}$$

for the first stage; and the adiabatic heat drop (I_1) for this stage is obtained from formula (3),

$$I_1 = \frac{(u/\alpha_1)^2}{224^2}. \quad . \quad . \quad . \quad (3a)$$

If the turbine is of the impulse type and consists of a two-row wheel followed by a number of one-row wheels, I_1 may vary between 20 and 30 per cent of I, the total adiabatic heat drop through the turbine. Similarly, for the one-row wheels,

$$v_2 = \frac{u}{\alpha_2} \text{ feet per second,}$$

and the adiabatic heat drop (I_2) in each one-row wheel

$$I_2 = \frac{(u/\alpha_2)^2}{224^2}. \quad . \quad . \quad . \quad (3b)$$

Since, neglecting the reheat factor, which is generally small, the B.Th.U. available in all the one-row wheels is $I - I_1$, the number of one-row wheels required (N_2) will be equal to $(I - I_1)/I_2$ to the nearest whole number. The velocity ratio (α_2) provisionally assumed for the one-row wheels can be checked by formula (16)

$$\alpha_2 = \sqrt{\frac{NR^2 d^2 \times 10^{-9}}{2{\cdot}65\,I}}.$$

In this formula N equals the equivalent number of one-row stages, or

$$N = 3 + N_2$$

for a two-row wheel followed by single rows, on the assumption already made, that a two-row wheel is equivalent to three one-row wheels.

(ii.) *Proportions of Nozzles.*—If it is desired to proceed further with the design and to calculate the proportions of the various nozzles, it is advisable to modify the above-described method of arriving at the heat drops, so as to conform more to conditions pertaining to actual practice.

The initial steam pressure and temperature are measured on the boiler side of the stop valve. As the steam passes through the valve and governor gear, there will be a drop in pressure before it reaches the first stage of the turbine. This is frequently assumed to be one atmosphere, or, say, 15 lbs. per square inch. The temperature, on the other hand, will remain the same, except for one or two degrees radiation loss.

The adiabatic heat drop will therefore be measured on a slightly higher entropy due to the increased superheat; but as the range of pressure is 15 lbs. less, the total heat available will be from 0·5 to 1·0 per cent less. The heat drop in the two-row wheel will be the same as before (I_1), so that there will be slightly less heat available for the one-row wheels. Supposing that instead of I heat units it is found that only I' heat units are available (where $I' = (1 - 0{\cdot}01)I$). Then the heat drop in the two-row wheel remaining the same leaves

$$I' - I_1 = I_2',$$

or I'_2/N_2 B.Th.U. available per stage in the one-row wheels.

Owing to causes pointed out in the paragraph on the reheat factor, the actual B.Th.U. through the turbine will be higher than I'; but as values of the reheat factor are difficult to estimate correctly, this adjustment is better left to the discretion of the individual designer. It will be found near enough for the purpose of deriving the nozzle areas if the reheat factor is omitted in estimating the pressure drop, and an approximation made in the probable increase in the volume of the exhaust steam due to reheating. If there had been no reheating, and therefore no increase in entropy (ϕ), the specific volume (V_0) of the steam at the exhaust pressure (p_0) corresponding to the adiabatic heat drop (I') can be found direct from the tables. If now an internal efficiency (η_i) for the whole turbine is estimated from the curves shown in § (6), then the actual heat drop through the turbine would equal $\eta_i I'$. (In the example such an efficiency would lie somewhere between the two-row and the one-row wheels, say 82 per cent.) The heat thrown away in the exhaust would be

$$\overline{\text{B.Th.U.}} - \eta_i I' = I_0',$$

where $\overline{\text{B.Th.U.}}$ represents the total heat at the beginning of the first stage of the turbine. From the temperature-entropy table, I'_0 and p_0 corresponds to an increased entropy and an increased specific volume V'_0. In the absence of more definite information the percentage increase in volume,

$$\frac{V_0' - V_0}{V_0} \times 100,$$

may be distributed gradually over the one-row wheels of the turbine. (For instance, if there is an estimated increase in specific volume of 9 per cent to be spread over 9 one-row wheels, then the adiabatic specific volume for the first one-row wheel should be increased 1 per cent; for the second, 2 per cent; and so on up to 9 per cent for the ninth or last one-row wheel.)

To estimate the nozzle areas it is necessary to know the output (kw. or h.p.) of the turbine and the over-all efficiency (η). Then, since 1 kilowatt-hour = 3412 B.Th.U., or 1 horse-power-hour = 2546 B.Th.U.,

$$Q = \frac{3412}{\eta I} \times \text{kw.} \quad . \quad . \quad . \quad (17)$$

or

$$\frac{2546}{\eta I} \times \text{h.p.} \quad . \quad . \quad . \quad (17a)$$

If the drop in pressure through the nozzle is beyond the critical, the theoretical area (a) in square inches follows directly from equation (5a). $a = Q_t/Mp_1$ sq. in. for saturated steam, or $Q_t/M_s p_1$ sq. in. for steam initially superheated. In practice this area would be increased about 3 per cent to allow for nozzle loss.

If the drop in pressure through the nozzle is less than the critical, equation (7) should be used:

$$a = \frac{QV'}{25c} \text{ sq. in.,} \qquad . \quad . \qquad (7a)$$

where V′ is the corrected specific volume in cubic feet at the exit from the nozzle.

Each row of nozzles should be set out to scale, to see that the above areas come out correctly. On the assumption that the whole of the expansion takes place in the nozzles, the approximate face area of each arc can be determined from the nozzle angle (θ). From *Fig.* 14 it will be seen that the distance o is equal to $l \sin \theta$. With due allowance for divergence and thickness, $o' = kl \sin \theta$ where k is a constant embodying the ratio o'/o and the thickness of the nozzle plate. It is most conveniently determined from the drawing board. The face area of the nozzle arc or segment (Σlh)

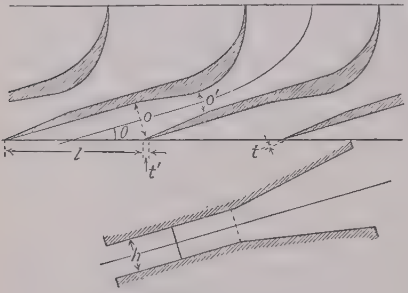

FIG. 14.

$$\Sigma lh = \frac{a}{k \sin \theta}.$$

Since the cross-sectional area of each nozzle is $o'h$, the theoretical number of nozzles in the arc for normal full load is $a/o'h$.

In practice, overload valves are generally fitted, making it possible for this number to be increased by about 50 per cent, to allow overloads to be taken, and to allow full output to be obtained in emergency under reduced steam conditions. When the length of the arc Σl exceeds πd, the mean circumference of the blade ring, the height h must be increased; but as it is not generally advisable to have h/d more than a certain ratio, it may be necessary to increase the blade angle towards the exhaust end of the turbine, to obtain sufficient area.

§ (16) THE AXIAL-FLOW REACTION TYPE (PARSONS).[1]—The reaction turbine has a large number of stages compared with the impulse type, and expansion of the steam takes place partly in the fixed and partly in the moving blades, instead of practically all in the fixed blades or nozzles, as is the case with the impulse type. For the purposes of design equal expansion in the fixed and moving blades is assumed; and the large number of stages ensures that the pressure drop through each stage is always less than the critical, and since it is not advisable generally to have a greater blade height than $d/5$ except at the exhaust end, the spindle diameter is stepped up as occasion requires, a common practice being to have three stages, which are then called the high-pressure, intermediate-pressure, and low-pressure drums respectively. Spindle diameters are arbitrarily fixed, usual proportions being in the ratio $1 : \sqrt{2}$ or $1 : \sqrt{3}$. The ideal turbine would have successively increasing blade heights in each cylinder to form what may be called a "conical" turbine; but in order to make machining practicable it is often arranged for the blades to form a series of parallel steps, though there may be several such steps on each drum. The fact that expansion takes place in both fixed and moving blades and the large number of stages makes it tedious to use the heat-drop method outlined for the impulse turbine. It is better to calculate the pressure throughout the turbine in the following way:

(i.) *The Pressure along an Axial-flow Reaction Turbine.*—For the sake of simplicity this is calculated for saturated steam. For an elemental difference of pressure dp with an adiabatic heat drop

$$v^2 = 2gH\frac{dp}{p}. \qquad . \quad . \quad . \qquad (18)$$

Since the pressure drop in each reaction stage is always less than the critical, the equation of continuity gives the relation $c = QV/3600A$, but $V = H/144p$,

so that $$v = \frac{QH}{3600 \times 144\, pA}$$

$$= \frac{QH}{51{\cdot}84 \times 10^4 pA},$$

and $$v^2 = \frac{Q^2H^2}{26{\cdot}9 \times 10^{10} p^2A^2}. \qquad . \quad . \qquad (19)$$

Equating (18) and (19),

$$64{\cdot}4\, p\, dp = \frac{Q^2H}{26{\cdot}9 \times 10^{10} A^2}.$$

Integrating between the limits of p_1 and p_2, and including N = the number of stages—that is, the number of rows on either the spindle or the casing that have the same mean diameter and height,

---

[1] See "Turbine, Development of the Steam," §§ (2), (3).

$$\frac{64{\cdot}4}{2}(p_1{}^2-p_2{}^2)=\frac{2\mathrm{NQ}^2\mathrm{H}}{26{\cdot}9\times10^{10}\mathrm{A}^2},$$

which on reduction gives

$$p_1{}^2-p_2{}^2=\frac{\mathrm{NQ}^2\mathrm{H}}{4{\cdot}33\times10^{12}\mathrm{A}^2}.$$

For a turbine working between the limits of p_1 and p_0 this becomes

$$p_1{}^2-p_0{}^2=\frac{\mathrm{Q}^2\mathrm{H}}{4{\cdot}33\times10^{12}}\Sigma\left(\frac{\mathrm{N}}{\mathrm{A}^2}\right);\quad.\quad(20)$$

p_0 is always small compared with p_1, and $p_0{}^2$ may be neglected. Equation (20) may then be written

$$p=\frac{\mathrm{Q}\sqrt{\mathrm{H}}}{208\times10^4}\sqrt{\Sigma\left(\frac{\mathrm{N}}{\mathrm{A}^2}\right)},\quad.\quad(20a)$$

which is the formula for the pressure along an axial-flow reaction turbine.

In practice it is more convenient to work with the mean ring diameter d and the blade height h than with the area in square feet A. The opening o' (see *Fig.* 14 *ante*) of a normal reaction blade may be taken as $0{\cdot}35l$; and since there is always full peripheral admission,

$$\mathrm{A}=\frac{0{\cdot}35\pi hd}{144}\quad.\quad.\quad.\quad.\quad(21)$$

$$=\frac{hd}{130{\cdot}9}.\quad.\quad.\quad.\quad.\quad(21a)$$

Substituting for A in equation (20*a*) gives

$$p=0{\cdot}63\times10^{-4}\mathrm{Q}\sqrt{\mathrm{H}}\sqrt{\Sigma\frac{\mathrm{N}}{h^2d^2}}.\quad.\quad(20b)$$

When allowance is made for carry-over in the exhaust, steam used to operate the governor (if used), leakage over the tips of the blades and in the dummies, and the steam used for packing the glands, the value for p would be about 80 per cent for small and 87½ per cent for large turbines of the theoretical value given by this expression. It may also be taken to increase in the ratio of the square root of the absolute temperature if superheated steam is used. This formula may be used either for one drum of uniform diameter or for a series of increasing steps in which each drum is of uniform diameter.

In some turbines, instead of the blades being on a series of steps, an approximation to the theoretical heights is obtained by making the whole or part of the turbine with increasing blade heights to approximate to a cone; for the conical parts of such a turbine $\Sigma(\mathrm{N}/h^2d^2)$ may be evaluated as follows: Consider a spindle of uniform diameter S with N stages of fixed and moving blades increasing evenly from a mean diameter d_1 to a mean diameter d_2, and let h_1 and h_2 be the blade heights at inlet and exit. Then

$$d_1=\mathrm{S}+h_1\text{ and }d_2=\mathrm{S}+h_2.$$

If $$a=\frac{h_2-h_1}{\mathrm{N}}=\frac{d_2-d_1}{\mathrm{N}},$$

then the average height $h=h_1+ax$, and the average mean diameter $d=d_1+ax$.

Hence $$\left[\Sigma\frac{\mathrm{N}}{h^2d^2}\right]=\int_0^{\mathrm{N}}\frac{dx}{(d_1+ax)^2(h_1+ax)^2}$$

$$=\frac{2}{a(d_1-h_1)^3}\int\left(\frac{1}{ax+d_1}-\frac{1}{ax+h_1}+\frac{d_1-h_1}{2(ax+d_1)^2}\right.$$
$$\left.+\frac{d_1-h_1}{2(ax+h_1)^2}\right)d(ax)$$

$$=\frac{2}{a\mathrm{S}^3}\left\{\log_\epsilon\frac{d_2h_1}{d_1h_2}+\frac{(h_2-h_1)a\mathrm{S}}{2}\left(\frac{1}{d_1d_2}+\frac{1}{h_1h_2}\right)\right\}\quad(22)$$

or $$\frac{2}{a\mathrm{S}^3}\left\{\log_\epsilon\frac{d_2h_1}{d_1h_2}+\frac{\mathrm{N}a\mathrm{S}}{2}\left(\frac{1}{d_1d_2}+\frac{1}{h_1h_2}\right)\right\}.\quad.\quad(22a)$$

The log term is negative and small compared with the second term within the bracket. $1/d_1d_2$ is also small compared with $1/h_1h_2$. The omission of these two terms, which to a large extent compensate one the other, would reduce equation (22*a*) to

$$\frac{\mathrm{N}}{\mathrm{S}^2h_1h_2}\quad.\quad.\quad.\quad.\quad(22b)$$

as an approximation. The expression for the pressure along a conical turbine then becomes

$$p=0{\cdot}63\times10^{-4}\mathrm{Q}\sqrt{\mathrm{H}}\sqrt{\frac{\mathrm{N}}{\mathrm{S}^2h_1h_2}}.\quad.\quad(20c)$$

Equation (20*b*) may be written

$$p_0=0{\cdot}63\times10^{-4}\mathrm{Q}\sqrt{\mathrm{H}}\sqrt{\Sigma_0},\quad.\quad.\quad(20d)$$

where Σ_0 represents an imaginary resistance of the turbine if the steam were expanded down to zero pressure from the exhaust pressure (p_0). By evaluating this expression for p_0 and then working backwards from the exhaust end it is possible to calculate the pressure drop (p) through each stage when the blade heights and diameters are known or assumed.

(ii.) *Relation between the Pressure and Parsons Design Coefficient K.*—For a conical turbine the design coefficient may be evaluated as follows, using the notation of the previous paragraph:

$$\mathrm{K}=\Sigma[\mathrm{N}d^2]\mathrm{R}^2\times10^{-9}$$
$$=\mathrm{R}^2\!\int(d_1+ax)^2dx\times10^{-9}$$
$$=\frac{\mathrm{R}^2}{3a}(d_2{}^3-d_1{}^3)\times10^{-9}$$
$$=\frac{\mathrm{NR}^2}{3}(d_2{}^2+d_1d_2+d_1{}^2)\times10^{-9}\qquad(23)$$
$$=\frac{\mathrm{NR}^2}{4}(d_1+d_2)^2\times10^{-9}\text{ nearly.}\quad(23a)$$

For the ordinary axial-flow reaction turbines with stepped spindles and blades, the design coefficient becomes

$$\mathrm{K}=[\mathrm{N}_1d_1{}^2+\mathrm{N}_2d_2{}^2+\mathrm{N}_3d_3{}^2]\mathrm{R}^2\times10^{-9},$$

where the suffixes represent the three cylinders. An arbitrary but convenient division is to assume that there is ½K on the low-pressure cylinder and ¼K in each of the other two cylinders. If a velocity ratio (a) is chosen from the curves or table in § (6) and assumed to be constant for the whole turbine, K can be obtained from formula (16a),

$$a=\sqrt{\frac{\mathrm{K}}{1\cdot32\,\mathrm{I}}}, \quad . \quad . \quad . \quad (16a)$$

when the total heat drop (I) is known from the steam conditions.

This enables the number of stages on each drum to be obtained when once the diameters are assumed.

Again,
$$\frac{v^2}{2g}=778i=\mathrm{H}\frac{dp}{p},$$

where i and dp are the heat drop and difference in pressure per stage. By integration

$$778\,\mathrm{I}=\mathrm{H}\log_\epsilon\frac{p_1}{p_2}=2\cdot3\mathrm{H}\log\mathrm{X},$$

whence
$$\mathrm{I}=\frac{2\cdot3\mathrm{H}\log\mathrm{X}}{778}.$$

Hence the velocity ratio may be written

$$a=\sqrt{\frac{778\mathrm{K}}{1\cdot3\times2\cdot3\mathrm{H}\log\mathrm{X}}}$$

$$=\sqrt{\frac{260\mathrm{K}}{\mathrm{H}\log\mathrm{X}}}. \quad . \quad . \quad . \quad . \quad (24)$$

If an average value of 65,000 is inserted for the homogeneous head H, equation (24) becomes

$$a=\sqrt{\frac{\mathrm{K}}{250\log\mathrm{X}}}, \quad . \quad . \quad . \quad (24a)$$

which means that if log p is plotted against K, assuming a constant H, the result will be a straight line. This forms a convenient check on a tabulated design, to see how near the velocity ratio remains constant through the turbine.

(iii.) *Proportions of Blading.*—The volume of steam flowing per second may be written

$$\frac{\mathrm{QV}'}{3600}\text{ cubic feet per second};$$

and the steam velocity

$$v=\frac{\mathrm{QV}'}{3600\mathrm{A}}\text{ feet per second.}$$

Since
$$\mathrm{A}=\frac{k\pi hd}{144}\text{ from equation (21),}$$

$$v=\frac{144\mathrm{QV}'}{3600k\pi hd}\text{ feet per second.}$$

Again, the blade speed u from formula (15) is

$$u=\frac{\mathrm{R}d}{230}\text{ feet per second,}$$

so that the velocity ratio (a) may be written

$$a=\frac{u}{c}=\frac{3600\pi k\mathrm{R}hd^2}{230\times144\mathrm{QV}'}$$

$$=\frac{k\mathrm{R}hd^2}{293\mathrm{QV}'}. \quad . \quad . \quad . \quad . \quad (25)$$

For a normal reaction blade the opening k may be taken as 0·35; but in order to prevent the blade heights from becoming excessive towards the exhaust end k may equal 0·5 (semi-wing blades) or 0·7 (wing blades). This is a convenient expression for determining the heights of the blades, particularly at the exhaust end, when due allowance has been made for reheating in the final specific volume.

G. S.
T. P.

STEFAN-BOLTZMANN LAW OF RADIATION: a law which states that the total radiation, of all frequencies, in unit volume, is a function of the absolute temperature T only, and is proportional to the fourth power of T. See "Radiation Theory," § (5) (i.); "Pyrometry, Total Radiation," § (2).

STEPHENSON'S "ROCKET." See "Steam Engine, Reciprocating," § (15).

STIFFNESS OF BEAMS. See "Structures, Strength of," § (9).

STIRLING'S CYCLE. See "Thermodynamics," § (27).

STOKES' FORMULA FOR THE RESISTANCES TO THE MOTION OF A SPHERE IN A VISCOUS LIQUID. See "Friction," §§ (8), (21).

STONE, BRICK, AND CONCRETE: GENERAL CONDITIONS FOR TESTING. See "Elastic Constants, Determination of," § (135).

STONE AND BRICK—TABULATED RESULTS OF CRUSHING, TRANSVERSE AND ABSORPTION TESTS. See "Elastic Constants, Determination of," § (135), Table 51.

STRAIN: a term used to denote a development of the purely kinematic conception of relative displacement; two kinds of strain must be distinguished:

(a) "Stretch" or "Extension," defined as the limit, when AB is indefinitely decreased, of the quantity

$$e=\left[\frac{\mathrm{A'B'}-\mathrm{AB}}{\mathrm{AB}}\right],$$

where AB is an element of length along the axis Ox, and A′, B′ the positions of the points A and B when the material is stretched.

(b) "Shear-strain" or "Slide," defined as the value of γ, in the expression

$$\gamma=\angle\mathrm{C'A'E'}-\angle\mathrm{CAE},$$

when the original ∠CAE is a right angle, ∠CAE being one of the angles of the elementary parallelepiped of the elastic solid, and C′, A′, E′ the new positions of the points C, A, E when the solid is stretched. See "Elasticity, Theory of," § (3).

STREAM-FLOW, GAUGING OF. See "Hydraulics," § (9).

STREAM-LINE MOTION

A "LINE OF MOTION," in Hydrodynamics, is a line drawn from point to point always in the direction of the velocity. A system of such lines, if drawn sufficiently close, gives an instantaneous picture of the whole state of motion of the fluid so far as direction only is concerned. In the case of an *incompressible* fluid the representation can be made to include the magnitude of the velocity as well. In two-dimensional motion, for instance, the velocity is indicated by the greater or less degree of closeness of the lines, if these are suitably spaced, being everywhere inversely proportional to the distance between adjacent lines. In three-dimensional cases we must imagine the fluid to be made up of filaments, or "tubes of flow," each containing the lines of motion which traverse a small area. If the sizes are adjusted so that the flux, *i.e.* the product of the velocity into the cross-section, is the same for each tube, the velocity will be everywhere inversely proportional to the area of the cross-section.

In general the configuration of the lines of flow is continually changing, so that the lines of motion may differ widely from the actual paths of the particles. It may happen, however, that the configuration is persistent, whether absolutely in space, or relatively to a moving solid. The latter case is reduced to the former by impressing on everything a velocity equal and opposite to that of the solid. The motion of the fluid is then said to be "steady," and the lines of motion are appropriately described as "stream-lines," since they are now the actual paths followed by the fluid particles.

The flow of a real fluid relation to an immersed solid is, however, seldom entirely "steady." Under the influence of friction eddies are formed which are continually detached from the surface, and drift away until they are finally extinguished by viscosity. To diminish the loss of energy and consequent increased resistance, due to this cause, it is important, in such questions as the design of aeroplanes, airships, and even aeroplane struts and wires, to choose such forms or sections as are found (empirically) to reduce the formation of eddies to a minimum. Such shapes are known as "stream-line" forms, since the flow past them can be represented to some extent by a permanent system of stream-lines. They are not unlike the contour of a fish, in that they are rather blunt in front and taper towards the rear.

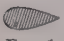

FIG. 1.

The tracing of the stream-lines in various typical cases of motion of an incompressible fluid is an important problem of theoretical hydrodynamics. In two dimensions this is facilitated by the use of Lagrange's "stream-function" ψ, which may be defined as follows. The motion being supposed everywhere parallel to the plane xy, we draw in this plane a line from a fixed point A to a variable point P. The quantity of fluid which in unit time crosses any two such lines (say from right to left as regards the direction from A to P) will be the same, provided the space between them is wholly occupied by fluid. It may therefore be regarded as a function of the position of P; we denote it by ψ_P, or simply by ψ. If P describes a stream-line, no change ensues in the value of ψ, and the equation of the family of stream-lines is therefore

$$\psi = \text{constant}. \qquad . \quad . \quad . \quad (1)$$

If δs be a line-element drawn in any direction, and $\delta\psi$ the corresponding variation of ψ, we have $\delta\psi = q\delta s$, where q is the component velocity at right angles to δs, reckoned positive when from right to left. Hence $q = \partial\psi/\partial s$. As particular cases, the component velocities at P parallel to the co-ordinate axes are

$$u = -\frac{\partial\psi}{\partial y}, \quad v = \frac{\partial\psi}{\partial x}, \qquad . \quad . \quad (2)$$

These satisfy, as they ought, the equation of continuity

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0. \qquad . \quad . \quad . \quad (3)$$

Any form (free from singularities) which we may choose to assign to ψ gives a state of motion which is geometrically possible as an instantaneous condition; but it does not follow that this can persist dynamically, even in a frictionless liquid subject only to the mutual pressure of its parts.

If the motion has been generated from rest, either by pressure only, or under the action of ordinary forces such as gravity, it may be shown that the component velocities can be expressed also in the forms

$$u = -\frac{\partial\phi}{\partial x}, \quad v = -\frac{\partial\phi}{\partial y}, \qquad . \quad . \quad (4)$$

where ϕ is a function called the "velocity-potential" from its analogy with the gravitational and electric potentials. The curves $\phi = \text{const.}$ are accordingly called "equipotential" lines. If δs be a linear element drawn in any direction, the velocity along δs is

$$u\frac{dx}{ds} + v\frac{dy}{ds} = -\frac{\partial\phi}{\partial s}, \qquad . \quad . \quad (5)$$

from (4). It follows that the component velocity along an equipotential line is zero, and therefore that the curves $\phi = \text{const.}$, $\psi = \text{const.}$ intersect at right angles. If these curves be drawn for a series of equal infinitesimal increments of ϕ and ψ they will, moreover, divide the plane into infinitesimal squares. For if δs be the distance between two consecutive stream-lines, and δn that between two consecutive equipotential lines, the velocity may be expressed (as regards magnitude) either by $\delta\psi/\delta s$, or by $\delta\phi/\delta n$. Hence if $\delta\psi=\delta\phi$ we have $\delta s=\delta n$.

A solution of the combined system of equations (2) and (4) is obtained by any assumption of the form

$$\phi+i\psi=f(x+iy), \quad . \quad . \quad (6)$$

where $i=\sqrt{(-1)}$. For this makes

$$\frac{\partial\phi}{\partial y}+i\frac{\partial\psi}{\partial y}=if'(x+iy)=i\left(\frac{\partial\phi}{\partial x}+i\frac{\partial\psi}{\partial x}\right), \quad (7)$$

whence, equating separately the real and imaginary parts,

$$\frac{\partial\psi}{\partial y}=\frac{\partial\phi}{\partial x}, \quad \frac{\partial\psi}{\partial x}=-\frac{\partial\phi}{\partial y}. \quad . \quad . \quad (8)$$

The particular case

$$\phi+i\psi=\mathrm{V}(x+iy) \quad . \quad . \quad . \quad (9)$$

gives a uniform flow with velocity V in the direction of x-negative.

Again, the assumption

$$\phi+i\psi=\mathrm{C}\log(x+iy)=\mathrm{C}\log(re^{i\theta}), \quad (10)$$

where r, θ are polar co-ordinates, gives

$$\phi=\mathrm{C}\log r, \quad \psi=\mathrm{C}\theta. \quad . \quad . \quad (11)$$

The stream-lines $\theta=$const. are straight lines radiating from the origin, which may be regarded as a fictitious "source" of fluid. Since the velocity is $-\partial\phi/\partial r=-\mathrm{C}/r$, the output of this source, as measured by the flux across any circle of radius r, is $-2\pi\mathrm{C}$.

If in (10) we put $\mathrm{C}=i\mathrm{C}'$, we have

$$\phi=-\mathrm{C}\theta, \quad \psi=\mathrm{C}\log r. \quad . \quad . \quad (12)$$

The stream-lines are now circles about the origin. We have the case of a "free vortex," but the region immediately about the origin must be excluded from the domain of the formulae, since the velocity there would be infinite.

Again, the assumption

$$\phi+i\psi=\frac{\mathrm{C}}{x+iy}=\frac{\mathrm{C}(x-iy)}{x^2+y^2}, \quad . \quad (13)$$

or

$$\phi=\frac{\mathrm{C}x}{x^2+y^2}=\frac{\mathrm{C}\cos\theta}{r},$$

$$\psi=-\frac{\mathrm{C}y}{x^2+y^2}=-\frac{\mathrm{C}}{r}\sin\theta \quad (14)$$

gives two systems of circles touching the axes of y and x, respectively. The circles $\psi=$const. are the lines of flow due to the motion of a cylinder parallel to the axis of x. This may be seen by superposing a uniform flow parallel to x, thus if

$$\psi=\mathrm{V}y\left(1-\frac{a^2}{r^2}\right), \quad . \quad . \quad (15)$$

the line $\psi=0$ consists partly of the axis of x ($y=0$), and partly of the circle $r=a$. The formula therefore represents the flow with the general velocity V past a stationary cylinder.

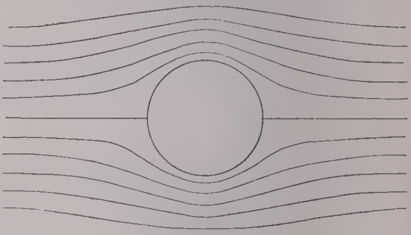

FIG. 2.

There is a similar theory for the case of symmetry about an axis. The motion being supposed to take place in a series of planes through Ox, and to be the same in each such plane, a line AP in the plane xy will represent an annular surface about Ox. If A is fixed the amount of fluid which in unit time crosses this annulus will depend only on the position of P; we denote it by $2\pi\psi$. If P be displaced parallel to Oy, we have $2\pi\delta\psi=-u\,.\,2\pi y\delta y$, where u is the velocity parallel to Ox. Again, considering a displacement parallel to Ox, we get $2\pi\delta\psi=v\,.\,2\pi y\delta x$, where v is the velocity at right angles to Ox. Thus

$$u=-\frac{1}{y}\frac{\partial\psi}{\partial y}, \quad v=\frac{1}{y}\frac{\partial\psi}{\partial x}. \quad . \quad . \quad (16)$$

In the same way the velocities along and at right angles to the radius vector OP are found to be, in polar co-ordinates,

$$u'=-\frac{1}{r^2\sin\theta}\frac{\partial\psi}{\partial\theta}, \quad v'=\frac{1}{r\sin\theta}\frac{\partial\psi}{\partial r}. \quad (17)$$

When a velocity-potential exists we have also

$$u=-\frac{\partial\phi}{\partial x}, \quad v=-\frac{\partial\phi}{\partial y} \quad . \quad . \quad (18)$$

or

$$u'=-\frac{\partial\phi}{\partial r}, \quad v'=-\frac{\partial\phi}{r\partial\theta}. \quad . \quad . \quad (19)$$

In the case of a uniform flow parallel to x we have obviously

$$\phi=\mathrm{V}x, \quad \psi=\tfrac{1}{2}\mathrm{V}y^2. \quad . \quad . \quad (20)$$

The next simplest case is that of radial flow from a source at the origin, viz.

$$\phi=\frac{\mathrm{C}}{r}, \quad . \quad . \quad . \quad . \quad (21)$$

whence

$$u'=\frac{\mathrm{C}}{r^2}, \quad v'=0. \quad . \quad . \quad . \quad (22)$$

Comparing with (16) we have

$$\psi=\mathrm{C}\cos\theta=\mathrm{C}\frac{x}{r}. \quad . \quad . \quad (23)$$

The case of a "double source," *i.e.* of a positive source and an equal negative source (or "sink") close together at the origin, is deduced by differentiation with respect to x. Thus

$$\phi = -\mathrm{C}\frac{x}{r^3}, \quad \psi = \mathrm{C}\frac{r^2 - x^2}{r^3} = \mathrm{C}\frac{y^2}{r^3} \quad . \quad (24)$$

This represents the case of a sphere moving through a liquid which is otherwise at rest. For, superposing a uniform flow parallel to x, we may write

$$\psi = \tfrac{1}{2}\mathrm{U}y^2\left(1 - \frac{a^3}{r^3}\right). \quad . \quad . \quad (25)$$

The line $\psi = 0$ consists partly of the axis of x and partly of the circle $r = a$.

The variation of pressure along a stream-line, in the steady motion of a frictionless liquid, is given by a simple formula due to D. Bernoulli (1738). Consider two cross-sections $\delta\mathrm{S}_1$, $\delta\mathrm{S}_2$ of a tube of flow. Let p denote pressure, v velocity, ρ the density, V the potential energy per unit mass relative to the field of force, if any. In the time δt a mass $\rho v_1 \delta\mathrm{S}_1 \delta t$ crosses the first section, and an equal mass $\rho v_2 \delta\mathrm{S}_2 \delta t$ crosses the other. Hence the portion of fluid which was initially included between the two sections has gained energy to the amount

$$\{(\rho\mathrm{V}_2 + \tfrac{1}{2}\rho v_2^2) - (\rho\mathrm{V}_1 + \tfrac{1}{2}\rho v_1^2)\}\rho v \delta\mathrm{S}\delta t.$$

An amount of work $p_1 \delta\mathrm{S}_1 . v_1 \delta t$ has been done on it at the first section, whilst it has done work on the adjacent fluid at the second, to the amount $p_2 \delta\mathrm{S}_2 v_2 \delta t$. The excess of work done on it is therefore $(p_1 - p_2)v\delta\mathrm{S}\delta t$. Equating this to the increment of energy, we find

$$p_1 + \rho\mathrm{V}_1 + \tfrac{1}{2}\rho v_1^2 = p_2 + \rho\mathrm{V}_2 + \tfrac{1}{2}\rho v_2^2. \quad (26)$$

The quantity

$$p + \rho\mathrm{V} + \tfrac{1}{2}\rho v^2$$

is called the "total head"; it is here proved to be constant along each stream-line, but not necessarily when we pass from one stream-line to another. This theorem is the basis of the method of measuring the velocity at any point of a stream by means of a "Pitot tube." The above proof assumes that friction may be neglected, but not that a velocity-potential exists. In the latter case the statement may be extended; the total head is then *everywhere* the same.

A simple proof of this statement may be given for the case of two dimensions. Let PQ be an element of a stream-line, and let PP′, QQ′ be normals drawn to an adjacent stream-line. Ultimately PP′ and QQ′ will be elements of two consecutive equipotential lines, so that the fall of potential (ϕ) from P to Q will be the same as from P′ to Q′. Hence, if v and $v + \delta v$ be the corresponding velocities,

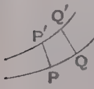

Fig. 3.

$$-\delta\phi = v . \mathrm{PQ} = (v + \delta v)\mathrm{P'Q'}. \quad . \quad (27)$$

But, writing R for the radius of curvature of the stream-line at P, and δn for PP′, we have

$$\frac{\mathrm{PQ}}{\mathrm{P'Q'}} = \frac{\mathrm{R}}{\mathrm{R} - \delta n} = 1 + \frac{\delta n}{\mathrm{R}},$$

ultimately. Hence

$$\frac{\partial v}{\partial n} = \frac{v}{\mathrm{R}}. \quad . \quad . \quad . \quad (28)$$

Again the normal force per unit volume at P is $\rho v^2/\mathrm{R}$, whence, resolving the forces on a small element,

$$\rho\frac{v^2}{\mathrm{R}} = -\frac{\partial p}{\partial n} - \rho\frac{\partial\mathrm{V}}{\partial n}. \quad . \quad . \quad (29)$$

Having regard to (28), this may be written

$$\frac{\partial}{\partial n}(p + \rho\mathrm{V} + \tfrac{1}{2}\rho v^2) = 0, \quad . \quad . \quad (30)$$

showing that the total head does not vary when we pass from one stream-line to another. The above proof could be extended to three dimensions with the help of elementary principles of the theory of curvature. H. L.

Stream-line Motion. See "Ship Resistance and Propulsion," § (16).

Stress:

A term used, in the theory of elasticity, to denote the limit to which the quantity p, defined by

$$p = \left[\frac{\mathrm{P}}{\text{area of ABDC}}\right],$$

tends, as the area is indefinitely reduced, where ABDC is one face of a small parallelepiped of the elastic solid under consideration, and P is the resultant force due to the total action across the face ABDC exerted *upon* the material contained in the parallelepiped. See "Elasticity, Theory of," § (3).

Alternating Stress Tests on Mild Steel. See "Elastic Constants, Determination of," § (68), Table 27.

Analysis of. See "Structures, Strength of," § (2).

Bairstow's Experiments on Elastic Ranges of Stress. See "Elastic Constants, Determination of," § (69).

Bauschinger's Theory of Failure under Repeated Stresses. See *ibid.* § (69).

The Criterion of Failure under Combined Stress. See *ibid.* § (75).

Effect of Rapid Changes of Section and Surface Condition on the Limiting Range of Stress. See *ibid.* § (73).

Effect of Speed of Testing on the Limiting Range of Stress. See *ibid.* § (72).

Experiments on Metals under Combined Stress. See *ibid.* § (76).

Experiments on the Repetition of Stresses. See *ibid.* § (68).

Formulae for Repeated Stress Tests. See *ibid.* § (70).
In a Structure due to External Loads. See "Structures, Strength of," § (4).
Intensity of. See *ibid.* § (5).
Maximum Shear Theory of Failure under Combined Stress. See "Elastic Constants, Determination of," § (75) (iii.).
Maximum Strain Theory of Failure under Combined Stress. See *ibid.* § (75) (ii.).
Maximum Stress Theory of Failure in Combined Stress. See *ibid.* § (75) (i.).
Repeated Applications of Combined Stress. See *ibid.* § (77).
Resistance of Materials to Combined Stresses. See *ibid.* § (74).
Tests on Rolled Aluminium Alloys, at varying temperatures, under alternating stresses. See *ibid.* § (121), Table 44.
Tests on Steel, at varying temperatures, under alternating stress. See *ibid.* § (121), Table 43.

Stress, Intensity of. See "Structures, Strength of," § (5).

Stress and Strain, Relations between. See "Elasticity, Theory of," § (5).

Stroboscopic Method of measuring Speed by Fork and Intermittent Illumination. See "Meters," § (13) (i.), Vol. III.

Stroboscopic Method of measuring Speed with Slit Fork. See "Meters," § (13) (i.), Vol. III.

Structure, Heavy, Limiting Size of a. See "Dynamical Similarity, The Principles of," § (43).

STRUCTURES, THE STRENGTH OF

I

§ (1) Factor of Safety.—The estimation of the strength of a structure, or the prediction whether a structure will be able to fulfil satisfactorily the purpose for which it is designed, is in many cases a matter of no small difficulty. Firstly, an estimation has to be made of the loads, moving and stationary, which the structure has to carry. Secondly, the strength and behaviour of the materials to be used have to be determined. Thirdly, the magnitudes of the actual internal stresses [1] and strains in the structure due to the applied loads have to be found, in order to see that the allowable amount, as determined from a test of the material, is not exceeded. With all three of these, in most cases there is considerable uncertainty, and to allow for this the engineer uses what is called a *factor of safety.*

The factor of safety is usually defined as the ratio of the stress which will just cause complete failure in a sample of the material divided by the maximum estimated stress of the same kind occurring in the structure. For example, a factor of safety frequently employed in steel bridges is about 4, but this must not be taken to mean that the structure is really four times stronger than required. To start with, the steel will be strained beyond its elastic limit if the tensile stress in it is only some 50 per cent of the tensile stress required to cause failure. If this occurs there will be permanent set, and the strain will be a function of the time during which the stress acts. For most purposes the structure will be useless, even if not completely broken. This reduces the so-called factor of safety to 2 instead of 4.

Again, the material is usually tested by applying a dead load to it, whereas, in many cases, part of the load on the structure is a moving one. In the case of a bridge, for example, allowance must be made for the dead weight of the structure itself, the moving load, and the force exerted on the bridge by this due to it coming on unsmoothly and thereby producing impacts; also the effect of wind on the structure has to be taken into account. The estimation of the value of all these can only be an approximate one, and the final estimate is liable to a fairly large error. The methods employed for estimating the applied loads hardly come into the scope of this article. For information on this subject the reader should refer to practical books on design.

The strength of the material used is not accurately known. It may vary somewhat from that of the test specimens experimented with, and whereas in the test the stress was of a simple character, in the actual structure the stress may be a compound one. Also various approximations have to be made in our estimation of the stresses induced by the external forces, and in certain parts of the design these may be considerably in error.

Lastly, allowance has to be made for errors of workmanship, effect of corrosion, and so on.

Taking all these points into consideration it is easy to see that our so-called factor of safety is largely a factor of ignorance. In most well-designed modern large structures it is probable that the margin of safety is not large, though sufficient. This is to be attributed as much to practical experience as to theory.

It has already been noted that in most cases a structure is useless if the material is strained beyond its elastic limit, and it is obviously of very great importance to know what actually is the factor which causes elastic breakdown in the different materials used in construction. Various and numerous attempts have been made to determine this.

The three main theories of elastic breakdown are that it occurs:

(i.) When the greatest tensile or compressive stress exceeds a certain amount.

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[1] For definitions see article "Elasticity, Theory of," §§ (2), (3).

(ii.) When the greatest stretch exceeds a certain amount.

(iii.) When the greatest slide exceeds a certain amount.

Since the greatest slide is a function of the greatest shearing stress, in the third theory we may read shear in place of slide. Moreover, since, as shown in § (2), the maximum shear is measured by half the difference of the principal stresses, this theory may also be described as a stress-difference theory.

Experiments so far have established the fact that no one of these theories is true for all materials. For ductile materials there is considerable experimental evidence that the governing factor is the greatest slide or shear. In the case of some brittle materials the greatest stretch theory appears to be fairly correct.

Another theory which has been suggested is that elastic breakdown occurs when the resilience, *i.e.* the energy per unit volume stored in the material due to strain, exceeds a certain amount.

Due to the uncertainty of the cause of elastic breakdown, the English and American method of design is generally to follow Rankine in assuming that the strength of a structure depends upon the greatest tensile or compressive stress to which the material is subjected. The ultimate strength of the material in simple tension or compression is found experimentally, and in the structure the greatest tensile or compressive stress is not allowed to exceed some fraction of the ultimate. In certain cases the greatest estimated shearing stress is kept to some fraction of the ultimate shearing stress found for the material.

§ (2) ANALYSIS OF STRESS.[1]—In order to analyse the state of stress existing at any point of a material it is usual to consider the equilibrium of a rectangular parallelepiped, of indefinitely small size, and containing the point considered. In general, the stress on each face may be resolved into a normal stress (R) and two tangential stresses (S), these stresses being parallel to the axes of the parallelepiped. See *Fig.* 1.

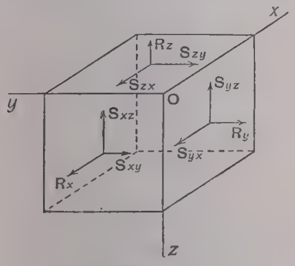

FIG. 1.

From the equilibrium of the parallelepiped it follows that $S_{xy}=S_{yx}$, $S_{yz}=S_{zy}$, and $S_{zx}=S_{xz}$.

By rotating the parallelepiped about the point we find that for one angular position of it the normal stresses on the faces are a maximum or a minimum. The planes of the faces are called the *principal planes* and the normal stresses on them are called the *principal stresses*. It is readily shown that on the principal planes there are no tangential stresses. Similarly it may be proved that if the faces of the parallelepiped be arranged at 45° to the principal planes the shearing stresses on the faces will be a maximum or minimum. Further, the intensity of greatest shearing stress at any point is equal in value to one-half the algebraic difference of the greatest and least principal stresses. We will examine the case of a two-dimensional state of stress, *i.e.* one in which there is no variation of stress in the direction perpendicular to the two dimensions considered.

Let ABCD be a rectangular parallelepiped of unit dimension in a direction along which there is no variation of stress. To find the normal and tangential stress on a plane inclined at an angle θ to the x axis, consider the equilibrium of the triangular wedge BCD.

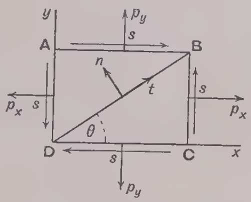

FIG. 2.

Revolving normally and tangentially to DB we have

$$n=p_x \sin^2\theta+p_y \cos^2\theta-S\sin 2\theta, \quad . \quad (1)$$

$$t=(p_y-p_x)\frac{\sin 2\theta}{2}-S\cos 2\theta. \quad . \quad (2)$$

For the principal planes, $dn/d\theta=0$. This gives

$$\theta=\tfrac{1}{2}\left\{\tan^{-1}\frac{2S}{p_x-p_y}+r\pi\right\},$$

where $r=0$, 1, 2, etc., *i.e.* there are two planes mutually at right angles.

Substituting the values of θ given by this equation in equation (1) we get

The principal stresses

$$=\frac{p_x+p_y}{2}\pm\frac{\sqrt{(p_y-p_x)^2+4S^2}}{2}.$$

Substituting in (2) we see that there is no shear stress on the principal planes.

Similarly we may show that the maximum shearing stress occurs on planes inclined at 45° to the principal planes and is of intensity

$$\frac{\sqrt{(p_y-p_x)^2+4S^2}}{2},$$

i.e. $\frac{1}{2}$ (the difference of the principal stresses).

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[1] See article "Elasticity, Theory of," § (4).

§ (3) STRAINS DUE TO PRINCIPAL STRESSES.—Let R_x, R_y, R_z be the principal stresses.

Let e_x, e_y, e_z be the stretches in the directions x, y, and z.

Let $1/m$ be Poisson's ratio, *i.e.* the ratio of the lateral contraction per unit breadth to the longitudinal extension per unit length, due to a single tensile stress.

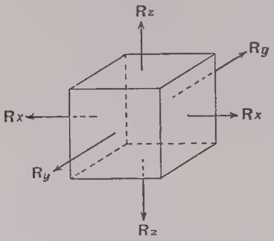

FIG. 3.

Let E equal the modulus of elasticity. Then

$$\left.\begin{aligned} e_x &= \frac{R_x}{E} - \frac{R_y}{mE} - \frac{R_z}{mE} \\ e_y &= -\frac{R_x}{mE} + \frac{R_y}{E} - \frac{R_z}{mE} \\ e_z &= -\frac{R_x}{mE} - \frac{R_y}{mE} + \frac{R_z}{E} \end{aligned}\right\}.$$

These are the principal stretches.

As an example of the different dimensions which the theories of elastic breakdown lead to in design, we will take the case of a thin circular tube subjected to an axial twisting moment of magnitude M inch-tons. The figure shows a cross-section perpendicular to the axis. It is obvious that the shearing stress on the section has to provide a couple to balance the twisting moment.

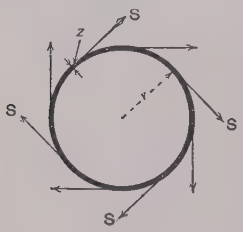

FIG. 4.

Since the thickness z in. is small compared with the mean radius r in. we may consider the shearing stress practically uniform. If S is the intensity of shearing stress in tons per sq. in. we have

$$M = 2\pi r^2 zS,$$

or $$S = \frac{M}{2\pi r^2 z} \text{ tons per sq. in.}$$

The three principal planes at any point will be two planes making angles 45° and 135° with the axis, and a plane perpendicular to the radius.

The principal stresses will be

$$S, \ -S, \ 0.$$

Greatest principal stress

$$= S. \qquad (1)$$

Greatest principal stretch

$$= \frac{S}{E}\left(1 + \frac{1}{m}\right). \qquad (2)$$

Greatest shearing stress (see § (2))

$$= S. \qquad (3)$$

Suppose a sample of the material when subjected to a simple tensile stress has an elastic limit p tons per sq. in.

The greatest principal stress $= p$.

The greatest principal stretch $= p/E$.

The greatest shearing stress $= p/2$.

With the same factor of safety k on the elastic limit, in the cases (1), (2), and (3) the thickness will be, respectively,

$$\frac{kM}{2\pi r^2 p}, \quad \frac{kM}{2\pi r^2 p} \times \frac{m+1}{m}, \quad \frac{kM}{\pi r^2 p}.$$

For steel we may take $m = 10/3$, and we then get the thicknesses as

$$1 : 1{\cdot}3 : 2.$$

§ (4) TOTAL STRESS INDUCED BY EXTERNALLY APPLIED LOADS.—In any loaded structure we have certain loads applied at definite points, and these are balanced by reactions usually acting at different points of the structure. This causes the material of the structure to be subjected to certain internal stresses induced by the external forces. The strength of any part of a structure will be some function of the resultant internal stress which the part has to carry, and also the dimensions of the part itself.

In order to estimate the resultant internal stress which occurs at any cross-section of a member of a structure, we imagine the member cut at the section considered, and we then see that the resultant of the internal stresses at the section must balance exactly the external forces acting on one portion of the member. The sense of this resultant will of course depend upon which part of the member we consider.

If we consider a member such as shown in *Fig.* 5 and take a plane section ABCD, say, it is obvious that, so far as the effect of the forces to the left of the section are concerned, we may replace them by a single force acting through any selected point O in the cross-section and a couple. This is easily seen

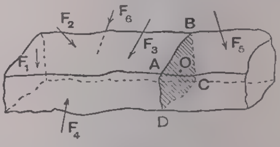

FIG. 5.

without any mathematical proof, since we may obviously prevent motion of the portion to the left by a ball and socket joint at O, combined with a couple to prevent any rotation about O.

The internal stresses at the section have to balance then a single force and a couple. For convenience we usually take O to be the centre of gravity of the area of cross-section, and we take three axes Ox, Oy, Oz, mutually at right angles, Ox being perpendicular to the cross-section and Oy and Oz being along the principal axes of inertia of the cross-section. We resolve the single force at O into its components X, Y, and Z along Ox, Oy, and Oz, and the couples into the three components M_x, M_y, M_z, about the axes Ox, Oy, Oz.

X = the algebraic sum of the components of all the external forces to the left of the section along the axis Ox, and similarly Y and Z.

M_x = the algebraic sum of the moments of the external forces to the left of the section about the axis Ox, and similarly M_y and M_z.

At the cross-section we see that the internal stresses have to balance, a normal force X, a shearing force Y, a shearing force Z, and the three couples M_x, M_y, and M_z.

The couples M_y and M_z are usually called bending moments, the couple M_x a twisting moment.

As noted above, we usually consider the resultant effect of external forces at any cross-section to consist of a single normal force passing through the centre of gravity of the section, two tangential forces and three couples. We assume the normal force produces a uniform normal stress and strain over the cross-section, and we have then to consider the shearing stresses and slides due to the tangential forces, and also the stresses and strains due to the bending moments and twisting moment.

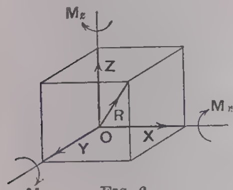

FIG. 6.

The problem then resolves itself into designing members to carry pulls, pushes, bending moments, and twisting moments.

In the case of a beam on which all the loads are vertical and lie in one plane we see immediately that at any cross-section the internal stresses have to balance a single vertical force, which is called the shearing force, at the section, and a single couple, which is called the bending moment.

Considering all the external forces on one side of the cross-section we have:

The *Shearing Force* is equal to the algebraic sum of all the external forces on one side of the section of the beam.

The *Bending Moment* is equal to the algebraic sum of the moments of all the external forces on one side of the section about a horizontal axis in the section.

We can establish a connection between the shearing force and bending moment at any section.

Consider a length δx of the beam.

Let F and M be the shearing force and the bending moment at the section AD, and $(F+\delta F)$ and $(M+\delta M)$ be the shearing force and the bending moment at the section BC.

Let $w\delta x$ be the distributed load on the length δx. For the equilibrium of ABCD, taking moments about a horizontal axis through C, we have

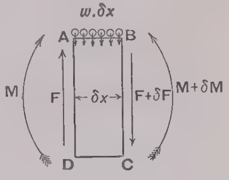

FIG. 7.

$$F\delta x + M - (M+\delta M) - \frac{w\delta x^2}{2} = 0.$$

Now $w\delta x^2/2$ is a term of the second order, and we may neglect it in the limit. Hence we get

$$F = \frac{dM}{dx}.$$

This is often very useful in finding the position of the maximum bending moment, since the latter occurs where the shearing force changes sign.

§ (5) INTENSITY OF INTERNAL STRESS.—Having investigated the resultant effects of the external forces which have to be balanced at each cross-section by the internal stress at the cross-section, we have next to investigate how the internal stresses are distributed over the cross-section and what is the intensity at each point.

At this stage we find that the principles of pure statics cannot supply us with sufficient information, since there may be an infinite variety of distributions of internal stresses which will give the required resultant forces and couples.

We have now to call in the aid of the mathematical theory of elasticity and also the aid of experiment. Even with the aid of both of these, we frequently find that we have to be content with an approximate solution of the problem, which is a mixture of theory and inexact experimental data. The engineer is at a disadvantage compared with the mathematician, since he is frequently *obliged* to find some solution of a problem in design which the mathematician can give up if it appears insolvable.

It may be said generally that the practical design of structures has always been ahead of the theory of design. If the engineer had waited for the theory of elasticity to be developed before building structures the strength of which he could not satisfactorily estimate, development would have been very slow indeed. To a large extent it may be said that designs have been made and the structures constructed, and subsequently these have been more or less subjected to theoretical analysis. The theory of elasticity has mainly been helpful in effecting economy of material, but in itself it has not, until quite recent times, had the effect of producing new designs. An article like the present one runs the risk of giving a quite false impression as to the way in which the true strength of a structure can be estimated. It emphasises the parts which are based on theory and omits the parts of the design which are based entirely on judgment, or past designs which have been successful. The engineer, apart from using all the help the mathematician and physicist can give him, has to employ what may be called his engineering instinct. Many of the parts of the design for which he has got no theory to help him are as important, from the point of view of the strength of the structure as a whole, as the parts which he can design with the aid of theory.

The subject of theory of structures, or strength of structures, differs from the pure theory of elasticity in so far as the solutions obtained are often only a rough approximation. The difference is similar to that between hydrodynamics and hydraulics. The one is the rigid mathematical theory dealing with an ideal fluid, the other is the engineers' treatment of practical problems based on mathematical theory so far as this agrees with practical results and also largely upon experiment.

In the theory of elasticity there are certain cases of bending, twisting, etc., which have been solved by the mathematician. These the engineer uses very largely. He also makes great use of the principle of superposition.[1] This assumes that the effects produced by the separate forces and couples are independent of one another, and that the resultant strain at any point is merely the resultant of the separate strains produced.

II

§ (6) MEMBERS SUBJECTED TO BENDING.—Consider the case of a beam of homogeneous and isotropic material subjected to pure couples applied at the ends tending to bend it in the plane of its longitudinal axis. In this case the internal stresses of any cross-section have only to produce a couple equal in magnitude to the bending moment.

We shall make an assumption, first made by Bernouilli, viz. that *plane sections perpendicular to the plane of bending remain plane.* Probably the best justification of this lies in the fact that results deduced from it are in agreement with practical experience. It can also be verified mathematically on certain assumptions as to the way the bending moment is applied.

Fig. 8 represents a small piece of the beam bounded at the ends by plane sections. Let C_0 and C be the centres of curvature before and after the bending moment is applied.

It is clear that there will be one layer of the material d_0g_0 which will not change in length during bending, the material above will be stretched, the material below this layer will be shortened. This layer is called the *neutral layer.* Let R_0 and R be the radii of curvature of this layer before and after bending.

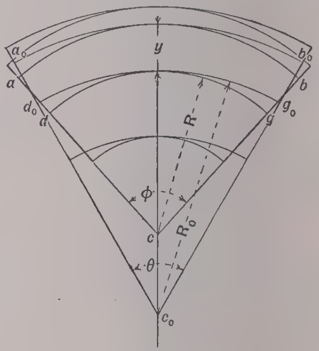

FIG. 8.

For a layer at a distance y above the neutral layer the stretch (e) is given by

$$\frac{ab-a_0b_0}{a_0b_0}=\frac{(R+y)\phi-c(R_0+y)\theta}{(R_0+y)\theta}=\frac{y(R_0-R)}{(R_0+y)R},$$

$$\because R\phi=R_0\theta.$$

If R_0 is large compared with y, we may write with sufficient accuracy for practical purposes,

$$e=\left(\frac{1}{R}-\frac{1}{R_0}\right)y.$$

If the beam is initially straight $R_0=\infty$, and

$$e=\frac{y}{R}.$$

In both these cases we get the stretch varying as the distance from the neutral layer.

Assuming that the material obeys Hooke's Law, and that each filament of the beam is free to expand and contract independently of the surrounding filaments, and also that the value of Young's modulus (E) is the same for tension and compression, we then have the normal stress varying as the distance from the neutral layer. For a beam initially straight we may write

$$p=\frac{E}{R}\cdot y$$

---

[1] See "Elasticity, Theory of," § (4).

The trace of the neutral layer on a cross-section is called the *neutral axis*.

Taking moments about the neutral axis for the normal stress on the cross-section, *Fig.* 9, we get for the resisting moment

$$M_r = \int bpydy = \frac{E}{R}\int by^2dy = \frac{E}{R} \, . \, I,$$

where I is the moment of inertia of the cross-section about the neutral axis.

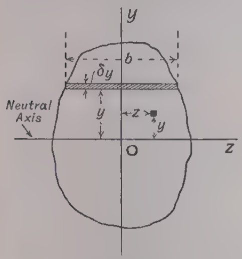

FIG. 9.

The resisting moment must balance the bending moment M, and we may write

$$\frac{p}{y} = \frac{M}{I} = \frac{E}{R}.$$

For the position of the neutral axis we note that the normal stress over the cross-section must have zero resultant, *i.e.*

$$\int bpdy = 0,$$

or

$$\int bydy = 0.$$

The neutral axis, therefore, passes through the centre of gravity of the cross-section.

We have, further, the fact that there can be no resultant couple about an axis oy perpendicular to the axis of bending oz, *i.e.* the neutral axis.

This gives

$$\iint zpdydz = 0,$$

i.e.

$$\iint zydydz = 0.$$

The product of inertia of the cross-section must be zero, and therefore the neutral axis must be a principal axis of inertia of the cross-section.

If f_1 and f_2 are the maximum tensile and compressive stresses respectively, at any cross-section, and y_1 and y_2 are the distances from the neutral axis to the top and bottom of the beam, then

$$M = \frac{I}{y_1} \, . \, f_1 = \frac{I}{y_2} \, . \, f_2.$$

The values I/y_1 and I/y_2 depend only upon the shape and size of the cross-section, and are called the *moduli of the section* (Z_1 and Z_2). If f_1 and f_2 represent the maximum tensile and compressive stresses allowed in the material, then the greatest bending moment the section may carry is given by whichever of the values f_1Z_1 or f_2Z_2 is least.

For standard rolled steel sections the values of Z_1 and Z_2 have been calculated and tabulated, thus simplifying the process of design.

In the case of built-up girders in which the flange area is large compared with the web area it will be found that the moment of resistance to bending contributed by the web is only a very small percentage of the whole, and in such cases we may assume that the flanges provide the whole resisting moment.

If A = the area of each flange,
f = the mean longitudinal stress in each flange,
h = the distance between the centres of gravity of the flanges,
the moment of resistance $= fAh$.

This is the usual formula employed in designing built-up I or box girders and also lattice-work girders.

§ (7) UNSYMMETRICAL BENDING. — When the plane of loading does not include a principal axis of inertia of the cross-section we resolve the bending moment into two components, each component bending the beam about a principal axis. In order to estimate the stress at any point we find the stress due to each component separately.

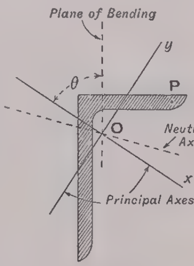

FIG. 10.

In *Fig.* 10 let M be the bending moment, Ox and Oy the principal axes of inertia of the section, I_x and I_y the moments of inertia about these axes, and O the centre of gravity of the cross-section.

The bending moments about Ox and Oy will be M cos θ and M sin θ respectively.

If x, y be the co-ordinates of a point P, the stress at P will be given by

$$p = \frac{M\cos\theta \, . \, y}{I_x} + \frac{M\sin\theta \, . \, x}{I_y}$$

for the neutral axis $p=0$, and the equation representing the neutral axis is

$$\frac{y\cos\theta}{I_x} + \frac{x\sin\theta}{I_y} = 0.$$

This generally will not be perpendicular to the plane of loading.

§ (8) SHEARING STRESS DOWN THE CROSS-SECTION OF A BEAM.—In most practical cases of beams not only do we get a bending moment, but we also have a shearing force at the different cross-sections of the beam. The resultant strains will not be of the simple character we have assumed in pure bending, since the shearing force will cause slides to occur. In general, plane sections do not remain plane if there is a shearing force. The usual method of design is to assume the formulae $p/y = M/I = E/R$ still hold, in spite of the existence of shear and a varying bending moment. St. Venant's investigations have shown that it is true when the shear is uniform. Having provisionally decided on a suitable beam by means of this formula, an investigation is then made of the shearing stress induced by shearing force. To investigate the distribution of shearing stress down the cross-section of a beam the following method is usually employed:

Consider a small slice, ABCD, of the beam, of length δx, and bounded by two planes perpendicular to the axis.

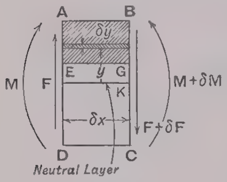

FIG. 11.

Let M and F represent the bending moment and shearing force at the section AD.

Take the case of a beam of rectangular cross-section. Assume the shearing stress on the cross-section uniform along a line parallel to the neutral axis, and also that it is vertical at each point. If S is the intensity of shearing stress at a height KG above the neutral axis, considering the equilibrium of the piece AEGB, we have, by resolving horizontally and remembering that the horizontal and vertical shearing stress at every point will be the same,

$$\int_{\mathrm{KG}}^{\mathrm{KB}}(p+\delta p)bdy-\int_{\mathrm{KG}}^{\mathrm{KB}}pbdy=\mathrm{S}b\delta x,$$

where b is the breadth of the cross-section, *i.e.*

$$\mathrm{S}b\delta x=\int_{\mathrm{KG}}^{\mathrm{KB}}\delta p\,.\,bdy.$$

But $$\delta p=\delta\mathrm{M}\,.\,\frac{y}{\mathrm{I}},$$

$$\therefore\ \mathrm{S}bdx=\frac{\delta\mathrm{M}}{\mathrm{I}}\int_{\mathrm{KG}}^{\mathrm{KB}}bydy,$$

or $$\mathrm{S}=\frac{\delta\mathrm{M}}{\delta x}\,.\,\frac{\mathrm{A}_0y_0}{b\mathrm{I}},$$

where A_0 is the area AEGB, and y_0 is the height of its centre of gravity above the neutral axis.

But $$\mathrm{F}=\frac{d\mathrm{M}}{dx},$$

$$\therefore\ \mathrm{S}=\frac{\mathrm{F}\,.\,\mathrm{A}_0\,.\,y_0}{b\mathrm{I}}\quad .\ .\ .\ .\ \ (1)$$

The maximum shearing stress occurs at the neutral axis, and for a rectangular cross-section is given by 3/2 . F/bh, where h is the height of the beam, *i.e.* $1\frac{1}{2}$ times the mean shearing stress for the cross-section.

It is obvious that this investigation cannot hold for sections other than rectangular, since the shearing stress at the boundary of the cross-section must be tangential to the boundary and therefore cannot be vertical as assumed. This follows as a deduction from the equality of the tangential stresses S_{xy} and S_{yx}, etc. The formula is not exactly true even for rectangular sections. It gives the average stress across the breadth of the section, which is a close approximation to the actual stress when the breadth is small compared with the depth.

Formula (1) is generally employed to estimate the shear at any point in the web of a rolled steel beam or built-up I girder.

In most practical cases it will be found that the total variation of shearing stress down the web is small, and no great error is made in assuming that the web carries the whole of the shearing force and that it is uniformly distributed over the web area.

In such cases the area of the web is often much larger than required to carry the shear, and the determining factor for its thickness is the tendency of the web to buckle, due to the principal compressive stress introduced by the shear stress and the horizontal stress.

The tendency to buckle in such a case cannot be determined satisfactorily by theory, and the required thickness of web and the necessary stiffeners have to be designed by empirical rules which are based on past experience.

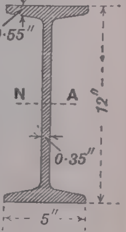

FIG. 12.

Take, as an example, a standard rolled steel joist of the dimensions shown in *Fig.* 12.

The total area of section is 9·4 sq. in., and the moment of inertia of this section about the neutral axis NA = 220 in.^{4}

Let F be the shearing force. This will produce the maximum shearing stress at the neutral axis. To determine the moment of area above the neutral axis we may, with sufficient accuracy, assume the flange as a rectangle 5″ × 0·55″, and the web of height (12 − 1·1)″ and thickness 0·35″.

Maximum shearing stress

$$=\frac{\mathrm{F}}{0{\cdot}35\times220}\left\{5\times0{\cdot}55\times5{\cdot}725-\frac{0{\cdot}35\times5{\cdot}45^2}{2}\right\}$$

$$=0{\cdot}27\mathrm{F}.$$

If we assume the shearing stress uniform over the whole section the magnitude

$$=\frac{\mathrm{F}}{10{\cdot}9\times0{\cdot}35}$$

$$=0{\cdot}26\mathrm{F}.$$

As will be seen, the error in assuming a uniform distribution is less than 4 per cent.

III

§ (9) DEFLECTION AND STIFFNESS OF BEAMS.—The stiffness of a beam is usually measured by the maximum deflection, when loaded, divided by the span. This frequently has to be considered in designing structures, particularly if the structure is subjected to a live load. Lack of stiffness may increase, by vibration, the stress produced by the load. Again in buildings with plastered ceilings, if the deflection produced by the changing load is not very small compared with the span, the plaster, being brittle, will be cracked and finally drop off. Stiffness has also to be considered when machinery is carried on beams or girders in order to make sure that a forced vibration will not be set up which synchronises with the periodicity of the machine.

To investigate the deflection of beams initially straight, or in which the curvature is very large, we make use of the formula

$$\frac{M}{I}=\frac{E}{R}.$$

Now $$\frac{1}{R}=\frac{d^2y/dx^2}{\{1+(dy/dx)^2\}^{3/2}},$$

and if the above condition is fulfilled dy/dx will everywhere be small. We may then write

$$\frac{1}{R}=\frac{d^2y}{dx^2},$$

and we get $$\frac{d^2y}{dx^2}=\frac{M}{EI}.$$

In this equation, if x is measured along the neutral layer to the right, and y is measured downwards say, M is a hogging bending moment.

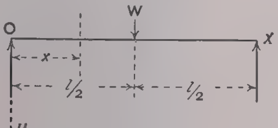

FIG. 13.

Take the simple case of a beam of uniform cross-section supported at the ends and loaded with a concentrated load W at the middle.

Let the origin be at the left-hand end (*Fig.* 13) and the axis of x to the right.

The bending moment at a distance x from O

$$=-\frac{Wx}{2},$$

$$\therefore\ EI\frac{d^2y}{dx^2}=-\frac{Wx}{2}.$$

Integrating we get

$$EI\frac{dy}{dx}=-\frac{Wx^2}{4}+A, \quad . \quad . \quad . \quad (1)$$

and $$EIy=-\frac{Wx^3}{12}+Ax+B. \quad . \quad . \quad (2)$$

Where A and B are constants, the value of which have to be determined by the end conditions.

We note that when $x=0$, $y=0$, and when $x=l/2$, $dy/dx=0$.

This gives $$B=0, \text{ and } A=\frac{Wl^2}{16},$$

$$EI\frac{dy}{dx}=-\frac{Wx^2}{4}+\frac{Wl^2}{16},$$

and $$EIy=-\frac{Wx^3}{12}+\frac{Wl^2x}{16}.$$

The maximum slope occurs when $x=0$, and is equal to

$$\frac{1}{16}\cdot\frac{Wl^2}{EI}.$$

The maximum deflection occurs when $x=l/2$, and is equal to

$$\frac{1}{48}\cdot\frac{Wl^3}{EI}.$$

For a load W uniformly distributed along the length, the maximum deflection is

$$\frac{5}{384}\,\frac{Wl^3}{EI}.$$

We will take a simple example of a timber bridge, which will illustrate how stiffness affects the design.

Strictly speaking, none of the formulae we have derived will hold for timber, since it is not isotropic. It is usual, however, to employ the formulae, and experiment shows that the errors introduced by so doing are not large. The degree of accuracy in estimating the stresses is probably greater than our knowledge of the strength of the actual piece of timber we may be employing.

A bridge has an effective span of 20 feet and has to carry a concentrated axle load of 14 tons. The road bearers are to be seasoned pitch pine with a cross-section 10 in. by 10 in. Assuming the load evenly distributed between the road bearers, how many will be required?

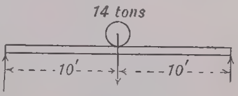

FIG. 14.

Allowable fibre stress for the timber 1 ton per sq. in.

Allowable shearing stress along the grain $\frac{1}{15}$ ton per sq. in.

Allowable deflection $\frac{1}{200}$th of the span.

Impact[1] factor $\frac{3}{2}$.

The weight of this type of bridge will be about $\frac{1}{4}$ ton per foot run.

From *Fig.* 14 we see that the maximum bending moment

$$=\tfrac{1}{4}\times\tfrac{3}{2}\times14\times20+\tfrac{1}{8}\times\tfrac{1}{4}\times20^2$$
$$=117{\cdot}5 \text{ foot-tons.}$$

The maximum shearing force will occur when the axle load is just on the bridge, therefore the maximum shearing force

---

[1] The impact factor is the factor by which the load is multiplied to allow for its motion.

$$=\tfrac{3}{2}\times 14+10\times\tfrac{1}{4}$$
$$=23{\cdot}5 \text{ tons.}$$

The moment of resistance of one road bearer

$$=\frac{10\times 10^2}{6}\text{ in.-tons}$$
$$=13{\cdot}9 \text{ foot-tons.}$$

For *bending*, the number of road bearers required

$$=\frac{117{\cdot}5}{13{\cdot}9}$$
$$=9.$$

The shearing force per road bearer

$$=\frac{23{\cdot}5}{9}$$
$$=2{\cdot}6 \text{ tons.}$$

The maximum shearing stress

$$=\frac{3}{2}\times\frac{2{\cdot}6}{100}$$
$$=0{\cdot}039 \text{ ton per sq. in.}$$

This is less than the allowable stress of $\frac{1}{15}$ ton per sq. in.

The deflection

$$=\frac{1}{48}\cdot\frac{Wl^3}{EI}+\frac{5}{384}\cdot\frac{W'l^3}{EI}.$$

We may take

$$E=1{\cdot}5\times 10^6 \text{ lbs. per sq. in.}$$

The deflection

$$=\left(\frac{21}{48}+\frac{5\times 5}{384}\right)\times\frac{20^3\times 12^3\times 12\times 2240}{1{\cdot}5\times 10^6\times 10^4\times 9}$$
$$=1{\cdot}4 \text{ in.}$$

The allowable deflection $=1{\cdot}2$ in.

In order to attain the necessary stiffness the number of road bearers must be increased to $9\times 1{\cdot}4/1{\cdot}2$, *i.e.* 11.

It will be observed that in dealing with stiffness we merely calculated the deflection due to the bending alone. In addition to this, except in the case of pure bending, there will also be a deflection produced by the shearing stress. This additional deflection may be calculated in certain simple cases by consideration of the potential energy of deformation which must be equal to the work done by the external forces in producing the deformation.

It is extremely rarely that this additional deflection is of importance in practical structures and it is usually neglected in considering stiffness.

§ (10) Several Loads.—If we have several concentrated loads at different points along the beam, or a series of discontinuous distributed loads, we shall not be able to determine the constants of integration in the simple way we have above, since we do not know where the slope is zero, and the equations (1) and (2) only hold for the portion of the beam from 0 up to the first load.

In general, we get two equations similar to (1) and (2), each pair with its own constants, for the different spans into which the loads divide the beam. We can, by using the two end conditions, eventually determine all the constants, and the curve of the loaded beam. This method is, however, long and wearisome, and the labour may be avoided by adopting a method due to W. H. Macaulay.[1]

To illustrate the method we will take an example of a beam loaded as shown below,

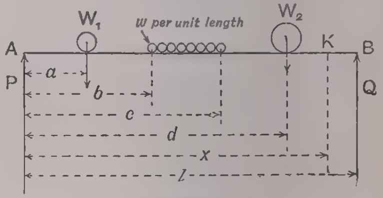

Fig. 15.

with two concentrated loads and a uniformly distributed load extending over a length $(c-b)$.

Imagine the distributed load extending to the end of the beam, and the part added to be neutralised by a load, acting upwards, of the same amount per unit length.

Let the origin be at A and the axis of x to the right.

Take a point K beyond the last applied load.

The sagging bending moment at K

$$=Px-W_1\{x-a\}-\frac{w}{2}\{x-b\}^2+\frac{w}{2}\{x-c\}^2-W_2\{x-d\}.$$

This expression will give the bending moment for any value of x, if the terms inside the brackets be omitted for values of x which make them negative.

We have then

$$-EI\frac{d^2y}{dx^2}=M.$$

Integrating each bracket twice, with respect to the term inside, we have

$$-EIy=\frac{Px^3}{6}-\frac{W_1}{6}\{x-a\}^3-\frac{w}{24}\{x-b\}^4$$
$$+\frac{w}{24}\{x-c\}^4-\frac{W_2}{6}\{x-d\}^3+Ax+B.$$

This equation will hold for all values of x between 0 and l, if the terms inside the brackets be omitted whenever they become negative, and we note that we have only two constants which can be found by the end conditions. In this case $y=0$ when $x=0$, and $y=0$ when $x=l$.

$$\therefore B=0,$$

and

$$\frac{Pl^3}{6}-\frac{W_1}{6}(l-a)^3-\frac{w}{24}(l-b)^4+\frac{w}{24}(l-c)^4$$
$$-\frac{W_2}{6}(l-d)^3+Al=0.$$

From this equation we can find A.

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[1] *Messenger of Mathematics*, No. 573, Jan. 1919, xlviii.

§ (11) CONTINUOUS BEAMS.—A beam is said to be *continuous* when it is carried on more than two supports. In such a beam we cannot, by the aid of statics, determine the reactions at the supports, or the shearing force and bending moment at different sections of the beam, since these depend upon the flexibility of the girder and the height of the supports, and are not merely a function of the applied loads.

In order to effect a solution, we may make use of the Macaulay method previously referred to, treating the reactions as loads.

Another method frequently employed is to make use of Clapeyron's theorem of three moments.

This may be established as follows:

Suppose the girder of the same cross-section throughout.

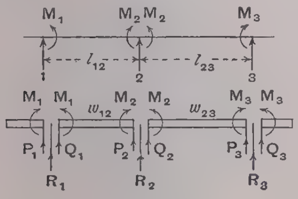

FIG. 16.

Consider span l_{12}. Take the origin at support (1) and to the right and y downwards,

$$M_x = M_1 - Q_1 x + w_{12}\frac{x^2}{2}, \quad . \quad . \quad . \quad (1)$$

$$EI\frac{d^2y}{dx^2} = M_x, \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

$$EI\frac{dy}{dx} = M_1 x - \frac{Q_1 x^2}{2} + \frac{w_{12}x^3}{6} + A, \quad . \quad . \quad (3)$$

$$EIy = \frac{M_1 x^2}{2} - \frac{Q_1 x^3}{6} + \frac{w_{12}x^4}{24} + Ax + B. \quad (4)$$

$$B = 0, \because y = 0 \text{ when } x = 0.$$

Also $\qquad y = 0$ when $x = l_{12}$.

$$\therefore \frac{M_1 l_{12}^2}{2} - \frac{Q_1 l_{12}^3}{6} + \frac{w_{12} l_{12}^4}{24} + A l_{12} = 0. \quad (5)$$

Let θ = the slope at support (2). From (3) and (5) we have

$$EI\theta = \frac{M_1 l_{12}}{2} - \frac{Q_1 l_{12}^2}{3} + \frac{w_{12} l_{12}^3}{8}. \quad . \quad (6)$$

Similarly taking the origin at support (3), and axis of x to the left, we get for the slope at support (2)

$$-EI\theta = \frac{M_3 l_{23}}{2} - \frac{P_3 l_{23}^2}{3} + \frac{w_{23} l_{23}^3}{8}. \quad . \quad (7)$$

From (6) and (7),

$$\frac{M_1 l_{12}}{2} + \frac{M_3 l_{23}}{2} + \frac{w_{12} l_{12}^3}{8} + \frac{w_{23} l_{23}^3}{8} - \frac{Q_1 l_{12}^2}{3} - \frac{P_3 l_{23}^2}{3} = 0. \quad (8)$$

Now from (1),

$$M_2 = M_1 - Q_1 l_{12} + \frac{w_{12} l_{12}^2}{2},$$

and for span l_{23},

$$M_2 = M_3 - P_3 l_{23} + \frac{w_{23} l_{23}^2}{2}.$$

From these and (8) we have

$$\frac{M_2 l_{12}}{3} + \frac{M_1 l_{12}}{6} + \frac{M_2 l_{23}}{3} + \frac{M_3 l_{23}}{6} - \frac{w_{12} l_{12}^3}{24} - \frac{w_{23} l_{23}^3}{24} = 0$$

or

$$(M_1 + 2M_2)l_{12} + (M_3 + 2M_2)l_{23} - \tfrac{1}{4}w_{12} l_{12}^3 - \tfrac{1}{4}w_{13} l_{23}^3 = 0.$$

This is for the particular loading taken. As will be observed, the formula connects the moments at the support of any two adjacent spans.

As an example of its use take the simple case of a beam uniformly loaded throughout and supported at the ends and middle, the tops of the supports all being at the same level.

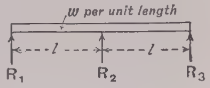

FIG. 17.

Here $\qquad M_1 = 0, \quad M_3 = 0, \quad l_{12} = l_{23} = l,$

$$w_{12} = w_{23} = w,$$

$$4M_2 l - \tfrac{1}{2}wl^3 = 0,$$

i.e. $\qquad M_2 = \tfrac{1}{8}wl^2.$

Taking moments about the middle support we have

$$R_1 l - \frac{wl^2}{2} = -M_2,$$

i.e. $\qquad R_1 = \tfrac{3}{8}wl = R_3$

and $\qquad R_2 = 2wl - \tfrac{3}{4}wl = \tfrac{5}{4}wl.$

§ (12) RELATIONS BETWEEN LOAD, SHEAR, BENDING MOMENT, SLOPE, AND DEFLECTION.—Taking the axis of x along the beam and the axis of y in the direction of the deflection, let

w = the load per unit length at any distance x from the origin.
F = the shearing force at distance x from the origin.
M = the bending moment at distance x from the origin.
i = the slope at distance x from the origin.
y = the deflection at distance x from the origin.

We have

$$F = \int w\,dx \quad \text{or} \quad w = \frac{dF}{dx},$$

$$M = \int F\,dx \quad \text{or} \quad F = \frac{dM}{dx},$$

i.e. $$w = \frac{d^2M}{dx^2},$$

$$i = \frac{dy}{dx}.$$

$$\therefore \frac{d^2y}{dx^2} = \frac{di}{dx} = \frac{M}{EI}$$

and $$\frac{d^4y}{dx^4} = \frac{d^2 . M/EI}{dx^2}.$$

If EI is constant, we get

$$\frac{d^4y}{dx^4}=\frac{1}{\mathrm{EI}}\cdot\frac{d^2\mathrm{M}}{dx^2}$$

or

$$\mathrm{EI}\frac{d^4y}{dx^4}=w.$$

If EI is constant, we have the following:

$$\mathrm{F}=\int wdx,$$

$$\mathrm{M}=\int \mathrm{F}dx,$$

$$i=\frac{\mathrm{I}}{\mathrm{EI}}\int \mathrm{M}dx,$$

$$y=\int idx.$$

Suppose we are given the distribution of load on a beam we may derive any of the quantities F, M, i, and y by integrating, the constants of integration being fixed by the end conditions of the beam.

Even in cases where w cannot be expressed mathematically as a known function of x we may still obtain results sufficiently accurate for practical purposes by graphical integration.

If I varies along the beam, in the graphical treatment, we may still proceed thus.

Having obtained the bending moment curve for the beam we divide each of the ordinates by the value of I corresponding, and then plot a curve M/I and x. By graphically integrating this curve we obtain Ei and integrating again y.

Frequently we are not interested in the slope, and in such cases, to obtain the deflection diagram, we may perform the double integration by treating the bending moment diagram as a load curve, and by drawing the funicular polygon for this load curve. The depth of the funicular polygon will then give the deflection.

§ (13) WHIRLING OF SHAFTS.—As an example of the use of the above equation we will consider the effect of centrifugal force on a rotating circular shaft. Due to the weight, vibration, or other causes, the centre of gravity of the cross-section of the shaft will not necessarily lie on the axis of rotation.

Let the distance between the centre of gravity and the axis of the bearings be y. Let ρ be the mass of the shaft per unit length. The centrifugal force per unit length will be $\rho(\omega^2y/g)$, where ω is the angular velocity of the shaft.

We shall have then

$$\frac{d^4y}{dx^4}=\frac{\rho\omega^2}{\mathrm{EI}g}\cdot y,$$

where E is the modulus of elasticity and I is the moment of inertia of a cross-section of the shaft about a diameter.

Let $$\frac{\rho\omega^2}{\mathrm{EI}g}=a^4.$$

The solution of the equation is

$$y=\mathrm{A}\cosh ax+\mathrm{B}\sinh ax+\mathrm{C}\cos ax+\mathrm{D}\sin ax.$$

Let l = the length of the shaft between the bearings, and let us take the case where the bearings simply support the shaft and do not prevent it bending.

Now the slope of the shaft is given by dy/dx, and the bending moment by EI (d^2y/dx^2).

We determine the constants as follows:

When $$x=0,\quad y=0,$$

$$\therefore\ \mathrm{A}+\mathrm{C}=0.$$

When $$x=0,\quad \frac{d^2y}{dx^2}=0,$$

$$\therefore\ \mathrm{A}a^2-\mathrm{C}a^2=0;$$

i.e. $$\mathrm{A}=0 \text{ and } \mathrm{C}=0.$$

From the further conditions that when $x=l$, $y=0$, and also $d^2y/dx^2=0$, we get

$$\mathrm{B}\sinh al+\mathrm{D}\sin al=0$$

and $$\mathrm{B}\sinh al-\mathrm{D}\sin al=0,$$

$$\therefore\ \mathrm{B}\sinh al=0$$

or $$\mathrm{B}=0.$$

Also $$\mathrm{D}\sin al=0,$$

$$\therefore\ \mathrm{D}=0, \text{ or } \sin al=0.$$

If D=0, there is no deflection of the shaft anywhere.

Taking $$\sin al=0,$$

we get $$al=r\pi,$$

where $r=0, 1, 2, 3$, etc.

Taking $r=1$, we see that the shaft will whirl in a single loop, the maximum deflection being at the middle.

The speed for this is given by

$$\left(\frac{\rho\omega^2}{\mathrm{EI}g}\right)^{\frac{1}{4}}\cdot l=\pi$$

or $$\omega=\frac{\pi^2}{l^2}\times\sqrt{\frac{\mathrm{EI}g}{\rho}} \text{ radians per second.}$$

If $r=2$, the shaft will whirl in two loops with a node at the middle, and we can calculate the corresponding speed.

In the same way we can find the whirling speeds for $r=3, 4$, etc.

It should be noted that for each of the whirling speeds the maximum deflection is not fixed. Any maximum deflection which may occur accidentally or be impressed upon the shaft will be maintained, provided it does not occur at a node, but it should also be remembered that by assuming the curvature to be given by d^2y/dx^2, we are really only considering the cases where $(dy/dx)^2$ may be neglected. This covers all practical cases.

IV

§ (14) Members in Tension.—The design of tension members is quite simple and direct, and we need only consider two cases: (1) in which the resultant of the applied loads at the ends passes through the centre of gravity of the cross-section of the member and is parallel to the axis, and (2) in which the resultant load is parallel to the axis but does not pass through the centre of gravity of the cross-section.

In the first case the intensity of stress induced will be uniform over the cross-section. If f is the allowable tensile stress in tons per sq. in. and P the applied load in tons, then the area of cross-section required will be equal to P/f sq. in.

In the second case we treat the applied force as equivalent to a single axial force passing through the centre of gravity of the cross-section and two couples tending to rotate the cross-section about the two principal axes of inertia.

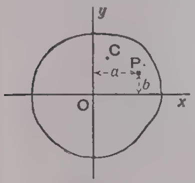

Fig. 18.

In *Fig.* (18) let O be the centre of gravity of the cross-section, and let the axes Ox and Oy be the principal axes of the section.

Suppose the line of action of the applied load P is parallel to the axis of the member, and intersects the cross-section at a point whose co-ordinates are (a, b). We shall take the equivalent system as consisting of

(1) a force parallel and equal to P acting at the centre of gravity O. This will produce a uniform tensile stress P/A, A being the area of cross-section.

(2) A bending moment about Ox equal to Pb, and a bending moment about Oy equal to Pa.

To find the resultant stress at any point C, co-ordinates (x, y), we superpose the stresses produced by (1) and (2).

We get for the resultant tensile stress (p) at C

$$p=\frac{P}{A}+\frac{Pby}{I_{ox}}+\frac{Pax}{I_{oy}},$$

where I_{ox} and I_{oy} are the moments of inertia of the cross-section about Ox and Oy respectively. We may write these as Ak_x^2 and Ak_y^2, where k_x and k_y are the radii of gyration about the axes Ox and Oy.

We get then

$$p=\frac{P}{A}\left(1+\frac{by}{k_x^2}+\frac{ax}{k_y^2}\right).$$

Suppose (u, v) are the co-ordinates of the point on the boundary which gives the maximum value of p. This value of p must not exceed f. If the member is perfectly designed,

$$f=\frac{P}{A}\left(1+\frac{bv}{k_x^2}+\frac{au}{k_y^2}\right).$$

It is seen immediately that the neutral axis of the section is given by $p=0$, and has the equation

$$1+\frac{by}{k_x^2}+\frac{ax}{k_y^2}=0.$$

If the neutral axis cuts the cross-section, it follows that there will be compression over part of the area.

In the above investigation we have omitted the effect of the deflection of the member due to the bending moments. In most cases this will be small and not seriously affect the distribution of stress. In the case of tension the deflection will be such as to displace the centre of gravity of the cross-section nearer to the line of action, thereby making the stress more uniformly distributed. This will not be the case when the member is a strut.

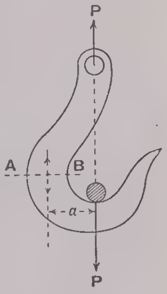

Fig. 19.

It should be carefully noted that this formula is only strictly applicable to cases where the axis of the member is straight. It will hold approximately if the radius of curvature is large compared with the dimensions of the cross-section. In the case of a crane hook, for example, as shown in *Fig.* 19, the section AB may be considered as subjected to a pull P applied through the centre of gravity of the section, and a bending moment of magnitude Pa.

Since the radius of curvature may even be less than the depth of the section, the ordinary formula for bending will not hold.

§ (15) Members in Compression.—The estimation of the true strength of members which are subjected to a compressive force in the direction of their length is, in all but the simplest cases, a matter of difficulty, and the results when arrived at can hardly be relied on to the same extent as those of the strength of ties, beams, and shafts. The chief reason for this will be understood if we refer back to the case of a tension member subjected to a load, the line of action of which did not pass through the centre of gravity of the cross-section. In such a case we noted that due to the eccentricity the member would be subjected to a bending moment, and further that this bending moment would cause a deflection tending to reduce the eccentricity and thereby making the stress more uniformly distributed. In the case of a member subjected to compression the reverse action occurs, and the bending moment tends to increase the eccentricity and thereby to increase the maximum stress induced. In actual practice it is impossible to obtain a perfectly straight strut or to obtain absolute axial loading, and even if this were the case at first, when the load was beginning to be applied, it would soon cease to be so, due to the fact that no material is perfectly homogeneous. The Young's modulus for compression at one part of the cross-section will vary slightly from that at another part, with the result that the strains will not be uniform, and some bending therefore is almost certain to occur. If the member is short in length relatively to its cross-dimensions the resistance to bending will be great, and the actual deflection produced by accidental eccentricity of loading or lack of homogeneity will have no appreciable effect. In such cases we may design the member in the same way as a tension member, by assuming that the stress is uniformly distributed. Also the crushing load of the member will be equal to A*f*, where A is equal to the area of cross-section and *f* the compressive stress required to cause crushing of the material.

If we take the opposite extreme, namely, that of a strut which is very long compared with its cross-dimensions, we get an entirely different phenomenon at failure. As the axially applied load is gradually increased, at first the member undergoes a shortening proportional to the load, and even if the strut is bowed out slightly it will return to its original straight condition if the load be removed. As we increase the load beyond a perfectly definite value, however, we find the member suddenly buckles out sideways and completely collapses if the load is continued. In other words, a condition of instability occurs, and this while the actual compressive stress in the material is well below the compressive stress required to crush the material. The material is either finally crushed or torn, but this is due to the stresses set up by the bending. This phenomenon was first theoretically investigated by Euler.

Consider a prismatic strut, perfectly hinged at the ends, and a load P applied axially.

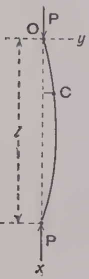

FIG. 20.

If the strut is bowed slightly, then at any point (x, y) there will be a bending moment of magnitude Py;

$$\therefore \frac{d^2y}{dx^2} = -\frac{Py}{EI},$$

where E is the linear modulus of elasticity and I is the moment of inertia for bending.

Write $$\frac{P}{EI} = a^2,$$

then $$\frac{d^2y}{dx^2} + a^2y = 0.$$

The solution of this is

$$y = A\cos ax + B\sin ax,$$

where A and B are constants depending upon the end conditions.

We have $y=0$, when $x=0$, and therefore A=0. Also $y=0$, when $x=l$, therefore $B\sin al=0$. This makes B=0, or $al=r\pi$, where $r=0, 1, 2$, etc. If B=0, the deflection everywhere is zero.

Taking $r=1$, we get

$$a = \frac{\pi}{l}$$

or $$P = \frac{\pi^2EI}{l^2}.$$

We note that P is independent of the amount of bowing.

The load π^2EI/l^2 is the load which will just keep the strut bowed and produce a state of neutral equilibrium. For a smaller load the strut would be in stable equilibrium, and for a greater load the deflection would increase without limit.

If we assume that the strut does not bend, but merely contracts under the load, it can be shown by considering the strain energy, that for a load greater than π^2EI/l^2 the equilibrium is unstable. (See Love's *Elasticity, Stability of Elastic Systems.*)[1]

The load π^2EI/l^2 is called the buckling load of the strut.

For values of r equal to 2, 3, etc., we get the loads required to maintain the strut when it is bent into segments of length $l/2$, $l/3$, etc.

Writing I equal to Ak^2, where A is the area of cross-section and k is the radius of gyration, we get

$$p = \frac{\pi^2Ek^2}{l^2},$$

where p is the load per unit area.

The ratio l/k is called the *slenderness ratio.* We will denote it by v.

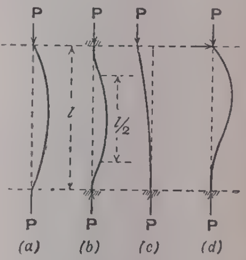

FIG. 21.

For the different end conditions represented in *Fig.* (21) we get the following buckling loads:

$$(a)\ \frac{P}{A} = \frac{\pi^2E}{v^2},\quad (b)\ \frac{P}{A} = \frac{4\pi^2E}{v^2},\quad (c)\ \frac{P}{A} = \frac{\pi^2E}{4v^2},$$

and $(d)\ \frac{P}{A} = \frac{2\pi^2E}{v^2}$ approximately.

If care is taken to ensure axial loading, and also that the end conditions are as assumed in the theory, experiment shows that these results are very closely true in the case of struts of homogeneous and isotropic material and with a large slenderness ratio, say greater than 150, but for struts of medium slenderness ratio, such as are met with in bridge work and building construction, the Euler formula gives altogether too high values.

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[1] *A Treatise on the Mathematical Theory of Elasticity,* by A. E. D. Love, F.R.S.

It is immediately obvious that the formula does not hold for short struts, as it gives a buckling load in excess of the crushing load of the material, and as l approaches zero the value of the buckling load approaches infinity.

The formulae which are actually used in the design of struts in building construction and bridgework are based on experiment. Some, such as Rankine's, have a more or less rational basis, but the constants in them ultimately have to be found by experiments on actual struts, and are not merely dependent upon the elastic constants of the material or the ultimate strength or yield stress in compression.

The Rankine formula is

$$p=\frac{f}{1+cv^2},$$

where f and c are two constants for the material, the constant c also depending upon the end conditions. p is the stress which will produce failure.

The values of f and c, adopted by different authorities, vary considerably, even for the same material.

For both ends hinged for mild steel many authorities take $f=21$ tons per sq. in. and $c=\frac{1}{7500}$.

It will be observed that if v is small the formula becomes $p=f$, f being the crushing stress of the material,[1] and if v is very large we may neglect the 1 in the denominator, and we get $p=f/cv^2$, which is of the same form as the Euler formula.

Actually with the most suitable values of f and c for struts of normal values of v, we find that for very large values of v the value of p exceeds that given by the Euler formula. The formula, therefore, should only be employed within the range of slenderness ratio of the experimental struts used for determining f and c.

In adjusting the formula for different end conditions the constant c is varied in exactly the same way as in the Euler formula. For example, for mild steel we should have:

Both ends hinged . . .	$c=\frac{1}{7500}$.
Both ends fixed	$c=\frac{1}{30000}$.
One end fixed and one end hinged but free to move laterally	$c=\frac{4}{7500}$.
One end fixed and one end hinged but kept in position	$c=\frac{1}{15000}$.

Many other strut formulae have been suggested, and used, amongst which should be mentioned Claxton Fidler's Formula, which is used by many British engineers.[2]

§ (16) STRAIGHT LINE FORMULAE.—If the various experimental values and the formulae values of the buckling load of struts of the range of v usual in practice be plotted against v, it will be seen that the curve is not very far from a straight line. Many authorities, particularly in America, at the present time design by means of a straight line formula. As an example, the American Railway Engineering Association formula is

$p=(16{,}000-70v)$ lbs. per sq. in., where p is the working stress.

The formula is for mild steel, ultimate tensile strength 55,000-65,000 lbs. per sq. in., and the strut is supposed to have hinged ends. The value of p is in no case to exceed 14,000 lbs. per sq. in. This gives a lower limit to the value of v. For smaller values of v than this lower limit p is kept equal to 14,000 lbs. per sq. in.

The formula is intended to apply to struts such as are normally used in structures and which are fastened at the ends, probably by bolts or rivets, but which cannot be considered *fixed*. For ends which are really fixed the formula

$p=16{,}000-56v$ may be used.

In British practice, structural steel has usually an ultimate strength in tension of 30 tons per sq. in., and we may use the formula

$$p=(8-0{\cdot}035v) \text{ tons per sq. in.}$$

for the working stress, for struts in which the ends cannot be considered fixed.

§ (17) STRUTS ECCENTRICALLY LOADED.—Very frequently a strut is intentionally loaded eccentrically, and the amount of eccentricity, apart from that caused by bending, is known. In such cases the common practice is to use the formula given above for members in tension, viz.

$$p=\frac{P}{A}\left\{1+\frac{bv}{k_x^2}+\frac{au}{k_y^2}\right\},$$

and to obtain from this the maximum value of p. Usually either a or b is zero.

This neglects the extra deflection due to bending, which in a well-designed strut will usually be small compared with the known eccentricity.

This maximum value of p has to be not greater than that obtained from the strut formula used on the assumption that p is axially applied.

In most practical cases this probably leads to a greater factor of safety than is required, since in the strut formula with axial loading the greatest stress is assumed to occur at the middle, whereas in many cases it will occur at the fixed end.

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[1] In the case of mild steel there is no definite crushing stress, since the material becomes plastic and flows. All we can measure is the stress at the yield point, and if f has to be fixed by compression tests of short lengths this yield stress should be taken.

[2] See *A Practical Treatise on Bridge Construction*, by Professor Fidler.

Allowing for the deflection due to bending, it may be shown that the end load P required to stress the material up to the yield stress at some point in the strut is given by

$$\mathrm{P} = \frac{f\mathrm{A}}{1 + (ay_c/k^2)\sec\{(l/2)\,.\,\sqrt{\mathrm{P/EA}k^2}\}},$$

where a is the eccentricity assumed in the line of the principal axis about which the radius of gyration is a maximum, k is the least radius of gyration, and y_c the distance from the centre of gravity to the edge of the section measured in the direction of a.

§ (18) BUILT-UP STRUTS.—When struts are built up of separate members connected together by bolts or rivets, the strength will depend very largely upon whether the various members are so connected that they behave like a single piece, *i.e.* there will be no yield at the joints, or whether there may be an appreciable yield at the joints. In steel struts the joints are usually riveted, and are such that we may neglect the slip at the joint. The ordinary formula may be employed, the radius of gyration of the whole section being used. In built-up timber struts, of such soft timber as is usually employed in construction, where the connections of the various members are usually made with bolts, there is considerable yield and slip at the joints, due to the relatively small compressive strength of the timber and the bending of the bolts. Experiment shows that such struts do not start to fail as a whole, but failure occurs in the individual members. The strength of such struts is merely the sum of strengths of the individual members in the cross-section.

In riveted joints in steel structures the members are really held together by friction, the necessary pressure between the members being produced by the tension in the rivets. In timber, although a bolted joint may temporarily be made to hold by friction by using large washers and tightening the nuts to give the required pressure, yet this cannot be relied on permanently. The expansion and contraction due to climatic changes is relatively large, and though a considerable tension may exist in the bolts one day, on another day there may be no appreciable tension at all.

If a composite strut, axially loaded, is free to buckle in any direction, the strength is determined by the least radius of gyration, and for economy in material the radii of gyration about the two principal axes should be as nearly equal as possible.

As will have been realised already, the design of struts cannot be direct as is the case in the design of ties, where the required area can be estimated. In dealing with struts the best we can do is to design the member, and then estimate the strength. If the latter is less or considerably more than required we have to alter our design, in the one case increasing the strength by additional material, and in the other reducing it so as to effect economy of material. The labour entailed in this process is considerably reduced by the standardisation of rolled steel sections, the geometrical properties of which, such as area, position of the principal axes, and the radii of gyration about these, have been calculated and tabulated.

Numerous experiments have also been made on struts consisting of standard rolled sections, and also struts built up of standard rolled section. The data of these experiments are to a very large extent used in selecting a suitable design for a strut, and also for estimating the strength of struts and column of different designs from those to which the experiments apply.

V

§ (19) JOINTS.—The strength of a composite structure, consisting of a number of members joined together, will depend as much upon the strength of the joints as upon the strength of the members themselves. Generally speaking, the stresses in the members can be more accurately determined than the stresses at the joints, and for this reason it is good practice to employ a somewhat greater factor of safety in designing the joints themselves than in the design of the members.

Three types of joints are commonly employed in which the members are connected by (1) rivets, (2) pins, (3) bolts. In ordinary structural work, as distinguished from machinery, the joints are almost entirely made by rivets, bolts being used only for temporary work, and pins in cases where a hinged joint is required.

To be satisfactory, a riveted joint should really be a friction joint, the members joined being pressed together by the tension in the rivets. This tension is produced by closing the rivets while at a red heat, the subsequent cooling causing a tension in the rivet, since the contraction is largely prevented by the members joined. If a riveted joint such as is shown below is tested by gradually increasing the pull, the joint will be found to be elastic at first, the actual extension being small and increasing uniformly with the load. At a certain load slip will occur, indicating that the frictional force has been overcome, and the actual amount of slip will depend upon how accurately the rivets filled the holes. After this initial slip the extension will vary somewhat irregularly until all the rivets are bearing against the plates, and have all reached their yield point, or until the stress in the plates has reached the yield point. It will readily be realised that the joint is not satisfactory if loaded beyond the load required to overcome the friction, since there would be backlash for reversal of stress; also the joint would leak, for example if used in a boiler; or, further, in ordinary structural work moisture would find its way in, with the result that rust would form.

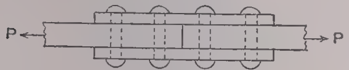


FIG. 22.

Owing to the difficulty of estimating the frictional force in a joint due to possible defective workmanship, etc., it is usual to design the joint simply on the assumption that there is no friction, and that the rivets transmit the stress from one member to the other by their shearing stress. This would appear to be a wholly illogical procedure, but it has this to be said in its favour, viz. that it produces a joint which, even if slip does occur, will still hold, and complete failure will not ensue.

Even when treating the riveted joint merely as a pin joint, as is usually done, the design depends more upon empirical rules, based on past experience, than upon theory, although there are a few theoretical considerations which are useful as a guide.

§ (20) WAYS IN WHICH A SIMPLE RIVETED JOINT MAY FAIL.—Let P be the load carried by one rivet.

Let p_1 and p_2 be the ultimate shearing stress of the rivets and plates respectively, p_3 be the ultimate tensile stress of the plates, p_4 be the ultimate compressive stress of the rivets and plates, d be the diameter of the rivets, t be the thickness of the plates joined.

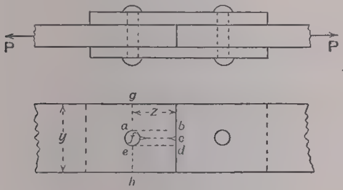


FIG. 23.

(1) The rivets may shear,

$$P = p_1 \times \frac{\pi d^2}{2}.$$

(2) The plate or rivet may be crushed at surface *afe*,

$$P = p_4 \times dt.$$

(3) The portion of the plate *baed* may shear out,

$$P = p_2 \times 2tz.$$

(4) The plate may tear across *ga* and *eh*,

$$P = p_3 \times (y - d)t.$$

(5) The plate may split along *fc*. The load cannot be calculated.

(6) Rivet may fail by bending. The load in this case cannot be calculated.

The diameter of the rivets is estimated by consideration of (1) and (2). In the case of the joint shown, for equality of strength we get

$$\frac{p_1 \pi d^2}{2} = p_4 dt,$$

or

$$d = \frac{p_4}{p_1}\left(\frac{2}{\pi}\right)t.$$

The sizes commonly adopted are:

Plates (thickness).	Rivets (diameter).	
$\frac{1}{4}''$ to $\frac{1}{2}''$	$\frac{1}{2}''$ to $\frac{3}{4}''$	Light structural work
$\frac{3}{8}''$ to $\frac{3}{4}''$	$\frac{3}{4}''$ to $\frac{7}{8}''$	Light bridge work
1″	1″	Heavy bridge work

Usually a single size is selected and used throughout, and the safe load per rivet is estimated by the shear or compression.

When there are more than two rows of rivets on either side of the joint, the distribution of load between the rivets has to be considered.

Take for example the joint shown below.

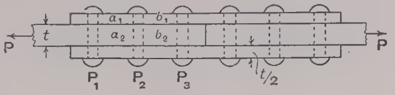


FIG. 24.

If we assume that the rivets are rigid in comparison with the plates, then the stretch between the rivets must be the same for the members joined and for the cover plates. Let P_1, P_2, P_3 be the load carried by the rivets.

For a_1 and a_2 we have

$$P - P_1 = P_1. \quad . \quad . \quad . \quad (1)$$

For b_1 and b_2 we have

$$P_1 + P_2 = P - P_1 - P_2. \quad . \quad . \quad (2)$$

Also

$$P_1 + P_2 + P_3 = P. \quad . \quad . \quad . \quad (3)$$

These give

$$P_2 = 0$$

and

$$P_1 = P_3 = P/2.$$

If we take the opposite extreme case and assume that the plates are rigid in comparison with the rivets, we see that we shall get equal distortion of the three rivets, *i.e.* each rivet will carry one-third the load. The true condition of things must be somewhere between these extremes and we shall get the middle rivet carrying less than one-third the load. For this joint, assuming the rivets rigid, the loads on them might be equalised by varying the breadth of the cover plates and the joined members, as shown below.

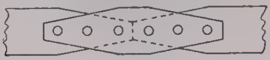


FIG. 25.

The above does not apply after the yield point of the rivets or plates has been reached. It is clear that if ultimately the joint fails by the shearing of the rivets the load will be evenly distributed.

Another factor which affects the distribution of load between the rivets is the arrangement of rivets with reference to the resultant line of action of the pull.

In the figure two T members are joined by two cover plates, one on each side of the vertical part of the T. The resultant pull will pass through the centre of gravity of the cross-section of the T, and it is obvious that the rivets will have to resist not only a pull P but also a couple of magnitude Pa. In order to secure uniform loading of the rivets the centre of gravity of the rivet holes should lie on the resultant line of the action of the pull.

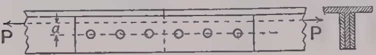


FIG. 26.

§ (21) ECCENTRICALLY LOADED RIVETED JOINT.—In many cases it is impossible to attain the condition that the resultant pull passes through the centre of gravity of the rivet holes, and in such cases it is important to estimate the load carried by the most severely stressed rivet. Consider the joint shown below in which the plate is supposed riveted to a large rigid member.

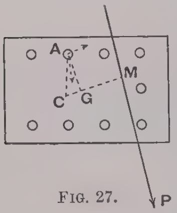


FIG. 27.

Let P be the resultant pull on the plate. We shall assume that the deformation of the plate is negligibly small compared with the yield of the rivets. This will generally be the case, since in joints of this type the stresses in the plate will usually be small. We shall further assume that the resistance of any rivet is proportional to the displacement of the portion of the plate around the rivet relative to the member.

Let there be n rivets and let G be the centre of gravity of the rivet holes. Draw GM perpendicular to the line of action of P.

We may consider the plate subjected to a pull P acting through G and parallel to the line of action of the pull on the plate, and also to a couple Pa, where a = GM.

The stress in each rivet due to pull P at G is equal to P/n. The displacement due to this in direction perpendicular to GM will be $k\,(\mathrm{P}/n)$, where k is some constant.

Let p be the load on any one rivet situated at a distance x from G due to the couple. Then for the couple we have

$$\Sigma px = \mathrm{P}a. \quad . \quad . \quad . \quad (1)$$

The displacement of each rivet will be proportional to its distance from G.

$kp = \theta x$, where θ is the angle of rotation of the plate relatively to the member.

Therefore, from (1),

$$\Sigma \frac{\theta}{k} x^2 = \mathrm{P}a,$$

i.e.

$$\frac{\theta}{k} = \frac{\mathrm{P}a}{\Sigma x^2}. \quad . \quad . \quad . \quad . \quad (2)$$

Any rivet A, say, will have a displacement $k(\mathrm{P}/n)$ in a direction perpendicular to GM and a displacement θ . AG perpendicular to AG.

Produce MG to C, then the displacement of C

$$= k\frac{\mathrm{P}}{n} - \theta \times \mathrm{CG}.$$

Make CG = $k\mathrm{P}/n\theta$, then C will have no displacement, *i.e.* it will be the instantaneous centre of rotation of the plate.

From (2),

$$\mathrm{CG} = \frac{\Sigma x^2}{na}.$$

The resultant displacement of A will be equal to θ . AC.

The load on rivet A will be $(\theta\,.\,\mathrm{AC})/k$

$$= \frac{\mathrm{P}a}{\Sigma x^2} \times \mathrm{AC}.$$

The most severely stressed rivet will obviously be the rivet most distant from C.

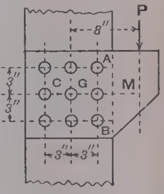


FIG. 28.

As an example take the joint shown below, *Fig.* 28.

From symmetry we see that the centre of gravity of the rivets is at rivet G.

$$\Sigma x^2 = 4 \times 3^2 + 4(3^2 + 3^2)$$

$$= 108,$$

$$\mathrm{CG} = \frac{\Sigma x^2}{na} = \frac{108}{9 \times 8} = 1{\cdot}5 \text{ in.}$$

The most severely stressed rivets will be A and B. Load carried by rivet A

$$= \frac{\mathrm{P}a}{\Sigma x^2} \times \mathrm{AC}$$

$$= \frac{\mathrm{P} \times 8}{108} \times \sqrt{4{\cdot}5^2 + 3^2}$$

$$= 0{\cdot}398\mathrm{P}.$$

VI

§ (22) FRAMES.—In designing framed structures such as roof trusses, lattice girders, etc., which are to be loaded at the joints, a determination of the total stress in each member has first to be made. The method of doing this depends upon whether the structure has just sufficient members to prevent it collapsing or has more than sufficient. In the former case the principles of pure statics enable us to estimate the total stress in each member, whereas in the latter case we have to employ the principles of the Theory of Elasticity.

In a frame in which all the members lie in one plane we have a triangle as the simplest form of a perfect frame, *i.e.* a frame which has just sufficient members to prevent a change in shape when loads are applied at the joints, except the slight deformation caused by the small change in length of each member due to the stress in it.

We note that the triangular frame has 3 joints and 3 members. If we add another joint to the frame we shall have to add two more members, and this will be the case for every joint added in building up a complex perfect frame. It follows that in a perfect frame of j joints the number of members will be 3 for the original triangle, and $2(j-3)$ for the added portion. This provides us with a useful test of a perfect frame. In the perfect frame in which all the members are in one plane, if j is the number of joints, the number of members must be $3+2(j-3)$, *i.e.* $2j-3$ members. If the number of members is greater than $(2j-3)$, the excess are redundant members.

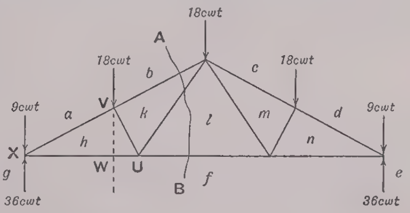
For frames in which the members do not lie in one plane the simplest perfect frame is the tetrahedron, and in this case in any perfect frame the number of members must equal $3j-6$.

There are two methods commonly employed in estimating the total stresses of a perfect frame: (1) Reciprocal figures; (2) Method of sections. We will briefly illustrate these by a simple example of a roof truss.

The truss with its loads is shown in *Fig.* 29. In such cases it is usual to assume that the joints are all frictionless pin joints, although in reality they are by no means so. Most of the joints will be riveted joints. It will be found, however, that the error introduced by this assumption in estimating the total stresses is usually small. As will be seen, Bow's system of notation has been adopted. This consists in indicating the spaces between the forces and the members by a single letter. In *Fig.* 29 the line $f'g'a'b'c'd'e'f'$ represents the force polygon for the external forces fg, ga, ab, etc.

To find the total stress in each member we draw a force polygon for each of the joints, going round the joint in the same direction, viz. clockwise, as in dealing with the external forces. Starting at the joint $fgah$ we have the polygon $f'g'a'h'f'$, and similarly for the other joints. The length of $a'h'$, for example, gives the total stress in member ah. The results are tabulated in the figure. This is the reciprocal figure method.

An alternative way of graphically finding the total stresses is to use the method of sections. Suppose we wish to find the total stress in the member bk. Call it a thrust T. Take a section AB and consider the equilibrium of the part of the truss to the left of the section. By taking moments about the joint U the only unknown force will be T, and we have



Member	Total Stress (cwt)
ah dn	-58
bk cm	-50·4
hk mn	-16
kl lm	+17·4
hf nf	+52
lf	+34·4

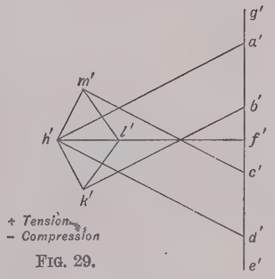


FIG. 29.

$$T \times UV + 18 \times UW + (9-36) \times UX = 0.$$

Substituting the distances measured from the diagram,

$$0\ 73T + 18 \times 0{\cdot}34 - 27 \times 1{\cdot}59 = 0,$$

$$\therefore\ T = 50{\cdot}4 \text{ cwt.}$$

§ (23) EFFECT OF STIFF JOINTS.—As illustrating the effect of estimating the total stress in the members of a frame by assuming the joints hinged, we will take the following simple example.

The roof truss shown below has a single concentrated load applied at the top. The sloping members are each of standard T section, 2 in. by 2 in. by $\frac{1}{2}$ in., and have an area of cross-section 0·947 sq. in. and a radius of gyration about the axis of bending 0·597 in. The tie is a round rod of diameter $\frac{1}{2}$ in. and length 10 ft. We will find the error in estimating the stress in the tie rod due to assuming the members rigid and the top joint hinged.

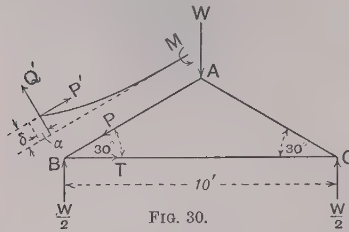


FIG. 30.

(1) Assume the members rigid and the joints free. Let P be the thrust in BA, and T the pull in BC. For the equilibrium of the pin at B we have

$$\mathrm{P}\sin 30^\circ=\frac{\mathrm{W}}{2},$$

$$\mathrm{P}\cos 30^\circ=\mathrm{T},$$

$$\therefore\ \mathrm{T}=\frac{\mathrm{W}}{2}\cot 30^\circ.$$

(2) Assume that the joint at A is rigid.

Let P′ and Q′ be the forces, along and perpendicular to BA at B, and let T′ be the pull in BC.

For the pin at B we have

$$\mathrm{P}'=\frac{\mathrm{W}}{2}\sin 30^\circ+\mathrm{T}'\cos 30^\circ$$

$$=\frac{\mathrm{T}}{2\sqrt{3}}+\frac{\sqrt{3}\mathrm{T}'}{2},$$

and

$$\mathrm{Q}'=\frac{\mathrm{W}}{2}\cos 30^\circ-\mathrm{T}'\sin 30^\circ$$

$$=\tfrac{1}{2}(\mathrm{T}-\mathrm{T}').$$

For the deflection of a beam such as AB, fixed at one end and loaded at the other, we have

$$\delta=\frac{\mathrm{Q}'l^3}{3\mathrm{EI}}$$

$$=\frac{(\mathrm{T}-\mathrm{T}')\times 10^3\times 1728}{6\times 3\times\sqrt{3}\times 0{\cdot}947\times 0{\cdot}597^2\times\mathrm{E}}$$

$$=\frac{(\mathrm{T}-\mathrm{T}')}{\mathrm{E}}\times 1{\cdot}64\times 10^5.$$

For the shortening of BA due to the compression P′, neglecting the shortening due to bending, we have

$$\alpha=\frac{\mathrm{P}'\times 120}{0{\cdot}947\times\sqrt{3}\times\mathrm{E}}$$

$$=\frac{(\mathrm{T}/2\sqrt{3}+\sqrt{3}\mathrm{T}'/2)}{\mathrm{E}}\times 73.$$

The extension of the rod BC

$$=\frac{\mathrm{T}'\times 16\times 120}{\pi\mathrm{E}}$$

$$=2\{\delta\cos 60^\circ-\alpha\cos 30^\circ\},$$

$$\therefore\ 610\mathrm{T}'=1{\cdot}64\times 10^5(\mathrm{T}-\mathrm{T}')-\left(\frac{\mathrm{T}}{\sqrt{3}}+\sqrt{3}\mathrm{T}'\right)\frac{\sqrt{3}}{2}\times 73,$$

$$610\mathrm{T}'-1{\cdot}64\times 10^5(\mathrm{T}-\mathrm{T}')+36{\cdot}5\mathrm{T}+109{\cdot}5\mathrm{T}'=0,$$

$$(1{\cdot}64\times 10^5-36{\cdot}5)(\mathrm{T}-\mathrm{T}')=756\mathrm{T}',$$

or

$$\frac{\mathrm{T}-\mathrm{T}'}{\mathrm{T}'}=4{\cdot}67\times 10^{-3}.$$

The error is about 5 in 1000.

The error in this simple case is extremely small, but the secondary stress induced due to stiff joints and the bending of the members may in many bridge trusses amount to as much as 20 per cent of the primary stress, and in certain cases to considerably higher values. In very large structures these secondary stresses are calculated and allowed for, but for small structures they are not directly estimated, but are provided for by keeping the working stress a small fraction of the breaking stress.

§ (24) DEFLECTION OF FRAMED TRUSSES. (i.) *The Deflection Diagram.*—In the case of framework in general, built-up lattice girders, and roof trusses, the deflection and deformation is most conveniently found graphically by drawing what is called a deflection diagram. We will illustrate the method by taking the simple frame shown in *Fig.* 31 and loaded with a load at a. The member dc may be considered rigid, and the joints pin joints.

Assuming the members rigid and the joints free, estimate the total stresses in each of the members. If P is the total stress in a member, A its area of cross-section, and l its length, then the total stretch will be equal to Pl/AE, and will be positive or negative according to whether P is a tension or a compression.

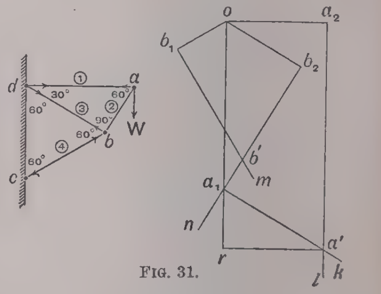


FIG. 31.

Now the relative motion of one end of a member to the other may be considered as consisting of two components, one along the member and the other perpendicular to the member. The component along the member may be found as shown above. We now draw the deflection diagram as follows:

From some point o draw ob_1, to represent the movement of b towards the fixed point c along bc. Draw b_1m perpendicular to ob_1. Then the displacement of b is given by ob', where b' lies on b_1m.

Similarly find the displacement of b by considering it as a point in the member db. Draw ob_2 parallel to db to represent the movement of b away from the fixed point d along db. Draw b_2n perpendicular to bd. Then b' is given by the intersection of b_1m and b_2n.

Similarly we find the displacement of a relative to b by drawing $b'a_1$ parallel to ba and to represent the shortening of the member *ba*. Draw a_1k perpendicular to *ab*. Lastly, draw oa_2 parallel to *da* to represent the lengthening of *da*. Draw a_2l perpendicular to *da*. The point of intersection of a_1k and a_2l gives a'.

The total displacement of *a* is given by oa', and the vertical and horizontal components by *or* and ra'.

In *Fig.* 31 we will assume that the areas of cross-sections of the members are so selected that all members in tension are stressed to 6 tons per sq. in. and all members in compression to 4 tons per sq. in. Take E = 12,000 tons per sq. in., then we get the following :

Member.	Stress (tons per sq. in.).	Length (feet).	Extension (inches).
(1)	6	$\frac{20}{\sqrt{3}}$	0·0693
(2)	−4	$\frac{10}{\sqrt{3}}$	−0·0231
(3)	6	10	0·06
(4)	−4	10	−0·04

From *Fig.* 31 we find :

The vertical deflection of $a = 0{\cdot}16$ in.

The horizontal deflection of $a = 0{\cdot}07$ in.

(ii.) *Maxwell's Method. Principle of Work.* —The deflection of any joint of a loaded frame may also be determined by the *principle of work* as follows :

Let F = the total stress in any member required to produce unit stretch in that member.

Let P = the actual total stress in the member due to the loads on the frame.

Let l = the length of the member.

Let k = the total stress in any member due to a unit pull in any specified direction applied at the joint considered.

Let δ = the displacement of the joint in the specified direction due to the loads on the frame.

Let z = the displacement of the joint in the specified direction due to a total stress k in the member.

Assuming all the members quite rigid except the member considered, we have, by the principle of work,

$$\tfrac{1}{2} \times 1 \times z = \frac{k}{2} \times \frac{kl}{\mathrm{F}},$$

since kl/F is the extension of the member considered due to the total stress k in it.

An extension of the member considered produces a displacement of the joint in the specified direction equal to $k \times$ extension of the member.

Therefore stress P in the member considered produces a displacement of the joint in the specified direction equal to $k \times \mathrm{P}l/\mathrm{F}$.

Due to the stresses in all the members of the frame, the displacement of the joint in the specified direction is given by

$$\delta = \Sigma k \times \frac{\mathrm{P}l}{\mathrm{F}}.$$

If we want the actual displacement of any joint we may find the displacement in two directions at right angles and then find the resultant.

Taking the previous examples, if $p = \mathrm{P/A}$ and $f = \mathrm{F/A}$, where A is the area of cross-section of the member considered, we may write the deflection of a joint in a specified direction equal to $\Sigma k(pl/f)$. We note that $f = \mathrm{E}$. Applying a unit load vertically at a we have for the vertical displacement :

Member.	k (tons).	p (tons per sq. in.).	l (in.).	$k\frac{pl}{\mathrm{E}}$.
(1)	$\frac{1}{\sqrt{3}}$	6	$\frac{240}{\sqrt{3}}$	0·04
(2)	$-\frac{2}{\sqrt{3}}$	−4	$\frac{120}{\sqrt{3}}$	0·0267
(3)	$\frac{2}{3}$	6	120	0·04
(4)	$-\frac{4}{3}$	−4	120	0·0533
		Total		0·16

Applying a unit load horizontally at a the value of k for member (1) will be 1 and for members (2), (3), (4) will be zero.

The deflection of a in a horizontal direction

$$= 1 \times 6 \times \frac{240}{\sqrt{3}} \times \frac{1}{12000}$$
$$= 0{\cdot}069 \text{ in.}$$

§ (25) FRAME WITH REDUNDANT MEMBERS. —Let the redundant members be denoted by *a*, *b*, *c*, etc., and the other members by 1, 2, 3, etc.

Let P_a represent the true total stress in member *a* when the frame is loaded, and similarly P_3 will represent the total stress in member 3 due to the loads.

Let Q_1, Q_2, Q_3, etc., represent the total stresses due to the loads which would exist in members 1, 2, 3, etc., if all the redundant members were absent.

Let k_{1a} represent the total stress in member 1 due to unit pull in member *a*, all the other redundant members being supposed removed.

Let F_3 represent the total stress in any member 3 required to produce unit stretch in that member, and so on.

Let l_3 represent the length of member 3, and so on.

We have

$$\mathrm{P}_1 = \mathrm{Q}_1 + k_{1a}\mathrm{P}_a + k_{1b}\mathrm{P}_b + k_{1c}\mathrm{P}_c + \ . \ . \ . \ ,$$
$$\mathrm{P}_2 = \mathrm{Q}_2 + k_{2a}\mathrm{P}_a + k_{2b}\mathrm{P}_b + k_{2c}\mathrm{P}_c + \ . \ . \ . \ ,$$

and similar equations for all the members which are not redundant.

Now let us confine our attention to member a, say, and employ the method of the last article. If δ_a is the shortening of the member a, we have

$$\delta_a = \Sigma\frac{Pkl}{F}.$$

But δ_a is also equal to $-P_a/F_a$.

$$\therefore\ -\frac{P_a}{F_a} = \frac{P_1 k_{1a} l_1}{F_1} + \frac{P_2 k_{2a} l_2}{F_2} + \ .\ .\ .\ .\ ,$$

$$i.e.\ -\frac{P_a}{F_a} = (Q_1 + k_{1a}P_a + k_{1b}P_b + k_{1c}P_c + \ .\ .\ .)\frac{k_{1a}l_1}{F_1}$$

$$+ (Q_2 + k_{2a}P_a + k_{2b}P_b + k_{2c}P_c + \ .\ .\ .)\frac{k_{2a}l_2}{F_2}$$

$$+ , \text{etc.}$$

We shall obtain a similar equation for each of the other redundant members. If there are n redundant members we shall obtain n equations containing only the unknowns P_a, P_b, P_c, etc.

From these equations we may find P_a, P_b, P_c, etc., and therefore P_1, P_2, P_3, etc.

As an example, take the frame used in the previous example, but with an additional member (5), as shown in *Fig.* 32.

Let $W=1$ ton, then from the previous example the areas of members (1), (2), (3), (4) are $1/6\sqrt{3}$, $1/2\sqrt{3}$, $1/9$, $1/3$ sq. in. respectively.

Let the area of the cross-section of (5) be $\frac{1}{2}$ sq. in.

Take the member (5) as the redundant member and consider the stress of the other members due to a unit pull in member (5). Draw bq parallel to da. Then

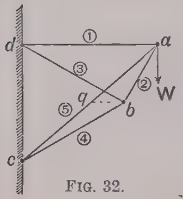


FIG. 32.

$$\left.\begin{aligned} k_{15} &= -\frac{qb}{aq} = -0{\cdot}388 \\ k_{25} &= -\frac{ab}{aq} = -0{\cdot}74 \\ k_{35} &= \ \ 0{\cdot}427 \\ k_{45} &= -0{\cdot}854 \end{aligned}\right\} \qquad \left.\begin{aligned} F_1 &= \frac{E}{6\sqrt{3}} \\ F_2 &= \frac{E}{2\sqrt{3}} \\ F_3 &= \frac{E}{9} \\ F_4 &= \frac{E}{3} \end{aligned}\right\}$$

$$\left.\begin{aligned} P_1 &= \frac{1}{\sqrt{3}} - 0{\cdot}388P_5 \\ P_2 &= -\frac{2}{\sqrt{3}} - 0{\cdot}74P_5 \\ P_3 &= \frac{2}{3} + 0{\cdot}427P_5 \\ P_4 &= -\frac{4}{3} - 0{\cdot}854P_5 \end{aligned}\right\}$$

The shortening of member (5), *i.e.* the displacement of a in direction ac, c being fixed,

$$= \Sigma\frac{Pkl}{F} = -\frac{((1/\sqrt{3}) - 0{\cdot}388P_5)\times 0{\cdot}388\times(240/\sqrt{3})}{12{,}000/6\sqrt{3}}$$

$$-\frac{(-(2/\sqrt{3}) - 0{\cdot}74P_5)\times 0{\cdot}74\times(120/\sqrt{3})}{12{,}000/2\sqrt{3}}$$

$$+\frac{((2/3) + 0{\cdot}427P_5)0{\cdot}427\times 120}{12{,}000/9}$$

$$-\frac{(-(4/3) - 0{\cdot}854P_5)\times 0{\cdot}854\times 120}{12{,}000/3}$$

$$= \frac{1}{100}(6{\cdot}74P_5 + 5).$$

But due to pull P_5 the shortening of member (5)

$$= -\frac{P_5\times 1{\cdot}525\times 120}{12{,}000/2} = -\frac{3{\cdot}05}{100}P_5,$$

$$\therefore 6{\cdot}74P_5 + 5 = -3{\cdot}05P_5,$$

i.e. $$P_5 = -0{\cdot}512 \text{ tons.}$$

Having obtained the thrust in member (5) we can now obtain the stress in each of the other members.

VII

§ (26) STRENGTH OF A 60-FOOT ROAD-BRIDGE.—We will illustrate in detail the method of estimating the strength of a structure by taking an actual example of a lattice-work bridge, designed originally for military purposes.

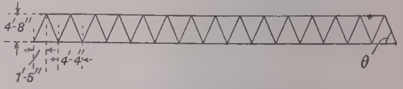


FIG. 33.—A Skeleton Elevation of a Girder.

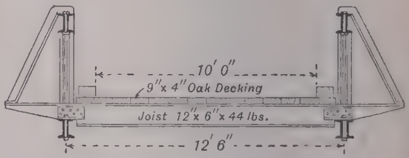


FIG. 34.—A Cross-section.

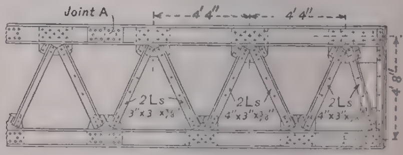


FIG. 35.—A Portion of a Girder in detail.

The bridge is not selected as specially representing modern highway bridges, but on account of the fact that the calculations involved are relatively simple and short, and at the same time illustrate the essential points which have to be considered in all steel bridges of this character. For a permanent structure the bolts used at the junctions in the flanges would be replaced by rivets.

The clear span is 60 ft., and the over-all length of the girders 64 ft. The bridge is designed to carry a continuous train of 16-ton tractors, with axle loads 5 tons and 11 tons, the axles being 10 ft. distant apart. The weight of the bridge is 32·6 tons. The distance between the centre lines of the girders is 12 ft. 6 in. The breadth of the roadway is 10 ft. The breadth of the wheel base of the tractors is 7 ft.

(i.) *Estimation of the Maximum Bending Moment.*—In this bridge the chords of the girders are the same throughout their length, so we need only consider the maximum bending moment produced by the loading.

We will allow 10 ft. between the rear axle of one tractor and the front axle of the tractor immediately behind. This leaves only some 3 ft. actual clearance. If the tractors close up more than this they will be going dead slow, and in that case the impact factor might be reduced.

We adopt an impact factor $\frac{5}{4}$.

The maximum live load which can be on the bridge is shown in *Fig.* 36, where the effective span is taken as 62 ft.

To estimate the maximum bending moment we may draw a series of bending moment diagrams for different positions of the train of tractors on the bridge and select from these the maximum value, or we may make use of the following facts:

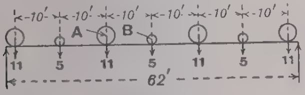


FIG. 36.

(1) The maximum bending moment for the bridge will occur under one or other of the loads.

A glance at the bending moment diagram for one position of the loads will show this to be true.

(2) The maximum bending moment for the bridge will occur near the middle of the bridge. This is found from experience to be the case.

(3) With a definite system of loads on the bridge the maximum bending moment under any one load occurs when that load and the centre of gravity of all the loads on the bridge are equidistant from the supports.

Assume the loads on the bridge are as shown in *Fig.* 36. The centre of gravity of the loads will be at load B. The maximum bending moment under B will be when it is at the centre of the bridge. Taking moments for the loads on the left-hand side about B, we have

$$M_B = 29{\cdot}5 \times 31 - 11 \times 30 - 5 \times 20 - 11 \times 10$$
$$= 374{\cdot}5 \text{ ft.-tons.}$$

For the maximum bending moment under A we find that, by fulfilling condition (3) above, the system of loads on the bridge changes and we get the system shown in *Fig.* 37.

The centre of gravity of the load on the bridge is now at A. For maximum bending moment under A it will have to be placed at the centre of the bridge. Taking moments about A for the loads on the left, we have

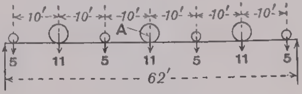


FIG. 37.

$$M_A = 26{\cdot}5 \times 31 - 5 \times 30 - 11 \times 20 - 5 \times 10$$
$$= 401{\cdot}5 \text{ ft.-tons.}$$

This is greater than the maximum under B, and therefore we must design for 401·5 ft.-tons. Suppose W is the load which, if uniformly distributed, would give the same maximum bending moment, then

$$\tfrac{1}{8}W \times 62 = 401{\cdot}5,$$

i.e. $$W = 51{\cdot}8 \text{ tons.}$$

We must now see how much of this load may have to be carried by one girder. The whole train may be on one side of the bridge, and we have the condition shown in *Fig.* 38.

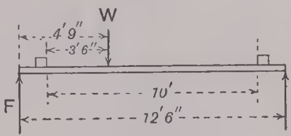


FIG. 38.

We get for the maximum moving load (F) carried by one girder

$$12{\cdot}5 \times F = 7{\cdot}75\ W,$$
$$\therefore\ F = 0{\cdot}62\ W.$$

Allowing for an impact factor of $\frac{5}{4}$, and for the weight of the bridge, each girder should be capable of withstanding a bending moment

$$= \tfrac{1}{8}(0{\cdot}62 \times 51{\cdot}8 \times \tfrac{5}{4} + 16{\cdot}3)62$$
$$= 438 \text{ ft.-tons.}$$

(ii.) *Longitudinal Stress in the Chords of the Girders.*—Each chord consists of two standard channels, each 10 in. × 4 in., weighing 30·16 lbs. per foot, and of area of cross-section

8·871 sq. in. To get the effective area we must consider a joint where it will be seen that there are two bolt-holes in a vertical cross-section. The bolts are $1\frac{1}{4}$ in. diameter, and the thickness of the web of the channel is 0·475 in.

Therefore the effective area

$$=2(8{\cdot}871-2\times\tfrac{5}{4}\times0{\cdot}475)$$
$$=15{\cdot}36 \text{ sq. in.}$$

If p is the mean stress in the chord, then

$$p\times15{\cdot}36\times4{\cdot}67=438,$$
$$\therefore\ p=6{\cdot}12 \text{ tons per sq. in.}$$

Now we have to bear in mind that in this bridge the roadway is supported on joists which rest on the bottom chord. There will therefore be an additional stress due to the fact that the portion of the chord between two panel points has to act as a beam. Since the chord is continuous through the panel points, and the load is carried on decking, which is also continuous over more than two joists, it is impossible to estimate exactly the maximum bending moment in the chord. We shall rather overestimate the maximum bending moment if we assume the chord merely supported at the panel points and a heavy axle midway between the panel points, as shown in *Fig.* 39.

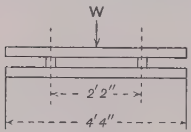


FIG. 39.

The maximum bending moment

$$=\frac{W}{2}\times\frac{13}{12}.$$

Now W may be $0{\cdot}62\times11\times\frac{5}{4}$ tons.

Therefore the maximum bending moment

$$=\frac{0{\cdot}62\times11\times5\times13}{2\times12\times4}$$
$$=4{\cdot}62 \text{ ft.-tons.}$$

For the chord the moment of inertia about the axis of bending is $2\times130{\cdot}7$ in.^{4}

The maximum longitudinal stress

$$=\frac{4{\cdot}62\times12\times5}{261{\cdot}4}$$
$$=1{\cdot}06 \text{ tons per sq. in.}$$

Adding this to the stress previously found we get the maximum longitudinal stress in the chord equal to 7·18 tons per sq. in. This is satisfactory, since the allowable stress may be 7·5 tons per sq. in.

We have not considered the possibility of buckling in the top chord. The chord itself has very little resistance to buckling in a horizontal direction, particularly at the joints. It will be seen in *Fig.* 34 that buckling has been prevented by means of side struts at frequent intervals, every 2 ft. 2 in. at the middle of the girder.

(iii.) *Joints in the Chord.*—At a joint in the chord only one channel has to be joined at one section. The most heavily stressed joint will be the one nearest the middle. This occurs at 2 ft. 2 in. from the middle, and the bending moment at this point will be practically the same as that at the middle.

The pull to be transmitted by the bolts of the joint is $438/2\times4{\cdot}67$, *i.e.* 47 tons.

There are seven $1\frac{1}{4}$-in. bolts on either side of the joint.

Therefore the bearing pressure

$$=\frac{47}{7\times1{\cdot}25\times0{\cdot}475}$$
$$=11{\cdot}3 \text{ tons per sq. in.}$$

The shearing stress

$$=\frac{47\times4}{7\times\pi\times1{\cdot}25^2}$$
$$=5{\cdot}5 \text{ tons per sq. in.}$$

These are satisfactory since, with an allowable tensile stress of 7·5 tons per sq. in., we may allow $\frac{3}{4}\times7{\cdot}5$, *i.e.* 5·7 tons per sq. in. in shear, and $1\frac{1}{2}\times7{\cdot}5$, *i.e.* 11·25 tons per sq. in. in bearing.

(iv.) *Diagonal Members.*—The first four of these at either end consist of two 4 in. × 3 in. × $\frac{3}{8}$ in. standard angles, and the intermediate ones of two 3 in. × 3 in. × $\frac{3}{8}$ in. standard angles. We need only examine the end one, which will have to carry the greatest load, and the fifth one from the end.

It is easy to see that the maximum shearing force will occur at one end of the bridge when an 11-ton axle has just got on to the bridge. From *Fig.* 36 we have the maximum shearing force F due to the moving load given by

$$F\times62=59\times32,$$
$$\therefore\ F=30{\cdot}4 \text{ tons.}$$

The maximum shearing force which may have to be carried by one girder

$$=30{\cdot}4\times0{\cdot}62\times\tfrac{5}{4}+8{\cdot}15$$
$$=31{\cdot}8 \text{ tons.}$$

Let P = the maximum thrust in the end diagonal, and θ the angle of inclination of the diagonal to the horizontal, then the maximum end reaction will be equal to $P\sin\theta$,

i.e. $P\sin\theta=31{\cdot}8$,

$$\therefore\ P=\frac{31{\cdot}8\times1{\cdot}8}{1{\cdot}72}=33{\cdot}2 \text{ tons.}$$

The length $=\dfrac{56}{\sin\theta}$

$$=\frac{56\times1{\cdot}8}{1{\cdot}72}$$
$$=58{\cdot}6 \text{ in.}$$

The diagonal is made of two 4 in. $\times$ 3 in. $\times \frac{3}{8}$ in. angles, arranged as shown in *Fig.* 40, (*a*) and (*b*).

Fig. 40 (*a*) shows the single angle. The area of cross-section is 2·485 sq. in. The moments of inertia about axes XX and YY through the centre of gravity are 3·89 in.^{4} and 1·87 in.^{4} respectively.

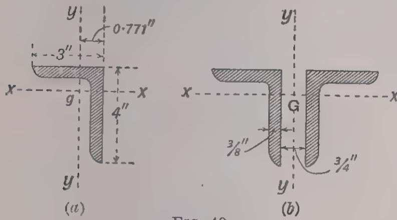


FIG. 40.

For the compound member we shall have for the moments of inertia about the axes XX and YY through the centre of gravity

$$I_{xx} = 2 \times 3{\cdot}89 = 7{\cdot}78 \text{ in.}^4,$$

$$I_{yy} = 2 \times 1{\cdot}87 + 2 \times 2{\cdot}485(0{\cdot}771 + 0{\cdot}375)^2 = 10{\cdot}23 \text{ in.}^4.$$

The least radius of gyration is

$$\sqrt{\frac{3{\cdot}89}{2{\cdot}485}} = 1{\cdot}25 \text{ in.}$$

The slenderness ratio

$$\left(\frac{l}{k}\right) = \frac{58{\cdot}6}{1{\cdot}25} = 46{\cdot}9.$$

Using the formula $(8 - 0{\cdot}035v)$ tons per in. for the working stress, we get the value $(8 - 0{\cdot}035 \times 46{\cdot}9)$, *i.e.* 6·38 tons per sq. in. The actual maximum stress is $33{\cdot}2/2 \times 2{\cdot}485$, *i.e.* 6·7 tons per sq. in.

The actual stress is high, even allowing for the fact that we have taken the length as measured from the centre of the chords, as is commonly done, whereas the actual length is considerably less than this.

We will now examine the fifth diagonal from the left-hand end. With sufficient accuracy we may take the maximum positive shear in this member to occur when a train of tractors is in the position shown in *Fig.* 41, the first load occurring at the joint between the third and fourth panel.

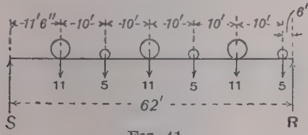


FIG. 41.

The positive shear in the third panel of the two girders is equal to the end reaction S.

$$S = \frac{33 \times 30{\cdot}5 + 15 \times 20{\cdot}5}{62} = 21{\cdot}2 \text{ tons.}$$

Of this, 0·62 S may come on one girder.

Due to the dead weight the positive shear in the first half of the third panel of one girder

$$= 8{\cdot}15 - \frac{126}{12} \times \frac{2}{3} \times \frac{16{\cdot}3}{64} - \frac{100}{12} \times \frac{1}{3} \times \frac{16{\cdot}3}{64} = 5{\cdot}7 \text{ tons.}$$

In this we have made the usual assumption, viz. that one-third of the weight is carried on the top chord and two-thirds on the bottom chord.

The maximum shear in the third panel

$$= 21{\cdot}2 \times 0{\cdot}62 \times \tfrac{5}{4} + 5{\cdot}7 = 22 \text{ tons.}$$

The length of the diagonal

$$= \sqrt{56^2 + 26^2} = 61{\cdot}6 \text{ in.}$$

The thrust in the diagonal

$$= \frac{22 \times 61{\cdot}6}{56} = 24{\cdot}2 \text{ tons.}$$

The diagonal consists of two 3 in. $\times$ 3 in. $\times \frac{3}{8}$ in. standard angles. Total area of cross-section, 4·222 sq. in., and least radius of gyration, 0·9 in.

The allowable stress

$$= 8 - \frac{0{\cdot}35 \times 61{\cdot}6}{9} = 5{\cdot}6 \text{ tons per sq. in.}$$

The actual maximum stress

$$= \frac{24{\cdot}2}{4{\cdot}222} = 5{\cdot}7 \text{ tons per sq. in.}$$

(v.) *Joints.*—In the end diagonal there are five $\frac{7}{8}$-in. rivets at each end.

The bearing pressure

$$= \frac{33{\cdot}2}{5 \times \frac{3}{4} \times \frac{7}{8}} = 10{\cdot}1 \text{ tons per sq. in.}$$

The shearing stress in the rivets

$$= \frac{33{\cdot}2 \times 4}{5 \times 2 \times \pi \times (\frac{7}{8})^2} = 5{\cdot}5 \text{ tons per sq. in.}$$

In the fifth diagonal we have four $\frac{7}{8}$-in. in place of five.

The bearing pressure

$$= \frac{24{\cdot}2}{4 \times \frac{3}{4} \times \frac{7}{8}} = 9{\cdot}2 \text{ tons per sq. in.}$$

The shearing stress in the rivets

$$=\frac{24 \cdot 2\times 4}{4\times 2\times \pi\times (\frac{7}{8})^2}$$

$$=5 \cdot 05 \text{ tons per sq. in.}$$

These are satisfactory, since the allowable bearing pressure is 11¼ tons per sq. in. and the allowable shearing stress 5·7 tons per sq. in. We have assumed the effective area for double shear equal to twice that for single shear. Engineers frequently only take 1¾ in place of 2 in estimating the strength in double shear. There appears no reason for so doing in a case such as this.

We must now examine the joints between the gusset plates and the chords, and we will consider the first from the left-hand end on the bottom chord.

The maximum stress due to the moving load will occur when there is no load to the left of the joint, an 11-ton axle at the joint,

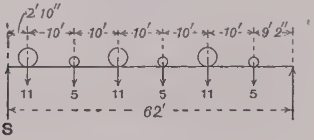


FIG. 42.

and the rest of the span covered by the train of tractors, as shown in *Fig.* 42.

Taking moments about the right-hand end, we have for the bridge

$$S=\frac{33\times 39 \cdot 17+15\times 29 \cdot 17}{62}$$

$$=27 \cdot 8 \text{ tons.}$$

Due to the dead weight of the girder, assuming one-third distributed at the joints of the top boom and two-thirds at the joints of the bottom boom, we have for the shear per girder

$$8 \cdot 15-\frac{1}{3}\times 2 \cdot 83\times\frac{16 \cdot 3}{64}-\frac{2}{3}\times 2 \cdot 83\times\frac{1}{2}\times\frac{16 \cdot 3}{64}$$

$$=7 \cdot 67 \text{ tons.}$$

The total vertical shearing force

$$=0 \cdot 62\times 27 \cdot 8\times \tfrac{5}{4}+7 \cdot 67$$

$$=29 \cdot 3 \text{ tons.}$$

The tension in the diagonal

$$=\frac{29 \cdot 3\times 61 \cdot 6}{56}$$

$$=32 \cdot 2 \text{ tons.}$$

By an exactly similar calculation we find for the third diagonal the maximum thrust is equal to 22 tons.

Resolving these vertically and horizontally we have for the resultant pull on the gusset plate

$$\sqrt{\left\{\frac{(32 \cdot 2+22)\times 26}{61 \cdot 6}\right\}^2+\left\{\frac{(32 \cdot 2-22)\times 56}{61 \cdot 6}\right\}^2}$$

$$=24 \cdot 6 \text{ tons.}$$

There are eleven ⅞-in. rivets connecting the ¾-in. gusset plate to the boom.

The bearing stress

$$=\frac{24 \cdot 6}{11\times\frac{7}{8}\times\frac{3}{4}}$$

$$=3 \cdot 4 \text{ tons per sq. in.}$$

The shearing stress

$$=\frac{24 \cdot 6\times 64\times 4}{11\times\pi\times 49\times 2}$$

$$=1 \cdot 85 \text{ tons per sq. in.}$$

These are considerably less than the allowable stresses and fewer rivets would have sufficed. It can readily be seen that the size of the gusset plate is more than sufficient so far as stress is concerned. It is determined by the space required by the joints at the ends of the diagonals.

(vi.) *Floor System.*—The roadway consists of 9 in. × 4 in. selected pitch pine planks running longitudinally, and supported on 12 in. × 6 in. × 44 lbs. standard steel joists. These rest on the bottom chords of the girders and are spaced 2 ft. 2 in. apart.

The greatest bending moment and shearing force in a joist will occur when a heavy axle is immediately over the joist.

The roadway occupies the middle 10 ft. of the distance between the girders. The distance apart of the wheels, centre to centre, is 68 in., and the breadth of each wheel 16 in.

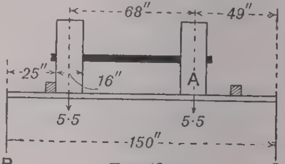


FIG. 43.

Fig. 43 shows the position of tractor wheels for maximum bending moment which occurs under wheel A.

$$\text{Reaction } Q=\frac{5 \cdot 5}{150}(101+33)$$

$$=4 \cdot 92 \text{ tons.}$$

The maximum bending moment, allowing an impact factor of $\frac{5}{4}$,

$$=4 \cdot 92\times\tfrac{5}{4}\times 49$$

$$=301 \text{ in.-tons.}$$

The dead load carried by a joist is its own weight and the weight of 2 ft. 2 in. length of the timber roadway This latter weighs

0·12 ton per foot run, and the joists weigh 44 lbs. per foot length.

Total dead load per joist

$$=\frac{44\times 12{\cdot}5}{2240}+\frac{0{\cdot}12\times 26}{12}$$

$$=0{\cdot}506 \text{ ton.}$$

The bending moment at A, *Fig.* 43, due to this

$$=0{\cdot}253\times 49-\frac{0{\cdot}253\times 49^2}{150}$$

$$=8{\cdot}35 \text{ in.-tons.}$$

The maximum bending moment for the joist = 309 in.-tons.

The moment of inertia of the joist for bending = 315·3 in.^{4}.

Therefore the maximum longitudinal stress

$$=\frac{309\times 6}{315{\cdot}3}$$

$$=5{\cdot}9 \text{ tons per sq. in.}$$

This is well within the limit of 7·5 tons per sq. in., and the joist could carry a heavier axle load.

The maximum shear will be equal to

$$\tfrac{5}{4}\text{P}+\frac{0{\cdot}506}{2}$$

$$=(11-4{\cdot}92)\tfrac{5}{4}+0{\cdot}253$$

$$=7{\cdot}85 \text{ tons.}$$

For this joist the shear may be as high as 20 tons, so it is quite safe to cut away part of the joists at the ends as it is done in this design.

We must now examine the timber decking. Since the breadth of the tractor wheel is 16 in. we see that this may rest on only two 9 in. × 4 in. boards. The maximum bending moment will be produced when a wheel is midway between the steel joists.

The maximum bending moment

$$=\tfrac{1}{4}\times 5{\cdot}5\times\tfrac{5}{4}\times 26$$

$$=44{\cdot}6 \text{ in.-tons.}$$

The fibre stress in the timber

$$=\frac{44{\cdot}6\times 2}{\frac{1}{12}\times 18\times 4^3}$$

$$=0{\cdot}93 \text{ ton per sq. in.}$$

$$=2080 \text{ lbs. per sq. in.}$$

The maximum shearing force in the timber

$$=5{\cdot}5\times\tfrac{5}{4}$$

$$=6{\cdot}88 \text{ tons.}$$

The maximum shearing stress equals one and a half times the mean shearing stress.

Therefore the maximum shearing stress

$$=\frac{3}{2}\times\frac{6{\cdot}88}{18\times 4}$$

$$=0{\cdot}143 \text{ ton per sq. in.}$$

$$=320 \text{ lbs. per sq. in.}$$

The stresses in the timber are high for permanent structures. For pitch pine of the best quality the fibre stress should not exceed some 1800 lbs. per sq. in., and the shearing stress along the grain some 225 lbs. per sq. in. The high stresses were justified in this case, since the timber decking could be easily and quickly repaired if any serious wear occurred, which would reduce the thickness of the decking and thus unduly increase the stresses.

VIII

§ (27) Arches.—In order to estimate the internal stress in arches we require first of all to determine for each normal cross-section the bending moment, the shearing force, and the normal thrust, produced by the externally applied loads. The methods employed to effect this analysis will depend upon the type of arch.

Let V_o and H_o be the vertical and horizontal components of the reaction at the abutment A, *Fig.* 44, and M_o the bending moment at the abutment.

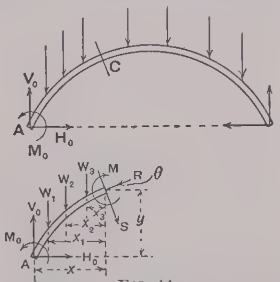


FIG. 44.

Let R, S, M be the normal thrust, the shearing force, and the bending moment produced by the internal stress at section C.

For the equilibrium of the portion AC we have

$$\left.\begin{aligned}&\text{R}\cos\theta-\text{S}\sin\theta-\text{H}_o=0\\&\text{R}\sin\theta+\text{S}\cos\theta+\text{W}_1+\text{W}_2+\text{W}_3-\text{V}_o=0\\&\text{M}-\text{M}_o+\text{V}_o x-\text{W}_1x_1-\text{W}_2x_2\\&\qquad\qquad-\text{W}_3x_3-\text{H}_o y=0\end{aligned}\right\}.\quad(1)$$

Supposing the loads W_1, W_2, etc., known, we see that we have only three equations from which to determine six unknown quantities. In the general case then we shall have to

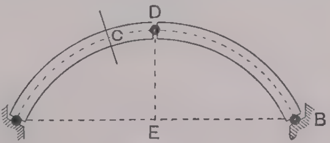


FIG. 45.

obtain three more equations by considering the strains produced.

We will consider three cases: (i.) Arch hinged at the abutments and at the crown; (ii.) Arch hinged only at the abutments; (iii.) Arch with no hinges.

(i.) *Arch hinged at the Abutments and at the Crown.*—The equations for the equilibrium of AC become

$$\left.\begin{aligned} R\cos\theta - S\sin\theta &= H_o \\ R\sin\theta + S\cos\theta &= -F \\ M &= H_o y - M' \end{aligned}\right\} , \quad . \quad . \quad (2)$$

where F and M′ are the shearing force and the bending moment for a straight beam loaded with the same loads as the arch. Considering the equilibrium of the whole arch, and taking moments about B, we can find the value of V_o. By considering the equilibrium of half the arch AD, and taking moments about D, we can find the value of H_o.

We then have the three equations above from which R, S, and M can be found.

For finding M, a graphical solution is usually the simplest. For the arch at D, since $M=o$, we have $M''=H_o \times DE$, where M″ is the bending moment at E of a similarly loaded beam AB.

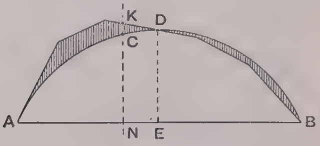


FIG. 46.

Let ADB be the centre line of the arch. On AB draw a bending moment diagram, assuming the loads W_1, W_2, etc., to act on a straight beam AB, and make the scale such that DE represents M″.

At any point C of the arch draw KCN perpendicular to AB; then

$$KN = M', \quad \text{and } CN = H_o y,$$
$$\therefore KC = KN - CN = -M.$$

The vertical distance between the arch and the bending moment diagram for the similarly loaded beam represents the bending moment in the arch.

(ii.) *Deformation of an Arch due to Bending.*—Suppose a small element *ds* of the arch situated at C is bent through an angle *di*. If the position of the arch to the left of the element remained fixed, and the portion to the right of the element rigid, then, due to the bending of *ds* at C, B would be displaced to some position such as B′, where

$$BB' = BC \times di.$$

Draw CN and B′E perpendicular to AB.

The horizontal displacement of B

$$\begin{aligned} &= BE \\ &= BB' \,.\, \sin E\hat{B}'B \\ &= BC \,.\, \sin C\hat{B}N \,.\, di \\ &= y \,.\, di. \end{aligned}$$

The vertical displacement of B

$$\begin{aligned} &= B'E \\ &= BB' \sin E\hat{B}B' \\ &= x \,.\, di. \end{aligned}$$

Now di/ds = the change of curvature at C due to the loading, *i.e.* M/EI, if the radius of curvature is always large compared with the thickness of the arch in the direction of the normal.

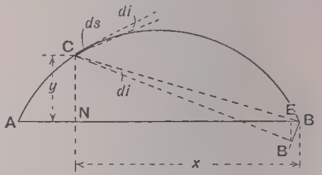


FIG. 47.

If the end B is fixed, there being no rotation or translation there, then, relative to B,

the horizontal displacement of C

$$= -\int_B^C \frac{My}{IE} ds,$$

the vertical displacement of C

$$= -\int_B^C \frac{Mx}{EI} ds,$$

the change of slope of C

$$= -\int_B^C \frac{M}{EI} ds.$$

These are the fundamental equations employed in the analysis of arches.

If the ends are fixed each of the three integrals, taken between the two ends of the arch, will be zero.

In the case of an arch with hinges at the abutments, assuming these do not move

$$\int_0^l \frac{My}{EI} ds = 0,$$

where l is the length of the arch.

Note in this case $\int_0^l \frac{Mx}{EI} ds$ does not give the relative vertical displacement of the two ends, since it does not allow for the rotation of the arch at the end A. Similarly, $\int_0^l \frac{M}{EI} ds$ does not represent the change of slope of the two ends.

(iii.) *Deformation due to Direct Thrust and Shear.*—The previous formulae do not allow either for the deformation produced by the shortening due to the direct thrust R, or for the deformation produced by shear. In practical cases these extra deformations will be quite small compared with the deformation due to bending, and they are usually omitted.

(iv.) *Arch hinged at the Abutments.*—In this case, as in the last, we can determine V_o, but the statical conditions of equilibrium will not enable us to find H_o. We have to make some further assumption. The assumption usually made is that there is no displacement of the abutments due to the application of the loads to the arch. By assuming this, we can determine H_o as follows:

Consider the effect of bending in the arch.

We have

$$\int_0^l \frac{My}{EI} ds = 0,$$

the integration being performed for the whole length l of the arch.

Now $$M = H_o y - M',$$

$$\therefore \int_0^l \frac{(H_o y - M')y}{EI} ds = 0,$$

i.e. $$H_o \int_0^l \frac{y^2 ds}{EI} = \int_0^l \frac{M'y}{EI} ds,$$

or $$H_o = \frac{\int_0^l (M'y/EI) ds}{\int_0^l (y^2/EI) ds}.$$

Usually the integrations cannot be performed directly, since M' and y cannot be expressed as a function of s, but the values of the numerator and denominator may be obtained graphically with sufficient accuracy by dividing the arch into a large number of equal small segments, and by finding the value of $M'y/EI$ and y^2/EI for each segment, M' and y being taken at the middle of the segment.

Having obtained the value of H_o we proceed, as in the case of the three-hinged arch, making use of a graphical construction to determine the bending moment in the arch at each point.

Now $$\frac{ds}{dx} = \sqrt{1 + \left(\frac{dy}{dx}\right)^2},$$

and if dy/dx is small everywhere we may, as a close enough approximation, write ds equal to dx.

(v.) *Arch without Hinges.*—In this case we cannot, as in the previous case, write $(V_o x - \Sigma wx)$ equal to the bending moment in a similarly loaded beam, since V_o may have a different value from that of a similarly loaded beam merely supported at A and B. We may, however, write $V_o = (V_o' + V_o'')$, where V_o'' is the vertical reaction at A of a similarly loaded beam.

Equations (1) become

$$\left.\begin{aligned} &R\cos\theta - S\sin\theta - H_o = 0 \\ &R\sin\theta + S\cos\theta + F - V_o' = 0 \\ &M - M_o + V_o'x + M' - H_o y = 0 \end{aligned}\right\}.$$

From the fundamental equations of the arch, if l = the length, assuming no yielding at the abutments, we have

$$\int_0^l \frac{My}{EI} ds = 0,$$

$$\int_0^l \frac{Mx}{EI} ds = 0,$$

$$\int_0^l \frac{M}{EI} ds = 0,$$

$$M_o \int_0^l \frac{y}{EI} ds - V_o' \int_0^l \frac{xy}{EI} ds - \int_0^l \frac{M'y}{EI} ds + H_o \int_0^l \frac{y^2}{EI} ds = 0,$$

$$M_o \int_0^l \frac{x}{EI} ds - V_o' \int_0^l \frac{x^2}{EI} ds - \int_0^l \frac{M'x}{EI} ds + H_o \int_0^l \frac{xy}{EI} ds = 0,$$

$$M_o \int_0^l \frac{1}{EI} ds - V_o' \int_0^l \frac{x}{EI} ds - \int_0^l \frac{M'}{EI} ds + H_o \int_0^l \frac{y}{EI} ds = 0.$$

From these we may find M_o, V'_o, H_o, and from equations (1) R, S, and M.

Here again we may now construct a bending moment diagram for the external forces and couples, with the exception of H_o, on a similarly loaded beam AB. By choosing suitable scales, the vertical distance between the arch and the bending moment diagram for the similarly loaded beam will give the bending moment in the arch itself.

(vi.) *Effects of Temperature Changes.*—In the case of the arch with hinges only at the abutments, and the arch with no hinges, a change of temperature will affect the stresses.

Let a be the coefficient of linear expansion, t the rise in the temperature, and L the distance between the abutments.

If the abutments were free to expand outwards, due to the increase of temperature, L would increase by atL. If the abutments do not yield, this amount of expansion has to be prevented by an additional horizontal thrust H' at the abutment and, in the case of an arch without hinges, by an additional moment M', and may be an additional force V' at the abutment.

The bending moment (M) at a section of the arch, the centre of gravity of which is at a height y above the abutments, due to the change of temperature, will be given by

$$M = M' + H'y - V'x.$$

For an arch without hinges, assuming no yielding at the abutments, we shall have

$$atL - \int_0^l \frac{My}{EI} ds = 0,$$

$$\int_0^l \frac{Mx}{EI} ds = 0,$$

and

$$\int_0^l \frac{M}{EI} ds = 0.$$

From these three equations we may calculate M', H', and V', and hence the stresses induced by the change in temperature.

If the arch is symmetrical about a vertical line through the centre of the span, V' becomes zero and the second and third equations become the same.

For an arch with hinges only at the abutments the second and third equations do not hold. M' and V' are zero, and M equals $H'y$. The first equation becomes

$$atL - \int_0^l \frac{H'y^2}{EI} ds = 0,$$

or

$$H' = \frac{atL}{\int_0^l (y^2/EI) ds}.$$

As illustrating the application of theory to practice we will take a simple example.

One span of a railway bridge built of circular arched ribs of cast iron is 62 feet in length, with a vertical rise of 8 feet. The ribs are of uniform cross-section, with the dimensions shown in *Fig.* 48, and are hinged at the abutments. After erection it was found that in consequence of uneven settlement the span had increased by 1 in. It was required to find the additional normal stress induced due to this.

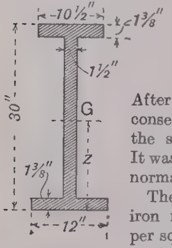


FIG. 48.

The modulus of elasticity for cast iron may be taken as 15×10^6 lbs. per sq. in.

The area of cross-section will be found $= 71 \cdot 7$ sq. in. The centre of gravity is 14·59 in. from the bottom of the section. The moment of inertia of the cross-section about a horizontal axis through the centre of gravity $= 8850$ in.^{4}.

Fig. 49 represents the centre line of the rib.

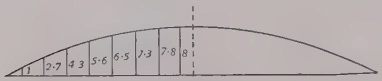


FIG. 49.

Let H = the horizontal thrust at the abutments, and M the bending moment in the arch due to the settlement, then $M = Hy$.

The decrease in span

$$= \int_0^l \frac{My}{EI} ds$$

$$= \frac{H}{EI} \int_0^l y^2 ds.$$

To find this we divide the span into 16 equal intervals and measure the value of y at the middle point of each interval. For the half-span the values of y and y^2 are given in the table below:

Interval . .	1	2	3	4	5	6	7	8
y (feet) . .	1	2·7	4·3	5·6	6·5	7·3	7·8	8
y^2	1	7·3	19·5	31·4	42·3	53·3	61	64

With sufficient accuracy we may take ds to equal dx.

$$\int_0^l y^2 ds = 2(64 + 61 + 53 \cdot 3 + 42 \cdot 3 + 31 \cdot 4 + 19 \cdot 5 + 7 \cdot 3 + 1) \times \frac{62}{16} = 2160 \text{ feet}^3,$$

$$H \times \frac{2160 \times 12^3}{15 \times 10^6 \times 8850} = -1,$$

$$\therefore H = -\frac{15 \times 10^6 \times 8850}{2160 \times 12^3} \text{ lbs.} = -15 \cdot 9 \text{ tons.}$$

The sign is negative, since the increase of span will cause a decrease in the total thrust at the abutments.

The maximum stress due to this will occur at the middle, where the normal pull will equal 15·9 tons and the sagging bending moment will be $15 \cdot 9 \times 8$ ft.-tons.

The normal tension will produce a uniform tensile stress equal to 15·9/71·7 or 0·22 ton per sq. in.

The maximum stress due to the bending will be

$$(1) \text{ Tensile stress} = \frac{15 \cdot 9 \times 8 \times 12 \times 14 \cdot 59}{8850}$$

$$= 2 \cdot 51 \text{ tons per sq. in.}$$

$$(2) \text{ Compressive stress} = \frac{15 \cdot 9 \times 8 \times 12 \times 15 \cdot 41}{8850}$$

$$= 2 \cdot 66 \text{ tons per sq. in.}$$

The settlement produces a compressive stress of 2·44 tons per sq. in. at the top of the rib, and a tension of 2·73 tons per sq. in. at the bottom of the rib.

With the data available we may examine the effect of a change of temperature. We will take the total change of temperature 100° F. The coefficient of linear expansion for cast iron $= 6 \cdot 2 \times 10^{-6}$ per 1° F.

Assuming the abutments do not yield, the abutment thrust has got to neutralise an expansion of $6 \cdot 2 \times 10^{-6} \times 100 \times 62 \times 12$ in., *i.e.* 0·46 in.

The change of stress induced will be about one-half the change due to the settlement.

IX

§ (28) SUSPENSION BRIDGES.—The ordinary simple suspension bridge, with a flexible roadway suspended from the cables, presents no special difficulties, as the stresses in the structures can readily be obtained by the use of pure statics. With such bridges, however, vibration is relatively large, and they are not suitable for heavy moving loads. To attain the necessary stiffness the roadway is usually carried on girders which are suspended from the cables. The girders may be continuous over the whole span, or they may be hinged in the middle.

(i.) *Girders hinged in the Middle and at the Ends.*—In this case it is usual to assume that the cables retain their parabolic shape when the bridge is loaded.

Let H be the horizontal component of the pull in the cables, V the vertical component at A, and T be the pull in the suspension rods. Consider any section KN. For the equilibrium of the portion of the chain AK, by taking moments about K we have

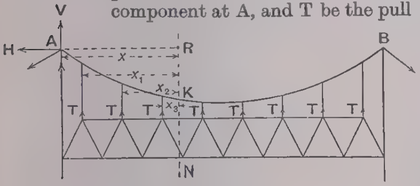


Fig. 50.

$$H \times KR = V \, . \, x - T \, . \, x_1 - T \, . \, x_2 - T \, . \, x_3,$$

$$\therefore \; KR = \frac{V \, . \, x - T \, . \, x_1 - T \, . \, x_2 - T \, . \, x_3}{H}.$$

The numerator of the right-hand side is the bending moment, at distance x from the end, of a straight beam similarly loaded to the cables. Hence the curve of the cables is a bending-moment diagram for a straight beam similarly loaded.

In order to estimate the bending moment in the girders, we draw two bending-moment diagrams, one due to the pulls in the suspending links between the cables and the girders, and the other due to the loads on the bridge, including the weight of the bridge itself. By graphically adding the two diagrams we obtain the bending moment in the girders. The scale to which the cable represents the bending moment diagram for the pulls T_1, T_2, etc., is fixed by the fact that there is no bending moment at the centre.

Let ADB represent the cables, and let ACEHDLB represent the bending moment due to the loads. The pulls in the suspension links will produce a hogging bending moment, whereas the loads will produce a sagging bending moment. The scale of ACEHDLB is adjusted so that it passes through the mid-point D of the cables.

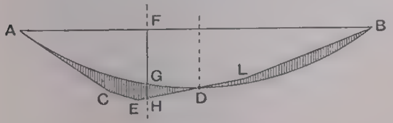


Fig. 51.

At any point F in the girders the bending moment is represented by the vertical distance GH.

The shearing force is easily found, since we know T_1, T_2, T_3, etc., and the loads.

The stresses in the girders may therefore be found.

(ii.) *Girders hinged only at the Ends.*—The full treatment of this case is beyond the scope of this article. It will be readily seen that the proportion of the load carried by the cables and girders respectively will depend upon the stiffness of the girders. One approximate method, which is employed, is to assume that the cables still remain parabolic in shape. The deflection in the cables at the centre is then calculated in terms of the pull in the suspension rods. The deflection of the girders at the middle, due to the applied loads and the pull of the suspension rods, is next calculated. By equating these two deflections the pull in the suspension rods may be found, and hence the external forces for the girders are known.

X. Torsion

§ (29) Circular Shaft. — In considering the stresses induced in a prismatic member by pure twisting couples applied about the axis, we shall deal only with the simplest, but the most commonly required case, viz. that of a circular shaft. In this case we can, with a minimum of mathematics, determine the stresses. Consider a circular shaft fixed at one end and subjected to an axial couple of magnitude T applied at the other end. The couple at the fixed end will also be equal to T. It follows, by symmetry, that plane sections perpendicular to the axis must remain plane. Suppose we view any particle first from one end and then from the other, it is obvious that there is no reason why the particle should be displaced towards one end more than towards the other end. Again it follows that a row of particles on a radius of a cross-section before twisting will also be on a radius after twisting.

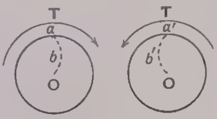


Fig. 52.

View the shaft from the right-hand end, say. We might possibly expect that a row of particles along a radius would be displaced into a position such as *oba* in *Fig.* 52. If we view the same particles from the left-hand end we should, for the same reason, expect them to move into a curve, *ob′a′* in *Fig.* 52. Since there is no reason why the particles should be displaced into a curved path, with the curvature one way rather than the other, we assume that they remain along a radius. This being so, it is obvious that there can be no slide between the surfaces of cylindrical tubes of which we may imagine the shaft built up. If there is no slide there will be no shearing stress, and therefore a shaft built up of perfectly fitting tubes will have the same strength as the solid shaft.

Consider an indefinitely thin tube of radius r, and suppose that all the twist occurs at one end, the other end being fixed.

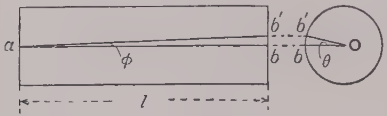


FIG. 53.

Since each elementary slice of the tube is subjected to the same twisting moment, it is obvious that the shearing stress acting horizontally will be the same at all points, and a line ab on the tube parallel to the axis will, after twisting, form a helix ab'. The angle bab' will be the angle of shear (ϕ). Now, remembering that ϕ is small, we get

$$\phi l = bb' = r\,.\,\theta,$$

i.e.

$$\phi = \frac{r\,.\,\theta}{l},$$

where θ is the angle of twist for a length l.

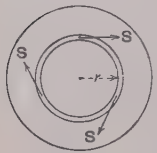


FIG. 54.

To determine the total resisting moment, which has to be equal to the twisting moment T, we divide the cross-section into an infinite number of elementary rings and integrate.

If S is the shearing stress at radius r, we have

$$S = C\,.\,\phi,$$

where C is the modulus of rigidity,

i.e.

$$\frac{S}{r} = C\,.\,\frac{\theta}{l}.$$

If d is the diameter of the shaft,

$$T = \int_0^{\frac{d}{2}} S\,.\,2\pi r^2\,.\,dr$$

$$= C\,.\,\frac{\theta}{l}\int_0^{\frac{d}{2}} 2\pi r^3 dr$$

$$= C\,.\,\frac{\theta}{l}\,.\,\frac{\pi d^4}{32}.$$

Now $\pi d^4/32$ = the polar moment of inertia of the cross-section. Call it J; then we get

$$\frac{T}{J} = C\,.\,\frac{\theta}{l} = \frac{S}{r}.$$

These are the formulae which are usually employed in the design of circular shafts. It will be seen that the formulae apply both to solid and hollow circular shafts.

In designing a shaft to transmit a given axial couple we have to find a suitable diameter such that the maximum shearing stress which occurs on the outside shall not exceed a certain amount, the amount allowable being determined by an experiment on the material to be used.

Suppose a shaft has to transmit H horse-power at a speed of N revolutions per minute. This means a twisting couple T given by

$$2\pi NT = 33{,}000H.$$

If f is the maximum allowable shearing stress,

$$\frac{f}{d/2} = \frac{32T}{\pi d^4},$$

i.e.

$$d^3 = \frac{16 \times 33{,}000 \times H}{2\pi^2 Nf},$$

or

$$d = 68{\cdot}5\sqrt[3]{\frac{H}{Nf}}.$$

In many cases the question of torsional stiffness has also to be considered in addition to the strength. This is measured by the angle of twist per unit length, θ/l.

$$\frac{\theta}{l} = \frac{T}{CJ}.$$

§ (30) HOLLOW SHAFTS.—Bearing in mind the fact that it is only the outside of the solid shaft which can be stressed up to the allowable amount, it is easily seen that economy of material will be effected by making the shaft hollow instead of solid, and thereby raising the mean stress in the material. For shafts to transmit large couples, and where economy of weight is important, this is usually done. It can easily be shown that maximum saving is made when the shaft is of infinite radius and infinitely thin. Practical considerations of space available and increased size of bearing required, etc., fix the outside diameter. Another factor which has to be borne in mind in the case of very thin tubes subjected to torsion is that there will exist in the tube a compressive stress inclined at 45° to the axis and equal in intensity to the shearing stress. This compressive stress may cause instability and wrinkling of the tube.

In the case of shafts subjected to end thrust, and possibly to bending as well as twisting, the method of superposition is employed, and finally the maximum principal stresses and greatest shearing stress are determined.

XI. STRENGTH OF CIRCULAR CYLINDERS AND SPHERES

§ (31) THIN SHELLS.—Consider a thin cylinder which is subjected to a uniform internal fluid pressure p. If the ends are fixed directly to the cylindrical body, and are not otherwise supported, the body of the cylinder is subjected to a circumferential, or hoop, tensile stress, and also to a longitudinal tensile stress. To determine the magnitudes of these stresses, which we may treat as being practically uniform throughout the thickness, we consider a longitudinal section passing through the axis, and also a cross-section.

If t is the thickness, f_1 and f_2 the circumferential and longitudinal stresses respectively, l the length, and d the diameter, for the longitudinal section the total push due to the pressure on one half equals pld. This must be balanced by the circumferential stress over the section of the metal, *i.e.* $2f_1tl$.

$$\therefore f_1 = \frac{pd}{2t}.$$

For the cross-section we have the total pressure on the ends equal to $p(\pi d^2/4)$, and this has to be balanced by the total longitudinal stress equal to πdtf_2.

$$\therefore f_2 = \frac{pd}{4t}.$$

The same method gives for the stress (f) in a thin spherical shell, due to an internal pressure, $f = pd/4t$, where d is the diameter and t the thickness. Steam boilers are usually designed by means of these formulae, due allowance being made for riveted joints.

In the case of the cylindrical shell, in which the pressure on the ends is carried by the cylindrical body, we see that the material at every point is subjected to three principal stresses f_1, f_2, f_3, the third stress being radial in direction. On the inside this stress must obviously be equal to p, the internal pressure, and on the outside it must be zero. Since, in general, p will be small compared with f_1 and f_2, by neglecting the radial stress altogether we are only making an error of the same order as we make by assuming f_1 and f_2 uniform throughout the thickness.

The greatest shearing stress, which is equal to one-half the difference of the greatest and least principal stresses, will be equal to $f_1/2$ In this case we shall obtain the same result whether we design for the greatest principal stress, or whether we design for the greatest shear stress.

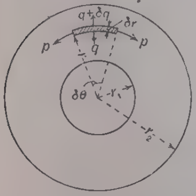


FIG. 55

§ (32) THICK CYLINDER.—When the thickness is not small compared with the radius, the variation of stresses with the radius must be allowed for. Consider a small element of unit length parallel to the axis of the cylinder situated at a radius r, and subtending an angle $d\theta$ at the centre.

Let p and q be the hoop and the radial tensile stresses. For the equilibrium of the element we have

$$pd\theta \,.\, dr = (q+dq)(r+dr)d\theta - q \,.\, rd\theta,$$

i.e.
$$p = r\frac{dq}{dr} + q. \quad . \quad . \quad . \quad (1)$$

We have now to make some assumptions, and firstly we shall assume that plane cross-sections perpendicular to the axis remain plane. This must be the case in long cylinders, except near the ends. Secondly, we will assume that there is no longitudinal stress. We have then the fact that the axial strain must be constant, *i.e.*

$$\frac{p}{m\mathrm{E}} + \frac{q}{m\mathrm{E}} = \text{constant}$$

or
$$p + q = 2\mathrm{A}, \quad . \quad . \quad . \quad (2)$$

where A is a constant for the particular cylinder.

From (1) and (2) we get

$$r\frac{dq}{dr} + 2q = 2\mathrm{A},$$

i.e.
$$\frac{dqr^2}{dr} = 2\mathrm{A}r,$$

or
$$q = \mathrm{A} + \frac{\mathrm{B}}{r^2},$$

where B is another constant.

And from (2) we get

$$p = \mathrm{A} - \frac{\mathrm{B}}{r^2}.$$

The constants A and B are determined by the fact that $q = -p_1$, the internal pressure, for $r = r_1$, and $q = o$ when $r = r_2$. Substituting the values of A and B obtained for these conditions, we get

$$p = p_1\left\{\frac{r_1^2 + (r_2^2r_1^2/r^2)}{r_2^2 - r_1^2}\right\},$$

and
$$q = p_1\left\{\frac{r_1^2 - (r_2^2r_1^2/r^2)}{r_2^2 - r_1^2}\right\}.$$

It is obvious that q is negative everywhere, and therefore p and q are the greatest and least principal stresses

The greatest shearing stress

$$= p_1 \,.\, \frac{r_2^2r_1^2}{r^2(r_2^2 - r_1^2)}.$$

This is a maximum when r is least, *i.e.* at the inside of the cylinder.

Assuming the criterion of elastic breakdown is that the maximum shearing stress must not exceed a certain amount f, say, then

$$f = \frac{p_1r_2^2}{(r_2^2 - r_1^2)}.$$

It is interesting to note that for a given value of f there is a limiting pressure for which a simple cylinder can be designed. This is given by making r_2 equal to infinity, which makes p_1 equal to f.

If it is desired to make a tube for an internal pressure greater than f, the method of shrinking on one tube over another, or winding the tube with wire under tension, is employed. By both these means the inside portions of the tube are initially thrown into a state of stress of the opposite kind from that produced by the internal pressure. Space will not permit of the detailed calculations required in these methods of construction. J. W. L.

STRUCTURES, STRENGTH OF, discussed by the Method of Dimensions. See "Dynamical Similarity, The Principles of," § (40).

STRUTS, FORMULAE FOR STRENGTH OF. See "Structures, Strength of," §§ (15)-(18).

SUCTION TUBES. See "Hydraulics," III. § (48) (iv.).

SULPHUR, BOILING-POINT OF:
- Determined since 1890 by a gas-thermometer and tabulated. See "Temperature, Realisation of Absolute Scale of," § (34), Table 9; "Resistance Thermometers," § (3).
- Used as third fixed point for defining the temperature scale between −40° and +500° C. and therefore studied exhaustively. See "Resistance Thermometers," § (15).

SULPHUR, VARIATION WITH PRESSURE OF THE BOILING-POINT OF. See "Resistance Thermometers," § (5).

SULPHUR BOILING-POINT APPARATUS, specifications of, proposed by Mueller and Burgess, to assist in obtaining standard practice in resistance thermometer calibration. See "Resistance Thermometers," § (16).

SULPHUR DIOXIDE, LATENT HEAT OF VAPORISATION OF, determined by Mathias. See "Latent Heat," § (8).

SUPERSATURATION OF A VAPOUR. See "Thermodynamics," § (39).

SURFACE TENSION, APPLICATION OF DYNAMICAL SIMILARITY TO. See "Dynamical Similarity, The Principles of," § (11).

SURFACE TENSION OF CURVED SURFACES, EFFECT ON CONDENSATION AND EBULLITION. See "Thermodynamics," § (54).

SURGE TANK. See "Hydraulics," § (46) (viii.).

SUSPENSION BRIDGES. See "Structures, Strength of," § (28).

SWISS STATE RAILWAY DYNAMOMETER CAR. See "Dynamometers," § (5) (vii.).

— T —

TACHEOMETERS. See "Meters," §§ (1) to (12), Vol. III.
- Centrifugal Pump and Pressure Gauge Pattern. § (5).
- Electrical: Magneto generator type. § (8).
- Magneto Generator. § (8).
- Methods of Calibration of. § (13).
- Resonance Instruments. § (11).

TANDEM HORIZONTAL GAS-ENGINE. See "Engines, Internal Combustion," § (8).

TEMPERATURE:
- Absolute Constant-pressure Scale of: defined by the relation $T' = 100v/(v_{100} - v_0)$, where v is the volume of a given mass of gas at given pressure at temperature T′, v_{100} and v_0 the volumes of the same mass at the same pressure at the "steam-point" and "ice-point" respectively. See "Temperature, Realisation of Absolute Scale of," § (11).
- Absolute Constant-volume Scale of: defined by the relation $T = 100\,p/(p_{100} - p_0)$, where p is the pressure of a given mass of gas in a given volume at temperature T, p_{100} and p_0 the pressures of the same mass in the same volume at the "steam-point" and "ice-point" respectively. See "Temperature, Realisation of Absolute Scale of," § (10).
- Absolute Gas Scale of, on which the temperature T is defined by the relation $T = pv/R$. See "Temperature, Realisation of Absolute Scale of," § (9); "Thermodynamics," §§ (4), (22).
- Absolute or Kelvin Thermodynamic Scale of: defined generally by the equation $T_1/T_2 = Q_1/Q_2$, where Q_1, Q_2 are the quantities of heat absorbed and heat rejected respectively at temperatures T_1 and T_2 by a perfectly reversible engine working through a Carnot's cycle consisting of two isothermals at temperatures T_1 and T_2 and two adiabatics. See "Thermodynamics," §§ (21), (22); "Temperature, Practical Scale of." Defined also by the two fixed and reproducible temperatures of: (1) The one-component system, water. Pressure: one standard atmosphere. Phases in equilibrium: liquid and crystalline. (2) The one-component system, water. Pressure: one standard atmosphere. Phases in equilibrium: liquid and vapour. The difference between the "ice-point" and the "steam-point," the temperatures of (1) and (2) respectively, is called 100°. See "Temperature, Realisation of Absolute Scale of," § (3).

Centigrade Gas Scales of: scales derived from the absolute gas scales of temperature in the interest of experimental precision and conveniences, since the ice-point has an independent value on each one of the absolute gas scales. The centigrade scales are defined by the relation $t_v = T - T_0$ or $t_p = T' - T_0'$, thus making the ice-point 0° by definition. See "Temperature, Realisation of Absolute Scale of," § (12); "Thermodynamics," § (4).
Centigrade Thermodynamic Scale of, experimental realisation of. See "Temperature, Realisation of Absolute Scale of," § (4).
Coefficient. The ratio of the change per unit of any physical quantity to the change of temperature to which it is due. Thus coefficient of linear expansion = increase in length per unit length per degree.
Cylinders, Pistons, and Valves, in Internal Combustion Engines. See "Engines, Thermodynamics of Internal Combustion," §§ (59) and (60).
Entropy Diagrams for Internal Combustion Engines. See "Engine, Thermodynamics of Internal Combustion," §§ (11) and (38); "Thermodynamics," § (24).
Errors in Measurement of the Mechanical Equivalent. See "Heat, Mechanical Equivalent of," § (3).
Filament, Determination of, by the Colour Match Method in pyrometry. See "Pyrometry, Optical," § (25).
Gas Scales of, Comparison of the Different, for ranges 0° to 100°, −273° to 0°, 100° to 500°, and above 500°. See "Temperature, Realisation of Absolute Scale of," §§ (19), (20).
Gas Scales of, Differences from Centigrade Thermodynamic Scale. See *ibid.* § (20).
Gas Scales of, in Common Use. See *ibid.* § (13).
Hydrogen Scale of. See "Temperature, Practical Scale of," (*a*).
Introduced as a dimension in the discussion, by the method of dynamical similarity, of properties of bodies which depend on temperature. See "Dynamical Similarity, The Principles of," § (27).
Measurement for Strength Tests. See "Elastic Constants, Determination of," § (115).
Measurement of, in Industrial Work, by thermocouples equipped with moving-coil indicators, forming instruments identical in construction with millivoltmeters. See "Thermocouples," § (7).
Measurement of, in terms of Total Intrinsic Brilliancy. See "Pyrometry, Optical," § (26).
Measurement of, in Thermal Conductivity Determinations. See "Heat, Conduction of," § (9) (ii.).
Scales of, tabulated. See "Temperature, Realisation of Absolute Scale of," § (14), Table 1.

TEMPERATURE, PRACTICAL SCALE OF

PRACTICALLY all measurements in heat involve a knowledge of temperature, and to that end the precise scale of temperature is of the first importance.

In the early part of the year 1914 an attempt was made to arrive at international agreement as to the adoption of a standard or fundamental temperature scale by the three national standardising laboratories—viz. the National Physical Laboratory, the Bureau of Standards, Washington, and the Reichsanstalt. The outbreak of war prevented formal acceptance of the Centigrade Thermodynamic Scale as the "*international*" *scale of temperature.* This scale has been adopted in the meantime at the National Physical Laboratory.

Lord Kelvin showed long ago the theoretical advantages of the thermodynamic (or absolute) scale, and also that a perfect gas (*i.e.* one which obeys Boyle's law and suffers no temperature change when subjected to free expansion with no external work) would give a scale identical with the thermodynamic. The practical advantage of the thermodynamic scale is that the high-temperature scale evaluated on the basis of the laws of radiation is consistent with that of the gas-thermometer at lower temperatures.

To promote the general use of the same temperature scale in both scientific and industrial circles, the following alternative methods have been agreed to as a means of attaining a "*Practical Scale*" of temperature which approximates to the thermodynamic scale. A statement of the exact relationship between the two scales is deferred until a sufficient degree of concordance has been reached in the measurements. There is, however, every reason to believe that the Practical Scale over the range 0° C. to 100° C. agrees within the limits of experimental error with the hydrogen scale of the International Bureau of Weights and Measures.

(*a*) *The Hydrogen Scale.*—In the interval between 0° C. and 100° C. the Practical Scale is realised with the exactness required for work of the highest precision in the scale of the constant-volume hydrogen thermometer, having for fixed points the temperature of pure ice melting under normal atmospheric pressure (0° C.) and that of the vapour of distilled water in ebullition under normal atmospheric pressure (100° C.).

(*b*) *The Platinum-resistance Thermometer Scale.*—In the interval between the freezing-point of mercury and the boiling-point of sulphur the Practical Scale is realised with sufficient exactness by the platinum-resistance thermometer standardised at the temperatures of melting ice (0° C.), of the vapour of water boiling under normal atmospheric pressure (100° C.), and of the vapour of sulphur boiling under normal atmospheric pressure in a specified form of apparatus and under specified conditions. The temperature of the vapour under these conditions is to be taken as 444°·5 C. The temperature t on the International Scale is deduced from the resistance of the platinum thermometer by the formula

$$t-t_{pt}=\delta\left[\left(\frac{t}{100}\right)^2-\frac{t}{100}\right]=\delta t(t-100)10^{-4},$$

where $t_{pt}=100\times(\mathrm{R}-\mathrm{R}_0)/(\mathrm{R}_{100}-\mathrm{R}_0)$, and R, R_0, and R_{100} are the observed resistances of the thermometer at temperatures $t°$, 0°, and 100° respectively. The platinum of which the thermometer is made shall be of such a degree of purity that the value of δ in this equation shall not be greater than 1·52, and $\mathrm{R}_{100}/\mathrm{R}_0$ shall not be less than 1·386.

The boiling-point of sulphur t_s at pressure p millimetres is connected with that at standard pressure, 760 millimetres, by the formula

$$t_s=444{\cdot}55+0{\cdot}0908(p-760)-0{\cdot}000047(p-760)^2.$$

A similar practical scale has been adopted by the Bureau of Standards, Washington, and the Reichsanstalt.

(*c*) *The Fixed Point Scale.*—The Practical Scale is also realised with sufficient exactness by the use of the following fixed points, in addition to the three fundamental points above specified :

	Temperature on the Centigrade Thermodynamic Scale.
Boiling-point of Oxygen . .	$-182^\circ{\cdot}9_5\ +0^\circ{\cdot}01258\ (p-760)\ -0^\circ{\cdot}0000079(p-760)^2$
Boiling-point of Carbon Dioxide	$-78^\circ{\cdot}5\ +0^\circ{\cdot}01595\ (p-760)\ -0^\circ{\cdot}0000111(p-760)^2$
Freezing-point of Mercury . .	$-38^\circ{\cdot}88$
Transformation point of Sodium Sulphate .	$32^\circ{\cdot}38_4$
Boiling-point of Naphthalene .	$217^\circ{\cdot}9_6\ +0^\circ{\cdot}058(p-760)$
Boiling-point of Benzophenone (pure) . .	$305^\circ{\cdot}9\ +0^\circ{\cdot}063(p-760)$

Melting or freezing point of—

Antimony	630°
Silver (in a reducing atmosphere) .	961°
Gold	1063°
Copper (in a reducing atmosphere) .	1083°

Fixed points of the second order are provided by the melting or freezing points of—

Tin	$231^\circ{\cdot}8_4$
Cadmium	320°·9
Zinc	419°·4
Common Salt (pure) . . .	801°

According to the official notice of the *Reichsanstalt-Zeits., Instrumentenk.*, 1916, xxxvi. 20, $\mathrm{R}_{100}/\mathrm{R}_0$ must not be less than 1·388 and δ must not be greater than 1·52 when the boiling-point of sulphur is taken as

$$444{\cdot}55+0{\cdot}0908(p-760)-0{\cdot}000047(p-760)^2.$$

TEMPERATURE,[1] REALISATION OF ABSOLUTE SCALE OF

I. Temperature Scales

§ (1) Definition of Temperature.—Notwithstanding the fact that temperature is one of the oldest and most familiar concepts of physics, it is only within comparatively recent times that this concept has been subjected to careful analysis and definition. Kelvin and others have shown [2] how temperature may be defined in terms of the availability of energy, and the definition made wholly independent of any property of any substance.

This definition may be put in the form

$$\frac{\theta_1}{\theta_2}=\frac{\mathrm{Q}_1}{\mathrm{Q}_2},$$

in which θ_1 and θ_2 are two different temperatures between which an ideal engine has operated (as, for example, with a Carnot cycle) in converting the quantity of heat $\mathrm{Q}_1-\mathrm{Q}_2$ into mechanical work.

§ (2) Measurement of Temperature-magnitudes.—It has been suggested (111) that the expression

$$\theta=\frac{d\mathrm{Q}}{d\mathrm{S}},$$

where θ is temperature and $d\mathrm{S}$ is the infinitesimal change in entropy [3] corresponding to the absorption of the heat energy $d\mathrm{Q}$ in a reversible process, be made to define temperature as a *derived* or secondary quantity, in terms of energy and entropy as the fundamental concepts. The usual method, however, and the one we shall follow here, is to consider entropy as a derived quantity, and temperature as one of the *fundamental* concepts.

Temperature differs from energy, length, mass, etc., in being an "intensive" magnitude, whereas energy, length, and mass are "extensive" magnitudes, *i.e.* are additive. To put our energy-definition of temperature into usable form, therefore, it becomes necessary to associate it with a continuous energy-change in some form of matter when that form of matter is subjected to change of temperature from one arbitrary standard value to another arbitrary standard value. The measurement is not influenced by the form of matter chosen.

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[1] The arabic numbers in parenthesis in the text refer to the Bibliography at the end of the article.

[2] For full discussion see the article on "Thermodynamics."

[3] For a definition and discussion of entropy see "Thermodynamics," § (24).

§ (3) ADOPTION OF NUMERICAL UNIT. THE THERMODYNAMIC SCALE.—The first problem is the finding of standard reproducible temperatures for adoption as standard values. A fixed and reproducible temperature is determined only if we have equilibrium[1] among $n+2$ phases in a system of n components; or among $n+1$ phases if the pressure is fixed; or among n phases if the pressure and one concentration are fixed, etc. From the immense number of such possible systems, each having a fixed temperature, there have been selected, by common consent, two systems for use in the definition of the unit of temperature; they are as follows:

(i.) The one-component system, water. Pressure: 1 standard atmosphere. Phases in equilibrium: liquid and crystalline.

(ii.) The one-component system, water. Pressure: 1 standard atmosphere. Phases in equilibrium: liquid and vapour.

The fixed temperature of system (i.) is usually called "the ice-point," and that of system (ii.) "the steam-point."

The *difference* between the temperatures (i.) and (ii.) is, by general agreement, called 100°, and the magnitude of a degree of temperature is thereby defined.

It should be noted at this point that the arbitrary assignment of any number, such as 0°, to temperature (i.) is not within the scope of the definition. The temperature of point (i.) is a "constant of nature" to be experimentally determined. If this temperature be called θ_0, then the temperature (ii.) is θ_0+100.

This scale of temperature may be called the *absolute thermodynamic scale* or the *Kelvin thermodynamic scale.* Temperatures expressed in terms of this scale are frequently indicated by the letter K or the abbreviation *Abs.* Meteorologists use *a.*

§ (4) EXPERIMENTAL REALISATION. THE CENTIGRADE THERMODYNAMIC SCALE.—Having our concept of a scale of temperature thus satisfactorily defined in terms of the laws of energy, and our unit of measurement defined by the adoption of the melting- and boiling-points of water at atmospheric pressure as arbitrary standard temperatures differing by 100° (but without numerical values), we are face to face with the fundamental practical problem of the experimental realisation of the scale. This problem evidently consists in determining (i.) the value of the temperature θ_0; (ii.) the variation with temperature of some property of some substance or object which may then be adopted as a working standard.

The determination of the numerical value of θ_0 is a matter of some experimental difficulty. Fortunately, the progress of exact thermometry need not wait for this determination, for we can define another scale thus:

$$t_\theta=\theta-\theta_0.$$

This derived scale is called the *centigrade thermodynamic scale.* Temperatures thereon are indicated[2] by the letter C. The ice-point becomes on this scale exactly 0 by definition.

The experimental determination of the true value of θ_0 will be considered elsewhere.

The second part of our experimental problem, as has been said, consists in selecting a particular property of a particular substance or object as a working standard, and determining the variation of that property with temperature. Logically, it makes no difference what property or what substance is studied, provided the *quantities of energy* involved are determinable.

§ (5) THE IDEAL GAS-THERMOMETER.—Practically, however, the substance chosen has been, almost of necessity, one of the so-called "permanent gases," and the property chosen has been either the pressure or the specific volume of the gas. The experimental apparatus for measuring this pressure or specific volume constitute what is generally known as the "gas-thermometer."

The reason why we turn to these particular substances and properties is found in the simple relation which holds *very nearly* between the pressure, specific volume, and temperature of a "permanent" gas when the temperature is that already defined (Kelvin scale). This relation is expressed in the equation

$$pv=k\theta,$$

in which p is pressure; v, specific volume; θ, temperature; and k a constant. An "ideal gas" may be defined, for thermometric purposes (15), as one which obeys this law at all values of p, v, and θ.

It is important at this point to recall that our initial measurements with the gas-thermometer tell us nothing about whether the gas in question obeys the law $pv=k\theta$ or not. Only measurements of the energy-relations of the gas can give us that information. But since such measurements involve the measurement of temperature, it is evident that the realisation of the temperature scale is logically a process of successive approximations. Practically, the first approximation is sufficient, so nearly do the gases commonly used in gas-thermometers conform to the "ideal" behaviour expressed in the law $pv=k\theta$.

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[1] See article on "Phase Rule," Vol. IV.

[2] As ordinarily employed the letter C may indicate almost any scale—mercurial, normal hydrogen, constant-pressure nitrogen, etc.—which approximates to the centigrade thermodynamic scale. Its current use is not as precise as it should be.

§ (6) The "Ideal-gas" Thermometer.—Let us suppose that a series of energy and pressure measurements have enabled us to find a gas which, within our working range and within the limits of accuracy of our observations, conforms exactly to the relation $pv=k\theta$. This gas may then be used, within that working range, as the standard substance of our fundamental thermometer, and we shall have what may be called an "ideal-gas thermometer."

§ (7) The "Perfect" Gas-thermometer.—Let us suppose, on the other hand, that no ideal gas has been found as the result of our series of approximations, but that we have succeeded in determining, within our working range, all the deviations of one particular gas from the relation $pv=k\theta$. This gas may then be used, within this range, as the standard substance to define the temperature scale, and our pressure or specific-volume measuring apparatus together with a table of these pressure or volume deviations will constitute what may be called a "perfect gas-thermometer."

§ (8) The Actual Gas-thermometer.—Historically, neither of the above-mentioned thermometers has been available at any given era in a form possessing the desired precision. Either the margin of error within which a gas could be considered "ideal" was too large, or the margin of possible error in defining a temperature with our imaginary "perfect gas-thermometer" was too large, to be in accord with the known precision of various secondary thermometers.

It has been found expedient, therefore, not only to make the first compromise, already mentioned, of adopting a "centigrade thermodynamic scale" (with ice-point at 0°) in place of the "Kelvin thermodynamic scale," but also to make a second compromise with experimental expediency, and *redefine temperature* in terms of an arbitrarily selected property of some arbitrarily selected gas at an arbitrarily selected initial pressure. This compromise is the more satisfactory the nearer the gas in question comes to being an "ideal gas."

This compromise, it may be noted, is of exactly the same character as those made in defining any working standard or unit (for instance, the electrical units) where *experimental precision* is the primary consideration and logical theoretical exactness is made secondary.

§ (9) The Absolute Gas Scales.—Accordingly, we will define a new temperature, T, by means of the relation

$$pv=k'\mathrm{T}, \quad i.e.\ \mathrm{T}=\frac{pv}{k'}.$$

It is obvious that this definition, unlike those that have preceded, involves three further choices, necessarily arbitrary in character and dictated by expediency or experience: (i.) what gas shall be used; (ii.) whether p or v shall be the variable; (iii.) what shall be the initial value of p. Evidently we may have several times as many scales as we have gases.

Certain further assumptions are necessary to give numerical values to any one of these scales; in other words, it is necessary to assign a value to k'. As before (see § (3)), the temperature interval between the ice-point and the steam-point will be called 100°. This will be adopted for all of the gas scales.

§ (10) Absolute Constant-volume Scale.—Let a certain gas be selected, at an initial pressure (*i.e.* pressure at the ice-point) represented by p_0, and let *pressure* be the property to be measured.

Let p_{100} represent the pressure of the same mass of the gas in the same volume at the steam-point. Let p represent its pressure in the same volume at any other temperature T. Call the ice-point and steam-point on the new scale T_0 and T_{100} respectively.

Then, by the definitions above, v and k' being constant,

$$\frac{\mathrm{T}}{\mathrm{T}_0}=\frac{p}{p_0} \text{ and } \frac{\mathrm{T}}{\mathrm{T}_{100}}=\frac{p}{p_{100}},$$

whence

$$\mathrm{T}=(\mathrm{T}_{100}-\mathrm{T}_0)\frac{p}{p_{100}-p_0}=\frac{100p}{p_{100}-p_0}.$$

Another similar "constant-volume" scale may be defined with the same gas, but at some other initial pressure. Still other similar scales may be obtained by substituting other gases.

The scales so defined are called, according to the gas used and according to its initial pressure:

Absolute constant-volume hydrogen scale at initial pressure x.

Absolute constant-volume nitrogen scale at initial pressure x.

Absolute constant-volume air scale at initial pressure x. Etc., etc.

§ (11) Absolute Constant-pressure Scale.—Again, let a certain gas be selected, at an initial pressure (at the ice-point) of p_0, but let *specific volume* be taken as the variable property to be measured.

Let v_{100} represent the specific volume of the gas at the steam-point. Let v represent its specific volume at any other temperature T′. Call the ice-point and the steam-point on the new scale T_0' and T_{100}' respectively.

Then, by the definitions, p and k' being constant,

$$\frac{\mathrm{T}'}{\mathrm{T}_0'}=\frac{v}{v_0} \text{ and } \frac{\mathrm{T}'}{\mathrm{T}_{100}'}=\frac{v}{v_{100}},$$

whence

$$\mathrm{T}'=(\mathrm{T}_{100}'-\mathrm{T}_0')\frac{v}{v_{100}-v_0}=\frac{100v}{v_{100}-v_0}.$$

Similar "constant-pressure" scales may be defined with the same gas at some other pressure, or with other gases.

These scales are called, according to the gas used and according to its pressure :

Absolute constant-pressure hydrogen scale at pressure x.

Absolute constant-pressure nitrogen scale at pressure x.

Absolute constant-pressure air scale at pressure x. Etc., etc.

It will be observed that the value of T_0 (or T_0') is not the same on any two of these scales, being dependent on the properties of the gas. On one of the constant-volume scales, for instance, the value of T_0 is given by $100p_0/(p_{100}-p_0)$; while on a constant-pressure scale the value of T_0' is $100v_0/(v_{100}-v_0)$. The value of T_0 (or T_0') does not depart by more than 1°, in most cases, from 273°.

§ (12) THE CENTIGRADE GAS SCALES.—A sub-compromise similar to that in paragraph (4) is now in order, again in the interest of experimental precision and convenience, since, as we have just seen, T_0 (or T_0'), the ice-point, has an independent value on each one of the absolute gas scales. This compromise consists in defining a derived scale by the relation

$$t_v = T - T_0 \text{ or } t_p = T' - T_0',$$

thus making the ice-point 0° by definition.

The scales so defined are called, according to (1) the variable, (2) the gas, and (3) the initial pressure :

Centigrade constant-volume hydrogen scale at initial pressure x.

Centigrade constant-pressure hydrogen scale at pressure x.

Centigrade constant-volume nitrogen scale at initial pressure x.

Centigrade constant-pressure air scale at pressure x. Etc., etc.

These scales differ among themselves by amounts which are never of importance industrially, but which are approximately determinable and of considerable significance in exact scientific work, as we shall see later.

§ (13) GAS SCALES IN COMMON USE.—The state of international agreement on temperature scales is not so far advanced as is agreement on standards of length, electrical quantities, etc. A conference between representatives of various national standardising laboratories, to draft an international agreement on temperature scales, had been called for September 1914, but was prevented by the outbreak of war.[1]

The only officially recognised international scale is the *centigrade constant-volume hydrogen scale at an initial pressure of* 1000 *mm. mercury*, adopted by the International Committee on Weights and Measures on October 15, 1887, and known as the "normal hydrogen scale" (*échelle thermométrique normale*).

At temperatures below 0° the normal hydrogen scale and the *centigrade constant-volume helium scale at initial pressure of* 1000 *mm.* have been most used. For very low temperatures it becomes necessary to use a constant-volume helium scale at a much lower initial pressure. For temperatures above 100° the scale which has been in widest use is the *constant-volume nitrogen scale at an initial pressure of about* 500 *mm. mercury*.

§ (14) INTERCOMPARISON OF THE GAS SCALES.—There are two ways of comparing the different gas scales with each other, and determining the corrections to be used in translating experimental data from one scale to another.

One way would be to put the two gas-thermometers side by side into a thermostat at uniform temperature and compare their readings, setting the thermostat at various temperatures. Suppose a constant-volume hydrogen thermometer at initial pressure 1000 mm. placed in a thermostat side by side with a constant-volume nitrogen thermometer at initial pressure 500 mm. The pressures might read, say 1183·132 mm. and 591·706 mm. respectively. The temperature of the thermostat is therefore defined by the first thermometer as 50·0012° and by the second as 50·0027°. The correction for translating data from the first scale to the second would be, therefore, +0·0015° at 50°.

This method of direct comparison has been used more frequently at temperatures below 0° than at temperatures above 0°.

The second method is indirect. It consists in determining the departure of each gas, under the given conditions, from the ideal gas relation $pv=k\theta$. It is obvious that from such a set of data the differences between the centigrade-thermodynamic temperature of the thermostat, t_θ, and its various centigrade gas-scale temperatures, $t_{H,\,v,\,1000}$, $t_{H,\,p,\,760}$, $t_{N,\,v,\,500}$, etc., could readily be tabulated, and the difference between any two of the gas scales could be found by subtraction of their centigrade thermodynamic corrections.

This is the method of comparison upon which dependence is now very generally placed, but the data thus far published are in many cases unduly divergent, and afford little ground for confidence that a final solution is within reach until considerably more experimental work has been done.

Intercomparison of the gas scales will be more fully considered in Part III. § (17).

For convenience of reference there is added, in Table 1, a list of the more commonly used scales.

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[1] See "Temperature, Practical Scale of."

II. Forms of Gas-thermometer

§ (15) We have seen that either pressure or specific volume of a given gas may be taken as the property to be used in defining a temperature scale. Three limiting forms of the gas-thermometer for carrying out such measurements may be distinguished, depending upon which of the three quantities—mass, volume, pressure—is permitted to vary. These forms are:

(i.) The thermometer with fixed mass of gas, fixed volume, and variable pressure; commonly known as the "constant-volume gas-thermometer."

Table 1

Examples of Temperature Scales

	Abbreviation.	Symbol.	Ice-point.
Absolute thermodynamic scale or Kelvin thermodynamic scale	K	θ	273·10 ± 0·05 *
Centigrade thermodynamic scale	C	t_θ	0 †
Absolute constant-volume hydrogen scale at initial pressure 1000 mm.	..	$T_{H, 1000}$, T_H or T	273·03 ± 0·05 *
Absolute constant-volume helium scale at initial pressure 1000 mm.	..	$T_{He, 1000}$ or T_{He}	273·11 ± 0·03 *
Absolute constant-pressure nitrogen scale at pressure 1000 mm.	..	$T'_{N, 1000}$	272·24 ± 0·05 *
Absolute constant-volume nitrogen scale at initial pressure 500 mm.	..	$T_{N, 500}$ or T_N	272·63 ± 0·05 *
Centigrade constant-volume hydrogen scale at initial pressure 1000 mm., or "normal hydrogen scale"	..	$t_{H, v, 1000}$; t_H or t	0 †
Centigrade constant-volume nitrogen scale at initial pressure 500 mm.	..	$t_{N, v, 500}$	0 †
Centigrade constant-pressure nitrogen scale at pressure 1000 mm.	..	$t_{N, p, 1000}$	0 †
Centigrade constant-volume helium scale at initial pressure 1000 mm. (proposed "international helium scale")	..	$t_{He, v, 1000}$ or t_{He}	0 †

* Experimentally determined. † Defined.

(ii.) The thermometer with fixed pressure and fixed volume, but variable mass of gas; commonly known as the "constant-pressure gas-thermometer."

(iii.) The thermometer with fixed mass of gas, fixed pressure, and variable volume.

The property measured in type (i.) is the pressure of the gas; in types (ii.) and (iii.) the specific volume of the gas. As already mentioned, slightly different scales result from using the gas at different initial pressures.

For reasons of experimental convenience type (iii.) has never been used, because of the difficulty of measuring a variable volume at temperatures considerably different from atmospheric.

The essential parts of a gas-thermometer of type (i.) are: (*a*) the containing vessel or "bulb"; (*b*) the manometer; (*c*) the tube connecting the bulb and the manometer, frequently called the "dead space" or "unheated space." [1]

The essential parts of a gas-thermometer of type (ii.) are: (*a*) the bulb; (*b*) the manometer; (*c*) the arrangements for measuring the amount of gas withdrawn from the bulb.

Any actual gas-thermometer is to a certain degree a combination of two or more of the limiting types. The volume of the containing vessel is never constant with changing temperature, but varies on account of thermal and elastic expansion. The mass of gas which is at the temperature θ is not always constant, for changing pressure forces some of it into or out of the tube that connects the containing vessel with the manometer. The pressure is never constant with changing temperature, on account of the necessity of adjustment, and must always be measured to ascertain its deviations from constancy. All these variations, however, are of the nature of corrections seldom exceeding 1 per cent, so that it remains true that any actual gas-thermometer conforms fairly closely to one or other of the limiting types.

§ (16) Relative Advantages of the two Principal Types.—If we were dealing with an ideal gas there would be no theoretical advantage in any one of these three limiting types of gas-thermometer over the other two, and the choice would become solely a matter of experimental convenience and precision.

From this standpoint type (i.), the "constant-volume thermometer," is generally admitted to have the advantage, and has been preferred by most investigators. Its manipulation is simple, requiring no transfer of gas or measurement of the amount transferred.

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[1] "Espace nuisible,' "schädlicher Raum."

In dealing with a gas which is not "ideal" we have to consider not only experimental convenience and precision in the gas-thermometer itself, but the state of the science as regards the accuracy of our knowledge of the deviations of the gas from the law $pv=k\theta$, for our goal is first the centigrade thermodynamic scale and ultimately the Kelvin thermodynamic scale, and our compromise on the various centigrade gas scales for reasons of experimental precision is to be regarded as only a passing stage in thermometric progress. From this standpoint type (ii.), the "constant-pressure thermometer," has been shown by Buckingham (14) to have a slight advantage at the moment, for the corrections necessary to translate readings on any of the constant-pressure gas scales into temperatures on the centigrade thermodynamic scale are a little more accurately known than the corresponding corrections for the constant-volume scales.

III. Basic Experimental Results with Gas-thermometers

§ (17) Experimental Ranges.—In dealing with the results of experimental investigation in gas thermometry it will frequently be convenient to recognise that the subject naturally divides itself, both historically and practically, according to the kind and extent of experimental facilities and experience necessary to realise different portions of the temperature scale. The most convenient range for experimental work is that from 0° to 100° C., and this range was the first to be studied in detail. From 0° downward a special kind of technique and experience is required, while from 100° upward a rather different, but again special, kind of technique and experience has had to be developed. The range above 100° may again be divided, on similar grounds, into the range 100°-500°, within which most of the ordinary materials of construction are available for experimental work; and the range above 500°, where available construction materials begin, one by one, to fail as the temperature rises, either through oxidation, mechanical weakness, or actual fusion.

§ (18) Pressure and Volume Coefficients of Gases.—As we have seen (§ (12)), the centigrade gas scales are defined in such a way that the temperature 0° is arbitrarily assigned to the ice-point under one atmosphere pressure, whence the temperature 100° becomes assigned to the steam-point under one atmosphere, by virtue of the definition of the unit of measurement in § (3).

These two fixed temperatures therefore become fundamental reference points for any type of gas thermometer, and all of its temperature indications are based upon measurements of pressure or volume at these two points.

We have seen that for each of the absolute constant-volume gas scales there is a value of T_0, the ice-point, given by the relation

$$T_0=\frac{100p_0}{p_{100}-p_0};$$

also that any other temperature, T, is defined by

$$T=T_0\frac{p}{p_0},$$

and that the corresponding centigrade temperature is defined by

$$t_v=T-T_0.$$

The constant $(p_{100}-p_0)/100p_0=1/T_0$ is called the "pressure-coefficient" of the gas at the initial pressure stated, and is usually represented by the symbol β.

Correspondingly, for each of the absolute "constant-pressure" gas scales there is a value of T_0', the ice-point, given by the relation

$$T_0'=\frac{100v_0}{v_{100}-v_0};$$

also, any other temperature, T′, is defined by

$$T'=T_0'\frac{v}{v_0},$$

and the corresponding centigrade temperature is defined by

$$t_p=T'-T_0'.$$

The constant $(v_{100}-v_0)/100v_0=1/T_0'$ is called the "volume-coefficient" of the gas at the pressure stated, and is represented by the symbol α.

Obviously, although it is necessary in a given gas-thermometer to measure p_0 or v_0, it is not necessary to measure with it β or α, since these are constants of the *gas* and not of the thermometer.[1]

Measurements of β and α for particular gases are listed in Tables 2 and 3.

§ (19) Comparison of the Different Gas Scales. Direct Method.—Since necessity or convenience has brought about the use of several different gas scales, it becomes of importance to know the corrections by which readings in any one of these can be reduced to a common standard, such as the normal hydrogen scale.

The direct method of determining these corrections, as has been said, is to put the two thermometers side by side into a thermostat, and compare the temperatures as defined by each of the two thermometers; or to measure a given temperature (controlled by a secondary thermometer or reproduced by a melting-point, for example) first with the one gas-thermometer and then with the other.

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[1] For a fuller discussion the reader is referred to the article on "Thermodynamics."

TABLE 2

PRESSURE-COEFFICIENTS OF VARIOUS THERMOMETRIC GASES. VALUES OF $10^6\beta$

$p_0=1000$ mm.	760 mm.	500 mm.	Author, Date, and References.
Hydrogen.			
3662·5	..	..	Chappuis (21), 1888.
3662·7	..	..	Onnes and Boudin (88), 1900.
..	3662·6	3662·7	Travers and Jaquerod (114), 1903
3663·0	..	..	Chappuis (22), 1907.
3662·2	..	..	
..	..	3661·8	Holborn and Henning (64), 1911.
Helium.			
..	3662·6	3662·3	Travers and Jaquerod (114), 1903.
3661·3	..	..	Onnes (87), 1907.
3661·6	..	..	
..	..	3661·8	Holborn and Henning (64), 1911.
Nitrogen.			
3674·7	..	..	Chappuis (21), 1888.
..	3671·4	3668·1	Chappuis and Harker (24), 1902.
..	..	3667·8	
3673	3670	3667·5	Day and Clement (34), 1908.
..	3671·2	3667·9	Holborn and Henning (64), 1911.
..	3670·2	3666·4	
..	..	3668·5	Day and Sosman (35), 1912.
..	..	3669·1	Chappuis (23), 1917.
Air.			
3674·4	..	..	Chappuis (22), 1907.
Oxygen.			
..	..	3671·7	Makower and Noble (80), 1903
Carbon Dioxide.			
3724·8	..	..	Chappuis (21), 1888.
3726·3	3712·3	3697·1	„ (22), 1907.

TABLE 3

VOLUME-COEFFICIENTS OF VARIOUS THERMOMETRIC GASES. VALUES OF $10^6\alpha$

$p=1000$ mm.	760 mm.	500 mm.	Author.
Hydrogen.			
3660·0	..	..	Chappuis (22), 1907.
Nitrogen.			
3673·1	..	..	Chappuis (22), 1907.
3673·2	..	..	
..	3669·8	3666·8	Eumorfopoulos (48), 1914.
Air.			
3672·8	..	..	Chappuis (22), 1907.
..	3670·8	..	Eumorfopoulos (47), 1908.
Carbon Dioxide.			
3741·0	3723·7	3706·1	Chappuis (22), 1907.

(i.) *Range* 0° *to* 100°.—In the range 0°-100° Chappuis (21) made in 1888 a direct comparison of the kind indicated, between constant-volume hydrogen, nitrogen, and carbon dioxide gas-thermometers. In 1907 (22) he extended the comparison to constant-pressure thermometers of the same three gases. He found the following differences (shown in Table 4) between the normal hydrogen scale (t_H) and the other scales.

TABLE 4

VALUES OF $t_H - t$ ACCORDING TO CHAPPUIS' COMPARISONS

	At 20°.	At 40°.
$t_H - t_{N, v, 1000}$	−0·010° (1888) −0·005 (1907)	−0·011° (1888) −0·008 (1907)
$t_H - t_{N, p, 1000}$	−0·021	−0·020
$t_H - t_{H, p, 1000}$	−0·003 0·000 *	+0·004 −0·001 *
$t_H - t_{CO_2, v, 500}$	+0·002	+0·010
$t_H - t_{CO_2, v, 1000}$	−0·039	−0·039
$t_H - t_{CO_2, p, 500}$	−0·027	−0·026
$t_H - t_{CO_2, p, 1000}$	−0·102	−0·133

* Compared with 1888 results at constant-volume through mercury thermometers.

Since the limit of reproducibility of Chappuis' results was about 0·005°, differences of less than that amount are of little significance. To this degree of exactness, therefore, the constant-volume and constant-pressure hydrogen scales are seen to be in exact agreement at 20° and 40°, while the other gas scales show measurable deviations from the normal hydrogen scale at those temperatures.

(ii.) *Range* −273° *to* 0°.—Olszewski (86) in 1896 compared constant-volume hydrogen and helium thermometers at −183° to −211°, and found that they agreed within 0·1° in that range.

In the same year Holborn and Wien (68) compared constant-volume hydrogen and air thermometers, and found that the air thermometer indicated a temperature 0·5° to 0·7° lower than the hydrogen at liquid air temperatures. Holborn confirmed this observation in 1901, finding a difference of 0·50° ± 0·06° between nitrogen and hydrogen thermometers at initial pressure 1000 mm.

Travers and Fox (112) observed a large effect of the value of the initial pressure on the deviations of the constant-volume oxygen thermometer, which reads lower than helium by an amount which varies with p_0, and is of the order of magnitude of 0·4° to 1·5°.

Travers, Senter, and Jaquerod (114) in 1903 compared constant-volume hydrogen and helium thermometers at initial pressures of 845-975 mm., through the intermediary measurement of the vapour pressures of oxygen and hydrogen. They found the hydrogen temperatures to be 0·10° lower than the helium in the neighbourhood of −190°, and 0·19° lower at −253°.

Kamerlingh Onnes and Holst (99) also made a careful comparison between the constant-volume hydrogen and helium thermometers, using a platinum-resistance control, at temperatures from 0° to −258°. The differences between the two did not exceed 0·06° down to −100°, and they may be considered as defining the same temperature within that range to an accuracy of 0·02°. At −258° the hydrogen thermometer yielded a temperature 0·12° to 0·15° lower than the helium.

(iii.) *Range* 100° *to* 500°.—Holborn and Henning's investigations (64) in 1911 constitute a comparison of the constant-volume hydrogen, helium, and nitrogen scales at initial pressures of 620-630 mm., using two platinum-resistance thermometers as intermediaries. In terms of one of these two the hydrogen and helium scales agreed exactly, to 0·01°, at 450°; in terms of the other, the helium-scale temperature was 0·07° lower than the hydrogen. The nitrogen scale was 0·11° to 0·12° lower than the hydrogen at 450°. The results indicate that the hydrogen and helium scales are in agreement at 450° within 0·05° or less, while the constant-volume nitrogen scale at initial pressure of about 600 mm. requires a correction of about 0·1° to convert its readings to the normal hydrogen scale.

Eumorfopoulos' determinations of the boiling-point of sulphur (48) in 1914, likewise, constitute a comparison of the constant-pressure nitrogen scale at pressure 792 mm. with the similar scale at 415 mm. The difference between the two was found to be 0·23° at 444·5°. Hence, on the fairly exact assumption that the correction is proportional to the pressure, the correction to convert the constant-pressure nitrogen scale at 1000 mm. to the thermodynamic scale is +0·60° ± 0·05°.

(iv.) *Range above* 500°.—The only exact comparison of gas-thermometers in this range is that by Jaquerod and Perrot (72). Using a bulb of fused silica, they determined the melting-point of gold on the constant-volume nitrogen, oxygen, air, carbon dioxide, and carbon monoxide scales at initial pressures of 182-252 mm.

Although some of their errors, particularly that due to non-uniformity of temperature, were abnormally large, nevertheless the fact that all of the gases were used under identical conditions makes the work a fairly reliable comparison between scales, even though their absolute value for the melting-point of gold is open to question.

Expressed in terms of corrections to convert readings on one of the gas scales to readings on the constant-volume nitrogen scale at initial pressure of about 250 mm., their results were :

$$t_{\mathrm{N},\,v,\,250} - t_{\mathrm{air},\,v,\,250} = 0{\cdot}0^\circ,$$
$$t_{\mathrm{N},\,v,\,250} - t_{\mathrm{CO_2},\,v,\,250} = +0{\cdot}9^\circ,$$
$$t_{\mathrm{N},\,v,\,250} - t_{\mathrm{O},\,v,\,250} = +0{\cdot}4^\circ,$$
$$t_{\mathrm{N},\,v,\,250} - t_{\mathrm{CO},\,v,\,250} = +0{\cdot}15^\circ.$$

§ (20) COMPARISON OF GAS SCALES BY THE INDIRECT METHOD, AND DIFFERENCES BETWEEN GAS SCALES AND THE CENTIGRADE THERMODYNAMIC SCALE.—The indirect method of comparison consists, as has been said, in calculating from existing energy-data of the thermometric gases the corrections necessary to reduce temperatures on one of the centigrade gas scales to temperatures on another centigrade gas scale or on the centigrade thermodynamic scale. Tables of such corrections have been compiled by Rose-Innes (109), Callendar (17), D. Berthelot (13), and Buckingham (14). Kamerlingh Onnes (87) and Braak (89, 90, 91) calculated the corrections to the constant-volume helium and hydrogen scales at temperatures below 0°. Hoxton (69) calculated corrections for the constant-volume air scale from his experiments on the Joule-Thompson effect. Keyes (75) recomputed the nitrogen corrections on the basis of his new equation of state, which receives considerable support from experimental data.

TABLE 5

COMPARISON OF GAS-SCALE CORRECTIONS AT 50°, IN THOUSANDTHS OF A DEGREE

	Rose-Innes.	Callendar (I.).	Callendar (II.).	Berthelot.	Buckingham (I.).	Buckingham (II.).	Onnes and Braak.	Hoxton.	Keyes.
$t_\theta - t_{\mathrm{H},\,v,\,1000}$	..	− 1·3	− 0·6	− 0·5	− 3·2	..	−2·0	..	..
$t_\theta - t_{\mathrm{N},\,v,\,1000}$	..	..	− 5·9	− 8·6	− 1·4	..	..	..	..
$t_\theta - t_{\mathrm{H},\,p,\,1000}$	− 3·3	− 3·5	− 1·8	− 1·1	− 3·9	−3·4	..	..	..
$t_\theta - t_{\mathrm{N},\,p,\,1000}$	−28·9	−22·6*	−26·0	−24 0	−20·9	..	..	−23·8	−18·0

* Calculated for air.

Limitations of space forbid any extended discussion in this article of the derivation of these corrections.[1] We will confine our attention to a brief indication of the present status of our knowledge of these corrections in relation to experimental gas-thermometry.

(i.) *Range* 0° *to* 100°.—The scales all coincide, by definition, at 0° and 100°. Some idea of the state of our knowledge of the inter-scale corrections in the intermediate range may be gained from Table 5, which compares the differences between centigrade thermodynamic temperature, t_θ, and gas-scale temperatures, calculated for the temperature 50° by some of the authors quoted.

It will be seen that the *normal hydrogen* scale, between 0° and 100°, is experimentally very nearly indistinguishable from the centigrade thermodynamic scale. To a lesser degree of certainty the same is true of the *constant-pressure hydrogen* scale from 0° to 100°, as is evident from both Table 4 and Table 5. The corrections for the *constant-pressure nitrogen* scale in this range are in fairly satisfactory agreement with Chappuis' directly determined differences (see Table 4). The *constant-volume nitrogen* correction is more uncertain ; Buckingham's figure is much smaller than the others and smaller than the results of direct comparison ; on the other hand, according to Keyes' equation, the constant-volume nitrogen correction is zero.

(ii.) *Range* −273° *to* 0°.—Table 6 gives a comparison, similar to that in a preceding paragraph, at the temperature −100°.

The agreement among the different estimates at −100° is very poor. Onnes and Braak's value for the *normal hydrogen* scale is undoubtedly the best, being based on experimentally determined isotherms at low temperatures. Onnes's data on the *constant-volume helium* scale show that it coincides with the thermodynamic scale within 0·01° at −100°, and has a positive correction of not over 0·03° at −250°. The differences between hydrogen and helium are satisfactorily checked by the direct comparisons (see an earlier paragraph).

(iii.) *Range* 100° *to* 500°.—As before, the estimates of various authors are compared at a single temperature, in this case 400°. (See Table 7.) From Table 7 it is evident that we know very little more of the magnitude of the *constant-volume* hydrogen and nitrogen corrections than we did seventy years ago, when Regnault concluded from his comparisons that the constant-volume air scale needed no correction. Even Holborn and Henning's direct comparison (see a preceding paragraph) leaves the matter somewhat in doubt.

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[1] For fuller discussion see the article on "Thermodynamics."

TABLE 6

COMPARISON OF GAS-SCALE CORRECTIONS AT − 100°, IN THOUSANDTHS OF A DEGREE

	Callendar (I.).	Callendar (II.).	Berthelot.	Buckingham (I.).	Buckingham (II.).	Onnes.
$t_\theta - t_{\mathrm{H},\,v,\,1000}$	+20	+ 5	+ 8	+ 32	..	+18·7
$t_\theta - t_{\mathrm{N},\,v,\,1000}$	..	+ 80	+125	− 59	..	..
$t_\theta - t_{\mathrm{H},\,p,\,1000}$	+66	+ 29	+ 21	+ 46	..	..
$t_\theta - t_{\mathrm{N},\,p,\,1000}$	..	+437	+469	+326	+348	..
$t_\theta - t_{\mathrm{He},\,v,\,1000}$	..	..	..	..	..	+ 3·4 − 6·0

The constant-pressure hydrogen scale is subject to a positive correction of about 0·1°, while the *constant-pressure nitrogen* correction is relatively the best known of all, amounting to about +0·45° at 400° for a thermometer at pressure 1000 mm. Eumorfopoulos' (48) direct determination of this correction as 0·60 ± 0·05° at 450° furnishes confirmation that the estimates are correct within 0·1°.

TABLE 7

COMPARISON OF GAS-SCALE CORRECTIONS AT 400°, IN THOUSANDTHS OF A DEGREE

	Callendar (I.).	Callendar (II.).	Berthelot.	Buckingham (I.).	Buckingham (II.).	Keyes.
$t_\theta - t_{\mathrm{H},\,v,\,1000}$	..	..	+ 12	+ 99	..	..
$t_\theta - t_{\mathrm{N},\,v,\,1000}$	..	..	+194	+139	..	0
$t_\theta - t_{\mathrm{H},\,p,\,1000}$	+68	+38	+ 21	+110	+ 68	..
$t_\theta - t_{\mathrm{N},\,p,\,1000}$	..	+550	+457	+433	+456	+436

(iv.) *Range above* 500°.—In this range no one has ventured to calculate the differences between the scales for any except the *constant-volume* and *constant-pressure nitrogen* thermometers at initial pressure 1000 mm. These are shown in Table 8.

TABLE 8

COMPARISON OF GAS-SCALE CORRECTIONS AT 1000°

	Callendar (II.).	Berthelot.	Buckingham (I.).	Buckingham (II.).	Keyes.
$t_\theta - t_{\mathrm{N},\,v,\,1000}$	+0·646°	+0·77°	+0·734°	..	0
$t_\theta - t_{\mathrm{N},\,p,\,1000}$	+2·047	+1·654	+1·706	+1·721	+1·72

In this range, as in the range 100°-500°, the constant-pressure nitrogen corrections are much better known than the constant-volume corrections. Jaquerod and Perrot's direct comparisons (see a preceding paragraph) are of no assistance as regards corrections to the thermodynamic scale.

§ (21) DETERMINATION OF θ_0.—The numerical value for θ_0, the ice-point on the Kelvin thermodynamic scale, is one of the basic data of thermometry. While its determination is a fundamental thermometric problem, and forms logically a part of the present article, the problem will be found fully discussed elsewhere in this dictionary.[1] The most reliable estimates vary from 273·08° to 273·14° and it may be conservatively placed at the value 273·1° ± 0·05°.

§ (22) SUMMARY FOR SECTION III.—*Centigrade thermodynamic* temperatures can now be measured with several different types of gas-thermometer, with an accuracy which is conservatively estimated to be as follows:

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[1] See article on "Thermodynamics."

At −250°	0·05°	At 200°	0·05°
At −100°	0·02°	At 400°	0·1°
At 0°	0·003°	At 1000°	1°
At 50°	0·006°	At 1500°	2°
At 100°	0·01°		

IV. Means of Comparison of Gas-thermometers with Secondary Standards

§ (23) Secondary Standards.—A gas-thermometer occupies so much space that it is seldom convenient for use in the direct measurement of temperature. For this reason the experimental realisation of the temperature scale and its translation into terms and measures of convenient application requires a further step, namely, the comparison of gas-thermometers with secondary standards available for general use.

Secondary standards may be classed as of three kinds:

(i.) Reproducible fixed temperatures (melting-points, boiling-points, etc.).

(ii.) Independent thermal properties of substances (vapour-pressure, index of refraction, etc.).

(iii.) Combination of fixed temperatures with a thermal property (resistance thermometer combined with ice, steam, and sulphur points, etc.).

The expression "thermal property" is used here in a broad sense to include any property which varies with temperature and at the same time can be measured with precision.

§ (24) Fixed Temperatures.—In accordance with the Phase Rule,[1] the temperature of a system is fixed if the pressure and the necessary number of concentrations are specified so that in a system of n components

$$P+F=n+2,$$

where P is the number of phases present and F the number of variables that have been specified.

For instance, take a one-component system such as pure water; the specifications that the pressure shall be one atmosphere and that the two phases, liquid water and ice, shall be present, determines that this system shall have a fixed and reproducible temperature. Or take the 2-component system—water and sodium chloride; the specifications that ice and solution shall be present, that the pressure shall be one atmosphere, and that the solution shall have a concentration of 10 parts salt to 90 parts water, again fixes a reproducible temperature. So also does the specification of the three phases, ice, salt, and solution, and a pressure of one atmosphere; or the specification of the three phases, salt, solution, and vapour, and a pressure of one atmosphere (boiling-point of the saturated solution).

The temperature of any such system may be measured with a gas-thermometer, and this fixed-temperature system then becomes available either for the direct indication of temperature or for the standardisation of any kind of secondary thermometer.

Customary practice in gas-thermometric work has quite needlessly restricted these fixed-temperature points to one-component systems—to the melting- and boiling-points of pure substances, usually elements.

The gas-thermometer need not be directly immersed in the fixed-temperature system. Many kinds of transferring devices may be used, such, for example, as a thermo-element. The thermo-element is immersed in the fixed-temperature system, say a melting metal, and its E.M.F. is read. Thermo-element and gas-thermometer are then placed side by side in a thermostat whose temperature is regulated until the thermo-element again gives the same E.M.F. A gas-thermometer reading then gives the desired temperature of the fixed point. There need be no sacrifice of accuracy in making the comparison in this way, for the sensitiveness of the thermo-element can easily be made equal to or greater than that of the gas-thermometer by multiplying junctions.

There are two considerations, either of which may dictate the choice of an indirect or transfer method, such as that described above, rather than the direct immersion method. In the first place, uniformity of temperature must be secured over the entire volume of the gas-thermometer bulb, and this frequently calls for the use of a secondary or auxiliary thermometric device of small volume, with which to explore the distribution of temperature. Chappuis' bulb, for example, was over a metre long. In a comparison of this character in which the dimensions of the two measuring instruments differ so extremely a considerable burden is placed on the thermostat to guarantee the constancy of temperature over so large an area with the full precision expected.

Or, to view the problem from a somewhat different angle, the thermostat may fulfil all expectations, but the metal bath or other fixed temperature system be too small or too uncertain in its performance to ensure the expected constancy of temperature over the entire volume of a large gas-thermometer bulb. In this case obviously the transfer method will yield better results than the direct method.

§ (25) Thermal Properties.—Any property of a substance which varies with temperature in such a way that it can be measured with accuracy and precision may be made the subject of comparison with a gas-thermometer. The measurements of the property so obtained may then be stated in terms of a centigrade gas scale or one of the thermodynamic scales, either by means of a table of data, or a graph, or a mathematical formula, and may thus become a secondary standard for the direct measurement of temperature.

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[1] See "Phase Rule," § (6), Vol. IV.

This method presupposes that the substance in question can be obtained of such constant and reproducible composition and structure that the thermal property in question is reproducible with a precision equal to that with which the comparison with the gas-thermometer was made. This is a requirement which is difficult to meet in the case of some properties, but is easily met in others.

Some of the properties that have been compared with gas-thermometers with a view to their independent use in secondary thermometry are: the vapour pressure of a pure liquid, the index of refraction of a gas, the specific heat capacity of a solid, and (most important of all) the wave-length and intensity of the radiation from solid bodies.

§ (26) COMBINATION OF FIXED TEMPERATURES WITH A THERMAL PROPERTY.—This method has been the one most commonly used for the realisation of gas-thermometer scales in the form of secondary standards. We may consider that what is really measured and certified to in this case by the gas-thermometer (in addition to the fixed temperatures themselves) is the *manner of variation* of the property in question. It is this manner of variation that is reproducible, not the absolute measure of the property itself.

Take, for example, the electrical conductivity of a particular metal. Electrical conductivity is so sensitive to small amounts of impurities in the metal that it cannot be considered sufficiently reproducible to be an independent secondary standard. But conductivity (or resistance) measurements on a particular sample, in conjunction with gas-thermometer measurements of temperature, may show that the resistance of that sample varies *continuously* and *reproducibly*, and that it can be represented within the limits of precision of the gas-thermometer by means of a mathematical formula containing as constants the resistance of the sample, say, at three fixed temperatures, whose value is independently known by other gas-thermometer measurements.

If this result is repeatedly obtained on various samples, it is reasonable to assume that any other sample, whose resistance does not vary greatly from those investigated, will likewise have a resistance which varies continuously and reproducibly and can be represented by a similar formula containing as constants the resistance at the selected fixed temperatures.

It will be recognised that measurements of this kind form the basis of the widespread and successful use of the platinum-resistance thermometer and the metal-alloy thermocouple — two very common kinds of secondary thermometer.

V. COMPARISON OF GAS-THERMOMETERS WITH SECONDARY STANDARDS IN THE RANGE 0° TO 100°

§ (27) FIXED TEMPERATURES.—The numerical values of the temperature of the melting-point of ice and the boiling-point of water are fixed by definition. The question of their reproducibility and precision are considered elsewhere.[1]

No fixed temperature within the range 0° to 100° has been made the subject of direct gas-thermometer evaluation.

The temperature of the system [Na_2SO_4 : $Na_2SO_4 . 10H_2O$: solution], under one atmosphere pressure, in the two-component system [sodium sulphate : water], has been proposed by Richards (105) as a fixed point in this part of the scale. Its temperature was accurately determined by Richards and Wells (106) by means of mercury thermometers calibrated against the standards of the International Bureau, and the determination may therefore be taken as an indirect determination in terms of the normal hydrogen scale. The temperature thus determined was 32·383°. A similar comparison was made by Dickinson and Mueller (43), using platinum-resistance thermometers which had been compared with mercurial standards checked by the International Bureau, and which were believed to represent the hydrogen scale within 0·002°, their figure for the point is 32·384°. The close agreement of these two is good evidence of the reproducibility of the point.

Similar determinations of the temperature of [$MnCl_2 . 4H_2O$: $MnCl_2 . 2H_2O$: solution], at 58·089°, have been made by Richards and Wrede (108); and of the temperature of [NaBr : $NaBr . 2H_2O$: solution], at 50·674°, by Richards and Wells (107). The NaBr point has the disadvantage that the salt is not easily obtained in a sufficiently pure state for exact work.

§ (28) THERMAL PROPERTIES.—No independent thermal property, which could be depended upon for the measurement of temperature without the aid of standardisation at fixed points, has been made the subject of gas-thermometer comparison in the range 0° to 100°.

§ (29) COMBINED THERMAL PROPERTIES AND FIXED TEMPERATURES. (i.) *Expansion of Mercury in Glass.*—The differential cubical expansion of mercury in a particular type of hard glass is a property capable of fairly exact reproduction, with the aid of two fixed temperatures. The use of this property in "mercurial thermometry" is made the subject of a special article in this dictionary and need not be referred to at length here.

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[1] See article on "Thermometry."

The comparison of this property with the constant-volume hydrogen thermometer has been the subject of some of the most exact work on record in gas-thermometry, namely, that of Chappuis (21).

Chappuis used a bulb of platinum-iridium alloy, having a capacity of about 1 litre, and filled with pure hydrogen at an initial pressure at 0° of about 1000 mm. mercury. This was immersed in a thermostat with four standard mercury-in-hard-glass thermometers of the International Bureau, Nos. 4428, 4429, 4430, and 4431. The true temperatures of these thermometers were then determined by the pressure of the hydrogen at about 5°, 10°, 15°, 20°, 25°, 30°, 35°, 40°, 45°, 50°, 61°, and 78°, and the corrections necessary to convert their scale-readings into true temperatures were tabulated.

These four thermometers thus became the custodians, so to speak, of the international temperature scale from 0° to 100°, and all other standard thermometers in the world's various national bureaus of standards have been calibrated by comparison with these four.

(ii.) *Electrical Resistance of Platinum Wire.*—In 1871 Siemens suggested the use of the electrical resistance of platinum wire as a secondary thermometric standard, but it was not put on a really exact basis until Callendar investigated it and revived the suggestion in 1886.[1] In 1891 he and E. H. Griffiths (19) published a direct comparison between a platinum-resistance thermometer and a constant pressure air thermometer at intervals of 5° over the range 0°-100° C. The two thermometers were placed side by side in an iron tank containing some 75 litres of water which was carefully stirred. The differences between the temperatures, as calculated by Callendar's quadratic formula with $\delta=1\cdot474$ and as given by the constant pressure air thermometer, were of the order of 0·01°; this value of δ was not, however, obtained by direct calibration at the sulphur point. The Kew Observatory authorities then took up the matter, and after a study of some of the characteristics of platinum-resistance thermometers had been made by Griffiths and others, requested the International Bureau of Weights and Measures to compare several such thermometers with the normal hydrogen scale, then on deposit at the Bureau in the form of eight mercury thermometers, as mentioned in preceding paragraphs of this section.

The plan was agreed to, and comparisons made by Chappuis and Harker (24) were published in 1902. Comparisons were made in the range −23° to 100° between two platinum-resistance thermometers and the standard mercury thermometers. It was found that temperatures interpolated by means of Callendar's quadratic formula based on calibrations at the ice, steam, and sulphur points deviated from the true temperature by about 0·02° or 0·03° at most (depending upon the value assumed for the sulphur boiling-point). The deviations were not, however, haphazard and accidental; the agreement was not wholly satisfactory, and was evidently not so regarded by the authors.

Except for the investigations of Callendar and Griffiths in 1891, and this comparison by Chappuis and Harker, no exact comparison of the platinum-resistance thermometer at temperatures between 0° and 100° has ever been made either directly with a gas-thermometer or indirectly through a single intermediary. The secondary indirect comparisons made by Griffiths (50) in 1893, Waidner and Mallory (124) in 1898, and by Dickinson, Waidner, and Mueller (43, 121) in 1907, however, gave what are probably more precise results than the primary indirect comparison by Chappuis and Harker, or the original direct work of Callendar and Griffiths, and must be considered the real basis for confidence in the platinum-resistance scale between 0° and 100°. Yet none of these comparisons warrants the statement that the resistance thermometer calibrated at ice, steam, and sulphur points is known with certainty to reproduce the normal hydrogen scale within 0·01°.

§ (30) SUMMARY FOR THE RANGE 0° TO 100°. (i.) *Mercurial and Resistance Thermometry.*—Although the normal hydrogen scale, as represented by a set of *verre dur* thermometers at the International Bureau, is still the only international standard of temperature, it has in recent years been less generally employed in consequence of the increasing use of and confidence in the platinum-resistance thermometer and the demonstrated variability of mercury-in-glass scales (52). Thus the Reichsanstalt (125) has officially announced its abandonment of the gas-thermometer as a fundamental experimental standard, and its adoption of the resistance thermometer (calibrated at ice, steam, and sulphur points, and having certain limiting values[2] for its constants) as its fundamental temperature scale for purposes of certification, from −39° to +445°.

Confidence in the resistance thermometer has been bred by its *convenience* and *precision* rather than by its demonstrated agreement with the normal hydrogen scale, for the demonstration in the range 0° to 100° really rests upon two or three relatively indirect comparisons with the aid of mercury thermometers. The second-degree equation for platinum resistance departs so radically from true temperatures below 0° that some departure, even though small, is rather to be expected above 0°. To cut loose completely from the gas-thermometer, therefore, seems to the writers a step backward toward the days of empirical and arbitrary thermometers and thermometric scales, for so little is known either about the fundamental laws of electrical conduction in metals and the effect of impurities thereon, or about the purification and analysis of platinum, that a new and unexpected impurity in platinum supplies could easily introduce much confusion into the thermometric situation. No objection, however, could be made to the adoption of particular platinum-resistance thermometers, deposited at a national standardising laboratory, as secondary standards to replace secondary mercurial standards.

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[1] For further details see the article on "Resistance Thermometers"; also *Phil. Trans. Roy. Soc.* A, 1887, clxxviii.

[2] R_{100}/R_0 must be not less than 1·388 and δ must be not greater than 1·52, when the boiling-point of sulphur is taken as $444\cdot55+0\cdot0908\ (p-760)-0\cdot000047\ (p-760)^2$.

(ii.) *Thermocouples.*—No general equation for any thermocouple, comparable with that for the resistance thermometer, has been developed. Reliance is placed rather upon tables of E.M.F. and temperature (as is done in the case of resistance thermometry below −40°), and the deviations of any particular thermo-element from the standard table are determined by calibration at fixed points.

(iii.) *Fixed Points.*—For those who do not wish to place full reliance either upon the reproducibility of platinum-resistance thermometers calibrated only at ice, steam, and sulphur points, or upon the reproducibility of thermocouples calibrated at ice, steam, and some other higher point, the following fixed points are available as fundamental reproducible standards (though the temperatures ascribed thereto may be changed by further investigation):

Transition temperature $Na_2SO_4 : Na_2SO_4 . 10H_2O$: solution, 32·384° C. ± 0·005°.

Transition temperature $MnCl_2 . 4H_2O : MnCl_2 . 2H_2O$: solution, 58·089° C. ± 0·005°.

VI. Comparison of Gas-thermometers with Secondary Standards in the Range −273° to 0° C.

At temperatures above 0° C. the gas-thermometer has seldom been used for the direct measurement of temperature, and its rôle has been confined to that of an ultimate standard against which secondary standards were compared. Below 0°, on the other hand, especially in the pioneer work on the liquefaction of gases, the use of gas-thermometers for the direct measurement of temperature has been the rule rather than the exception.

We shall not undertake here a discussion of all the researches in which gas-thermometers have been used as measuring instruments, but shall confine our attention to a few researches which have a direct bearing on thermometry.

§ (31) Fixed Points.—The following fixed points have been made the subject of investigation with gas-thermometers with a view particularly to their use as standard thermometric points: melting-point of mercury; sublimation-point of carbon dioxide; boiling-points of oxygen, nitrogen, hydrogen, and helium.

(i.) *Melting-point of Mercury.*—Chappuis (25) in 1894 extended his comparisons of mercury thermometers with the normal hydrogen thermometer down to the freezing-point of mercury, which he thus determined to be −38·80°. Henning (55) placed it at −38·89°, using platinum-resistance thermometers as intermediaries between the mercury and a constant-volume hydrogen thermometer.

(ii.) *Sublimation-point of Carbon Dioxide.*—The temperature of equilibrium between the solid and gaseous phases of carbon dioxide under a pressure of one atmosphere constitutes a fixed point that has been much used in low-temperature research.

Its temperature was placed by Regnault in 1863 at −78·2°; by Pictet in 1878 at −80°; by Villard and Jarry in 1895 at −79°; by Ladenburg and Krügel in 1899 (hydrogen thermometer) at −78·6°.

A more exact determination, with a constant-volume hydrogen thermometer, was made in 1901 by Holborn (60), who placed the temperature at −78·34° for a mixture of alcohol and solid carbon dioxide.

As usually employed, the solid carbon dioxide is mixed with ether or with alcohol in order to obtain uniformity and good contact with the object to be cooled. If no alcohol or ether were dissolved in the solid CO_2, and if the vapour pressures of these liquids were inappreciable in comparison with one atmosphere, the equilibrium temperature would be unaffected by their presence. Holborn looked into this question particularly, and found that the temperature of pure, dry, solid CO_2 in equilibrium with the gas at one atmosphere is −78·44°, or 0·10° lower than that of the alcohol mixture. This difference was confirmed by Kaspar (74) in 1913.

Henning (55) found by the static method, using platinum-resistance as intermediary between pure CO_2 and the constant-volume hydrogen thermometer, the value −78·52°.

(iii.) *Boiling-point of Oxygen.*—The early determinations (before 1900) by Wroblewski (127), Olszewski, Estreicher (46), Holborn and Wien, and Ladenburg and Krügel gave results, referred to the hydrogen thermometer, which ranged from −181·6 to −182·56°.

Holborn's determination (60) in 1901, using a platinum-resistance thermometer as intermediary from a hydrogen thermometer, gave −182·7°.

Baly's series of experiments (4) on the composition of the gaseous and liquid phases in the system oxygen-nitrogen gave −182·0° on the constant-pressure hydrogen thermometer. Travers believes the results to have been affected by a constant error causing the boiling-point to appear high.

Dewar (41) in 1901 placed the boiling-point at −182·5°, based on direct measurements with constant-volume hydrogen, oxygen, and helium thermometers.

Travers and Fox (112) in 1903 found −183·3° on the scale of a constant-volume oxygen thermometer filled at various initial pressures from 659 to 336 mm. and extrapolated to zero pressure. Travers, Senter, and Jaquerod (114) found that liquid oxygen easily superheats as much as 1°, and a reliable boiling-point is obtained only if gas is bubbled through the liquid, or if the point is determined by the static method. Previous measurements might therefore be expected to prove too high. Their final boiling-point (calculated from the published data, which are given directly in absolute gas scale temperatures) is −182·93° by the constant-volume hydrogen thermometer, and −182·83° by the constant-volume helium thermometer, at initial pressures of 845-975 mm. Their determinations were much more exact than any that had preceded.

Another exact determination was made by Kamerlingh Onnes and Braak (92) in 1908, using a platinum-resistance thermometer as intermediary between the boiling oxygen and two constant-volume hydrogen thermometers, which agreed to within 0·04°. The boiling-point is placed at −183·04 ± 0·03° on the normal hydrogen scale, or −182·99° on the centigrade thermodynamic scale.

The most recent determination was by Henning (55) in 1914. Using platinum-resistance thermometers previously compared with a constant-volume hydrogen thermometer, he found the value −183·01°, or −182·97° C. This is in excellent agreement with the preceding value.

(iv.) *Other Fixed Points.*—While a discussion of the boiling-points of nitrogen, hydrogen, and helium, as well as of other low-temperature points which have been suggested as thermometric standards, might properly form a part of this section, they have been fully covered elsewhere,[1] so that there is no need of duplicating the discussion here. We could only urge that, when these data are being considered from the standpoint of fundamental thermometry, care be taken to distinguish those determinations which are made by direct or indirect comparison with a gas-thermometer from those which are based on secondary standards or secondary fixed points; they should not be indiscriminately lumped together without any indication as to their degree of independence, as is too often done in tables of physical constants.

§ (32) THERMAL PROPERTIES. (i.) *Vapour Pressure of Oxygen.*—Stock and Nielsen (110) in 1906 described a low-temperature thermometer using the vapour pressure of liquid oxygen. The fundamental data for such a thermometer have been provided by the vapour pressure measurements of Onnes and Braak, and Henning, in which temperatures were measured with hydrogen or helium gas-thermometers.

Possessing some of the disadvantages of the gas-thermometer, this vapour-pressure thermometer has not been widely used as a secondary standard, since investigators have usually preferred a secondary thermometer of smaller volume and easier manipulation.

§ (33) COMBINATION OF THERMAL PROPERTIES AND FIXED POINTS. (i.) *Expansion of Mercury in Glass.*—The work of Chappuis on the comparison of mercury thermometers with the hydrogen thermometer, already referred to in the preceding section, covered the range down to −24° C. Four standard thermometers of the International Bureau, Nos. 4479, 4480, 4481, and 4482, are the depositories, so to speak, of this part of the international hydrogen scale. Chappuis' comparisons indicate that other mercury thermometers, made of glass similar to the "verre dur" standards and calibrated at 0° and 100°, can reproduce the normal hydrogen scale between 0° and −24° within 0·01°.

(ii.) *Expansion of Organic Liquids in Glass.*—Holborn (60) compared the petroleum-ether-in-glass thermometer with the nitrogen thermometer to −190° and showed that it could be used for measurements with an accuracy of 1°, but difficulties with the meniscus and the adherence of liquid to the walls have prevented the use of this and similar thermometers (pentane, toluene, alcohol) for accurate thermometry.

(iii.) *Electrical Resistance of Platinum Wire.*—Early investigations by Dewar and Fleming and by Olszewski gave rather variable deviations between the temperature extrapolated by means of the resistance of platinum and the true temperature. This variability was probably due to impure metal.

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[1] See article on "Liquefaction of Gases."

Holborn and Wien (68) in 1896 made the first real comparison of the modern platinum-resistance thermometer with a gas-thermometer. Their resistance thermometers were in some cases sealed directly into the glass bulb of the hydrogen thermometer, and, in others, placed alongside the bulb. Comparisons at various temperatures between −18° and −190° indicated that the resistance of the wire could be represented in terms of t by a second-degree equation, with an accuracy of 1°.

Continuing this work in 1901, Holborn showed that the Callendar quadratic formula for a platinum-resistance thermometer, calibrated at the ice, steam, and sulphur points in the usual manner, gives the hydrogen-thermometer temperature at −78° within 0·15°, but is 2° off at −190°.

A similar comparison was made in 1902–1904 by Meilink (81, 82) at Leiden, who showed by measurements at various points between −51° and −210° that the representation of the resistance by a quadratic formula (based only on points below 0°) is only permissible when the desired accuracy is less than 0·2°, and then only down to −197°. For an accuracy of 0·05° down to −197° a cubic formula is required, with calibrations recommended at at least six points.

Finding the gas-thermometer rather cumbrous for their low-temperature work, Travers and Gwyer (113) in 1905 undertook a re-examination of the dependence of platinum resistance on temperature below 0°. As before, the Callendar quadratic formula extrapolation proved to be low both at −78° and −190°. In order to obtain a formula which would yield hydrogen-scale temperatures, the term δ in Callendar's formula, which has a value of about 1·50 above 0°, had to be made 1·84 to 1·90. This variability in the value of δ (equivalent to considerable deviations from a quadratic formula) led them to conclude that the platinum-resistance thermometer could only be used for interpolation between fixed points, and not for extrapolation or for the independent definition of a scale.

An investigation of higher accuracy than had yet been made on the platinum-resistance thermometer at low temperatures was carried out by Kamerlingh Onnes and Clay (94) in 1906. Measuring the temperature at thirteen points between −30° and −259° with a constant-volume hydrogen thermometer whose accuracy was estimated at 0·02°, they found that the resistance of the platinum wire used could be satisfactorily represented only by a cubic formula to which was added also reciprocal powers of the absolute temperature.

As the use of so cumbrous a formula is impractical, resistance-thermometric practice soon settled down to the use of a standard table of resistances and temperatures, deviations from which might be determined for any particular platinum-resistance thermometer. Kamerlingh Onnes, Braak, and Clay (93) considered in 1907 that the definition of a low temperature by a single determination on a standardised platinum-resistance thermometer has about the same probable error as a determination on the constant-volume hydrogen thermometer, namely, 0·02° or less.

The most recent exact determination of the relation is that by Henning (53, 54) in 1913. He used a constant-volume hydrogen thermometer with glass bulb, at initial pressures of 740 and 822 mm. The Callendar quadratic formula was found to hold down to −40°, but gave temperatures which were 0·08° low at −78°, and rapidly increased its deviation below that temperature. Different platinum thermometers were found to give "platinum temperatures" which differed by as much as 0·50°. Their tabulated indications could, however, be reduced to those of one standard platinum thermometer by means of a "difference formula" (similar to one originally suggested by Nernst), of the form

$$t'_p - t_p = c t_p (t_p - 100),$$

in which t_p is the "platinum temperature" of the standard, and c a constant of the new thermometer. According to Henning's data, comparison of a new thermometer with the standard at one low temperature, such as that of liquid air, determines c with sufficient accuracy.

The difficulty of defining a low-temperature scale based on resistance of platinum is recognised in the Reichsanstalt's thermometric definitions adopted in 1916 (125), in which the temperature scale below −39° is "defined by the platinum-resistance thermometer which Henning has compared with the hydrogen thermometer." This is the only one of the definitions which is based on a particular existing instrument.

(iv.) *Electrical Resistance of Gold Wire.*—The gold-resistance thermometer has characteristics very much like those of the platinum thermometer. It has, furthermore, certain advantages over platinum for low-temperature work; it is obtainable commercially in much higher purity than platinum, has a resistance-temperature curve which is better adapted to interpolation than platinum, and has a lower inflection point in the resistance curve.

Kamerlingh Onnes and Clay (95) made in 1906 a very exact comparison of the resistance of pure gold wire with that of a standard platinum thermometer, and continued the work in 1907 (96) by studying the effect of small amounts of impurities on the characteristic constants of the wire.

(v.) *Electrical Resistance of other Metals.*—The electrical resistance of mercury, silver, bismuth, lead, and tin has been measured at various low temperatures determined by gas-thermometers, and wires of these metals have then been used as secondary thermometers.[1]

(vi.) *The Thermocouple Constantan : Iron.*—This couple was first compared with the hydrogen thermometer by Holborn and Wien (68), who found from measurements at −78° and −190° that the E.M.F. could be satisfactorily represented by a quadratic formula if the constantan was homogeneous. Inhomogeneous wire varied as much as 2° from the normal. Ladenburg and Krügel (76) in 1899, from measurements at four points with a hydrogen thermometer, found a third-degree equation necessary to represent the results within 1° from 0 to −190°.

The couple was further studied by Kamerlingh Onnes and Crommelin (98) in 1906, who compared it with a constant-volume hydrogen thermometer at various points between −30° and −259°. Down to −217° the E.M.F. could be represented by a four-term formula, which could be established for a new element by calibration at four fixed points. No formula was devised which would represent the entire range accurately.

(vii.) *Other Thermocouples.*—Dewar in 1905 showed that the couple platinum : silver was suitable for measurements of temperature below −250°, where the resistance thermometer becomes relatively insensitive.

The couple gold : silver, though not sensitive at 0°, increases rapidly in sensitiveness at low temperatures, and is better than either constantan : iron or platinum : German-silver (42) below −200°. The gold : silver couple was investigated by Kamerlingh Onnes and Clay (97) in 1908.

§ (34) SUMMARY FOR THE RANGE −273° TO 0°. (i.) *Fixed Points.*—The following fixed points have been determined by measurement with gas-thermometers and form a reliable basis for the establishment and maintenance of the low-temperature centigrade thermodynamic scale :

Melting-point of mercury	$-38{\cdot}9\pm0{\cdot}05^\circ$
Sublimation point of carbon dioxide	$-78{\cdot}5+0{\cdot}016\ (p-760)\pm0{\cdot}05^\circ$
Boiling-point of oxygen	$-183{\cdot}0+0{\cdot}0126\ (p-760)\pm0{\cdot}05^\circ$
Boiling-point of hydrogen	$-252{\cdot}7+0{\cdot}005\ (p-760)\pm0{\cdot}05^\circ$

(ii.) *Resistance Thermometers and Thermo-elements.*—The platinum-resistance thermometer calibrated at ice, steam, and sulphur points will give temperatures accurate to 0·03° down to −40°. Below that temperature additional calibrations are necessary and recourse must be had to a standard table of resistance and temperature, with the aid of which it is possible to measure temperature at −200° within 0·05° or less. Gold is preferable to platinum below −200°. Below −250° both are relatively insensitive; manganin or constantan may be used in this region.

Thermo-elements of constantan : iron, platinum : silver, gold : silver, and other combinations, may be used over various parts of the range with the aid of standardised tables. Their sensitiveness varies greatly and the data cannot be briefly summarised.

VII. COMPARISON OF GAS-THERMOMETERS WITH SECONDARY STANDARDS IN THE RANGE 100° TO 500°

§ (35) FIXED TEMPERATURES.—We shall consider here the fixed temperatures which have been the subject of comparison with gas-thermometers, either directly or through the intermediary of some temperature-indicating device whereby the temperature was directly transferred without reference to the device as an independently operating thermometer.

(i.) *Melting-point of Zinc.*—No melting-points of metals have been determined directly by measurement with a gas-thermometer. The nearest to a direct determination that has been made is Day and Sosman's determination of the melting-point of zinc by what might be called a "substitution" method (35). The temperature of a stirred nitrate bath was first measured near 419° with a constant-volume nitrogen thermometer at initial pressure 500 mm., to the bulb of which three thermocouples were attached. The thermometer bulb was then replaced by a steel vessel of the same shape, containing a charge of zinc in a graphite crucible, with one of the three thermocouples immersed in the metal (this one having been previously at the centre of the bulb in a re-entrant tube). The melting-point thus determined was 419·28°.

An indirect determination, using platinum : platinrhodium thermocouples as intermediaries, was made by Day and Sosman (37) in 1910, in connection with a series from 400° to 1550°, and gave the value 418·2° on the constant-volume nitrogen scale. Subsequent investigations, however, showed that the lack of uniformity in the air-bath designed and used for the high-temperature series caused low readings at 400°, although the difference disappeared above 630°. No weight is attached, therefore, to the first value of 418·2.

(ii.) *Boiling-points of Naphthalene, Diphenyl, and Benzophenone.*—The only direct determinations of these points are those by Jaquerod and Wassmer (73) in 1904, and Crafts (33) in 1913. Jaquerod and Wassmer measured the boiling-points under various pressures by means of a constant-volume hydrogen thermometer with glass bulb, obtaining the following boiling-points under atmospheric pressure :

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[1] See article on "Resistance Thermometers."

Naphthalene	$217{\cdot}68° + 0{\cdot}057\,(p-760)$
Diphenyl	$254{\cdot}93° + 0{\cdot}061\,(p-760)$
Benzophenone	$305{\cdot}44° + 0{\cdot}063\,(p-760)$

A later determination on naphthalene by Jaquerod and Perrot (72), with a constant-volume hydrogen thermometer with fused silica bulb, gave 217·84° at 760.

Crafts determined the boiling-point of naphthalene directly on the constant-volume nitrogen thermometer at initial pressure 757 mm., obtaining the value 218·06°.

Day and Sosman (35) made an indirect determination of the benzophenone point in 1912, by transfer with thermocouples. The boiling-point on the constant-volume nitrogen scale at initial pressure 500 mm. was found to be 305·87°.

(iv.) *Boiling-point of Sulphur.*—The boiling-point of sulphur has been a rallying-point for all gas-thermometer investigators working in the range 100° to 500°. The determinations before 1890 were not sufficiently accurate to require discussion here. In 1890 Callendar and Griffiths (19) determined the point indirectly, with the platinum-resistance thermometer as intermediary. Chappuis and Harker (24) determined it in the same manner in 1902. Eumorfopoulos (47) made a direct determination in 1908. Another indirect determination with a platinum-resistance intermediary was made by Holborn and Henning (64) in 1911. Finally, direct determinations were made by Day and Sosman (35) in 1912, by Eumorfopoulos (48) in 1914, and by Chappuis (23) in 1917; and an indirect determination by Dickinson and Mueller in 1912 in co-operation with Day and Sosman, using Day and Sosman's nitrogen thermometer (44).

Both constant-pressure and constant-volume thermometers have been used in determining the point. The gases employed and their initial pressures, together with the final results calculated in terms of several scales with the aid of Buckingham's corrections, are shown in Table 9.

The value by Chappuis and Harker was subsequently corrected by the authors themselves in view of new and more accurate determinations of the expansion coefficient of their porcelain bulb. The value by Eumorfopoulos in 1908 was corrected by Callendar and Moss (20) in 1910 with the aid of new determinations of the expansion of mercury, used by Eumorfopoulos to obtain the expansion of his glass bulb. The determinations by Eumorfopoulos in 1914 were made with the constant-pressure nitrogen thermometer at

TABLE 9.—GAS-THERMOMETER DETERMINATIONS OF THE BOILING-POINT OF SULPHUR SINCE 1890

Date.	Author.	Thermometer.	Gas.	Pressure, mm.	Original Figure.	Corrected Figure.	Const. Pr. Nitrogen Scale p=1000.	Const. Vol. Nitrogen Scale.		Thermo-dynamic Scale.	Notes.
								p_0=1000.	p_0=500.		
1890	Callendar and Griffiths	Const. pr.	Air	760	444·53	..	444·41	444·71	444·81	444·91	Indirect through Pt res. therm.
1902	Chappuis and Harker	Const. vol.	Nitrogen	530	445·2	444·7 *	444·30	444·60	444·70	444·80	
1908	Eumorfopoulos	Const. pr.	Air	760	443·58	444·55	444·43	444·73	444·83	444·93	Direct.
1911	Holborn and Henning	Const. vol.	Hydrogen	623	444·51	..	444·01	444·31	444·41	444·51	Indirect through Pt res. therm.
			Helium	612							
			Nitrogen	625	(444·39) †						
1912	Day and Sosman	Const. vol.	Nitrogen	502	444·45	..	444·05	444·35	444·45	444·55	Direct.
1912	Dickinson and Mueller ‡	Const. vol.	Nitrogen	502	444·28	..	443·88	444·18	444·28	444·38	Indirect through Pt res. therm.
1914	Eumorfopoulos	Const. pr.	Nitrogen	415	444·36	..	444·07	444·37	444·47	444·57	Direct.
				792	444·13	..	444·03	444·33	444·43	444·53	
1917	Chappuis	Const. vol.	Nitrogen	500-564	444·48	444·49	444·09	444·39	444·49	444·59	

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* *Trav. Mem. Bur. Int.*, 1902, xii. 90.

† The value given for "Nitrogen" is calculated from the authors' table of experimental results. They give in their paper only the final mean value on the thermodynamic scale.

‡ Using Day and Sosman's nitrogen thermometer.

two pressures, 792 and 415 mm., and thus permitted a direct calculation of the thermodynamic temperature, which the author places at 444·61°. However, to avoid confusion in the comparisons in the table, we have used a uniform set of corrections for these as well as the other determinations.

It is now fairly certain that the first three determinations in the table are too high. By an unexpected coincidence the original Callendar value on the *constant-pressure air* scale is practically the same as the value on the *thermodynamic* scale now most widely used, but it should be emphasised, in order to avoid possible confusion, that this is a coincidence and not an agreement.

§ (36) COMBINATION OF THERMAL PROPERTIES AND FIXED POINTS. (i.) *Electrical Resistance of Platinum Wire.* — The experiments of Callendar (16) in 1886 showed that the platinum-resistance thermometer could be made a very precise and convenient secondary thermometer. He used a constant-volume air thermometer, at an initial pressure of about 1 atmosphere, with soft glass, hard glass, and porcelain bulbs, and determined the resistance of platinum wires, which were sealed into the glass bulbs, at various temperatures from 207° to 624° (671° with porcelain bulb), together with a few check results in a preliminary series at lower temperatures. The results as a whole showed that the resistance could be expressed by the quadratic formula :

$$\frac{R}{R_0} = 1 + \alpha t + \beta t^2,$$

which was put into the form, more convenient for calculation and use, of the two equations :

$$t_{pt} = 100 \times \frac{R - R_0}{R_{100} - R_0}$$

$$\text{and } t - t_{pt} = \delta\left[\left(\frac{t}{100}\right)^2 - \frac{t}{100}\right].$$

The comparison made in 1891 by Callendar and Griffiths (19) has been referred to in § (29) ii. Except in three cases the agreement with the constant pressure air thermometer was within 0·012°.

A further exact test of the platinum-resistance formula was made by Chappuis and Harker (24) in 1902. Having found that the pressure of hydrogen in a glass bulb decreased progressively at 180° and higher, the authors abandoned hydrogen and used nitrogen at initial pressures of 392 to 793 mm. One series was also made with a porcelain bulb. Three resistance thermometers were used. As a result of comparisons at a large number of points, it appeared that the Callendar formula would represent the true temperature within 0·1° between 100° and 250°, and within 0·3° between 250° and 450°. The authors conservatively stated that "rigorous conclusions cannot be drawn concerning the application of the Callendar method to the calibration of platinum thermometers. Further comparisons are needed with the gas thermometer and of various resistance thermometers with each other."

The desired recomparison was not completely forthcoming until 1911, when Holborn and Henning (64) published their work on nitrogen, hydrogen, and helium thermometers from 200° to 450°, although comparisons at 150° and 200° were published by these authors (63) in 1908.

The 1908 comparisons showed that, when the sulphur point was taken as 445·0°, temperatures near 150° and 200°, interpolated by the resistance thermometer, agreed with the centigrade thermodynamic scale within 0·05°. Recalculation on the basis of 444·55° for the sulphur point, however, makes the deviations 0·04° to 0·07° larger, but still leaves the average agreement within 0·1°.

In the 1911 comparisons differences of 0·05° to 0·11° were found between bulbs of Jena glass and fused silica containing nitrogen, but these differences were ascribed to uncertainty in the expansion of the Jena glass. The nitrogen-in-silica temperatures differed from those with helium and hydrogen by about the amount to be expected from Berthelot's and Buckingham's gas-thermometer corrections. Taking the temperatures obtained with hydrogen and helium in glass and silica as standard, therefore, it appeared that these temperatures could be interpolated with an accuracy within 0·03° at 200° and 0·05° at 400° with the aid of the Callendar formula and calibration at ice, steam, and sulphur points.

(ii.) *Platinum-metal Thermocouples.* — Holborn and Day (61) made in 1899 the first thorough-going comparison of the platinum : 90 platinum, 10 rhodium couple with a gas-thermometer. They continued the work in 1900, revising the earlier data with the aid of new values for the expansion of the bulbs. Their comparisons with the constant-volume nitrogen thermometer extended from 195° up. It was found possible to represent the E.M.F. in terms of temperature by an equation of the form

$$e = -a + bt + ct^2$$

over the range 250° to 1100°, with an accuracy of 1° or better. A similar formula held for couples of platinum : 90 platinum, 10 ruthenium and platinum : rhodium over the same range. The couple platinum : iridium, however, deviated from this formula below 400° by several degrees.

The nitrogen-thermometer comparisons by Day, Clement, and Sosman (36) (1908–1912) showed that a quadratic equation would represent the E.M.F. of the platinum : platinrhodium couple from 300° to 1100° within 0·3°.

(iii.) *Copper-constantan Thermocouple.*—Although this couple has not been directly compared with a gas-thermometer at a series of temperatures between 100° and 500°, Adams and Johnston (3) were able, by utilising the measurements of Day and Sosman in 1912 at 300°, and by measuring the E.M.F. at fixed points which had been otherwise interpolated, to show that the couple will reproduce temperatures between 100° and 360° with an accuracy of 0·02°.

(iv.) *Interpolated Fixed Points.*—Day and Sosman determined the melting-point of *cadmium* by interpolation with thermocouples calibrated against the nitrogen thermometer.

Holborn and Day, after calibrating platinum : platinrhodium thermocouples against the nitrogen thermometer and obtaining their law of variation, used these couples as secondary thermometers to determine the melting-points of *cadmium*, *lead*, and *zinc*.

Holborn and Henning, after satisfying themselves that their resistance thermometers, with the Callendar formula, gave constant-volume hydrogen and helium temperatures with an accuracy of 0·03° to 0·05°, interpolated the melting-points of *tin*, *cadmium*, and *zinc*, and the boiling-points of *naphthalene* and *benzophenone*, with their resistance thermometers.

Crafts (32) determined the boiling-points of *naphthalene* and *benzophenone* by means of mercury thermometers standardised against a constant-volume hydrogen thermometer, obtaining the values 218·08° and 306·1° respectively.

In addition to the foregoing data, which may be called "primary" interpolations, there have been various "secondary" interpolations of fixed points not directly connected with any gas-thermometer observations, but based solely upon other fixed points which had been determined by means of gas-thermometers, taken in conjunction with a law of variation made probable by still other gas-thermometer observations. Such interpolations, including those by Heycock and Neville (57), Callendar and Griffiths (19), Waidner and Burgess (122), Travers and Gwyer (113), and Adams and Johnston (3), are, strictly speaking, outside the scope of this article, but are included here for the sake of comparison. All of the interpolated data, together with the direct and indirect gas-thermometer determinations, are collected in Table 10.

§ (37) THERMAL PROPERTIES. (i.) *Index of Refraction of Gases.*—In 1895 D. Berthelot (10) suggested that the density of a gas at a known pressure (atmospheric), and hence its temperature, could be determined by measuring its index of refraction. Tests of the method at the boiling-points of alcohol (78°), water, and aniline (184°) gave deviations of 0·05° to 0·12° at 78°, 0·03° to 0·12° at 100°, and 0·08° to 0·33° at 184°, from the known boiling-points of these substances on the normal hydrogen scale. The method was therefore considered promising enough to be extended to temperatures above 500°. A fuller discussion of it will be found in Part VIII. of this article. From the results given above it appears that in the range just above 100° the method is not as precise as mercurial or resistance thermometry.

(ii.) *Total Radiation of a Black Body.*—Valentiner (115) has obtained values of σ, the constant of the Stefan-Boltzmann law, from measurements at the boiling-points of water and of sulphur. These are fully discussed elsewhere.[1]

§ (38) SUMMARY FOR THE RANGE 100°-500°. (i.) *Resistance Thermometry.*—The Callendar formula applied to a platinum-resistance thermometer meeting the specifications of the Reichsanstalt,[2] calibrated at ice, steam, and sulphur points, will give either normal hydrogen scale temperatures or centigrade thermodynamic temperatures, depending upon the scale in which the sulphur point is taken, within 0·05° at 200° and within 0·1° at 400°.

(ii.) *Thermocouples.*—A cubic formula for e in terms of t, applied to the copper-constantan couple calibrated at three fixed points such as the boiling-points of water, naphthalene, and benzophenone, will give normal hydrogen or thermodynamic scale temperatures within 0·05° in the range 100° to 350°. A platinum : platinrhodium couple calibrated at two points to determine its deviation curve will, when used with Adams' table (1), give temperatures within 0·2° at 200° and 0·3° at 500°.

(iii.) *Fixed Points.*—The following fixed points are suitable for the establishment and maintenance of a temperature scale with the aid of secondary thermometers, and are known on the thermodynamic scale with the accuracy indicated, assuming the materials to be the purest now obtainable :

Boiling-point of naphthalene	$217{\cdot}95 + 0{\cdot}058(p - 760)$, ±0·05°
Melting-point of tin . .	231·85±0·1°
Boiling-point of benzophenone	$305{\cdot}9 + 0{\cdot}063(p - 760)$ ±0·1°
Melting-point of cadmium	320·9 ±0·1°
Melting-point of zinc . .	419·4 ±0·2°
Boiling-point of sulphur	$444{\cdot}55 + 0{\cdot}0908(p - 760) - 0{\cdot}000047(p - 760)^2$ ±0·1°

Some of these substances are now obtainable as certified samples from the national standardising bureaus (for example, tin and zinc from the National Bureau of Standards of the United States).

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[1] See article on "Radiation, Determination of Constants," Vol. IV.; also Part VIII. of the present article.

[2] See summary of Part V. of this article.

TABLE 10

INTERPOLATED VALUES OF FIXED POINTS IN THE RANGE 100°-500° COMPARED WITH GAS-THERMOMETER DETERMINATIONS (ALL ON THERMODYNAMIC SCALE)

Author.	Date.	B.P. Naphthalene.	M.P. Tin.	B.P. Benzophenone.	M.P. Cadmium.	M.P. Lead.	M.P. Zinc.	Basis.
				Direct or Indirect Gas-thermometer Determinations.				
Jaquerod and Wassmer	1904	217·68	..	305·44	..	..	..	Constant-volume hydrogen.
Jaquerod and Perrot	1905	217·84	..	..	..	..	..	
Day and Sosman	1912	..	..	305·91	..	..	419·4	Constant-volume nitrogen.
Crafts	1913	218·08	..	..	..	..	..	
				Primary Interpolations.				
Crafts	1883	218·08	..	306·1	..	..	..	Mercury thermometers standardised by constant-volume hydrogen thermometer.
Holborn and Day	1900	..	..	..	321·7	326·9	419·0	Pt: PtRh couple and nitrogen thermometer.
Holborn and Henning	1911	217·96	231·83	305·89	320·92	..	419·40	Pt res. thermometer and helium thermometer.
Day and Sosman	1912	..	..	..	320·9	..	..	Pt: PtRh couple and nitrogen thermometer.
				Secondary Interpolations.				
Griffiths	1891	218·04	..	..	..	..	..	Pt res. thermometer standardised at 0, 100, and 444·55.
Callendar and Griffiths	1891	217·94	231·7	305·82	320·7	327·7	417·6	
Heycock and Neville	1895	..	231·9	..	..	..	419·0	
Travers and Gwyer	1905	218·04	..	305·80	..	..	..	
Waidner and Burgess (123), (122)	1910	217·95	231·88	305·95	320·92	327·33	419·23	
Adams and Johnston	1911	..	231·73	..	320·92	327·39	..	Copper - constantan thermoelements between 217·95, 305·9, and 419·4.

VIII. COMPARISON OF GAS-THERMOMETERS WITH SECONDARY STANDARDS IN THE RANGE 500° TO 1600°

§ (39) HISTORY. (i.) *Prinsep*, 1828.—The first high-temperature thermometer based on the expansion of gases, so far as we now know, was made by Prinsep (102) and described by him in 1828. He used a bulb of gold, connected with a sensitive manometer with which to maintain the gas (air) at constant pressure within, and connected also with a reservoir of olive oil; the expansion of the air in the bulb displaced a proportionate amount of oil, which was caught and weighed. With this apparatus Prinsep made excellent temperature measurements, chiefly of the melting-points of the alloys of gold, silver, and platinum, which bear his name, and are still sometimes used.

(ii.) *Pouillet*, 1836.—Prinsep was quickly followed by Sir Humphry Davy and several others, all employing the expansion of air at constant pressure, but none contributing materially to the improvement of Prinsep's apparatus until Pouillet (101) constructed his instrument in 1836. Pouillet's bulb was of platinum, which enabled him to reach the highest temperatures, and his experimental procedure, with but slight modifications, is that employed in later years by Callendar and his associates. It was Pouillet also who made and calibrated the first practical thermo-element (platinum-iron), who anticipated the method of measuring temperature through determinations of the specific heat of platinum subsequently developed by Violle, and who made some study of the radiant energy sent out by glowing solids. In varying degree, and with many of the inevitable limitations of the pioneer, Pouillet not only established gas-thermometry upon a sound basis, but introduced several of the important practical methods of pyrometry which have been in use since his time.

Following Pouillet, therefore, the advancement of pyrometric measurement became to a considerable degree a question of perfection of experimental detail rather than of the development of new principles, and so, with one or two exceptions which will be noted presently, it has since remained. Regnault in particular made in 1847 a number of improvements in the Pouillet instrument (103).

The first gas-thermometer which measured the expansion of the gas under constant-volume appears to have been built by Silbermann and Jacquelin in 1853, but it was only indifferently successful. Effective use was first made of the method in the work of Becquerel, described below.

(iii.) *St. Claire-Deville and Troost* (39), 1857.—It was soon after this that a real catastrophe occurred in the development of the gas-thermometer. Deville and Troost (1857), desiring to use a heavier gas in place of air, introduced iodine into a bulb of porcelain and made determinations of a number of constant temperatures, most conspicuous among which, in the discussion which followed, was the boiling-point of zinc, which they ascertained to be 1040°.

(iv.) *Edmond Becquerel* (9), 1863.—Becquerel followed in 1863, using the Pouillet apparatus with platinum bulb and air as the expanding gas, and reached the conclusion that zinc boiled at 932°, more than 100° lower. In the controversy which followed, and which was maintained from both sides with considerable bitterness, these observations were repeated by both observers with substantial confirmation of the first results, Deville and Troost maintaining from experiments of their own that Becquerel's platinum bulb was permeable to hot gases, and that his results must of necessity be too low. To this contention Becquerel replied convincingly by using a porcelain bulb himself (still retaining air as the expanding gas), with both the constant-volume and constant-pressure methods of measurement, and announced a result (891°) even lower than his previous determination. Notwithstanding this, Deville and Troost were unwilling to regard the result as conclusive, and looked upon the discrepancy between Becquerel's earlier and later results (932° and 891°) with unconcealed suspicion. They reiterated their belief that the platinum bulb was permeable and that Becquerel's results with porcelain bulbs were still too low through failure to expose the bulb directly to the zinc vapour. Becquerel's bulb had been shielded from the direct action of the zinc by a protecting tube. Deville and Troost then repeated their own measurements and again obtained their earlier result. Becquerel, following, insisted that his measurements with the platinum bulb were not seriously affected by permeability to hot gases, a property with which he appeared to be familiar, and explicitly criticised the use of iodine by Deville and Troost.

The discussion ended here for the moment without a decisive issue, but subsequent experience has substantially confirmed Becquerel in his contention and his numerical results. The high value obtained by Deville and Troost was undoubtedly due chiefly to the dissociation of the iodine at high temperatures.

The real catastrophe in the development of the gas-thermometer, however, does not lie in the uncertainty of the results obtained with it by these distinguished observers, nor yet in the subsequent discovery that iodine is an inappropriate expanding medium with which to measure temperature; but rather in the discredit in which the platinum bulb came to be held and the universal substitution therefor of porcelain—a material of wholly uncertain chemical composition and physical characteristics. This was a backward step which was not retrieved for more than thirty years.

Deville and Troost (40) then entered upon a long series of experiments with porcelain glazed inside and outside with feldspar, in the course of which it appeared that the expansion of the bulb, a factor of great importance both then and now in gas-thermometry, was variable with the temperatures to which it had previously been exposed. These irregularities diminished with continued use, and were thought to become negligible in bulbs of Bayeux porcelain after a few heatings to a very high temperature.

(v.) *Regnault* (104), 1861.—During the progress of the above investigation Regnault was at work upon a displacement method (boiling mercury in an iron flask and estimating the temperature from the quantity remaining in the flask after cooling), which did not prove satisfactory. Schinz, Berthelot, and Weinhold suggested some modifications of this and other contemporary methods, but none of them proved of permanent value.

(vi.) *Erhard and Schertel* (45), 1879.—Erhard and Schertel redetermined the melting temperatures of the Prinsep alloys in 1879, using a bulb of Meissen porcelain and air as expanding gas with considerable success. Their work contributed little of novelty, but was carefully done, and the results have since been extensively used.

In 1880 Deville and Troost reappeared in the field, after a long silence, and also proposed a displacement scheme containing some improvements over the apparatus proposed by Regnault. Nitrogen was here used in place of air, but otherwise the method possessed insufficient accuracy to secure for it general approval. In the same year they published a summary of all their work on boiling zinc, giving 942° as the mean of 27 determinations, which was (for that time) in good agreement with Becquerel's first value, (1882).

(vii.) *Violle* (120), 1882.—In the same year Violle, using Deville and Troost's methods and apparatus, found zinc to boil at 930°, and thus added a further degree of probability to the determination of Becquerel. Violle continued his researches by determining with the gas-thermometer the specific heat of platinum for a number of temperatures up to 1200°.

In the decade between 1882 and 1892 contributions to gas-thermometry and the measurement of high temperatures are few and unimportant, but investigations were begun in those years on both sides of the Atlantic, namely, those of Barus at the U.S. Geological Survey in Washington and of Holborn and his colleagues at the Reichsanstalt in Charlottenburg.

(viii.) *Barus* (5, 7), 1889, recognised, as no observer who preceded him had done, the superlative importance of a uniform temperature distribution about the gas-thermometer bulb for purposes of high-temperature measurement. He sought to avoid irregularities due to the shape of the apparatus and the use of gas flames by a method of great ingenuity, but also of great technical difficulty. He enclosed his bulb within a rapidly revolving muffle (5, 6), which by its motion protected every portion of the bulb from direct exposure to a particularly hot or a particularly cold portion of the adjacent furnace. This complicated furnace structure and consequently inaccessible position of the bulb made it necessary to use thermo-elements which were first calibrated by exposure in the furnace with the bulb and then used independently to measure other desired temperatures. The thermo-element has continued in general use in this intermediary rôle since that time.

(ix.) *Holborn and Wien*, 1892.—In the same year in which Barus published his final memoir on the gas-thermometer and the thermo-element (1892), Holborn and Wien published a paper (66) covering nearly the same ground in the same general way, but with somewhat different results. Both used air as the expanding gas, both used thermo-elements to transfer the standard gas temperatures over to the substance to be measured; but Holborn and Wien attained to higher temperatures (above 1300°), while Barus took much greater precautions than his German contemporaries to secure a uniform temperature about his bulb. The arrangement adopted by Holborn and Wien possessed the further advantage that the thermo-element was entirely enclosed within the bulb itself and so was well protected against the contaminating influence of furnace gases, besides giving a truer record of the actual temperature of the expanding gas. Over against this it should be stated that the volume of the unheated portions of their bulb and manometer connections, which then constituted the chief source of error in all gas-thermometer measurements, was dangerously large.

After 1892 Barus turned his attention to other things, but Holborn and Wien published a second article (67) in 1895 confirming and extending their earlier results. By employing a specially refractory porcelain bulb they were able to continue the gas measurements nearly to the melting-point of nickel.

(x.) *Holborn and Day* (61), 1899.—With the advancing demands of science for trustworthy high-temperature measurements, the existing differences in the absolute temperature of the melting-point of gold, which is an ideal substance for a temperature constant, soon came to be regarded as unsatisfactory, and the whole problem was again taken up at the Reichsanstalt by Holborn and Day, with a view to clearing up these differences. At that time the gas-thermometer was in serious danger of falling into disrepute as a physical instrument of precision. Holborn and Day began by using bulbs of Royal Berlin porcelain, but, after the investigation had proceeded for a year or more, abandoned them definitely and permanently to return to the old platinum bulb of Pouillet, with an appropriate gas (nitrogen) which could not penetrate the bulb wall. A further improvement of inestimable value in attaining constant and reproducible conditions was made when electric heating-coils were substituted for gas. With this change the contamination of the thermo-elements through the action of combustion gases, the danger of one or other of these gases penetrating the bulb wall itself, irregularities of temperature about the bulb, and inadequate control of the heat supply, were all eliminated or much reduced in magnitude at a single stroke. Several metal melting-points were established as points of reference for the high-temperature scale, which soon found general acceptance and were almost universally used until within a few years.

Since the beginning of the present century, but four attempts have been made to reach 1000° C. with the gas-thermometer. These may be taken up in the order of their publication as follows: (1) J. A. Harker (1904), using a porcelain bulb and nitrogen; (2) Jaquerod and Perrot (1905), using a bulb of "quartz glass" and various gases; (3) Holborn and Valentiner (1906), using one bulb of platinum containing 20 per cent of iridium and one of pure iridium, both with nitrogen as the expanding gas; and finally (4) Day and Clement (1908), and Day and Sosman (1910), using bulbs of platinum containing 10 per cent of iridium and 20 per cent of rhodium respectively.

(xi.) *Harker* (51).—The work of J. A. Harker at the National Physical Laboratory (England) does not differ in any important particular from the work of Holborn and Day which immediately preceded it at the Reichsanstalt. His instrument was an exact duplicate of the Reichsanstalt instrument by the same maker, except that the bulb was of porcelain instead of platiniridium.

(xii.) *Jaquerod and Perrot* (72), 1905.—Jaquerod and Perrot sought to establish a high-temperature scale from which two of the important sources of uncertainty in previous work should be eliminated: (1) the uncertainty due to differences in the expansion of the various available gases; (2) any uncertainty which might enter the problem through the expansion of the containing vessel (bulb). Their results toward the accomplishment of their first object have been discussed in Part III. To accomplish the second they selected for the material of their bulb a substance whose expansion coefficient was less than one-tenth as great as any which had been employed for the purpose up to that time. Both improvements afforded most valuable information.

(xiii.) *Holborn and Valentiner* (65), 1906.—The experiments of Holborn and Valentiner contemplated another definite and important step in advance. Theirs was the first serious effort to extend the gas scale itself from 1150° C., where all previous investigations had been halted, to 1600° C. The difficulties confronting such an undertaking are obvious and of an insistent kind. Of the limited number of substances available for use as bulbs none is without serious limitations at these extremely high temperatures. Porcelain becomes soft and its walls both absorb and generate gas in prohibitive quantities; silica glass devitrifies; pure platinum is very soft and is permeable to hydrogen; when stiffened with iridium or rhodium it is the best material available but the iridium is destructive to the thermo-elements, and the bulb is likely to develop leaks and is permeable always to hydrogen if but a trace of the gas or of water-vapour is about. Furthermore, the difficulty of maintaining a constant temperature about a bulb of 200 c.c. capacity increases at these temperatures, and the difficulty of measuring with thermo-elements within the furnace is greatly increased by the conductivity of all insulating material. It is also a matter of no inconsiderable difficulty to generate and to regulate accurately the quantity of heat required for a bulb of this size under conditions where all electrical insulation begins to break down, and to protect the mercury manometer from so hot a furnace without removing it to an impracticable distance.

Their effort demonstrated beyond peradventure that the extension of the gas scale to 1600° was practicable.

(xiv.) *Day, Clement, and Sosman* (1908–1912).—Having in mind the diminution of errors due to lack of uniformity of temperature in an air-bath, Day and Clement (34) in 1908 constructed a gas-thermometer with a platinum-wire-wound furnace especially designed to produce uniformity, and with a bulb made of an alloy of 90 parts platinum and 10 parts iridium. They also prevented both variable and constant errors caused by the difference in pressure between the inside and outside of the bulb, by enclosing it in a furnace jacket containing nitrogen or air at the same pressure as that within the bulb. They further reduced the magnitude of the constant correction due to the "unheated space" to a much smaller value than had been attained before. By means of a cap having the shape of the mercury meniscus, the unheated space ratio was reduced to 0·0015.

To avoid the errors due to contamination of the intermediary thermocouple by iridium volatilising out of the bulb, this gas-thermometer was then further improved by the substitution of a bulb made of an alloy of 80 parts platinum and 20 rhodium in place of the platiniridium bulb. With this apparatus Day, Sosman, and Allen (37, 38, 36) made in 1908–1910 an extended series of gas-thermometer observations, using nitrogen at initial pressures of 217-347 mm., and platinum-platinrhodium thermocouples to transfer the temperatures to a series of melting-points of metals and silicates as fixed temperatures. All of the materials used were analysed to determine their purity.

§ (40) Fixed Temperatures. (i.) *Melting-points of Metals.*—Several of the earlier investigations of the high-temperature scale made comparisons between the gas-thermometer and the melting-points of metals. Erhard and Schertel's measurements in 1879 on gold and silver were direct comparisons, made by placing the melting metal and the air-thermometer bulb side by side in the furnace. Becquerel's determination of the silver point in 1863 was an indirect comparison, using a platinum-ruthenium thermocouple as intermediary. These earlier measurements were admittedly of the order of accuracy of ±5°, and therefore need not be considered in comparison with the more exact measurements made since 1890.

Between 1890 and 1910 most of the high-temperature gas-thermometer observations were directed toward determining the law of variation of secondary thermometric properties, such as electrical resistance, thermal E.M.F., and radiation, and the fixed temperatures determined in connection therewith were, so to speak, tertiary standards. A discussion of these measurements belongs properly in the third part of this section.

The only exception was the series of measurements of the melting-point of gold by Jaquerod and Perrot (72) in 1905. They determined the temperature of the melting metal directly in terms of constant-volume thermometers containing air, nitrogen, oxygen, carbon monoxide, and carbon dioxide, obtaining as a mean value for the melting-point 1067·4 ±1·8°.

The most accurate determinations of metal melting-points above 500° in terms of a gas-thermometer are those of Day, Sosman, and Allen (37) in 1910. They first measured the E.M.F. of several platinum-platinrhodium couples at a fixed temperature, then placed these couples together with the thermometer bulb in an electric resistance furnace and brought the temperature as nearly as possible to the same point. The determinations are thus nearly independent of the law of variation of the thermocouples with temperature.

Their melting-points, in the scale of the constant-volume nitrogen thermometer at initial pressure 350 mm., are given in Table 11.

Table 11

Melting-points of Metals as determined by Day, Sosman, and Allen

Metal.	Point.	Atmosphere.	Crucible.	Total Impurities in Metal, per cent.	Temperature.
Antimony	Melting and freezing	Carbon monoxide	Graphite	0·031	629·85 ± 0·3
Silver	,,	,,	,,	0·003	960·0 ±0·7
Gold	,,	,,	,,	<0·005	1062·4 ±0·8
Copper	,,	,,	,,	0·008	1082·6 ±0·8
Nickel	,,	Hydrogen and nitrogen	Magnesia and magnesium aluminate	0·165	1452·3 ±2·0
Cobalt	,,	,,	Magnesia	0·049	1489·8 ±2·0
Palladium	,,	Air	Pure magnesia	0·025	1549·2 ±2·0

(ii.) *Boiling-points of Metals.*—Barus, in beginning his work in 1882, realised the importance of securing uniformity of temperature about the gas-thermometer bulb, and was impressed by the advantages of employing vapours of boiling liquids for this purpose. He accordingly devoted considerable time to the study of vapour baths containing boiling mercury, sulphur, cadmium, zinc, and bismuth.

Of the higher-boiling metals only zinc has been studied directly with a gas-thermometer. Holborn and Day in 1899 made measurements with one of their porcelain bulbs immersed in the vapour of boiling zinc, and obtained 916°-920° as the boiling-point.

The difficulties of handling an apparatus containing a considerable volume of boiling metal led to its abandonment for work above 500°, and no high-temperature metal boiling-points have been determined with the gas-thermometer since 1899, either directly or with a transfer device.

(iii.) *Melting-points of Inorganic Compounds.*—The work of Day, Clement, Sosman, and Allen at the Geophysical Laboratory was inspired by the need of a more exact thermometric scale above 1100° for use in investigations on the silicates, and several silicates were accordingly used as fixed temperatures of reference. These are contained in Table 12, together with the melting-points of two salts determined by White with thermocouples intercompared with the gas-thermometer couples.

TABLE 12

MELTING-POINTS OF INORGANIC COMPOUNDS AS DETERMINED BY DAY AND SOSMAN

Compound.	Formula.	Temperature.
Sodium chloride .	$NaCl$	800 ± 1
Sodium sulphate .	Na_2SO_4	884 ± 1
Lithium metasilicate	Li_2SiO_3	1201 ± 1
Diopside . . .	$CaMgSi_2O_6$	$1391 \pm 1{\cdot}5$
Anorthite . . .	$CaAl_2Si_3O_8$	1550 ± 2

§ (41) THERMAL PROPERTIES.—The following reproducible thermal properties have been studied with the aid of gas-thermometers in the range above 500° C.:

The specific heat of platinum; the index of refraction of air; the total radiation from a black body; the radiated energy corresponding to a given wave-length in the radiation from a black body.

(i.) *Specific Heat of Platinum.*—The dependence upon temperature of the mean specific heat of platinum from 0° was found by Violle (117) with the aid of Deville's constant-volume air-thermometer with porcelain bulb, to be given by the equation

$$C = 0{\cdot}0317 + 6 \times 10^{-6}t$$

from 100° to 1177°. The specific heat of platinum was then used by Violle as a secondary thermometric property for the determination of the melting-points of silver, gold, and copper (119); also, by extrapolation, for the melting-points of palladium (118), platinum (117), and iridium. The values are now only of historical interest.

The measurement of heat energy is logically one of the simplest of physical measurements. Practically, it is one of the most difficult, so elusive and unconfinable is the thing which it is desired to measure. For this reason, and also because of difficulties in the manipulation, thermometry by calorimetric methods has never become popular.

(ii.) *Index of Refraction of Air.*—Experiments at pressures of 1 to 19 atmospheres and at temperatures of 0° to 80° C. had shown that within the limits of error of the measurements the refraction $n-1$ of air and of cyanogen gas was proportional to the specific volume of the gas whether the change in specific volume was produced by pressure or by heat. The measurement of refractive index thus offered a means of realising a secondary constant-pressure gas-thermometer.

D. Berthelot in 1898 utilised this fact in a "gas-thermometer" (10, 11), with which he measured the melting-points of silver and gold by direct comparison (12).

Strictly speaking, the method is not only secondary, but as used by Berthelot constituted a wide extrapolation of a law established experimentally only at low temperatures. Investigations made since the date of Berthelot's experiments indicate that gases follow the Lorenz-Lorentz formula for the relation of refractive index to density at constant temperature; when the temperature is changed, an effect independent of the diminution in density with rising temperature is indicated, but its amount and magnitude are by no means certain.[1]

The results obtained by Berthelot seem to have had a precision of about ±1·5° at 1060°. The "end corrections" of the furnace tube constitute the most serious source of error. The method has not been further used since the time of his experiments.

(iii.) *Total Radiation of a Black Body.*—E, the total energy radiated by a black body per second per unit of surface, and θ, the absolute temperature, are related according to the Stefan-Boltzmann law:

$$E = \sigma\theta^4,$$

in which σ is a constant.

Although originally discovered by Stefan as an empirical relation, this law is from one point of view not in need of confirmation by comparison with a gas-thermometer, for Boltzmann subsequently showed that this relation between temperature and radiated energy depends only upon the principles of thermodynamics and the electromagnetic theory. If these be taken as fundamental, the Stefan-Boltzmann law might therefore itself be used as an independent definition of θ.

For this purpose it is necessary only to go back to Part I. and, starting with the definition of the absolute thermodynamic scale and the arbitrary numerical difference of 100° between melting ice and boiling water, to make measurements of radiation at those temperatures. The scale is then completely determined.

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[1] See discussion in Loria, *Die Lichtbrechung in Gasen*, pp. 29-39 (Braunschweig, 1914).

The experimental precision attainable in radiation measurements at 0° and 100° is, however, too low to permit of a successful independent definition of a temperature scale by this means. It is necessary, in order to obtain an accurate value for σ or an accurate ratio of θ's, to make absolute energy measurements at higher temperatures, or relative energy measurements at temperatures widely separated, such as 1500° and 1000°. The total radiation scale thereupon becomes, in effect, a secondary scale, dependent upon the gas-thermometer for the definition of its basic temperatures.

But the radiation scale is not dependent upon the gas-thermometer to the extent that the platinum-resistance thermometer is dependent. It has a sufficiently sound and well-established theoretical basis to have attained a practically independent status. If the gas-thermometer ventures too far into the high-temperature region where it is losing its precision, but where the total radiation thermometer is gaining precision or at least holding its own, the gas-thermometer will reach a point where it can no longer compete with its rival. Part of the present disagreement over the temperature scale above 1200° can be reduced to this simple question: Is the gas-thermometer or the radiation thermometer ahead in this race at the 1500° level? It is generally admitted that the gas-thermometer scale has no competitors at 1000° and the radiation-thermometer scale none at 2000°. In the intervening range the favour of the experimental physicist will turn toward the scale which possesses the greater experimental precision.

In answer to this question, an examination of the experimental data shows that the gas-thermometer still has the advantage at 1500°. Its indications certainly are capable of an accuracy at that temperature of better than 2°. To obtain this accuracy on the radiation scale calls for measurements equivalent to the determination of σ to within 0·45 per cent, or to within $0{\cdot}026 \times 10^{-12}$, taking the value of σ as $5{\cdot}70 \times 10^{-12}$. The value of σ or the measurements of intensity-ratios are hardly yet certain to that degree of accuracy.

As for the constant itself, no direct determination of σ at high temperatures with direct reference to a gas-thermometer has ever been made. Valentiner's determinations (115) are perhaps the most closely related to gas-thermometer temperatures, being in terms of platinum - platinrhodium thermocouples standardised by comparison with Holborn and Valentiner's constant-volume nitrogen thermometer up to 1600°. Valentiner's first result was admittedly inaccurate by 4 per cent (116), and Coblentz believes that "a conservative estimate of the total correction to Valentiner's original data is 5 to 6 per cent."

A discussion of the twelve or more other determinations of σ need hardly be undertaken here, as the subject is more fully considered in the article on Radiation Laws. Reference need only be made to Coblentz's detailed summary (30) of the status of the constant. The original determinations of $10^{12}\sigma$ quoted by Coblentz range from 5·30 to 6·51. These data he has recalculated, using corrections obtained from his own extensive experiments (26, 29), thereby reducing the range to 5·48-5·9, leaving several uncertain. The mean, 5·7, of so motley a collection would have very little authority did it not happily coincide with the much more accurate value, 5·722 $\pm 0{\cdot}012$, determined by Coblentz[1] (31); this is, again, in excellent agreement with several fundamental physical constants, as shown by Millikan (84).

(iv.) *The Energy corresponding to a given Wave-length in the Radiation from a Black Body.*—From the Wien-Planck law, as revised by Planck (100), for the distribution of energy in the spectrum of a black body, it follows that if E_1 represents the energy intensity, corresponding to the wave-length λ, radiated from a black body at absolute temperature θ_1, and E_2 the corresponding energy for the same wave-length, but at temperature θ_2, then

$$\frac{E_1}{E_2} = \frac{e^{C_2/\lambda\theta_2} - 1}{e^{C_2/\lambda\theta_1} - 1}.$$

For values of λ in the visible portion of the spectrum and temperatures attainable in the laboratory, the effect of neglecting the term -1 is less than 1 per cent in the ratio, corresponding to less than 1° at 1500°; this simplification is therefore usually made, and the relation put into the form

$$\frac{1}{\theta_2} - \frac{1}{\theta_1} = \frac{\lambda}{C_2} \log_e \frac{E_1}{E_2},$$

which is the usual equation for the optical pyrometer.

To what extent is the Planck radiation law a direct deduction from the Stefan-Boltzmann radiation law? Both Wien and Planck derived their original law from purely theoretical grounds, but experimental results at long wave-lengths showed that the "constant" C_2 in the original form was not a constant (78). Planck then revised the derivation, again on a purely theoretical basis, but not without assumptions which may or may not be true. The conservative view that should be taken in defining a temperature scale must therefore consider the Planck radiation law to be strictly experimental, valid over the range and to the degree of accuracy for which it has been experimentally proved.

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[1] See article "Radiation, Determination of the Constants," Table I. and § (10), Vol. IV.

From this standpoint any temperature scale based on the law is a purely secondary scale, no more accurate and entitled to no greater validity than the gas-thermometer temperatures upon which the value of the constant C_2 is based. It is evident, furthermore, that when they are viewed as secondary scales the optical scale (with constant C_2) is logically in a much less favourable position than the total radiation scale (with constant σ), since ratios of total radiation can be determined with a temperature range of say 100° to 1200°, whereas the corresponding range for C_2 must begin within the region of visible radiation due to temperature, or at about 700°, and must be further handicapped by the rapid fall in sensitivity of the optical measurement when the temperature is lowered from 1200° to 700°.

There has been no determination of C_2 accompanied by direct measurement of the temperature by a gas-thermometer, but several determinations have been closely related to gas-thermometer temperatures by the transfer of temperature with thermocouples.

The first of these was by Lummer and Pringsheim in 1900 at the Reichsanstalt (79), using temperatures transferred by platinum-platinrhodium couples from Holborn and Day's nitrogen thermometer. They obtained values of C_2 for different wave-lengths, from measurements at 790° to 1430° C., varying from 1·450 to 1·469, with a mean of 1·458.

Holborn and Valentiner (65) in 1907 likewise transferred their temperatures thermoelectrically from their nitrogen thermometer to the black body. Their temperatures ranged from 1100° to 1685°, and the values of C_2 obtained ranged from about 1·38 to about 1·46; mean, $1{\cdot}420 \pm 0{\cdot}014$.

Another step removed from the gas-thermometer are those determinations based upon fixed temperatures, such as melting-points, which have been previously determined by a gas-thermometer.

Nernst and von Wartenberg's photometric measurements (85) in 1906, at wave-length 0·590 μ and at the melting-points of gold and palladium (1063° and 1550° C., respectively, by the nitrogen thermometer), correspond to a value of 1·438 for C_2.

Hoffmann and Meissner (58, 59) in 1912 and 1913 made spectrophotometric observations in various wave-lengths of light at the melting-points of gold and palladium. On the same temperature basis as above, their data correspond to values of C_2 from 1·440 to 1·447.

Hyde, Cady, and Forsythe (70, 71), likewise, have measured with a Holborn-Kurlbaum pyrometer the ratio of intensities at the gold and palladium points, at an effective wave-length of 0·666 μ. The result corresponds to $C_2 = 1{\cdot}447$.

Similar measurements by Mendenhall (83) correspond to a value of 1·439. Another series, based upon the gas-thermometer temperature of 1330° C., and a second temperature 2460° C. based upon total radiation, also gave the value 1·439.

Mendenhall's second series is a step further removed from direct dependence on the gas-thermometer, being based on only one fixed temperature. The same is true of the series of measurements by Warburg, Leithaeuser, Hupka, and Mueller (126) at the Reichsanstalt in 1913. Their basic point was the gold melting-point, but higher temperatures (1400° and 1970° C.) were obtained through the use of Wien's "displacement law," in the form

$$E_{\max.} = B\theta^5.$$

These measurements, at wave-lengths of 0·656 to 2·172 μ, gave a value for C_2 of $1{\cdot}437 \pm 0{\cdot}004$.

Coblentz's determinations (26) in 1913 were based essentially on 1550° as the melting-point of palladium, although the original temperature determinations were made with thermocouples standardised at 1083° and lower, and used for higher temperatures by extrapolation with assumed corrections. Subsequent standardisation at the palladium point furnished a sound temperature basis. The original value for C_2, 1·447, has been modified by more recently determined corrections for refractive index and bolometer setting and is now[1] (28) placed at 1·432.

The mean of all the above determinations is about $1{\cdot}438 \pm 0{\cdot}008$; or $1{\cdot}439 \pm 0{\cdot}003$ omitting the abnormally high value of Lummer and Pringsheim and the abnormally low value of Holborn and Valentiner. An accuracy of 2° at 1550° (melting-point of palladium) corresponds to 0·0049 in C_2, or 0·34 per cent. The average deviation of the more recent values from their mean is somewhat less than this.

The possibilities for error in C_2 when determined on the basis of fixed temperatures already standardised by the gas-thermometer are of two kinds: (1) those due to the fixed temperature reproduction, (2) those due to the intensity measurements.

Gold and palladium are the metals commonly used for the fixed melting-points. Gold is easily obtained pure, and accumulated experience has shown that its temperature is readily reproducible within the limits of error of the measurements. The same is not true of palladium. It is difficult to purify, and samples of high chemical purity are not readily obtainable.

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[1] See article "Radiation, Determination of the Constants," §§ (11), (12), Vol. IV.

Impure palladium usually melts higher than pure, as pointed out by Adams (2), and the tendency of impure samples is therefore toward lower values for C_2. In none of the determinations cited above was the palladium analysed. Only in connection with Day and Sosman's nitrogen-thermometer measurement of the melting-point was its purity determined.

This disadvantage can to a certain extent be overcome by actual comparison of the samples of palladium used with the nitrogen-thermometer sample. This has been done in the cases of Warburg's and Forsythe's measurements. A new platinum-platinrhodium thermocouple, calibrated at the palladium point at the Geophysical Laboratory, agreed satisfactorily with Warburg's temperature scale.

A sample of the nitrogen-thermometer palladium, sent to Dr. Forsythe, proves to have a melting-point averaging within 1° of Forsythe's sample, the range of 15 determinations on each being about 5°. The two samples are therefore identical, as nearly as can be told by this comparison.[1]

The other class of errors, those arising from the intensity measurements, includes many small uncertainties that must be taken account of in fundamental work, though often overlooked in past determinations of the constant. An excellent discussion of these errors, particularly with reference to the disappearing-filament type of pyrometer, has been given by Forsythe (49).

We have seen that the average deviation of recent measurements of C_2 from the mean value is of an order of magnitude equivalent to about 2° at 1550°, and that the uncertainties due to the normal variability of the palladium melting-point and to the various sources of error in the optical observations are more than sufficient to account for such a deviation in any one measurement. There appears to be no reason from the thermometric standpoint for adopting any other value for C_2 than the mean value given above, namely, 1·439, nor for adopting any other value for the palladium point than that derived from the nitrogen thermometer,[2] namely, 1550° C.

The real pressure toward adopting a lower value of C_2 (1·420 to 1·435) arises from the better theoretical agreement between the lower value and the accepted values for Planck's "Quantum" h, the charge of an electron e, the Stefan-Boltzmann radiation constant σ, and other related constants. Whether this is a valid reason for changing the value of C_2 depends to a large extent upon the point of view.

The difference is not large—far too small to be of any industrial importance as yet—but is important enough to deserve considerable further investigation from all points of view, thermometric, theoretical, and radiometric. The investigations of the past few years have been too exclusively occupied with the latter two.

As to the best choice of C_2 in the meanwhile, the history of research furnishes little guidance, for instances can be brought forward in about equal number, on the one hand of good theoretical judgment which disregarded existing data and was afterwards justified by the results of more exact measurement, and on the other hand of apparent discrepancies in experimental results which, though carefully explained away by the experimenter himself, subsequently proved his experimental accuracy to have been better than his judgment.

§ (42) COMBINED THERMAL PROPERTIES AND FIXED TEMPERATURES.—There are two thermal properties which, while not *independently* reproducible with the accuracy desired for thermometry, are nevertheless, when combined with standard fixed temperatures, the main reliance of investigators in the range above 500°. These are (1) the thermal E.M.F. of platinum-group metal thermocouples, among which the Le Chatelier couple, platinum against an alloy of 90 parts platinum and 10 parts rhodium, is preeminent; the thermocouples are limited in working range mainly by the melting-temperature of the wires, but also by the contamination of the wires with impurities due to reduction or volatilisation of neighbouring materials; (2) the electrical resistance of pure platinum, limited in its working range to a maximum temperature of 1100° C.

(i.) *Platinum-metal Thermocouples.*—In the preparation and use of thermo-elements Barus made much more extensive and elaborate studies than any one who has followed him. He first investigated a great number of substances, both pure metals and alloys, and measured and tabulated their electromotive forces for different absolute temperatures. From these a couple made from pure platinum and an alloy containing 90 parts platinum and 10 parts of iridium was finally selected for his standard work with the gas-thermometer (5, 8).

It is an unfortunate accident that history has failed to record Barus's name along with that of Le Chatelier (77) in the development of the thermo-element for purposes of high-temperature measurement. There is no question that Barus contributed enormously to our knowledge of the thermoelectric properties of the different metals and their use, but the 10 per cent iridium alloy which he finally selected proved to be less serviceable than the 10 per cent rhodium alloy developed by Le Chatelier, probably by reason of the greater volatility of the iridium and a consequent slow change in its readings. And so we find the Le Chatelier platinrhodium thermo-element in use to-day the world over, while the magnificent pioneer work of Barus remains but little known.

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[1] Private communication.

[2] The actual determination is 1549°, but the probabilities favour the existence of a small positive correction of the order of 1° or less to convert constant-volume nitrogen temperatures to thermodynamic.

The first calibration of the platinum-platinrhodium couple was made by Holborn and Wien (66, 67) in 1892. They found that the E.M.F. could be represented in terms of temperature by a cubic formula, and that the scale could be reproduced with the aid of melting-points of metals, with an accuracy of 5°, over the range 400° to 1440°.

Holborn and Day (62), though their maximum temperature (1100°) was not as high as that of Holborn and Wien, considerably improved the accuracy of the comparison, and were able to show that the formula $e = -a + bt + ct^2$ holds within 1° over the entire range from 400° to 1100°. Similar comparisons by Harker (51) in 1904 confirmed this empirical relation for the range 400° to 1000°.

Day and Clement's measurements (34) still further improved the accuracy of the measurements, in showing that the quadratic formula is followed from 400° to 1100°, within 0·5°. A certain drift of the deviations was also observed which may indicate that a more complex formula is needed to represent the relation of e and t even more closely than to 0·3°. Data are not yet available for testing this question to a higher degree of accuracy.

The measurements of Day and Sosman were devoted primarily to determining the values of chosen fixed points, using the thermocouple only as a transfer device. The large number of points at small intervals of temperature, which are called for to really test the possible deviation (within 0·3°) from the quadratic formula, is lacking in the data of Day and Sosman because of this concentration of most of the measurements at fixed points. With this limitation (which is equally true of most of the other experimental confirmations of empirical laws in thermometry) the data of Day and Sosman furnish a good test of the quadratic equation, since the substitution of a platinrhodium alloy in place of platiniridium as the bulb material considerably increased the attainable precision with thermocouples, removing as it did the serious source of error due to contamination of the couples with iridium. The data show that the quadratic formula represents the mode of variation of e with t over the range from 300° to 1100° with an accuracy of 0·3°, while a similar formula with different constants will cover the range 1000° to 1550° within 1°. A cubic equation can be made to fit all the points from 300° to 1550° with a deviation of a little over 1°.

(ii.) *Electrical Resistance of Platinum Wire.*—The only direct calibration of a platinum-resistance thermometer by reference to a gas-thermometer that has ever been made in the range above 500° is the comparison by Harker (51) in 1904. Using a constant-volume nitrogen thermometer with porcelain bulbs, Harker showed that between 400° and 1000° the quadratic formula for the platinum resistance yielded temperatures that were within 1° of the gas-thermometer at the lower temperatures, but deviated by more than the probable error in the upper part of the range, the difference reaching a maximum of 6·3°.

In the ultimate analysis, then, confidence in the platinum-resistance thermometer above 500° depends on the degree of accuracy with which it can interpolate the melting-points of antimony and silver between the two base-points, 444·55° (sulphur) and 1063° (gold). A really exact standardisation, having a precision comparable with that of which both the gas-thermometer and platinum-resistance thermometer are now capable, has yet to be made.

(iii.) *Interpolated Fixed Points.*—As already pointed out in preceding sections, determinations of fixed temperatures may be roughly divided into four classes: (1) those determined directly by immersion of the gas-thermometer in the constant-temperature system (sulphur vapour, for example); (2) those determined indirectly by transferring the temperature from gas-thermometer to fixed point by means of an intermediary transfer device; (3) those determined by first establishing the law of variation of a secondary thermometer (thermo-element, for example) and then using this thermometer to interpolate the fixed points; (4) those "secondary interpolations" made by calibrating a secondary thermometer at several known fixed points determined by methods (1), (2), or (3), and then using it for further interpolation.

"Primary interpolations" of class (3) were made by Holborn and Wien, Holborn and Day, and Day, Clement, and Sosman by means of thermocouples.

Several "secondary interpolations" of class (4) (Heycock and Neville, Waidner and Burgess) have been made with platinum-resistance thermometers. These were originally made as *extrapolations* of the Callendar formula over the range from 500° to 1100°, an extrapolation more than equal to the range for which the law had been established. The melting-point data so obtained are often quoted as independent determinations of the points, side by side with gas-thermometer determinations. Strictly speaking, they are not determinations at all, but merely hopeful estimates. Such an extrapolation of an empirical law to cover an additional range more than equal to that for which it has been established is justifiable only as a temporary measure. That it does sometimes succeed is shown by the unexpectedly close agreement between the melting-points extrapolated with the platinum-resistance thermometer and the actual determinations made later with the gas-thermometer. Usually the attempt has failed, as in the case of platinum resistance below 0°, and of the thermo-element above 1100°, but the erroneous results have been quickly forgotten, and have done harm only by producing temporary confusion. The original comparisons have permanent value, furthermore, when the extension of the gas-thermometer scale establishes the true law of variation.

A number of primary and secondary interpolated melting-points are shown in Table 13, compared with direct or indirect gas-thermometer determinations of the same points.

TABLE 13

INTERPOLATED VALUES OF FIXED POINTS IN THE RANGE 500° TO 1600° COMPARED WITH GAS-THERMOMETER DETERMINATIONS

Author.	Date.	M.P. Antimony.	M.P. Aluminium.	M.P. Silver.	M.P. Gold.	M.P. Copper.	Basis.
Direct or Indirect Gas-thermometer Determinations.							
Jaquerod and Perrot .	1905	..	..	..	1067·4	..	Constant-volume N, O, CO, and CO_2.
Day, Sosman, and Allen	1910–12	630·0	..	960·0	1062·4	1082·6	Constant-volume nitrogen.
Primary Interpolations.							
Holborn and Wien .	1892	..	..	968	1072	1082	Pt : PtRh couple and constant-volume air thermometer.
Holborn and Day .	1900	630·6	..	961	1064	1084	Pt : Pt Rh couple and constant-volume nitrogen thermometer.
Day and Sosman .	1911	..	658·7		..	..	
Secondary Interpolations.							
Heycock and Neville .	1895	630·2	655·3	962	1063·7	..	Pt res. thermometer between 444·55° and 1083·0°.
Waidner and Burgess .	1910	630·7	658·0	960·9	..	..	

§ (43) SUMMARY FOR THE RANGE 500° to 1600°. (i.) *Thermoelectric Thermometry.*—The best secondary thermometer for use in the range 500° to 1600° is the platinum : platinrhodium thermo-element. The E.M.F. of the couple can be represented in terms of the constant-volume nitrogen scale by a quadratic equation over limited parts of the range (300° to 1100° and 1000° to 1550°) with an accuracy of within 0·3° in the lower part of the range and within 2° at 1500°. A cubic equation will represent all temperatures from 300° to 1550° with somewhat less accuracy. By calibration at three points (such as the melting-points of zinc, gold, and palladium), and the use of a standard table such as that of Adams (1), nitrogen scale temperatures can be measured within 0·3° at 500° and 3° at 1500°.

(ii.) *Platinum-resistance Thermometry.*—Nitrogen scale temperatures between 500° and 1100° can be measured with a platinum-resistance thermometer made of pure platinum and standardised at the sulphur boiling-point and the gold melting-point within 0·3° at 500° and 1° at 1000°.

(iii.) *Optical and Radiation Thermometry.*—By calibration at the melting-points of gold and palladium, nitrogen scale temperatures can be measured optically or by total radiation, when the various correction factors for absorption, etc., are taken into account, within 2° at 1000° and 5° at 1600°.

(iv.) *Fixed Points.*—The following fixed points may be used to establish and maintain the temperature scale from 500° to 1600°, and are known on the centigrade thermodynamic scale with the accuracy indicated. It is assumed that the substances used are the purest obtainable commercially.

Melting-point of antimony . . .	630 ± 0·5°
" " aluminium . . .	659 ± 0·5°
" " silver	960 ± 0·5°
" " gold	1063 ± 0·5°
" " copper	1083 ± 1°
" " diopside ($CaMgSi_2O_6$) .	1391 ± 2°
" " palladium . . .	1550 ± 3°

Some of these substances are now obtainable as certified standards for thermometric calibration (for example, aluminium from the Bureau of Standards of the United States).

(v.) *Thermodynamic Scale.*—The corrections to convert readings on the constant-volume gas scales to the thermodynamic scale at 1000° and higher are very uncertain. The correction to the constant-volume nitrogen scale at initial pressure 500 mm. is probably of the order of magnitude of +0·5° or less at 1200°.

IX. THE SOURCES OF ERROR IN GAS THERMOMETRY

§ (44) CONSTANT-PRESSURE THERMOMETER.—The development of the constant-pressure gas-thermometer has remained almost wholly in the hands of Callendar and his associates. From its inception in 1886 to the last important publication in 1914 no serious attempt has been made to extend the use of the instrument to the higher temperatures (above the sulphur boiling-point). For this reason, perhaps, the sources of error in this system of procedure have remained undeveloped and cannot be examined with the same sharp scrutiny which may now be applied to the sources of error in the constant-volume instrument. In principle the apparatus is sound, and when judged from theoretical considerations alone has been accorded the preference by several writers on the subject. It has also been stoutly defended by Callendar on several explicit grounds: (1) both the apparatus and the calculation are simple; (2) the internal pressure upon the bulb does not increase with the temperature to be measured; (3) the accuracy is limited only by the precision with which weighings can be made.

TABLE 14

CONSTANT CORRECTIONS TO THE CONSTANT-VOLUME GAS-THERMOMETER (DEGREES)

Temp. ° C.	Unheated Space Ratio assumed $v/V_0=0·01$.		Thermal Expansion.						Elastic Expansion assumed $=10^{-7}$ per mm.	
			Pt – Ir.		Glass 59 III.		Fused Silica.			
	Δ.	Δ'.	Δ.	Δ'.	Δ.	Δ'.	Δ.	Δ'.	Δ.	Δ'.
− 270	− 0·03	+ 3·4	..	..	..	..	..	..	−0·00	+0·04
− 200	− 0·50	+ 2·1	..	..	..	..	..	..	−0·01	+0·02
− 191	− 0·54	+ 1·9	− 0·38	+ 1·4	−0·22	+0·78	+0·01	−0·04	−0·01	+0·02
− 100	− 0·59	+ 0·69	..	..	..	..	..	..	−0·01	+0·01
0	0·00	0·00	0·00	0·00	0·00	0·00	0·00	0·00	0·00	0·00
+ 100	+ 1·3	0·00	+ 0·92	0·00	+0·61	0·00	+0·06	0·00	+0·01	0·00
+ 200	+ 3·3	+ 0·69	+ 2·4	+ 0·54	+1·7	+0·38	+0·15	+0·03	+0·03	+0·01
+ 500	+13	+ 6·9	+10	+ 5·7	+7·3	+4·0	+0·63	+0·32	+0·14	+0·07
+1000	+44	+31	+37	+27	..	..	+2·1	+1·5	+0·47	+0·33
+1500	+91	+72	+82	+68	..	..	..	..	+0·98	+0·77

Over against this may be set the fact that results obtained with the instrument have not been satisfactorily concordant, even though observations with it have been confined to a short range of easily accessible temperatures. The sources of error thus far recognised by Callendar and his active coadjutor Eumorfopoulos are mainly three: (1) a permanent change in the volume of the bulb after each exposure at the higher temperatures; (2) changes and uncertainty in the expansion coefficient of the bulb material (18) (chiefly glass and porcelain); (3) inexact knowledge of the expansion coefficient of mercury through an adequate temperature range. These errors are of a kind which can be greatly reduced in magnitude by a judicious selection of bulb material and careful study, and the constant-pressure gas-thermometer may yet prove to be a most valuable instrument for the determination of standard high temperatures. At the moment, however, one must regard it as an untried instrument in the region where further work is mainly needed (above 1200°) and no experience is available through which to forecast its behaviour there.

§ (45) CONSTANT-VOLUME THERMOMETER.—The sources of error in the constant-volume gas-thermometer, on the contrary, are now so well known that no more than a brief review of them is needed here. In the comprehensive treatment of gas thermometry by Henning (56) three primary correction factors in which lie sources of error have been emphasised and their magnitude tabulated for various temperatures. The data are contained in Table 14, which will serve to show the magnitude to which these correction factors attain in common practice; the errors arising therefrom are in most cases naturally a small fraction of the correction stated. The corrections marked Δ in the table are on the assumption that the pressure-coefficient of the gas, β, is independently known; those marked Δ′ are on the assumption that β has been determined in the same apparatus.

Of these magnitudes it should be stated that the first, v/V_0, the ratio of the volume of the connecting tube between bulb and manometer to that of the bulb, was reduced by Day and Sosman to about one-sixth of the value assumed by Henning in the above computation, and its temperature and volume are determinable with such accuracy that this can hardly be accounted a major source of error now.

The expansion coefficient of the bulb material will always be a factor requiring precise determination. To determine the expansion coefficient of a stable and well-defined solid to a few parts in a thousand even over a very great temperature range offers no difficulty to-day, but the solids which are chosen for their refractory quality or for their impermeability to certain gases may be neither stable nor well-defined. Witness, for example, the materials cited in the above table which are those now most commonly used for high-temperature gas-thermometry. Silica glass, and indeed all glasses, are in unstable equilibrium in which normal molecular forces are held in check by extreme viscosity. Diminish this viscosity by exposure to a sufficiently high temperature and the instability will appear as a more or less complete change of physical state. Silica glass, for example, begins to crystallise at an appreciable rate at about 1100°, while at higher temperatures the change becomes rapid. More complicated glasses such as Jena 59^{III} develop various crystalline compounds when heated. Platiniridium is a stable alloy at low temperatures but in the higher ranges loses iridium and may take on iron or silicon if present in a reducing atmosphere. With adequate protection against these contingencies, that is to say, in an appropriate atmosphere and temperature range, each of these substances has a determinable expansion coefficient through which no error of appreciable magnitude need enter the temperature determination. The effort to stretch the range of measurements with a particular bulb to temperatures within the region where the material becomes measurably weak or incipient crystallisation occurs (in glasses) or a finely crystalline alloy becomes coarsely crystalline, brings uncertain expansion, permanent changes of the ice-point, deformation, and like uncertainties impossible to measure and difficult to appraise.

The elastic expansion of the bulb under change of internal pressure need give no further concern. Theoretically it does not arise in the constant-pressure thermometer, though Callendar's observations (with glass bulbs) appear not to have been entirely free from the effects of volume-changes at any time. Practically it may be completely avoided in the constant-volume thermometer by enclosing the bulb in a bomb in which the pressure outside the bulb is regulated to correspond to the pressure within, as was done by Day and Sosman. The principal uncertainties met with in constant-volume gas thermometry to-day, assuming that the bulb has been chosen appropriately for the temperature range to be measured and full advantage is taken of existing experience, are not sources of error properly inherent in the gas-thermometer system. They are (1) the temperature distribution about the bulb, *i.e.* the *uniformity of temperature* in the space to be measured; and (2) the uniformity of temperature distribution about the mercury manometers. After a long experience with high-temperature measurement it is our impression that a space sufficient to enclose a 200 c.c. bulb, *all* of which has a temperature of 1500° C. ±5°, has perhaps never been available to a student of gas-thermometry. How then shall we measure such a temperature with this precision? Holborn and Valentiner record the fact that in their attempt to reach 1600° with a bulb of pure iridium, differences of temperature were observed on the bulb surface amounting to as much as 60°. How much greater than this the temperature variations in the zone of measurement might have been without the integrating effect of the bulb wall of metallic iridium it is impossible to say. The first necessity in the attainment of precise temperature definition at 1500°, which happens to be the temperature region in which most of the recent gas-thermometry has been carried out, is a suitable space *uniformly* heated to 1500°. Given this, and the errors discussed above become relatively insignificant magnitudes.

In the same sense, future gas-thermometry must place the long columns of mercury forming the manometer in a thermostat where the temperature can be properly controlled. It is no longer necessary at the highest temperatures that the gas pressure within the bulb should not exceed one atmosphere lest the bulb become strained and the "constant volume" be jeopardised. Two or three times this pressure may be used equally well and a corresponding increase of sensitiveness attained, but the increased length of the measuring column of mercury must not introduce errors through uncertainties in the mercury temperature. A further advantage in simplified technique would also be attained if a closed-tube manometer were substituted for the more usual open-tube manometer; in many cases this has not been done hitherto, chiefly because it would have added the barometric height to the already long mercury column and the difficulty of temperature control of the column would have been increased.

The relative magnitude of the accidental sources of error in a typical set of measurements with the constant-volume gas-thermometer are shown in Table 15, compiled by Day and Sosman in connection with their work on the nitrogen thermometer in the range 500° to 1550°.

TABLE 15

EXAMPLE SHOWING ACCIDENTAL ERRORS IN GAS-THERMOMETRY. ESTIMATED ERRORS OF A CONSTANT-VOLUME NITROGEN-THERMOMETER AT INITIAL PRESSURE OF ABOUT 200 MM. MERCURY (37)

Bulb material: 80 platinum, 20 rhodium.
Volume of bulb: $V_0 = 206$ c.c.
Volume of unheated space: $v_1 = 0{\cdot}31$ c.c.
Ratio $v_1/V_0 = 0{\cdot}0015$.
Furnace: electrically heated air-bath.
Manometer: open type. Barometer read separately.
Pressure-coefficient of gas and expansion-coefficient of bulb assumed known.
Temperature transferred to fixed points by platinum : platinrhodium thermocouples.

Quantity Affected.	Source of Error.	Amount of Error.		Effect on *t*.	
		At 400°.	At 1500°.	At 400°.	At 1500°.
(A) Temperature of gas	Temperature differences over bulb surface	2 mv.	5 mv.	±0·2°	±0·4°
	Variability	0	1 „	0	±0·1
(B) p_0	Reference point	0·02 mm.	0·02 mm.	±0·04	±0·15
	Manometer setting	0·02 „	0·02 „	±0·04	±0·15
	Scale corrections	0·01 „	0·01 „	±0·02	±0·07
	Temperature of mercury	0·03 „	0·03 „	±0·06	±0·23
	Barometer setting	0·03 „	0·03 „	±0·06	±0·23
	Temperature of barometer	0·03 „	0·03 „	±0·06	±0·23
	Permanent variations in p_0	0	0 to ·05 ,	0	0 to ±0·3
(B) p	Reference point	0·02 „	0·02 „	±0·02	0
	Manometer setting	0·02 „	0·02 „	±0·02	0
	Scale corrections	0·02 „	0·02 „	±0·02	0
	Temperature of mercury	0·07 „	0·20 „	±0·07	±0·05
	Barometer setting	0·03 „	0·03 „	±0·03	±0·01
	Barometer temperature	0·03 „	0·03 „	±0·03	±0·01
	Unheated space, v_1	0·020 c.c.	0·020 c.c.	±0·07	±0·5
	Unheated space, t_1	0·5-50°	0·5-100°	±0·01	±0·1
(C) E.M.F. . . .	Instrumental corrections	1 mv.	2 mv.	±0·1	±0·2
	Contamination	0	0-12 mv.	0	0 to +1·0
	Integration over bulb	5 mv.	12 mv.	±0·5	±1·0
(D) Fixed points .	Instrumental corrections	1 mv.	2 mv.	±0·1	±0·2
	Contamination	0	0-10 mv.	0	0 to −1·0
	Variation in given charge	Specific, 1-10 mv.		Specific, 0·1 to 1·0	
	Variation between different charges	Specific, 1-20 „		Specific, 0·1 to 2·0	

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TEMPERATURE, SECONDARY STANDARDS COMPARED WITH GAS-THERMOMETERS FOR VARIOUS RANGES. See "Temperature, Realisation of Absolute Scale of," §§ (27), (34), (38), (39).

TEMPERATURE, STANDARDS OF:
  Primary: mercury thermometers of *verre dur*, constructed by Baudin of Paris for the International Bureau of Weights and Measures at Sèvres, the corrections to which over the range 0° to 100° C. have been carefully ascertained, so that they may serve as fundamental temperature standards over this range. See "Thermometry," § (4).
  Secondary, available for general use and compared with gas-thermometers. See "Temperature, Realisation of Absolute Scale of," § (23).
  Tests of, by Comparison with Standards. See "Thermometry," § (11).

TENSION MEMBERS IN STRUCTURES. See "Structures, Strength of," § (14).

TENSION TEST. See "Elastic Constants, Determination of."
  Data ordinarily observed. § (20).
  Definition of the Yield Point. § (21).
  The Maximum Load. § (22).
  Method of reporting Results. § (25).
  Quality Factors. § (28).
  Results of Tests on Alloy Steels at High Temperatures. § (117), Table 41.
  Results of Tests on Mild Steel at Varying Temperatures. § (117), Table 40.
  Results of Tests at Varying Temperatures. §§ (117) and (118).

TEST BARS:
  Forms of Bar for Tension Tests. See "Elastic Constants, Determination of," § (26).
  Forms of Enlarged Ends for Tension Tests. See *ibid.* § (27).

TEST PIECES—PREPARATION AND SELECTION OF. See "Elastic Constants, Determination of," § (19).

TEST RESULTS:
  Apparatus for determining the Effect of Temperature on Test Results. See "Elastic Constants, Determination of," § (113).
  Influence of Form of Test Piece on Results. See *ibid.* § (41).
  Influence of Time of Testing on Results. See *ibid.* § (42).

TESTING MACHINES. See "Elastic Constants, Determination of."
  Amsler Testing Machine. § (9) (i.).
  Arnold Machine for Alternate Bending Tests. § (78) (i.).
  Arrangement of the Lever for Lever Testing Machines. § (5).
  Avery 700,000 lbs. Horizontal Machine. § (7) (vi.).
  Brinell Hardness. §§ (81)-(84).
  Calibration by Means of a "Standardising Box." § (13) (v.).
  Calibration by the Use of a Series of Test Pieces. § (13) (iii.).
  Calibration by the Use of Crushers. § (13) (ii.).
  Calibration by a Standard Test Bar. § (13) (i.).
  Methods of Calibration. § (13).
  The Charpy Pendulum Impact Machine. § (100) (ii.).
  Compression Shackles. § (12) (viii.).
  Dead Load Calibration of the Emery Machine. § (8) (v.).
  Dynamic Hardness. § (89).
  Emery Testing Machine. § (8) (i.).
  General Methods of Testing. § (2).
  Grips for holding Test Bars. § (12).
  Grips for testing Chain in Tension. § (12) (vii.).
  Grips for testing Rope. § (12) (vi.).
  Guillery Impact Machine. § (100) (iii.).
  Horizontal Compound Lever Machines. § (7) (iv.).
  Horizontal and Vertical Arrangement of. § (3).
  Izod Impact Tester. § (100) (i.).
  Landgraf-Turner Alternating Bending Impact Machine. § (78) (ii.).
  Machines using Fluid Pressure for estimating the Load. § (9).
  Modern Types of Machines. § (4).
  Olsen's 200,000 lb. Testing Machine. § (7) (ii.).
  Sankey Hand Bending Testing Machine. § (78) (iii.).
  Self-aligning Grips for. § (12) (iii.).
  Shackles for Torsion Tests in. § (12) (xiv.).
  Shear Shackles for Tests on Bar and Plate. § (12) (x.) to (xii.).
  Torsion Tests. § (10).
  Transverse Tests. § (11).

  - Transverse Test Tools. § (12) (ix.).
  - Using a Diaphragm as a Means of Record. § (8).
  - Vertical Compound Lever Machines. § (7).
  - Vertical Single Lever Machines. § (6).
  - Werder Single Lever Horizontal Machine. § (6) (v.).
- Testing Machines for Repeated Stress Tests. See "Elastic Constants, Determination of."
  - Direct Stress Machines employing an Alternating Current. § (71) (iv.).
  - Direct Stress Machines employing an Unbalanced Weight. § (71) (iii.).
  - General Methods. § (71).
  - Rotating Cantilever (Wohler) Type. § (71) (i.).
- Thermal Capacity of a Substance. The quantity of heat required to raise the temperature of unit mass of the substance 1°. It depends to some extent on the range selected.

THERMAL EXPANSION

Most properties of material bodies vary with temperature. Amongst these must be included the size of a body. In the majority of cases this increases with rise in temperature. Notable exceptions are found, however, in the case of water and some aqueous solutions which undergo contraction in the range from 0°-4° C. (approximately); and in that of iodide of silver (resolidified) which has a minimum volume at about 142° C.

In making these statements we are tacitly taking for granted that we possess a satisfactory way of measuring temperature. The methods in use are described in the article on "Thermometry."

Each dimension of the expanding body changes. In the case of isotropic solids (which, as their name implies, have the same properties in all directions) the expansion of a line of unit length drawn in the body is independent of the direction in which it is drawn. Bodies which have this property are either amorphous (*e.g.* glass) or belong to the regular system of crystals (*e.g.* rock-salt, diamond). Metals may usually be treated as isotropic, because although they are partly crystalline the crystals are not arranged in any selective way and the *average* properties are independent of direction.

I. Isotropic Solids

§ (1) Linear Expansion.—Let the distance between two points in the body at 0° C. be l_0 and at t° C. be l_t. If we write

$$l_t = l_0(1 + \lambda t),$$

then λ is usually a positive quantity and is called the mean coefficient of linear expansion between 0 and t° C. It is not a constant, but varies with the range of temperature selected. The value of λ is quite small, and usually special devices must be adopted to enable it to be determined. It is most easily determined in the case of a long rod or wire or strip of the material. The change of length of an iron rod one metre long when raised from 20° C. to 100° C. is about 1 mm. The proportional accuracy with which λ can be obtained is about the same as that with which the change equal to 1 mm. can be measured. Many methods (once regarded as standard ones) have now become historic, having been superseded (methods of Lavoisier and Laplace, Ramsden, etc.). The most satisfactory direct methods are the two following.

§ (2) Material in the form of a Long Rod or Wire or Strip. Comparator Method. —If the body is a rod it may be supported horizontally in such a way that its expansion is not resisted. If a wire or strip, it should be suspended vertically (so as to avoid sagging), a small load being affixed to keep it taut. In each case two fiducial marks are made on the body and the distance apart measured at an observed temperature t_1, either by a cathetometer or by rougher means, according to the accuracy required.

In order that the temperature may be adjusted the body must be surrounded by a jacket through which steam or other vapour may be passed. This jacket must be provided with windows of plane parallel glass or mica placed so that the fiducial lines may be observed through them. Each such line is observed through a separate microscope, provided with an eyepiece micrometer or capable of a small parallel motion in the direction of the expansion, this motion being indicated by a micrometer screw. If d_1 and d_1' are the micrometer readings at a temperature t_1, and d_2 and d_2' are the readings at t_2 (reduced if necessary to the units in which l_1 is measured), then the distance between the fiducial lines at t_2 is $l_1 + (d_2' - d_1') - (d_2 - d_1)$. Calling this l_2, we have

$$l_1 = l_0(1 + \lambda t_1), \quad l_2 = l_0(1 + \lambda t_2),$$

$$l_2 = l_1\frac{1 + \lambda t_2}{1 + \lambda t_1} = l_1(1 + \lambda t_2)(1 - \lambda t_1 +, \text{etc.}).$$

The terms λt_2 and λt_1 are usually so small that this can be written approximately

$$l_2 = l_1[1 + \lambda(t_2 - t_1)].$$

§ (3) Fizeau's Interference Method.[1]—This is an optical method depending upon the colours of thin plates and having the advantage that only small quantities of material are required. It is especially useful therefore for the investigation of crystals. The substance to be investigated is cut into a plate with parallel faces, from 1 to 10 mm. thick. This plate P (*Fig.* 1) rests on a plane metal disc AB, which is supported on three metal screws which pass through it, and in their turn support a second plate (of glass) CD, which can be brought very near the test plate P by adjusting the screws. A beam of light falls perpendicularly on the glass plate, at the second surface of which it is partially reflected; the remainder passing into the air-gap suffers successive reflections backwards and forwards within it, part of it escaping upwards at each incidence upon the glass plate. Interference between these emerging rays produces interference bands formed in the same general way as Newton's rings. Fizeau used white light, and determined the position of the fringes with respect to lines drawn on the under side of the glass plate. Their positions depend upon the thickness of the air-gap between P and CD. In later forms of the device, due to Abbe, the distance screws are replaced by a hollow cylinder (or ring) of quartz cut with its generating lines parallel to the optic axis. The specimen is placed inside this cylinder. The metal and glass plates are replaced by quartz plates.

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[1] *Ann. de Chim. et de Phys.*, 1864, 4e, ii.; 1866, viii. 335.

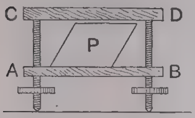

FIG. 1.

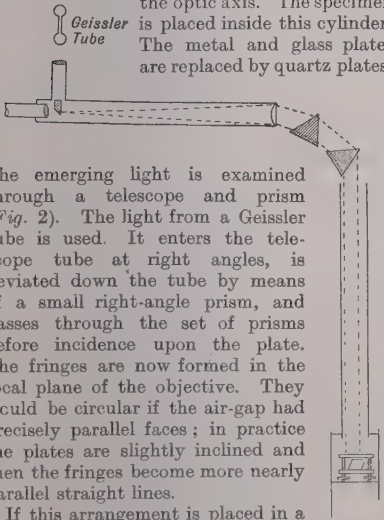

FIG. 2.

The emerging light is examined through a telescope and prism (*Fig.* 2). The light from a Geissler tube is used. It enters the telescope tube at right angles, is deviated down the tube by means of a small right-angle prism, and passes through the set of prisms before incidence upon the plate. The fringes are now formed in the focal plane of the objective. They would be circular if the air-gap had precisely parallel faces; in practice the plates are slightly inclined and then the fringes become more nearly parallel straight lines.

If this arrangement is placed in a chamber which can be heated the fringes are displaced, because the air-gap changes in thickness owing to the differential expansion of the screws and test piece.

By this means very small changes of thickness can be observed. Thus, if the centre of one band is displaced through the distance between two bright bands, the air-gap has changed through half a wave-length of the light employed; *i.e.* for sodium light about ·000029 cm. If the specimen is transparent its lower face should be blackened to prevent reflection at that face.

The expansion of the screws is determined by making observations without the plate of crystal in between. The use of the quartz ring instead of the screws requires a previous careful study of the expansion of quartz. Tutton has modified the apparatus by placing the specimen on a plate of aluminium, the thickness of which is made such that the expansions of the quartz and aluminium just cancel each other. It is not likely, however, that this compensation will be complete at all temperatures.

§ (4) TEMPERATURE VARIATIONS OF THE COEFFICIENT.—The mean coefficient, as has been said, varies according to the temperature range to which it relates; that is to say, it is itself a function of the temperature; or, in other words, the relation between length and temperature is not a linear one. A more complete representation can be made by writing

$$l=l_0(1+\lambda_1 t+\lambda_2 t^2+,\ \text{etc.}),$$

when λ_1, λ_2, etc., are constants to be determined from experiment. If no powers of t higher than the second are retained the relation is parabolic, corresponding to concave or convex curvature upwards according as λ_2 is positive or negative. In most cases it is positive; a body becoming more expansible as the temperature rises.

To find l_0, λ_1, λ_2 from experiment the length l at each of three temperatures t_1, t_2, t_3, must be measured. Thus three equations are obtained,

$$l_1=l_0+l_0\lambda_1 t_1+l_0\lambda_2 t_1^2,$$
$$l_2=l_0+l_0\lambda_1 t_2+l_0\lambda_2 t_2^2,$$
$$l_3=l_0+l_0\lambda_1 t_3+l_0\lambda_2 t_3^2,$$

which must be solved simultaneously for l_0, $l_0\lambda_1$, and $l_0\lambda_2$. They can be reduced to two by subtraction, whence

$$l_2-l_1=l_0\lambda_1(t_2-t_1)+l_0\lambda_2(t_2^2-t_1^2),$$
$$l_3-l_1=l_0\lambda_1(t_3-t_1)+l_0\lambda_2(t_3^2-t_1^2);$$

whence

$$l_0\lambda_1=\frac{(l_2-l_1)(t_3^2-t_1^2)-(l_3-l_1)(t_2^2-t_1^2)}{(t_2-t_1)(t_3^2-t_1^2)-(t_3-t_1)(t_2^2-t_1^2)},$$

and

$$l_0\lambda_2=\frac{(l_2-l_1)(t_3-t_1)-(l_3-l_1)(t_2-t_1)}{(t_2-t_1)^2(t_3-t_1)-(t_3-t_1)^2(t_2-t_1)}.$$

The value of l_0 is next found by inserting these values in any *one* of the initial equations, and thus λ_1 and λ_2 can ultimately be found. Such an equation is quite empirical; molecular theory is not sufficiently advanced to indicate a satisfactory theoretical formula. Various other empirical formula may be employed; *e.g.* instead of the simple formula $l=l_0(1+\lambda t)$, it is usually more satisfactory to write $l=l_0/(1-\lambda t)$, where λ must be chosen to fit experimental observations best.

§ (5) AREA AND VOLUME EXPANSION.—The change of area of any cross-section or of the surface of the body can easily be deduced. If we take a rectangle, as typical, the sides of which are l_0 b_0 and change to l and b, where

$$l = l_0(1 + \lambda t), \quad \text{and } b = b_0(1 + \lambda t),$$

then

$$lb = l_0 b_0 (1 + \lambda t)^2, \quad \text{or } A = A_0(1 + \lambda t)^2,$$

where A_0 and A are the original and new areas. Since in practice λt is usually a small quantity, we can write

$$A = A_0(1 + \lambda t)^2 = A_0(1 + 2\lambda t) \text{ approximately.}$$

The change of area follows approximately the same law as the change of length, but with double the coefficient. In the same way the change of volume of a parallelepiped is given by

$$V = V_0(1 + \lambda t)^3,$$

or approximately

$$V = V_0(1 + 3\lambda t).$$

The value 3λ is called the coefficient of volume expansion (or cubical expansion). These formulae are only approximate. When the required precision justifies the use of more exact expressions for the linear expansion they must be modified accordingly. Thus, adding a term containing the square of the change of temperature,

$$V = V_0(1 + \lambda_1 t + \lambda_2 t^2)^3$$
$$\doteq V_0(1 + 3\lambda_1 t + 3(\lambda_2 + \lambda_1{}^2)t^2).$$

Such formulae are easily worked out when required. We will be content with pointing out that the terms of higher powers become increasingly important at higher temperatures.

It should be mentioned that the expansion of a hollow vessel is the same as if it were solid throughout. For the expansion of any part of the material is in a fixed proportion independent of the other parts; hence the whole of the central part may be removed without altering the change in the peripheral part. This assumes that the shell is perfectly homogeneous.

§ (6) Expansion of Silica Glass.—The importance of silica glass (*i.e.* quartz which has been decrystallised by fusion and subsequent solidification) in thermometric work has caused considerable attention to be paid to it. The extreme smallness of its expansion-coefficient makes the optical method (Fizeau) the most feasible one. This has been employed by Chappuis,[1] Scheel and Heuse,[2] Randall,[3] and Dorsey.[4] Callendar [5] obtained the value $\cdot 59 \times 10^{-6}$ for the mean coefficient of expansion (linear) between room temperature and 1000° C. of a silica rod 40 cm. long. He stated that the expansion is uniform up to 1000° C., increases rapidly from 1000° to 1400° and changes to a contraction beyond that point. The length was measured by a micrometer microscope, and the temperature was measured by the expansion of a surrounding platinum cylinder which was heated by passing an electric current through it, and thus heated the silica in turn.

Holborn and Henning used a rod of silica 52 cm. long and, by a microscopic method,[6] measured the length at room temperature, 250°, 500°, 750°, and 1000°, using electrical heating inside a porcelain jacket. The temperatures at different points of the rod were measured by thermocouples.

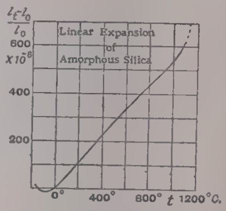

Fig. 3.

The results of all these investigators have been co-ordinated by G. W. C. Kaye,[7] and are shown on the subjoined curve.

This curve was drawn out on a large scale, and from it the following mean coefficients were derived by him:

Temperature Range, ° C.	Mean Coefficient λ.	Temperature Range, ° C.	Mean Coefficient λ.
− 160 to − 120	− 0·43 × 10⁻⁶	30 to 100	0·53 × 10⁻⁶
− 120 to − 80	− 0·11 ,,	100 to 500	0·58 ,,
− 80 to − 40	+ 0·14 ,,	500 to 900	0·50 ,,
− 40 to 0	0·31 ,,	900 to 1100	0·80 ,,
0 to 30	0·42 ,,		

The negative coefficient below − 80° C. is of great interest.

An investigation by Callendar on a silica rod, using a Newton's ring method, gave results between 20° and 300° C., which he represents by the formula

$$\lambda = \left(78\cdot 0 - \frac{8650}{t + 175}\right)10^{-8},$$

where λ is the mean coefficient between 0° and t° C.

Some experiments by Callendar [8] appear to show that silica glass, when drawn into tubular form, is not isotropic. Further investigations are required under this head.

§ (7) Experimental Results.—We give next some experimental values for different substances obtained by various methods. Writing

$$l = l_0(1 + At + Bt^2),$$

the values of A and B are as follows: ${}_0\lambda_{100}$ is the *mean* coefficient between 0° and 100° C.

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[1] Chappuis, *Procès Verbaux Inter. Comm. des Poids et Mesures*, 1903, p. 75.

[2] Scheel and Heuse, *Verh. d. D. Phys. Ges.*, 1907, ix. 718-721; 1914, xvi.

[3] Randall, *Physical Review*, 1910, xxx. 216.

[4] Dorsey, *Physical Review*, 1907, xxv. 88; 1910, xxx. 271.

[5] *Chemical News*, 1901, lxxxiii. 151.

[6] H. and H., *Ann. der Phys.*, 1903, x. 447.

[7] Kaye, *Phil. Mag.*, 1910, cxv. 723.

[8] Callendar, *Phil. Mag.*, 1912, xxiii. 998.

	$A \times 10^8$.	$B \times 10^{10}$.	$10^8{}_0\lambda_{100}$.	Author.
Cadmium	2693	466	3159	Matthiessen.
Lead	2726	74	2800	
Copper	1481	185	1666	
Palladium	1011	93	1104	
Platinum	851	35	886	
	880·6	19·5	..	Scheel.
	886·8	13·24	..	Holborn and Day.
	890·1	12·1	..	Benôit.
Palladium	1161·2	32·3	..	Scheel.
	1167·0	21·87	..	Holborn and Day.
Porcelain	272·1	30·6	..	Scheel.
Jena Glass 59‴	560·8	29·0	..	

Measurements of the linear coefficient $\lambda = 1/l_0 \cdot (dl/dT)$ at very low temperatures have been made by many investigators.[1]

These determinations are of great interest, because E. Gruneisen[2] has shown that there is almost strict proportionality in the case of metals between the coefficient of expansion and the specific heat of constant pressure C_p. Data are given below, together with the value of the ratio λ/C_p.

t° C.	$\lambda + 10^6$.	$\lambda 10^6/C_p$.
	Aluminium	
−173°	13·6	(107)
−100	18·2	109
0	23·0	110
100	24·9	112
300	29·0	119
438	29·8	112
	Wrought Iron	
−87°	9·0	105
0	11·7	109
100	12·7	109
300	14·8	104
500	17·0	89
700	16·0	50
880°−950°	24·5	112
	Nickel	
−87°	10·1	117
0	12·5	120
100	14·0	121
300	16	114
500	16·8	126
	Copper	
−87°	14·1	174
0	16·1	177
100	16·9	180
400	19·3	179
600	20·9	182

t° C.	$\lambda \times 10^6$.	$\lambda 10^6/C_p$.
	Palladium	
−150°	9·2	192
−100	10·1	191
0	11·5	198
100	12·2	197
875	15·5	204
	Silver	
−167°	15	(319)
−87	17·1	329
0	18·3	327
100	19·2	331
500	23·1	350
800	26·0	342
	Iridium	
0°	6·36	205
100	7·0	212
1200	9·5	207
	Platinum	
−150°	7·4	269
−100	7·9	268
0	8·9	280
100	9·2	277
875	11·2	267

It will be seen that the linear coefficient diminishes with fall in temperature. It is certain now that the specific heat falls at an increasing rate until near the absolute zero, but finally varies as the cube of the absolute temperature. It may be expected therefore that the coefficient of expansion will do likewise, becoming zero at the absolute zero itself.

II. Expansion of Non-isotropic Crystals

§ (8) Theoretical Consideration.—When the crystal is non-isotropic, its expansion with temperature is different in different directions. This was first shown to be the case by Mitscherlich, who discovered that the angles between the faces of a cleave of Iceland spar vary with a change in temperature. Let us assume that three mutually perpendicular directions in the crystal can be selected, such that if two particles lie at one temperature on any one of these axes they will continue to do so if the temperature is altered. To each of these directions, x, y, z, corresponds a coefficient of linear expansion, λ_x, λ_y, λ_z respectively. These are called the principal coefficients. A cube, each edge of which is l_0, at zero temperature with its edges parallel to one or other of these directions will, on change of temperature, become a parallelepiped whose edges are given by

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[1] H. D. Ayres, *Phys. Rev.*, 1905, xx. 38; F. Henning, *Ann. d. Phys.*, 1907, xxii. 631; K. Scheel, *Verh. d. Deutsch. Physik. Ges.*, 1907, ix. 3; K. Scheel and Heuse, *Verh. d. Deutsch. Physik. Ges.* p. 449; J. S. Shearer, *Phys. Rev.*, 1905, xx. 52.

[2] *Ann. der Phys.*, 1908, xxvi. 211.

$$l_x = l_0(1+\lambda_x t),$$
$$l_y = l_0(1+\lambda_y t),$$
$$l_z = l_0(1+\lambda_z t),$$

and the volume becomes

$$V = l_x l_y l_z = l_0^3[1+(\lambda_x+\lambda_y+\lambda_z)t].$$

Let us now take any straight line of length r, making angles θ, ϕ, and ψ with the axes x, y, z, and let it change in temperature so as to become of length ρ at $t°$ C where

$$\rho = r(1+\beta t).$$

Then since the square on ρ is equal to the sum of the squares of its projections on x, y, z,

$$\rho^2 = r^2\cos^2\theta\,(1+\lambda_x t)^2 + r^2\cos^2\phi\,(1+\lambda_y t)^2 + r^2\cos^2\psi\,(1+\lambda_z t)^2$$
$$\doteq r^2 + 2r^2 t(\lambda_x\cos^2\theta + \lambda_y\cos^2\phi + \lambda_z\cos^2\psi)$$
$$\doteq r^2(1+2\beta t),$$

whence

$$\beta = \lambda_x\cos^2\theta + \lambda_y\cos^2\phi + \lambda_z\cos^2\psi.$$

Now take *any* three mutually perpendicular directions of which that of ρ is one. Then

$$\beta_1 = \lambda_x\cos^2\theta + \lambda_y\cos^2\phi + \lambda_z\cos^2\psi,$$
$$\beta_2 = \lambda_x\cos^2\phi + \lambda_y\cos^2\psi + \lambda_z\cos^2\theta,$$
$$\beta_3 = \lambda_x\cos^2\psi + \lambda_y\cos^2\theta + \lambda_z\cos^2\phi.$$

But since $\cos^2\theta + \cos^2\phi + \cos^2\psi \equiv 1$,

it follows that

$$\beta_1+\beta_2+\beta_3 = \lambda_x+\lambda_y+\lambda_z.$$

Hence this sum is an invariant for all such sets of mutually perpendicular axes. We have seen that this sum is equal approximately to the coefficient of volume expansion.

If we select a direction which makes the same angle ω with each of the chief axes so that

$$\cos^2\omega = \cos^2\theta = \cos^2\phi = \cos^2\psi = \tfrac{1}{3},$$

and for which ω is therefore 54° 44′, then for this direction

$$\beta = (\lambda_x+\lambda_y+\lambda_z)\cos^2 w$$
$$= \tfrac{1}{3}(\lambda_x+\lambda_y+\lambda_z),$$

so that the linear coefficient for this direction is the arithmetic mean of the three chief coefficients.

§ (9) EXPERIMENTAL RESULTS.—The best values of the coefficients have been determined by Fizeau's method. In some cases linear contraction takes place (in particular directions) instead of expansion; but this contraction is always very small. The volume coefficient is always positive. In the hexagonal system Pfaff found that in optically negative crystals the expansion along the axis is always greater than at right angles thereto; while the reverse holds for optically positive crystals. The values for quartz and Iceland spar have been very carefully determined by Benoît, using Pulfrich's modification of Fizeau's apparatus. For quartz the length along the axis between 0° C. and 80° C. is given by

$$l_t = l_0(1+7\cdot1614\times10^{-6}t + \cdot00801\times10^{-6}t^2),$$

and perpendicular to the axis by

$$l_t = l_0(1+13\cdot2546\times10^{-6}t + \cdot01163\times10^{-6}t^2).$$

For Iceland spar the corresponding values between 0° C. and 80° C. are:

Along the axis

$$l_t = l_0(1+25\cdot1353\times10^{-6}t + \cdot01180\times10^{-6}t^2).$$

Perpendicular to the axis

$$l_t = l_0(1-5\cdot5782\times10^{-6}t + \cdot00138\times10^{-6}t^2).$$

The last equation indicates a contraction with increase in temperature.

Federow [1] has employed a new method for the examination of crystals. The surface of a section of the crystal (cut in any desired plane) is coated with a thin layer of a noble metal and a diffraction grating is ruled thereon. If the grating space changes with temperature the deflections of the spectra change in about the same proportion. These shifts enable the thermal expansion to be studied.

Even an apparently homogeneous mass like Carrara marble exhibits crystalline properties, *i.e.* the expansion is different in different directions. Very careful measurements have been made on a specimen obtained from the Japanese Imperial Museum by H. Nisi, by means of the Fizeau-Pulfrich method. Hollow cylindrical rings were cut in different azimuths, and formed the distance pieces of the two plates between which interference figures were obtained. At 20° the coefficients for three mutually perpendicular directions were:

	x.	y.	z.
1st Specimen	$2\cdot05\times10^{-6}$	$4\cdot48\times10^{-6}$	$8\cdot09\times10^{-6}$
2nd Specimen	$2\cdot05\times10^{-6}$	..	$8\cdot14\times10^{-6}$

Values were also obtained for other azimuths; those in the x, y plane lie sensibly upon an ellipse. Near 0° C. the expansion was negative in and near the x direction. In connection with this crystalline property it may be added that marble exhibits definite cleavage planes.

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[1] *Lehrbuch der Krystallographie* (St. Petersburg, 1901).

III. Expansion of Liquids

§ (10) Measurement of the Relative Expansion. — In the case of liquids the volume expansion alone has any significance, for a liquid has no intrinsic shape, but takes the shape of that part of a vessel which it occupies. We can again express the volume in powers of the temperature, writing

$$V=V_0(1+\alpha_1 t+\alpha_2 t^2+\ .\ .\ .),$$

or approximately

$$V=V_0(1+\alpha_1 t).$$

In most of the methods of determining α_1 experimentally the expansion of the vessel (in which the liquid is necessarily contained) enters as a complication, and it is therefore necessary, in such cases, to have a previous knowledge of the coefficient of the solid of which the vessel is made. Two methods will be described which fall into this class.

(i.) *Determination of α_1 by finding the Apparent Weight of a Solid suspended in the Liquid at Various Temperatures.*—By Archimedes' principle the apparent weight of the solid W_a is equal to the real weight W minus the weight w of an equal volume of the liquid, or

$$\begin{aligned} w=W-W_a&=\text{vol.}\times\text{density of liquid}\\ &=VD.\end{aligned}$$

Let w be determined at two temperatures, then

$$w_1=V_1D_1,$$

$$w_2=V_2D_2,$$

or

$$\frac{w_1}{w_2}=\frac{V_1}{V_2}\frac{D_1}{D_2}.$$

Now the ratio of the densities of the liquid must be inversely as the ratio of the volumes *of a given mass* of liquid, *i.e.*

$$\frac{D_1}{D_2}=\frac{1+\alpha t_2}{1+\alpha t_1}.$$

Also considering the solid,

$$\frac{V_2}{V_1}=\frac{1+\gamma t_2}{1+\gamma t_1},$$

where γ is coefficient of volume expansion of the solid. Hence

$$\frac{w_1}{w_2}=\frac{1+\alpha t_2}{1+\alpha t_1}\cdot\frac{1+\gamma t_1}{1+\gamma t_2},$$

so that a knowledge of γ enables α to be calculated from the observed values of w and t. Approximately, in usual cases this can be written

$$\frac{w_1}{w_2}\doteq\frac{1+\alpha(t_2-t_1)}{1+\gamma(t_2-t_1)}$$

$$\doteq 1+(\alpha-\gamma)(t_2-t_1),$$

so that the alteration of apparent weight is the same as if the volume of the solid had remained unaltered and the coefficient of expansion of the liquid had been $\alpha-\gamma$. The value of $\alpha-\gamma$ is called the coefficient of apparent expansion. When the difference of temperature is too great to justify the last approximation, we can write

$$\alpha(t_2-t_1)\doteq\frac{w_1}{w_2}-1+\frac{w_1}{w_2}\gamma(t_2-t_1)$$

or

$$\frac{w_1-w_2}{w_2(t_2-t_1)}\doteq\alpha-\frac{w_1}{w_2}\gamma.$$

As a rule the last term is very nearly equal to γ. If amorphous silica is used as the solid the correction for γ is exceedingly small, and it may often be neglected entirely.

(ii.) *The Weight-thermometer Method.* — A vessel is taken of the shape shown, usually of glass or silica. This can be completely filled with the liquid under examination by alternately heating and cooling it while the open end dips under the liquid in a porcelain crucible. There is often a difficulty in removing the last trace of air which sticks in the neck. This can only be done by boiling the liquid which has entered until half (or more) is evaporated and then letting it cool, repeating the operation if it does not succeed the first time. The course of the experiment consists in letting the filled thermometer attain two temperatures in succession, the open end being kept immersed in the liquid in the crucible, and weighing it after each adjustment. If the weight of the empty thermometer has been determined once for all, the weighings give the weight of liquid contained at the respective temperatures. The calculation of the coefficient follows the same lines as in case (i.); in fact no further change is necessary, if w_1, w_2 are taken as the weights of liquid contained at the two temperatures, and V_1, V_2 are the corresponding volumes of the containing vessel. Hence, finally,

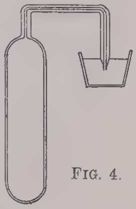

Fig. 4.

$$\alpha-\frac{w_1}{w_2}\gamma\doteq\frac{w_1-w_2}{w_2(t_2-t_1)}.$$

The necessity of knowing γ before α can be calculated is a defect of both these methods. It is practically impossible to determine γ directly for the actual vessel employed. Moreover, there is often doubt as to whether the material of the vessel can be treated as isotropic, even when very well annealed. Owing to the introduction of silica that has been fused and resolidified in the non-crystalline form the importance of γ is much diminished. Its value for glass is about

$3 \times \cdot000009$ or $\cdot000027$; while for silica it is only $\cdot0000005 \times 3$ or $\cdot0000015$. The value of γ for mercury is about $\cdot000182$, so that the apparent coefficient in glass is about $\cdot000155$, while in silica it is $\cdot0001805$. It is evident that even an approximate value in the case of silica will be sufficiently good except in investigations of the highest precision.

The introduction of silica has the additional advantage that the manipulation is much simplified. With glass it is necessary to heat the vessel and incoming liquid nearly to the same temperature in the process of filling in order to prevent fracture from a sudden change in temperature. In the case of a silica thermometer no such precaution is required.

This method is of great use for determining the coefficient for a solid indirectly. In this case the coefficient for the liquid must first be known. We describe in § (11) the only known method by which it can be determined. It is not usually possible to make the thermometer out of the material under test. A thermometer of glass (or silica) can be made containing a weighed block of the test specimen. The volume occupied by the liquid, v_1, is now the difference between the volumes of the vessel V and of its solid contents (v'). As before, we have for the liquid, at two temperatures,

$$\frac{w_1}{w_2} = \frac{v_1}{v_2} \cdot \frac{D_1}{D_2} = \frac{v_1}{v_2}(1 + at) \text{ approximately,}$$

or

$$\frac{v_2}{v_1} = \frac{V_2 - v_2'}{V_1 - v_1'} = \frac{w_2}{w_1}(1 + at).$$

Now if

$$V_2 = V_1(1 + \gamma t)$$

and

$$v_2' = v_1'(1 + st),$$

this becomes

$$1 + \frac{(V_1\gamma - v_1's)t}{v_1 - v_1'} = \frac{w_2}{w_1}(1 + at).$$

The volumes v_1' and V_1 can be calculated from the weights of the solid and of the liquid at t_1 if their densities at that temperature are known. Hence a further previous knowledge of a and γ enables the coefficient s for the solid to be determined.

Any form of specific gravity bottle is essentially a weight thermometer. The removable stopper facilitates the introduction of any liquid or solid.

(iii.) *Volume Thermometer Method.* — The principle of this method is very similar to that of the weight thermometer, but volumes are observed instead of weights. A bulb is taken with a stem of uniform bore attached. A scale can be graduated upon the stem, or alternatively a single fiducial mark can be made thereon, and by means of a cathetometer the distance of the level of the liquid from this mark can be read. It is necessary first to find the volume of the bulb up to the zero of the scale; this is done by weighing the thermometer empty and containing mercury up to the zero when in melting ice. If now an additional amount of mercury is admitted, so that the surface at 0° C. is at the level a, and the whole is reweighed, the extra weight gives the volume of a divisions of scale in the same units as the weight of the original mercury gives the volume of the bulb; and from these data the volume corresponding to one scale division when at 0° C. can be calculated; let it be v_0.

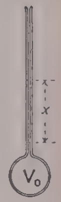

FIG. 5.

Now expel the mercury and replace it by the liquid under test. Observe the positions x_0 x_1 of the surface at 0° C. and at t° C. Then the volumes occupied by the liquid are

$$V_0 + x_0v_0 \text{ and } (V_0 + x_1v_0)(1 + \gamma t),$$

(γ = vol. coefficient for vessel), so that

$$(V_0 + x_1v_0)(1 + \gamma t) = (V_0 + x_0v_0)(1 + at);$$

whence, if γ is known, a is calculable.

If the alternative method is employed, *i.e.* the lengths are read by means of an independent scale (*e.g.* the scale of a cathetometer) at the temperature t_a, and if λ_s is the linear coefficient of expansion of the scale and λ_g that of the material of the bulb, the volume of a true centimetric length of stem at a temperature t is $v_t = v_0(1 + 2\lambda_g t)$. Hence if the scale readings are in every case reduced to true centimetres by multiplying by $1 + \lambda_s t_a$, the formula becomes

$$V_0(1 + 3_g\lambda t) + x_1v_0(1 + 2\lambda_g t) = (V_0 + x_0v_0)(1 + at),$$

where x_1 and x_0 are the scale readings *after reduction to zero.*

§ (11) ABSOLUTE EXPANSION OF A LIQUID.— There is only one method of determining a which does not require a knowledge of the expansion of the containing vessel. This was devised by Dulong et Petit and employed by them and, with various modifications, by Regnault, Callendar, and others. It is a hydrostatic method, use being made of the fact that the increase of pressure with depth in a liquid is proportional to its density as well as to the difference in depth. If two liquids (which do not mix) are placed in the two limbs of a U-tube the interface A between them adjusts itself so that the hydrostatic pressure thereat is the same as that at the same level in the other limb. If the heights of liquid in the two limbs (measured from the *level* of the

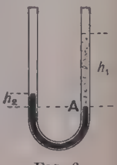

FIG. 6.

interface) are h_1 and h_2 and the densities of the liquids are D_1 and D_2, then

$$D_1 h_1 = D_2 h_2.$$

Now, instead of taking two liquids, let a single liquid be contained in both limbs so that the free surface in both comes to the same level when both limbs are at one temperature; they will fail to do so when a difference of temperature exists between them, for they will then be of different densities. Let the lengths of the heated and unheated columns be h_2 and h_1. Then

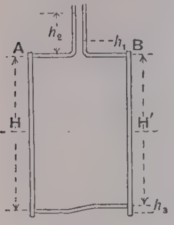

FIG. 7.

$$\frac{h_2}{h_1} = \frac{D_1}{D_2},$$

and this equals

$$\frac{1 + \alpha t_2}{1 + \alpha t_1},$$

as in the previous cases; or

$$\alpha\left[t_2 - \frac{h_2}{h_1} t_1\right] = \frac{h_2 - h_1}{h_1}.$$

N.B. If a stopcock had been inserted in the horizontal connecting piece, and at uniform temperature the stopcock were turned off, the height on the right would remain unvaried while the height on the left would increase by an amount depending upon the expansion of both the liquid and the vessel. In fact, the volume occupied would be changed according to the equation $V = V_0(1 + \alpha(t_2 - t_1))$; the cross-sectional area according to the equation $A = A_0(1 + 2\gamma(t_2 - t_1))$; and therefore the height (supposed to be read on an independent true scale) according to

$$\frac{h_2}{h_1} = \frac{1 + \alpha(t_2 - t_1)}{1 + 2\gamma(t_2 - t_1)}.$$

This is less than in the actual experiments; in other words, in the actual case a little liquid flows through the connecting tube from the cold to the hot side during the attainment of the equilibrium state.

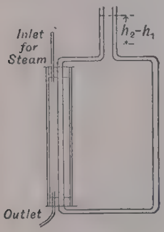

FIG. 8.

(i.) *Regnault's Experiments.*—In Dulong et Petit's original experiments the heated and unheated columns were at some distance apart. The difficulty of measuring the difference of level is thereby increased. Regnault improved the method by bringing the columns together at the top as indicated in the sketch.

To obtain accurate values the temperatures of all parts of the liquid columns must be observed. If these temperatures are represented by letters of the same type as the columns themselves the final equation becomes

$$\frac{H}{1 + \alpha_T T} + \frac{h_2}{1 + \alpha_2 t_2} = \frac{H'}{1 + \alpha' t'} + \frac{h_1}{1 + \alpha_1 t_1} + \frac{h_3}{1 + \alpha_3 t_3}.$$

Regnault determined the value of α_T by means of the formula by the method of successive approximations; that is, by first assuming α to be constant, and finally inserting in each term the approximate value found for the respective temperature range.

Regnault also devised another modification, the general nature of which is shown in *Fig.* 9. The pressure is equalised at the upper level by a connecting tube AB; at the lower end the connection is by an inverted ∪-tube, the upper part of which is connected to a reservoir and pump, and contains air at high pressure. The pressure is the same at the surface of both the surfaces of mercury in the limbs of this ∪-tube. We have therefore

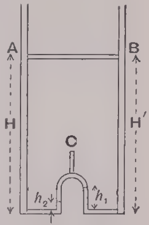

FIG. 9.

$$\frac{H}{1 + \alpha_T T} - \frac{h_2}{1 + \alpha_2 t_2} = \frac{H'}{1 + \alpha' t'} - \frac{h_1}{1 + \alpha_1 t_1}.$$

Regnault represented his results for mercury by the empirical formula

$$\alpha_T = 0{\cdot}00017905 + 2{\cdot}52 \times 10^{-8}\,T,$$

where α_T is the mean coefficient between 0° and T° C. The mean coefficient between 0° and 100° is accordingly 0·00018157.

These values which Regnault deduced from his experiments have been subjected to considerable criticism. Recknagel[1] showed that a parabolic formula was necessary adequately to represent the experimental results. Bosscha[2] assumed that if α', the coefficient at a temperature t, is defined as $1/v \, . \, dv/dt$, then its value is constant. Hence by integration $v = v_0 e^{\alpha/t}$, and he deduced the value $\alpha' = 00018077$.

Wüllner, Levy, and Broch also fitted three-constant formulae to the observations. Broch's formula is

$$v = v_0(1 + at + bt^2 + ct^3),$$

where $a = 0{\cdot}000181792$,
$b = 0{\cdot}000000000175$,
$c = 0{\cdot}000000000035116$,

which gives, for the mean coefficient between 0° and 100° C., the value

$$0{\cdot}00018216.$$

Thiessen, Scheel, and Sell, from the expansion of mercury in glass vessels, found the value

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[1] Recknagel, *Pogg. Ann.*, 1864, cxxiii. 115.

[2] Bosscha, *Pogg. Ann.*, 1871, Ergänzungsband v. 276.

$$0{\cdot}00018245.$$

More recently Chappuis by the weight-thermometer method finds for the range from $-20°$ to $0°$ the equation

$$v_t = v_0(1 + 1{\cdot}815405 \times 10^{-4}t + 1{\cdot}95130 \times 10^{-9}t^2 + 1{\cdot}00917 \times 10^{-10}t^3 - 2{\cdot}03862 \times 10^{-13}t^4),$$

and from the range 0° to 100° C. the equation

$$v_t = v_0(1 + 1{\cdot}816904 \times 10^{-4}t - 2{\cdot}951266 \times 10^{-9}t^2 + 1{\cdot}14562 \times 10^{-10}t^3).$$

This last gives as a mean coëfficient between 0° and 100° C. the value

$$0{\cdot}000182541.$$

(ii.) *Callendar's Experiments.*—These values show differences which are considerable when the importance of this coefficient is borne in mind.

Regnault's results "though as perfect as the methods and apparatus available in his time would permit, and obtained with all the care he used to secure accuracy, left a much greater margin of uncertainty than is admissible at the present time in many cases to which they have been applied. The order of uncertainty may be illustrated by comparing the value of the fundamental coefficient of expansion given by Regnault himself with the values since deduced from his observations by Wüllner and Broch. The discrepancy amounts to 1 in 180, . . . and would be equivalent to an uncertainty of about 4 per cent in the expansion of a glass bulb determined with mercury by the weight-thermometer method. The uncertainty of the mean coefficient is naturally greater at higher temperatures. If in place of the mean coefficient we take the actual coefficient at any temperature the various reductions of Regnault's work are still more discordant, and the rate of variation of the coefficient with temperature, which is nearly as important as the value of the mean coefficient itself in certain physical problems, becomes so uncertain that the discrepancies often exceed the value of the correction sought. It is only fair to Regnault to say that these discrepancies arise to some extent from the various assumptions made in reducing his results, and are not altogether inherent in the observations themselves. With regard to the weight-thermometer method, although the accuracy of weighing permits an accuracy in the final result of 1 in 20,000 of the weight of mercury expelled corresponding to the fundamental interval, yet it leaves the absolute value of the fundamental interval uncertain, because it is rather gratuitous to assume that the expansion of the containing bulb is the same in all directions, however carefully it may have been annealed. Chappuis' results with the weight thermometer nearly agree with Wüllner's reduction of Regnault's by the hydrostatic method, but they can be brought into line with Broch by assuming that the expansion of the bulb along a diameter was about 2 per cent less than in the direction of its length." These considerations led Callendar and Moss to repeat Regnault's investigation on a larger scale with modern appliances, the whole experiment being so designed as to give, if possible, the same order of accuracy in the absolute expansion that is obtainable in the relative expansion by the weight-thermometer method.

Instead of the single pair of hot and cold columns, each 1·5 metres long, employed by Regnault, six pairs of hot and cold columns, each nearly 2 metres long, were connected in series, giving nearly eight times the expansion obtainable with Regnault's apparatus.[1]

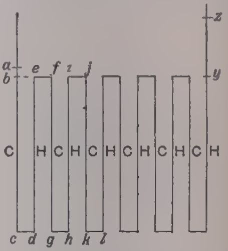

FIG. 10.

The hot and cold columns are labelled H and C respectively. The difference of height $z - a$, will be six times the difference due to a single pair. In the actual apparatus the cross-tube *ef* was doubled back so as to be behind *bc*; similarly for the others. All the hot columns were placed together in one limb of a rectangle of iron tube, 5 cm. in bore, filled with circulating oil and lagged with asbestos. All the cold columns were placed in one limb of a similar rectangle. The second limbs of the rectangles were utilised for the electric heating coils and the ice-cooling bath respectively. Centrifugal circulators continuously driven by an electric motor were provided for maintaining the oil in rapid circulation through the rectangles, so that the temperatures of the hot and cold columns were each nearly uniform. The mean temperatures of these columns were observed by means of a pair of platinum thermometers contained in tubes similar in size to the tubes containing the mercury columns. The lengths of the loops of platinum wire forming the " bulbs " of the thermometer were made as nearly as possible equal to the lengths of the columns, and were fixed at the same level in the tubes, so as to give the true mean temperature, in case there were any appreciable variation throughout its length. Measurements were made up to 300° C., and the total assemblage of the results obtained were fitted to a parabolic formula

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[1] Similar increased expansion could be obtained by using a single pair of great length. But in this case the average pressure of the mercury would be very different, and since the coefficient may be expected to vary with the pressure an additional source of variation would thereby be introduced. Even in the actual experiment as carried out by Callendar the mean pressure was about 2½ atmospheres. According to Bridgman the change of the compressibility of mercury per degree is about 7×10^{-9} (1/atmos.° C.) between 0° and 22° C. We may deduce from this that the change of α per atmosphere should be about 7×10^{-9} also. For 1½ atmospheres excess this makes about 1×10^{-8}; that is, only the fifth significant figure should be affected. But with ten times this pressure the fourth would begin to be affected. Values of the coefficient are usually given to at least five significant figures. When the determination of the coefficient comes to be perfected so as to justify a retention of the fifth figure it will be necessary to specify the pressure of the mercury to which the values refer. The theory of the influence of pressure is given in § (12).

$$_0\alpha_t = \left[1805553 + 12444\frac{t}{100} + 2539\left(\frac{t}{100}\right)^2\right] \times 10^{-10},$$

to which corresponds as value of the fundamental coefficient

$$_0\alpha_{100} = 0{\cdot}0001820536,$$

which they considered to be accurate to about 1 part in 10,000. The value is, however, 1 part in 1800 less than Broch's value based on Regnault's experiments, and 5 parts in 1800 less than Chappuis' value deduced from weight-thermometer measurements.

This discordance led F. J. Harlow (under Callendar's guidance) to make a series of weight-thermometer determinations, using a silica weight thermometer. These appear to have been made with great care, and the values they give for the coefficient of *apparent expansion in silica* may be taken as amongst the best measurements made. Since the discussion that has taken place turns on the deduction of the coefficient of absolute expansion from these it is important to give the results obtained.

Temperature Range.	Coefficient of Apparent Expansion $\times 10^8$.	Temperature Range.	Coefficient of Apparent Expansion $\times 10^8$.
0° – 30°	18060	184	18220
50	18068	200	18248
75	18081	250	18356
100	18102	300	18489
140	18150		

These data are the mean of values obtained with a large bulb thermometer, a small one (both cylindrical), a value at 100° with a spherical bulb, and values at 100° and 184° C. taken previously. The percentage accuracy is not so great at the lower temperatures as at the higher, owing to the smallness of the overflow, but above 50° they are regarded as being accurate to 1 part in 18,000. The close correspondence which he obtained when he employed both a cylindrical bulb and a spherical one seemed to justify the conclusion that the silica was isotropic.

If these values be combined with the values for silica obtained by Callendar on a rod of silica procured from the same firm, which are given by the formula

$$_0\lambda_t = \left\{78{\cdot}0 - \frac{8650}{t+175}\right\} \times 10^{-8},$$

for the mean linear coefficient, the coefficient of absolute expansion of mercury takes the values given under the head Harlow (Callendar) in the table below. If, on the other hand, the values for silica deduced by Kaye from Randall and Chappuis, etc., be employed, those given under the heading Harlow (Kaye) are calculated.

Temperature Range.	1. Chappuis.	2. Callendar and Moss.	3. Harlow, 1913. (Callendar.)	3. Harlow, 1913. (Kaye.)	4. Sears' Quartic.
0° – 30° C.	18,171	18,095	18,168	18,187	(18,174)
50	18,183	18,124	18,188	18,201	18,194
75	18,211	18,163	18,213	18,223	18,221
100	18,254	18,205	18.244	18,251	(18,251)
140	..	18,280	18,305	18,305	18,306
184	..	18,371	18,387	18,380	18,379
200	..	18,406	18,419	18,410	(18,410)
250	..	18,525	18,537	18,523	18,522
0° – 300°	..	18,657	18,678	18,660	(18,663)

Finally, in column 4, are values calculated by Sears from a quartic formula made to fit exactly the four values enclosed in brackets. The values up to 100° C. were calculated by taking a mean of the Harlow-Callendar and the Harlow-Kaye values and then a mean of this and Chappuis' value. Above 100° C. the mean of the three quoted values was taken. It is probable that column 4 represents the best that is known at present concerning the absolute expansion of mercury. The formula from which the intermediate values in column 4 have been calculated is

$$\frac{V - V_0}{V_0} = \begin{pmatrix} 181{\cdot}456t + 0{\cdot}009205t^2 + 0{\cdot}000006608t^3 \\ + 0{\cdot}00000067320t^4 \end{pmatrix} \times 10^{-6}.$$

The reason of the very considerable differences between the values of Callendar and Moss at low temperatures and the consensus of those obtained by others has not yet been cleared up. The fact that hard glass and silica bulbs give concordant results is against the assumption of the bulbs being non-isotropic; so also is the fact that very fair agreement is obtained at higher temperatures. If we proceed conversely and calculate the expansion of silica from Harlow's values for the apparent expansion and Callendar and Moss's absolute values, impossible values for it are obtained unless considerable non-isotropy is called in as an explanation.

The suggestion has also been made that the weight-thermometer method is less accurate than the hydrostatic method on account of the films of air which cling obstinately to the vessel. Experiments, however, made with weight thermometers containing blocks of silica presenting a large surface do not support the suggestion.

(iii.) *Expansion of Water.* — Water is anomalous in its behaviour in the region between 0° C. and 4° C. The following are relative volumes of water at various temperatures: [1]

−10° C.	−5°.	0°.	5°.	10°.
1·0017	1·0006	1·0000	0·9999	1·0001
15°.	20°.	25°.	30°.	35°.
1·0007	1·0016	1·0028	1·0041	1·0057
40°.	45°.	50°.	55°.	60°.
1·0076	1·0096	1·0118	1·0143	1·0168
65°.	70°.	75°.	80°.	
1·0195	1·0224	1·0255	1·0287	..

The following values, in the neighbourhood of 4° C., are relative to the values at 4° C.:

0° C.	2°.	4°.	6°.	8°.
·99987	·99997	1·00000	·99997	·99988

The anomaly is often attributed to the existence of three types of molecule, H_2O, $2H_2O$, $3H_2O$, the first (monohydrol) being present alone in steam, and the third (trihydrol or ice molecules) present by itself in ice, and water being a mixture of all three. If each kind has its own specific volume (for the first and third, those of steam and ice respectively), mixtures of them can be imagined, *if the additive law be assumed,* which would have the actual specific volumes of water at various temperatures. Although it may be true that these various types exist no confidence can be placed in the proportions that are calculated, because there is no justification for assuming the additive law to be true. The variation of the specific heat of water with temperature is also explained along the same lines, but with similar uncertainty.

§ (12) INFLUENCE OF PRESSURE.—We have discussed the thermal expansion of solids and liquids without considering it necessary to contemplate the possible influence of pressure. For moderate pressures this course is certainly justifiable because of the very small compressibility. But, in reality, all solids and liquids diminish in volume with increase of pressure, and if the diminution is not the same at different temperatures the coefficient of thermal dilatation must also change.

Consider unit volume of this substance. Any small change in it brought about jointly by change of temperature and change of pressure can be written

$$dV = a dT - b dp,$$

where a is the coefficient of volume expansion and b (which obviously stands for $-(\partial V/\partial p)_T$ is the compressibility. Now, assuming that the volume always returns to the same value when T and p do so, it follows that dV must be a perfect differential, and therefore

$$\left(\frac{\partial a}{\partial p}\right)_T = -\left(\frac{\partial b}{\partial T}\right)_p.$$

That is, if the compressibility increases with temperature then it follows that the coefficient of thermal expansion diminishes with increase in pressure.

The following values for water and mercury are derived from the investigations of P. W. Bridgman in America.[2] The volumes were measured at a series of pressures at different temperatures.

WATER

Pressure in kg./cm.².	$a \times 10^6$				
	0°.	20°.	40°.	60°.	80°.
0	− 55	+195	388	530	660
500	+128	250	392	529	600
1,000	215	300	398	506	540
2,000	310	356	400	460	480
3,000	350	375	399	434	445
4,000	355	375	390	420	422
5,000	332	365	390	409	404
7,500	..	458	399	392	368
10,000	..	..	395	380	348
12,000	..	..	390	378	338

RELATIVE VOLUMES OF MERCURY

(Bridgman)

Pressure. kg./cm.².	−30° C.	−20° C.	−10° C.	0° C.	+10 C.	+20° C
0	0·99457	0·99638	0·99819	1·00000	1·00181	1·00362
1,000	99207	99280	99553	0·99626	0·99799	0·99972
2,000	98760	98927	99094	99261	99428	99593
3,000	..	98581	98743	98905	99067	99232
4,000	..	98245	98403	98561	98719	98877
5,000	..	..	98077	98231	98385	98540
6,000	..	..	97763	97914	98065	98216
7,000	..	..	..	97607	97756	97904
8,000	..	..	..	..	97461	97608
9,000	..	..	..	..	97182	97327
10.000	..	..	..	..	96915	97059
11,000	..	..	..	..	..	96806
12,000	..	..	..	..	..	96567

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[1] P. W. Bridgman, *Z. Anorg. Chem.* lxxvii. 384-385.

[2] *Proc. Am. Acad.* xlviii. 344 (water); xlvii. 432 (mercury).

The temperature of maximum density of water decreases as the pressure increases.

For $p = 1000$ kg./cm.^{2}, the maximum density of water is at about 0° C.; for $p = 2000$ it is below $-20°$ C.

In a similar way for the case of the stretching of a wire or tie-rod for unit length

$$dl = \lambda dT + b dF,$$

where b is the reciprocal of Young's modulus and F is the stretching force per unit cross-sectional area, whence $d\lambda/dF = \partial b/dT$. The following values are calculated by means of this equation:

$$\frac{d\lambda}{dF}\left(\text{per } 10^6 \ \frac{\text{dynes}}{\text{cm.}^2}\right).$$

Aluminium	3×10^{-9}	Steel	0·11
Copper	0·3	Platinum	0·05
Gold	0·6	Silver	1·0
Iron	0·1	Brass	0·37

Since 10^6 dynes/cm.^{2} is approximately one kilogram wt./cm.^{2}, 1000 kilograms wt./cm.^{2} must increase the linear expansion of copper by $\cdot 3 \times 10^{-6}$, that is, it must increase it from ·000017 to ·0000173.

IV. EXPANSION OF GASES

§ (13) THE GASEOUS LAWS. (i.) *Charles' Law.*—In the expansion of liquids and solids which are only slightly compressible, the influence of pressure has been treated as of quite secondary importance. It is quite otherwise in the case of gases. The volume varies so fast with pressure as well as with temperature that unless the pressure variation is carefully specified the results have no significance. It is obviously simplest to consider cases in which the *pressure keeps constant* and for which, therefore, any changes of volume may be attributed to the influence of temperature alone. In such a case it is found that it is valid to write

$$v = v_0(1 + at),$$

where a is approximately constant (at least for the more permanent gases), and is called the coefficient of expansion at constant pressure. This is known as the law of Charles or Gay-Lussac. The value of a is nearly the same for all the more permanent gases—a fact which has great theoretical importance. It must be pointed out that, since the volume changes with temperature much more rapidly than for liquids and solids, it is not possible to employ the approximate expressions applicable to them.

(ii.) *Boyle's Law.*—In practice it is not usually possible to maintain the pressure constant during an investigation, and it is necessary, therefore, to take its effect into account.

This can be done by investigating how the volume changes with change in pressure when the temperature keeps constant. The law obeyed in this case was first of all investigated by the Honourable Robert Boyle[1] in 1662.

Taking a "goodly tube" of the shape shown in *Fig.* 11, with its short limb closed and its long one open to the air, he poured mercury into it so as to fill the lower part and thereby enclose a mass of air which kept constant throughout the experiment. The pressure of this enclosed air is the barometric pressure plus that due to a column of mercury of height equal to the difference of levels of the mercury in the two limbs. If the short limb is cylindrical the volume of the enclosed air is proportional to the length l which it occupies. Thus both the pressure and the volume are measurable. If more mercury is now poured into the open limb the height h will change; so also does the volume l.

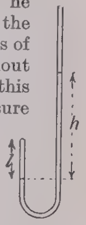

FIG. 11.

In this way a series of corresponding values of pressure and volume are obtained. Boyle found that when the temperature remains constant the product of the pressure and volume *for a constant mass of gas* is itself a constant; or, in symbols,

$$pv = \text{constant (at const. temp.)}.$$

This is known as Boyle's Law.

(iii.) *Perfect Gases.*—We can obtain a single expression which includes both of these laws by writing

$$pv = \text{constant } (1 + at),$$

for if the temperature is constant the right hand is so also, and we have Boyle's Law; while if the pressure is constant

$$v = \frac{\text{constant}}{p}(1 + at) = \text{constant } (1 + at),$$

and this new constant is obviously the volume (at p) when the temperature is 0° C. This equation shows us that if the *volume* be kept constant then the pressure must be

$$p = \frac{\text{constant}}{v}(1 + at) = \text{constant } (1 + at),$$

and this new constant is clearly the value of p (at the volume v) when the temperature t is zero, or

$$p = p_0(1 + at).$$

Hence, under constant volume, the pressure varies in the same way as the volume does under constant pressure. The coefficient a may therefore be regarded as a pressure coefficient. This is only true if Boyle's law and the law of Charles are both *exactly* followed. In reality they are only approximate laws, and the pressure coefficient is found not to be identical with the volume coefficient. When it is intended to emphasise the difference we denote the pressure coefficient by β. The equation can be written in the alternative form

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[1] Boyle, *Nova experimenta physico-mechanica de vi aeris elastica.*

$$pv=\mathrm{R}\left(\frac{1}{a}+t\right),$$

where R is a constant, and is called the characteristic constant of the gas. The value of $1/a$ is about 273. Hence if we measure temperature in centigrade degrees from a zero which is 273 such degrees below 0° C., and represent it by the symbol T, then

$$\mathrm{T}=\frac{1}{a}+t,$$

and the equation becomes

$$pv=\mathrm{RT}.$$

The temperature so measured is known as absolute temperature. The utility of this change is due to the fact that a, and therefore $1/a$, is practically the same for all the more permanent gases. The value of R is different for different gases; its value is pv/T. If we take v as referring to unit mass so that it is the specific volume or the reciprocal of the density at 0° C., it is clear that at a given temperature and pressure R for different gases is inversely as the density; its value can be calculated when the density of the gas under any pressure and temperature is known. The state usually taken as a standard is 0° C. and 1 atmosphere pressure. The value of R for hydrogen expressed in various units is given below.

Characteristic Constants of Hydrogen

Gas.	Density at 1 Atmo. and 0° C.	R.			
	$\frac{\text{Grms.}}{\text{cm.}^3}$	$\frac{\text{Dyne-cm.}}{\text{gr. deg. C.}}$	$\frac{\text{Atmos. cm.}^3}{\text{gr. deg. C.}}$	$\frac{\text{Atmos. ft.}^3}{\text{lb. deg. C.}}$	$\frac{\text{Atmos. ft.}^3}{\text{lb. deg. F.}}$
Hydrogen .	·00009	$41{\cdot}57\times10^6$	41·035	·6574	·3652

For other gases it is inversely proportional to the relative density, *i.e.* to the molecular weight compared with that of hydrogen. If v_m is the volume of gas containing one gram-molecule,[1] then

$$pv_m=\mathrm{R}_0\mathrm{T},$$

where R_0 is a universal gas constant, $83{\cdot}14\times10^3$ ergs per gram-molecule per degree C., or about 1·989 calories per gram-molecule per degree C., if thermal units be employed.

§ (14) Kinetic Theory.—These laws are readily accounted for by the kinetic theory of matter. If a gas consists of a large assemblage of massive points moving at random in all directions, and impinging upon the walls of the vessel from which they rebound elastically, they must exert a pressure upon the walls. Let the vessel be a cube of unit side. A single molecule of mass m moving to and fro between two opposite walls with a velocity u has its velocity reversed at each impact, *i.e.* its momentum is changed by the amount $2mu$. The time taken for it to travel from a face and back again is $2/u$, and therefore the number of impacts in unit time is $u/2$. The total change of its momentum at the face in unit time is therefore mu^2. If all the molecules in the unit cube (n per unit volume) were moving similarly in parallel lines the change per unit time would be nmu^2, and since the face has unit area this would equal the pressure p upon it. In reality the molecules are moving in all directions; but their velocity V can be resolved into three mutually perpendicular components, u, v, w, where

$$\mathrm{V}^2=u^2+v^2+w^2.$$

Now, at the surface which we have considered it is only the normal component of velocity (say u) which is reversed at an impact, and the time between two impacts is determined by the same component. So that, taking three faces at right angles to each other,

$$p_x=nm\overline{u^2},\quad p_y=nm\overline{v^2},\quad p_z=nm\overline{w^2},$$

where we only need to consider average values because it is only an average pressure that can be observed experimentally.

But from hydrostatics we know that for a very small cube the pressure is the same on each of its surfaces. Hence

$$\overline{u^2}=\overline{v^2}=w^2=\tfrac{1}{3}\overline{\mathrm{V}^2},$$

whence each of these pressures is

$$p=\tfrac{1}{3}nm\overline{\mathrm{V}^2}.$$

The density D of the gas (*i.e.* the mass per unit volume) is nm. Hence

$$\frac{p}{\mathrm{D}}=pv=\tfrac{1}{3}\overline{\mathrm{V}^2}.$$

To yield Boyle's law $\overline{\mathrm{V}^2}$ must be constant at constant temperature. To yield Charles's law it must be proportional to the absolute temperature.

§ (15) Experimental Determination of the Coefficient of Increase of Pressure at Constant Volume for a Gas.—To determine β, the pressure coefficient of a gas, it is essential that the volume be kept practically constant during the investigation. A simple apparatus due to J. Joly will illustrate the method. A bulb about 10 cm. diameter is connected to a stem bent twice at right angles. This, in turn, is connected with a long straight glass tube by means of a flexible tube. The bulb contains dry air (or other gas) which is enclosed by means of mercury. The mercury can be brought up to a fiducial mark A on the horizontal limb by raising the straight tube; the air will then have a definite volume. The pressure in the bulb is given by the barometric height + the difference of levels of the mercury h; *i.e.* $p=\text{B}+h$. The bulb is heated in a water bath, and at successive temperatures the original volume is restored, and the fresh pressure determined. Putting $p=p_0(1+\beta t)$, any two temperatures with the corresponding pressures enable β to be calculated. In reality the volume changes slightly owing to the expansion of the glass (volume coefficient $=\gamma$), and the true coefficient is given by

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[1] The gram-molecule is the quantity of the gas which has a mass equal to the molecular weight expressed in grams.

$$p(1+\gamma t)=p_0(1+\beta t),$$

or

$$p=p_0(1+(\beta-\gamma)t)$$

nearly for moderate temperatures.

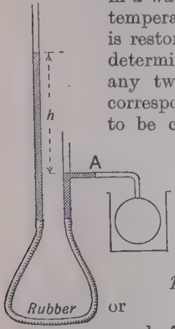

Fig. 12.

When the temperature range is great it may be necessary to use the more exact formula (the last but one). When precise values are sought for, attention must be given to several sources of error. The stem between A and the bulb, being outside the water bath, does not get heated to the same extent as the bulb. To correct for this it is necessary beforehand to determine the relative value of the volume v of this portion compared with that of the bulb V, and in the course of the experiment to observe its mean temperature τ. The true pressure requires the heights of the mercury columns and of the barometer to be "corrected for temperature." Two temperatures and pressures then give the equations

$$p_1\left[\frac{\text{V}_0(1+\gamma t_1)}{1+\beta t_1}+\frac{v_0(1+\gamma\tau_1)}{1+\beta\tau_1}\right]=p_0(\text{V}_0+v_0),$$

$$p_2\left[\frac{\text{V}_0(1+\gamma t_2)}{1+\beta t_2}+\frac{v_0(1+\gamma\tau_2)}{1+\beta\tau_2}\right]=p_0(\text{V}_0+v_0),$$

from which β can be calculated.

The following values were obtained by Regnault for air at various pressures:

Pressure Coefficient (β) for Air

Pressure at 0° C.	Pressure at 100 ° C.	β.
mm.		
109·72	149·31	0·0036482
174·36	237·17	36513
266·06	395·07	36542
374·67	510·35	36587
375·23	510·97	36572
760·00	..	36650
1678·40	2286·09	36760
1692·53	2306·23	36800
2144·18	2924·04	36894
3655·56	4992·09	37091

§ (16) **Determination of the Coefficient of Expansion at Constant Pressure.**—The methods of the determination of this coefficient can be illustrated in various ways. The apparatus consists essentially of a volume thermometer, but with special provision for keeping the pressure constant. Regnault employed the apparatus shown in *Fig.* 13. The bulb was well dried and filled with dry air (or other gas) at the pressure of the atmosphere. The adjustment of pressure was made by altering the amount of mercury until it came up to the level a in both limbs, while a side tube p was open to the atmosphere, the bulb being meanwhile in melting ice. This lateral tube was then sealed off. Let V_0 be the volume of the bulb at 0° C., v the volume of the stem as far as a at 0°, and H the initial pressure. The bulb is then placed in a steam jacket and raised to 100° C. Due to the rise the pressure changes, but it can be once more brought back to atmospheric (or near it) by running mercury out through the stop-cock. Let it reach the level b. If the level is rather higher on the left the difference is measured and the pressure calculated; let it be H′. If the stem has been previously calibrated the extra volume v' of the stem between a and b can be found. If the temperature of this part is t, we have

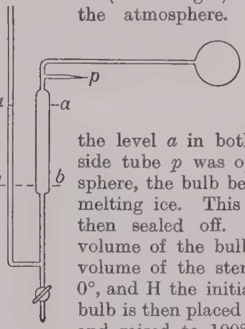

Fig. 13.

$$\text{H}'\left[\frac{\text{V}_0(1+\gamma 100)}{1+\alpha 100}+\frac{v(1+\gamma\tau')}{1+\alpha\tau'}+\frac{v'(1+\gamma t)}{1+\alpha t}\right]$$

$$=\text{H}\left[\text{V}_0+\frac{v(1+\gamma\tau)}{1+\alpha\tau}\right],$$

where τ and τ' are the initial and final volumes of the portion v of the stem, and γ the coefficient of volume expansion of glass. From this equation α is calculated; first, the small terms containing v and v' are neglected, and then the approximate value is inserted in the small terms, which are now taken into account, and a more accurate value calculated.

The values obtained by Regnault are given below; they apply to the case when the pressure is one atmosphere:

Air	0·0036706
Hydrogen	·0036613
Carbon dioxide	·0037099
Sulphur dioxide	·0039028
Carbon monoxide.	·0036688
Nitrous oxide	·0037195
Cyanogen	·0038767

For the more permanent gases nearly the same value is obtained for β, and it is also nearly the same as α. The simple theory of gases requires (as we have seen) a constant, identical in value in the two cases.

Another method was used by Regnault. A bulb, capable of holding 800 to 1000 grams of mercury, was taken empty and connected through drying tubes with a pump. While the bulb itself was immersed in a steam bath it was evacuated and refilled with dry air several times. Finally it contains dry air at atmospheric pressure (B) and temperature (t_0). The tip of the stem is now sealed by being melted in a blowpipe flame. It is taken out of the bath and allowed to cool. It is then placed inverted with its tip dipping in a vessel of mercury, the bulb itself being surrounded with melting ice. In this position the tip is broken off with pincers and the mercury rises and partly fills the bulb. After standing thus for sufficient time to ensure equilibrium of temperature the height of mercury inside above that outside (h) is read. A small metal cap containing soft wax is slid over the tip so as to close it, and the bulb with its contents is taken and weighed (w_1). It is then completely filled with mercury and weighed again (w_2). Let the weight empty be w_0. Then the volume occupied at 0° C. by air that fills the bulb at t_0 is $(w_2-w_1)/(w_2-w_0)$ of the volume at 100°. Allowing for the small difference of pressure, we have

$$\left(\frac{\mathrm{B}(w_2-w_0)(1+\gamma t_0)}{1+\alpha t_0}\right)=(\mathrm{B}-h)(w_2-w_1).$$

If h is only small the value of α should be nearly the same as for constant pressure.

Regnault found that when the mercury rushed in on opening the tip it was apt to carry air in with it. He attributed this to the fact that mercury does not wet glass and a film of air enclosed between the mercury and the glass is carried in along the stream. In order to avoid this he surrounded the stem with small brass collars which amalgamated and made perfect contact with the mercury.

Full details as to modern work connected with the expansion of gases will be found in the article "Temperature, Realisation of the Absolute Scale of."

FLUIDS UNDER HIGH PRESSURES

§ (17) EXPERIMENTAL RESEARCHES.—Although fluids follow approximately the "perfect" law under low pressures, under high pressures this is markedly not the case; neither Boyle's nor Charles's law is then even roughly followed. As a particular case may be cited the distinction between a liquid and its gaseous vapour. They exist in equilibrium together under the same temperature and pressure; but the densities of the two parts (or *phases*) are very different. Nothing of this kind is indicated by the perfect gas law.

(i.) *Natterer and Cailletet.*—The first investigations for high pressures were made by Natterer,[1] who forced successive quantities of hydrogen, oxygen, or nitrogen into a closed vessel and measured the pressures by means of a weighted valve. His object was to produce liquefaction by pressure alone, as Faraday had liquefied chlorine and Thilorier carbon dioxide, but he did not succeed, although the pressure was increased in the case of nitrogen to 2790 atmospheres. He found from 100 atmospheres upwards the volume became greater than given by Boyle's law; that is, that at constant temperature pv increased. Cailletet[2] in 1870 showed that in the case of hydrogen pv regularly increases (for unit mass), while for air there is first a decrease, then an increase, with a minimum at about 80 atmospheres.

(ii.) *Andrews.*—In 1863 Andrews[3] observed that on liquefying carbon dioxide by pressure and gradually raising the temperature to 88° F. (31° C.) the surface of demarcation between the liquid and gas became fainter, lost its curvature, and at last disappeared. The space was then occupied by a homogeneous fluid, which became cloudy when the pressure was suddenly diminished or the temperature slightly lowered. Above 88° F. he could get no liquefaction. Nitrous oxide gave analogous results. He then proceeded to extend his investigations so as to determine a set of corresponding values of p, v, T, which he plotted as isothermals on a p, v diagram. The gas was contained in a thick-walled tube the upper third of which had a capillary bore, and was kept enclosed by mercury filling the lower part, and placed in a water reservoir. A similar tube containing air was in communication with it. Pressure was applied by means of a plunger through the water and its value read by the change of volume of the air, which was supposed to follow Boyle's law. The CO_2 tube had been carefully calibrated beforehand, and from the upper level of the

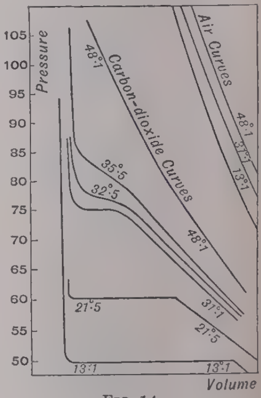

FIG. 14.

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[1] *Sitzungsb. d. Wien. Akad.* v. vi. vii., and *Pogg. Ann.*, 1844, lxii. 139; 1855, xciv. 436.

[2] *Comptes Rendus*, 1870, lxx. 1131.

[3] Miller's *Chemical Physics*, 3rd edit. p. 328; *Phil. Trans.*, 1869, Part II. 575.

mercury the volume of the gas was determined. The values obtained are sketched in *Fig.* 14. A small quantity of air was mixed with his CO_2, so that the results require some modification on this account. But they constitute the first determinations of the conditions under which CO_2 can be liquefied by pressure alone and of the deviations from the gas laws. While at 48° C. it behaves somewhat as an ideal gas, yet at 35° C. the isothermal already assumes a different character; at 31° C. it becomes horizontal at one point, and below this temperature the fluid divides into two parts, liquid and vapour. While liquid and vapour coexist at a given temperature, the pressure keeps constant (at the "saturation" pressure) whatever the relative proportions of the liquid and vapour. At a certain volume, v_2, the fluid is all gas, while at a certain smaller volume, v_1, it is all liquid. Further compression of the liquid requires greatly increasing pressure, *i.e.* the liquid is nearly incompressible. A dotted curve (the "border" curve) can be drawn connecting all the points v_1 for the liquid, and another for the points v_2 for the gas. These dotted curves meet at about 31° C., where the isothermal becomes horizontal, so that the border curve is dome-shaped. The summit is called the critical point because above it the heterogeneous region cannot exist. These are the main features of the results obtained. It is customary to regard as gas any state of the fluid above the critical temperature, and as liquid any state below the critical temperature, which is also on the less compressible side. There is, however, no discontinuity, *no way of detecting the passage from gas to liquid so defined,* except when the path of transformation passes through the heterogeneous region, and the fluid then splits into two parts which have different properties and can therefore be distinguished one from the other.

These properties are not peculiar to carbon dioxide; they are possessed by all fluids. But the values of p, v, and T at the critical point are different for different fluids.

(iii.) *Amagat.*—In 1878 Amagat began a series of elaborate investigations in the high-pressure region. His apparatus was installed at the bottom of a shaft 327 metres deep. It comprised a glycerine pump which could force mercury up a tube extending up the shaft and at the same time into an air manometer made of steel. The working tube was of glass 1 mm. internal diameter and 1 cm. external. It was surrounded by a jacket of glass through which water was circulated, and this in turn was surrounded by a copper jacket (for safety) which had narrow windows extending down opposite generating lines. Thermometers were placed every 30 metres. With such apparatus measurements were made up to 430 atmospheres on nitrogen.

In 1893 investigations were extended in some cases to 3000 atmospheres. For these it was necessary to enclose the working tube in a metallic one and to produce the same pressure outside as inside the glass tube so as to prevent the latter from bursting. It became necessary to read the height of the mercury by an indirect method, either using electric contacts (devised by Tait) or the method of "regards" devised by Amagat himself. He applied the method of electric contacts to oxygen, hydrogen, nitrogen, and air up to 3000 atmospheres at temperatures of 0°, 15°, and 45° C., and that of "regards" to oxygen, hydrogen, nitrogen, and air from 1 to 1000 atmospheres (by steps of 50 atmospheres) at temperatures of 0°, 15°, 100°, and 200° C. Carbon dioxide and ethylene were examined, at closer intervals, up to 1000 atmospheres every 10° from 0° to 100°, then at 137° C. and (in the case of CO_2) even 258° C. A great number of observations besides were made on CO_2 in the neighbourhood of the minimum value of pv. The general character of the results is exhibited in *Fig.* 15 for hydrogen and *Fig.* 16 for carbon dioxide In these diagrams the value of pv is plotted against p. The chief point of distinction is that the curve for hydrogen *ascends* from the start, whereas for carbon dioxide it first descends, reaches a minimum, and then ascends. Hydrogen came to be known thence as a "pluperfect" gas, CO_2 being called an "imperfect" gas. A perfect gas would of course have given a horizontal straight line at each temperature. At the minimum point the real curves are horizontal, and it will be observed that *ultimately,* as the temperature rises, the minimum approaches $p=0$. At higher temperatures CO_2 might be expected to

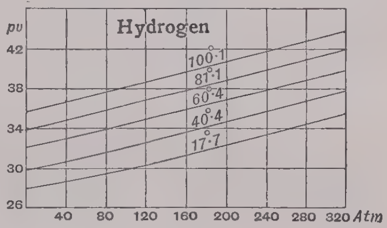

Fig. 15.

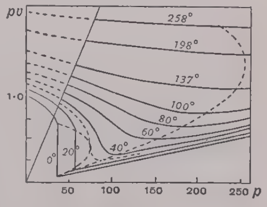

Fig. 16.

behave like H_2 at ordinary temperature, so that there is no essential distinction between the two gases. We return to this point later.

Amagat's investigations are all related to temperatures of 0° C. and upwards.

A table of relative values of the volume for various gases is given on p. 889.

(iv.) *Witkowski.*—In 1891 Witkowski[1] published an account of an investigation on air comprising temperatures below 0° C. These experiments were of two kinds: in one the quantity of gas was kept constant (as in Andrews' and Amagat's cases); in the other the volume was kept constant (as in Natterer's investigations). His results are given on p. 889.

From these he calculates the pressures at which pv is a minimum for air:

Temperature.	Pressure.	Minimum pv (approximately).
+100	< 10 atm.	1·367
+16	79	1·0379
0	95	0·9680
−35	115	0·8004
−78·5	123	0·5519
−103·5	106	0·3873
−130	66	0·1985
−135	57	0·1551

In 1904 A. W. Witkowski[2] published the corresponding results for hydrogen. These were obtained with the most scrupulous care

The relative values of pv are given on p. 889.

The minima of pv occur at the following pressures (approximate for hydrogen):

Temperature.	Pressure Atm.
−183° C.	32
−190	43·5
−205	55
−212	54·8

The value at about −183° C. does not correspond with what would be required by the law of corresponding states (even approximately), to which reference is made later in this article. Witkowski considers it to be not too risky to conclude that before hydrogen reaches its critical temperature a polymerisation takes place. What takes place would be explained if for temperatures above −190° C. the gas behaved as though its critical pressure were about double that which results from experiments on liquid hydrogen.

(v.) *Kamerlingh Onnes.*—Kamerlingh Onnes and C. Braak[3] have also made an extensive investigation on hydrogen at low temperatures. They find for the minima of pv the following positions and values:

$t°$ C.	p Atm.	$\frac{\text{Density}}{\text{Density at 0° and 1 Atm.}}$	$p\times\frac{\text{Vol.}}{\text{Vol. at 0° and 1 Atm.}}$
−182·81	32·57	99·70	·32663
−195·27	50·07	183·10	·27348
−204·70	54·67	238·27	·22945
−212·82	54·09	287·99	·18782
−217·41	42·54	321·51	·16342

§ (18) THEORETICAL CONSIDERATIONS.—The departure of the behaviour of real fluids from the laws of perfect gases indicates that other properties require to be taken into account. In the first place, it is insufficient to regard the particles of the fluid as merely moving points—each must have a certain volume. In the second place, it is necessary to allow for the effect of attractive forces between them. The existence of such forces is made very obvious in the case of solids. They are overcome whenever a solid is broken. Work is done against them whenever it is stretched or twisted. Similarly in a liquid the particles are kept from flying apart (as in a gas) by the presence of such forces. The fact that in a gas at moderate pressures they are not so conspicuous in their effects is explained by the assumption that the forces diminish with increased distance between the particles. The rate of diminution with alteration of their separation must indeed be very great: for if after a solid is broken the two parts are placed in close juxtaposition with moderate pressure they do not reunite: great pressures are necessary for the approximation of the surfaces to become sufficiently near for reunion to occur. Such reunion takes place when powders are formed into blocks under pressure, as in lead pencils, crayons, etc. Any attractions assumed to be present must therefore become practically insensible at even very small distances—say 10^{-6} cm.; though it can be shown that at closer approach they become very large indeed, giving rise to an effective pressure inside a liquid of the order of 10,000 atmospheres. The molecules are prevented from approaching into permanent absolute contact with each other by their thermal agitation in much the same way as planets are kept from falling into the sun by their motions round it. In other words, a state of thermal equilibrium is set up.

(i.) *Clausius, The Virial.*—Some advance can be made toward allowing for these additional complications by means of a theorem due to Clausius. Let x, y, z be co-ordinate positions of a single particle. If x^2 be

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[1] *Proc. Acad. Sci. Cracow*, 1891, xxiii.; *Phil. Mag.*, 1896, xli. 288, xlii. 1.

[2] *Bull. de l'Acad. de Sc. de Cracovie*, June 1905.

[3] *Comm. Phys. Lab. Leiden*, No. 100, 1907.

Relative Volumes (Amagat) (Volume at 0° C. and 1 Atm. ≡ 1,000,000)

Atm.	Oxygen.			Air.			Nitrogen.			Hydrogen.		
	0°.	99°·5.	199°·5.	0°.	99°·4.	200°·4.	0°.	99°·5.	199°·6.	0°.	99°·3.	200°·5.
100	9265	..	..	9730	..	..	9910	..	..	..	..	..
200	4570	7000	9095	5050	7360	9430	5195	7445	9532	5690	7567	9420
300	3208	4843	6283	3658	5170	6622	3786	5301	6715	4030	5286	6520
400	2629	3830	4900	3036	4170	5240	3142	4265	5331	3207	4147	5075
500	2312	3244	4100	2680	3565	4422	2780	3655	4515	2713	3462	4210
600	2115	2867	3570	2450	3180	3883	2543	3258	3973	2387	3006	3627
700	1979	2610	3202	2288	2904	3502	2374	2980	3589	2149	2680	3212
800	1879	2417	2929	2168	2699	3219	2240	2775	3300	1972	2444	2900
900	1800	2268	2718	2070	2544	3000	2149	2616	3085	1832	2244	2657
1000	1735	2151	..	1992	2415	2828	2068	..	..	1720	2093	..

Values of *pv* for Air (Witkowski)

Pressure in Atm.	Temperatures Centigrade.									
	+100.	+16.	0.	−35.	−78·5.	−103·5.	−130.	−135.	−140.	−145.
1	1·367	1·0587	1·0000	0·8716	0·7119	0·6202	0·5229	0·5046	0·4862	0·4679
10	1·3678	550	0·9951	..	..	..	..	..	..	..
15	685	529	923	..	..	..	..	..	0·4095	0·3786
20	691	509	897	..	0·6778	0·5697	0·4410	..	3808	447
25	698	488	869	..	689	559	183	..	476	015
30	704	468	842	..	599	417	0·3936	0·3502	063	2444*
35	713	449	816	..	510	270	650	115	2419	..
40	725	433	793	..	423	125	329	0·2598	1128	..
45	738	419	772	..	335	0·4980	0·2963	1942	..	..
50	754	408	754	0·8288	252	839	544	1605	..	..
55	770	399	738	253	170	701	171	1553	..	..
60	784	390	723	219	089	567	013	1556	..	..
65	802	384	710	187	011	439	0·1985	1576	..	..
70	821	381	701	158	0·5937	318	989	..	..	..
75	842	379	694	132	863	206	0·2013	..	..	..
80	866	379	688	105	796	103	043	..	..	..
85	887	380	684	081	734	014	..	..	..	..
90	908	382	681	058	680	0·3948	..	..	..	..
95	929	386	680	038	634	903	..	..	..	..
100	951	390	681	023	600	881	..	..	..	..
105	977	397	685	013	568	874	..	..	..	..
110	1·4004	406	690	006	544	877	..	..	..	..
115	34	418	699	004	530	892	..	..	..	..
120	65	432	710	006	520	914	..	..	..	..
125	..	448	722	0·8012	520	944	..	..	..	..
130	..	1·0467	0·9738	..	0·5528	981	..	..	..	..

* Corresponds to 29 atm.

Values of *pv* for Hydrogen (Witkowski) (Value at 0° C. and 1 Atm. ≡ 1)

Pressure in Atm.	+100° C.	−0°.	−77°.	−104°.	−147°.	−183°.	−190°.	−205°.	−212° C.
1	1·3661	1·0000	0·7180	0·6189	0·4611	0·3283	0·3023	0·2470	0·2207
5	688	024	201	208	622	284	020	452	180
10	721	055	228	232	635	284	015	427	145
15	755	086	255	255	648	279	004	401	106
20	789	118	282	279	661	272	0·2991	373	065
25	823	150	309	303	674	270	84	345	029
30	858	181	336	327	689	270	77	321	0·1997
35	892	213	364	352	705	270	73	301	68
40	927	245	391	376	721	270	70	288	46
45	961	277	418	402	739	273	70	280	33
50	996	309	445	427	758	278	72	275	28
55	1·4030	341	473	452	779	286	77	273	26
60	1·4064	1·0373	0·7501	0·6478	0·4801	296	0·2984	0·2275	0·1928

differentiated twice with respect to the time t, we get

$$\frac{d^2}{dt^2}(x^2)=2x\frac{d^2x}{dt^2}+2\left(\frac{dx}{dt}\right)^2.$$

Now if m is the mass, $m(d^2x/dt^2)$ is the component force X in the direction of x, whence

$$-\tfrac{1}{2}\mathrm{X}x=\tfrac{1}{2}m\left(\frac{dx}{dt}\right)^2-\frac{m}{4}\frac{d^2}{dt^2}(x^2).$$

If similar equations be written down for the other two co-ordinate directions and all three be added,

$$-\tfrac{1}{2}(\mathrm{X}x+\mathrm{Y}y+\mathrm{Z}z)=\tfrac{1}{2}mv^2-\frac{m}{4}\frac{d^2(r^2)}{dt^2},$$

where v is the resultant velocity and r the distance of the particle from the origin. By adding together such equations for all the particles an expression for the total kinetic energy $\mathrm{E}=\Sigma(\tfrac{1}{2}mv^2)$ is obtained. Now in the kind of motion with which we have to deal it is only mean values that have any significance. The mean value of the last term in any period t is defined by

$$\frac{1}{t}\int_0^t \frac{m}{4}\frac{d^2r^2}{dt^2}dt=\frac{m}{4t}\left[\left(\frac{dr^2}{dt}\right)_t-\left(\frac{dr^2}{dt}\right)_0\right].$$

Now this mean value fluctuates, but, provided the velocities are always confined between definite limits, it will always vanish if a sufficient time elapses because t occurs in the denominator. The steady average value of the kinetic energy E is therefore given by

$$\mathrm{E}=-\Sigma\tfrac{1}{2}(\overline{\mathrm{X}x}+\overline{\mathrm{Y}y}+\overline{\mathrm{Z}z}),$$

where the barred quantities on the right-hand side indicate average values.

Let the forces be purely external forces (as in the simple kinetic theory of gases) and consist of a uniform pressure p acting normally to the surface. Let θ be the angle between the direction of p and the axis of x, $d\mathrm{A}$ an element of the surface. Then

$$\mathrm{X}=-pd\mathrm{A}\cos\theta-\mathrm{X}x=pxd\mathrm{A}\cos\theta$$
$$=px \text{ element of volume},$$

as shown by dotted lines (*Fig.* 17), and $-\Sigma\mathrm{X}x$ is equal to $p\times$total volume. Similarly for the y and z components. Hence

Total kinetic energy $=\tfrac{3}{2}p\times\text{Vol.}$,

which is the equation obtained from the simple theory.

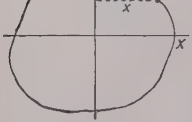

FIG. 17.

(ii.) *Van der Waals' Equation.*—When forces operative between the molecules are to be included they can be divided into two groups: (i.) Attractive forces acting sensibly over short ranges, and (ii.) repulsive forces which only act during contact. The effect of the former is to increase the pressure inside over that at the surface. Neglecting a thin layer at the surface (whose thickness may be taken as equal to the distance of sensible molecular attraction), the pressure will be uniform of value $\mathrm{P}=p+\mathrm{K}$, where K is called the intrinsic pressure. Van der Waals showed on certain assumptions that K varies directly as the square of the density (*i.e.* inversely as the square of the specific volume), and further that the forces due to contact could be taken into account by subtracting a constant b from the specific volume. Hence finally, if K is written a/v^2, where a is a constant, the equation

$$\left(p+\frac{a}{v^2}\right)(v-b)=\mathrm{RT}$$

was obtained as the form holding under wider ranges than the perfect gas equation. This is known as van der Waals' equation. Since it only professed to be a second approximation, it is not surprising that it is insufficient to represent completely the behaviour of a fluid at various temperatures and pressures. It is rather a matter of surprise that any such simple equation can represent the general properties of a fluid as well as it does. When fairly precise numerical values are considered it is, however, found that it must be used with considerable caution. In any case it does not give the constant pressure inside the heterogeneous region, but replaces it by an S-shaped curve. This will be dealt with later.

The significance of the equation can best be seen by writing it

$$v^3-\left(b+\frac{\mathrm{RT}}{p}\right)v^2+\frac{av}{p}-\frac{ab}{p}=0.$$

Any isothermal (T=constant) is therefore given by a cubic equation in v, and hence for a given pressure there will be possible either one volume or three according to the values of a, b, T, p.

Any cubic equation can be written

$$(v-v_1)(v-v_2)(v-v_3)=0,$$

where v_1, v_2, v_3 are the three roots. Comparing this with van der Waals' equation,

$$v_1+v_2+v_3=b+\frac{\mathrm{RT}}{p},$$

$$v_1v_2+v_2v_3+v_3v_1=\frac{a}{p},$$

$$v_1v_2v_3=\frac{ab}{p}.$$

The critical point is represented by the case in which the three roots coalesce into one; in this case

$$3v_c = b + \frac{RT_c}{p_c}, \quad . \quad . \quad . \quad \text{(i.)}$$

$$3v_c^2 = \frac{a}{p_c}, \quad . \quad . \quad . \quad . \quad \text{(ii.)}$$

$$v_c^3 = \frac{ab}{p_c}, \quad . \quad . \quad . \quad . \quad \text{(iii.)}$$

where the critical values are distinguished by the suffix *c*. Whence from (iii.) and (ii.)

$$v_c = 3b;$$

whence (ii.) gives

$$p_c = \frac{a}{3v_c^2} = \frac{a}{27b^2},$$

and then (i.) gives

$$T_c = \frac{8a}{27bR}.$$

The coincidence of the three values of v is equivalent to making $\partial p/\partial v = 0$ and $\partial^2 p/\partial v^2 = 0$; these conditions can still be applied for equations of state for which the above method cannot be used. The isothermals have the same general characteristics as those in Andrews' diagram for carbon dioxide. One way of showing that considerable numerical discrepancy exists, however, is to calculate the expression RT_c/p_cv_c, and compare with the experimental values. The expressions for the critical values give the value 2·66 for this fraction, whereas the experimental values give usually about 3·7 or still greater. The following table is abridged from one collated by S. Young, who takes the mean value for a large number of substances as 3·75:

Substance.	$\frac{Rt_e}{p_cv_c}$.
Benzene	3·750
Isopentane	3·732
Carbon tetrachloride	3·674
Ether	3·811
Ethyl acetate	3·944
Acetic acid	4·986
Ethyl alcohol	4·024
Methyl alcohol	4·549

Kamerlingh-Onnes,[1] however, finds the value 3·424 for argon, 3·419 for oxygen, and for nitrogen 3·421. Patterson, Cripps and Gray [2] find the value 3·605 for xenon. These values make it impossible to connect the atomicity and the magnitude of the ratio; though the large values are for the more complicated molecules. A further discrepancy is in the value of the critical volume. If b be taken as the least volume into which unit mass can be compressed by an indefinitely large increase of pressure, van der Waals makes it one-third of the critical volume. Now the least volume cannot differ much from the volume of the liquid under ordinary conditions (especially when the ordinary state is far below the critical), but will be slightly less; yet the experimentally determined critical volume is, by experiment, about *four* times the liquid volume. Again, Clausius showed that the "constant" a is really a function of the absolute temperature; he wrote a/T instead of a. Others have substituted $ac^{-\alpha T}$ instead of a. In either of these ways a better fit with experiment is obtained.

§ (19) LAW OF CORRESPONDING STATES.—If van der Waals' equation is taken, it is convenient to measure p, v, and T, not in absolute measure, but as fractions of the corresponding critical data for the particular substance under consideration; *i.e.* to write $p = \alpha p_c$, $v = \beta v_c$, $T = \gamma T_c$. Inserting these values and cancelling common factors, the equation becomes

$$\left(\alpha + \frac{3}{\beta^2}\right)(3\beta - 1) = 8\gamma,$$

which contains no constants peculiar to the particular substance. The values α, β, γ are called "reduced values"; the equation is known as the "reduced equation of state." So far as it is true it implies that all substances should behave alike for the same values of α, β, γ. This is the law of corresponding states—a law which is at least approximately true. The validity of this law is not peculiar to van der Waals' equation, however; it can be shown that it must also be true if the equation of state only contains three constants (such as a, b, R), and at the same time indicates a critical state; for the three constants can be eliminated by means of the values for the *three* critical data. For example, if a is replaced by a/T^n, where n is any power, the reduced equation is

$$\left(\alpha + \frac{3}{\gamma^n\beta^2}\right)(3\beta - 1) = 8\gamma.$$

Clausius took $n = 1$ (Clausius I.).

Another test of the equation is to find the value of $(\partial\alpha/\partial\gamma)_\beta$ at the critical point, *i.e.* the pressure coefficient at constant volume. Taking the last more general equation, the value of the pressure coefficient is found to be

$$\frac{\partial\alpha}{\partial\gamma} = \frac{8}{3\beta - 1} + \frac{3n}{\gamma^{n+1}\beta^2},$$

or at the critical point ($\alpha = \beta = \gamma = 1$)

$$\left(\frac{\partial\alpha}{\partial\gamma}\right)_{\text{crit.}} = 4 + 3n.$$

Now, experimental results give a value 7 or thereabouts; so that n is near unity as Clausius supposed.

A still further test is to examine the slope on an Amagat diagram (pv against p or the corresponding reduced diagram, $\alpha\beta$ against α). Multiplying the equation by β we get

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[1] *Comm. Phys. Lab. Leiden*, No. 145.

[2] *Roy. Soc. Proc.* A., 1912, lxxxvi. 579.

$$\alpha\beta=\frac{8\gamma\beta}{3\beta-1}-\frac{3}{\gamma^n\beta}.$$

Now $$\frac{\partial(\alpha\beta)}{\partial\alpha}=\frac{\partial(\alpha\beta)}{\partial\beta}\cdot\frac{\partial\beta}{\partial\alpha},$$

whence

$$\frac{\partial(\alpha\beta)}{\partial\alpha}=\left(-\frac{8\gamma}{(3\beta-1)^2}+\frac{3}{\gamma^n\beta^2}\right)\div\left(-\frac{24\gamma}{(3\beta-1)^2}+\frac{6}{\gamma^n\beta^3}\right).$$

Considering specially the region of very low pressures for which β is very large, this is approximately

$$\frac{\partial(\alpha\beta)}{\partial\alpha}=\frac{8-(27/\gamma^{n+1})}{24}.$$

The isothermal should therefore start out horizontally ($\alpha=0$) when $\gamma^{n+1}=27/8$. Now it is found that for nitrogen this horizontality occurs when $\gamma=2{\cdot}56$, which would require n to be 0·29. Moreover, for helium it occurs at ordinary temperatures, *i.e.* for $\gamma=50$ or thereabouts, which would require $n=-{\cdot}7$.

These results indicate that the properties of gases are more complicated than implied by van der Waals' equation or simple modifications thereof. Berthelot has made use of the following modification *for low pressures only*:

$$\left(\alpha+\frac{16}{3\gamma\beta^2}\right)(\beta-\tfrac{1}{4})=\frac{32}{9}\gamma.$$

It clearly does not fit at the critical point.

§ (20) THE GENERAL EQUATION OF STATE.—The imperfections of van der Waals' equation have led to innumerable other attempts.

(i.) *Dieterici.*—Dieterici has discussed the more general equation

$$\left(p+\frac{a}{v^k}\right)(v-b)=\text{RT},$$

and finds that $v_c=4b$ (as required) if $k=5/3$ (Dieterici I.).

One objection to this equation is that $\partial(pv)/\partial p$ cannot be zero at $p=0$ unless T is infinite.

(ii.) *Clausius.*—Clausius has examined in detail the equation

$$\left(p+\frac{a}{\text{T}(v+c)^2}\right)(v-b)=\text{RT}.$$

(Clausius II.)

This contains a fourth constant c, so that it is capable of representing deviations from the law of corresponding states.

(iii.) *Dalton.*—J. P. Dalton writes the reduced equation

$$\left(\alpha+\frac{3e^{(1-\gamma)}}{\beta^2}\right)(3\beta-1)=8\gamma,$$

(J. P. Dalton)

and calculates very satisfactory values for vapour pressures from it, although it suffers from many of the defects of the original equation.

(iv.) *Dieterici's Second Method.*—Dieterici approached the question also by a different method. Regarding the surface film of a fluid as being a layer of transition in which the pressure changes *gradually* from the external pressure to that inside, and assuming that the density in this layer follows the law of perfect gases, he finds the equation

$$p=\frac{\text{RT}}{v-b}e^{-\frac{\text{A}}{\text{RT}v}},$$

(Dieterici II.)

(e = base of natural logarithms), which in the reduced form becomes

$$\alpha=\frac{\gamma}{2\beta-1}e^{2\left(1-\frac{1}{\gamma\beta}\right)}.$$

This can be made more general by taking T^n instead of T in the index of e. The value of RT_c/p_cv_c at the critical point becomes $e^2/2=3{\cdot}69$, which is a great improvement on van der Waals'. But $v_c=2b$, which is hopelessly wrong. Dieterici then takes b as being a function of the volume instead of a constant; and is thereby able to get a closer correspondence with facts; but this is effected only by making the equation too complicated to be convenient. With constant b it fits very much better than van der Waals' equation in the region of low pressures, but is hopeless for high pressures — a fact that is illustrated by the accompanying table of values for the critical isotherm for isopentane.

CRITICAL ISOTHERM OF ISOPENTANE

$\text{T}_c=460{\cdot}8$ abs. $v_c=4{\cdot}266$ cm.^{3}.

v in cm.^{3}.	p (calculated) mm. Mercury (Dieterici).	p (observed). mm. Mercury.
2·4	42,730	49,080
2·5	35,810	40,560
2·6	32,090	34,980
2·8	28,390	28,940
3·0	26,780	26,460
4·0	25,320	25,020
5·0	25,240	24,940
6·0	24,880	24,840
8·0	23,400	23,710
10·0	21,590	22,040
20·0	14,560	14,840
40·0	8,508	8,570
60·0	5,978	6,001
80·0	4,604	4,614
90·0	4,127	4,132
100·0	3,740	3,750

If the power of T in the index is taken as 3/2 good correspondence is obtained in the region of moderate pressures for gases as diverse as nitrogen, isopentane, and hydrochloric acid.

That v should occur to the first power in the denominator of the index was not absolutely determined by the theory. By calculating, from Young's experimental data for isopentane, the numerical values of $T/p \,.\, (\partial p/\partial T) - 1$, I find that it is approximately a parabolic function of $1/v$. Dieterici's equation requires the value $nA/(RT^n v)$.

§ (21) COMPARISON OF RESULTS.—A summary of the results of the tests above given for various equations of state is exhibited in the accompanying table:

Author.	$\frac{RT}{v-b}$.	$\frac{RT_c}{p_c v_c}$.	v_c.	$\left(\frac{T}{p}\frac{\partial p}{\partial T}\right)_{\text{crit.}}$	γ for $\frac{\partial(\alpha\beta)}{\partial\alpha}=0$ at $p=0$.
Van der Waals . .	$p+\frac{a}{v^2}$	2·67	$3b$	4	3·375
Clausius I. . . .	$p+\frac{a}{Tv^2}$	2·67	$3b$	7	1·837
Dieterici I. . . .	$p+\frac{a}{v^k}$ $(k=5/3)$	3·75	$4b$	5	Infinity
„ (modified)	$p+\frac{a}{T^{\frac{1}{2}}v^{5/3}}$	3·75	$4b$	7	Infinity
J. P. Dalton . . .	(page 892)	2·67	$3b$	7	1·69
Dieterici II. . . .	$pe^{\frac{A}{RTv}}$	2·69	$2b$	3	4
„ (modified)	$pe^{\frac{A}{RT^n v}}$	2·69	$2b$	$1+2n$	$(4)^{\frac{1}{n}}$
„ $n=\frac{3}{2}$.	„	„	„	4	2·52
Clausius II. . . .	$p+\frac{a}{T(v+c)^2}$ $c=b/2$	3·00	$4b$	7	2·25
Experimental . .	. .	3·5 upwards	$4b$	7	Nitrogen 2·56 Helium 50

§ (22) THE JOULE-THOMSON EFFECT.—An additional method of testing a gas equation is provided by the thermodynamic equation for the Joule-Thomson effect.[1] This effect will be discussed in detail in the article on "Thermodynamics." The inversion points for this effect can be calculated from any experimental data for p, v, T, and can then be compared with the theoretical positions (for which $T(\partial v/\partial T)_p - v$ is zero). Curves are given (*Fig.* 18) of reduced temperatures plotted against reduced pressures for inversion points, while the points determined from experimental data are shown as small circles (for CO_2) and crosses (for nitrogen). It will be seen that Dieterici's equation II. gives far better correspondence with experiments than van der Waals'. The value of n taken in the calculation is 3/2; better agreement still is obtained if $n=1\frac{2}{3}$. The test made in this way involves a large range of pressure and of temperature It is not confined to one small region, as in many of the other tests that we have described.

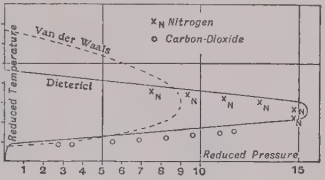

FIG. 18.

§ (23) CALLENDAR'S EQUATION. STEAM.—Besides the equations already given, which attempt to represent the behaviour of fluids over long ranges, equations have also been developed which are of more limited applicability. Most important of these is the equation of Callendar, which is

$$v-b=\frac{RT}{p}-c,$$

where c is taken as varying according to the inverse n^{th} power of the absolute temperature, *i.e.* $c=c_0T_0^n/T^n$. The equation, it is claimed, holds good for *moderate pressures*; for higher pressures it is necessary to regard c as being also a function of the pressure. The equation has been applied by him particularly to the case of steam; and since, from its simplicity, the various properties of steam can be coordinated by manageable expressions when use is made of the thermodynamic equations which connect these properties, it is rapidly coming into general use. The general nature of Callendar's equation can be best described by pointing out that the isothermals on a p, v diagram given by it will be rectangular hyperbolas as for a perfect gas, but shifted sideways by amounts, $c-b$, depending upon the temperatures. It makes no attempt to represent the critical data, nor does its applicability extend to the liquid state at all; for that reason it would be better described as applying to moderate densities rather than to moderate pressures. For steam the value of n, between 100° and 150° C., is about 10/3. At 160° C. the value of c is 16 cm.^{3}, and since it increases with fall in temperature it is always, through the range in which the equation is used, greater than b, which is taken as 1 cm.^{3}. The existence of c is taken to imply coaggregation of single molecules into double and treble molecules; but it may equally well represent the effect of the attractions between molecules which may be regarded as a potential coaggregation. It is safer so to interpret it; for whatever may happen in the case of steam there is no evidence that molecular association takes place when a substance like isopentane liquefies; yet a term like c is necessary in this case also.

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[1] See Porter, *Phil. Mag.*, 1906, xi. 554; 1910, xix. 888.

Its relation to van der Waals' expression is shown by first neglecting the product of small quantities in the latter, whence

$$p(v-b)+\frac{a}{v}=RT,$$

or

$$v-b=\frac{RT}{p}-\frac{a}{pv}.$$

If, next, pv in the small term is replaced by its approximate equivalent RT and a is taken as a function of temperature alone, Callendar's equation is obtained.

§ (24) METHODS OF EXPRESSING RESULTS.—Deviations from Boyle's and Charles' Laws are usually represented by a diagram of pv against p. Kamerlingh-Onnes has also made use of a pv against density diagram. Another very useful way, for some purposes, is to plot against temperature the discrepancy $\Delta=v-Tv_0/T_0$, where v is the actual specific volume at temperature T, v_0 the specific volume at any standard temperature T_0; each curve obtained being a constant pressure line. As examples, the curves for hydrogen at 60 and 30 atmospheres respectively are drawn in *Fig.* 19. These curves represent Witkowski's determinations, which were obtained with the most scrupulous care. Throughout the whole range of his observations these curves can be fitted with hyperbolas with one asymptote *nearly* vertical. It should be noted that if the product term ab in van der Waals' equation is neglected one obtains

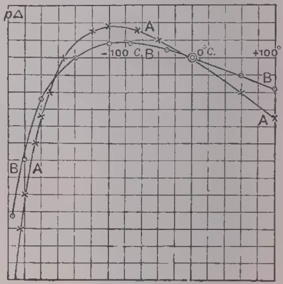

FIG. 19.

$$v^2-\left(b+\frac{RT}{p}\right)v+\frac{a}{p}=0,$$

i.e. a hyperbola with a vertical asymptote.

One use to which such a diagram can be put is to an examination of specific heats. (See article "Thermodynamics.") It is shown thermodynamically that

$$\frac{\partial C_p}{\partial p}=-T\frac{\partial^2 v}{\partial T^2},$$

where C_p is the specific heat at constant pressure p, v is the specific volume. An examination of the discrepancy Δ shows that the right-hand side can also be written $-T(\partial^2\Delta/\partial T^2)$; whence it follows that when the constant pressure line is convex upwards the value of Cp increases with pressure, and it

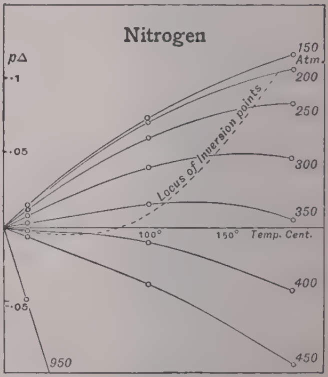

FIG. 20.

diminishes when the curve is concave upwards. Since the discrepancy is equal to b—*i.e.* the least volume of the liquid—when T is zero, curves such as those shown for hydrogen must turn back somewhere between the lowest temperature shown and absolute zero so as to cross over to the positive side; this can only take place by the curvature reversing in sign—the curves becoming concave upwards at low temperatures. In this latter region the value of C_p must diminish with increase in pressure. Discrepancy curves are also shown (*Figs.* 20, 21) for nitrogen and carbon-dioxide based on Amagat's investigations.[1]

The curves for CO_2 are concave upwards in the region shown; in this region C_p diminishes with increase in pressure.

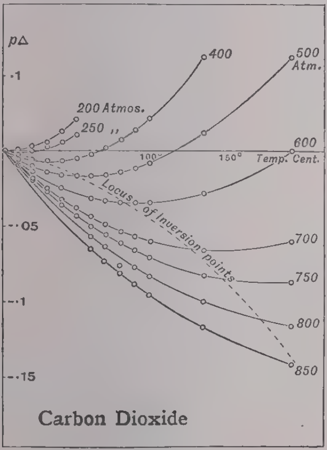

FIG. 21.

§ (25) THE QUANTUM THEORY.—In recent years initial attempts have been made to introduce the consequences of the quantum theory; that is, the theory that in the interchanges of energy that take place between molecules certain integral relations are always preserved between the amounts transferred. There is no doubt that if this theory is true modifications will be required even in the equation for a "perfect" gas; for the ordinary theory allows energy to be transferred in any proportion whatever, while according to the quantum theory it may not be transferred because (for example) it is smaller than the unitary amount (the quantum) which is transferable. This unitary amount is proportional to the frequency of the atom, say $h\nu$, where h is a universal constant.[2] If we write $\beta=(hN/R)$, where N is the number of atoms per gram atom, and R is the ordinary gas constant, then Nernst has shown that the gas equation becomes

$$pV\left(1-\frac{\beta\nu}{1.2T}+\frac{(\beta\nu)^2}{1.2.3.T^2}-, \text{ etc.}\right)=RT.$$

For hydrogen gas $\beta\nu=0°·1417$, where V is the normal molecular volume.[3]

Again, Planck has started from considerations of entropy and energy, the entropy being defined as $S=k\log W+\text{const.}$, W being the probability of any one state of a given monatomic gas, and k is the gas constant per atom. He deduces finally the equation

$$p=-\frac{RT}{c}\log\left(1-\frac{c}{v}\right)-\frac{a}{v^2},$$

where c is theoretically equal to eight times the volume of an atom regarded as spherical, *i.e.* $2b$ as calculated by van der Waals. This equation could only be applicable throughout a limited range because the least volume is clearly c itself; so that it is really c which takes the place of b in van der Waals' equation, but the value calculated for it from theory is greater than van der Waal's value. This equation yields critical values

$$v_c=2c,\quad RT_c=\frac{a}{2c},\quad p_c=\frac{a}{2c^2}(\log_e 2-\tfrac{1}{2});$$

$$\frac{RT_c}{p_c v_c}=2·6 \text{ nearly.}$$

The value of $T/p\,.\,\partial p/\partial T$ at the critical point is 3·6. The probability is that a should be taken as varying inversely as the temperature, and then this number becomes 6, which is much closer to the experimental value. But it fails also at low pressures to give pv constant at any temperature.

§ (26) FURTHER THEORETICAL DEVELOPMENTS.—We have quoted some of the chief equations of state that have been proposed. The following may also be selected for mention out of the very large numbers that have been proposed:

Hirn I.: $$(p+a)(v-b)=RT.$$

(This was given in 1865.)

Hirn II. and G. Schmidt:

$$pv=BT-Cv^{-n}.$$

Kamerlingh-Onnes:

$$\left(p+\frac{a}{v^2}\right)v\psi\left(\frac{b}{v}\right)=RT,$$

where $\psi(b/v)$ is a series of powers of b/v.

Amagat:

$$p(v-a)=\phi(v)\left[T-\frac{M(v-a)}{v^m+av^{m-1}}+\ .\ .\ .\right].$$

Tait:

$$p(v-b)=RT-\frac{A}{v-\gamma}+\frac{E}{v-a}.$$

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[1] Porter, *Phil. Mag.*, 1910, xix. 890.

[2] See article "Quantum Theory," Vol. IV.

[3] Other equations are proposed by W. H. Keesom. *Comm. Phys. Lab. Leiden Supplement* 30.

Rankine, Thomson, and Joule:

$$pv = \mathrm{RT} - \frac{a}{\mathrm{T}v},$$

Dupré:

$$p(v+c) = \mathrm{RT}.$$

(If c is a function of the temperature this may become Callendar's equation.)

Sarrau:

$$p = \frac{\mathrm{RT}}{v-b} - \frac{\mathrm{C}a^{-\mathrm{T}}}{(v+c)^2}$$

Boltzmann and Mache:

$$\left(p + \frac{a}{v^2}\right)\left(v - b + \frac{c}{v^2+d}\right) = \mathrm{RT}.$$

Reinganum:

$$\left(p + \frac{\theta}{v^2}\right)\left(v - be^{\frac{c}{\mathrm{T}}}\right) = \mathrm{RT},$$

where θ is a function of temperature.

Rose-Innes and Young:

$$pv = \mathrm{RT}\left(1 + \frac{e}{v+k-gv^{-2}}\right) - \frac{l}{v+k}.$$

This equation was based upon S. Young's experimental values for isopentane. It contains five constants. It fits exceedingly well. Each term was introduced as it was found to be required. It is probable that the term with g as a factor could be replaced by a term involving the pressure. Thomson and Joule also put forward an equation based upon the Joule-Thomson effect (porous plug experiments). If K is the cooling per atmosphere

$$\mathrm{K} = \frac{\mathrm{T}(\partial v/\partial \mathrm{T})_p - v}{\mathrm{C}_p},$$

where v = specific volume, and C_p the specific heat at constant pressure. K was found to be given by a/T^2, where a is a constant. If C_p be taken as a constant the equation when integrated gives

$$\frac{v}{\mathrm{T}} = -\frac{\mathrm{C}_p a}{3\mathrm{T}^3} + f(p),$$

where $f(p)$ must be identified with R/p in order that the equation may reduce to that for a perfect gas when the temperature is great. If v be replaced by $v-b$, and the power of T in the small term be altered to suit requirements, Callendar's equation is obtained.

§ (27) THE RELATION BETWEEN PRESSURE AND TEMPERATURE IN THE HETEROGENEOUS REGION. MAXWELL'S THEOREM. VAPOUR PRESSURES.—All such equations as we have given are continuous functions of the variables. Now examination of Andrews' diagram shows that in the mixed region where both gas and liquid exist in equilibrium with each other the pressure keeps constant at constant temperature whatever the proportion between the two phases may be. This is not indicated by any of the suggested equations. If a gas below its critical temperature is compressed, a point is reached at which it no longer remains homogeneous but breaks up into two parts of different density—gas and liquid respectively. It is in this region that the pressure is a function of the temperature alone. This pressure is the *saturation pressure*. As to what the pressure will be, at any particular temperature, we have no theoretical guide except a somewhat doubtful theorem due to Maxwell. Maxwell imagines a cycle of operations performed throughout at one temperature; in the forward part of the cycle the transformation from liquid to gas takes place in accordance with an equation of state—say van der Waals' equation (shown by dotted line)—the return path being along the constant saturation pressure line. If this cycle is performed reversibly at constant temperature it should be impossible to gain work by means of it—or, in other words, the work done in the forward part must be cancelled by the negative work in the return transformation. Doubt as to the validity of this theorem arises from the fact that transformation in accordance with van der Waals' equation in the heterogeneous region must be unstable, otherwise the fluid would not so regularly take another path (the heterogeneous one) instead. In fact, in one part of the homogeneous path the pressure *increases* with *increase* in volume: which is a completely unstable state of things. Attempts have been made to get over this difficulty, and not without some success.[1] Hence, since the theorem appears to be, in practice, a reliable guide in spite of misgivings concerning its validity, it is worth while to give examples of its application. Let v_1, v_2 be volumes of liquid and vapour respectively at temperature T, and let π be the saturation pressure. We will, in the first place, assume van der Waals' equation. Along the dotted curve the work done is $\int_{v_1}^{v_2} p\,dv$, where

$$p = \frac{\mathrm{RT}}{v-b} - \frac{a}{v^2},$$

while backwards along the saturation line it is $-\pi(v_2 - v_1)$. Hence the total work is

$$\mathrm{RT}\log\frac{v_2-b}{v_1-b} + a\left(\frac{1}{v_2} - \frac{1}{v_1}\right) - \pi(v_2 - v_1) = 0$$

by Maxwell's theorem. Moreover, since the

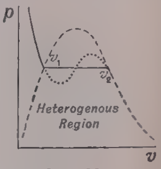

FIG. 22.

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[1] See Bryan, *Thermodynamics* (B. G. Teubner, 1907).

points v_1 and v_2 are both on the van der Waals isotherm,

$$\pi = \frac{RT}{v_1 - b} - \frac{a}{v_1^2},$$

also

$$\pi = \frac{RT}{v_2 - b} - \frac{a}{v_2^2}.$$

If we had employed the reduced equation the three equations would have been

$$\frac{8}{3}\gamma \log \frac{3\beta_2 - 1}{3\beta_1 - 1} + 3\left(\frac{1}{\beta_2} - \frac{1}{\beta_1}\right) - \pi(\beta_2 - \beta_1) = 0,$$

$$\pi = \frac{8\gamma}{3\beta_1 - 1} - \frac{3}{\beta_1^2},$$

$$\pi = \frac{8\gamma}{3\beta_2 - 1} - \frac{3}{\beta_2^2},$$

where the symbol π is retained for the *reduced* vapour pressure. From the last two equations we can eliminate either γ or π, giving

$$\pi = \frac{3\beta_1\beta_2 + \beta_1 + \beta_2}{\beta_1^2\beta_2^2} = (3 + \Delta_1 + \Delta_2)\Delta_1\Delta_2,$$

$$\gamma = \frac{(\beta_1 + \beta_2)(3\beta_1 - 1)(3\beta_2 - 1)}{\beta_1^2\beta_2^2}$$

$$= (\Delta_1 + \Delta_2)(3 - \Delta_1)(3 - \Delta_2),$$

where $\Delta_1 = (1/\beta_1)$, $\Delta_2 = (1/\beta_2)$ are the reduced densities.

From these, together with the first equation, β_1, β_2, π can all be expressed in terms of γ (the reduced temperature). The elimination of the variables is a somewhat cumbrous process; a method for effecting it has been given by Max Planck,[1] who applied it to Clausius' equation I. J. P. Dalton has applied a similar method of reduction,[2] starting from his own equation

$$\left(\alpha + \frac{3}{\beta^2}e^{1-\gamma}\right)(3\beta - 1) = 8\gamma,$$

which gives most excellent results. If we write

$$3\beta_1 - 1 = re^{-x},$$

$$3\beta_2 - 1 = re^{x},$$

where r and x are auxiliary variables, and apply Maxwell's theorem, we find

$$2r = \frac{\sinh 2x - 2x}{x \cosh x - \sinh x}.$$

Starting from arbitrary values of x the values of r are calculated; thence the corresponding values of β_1 and β_2; the remaining procedure is straightforward but cumbrous. Results calculated from van der Waals' equation and from Dalton's [2] are given in *Fig.* 23, and compared with experimental values for isopentane (a non-associated substance) and methylalcohol (associated). If the law of corresponding states were rigorously obeyed these experimental curves would be coincident; as long as such differences as these occur it is obviously impossible to represent the thermal behaviour of different substances by a single equation, and all that one can do is to construct an equation for a fictitious substance to serve as a type from which particular substances deviate more or less according to their molecular complexity.

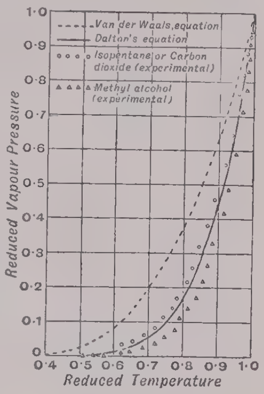

FIG. 23.

It will be seen that Dalton's equation gives practically the same slope near the critical point as both experimental curves. Van der Waals put forward an empirical equation

$$\log_{10} \pi = f\left(1 - \frac{1}{\gamma}\right)$$

connecting saturation pressure and temperature for reduced vapour pressures, where f is 2·97 for CO_2 and 2·95 for isopentane. Dalton's equation gives 3·04 down to a reduced temperature 0·83, after which it gradually increases. When natural logarithms are used Dalton's factor is 3·04 × 2·303 = 7·00.

§ (28) LAW OF RECTILINEAR DIAMETERS.—If the *densities* of vapour and of the liquid at the border curve are both plotted against the temperature the two curves meet at the critical point, thus jointly forming a curve roughly parabolic in shape. A curve drawn on the same diagram, whose ordinate is the arithmetic mean of these densities, is nearly a straight line also passing through the critical point. This fact, discovered by Mathias, is known as the law of rectilinear diameters. It leads to one of the most accurate methods of determining the critical volume which is the least easily measured of all the critical data, owing to the rapid change of volume at that point with change in pressure,

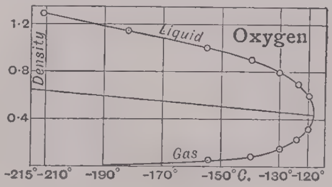

FIG. 24.

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[1] *Wied. Ann.*, 1881, xiii. 535.

[2] Dalton, *Trans. R. Soc. of S. Africa*, 1914, iv. part 2, 128.

For if the densities have been determined at temperatures not too remote from the critical point the diameter can be drawn and extrapolated to the critical temperature.

To determine these densities a tube, such as Andrews', is used and filled with the gas: the mass of the gas becomes known from an observation of its pressure, volume, and temperature. This gas is then compressed till condensation begins; the volume is then noted, and from this the density of the saturated gas is calculated. The mercury is then caused to rise in the tube till the whole gas is liquefied; again, the observation of the new volume enables the density of the liquid to be calculated. To determine the critical temperature and pressure the temperature is adjusted until the meniscus separating the liquid and gas disappears or reappears due to a minute change. The temperature and pressure can then be read off. If the substance attacks glass, and thereby makes the observation of the meniscus impossible, a steel tube is taken (platinised inside, if necessary, to prevent attack of the vapour on the steel), and observations of pressure and temperature are taken until the critical temperature has been passed. The experiment is then repeated *with a different quantity of substance present.* The pressure-temperature curve will be the same as before (viz. the saturation pressure curve) until the critical temperature is reached; after this point the pressure will change differently from the previous case. The point of separation of the two curves gives the critical values.

§ (29) Experimental Results. — Values are given in the subjoined table. The critical volume is only given when it was experimentally determined. The unit of volume is the volume of the gas under one atmosphere pressure at 0° C. The critical density is also given taking the density of water at 4° C. as unit.

	t_c.	p_c. Atm.	v_c.	d_c.	Observer.
Acetone	233 – 246	52 – 60	..	..	..
Ethyl ether	194·4	35·61	..	·2622	Young
Ethylene	10	51·7	..	..	Olszewski
Ethyl alcohol	243·6	62·76	0·00713	0·288	Ramsay and Young
Argon	– 117·4	52·9	..	..	Ramsay and Travers
„	– 121	50·6	..	..	Olszewski
Benzol	+288·5	47·89	..	0·3045	Young
Bromine	302·2	..	0·00605	1·18	Nadejdine
Chlorine	141·0	83·9	..	..	Dewar
Carbon tetrachloride. .	283·15	44·97	..	0·5576	Young
Hydrogen chloride (HCl)	52·3	86·0	..	0·61	Dewar
Neon	< – 205	..	..	..	Ramsay and Travers
Octane (normal) . . .	296·2	24·645	..	0·2327	Young
Pentane (normal). . .	197·2	33·03	..	0·2323	Young
Isopentane	187·8	32·92	..	0·2343	Young
Phosphorus trichloride .	285·5	..	..	..	Pawlevsky
Oxygen	– 118·8	50·8	..	0·6044 (von Wroblewski)	Olszewski
„	– 118·82	49·713	..	..	Kamerlingh Onnes, etc.
Carbon disulphide . .	273·05	72·868	0·009011	..	Batelli
Sulphur dioxide . . .	156·0	78·9 (Sajotschewsky)	..	0·52	Cailletet et Mathias
Silica tetrachloride . .	230·0	..	..	..	Mendelejew
Nitrogen	– 146	35·0	..	0·44 (von Wroblewski)	Olszewski
„	– 147·13	33·490	..	..	Kamerlingh Onnes, etc.
Toluol	320·6	41·6	..	..	Altschul
Water	358·1	..	0·001874	0·429	Nadejdine
„	364·3	194·61	0·003864	..	Battelli
„	365·0	200·5	..	..	Cailletet et Collardeau
Xenon	14·7	57·2	..	..	Ramsay and Travers
Air	– 140·0	39·0	..	..	Olszewski
„	– 141·0	39·6	..	..	v. Wroblewksi

H. Davies[1] has shown that the coefficient of expansion of liquids at any temperature sufficiently remote from the critical can be calculated from the formula $\alpha = 1/(2T_c - T)$. The following values at 0° C. illustrate the application of this rule—which is based upon the law of rectilinear diameters:

	Critical Temperature ° C.	α Calculated.	α Observed.
Acetone . .	237	0·001339	0·001324
Aniline . .	426	869	·00082
Bromine . .	302	1167	·00104
Benzene . .	290	1172	·001176
Pentane . .	197	1499	·001464
Phenol . .	419	·00088	·000834
Toluol . .	321	·001062	·001028
Naphthalene.	468	·00082	·00075
o-Xylol . .	358	0·00101	0·00092

Thorpe and Rucker[2] had previously shown that

$$T_c = \frac{T_2D_1 - T_1D_2}{A(D_1 - D_2)},$$

where D_1 and D_2 are the densities of a liquid at temperatures T_1 and T_2 respectively, and A is a constant which they take to be 1·995. The two expressions are equivalent to one another.

§ (30) MIXED GASES.—The equations of state which have been considered are all intended to apply only to pure fluids. The question of mixed gases is in a much more imperfect state. On the experimental side the mixture which has been most studied is that of air. Amagat's[3] investigations extend up to 3000 atmospheres, and throughout this range he finds the following law to be true:

The volume of a gaseous mixture is equal to the sum of the volumes that the components would occupy if they were separate and each at the pressure of the mixture.

Sacerdote[4] found the law to be sensibly exact in the neighbourhood of atmospheric pressure for a mixture of carbon dioxide and nitrogen protoxide. On the contrary, it is in default for a mixture of CO_2 and SO_2,[5] of oxygen and hydrogen,[6] and in some other cases.[7] Experimentally it is, of course, much better not to study the law directly (by making various mixtures at a given pressure and finding the pressure of the mixture when the total volume is preserved constant) but to take each of several mixtures and investigate the p, v, T relations for it.

On the theoretic side it is natural to endeavour to represent the results by an equation of which van der Waals' is the type:

$$\left(p + \frac{A}{v^2}\right)(v - B) = RT.$$

If x and $1 - x$ are the relative numbers of molecules of the two species present, theoretical considerations led van der Waals to write

$$A = a_1(1-x)^2 + a_2x^2 + 2a_{12}x(1-x),$$
$$B = b_1(1-x)^2 + b_2x^2 + 2b_{12}x(1-x),$$

where a_{12} and b_{12} are new constants. Kuenen has verified this by direct experiment on mixtures of carbon dioxide and methyl chloride, Verschaffelt on carbon dioxide and hydrogen, and Quint on ethane and hydrochloric acid. Berthelot has concluded from his experiments, in the neighbourhood of atmospheric pressure, that

$$a_{12} = \sqrt{a_1a_2} \quad \text{and } B = b_1(1+x) + b_2x.$$

These conclusions must, however, be accepted with reserve.

During the liquefaction of a gaseous mixture the pressure does not keep constant during the co-existence of the two phases, but progressively changes with increase in the quantity of liquid formed. The isothermals in this region are not only inclined (instead of horizontal as for a pure substance), but they are curved. The composition of the liquid phase is in general different from that of the gaseous.

A commonly occurring case is illustrated by mixtures of acetone and ether (*Fig.* 25).[8] The abscissae on the diagram represent molar fractions of acetone, *i.e.* the number of normal molecules of acetone in the liquid mixture divided by the total number of normal molecules of both kinds; the ordinates are the partial vapour pressures of acetone (A) and ether (B). These curves can be represented very accurately indeed by the equations

$$\pi_a = \Pi_a\mu_a e^{\beta(1-\mu_a)^2},$$
$$\pi_b = \Pi_b(1-\mu_a)e^{\beta\mu_a^2},$$

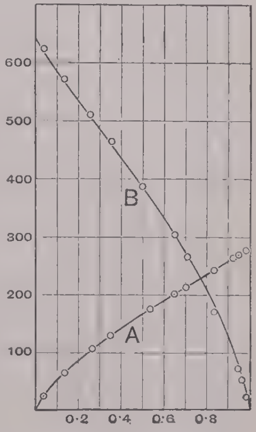

FIG. 25.

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[1] *Phil. Mag.*, 1912, xxiii. 657.

[2] *Trans. Chem. Soc.*, 1884, xlv.

[3] Amagat, *Comptes Rendus*, 1898, cxxvii.

[4] Sacerdote, *Comptes Rendus*, 1899, cxxviii. 991.

[5] Leduc, *Comptes Rendus*, Jan. 17, 1889; Sacerdote, *Comptes Rendus*, Jan. 24, 1898.

[6] Berthelot and Sacerdote, *Comptes Rendus*, March 27, 1899.

[7] Braun, *Wied. Ann.* xxxiv. 948.

[8] Porter, *Trans. Farad. Soc.*, 1920, xv.

where the suffixes a and b refer to the two constituents; π is partial vapour pressure; Π the vapour pressure of a constituent when the other is absent; μ_a is the molar fraction of a, and β is a characteristic constant. For the particular mixture $\beta = 0{\cdot}7414$.

It will be seen that for the ratio π/Π there is only one characteristic constant. The form of the curve for various values of β is shown in *Fig.* 26. For methyl-alcohol and glycerol $\beta = 1$; for acetone and oleic acid $\beta = 0{\cdot}5$; for ether and oleic acid $\beta = -0{\cdot}2$; for ether and sulphuric acid $\beta = -6$.

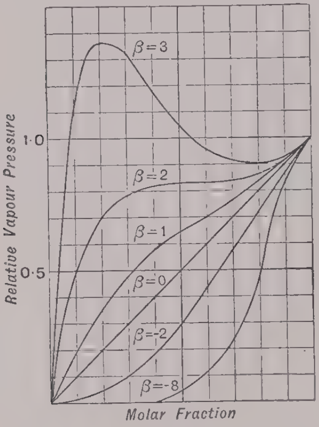

FIG. 26.

§ (31) THE SOLID STATE.—There is no satisfactory equation yet available for the solid state. The transition from liquid to solid implies much more radical changes than from gas to liquid. In a true solid the atoms become practically fixed in position (except for slight thermal agitation) under the action of *crystalline* forces so as to be distributed upon an imaginary framework or lattice. An amorphous body like ordinary glass or silica glass is in reality a subcooled liquid of very high viscosity of the same general character as (only much more viscous than) water which at ordinary pressure may have been cooled down to −20° C. without crystallisation into the solid form taking place. Solidification (*i.e.* crystallisation) is marked by taking place at a definite temperature for a definite pressure. At such temperature and pressure the solid and liquid can exist together *in any proportion* just as for a liquid and vapour. There is a change in specific volume in the act of solidification or in the reverse process of melting; as a rule the specific volume is greater for the liquid, though this rule has notable exceptions (*e.g.* water). The following are examples of the densities of the liquid and that of the solid:

Substance.	Density.	
	Solid.	Liquid.
Aluminium (hammered)	2·75	2·426
Arsenic	5·73	5·71
Lead	11·00	10·64
Iron (pure)	7·85 – 7·88	6·88
Quicksilver	14·19 at 38°·8 14·38 at −188°	13·6 at 0°
Ice-water at 0° C. . .	0·91674	0·99988
Bismuth at 271° C. .	9·673	10·004

The change of specific volume in passing either from solid to liquid or from liquid to vapour is important in Thermodynamics (which see).

A gas *may* pass directly into the solid state (or the solid may form vapour) instead of into the liquid state. The conditions which determine the phases that can be in equilibrium with one another at a given pressure and temperature form a chapter in Chemical Dynamics.

A. W. P.

THERMAL PROPERTIES, used as secondary standards of temperature and compared with a gas-thermometer in the range −273° to 0° C. See "Temperature, Realisation of Absolute Scale of," § (32).

THERMAL PROPERTIES which can be reproduced and used as secondary standards of temperature above 500° C. (1) The specific heat of platinum; (2) The index of refraction of air; (3) The total radiation from a black body; (4) The radiated energy corresponding to a given wave-length in the radiation from a black body. See "Temperature, Realisation of Absolute Scale of," § (41).

THERMAL PROPERTIES AND FIXED TEMPERATURES, COMBINATION OF, used as a secondary standard of temperature in the range above 500° C. and compared with a gas-thermometer. See "Temperature, Realisation of Absolute Scale of," § (42).

THERMAL PROPERTIES OF SUBSTANCES, used as secondary standards of temperature, by comparison with a gas-thermometer. See "Temperature, Realisation of Absolute Scale of," § (25).

THERMAL UNIT, BRITISH. One 180th part of the heat required to raise one pound of water from the melting-point to the boiling-point at a pressure of one atmosphere. See "Thermodynamics," § (2).

THERMOCOUPLES

§ (1) INTRODUCTION. — The thermoelectric pyrometer is the most generally used of all appliances for the measurement of moderately high temperatures, and in the course of time it has been developed into an instrument of precision.

Starting from the basic discovery of Seebeck in 1828 that at the junction of two dissimilar metals when heated an electromotive force was generated, innumerable attempts were made during the subsequent half-century to construct a practical form of pyrometer upon this principle.

These attempts generally ended in failure, not on account of any inherent defect in the methods, but because the extreme susceptibility of the thermoelectric properties of metals to slight chemical and physical changes was not at that time fully realised. Pouillet, for example, adopted the combination of a platinum wire enclosed in an iron gun-barrel. It is now known that the platinum is rapidly deteriorated at high temperatures by reducing gases and metallic vapour, so that the choice of an iron barrel by Pouillet was very unfortunate.

Another serious disability that the early investigators laboured under was inadequate electrical instruments for measuring the small E.M.F. generated; it is only within comparatively recent years that thermoelectric potentiometers and high resistance moving coil indicators of the pivoted type have been evolved.

At the present day the thermoelectric method of measuring temperature has attained a degree of precision second only to the resistance thermometer, and for temperatures exceeding 1100° C. it is the only convenient and sensitive electrical method available.

It must be emphasised, however, that the thermoelement is only a subsidiary instrument whose scale will not stand extrapolation over extended temperature ranges, and each individual pyrometer requires calibration.

A thermoelectric pyrometer outfit is made up of the following elements:

1. The two metals constituting the thermoelement.
2. The electrical insulation of these wires and the protecting tubes.
3. The indicator or potentiometer for measuring the thermal E.M.F.
4. The provision for controlling the cold junction temperature.
5. The wiring system, switches, etc., when the installation consists of more than one pyrometer.

The choice of pure metals and alloys for use in the construction of thermocouples is primarily determined by the temperature which has to be measured.

But for the question of cost platinum and its alloys would be universally used, as their non-oxidisability and high melting-point make them ideally suitable for thermoelectric work.

The necessity of substituting some less expensive material for these rare metals has led to an extended study of other metals and alloys, with the result that it is now possible to measure temperatures up to 1200° C. with base metal couples with a moderate degree of accuracy, but for scientific work at high temperatures there is no alternative to the rare metal thermoelement.

§ (2) BASE METAL THERMOELEMENTS.—For low-temperature work up to about 300° C. copper, iron, or silver versus constantan[1] are quite satisfactory, possessing a large E.M.F. per degree of the order of 40 to 60 microvolts.

(i.) *Copper Constantan.*—Both copper and constantan are obtainable in any size wires and the thermocouples maintain their calibration if not overheated. At temperatures above 300° C. rapid deterioration occurs unless the couple is made of heavy section wires, and experience has shown that a precision of only 5° to 10° C. can be expected in measuring temperatures in the neighbourhood of 500° C.

(ii.) *Iron Constantan.*—The E.M.F. temperature relationship of iron constantan is a closer approximation to a straight line than is the case with copper constantan. For work at low temperatures the combination has the practical drawback that the iron is liable to rust in a humid atmosphere.

Iron constantan thermocouples are employed in technical work up to 800° C. and are then made of very heavy section wires. After prolonged exposure to high temperatures iron is subject to the development of parasitic currents. These may have their origin in a variety of sources. It is believed that segregation and cavities of occluded gas in the casting can give rise to a want of homogeneity, whilst changes of crystal structure on prolonged heating are also a frequent source of trouble.

(iii.) *Iron Nickel.*—This combination of two common metals was once extensively used in industrial work, but has since been largely displaced by alloys more resistant to oxidation and to change at high temperatures.

Nickel undergoes a molecular transformation between 230° C. and 390° C. which renders the pyrometer unsuitable for use over this range, but it gives fairly satisfactory results between 400° and 800° C.

The E.M.F. temperature relationship of the couple is nearly linear over the working range.

A peculiar fact has been observed about nickel: the pure metal is oxidised and rendered brittle by heating in air, whilst its alloys with chromium and aluminium resist oxidisation fairly well and do not deteriorate rapidly.

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[1] An alloy containing 60 per cent copper, 40 per cent nickel.

(iv.) *Nickel-Chromium, Nickel-Aluminium.*—Nickel-chromium: 90 per cent Ni, 10 per cent Cr; nickel-aluminium: 98 per cent Al, about 2 per cent Ni, with Si and Mn.

This combination was introduced by Hoskins and is sometimes known by the trade name of chromel-alumel thermoelement. It originated in a search for a metal to replace the iron element in the iron-nickel thermoelement. The alloy "chromel" proved so successful that a substitute for nickel was found in the "alumel" alloy. The presence of small quantities of silicon and manganese appears essential, for it was found that although the pure nickel aluminium alloy stood up well at high temperatures it became brittle with use at lower temperatures.

The chromel-alumel couple can be used up to 1100° C. continuously, and will stand for short periods a temperature 200° C. higher.

The E.M.F. temperature curve above 100° C. is nearly a straight line.

It should be remembered that the E.M.F. of base metal couples decrease with prolonged exposure to high temperatures, so frequent re-calibration is necessary.

The fact that base metal thermocouples generate about four times the E.M.F. per degree of a platinum-rhodium couple is an advantage from the point of view of the construction of robust pivoted indicators for portable outfits, but since they are less stable than the platinum couples they cannot be employed for work where permanency of calibration is essential.

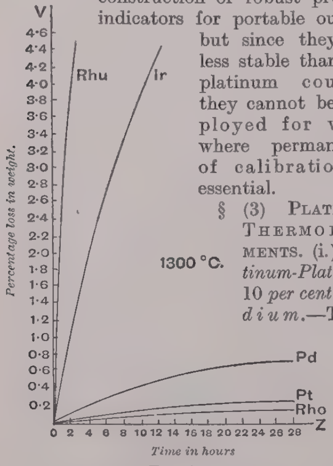

FIG. 1.

§ (3) PLATINUM THERMOELEMENTS. (i.) *Platinum-Platinum 10 per cent Rhodium.*—This thermoelement was introduced by Le Chatelier in 1886, and has since remained the most reliable of all combinations tested.

It is generally employed in scientific work at high temperatures and in a great many industrial installations. Such couples will stand 1500° C., but care must be taken to shield them from reducing gases, silicon, or metallic vapours. Platinum couples must be thoroughly protected from the vapours distilled from graphite or carbon at high temperatures.

(ii.) *Platinum-Platinum Iridium.*—The platinum-iridium alloy, first used by Barus, works satisfactorily up to about 1000° C., but at higher temperatures the iridium volatilises and prolonged exposure causes contamination of the pure platinum limb of the element.

The relative volatility of the rare metals is shown by the curves in *Fig.* 1, which are based on some experiments made by Sir William Crookes. The metals were kept at a temperature of 1300° C.

§ (4) PROTECTING TUBES.—The choice of protecting sheath for a thermoelement is determined primarily by the conditions under which the pyrometer is to be used. For experimental work where there is no danger of contamination it is possible to use a thermoelement without a protecting tube and thus diminish "lag" enormously.

A convenient method of mounting for laboratory use is shown in *Fig.* 2. The parts to be inserted in the hot region are insulated with capillaries of fireclay, while the junctions with the copper leads are inserted in glass tubes which are maintained at 0° C. by immersion in powdered ice. The wide-necked form of commercial vacuum flask is a useful receptacle for the ice. The two leads on the right of *Fig.* 2 are of thin guttapercha-covered copper and are connected to the indicator or potentiometer. Where there is risk of contamination of the element it is advisable to protect it by enclosing in a hard glass, fused silica, or glazed porcelain tube.

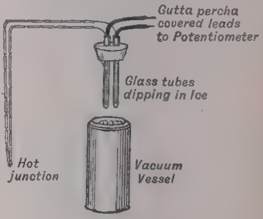
Gutta percha covered leads to Potentiometer
Glass tubes dipping in Ice
Hot junction
Vacuum Vessel

FIG. 2.

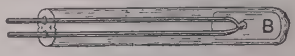

FIG. 3.

For temperatures up to 500° C. hard glass capillaries and sheaths can be employed. Similarly, fused silica is available for temperatures up to 1000° C. for prolonged periods.

A compact type of covering is shown in *Fig.* 3. Two silica capillaries are fused into a sheath terminating in a bulb at the bottom.

The wires are threaded through the end B, which is then sealed up with the junction enclosed in the bulb, as shown. Since the outside diameter is only about 5 millimetres the couple can be inserted into a small space.

When necessary the tube can be bent with the wire *in situ* by means of the oxy-coal gas blow-pipe.

For work at comparatively low temperatures cotton or silk insulation is quite satisfactory, provided it is paraffined or shellac-varnished.

Twin wires of iron constantan are obtainable with the two wires separately insulated and enclosed in a double covering of cotton or silk. In this form the wires can be laid in grooves a few millimetres deep.

(i.) *Porcelain Tubes.*—For industrial use it is generally necessary to enclose the thermocouple in a tube of glazed porcelain hemispherically closed at one end. The best grades of refractory porcelain have a melting-point above that of platinum, but since the material is porous to gases it cannot be used without a coating of glaze. The tubes are then serviceable to about 1400° C. only. The term porcelain comprises a variety of materials from vitrified porcelains to highly refractory porcelains.

Vitrified porcelain will not stand rapid changes of temperature, but can be used continuously up to 1200° C. Such tubes are usually gas-tight without being glazed, but are frequently glazed as an added precaution.

Refractory porcelain tubes will stand up to 1400° C., but prolonged exposure causes absorption of the softened glaze into the body of the tube.

(ii.) *Fused Silica Tubes.*—Fused silica tubes can be used up to 1000° C. in an oxidising atmosphere free from alkalis.

Prolonged exposure to temperatures above 1000° C. causes devitrification of the quartz: the material becomes crystallised, loses its mechanical strength, and is then permeable to gases. It appears that silica is slightly permeable to hydrogen at high temperatures, which is a serious drawback, since the presence of hydrogen within the protecting tube of a platinum thermoelement results in the reduction of silica to silicon, which attacks the platinum.

The principal advantage of quartz as a thermoelement protection lies in its extremely low coefficient of thermal expansion, which enables it to withstand violent temperature changes without fracture.

(iii.) *Alundum.*—This is the trade name for a tubing composed of fused alumina (Al_2O_3) with fireclay as binder. The alumina is shrunk into a dense mass in an electric furnace and then ground and made into tubes by the addition of a little clay.

The unglazed tubing will stand 1550° C., but is porous, consequently the tubes are frequently given a glaze coating and this again covered with a layer of alumina.

This method permits the tubes to be used to temperatures exceeding the softening point of the glaze. The limiting temperature of the glazed tubes appears to be about the same as that of porcelain tubes.

§ (5) Outer Protecting Tubes.—In industrial installations further mechanical protection is required by the porcelain or quartz sheath, and for this purpose carborundum, graphite, and a variety of metallic tubes are employed.

(i.) *Carborundum.* — Carborundum is an electric furnace product primarily composed of carbon and silicon. It is highly refractory and possesses many of the physical properties desirable in a protecting tube for the porcelain sheath.

When heated in an oxidising atmosphere the oxidisation of the silicon to silica begins at 1200° C. At 1500° C. the silica formed on the surface of the tube fuses and protects the tube from further oxidation. Very finely crystalline carborundum made into tubing known as "silfrax" is extensively used for furnace work. Carborundum tubes are permeable to gases and are readily attacked by basic slag. Since carborundum reacts at high temperatures with practically all metals it is essential to employ an inner protecting tube around the thermocouple.

(ii.) *Graphite.* — In reducing atmospheres graphite tubes form an excellent protection to porcelain and quartz sheaths. They are particularly useful in the case of molten aluminium, which readily attacks porcelain.

Plumbago tubes, which are made of graphite and fireclay, will withstand much higher temperatures than fireclay alone, and are frequently used in permanent installations.

(iii.) *Iron and Steel.*—The use of wrought iron or steel tubing as mechanical protection is very common in the industries. They are satisfactory up to about 900° C., and the cost of replacement is not a serious item.

Calorising the surface, which consists in impregnating the surface with aluminium, increases the resistance to oxidation and so prolongs the life of the tube two- or threefold.

(iv.) *Nichrome.*—As a substitute for iron and steel tubes cast nichrome tubes are used to some extent. These tubes withstand oxidation remarkably well, so that their greater cost is more than balanced by their longer life. Up to the present it has not been found possible to draw nichrome into tubes, so castings have to be employed, and with these extreme care has to be exercised to avoid pinholes.

(v.) *Molybdenum.* — This element has a

melting-point of about 2500° C. Experiments have proved that it can be dipped into molten brass, copper, etc., without being attacked, hence there are possibilities for this material when it becomes available on a commercial basis.

(vi.) *Zirconia.*—Fused zirconia is an excellent refractory and will probably be extensively used when the difficulties of manufacture have been overcome.

§ (6) ELECTRICAL INSULATION OF THE WIRES.—The electrical insulation of the wires constituting a thermoelement can be effected by the use of porcelain or fireclay capillary tubing in the case of platinum thermocouples.

Base metal couples of heavy wire are sometimes insulated with fireclay tubing. A slight fluxing of the oxides with the fireclay is noticeable on the constantan wire when heated to 1000° C., but the nickel-chromium alloys seem to be free from this trouble.

The alternative method of insulating base metal thermocouples is to employ asbestos string or tubing painted with a solution of carborundum, firesand, and sodium silicate mixed to a thick paste. When heated above 600° C. this insulation rapidly disintegrates. If the iron element of a thermocouple is protected this way and heated for some time to about 1000° C., it will be found that the wire grows to nearly twice its original diameter and can be easily broken by the fingers. In fact the iron, asbestos, and carborundum will combine chemically, and no free iron will be left. Carborundum, of course, attacks metals vigorously at high temperatures.

§ (7) TEMPERATURE INDICATORS OF THE MILLIVOLTMETER TYPE.—The majority of the thermocouples used in industrial work are equipped with moving coil indicators. The instruments are identical in construction with millivoltmeters, while the scales are generally graduated to read temperatures directly.

The calibration of such an instrument is correct so long as the total resistance of the circuit remains unaltered.

When the indicator has a resistance of from 100 to 500 ohms, small variations in the resistance of the leads or of the couple are of no consequence.

An idea of the high sensitivity necessary in the indicator may be formed when a comparison is made with ordinary voltmeters. With a thermocouple outfit it is often desirable to have a full scale deflection for 10 millivolts, which is only one ten-thousandth of the voltage which the switch-board instrument has to measure on a 100-volt system.

In order to obtain a robust moving-coil system the indicators fitted to base metal couples sometimes have a resistance as low as 2 ohms, and if the couples are of heavy section wire, protected from oxidation, the outfits work fairly satisfactorily.

Especial care must be taken with such installations not to cause any alteration of the total resistance of the circuit.

It must be borne in mind that the indications of such an outfit will also be affected by changes of resistance of the leads caused by variations of temperature of the furnace room. It was observed in the case of a 5-ohm resistance indicator that changes of temperature from 0° to 35° C. along 50 ft. of wiring from the thermocouple to the instrument caused the indicator to read 10° C. low at 650° C. So apart from difficulties due to oxidation and varying depths of immersion high accuracy cannot be obtained with a low-resistance millivoltmeter. With platinum thermocouples the cost of the material prohibits the use of thick wire, and it is therefore necessary to employ high-resistance indicators.

Millivoltmeters are now obtainable of 500 ohms resistance, giving a full scale deflection for 40 millivolts, and the indications of such an outfit can generally be relied upon within ± 5° C. When higher accuracy is desired it is necessary to use a potentiometer.

§ (8) COMPENSATION FOR VARYING CIRCUIT RESISTANCE.—Harrison and Foote have described a scheme of circuits by means of which it is possible to arrange that the total resistance of galvanometer, leads, and thermoelement is periodically adjusted to a fixed value. The principle of the method will be understood from *Fig.* 4. There is an adjustable resistance r_x in series with the moving coil and swamping resistance of the millivoltmeter or temperature indicator.

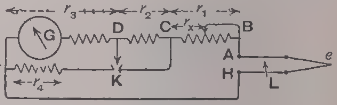

FIG. 4

On depressing a key, part r_2 of the swamping resistance is short-circuited and the remaining part together with the moving coil r_3 is shunted by the resistance r_4. The instrument is calibrated in terms of the potential drop across its terminals for a maximum value of $r_x = r_1$. In the construction the resistances are proportioned according to the relation $r_2 r_4 = r_1 r_3$. If the resistance r_x is so adjusted that the deflection of the pointer is unchanged by depressing the key K it can readily be proved that the total resistance of the circuit is that for which the instrument is calibrated, the sum of r_x and all external resistance being thus made equal to r_1. Hence it follows that the instrument measures the true E.M.F. in a simple circuit, or if connected across a resistance or network through which a current flows it indicates the potential drop which would have existed had the instrument not been connected. In this respect it functions as a potentiometer, yet it does not operate on the potentiometric principle, since it does not require a standard cell or an auxiliary battery, the only E.M.F. employed in the adjustment being that of the source measured.

By constructing the apparatus so that the ratio r_3/r_4 is equal to from 5 to 10 it is possible to adjust r_x with 5 to 10 times the precision necessary. Thus if the galvanometer can be read to $\frac{1}{10}$ of a scale division the line resistance may be adjusted with a precision equivalent to $\frac{1}{100}$ of a scale division, which is at least 10 times the accuracy possible with an indicating instrument. This principle of magnification of errors greatly facilitates the proper adjustment of r_x. By varying the copper to manganin ratio in r_4 it is possible to produce a compensated instrument of zero temperature-coefficient from a millivoltmeter having an excessive copper content.

Such an instrument should be serviceable in thermocouple work. The compensated millivoltmeter may also be used in multiple installations of thermocouples having different line resistances, as many resistances r_x being employed as there are couples. These may be inexpensive rheostats, one located in each line between the couple and selective switch, and still the accuracy of adjustment will be as high as though precision rheostats were employed.

§ (9) POTENTIOMETERS FOR THE MEASUREMENT OF THERMAL E.M.F.'S.—Since the electromotive forces to be measured are of millivolt order the potentiometer has also to be specially designed for the work.

With a low-resistance potentiometer and a sensitive moving coil galvanometer, it is possible to measure to 1 microvolt with certainty, and with greater precaution measurements to 0·1 microvolt are possible, but rarely necessary.

Essentially, a potentiometer is a row of resistances in series through which steady current is passed. This current is kept constant by occasionally adjusting it so that the fall of potential through a fixed resistance balances the E.M.F. of a standard (cadmium) cell.

Then, since in a simple circuit the fall of potential is proportional to the resistance, any E.M.F. within the range of the instrument can be measured by balancing it against the drop over the appropriate known resistances.

In *Fig.* 5 the rheostat R is adjusted until the difference in potential across K is balanced against the E.M.F. of the standard cadmium cell C (1·0185 volts[1]). The E.M.F. of the thermocouple is balanced on the resistance r, shown here for simplicity as a uniform wire.

The potentiometer can be made direct-reading in microvolts, by so constructing that the fall in potential per ohm resistance is some definite value, for example, 1 volt per 100 ohms. Hence to measure in steps of millivolts requires subdivision of the resistances into 0·1 ohm coils. A bridge wire in series (of the same resistance), having a scale divided into a hundred parts, would enable readings to be taken to 10 microvolts or by estimation to 2. In such a case the point P could move over the studs of the coils, while Q would traverse the bridge wire.

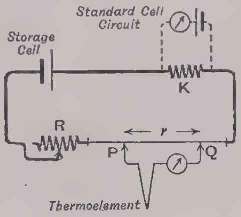

FIG. 5.

Since potential contacts only are necessary, there is no objection to the use of low resistances in the circuits.

§ (10) SIMPLE CIRCUIT POTENTIOMETER.—A practical form of the diagram above is shown in *Fig.* 6.

The current through the potentiometer is adjusted to be 0·01 amp. by balancing the cadmium cell E.M.F. 1·0185 volts at 15° C. across 101·85 ohms (circuit shown dotted in diagram).

The working range of the potentiometer consists of 100 equal coils arranged in four dials of 25 coils each.

The resistance of each coil is 0·01 ohm, hence the total fall of potential across the hundred coils is 10 microvolts, and across each coil 100 microvolts.

The bridge wire in series with the coils is of 0·01 ohm, and the scale is subdivided into 100 parts, consequently the value of 1 division is 1 microvolt.

By inserting a plug which short-circuits half the resistance across which the standard cell is balanced, the range of the instrument is doubled.

An alternate way of obtaining higher ranges is to employ two or three secondary cells in series and balance against 2, 3, or 4 cadmium cells.

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[1] E.M.F. of standard cadmium cell at 15° C. = 1·0185 International volts, or 1·0183 at 20° C.

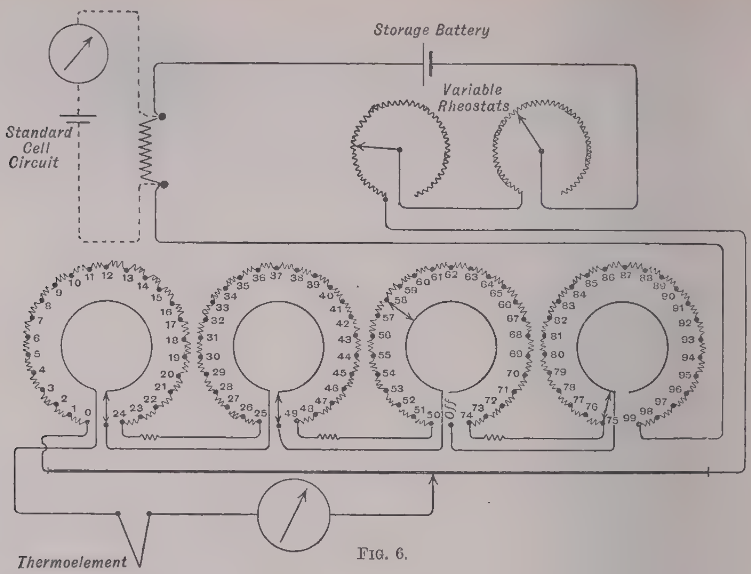

FIG. 6.

§ (11) VERNIER FORMS OF POTENTIOMETERS.—The above-described potentiometer, which is representative of a large class, has the disadvantage of only permitting steps of equal value. The decade principle common in P.O. resistance boxes is not possible with the simple circuit of series coils.

This difficulty was surmounted in a very ingenious manner by Thomson and Varley in their vernier type of potentiometer, which was designed for resistance and cell comparison work. In this type the slide wire of the ordinary Clark potentiometer is replaced by a subdivided resistance coil, which can be placed in parallel with any two coils in series of the main circuit, as shown in *Fig.* 7.

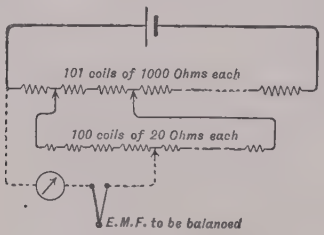

FIG. 7.

The storage battery is connected through a rheostat with a dial consisting of 101 coils of 1000 ohms, each in series. A second dial of 100 coils of 20 ohms each can be placed in parallel with any two adjacent coils of the main dial by a movable contact-maker.

Now, the effect of the second dial as shunt on the two coils of the main dial is to make the effective resistance between the points of contact equal to 1000 ohms, *i.e.* that of the individual steps. Hence the fall in potential over the 100 coils in the shunt dial is the same as that over a single coil of the main dial, so that the effect of the shunt dial coils is to give 100 intermediate steps between any two points on the main dial.

It will be observed that the contact resistances at the points where the shunt dial is connected to the main dial are assumed to be negligible.

The drawback to the application of the vernier principle to thermoelectric potentiometers is that the coils of the latter have to be of low resistance in order to obtain sensitivity. Hence contact resistances require careful elimination when the principle is applied to low-resistance potentiometers.

Harker (1903) described a simple form of vernier potentiometer in which the second dial above referred to was replaced by a plain bridge wire.

The connections are shown in *Fig.* 8. The standard cell is balanced across 101·85 ohms, hence the current is 0·01 amp.

The 20 coils AB each have a resistance of 0·1 ohm, so that the potential drop per coil is 1 millivolt or 1000 microvolts. Each of the 11 coils in the row CD has a resistance of 0·01 ohm, consequently the potential difference per coil is 100 microvolts. The bridge wire EF can be placed in parallel with any two coils CD, and is 0·02 ohm in resistance. The scale is subdivided into 100 parts, each of which corresponds to 1 microvolt. All contacts to the coils AB and CD are made by means of mercury cups. Manganin bars are laid alongside the cups and drilled with a corresponding series of holes, so that the connection at any point may be effected by a ∩-shaped copper bar.

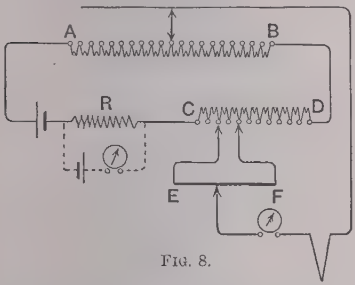

FIG. 8.

Tinsley has devised a form of vernier potentiometer in which the slide wire is replaced by a dial with 100 stud contacts, between each of which there is a resistance of one-thousandth of an ohm. The potentiometer consists essentially of 1 main dial of 20 coils of 10 ohm. each, across which is inserted the shunt dial, consisting of 100 coils of 0·2 ohms each, see *Fig.* 9. This dial is arranged so that it is always shunted across two coils of the main dial in the usual manner. Thus with a current of one milliampere through the circuit the main dial reads ·01 volt per stud. The vernier dial reads ·0001 volt per stud, whilst the fine adjustment dial reads ·000001 or 1 microvolt per step. Hence the range of the potentiometer lies between ·1901 volt and 1 microvolt. A range of 10 times this value is obtained by moving the plug from B to A. The current through the potentiometer is set by balancing a cadmium standard cell against 101·85 ohms on the dials and with the plug in position A. When the plug is placed in the B current through the potentiometer is reduced to one-tenth its original value, whilst the total resistance in the battery circuit remains unaltered.

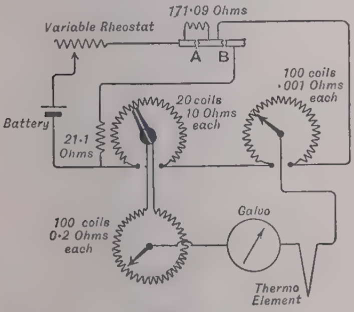

FIG. 9.

The design would be improved if the standard cadmium cell was balanced against a separate resistance, as then the adjustments of the dials would not have to be altered when it was desired to check the current in the circuit.

Mr. R. Paul has also designed a vernier potentiometer with two ranges; on the lower range E.M.F.'s from 1 microvolt to 17 millivolts can be measured, while the second range is 10 times that of the lower.

Fig. 10 shows the connections diagrammatically: in the actual instrument the coils are carried on a series of drums set alongside each other. The standard cell is balanced by varying X[1] with the plug inserted in A, the position corresponding to the higher range. It will be seen that the values of the various resistances are so adjusted that the effect of transferring the plug from A to B is to reduce the E.M.F. on the working portion of the potentiometer to one-tenth the original value, while leaving the resistance of the complete circuit in series with the storage cell unchanged. The plug in the position B gives the lower range.

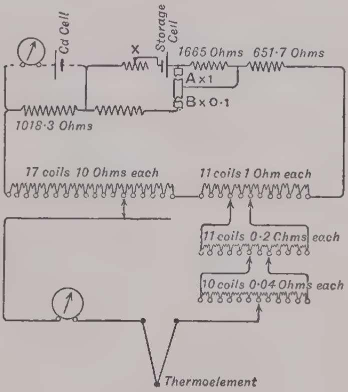

FIG. 10.

The current through the potentiometer coils is 0·001 amp. for the higher range, and 0·0001 amp. for the lower range. The high resistance of this potentiometer necessitates the use of a galvanometer of high resistance (of the order of 1000 ohms) and high sensitivity. In fact, for working on the lower range the best class of moving coil galvanometer is required. Also attention must be paid to the elimination of contact resistances at the double brushes of the shunt dials.

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[1] The adjustable resistance X brings the potentiometer resistance up to 1850 ohms when the E.M.F. of the storage cell is 1·85 volt.

§ (12) EFFECT OF CONTACT RESISTANCE ON THOMSON VARLEY SHUNT.—Since the shunt dial of the Thomson Varley arrangement is connected to the main circuit by two movable contacts it is necessary to consider the effect on the readings of contact resistance at these switches.

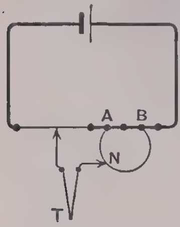

FIG. 11.

Taking the elementary circuit shown in *Fig.* 11, and assuming the steps in the main circuit to be one ohm each and the shunt circuit two ohms divided into any number of steps, then if c is the current in the battery circuit, when there is *no contact resistance* the current in the shunt is $c/2$.

If the resistance between A and N be $2/n$ the potential difference between A and N is $c/2 \times 2/n = c/n$. Let us suppose now that there is a *contact resistance of magnitude r at A*. The resistance of the main circuit between A and B is still 2 ohms whilst that of the shunt circuit is $2+r$. Assuming the current in the battery circuit unaltered, then the current between A and B in the main circuit is $c\{(2+r)/(4+r)\}$, and the current between A and B in the shunt circuit is $c \times 2/(4+r)$, which may be approximately expressed as $c(1-r/4)/2$ if r is small. Then potential difference between A and N is

$$\frac{c}{2}\left(1-\frac{r}{4}\right)\left(\frac{2}{n}+r\right)=\frac{c}{n}\left(1-\frac{r}{4}\right)\left(1+\frac{nr}{2}\right)$$

$$=\frac{c}{n}\left[1+\frac{r}{2}(n-\tfrac{1}{2})-\frac{nr^2}{8}\right].$$

Hence the error is

$$\frac{c}{n}\,\frac{r}{2}(n-\tfrac{1}{2})=\frac{cr}{2}\cdot\left(1-\frac{1}{2n}\right)$$

since the r^2 term is negligible.

Now in the case when the point of contact N is close to A, the resistance between A and N is small, *i.e.* n is large, hence the error would be $cr/2$. When the point N is close to B the resistance between A and N is practically 2, so the error would be $cr/4$. When there is a *contact resistance r at B* the current in the shunt circuit as before is

$$\frac{c}{2}\left(1-\frac{r}{4}\right).$$

The potential difference between A and N is

$$\frac{c}{2}\left(1-\frac{r}{4}\right)\frac{2}{n}=\frac{c}{n}\left(1-\frac{r}{4}\right).$$

So the error will be $-cr/4n$, which has its greatest value when the contact N is near B (*i.e.* $n=1$), then the error is $-cr/4$.

From the above it will be seen that when the contact resistance is at A the error produced varies from one-half to one-quarter that which would be caused by the same resistance in the main circuit. When the contact resistance is at B the error produced is less, and affects the observations in the opposite direction.

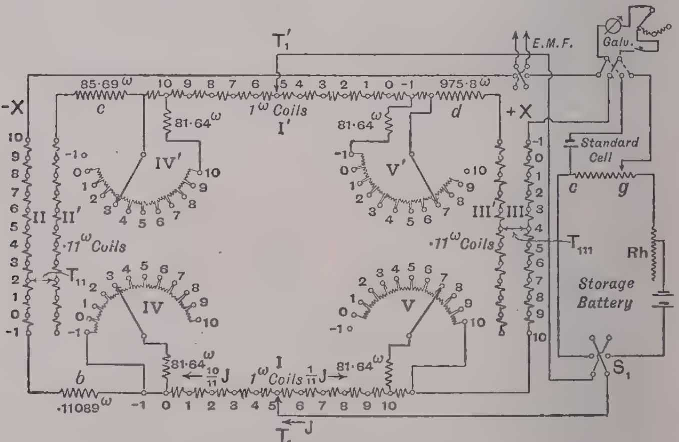

FIG. 12.

§ (13) THE HAUSRATH, WHITE, DIESSELHORST TYPE POTENTIOMETER.—This potentiometer has been designed with a view to the reduction of thermoelectric effects at brush contacts to a minimum. The potentiometers previously described have been series arrangements, that is, the potential difference which balances the E.M.F. of the thermoelement is the sum of the potential differences existing between the terminals of various groups of coils of which the circuit is composed. In this potentiometer a divided circuit takes the place of coils in series. In which case the potential difference for balancing the thermoelement is due to the difference of the potential drops along the two branches measured from the point where the current enters the potentiometer circuit. *Fig.* 12 shows diagrammatically the connections of the circuits.

The potentiometer current is adjusted to ·01 amp. when the last dial is to read in microvolt steps, and generally provision is made for working with a current of one-tenth this amount if desired. The current enters the potentiometer at T_I and leaves at T_I'. On entering it divides into the two branches, the resistances of which are so arranged that ten-elevenths of the total current flows to the left and one-eleventh to the right, and the design is such that these relative values are maintained for all positions of the switches.

Each coil in the decade I. and in the group of compensating coils I′. has a resistance of 1 ohm, the coil contacts are arranged on the circumference of a circle, and the sliding brushes T_I and T_I' are so linked that if T_I is shifted to the right, T_I' is shifted an equal number of coils to the left. The resistances of both paths between the terminals T_I and T_I' are thus kept constant.

As all the coils in decades II. and III. and all the compensating coils in II′. and III′. are alike, the resistances of both the left-hand and the right-hand branches of the circuit are independent of the positions of T_{II} and T_{III}. The coils are grouped in dials so that the single movement of the brush contacts suffices to bring in the coil and its compensator. The resistances of the coils are 0·11 ohm each. The group of coils IV. consists of a 1-ohm coil (*i.e.* the 1-ohm coil between −1 and 0 of decade I.), shunted by a variable resistance which consists of a fixed portion 81·64 ohm and a variable part included between −1 and the position of the sliding contact. The coils in the variable portion of IV. are:

Between	Ohms.	Between	Ohms.
−1 and 0	8·264	4 and 5	30·30
0 and 1	10·101	5 and 6	45·41
1 and 2	12·626	6 and 7	75·80
2 and 3	16·234	7 and 8	151·51
3 and 4	21·645	8 and 9	454·54
		9 and 10	∞

In the group IV′. the order is reversed, for example, the resistance coil of 8·264 ohms is between contacts 9 and 10 instead of between −1 and 0. The resistances are given these particular values in order that, when the contact in IV. is moved one step, the resistance between T_I and T_{II} may always be altered by a definite amount, 0·0011 ohm, which is $\frac{1}{100}$ of the alteration which would be obtained by moving T_{II} one number. For instance, if the decade contact is on number 4, the resistance f group IV. is

$$R_{IV} \quad \frac{1 \times 150{\cdot}51}{} \quad {\cdot}342.$$

When it is on number 5, this becomes

$$R_{IV} = \frac{1 \times 180{\cdot}81}{1 + 180{\cdot}81} = 0{\cdot}99452.$$

The difference of these two values is 0·0011 ohm, while the value of a single step in II. is 0·11 ohm.

The minimum value of R_{IV} is

$$R_{IV} = \frac{1 \times 89{\cdot}904}{1 + 89{\cdot}904} = 0{\cdot}9890,$$

so the general value is

$$R_{IV} = 0{\cdot}9890 + 0{\cdot}0011 \times n_{IV},$$

where n_{IV} is the number of the contact on dial IV.

Similarly, the general value of $R_{IV'}$ is

$$R_{IV'} = 0{\cdot}9989 - 0{\cdot}0011 \times n_{IV'}.$$

In decade V. the coil of 8·264 ohms is between contacts 9 and 10: in the group of compensating coils V′. it is between −1 and 0, so in general

$$R_V = 0{\cdot}9989 - 0{\cdot}0011 n_V,$$
$$R_{V'} = 0{\cdot}9890 + 0{\cdot}0011 n_{V'}.$$

The coils *b*, *c*, *d* have such values that, taken in conjunction with the other resistances, they divide the current flowing in at T_I in the ratio 10 to 1. The resistance of *b* is 0·11089.

The voltage between T_{II} and T_{III} is the difference of the ohmic drops measured from T_I, so for any setting

$$\text{(1)}\quad \text{P.D.} = \frac{10}{11} \times J[n_I \times 1 + 0{\cdot}9890 + 0{\cdot}0011 n_{IV} + 0{\cdot}11089 + 0{\cdot}11(n_{II} + 1)]$$
$$- \frac{1}{11} \times J[(10 - n_I)1 + 0{\cdot}9989 - 0{\cdot}0011 n + 0{\cdot}11(10 - n_{III})]$$
$$= J\left(n_I + \frac{n_{II}}{10} + \frac{n_{III}}{100} + \frac{n_{IV}}{1000} + \frac{n_V}{10{,}000}\right),$$

where J is the current in the potentiometer circuit.

Therefore, when the dials are properly graduated, the unknown P.D. is measured by the sum of the dial readings in the customary manner.

A study of the network shows that if the battery circuit is open the resistance betwee
the galvanometer terminals (+X and
to a good degree of approximation,
and when the battery circuit is
a series resistance B, which
potentiometer, the resista
a second approximatio

between T_I and T_I' with the galvanometer circuit open.

The resistance of the right-hand path between T_I and T_I' is 990 ohms: that of the left-hand path 99 ohms: the resistance of the whole apparatus is therefore $R=90$ ohms.

If a storage cell is used and the potentiometer current is 0·001 amp., $R+B$ must be approximately 2000 ohms: therefore, the maximum variation in the resistance of the galvanometer circuit will be only 0·05 ohm, or about 0·3 per cent. For a current of ·01 amp. the variation will be 3 per cent. This approximate constancy of the resistance allows one to obtain the last figure in the P.D. under measurement, by the deflection method, the reading of the last decade, n_V, being kept at zero. By properly setting up the apparatus, the full reading of the last decade, $n_V=10$, may be made to correspond to 1, 10, or 100 divisions on the galvanometer scale, and the necessity for exact balancing may be thus obviated. Where the P.D. to be measured is fluctuating slightly, this is a decided advantage.

Another advantage if a moving coil galvanometer is used is that the damping remains constant, irrespective of the setting of the potentiometer.

§ (14) Effect of Thermoelectromotive Forces.—The magnitude of the E.M.F. set up by manipulating any switch is less than one microvolt.

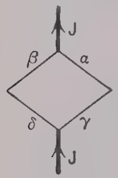

Fig. 13.

Any E.M.F.'s arising from manipulating I. and I′. are added to the battery E.M.F. (2 volts) and will be negligible. The effect of a thermoelectromotive force of magnitude ϵ due to moving contact II. will be very small.

Referring to *Fig.* 13 by Kirchhoff's laws the current in the left-hand branch, if J is the total battery current coming to the potentiometer, is

$$J_L'=\frac{I(\beta+\delta)\mp\epsilon}{\alpha+\beta+\gamma+\delta},$$

and in the right-hand branch

$$J_R'=\frac{I(\alpha+\gamma)\pm\epsilon}{\alpha+\beta+\gamma+\delta}.$$

Therefore, the change in the P.D. between the terminals $+X$ and $-X$ due to $\pm\epsilon$ will be

$$\frac{\pm\epsilon(\gamma+\delta)}{\alpha+\beta+\gamma+\delta}.$$

[illegible] of γ and δ is 14·42 ohms and the [illegible] 1089 ohms, so

$$[illegible] \times 0{\cdot}013.$$

[illegible]

Similar considerations show that the error introduced by manipulating IV. and V. is only about 1·2 per cent of ϵ. These errors having been reduced to negligible amounts, the apparatus is said to be free from thermoelectromotive forces.

It can also be shown that contact resistances of the brushes only produce negligible errors. Thus contact resistances at T_I and T_I' merely add on to the total resistance in the battery circuit, while such resistances in the decades IV., IV′., V., and V′. are obviously less important than in the case of decades II. and III. Of these II. is the more important since it is the low-resistance arm 99 ohms as compared with the other arm 990 ohms. A good brush has a resistance of the order of 2×10^{-4} ohm, and if this resistance is assumed at II. the current proportion in the two arms instead of being $\frac{1}{10}$ will be $\frac{1}{10}(1+2\times10^{-6})$. Using the equation (1) above it can easily be shown that the error in the E.M.F. value is $2{\cdot}2\times10^{-6}$, or 0·022 of one step of the smallest dial. The contact resistance in II. would have to be as great as $\frac{1}{100}$ ohm to make a difference of one step on the smallest dial.

Nearly 120 resistances are required in the construction of the potentiometer described above, and some of them require very accurate adjustment. Thus the 1-ohm coils of decade I. should be correct to one part in 50,000 at the very least. The coils I′. and II. should not depart by more than $\frac{1}{10,000}$: for the coils II′. and III. $\frac{1}{10,000}$ is ample and for the rest $\frac{1}{1000}$.

§ (15) Deflection Potentiometers.—A type of instrument, which occupies a position intermediate between the null potentiometer on the one hand and the moving coil deflection instrument on the other hand, is the portable deflection potentiometer.

In one of this class of instruments the thermal E.M.F. is balanced to the nearest two millivolts and the deflection of the pointer observed. The galvanometer has a central zero, and 10 scale divisions correspond to one millivolt.

To maintain a constant sensitivity of the indicator for all potentiometer readings, it is necessary to keep the total resistance in the galvanometer circuit to a constant value. This is effected by arranging that the switch on the millivolt studs also cuts out from the galvanometer circuit a resistance equivalent to that added in the potentiometer circuit.

Referring back to *Fig.* 5, let

G be the resistance of the galvanometer and that of a series resistance.

T be the resistance of the thermoelement.

r the resistance in the potentiometer circuit, across which the thermal E.M.F. is balanced.

$Z+r$ is the total resistance of the main circuit of the potentiometer.

Let Δe be the residual unbalanced E.M.F. of the thermoelement which produces the galvanometer deflection. Now, the deflection of the galvanometer is proportional [illegible] urrent thro [illegible] to obtain [illegible] tude of this current we require the total resistance in the circuit P to Q.

The resistance between the points P and Q is that of the resistance r shunted by the main circuit of the potentiometer outside these points, viz. Z.

Hence the effective resistance is $rZ/(r+Z)$; so the deflection of the galvanometer is proportional to

$$\frac{\Delta e}{\{rZ/(r+Z)\}+T+G}.$$

For the small E.M.F. generated by the couple r is small compared with Z. Hence r in the denominator of $rZ/(r+Z)$ may be neglected. So the deflection is proportional to $e/(r+T+G)$. From this it follows that to obtain deflections proportional to e and independent of r it is necessary to diminish the resistance in series with the galvanometer by an amount r, and this can be automatically effected by a simple brush arrangement.

Portable potentiometers are also made working on the "null" principle, and with a sensitive pivoted galvanometer it is possible to read to within 10 microvolts.

§ (16) Recording Pyrometers.—In manufacturing processes where it is desirable to keep a continuous record of the temperature of the furnace, it is either necessary to fill up charts at periodic intervals, or arrange that the instrument gives a permanent record. Frequently when a recording indicator is installed, a direct reading indicator is situated near the furnace to aid the operator, both being connected to the same thermoelement. The fundamental feature necessary in such recorders is reliability, and this has only obtained after lengthy experiments, in which many practical difficulties had to be surmounted. The usual form of record desired is that in which temperature appears as one co-ordinate, and time as the other. Recorders may be divided into two classes; the one operating on the same principle as a deflection galvanometer, and the other as a potentiometer. The second class of recorders, which belong to the category of "null" instruments, are considerably more complicated than the first, but they have the advantage that variations in the resistance of the circuit are of no importance.

§ (17) Deflection Instruments.—It has not been found practicable to construct a recording millivoltmeter operating a pen in contact with the paper, as in the case of the ordinary switchboard voltmeter. The forces are so much smaller that the friction between pen and paper would introduce serious errors. Hence the usual practice is to employ an arrangement which periodically depresses a pointer into contact with a chart.

(i.) *Methods of marking the Chart.* — The earliest successful mechanism was the "chopper bar," which is now widely used. In this recording arrangement the paper is unwound by clockwork at a uniform speed from the roll. An inked ribbon lies on the metal table beneath the paper. At periodic intervals of 10 to 30 seconds the chopper bar falls, pressing a point on the end of the galvanometer boom into contact with the paper and against the ribbon and plate underneath. This produces a small dot on the under side of the thin paper which shows through. The record obtained is a series of dots.

A modification of this device is shown in *Fig.* 14. In this an inked thread is stretched between the pointer and the paper. At periodic intervals the pointer is depressed and strikes the thread against the paper, so producing a dot. The thread is slowly carried around inked rollers, so as to expose fresh portions to the pointer and to replenish the ink.

The above recorders are usually operated by powerful clockwork mechanism, but small electric motors are also employed for this purpose. This is a considerable advantage when it is desired to work switches in a multiple unit installation with one recorder.

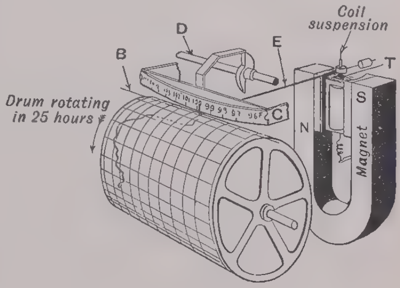

Fig. 14.

The usual plan is to provide the motor with a centrifugal governor, which keeps the speed constant within narrow limits. The motor tends to run too fast, this tendency being controlled by the governor, which, flying out, closes a shunt circuit across the armature.

Power is transmitted from the motor to the recording mechanism through a reducing worm gear.

A diagrammatic view of a recording mechanism in which a typewriter ribbon is employed instead of an inked thread is shown in *Fig.* 15.

A shaft will be observed carrying the two cams, this shaft being driven by the governed motor. The left-hand or front cam causes a light frame to be intermittently lifted and then dropped upon the pointer, below which is a typewriter ribbon; under the ribbon lies a fixed bar having a narrow straight edge, over which the chart is continuously driven. The intersection of the pointer and straight edge represents the point at which the mark is made. A swinging frame carries the ribbon bobbins, and the ribbon is fed intermittently from the left-hand to the right-hand bobbin at a very slow rate, so as to present always a fresh surface of ribbon for use. The swinging frame is actuated from the right-hand or near cam, and the ribbon feed is driven from a racket and worm wheel. The to-and-fro movement of the swinging frame serves also another purpose, since when two records or more are to be made on one chart the typewriter ribbon has two colours; in the case, for example, of the two records a two-way switch is added to the mechanism, so that when the switch is on one couple the record is in black, and when on the other couple in red.

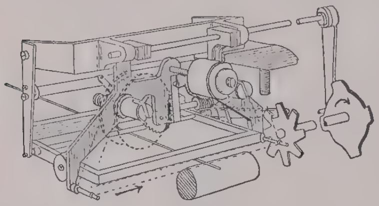

FIG. 15.

An alternate arrangement for recording is to have an inked roller at the side of the chart, and a mechanical device which deflects the pointer so that its tip strikes the pad at intervals. It then swings back and attains its equilibrium position over the scale, and the chopper bar comes down in the usual manner. The inked roller rotates slowly on its axis so as to expose a fresh surface to the pointer.

A totally different recording arrangement is employed in some instruments. With these the pointer is not periodically depressed, but swings free with its tip close to the record. At half-minute intervals an electric spark passes from the pointer to the chart, puncturing the paper. The record is a series of holes with seared edges, which are easily seen. There is a tendency of the spark to jump at an angle, causing a slight error, which, however, is not serious.

§ (18) RECORDING POTENTIOMETER.—The first instrument to work on the "null principle" was the Callendar Recorder. This was originally designed, about twenty-five years ago, for use with recording resistance thermometers, and is practically an automatic Wheatstone's bridge, but the same mechanical devices will operate a recording potentiometer. A description of this recorder will be found in the section on Resistance Thermometry. In recent years the Leeds & Northrup Co. of Philadelphia have developed a potentiometer recorder which is used extensively in America. In this instrument a number of thermocouples can also be recorded on the same chart, a print wheel being then used in place of a pen and the couples switched in by an automatic commutator. The essential part of the recorder is the mechanical device for automatically moving the slide-wire contact, and moving the pen across the chart (*Fig.* 16). The mechanism of this device may be briefly described as follows:

The essential point, of course, is that the deflection of a galvanometer results in a movement of the slide-wire contact maker and pen without requiring the galvanometer to do any work.

The disc A is mounted on a shaft and operates the slide-wire contact by a cord wound on its circumference, visible in *Fig.* 16. The power, supplied by a small continuously running electric motor, enters the mechanical system through the shaft B carrying the large cams C and the small cams D and E. At each revolution of the shaft B, the cams C straighten out the arm F, which perchance has been tilted a moment before, and in doing this rotates the disc A, arm F being pressed at this time against the disc A by the spring G. The arm F is pivoted on the spring G, which is fast to the frame of the instrument. When the cams C have rotated until their longest radii are passing the extensions of arm F, the cam E begins to raise G, lifting F away from the disc. When F is free the cam D raises the rocker-arm H, which, in case the galvanometer is unbalanced, catches the pointer under one of the right-angle levers J pivoted at K. One lever is thus made to swing the arm F by pressing against one of the eccentrically located lugs L. The rocker-arm H is then immediately lowered to allow the galvanometer to swing freely. Cam E is so shaped and fixed on the shaft B that it will recede from the spring G, allowing

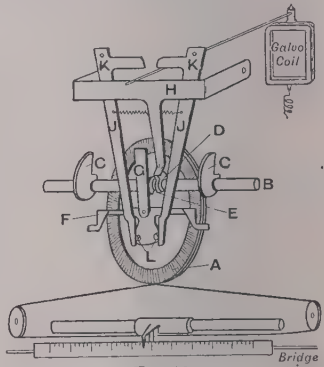

FIG. 16.

G to press F against the disc just before the cams C begin once more to straighten F.

This mechanism, in its cycle of operations, moves the contact on the slide wire whenever the potentiometer is out of balance with the thermocouple, and in so doing operates to obtain or restore the balance. The shaft B rotates once in about 2 seconds, which is slow enough to allow the galvanometer time to come to rest or nearly so. This design is such that the amount of rotation of the arm F increases with the extent of the galvanometer deflection, since the pointer approaches the fulcrum of the levers J as the deflection increases. The motion of H is adjusted so that the rotation of F will correspond to a rebalancing step of the pen, of $\frac{3}{4}$ inch (19 mm.) when the deflection is a maximum, and decreases uniformly to about $\frac{1}{50}$ inch when the deflection is just sufficient to catch the boom under one of the right-angle levers. This gives sufficient rapidity of the various actions to take the pen the width of the scale in somewhat less than 1 minute. A record is made once a minute on the multiple-point recorders of standard design. The position of the pen, when a balance has been obtained just before each record, corresponds to a definite point on the slide wire, for the pen is fixed to the slide-wire contact.

Once during a revolution of the commutator, the thermocouple is disconnected and the standard-cell connection made. At the same time the potentiometer slide wire is let loose from its shaft and the clutch engages a second resistance. Movements of the disc then result in changing the resistance of the battery circuit and the current is thus set to its proper value. The pen does not follow this adjustment and no record is made of variations in the current. With batteries in fair condition, the current is easily maintained constant; but if there arises any doubt of this constancy, the recorder may be watched for a few minutes and, when the standard-cell is made, the first deflection of the galvanometer is an indication of the change in the current since the last adjustment. A short-circuiting contact on the slide wire carries the pen to zero on the chart when the battery has run down, thus providing ample warning under most circumstances.

The scale of this recorder is uniform when graduated in millivolts, and departs from uniformity for a temperature graduation according to the temperature electromotive force relation of the thermocouple. The standard galvanometer is sufficiently sensitive to work satisfactorily with a full-scale range of 10 millivolts, which gives a very open scale, particularly for base-metal couples, when $\frac{1}{10}$ inch (2·5 mm.) of scale corresponds to $2\frac{1}{2}$° C.

§ (19) COLD JUNCTION CORRECTION.—For accurate work the cold junctions should be maintained at 0° C. by inserting the junctions with the copper leads into two tubes standing in ice.

The recent development of the "all steel" Dewar vacuum vessel is likely to remove many of the troubles encountered with varying cold junction temperatures.

When these vacuum vessels are employed the platinum-rhodium thermoelement is made with leads sufficiently long to reach from the couple down into the steel bottle and connections made from this point by means of copper leads. In the vicinity of a hot furnace the bottles will preserve crushed ice for periods up to twenty-four hours, but it is advisable to use tubes of low thermal conductivity for protecting the junctions in the ice, otherwise the heat conduction along the tube melts it rapidly.

If it is not feasible to have the cold junction at 0° a correction must be applied. This cold junction correction in general is *not* equal to the temperature of the cold junction, but depends on the temperatures of both hot and cold junctions.

(*a*) If the temperature of the cold junction is determined by the aid of a mercury thermometer, then the E.M.F. corresponding to this temperature is to be added directly to the observed E.M.F. of the hot junction.

(*b*) With a pyrometer provided with a direct reading instrument this correction may be accomplished mechanically by changing the zero of the instrument so that when short-circuited it indicates the temperature of the cold junction. An alternate plan which is convenient when it is desirable not to alter the zero of the indicator is to correct the observed reading as follows:

When the cold junction is at a temperature of (t_0) the true temperature may be obtained by adding to the observed temperature (t_1) the quantity obtained by multiplying the value of the temperature of the cold junction (t_0) by a factor which is the ratio of the slopes of the calibration curve (E.M.F. against temperature) at the origin and at the temperature (t_1); or putting it into symbols:

$$\text{Correction} = \frac{(de/dt)_{0^\circ \text{ to } 30^\circ}}{(de/dt)_{t_1}} \times t_0.$$

(*c*) The third method of fixing the cold junction correction is a graphical one. By means of a series of curves such as those shown in *Fig.* 17, the correction may be determined by inspection. In this diagram the number of degrees to be added to the hot junction temperature are plotted as ordinates and the cold junction temperature as abscissae. In the diagram curves are drawn for several temperatures (uncorrected) of the hot junction, between 400° and 1700° C.

In industrial installations it is inconvenient to have to apply cold junction corrections or to adjust the zero of the indicator, and a variety of devices have been evolved to minimise the trouble to variations of cold junction temperature.

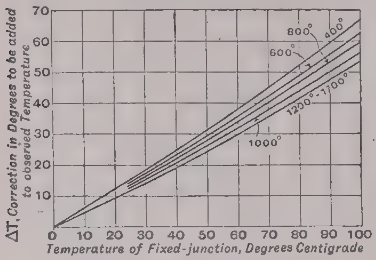

FIG. 17.

It is now customary to remove the cold junction from the head of the pyrometer, which may vary a hundred degrees or so if near the furnace wall, to a distant point by means of compensating leads.

§ (20) COMPENSATING LEADS. — For base-metal couples these lead wires are of the same materials as those employed in the couple, small-stranded wires being used for flexibility. Thus the cold junction is transferred to a point where the temperature is reasonably constant, and from this point copper wires lead to the indicator. The compensating wires may terminate in a cold junction box fitted with a thermostat, or may be buried underground. At a depth of 10 feet beneath the floor of a large building, the temperature remains constant to within 2° C. throughout the year; usually this mean temperature is about 12° C. for temperate climates, but may differ somewhat in the immediate vicinity of a large furnace. To apply this method of control, an iron pipe of the proper length, closed at the bottom, is driven into the ground, and the two cold junctions, well soldered and carefully insulated, are threaded to the bottom of the pipe in such manner as to be conveniently removable when necessary. The top of the pipe may be plugged with asbestos or waste, and covered with pitch to keep water away from the insulation. The scale of the indicator is set to read the mean temperature of the bottom of the tube. It is convenient to have an extra pair of compensating leads or an extra thermocouple with its junction at the bottom of the pipe, to measure this temperature occasionally. A disadvantage of the buried pipe is that the moisture or water may accumulate in the pipe. When this occurs it generally gives rise to galvanic effects which result in greater error than those caused by changes of cold junction temperature. So the thermostat box arrangement is preferable when possible.

Usually the compensating leads of a base-metal couple are marked, or are equipped with one-way terminals, so that they are easily connected properly to the head of the couple. If reversed at the couple, the leads will cause an error double the amount of the compensation. When compensating leads of a base-metal couple are properly connected to the couple no deflection of the indicator is registered by heating the head of the couple.

The high cost of platinum prevents the use of compensating leads of that metal, but inexpensive wires of copper and nickel-copper alloy are now available for use with the platinum and platinum-rhodium couples. These lead wires do not compensate individually, but taken together they compensate to within 5° C. for a variation of 200° C. at the junction of the couple and lead wires. Both terminals on the head of the couple should be kept as nearly as possible at the same temperature. The copper compensating lead is connected to the platinum-rhodium wire of the couple, and the copper-nickel wire is connected to the platinum wire of the couple, *i.e.* alloy wire to pure metal in each case. The cold junction is then located at the indicator end of the compensating leads; the temperature at this end may be controlled by one of the methods described.

In the majority of cases the compensating leads terminate in the indicator box, and then it is merely necessary to correct for the changes of temperature at the indicator, which is usually favourably situated as regards uniformity of temperature.

§ (21) AUTOMATIC COMPENSATION FOR COLD JUNCTION TEMPERATURES OF THERMOCOUPLES. —One simple method of automatically correcting for cold junction temperature is that devised by Darling, in which the control spring of the moving coil is coupled to a compound strip which coils or uncoils when cooled or heated, thereby moving the pointer over the scale.

The length of the spiral is such that an alteration of a given number of degrees in its temperature moves the pointer the same number of degrees on the scale, or, in other words, the temperature scale of the pyrometer is identical with that of the spiral.

Numerous other methods for automatically compensating for cold junction temperature changes of thermocouples have been proposed and used in connection with millivoltmeters. One of the earliest was a bare resistance wire immersed in a column of mercury located near the cold junction of the thermocouple. An increase in temperature near the cold junction resulted in a rise of the mercury column, which short-circuited more of the resistance wire, causing an increase in the potential difference across the millivoltmeter, and this compensated for the decreased electromotive force of the thermocouple.

The method is open to the objection that the compensation is accurate at only one temperature of the hot junction, for the voltage change across the compensating resistance is a function not only of the compensating resistance, but also of the current passing through it. As the current changes with the hot junction temperature, it is obvious that accurate compensation for cold junction temperature can be obtained only for one temperature of the hot junction.

Another device operating on the same principle consists of thin carbon discs tightly packed in a porcelain cylinder which has a small coefficient of expansion. These are pressed together by a metal rod (zinc) possessing a large coefficient of expansion. As the temperature rises the rod expands and increases the pressure between the carbon discs, thus decreasing the resistance. The device is connected in series with the thermocouple and indicator. This method of compensation suffers from the same defect as the previous one in giving exact compensation at only one hot junction temperature.

Fig. 18 shows another scheme of automatic cold junction compensation. The resistors *a*, *b*, *c*, and *d* are connected in the form of a Wheatstone bridge, as shown, and placed near the cold junction of the thermocouple. Three arms of this bridge are made of resistances having a zero temperature coefficient, such as manganin. The fourth arm is made of a metal having a high temperature coefficient, such as nickel. These resistances are adjusted so that they have equal resistances at some reference temperature, such as 0° C. The bridge consequently is balanced at this temperature, and no difference of potential due to the battery B*a* appears at the terminals *e* and *f*. If the temperature of the cold junction changes, the resistance of the nickel changes and throws the bridge out of balance. This changes the E.M.F. across the terminals *e* and *f*, and if the coils are properly adjusted will exactly neutralise the change in E.M.F. at the cold junction of the thermocouple. The nickel resistance *c* is made low in comparison to the total resistance in the millivolt circuit, so that resistance changes of the nickel coil do not materially change the total resistance in the millivoltmeter circuit. Consequently the compensation is practically correct for all temperatures of the hot junction, provided the voltage at the terminals of the bridge is kept constant. Changes in the voltage of the battery can be corrected for by means of a rheostat in the battery circuit.

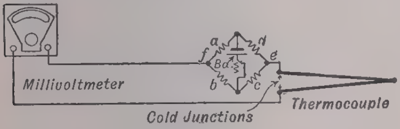

FIG. 18.

The value of the resistance is so chosen that the scale starts at 0° or at any other desired temperature.

§ (22) STANDARDISATION OF THERMOCOUPLES. —For low temperature work up to 450° C. the following fixed points are available for standardisation purposes:

Substance.	Boiling Point of the Organic Compound or Freezing Point of the Metal, ° C.
1. Steam	100°
2. Napthalene . . .	$217°{\cdot}9_6 + 0{\cdot}058$ (p. − 760)
3. Aniline . . .	$184°{\cdot}1 + 0{\cdot}05$ (p. − 760)
4. Tin	231°·9
5. Benzophenone .	$305°{\cdot}9 + 0{\cdot}063$ (p. − 760)
6. Cadmium . . .	320°·9
7. Lead	327°·4
8. Zinc	419°·4
9. Sulphur . . .	$444°{\cdot}5 + 0{\cdot}09$ (p. − 760)

In this table p. is the pressure in mm. of mercury.

When great accuracy is not desired, the boiling-point of diphenylamine (302° C.) may replace that of benzophenone, which is costly and difficult to obtain pure.

For temperatures above 450° C., the reference temperatures are freezing-points of metals or melting-points of salts.

Most of the commoner metals can be melted in small gas-fired crucible furnaces, the material of the crucible depending on the particular metal which is to be melted.

The value obtained for the melting-point should be in agreement with the freezing-point, and both should be independent of the rate of heating and the depth of immersion (within limits) of the thermoelement.

As a general rule, the end of the sheath protecting the thermoelement should be immersed to within about ½-inch of the bottom of the crucible.

The chemical action of the surrounding atmosphere on the heated metal is an important factor, since the solution of the oxide or a gas in the molten metal lowers its freezing-point quite considerably.

For example, the solution of cuprous oxide to form an eutectic (3·5 per cent of Cu_2O) in copper lowers the freezing-point by 20°, while the absorption of oxygen by silver has a similar influence on its freezing-point.

(i.) *Metals which require a Reducing Atmosphere.* — Antimony, aluminium, silver, and copper must be melted in a reducing atmosphere. The thermoelement requires careful protection, and this can be readily effected by means of a glazed porcelain tube. In the case of antimony Day and Sosman recommend the addition of a thin sheath of graphite over the porcelain.

With aluminium it is necessary to protect the metal from the pyrometer sheath as in the case of antimony.

When an electric furnace is employed for heating, it is possible to maintain an atmosphere of carbon monoxide inside as a reducing agent.

The eutectics of

Aluminium copper . .	542°
Aluminium iron . . .	649°
Nickel carbon . . .	1330°

are sometimes used for standardising purposes. These should be melted in a reducing atmosphere. It is as well not to attempt to make up the alloy of eutectic composition, as, just failing to do this, the liquidus would not be distinguishable from the eutectic. By making the alloy so as to contain a few per cent more of one of the metals than the eutectic alloy, the liquidus is easily distinguished from the lower point, which is the one required.

(ii.) *Metals which require a Neutral Atmosphere.*—The metals nickel and cobalt require a neutral or reducing atmosphere free from carbon compounds.

For nickel freezing-point determinations, the writer has found a refractory clay crucible satisfactory with fused borax as flux to prevent oxidation. The thermoelement should be protected by a hard porcelain sheath.

It is advisable to heat the crucible with a layer of borax glass in the bottom up past the melting-point of nickel, then slowly pour in the nickel in the form of shot.

A large-sized crucible of metal is desirable, and to diminish the rate of fall of temperature, the blow-pipe should not be turned completely off when the cooling curve is taken.

Covering the exterior surface of the crucible with a layer of carborundum powder, admixed with a percentage of fireclay, greatly increases its resistance to the cutting action of the blast flame.

Day and Sosman, in their melting determinations, employed an electric furnace with an atmosphere of hydrogen, which was replaced by nitrogen previous to inserting the thermoelement, since the hydrogen would be occluded by the platinum at high temperatures. The nickel was contained in an unglazed porcelain crucible lined with 90 per cent Al_2O_3 and 10 per cent MgO.

They found that, in taking the freezing-point of nickel, with oxide present, a fairly sharp halt was obtained 10° below freezing-point, which may represent the eutectic of nickel and nickel oxide. The break disappeared when the nitrogen was replaced by hydrogen for a few minutes.

For cobalt, "pure magnesia" crucibles had to be employed, otherwise the metal would have penetrated through the lining and attacked the porcelain beneath.

Glazed marquardt or pure magnesia was used to protect the thermoelement, but it was found practically impossible to prevent some contamination of the thermocouple.

The difficulties in the determination of the freezing-points of nickel and cobalt are very considerable, owing to the high temperature required.

(iii.) *Metals which can be melted in Air.*—Tin, zinc, and gold require no special atmosphere, graphite crucibles should be used and the couple protected by a hard paste porcelain tube glazed. The oxides of tin and zinc are practically insoluble in their metals, so have little if any effect on the freezing part of these metals.

But for its cost gold would be an excellent fixed point to employ for standardising purposes, on account of its non-oxidisable nature and purity. It has, however, a tendency to volatilise.

If moderate accuracy is sufficient, the wire method may be used. In this method the junction between the couple wires is effected by a short length of gold wire, and the E.M.F. observed as the furnace rises slowly in temperature. The break in the circuit corresponds to the melting-point of the bridging metal.

For the standardisation of base-metal couples in the vicinity of 800° C., the freezing-point of common salt is very convenient.

The salt is contained in a large pot which is heated in a gas furnace.

The freezing-point of NaCl [1] is 801° C., while the ordinary domestic material has a freezing-point from 1° to 3° lower. The salt is very volatile at these temperatures.

The sulphates and carbonates of sodium and potassium are not entirely satisfactory as fixed points. They are rapidly acted on by the water vapour and reducing gases of the furnace. Fused sulphates are slightly reduced to sulphides, and carbonates to hydrates, the magnitude of the change being dependent on the duration of the heating.

Heycock and Neville found that the freezing-point of sodium carbonate became lower the longer it was heated. The deterioration of the salt could be watched by looking into the crucible; the first two or three times the salt was fused it remained quite transparent, but afterwards it became opaque, in consequence of some chemical change.

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[1] Pure salt guaranteed to be 99·98 per cent purity is manufactured on a large scale for dairy purposes by C. Moore & Co., Lymm, near Warrington, Cheshire.

They found that the sulphates were more stable, and no progressive change could be observed in the freezing-points.

For high temperature salt-points platinum crucibles must be employed, with the thermocouple directly immersed in the salt, or if sheathed with glazed porcelain, this should be protected with a thin-walled platinum tube fitting it closely. This procedure was employed by Neville and Heycock in their resistance-thermometer work.

Since there is no metal freezing-point available in the region between copper (1083° C.) and nickel (1452° C.), attention has been directed towards the silicates to find some with melting-points in this gap. Jaeger has suggested the melting-point of lithium silicate (1201° C.) as a transition point for calibration purposes. The other two silicates, diopside (1391° C.) and anthorite (1549·5° C.), have also been proposed.

The following method was employed by White in the determination of the melting-points of silicates:

About 3 grams of the salt was contained in a platinum crucible, 10 mm. in diameter and 18 mm. deep, which was suspended by a platinum sleeve from the open end of a porcelain tube (unglazed).

The supporting tube was surrounded with pure platinum foil, to prevent contamination of the thermoelement by iridium vapour volatilised from the heating coil of the furnace.

The thermocouple dipped directly into the molten silicate.

It was found that the values of the melting-point were reproducible to about 1°. Freezing-points could not be determined, since the fused silicates under-cooled considerably, sometimes to a glass, in which case of course no melting-point would be observed on reheating.

The writer has found that the nickel carbon eutectic (1330° C.) is a very convenient standardising point in the region between copper and nickel, and it has the practical advantage that the usual apparatus for metal freezing-points can be used.

(iv.) *Palladium.*—The melting-point of this metal (1549° C.) represents the upper limit of the gas thermometer at the present time. Owing to the cost of the metal the melting-point is generally obtained by the wire method as above described. The writer has employed a vertical graphite furnace with a liner-tube of hard porcelain. The couple was symmetrically placed in the centre of the furnace, with a small connecting piece of palladium between the junctions.

It is also possible to carry out the same experiment with a platinum-foil-wound furnace, provided with two heaters in cascade. Two concentric tubes are used; the outer heating element brings the temperature up to about 1350° C., while the inner one takes it beyond 1550° C. The space between the two heater tubes is packed with shrunk alumina. Alternating current is employed for heating to avoid trouble due to leakage.

Day and Sosman employed a charge of 120 to 200 grams of the metal contained in a crucible of pure shrunk magnesia bound with magnesium chloride. The thermocouple was protected by a pure magnesia tube. The heating was effected in the internally wire-wound furnace.

The extreme temperature involved imposed a severe strain on platinum-wound furnaces. They found that the danger of contamination by the vapour of the palladium was very considerable.

(v.) *Melting and Freezing Points of Salts.*—The melting and freezing-points of salts present greater practical difficulties to accurate determination than those of metals at medium temperatures.

The latent of fusion of most salts is small, and as the solid salt is deposited on the pyrometer tube, it forms a poorly conducting layer, which renders the freezing-point less well defined than is the case with metals.

§ (23) FORMULAE FOR THE REPRESENTATION OF THE TEMPERATURE E.M.F. RELATIONSHIP.—When a large number of temperature observations have to be taken by means of a thermoelement, it is advisable to draw up a table on some such scheme as that below, by means of which the observed E.M.F. may be readily converted to temperatures.

The first step towards the construction of a table is to interpolate between the fixed points by means of an empirical equation.

Experience has shown that certain classes of equations represent closely the actual form of the curve characteristic of the thermoelement.

For example, Adams found that the relationship

$$E = 74{\cdot}672t - 13892(1 - e^{-0{\cdot}00261t}),$$

where E is the E.M.F. in microvolts,
t ,, temperature ° C.,
e ,, base of the natural logarithms,
represented the temperature E.M.F. curve of a batch of copper-constantan correctly to a fraction of a microvolt over the range 0° to 350° C., whilst above 350° the equation

$$E = 92{\cdot}20t - 29770(1 - e^{-0{\cdot}0018t})$$

was applicable. It must be remembered that the couple deteriorates rapidly above 300°.

By a slight variation in the numerical coefficients the same equation could be employed for any other copper-constantan thermoelement.

In the case of Pt-Pt Rh couples no single equation, with a reasonable number of constants, has been found that will represent the curve over the whole range from 0° to 1750° C.

Hence recourse has to be made to three equations covering overlapping sections:

1st, 0° to 400° C.,

$$E = At + B(1 - e^{ct});$$

2nd, 300° to 1200° C.,

$$E = -A_1 + B_1 t + C_1 t^2;$$

3rd, 1100° to 1750° C.,

$$E = -A_2 + B_2 t + C_2 t^2.$$

It should be ascertained, in a particular case, that there are no discontinuities in the slope of the curve at the points of transition from one section to another, and any slight inequalities should be evened out by adjustment of the successive differences.

The formula proposed by Holman,

$$E = Bt^A \text{ or } \log E = A \log t + \log B,$$

where E = E.M.F. in microvolts,
t = temperature ° C.,
A and B constants,
is occasionally used but does not represent the curve as accurately as the above.

From such equations a table can be calculated giving the temperature corresponding to every 100 microvolts or the E.M.F. corresponding to every 10°.

Typical specimens of such tables are shown below:

(i.) *Copper-Constantan.*—

EVEN HUNDREDS OF E SYSTEM

E (Microvolts).	t ° C.	1st Difference.
0	0	..
100	2·60	2·60
200	5·17	2·57
300	7·73	2·56
400	10·28	2·55
500	12·81	2·53
600	15·33	2·52
etc.		

(ii.) *Pt-Pt Rh Couple.*—

EVEN TENS OF t SYSTEMS

t.	E.	1st Difference.
0	0	55
10	55	57
20	112	60
30	172	62
40	234	63
50	297	65
60	362	..
etc.		

In connection with the difference column it is of interest to study the curves given by Adams and reproduced in *Fig.* 19 for the sensitivity of three kinds of couples at various temperatures.

The line for the Hoskins alloys is broken in the region where the couple is not particularly suitable for temperature measurements.

Once a table has been calculated out it can serve as a basis of other tables for thermoelements of the same group by the aid of a "difference curve." It will be found that but little labour is involved in the construction of a second table, taking the first as basis and amending it in accordance with the requirements shown by the "difference curve." Typical tables for platinum, rhodium, copper-constantan, and Hoskins alloys have been published by Adams in *Tables and Curves for Use in measuring Temperatures with Thermocouples* (Symposium on Pyrometry, American Instit. of Mining and Metall. Engineers), Sept. 1919, p. 165.

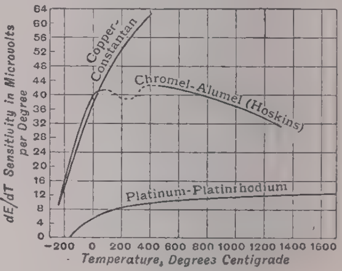

FIG. 19.

§ (24) CALIBRATION BY COMPARISON WITH A STANDARD THERMOELEMENT. — The method of standardising by freezing-point determinations above referred to should only be resorted to in the case of primary standards or as a check-point in case of doubt. The calibration of one couple by direct comparison with another is both simple and expeditious.

In the case of platinum alloy couples the usual procedure is as follows:

The heavy porcelain sheath is removed and the wires insulated by capillary fireclay tubing or quartz. The junctions of the two couples are tied together by a piece of pure platinum wire and inserted at the mid-point of an electric furnace.

The couples are carried in a liner-tube supported from the cold ends of the furnace to avoid leakage from the heating circuit into that of the potentiometer. At temperatures exceeding 1000° C. the electric resistivity of porcelain, quartz, etc., falls off rapidly with temperature, and quite minute electric leakages are noticeable with apparatus designed to measure E.M.F.'s of the order of one hundred-millionth that of the heating circuit (*i.e.* 100 volts to 1 microvolt).

§ (25) CONTAMINATION TESTS ON THERMOCOUPLES.—Despite all the care taken in the use of platinum thermoelements at high temperatures, contamination troubles occur in the course of time. Even if the couple is carefully protected from external influences by sheathing in glazed tubing it is impossible to eliminate completely an internal source of trouble, namely, the volatilisation of rhodium or iridium from the alloy limb on to the pure platinum limb.

It must be remembered that recalibration of a suspected thermocouple will not show whether the portion inserted in the furnace is contaminated or not. Should the affected length be within the uniformly heated region, it is immaterial what the composition of the wire may be.

A variety of methods have been devised for detecting the contaminated region of a thermoelement. The simplest is to stretch the wire between supports, connect the ends to a galvanometer, and run a small Bunsen flame along beneath. In the contaminated region the galvanometer will show marked deflection from the normal position. Each limb of thermoelement should be tested separately.

There is no known method of restoring a contaminated couple to its original state, and the only solution is to cut off the contaminated portions.

Base-metal thermocouples are particularly liable to develop heterogeneity after prolonged exposure to high temperatures, this being usually due to structural changes in the alloy. When this occurs the readings obtained will depend upon the depth of immersion if the affected part happens to be in a region where there are temperature gradients. Hence it is advisable to keep a check on the pyrometers permanently installed by inserting a standard couple into the furnace alongside it with the hot junctions in close proximity. Observations should be taken at a series of temperatures. If the conditions prevailing in the furnace are fairly definite and the heterogeneity effect small, consistent values should be obtained for the difference between the two couples. If, however, the differences are of variable magnitude and not reproducible it would be advisable to discard the suspected couple.

It is, of course, useless to expect the same accuracy in a test of this character as would be obtained under laboratory conditions. But the data should prove the accuracy of the temperature observations. It is advisable to use as check couple one of small cross-section and protected by a thin tube. It would then serve to show whether the depth of immersion of the working couple is sufficient. When heavy iron protecting tubes are employed it may happen that the conduction along the tube is so considerable as to keep the temperature of the hot junction below that of the region in which it is placed.

§ (26) THE INSTALLATION OF A POTENTIOMETER OUTFIT.—In the installation of a potentiometer for thermoelectric work two points require attention :

(1) The elimination of parasitic E.M.F.'s,
(2) The prevention of leakage into the potentiometer circuit from neighbouring lighting or furnace circuits.

Parasitic E.M.F.'s can be largely eliminated by a suitable choice of metals for the resistance-coils, binding-screws, and leads.

Manganin has a comparatively small thermal E.M.F. against copper, and copper terminals are now obtainable. All keys should be protected from temperature fluctuations.

It is obvious that a leakage current must enter the circuit at one point and leave at another, after passing through the galvanometer, if it is to produce errors in the observations.

Leakage into the potentiometer can be detected by observing the deflection when the thermoelement is short-circuited.

When a thermocouple is used in connection with an electric furnace, the heating circuit should be provided with a reversal switch, and the deflection of the galvanometer spot, on quickly changing over, noted from time to time.

It is a difficult matter to provide an effective insulation of the thermocouple from the high-voltage heating circuit of a furnace at temperatures exceeding 1400° C., since the ionised atmosphere conducts slightly.

White recommends a system of shielding, by which a good conductor is interposed at all points between the potentiometer system and the source from which the leakage occurs.

For details of the method of application of the equipotential shields, reference should be made to the original papers, a list of which is given in the bibliography.

Good electrical insulation of the apparatus should be the first consideration when leakage troubles occur.

§ (27) APPLICATION OF THERMOELEMENTS TO THE MEASUREMENT OF EXTREMELY LOW TEMPERATURES.—The use of thermoelements in practical work at low temperatures has been studied by Onnes and Dewar. Onnes favours constantan and steel on account of the large E.M.F. developed. Dewar found cupro-nickel and gold to be satisfactory down to liquid hydrogen temperatures.

During the course of his work he observed some curious changes after exposure of the couples to low temperature. If the junction had been soft-soldered the E.M.F. would not be reproducible after exposure to liquid hydrogen temperatures. Hard-soldering was found to be quite satisfactory. He found it advisable to cool the couple repeatedly to the lowest temperature before calibration in order to ensure that it had settled to an equilibrium state.

For low-temperature work the couples are generally calibrated by a direct comparison with a hydrogen or helium gas thermometer.

A study of the curves in *Fig.* 19 shows that the thermoelectric method lacks sensitivity at low temperatures, and for any work requiring accurate measurements of temperature the resistance method should be employed.

§ (28) Wiring the Thermoelements to the Indicator. (i.) *Wiring and Switches.*—The wiring of a thermocouple installation demands more care than is usually bestowed on ordinary lighting installations.

Indicators, especially of the low resistance type, are calibrated for a definite lead resistance and the size of wire for the connections should be chosen with this fact in mind. The switches should be selected to have a low thermoelectric effect at the contacts and also low contact resistance.

The situation of the leads should be chosen so as to avoid localities subject to large fluctuations of temperature, such as the vicinity of furnaces, etc., otherwise the temperature coefficient of resistance of the leads, especially if of copper, may introduce appreciable errors. In order to obtain mechanical protection it is advisable to enclose the leads in metal conduits. These conduits should be "earthed" so as to prevent leakages from the power circuits in the vicinity affecting the sensitive indicators of the thermoelectric installation.

The use of the common return for a multiple element installation is not to be recommended, as it is a frequent source of trouble due to leakages. It is impossible to insulate base-metal thermocouples at high temperatures from the ironwork of the furnace, and frequently the hot junction is actually welded to the end of its iron protecting tube in order to reduce thermal lag. The consequence of this is that when the tubes are in contact with the casing of the furnace the limb of the element which is attached to the common lead is shunted by a differential couple composed of the corresponding limb of the other couple and the ironwork. This shunt effects the readings of both thermocouples whenever the temperatures of the hot junctions differ.

(ii.) *Use of a "Common" Cold Junction.*—When a number of couples are read on the same meter it is inconvenient to bury each cold junction, so the practice is frequently followed of employing a "common" cold end for the installation. In this case one couple extension circuit is run into the well, as shown in *Fig.* 20. All the thermocouple extensions terminate at one point called a "junction box," ensuring the same temperature to all connections within. The couples are connected by compensating leads to the junction box, and leads of the same alloy also connect the junction box to the well. The connections from the junction box through the multipoint switch to the indicator are made of copper.

(iii.) *Indicator and Recorder.*—It is sometimes desirable to have an indicator situated in the neighbourhood of the furnace and a recorder connected in parallel situated in an office.

In such installations due regard must be given to the fact that the recorder is a shunt across the indicator. If the indicator or the recorder has been calibrated to read correctly when alone connected to the couple, then when the other is added both instruments will read low. On the other hand, if the two instruments are calibrated to read correctly in parallel they will both read high when connected to different couples in a multiple installation with commutating switches.

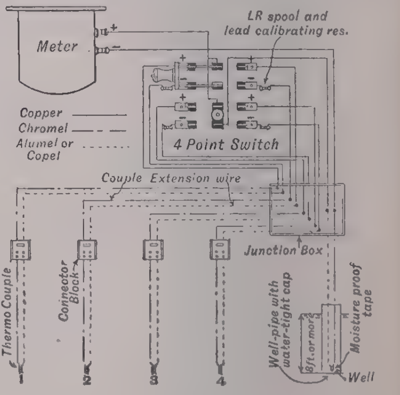

Fig. 20.

Consequently, the practice of installing indicators and recorders in parallel can only be safely adopted when the instruments are of the high-resistance type.

§ (29) Notes on Differential Couples.—When small differences of temperature have to be measured, a battery of thermoelements may be employed and connected together in series.

When the battery is composed of a large number of elements, it may conveniently be divided into two equal groups of couples. Then, by connecting the two groups in opposition and inserting one end of each in ice, while the other is at room temperature, the E.M.F. should be zero, provided the two groups are exactly symmetrical and there are no internal short circuits or leaks.

Generally in differential work the battery of thermoelements has to be made as compactly as possible and with the minimum of insulation consistent with safety. The copper wire, owing to its greater specific conductivity, may be of less diameter than the constantan wire without loss of sensitivity, a combination of 0·15 mm. diameter copper wire with 0·25 mm. diameter constantan being quite satisfactory. The junctions should be soldered with silver solder, using a little anhydrous borax as flux. This operation is facilitated by using a minute gas-jet rather than an ordinary blow-pipe.

Insulation of the bared parts is effected by repeatedly dipping into a solution of celluloid in acetone (freed from water); this coating is suitable for work at room temperatures.

The following method of insulating by means of hard rubber has been recommended by Adams: Enough precipitated sulphur, or still better, insoluble sulphur, is stirred with rather thick rubber cement (pure gum rubber dissolved in benzene or CO_2) to equal 20 to 25 per cent of the solid rubber. The junctions are dipped in the mixture, and after drying in the air are maintained at a temperature of 140° C. for fifteen hours.

[The writer desires to acknowledge his indebtedness to Messrs. Charles Griffin for permission to utilise in the preparation of the articles on pyrometry some of the material contained in *Methods of Measuring Temperature.*]

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E. G.

THERMOCOUPLES:

Cold Junction Temperatures of, Automatic Compensation for. See "Thermocouples," § (21).

Contamination Tests on. See *ibid.* § (25).

Standardisation of, for low temperature work up to 450° C. See *ibid.* § (22).

Standardisation of, Table of Fixed Points suitable for, up to 450° C. See *ibid.* § (22).

THERMODYNAMIC PERFORMANCE IN STEAM ENGINES, TESTS OF. See "Steam Engine, Theory of," § (13).

THERMODYNAMIC RELATIONS BETWEEN FUNCTIONS OF THE STATE IN ANY FLUID. See "Thermodynamics," §§ (45), (46), (47), (49).

THERMODYNAMIC SCALE FOR ADOPTION OF NUMERICAL UNIT OF TEMPERATURE, by finding standard reproducible temperatures for adoption as standard values. See "Temperature, Realisation of Absolute Scale of," § (3).

THERMODYNAMICS

§ (1) Introductory.—Thermodynamics is primarily that branch of science which deals with the relation of heat to mechanical work, but in a more extended sense it deals with the relation of heat to other forms of energy. An important part of it is the theory of Heat-Engines, which are devices for getting work done through the agency of heat. It is also concerned with the theory of Refrigerating Machines, which are devices for removing heat from bodies that are colder than their surroundings. Such devices may also be called Heat-Pumps, for their function is to take in heat at a relatively low temperature and discharge heat at a higher temperature.

A Heat-Engine, on the other hand, takes in heat, which is generally supplied by the combustion of a fuel, at a high temperature, and discharges heat at a lower temperature. A condensing steam-engine, for example, takes in heat at the temperature of the boiler and discharges heat to the cooling water of the condenser. In any heat-engine the heat is let down, within the engine, from a high level of temperature to a lower level of temperature, and it is by so letting heat down that the engine is able to do work, as a water-wheel is able to do work by letting water down from a high level of position to a lower level. But there is this important difference, that some of the heat disappears in the process of being let down; it is converted into the work which the engine does. From the point of view of practical thermodynamics, the object of a heat-engine is to get work done with the least possible expenditure of fuel. It is desired, subject to other considerations of economy or convenience, that the ratio of the work done to the heat taken in should be as large as is practicable. This ratio is called the Efficiency of the engine as a heat-engine. The theory of heat-engines discusses the conditions that affect efficiency, and shows what is the limit of efficiency that can be approached when the conditions are most favourable.

In a mechanically driven refrigerating machine or heat-pump the action is essentially the reverse of that of a heat-engine. The object is to remove heat from the cold body and raise it to the higher level of temperature at which it can be discharged, with the least possible expenditure of work. The ratio of the heat that is taken in from the cold body to the work that is spent in driving the machine, which is called the Coefficient of Performance, should be as large as is practicable, and the theory of refrigeration discusses the conditions that will make it so.

§ (2) Units of Heat.—A convenient and precise unit in which quantities of heat may be reckoned is the quantity that is required to warm a unit mass of water from the temperature of melting ice to the temperature at which water boils under a pressure of one atmosphere. These two points serve to determine two definite states of temperature that can easily be reproduced. A unit of heat which is obtained by taking a certain fraction of this quantity of heat is described as a *mean thermal unit.* The mean thermal unit called the Gramme-calory is one-hundredth part of the heat which will serve to warm one gramme of water from the melting-point to the boiling-point at a pressure of one atmosphere. One-hundredth part is taken because on the Centigrade scale of temperature the interval between these fixed points is divided into 100 degrees: consequently the gramme-calory is the average amount of heat required to warm a gramme of water through one degree Centigrade, between the melting-point and the boiling-point as limits. The actual amount required per degree need not be the same for each degree of the scale, and in fact is not the same, for the specific heat of water is not quite constant.

What is commonly called the British Thermal Unit (when the Fahrenheit scale is employed) is $\frac{1}{180}$ of the quantity of heat required to warm one pound of water from the melting-point to the boiling-point, because on the Fahrenheit scale there are 180 degrees between the two fixed points.

A much preferable unit of heat for British measures is the mean Pound-calory, which is one-hundredth of the amount of heat required to warm one pound of water from the melting-point to the boiling-point.

Instead of the mean calory a unit of heat is often taken called the 15° calory,[1] which is the quantity of heat required to raise one gramme of water through 1° C. at 15°, or from $14\frac{1}{2}$° to $15\frac{1}{2}$°. The mean calory is greater than the 15° calory in the ratio of 1·0002 to 1.

§ (3) Mechanical Equivalent of Heat.—The experiments of Joule, begun in 1843 and continued for several years, demonstrated that when work is expended in producing heat a definite relation holds between the amount of heat produced and the amount of work spent. By causing the potential energy of a raised weight to be used up in turning a paddle which generated heat by stirring water in a vessel, and observing the rise of temperature so produced, Joule determined the number of units of work that are spent in producing a unit of heat. This number is called the mechanical equivalent of heat. In early experiments Joule found that 772 foot-pounds were required to raise the temperature of one pound of water through one degree Fahrenheit on the thermometer he employed, at a particular part of the scale.

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[1] In some cases the mean for the range 15° C. to 20° C. is taken, giving a value at 17°·5. See "Heat, Mechanical Equivalent."

Many later and more exact determinations were made by Joule himself and by other observers, using various methods of experiment. They agree in showing that Joule's original figure was rather low. The general result is to fix 1400 as the number of foot-pounds (in the latitude of London) that are equivalent to one mean pound-calory. The corresponding value of the mechanical equivalent of the "British Thermal Unit" is 777·8 foot-pounds, and that of the gramme-calory is 426·7 gram-metres. In absolute (C.G.S.) units the gramme-calory is equivalent to 4·1868 Joules or $4{\cdot}1868 \times 10^7$ ergs, the Joule being 10^7 ergs.

The mechanical equivalent of heat enters into many of the formulas of thermodynamics. It is often called Joule's Equivalent, and is generally represented by the symbol J. The symbol A is used for the reciprocal of Joule's equivalent, or $1/J$.

§ (4) SCALES OF TEMPERATURE.—In the construction of an ordinary thermometer a fine tube of uniform bore is chosen, and a bulb is formed on it to contain the mercury or other liquid whose expansion is to be used as an indication of temperature. When it is filled the two fixed points are determined by placing the instrument (*a*) in melting ice, and (*b*) in the steam coming from water boiling under a pressure of one atmosphere. The position taken by the end of the column of liquid in the tube is marked for each of these two points. The distance between them is then divided into equal parts which are called degrees, 100 parts for the Centigrade scale and 180 for the Fahrenheit scale. By this construction equal steps in temperature are defined by equal amounts of expansion on the part of the selected liquid, or rather by equal amounts of difference between the expansion of the liquid itself and that of the glass in which it is contained, for it is the difference of expansion that determines the rise of the column in the tube. This common method of measuring temperature gives results that vary for different liquids and for different sorts of glass. Each of two mercury thermometers, for example, may have the fixed points correctly marked, and be of uniform bore, and yet if they are made of different sorts of glass they may give readings that differ by as much as half a degree at the middle of the range between the fixed points, and may show still more serious discrepancies when they are applied to measure higher temperatures. This illustrates the fact that the measurement of temperature by an ordinary thermometer gives an arbitrary scale, which cannot even be relied on to be the same in different instruments.

Measurements of temperature are much less capricious if we select for the expanding substance any one of the so-called permanent gases such as air, or nitrogen, or hydrogen, taking care to keep the pressure of the gas constant while it is employed to measure temperature by its changes of volume. Such an instrument is called a constant-pressure gas thermometer. It would be inconvenient for ordinary use; but it serves to supply a scale with which the readings of an ordinary thermometer can be compared. Thus the readings of any thermometer can be corrected to bring them into agreement with the scale of a gas thermometer if that scale be adopted as the standard scale in stating temperatures.

Experiments on the expansion of various gases by heat have shown that all gases which are far from the conditions that would cause liquefaction expand very nearly alike. If we compare an air thermometer with a nitrogen or a hydrogen thermometer we get practically the same scale except at extremely low temperatures such as those at which the gas is approaching the liquid state. Gases expand by almost exactly the same amount between the two fixed points, and at intermediate points, or at points beyond the range, their agreement with one another is almost perfect. Hence the scale of the gas thermometer is much to be preferred to that of any mercury thermometer as a means of stating temperature. But there is another and even stronger reason for this preference. We shall see later that it is possible to imagine a scale of temperature, based on general thermodynamic principles, which does not depend on the properties of any particular substance: that scale is called the *thermodynamic scale* of temperature, and much use is made of it in thermodynamic reasoning. The scale of a gas thermometer is practically identical with the thermodynamic scale. This is true whether we use a constant-pressure gas thermometer, or what is called a constant-volume gas thermometer, in which increments of temperature are measured by the increments of pressure that are required to keep the volume of the gas constant while it is heated.

Experiment shows that the amount by which air or hydrogen or any other so-called permanent gas expands between the two fixed points is about 100/273 of the volume at the lower fixed point, care being taken that the pressure does not change. Hence, if we adopt the scale of the gas thermometer as our scale of temperature, and use Centigrade divisions, this result may be expressed by saying that when 273 cubic inches of gas at 0° C. are heated under constant pressure to 1° C. the volume alters to 274 cubic inches. When the gas is heated to 2° C. its volume becomes 275 cubic inches, at the upper fixed point it becomes 373, and so on. Similarly, if the gas be cooled from 0° C. to $-1°$ C. its volume changes from the original 273 cubic inches to 272, and so on. Hence the volume would become zero at $-273°$ C., if the same law could be held to apply down to the lowest temperatures.

This result may be concisely expressed by saying that if temperature be reckoned not from the ordinary zero but from a zero which is about 273 Centigrade degrees below it (more exactly 273·1), the volume of a gas, heated under constant pressure, is proportional to the temperature reckoned from that zero. The zero in question is spoken of as the Absolute Zero of temperature. Denoting any temperature on the ordinary scale by t and the corresponding temperature reckoned from the absolute zero by T, we have (using Centigrade degrees)

$$T = t + 273{\cdot}1.$$

The absolute zero has been defined here by reference to the expansion of a gas. But it will be seen later that the thermodynamic scale of temperature starts from a zero which is absolute in the sense that no lower temperature can possibly exist, and that the zero of the thermodynamic scale coincides with the zero of the gas scale as defined above.[1]

§ (5) PROPERTIES OF A GAS: CHARLES'S LAW AND BOYLE'S LAW.—The experimental fact that all "permanent" gases expand by very nearly the same fraction of their volume for a given increase of temperature, the pressure being kept constant, is known as Charles's Law. Another fundamental property of gases, discovered by the experiments of Boyle, is that when the volume of a gas is altered by altering the pressure, the temperature being kept constant, the volume varies inversely as the pressure.

Thus if V be the volume of a given quantity of any gas, and P the pressure, then so long as the temperature remains unchanged V varies inversely as P, or PV = constant. This is Boyle's Law. It is very nearly though not exactly true in gases such as air or oxygen, or nitrogen or hydrogen, at ordinary temperatures and pressures; the deviations from it become serious at high pressure or when the gas approaches conditions that would produce liquefaction.

§ (6) NOTION OF A "PERFECT" GAS.—In formulating the principles of thermodynamics it is convenient to imagine a gas which exactly conforms to laws that are only very nearly true of real gases. Such a gas is called a "perfect" gas. The properties of real gases are most easily treated as involving small deviations from those of imaginary "perfect" gases obeying simple laws. Among real gases hydrogen probably comes nearest to the ideal of a perfect gas, but no real gas is in this sense strictly perfect.

In a gas which is perfect in the sense of conforming exactly to Boyle's Law we should find PV strictly constant, so long as the temperature is constant. If we define the temperature scale by reference to the expansion of the gas we should also have V varying as the temperature T (reckoned from the absolute zero) under any constant pressure. Combining these two statements we should have

$$PV = RT$$

where R is a constant. For the present it is to be understood that the symbol T stands for temperature measured on the scale of a gas thermometer, from a zero which is 273·1° C. below the melting-point of ice.

When a gas satisfying this equation is heated under constant pressure and consequently expands, R is a measure of the amount of work done by the gas for each degree through which the temperature rises. Let the original temperature of the gas be T_1 and its volume V_1, and let it be heated under constant pressure P till the temperature is T_2 and the volume V_2. Then we have $RT_1 = PV_1$ and $RT_2 = PV_2$, from which

$$R(T_2 - T_1) = P(V_2 - V_1),$$

which is the work done by the gas in expanding from V_1 to V_2. Let the interval of temperature be 1°, then R is equal to the work done.

§ (7) WORKING SUBSTANCE.—In the action of any heat-engine or refrigerating machine there is always a working substance which in consequence of its capacity for heat forms the vehicle by which heat passes through the machine. In the course of its action the working substance takes in heat and gives out heat, and thereby conveys heat from one level of temperature to another. Its volume also changes, and it is through changes of volume on the part of the working substance that the engine does work, if it be a heat-engine, or has work spent upon it if it be a refrigerating machine. Accordingly an important part of the science of thermodynamics deals with the properties of substances in relation to heat, and the connection between such properties in any substance, especially in the fluid state, whether gaseous or liquid. Any fluid has its individual characteristics, but between its various properties, pressure, volume, temperature, internal energy, specific heat, and so forth, certain general relations hold which are true of all fluids, and are deduced from the fundamental laws of thermodynamics. These relations will be considered in a later part of this article.

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[1] The exact position of the absolute zero is uncertain to the extent of about one-tenth of a degree. Callendar places it at $-273{\cdot}1°$ C., and that figure is generally adopted.

In general the working substance is a fluid, but it is easy to imagine a heat-engine whose working substance is a solid body, say a long rod of metal arranged to act as the pawl of a ratchet-wheel with closely pitched teeth. Let the rod be heated so that it lengthens sufficiently to drive the wheel forward through the space of one tooth. Then let the rod be cooled, say by applying cold water, the ratchet-wheel being meanwhile held from returning by a detent. The rod on cooling will retract so as to engage itself with the next succeeding tooth, which may then be driven forward by heating the rod again, and so on. To see that such an engine could do work we have only to suppose that the ratchet-wheel carries round with it a drum by which a weight is wound up. The device forms a complete heat-engine, in which the working substance is a solid rod, doing work in this case not through changes of volume but through changes of length. While its length is increasing it is exerting force in the direction of its length. It receives heat by being brought into contact with some source of heat at a comparatively high temperature; it transforms a small part of this heat into work; and it rejects the remainder to what we may call a receiver of heat, which is kept at a comparatively low temperature. The greater part of the heat may be said simply to pass through the engine, from the source to the receiver, *becoming degraded as regards temperature in the process.* This is typical of the action of all heat-engines: they convert some heat into work only by letting down a much larger quantity of heat from a high temperature to a relatively low temperature. The engine we have just imagined would not be at all efficient in the thermodynamic sense; the fraction of the heat supplied to it which it could convert into work would be very small. Much greater efficiency can be obtained by using a fluid for working substance and by making it act so that its own expansion of volume not only does work but also causes it to fall in temperature before it begins to reject heat to the cold receiver.

§ (8) CYCLE OF OPERATIONS OF THE WORKING SUBSTANCE.—In the action of many heat-engines and refrigerating machines the working substance returns periodically to the same state of temperature, pressure, volume, and physical condition. Each time this has occurred the substance is said to have passed through a complete cycle of operations. For example, in a condensing steam-engine water taken from the hot-well is pumped into the boiler; it then passes into the cylinder as steam, then from the cylinder into the condenser, and finally from the condenser back to the hot-well; it completes the cycle by returning to the same condition in all respects as at first, and is ready to go through the cycle again.

In the theory of heat-engines it is convenient to consider the cycle of operations as a whole. If the cycle is complete we know that whatever quantity of heat or other energy the substance contains within itself is equal to the quantity that was there to begin with, for the state of the substance is the same in all respects, and consequently any work that it has done must have been done at the expense of heat which it has taken in during the cycle. We can at once apply the principle of the Conservation of Energy and say that for the cyclic process as a whole the work done must be equivalent to the difference between the heat taken in and the heat discharged.

§ (9) THE FIRST LAW OF THERMODYNAMICS.—The principle of the Conservation of Energy in relation to heat and work may be expressed in the following statement, which constitutes the First Law of Thermodynamics. When work is done by the expenditure of heat a definite quantity of heat goes out of existence for every unit of work done; and, conversely, when heat is produced by the expenditure of work the same definite quantity of heat comes into existence for every unit of work spent. The word " work " is to be understood here in a comprehensive sense: it includes electrical work as well as work done against a mechanism or in raising weights. Electrical work may be done directly by the expenditure of heat in a thermoelectric circuit, which is a true heat-engine though it acts without exhibiting any mechanical movement.

§ (10) INTERNAL ENERGY.—No means exist by which the whole stock of energy that a substance contains can be measured. But we are concerned only with changes in that stock, changes which may arise from the substance taking in or giving out heat, or doing work, or having work spent upon it. If a substance takes in heat without doing work, its stock of internal energy increases by an amount equal to the heat taken in. If it does work without taking in heat, it does the work at the expense of its stock of internal energy, and the stock is diminished by an amount equal to the work done. In all cases, when heat is being taken in and the substance is at the same time doing work, we have

Heat taken in = Thermal equivalent of work
done + Increase of Internal Energy.

For any infinitesimally small step in the process, this equation may be written in the form

$$dQ = A\,dW + dE, \quad . \quad . \quad . \quad (1)$$

where dQ is the heat taken in during the step, AdW is the thermal equivalent of work done, and dE the increase of internal energy. A is the reciprocal of Joule's equivalent.

In a complete cycle there is, at the end, no change of the internal energy E, and consequently for the cycle as a whole,

$$Q_1 - Q_2 = AW,$$

where Q_1 is the heat taken in; Q_2 is the heat rejected, and AW is the thermal equivalent of the work done.

§ (11) WORK DONE IN CHANGES OF VOLUME OF A FLUID.—In an engine of the usual cylinder and piston type the working fluid does mechanical work by changes of its volume. The amount of work done depends only on the relation of the pressure to the volume in these changes, and not on the form of the vessel or vessels in which the changes of volume take place. Let the intensity of pressure of the fluid (that is to say the pressure on unit of area) be P while the piston moves forward through a small distance δx. If the area of the piston is S the total force on it is PS and the work done is $PS\delta x$. But $S\delta x = \delta V$, the change of volume: hence the work done is $P\delta V$ for the small change of volume δV, or $\int_{V_1}^{V_2} PdV$ for a finite change of volume from a volume V_1 to a volume V_2 during which the pressure may vary.

In any complete cycle of operations the volume at the finish is the same as at the start, and the work done is $\int PdV$ taken round the cycle as a whole.

It is very useful to represent graphically the work which a fluid does in changing its volume. Let a diagram be drawn in which the relation of the pressure of any supposed working substance to its volume is shown by rectangular co-ordinates as in *Fig.* 1.

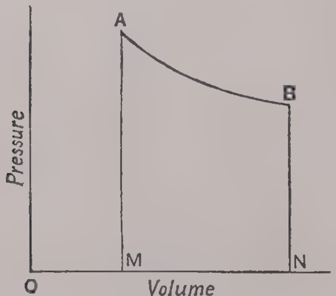

FIG. 1.

Beginning with the state represented by the point A, where the pressure is AM and volume OM, suppose the substance to expand to a state B, where the pressure is BN and the volume ON, and let the curve AB represent the intermediate states of pressure and volume. Then the work done by the substance in this expansion, which is $\int_{OM}^{ON} PdV$, is represented by the area MABN under the curve AB.

Again, if the substance undergoes any complete cycle of change (*Fig.* 2) by expanding from A through B to C and by being compressed back through D to A, work is done by it while it is expanding from A to C, equal to the area MABCN, and work is spent upon it while it is being compressed from C through D to A, equal to the area NCDAM. The net amount of work which the substance does during the cycle is equal to the difference between those areas: in other words, it is equal to the area of the closed figure ABCDA representing the complete cyclic operation, which area is $\int PdV$.

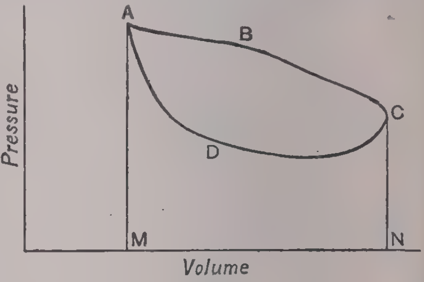

FIG. 2.

If, on the other hand, the operation were such as to trace the figure in the opposite direction, the substance being expanded from A to C through D and compressed from C to A through B, the enclosed area would be a measure of the work expended upon the substance in the cycle.

This method of representing the work done during the operation of an engine was used by James Watt, who invented an instrument called the Indicator, by means of which the engine might automatically draw a diagram representing the changes of pressure in relation to changes of volume. Such a diagram is accordingly called an Indicator Diagram.

Watt's Indicator consists of a small cylinder containing a piston which can move in it without sensible friction but is controlled by a stiff spring. This is put in free communication with one end of the working cylinder of the engine, so that the working substance presses on the indicator piston and displaces it, against the spring, through distances that are proportional to the pressure at every instant. Connected with the indicator piston is a pencil which rises or falls with it, the connection being made, in modern instruments, through a lever that gives the movements of the indicator piston a convenient magnification. A sheet of paper on which the pencil marks its movements is caused to move through distances proportional to the motion of the engine piston, and at right angles to the path of the pencil. Thus a diagram is drawn like that of *Fig.* 2, exhibiting a closed curve for each double stroke of the engine piston, and with co-ordinates which represent the changes of pressure and changes of volume. The enclosed area, when interpreted by reference to the appropriate scales of pressure and volume, measures the net amount of work done in the engine cylinder during the double stroke, so far as one side of the piston is concerned. If the engine is double-acting—that is to say, if the working substance acts successively on the two sides of the engine piston during successive strokes—a similar indicator diagram is taken for the other end of the cylinder as well.

In some modern indicators the motion which measures the variation of pressure causes a small mirror to be tilted, from which a beam of light is reflected on to a photographic plate carried in a frame which copies the motion of the engine piston. This optical method of magnifying and recording the motion has the advantage of avoiding those errors which in mechanical indicators arise from the inertia of the magnifying lever and from the friction of the pencil point.

§ (12) INTERNAL ENERGY OF A GAS: JOULE'S LAW.—The Internal Energy of a given quantity of a gas depends only on the temperature.

This is an inference from the fact established by experiments of Joule, that when a gas expands without doing external work and without taking in or giving out heat, and therefore without changing its stock of internal energy, its temperature does not change.

Joule's Law is to be regarded as strictly true only of imaginary perfect gases: in any actual gas there is a slight departure from it, which is very small indeed in a nearly perfect gas such as hydrogen. The law was originally established by means of an experiment in which Joule connected a vessel A (*Fig.* 3) containing compressed gas with another vessel B which was empty, by means of a pipe with a closed stop-cock C. Both vessels were immersed in a bath of water and were allowed to assume a uniform temperature. Then the stop-cock was opened, and the gas distributed itself between the two vessels, expanding without doing external work. After this the temperature of the water in the bath was found to have undergone no appreciable change. The temperature of the gas appeared unaltered, and no heat had been taken in or given out by it, and no work had been done by it. Since the gas had neither gained nor lost heat, and had done no work, its internal energy was the same at the end as at the beginning of the experiment. The pressure and volume had changed, but the temperature had not. The conclusion follows that the internal energy of a given quantity of gas depends only on its temperature, and not upon its pressure or volume; in other words, a change of pressure and volume not associated with a change of temperature does not alter the internal energy. Hence in any change of temperature the change of internal energy is independent of the relation of pressure to volume during the operation: it depends only on the amount by which the temperature has been changed.

It is now, however, known that a very slight change of temperature does in fact take place when a real gas expands without doing work. In later experiments by Joule and W. Thomson (Lord Kelvin) a more delicate method was adopted of detecting whether there is any change of internal energy when the pressure and volume change under conditions such that external work is not done. The gas was forced to pass through a porous plug by maintaining a constant high pressure on one side of the plug and a constant low pressure on the other. Care was taken to prevent any heat being gained or lost by conduction from outside. In this operation work was done upon the gas in forcing it up to the plug, and work was done by it when it passed the plug, by displacing gas under the lower pressure on the side beyond the plug. If no change of temperature took place, and if the gas conformed to Boyle's Law, these two quantities of work would be exactly equal, and consequently no external work would be done on the whole. For let P_1 be the pressure and V_1 the volume before passing the plug, and P_2 the pressure and V_2 the volume after passing the plug, the volumes being in both cases stated per unit quantity of the gas. Then the work done upon the gas as it approaches the plug is P_1V_1, and the work done by it as it leaves the plug is P_2V_2. If the temperature is the same on both sides these quantities are equal in a gas for which PV is constant

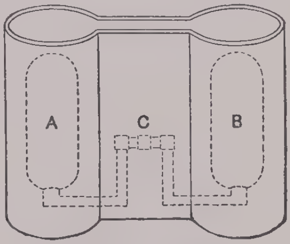

FIG. 3.—Joule's Apparatus.

at any one temperature. Thus a gas which is "perfect" in the sense that it conforms strictly both to Boyle's Law and to Joule's would in its passage of the plug have expanded without (on the whole) doing any work, and therefore without changing its internal energy, no heat being gained or lost. In such a gas no change of temperature should accordingly be found, as it passes the plug, and if a change of temperature is observed in any real gas it is due to the fact that real gases are not strictly "perfect."

In the experiments of Joule and Thomson small changes of temperature were in fact detected and measured in air and other real gases, on passing the porous plug. This Joule-Thomson effect, as it is called, is in general a cooling. Observations of the Joule-Thomson effect are of great value in determining exactly the properties of gases and vapours which are not perfect; and certain practical methods of liquefying gases under extreme cold depend upon the existence of this effect.

In the imaginary perfect gas, however, the Joule-Thomson effect is entirely absent. There is no change of temperature in passing the plug, and there is also no change of internal energy, for no work is done and (by assumption) no heat is taken in or given out.

It is important to notice that we assume the imaginary perfect gas to satisfy two conditions: it obeys Boyle's Law exactly and also Joule's Law exactly. These characteristics are independent of one another: it would be possible to have a gas satisfy one and not the other, but a gas is said to be perfect in the thermodynamic sense only when it satisfies both, and in that case certain other properties follow which will now be pointed out.

§ (13) Specific Heats.—The Specific Heat of any substance means the amount of heat required per degree to raise the temperature of unit quantity of the substance, under any assumed mode of heating. Thus when a substance is heated through a small interval of temperature dT the heat taken in (per unit quantity) is KdT, where K is the specific heat for the particular conditions and mode of heating. In dealing with gases or other fluids two important modes of heating must be distinguished: we may heat them under conditions of constant pressure or of constant volume. We shall use the symbol K_p to represent specific heat at constant pressure, and K_v to represent specific heat at constant volume.

Suppose any fluid to be heated through a small interval of temperature dT. If the heating is at constant volume, the heat taken in is $K_v dT$ and all of it goes to increase the stock of internal energy. Hence in heating at constant volume the increase of internal energy $dE = K_v dT$. If the heating is at constant pressure P, through the same interval of temperature, the heat taken in is $K_p dT$. Only part of this heat goes to increase the stock of internal energy, for part of it is used in doing external work, since, to keep the pressure constant, the fluid has to expand through some amount dV. The external work done is PdV and its thermal equivalent is $APdV$. Hence in heating at constant pressure the increase of internal energy is given by $dE = K_p dT - APdV$.

Apply these results to a perfect gas. By Joule's Law the internal energy of a perfect gas depends only on the temperature, and therefore dE is the same in both modes of heating. Further, since in such a gas $PV = RT$, $PdV = RdT$ when P is constant. Hence in a perfect gas

$$K_v dT = K_p dT - ARdT$$

or

$$K_v = K_p - AR.$$

Thus for a perfect gas we have the important relation between the two specific heats

$$K_p - K_v = AR, \quad . \quad . \quad . \quad (2)$$

which follows from the Laws of Boyle and of Joule.

It follows also that in a perfect gas both the specific heats are constant if one of them is constant. To be constant the specific heat has to be independent both of the pressure and of the temperature. By Joule's Law the change of internal energy in a perfect gas for a given change of temperature, is the same whatever be the pressure: hence in such a gas the specific heats are the same at all pressures. There is, however, nothing in the Laws of Boyle or of Joule to determine whether the specific heats are the same at all temperatures. Experiment shows that in some gases there is much variation of specific heat with temperature (although the gas may be nearly perfect) and in others there is practically none. For the purpose of thermodynamic reasoning it is very useful to think of a gas which is not only perfect in the sense of conforming exactly to the Laws of Boyle and of Joule but also has constant specific heats. Such an imaginary gas forms a convenient scaffolding by the help of which we may most easily build up the theory of thermodynamics, but it is nothing more than a convenient scaffolding, which has nothing to do with the stability of the completed work. In what follows we shall make this temporary use of an ideal gas, as the supposed working substance of an ideal heat-engine, in order to arrive at conclusions which, as will be shown, have a quite general application.

§ (14) REVERSIBLE ACTIONS.—The next step is to consider particular modes in which a working substance may be expanded or compressed and may take in or give out heat, and at the outset it is important to distinguish between actions that are *reversible* and those that are *irreversible.*

An expansion or compression is reversible if it is carried out in such a manner that the operation can be reversed, with the result that the substance will pass back through all the stages through which it has passed during the expansion or compression and be in the same condition in all respects at each corresponding stage in both processes.

This implies that the substance must expand smoothly, without setting up any motions within itself of a kind such that their kinetic energy is frittered down into heat through internal friction. The whirls and eddies which occur in the cylinder of an engine are irreversible, and in ideal reversible expansion we must suppose them absent. Reversible expansion implies that there are no losses of mechanical effect from any sort of internal friction. It excludes throttling, such as occurs when a substance expands through a valve or other constricted opening into a region of lower pressure where the kinetic energy of the stream and eddies is dissipated. In such cases the motion of the stream and eddies cannot be reversed. To get the substance back to the region of higher pressure would require an expenditure of more work than was done by the substance during its expansion, and if we were to force it back we should find it had gained heat through the subsidence of the internal eddying motions, though no heat had come in from outside.

A transfer of heat to or from any substance is reversible only if the substance is at the same temperature as the body from which it is taking heat or to which it is giving heat. Suppose, for instance, that a substance is taking in heat from a hot source and is expanding as it does so. The expansion may be reversible in itself, that is to say, it may involve no internal friction, but unless the temperature of the substance be the same as that of the source, the operation as a whole—considered in its relation to the source—cannot be reversed. So considered it is reversible only when the further condition is fulfilled that compression of the substance will reverse the transfer of heat, giving back to the source the heat that was taken from it. Any thermal contact between bodies at different temperatures involves an irreversible transfer of heat.

The expansions and compressions and the transfers of heat that occur in a real engine are never strictly reversible, some of them indeed are far from being reversible. But the study of an ideal engine, in which all the operations are reversible, is of fundamental importance in the science of thermodynamics, and it furnishes a basis for the critical analysis of actions in a real engine.

§ (15) ADIABATIC EXPANSION.—There are two specially important kinds of reversible expansion; (1) Adiabatic and (2) Isothermal.

Adiabatic expansion or compression means expansion or compression, carried out reversibly, in which no heat is allowed to enter or leave the substance. A curve drawn to show the relation of pressure to volume during the process is called an adiabatic line. Adiabatic action would be realised if we had a substance expanding, or being compressed, without change of chemical constitution, and without any eddying motions, in a cylinder which (along with the piston) was totally impervious to heat. From this definition it follows that the work which a substance does while it is expanding adiabatically is all done at the expense of its stock of internal energy; and the work which is spent upon a substance while it is being compressed adiabatically all goes to increase its stock of internal energy.

In actual heat-engines the action is never strictly adiabatic, for there are always some exchanges of heat between the working substance and the surface of the cylinder and piston. Very rapid compression or expansion may come near to being adiabatic by giving little time for any transfer of heat to occur. Expansion through a throttle-valve is not adiabatic, because it is not reversible, though it may occur in such a way that no heat enters or leaves the substance.

In the adiabatic expansion of any substance work is done, and since no heat is taken in or given out, there must be a decrease of internal energy equivalent to the amount of the work done by the substance.

Accordingly, in the adiabatic expansion of any fluid

$$dE = -AdW = -APdV.$$

Here, as before, dW is the work done, and A is the factor required to convert an expression for work units into heat units. When this is applied to a perfect gas, in which $dE = K_v dT$ and $PV = RT$, we obtain the equation

$$AR\frac{dV}{V} + K_v\frac{dT}{T} = 0.$$

If we assume the specific heat to be constant, this gives on integration

$$AR \log_e V + K_v \log_e T = \text{constant}.$$

Writing $K_p - K_v$ for AR (§ (13)) and dividing by K_v this becomes

$$\left(\frac{K_p}{K_v} - 1\right) \log_e V + \log_e T = \text{constant},$$

or, using γ to represent the ratio of the specific heats K_p/K_v,

$$(\gamma - 1) \log_e V + \log_e T = \text{constant}. \quad (3)$$

Further, since PV/T is constant we have

$$\log_e P + \log_e V - \log_e T = \text{constant},$$

and adding the two equations

$$\log_e P + \gamma \log_e V = \text{constant},$$

which gives

$$PV^{\gamma} = \text{constant} \quad (4)$$

as the equation connecting pressure and volume in the adiabatic expansion of a perfect gas with constant specific heat. In other words, this is the equation of an adiabatic line, for such a gas, on the pressure-volume diagram.

In adiabatic expansion the temperature of the gas falls and in adiabatic compression it rises. By integrating equation (3),

$$TV^{\gamma - 1} = \text{constant}, \quad (5)$$

and by combining that with equation (4), and eliminating V,

$$T/P^{\frac{\gamma - 1}{\gamma}} = \text{constant}. \quad (6)$$

The work done in expanding from an initial volume V_1 to a volume V_2, which for any fluid is $\int_{V_1}^{V_2} PdV$, becomes, for the adiabatic expansion of a perfect gas,

$$W = P_1V_1^{\gamma} \int_{V_1}^{V_2} \frac{dV}{V^{\gamma}} = \frac{P_1V_1 - P_2V_2}{\gamma - 1}, \quad (7)$$

since $PV^{\gamma} = P_1V_1^{\gamma} = P_2V_2^{\gamma}$.

It follows also that this expression, multiplied by A, measures the decrease of internal energy that results from the expansion. In adiabatic compression from volume V_2 to volume V_1 this measures the work spent in compressing the gas.

§ (16) Isothermal Expansion.—Isothermal expansion or compression means expansion or compression carried out reversibly (as regards internal action) and without change of temperature. A curve drawn to show the relation of pressure to volume during isothermal expansion or compression is called an isothermal line.

When a substance is expanding isothermally it takes in heat to maintain its temperature constant; it therefore must be in contact with a source of heat. When it is being compressed isothermally it gives out heat, and must be in contact with a receiver which can take heat from it.

When a perfect gas expands isothermally PV is constant, and hence its isothermal line on the pressure-volume diagram, for any assigned temperature, is a rectangular hyperbola. The work done in the expansion is

$$W = \int_{V_1}^{V_2} PdV = P_1V_1 \int_{V_1}^{V_2} \frac{dV}{V} = P_1V_1 \log_e \frac{V_2}{V_1},$$

which may also be written

$$W = RT \log_e \frac{V_2}{V_1}, \quad (8)$$

where T is the constant temperature at which the process takes place. If we denote the ratio of isothermal expansion by r, this may be written

$$W = RT \log_e r.$$

This also measures the work done in isothermal compression from volume V_2 to volume V_1. There is no change of internal energy, and consequently during isothermal expansion the gas must take in a quantity of heat equivalent to the work it does; and during isothermal compression the gas must give out a quantity of heat equivalent to the work spent upon it.

§ (17) The Second Law of Thermodynamics.—We are now in a position to deal with the following fundamental questions of heat-engine efficiency:

(1) Having given a hot source from which heat may be taken, and a cold sink or receiver to which heat may be rejected, how may the greatest amount of mechanical work be done by each unit of heat that is taken from the hot source?

(2) What fraction of the heat taken from the hot source is it theoretically possible to convert into work?

So far as the First Law of Thermodynamics goes, it is not obvious that there is anything to prevent all the heat which the source can supply from being converted into work. But a limit is imposed as a consequence of the following principle, which is known as the *Second Law of Thermodynamics*:

It is impossible for a self-acting machine, unaided by any external agency, to convey heat from one body to another at a higher temperature.

The Second Law says, in effect, that heat will not pass up automatically from a cold body to a warmer one. We can force it to pass up, as in the action of a refrigerating machine, but only by applying an "external agency" to drive the machine. A heat-engine acts by letting heat pass down from a hot body to a colder body, by making the working substance alternately take in heat from the hot body and reject heat to the cold body, and thereby undergo expansions and contractions in which its pressure is on the whole greater during expansion than during contraction, with the result that a part of the heat that is passing down through the engine is converted into work. In consequence of the Second Law it is only a certain fraction of the whole heat supplied by the hot body that can be converted into work by any such process.

§ (18) Reversible Heat-engine. *Carnot's Cycle of Operations.*—To the first of the above two questions a correct answer was given by Sadi Carnot in a remarkable essay, published in 1824, entitled "Réflexions sur la puissance motrice du feu et sur les machines propres à développer cette puissance," which may be said to have laid the foundation of thermodynamics. Carnot pointed out that the greatest possible amount of work was to be obtained by letting the heat pass from the source to the receiver through an engine working in a strictly reversible manner, not only as regards its own internal actions but also as regards the transfer of heat to it from the source and from it to the receiver. The engine conceived by Carnot is an ideal engine every one of whose operations is reversible. Its cycle consists of these four reversible operations:

(1) Isothermal expansion during which the working substance is at the temperature of the hot source (T_1). During this operation heat is taken in reversibly from the hot source.

(2) Adiabatic expansion, during which the temperature of the working substance falls from T_1 to T_2 (the temperature of the receiver).

(3) Isothermal compression at the temperature of the receiver. During this operation heat is rejected reversibly to the receiver.

(4) Adiabatic compression by which the temperature of the working substance is raised from T_2 to T_1. This completes the cycle by bringing the substance back to the condition in which it was assumed to be at the beginning of the first operation.

In the cycle as a whole work is done by the substance, the average pressure in (1) and (2) being greater than in (3) and (4).

This cycle of operations, which is known as Carnot's Cycle, is entirely reversible. The working substance might be forced to go through it in the reversed direction, taking in heat from the cold body and giving out heat to the hot body. The transfers of heat would be exactly reversed, and at every stage the pressure and volume and temperature of the substance would be the same as when working direct. The work spent upon it would be equal to the work got from it in the direct action. Carnot's ideal engine accordingly affords a strictly reversible means of letting heat down from the hot source to the cold receiver. The reasoning by which he showed that no heat-engine can utilise heat more completely is substantially as follows.

To prove that no other heat-engine, working between the same source and receiver of heat, can do the same amount of mechanical work as a reversible engine by taking in a smaller quantity of heat, suppose there are two heat-engines R and S, one of which (R) is reversible, working between the same hot body or source of heat and cold body or receiver of heat, and each producing the same amount of mechanical work. Let Q be the quantity of heat which R takes in from the hot body. Now if R be reversed it will, by the expenditure on it of the same amount of work, give to the hot body the amount of heat it formerly took from it, namely Q. For this purpose set the engine S to drive R reversed. The work which S produces is just sufficient to drive R, and the two machines (S driving R) form together a self-acting machine unaided by any external agency. One of the two, namely S, takes heat from the hot body, and the other, R, which is reversible, gives back to the hot body the amount of heat Q. Now if S could do its work by taking less heat than Q from the hot body the hot body would on the whole gain heat. No work is being done on the system from outside, nor is any heat supplied from other sources, so whatever heat the hot body gains must come from the cold body. Therefore, if S could do as much work as the reversible engine R, with a smaller supply of heat, we should be able to arrange a purely self-acting machine through which heat would continuously pass up from a cold body to a hot body. This would be a violation of the Second Law of Thermodynamics.

The conclusion is that S cannot do the same amount of work with a smaller supply of heat than a reversible engine; or, in modern language, that no other engine can be more efficient than a reversible engine, when they both work between the same two temperatures in source and receiver.

Further, let both engines be reversible. Then the same argument shows that each cannot be more efficient than the other. Hence all reversible engines taking in and rejecting heat at the same two temperatures are equally efficient.

§ (19) Reversibility the Criterion of Perfection in a Heat-engine.—These results imply that, in the thermodynamic sense, reversibility is the criterion of what may be called perfection in a heat-engine. A reversible heat-engine is perfect in the sense that it cannot be improved on as regards efficiency: no other engine taking in and rejecting heat at the same two temperatures can obtain from the heat taken in a greater proportion of mechanical effect. Moreover, if this criterion be satisfied, it is, as regards efficiency, a matter of complete indifference what is the nature of the working substance or what, in other respects, is the mode of the engine's action.

It is therefore a complete answer to the first question in § (17) to say that the greatest amount of work that is theoretically obtainable from each unit of heat will be got by supplying the heat to an engine which works in such a way that every one of its operations is reversible.

§ (20) Efficiency of a Reversible Heat-engine.—The second question could not be answered by Carnot because in his time the doctrine of the Conservation of Energy was unknown, and it was not recognised that part of the heat disappears, as heat, in passing through the engine. Carnot realised that work is done by an engine through the agency of heat, but he did not know that it is done by the conversion of heat. It is remarkable that he was nevertheless able to conceive the idea of a reversible engine and to see that it is the most effective possible means of getting work done through the agency of heat. His argument as to this is perfectly valid though it makes no use of the First Law of Thermodynamics. It is, moreover, quite general. There is no assumption in it as to the properties of any substance, nor as to the nature of heat, nor as to the way in which temperature is to be measured. All that he assumes about the temperatures of the source and the receiver is that one is hotter than the other.

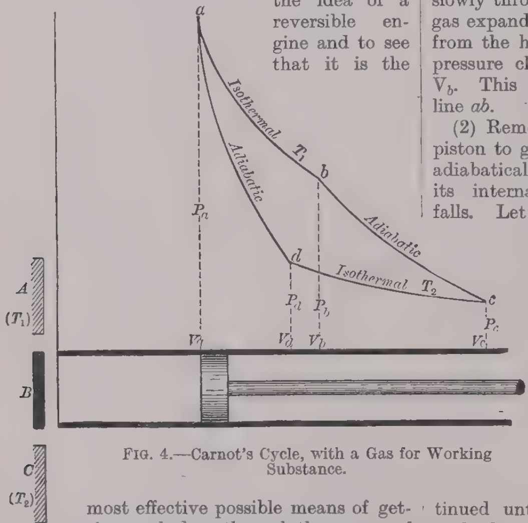

Fig. 4.—Carnot's Cycle, with a Gas for Working Substance.

But for the purpose of answering the second question we shall in the first place deal with one particular reversible heat-engine, namely a reversible engine which has for working substance a perfect gas with constant specific heat, and shall calculate its efficiency with the help of the results already obtained for such a gas.

Suppose that the cycle is performed in an ideal engine with a cylinder and piston composed of perfectly non-conducting material, except as regards the bottom of the cylinder, which is a conductor. Imagine also a hot body or indefinitely capacious source of heat A, kept always at a temperature T_1, also a perfectly non-conducting cover B, and a cold body or indefinitely capacious receiver of heat C, kept always at some temperature T_2 which is lower than T_1. It is supposed that A or B or C can be applied at will to the bottom of the cylinder. Let the cylinder contain unit quantity of the gas at temperature T_1, volume V_a, and pressure P_a to begin with. The suffixes refer to the points *a*, *b*, *c*, and *d* on the indicator diagram, *Fig.* 4. There are four successive operations:

(1) Apply A, and allow the piston to advance slowly through any convenient distance. The gas expands isothermally at T_1, taking in heat from the hot source A and doing work. The pressure changes to P_b and the volume to V_b. This operation is represented by the line *ab*.

(2) Remove A and apply B. Allow the piston to go on advancing. The gas expands adiabatically, doing work at the expense of its internal energy, and the temperature falls. Let this go on until the temperature is T_2. The pressure is then P_c, and the volume V_c. This operation is represented by the line *bc*.

(3) Remove B and apply C. Force the piston back slowly. The gas is compressed isothermally at T_2, since the smallest increase of temperature above T_2 causes heat to pass into C. Work is spent upon the gas, and heat is rejected to the cold receiver C. Let this be continued until a certain point *d* is reached, such that the fourth operation will complete the cycle.

(4) Remove C and apply B. Continue the compression, which is now adiabatic. The pressure and temperature rise, and if the point *d* has been properly chosen, when the pressure is restored to its original value P_a, the temperature will also have risen to its original value T_1. This completes the cycle.

To find the proper place at which to stop the third operation, we have, by equation (5), for the cooling during adiabatic expansion in the second operation,

$$(V_c/V_b)^{\gamma-1}=T_b/T_c=T_1/T_2,$$

and also, for the heating during the fourth operation,

$$(V_d/V_a)^{\gamma-1}=T_a/T_d=T_1/T_2.$$

Hence $V_c/V_b = V_d/V_a$,

and therefore also

$$V_c/V_d = V_b/V_a.$$

That is to say, the ratio of isothermal compression in the third operation of the cycle is to be made equal to the ratio of isothermal expansion in the first operation, in order that an adiabatic line through *d* shall complete the cycle. For brevity we shall denote the ratio of isothermal expansion or compression by *r*.

The following are the transfers of heat to and from the working gas, in the four operations of the cycle:

(1) Heat taken in from $A = ART_1 \log_e r$.
(2) No heat taken in or rejected.
(3) Heat rejected to $C = ART_2 \log_e r$.
(4) No heat taken in or rejected.

Hence the net amount of heat converted into work, being the excess of the heat taken in above the heat rejected, in the complete cycle, is

$$AR(T_1 - T_2) \log_e r,$$

and the efficiency of the cycle, namely the fraction

$$\frac{\text{Heat converted into work}}{\text{Heat taken in}},$$

is accordingly

$$\frac{AR(T_1 - T_2) \log_e r}{ART_1 \log_e r} = \frac{T_1 - T_2}{T_1}.$$

But by Carnot's principle all reversible heat-engines taking in and rejecting heat at the same two temperatures are equally efficient. Hence the expression

$$\frac{T_1 - T_2}{T_1}$$

measures the efficiency of *any* reversible heat-engine and therefore also expresses the largest fraction of the heat supplied that can possibly be converted into work by any engine whatever, operating between these limits of temperature, whatever be the working substance.

This is the measure of perfect efficiency: it is the theoretical limit beyond which the efficiency of a heat-engine cannot go. No engine can conceivably surpass this standard, and as a matter of fact any real engine falls short of it, because no real engine is strictly reversible.

The conclusion may also be stated, with equal generality, for any reversible engine, in the form

$$\frac{Q_1}{T_1} = \frac{Q_2}{T_2}, \quad . \quad . \quad . \quad . \quad (9)$$

where Q_1 is the heat taken in by the engine from the source at T_1, and Q_2 is the heat rejected by it to the receiver at T_2.

The efficiency of any heat-engine may be written

$$\frac{Q_1 - Q_2}{Q_1} \text{ or } 1 - \frac{Q_2}{Q_1},$$

whether the engine be reversible or not. In a reversible engine, or as we may call it a thermodynamically perfect engine, this becomes

$$1 - T_2/T_1.$$

In an engine which falls short of reversibility a smaller fraction of the heat supplied is converted into work, and the heat rejected is relatively larger; Q_2/T_2 is greater than Q_1/T_1.

§ (21) ABSOLUTE ZERO OF TEMPERATURE.—The zero from which T_1 and T_2 are measured is the zero of the gas thermometer, which was defined as the temperature at which the volume of the gas would vanish if the same law of expansion continued to apply. But we can now give it another meaning. Taking the above expression for the efficiency of a reversible heat-engine, $1 - T_2/T_1$, we see that if the cold receiver were at the temperature of the absolute zero (so that $T_2 = 0$) the efficiency would be equal to 1; in other words, all the heat supplied to the engine would be converted into work. It is clearly impossible to imagine a receiver colder than that, for it would make the efficiency greater than 1 and thereby violate the First Law of Thermodynamics by making the amount of work done greater than the heat supplied. Hence the zero of the perfect gas scale is also an absolute thermodynamic zero, a temperature so low that it is inconceivable on thermodynamic grounds that there can be any lower temperature (compare § (4)).

§ (22) THERMODYNAMIC SCALE OF TEMPERATURE.—It was first pointed out by Lord Kelvin that thermodynamic principles allow a scale of temperature to be defined which is independent of the properties of any particular substance, real or imaginary. In the foregoing argument we have based the measurement of temperature on the properties of a perfect gas, taking a scale in which the degrees, that is to say the intervals of temperature which are called equal, correspond to equal amounts of expansion on the part of a perfect gas kept at constant pressure. Using this scale we have seen that a reversible engine which works between the limits T_1 and T_2, and takes in any quantity of heat Q_1 at T_1, rejects at T_2 a quantity Q_2 equal to Q_1T_2/T_1, and has an efficiency equal to $(T_1 - T_2)/T_1$.

Now imagine that the heat Q_2 which is rejected by this engine forms the supply of a second reversible engine taking in heat at T_2 and rejecting heat at a lower temperature T_3, such that the interval of temperature through which it works $(T_2 - T_3)$ is the same as the interval through which the first engine works

$(T_1 - T_2)$. Call each of these intervals ΔT. Let the heat Q_3 rejected by this second engine pass on to form the supply of a third reversible engine, working through an equal interval ΔT and rejecting heat Q_4 to a fourth reversible engine, and so on. We imagine a series of engines, every one of which is reversible, each passing on its rejected heat to form the supply of the next engine in the series, and each working through the same number of degrees on the perfect gas thermometer, ΔT. The efficiencies of the successive engines are

$$\Delta T/T_1,\ \Delta T/T_2,\ \Delta T/T_3,\ \text{etc.},$$

The amounts of heat supplied to them are

$$Q_1,\ Q_2 = Q_1T_2/T_1,\ Q_3 = Q_2T_3/T_2 = Q_1T_3/T_1,\ \text{etc.}$$

Multiply in each case the heat taken in by the efficiency to find the amount of work done by each engine in the series, and we find that the amount of work done is the same for all the engines, namely

$$\frac{Q_1\Delta T}{T_1}.$$

Accordingly, we might define the interval of temperature for each engine, without reference to a perfect gas or to any other thermometric substance, as that interval which makes every engine in the series do the same amount of work; and if we do so we get a scale of temperature which is identical with the scale of the perfect gas thermometer. Thus we not only get a thermodynamic scale of temperature, but we find that its divisions agree exactly with those which correspond to equal amounts of expansion on the part of a perfect gas.

The above method of obtaining a thermodynamic scale of temperature may be put thus: Starting from any arbitrary condition of temperature at which we may imagine heat to be supplied, let a series of intervals be taken, such that equal amounts of work will be done by every one of a series of reversible engines, each working with one of these intervals for its range, and each handing on to the engine below it the heat which it rejects, so that the heat rejected by the first forms the supply of the second, and so on. Then call these intervals of temperature equal. What the above proof shows is that the intervals so defined to be equal are also equal when measured on the scale of the perfect gas-thermometer; in other words, the thermodynamic scale and the perfect gas scale coincide at every step. The symbol T, which up to this point has stood for temperature measured on the scale of a perfect gas thermometer, may now be interpreted as representing absolute temperature on the thermodynamic scale, and is to be so understood in what follows.

§ (23) REVERSIBLE CYCLE IN WHICH HEAT IS TAKEN IN AT VARIOUS TEMPERATURES.—In Carnot's cycle it was assumed that there was only one source and one receiver of heat. All the heat that was taken in was taken in at T_1; all the heat that was rejected was rejected at T_2. But an engine may take in heat in stages, at more temperatures than one, and may also reject heat in stages. With regard to every quantity of heat so taken in the result still applies that the greatest fraction of it that can be converted into work is represented by the difference between its temperatures of reception and rejection, divided by the absolute temperature of reception. And this is the fraction that will be converted into work provided the processes within the engine are reversible.

Thus let Q_1 represent that part of the whole supply of heat which is taken in at T_1 and let Q_2 represent what is taken in at some other temperature T_2, Q_3 what is taken in at T_3, and so on; and let T_0 be the temperature at which the engine rejects heat; then the whole work done, if the engine be reversible, is

$$W = \frac{Q_1(T_1 - T_0)}{T_1} + \frac{Q_2(T_2 - T_0)}{T_2} + \frac{Q_3(T_3 - T_0)}{T_3} + \text{etc.}$$

We have taken here, for simplicity of statement, a single temperature of rejection T_0.

Another way of putting the matter has a wider application. Let the engine as before take in quantities of heat represented by Q_1, Q_2, Q_3, etc., at T_1, T_2, T_3, and let it reject heat at T', T'', T''', etc., the quantities rejected being respectively Q', Q'', Q''', etc. Then by the principle that in a reversible cycle the heat rejected is to the heat taken in as the absolute temperature of rejection is to the absolute temperature of reception, we have

$$\frac{Q'}{T'} + \frac{Q''}{T''} + \frac{Q'''}{T'''} + \ \ldots \ = \frac{Q_1}{T_1} + \frac{Q_2}{T_2} + \frac{Q_3}{T_3} + \ \ldots,$$

from which $$\Sigma\frac{Q}{T} = 0,$$

when the summation is effected all round the reversible cycle. In this summation heat taken in is reckoned as positive and heat rejected as negative. If the cycle be not reversible, the heat rejected will be relatively greater, and therefore, for a non-reversible cycle, $\Sigma(Q/T)$ will be a negative quantity.

Some of the processes may be such that changes of temperature are going on continuously while heat is being taken in or given out, and if so the reception or rejection of heat cannot be divided into a limited number of steps, as has been done above. But the equation for a reversible cycle may be adapted to the most general case by writing it

$$\int \frac{dQ}{T} = 0, \quad . \quad . \quad . \quad (10)$$

integration being performed round the whole cycle.

This holds for any internally reversible cycle. It means that when a substance has passed through any series of reversible changes which cause it to return to its initial state, the quantities of heat which it has taken in and given out are so related to the temperature of the substance at each stage as to make this integral vanish for the cycle as a whole. When the cycle is not reversible, $\int dQ/T$ is a negative quantity, because the amount of heat rejected is relatively larger than when the cycle is reversible.

§ (24) Entropy.—The Entropy of a substance is a function of its state which is most conveniently defined by reference to the heat taken in or given out while the state of the substance undergoes change in a reversible manner. In any such change the heat taken in or given out, divided by the absolute temperature of the substance, measures the change of entropy. Thus if a substance which is either expanding reversibly or not expanding at all takes in heat δQ when its temperature is T, its entropy increases by the amount $\delta Q/T$. We shall see that the entropy of any substance in a definite state is a definite quantity, which has the same value when the substance comes back again to the same state after undergoing any changes. To give the entropy a numerical value we must start from some arbitrary point where, for convenience of reckoning, the entropy is taken as zero. We are concerned only with *changes* of entropy, and consequently it does not matter, except for convenience, what zero state is chosen for the purpose of calculating the entropy.

Starting then from any suitable zero, let the substance undergo any reversible change of state. Let each element δQ of the heat taken in be divided by T, which is the absolute temperature of the substance when δQ was being taken in. Then the sum

$$\Sigma \frac{\delta Q}{T}$$

measures the entropy of the substance, on the assumption that no irreversible change of state has occurred during the process. We shall denote the entropy of any substance by ϕ. If the temperature is changing continuously while heat is being taken in, the change of entropy from any state a to any other state b is

$$\phi_b - \phi_a = \int_a^b \frac{dQ}{T}, \quad . \quad . \quad . \quad (11)$$

provided there is no irreversible action within the substance during its change of state.

Thus, if we write $\delta\phi$ for the increment of entropy, we have

$$\delta\phi = \frac{\delta Q}{T},$$

whence $$T = \frac{dQ}{d\phi}$$

ultimately, an expression which has been suggested as a definition of temperature.

The definition of the entropy of a substance as a quantity which changes by the amount $\int_a^b \frac{dQ}{T}$ while the substance passes, by a reversible process, from any state a to any other state b, is consistent with the fact that the entropy is a definite function of the state of the substance, which means that it has only one possible value so long as the substance is in the same state. To prove this we must show that the same value is obtained for the entropy no matter what reversible operation be followed in passing from one state to the other: in other words, that $\int_a^b \frac{dQ}{T}$ is the same for all reversible operations by which a substance might pass from state a to state b. Consider any two reversible ways of passing from state a to state b. If we suppose one of them to be reversed, the two together will form a complete cycle which is completely reversible, and for which consequently the cyclic integral $\int \frac{dQ}{T} = 0$. Hence $\int_a^b \frac{dQ}{T}$ for one of them must have the same value as for the other. It is therefore a matter of indifference, in the reckoning of entropy, by what "path" or sequence of changes the substance passes from a to b provided it be a reversible path: starting from any zero state the reckoning of the entropy in a given state will always give the same value, which shows that the entropy is simply a function of the actual state and does not depend on previous conditions.

It is chiefly because the entropy of a substance is a definite function of the state, like the temperature, or the pressure, or the volume, or the internal energy, that the notion of entropy is important in thermodynamic theory. The entropy of a substance is usually reckoned per unit of mass, and numerical values of it reckoned in this sense are given in tables of the properties of steam and of the other substances which are used in heat-engines and refrigerating machines.

But we may also reckon the entropy of a body as a whole when the state of the body is fully known, or the change of entropy which a body undergoes as a whole when it takes in or gives out heat. And we may also reckon the total entropy of a system of bodies by adding together the entropies of the several bodies that make up the system.

As a simple illustration of the uses to which the idea of entropy may be put, consider the changes of entropy which a substance undergoes when it is taken through Carnot's cycle (§ (18)). All four operations are reversible. In the first, which is isothermal expansion at T_1, the entropy of the substance increases by the amount Q_1/T_1, where Q_1 is the amount of heat taken in from the hot source. In the second operation no heat is taken in or given out and there is no change of entropy. In the third operation a quantity of heat Q_2 is rejected at T_2; the entropy of the substance accordingly falls by the amount Q_2/T_2. In the fourth operation there is again no transfer of heat and no change of entropy. It is only in the first and third operations that changes of entropy occur. Moreover, they are equal, for $Q_1/T_1=Q_2/T_2$, which shows that the substance has the same entropy as at first, when it has returned to the original state.

During the first operation, while it was taking in heat its entropy rose from the initial value, which we may call ϕ_a, to a value ϕ_b, such that $\phi_b=\phi_a+Q_1/T_1$. During the third operation, while the substance was rejecting heat, its entropy fell again from ϕ_b to ϕ_a, and $\phi_a=\phi_b-Q_2/T_2$. Taking the cycle as a whole, the thermal equivalent of the work done by the substance is Q_1-Q_2, and is accordingly equal to $(T_1-T_2)(\phi_b-\phi_a)$.

Further, the source of heat has lost an amount of entropy equal to Q_1/T_1, and the receiver has gained an equal amount of entropy, namely Q_2/T_2. We may therefore regard the reversible engine of Carnot as a device which transfers entropy from the hot source to the cold receiver without altering the amount of the entropy so transferred. The amount of heat alters in the process of transfer, for an amount of heat, Q_1-Q_2, disappears, which is the thermal equivalent of the work done; but the amount of entropy in the system as a whole does not change.

It is instructive to represent the changes of entropy in a Carnot cycle by means of a diagram, the two co-ordinates of which are the entropy of the working substance and its temperature (*Fig.* 5).

The first operation (isothermal expansion) is represented by *ab*, a straight line drawn at the level of temperature T_1: during this operation the entropy of the substance rises from ϕ_a to ϕ_b. This is followed by adiabatic expansion *bc*, during which the temperature falls, but the entropy does not change. Then comes isothermal compression *cd* at temperature T_2, during which the entropy returns to the initial value. Finally adiabatic compression *da* completes the cycle.

The area of the closed figure *abcd* measures (in heat units) the work done during the cycle. The area *mabn*, measured to the base line, which is the absolute zero of temperature, is the heat taken in from the source. The area *mdcn* is the heat rejected to the receiver. These figures are rectangles.

All this is true whatever be the working substance. The diagram applies to any engine going through the reversible cycle of Carnot whether it use a gas or any other substance.

It follows from the definition of entropy given above that when a substance is expanded or compressed in an adiabatic manner its entropy does not change. An adiabatic line is consequently a line of constant entropy, or, as it is sometimes called, an *isentropic* line. We might accordingly define the entropy of a substance as that characteristic of the substance which does not change in adiabatic expansion or compression, and this definition would be consistent with the method of reckoning entropy already described.

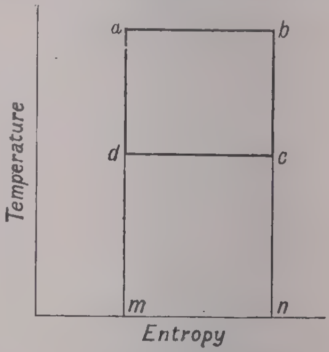

FIG. 5.—Entropy-temperature Diagram for Carnot's Cycle.

It is only in a reversible process that the change of entropy of a substance is to be determined by reference to the heat it takes in or gives out. When a substance expands in an irreversible manner, as by passing through a throttle-valve from a region of high pressure to a region of lower pressure, it gains entropy although no heat is taken in. Work is then done on each portion of the substance, by the substance behind, in giving it energy of motion as it passes through the valve, and the energy of motion then acquired is frittered down into heat as the motion subsides through internal friction. Expansion through a throttle-valve may be regarded as consisting of two stages. The first stage is a more or less adiabatic expansion during which the substance does work in setting a portion of itself in motion: the second stage is the loss of this motion and the consequent generation within the substance itself of an equivalent amount of heat. There is accordingly a gain of entropy, which occurs because the process as a whole is not reversible.

When a substance undergoes any change, and the final state is known, it is in general easy to calculate the entropy corresponding to that state, by considering the amount by which the entropy would have changed if the substance had come to that state by a reversible operation, for which $\int dQ/T$ measures the change.

When a substance has passed through any complete cycle of operations its entropy is the same at the end as at the beginning, for the original state has been restored in all respects. This is true of an irreversible cycle as well as of a reversible cycle. But for an irreversible cycle $\int dQ/T$ does not vanish. It has a negative value and it does not measure change of entropy, for it is only in an internally reversible action that the change of entropy is dQ/T. If any step in the action of a substance, during which it takes in a quantity of heat dQ, is irreversible, the increase of entropy during that step is greater than dQ/T.

§ (25) SUM OF THE ENTROPIES IN A SYSTEM.—It is instructive to inquire how the sum of the entropies of all parts of a thermodynamic system is affected when we include not only the working substance but also the source of heat and the sink or receiver to which heat is rejected. Consider a cyclic action in which the working substance takes in a quantity of heat Q_1 from a source at T_1 and rejects a quantity Q_2 to a sink at T_2. When the cycle is completed the source has lost entropy to the amount Q_1/T_1: the working substance has returned to the initial state, and therefore has neither gained nor lost entropy: the sink has gained entropy to the amount Q_2/T_2. If the cycle is a reversible one, $Q_1/T_1=Q_2/T_2$, and therefore the system taken as a whole, consisting of source, substance, and sink, has suffered no change in the sum of the entropies of its parts. But if the cycle is not reversible, the action is less efficient, Q_2 bears a larger proportion to Q_1 and Q_2/T_2 is greater than Q_1/T_1. Hence in an irreversible action the sum of the entropies of the system as a whole becomes increased. This conclusion has a very wide application: it is true of any system of bodies in which thermal actions may occur. It may be expressed in general terms by saying that when a system undergoes any change, the sum of the entropies of the bodies which take part in the action remains unaltered if the action is reversible, but becomes increased if the action is not reversible. No real action is strictly reversible, and hence any real action occurring within a system of bodies has the effect of increasing the sum of the entropies of the bodies which make up the system. This is a statement, in terms of entropy, of the principle that in all actual transformations of energy there is what Lord Kelvin called a universal tendency towards the dissipation of energy.[1] Any system, left to itself, tends to change in such a manner as to increase the aggregate entropy, which is calculated by summing up the entropies of all the parts. The sum of the entropies in any system, considered as a whole, tends towards a maximum, which would be reached if all the energy of the system were to take the form of uniformly diffused heat; and if this state were reached no further transformations would be possible. Any action within the system, by increasing the aggregate entropy, brings the system a step nearer to this state, and to that extent diminishes the availability of the energy in the system for further transformations.

This is true of any limited system. Applied to the universe as a whole, the doctrine suggests that it is in the condition of a clock once wound up and now running down. As Clausius, to whom the name entropy is due, has remarked, "the energy of the universe is constant: the entropy of the universe tends towards a maximum."

An extreme case of thermodynamic waste occurs in the direct conduction of a quantity of heat Q from a hot part of the system, at T_1, to a colder part at T_2, no work being done in the process. The hot part loses entropy by the amount Q/T_1: the cold part gains entropy by the amount Q/T_2, and as the latter is greater there is an increase in the aggregate quantity of entropy in the system as a whole.

§ (26) ENTROPY-TEMPERATURE DIAGRAMS.—We shall now consider, in a more general manner, diagrams in which the action of a substance is exhibited by showing the changes of its entropy in relation to its temperature. Such a diagram forms an interesting and often useful alternative to the pressure-volume or indicator diagram. One example, namely the entropy-temperature diagram for a Carnot cycle, has already been sketched in *Fig.* 5.

Let $d\phi$ be the small change of entropy which a substance undergoes when it takes in the small quantity of heat dQ at any temperature T, it being assumed that in the process the substance undergoes only a reversible change of state. Then, by the definition of entropy

$$d\phi=\frac{dQ}{T},$$

whence $$Td\phi=dQ$$

and $$\int Td\phi=\int dQ,$$

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[1] *Mathematical and Physical Papers*, i. 511.

the integration being performed between any assigned limits. Now if the curve be drawn with T and ϕ for co-ordinates, $\int T d\phi$ is the area under the curve. This by the above equation is equal to $\int dQ$ which is the whole amount of heat taken in while the substance passes through the states which that portion of the curve represents. Let *ab*, *Fig.* 6, be any portion of the curve of ϕ and T. The area of the cross-hatched strip, whose breadth is $\delta\phi$ and height T, is $T\delta\phi$ which is equal to δQ, the heat taken in during the small change $\delta\phi$.

The whole area *mabn*, or $\int T d\phi$ between the limits *a* and *b*, is the whole heat taken in while the substance changes in a reversible manner from the state represented by *a* to the state represented by *b*. Similarly, in changing reversibly from state *b* to state *a* by the line *ba* the substance rejects an amount of heat which is measured by the area *nbam*. The base line *ox* corresponds to the absolute zero of temperature.

When an entropy-temperature curve is drawn for any complete cycle of changes it forms a closed figure, since the substance returns to its initial state. To find the area of the figure we have to integrate throughout the complete cycle, and provided there has been no irreversible action within the substance

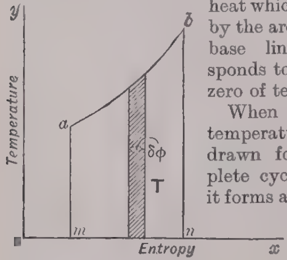

FIG. 6.—Entropy-temperature Curve.

$$\int T d\phi = Q_1 - Q_2,$$

Q_1 being the heat taken in and Q_2 the heat rejected. But the difference between these quantities of heat is the heat converted into work, hence

$$\int T d\phi = AW,$$

when the integration extends round a complete cycle. Thus an entropy-temperature diagram, so long as it represents changes of state all of which are reversible, but not otherwise, has the important property, in common with a pressure-volume diagram, that the enclosed area measures the work done in a complete cycle.

But the entropy-temperature diagram has an advantage not possessed by the pressure-volume diagram, in that it exhibits not only the work done, but also the heat taken in and the heat rejected, by means of areas under the curves. An illustration of this has been given in speaking of the Carnot cycle (§ (24)).

§ (27) PERFECT ENGINE USING REGENERATOR.—Besides the cycle of Carnot there is (theoretically) one other way in which an engine can work between a source and receiver, so as to make the whole action reversible, and thereby transform into work the greatest possible proportion of the heat that is supplied. Suppose there is, as part of the engine, a body (called a "regenerator") into which the working substance can temporarily deposit heat, while the substance falls in temperature from the upper limit T_1 to the lower limit T_2, and suppose further that this is done in such a manner that the transfer of heat from the substance to the regenerator is reversible. This condition implies that there is to be no sensible difference in temperature between the working substance and the material of the regenerator at any place where they are in thermal contact. Then when we wish the substance to pass back from T_2 to T_1 we may reverse this transfer, and so recover the heat which was deposited in the regenerator. This alternate storing and restoring of heat serves instead of adiabatic expansion and compression to make the temperature of the working substance pass reversibly from T_1 to T_2 and from T_2 to T_1 respectively. It enables the temperature of the substance to fall to T_2 before heat is rejected to the receiver, and to rise to T_1 before heat is taken in from the source.

This idea is due to the Rev. Robert Stirling, who in 1827 designed an engine to give it effect. For the present purpose it will suffice to describe the regenerator as a passage (such as a group of tubes) through which the working fluid can travel in either direction, whose walls have a very large capacity for heat, so that the amount alternately given to or taken from them by the working fluid causes no more than an insensible rise or fall in their temperature. The temperature of the walls at one end of the passage is T_1, and this falls continuously down to T_2 at the other end. When the working fluid at temperature T_1 enters the hot end and passes through, it comes out at the cold end at temperature T_2, having stored in the walls of the regenerator a quantity of heat which it will pick up again when passing through in the opposite direction. During the return journey of the working fluid through the regenerator from the cold to the hot end its temperature rises from T_2 to T_1 by picking up the heat which was deposited when the working fluid passed through from the hot end to the cold. The process is strictly reversible, or rather would be so if the regenerator had an unlimited capacity for heat, if no conduction of heat took place along its walls from the hotter parts towards the cold end, and if there were no loss by conduction or radiation from its external surface. A regenerator satisfying these conditions is of course an ideal impossible to realise in practice.

Using air as the working substance, and employing his regenerator, Stirling made an engine which, allowing for practical imperfections, is the earliest example of a reversible engine. The cycle of operations in Stirling's engine was substantially this (in describing it we shall treat air as a perfect gas):

(1) Air, which had been heated to T_1 by passing through the regenerator, was allowed to expand isothermally through a volume-ratio r, taking in heat from a furnace and raising a piston. Heat taken in (per unit quantity of air) $= RT_1 \log_e r$.

(2) The air was caused to pass through the regenerator from the hot to the cold end, depositing heat and having its temperature lowered to T_2, without change of volume. Heat stored in regenerator $= K_v(T_1 - T_2)$. The pressure of course fell in proportion to the fall in temperature.

(3) The air was then compressed isothermally at T_2, through the same ratio r to its original volume, in contact with a receiver of heat. Heat rejected $= RT_2 \log_e r$.

(4) The air was again passed through the regenerator from the cold to the hot end, taking up heat and having its temperature raised to T_1. Heat restored by the regenerator $= K_v(T_1 - T_2)$. This completed the cycle.

The efficiency of this ideal cycle is

$$\frac{RT_1 \log_e r - RT_2 \log_e r}{RT_1 \log_e r} = \frac{T_1 - T_2}{T_1}.$$

The indicator diagram of the action is shown in *Fig.* 7. Stirling's regenerative engine is theoretically important because it is typical of the only mode, other than Carnot's plan of adiabatic expansion and adiabatic compression, by which the action of a heat-engine can be made reversible.

A modified form of regenerative engine was devised later by Ericsson, who kept the pressure instead of the volume constant while the working substance passed through the regenerator, and so got an indicator diagram made up of two isothermal lines and two lines of constant pressure.

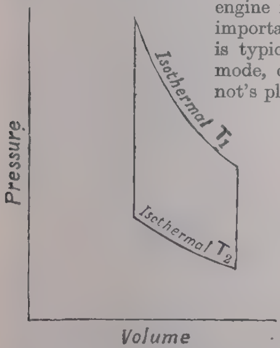

FIG. 7.—Ideal Indicator Diagram of Air-engine with Regenerator (Stirling).

The entropy-temperature diagram of any regenerative engine is of the type shown in *Fig.* 8.

The isothermal operation of taking in heat at T_1 is represented by *ab*; *bc* is the cooling of the substance from T_1 to T_2 in its passage through the regenerator, where it deposits heat: *cd* is the isothermal rejection of heat at T_2; and *da* is the restoration of heat by the regenerator while the substance passes through it in the opposite direction, by which the temperature of the substance is raised from T_2 to T_1. Assuming the action of the regenerator to be ideally perfect, *bc* and *ad* are precisely similar curves whatever be their form. The area of the figure is then equal to the area of the rectangle which would represent the ordinary Carnot cycle (*Fig.* 5). The equal areas *pbcq* and *ndam* measure the heat stored and restored by the regenerator.

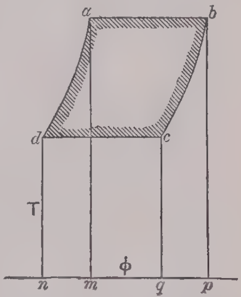

FIG. 8.—Entropy-temperature Diagram of Perfect Engine using a Regenerator.

§ (28) STATES OF AGGREGATION OR PHASES.[1]—In the foregoing sketch of general principles the only substances whose properties were discussed were imaginary ones, namely perfect gases. We have now to treat of real substances, such as may exist in three states of aggregation, solid, liquid, and gaseous. These states are now generally called phases. Some substances, such as sulphur or iron, have more than one solid phase. We are mainly concerned with the liquid and gaseous phases, in either of which the substance is spoken of as a fluid. The working fluid in an engine is often a mixture of the same substance in the two phases of liquid and vapour; but in some stages of the action it may consist entirely of liquid, in others entirely of vapour. The vapour of a substance may be either saturated or superheated. A vapour mixed with its liquid, and in equilibrium with it, must be saturated. Any attempt to heat the mixture would result in more of the liquid turning into saturated vapour. But when a vapour has been removed from its liquid it may be heated to any extent, thereby becoming superheated. Thus when steam is formed in a boiler it is saturated as it leaves the water, but it may be superheated on its way to the engine by passing through hot pipes which cause its temperature to rise above that of the boiler.

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[1] See article "Phase Rule."

A gas such as hydrogen, or oxygen, or nitrogen, is a superheated vapour which can be reduced to the saturated condition by greatly lowering its temperature. At any one pressure the saturated vapour of a substance can have but one temperature: the superheated vapour at the same pressure may have any temperature higher than that.

In the change of phase from solid to liquid, and again in the change from liquid to vapour, heat is taken in, though the substance does not rise in temperature while the change is going on. The heat so taken in was said in the phraseology of old writers to become latent, and the name Latent Heat is still applied to it. Thus the heat taken in by unit mass of a substance in passing, without change of pressure, from the solid to the liquid phase is called the latent heat of the liquid, and the heat taken in by unit mass in passing, without change of pressure, from liquid to vapour is called the latent heat of the vapour. The latent heat of water, for example, is 80 thermal units, which means that unit mass of ice takes in 80 thermal units while it melts. If we assume the pressure to be one atmosphere, this happens at the temperature which is taken for the lower fixed point (0° C.) in graduating a thermometer.

Changes of phase from solid to liquid or from liquid to vapour, in pure substances, are reversible. Under the same condition as to pressure, for example, water vaporises, and steam is condensed, at the same temperature, and the same quantity of heat is taken in in the one process as is given out in the other.

The temperature at which a solid melts is only slightly affected by the pressure, and this is also true of the latent heat of melting.

At a pressure of one atmosphere water boils at the temperature which is taken for the upper fixed point of the thermometer (namely 100° C.); and the latent heat of the vapour is 539·3 thermal units. We shall see immediately that the temperature at which any fluid changes from liquid to vapour, and also the amount of heat taken in during the change, depend greatly on the pressure. At high pressures the temperature of boiling is higher and the amount of latent heat is less.

§ (29) FORMATION OF A VAPOUR UNDER CONSTANT PRESSURE.—The properties of a vapour are most conveniently stated by referring in the first instance to what happens when it is formed *under constant pressure.* This is substantially the process which occurs in the boiler of a steam-engine when the engine is at work. To fix the ideas we may suppose that the vessel in which vapour is to be formed is a long upright cylinder fitted with a frictionless piston which may be loaded so that it exerts a constant pressure on the fluid below. Let there be, to begin with, at the foot of the cylinder, a quantity of the liquid (which for convenience of statement we shall take as one unit of mass), and let the piston rest on the surface of the liquid with a pressure P, expressed per unit of area. Let heat now be applied to the bottom of the cylinder. As heat enters it produces the following effects in three stages:

(1) The temperature of the liquid rises until a certain temperature T_s is reached, at which vapour begins to be formed. The value of T_s depends on the particular pressure P which the piston exerts. Until the temperature T_s is reached there is nothing but liquid below the piston.

(2) Vapour is formed, more heat being taken in. The piston, which is supposed to continue to exert the same constant pressure, rises. No further increase of temperature occurs during this stage, which continues until all the liquid is converted into vapour. During this stage the vapour which is formed is *saturated.* The volume which the piston encloses at the end of this stage—the volume, namely, of unit mass of saturated vapour at pressure P and consequently at temperature T_s—will be denoted by V_s.

(3) If more heat be allowed to enter after all the liquid has been converted into steam, the volume will increase and the temperature will rise. The vapour is then superheated: its temperature is above the temperature of saturation.

The difference between saturated and superheated vapour may be expressed by saying that if some of the liquid (at the temperature of the vapour) be mixed with the vapour, part of that liquid will be evaporated if the vapour is superheated, but none will be evaporated if the vapour is saturated. A vapour in contact with its liquid, and in thermal equilibrium with it, is saturated. When saturated its properties differ considerably, as a rule, from those of a perfect gas, but when superheated they approach those of a perfect gas more and more closely the farther the process of superheating is carried, that is to say, the more the temperature is raised above T_s, the temperature of saturation corresponding to the given pressure P.

In the first stage of this process the substance is wholly in the condition of liquid which is being heated from the initial temperature to T_s, the temperature at which the second stage begins. If, for example, the fluid is water, during this first stage the heat taken in (per unit quantity of the water) is approximately equal to one thermal unit for each degree by which the temperature of the water rises. It would be exactly equal to that if the specific heat of water were constant and equal to unity, but this is not the case. At about 30° C. the specific heat of water is less than unity; it passes a minimum value thereabouts of 0·9967, and then increases, becoming appreciably greater than unity at such temperatures as are found in steam boilers.

During this first stage, while the substance is still liquid, nearly all the heat that is taken in goes to increase the stock of internal energy. There is scarcely any external work done, for the volume is only slightly increased. Thus, for example, in heating water from 0° C. to 200° C. (under a pressure of 225·24 pounds per square inch) the volume of the water changes from 0·0160 cubic feet per pound to 0·0185. The external work done during this heating is therefore $225{\cdot}24 \times 144 \times 0{\cdot}0025$ or 81 foot-pounds. This is equivalent to barely 0·06 thermal unit (pound-calory), and is negligible in comparison with the quantity of heat that is taken in, which amounts to 203·2 units.[1]

In the second stage, the liquid changes into saturated vapour without change of temperature. The heat that is taken in during this stage constitutes what is called the latent heat of the vapour. We shall denote it by L. At the end of the second stage the substance contains no liquid; it is spoken of as dry saturated vapour: at any earlier point the substance is a wet mixture consisting partly of saturated vapour and partly of liquid.

The latent heat of a vapour may be defined as the amount of heat which is taken in by unit mass of the liquid while it all changes into saturated vapour under constant pressure, the liquid having been previously heated up to the temperature at which the vapour is formed.

A considerable part of the heat taken in during this process is spent in doing external work, since the substance expands against the constant pressure P. It is only the remainder of the so-called latent heat L that can be said to remain in the fluid and to constitute an addition to its stock of internal energy. The amount spent in doing external work during the second stage is

$$AP(V_s - V_w),$$

where V_s is the volume of the saturated vapour and V_w is the volume of the liquid at the same temperature and pressure. As previously, A is the factor for converting units of work into thermal units. The excess of L above this quantity measures the amount by which the internal energy increases during the second stage.

Thus, for instance, when water at 200° C. and a pressure of 225·24 pounds per square inch is converted into steam, the volume changes from 0·0185 cubic feet to 2·0738; 467·41 thermal units are taken in, of which 47·61 units are spent in doing external work and 419·8 units go to increase the stock of internal energy.

In the two stages together the whole amount of external work done is to be found by taking the whole increase of volume and multiplying it by the pressure. The whole increase of internal energy is equal to the whole amount of heat taken in less the equivalent of the external work done. This is only a particular example of the general principle that when any substance expands in any manner, taking in heat and doing work, the heat taken in is equal to the work done plus the increase of internal energy. In the case here considered the action is going on under constant pressure, but the statement applies to any change of state whatever, whether or no the substance changes in phase during the operation.

§ (30) THE INTERNAL ENERGY OF A FLUID.—No matter what changes a substance may undergo, its internal energy will return to the same value when the substance returns to the same condition in all respects. In other words, the internal energy is a function of the actual state of the substance and is independent of the way in which that state has been reached.

We have no means of measuring the total stock of internal energy in a substance, and can deal only with changes in the stock. But by taking some arbitrary starting-point as a zero from which the internal energy E is reckoned we can give E a numerical value for any other state of the substance. That value really expresses the difference from the internal energy in the zero state. The usual convention is to write $E=0$ when the substance is in the liquid condition at a temperature of 0° C., and at a pressure equal to the vapour-pressure corresponding to that temperature. We may call this, for brevity, the zero state of the substance.

Following this convention we take $E=0$ for water at 0° C. The value of E for saturated water-vapour at 0° C. will then be 564·21 thermal units.

Values of E for saturated steam at various temperatures are given in the Steam Tables. It will be seen that they increase slowly with the temperature.

§ (31) THE "TOTAL HEAT" OF A FLUID.—We come now to another function of the state of any substance, a function which is of very great use in thermodynamic calculations. It is generally called the "Total Heat," and is represented[2] here by the letter I.

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[1] For numerical values of the thermodynamical quantities relating to steam, reference should be made to *The Callendar Steam Tables* (E. Arnold, 1915), or to *Properties of Steam Callendars* (E. Arnold, 1920).

[2] Callendar in his *Steam Tables* and in his book uses H to represent this function.

The "total heat" I is defined for any state of the substance by the equation

$$I = E + APV. \quad . \quad . \quad . \quad (12)$$

That is to say, I is equal to the sum of the internal energy and the external work which would be done if the substance could be imagined to start from no volume at all and to expand to its actual volume, under a constant pressure equal to its actual pressure. Since the pressure, volume, and internal energy are all functions of the actual state, I is also a function of the actual state: its value is independent of how the state has been reached. In steam, for example, although the heat taken in during formation depends on how the steam is formed, the "total heat" I depends only on the final condition. The total heat can be calculated for any condition of a substance, whether in the state of liquid or of saturated or superheated vapour. It is measured in thermal units per pound. Values of the total heat of saturated steam, and also of water under saturation-pressure at various temperatures, are given in Steam Tables. The total heat of steam increases progressively with the temperature rather more rapidly than does the internal energy.

It follows from the definition of I that in the zero state of any substance, at which E is reckoned to be zero, I is not equal to zero, but has a small positive value depending on the volume of the liquid and its pressure at that state. Since E is then zero I is equal to AP_0V_0 where P_0 is the pressure at the zero state, namely, the vapour-pressure at 0° C., and V_0 is the volume of the liquid at 0° C. and pressure P_0. For water this quantity AP_0V_0 is quite negligible, amounting as it does to 0·000146 thermal unit. For carbonic acid it is about 1 thermal unit, for ammonia and sulphurous acid it is much less.

An important property of the function I is that when any substance is heated under constant pressure the change of I is equal to the amount of heat taken in. To prove this, let Q be the amount of heat taken in while the substance expands under constant pressure P from a state in which the volume is V_1 and the internal energy is E_1 to another state in which the volume is V_2 and the internal energy is E_2. Then the amount of external work done is $P(V_2 - V_1)$ and, by the conservation of energy,

$$Q = E_2 - E_1 + AP(V_2 - V_1),$$

which may be written

$$Q = E_2 + APV_2 - (E_1 + APV_1)$$

or

$$Q = I_2 - I_1,$$

where I_1 is the total heat in the first state and I_2 is the total heat in the second state.

It follows that while a liquid is being converted into vapour, under constant pressure, the total heat I increases in proportion to the amount of vapour that is formed. At any intermediate stage in the process, let q represent the fraction that is vaporised and $1 - q$ the fraction that is still liquid, then the total heat of the mixture is

$$qI_s + (1 - q)I_w,$$

which may be written

$$I_w + qL.$$

Similarly, while a vapour is being condensed under constant pressure I becomes less by an amount which is measured by the heat given out. This is proportional, at any intermediate stage, to the fraction then condensed.

It follows also that the total heat of a superheated vapour exceeds that of the saturated vapour at the same pressure by an amount which is equal to the heat that would be taken in if the process of superheating were carried out at constant pressure. It is, however, important to recognise that the total heat of a superheated vapour at any assigned state of the fluid, as to pressure and temperature, is a definite quantity which depends simply on the actual state and is independent of how the process of superheating may have been effected, or of what states the substance may have passed through before the actual state is reached. This is also true of the internal energy and the entropy, both of which, like the total heat, are functions of the actual state.

§ (32) CONSTANCY OF THE TOTAL HEAT IN A THROTTLING PROCESS.—An important property of the function I, in any substance, is that it does not change when the substance passes through a valve or other constricted opening, such as the porous plug of the Joule-Thomson experiment, by which it becomes throttled or "wire-drawn," so that its pressure drops. A practical instance of this kind of action occurs when steam passes through a partially closed orifice or "reducing valve." Eddies are formed in the fluid as it rushes through the constricted opening, and the energy expended in forming them is frittered down into heat as they subside.

To prove that I is constant in such an operation we shall consider what happens while a unit quantity of the substance passes through a constricted opening (as in *Fig.* 9), and, to make the matter clear, imagine this unit quantity to be separated from the rest of the substance by two frictionless pistons, one of which (A) slides in the pipe that leads to the constriction, and the other (B) slides in the pipe that leads away from it. On one side, as the substance comes up, let its pressure be P_1, volume V_1, and internal energy E_1.

On the other side, after passing the constriction, let its pressure be P_2, volume V_2, and internal energy E_2. As each portion approaches the constriction work is done upon it by the substance behind pushing in the imaginary piston A, and the amount of that work done while unit quantity is passing is P_1V_1. After each portion has passed the constriction it does work upon the substance in front by pushing out the imaginary piston B, and the amount of that work is P_2V_2 for the whole unit quantity. Any excess of the work done by the substance on piston B over the work done upon it by piston A must be supplied by a reduction in its stock of internal energy. Hence

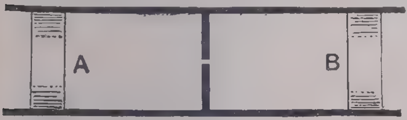

FIG. 9.

$$AP_2V_2 - AP_1V_1 = E_1 - E_2,$$

from which

$$E_2 + AP_2V_2 = E_1 + AP_1V_1$$

or

$$I_2 = I_1.$$

Thus the total heat does not change in consequence of the throttling. The imaginary pistons were introduced only to make the reasoning more intelligible; the argument holds good whether they are there or not. It applies to any fluid, and to any action in which there is a frictional fall of pressure.

We might accordingly describe the quantity I as that property of a substance which does not change in a throttling process. In this argument it is assumed that no heat is taken in or given out, and also that the velocity in the pipes is so small that no account need be taken of any difference in the kinetic energy of the stream in the pipes before and after passing the constriction, once the eddies have subsided. If the stream has acquired an appreciable amount of kinetic energy after the process, there will be a corresponding reduction in I.

§ (33) ENTROPY OF A FLUID.—In reckoning the entropy of a fluid the same convention is followed as in reckoning internal energy: the entropy of the liquid at 0° C. is taken as zero. Consider, as before, a process in which the liquid is first heated under constant pressure and then vaporised at that pressure. During the heating of the liquid from an initial temperature T_0 to any temperature T (on the absolute scale) the entropy increases by the amount

$$\int_{T_0}^{T} \frac{dQ}{T} = \int_{T_0}^{T} \frac{\sigma dT}{T},$$

where σ is the specific heat at constant pressure.

If σ could be treated as constant this would give on integration

$$\sigma(\log_e T - \log_e T_0).$$

In the case of water σ is not far from constant and equal to unity.

During vaporisation an additional amount of heat L is taken in at constant temperature T_s, namely, the temperature at which vapour is formed under the given pressure. Hence the entropy increases by the amount L/T_s, and we have, for the entropy of any saturated vapour,

$$\phi_s = \phi_w + \frac{L}{T_s}. \quad . \quad . \quad . \quad (13)$$

During superheating there is a further increase of entropy as the substance takes in more heat.

§ (34) MIXED LIQUID AND VAPOUR: WET VAPOUR.—In many of the actions that occur in steam-engines and refrigerating machines we have to do, not with dry saturated vapour but with a mixture of saturated vapour and liquid. In the cylinder of a steam-engine, for example, the steam is generally *wet*; it contains a proportion of water which varies as the stroke proceeds. When any such mixture of two phases of the same substance is in a state of thermal equilibrium the liquid and vapour have the same temperature, and the vapour is saturated. What is called the dryness of a vapour is measured by the fraction q of vapour which is present in unit mass of the mixture. When the dryness is known it is easy to determine other quantities. Thus, reckoning in every case per unit mass of the mixture, we have:

Latent heat of wet vapour

$$= qL = q(I_s - I_w). \quad . \quad . \quad (14)$$

Total heat of wet vapour,

$$I_q = I_w + qL = I_s - (1-q)L. \quad . \quad (15)$$

Volume of wet vapour,

$$V_q = qV_s + (1-q)V_w, \quad . \quad . \quad (16$$

which is very nearly equal to qV_s, unless the mixture is so wet as to consist mainly of liquid.

Entropy of wet vapour,

$$\phi_q = \phi_w + \frac{qL}{T_s} = \phi_s - \frac{(1-q)L}{T_s}. \quad . \quad (17)$$

From (15) it follows that when the total heat I_q of a wet vapour is known, the dryness may be found by the equation

$$q = \frac{I_q - I_w}{I_s - I_w}. \quad . \quad . \quad . \quad (18)$$

Combining (15) and (17), and eliminating q, we have

$$I_q = I_w + T_s(\phi_q - \phi_w), \quad . \quad . \quad (19)$$

which is a convenient expression for finding the total heat of a wet vapour when the data are the temperature and the entropy. An alternative form is

$$I_q = I_s - T_s(\phi_s - \phi_q). \qquad (20)$$

In these expressions I_w is the total heat of the liquid and I_s that of the dry saturated vapour at the temperature of the wet mixture.

§ (35) Specification of the State of any Fluid.—We have now spoken of the following quantities which are functions of the state of the substance. They all depend on the actual state, not on how that state has been reached:

The temperature, T.
The pressure, P.
The volume, V.
The internal energy, E.
The total heat, I.
The entropy, ϕ.

(The volume, internal energy, total heat and entropy:) These four are to be reckoned per unit quantity of the substance.

A substance may change its state in many different ways: it may, for instance, take in heat at constant volume or while expanding; it may expand or be compressed with or without taking in heat; expansion may take place through a throttle-valve or under a piston; there may or may not be a change of phase. But in any change of state whatever, the amount by which each of these quantities is altered depends only on what the initial and final states are, and not at all on the particular process by which the change of state has been effected.

There are other quantities, such as the heat taken in, or the work done, which depend on how the change of state has taken place. In dealing with them we have to distinguish between one process of change and another, even when both processes bring the substance from the same initial to the same final condition.

Any two of these six quantities will serve as data for completely specifying the state of a fluid, so long as it is homogeneous, that is to say, so long as it is not only the same chemically but all in the same phase, either all liquid or all gas. When it is a mixture of two phases, namely, of liquid and saturated vapour, the pressure and temperature do not suffice without some other particular, such as the dryness q.

§ (36) Isothermal Expansion of a Fluid: Isothermal Lines on the Pressure-volume Diagram.—It is instructive to consider the general form of the isothermal lines on the diagram whose co-ordinates are the pressure and the volume, when the fluid passes successively through the stages of being (1) entirely liquid, (2) a mixture of vapour and liquid, (3) entirely vaporous, by having its pressure gradually reduced under conditions such that the temperature remains constant throughout the process. Imagine, for instance, a cylinder to contain a quantity of the liquid under pressure applied by a loaded piston, and let the cylinder stand on a body at a definite constant temperature, which will supply enough heat to it to maintain the temperature unchanged when the pressure of the piston is gradually relaxed and the volume consequently increases. Starting from a condition of very high pressure, say at A_1 (*Fig.* 10), when the contents of the cylinder are wholly liquid, let the load on the piston be slowly reduced, so that the pressure gradually falls. The contents at first remain liquid, until the pressure falls to the saturation value for the given temperature, namely the pressure at which vapour begins to form. Thus we have in the pressure-volume diagram a line A_1B_1 to represent what happens while the pressure is falling during this first stage; the contents are then still liquid. The volume of the liquid increases, but only very slightly, in consequence of the pressure being relaxed, and hence A_1B_1 in the diagram is nearly, but not quite, vertical. At B_1 vapour begins to form, and continues forming until all the liquid becomes vapour. This is represented by B_1C_1, a stage during which there is no change of pressure. At C_1 there is nothing but saturated vapour. Then, if the fall of pressure continues, a line C_1D_1 is traced, the progressive fall of pressure being associated with a progressive increase of volume. The temperature, by assumption, is kept constant throughout. At D_1, or at any point beyond C_1, the vapour has become superheated, because its pressure is lower than the pressure corresponding to saturation, and hence its temperature is higher than the temperature

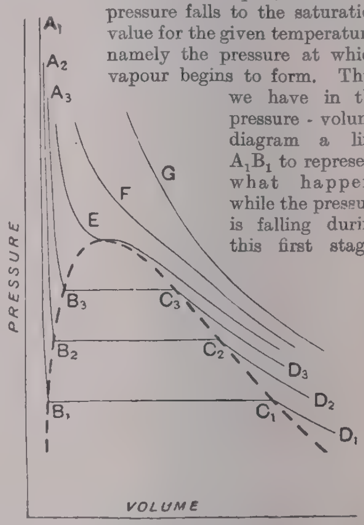

Fig. 10.—Isothermal Lines on the Pressure-volume Diagram.

corresponding to saturation at the actual pressure. Any such line ABCD is an isothermal for the substance in the successive states of liquid (A to B), liquid and vapour mixed (B to C), saturated vapour (at C), superheated vapour (C to D). Now take a much higher temperature. We get a similar isothermal $A_2B_2C_2D_2$; and at a still higher temperature another isothermal $A_3B_3C_3D_3$, and so on. The higher the temperature the nearer do B and C approach each other, and if the temperature be made high enough the horizontal portion of the isothermal line vanishes.

§ (37) THE CRITICAL POINT: CRITICAL TEMPERATURE AND CRITICAL PRESSURE.—A curve (shown by the broken line) drawn through $B_1B_2B_3$, etc., is continuous with one passing through $C_1C_2C_3$, and it is only within the region of which this curve is the upper boundary that any change from liquid to vapour takes place. The branch $B_1B_2B_3$, which shows the volume of the liquid, meets the branch $C_1C_2C_3$, which shows the volume of the saturated vapour, in a rounded top. The summit of this curve represents a state which is called the Critical Point. The temperature for an isothermal line E that would just touch the top of this curve is called the Critical Temperature. We might define the critical temperature in another way by saying that if the temperature of a vapour is above the critical temperature no pressure, however great, will cause it to liquefy. The pressure at the critical point is called the Critical Pressure; at any higher pressure the substance cannot exist as a non-homogeneous mixture of two phases, partly liquid and partly vapour. At the critical point the distinction between liquid and vapour disappears.

Starting from D and increasing the pressure, the temperature being kept constant, we may trace any of the isothermals backwards. The initial state is then that of a gas (a superheated vapour). If the temperature is low enough we have a discontinuous process DCBA: as the pressure increases C is reached when the vapour is saturated and condensation begins: at B condensation is complete, and from B upwards towards A we are compressing liquid. At any point between C and B the substance exists in two phases or states of aggregation; part is liquid and part is vapour. But if the temperature is above the critical temperature the isothermal is one that lies altogether outside of the boundary curve, shown by the broken line; in that case the substance does not suffer any sharp change of phase as the pressure rises. It passes from a state that would be called gaseous to one that would be called liquid, in a continuous manner, following a course such as is indicated by the lines F or G, and at no stage in the process is it other than homogeneous. The continuity of the liquid and gaseous states, in any substance, may be more clearly realised if one thinks of a process by which the substance may pass from a state that is obviously liquid, to another that is obviously gaseous, without any abrupt change, such as that which occurs in the boiling of a liquid. Starting from B (*Fig.* 11), where the substance is a liquid, it might be heated at constant volume to a temperature equal to the critical temperature. This brings it to H. Then it might expand isothermally along the line HI, and then be cooled at constant volume from I to C. At C it is a saturated vapour. During each of these steps the substance has remained homogeneous; the passage from liquid to vapour has taken place in a continuous manner, and it would be impossible to point to any stage of the process as the stage of transition from one phase to the other. In this process any isothermal higher than the critical isothermal E would serve equally well for the step in which the substance expands.

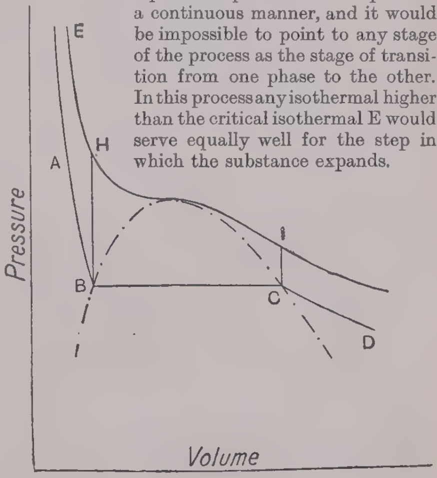

FIG. 11.

The critical temperature of water vapour is about 374° C., a temperature much higher than is reached in the action of an ordinary steam engine. But with carbonic acid, whose critical temperature is only about 31° C., the behaviour in the neighbourhood of the critical point, and above it, is of great practical importance in connection with refrigerating machines which employ carbonic acid as working substance.

The so-called permanent gases, such as air, hydrogen, oxygen, and so forth, are vapours which under ordinary conditions are very highly superheated. Their critical temperatures are so low that it is only by extreme cooling that they can be brought into a condition which makes liquefaction possible. The critical temperature of hydrogen is −241° C. or 32° absolute. Even helium, the most refractory of the gases, has been liquefied, but only by cooling it to a temperature less than 5 degrees above the absolute zero.

§ (38) ADIABATIC EXPANSION OF A FLUID: HEAT-DROP.—When a fluid expands adiabatically it does work at the expense of its stock of internal energy, and its total heat I consequently becomes reduced. To calculate the amount of this reduction we have, by definition of the total heat,

$$I = E + APV.$$

Hence by differentiation

$$dI = dE + APdV + AVdP.$$

But by the conservation of energy the increase of internal energy plus the work done by the fluid is equal to the heat taken in. Applying this principle to a small change of state, we have

$$dE + APdV = dQ,$$

where dQ is the heat taken in during the change. Hence in any small change of state

$$dI = dQ + AVdP.$$

In an adiabatic operation $dQ = 0$, and consequently $dI = AVdP$. Therefore, if the fluid expands adiabatically from any state a to any state b, the resulting decrease in its total heat, namely,

$$I_a - I_b = A\int_b^a VdP. \quad . \quad . \quad (21)$$

This integral is the area *eabf* of the pressure-volume diagram (*Fig.* 12). It is the whole work done in a cylinder when the fluid is admitted at the pressure corresponding to state a, then expanded adiabatically to state b, and then discharged at the pressure corresponding to state b.

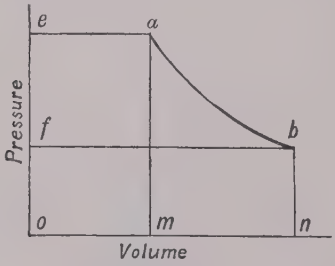

FIG. 12.

The decrease of total heat in expansion $I_a - I_b$ is called the "Heat-drop." It is a quantity of much importance in the theory of heat-engines. The above equation shows that under adiabatic conditions the whole work done in the cylinder, namely the area *eabf*, when expressed in heat units, is measured by the heat-drop.

This quantity must not be confused with the work which the fluid does during expansion, namely the area *mabn*. Under adiabatic conditions that area is equivalent to the loss of internal energy during expansion, or

$$E_a - E_b = A\int_b^a PdV. \quad . \quad . \quad (22)$$

In the adiabatic expansion of any fluid its temperature, pressure, energy, and total heat fall, but the entropy remains constant since the operation is reversible and no heat is communicated to or taken from the substance. This consideration enables the form of the pressure-volume curve to be determined when the relation of the entropy to other properties of the fluid is known. A case which is important in practice is that of a wet vapour, such as the mixture of steam and water which expands in the cylinder of an engine. Usually, though not always, a wet vapour becomes wetter as expansion proceeds. The curve of its expansion may be traced as follows, if we assume that the liquid and the vapour which constitute the mixture are in thermal equilibrium throughout the process.

For greater generality we shall suppose the vapour to be wet to begin with. Let the initial temperature be T_1 and the initial dryness q_1. In this state the entropy is

$$\phi = \phi_{w1} + \frac{q_1 L_1}{T_1},$$

L_1 being the latent heat of the vapour and ϕ_{w1} the entropy of the liquid, both at the temperature T_1. Let the substance expand adiabatically to any lower temperature T_2, at which the latent heat is L_2 and the entropy of the liquid is ϕ_{w2}: we have to find the resulting value of the dryness q_2. The entropy may now be expressed as

$$\phi_{w2} + \frac{q_2 L_2}{T_2},$$

and since there has been no change of entropy this is equal to the initial value ϕ. Hence

$$q_2 = \frac{T_2}{L_2}(\phi - \phi_{w2}). \quad . \quad . \quad (23)$$

This equation serves to determine the dryness after expansion, and once it is known the volume V_q is readily found. The exact value of V_q is $q_2V_{s2} + (1 - q_2)V_{w2}$, which is practically equal in ordinary cases to q_2V_{s2}, V_{s2} being the volume of saturated vapour at the temperature T_2. The pressure is the saturation pressure corresponding to T_2. Thus the calculation fixes a point in the adiabatic line of the pressure-volume diagram for expansion from the initial conditions. A series of points may be found in the same way, corresponding to successive assumed temperatures which are reached in the course of the expansion, if it is desired to trace the line.

In the special case when the vapour is dry and saturated to begin with, the constant entropy ϕ is equal to ϕ_{s1}, and the expression for the wetness after expansion to any temperature T_2 becomes

$$q_2 = \frac{T_2}{L_2}(\phi_{s1} - \phi_{w2}).$$

Similarly, if the substance is entirely liquid in the initial state, the pressure being sufficient to prevent vapour from forming, adiabatic expansion will cause some of it to vaporise. Its initial entropy is ϕ_{w1}, and since this does not change,

$$q_2 = \frac{T_2}{L_2}(\phi_{w1} - \phi_{w2})$$

after expansion to a temperature T_2.

When a homogeneous gas expands adiabatically, without liquefaction, the relation of pressure to volume is approximately expressed by the equation $PV^\gamma = \text{constant}$, where γ is (as before) the ratio of the specific heats K_p/K_v. This expression is, as we saw in § (15), rigorously true of a perfect gas, where isothermal expansion makes PV constant, and adiabatic expansion makes PV^γ constant. In any homogeneous fluid, as will be shown later, the slope of the pressure-volume curve, namely dP/dV, is γ times as great in adiabatic expansion as in isothermal expansion. In water vapour the value of γ is found to be 1·3, and accordingly that value of the index serves to determine the expansion curve for superheated steam down to the point at which liquefaction begins, after which the curve may be traced in the manner described above, by determining the dryness at successive stages.

In the adiabatic expansion of an initially superheated vapour the point at which liquefaction may be expected to begin is determined from the fact that the entropy is constant, by finding at what temperature, or pressure, the entropy of the saturated vapour is equal to that of superheated vapour in the given initial state. This comparison is readily made when tables or charts giving the properties of the substance are available, as they are for steam, ammonia, carbon dioxide, and certain other fluids. With a suitable chart the progressive effect of any adiabatic expansion or compression is readily traced throughout its whole course.

§ (39) SUPERSATURATION.—In the above discussion of adiabatic expansion it has been assumed that at every step in the expansion there is a condition of equilibrium between the part that is vapour and the part that is liquid. But it is known, as a result of experiment, that when a vapour is suddenly cooled by adiabatic expansion the condition of equilibrium is not reached at once. Suppose a vapour such as steam to be initially dry and saturated; on expansion a part of it must condense if equilibrium is to be established. This condensation takes an appreciable time; it is a surface phenomenon, taking place partly on the inner surfaces of the containing vessel and partly by the growth of drops throughout the volume. Consequently the sudden expansion of a vapour may produce temporarily what is called *supersaturation*, a state in which the substance continues for a time to exist as a homogeneous vapour, although its pressure and temperature are such that the condition of equilibrium would require a part of it to be condensed. Such a state is often described as a *metastable state*. In the supersaturated state the density of the vapour is abnormally high, higher than the density of saturated vapour at the actual pressure. The temperature is also abnormally low, lower than the temperature of saturation at the actual pressure; for this reason a supersaturated vapour might be called *supercooled*. The supersaturated condition is not stable; it disappears through the condensation of a part of the vapour, and the resulting mixture of vapour and liquid has its temperature raised by the latent heat which is given out in this condensation. An important practical example of this kind of action is found when a jet of steam is formed by sudden expansion in a nozzle such as that of a De Laval steam turbine. There the expansion is nearly adiabatic and the work done during expansion is employed in giving kinetic energy to the issuing stream. It is found that in the early stages of this expansion there is no condensation even if the steam is saturated when it enters the nozzle. In these stages the steam becomes supersaturated, and, as Callendar has pointed out (*Proc. Inst. Mech. Eng.*, Feb. 1915), the relation of its volume to its pressure during these stages is that which holds for a dry vapour, not for a wet mixture.

The supercooling of a vapour without condensation is analogous to the supercooling of a liquid without crystallisation—another example of a metastable state. In both processes there is a departure from the state of equilibrium, and in both the restoration of equilibrium involves an *irreversible* action within the substance. The adiabatic expansion of a vapour under conditions of thermal equilibrium throughout is reversible, but if there has been supercooling there is an irreversible development of heat within the fluid when the supercooled vapour passes from the metastable state into the stable state of a mixture of liquid and saturated vapour.

§ (40) CARNOT'S CYCLE WITH A WET VAPOUR FOR WORKING SUBSTANCE.—Returning now to processes in which the adiabatic expansion or compression of a vapour is imagined to be carried out under conditions of thermal equilibrium, we may think of an ideal engine, such as was shown in *Fig.* 4, but with a liquid and its vapour for working substance. Carnot's reversible cycle of four operations can then be performed as follows Suppose that the cylinder contains unit mass of the substance, in the liquid state, at the upper limit of temperature T_1 and at the corresponding pressure of saturation P_1.

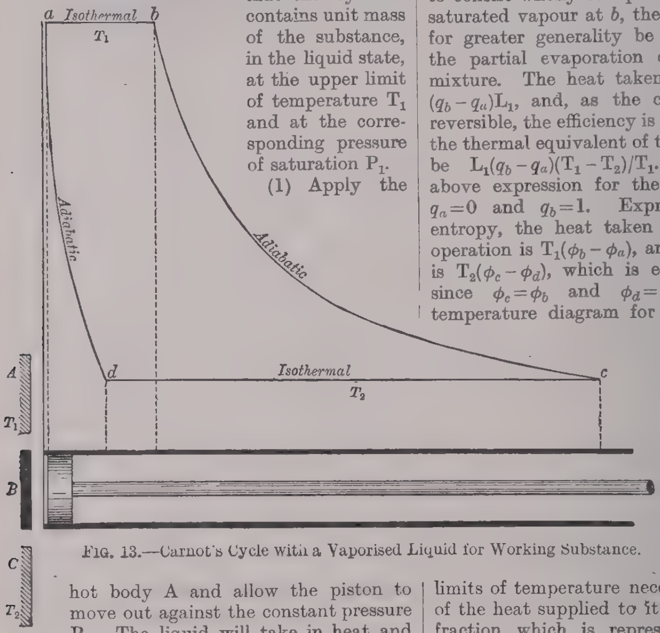

FIG. 13.—Carnot's Cycle with a Vaporised Liquid for Working Substance.

(1) Apply the hot body A and allow the piston to move out against the constant pressure P_1. The liquid will take in heat and be converted into vapour, expanding isothermally at the temperature T_1. This operation is shown by the line *ab* (*Fig.* 13).

(2) Remove A and apply the non-conducting cover B. Allow the expansion to continue adiabatically (*bc*), with falling pressure, until the temperature falls to that of the cold body, T_2. The pressure will then be P_2, which is the pressure of saturation at T_2.

(3) Remove B, apply the cold body C, and compress. Vapour is condensed by rejecting heat to C. The action is isothermal and the pressure remains P_2. Let this operation (*cd*) be continued until a certain point *d* is reached, which is chosen so that adiabatic compression will complete the cycle.

(4) Remove C and apply B. Continue the compression, which is now adiabatic. If the point *d* has been rightly chosen, this operation (*da*) will complete the cycle by restoring the working fluid to its original completely liquid state at temperature T_1.

Since the cycle is reversible, and since heat is taken in only at T_1 and rejected only at T_2, it follows from the argument of § (20) that the efficiency is $(T_1 - T_2)/T_1$. The heat taken in is L_1, and therefore the thermal equivalent of the work done is

$$\frac{L_1(T_1 - T_2)}{T_1}.$$

Instead of supposing the working substance to consist wholly of liquid at *a* and wholly of saturated vapour at *b*, the operation *ab* might for greater generality be taken to represent the partial evaporation of an initially wet mixture. The heat taken in would then be $(q_b - q_a)L_1$, and, as the cycle would still be reversible, the efficiency is still $(T_1 - T_2)T_1$, and the thermal equivalent of the work done would be $L_1(q_b - q_a)(T_1 - T_2)/T_1$. This gives the above expression for the special case when $q_a=0$ and $q_b=1$. Expressed in terms of entropy, the heat taken in during the first operation is $T_1(\phi_b - \phi_a)$, and the heat rejected is $T_2(\phi_c - \phi_d)$, which is equal to $T_2(\phi_b - \phi_a)$, since $\phi_c = \phi_b$ and $\phi_d = \phi_a$. The entropy-temperature diagram for the complete cycle is a rectangular figure (as in *Fig.* 5) whose width is $\phi_b - \phi_a$ and whose height is $T_1 - T_2$.

In the article "Steam Engine, Theory of," reasons will be found why any real vapour engine working between assigned limits of temperature necessarily converts less of the heat supplied to it into work than the fraction which is represented by the ideal Carnot efficiency, namely $(T_1 - T_2)T_1$.

§ (41) CLAPEYRON'S EQUATION.—This name is given to an important relation between the latent heat of any vapour, the change of volume which it undergoes in being vaporised, and the rate at which the saturation pressure varies with the temperature. Imagine a Carnot engine, with complete vaporisation from liquid at *a* to saturated vapour at *b*, (*Fig.* 14), and suppose the engine to work between temperatures which differ by only a small amount δT. Call the upper tempera-

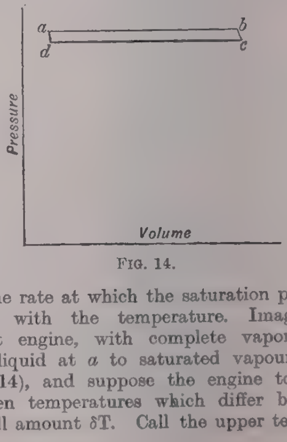

FIG. 14.

ture T and the lower temperature $T - \delta T$. The heat taken in is L, the efficiency is $\delta T/T$, and the work done is $JL\delta T/T$, the factor J being introduced to convert from heat units to work units. The indicator diagram, whose area measures the work done, is now a long narrow strip. Its length is $V_s - V_w$, V_s being the volume of the saturated vapour and V_w that of the liquid. Its height is δP, the fall in saturation pressure corresponding to the fall in temperature from T to $T - \delta T$. When δP is made very small, by taking the two temperatures very near together, the area of the diagram becomes more and more nearly equal to the product of the length by the height, or $(V_s - V_w)\delta P$. Hence, in the limit,

$$(V_s - V_w)\delta P = \frac{JL\delta T}{T}$$

or

$$V_s - V_w = \frac{JL}{T}\left(\frac{dT}{dP}\right)_s, \quad . \quad . \quad (24)$$

where $(dT/dP)_s$ means the rate at which the temperature of the saturated vapour changes relatively to the pressure. This is Clapeyron's equation. The reasoning by which it is established would be valid for any reversible change of phase which occurs isothermally at constant pressure, such as that which occurs when a solid melts, as well as that which occurs when a liquid is vaporised. It may accordingly be written in the more general form

$$V'' - V' = \frac{J\lambda}{T} \cdot \frac{dT}{dP}, \quad . \quad . \quad (24a)$$

where V' is the volume of the substance in its first state, V'' is its volume in the second state, λ is the heat absorbed in the transformation, and dT/dP is the rate at which the temperature of the transformation (say the melting-point or the boiling-point) alters with the pressure.

Applied to the melting of ice, where there is contraction of volume on melting, Clapeyron's equation accordingly shows that the melting-point would be lowered by applying pressure. From the known amount by which ice contracts when it melts, James Thomson predicted that the melting-point of ice would be lowered about 0·0074° C. for each atmosphere of pressure, a result which afterwards was verified experimentally by his brother, Lord Kelvin (*Collected Papers*, i. 156, 165).

The lower of the two fixed points used in graduating a thermometer is the temperature at which ice melts under a pressure of one atmosphere. If this pressure were removed—as it might be by putting the ice in a jar exhausted of air by means of an air-pump—the temperature of melting would be raised. The water-vapour given off at the melting-point has a pressure of only 0·09 pound per square inch, and consequently if no air were present, and if the only pressure were that of its own vapour, ice would melt at approximately 0·0074° C., for the pressure would be reduced by nearly one atmosphere. The temperature at which ice melts under these conditions is called the *Triple Point*, because (in the absence of air) water-stuff can exist at that particular temperature in three phases, as ice, as water, and as vapour, in contact with one another and in equilibrium.

§ (42) CHARTS EXHIBITING THE PROPERTIES OF A FLUID.—Charts for exhibiting the properties of a fluid are drawn by selecting two functions of the state (such as pressure and volume, or entropy and temperature, or entropy and total heat) as co-ordinates, and drawing a family of lines each of which shows the relation between these co-ordinates when some third function of the state is kept constant. One example of such a chart has been illustrated in *Fig.* 10, where lines of constant temperature were drawn with the pressure and the volume as the two co-ordinates. In the engineering applications of thermodynamics two other types of chart are specially useful: in one of these the co-ordinates are the temperature and the entropy; in the other (a chart introduced by R. Mollier) the co-ordinates are the total heat and the entropy.

The entropy-temperature chart has the valuable property, already pointed out, that the area under any line which represents a reversible process measures the heat taken in or given out during that process (since $Q = \int T d\phi$), the area being measured down to a base line drawn at the absolute zero of temperature. Any reversible cycle is represented on the chart by a closed system of lines, and the area enclosed by them is the thermal equivalent of the work done in the cycle. For the purpose of exhibiting the properties of a substance on an entropy-temperature chart, lines of constant pressure are drawn as in *Fig.* 15. There AB represents

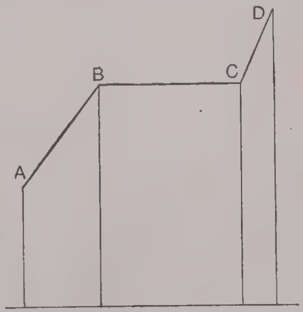

FIG. 15.

the process of heating a liquid under constant pressure up to the temperature at which vaporisation begins. BC represents the process of vaporisation, and CD represents the process of superheating, still at the same pressure. The heat absorbed in the three stages is represented by the areas under AB, BC, and CD respectively. When a group of such lines is drawn for various constant pressures, the locus of B forms a curve called the liquid boundary and that of C forms a curve called the vapour boundary. These curves meet in a rounded top at the critical point, as will be seen in *Fig.* 16, which shows the general character of an entropy-temperature chart for water and steam. In that figure lines of constant pressure are drawn for various pressures, also (in the region of superheat) lines of constant total heat. In the wet region, between the two boundary curves, lines of constant dryness are also shown, which allow the wetness resulting from any stated amount of adiabatic expansion to be readily determined by drawing a vertical straight line through a point which represents the initial state. Examples of the use of the entropy-temperature chart will be found in the article "Steam Engine, Theory of."

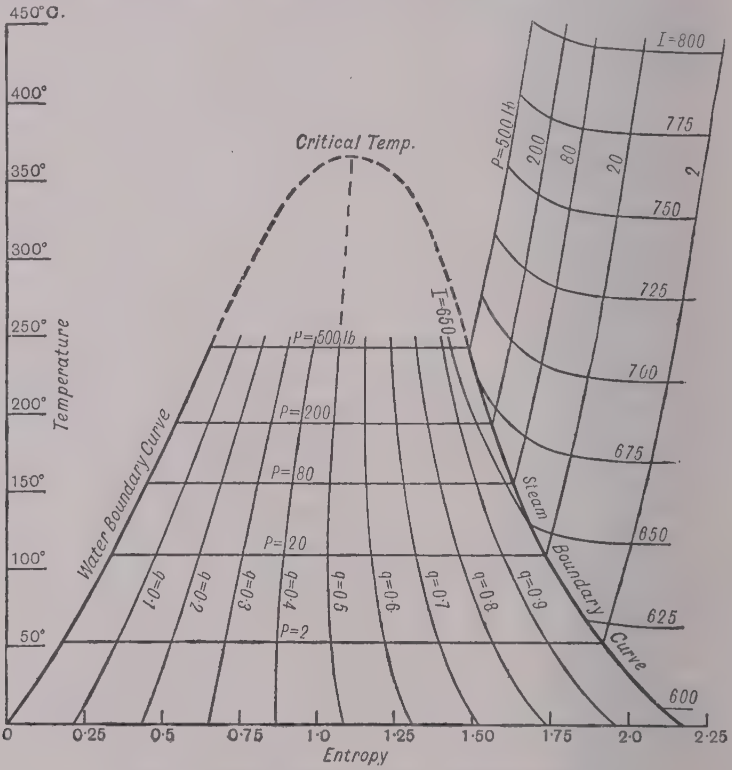

FIG. 16.—Entropy-temperature Chart for Water and Steam.

Even more useful for practical purposes is Mollier's chart of entropy and total heat, which allows the "heat-drop" to be directly measured. A chart of this kind (for water and steam) is sketched in outline in *Fig.* 17, showing lines of constant pressure, lines of constant temperature (in the region of superheat), and lines of constant dryness (in the wet region). The slope of any line of constant pressure is a measure of the absolute temperature, for at constant pressure $dI = dQ = Td\phi$, and consequently $dI/d\phi = T$. Hence in the wet region each constant pressure line is straight. It crosses the vapour boundary without change of slope, but gradually bends upwards as superheating proceeds. The liquid and vapour boundary curves are connected by a broken line passing through the critical point. The critical point is not at the summit of that line, but considerably lower, between the summit and the point of inflection.

In the sketch the line *bb′* illustrates the process of superheating steam, at a constant pressure of 200 pounds per square inch, to a temperature of 400° C. The line *b′c* illustrates adiabatic expansion, from that initial state, to a pressure of 1 pound per square inch. The length of *b′c* (measured on the scale of I) is the adiabatic heat-drop, which is the thermal equivalent of the work obtainable from steam, under adiabatic conditions, when supplied to an engine in the state *b′* and discharged in the state *c*. From the position of *c*, below the steam boundary curve, it is seen that the steam is then wet, the dryness being about 0·88. The most useful part of the chart is the part immediately above and below the steam boundary curve; large scale drawings of the chart in this region are obtainable as aids in engineering calculations.

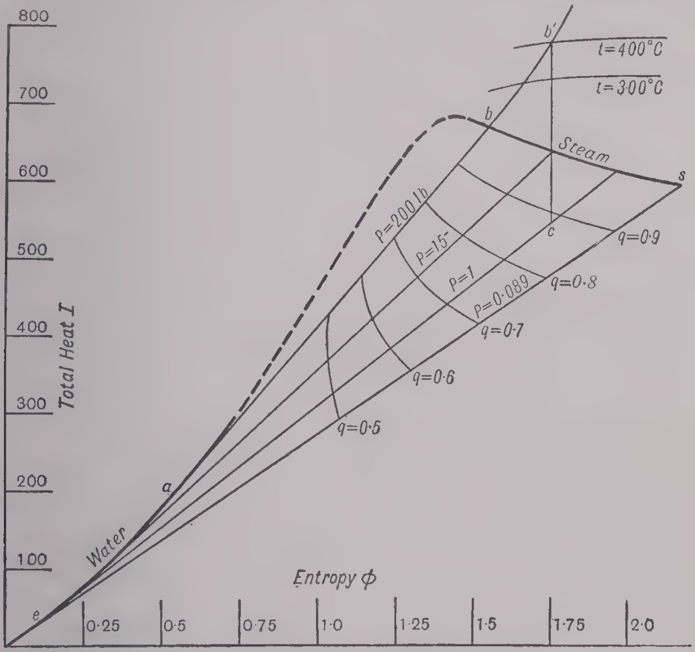

FIG. 17.—Mollier Chart for Water and Steam.

§ (43) EFFECTS OF THROTTLING: THE JOULE-THOMSON COOLING EFFECT.—We have already seen that when a throttling process is carried out under conditions that prevent heat from entering or leaving the substance the total heat I does not change. Lines of constant total heat on any of the diagrams accordingly show the changes in other quantities which are brought about by throttling. It is the process that occurs when a fluid passes through a "reducing valve" or other constricted orifice such as the porous plug of the Joule-Thomson experiment.

In a perfect gas throttling produces no change of temperature, but in steam and other vapours it produces a cooling effect which is measured as the fall of temperature per unit fall of pressure under the condition that I is constant, or

$$\text{Cooling effect} = \left(\frac{dT}{dP}\right)_I.$$

This is called the Joule-Thomson cooling effect. Under certain conditions (which will be discussed later) it may be negative, that is to say a gas may be slightly heated by throttling, but in general the effect of throttling a fluid is to cool it. This cooling effect has been applied by Linde to bring about the liquefaction of air and other not easily liquefiable gases, the apparatus being arranged in such a way that the escaping gas, which has been slightly cooled by passing the throttling orifice, takes up heat from the warmer gas that is entering, with the result that a cumulative cooling takes place and the temperature at the orifice is reduced below the critical point. (See the article "Liquefaction of Gases.")

When a vapour which is initially saturated has its pressure lowered by throttling, it becomes superheated, for the saturation temperature is reduced by a greater amount than the actual fall of temperature. Similarly a wet vapour tends to become drier. The process of throttling is very simply shown by drawing horizontal lines (I = constant) on the Mollier chart of entropy and total heat. It is easy in this way to trace the extent to which a vapour may be superheated by throttling, or the extent to which a wet mixture, or the liquid itself, will be vaporised.

§ (44) Heat-account for a Real Process.—With the help of the total-heat function I, we may now write out, in general terms, a thermodynamic balance-sheet which will express the giving and taking of heat in any real process, during which the working fluid passes through any apparatus, such as a pipe, or an engine or a turbine, considered as a whole. The processes hitherto considered have been *adiathermal*, in the sense that there was no transmission of heat to or from the working substance in the course of its passage through the apparatus, but we may now discard this limitation, and include thermal losses to the space outside and also irreversible actions within the apparatus.

Whatever the nature of the apparatus, we may imagine a steady flow of the working fluid through it, and compare the state of the fluid (1) at entry and (2) at exit, as for example in (1) the admission pipe of an engine, and (2) the exhaust pipe. At entry let its pressure be P_1, its volume (per unit of mass) V_1, and its internal energy E_1. At exit let its pressure be P_2, its volume V_2, and its internal energy E_2. To make the comparison complete we may write K_1 for the kinetic energy (in thermal units) of the stream as it enters, and K_2 for its kinetic energy as it leaves. In passing through the apparatus the fluid will, in general, do external work, and also lose by conduction some heat to external space. Let W represent, in thermal units, this output of work, and let Q_l represent the heat lost by conduction to external space, all of these quantities being reckoned per unit of mass of the fluid that passes through.

Each unit that enters the apparatus represents a supply of energy which is equal to $K_1 + E_1 + AP_1V_1$, for E_1 is the internal energy it carries, and P_1V_1 is the work done by the fluid behind in pushing it in. But $E_1 + AP_1V_1$ is equal to I_1, the total heat per unit of the fluid in its actual state at entry. Similarly, each unit that leaves the apparatus represents a rejection of energy amounting to $K_2 + E_2 + AP_2V_2$, for E_2 is the internal energy which the fluid carries out, and P_2V_2 is the work spent upon it by the fluid behind in pushing it out. $E_2 + AP_2V_2$ is equal to I_2, the total heat per unit of the fluid in its actual state at exit. Hence, by the conservation of energy, for the action as a whole,

$$K_1 + I_1 = K_2 + I_2 + W + Q_l.$$

The terms on the left of this equation represent the energy that enters the apparatus; the terms on the right show how it is disposed of in the issuing stream, in output of useful work, and in leakage of heat. This equation may be written

$$I_1 - I_2 = K_2 - K_1 + W + Q_l. \quad . \quad (25)$$

The terms K_1 and K_2 are usually very small, except when the apparatus is one for forming a steam jet, in which case K_2 is the useful term. When the change of kinetic energy in the stream is practically negligible, as it is between the admission pipe and exhaust pipe of an ordinary engine, we have

$$I_1 - I_2 = W + Q_l. \quad . \quad . \quad (26)$$

When the apparatus does not allow any appreciable amount of heat to escape to the outside ($Q_l = 0$), we have

$$I_1 - I_2 = W + K_2 - K_1. \quad . \quad (27)$$

This means that when there is a steady flow of a working substance through any thermodynamic apparatus, the output of work together with any gain of kinetic energy is measured by the actual heat-drop, whether the internal action is or is not reversible, provided there is no loss of heat to the outside by conduction through the walls.

The actual heat-drop must not be confused with the adiabatic heat-drop, which is the difference between I_1 and that value which the total heat would reach if there were adiabatic expansion to the exit pressure P_2. The actual heat-drop $I_1 - I_2$ is identical with the adiabatic heat-drop only when there is no loss of heat to the outside and when, in addition, the internal action is wholly reversible.

Any irreversible feature in the internal action will increase I_2 above the value which would be reached by adiabatic expansion, and will consequently diminish the output of work.

In the extreme case of a throttling process there is no output of work, and therefore $I_2 = I_1$, provided there is no loss of heat to the outside. Any loss of heat to the outside in a throttling process will make I_2 correspondingly less, for we then have $I_2 = I_1 - Q_l$.

The losses of thermodynamic effect in a real engine, which make W less than the ideal output, that is the value corresponding to the adiabatic heat-drop, arise partly from loss of heat to the outside and partly from two kinds of irreversible internal action. One of these two kinds is mechanical; the other is thermal. In the mechanical kind, the action involves fluid friction within the working substance. It is of the same nature as that which occurs in throttling: there is irreversible passage of the working substance from one part of the engine to another where the pressure is lower, as for instance the passage of steam through somewhat constricted openings into the cylinder, or its passage, on release after incomplete expansion, into the exhaust pipe, with a sudden drop of pressure: or again, there is the same kind of irreversibility in a turbine in the frictional losses that attend the formation of steam jets or in the friction of the jets on the turbine blades. These are all instances of mechanical irreversibility. In the second kind of irreversible action there is exchange of heat between the working substance and the internal surface of the engine walls. The hot steam, on admission to a cylinder which has just been vacated by a less hot mixture of steam and water, finds the surfaces colder than itself. A part of it is accordingly condensed on them, which re-evaporates after the pressure has fallen through expansion. This alternate condensation and re-evaporation involves a considerable deposit and recovery of heat in a manner that is not reversible, for it takes places by contact between fluid and metal at different temperatures. The action may occur without loss of heat to the outside: it would occur, for instance, in an engine with a conducting cylinder covered externally with non-conducting material. Its effect, like that of throttling or fluid friction generally, is to reduce the output of work below the limit that is attainable only in a reversible process, and it does this by making the actual heat-drop $I_1 - I_2$ less than the adiabatic heat-drop.

The full statement of the heat-account in a real process may accordingly be expressed as follows: When there is a steady flow of a working substance through any thermodynamic apparatus, the output of work is measured by the actual heat-drop from entrance to exit, less any heat that escapes by conduction to the outside, and less any gain of kinetic energy of the issuing stream over the entering stream.

This statement applies to any type of heat-engine and also to reversed heat-engines, or heat-pumps, but in them the quantity W is negative: work is expended on the machine instead of being produced by the machine. In such machines Q_l is also generally negative, for as a rule the apparatus is colder than its surroundings and the leakage of heat is inwards.

In an apparatus such as the convergent-divergent nozzle of De Laval[1] the heat-drop is utilised in giving kinetic energy to the stream. No other work is done. Let the fluid enter such an apparatus with the velocity v_1 and leave it with the velocity v_2. If we assume that the loss of heat by conduction is negligible, then the gain of kinetic energy is equal to the actual heat-drop, or

$$\frac{v_2^2 - v_1^2}{2g} = J(I_1 - I_2). \quad . \quad . \quad (28)$$

With given initial conditions, and a given fall of pressure, this quantity reaches its highest value when $I_1 - I_2$ is equal to the adiabatic heat-drop. Any dissipation of energy through friction or eddies within the apparatus reduces the actual heat-drop, by increasing the value of I_2.

§ (45) General Thermodynamic Relations between Functions of the State in any Fluid.—We have now to consider, from a more mathematical standpoint, the thermodynamic relations which hold, in any fluid, between the various functions of its state. By a function of the state is meant a quantity which depends only on the actual state and not on any changes through which the fluid may have passed in reaching the actual state. Six such quantities have already been mentioned, namely P, V, T, E, I, and ϕ. When a fluid passes in any manner from one state to another, each of these quantities is altered by a definite amount which does not depend on the nature of the operation, but only on what the state was before the operation and what the state is at the end of the operation. In mathematical language this fact is expressed by saying that the differential of any of these quantities is a "perfect" differential. It is convenient to include two other quantities in this list, both of which are also functions of the state, namely a quantity ζ, which is defined by the equation

$$\zeta = I - T\phi, \quad . \quad . \quad . \quad (29)$$

and a quantity ψ, which is defined by the equation

$$\psi = E - T\phi. \quad . \quad . \quad . \quad (30)$$

We defined the entropy ϕ by the equation $d\phi = dQ/T$ in a reversible operation; and the fact that ϕ is a function of the state was proved as a consequence of the result that $\int dQ/T = 0$ for a reversible cycle, a result which follows from the Second Law of Thermodynamics. This is expressed mathematically by the statement that dQ/T or $d\phi$ is a perfect differential. The Second Law is therefore involved in treating ϕ as a function of the state. Hence the fact that $d\phi$ is a perfect differential is sometimes spoken of as a mathematical expression of the Second Law. It is important to notice that while dQ/T, which is $d\phi$, is a perfect differential, dQ itself is not a perfect differential, for the amount of heat involved in a change is not a function of the state alone. When a substance changes from one state to another, the amount of heat taken in depends not simply on what the two states are, but also on the nature of the operation by which the change occurs. For the same reason, if W represent the work done during a change of state, dW is not a perfect differential.

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[1] See "Turbine, Development of the Steam."

Since E, P, and V are all functions of the state, it follows that the total heat I, which is equal to $E+APV$, is also a function of the state. And since T and ϕ are also functions of the state, it follows that this is also true of ζ, which is $I-T\phi$, and of ψ, which is $E-T\phi$. Hence dI, $d\zeta$, and $d\psi$, as well as $d\phi$, dE, dP, dV, and dT, are perfect differentials.

§ (46) Relation of any one Function of the State to two others.—The state of the fluid (assumed to be homogeneous) is completely specified when any two of the functions of the state are known. Any third function is then determinate; that is to say, it can have only one value in any particular substance. Thus if any two functions (such, for example, as the pressure and the volume) be selected as "independent variables," by reference to which the state is to be specified, then any third function (such, for example, as the temperature, or the total heat) may be represented in relation to them by the familiar device of drawing a figure in which the two functions selected as independent variables are represented by rectangular co-ordinates X and Y, and the third function is represented by a third co-ordinate Z, perpendicular to the plane of X and Y. This gives a solid figure, the height of which shows, for any given state of the substance, the value of the function Z in relation to the values of the functions X and Y which serve to specify that state. The surface of such a figure may be called a thermodynamic surface.

Suppose now that the substance undergoes an infinitesimal change of state, so that the independent variables change by dX and dY respectively. That is to say, we suppose X to change to $X+dX$ and Y to change to $Y+dY$. Then the third function changes from Z to $Z+dZ$, by an amount dZ which may be expressed thus—

$$dZ=MdX+NdY, \quad . \quad . \quad (31)$$

where M and N are quantities depending on the relations of the functions to one another, and are therefore also functions of the state.

This expression applies whether both functions X and Y vary, or only one of them. If X varies but not Y, then $dY=0$ and $dZ=MdX$: similarly if Y varies but not X, $dX=0$ and $dZ=NdY$. Hence

$$M=\left(\frac{dZ}{dX}\right)_Y \text{ and } N=\left(\frac{dZ}{dY}\right)_X. \quad . \quad (32)$$

In this notation, $(dZ/dX)_Y$ means the rate of variation of Z with respect to X when Y is constant. In the language of the calculus, $(dZ/dX)_Y$ is the partial differential coefficient of Z with respect to X when Y is constant, and $(dZ/dY)_X$ is the partial differential coefficient of Z with respect to Y when X is constant.

We might regard the change of Z as occurring in two steps. In the first step suppose X to change and Y to keep constant. The corresponding part of the change of Z is MdX, and M is the slope of the thermodynamic surface in a section-plane ZX. In the second step X is constant and Y changes. The corresponding part of the change of Z is NdY, and N is the slope of the thermodynamic surface in a section-plane ZY. The whole change of Z is the sum of these two parts, as expressed in equation (31). The slopes along the two section-planes are expressed in equation (32).

Combining these equations we have

$$dZ=\left(\frac{dZ}{dX}\right)_Y dX+\left(\frac{dZ}{dY}\right)_X dY. \quad . \quad (33)$$

These equations apply when X, Y, and Z are interpreted as any three functions of the state of a fluid. Thus, for instance, if we think of a small change of state in which the temperature changes from T to $T+dT$, and the pressure from P to $P+dP$, the consequent change of volume will be

$$dV=\left(\frac{dV}{dT}\right)_P dT+\left(\frac{dV}{dP}\right)_T dP.$$

Similarly, if the volume and pressure change, the consequent change of temperature is

$$dT=\left(\frac{dT}{dV}\right)_P dV+\left(\frac{dT}{dP}\right)_V dP.$$

Or again, the change of entropy consequent on a change of temperature and pressure is

$$d\phi=\left(\frac{d\phi}{dT}\right)_P dT+\left(\frac{d\phi}{dP}\right)_T dP,$$

and so on. It will be obvious that a very large number of similar equations might be written out, each using one pair of functions of the state as independent variables, and expressing in terms of their variation the variation of some third function of the state. These are merely forms of the general equation (33).

Returning now to the general form in X, Y, and Z, suppose a small change of state to occur of such a character that the function Z undergoes no change. In that special case $dZ=0$; the steps MdX and NdY cancel one another. Consequently

$$\left(\frac{dZ}{dX}\right)_Y dX=-\left(\frac{dZ}{dY}\right)_X dY,$$

when dX and dY are so related that there is no variation of Z.

Hence the general conclusion follows that

$$\left(\frac{dZ}{dX}\right)_Y=-\left(\frac{dZ}{dY}\right)_X\left(\frac{dY}{dX}\right)_Z. \quad . \quad (34)$$

This relation between the three partial differential coefficients holds, in all circumstances, for any three functions of the state of any fluid. It may be expressed in these alternative forms:

$$\left(\frac{dX}{dY}\right)_Z\left(\frac{dY}{dZ}\right)_X\left(\frac{dZ}{dX}\right)_Y=-1, \quad . \quad (34a)$$

or

$$\left(\frac{dX}{dY}\right)_Z=-\frac{\left(\frac{dX}{dZ}\right)_Y}{\left(\frac{dY}{dZ}\right)_X}. \quad . \quad . \quad (34b)$$

Returning now to equation (31),

$$dZ=MdX+NdY,$$

the principles[1] of the calculus show that *when dZ is a perfect differential*, but not otherwise,

$$\left(\frac{dM}{dY}\right)_X=\left(\frac{dN}{dX}\right)_Y. \quad . \quad . \quad (35)$$

In dealing with functions which depend only on the actual state of the fluid, the condition that dZ is a perfect differential is satisfied, and consequently equation (35) applies. We shall see immediately some of the results of its application.

§ (47) ENERGY EQUATIONS AND RELATIONS DEDUCED FROM THEM.—Consider now the heat taken in when a small change of state occurs in any fluid. Calling the heat dQ we have, by the First Law,

$$dQ=dE+dW, \quad . \quad . \quad . \quad (36)$$

where dE is the gain of internal energy and dW is the work which the fluid does through increase of its volume. Since $dW=PdV$ the equation may be written

$$dE=dQ-PdV. \quad . \quad . \quad . \quad (37)$$

Here and in what follows we shall assume that quantities of heat are expressed in work units. This simplifies the equations by allowing the factor J or A to be omitted.

We are concerned for the present only with reversible operations. In any such operation $dQ=Td\phi$; hence

$$dE=Td\phi-PdV. \quad . \quad . \quad (38)$$

Again, $I=E+PV$, by definition of I.

Hence
$$\begin{aligned} dI&=dE+d(PV)\\ &=Td\phi-PdV+PdV+VdP\\ &=Td\phi+VdP. \quad . \quad . \quad . \quad . \quad . \quad (39)\end{aligned}$$

Again, $\zeta=I-T\phi$, by definition of ζ.

Hence
$$\begin{aligned} d\zeta&=dI-d(T\phi)\\ &=Td\phi+VdP-(Td\phi+\phi dT)\\ &=VdP-\phi dT. \quad . \quad . \quad . \quad . \quad (40)\end{aligned}$$

Again, $\psi=E-T\phi$, by definition of ψ.

Hence
$$\begin{aligned} d\psi&=dE-d(T\phi)\\ &=Td\phi-PdV-(Td\phi+\phi dT)\\ &=-PdV-\phi dT. \quad . \quad . \quad . \quad . \quad (41)\end{aligned}$$

But dE, dI, $d\zeta$, and $d\psi$ are all perfect differentials. Hence, applying equation (35) in turn to equations (38), (39), (40), and (41) we obtain at once the following four relations between partial differential coefficients:

From (38), $$\left(\frac{dT}{dV}\right)_\phi=-\left(\frac{dP}{d\phi}\right)_V. \quad . \quad . \quad (42)$$

From (39), $$\left(\frac{dT}{dP}\right)_\phi=\left(\frac{dV}{d\phi}\right)_P. \quad . \quad . \quad (43)$$

From (40), $$\left(\frac{dV}{dT}\right)_P=-\left(\frac{d\phi}{dP}\right)_T. \quad . \quad . \quad (44)$$

From (41), $$\left(\frac{dP}{dT}\right)_V=\left(\frac{d\phi}{dV}\right)_T. \quad . \quad . \quad (45)$$

These are known as Maxwell's four thermodynamic relations.

The following further relations are immediately deducible from equations (38) to (41). Taking equation (38), imagine the fluid to be heated at constant volume. Then $dV=0$ and $dE=Td\phi$; hence

$$\left(\frac{dE}{d\phi}\right)_V=T.$$

Again, imagine the fluid to expand adiabatically. Then $d\phi=0$ and $dE=PdV$; hence

$$\left(\frac{dE}{dV}\right)_\phi=-P.$$

Similarly from equation (39) we obtain

$$\left(\frac{dI}{d\phi}\right)_P=T, \text{ and } \left(\frac{dI}{dP}\right)_\phi=V;$$

from equation (40)

$$\left(\frac{d\zeta}{dP}\right)_T=V, \text{ and } \left(\frac{d\zeta}{dT}\right)_P=-\phi;$$

from equation (41)

$$\left(\frac{d\psi}{dV}\right)=-P, \text{ and } \left(\frac{d\psi}{dT}\right)_V=-\phi.$$

Collecting these results,

$$\left(\frac{dI}{d\phi}\right)_P=T=\left(\frac{dE}{d\phi}\right)_V, \quad . \quad . \quad (46)$$

$$\left(\frac{dI}{dP}\right)_\phi=V=\left(\frac{d\zeta}{dP}\right)_T, \quad . \quad . \quad (47)$$

$$\left(\frac{d\psi}{dV}\right)_T=-P=\left(\frac{dE}{dV}\right)_\phi, \quad . \quad . \quad (48)$$

$$\left(\frac{d\psi}{dT}\right)_V=-\phi=\left(\frac{d\zeta}{dT}\right)_P. \quad . \quad . \quad (49)$$

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[1] We have Z as a function of X and Y only. We may therefore write

$$Z=F(XY),$$

where F(XY) means some function of X and Y.

Hence $$dZ=\left(\frac{dF}{dX}\right)_Y dX+\left(\frac{dF}{dY}\right)_X dY.$$

Thus $$M=\left(\frac{dF}{dX}\right)_Y \text{ and } N=\left(\frac{dF}{dY}\right)_X.$$

Hence $$\left(\frac{dM}{dY}\right)_X=\frac{d^2F}{dYdX}=\left(\frac{dN}{dX}\right)_Y.$$

§ (48) Expressions for the Specific Heats K_v and K_p.—In general the specific heats of a fluid are not constant; they are functions of the state of the fluid. We shall proceed to find differential expressions connecting them with the temperature, volume, and pressure. Such expressions enable other properties to be calculated when the relation between T, V, and P is known.

Consider, as before, a small change of state during which the fluid takes in an amount of heat dQ while it expands in a reversible manner. Its entropy accordingly increases by an amount $d\phi$ such that $Td\phi = dQ$. Its temperature changes from T to $T + dT$ and its volume from V to $V + dV$. Take, in the first place, the temperature and volume as the two independent variables by means of which the state of the fluid is specified. The change in any third quantity may be stated with reference to the changes in T and in V. Thus the heat taken in may be written

$$dQ = K_v dT + l dV. \quad . \quad . \quad (50)$$

Here K_v, which is the specific heat at constant volume, is $(dQ/dT)_V$ and l is a symbol for $(dQ/dV)_T$.

Since $$dQ = Td\phi,\ l = T\left(\frac{d\phi}{dV}\right)_T.$$

But by equation (45)

$$\left(\frac{d\phi}{dV}\right)_T = \left(\frac{dP}{dT}\right)_V.$$

Hence $$l = T\left(\frac{dP}{dT}\right)_V, \quad . \quad . \quad (51)$$

and $$dQ = K_v dT + T\left(\frac{dP}{dT}\right)_V dV.$$

Dividing both sides by T, we have

$$d\phi = \frac{K_v}{T}dT + \left(\frac{dP}{dT}\right)_V dV. \quad . \quad . \quad (52)$$

This is a perfect differential, and, therefore, by equation (35),

$$\left(\frac{d}{dV}\right)_T\left(\frac{K_v}{T}\right) = \left(\frac{d}{dT}\right)_V\left(\frac{dP}{dT}\right)_V.$$

Hence $$\frac{1}{T}\left(\frac{dK_v}{dV}\right)_T = \left(\frac{d^2P}{dT^2}\right)_V,$$

or $$\left(\frac{dK_v}{dV}\right)_T = T\left(\frac{d^2P}{dT^2}\right)_V. \quad . \quad . \quad (53)$$

This is an important property of K_v.

To obtain a corresponding property of K_p, take the temperature and pressure as the two independent variables and express the heat taken in with reference to them. The heat taken in, dQ, is the same as before, being still equal to $Td\phi$. We may write

$$dQ = K_p dT + l' dP. \quad . \quad . \quad (54)$$

Here K_p, which is the specific heat at constant pressure, is $(dQ/dT)_P$ and l' is a symbol for $(dQ/dP)_T$.

Since $$dQ = Td\phi,\ l' = T\left(\frac{d\phi}{dP}\right)_T.$$

But by equation (44)

$$\left(\frac{d\phi}{dP}\right)_T = -\left(\frac{dV}{dT}\right)_P.$$

Hence $$l' = -T\left(\frac{dV}{dT}\right)_P \quad . \quad . \quad . \quad (55)$$

and $$dQ = K_p dT - T\left(\frac{dV}{dT}\right)_P dP.$$

Dividing both sides by T, we have, since $dQ = Td\phi$,

$$d\phi = \frac{K_p}{T}dT - \left(\frac{dV}{dT}\right)_P dP \quad . \quad (56)$$

And by equation (35), since this is a perfect differential,

$$\left(\frac{d}{dP}\right)_T\left(\frac{K_p}{T}\right) = -\left(\frac{d}{dT}\right)_P\left(\frac{dV}{dT}\right)_P.$$

Hence $$\frac{1}{T}\left(\frac{dK_p}{dP}\right)_T = -\left(\frac{d^2V}{dT^2}\right)_P$$

or $$\left(\frac{dK_p}{dP}\right)_T = -T\left(\frac{d^2V}{dT^2}\right)_P, \quad . \quad . \quad (57)$$

which is the property of K_p corresponding to that of K_v in equation (53).

Further, from equations (50) and (54)

$$K_p dT + l' dP = K_v dT + l dV$$

or $$(K_p - K_v)dT = l dV - l' dP.$$

By writing $dP = 0$ it follows that

$$K_p - K_v = l\left(\frac{dV}{dT}\right)_P.$$

Or by writing $dV = 0$,

$$K_p - K_v = -l'\left(\frac{dP}{dT}\right)_V.$$

By equation (51) or (55), either of these gives this important expression for the difference between the two specific heats,

$$K_p - K_v = T\left(\frac{dP}{dT}\right)_V\left(\frac{dV}{dT}\right)_P. \quad . \quad (58)$$

And since by equation (34)

$$\left(\frac{dV}{dT}\right)_P = -\left(\frac{dV}{dP}\right)_T\left(\frac{dP}{dT}\right)_V,$$

this result may be written

$$K_p - K_v = -T\left(\frac{dV}{dP}\right)_T\left(\frac{dP}{dT}\right)_V^2. \quad . \quad (58a)$$

From this it will be seen that K_p can never be less than K_v, for $(dV/dP)_T$ is essentially negative, increase of pressure causing decrease of volume in any fluid, and therefore the whole expression on the right is positive. Accordingly K_p is always greater than K_v, except in the special case when one of the factors on the right-hand side is equal to zero, in which case K_p is equal to K_v. This is possible in a fluid which has a temperature of maximum density (as water has at about 4° C.). At the temperature of maximum density $(dV/dT)_p=0$, and consequently at that point $K_p-K_v=0$.

Return now to equations (52) and (56). In heating at constant volume $dV=0$; hence by equation (52)

$$K_v=T\left(\frac{d\phi}{dT}\right)_V . \quad . \quad . \quad (59)$$

In heating at constant pressure $dP=0$; hence by equation (56)

$$K_p=T\left(\frac{d\phi}{dT}\right)_P . \quad . \quad . \quad (60)$$

In an adiabatic operation $d\phi=0$; hence by equation (52)

$$\frac{K_v}{T}\left(\frac{dT}{dV}\right)_\phi=-\left(\frac{dP}{dT}\right)_V \quad . \quad . \quad (61)$$

and by equation (56)

$$\frac{K_p}{T}\left(\frac{dT}{dP}\right)_\phi=\left(\frac{dV}{dT}\right)_P . \quad . \quad . \quad (62)$$

Further, by equation (34*b*)

$$\left(\frac{dV}{dT}\right)_T=-\frac{\left(\frac{dV}{dT}\right)_P}{\left(\frac{dP}{dT}\right)_V}=\frac{\frac{K_p}{T}\left(\frac{dT}{dP}\right)_\phi}{\frac{K_v}{T}\left(\frac{dT}{dV}\right)_\phi}=\frac{K_p}{K_v}\left(\frac{dV}{dP}\right)_\phi$$

or

$$\frac{K_p}{K_v}=\left(\frac{dV}{dP}\right)_T\left(\frac{dP}{dV}\right)_\phi . \quad . \quad . \quad (63)$$

This is the ratio usually called γ.

Thus in the adiabatic expansion of any fluid the slope of the pressure-volume line is γ times its slope in isothermal expansion, for

$$\left(\frac{dP}{dV}\right)_\phi=\gamma\left(\frac{dP}{dV}\right)_T \quad . \quad . \quad (63a)$$

§ (49) OTHER GENERAL RELATIONS.—By equation (37)

$$dE=dQ-PdV.$$

Hence by equation (50)

$$\begin{aligned}dE&=K_vdT+ldV-PdV\\&=K_vdT+(l-P)dV.\end{aligned}$$

In heating at constant volume $dV=0$; hence

$$\left(\frac{dE}{dT}\right)_V=K_v. \quad . \quad . \quad . \quad (64)$$

In isothermal expansion $dT=0$; hence, using equation (51),

$$\left(\frac{dE}{dV}\right)_T=l-P=T\left(\frac{dP}{dT}\right)_V-P. \quad . \quad (65)$$

We may therefore write

$$dE=K_vdT+\left[T\left(\frac{dP}{dT}\right)_V-P\right]dV. \quad (66)$$

Again, by equation (39)

$$dI=dQ+VdP.$$

Hence by equation (54)

$$\begin{aligned}dI&=K_pdT+l'dP+VdP\\&=K_pdT+(l'+V)dP.\end{aligned}$$

In heating at constant pressure $dP=0$; hence

$$\left(\frac{dI}{dT}\right)_P=K_p. \quad . \quad . \quad . \quad (67)$$

In isothermal compression $dT=0$; hence, using equation (55),

$$\left(\frac{dI}{dP}\right)_T=l'+V=V-T\left(\frac{dV}{dT}\right)_P . \quad . \quad (68)$$

We may therefore write

$$dI=K_pdT+\left[V-T\left(\frac{dV}{dT}\right)_P\right]dP. \quad . \quad (69)$$

A few other general relations may be mentioned which are easily derived from those already given:[1]

$$\left(\frac{dI}{d\phi}\right)_T=T+V\left(\frac{dP}{d\phi}\right)_T=T-V\left(\frac{dT}{dV}\right)_P; \quad (70)$$

$$\left(\frac{dI}{d\phi}\right)_V=T+V\left(\frac{dP}{d\phi}\right)_V=T-V\left(\frac{dT}{dV}\right)_\phi; \quad (71)$$

$$\left(\frac{dI}{dP}\right)_V=V+K_v\left(\frac{dT}{dP}\right)_V=V-T\left(\frac{dV}{dT}\right)_\phi; \quad (72)$$

$$\left(\frac{dI}{dV}\right)_P=T\left(\frac{dP}{dT}\right)_\phi; \quad . \quad . \quad (73)$$

$$\left(\frac{dE}{dP}\right)_T=-T\left(\frac{dV}{dT}\right)_P-P\left(\frac{dV}{dP}\right)_T \quad (74)$$

§ (50) THE JOULE-THOMSON COOLING EFFECT.—In a throttling process $dI=0$: hence, from equation (69),

$$\left(\frac{dT}{dP}\right)_I=\frac{1}{K_p}\left[T\left(\frac{dV}{dT}\right)_P-V\right]. \quad . \quad (75)$$

This is the "cooling effect" in the Joule-Thomson porous plug experiment of § (12); the cooling effect which the working fluid of a refrigerating machine undergoes in passing the expansion valve; the cooling effect used cumulatively by Linde for the liquefaction of gases. It expresses the fall of temperature per unit fall of pressure when any fluid suffers a throttling operation, during which it receives no heat from outside, nor takes in any.

From equation (75) it follows that the cooling effect vanishes when

$$\left(\frac{dV}{dT}\right)_P=\frac{V}{T}.$$

This occurs in any ideal "perfect" gas under all conditions, that is to say, in a gas which exactly satisfies the equation $PV=RT$, for then

$$\left(\frac{dV}{dT}\right)_P=\frac{R}{P}=\frac{V}{T}.$$

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[1] See Ewing's *Thermodynamics for Engineers*, vii.

But it also occurs in real gases under particular conditions of temperature and pressure. A gas tested for the Joule-Thomson effect at moderate pressure, and at various temperatures, will be found to become warmer instead of colder on passing the plug if the temperature exceeds a certain value. At that temperature, which is called the temperature of inversion of the Joule-Thomson effect, throttling produces no change of temperature. Above the temperature of inversion the effect of passing the plug is to heat the gas; $(dV/dT)_P$ is then less than V/T and the expression for the "cooling effect" is negative. Below the temperature of inversion the cooling effect is positive. The temperature of inversion depends to some extent on the pressure, in any one gas. It differs widely in different gases. In air, oxygen, carbonic acid, steam, and most other gases it is so high that the normal effect of throttling is to make the gas colder; in hydrogen, on the other hand, the normal effect of throttling is to make the gas warmer, for the temperature of inversion is exceptionally low, about −80° C. In the Linde process of liquefaction it is essential that the gas to be liquefied should enter the apparatus at a temperature below its temperature of inversion: the process can be applied to hydrogen only by cooling the gas beforehand to a suitably low temperature.

Taking equations (68) and (75) together we have

$$K_p\left(\frac{dT}{dP}\right)_I = T\left(\frac{dV}{dT}\right)_P - V = -\left(\frac{dI}{dP}\right)_T. \quad (76)$$

This product, $K_p(dT/dP)_I$, is the quantity of heat that would just suffice to neutralise the Joule-Thomson cooling effect per unit drop in pressure, if it were supplied to the fluid in the process of throttling. It may conveniently be represented by the single symbol ρ. It measures the cooling effect, per unit drop in pressure by throttling, as a quantity of heat (expressed in work units), while $(dT/dP)_I$ measures that effect as a change in temperature.

It follows that if the range through which the pressure falls in a throttling process is from P_A to P_B, the whole quantity of heat that would have to be supplied to neutralise the cooling effect is

$$\int_{P_B}^{P_A} \rho dP = \int_{P_B}^{P_A}\left[T\left(\frac{dV}{dT}\right) - V\right]dP.$$

Since $I = E + PV$, we may write equation (76) in the form

$$\rho = -\left(\frac{dE}{dP}\right)_T - \left(\frac{d(PV)}{dP}\right)_T. \quad . \ (76a)$$

This is instructive as showing the analysis of the Joule-Thomson effect into two parts. When an imperfect gas or vapour is throttled, that part of the effect which is measured by the first term arises from the fact that the internal energy is not constant at any one temperature but depends to some extent on the pressure. In other words, the first term is due to departure from Joule's Law. There is in general an additional part of the effect, measured by the second term. It is due to departure from Boyle's Law, according to which PV should be constant for constant temperature. A gas may conform to Boyle's Law at a particular temperature and still be imperfect: in that case it will show a cooling effect due to the first term alone. It is only when both terms vanish that the gas is perfect in the thermodynamic sense.

Experiments on the compressibility of gases by Amagat and others, which will be further referred to later (§ (59)), show that in an imperfect gas the term $(d(PV)/dP)_T$ may be either negative or positive according to the conditions of pressure and temperature. Hence that part of the Joule-Thomson effect which is due to deviation from Boyle's Law will under some conditions assist, and under other conditions oppose, that part of the effect which is due to deviation from Joule's Law. The latter part is always a cooling effect; the former may be either a cooling or a heating effect. At the temperature of inversion the two parts cancel one another.

§ (51) THE THERMODYNAMIC POTENTIALS. HETEROGENEOUS SYSTEMS.—We saw in § (25) that the entropy of a system of bodies might be treated as a whole by summing up the entropies of the various parts. This is also true of the total heat, the energy, and the functions ψ and ζ. In physical chemistry it is important to consider the functions ψ and ζ not only for a single substance, or a mixture of phases of a single substance, but for a heterogeneous system made up of different substances that may be forming solutions or reacting chemically on one another, such, for example, as the substances in a galvanic cell. The action of a galvanic cell will be considered more particularly in a later paragraph (§ (64)), but certain properties of these functions may be mentioned here. It is essential to notice, in dealing with a heterogeneous system, that it may do work otherwise than by change of volume. Changes may take place within the system which do work against gravity, by raising part of the system to a higher level; or they may do work electrically by producing a current in an external circuit. It will be obvious that in any such case dW must not be taken as equal to PdV. There may even be no change of volume. But in whatever way work is done the energy equation holds

$$dW = dQ - dE,$$

where dW is the work done by the system,

dQ is the heat it takes in, and $-dE$ is its loss of internal energy.

Since
$$\zeta = I - T\phi,$$
$$d\zeta = dI - Td\phi - \phi dT.$$

This is true of any change, reversible or not. Let the imposed conditions be such that the temperature is kept constant during the change, then $\phi dT = 0$. This condition would be realised if the system were surrounded by a capacious reservoir of heat, and the change were to take place very slowly. If the pressure also be kept constant dI is equal to dQ, the heat taken in. Hence under these conditions we have

$$d\zeta = dQ - Td\phi.$$

Now $d\phi$ is equal to dQ/T in any reversible change, but is greater than dQ/T in an irreversible change (§ (24)). It follows that in a change under the prescribed conditions of constant temperature and constant pressure $d\zeta$ vanishes only if the change be reversible, and is a negative quantity if the change be irreversible, $Td\phi$ being then greater than dQ. Hence if the system were to be a little disturbed from thermal equilibrium, with the result that for a time there is irreversible action, ζ is diminishing while the system adjusts itself so that stable equilibrium is restored. Accordingly the fact that $d\zeta$ is zero in the condition of equilibrium is to be interpreted as meaning that ζ is then a minimum. Thus under the stated conditions of constant temperature and constant pressure the criterion of stable equilibrium is that the function ζ for the system as a whole shall be a minimum.

Returning to the equation

$$d\zeta = dI - Td\phi - \phi dT,$$

in a reversible change at constant pressure $Td\phi = dI$ and, by equation (49), $(d\zeta/dT)_P = -\phi$. Substituting this in the expression for ζ, we have

$$\zeta = I + T\left(\frac{d\zeta}{dT}\right)_P . \quad . \quad . \quad . \quad (77)$$

Again, since
$$\psi = E - T\phi,$$
$$d\psi = dE - Td\phi - \phi dT.$$

Hence in a reversible change at constant temperature
$$d\psi = dE - dQ$$
or
$$-d\psi = dW . \quad . \quad . \quad . \quad (78)$$

Therefore, if a system changes reversibly by a finite amount, at constant temperature, from state (a) to state (b),

$$\psi_a - \psi_b = W, \quad . \quad . \quad . \quad (78a)$$

where W is the work done (in any manner) during the change. In other words, the decrement of ψ measures the amount of energy actually converted into external work by a system, whether by expansion of volume, or by generating electricity, or otherwise, during any isothermal reversible process. If the process were not reversible the work done would be less. Thus the decrement of ψ measures how much of the energy of the system can, in the most favourable case, be converted into work while the system undergoes a change at constant temperature. It consequently also measures how much the energy of the system has lost of *availability* for further conversion into work under isothermal conditions. For this reason Helmholtz (*Wied. Ann.*, 1879, vii. 337) called the function ψ the Free Energy of the system, regarding the whole energy E as made up of two parts, namely the "free" or available energy ψ and the "bound" energy $T\phi$. It should, however, be borne in mind that during the conversion some heat may be taken in from or given out to the isothermal envelope, in keeping the temperature of the system constant. Following Helmholtz, many writers on the thermodynamics of chemical processes speak of ψ as the "free energy."

From the above equation it also follows that in a system which is maintained at constant temperature $d\psi$ is zero for a change that does not involve the doing of any external work, when the change is reversible, but is negative when the change is irreversible. Hence if the system be such that work can be done only by expansion of volume, its criterion of stable equilibrium at constant temperature and constant volume is that the function ψ for the system as a whole shall be a minimum.

To these functions Willard Gibbs (*Collected Works*, i. 93) gave the name of "Potentials," from their analogy to the Potential function in statics. On account of the properties which have been indicated above, ζ is called the Thermodynamic Potential at constant pressure, and ψ is called the Thermodynamic Potential at constant volume.

Returning to the equation

$$d\psi = dE - Td\phi - \phi dT,$$

since in a reversible change

$$dE - Td\phi = dE - dQ = -dW,$$
$$d\psi = -dW - \phi dT.$$

Hence if the change is such that no external work (of any kind) is done,

$$d\psi = -\phi dT$$

or

$$\left(\frac{d\psi}{dT}\right)_W = -\phi,$$

where the suffix implies that the partial differential coefficient $d\psi/dT$ is the rate at which ψ increases with the temperature when no work is being done. Substituting this in the expression $\psi = E - T\phi$, we have

$$\psi = E + T\left(\frac{d\psi}{dT}\right)_W. \qquad (79)$$

In applying this equation, or the foregoing one for ζ, to a system which undergoes any reversible transformation at constant temperature—as, for instance, to a galvanic cell (§ (64))—we are concerned only with changes in E and in ψ or ζ, not with the absolute values of these quantities. In such applications the symbols E, ψ and ζ may accordingly be taken as standing for corresponding finite changes in the quantities they name.

§ (52) Change of Phase.—The function ζ has the important property of remaining unchanged during any change of phase which a substance may undergo at constant temperature and constant pressure, as for instance during the melting of a solid or the vaporisation of a liquid under constant pressure. This follows from equation (40), according to which, in a reversible operation,

$$d\zeta = VdP - \phi dT.$$

In a change of phase at constant pressure and temperature the operation is reversible and dP and dT are both zero, therefore $d\zeta$ is zero. Thus the function ζ has the same value for unit mass of a saturated vapour as for unit mass of the liquid at the same pressure and temperature, or for unit mass of a wet mixture made up of the liquid and the vapour in equilibrium. Comparing the states before and after vaporisation we have $\zeta_s = \zeta_w$. Hence if the condition as to pressure and temperature under which the fluid is vaporised be slightly altered, the pressure changing from P to $P + dP$ and the saturation temperature accordingly changing from T to $T + dT$, we have

$$d\zeta_s = d\zeta_w.$$

By equation (40) this gives

$$V_s dP - \phi_s dT = V_w dP - \phi_w dT$$

or

$$V_s - V_w = (\phi_s - \phi_w)\frac{dT}{dP}. \qquad (80)$$

But $\phi_s - \phi_w$, the change of entropy during vaporisation under constant pressure, is equal to L/T. We thus obtain Clapeyron's Equation, already given in § (41),

$$V_s - V_w = \frac{L}{T}\frac{dT}{dP}.$$

Here the differential coefficient dT/dP is to be understood as meaning the rate at which the saturation temperature changes with the pressure, or more generally, the rate at which the temperature, in any isothermal change of phase, varies with the pressure under which that change of phase occurs, V_s and V_w being the specific volumes in the two phases.

§ (53) Specific Heat of Saturated Vapour.—Rankine, in 1850, introduced a quantity K_s which he named the specific heat of saturated vapour. This is the quantity of heat required to raise the temperature of unit mass of the saturated vapour by one degree while the pressure and the volume alter so that the vapour is maintained in a state of saturation. Usually K_s is a negative quantity, that is to say, heat has to be removed to keep the vapour saturated as the temperature rises, but in certain fluids it may be positive over a limited range of temperatures. Its character will be most readily understood by referring to the boundary curve of the entropy-temperature diagram (such as *Fig.* 16). Imagine a saturated vapour, the state of which is represented by a point on the saturation curve forming the right-hand limb of the boundary curve, to have its temperature raised by the small step dT but to remain saturated, so that the step occurs along the saturation line. By definition of the specific heat of saturated steam, the quantity of heat taken in is $K_s dT$. That this is usually negative will be obvious from the fact that in general the line for saturated vapour slopes back to the left as it ascends. When that is the case the entropy of the saturated vapour becomes less as the temperature rises, and the area under the step represents removal, not supply, of heat. The step being reversible the heat communicated is $Td\phi$, which in this case is $Td\phi_s$. Hence we have $K_s dT = Td\phi_s$, from which

$$K_s = T\frac{d\phi_s}{dT}. \qquad (81)$$

The specific heat of saturated vapour is therefore negative under all conditions that make the entropy of the saturated vapour decrease with increasing temperature. In many substances, such as water, as the form of the entropy-temperature diagram in *Fig.* 16 shows, this is true at all temperatures from the critical point downwards. K_s becomes positive only when the saturation line of the entropy-temperature diagram slopes up to the right, making the entropy of the saturated vapour increase with rising temperature. In some other fluids the saturation line bends in such a manner as to make K_s positive throughout a part of its course. This is exemplified in the entropy-temperature diagrams (*Figs.* 18 and 19) for benzene and acetic acid (*Phil. Mag.*, June 1920). In all fluids the rounded top of the diagram shows that K_s is negative as the critical point is approached; it increases in the negative sense as the temperature rises, and becomes equal to minus infinity at the critical temperature.

By drawing adiabatic lines (ϕ = const.) on the entropy-temperature diagram, starting from points on the saturation curve, it will be obvious that when K_s is negative (as in steam at any temperature) a vapour becomes supersaturated or partially condensed when it suffers adiabatic expansion, and becomes superheated when it suffers adiabatic compression; conversely when K_s is negative (as for example in benzene vapour at about 160°) adiabatic expansion causes superheating, and adiabatic compression supersaturates the vapour or makes part of it condense. Experiments by Cazin (*Ann. de Chim. et de Phys.*, 1868, xiv. 374) show that in vapours such as benzene this in fact occurs within a limited range of temperature.

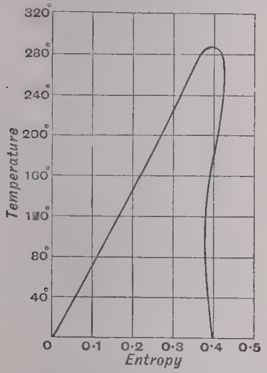

FIG. 18.—Entropy-temperature Diagram for Benzene.

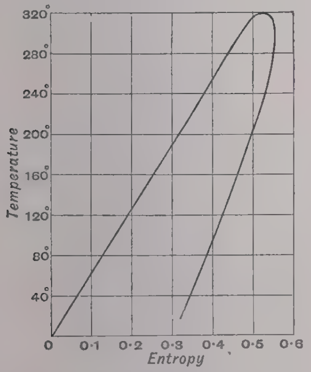

FIG. 19.—Entropy-temperature Diagram for Acetic Acid.

Analogous to the quantity K_s for saturated vapour there is a quantity K_w which may be called the *specific heat of saturated liquid.* It is the quantity of heat required to raise the temperature of unit mass of the liquid by one degree while the pressure and the volume alter so that the liquid is maintained on the point of boiling throughout the operation, though no vapour is formed. In other words, it is that specific heat which would correspond to a step up along the liquid boundary curve of the entropy-temperature diagram. In any such step the quantity of heat taken in would by definition be $K_w dT$ and would be equal to $Td\phi_w$. Thus

$$K_w = T\frac{d\phi_w}{dT}. \quad . \quad . \quad . \quad (82)$$

K_w is positive in all liquids at all temperatures. When the temperature is much lower than the critical temperature the numerical value of K_w does not differ greatly from that of the specific heat of the liquid at constant pressure, K_p: as the critical point is approached K_w increases, and at the critical temperature it is infinite. A relation between K_s and K_w, originally given by Clausius, may be obtained very directly as follows. In any fluid the entropy increases during vaporisation at constant temperature by the amount L/T. Thus

$$\phi_s = \phi_w + \frac{L}{T}.$$

On differentiating this with respect to T and multiplying by T we have

$$T\frac{d\phi_s}{dT} = T\frac{d\phi_w}{dT} + \frac{dL}{dT} - \frac{L}{T}$$

or

$$K_s = K_w + \frac{dL}{dT} - \frac{L}{T}. \quad . \quad . \quad (83)$$

To obtain relations between the saturation specific heats K_w and K_s and the specific heats at constant pressure and at constant volume, for liquid and vapour respectively, we may proceed thus. In any heating operation, the heat taken in (by equations (54) and (55)) is

$$dQ = K_p dT - T\left(\frac{dV}{dT}\right)_P dP.$$

Hence in a step up along the boundary curve, whether on the liquid or vapour branch,

$$\frac{dQ}{dT} = K_p - T\left(\frac{dV}{dT}\right)_P \frac{dP}{dT},$$

giving for the liquid

$$K_w = K_p - T\left(\frac{dV_w}{dT}\right)_P \frac{dP}{dT}, \quad . \quad (84)$$

and for the vapour

$$K_s = K_p - T\left(\frac{dV_s}{dT}\right)_P \frac{dP}{dT}, \quad . \quad . \quad (85)$$

V_s being, as before, the volume of the saturated vapour and V_w that of the liquid.

Again, in any heating operation (by equations (50) and (51))

$$dQ - K_v dT + T\left(\frac{dP}{dT}\right)_V dV.$$

Hence in a step up along either branch of the boundary curve

$$\frac{dQ}{dT}=K_v+T\left(\frac{dP}{dT}\right)_V\frac{dV}{dT},$$

where dV/dT is the rate of change of the saturation volume (of liquid or vapour) with the temperature.

This gives, for the liquid,

$$K_w=K_v+T\left(\frac{dP}{dT}\right)_V\frac{dV_w}{dT}, \quad . \quad (86)$$

and for the vapour

$$K_s=K_v+T\left(\frac{dP}{dT}\right)_V\frac{dV_s}{dT}. \quad . \quad (87)$$

E. Mathias (*Comptes Rendus*, vol. cxix.) has described a calorimetric method of determining in any fluid the mean value of both K_s and K_w over a given range of temperature, by partially filling a closed vessel of constant volume with the liquid, and then heating the resulting mixture of liquid and vapour in the vessel through the given range, observing the heat that is taken in. On repeating the experiment after varying the quantity of the mixture in the closed vessel, data are obtained for evaluating both of these specific heats. He has applied his method to sulphur dioxide and has found that in the vapour of that substance K_s is positive throughout a certain short range of temperature, as it is in benzene, ether, chloroform, carbon tetrachloride, and various other fluids.

§ (54) INFLUENCE OF SURFACE TENSION ON CONDENSATION AND EBULLITION.—In connection with the subject of change of phase, reference should be made to certain thermodynamic effects of surface tension. In consequence of surface tension, the skin of a liquid at any surface which separates liquid from vapour is the seat of a definite amount of potential energy, and this affects the formation of drops in the process of condensing a vapour, or of bubbles in the process of boiling a liquid. One result of surface tension is to make the conditions of equilibrium between the liquid and the vapour depend on the curvature of the liquid surface. It is only when the surface is flat that the normal conditions of equilibrium between the vapour and the liquid apply, namely that when both are at the same temperature the vapour should have what we are accustomed to call the saturation pressure corresponding to that temperature. In consequence of surface tension a small drop will evaporate into an atmosphere that would be saturated or even supersaturated with respect to large drops or large quantities of the liquid, because the vapour pressure which is required to prevent evaporation from the curved surface of a small drop is greater than the vapour pressure which will prevent evaporation from a flat surface of the same liquid at the same temperature. In other words, the equilibrium vapour pressure for a small drop is higher than the normal saturation pressure.

The cohesive forces which the molecules of any liquid exert upon one another make the free surface of the liquid behave as if it were a stretched elastic skin. It is to this that the phenomena of capillarity are due—the rise of a liquid column in a tube when the liquid is one that wets it, and the depression of the column when the liquid does not wet the tube. To this also are due the forms assumed by liquid films and by drops. It is the tension of the surface layer that makes a drop take a spherical shape when there are no disturbing forces: the drops of molten metal in a shot-tower, for example, become spheres as they fall freely, and solidify into spherical shot before they reach the bottom. A drop of mercury on a plate, or of dew on a leaf which it does not wet, would be spherical were it not for the upward pressure of the support on which the drop rests; the smaller the drop is the nearer does it come to being a sphere, for the disturbing forces due to gravity are relatively unimportant in a small drop. As a result of surface tension, the energy contained in a drop of liquid is greater than the energy contained in an equal quantity when that forms part of a big mass of the same liquid at the same temperature, for energy is stored in the surface layer in much the same way as it would be stored by the stretching of an elastic skin. For the same reason a small drop contains more energy per unit of mass than a large one (of the same liquid at the same temperature), for the surface of the small drop has a relatively larger area. The energy stored in the surface layer is directly proportional to the area of the layer. To see that this is so, consider the amount of work that has to be spent in forming a thin film of a liquid. The film that is formed when a bubble is blown, or when a soapy solution is smeared over a ring or hoop of wire, consists of two surface layers, back to back, with some of the liquid between. When the film is very thin, as, for instance, when it looks black in reflected light just before it breaks, it may be said to consist of two surface layers only; but it can be made a hundred or more times thicker than that and have the same tension, for the state of tension exists in the surface layers only. The tension of such a film, whether thick or thin, is the tension of two surface layers; in other words, it is twice the surface tension. It does not change when the film contracts or is stretched, and it has necessarily the same value in all directions along the surface.

Let a liquid film be formed on a U-shaped frame (*Fig.* 20) by wetting a wire AB with the liquid, placing it over C, and then drawing it away in the direction of the arrow. The force that will have to be applied to draw it away or to hold it from coming back is $2Sl$, where l is the length AB and S is the tension of the surface layer on each side of the film per unit of length. The quantity S so defined measures the surface tension of the liquid. In drawing the rod away through any distance x in the direction of the arrow the work done in forming the film is $2Slx$, and this work is stored in the two surface layers of the film, for it is recoverable by letting the rod slip back. Hence the energy stored in a single surface layer, in consequence of surface tension, is numerically equal to S per unit area of surface. It follows that the surface energy of a spherical drop of radius r is $4\pi r^2S$. S is a quantity to be determined by experiment in any liquid; it is a function of the temperature, becoming smaller when the temperature is raised.

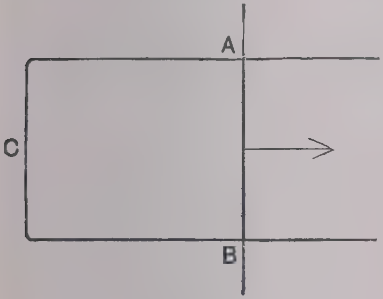

FIG. 20.

The spherical form which a free drop assumes is the form which will make the surface energy (for a given volume) a minimum. A drop resting on a support takes such a form as will make its total potential energy a minimum, namely the sum of the energy of surface tension and the energy of position which the drop possesses in consequence of the height of its centre of gravity above the level of the support.

Imagine now a drop to be evaporating under conditions that keep its temperature constant. Energy has to be supplied in proportion to its loss of mass to provide for the latent heat of the vapour that is formed. But the drop is losing surface energy because its surface is getting less, and to some extent this reduction of surface energy supplies the latent heat that is required; only the remainder has to be supplied from outside the drop. Consequently a drop will continue to evaporate into an atmosphere which would be saturated with respect to the same liquid in bulk. There can be no equilibrium between a drop and a surrounding atmosphere of saturated vapour. As the drop gets smaller, a stage is reached when the loss of potential energy due to contraction of the surface is sufficient to supply all the latent heat of the vapour that is passing off. After that the evaporation of the drop would complete itself without any further supply of heat, if surface tension continued to operate in the same way.

For the same reason a drop cannot form except around a nucleus, and the larger the nucleus the more readily it forms. To make drops form, the surrounding vapour must be supersaturated to an extent which depends on the smallness of the nuclei. When particles of dust are present in expanding vapour, the first drops to be formed use them for nuclei, as was shown by Aitken (*Trans. R.S.E.* vol. xxx.), and only a small amount of supersaturation is required before such drops begin to form.

Experiments by C. T. R. Wilson (*Phil. Trans.*, 1887, 1889) show that when dust-free air containing water-vapour is suddenly expanded clouds are formed, but only when there is much supersaturation of the vapour. The water particles composing these clouds are formed around nuclei which may consist of accidental coaggregations of the molecules of the gas itself, or of electrically charged molecules, such as are always present in small numbers. The presence of an electric charge greatly favours condensation of the vapour upon any nucleus. As an electrified drop evaporates, the charge remains behind; the potential energy due to electrification therefore increases as the drop becomes smaller, for the energy due to a constant electric charge varies inversely as the radius of the sphere that carries it. In this respect the effect of an electric charge is opposite to that of surface tension. Hence when a drop is charged more energy has to be supplied from outside to make it evaporate than would be required if it were uncharged. An electrically charged drop will therefore evaporate less readily than an uncharged drop of the same size, and may grow larger in an atmosphere that is but little supersaturated or even not supersaturated at all. In vapour which is slightly supersaturated it is found that any ionising action, such as that of an electric spark, or of Röntgen rays, or of ultraviolet light, brings about a cloud of condensation, by creating fresh nuclei, or by stimulating the powers of existing nuclei through causing them to acquire an electric charge.

Confining ourselves, however, to cases in which there is no electrification, we may consider how, as a consequence of surface tension, the equilibrium of liquid and vapour

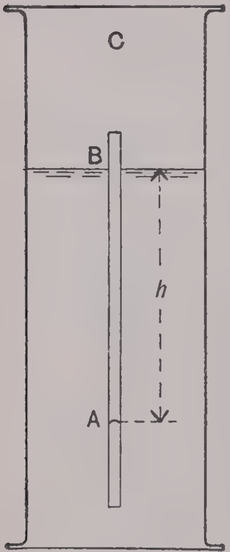

FIG. 21.

depends on the curvature of the surface between them. Assume the liquid and the vapour to be at the same temperature. Liquid with a flat surface is in equilibrium with the vapour above it when the vapour is at the pressure of saturation: there is then no tendency on the whole for the liquid to evaporate or for the vapour to condense, any evaporation that occurs being exactly balanced by an equal amount of condensation. Liquid with a convex curved surface is in equilibrium with the vapour only when the pressure of the vapour exceeds the normal saturation pressure by a definite amount; in other words, only when the vapour is sufficiently supersaturated. The degree of supersaturation necessary for equilibrium depends on the curvature of the surface, in a manner first established by Kelvin (*Proc. R.S.E.*, 1870). Let P_s be the normal pressure of saturation (namely, the saturation pressure for any assigned temperature as given in tables of the properties of saturated steam, which is the equilibrium vapour pressure over the flat surface of a liquid) and let P' be the equilibrium vapour pressure over a curved surface. Take for this purpose the curved surface at A, *Fig.* 21, which is formed by holding in the liquid a capillary tube of a material such that the liquid does not wet it. The column of liquid in the tube is accordingly depressed through some distance h, and if the bore is small enough the free surface at A is sensibly part of a sphere. Imagine the liquid to be contained in a closed vessel, and that the space C above it contains nothing but the vapour of the liquid. Let all be at one temperature T, and in equilibrium. Over the flat surface at B there is vapour whose pressure is P_s: over the curved surface at A there is vapour of a higher pressure P'. The difference $P'-P_s$ is equal to the weight of the column of vapour in the tube (per unit area of cross section) from the level of A to the level of B. Let σ be the weight of unit volume of the vapour. If this were constant, the weight of the column of vapour in the tube (per unit area of section) would be simply σh. But σ depends upon the pressure P; it is equal to $1/V$ and may therefore be written

$$\sigma=\frac{P}{RT},$$

if we take the equation $PV=RT$ to apply, as it approximately does. The difference in the two vapour pressures is

$$P'-P_s=\int\sigma dh,$$

integrated between the level at B and the level at A.

Compare next the hydrostatic pressures within the liquid just under the surface at B and just under the surface at A. Just under the flat surface at B the hydrostatic pressure in the liquid is equal to the pressure of the vapour over the surface; it is therefore equal to P_s. As we go down through the liquid to the level of A, the hydrostatic pressure increases by the amount ρh, where ρ is the weight of unit volume of the liquid. Therefore just under the curved surface at A its value is $P_s+\rho h$. But the hydrostatic pressure under the curved surface at A may also be calculated in another way. The top of the liquid column at A, which has a surface layer in tension, may be treated as a segment of a sphere of radius r. Its surface layer forms a cap whose surface tension S causes it to press down upon the liquid below with a pressure p such that $\pi r^2 p=2\pi r S$. That this is so will be seen by considering the equilibrium of a complete hemisphere of the same curvature and with the same surface tension. Round the circumference ($2\pi r$) of the horizontal plane forming the base of such a hemisphere there would be a vertical force $2\pi r S$ balancing the resultant force due to the base pressure p acting on the area of πr^2. Hence

$$p=\frac{2S}{r},$$

and the hydrostatic pressure just under the curved surface is therefore equal to

$$P'+\frac{2S}{r}.$$

Equating the two expressions for this hydrostatic pressure, we have

$$P'+\frac{2S}{r}=P_s+\rho h$$

or

$$\frac{2S}{r}=\rho h-(P'-P_s).$$

Hence, since $P'-P_s=\int\sigma dh$,

$$\frac{2S}{r}=\rho h-\int\sigma dh=\int(\rho-\sigma)dh.$$

And since $dP = \sigma dh$,

$$\frac{2S}{r} = \int_{P_s}^{P'} \frac{\rho - \sigma}{\sigma} dP = \int_{P_s}^{P'} \frac{\rho}{\sigma} dP \text{ very nearly,}$$

because σ is small compared with ρ.

On substituting P/RT for σ this approximation gives

$$\frac{2S}{r} = RT\rho \int_{P_s}^{P'} \frac{dP}{P} = RT\rho \log_e \frac{P'}{P_s}$$

or

$$\log_e \frac{P'}{P_s} = \frac{2S}{RT\rho r}.$$

Applied to a spherical drop of radius r this (approximately) expresses how big must be the pressure P′ in the supersaturated vapour around the drop in relation to the normal pressure of saturation P_s for the same temperature, if the drop is just to escape shrinking by evaporation. Any increase of P′ above the value so calculated would cause the drop to grow. The expression also shows what is the least size of drop that can exist in an atmosphere with a given degree of supersaturation: any drop for which r is smaller would disappear by evaporation; on the other hand any drop for which r is larger would grow.

It is only when the drop is very small that the excess of P′ over P_s is considerable. With water-vapour at 10° C. RT (which is treated as equal to PV) is $1{\cdot}30 \times 10^9$ in C.G.S. units. The surface tension of water at that temperature is about 76 dynes per linear centimetre, and ρ is 1 gramme per cubic centimetre. Hence

$$\log_{10} \frac{P'}{P_s} = \frac{1{\cdot}01}{D},$$

where D is the diameter of the drop in millionths of a millimetre. The formula accordingly gives these results, for drops of water:

Diameter of Drop in Millionths of a Millimetre.	Ratio of P′ to P_s.
100	1·02
50	1·05
10	1·26
5	1·59
2	3·2
1	10·2

This means, for instance, that a drop of water two-millionths of a millimetre in diameter will grow if the ratio of supersaturation in the vapour around it is greater than 3·2, but will evaporate if that ratio is less. Hence when the ratio is 3·2, drops will not form unless there are nuclei present which are at least big enough to be equivalent to spheres with a diameter of two-millionths of a millimetre.

Similar considerations govern the formation of bubbles in a boiling liquid. Any small bubble may be treated as a spherical space of radius r, containing gas, bounded by a spherical envelope in which there is surface tension. Outside of that is the liquid at a pressure P. In consequence of the surface tension in the envelope, the pressure P_i inside the bubble must exceed P by the amount 2S/r, where S is the surface tension in the boundary surface of the bubble, making

$$P_i - P = \frac{2S}{r}.$$

When r is very small this implies a great excess of pressure within the bubble. If no particles of air or other nuclei were present to start the formation of bubbles, boiling would not begin until the temperature were raised much above the point corresponding to the external pressure, and then would occur with almost explosive violence. Once formed a bubble would be highly unstable, for as the radius increases the tension of the envelope becomes less and less able to balance the excess of pressure within it. This happens, to some extent, when water is boiled after being freed of air in solution: it is then said to boil with bumping.

It follows that a pure liquid may be superheated, that is to say, raised above the temperature of saturation corresponding to the actual pressure. This is an example of a metastable state like the state that is produced when a vapour is supercooled without condensing, or when a liquid is supercooled without solidifying. Water at atmospheric pressure may be heated to 180° C. or more when it has been freed of air and when it is kept from contact with the sides of the vessel by supporting it in oil of its own density, so that the water takes the form of a large globule immersed in oil.

In the ordinary process of boiling, a bubble contains in general some air or other gas besides the vapour of the liquid itself. Without gas in it, the bubble could not exist in stable equilibrium. With gas in it, the bubble will be in stable equilibrium when the partial pressure due to the gas provides the necessary excess of the whole internal pressure P_i over the external pressure P. Any reduction of the bubble's size would then raise the pressure of the gas more than enough to balance the increase of 2S/r. Let P_v be the vapour pressure inside the bubble. If we assume that the external pressure and temperature remain constant, the partial pressure due to the gas may be expressed as a/r^3 where a is a constant. Then $P_i = P_v + a/r^3$, and the equation

$$P_v + \frac{a}{r^3} = P + \frac{2S}{r}, \text{ or } P_v - P = \frac{2S}{r} - \frac{a}{r^3},$$

determines the value of r at which the bubble is in equilibrium. The quantity $P_v - P$ is the excess of the vapour pressure in the bubble over the pressure in the liquid. Differentiating this with respect to r, we have

$$\frac{d(\mathrm{P}_v - \mathrm{P})}{dr} = 0 \text{ when } \frac{a}{r^3} = \tfrac{1}{3}\,\frac{2\mathrm{S}}{r},$$

and therefore when

$$\mathrm{P}_v - \mathrm{P} = \frac{4\mathrm{S}}{3r}.$$

Hence if a liquid containing bubbles of radius r is heated, the temperature will rise until the vapour pressure within the bubbles exceeds the pressure in the liquid by the amount $4\mathrm{S}/3r$, but when that point is reached the bubbles will become unstable and ebullition will begin. Callendar (*Ency. Brit.*, article "Vaporisation") calculates on this basis that with bubbles one millimetre in diameter, water (under one atmosphere) will boil at a temperature of 100·05° C., and that to produce 10° of superheat the diameter of the bubbles must not be more than about 1/200 mm.

§ (55) Compressibility and Elasticity of a Fluid.—Let any homogeneous fluid be subjected to an increase of pressure $d\mathrm{P}$, with the result that the volume is reduced from V to $\mathrm{V} - d\mathrm{V}$. Then $-d\mathrm{V}/\mathrm{V}$ measures the volume strain, and the ratio of this strain to $d\mathrm{P}$ measures the compressibility.

The reciprocal of the compressibility or $-\mathrm{V}(d\mathrm{P}/d\mathrm{V})$ measures what is called the elasticity of the fluid. Its value will obviously depend on the circumstances under which the compression takes place. Suppose for instance that the temperature is kept constant during the compression. In that case the expression for the elasticity becomes $-\mathrm{V}(d\mathrm{P}/d\mathrm{V})_\mathrm{T}$. This is called the isothermal elasticity of a fluid, and will be denoted here by E_t. Suppose on the other hand that no heat is allowed to leave or to enter the fluid during the compression. In that case the expression becomes $-\mathrm{V}(d\mathrm{P}/d\mathrm{V})_\phi$. This, which is called the adiabatic elasticity of a fluid, will be denoted here by E_ϕ. We have accordingly the two elasticities

$$\mathrm{E}_t = -\mathrm{V}\left(\frac{d\mathrm{P}}{d\mathrm{V}}\right)_\mathrm{T}, \quad . \quad . \quad . \quad (88)$$

$$\mathrm{E}_\phi = -\mathrm{V}\left(\frac{d\mathrm{P}}{d\mathrm{V}}\right)_\phi. \quad . \quad . \quad . \quad (89)$$

Hence

$$\frac{\mathrm{E}_\phi}{\mathrm{E}_t} = \frac{\left(\dfrac{d\mathrm{P}}{d\mathrm{V}}\right)_\phi}{\left(\dfrac{d\mathrm{P}}{d\mathrm{V}}\right)_\mathrm{T}} = \frac{\mathrm{K}_p}{\mathrm{K}_v}, \quad . \quad . \quad (90)$$

by equation (63). That is to say; the ratio of the adiabatic to the isothermal elasticity is equal to γ, the ratio of the specific heats. Since K_p is greater than K_v, E_ϕ is greater than E_t.

§ (56) Applications to Particular Fluids: Characteristic Equation.—The thermodynamic relations given in equations (29) to (90) are general in the sense that they are true of any fluid. They can be applied to determine various properties of any individual fluid when an equation connecting one of the properties of the fluid with two others is known. Such an equation is called a "Characteristic Equation" or "Equation of State." The most usual type of characteristic equation is one connecting the volume (of a given mass) with the pressure and the temperature. When this is known, and when, in addition, there are data as to the variation of the internal energy with the temperature, numerical values of the thermodynamic properties of the given fluid in various states may be calculated by help of the general relations which have been detailed.

§ (57) Properties of a Perfect Gas.—The simplest example of a characteristic equation is that of an ideal gas, namely (§ (6))

$$\mathrm{PV} = \mathrm{RT},$$

where R is constant. We may illustrate the use of some of the general thermodynamic relations by applying them to this ideal substance. On differentiating the characteristic equation of an ideal gas, we have

$$\mathrm{P}d\mathrm{V} + \mathrm{V}d\mathrm{P} = \mathrm{R}d\mathrm{T}.$$

Hence in such a gas

$$\left(\frac{d\mathrm{P}}{d\mathrm{T}}\right)_\mathrm{V} = \frac{\mathrm{R}}{\mathrm{V}} = \frac{\mathrm{P}}{\mathrm{T}}; \quad \left(\frac{d\mathrm{V}}{d\mathrm{T}}\right)_\mathrm{P} = \frac{\mathrm{R}}{\mathrm{P}} = \frac{\mathrm{V}}{\mathrm{T}};$$

$$\left(\frac{d\mathrm{P}}{d\mathrm{V}}\right)_\mathrm{T} = -\frac{\mathrm{P}}{\mathrm{V}}; \quad \left(\frac{d^2\mathrm{P}}{d\mathrm{T}^2}\right)_\mathrm{V} = 0; \quad \left(\frac{d^2\mathrm{V}}{d\mathrm{T}^2}\right)_\mathrm{P} = 0.$$

By equations (53) and (57), in any fluid,

$$\left(\frac{d\mathrm{K}_v}{d\mathrm{V}}\right)_\mathrm{T} = \mathrm{T}\left(\frac{d^2\mathrm{P}}{d\mathrm{T}^2}\right)_\mathrm{V} \text{ and } \left(\frac{d\mathrm{K}_p}{d\mathrm{P}}\right)_\mathrm{T} = -\left(\frac{d^2\mathrm{V}}{d\mathrm{T}^2}\right)_\mathrm{P}.$$

Hence in the ideal gas

$$\left(\frac{d\mathrm{K}_v}{d\mathrm{V}}\right)_\mathrm{T} = 0 \text{ and } \left(\frac{d\mathrm{K}_p}{d\mathrm{P}}\right)_\mathrm{T} = 0.$$

Thus it follows from the characteristic equation that both K_v and K_p are constant at any one temperature; in other words, they are independent of the pressure. They may, however, vary with temperature: the characteristic equation gives no information on that point.

By equation (58), in any fluid,

$$\mathrm{K}_p - \mathrm{K}_v = \mathrm{T}\left(\frac{d\mathrm{P}}{d\mathrm{T}}\right)_\mathrm{V}\left(\frac{d\mathrm{V}}{d\mathrm{T}}\right)_\mathrm{P}.$$

Hence in the ideal gas

$$\mathrm{K}_p - \mathrm{K}_v = \mathrm{T} \cdot \frac{\mathrm{R}}{\mathrm{V}} \cdot \frac{\mathrm{R}}{\mathrm{P}} = \mathrm{R},$$

in agreement with equation (2). The factor A is omitted because quantities of heat are here expressed in work units.

By equation (75), in any fluid the cooling effect in the Joule-Thomson porous plug experiment is

$$\frac{1}{K_p}\left[T\left(\frac{dV}{dT}\right)_P - V\right].$$

In the ideal gas $(dV/dT)_P = V/T$; hence the quantity in square brackets vanishes and there is no cooling effect.

By equation (66), in any fluid,

$$dE = K_v dT + \left[T\left(\frac{dP}{dT}\right)_V - P\right]dV.$$

In the ideal gas $T(dV/dT)_V = P$, hence

$$dE = K_v dT,$$

and since K_v is independent of the pressure it follows that the internal energy of the ideal gas depends upon the temperature alone. The ideal gas exactly obeys Joule's Law.

By equation (69), in any fluid,

$$dI = K_p dT + \left[V - T\left(\frac{dV}{dT}\right)_P\right]dP.$$

In the ideal gas $T(dV/dT)_P = V$, hence

$$dI = K_p dT,$$

and since K_p is independent of the pressure it follows that the total heat of the ideal gas also depends upon the temperature alone.

These results show that a gas which conforms exactly to the characteristic equation $PV = RT$ (T being the temperature on the thermodynamic scale) conforms exactly both to Boyle's Law (PV constant for any one temperature) and to Joule's Law (E a function of the temperature alone). It is therefore "perfect" in the sense of § (12).

When the equation $PV = RT$ was introduced in § (6) the symbol T denoted temperature on the scale of the gas thermometer, that is to say, a scale defined by the expansion of the gas itself, and the gas was assumed to conform exactly to Boyle's Law. But if it also conforms exactly to Joule's Law, the scale of the gas thermometer coincides with the thermodynamic scale (§ (22)).

By equation (63*a*) for the adiabatic expansion of any fluid,

$$\left(\frac{dP}{dV}\right)_\phi = \gamma\left(\frac{dP}{dV}\right)_T.$$

Hence in the ideal gas

$$\left(\frac{dP}{dV}\right)_\phi = -\gamma\frac{P}{V}.$$

So that in the adiabatic expansion of an ideal gas

$$\frac{dP}{P} + \gamma\frac{dV}{V} = 0.$$

If now we make the further assumption that γ is constant, which is equivalent to assuming that the specific heat does not vary with temperature, this gives on integration $PV^\gamma = \text{const.}$, as in equation (4), § (15).

For the entropy, energy, and total heat of the ideal gas we have, by equations (38) and (39), in any fluid,

$$d\phi = \frac{dE + PdV}{T} = \frac{dI - VdP}{T}.$$

In the ideal gas

$$dE = K_v dT \text{ and } dI = K_p dT,$$

and since $PV = RT$,

$$d\phi = K_v\frac{dT}{T} + R\frac{dV}{V}$$

$$= K_p\frac{dT}{T} - R\frac{dP}{P}.$$

If, as before, we assume that the specific heat does not vary with the temperature, these results give on integration

$$E = K_v T + \text{const.},$$

$$I = K_p T + \text{const.},$$

$$\phi = K_v \log_e T + R \log_e V + \text{const.}$$

$$= K_p \log_e T - R \log_e P + \text{const.}$$

The values of the constants of course depend on what initial state is chosen as the starting-point of the reckoning. When we are concerned only with changes of E, I, or ϕ the integration is between limits and the constants disappear.

As an example, consider the change of entropy which occurs in Joule's experiment (§ (12)) when a gas, originally contained in one vessel, expands without doing work and without taking in or giving out heat, so as to distribute itself between that vessel and another. Let V be the original volume and V′ the greater volume after expansion. With an ideal gas there is no change in T or in E or in I. But the above expression for ϕ shows that, as a result of the irreversible expansion, the entropy has increased by the amount

$$\phi' - \phi = R(\log_e V' - \log_e V)$$

or

$$R(\log_e P - \log_e P').$$

Though the system has lost no energy it has lost availability for conversion into work. A quantity of energy has been *dissipated* which is equal to the work that might have been done had the gas expanded reversibly from the same initial to the same final state without change of temperature, namely,

$$\int_V^{V'} PdV = \int_V^{V'} \frac{RT}{V}dV = RT(\log_e V' - \log_e V).$$

We may regard that amount of work as done internally, in giving kinetic energy to the gas as it streams from one vessel to the other, and then as converted into heat within the gas. From this point of view the effect on the entropy is the same as if the gas were to take in, at temperature T and without irreversible action, a quantity of heat equivalent to that amount of work. The entropy would thereby be increased by $RT(\log_e V' - \log_e V)/T$, which is equal to the above expression for $\phi' - \phi$. The loss of availability on the part of the energy of the system for conversion into work is measured by $T(\phi' - \phi)$.

Similarly when an ideal gas streams continuously through a porous plug or other constricted orifice from a region of pressure P to a region of pressure P′, without taking in or giving out heat, as in the Joule-Thomson experiment, there is again no change in T or in E or in I, but the entropy increases by the amount

$$\phi' - \phi = R(\log_e P - \log_e P').$$

Here also, as a result of the irreversible process, the energy of the gas loses availability for conversion into work to an extent which is measured by the product of that quantity into T.

It should be noted that these conclusions are in agreement with § (51), where it was shown that the decrement of the function ψ, namely $\psi - \psi'$, measures the amount by which the energy of any system loses availability for conversion into work in an isothermal process. For since $\psi = E - T\phi$, $d\psi = dE - Td\phi - \phi dT$. In either the Joule experiment or the Joule-Thomson experiment dE and dT are zero for an ideal gas, hence $d\psi = -Td\phi$, making the loss of availability $\psi - \psi' = T(\phi' - \phi)$, as above.

§ (58) Ratio of the Specific Heats in Gases.—We saw (§ (55)) that in any fluid the ratio γ of the two specific heats K_p and K_v is equal to the ratio of the adiabatic elasticity E_ϕ to the isothermal elasticity E_t; and also that $E_t = -V(dP/dV)_T$. Hence in a gas for which $PV = RT$,

$$E_t = V\left(\frac{P}{V}\right) = P \text{ and } E_\phi = \gamma P.$$

This relation has been used as a means of finding γ experimentally in air and other gases which at ordinary temperatures and pressures very nearly conform to the equation $PV = RT$. The method is based on Newton's theory of the transmission of waves of sound. Newton showed that waves of compression and dilatation, such as those of sound, travel through any homogeneous fluid with a velocity which may be expressed as $\sqrt{EV}$, where V is as usual the volume of the fluid per unit mass (the reciprocal of the average density) and E is the elasticity, in kinetic units. It was afterwards pointed out by Laplace that in applying this result to the passage of sound through air or other gases E should be taken as the adiabatic elasticity E_ϕ, for the compressions and dilatations follow one another so fast as to leave no time for any substantial transfer of heat from the portions that are momentarily heated by compression to those that are momentarily cooled by expansion. Hence in air under atmospheric conditions, or in any other nearly perfect gas, sound travels at a rate equal to $\sqrt{\gamma PV}$. This fact is used as a means of determining γ by measuring the velocity of sound or (what comes to the same thing) by measuring the wave-length in sound of a known pitch.

In air at 0° C. and a pressure of one atmosphere the values given by various observers for the velocity of sound range from 33,060 to 33,240 centimetres per second. Under these conditions the volume of one gramme of air is 773·1 cubic cm., and P is $1{\cdot}0133 \times 10^6$ dynes per sq. cm. Hence, taking an average of 33,150 for the velocity,

$$33{,}150 = \sqrt{\gamma \times 1{\cdot}0133 \times 10^6 \times 773{\cdot}1},$$

which makes $\gamma = 1{\cdot}403$.

Another experimental method of determining the value of γ in a gas is that of Clément and Desormes. A quantity of the gas is contained in a large vessel at a pressure somewhat higher than that of the atmosphere, and at atmospheric temperature. There is a pressure-gauge attached, and a tap which may be opened to allow some of the gas to escape quickly. On opening the tap, the pressure falls suddenly to that of the atmosphere: when this has happened the tap is at once closed. Then the pressure of the gas that remains in the vessel slowly rises, because the temperature, which had been reduced by the sudden expansion of the gas in the vessel while the tap was open, rises gradually to the value which it had at first, namely the temperature of the surrounding atmosphere. When this process is complete the final pressure is noted. Let the original pressure be P_1, the pressure of the atmosphere P_2, and the final pressure P_3. The change from P_1 to P_3 is approximately adiabatic on account of its suddenness: the change from P_2 to P_3 occurs at constant volume. Let V_1, V_2, and V_3 be the volumes of the gas *per unit mass*, at the three corresponding stages. Then $V_2 = V_3$. It is assumed that the gas conforms, very nearly, to the equation $PV = RT$. We accordingly have, in the adiabatic expansion,

$$P_1 V_1^{\gamma} = P_2 V_2^{\gamma},$$

and since the initial and final temperatures are the same,

$$P_1 V_1 = P_3 V_3 = P_3 V_2.$$

Hence $$\frac{P_1}{P_2}=\left(\frac{V_2}{V_1}\right)^{\gamma}=\left(\frac{P_1}{P_3}\right)^{\gamma},$$

or $$\gamma=\frac{\log P_1-\log P_2}{\log P_1-\log P_3}.$$

Values of γ are accordingly found by observing these three pressures. Experiments by Lummer and Pringsheim, using this method in an improved form, give 1·4025 as the value of γ for normal air.

Similar methods of experiment applied to other gases have shown that in all the light diatomic cases, such as hydrogen, oxygen, nitrogen, carbonic oxide, nitrous oxide, the value of γ is approximately 1·4 at ordinary temperatures. It becomes somewhat less when the gas is strongly heated. They have also shown that in monatomic gases, such as helium or argon or mercury-vapour, the value of γ is very nearly $1\frac{2}{3}$ and does not change when the gas is heated. In triatomic and polyatomic gases, on the other hand, the experimentally found values of γ do not exceed $1\frac{1}{3}$ and are generally less, especially when the gas is heated. In water-vapour, for example, and in carbonic acid gas, both of which are triatomic, γ is about 1·3.

These experimental results agree with what is to be expected from the molecular theory of gases. According to that theory the energy E in a gas is due to movements on the part of the molecules. In a monatomic gas substantially all the energy consists of the kinetic energy which the molecules have in consequence of their movements of translation. In any gas the molecules have three degrees of freedom of translation and each of these degrees of freedom contributes to the whole energy E a quantity equal to $\frac{1}{2}$RT (see § (66)). Consequently in a monatomic gas the whole energy is equal to $\frac{3}{2}$RT. This makes K_v equal to $\frac{3}{2}$R, with the result that (since $K_p=K_v+R$) K is $\frac{5}{2}$R and γ is $1\frac{2}{3}$. In a diatomic gas the molecules still have energy of translation equal to $\frac{3}{2}$RT, but in addition they have energy of rotation about axes transverse to the line joining the two atoms of the molecule. There are two effective degrees of freedom about such axes, and it follows from the theory that each effective degree of freedom of rotation takes up the same amount of energy as each degree of freedom of translation. This brings the value of E up to $\frac{5}{2}$RT, making K_v equal to $\frac{5}{2}$R, K_p to $\frac{7}{2}$R, and γ to $1\frac{2}{5}$. If the molecules have any appreciable energy of vibration the effect of that is to increase K_v and K_p and to reduce γ. This is found to occur when the gas is strongly heated, but at ordinary temperatures the observed values of the specific heats and of γ show that vibration does not contribute any substantial part of the whole energy. In heavy diatomic gases, on the other hand, such as chlorine or the vapours of the other halogen elements, there is considerable energy of vibration even at moderate temperatures, which increases both of the specific heats and makes γ less than 1·4. The energy of vibration that affects the specific heat consists of to and fro movements on the part of the atoms that make up the molecule. In a monatomic gas there is no possibility of this kind of movement. In most diatomic gases it is negligible until the gas is strongly heated, but with heavy atoms like those of chlorine, vibrating with a comparatively long period, it forms a sensible part of the whole energy even when the gas is cold. In gases that have more than two atoms in the molecule there are three effective freedoms of rotation. The energy due to translation and rotation together is therefore equal to 3RT, which, if there were no vibration to be taken account of, would make K_v equal to 3R, K_p equal to 4R, and γ equal to $1\frac{1}{3}$. The effect of vibration, even at ordinary temperatures, is to make γ less, especially in gases with complex molecules where there may be many kinds of to and fro movements, with various periods, on the part of the atoms within the molecule. The general principle holds that those vibrations which have a long period are excited at comparatively low temperatures, contributing to the energy and augmenting the specific heat, whereas those of short period are not excited and do not contribute appreciably until the temperature is high.

This principle finds expression in a formula[1] devised by Planck to connect the energy which is contributed by vibrations of any particular frequency with the temperature, when a state of equilibrium has been reached through the mutual encounters of the molecules. According to Planck's theory the vibratory energy per gramme-molecule of any gas, corresponding to any given frequency ν, is

$$E\nu=\frac{Nh\nu}{e^{Nh\nu/RT}1},$$

where N is the number of molecules in a gramme-molecule, h is a constant known as Planck's constant, which is the same for all gases and is approximately equal to $6{\cdot}55\times10^{-27}$ in C.G.S. units. R, as usual, is the gas-constant, whose value per gramme-molecule is 1·985 thermal units or $83{\cdot}1\times10^{6}$ ergs, and e is the base of the Napierian logarithms. In a gas whose molecules are capable of more than one mode of vibration the whole vibrational energy would be the sum of as many separate terms, in this form, as there are modes. At any one frequency let the quantity $Nh\nu/RT$ be represented by x. Then Planck's formula becomes

$$E_v=\frac{x}{e^{x}-1}RT.$$

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[1] See article "Quantum Theory," Vol. IV.

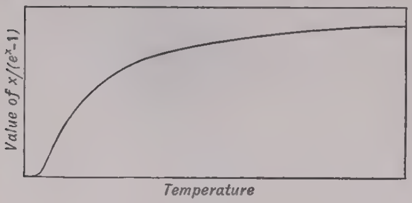

FIG. 22.

The factor $x/(e^x-1)$ tends towards the value 1 as T is indefinitely increased, for then x becomes indefinitely small, but is insignificantly small when T is low. Plotted in relation to T it gives a curve (*Fig.* 22) with a scarcely preceptible rise at the beginning, then a steep ascent, and finally an asymptotic approach to the limit, at which its value is 1. Accordingly, in the heating of the gas whose molecules may vibrate with a particular frequency, the amount of energy which they take up in respect of that vibration is nearly nil at first, then increases rapidly, and finally approaches a limiting value equal to RT when the temperature is indefinitely raised. The temperature at which the rapid rise begins is relatively high for vibrations of short period.

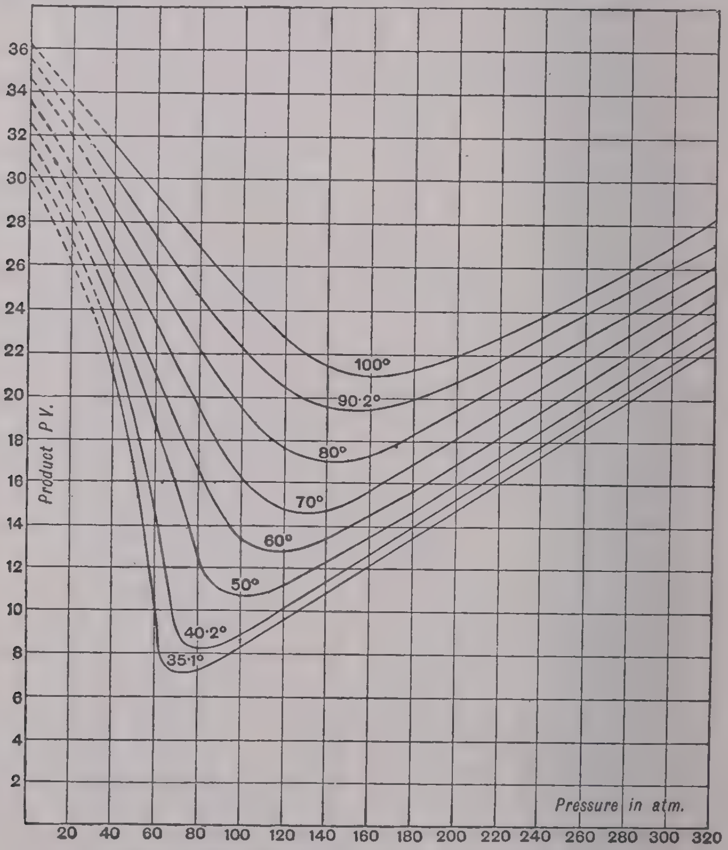

FIG. 23.—Amagat's Isothermals of PV and P for Carbonic Acid.

§ (59) IMPERFECT GASES. AMAGAT'S CURVES. VAN DER WAALS' EQUATION.[1]—No real gas strictly conforms to the simple characteristic equation PV = RT. Experiments by Andrews, Amagat, and others[2] show that at any one temperature the product PV is not quite constant. It generally varies with the pressure in the manner of the curves in *Fig.* 23, first decreasing and then increasing as the pressure is raised to high values.

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[1] See article "Thermal Expansion," § (18) ii.

[2] See particularly "Thermodynamic Properties of Air," also Witkowski, *Phil. Mag.*, 1896, xli. 288.

This is true even at temperatures much above the critical temperature, and the deviation from Boyle's Law becomes more and more marked as the critical point is approached. At any constant temperature below the critical temperature the product PV diminishes with increasing pressure until the pressure of saturation is reached, at which the gas liquefies. Above the critical temperature an isothermal generally has a minimum of PV at a particular pressure, the value of which depends on the temperature T for which the isothermal is drawn. This will be seen in the figure, which gives Amagat's isothermals for carbonic acid at various temperatures above the critical temperature. With rising values of T in any gas the position of the minimum of PV on the isothermal shifts first to the right, and then (as the temperature is further raised) to the left. Accordingly, when T is much above the critical temperature the whole isothermal may consist of a line sloping upwards with increasing P. This is the case with hydrogen at ordinary temperatures (*Fig.* 24), though at much lower temperatures the isothermals would at first slope down towards a minimum, and consequently (see § (50)) the Joule-Thomson effect, which depends in part on the value of $(d(\mathrm{PV})/d\mathrm{P})_{\mathrm{T}}$, suffers inversion.

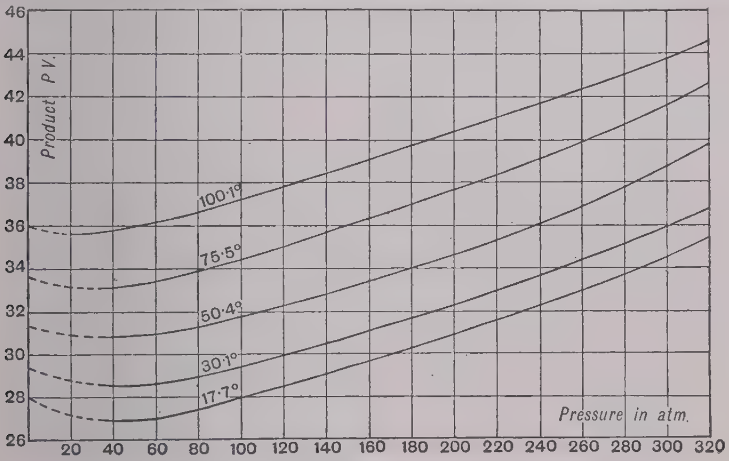

FIG. 24.—Amagat's Isothermals for Hydrogen.

The molecular theory shows that a gas cannot be expected to conform to the equation PV = RT unless (1) the size of the molecules is indefinitely small compared with the distances traversed by them between their encounters, and (2) no appreciable part of the energy of the gas is due to the mutual attraction of the molecules for one another. In a real gas neither of these conditions holds. The volume of the molecules constitutes an appreciable part of the whole volume V occupied by the gas, and it is only after making a deduction for it that we have the volume that can be reduced by applying more pressure. There is some attraction between the molecules, which causes a small part of the energy of the gas to be due to their mutual attractions and assists the external pressure P in preventing the gas from expanding. Van der Waals has endeavoured to frame a characteristic equation which will take account of these two effects. If the first effect stood alone we should have $\mathrm{P}(\mathrm{V}-b)=\mathrm{RT}$ where b, which is called the co-volume, represents the deduction due to the volume of the molecules. The attraction between molecules will depend on the number which are at any moment so near as to be exercising mutual forces: on any unit plane within the gas this will be proportional to the square of the density. Accordingly Van der Waals takes a/V^2 as the term to be added to P. He treats a and b as constants for any particular fluid, and so obtains the equation

$$\left(\mathrm{P}+\frac{a}{\mathrm{V}^2}\right)\left(\mathrm{V}-b\right)=\mathrm{RT} \quad . \quad . \quad (91)$$

as a characteristic equation applicable to any homogeneous state, gaseous or liquid. It does in fact represent comprehensively the chief phenomena of both states, and also those of the critical point, but when examined in detail it fails to give exact results. If the constants are adjusted to bring the formula into close agreement with one set of observed phenomena, such as the relation of volume to pressure along an isothermal, there are quantitative discrepancies in other phenomena such as the Joule-Thomson cooling effect, between the calculated and observed figures. The apparent reason of this failure is that the "constants" of the formula are not strictly constant: they are functions of the temperature or the density or both. Attempts by Clausius, Dieterici, and others to modify the Van der Waals equation have been only partially successful. Though characteristic equations of the Van der Waals type serve well as a broad guide to the behaviour of fluids, none of them can be accepted as strictly accurate. It appears that the properties of a fluid are too complex to be expressed by means of a formula containing so small a number of adjustable constants.

Before any equation of the kind had been framed James Thomson had suggested that in drawing the separate isothermal curves of P and V for a liquid and its vapour, at any temperature below the critical temperature, as in *Fig.* 10, we might think of an imaginary connection between them such as is sketched in *Fig.* 25, where AB, which relates to the liquid, and CD, which relates to the vapour, are regarded as forming parts of a continuous curve AJLD. The connecting curve JL is a mathematical abstraction, but it allows a continuous algebraic expression for P in relation to V (such as that of Van der Waals) to be interpreted for isothermals below the critical temperature as well as for isothermals above that temperature. The straight line BC, which represents the actual change from liquid to vapour by vaporisation at constant pressure, stands at such a level that the area BJK is equal to the area KLC; for we may conceive the fluid to be taken through a complete cycle from B through JKL to C and then back to B by the straight line CB. During this imaginary cycle its temperature does not change, and therefore, by Carnot's principle, the work done in the cycle as a whole is zero, or $\int P dV = 0$.

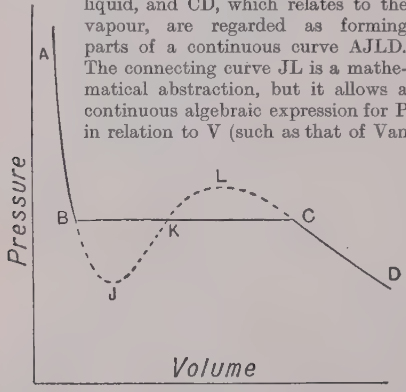

FIG. 25.—James Thomson's Ideal Isothermal.

A characteristic equation such as that of Van der Waals expresses in algebraic form this imaginary continuity between the isothermal curves for the liquid and for the vapour, at temperatures below the critical temperature, as well as the real continuity in isothermal curves above it. Van der Waals' equation may be written thus, as a cubic in V,

$$V^3 - \left(b + \frac{RT}{P}\right)V^2 + \frac{aV}{P} - \frac{ab}{P} = 0.$$

For any one isothermal, this gives three roots, real or imaginary, for V, corresponding to any assigned value of P. When the temperature for which this isothermal is drawn is higher than the critical temperature only one of the three roots is real: that is to say, there is only one value of V for each value of P on any isothermal that passes through the critical point. At any temperature below the critical temperature all three roots are real in the mathematical sense. The isothermal curve calculated from the equation then takes the continuous form conceived by James Thomson and illustrated in *Fig.* 25. One of the three roots corresponds to a point on the curve AJ, another to a point on DL, and the third to a point (not realisable experimentally) on JL.

Van der Waals' equation makes the product PV, for constant T, vary in the manner indicated by Amagat's isothermals, showing a minimum at a particular value of the pressure that depends on the temperature for which the isothermal is drawn. Writing the equation in the form

$$PV = \frac{RTV}{V - b} - \frac{a}{V},$$

and differentiating with respect to P, keeping T constant, we have

$$\left(\frac{d(PV)}{dP}\right)_T = \left[\frac{-RTb}{(V-b)^2} + \frac{a}{V^2}\right]\left(\frac{dV}{dP}\right)_T.$$

Since on any isothermal $d(PV)/dP$ is zero at the minimum of PV, the quantity within the square brackets must vanish at that point. Hence, on any isothermal, the minimum of PV is found when the volume is such that

$$\left(1 - \frac{b}{V}\right) = \frac{2bRT}{a}.$$

This shows that the volume at which the minimum of PV occurs on any isothermal becomes greater as the temperature is raised. When the temperature is so high that the minimum occurs at zero pressure, V is indefinitely large; $1 - (b/V)$ then becomes equal to 1, and T is given by the equation $T = a/bR$. Hence in a fluid which satisfies Van der Waals' equation an Amagat isothermal for a temperature equal to or greater than a/bR will slope up along its whole course, with increasing P, but an isothermal for any temperature lower than this will first dip towards a minimum of PV and then rise.

To find the critical point of a fluid which satisfies Van der Waals' equation, write the equation in the form

$$\mathrm{P}=\frac{\mathrm{RT}}{\mathrm{V}-b}-\frac{a}{\mathrm{V}^2},$$

from which

$$\left(\frac{d\mathrm{P}}{d\mathrm{V}}\right)_\mathrm{T}=\frac{-\mathrm{RT}}{(\mathrm{V}-b)^2}+\frac{2a}{\mathrm{V}^3}$$

and

$$\left(\frac{d^2\mathrm{P}}{d\mathrm{V}^2}\right)_\mathrm{T}=\frac{2\mathrm{RT}}{(\mathrm{V}-b)^3}-\frac{6a}{\mathrm{V}^4}.$$

At the critical point, in any fluid,

$$\left(\frac{d\mathrm{P}}{d\mathrm{V}}\right)_\mathrm{T}=0 \text{ and } \left(\frac{d^2\mathrm{P}}{d\mathrm{V}^2}\right)_\mathrm{T}=0.$$

Hence, writing T_c, P_c, and V_c for the critical temperature, pressure, and volume, we should have, in a Van der Waals fluid,

$$\frac{\mathrm{RT}_c}{(\mathrm{V}_c-b)^2}=\frac{2a}{\mathrm{V}_c{}^3} \text{ and } \frac{2\mathrm{RT}_c}{(\mathrm{V}_c-b)^3}=\frac{6a}{\mathrm{V}_c{}^4}.$$

This gives

$$\frac{2}{\mathrm{V}_c-b}=\frac{3}{\mathrm{V}_c},$$

from which

$$\mathrm{V}_c=3b$$

It follows also that

$$\mathrm{T}_c=\frac{8}{27}\frac{a}{\mathrm{R}b} \text{ and } \mathrm{P}_c=\frac{a}{27b^2}.$$

Thus if the constants a and b as well as R were known for a gas which strictly satisfied Van der Waals' equation, the critical volume, temperature, and pressure might be calculated: or conversely the constants might be inferred from known values of T_c, P_c, and V_c.

§ (60) CORRESPONDING STATES.[1]—If we have two or more different fluids to which Van der Waals' equation applies, with different constants for each fluid, an important relation between them can be established by selecting scales of temperature, pressure, and volume such that the critical temperatures of the different fluids are expressed by the same number, the critical pressures by the same number, and the critical volumes by the same number. Isothermal curves drawn to these scales for the different fluids will then coincide: in other words, a single diagram will show the relation of P to V in all the fluids, when it is read by reference to the appropriate scales. Similarly a single diagram will show the Amagat curves for all. Any point taken in such a diagram, interpreted on the proper scale, marks a definite state for each fluid; and for the different fluids it marks what are called "corresponding states." Thus fluids are said to be at corresponding pressures when their pressures bear the same ratio to the respective critical pressures: they are said to be at corresponding volumes when their volumes bear the same ratio to the respective critical volumes, and at corresponding temperatures when their temperatures bear the same ratio to the respective critical temperatures. If substances conform to a characteristic equation of the Van der Waals type all three quantities P, V, and T, simultaneously have corresponding values in the sense here defined. To put this statement in another form, let the unit of temperature chosen for each fluid be its (absolute) critical temperature, the unit of volume its critical volume, and the unit of pressure its critical pressure. Then one family of curves, either on the pressure-volume diagram or the Amagat diagram, will serve to represent the isothermals for all fluids that conform to a characteristic equation of the Van der Waals type.

That this is true of any fluid to which the Van der Waals equation applies will be seen by reducing the equation to a more general form. Take any such fluid, in any given state, and write its pressure P as $p_r\mathrm{P}_c$ where p_r is the number by which the pressure is stated when we use the critical pressure P_c as the unit of pressure. Similarly for V write $v_r\mathrm{V}_c$ where v_r is the number by which the volume is stated when we use the critical volume V_c as the unit of volume; and for T write $t_r\mathrm{T}_c$ where t_r is the number that expresses the (absolute) temperature when we use the critical temperature T_c as unit of temperature. The quantities p_r, v_r, and t_r are called the "reduced" pressure, volume, and temperature respectively.

Then

$$\mathrm{P}=p_r\mathrm{P}_c=p_r\frac{a}{27b^2},$$

$$\mathrm{V}=v_r\mathrm{V}_c=v_r\,.\,3b,$$

$$\mathrm{T}=t_r\mathrm{T}_c=t_r\frac{8}{27}\frac{a}{b\mathrm{R}}.$$

On substituting these values in Van der Waals' equation,

$$\left(\mathrm{P}+\frac{a}{\mathrm{V}^2}\right)(\mathrm{V}-b)=\mathrm{RT},$$

it will be seen that the constants a, b, and R cancel out, and the equation becomes

$$\left(p_r+\frac{3}{v_r{}^2}\right)(v_r-\tfrac{1}{3})=\tfrac{8}{3}t_r. \quad . \quad . \quad (92)$$

The constants that characterised a particular fluid have disappeared. Accordingly this "reduced" characteristic equation, as it is called, is true of any substance that satisfies a Van der Waals equation, and consequently the forms of the curves connecting p_r, v_r, and t_r are the same for all such substances.

This is the theorem of corresponding states, first enunciated by Van der Waals. It was tested by Amagat and found by him to be nearly true of a number of fluids which he examined through a wide range of conditions, and it has been shown by Young to hold approximately in many substances. The validity of the principle does not depend on the precise form of the characteristic equation: Van der Waals' equation is by no means the only one that would lead to the same conclusion. Any characteristic equation connecting P, V, and T with no more than three independent constants (two adjustable constants in addition to R), and giving a critical point, can be brought in like manner to the form of a "reduced" equation in which the constants peculiar to the fluid have disappeared. Hence any such equation serves equally well as a basis for the theorem of corresponding states.

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[1] See also "Thermal Expansion," § (19).

§ (61) CALLENDAR'S EQUATION FOR STEAM.[1]—One of the uses of a characteristic equation is to give, in conjunction with the general thermodynamic relations, a set of numerical values of the properties of the fluid which will be thermodynamically consistent with one another. For the practical purpose of enabling tables to be calculated which will show the properties of a fluid throughout a limited range of variation of state, it is not impossible to frame a characteristic equation which, by empirical adjustment of the constants, will serve well within that range, though it may fail entirely outside the range. A conspicuous example is the equation which Professor H. L. Callendar has devised as a characteristic equation, applicable to any substance in the gaseous state at low or moderate pressures, and has used in calculating his Tables of the properties of Steam. Callendar's equation gives results which agree well with the experimental data for water vapour throughout the range of temperatures and pressures which are met with in engineering practice, but it does not apply at higher pressures, and it makes no attempt to express the continuity of the gaseous and liquid states. The Callendar equation is

$$V=\frac{RT}{P}-c+b, \quad . \quad . \quad . \quad (93)$$

where RT/P is, as usual, the ideal volume of a perfect gas; b is a constant representing the co-volume, as in the equation of Van der Waals, and c is a term which is not constant but is a function of the temperature. Callendar takes $c=C/T^n$, where C is a constant and n is a number which depends on the nature of the gas and is taken as $\frac{10}{3}$ for steam. The quantity c is called the "coaggregation volume": it represents the effect of inter-molecular forces in reducing the volume below its ideal value in consequence of a temporary interlinking of some of the molecules during their encounters. The equation is treated as applying to dry steam in any state (saturated or superheated) provided the pressure is less than say 600 pounds per square inch and the temperature lower than 500° C. When appropriate values are assigned to the constants, the equation, within these limits of applicability, takes for water vapour the form

$$V=\frac{154{\cdot}17T}{P}-\frac{157{\cdot}52\times10^6}{T^{\frac{10}{3}}}+0{\cdot}01602,$$

where V is the volume of 1 lb. of water vapour in cubic feet, P is the pressure in pounds per square foot, and T is the absolute temperature in Centigrade degrees. Callendar further assumes that the specific heat of water-vapour at an indefinitely low pressure is independent of the temperature, within the range of temperature in which the equation is applied. On this basis, and by making use of the general thermodynamic relations that have been given above, he obtains working formulas by means of which the values of the various properties are calculated which will be found in his Tables. The fact that the Callendar equation applies only at comparatively low pressures is apparent when one considers the form of any isothermal line given by it in the Amagat diagram whose co-ordinates are PV and P. The equation may be written

$$PV=RT-cP+bP.$$

Since c is a function of T only, this gives

$$\left(\frac{d(PV)}{dP}\right)_T=-c+b. \quad . \quad . \quad (94)$$

Hence in a gas which obeys Callendar's equation the isothermal lines on the Amagat diagram would be straight lines each with a constant slope, inclined downwards (as P increases) when c is greater than b, and inclined upwards when c is less than b. There would be no minimum of PV nor change of inclination along any isothermal. The equation therefore applies only at pressures for which these isothermals are substantially straight. The Joule-Thomson cooling effect, which by equation (76) is $T(dV/dT)_P-V$, becomes by Callendar's equation $(n+1)c-b$. It is positive in steam under all conditions to which the equation applies, $(n+1)c$ being considerably greater than b; that is to say the effect of throttling is to cool the vapour. Callendar relies largely on the observed amount of the Joule-Thomson cooling effect in assigning values to the constants of his characteristic equation (See *Phil. Mag.*, Jan. 1903). The Callendar formula provides for the inversion of the cooling effect which is known to occur in real gases, since in any gas c is reduced by raising the temperature. So long as $(n+1)c$ is greater than b the gas is cooled by throttling, but when $(n+1)c$ is less than b the gas is warmed by throttling, as hydrogen is at ordinary temperatures, and as any gas will be when T is sufficiently high. At the temperature of inversion the slope of the Amagat isothermal becomes equal to *nc*. In the usual case, when the gas is cooled by throttling, the isothermal slopes up less steeply than this, or slopes down.

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[1] See *Properties of Steam*, H. L. Callendar (E. Arnold, 1920). Also Ewing's *Thermodynamics for Engineers*, chap. viii.

§ (62) MIXTURE OF GASES.—When a vessel of constant volume contains a mixture of two or more gases in equilibrium, the pressure on the containing walls is equal to the sum of what are called the partial pressures of the constituents. The partial pressure of each constituent gas is the pressure which it would exert on the walls if all other gas were absent. This was experimentally discovered by Dalton and is known as Dalton's Law. It is very nearly true of real gases and vapours at moderate pressures, and is exactly true of the ideal perfect gases of thermodynamic theory.

Dalton's Law serves to determine the amount of water-vapour that will be present in air, at any assigned temperature, when the atmosphere is "saturated," that is to say when there is equilibrium in respect of evaporation, between the atmosphere and a flat surface of water at the same temperature. The partial pressure of the water-vapour in the air will be equal very nearly to the pressure of saturated water-vapour at the same temperature, and the quantity of vapour present, per unit volume of the air, will be equal to the density of saturated water-vapour at that temperature.

The principle, of which Dalton's Law is one manifestation, may be comprehensively stated by saying that in a mixture of perfect gases each constituent behaves as if the others were not there. For any given volume and temperature of the mixture, each constituent quantity that is present contributes to the pressure, to the energy, to the total heat, and to the entropy, just what it would contribute if it alone occupied the given volume at the given temperature.

Imagine two vessels A and B of constant volume, containing two different gases α and β, both at the same temperature and pressure, with a partition between them through which an opening can be made, say by having a slide-valve in the partition. When communication is opened a process of diffusion begins which, after a sufficient time, causes both vessels to contain one homogeneous mixture. It is assumed that the gases do not exert any chemical action on one another. If no heat enters or leaves the apparatus during the process, the temperature and the pressure are found to have undergone no change. From this it may be inferred that the gas α originally in A expands into B as if the other gas β were not there, and the gas β expands into A as if the gas α were not there. Each gas behaves like the gas in Joule's experiment (§ (12)); it expands without doing work and neither its temperature nor its internal energy is changed: consequently the mixture keeps the same temperature, and the energy of the mixture is equal to the sum of the energies which the constituents had at first. The partial pressure P_α of one constituent has (by Boyle's Law) changed from the original pressure P to PV_A/V', where $V'=V_A+V_B$, and the partial pressure P_β of the other constituent has changed from P to PV_B/V'. Hence $P=P_\alpha+P_\beta$ as Dalton's Law asserts. Thus the observed fact that when gases become mixed by diffusion there is no change of temperature, provided no heat is taken in or given out and no external work is done, allows Dalton's Law to be anticipated from the other properties of perfect gases.

Though the process of diffusion does not alter the energy of the system it is an irreversible process, and therefore must be expected to increase the entropy. That it does so is clear from a comparison of the entropy before and after mixture, using the expression for ϕ in § (57). Say that there are M_α units of α and M_β units of β in the mixture. The specific volume of the α constituent changes from V_A/M_α to V'/M_α. Before mixture took place its entropy, per unit of mass, was

$$\phi_\alpha=K_v\log_e T+R\log_e\frac{V_A}{M_\alpha}+\text{const.}$$

After mixture it is

$$\phi'_\alpha=K_v\log_e T+R\log_e\frac{V'}{M_\alpha}+\text{const.}$$

Hence the increase of entropy for the whole constituent α

$$M_\alpha(\phi'_\alpha-\phi_\alpha)=M_\alpha R(\log_e V'-\log_e V_A).$$

Similarly for the constituent β,

$$M_\beta(\phi'_\beta-\phi_\beta)=M_\beta R(\log_e V'-\log_e V_B),$$

and adding these terms we have the increase of entropy that results from mixture, for the system as a whole. The calculation may obviously be extended to a mixture of more than two gases.

This increase of entropy implies that energy is dissipated when gases mix by diffusion or otherwise. When the gases are separate, in A and B respectively, the availability of the system for doing work is greater than when they are mixed, though there is no change in temperature or pressure or energy. To realise this we have to think of some way by which the system, with separate gases, can be made to do work. Imagine the partition to be made of some porous material but to include what chemists call a *semi-permeable membrane*, such as will allow one of the gases to pass but will hold the other gas from passing. Membranes that have the property of being permeable to one substance and not to another are well known, and their action involves no breach of thermodynamic laws. Assume then, that the partition allows the gas α to pass but not the gas β. The result is some of the gas α passes into B and a difference of pressure is set up in the two vessels. The gas α will continue to diffuse into the gas β against this difference of pressure, until its partial pressure in vessel B is equal to the pressure of what is left of that gas in vessel A. The total pressure in B will then be $P+P_\alpha$, and the pressure in A will be P_α, where P_α is, as before, PV_A/V'. From the system in this state it is obvious that work could be obtained, by allowing the pressures to become equalised through an engine. The change from the original condition of the system took place without any interference from outside: it was thermodynamically "self-acting." Hence the system, in its original condition, had an availability for doing work which is not possessed when the gases are completely mixed. If a semi-permeable membrane were fixed between the vessels after complete mixture had taken place it would be without effect, for the partial pressure of the gas capable of passing it would then be the same on both sides, and any diffusion through it would go on equally in both directions.

Planck (*Thermodynamics*, *Trans.* p. 211) has described an imaginary device for separating the constituents of a mixture of two gases without taking in heat or doing work and without change of temperature. There are two semi-permeable partitions, one fixed and one in the form of a moving piston which traverses the mixed gases while another piston enlarges the capacity of the containing vessel. One of the membranes is permeable to gas α and the other to gas β. When the mixed gases have been separated by this device each of them occupies a volume equal to that of the original mixture. The process is reversible: there is no change of entropy, and therefore the system recovers no availability for doing work. The extent to which the aggregate volume has been increased neutralises the thermodynamic advantage of the separation. Planck uses the action of this device to establish the proposition that the entropy of a mixture of (perfect) gases at a given temperature is equal to the sum of the entropies which the constituents would have, if, at the same temperature, each of them separately occupied a volume equal to the volume of the mixture.

By § (51) the loss of availability on mixing the gases is $M_\alpha(\psi_\alpha-\psi'_\alpha)+M_\beta(\psi_\beta-\psi'_\beta)$. In each constituent, since there is no change in E, $\psi-\psi'=T(\phi'-\phi)$. Hence this expression for the loss of availability is equal to the gain of entropy multiplied by T:

$$M_\alpha(\psi_\alpha-\psi'_\alpha)+M_\beta(\psi_\beta-\psi'_\beta)$$
$$=M_\alpha T(\phi'_\alpha-\phi_\alpha)+M_\beta T(\phi'_\beta-\phi_\beta).$$

§ (63) SOLUTIONS.—The application of thermodynamic reasoning to the study of solutions is now an important part of the science of Physical Chemistry. Only a few of the salient points can be noticed here. In the theory of solutions much use is made of the notion of hypothetical semi-permeable membranes, and on the experimental side real semi-permeable membranes serve to exhibit fundamental facts and to furnish necessary data. Chemists can cause partitions, otherwise porous, to contain and support a "membrane" which will, for example, allow water to pass freely but will not allow a substance dissolved in the water to pass; and such semi-permeable partitions can be made strong enough to stand, without damage, a large difference of hydrostatic pressure on the two sides. Thus it is mechanically possible to have such a partition separate a quantity of the solution at one pressure from a quantity of the solvent, or pure liquid, at a lower pressure. The constituents of a solution may be present in various proportions, but in what follows we shall confine ourselves to considering solutions with two constituents, one of which makes up nearly all the mass and is called the solvent.

Imagine now two vessels W and S (*Fig.* 26) separated by a fixed semi-permeable partition. The vessel S contains a quantity of a solution, and W contains a quantity of the pure solvent, at the same temperature. The partition is permeable by the solvent, but not by the dissolved substance. It is found that some of the solvent tends to pass through the partition from W into S, weakening the solution. This can only be prevented by increasing the pressure in S by a certain definite amount P_0. We may think of the two vessels as having pistons by means of which pressure may be applied to the liquid in each. Whatever be the fluid pressure P on the side of the membrane that faces the pure solvent, there must be a greater fluid pressure $P+P_0$ on the other side if equilibrium is to be maintained. The excess fluid pressure P_0 on the side that faces the solution, when the solution is in equilibrium with the pure solvent on the other side, is called the *Osmotic Pressure.*

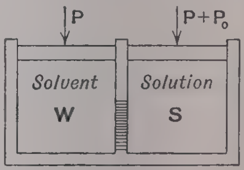

FIG. 26.

The amount of the osmotic pressure depends, for a given dissolved substance and a given solvent, on the "concentration" or quantity of the dissolved substance that is present per unit of volume of the solution. It is increased by increasing the concentration: it is also increased by raising the temperature of the system. We may think of it as a "partial pressure" due to the presence, in that volume, of the particles of the dissolved substance. This partial pressure is to be added to the partial pressure due to the other component of the solution, namely, the solvent, in determining the total pressure. From this point of view the semi-permeable membrane is exposed on one side to the pressure of the solvent alone, and on the other side to the sum of two partial pressures, one due to the dissolved substance and the other due to the solvent. Hence if the total pressure P were made the same on both sides, that part of it which is due to the solvent would be less on the solution side, and consequently the solvent would tend to flow from W to S in the effort to bring its partial pressure in S up to equality with its pressure in W. This explains why under equilibrium conditions the total pressure in S must be greater, by the amount of the partial pressure of the dissolved substance, which excess constitutes the osmotic pressure.

If the excess pressure actually applied to the solution in the vessel S is less than the osmotic pressure P_0 some of the solvent will flow from W to S. On the other hand, if an excess pressure greater than P_0 be applied, some of the solvent will pass out of the solution into W. These changes will go on until the solution becomes sufficiently less or more concentrated to allow equilibrium to be again attained.

It was pointed out by Van't Hoff (*Phil. Mag.*, Aug. 1888) that in dilute solutions the molecules of the dissolved substance act, in solution, like the molecules of a gas in this sense that the partial pressure which they exert is the same as would be exerted by an equal quantity of the same substance in the gaseous state, occupying the same volume, namely, the volume of the solution, at the same temperature. Thus the osmotic pressure in a dilute solution may be approximately calculated at any temperature and for any (small) concentration by inference from the gas equation $PV = RT$, on the basis that the dissolved substance contributes pressure like a gas whose density is the quantity of dissolved substance divided by the volume of the solution. This applies whether the dissolved substance is itself a gas, a liquid, or a solid; it may, for instance, be a substance that is non-volatile at the given temperature. It follows that the osmotic pressure in weak solutions varies in direct proportion to the absolute temperature. Also that, at any one temperature, the osmotic pressure varies in direct proportion to the quantity of dissolved substance in the solution. Also that when solutions of different substances have the same osmotic pressure at the same temperature they contain the same number of molecules of dissolved substance per unit of volume. These remarkable conclusions of Van't Hoff are found to be true of very weak solutions, in which the osmotic pressure is not so great as to make the deviation from the gas law considerable, provided the molecules of the dissolved substance do not undergo dissociation but retain their chemical character. They are closely true, for example, in dilute solutions of sugar. In solutions of electrolytic salts or other electrolytes, however, there is, as was shown by Arrhenius, much separation of the dissolved molecules into their constituent ions, with the result that the salt contributes more than one partial pressure, and the osmotic pressure is consequently greater than it would be if there were no such chemical change.

It may naturally be asked why, if a substance dissolved in water behaves there like a gas, it does not escape into the atmosphere when the solution lies in an open vessel. The answer is that at the free surface of the solution the effects of surface tension make the free surface virtually act as a semi-permeable membrane, through which molecules of the water may pass while those of the dissolved substance are held back. Similarly, a gas may be absorbed into solution by a non-volatile liquid through a free surface which is exposed to contact with the gas, because the surface is equivalent to a membrane permeable by the gas, and not by the liquid.

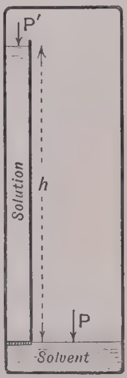

Fig. 27.

The vapour given off by a solution of a non-volatile substance is composed entirely of the solvent. At any given temperature its pressure is lower than the vapour-pressure of the pure solvent, to an extent that depends as follows on the osmotic pressure and the relative density of the vapour and the liquid. Let a tall vertical column of homogeneous solution with a free surface (*Fig.* 27) be in equilibrium, through a semi-permeable partition at its base, with a quantity of the pure solvent, the whole being enclosed in a vessel in which the only atmosphere is the vapour of the solvent. The whole system is at a uniform temperature T. Since it is in equilibrium the height *h* of the column of solution must be such that the hydrostatic pressure on the upper side of the partition exceeds the hydrostatic pressure on the under side by an amount equal to the osmotic pressure P_0; also the vapour pressure P' at the level of the top of the column must be the saturation vapour pressure of the solution, and the vapour pressure P at the free surface of the solvent must be the saturation vapour pressure of the solvent. Since the vapour has weight, P exceeds P'. Writing σ for the weight of the vapour per unit of volume, we have $P-P'=\sigma h$ if the column is so short that σ may be treated as constant, or $P-P'=\int\sigma dh$ if the variation of density with level is taken into account. The pressure on the upper side of the semi-permeable base is $P'+\rho h$, where ρ is the weight of the solution per unit of volume, and on the under side it is equal to P, if (as is shown in the sketch) the column is held with its base just level with the free surface of the solvent. Hence

$$P'+\rho h-P=P_0.$$

Taking $P-P'=\sigma h$, this gives $h=P_0/(\rho-\sigma)$, and $P-P'=P_0\sigma/(\rho-\sigma)$, or $P_0\sigma/\rho$ nearly, since σ is small compared with ρ. This may also be written $P_0/\rho V_s$, where V_s is the specific volume of the saturated vapour. But unless the solution is exceedingly dilute h is large and $P-P'$ should be taken as $\int\sigma dh$. An approximate relation is then obtained, exactly as in § (54),

$$\log_e\frac{P}{P'}=\frac{P_0}{\rho RT}=\frac{P_0}{\rho PV_s}. \quad . \quad . \quad (95)$$

Since the vapour pressure of the solution is less than that of the pure solvent at the same temperature, its boiling-point must be higher, that is to say, a higher temperature is required in the solution than in the solvent to make it give off vapour at the same pressure. To find a relation between the rise of the boiling-point and the osmotic pressure in a weak solution, let the curves C and C′ of *Fig.* 28 represent curves of vapour pressure in relation to temperature for solvent and solution respectively. If the pure solvent were boiling alone under a vapour pressure P' its temperature would be T. But when the solution is boiling under the same vapour pressure P' its temperature is T', a temperature which would correspond to a higher pressure P in vapour coming from the pure solvent. Hence from the geometry of the figure, when the change of boiling-point is small, as it is in a very weak solution,

$$\frac{P-P'}{T'-T}=\left(\frac{dP}{dT}\right)_S,$$

where $(dP/dT)_S$ is the slope of the saturation curve C for the pure solvent. But by Clapeyron's equation (§ (41)), $(dP/dT)_S=(L/(V_s-V_w))T$. Hence $(P-P')/(T'-T)=L/TV_s$ nearly, since V_w is small compared with V_s. But, as we have seen above, in a very weak solution $P-P'=P_0/\rho V_s$. Hence in such a solution

$$T'-T=\frac{P_0T}{\rho L}, \quad . \quad . \quad . \quad (96)$$

or the boiling-point is raised from T to $T(1+(P_0/\rho L))$ by the presence of the dissolved substance.

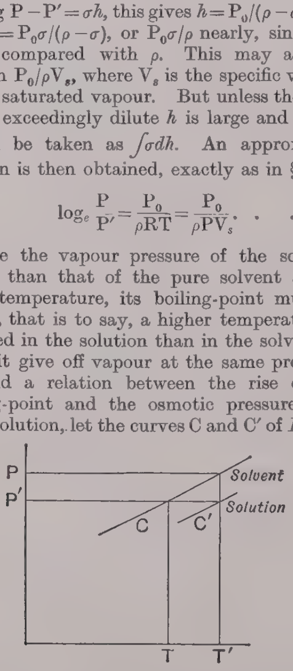

FIG. 28.

It may readily be shown that the freezing-point is lowered by a corresponding amount, namely, $P_0T/\rho\lambda$, where λ is the latent heat of the freezing solvent.

We are here dealing with a solution which when it is cooled begins to freeze by forming crystals of the pure solvent, and when it is heated (under constant pressure) begins to vaporise by giving off vapour of the pure solvent. The vapour is given off at a definitely higher temperature, and the crystals begin to form at a definitely lower temperature, than if the liquid were pure. In either case the difference $T'-T$ depends on the concentration. The vapour of the boiling solution is superheated to that extent, at the moment it comes off. The efficacy of a "freezing mixture," such as a mixture of ice and salt, depends on this lowering of the freezing-point: equilibrium is reached only when some of the ice has melted to form a solution, and the temperature of the whole, including the unmelted ice, has assumed the lower value which corresponds to the freezing-point of the solution. Conversely, when saturated steam at temperature T is passed into a solution of salt in water it will condense until equilibrium is reached, namely, until the temperature of the solution rises to the higher value T', for steam can only be given off by the solution at that higher temperature. The apparent anomaly of steam being condensed in a liquid warmer than itself is no more remarkable than what occurs during the approach to equilibrium on the part of a freezing mixture, when ice melts in a liquid that is colder than itself: both are effects of osmotic pressure.

To illustrate the thermodynamic reasoning by which Van't Hoff's principle is established, we may consider the solution of a gas in a liquid. According to that principle the osmotic pressure, at any temperature, should be equal to the pressure which the gas would have, at the same temperature, if it alone occupied a space equal to the volume of the solution. To prove that the osmotic pressure actually has that value, imagine a very long cylinder (*Fig.* 29) with a fixed partition *a* and two movable pistons *b* and *c*. Both *a* and *c* are semi-permeable: *a* is permeable only to the gas, and *c* only to the solvent.

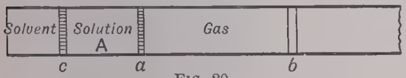

FIG. 29.

Behind *c* there is pure solvent; in the space between *a* and *b* there is gas; in the space between *c* and *a* is the liquid which dissolves the gas as the operation proceeds. Suppose the whole system to be at one temperature T and to be kept at that temperature (say by a water-jacket). At the beginning suppose *c* to be fixed and *b* to have been drawn so far away to the right that the pressure of the gas is negligibly small. Then equilibrium requires that the liquid in the space A shall contain no gas, or, to be exact, a negligible quantity of gas, for it is known as an experimental result (called Henry's Law) that the quantity of gas which a liquid will dissolve is directly proportional to the pressure. We begin therefore with practically pure solvent in the space A, whose volume we shall call V_A. Now imagine *b* to be slowly pressed in, compressing the gas isothermally and causing it to be gradually absorbed by the liquid in A. This is a reversible process: if, at any stage, *b* were stopped and slowly moved out again the action would be exactly reversed. When *b* reaches *a* all the gas is dissolved. The work spent in forcing the piston home is $\int_0^{P_1} PdV$, where P_1 is the pressure that has to be applied at the finish, under which the last part of the gas passes the partition *a* into the liquid. Now, keeping *b* with an external pressure P_1 still applied to it, suppose *c* to be forced slowly towards *a*. To do this will require that a pressure equal to the osmotic pressure P_0 be applied to *c*. The solution will thereby be separated into its components, the solvent passing behind *c*, and the gas passing through *a* and pushing out the piston *b* with the constant pressure P_1. P_1 does not change, for, as *c* advances, there is no change in the concentration of the remaining part of the solution. When *c* reaches *a* all the gas has left the solution, and is now behind the piston *b*, still at pressure P_1 and occupying a volume which we shall call V_1. The work done by the gas on *b* is P_1V_1, and the work that has been spent in forcing in the semi-permeable piston *c* is P_0V_A. Now let the gas expand isothermally till the pressure of the gas is again negligibly small: the work done in that expansion is the same as was originally spent, namely, $\int_0^{P_1} PdV$. To complete a cycle of operations we have only to withdraw *c* to its original position, which requires no work to be done, for it now has pure solvent on both sides. Since the cycle is isothermal the work done must be equal to the work spent; hence

$$P_0V_A = P_1V_1,$$

which proves the osmotic pressure P_0 to be equal to the pressure the gas would have if it alone occupied the space V_A, as Van't Hoff's principle requires.

The consideration of solutions which are not dilute, but in which the constituents may be present in any proportions, and of *solid solutions*, such as are found in metallic alloys, is beyond the scope of this article.

§ (64) ELECTROLYTIC TRANSFORMATIONS.[1]—It was pointed out in § (51) that when the fundamental equation of energy

$$dW = dQ - dE$$

is applied to a complex system, dQ being the heat taken in from outside, and $-dE$ the decrease of internal energy, the external work dW may be done in other ways than by expansion of volume. In an electrolytic system, such as a galvanic cell, the transformation which goes on within the system results in the doing of electric external work, the measure of which (in a small transformation) is Fde, where F is the electromotive force and de is the quantity of electricity generated. In many electrolytic actions the amount of mechanical work due to change of volume, or to alterations of level of substances within the cell, is negligibly small. This is the case when electric energy is produced by a battery such as Daniell's, or when it is stored and restored by the chemical action on the lead plates of a storage battery. In what follows regarding electrolytic action we shall confine our attention to those cases in which sensibly all the external work is electrical. The action may take place under reversible conditions: the deposit of copper from a copper sulphate solution, for example, such as occurs in a Daniell cell, is exactly reversed when a reversed current is caused to pass through the cell. The energy equation for a reversible electrolytic action, involving no appreciable change of volume, may accordingly be written

$$Fde = dQ - dE.$$

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[1] See also article "Batteries, Primary," Vol. II.

Here dQ represents heat taken in reversibly from outside of the cell, and dE, the change of internal energy, represents what chemists call the heat of reaction in the chemical changes which are associated with the passage of the current. The heat of reaction is the quantity of heat that would be generated (say in a calorimeter) if the same chemical action were to take place without giving out electrical energy. If the electrical energy given out by a galvanic cell were dissipated within the cell itself, instead of being employed to do work outside of it, dE is the quantity of heat which would appear. In a Daniell cell, for example, dE is the quantity of heat which would appear if zinc were consumed to form zinc sulphate in solution, less the quantity of heat which would appear if an equivalent amount of copper were consumed to form copper sulphate in solution, without the production of any external electrical effect. It is the difference between these quantities that measures the "heat of reaction" in the Daniell cell as a whole, and this is numerically equal to the loss of internal energy that occurs when the cell is employed to do external work by producing electrical energy reversibly.

In this action heat may or may not be taken in from outside. Suppose a galvanic cell to be placed in a bath of water or other isothermal enclosure so that its temperature is kept uniform. Experiment shows that the quantity of heat taken in during its action, namely dQ, may be either positive or negative. In other words, the reversible chemical action which goes on within the cell may tend either to make it colder or to make it warmer. In the former case some heat, dQ, will be taken in from the isothermal enclosure in which we have imagined the cell to be placed; in the latter case some heat will be given out to the enclosure.

A Daniell cell working reversibly, and therefore with an internal resistance so low that no sensible amount of the electric energy which it produces is dissipated within the cell by the heating effect of the current, must take in a small quantity of heat from outside if its temperature is not to fall. In the Daniell cell dQ has a positive value amounting to rather less than one per cent of the output of electrical energy. The ordinary storage battery also requires a small addition of heat to maintain its temperature constant while it discharges. In the Clark cell, on the other hand, dQ is negative and its numerical amount is greater. If dQ were zero, which is nearly true of the Daniell cell, we should have $Fde = -dE$, which would furnish a very simple means of calculating the electromotive force when the heat of reaction is known. By Faraday's Laws one and the same quantity of electricity (about 96,540 coulombs) is required for the deposition of one gramme-equivalent of any substance. From the known heats of reaction of the active substances in a given cell it is therefore easy to calculate the change of the internal energy E which occurs in the passage of one unit of electricity, at constant temperature, and the value so obtained would be numerically equal to F if no heat were taken in. On this basis Kelvin in 1851 calculated the electromotive force of a Daniell cell, obtaining a number which is a little short of the actual value as determined by experiment. When account is taken of the dQ term a correct value is deduced.

Direct measurements of the quantity of heat which is reversibly taken in or given out during the action of a cell are difficult, for the effect is inevitably mixed up, in any experiment, with the irreversible development of heat within the cell that arises from its electrical resistance. But the quantitative influence of the dQ term may be inferred, without direct measurement of the heat taken in or given out, from observations of the extent to which the electromotive force of the cell is affected by changing the constant temperature at which the cell works. This was shown independently by Willard Gibbs and Helmholtz, who thereby applied the necessary correction to the original calculation of Kelvin.

For this purpose it is convenient to make use of the function ψ, or $E - T\phi$, for the system as a whole. We saw (§ (51)) that in any system undergoing a reversible transformation at constant temperature the decrement of ψ measures the external work done. Let the amount by which ψ is diminished while one unit of electricity is generated be represented by ψ_1: for e units the amount will be $e\psi_1$, and this is equal to the external work. The amount of electrical work done while e units of electricity are generated is eF. Hence if all the external work is electrical (a condition substantially satisfied in a cell where volumetric or gravitational work is negligible),

$$eF = e\psi_1, \text{ or } F = \psi_1.$$

Then from equation (79) (§ (51))

$$\psi = E + T\left(\frac{d\psi}{dT}\right)_W$$

we have at once

$$F = E_1 + T\frac{dF}{dT}, \qquad (97)$$

where E_1 is the "heat of reaction" corresponding to the passage of one unit of electricity, and dF/dT is the rate of change of the electromotive force with temperature as observed when the cell is on "open circuit," doing no work. The term $T(dF/dT)$ corresponds to Q_1, the amount of heat that is taken in, reversibly, during the passage of one unit of electricity, when the cell works at constant temperature T. This important expression for the electromotive force of a cell is known as the Gibbs-Helmholtz equation. We may illustrate it by numerical values for the Daniell cell. When zinc replaces copper in a (moderately strong) solution of the sulphate the "heat of reaction" is found, by measurements in a calorimeter, to be 104,900 joules per gramme-equivalent of either metal. The corresponding quantity of electricity is 96,540 coulombs. Thus E_1 is $\frac{104900}{96540}$ or 1·0866 joules per coulomb. The electromotive force of a Daniell cell would therefore be equal to 1·0866 volts if the temperature coefficient were zero. But dF/dT is observed to be positive, and equal to 0·000034 volts per degree. Hence at, say, 15° C. the term $T(dF/dT)$ or Q, namely the heat which the cell takes in to keep its temperature from falling, is $288 \times 0{\cdot}000034$ or 0·0098 joules per coulomb. With these data the calculated electromotive force of the Daniell cell is accordingly

$$F = 1{\cdot}0866 + 0{\cdot}0098 = 1{\cdot}0964 \text{ volts.}$$

The Gibbs-Helmholtz equation may obviously be applied in the converse manner, to calculate the aggregate heat of reaction for the chemical changes which go on in a reversible cell, from observations of the electromotive force and its temperature coefficient.

Readers unfamiliar with the use of the function ψ may find it more satisfying to have the Gibbs-Helmholtz equation established by another method, namely, by considering a cyclic process of four operations in which the cell does electrical work during one part of the cycle and has electrical work spent upon it during another part. We shall assume, as before, that the action of the cell is reversible, and that it is surrounded by an isothermal jacket containing a fluid which will serve as source or receiver of heat. We shall further imagine that the temperature of the jacket, and therefore of the system as a whole, can be reversibly altered through some small amount δT, say by means of adiabatic expansion, so that a part of the cycle of the cell's action can be performed at temperature T and another part at temperature $T - \delta T$, the heat taken from the system in lowering its temperature from T to $T - \delta T$ being returnable to the system without loss, with the effect of restoring the temperature of the system to T. Suppose that the cell first produces electric energy at temperature T; then has its temperature lowered to $T - \delta T$; then has enough electric energy spent upon it at that lower temperature to cause the chemical changes which took place in the first operation to be exactly reversed in this third operation; and finally has its temperature restored to T. By Faraday's Laws the same quantity of electricity must pass through the cell in the third operation as in the first, in the reverse direction. But the electromotive force depends on the temperature: call it F in the first operation and $F - \delta F$ in the third. We assume that each of the four operations is reversible in the thermodynamic sense, and also that no appreciable amount of work is done by or upon the cell except the electric work. There is no chemical action, and no passage of electricity, in the second operation or in the fourth. Let e represent the quantity of electricity that passes in each of the first and third operations, and as before let E_1 represent the "heat of reaction" for the chemical changes that are associated with the passage of one unit of electricity. Let Q be the heat taken in (reversibly) from the jacket during the first operation, per unit of electricity that passes, and let $Q_1 - \delta Q$ be the quantity of heat returned to the jacket during the third operation, also per unit of electricity. Then the energy equation for the first operation is

$$eF = eE_1 + eQ_1,$$

and for the third operation it is

$$e(F - \delta F) = eE_1 + e(Q_1 - \delta Q).$$

The quantity of heat reversibly abstracted from the jacket in lowering its temperature in the second operation is returned to it in the fourth, and may therefore be omitted in summing the energies for the cycle as a whole. The cell is now restored exactly to its original state, and for the cycle as a whole, by adding the above expressions, we have

$$e\delta F = e\delta Q,$$

where $e\delta F$ is the net amount of electrical work done by the cell, and $e\delta Q$ is the net amount of heat taken in from the isothermal jacket. The energy of the cell is the same as at first. The result of the cycle as a whole is to convert an amount of heat $e\delta Q$ into electrical work $e\delta F$, and this conversion has been effected in a reversible process, by taking in heat eQ at temperature T and rejecting heat at the lower temperature $T - \delta T$. Hence by the Second Law the work done is equal to $\delta T/T$ times the heat taken in, or

$$e\delta F = \frac{eQ_1\delta T}{T},$$

from which

$$Q_1 = T\frac{\delta F}{\delta T} = T\frac{dF}{dT}.$$

Substituting this in the energy equation $F = E_1 + Q_1$ we have the Gibbs-Helmholtz result

$$F = E_1 + T\frac{dF}{dT}.$$

It should be added that to simplify this argument no account was taken (1) of any difference there may be in the heat of reaction E at T and at $T-\delta T$, also (2) of any difference there may be in the thermal capacity of the substances in the cell before and after the reaction. These two small quantities in fact cancel out. For in a calorimetric experiment in which the reaction was allowed to develop heat, it would be a matter of indifference whether the reaction took place at a temperature T and the products were then cooled to $T-\delta T$, or whether the substances were first cooled to $T-\delta T$ and the reaction were then to take place. In both cases the same total quantity of heat would be given out. Thus in ignoring both (1) and (2) we ignored quantities whose sum amounts to zero in the cyclic operation as a whole, and the validity of the argument was not affected.

§ (65) THERMO-ELECTRIC CIRCUITS.—When a circuit is made up of two different metallic conductors joined at their ends, and the junctions are kept at different temperatures, electricity is continuously generated. The circuit is a heat-engine which converts heat into electrical energy. Its thermodynamic action depends on the following facts:

(1) When two metals are in contact and electricity passes from one to the other, the junction being kept at any constant temperature, heat is taken in or given out at the junction. This action, which was discovered by Peltier soon after thermo-electric phenomena were first observed, and is called the *Peltier Effect*, is thermodynamically reversible. If a quantity of electricity passes through the junction in one direction heat is taken in; if the same quantity of electricity passes in the opposite direction, an equal quantity of heat is given out. The reversible character of the Peltier effect distinguishes it at once from any generation of heat through imperfect electric conductivity. The quantity of heat taken in or given out depends on the nature of the metals in contact and on the temperature: for a given junction at a given temperature it is directly proportional to the quantity of electricity that passes. The Peltier coefficient, which is represented by Π, may accordingly be expressed in joules per coulomb, and the quantity of heat that is reversibly taken in or given out at a junction when e units of electricity pass is $e\Pi$.

(2) When electricity passes along a metallic conductor of uniform quality along which a gradient of temperature is maintained, heat is taken in or given out. This again is a reversible action, quite distinct from any heating of the conductor through imperfect conductivity. It was predicted by W. Thomson (Lord Kelvin) in 1854 as a thermodynamic consequence of the observed properties of circuits in which electric currents flowed from hot parts to cold parts of a conductor, and is called the *Thomson Effect*. In a given metal, electricity passing from hot to cold will cause heat to be given out in any part along which there is a gradient of temperature; the same quantity of electricity passing in the reverse direction will cause an equal amount of heat to be absorbed, in the same part of the conductor. The Thomson effect is proportional to the quantity of electricity that passes: it may be expressed as $e\sigma\delta T$ for any length of a conductor between points whose temperatures differ by δT, σ being a coefficient which depends on the nature of the conductor and differs in amount and even in sign in different metals. In the conductor as a whole the Thomson effect per coulomb is therefore equal to $\int_{T_2}^{T_1}\sigma dT$, where T_1 and T_2 are the temperatures of the ends.

Imagine now an ideal circuit of two metallic conductors a and b—ideal in the sense that the conductors have no appreciable electrical resistance, so that we may omit consideration of irreversible heating effects. Let one junction be kept at a temperature T_1 and the other at a lower temperature T_2. If the metals are different, there will in general be a Peltier effect, as well as a Thomson effect, and a current will pass. Assume that in the circuit there is a perfectly efficient electric motor by which the electrical energy which is produced by the agency of heat is utilised as external work. Calling F the electromotive force produced in the circuit and employed to drive the motor, the work done by e units of electricity is eF, and the energy equation is

$$eF = e\Pi_1 - e\Pi_2 + e\int_{T_2}^{T_1}\sigma_a dT - e\int_{T_2}^{T_1}\sigma_b dT.$$

Here $e\Pi_1$ is the Peltier effect at the hot junction, namely, the heat taken in there, $e\Pi_2$, is the heat given out at the cold junction; and the two other terms are the Thomson effects in the two conductors, a taking in of heat in conductor a and a giving out of heat in conductor b. On dividing by e we have

$$F = \Pi_1 - \Pi_2 + \int_{T_2}^{T_1}(\sigma_a - \sigma_b)dT. \quad . \quad (98)$$

The whole system forms a reversible heat-engine in which the heat taken in does work as in a Carnot engine, by being let down in temperature. Hence by the Second Law $\Sigma(\delta Q/T)=0$ for the action as a whole, and

$$\frac{\Pi_1}{T_1} - \frac{\Pi_2}{T_2} + \int_{T_2}^{T_1}\frac{(\sigma_a - \sigma_b)}{T}dT = 0. \quad . \quad (99)$$

Differentiating this with respect to T,

$$\frac{d}{d\mathrm{T}}\frac{\Pi}{\mathrm{T}}+\frac{\sigma_a+\sigma_b}{\mathrm{T}}=0$$

or $$\sigma_a-\sigma_b=\frac{\Pi}{\mathrm{T}}-\frac{d\Pi}{d\mathrm{T}}. \quad . \quad . \quad (100)$$

On substituting this expression for $\sigma-\sigma_b$ in equation (98) we obtain

$$\mathrm{F}=\int_{\mathrm{T}_2}^{\mathrm{T}_1}\frac{\Pi}{\mathrm{T}}d\mathrm{T}.. \quad . \quad . \quad (101)$$

Apply these equations to a circuit in which the temperatures of the junctions differ by only an infinitesimal quantity $\delta\mathrm{T}$, and write $\delta\mathrm{F}$ for the corresponding electromotive force, and $\delta\Pi$ for the difference between the two Peltier coefficients. Then equation (98) becomes

$$\delta\mathrm{F}=\delta\Pi+(\sigma_a-\sigma_b)\delta\mathrm{T}$$

or, since $$\delta\Pi=\frac{d\Pi}{d\mathrm{T}}\delta\mathrm{T},$$

$$\delta\mathrm{F}=\left\{\frac{d\Pi}{d\mathrm{T}}+(\sigma_a-\sigma_b)\right\}\delta\mathrm{T}.$$

By equation (100) this gives

$$\delta\mathrm{F}=\frac{\Pi}{\mathrm{T}}\delta\mathrm{T},$$

from which $$\Pi=\mathrm{T}\frac{d\mathrm{F}}{d\mathrm{T}}, \quad . \quad . \quad . \quad (102)$$

a result which might have been got more shortly by differentiating equation (101). On substituting this expression for Π in equation (100) we have

$$\sigma_a-\sigma_b=\frac{d\mathrm{F}}{d\mathrm{T}}-\frac{d}{d\mathrm{T}}\left(\mathrm{T}\frac{d\mathrm{F}}{d\mathrm{T}}\right)=-\mathrm{T}\frac{d^2\mathrm{F}}{d\mathrm{T}^2}. \quad (103)$$

Equation (102) allows the Peltier effect for any given pair of metals to be calculated from the observed value of what is called the "thermo-electric power" of the pair, namely $d\mathrm{F}/d\mathrm{T}$ or the ratio of $\delta\mathrm{F}$, the observed electromotive force for a small difference of temperature between the junctions, to $\delta\mathrm{T}$ the amount of that difference. Equation (103) allows the difference of Thomson effects for the two metals to be calculated when the relation of the thermo-electric power to the temperature is ascertained. The Thomson effect is positive in some metals and negative in others; it is believed to be sensibly nil in lead at all temperatures. Consequently, in tabulating values of the Peltier and Thomson effects, lead is usually taken as one metal of the pair.

The thermo-electric power $d\mathrm{F}/d\mathrm{T}$ of any pair is a function of the temperature: in most metals its rate of variation with temperature is constant or nearly constant,[1] so that a line drawn on what is called a thermo-electric diagram to exhibit the values of the thermo-electric power of a given metal with respect to lead, in relation to the temperature, is straight or nearly straight. The line for lead is taken as a horizontal straight line. When the lines for two metals cross one another, it means that the thermo-electric power of that pair vanishes at the corresponding temperature, and has opposite signs at temperatures above and below that "neutral point." The thermo-electric power of a given pair was in fact discovered very early by Cumming to suffer inversion of sign at a particular temperature. Thus when the temperature of one junction is fixed a maximum of electromotive-force is obtained by bringing the temperature of the other junction to the temperature of inversion. With a copper-iron pair, for example, when the hot junction is raised to about 275° C. the thermo-electric power vanishes and the electromotive-force of the circuit is a maximum. It was this fact of inversion that led W. Thomson to the discovery of the Thomson effect. If the Peltier effect were the only reversible thermal phenomenon in the action of a thermo-electric circuit no inversion could occur. The circuit would then be a very simple reversible heat-engine taking in heat only at T_1 and rejecting heat only at T_2. With any assigned value of T_1 the amount of heat converted into electrical work would necessarily, by the Second Law, be proportional to $\mathrm{T}_1-\mathrm{T}_2$. Hence the electromotive-force would also be proportional to $\mathrm{T}_1-\mathrm{T}_2$, for all values of T_2, and there would be no inversion. Thomson inferred that the passage of the current must cause, in addition, some other kind of reversible thermal effect, and that it could only occur in the conductors as a consequence of the fact that along each of them there was a temperature gradient.

§ (66) Molecular Theory.—The principles of thermodynamics are not based on any assumption as to the structure of matter: their validity is independent of molecular theory. The action of ideal gases, for example, through which it is convenient to approach the thermodynamic scale of temperature and other fundamental notions, can be sufficiently described without mention of the fact that a gas is made up of moving molecules whose movements furnish a key to its properties. But though there is no need to rely on the theory of molecules and their movements—a theory which is well established not only for gases but for all fluids—there are few thermodynamic phenomena on which it does not throw light. In the course of this article it has already been referred to from time to time, as, for instance, in speaking of Van der Waals' equation (§ (59)), and of the specific heats of gases (§ (58)).

According to the molecular theory any

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[1] Unless the metal—like iron or nickel—undergoes allotropic modification (change of phase), in which case there is a sharp bend in the region of temperature where the change occurs.

substance that is chemically homogeneous (not a mixture) is made up of molecules of equal mass. They are particles which cannot be broken up without changing the chemical nature of the substance. The number of them in a gramme of any substance is equal to a constant N_a divided by the "molecular weight" w. Thus if we take a quantity of any substance equal to w grammes—what chemists call a "gramme-molecule" or "mol" of the substance—it contains the same number of molecules N_a whatever be the substance. The number N_a of molecules in a gramme-molecule is called Avogadro's constant. It has been determined in various ways with results that agree fairly well. We may take it provisionally as $6 \cdot 16 \times 10^{23}$, which makes the mass of an oxygen molecule 52×10^{24} grammes, of a hydrogen molecule about $3 \cdot 3 \times 10^{-24}$ grammes, and of a water molecule $29 \cdot 3 \times 10^{-24}$ grammes. When substances are in a gaseous state it is found, by experiment, that the volume of a gramme-molecule is nearly the same for all, under the same conditions as to temperature and pressure. This equality of volumes becomes more and more exact the lower the pressure at which the comparison is made, for the gases then become more nearly "perfect." At any one temperature and pressure equal volumes of different perfect gases contain the same number of molecules. This is known as Avogadro's law, and it is approximately true of real gases.

In a gas each molecule moves freely, with uniform velocity in a straight line, except when it encounters another molecule or the wall of the containing vessel. In an encounter the velocity changes in direction, and generally in amount, but there is no dissipation of energy; the molecules behave like perfectly elastic bodies. As a result of many encounters, a stable distribution of speed among the molecules is established, but the speed of any one molecule is being constantly changed, by its encounters, within very wide limits. The length of the free path, which it traverses between one encounter and the next, is also quite irregular. The average of that length, or what is called the "mean free path," is very long compared with the dimensions of the molecule itself. This characteristic distinguishes a gas from a liquid, but the distinction is entirely one of degree. In any fluid, whether gaseous or liquid, mechanical effects of the molecular blows can be seen in their producing what are called Brownian movements on the part of small bodies suspended in the fluid, if these are big enough to be individually visible. Observations of such movements have in fact been one means of determining N (see Perrin's *Les Atomes*). In a gas the average time during which a molecule is moving in its free path is very large compared with the time of an encounter. By the "time of an encounter" is meant the time during which the molecule is so near another molecule that there is a sensible force acting between them. When a gas is compressed the mean free path is reduced, and the encounters become more frequent between one molecule and another, and also between the molecules and the walls of the vessel. When a gas is heated the speed with which the molecules move is increased; we shall see immediately that their average kinetic energy is proportional to the absolute temperature.

The pressure of the gas, that is to say, the pressure which the gas exerts on every unit of surface of the containing vessel, is due entirely to the blows of the molecules upon the surface: the momentum given to the surface by their blows, per unit of area and per unit of time, measures the pressure in kinetic units.

Taking a gas that is chemically homogeneous, in which all the molecules have the same mass, call that mass m. Let N be the number of molecules present in unit of volume of the gas in any actual state as to pressure and temperature. Then mN represents the density, namely, the whole mass per unit of volume, and V, the volume per unit of mass, is equal to $1/m\mathrm{N}$.

Before proceeding to consider the pressure caused by molecular blows, we shall make the following postulates:

(1) That the molecules are perfectly free except during encounters, and therefore move in straight lines with uniform velocity, from one encounter to the next.

(2) That the time during which an encounter lasts is negligibly small in comparison with the time during which the molecule is free.

(3) That the dimensions of a molecule are negligibly small in comparison with the free path.

These three postulates are equivalent to assuming that the gas is perfect in the sense of § (12). They are not strictly true of any real gas; but we shall assume them to be true in what immediately follows, and shall thereby deduce from the molecular theory a result which corresponds to the formula of an ideal gas, $\mathrm{PV} = \mathrm{RT}$.

Suppose the gas to be in equilibrium in a vessel at rest, and let the velocity v of any molecule be resolved into rectangular components v_x, v_y, and v_z along three fixed axes.

Consider the pressure due to molecular blows upon a containing wall, of area S, forming a plane surface at right angles to the direction of x. The contribution which any molecule makes to the pressure on that wall is due entirely to the component velocity v_x; nothing is contributed by the components v_y or v_z. Any molecule which strikes the wall has the normal component of its velocity reversed by the collision. Hence the momentum due to the blow is $2mv_x$ where v_x is the normal component of the velocity and m is the mass of the molecule.

Consider next how to express the sum of the effects of such blows in a given time. For this purpose we may think of the molecules as divided into groups according to their velocities at any instant. Let n be the number, in unit volume of the gas, whose x-component of velocity, v_x, has the same numerical value, or does not differ from it by more than some assigned very small quantity. Since the number of molecules is very great, we may take the number to be the same in one cubic centimetre (say) as in another. There will of course be very many such groups, each with a different value of v_x. Think, in the first place, only of those in the group n. Half of the whole number of molecules in the group are moving towards S; the other half are moving away from it. At any instant of time there will therefore be within a small distance δx of the surface S, and moving towards it with component velocity v_x, a number of molecules of that group equal to $\frac{1}{2}n\mathrm{S}\delta x$. A molecule distant δx from S, and having a component velocity v_x towards S, would reach S in a time $\delta t = \delta x/v_x$, provided it did not encounter any other molecule on its way. Hence the number of blows delivered to S by molecules of that group, in the time δt, would (on the same proviso) be equal to the number of such molecules as originally lay within a distance x, namely, the number $\frac{1}{2}n\mathrm{S}\delta x$.

Hence also the momentum due to the blows on the area S in the time δt would be equal to $\frac{1}{2}n\mathrm{S}\delta x \times 2mv_x$, which becomes, per unit of area and per unit of time,

$$nmv_x\frac{\delta x}{\delta t} = nmv_x^2,$$

since $$v_x = \frac{\delta x}{\delta t}.$$

This is the momentum contributed by one group only. The pressure P is made up of the sum of the quantities of momentum contributed by all the groups; hence

$$\mathrm{P} = \Sigma nmv_x^2 = m\Sigma nv_x^2$$

or $$\mathrm{P} = m\mathrm{N}\overline{v_x^2},$$

where N is as before the whole number of molecules per unit of volume, and $\overline{v_x^2}$ is the average of v_x^2 for all the molecules.

Now the velocity v of any molecule is related to its components by the equation

$$v^2 = v_x^2 + v_y^2 + v_z^2.$$

Hence, if we write $\overline{v^2}$ for the average value of v^2 for all the molecules,

$$\overline{v^2} = \overline{v_x^2} + \overline{v_y^2} + \overline{v_z^2} = 3\overline{v_x^2},$$

since the motions take place equally in all directions.

The square root of $\overline{v^2}$ is called the "velocity of mean square." It is not the same thing as the average velocity, but is the velocity a molecule would have whose kinetic energy is equal to the average kinetic energy of all the molecules. Maxwell has shown (*Collected Papers*, i. 381) that in consequence of the encounters the distribution of velocity among the molecules is such that their average velocity is $\sqrt{8/3\pi}$ or 0·921 times the velocity of mean square. In calculating the pressure we are only concerned with the velocity of mean square.

The above expression for P may be written

$$\mathrm{P} = \tfrac{1}{3}m\mathrm{N}\overline{v^2}. \quad . \quad . \quad . \quad (104)$$

Further, since mN is the quantity of gas in unit volume, or 1/V, where V is (as usual) the volume of unit mass, this gives

$$\mathrm{PV} = \tfrac{1}{3}\overline{v^2}. \quad . \quad . \quad . \quad (105)$$

In obtaining this result we made (in order to simplify the argument) a proviso that each molecule of a particular group, lying initially within the distance δx of the wall, struck the wall without encountering other molecules on the way. This is not true, but any encounter on the way does not affect the final result in a gas to which the three postulates apply. For in any encounter, occurring in a gas that satisfies these postulates, some momentum, perpendicular to the wall, is simply transferred to another molecule, and reaches the wall without loss. The molecule which takes it up has to travel the full remainder of the distance in the direction of x, neither more nor less, since the dimensions of the molecules are negligibly small (Postulate 3), and no time is lost in the encounter (Postulate 2). Hence the general result of the encounters is not to alter the amount of momentum which reaches the wall in any given time, and the conclusion remains valid that

$$\mathrm{PV} = \tfrac{1}{3}\overline{v^2}.$$

Comparing this with the perfect-gas equation

$$\mathrm{PV} = \mathrm{RT},$$

we see that $\overline{v^2}$ is proportional to the absolute temperature; and consequently the average kinetic energy which the molecules possess in virtue of their velocity of translation is proportional to the absolute temperature. We shall call their energy of translation E′; they may, in addition, have energy of other kinds, as was pointed out in § (58).

The energy of translation of the molecules

E′ is equal to $\frac{1}{2}\overline{v^2}$ per unit mass of the gas. Hence by the molecular theory

$$PV=\tfrac{2}{3}E', \quad . \quad . \quad . \quad (106)$$

and the pressure is equal to two-thirds of the energy of translation per unit volume of the gas. Further, since PV = RT, this result may be written in the form

$$E'=\tfrac{3}{2}RT. \quad . \quad . \quad . \quad (107)$$

It may be noted in passing that the molecular theory explains why a gas is heated by compression. Think of the gas as contained in a cylinder, and being compressed by the pushing in of a piston in the direction of x. Then any molecule which strikes the piston recoils with an increased velocity because it has struck a body that is advancing towards it. The component velocity v_x normal to the piston is not simply reversed by the blow, but is increased by an amount $2v'$, where v' is the velocity with which the piston is moving when the molecule strikes it, for the quantity which is reversed is the relative velocity v_x+v'. The result is that the motion of the piston in compressing the gas augments the average velocity of the molecules, and consequently increases $\overline{v^2}$, on which the temperature depends.

The laws of Boyle, Avogadro, and Dalton follow immediately from the molecular theory, for gases which obey the three postulates. Keeping $\overline{v^2}$ constant, which implies constant temperature, we at once have Boyle's Law PV = constant, since $PV=\frac{1}{3}\overline{v^2}$.

If there are two gases at the same pressure, since $P=\frac{1}{3}mN\overline{v^2}$ in each,

$$m_1N_1\overline{v_1{}^2}=m_2N_2\overline{v_2{}^2}. \quad . \quad . \quad (108)$$

Maxwell has shown that if two gases are at the same temperature, the average kinetic energy of a molecule is the same in both, or

$$m_1\overline{v_1{}^2}=m_2\overline{v_2{}^2}. \quad . \quad . \quad (109)$$

Hence if they are at the same pressure and the same temperature

$$N_1=N_2, \quad . \quad . \quad . \quad (110)$$

that is to say, the number of molecules in unit volume is the same for both, which is Avogadro's Law. It follows that the density, or mass of unit volume, differs in the two gases in the ratio of the masses of their molecules; in other words, the density is proportional to the molecular weight.

Again, the molecular theory shows that in a mixture of two or more gases, each of which obeys the three postulates,

$$P=\tfrac{1}{3}m_1N_1\overline{v_1{}^2}+\tfrac{1}{3}m_2N_2\overline{v_2{}^2}+\text{etc.} \quad (111)$$

In other words, the partial pressure due to each constituent of the mixture is the same as it would be if the other constituents were not there. This is Dalton's Law (§ (60)). Thus the molecular theory, for gases which satisfy the three postulates, gives results identical with those we already know as laws of ideal perfect gases.

In a real gas the postulates do not strictly hold. The size of the molecules is not negligible, and in any encounter there is an appreciable time during which the molecules concerned exert forces on one another. There may even be temporary pairing or co-aggregation on the part of some molecules. It is interesting to inquire, in a general way, how these departures from the ideal conditions affect the calculation of the pressure.

For this purpose, consider the simple case in which one of a group of molecules, advancing towards the wall, meets another molecule, initially at rest, to which it passes on the whole of its momentum, and the other molecule then completes the journey and delivers the blow. If there were no loss of time in the encounter, and if the second molecule could be regarded as travelling over exactly the remainder of the distance, the rate at which the wall receives momentum would be exactly the same as if the encounter had not taken place. But if there were loss of time in the encounter, such, for example, as would occur if the two colliding molecules moved together for any appreciable time, with a common velocity, then the rate at which the wall receives momentum would be reduced, with the result of reducing P. On the other hand, if the molecules have a finite size, so that the one which was initially at rest had less distance to travel in completing the journey, the rate at which the impacts succeed one another on the wall would be increased, with the result of increasing P. This indicates that the pressure in a real gas will differ from the ideal pressure, which is given by the equation $PV=\frac{1}{3}\overline{v^2}$, by two small terms, one positive, depending on the size of the molecules, and one negative, depending on their cohesion. Such, in effect, is the kind of modification which finds expression in characteristic equations like those of Van der Waals, Clausius, or Callendar.

From the equation $P=\frac{1}{3}mN\overline{v^2}$, which may be taken as very nearly true of real gases at moderate pressures, it is easy to calculate the molecular velocity when the pressure and density are known. The product mN is the density, and there is no need to know m or N separately to find v. In oxygen, for example, at 0° C., the density is 0·001429 grammes per cubic cm. when the pressure is one atmosphere or $1{\cdot}0133\times10^6$ dynes per sq. cm. Hence $\bar{v}$ is 461 metres per second; in hydrogen at the same pressure and temperature it is 1839 metres per second.

In the ideal gas the energy of translation of the molecules, E′, is $\frac{3}{2}$PV or $\frac{3}{2}$RT. Consequently each of their three degrees of freedom of translation accounts for an amount of internal energy equal to $\frac{1}{2}$RT. By the dynamical principle of equipartition each *effective* degree of freedom of rotation must take up an equal amount, and we thereby obtain the results which were stated in § (58), as to the energy and specific heats of monatomic, diatomic, and polyatomic gases. So long as there is no appreciable energy of vibration within the molecule the whole energy of a gas which obeys the three postulates is made up of terms each of which is equal to $\frac{1}{2}$RT; the sum therefore depends only on T. Hence such a gas satisfies Joule's Law (§ (12)). Since the sum is simply proportional to T, it follows also that the specific heat is constant. It is only when account is taken of energy of vibration, which is not directly proportional to T, that the increase of specific heat is explained which is known to occur at high temperatures in gases that have more than one atom in the molecule.

J. A. E.

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THERMODYNAMICS:

- First Law of. When work is done by the expenditure of heat a definite quantity of heat goes out of existence for every unit of work done; and conversely, when heat is produced by the expenditure of work the same definite quantity of heat comes into existence for every unit of work spent. See "Thermodynamics," § (9); "Engines, Thermodynamics of Internal Combustion," § (4).
- Second Law of. It is impossible for a self-acting machine, unaided by any external agency, to convey heat from one body to another at a higher temperature. See "Thermodynamics," § (17); "Engines, Thermodynamics of Internal Combustion," § (4).

THERMO-ELECTRIC EFFECT. The passage of electricity round a circuit consisting of two dissimilar metals when the junctions are maintained at different temperatures. See "Thermodynamics," § (65).

THERMOELEMENT:

- Calibration of, by comparison with a standard thermoelement. See "Thermocouples," § (24).
- Copper-constantan, for temperatures up to 300° C. See *ibid.* § (2) (i.).
- Iron-constantan, for temperatures up to 800° C. See *ibid.* § (2) (ii.).
- Iron-nickel, for temperatures between 400° C. and 800° C. See *ibid.* § (2) (iii.).
- Nickel-chromium, Nickel-aluminium, for temperatures up to 1100° C. See *ibid.* § (2) (iv.).
- Platinum-Platinum 10 per cent rhodium, introduced by Le Chatelier in 1886, the most reliable of all combinations tested; generally employed in high-temperature work up to 1500° C. See *ibid.* § (3) (i.).
- Platinum-Platinum iridium, introduced by Barus in 1889, reliable for temperatures up to 1000° C. See *ibid.* § (3) (ii.).
- Temperature-E.M.F., Relationship of, represented by formulae. See *ibid.* § (23).

THERMOELEMENTS:

- Base Metal, for low-temperature work up to about 300° C. See "Thermocouples," § (2).
- Differential. See *ibid.* § (29).
- Temperature-E.M.F., Relationships of, for every 100 microvolts or every 10°, tabulated. See *ibid.* § (23).
- Wiring of, to Indicator. See *ibid.* § (28).

THERMOMETER:

- Calorimetric and Beckmann: instruments of short range capable of being read to a high degree of accuracy. See "Thermometry," § (8) (viii.).
- Chemical. See *ibid.* § (8) (iii.).
- Clinical: a mercury thermometer of short range, of the maximum type, used for the determination of the temperature of the human body or of animals. See *ibid.* § (8) (ix.).
- Comparison of, above 100° C. See *ibid.* § (11) (ii.).
- Constant-pressure, sources of error in. See "Temperature, Realisation of Absolute Scale of," § (44).
- Constant-volume, sources of error in. See *ibid.* § (45).
- Constant-volume Gas, constant corrections to (degrees), tabulated. See *ibid.* § (45), Table 14.
- Construction of Comparison Baths for. See "Thermometry," § (11) (i.).
- Corrections, applicable to mercury-in-glass thermometers, to reduce their readings to

those of the hydrogen gas-thermometer, tabulated. See *ibid.* § (5), Tables IV., V.

Deep-sea, used in the determination of the temperature of the sea at varying depths. See *ibid.* § (8) (vi.).

Determination of Upper Fixed Point of. See *ibid.* § (3) (v.).

Earth, used in the determinations of the temperature of the soil. See *ibid.* § (8) (v.).

High-range. See *ibid.* § (8) (ii.).

Liquid-in-glass Types of. See *ibid.* § (8).

Maximum. See *ibid.* § (8) (iv.*a*).

Mercury in Glass, used as a secondary standard of temperature and compared with a gas-thermometer in the range 0°-100°. See "Temperature, Realisation of Absolute Scale of," § (29).

Mercury in Glass and Organic Liquids in Glass, used as secondary standards of temperature and compared with a gas-thermometer in the range −273° to 0° C. See *ibid.* § (33).

Meteorological. See "Thermometry," § (8) (iv.).

Minimum. See *ibid.* § (8) (iv.*b*).

Pentane: a useful instrument for work at low temperatures. See *ibid.* § (8) (vii.).

Platinum, used as a secondary standard of temperature and compared with a gas-thermometer in various ranges. See "Temperature, Realisation of Absolute Scale of," §§ (29), (33) (iii.), (42) (ii.). See also "Resistance Thermometers."

Sodium-potassium. See "Thermometry," § (8) (x.).

Transmitting. See *ibid.* § (8) (xiii.).

Zero Changes of. See *ibid.* § (7).

THERMOMETRY

§ (1) INTRODUCTION.—The determination of temperature is one of the most frequently occurring operations in pure or applied science, and a very simple and convenient method of carrying out this observation is afforded by the expansion of a liquid contained in a closed receptacle. In the first place the mercury-in-glass thermometer will be dealt with. This instrument consists of a glass tube, generally referred to as the stem, closed at one end, while the other end terminates in a spherical or cylindrical enlargement, known as the bulb. The bulb and a portion of the stem are filled with mercury, the remaining part of the stem being vacuous or filled with gas. The stem is graduated, and readings of the temperature are obtained by noting the position of the end of the mercury column when the instrument is immersed in the medium the temperature of which is required.

Mercury offers many advantages over other liquids in the construction of a thermometer; these may be summarised briefly as follows:

(*a*) Mercury exists in a liquid state over a wide range of temperature; its freezing-point is −40° C., while the boiling-point is 356° C. under normal pressure. Under higher pressure the boiling-point may be raised considerably, and the use of mercury in thermometers is extended by an application of this to temperatures up to 550° C. This point will be dealt with in connection with high-range thermometers.

(*b*) The expansion of mercury is regular, that is to say, mercury does not exhibit any anomaly as in the case of water, which has a maximum density at 4° C.; this would lead to ambiguity if the latter substance were used in a thermometer. The expansion of mercury is not, however, accurately proportional to the temperature measured on the absolute scale, although it is approximately so; hence the normal scale of a mercury thermometer departs from the gas or absolute scale [1] as dealt with in a later paragraph.

(*c*) Mercury does not "wet" the glass, and consequently can be employed in fine capillaries.

(*d*) Mercury is opaque, and therefore can be easily seen in a fine-bore tube.

(*e*) Mercury may readily be obtained in a high state of purity by distillation.

The glass employed for the construction of a thermometer must be chosen with regard to several characteristics. The main consideration is that the glass shall rapidly return to its normal state after being exposed to an increase in temperature, and furthermore that it shall be stable. The first condition is never exactly realised in practice, for if the zero of a thermometer be accurately determined before and after exposure to a higher temperature it will be noticed that the original reading is not exactly reproduced. Furthermore, if a thermometer be kept at a constant temperature over a long period its readings will be found to have changed slightly; this matter is dealt with in the section on zero changes of thermometers. It is, of course, necessary that the glass used for the construction of thermometers, particularly those of higher range, shall have a high softening-point; the glass must be such that it can be worked in the blowpipe flame in the process of construction of the instrument without devitrifying and becoming cloudy. The glasses used in the construction of high-range thermometers offer greater difficulty in working than those employed for the ordinary types of instrument. The glass used for the stem of a thermometer (which need not necessarily be of the same material of which the bulb is constructed) should give clean-cut divisions when etched by hydrofluoric acid in order that an accurate scale may be obtainable. The glass, further, should be homogeneous and free from striae or inclusions which would lead to distortion of the position of the mercury column, besides rendering the bore of the capillary irregular.

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[1] See "Temperature, Realisation of Absolute Scale of," § (5).

Two classes of thermometers may be distinguished: firstly, standard instruments which are provided with both fixed points (viz. 0° C. and 100° C.), and for which the corrections may be determined by direct calibration; and secondly, those of restricted range which do not embrace both fixed points, and which are calibrated by comparison with standard instruments.

The use of other liquids may be mentioned here, although detailed descriptions of the thermometers will be given later. For temperatures lower than $-40°$ C. it is necessary to replace mercury by liquids of lower freezing-point. Alcohol is employed to a large extent and enables thermometers to be constructed for temperatures as low as $-80°$ C. Petroleum ether may be employed for still lower ranges, but a more satisfactory substance is pentane, which can be used down to the temperature of liquid air. For higher temperatures various metals, such as tin or a mixture of sodium and potassium, have been suggested; so far, however, these have not been found very satisfactory, and for temperatures above 500° C. recourse should be had to electrical pyrometers and radiation instruments.

§ (2) HISTORICAL.—The thermometer appears to have been first used at the beginning of the seventeenth century, but it is not known for certain who invented the instrument. D'Alencé, in his book on *Barometers, Thermometers, and Hygrometers*, published in 1688, probably the first separate treatise written on this subject, attributes the invention to the Dutch scientist Drebbel, about the year 1608; but it would appear that Galileo constructed similar instruments a few years earlier. Such a thermometer should more correctly be called a thermoscope; it consisted of a glass vessel provided with a long neck inverted in a basin of coloured liquid; by warming the vessel some of the air was expelled, and on subsequently cooling it the liquid rose in the tube; changes of temperature were made manifest by the rise and fall of the liquid. An arbitrary scale attached to the tube served to give a rough indication of the changes of temperature to which the instrument was subjected. In all these early instruments the expansion and contraction of air was relied on to give the indications. The employment of a liquid receives first mention some sixty to seventy years later in the *Proceedings of the Florentine Academy*, published in 1667, in which are described thermometers[1] consisting of a long tube with a spherical bulb filled with spirit, thereby rendering the instrument independent of changes of atmospheric pressure. Scales were attached to the thermometers, and an effort was made to secure uniformity between different instruments by figuring the scales in accordance with a definite scheme; the values chosen were 20 for the coldest temperature experienced in winter, and 80 for the hottest at midsummer. The resulting scale is, of course, very rough, but it affords evidence of an attempt to construct instruments whose readings would be comparable. Both Boyle and Hooke realised the deficiencies of these scales and independently suggested the employment of more definite reference points; Boyle used as his datum mark the height of the column of liquid at the freezing-point of oil of aniseed. Hooke, on the other hand, used the freezing-point of distilled water as a zero and, further, marked his degree-scale as proportional parts of the volume at the zero point. Sir Isaac Newton constructed a thermometer of linseed oil in glass,[2] his scale being based upon the melting-point of ice as zero and the temperature of the human body as 12°; on this scale he found that the boiling-point of water was 34° and the melting-point of tin 71°; the extended use of a liquid-in-glass thermometer to higher ranges is worthy of note.

A few years later considerable advances were made by Fahrenheit, who introduced mercury as a thermometric liquid, thereby constructing the forerunner of the mercury-in-glass thermometer. Fahrenheit introduced cylindrical bulbs in order to increase the sensitivity of his instruments, and also laid the foundations for the employment of a rational scale of temperature. His scale was based on three fixed points: the temperature of a mixture of ice, water, and salt was taken as zero, that of a mixture of ice and water as 32°, and the normal temperature of the human body as 96°. Fahrenheit observed that on his scale the boiling-point of water was 212°, although he did not take this as a reference point. The origin of the numbers appears to have been quite arbitrary. The numbers 32 and 212 are retained to the present day for the freezing- and boiling-points of water on the "Fahrenheit scale," although the modern Fahrenheit scale is not exactly the same as the original. More accurate observations show the temperature of the human body to be 98·4° instead of the value 96° assigned to it by Fahrenheit himself.

Mention must be made of the work of

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[1] One of these instruments is in the Museum at the Cavendish Laboratory, Cambridge.

[2] *Phil. Trans.*, 1701.

Réaumur, who studied the alcohol thermometer. He found that a certain mixture of alcohol and water expanded from 1000 parts to 1080 between the freezing- and boiling-points of water; he consequently divided his scale into 80 divisions, taking zero as the freezing-point of water and 80 for the boiling-point. His nomenclature survives to the present day, but the later scale does not agree with the original owing to the fact that Réaumur overlooked the serious error introduced by the assumption that the alcohol thermometer could be used up to the temperature of boiling water. His scale was, however, applied to mercury thermometers by other workers, giving the modern scale which is still extensively used in parts of Central Europe and Russia.

The centigrade scale was derived from a scale suggested by Celsius, who divided the interval between the boiling- and freezing-points of water into 100. He, however, inverted the usual order, and designated the freezing-point by 100 and the boiling-point by 0. Shortly afterwards this scale was reversed, producing the modern centigrade scale.

Much of the above information is given in a series of essays by Dr. Martine on the *Construction and Graduation of Thermometers*, published in 1792. Martine calls attention to the following points:

(*a*) The variation of boiling-point of water with pressure.

(*b*) The calibration of a thermometer tube by means of a detached thread of mercury.

(*c*) The difference in scales resulting from the use of different kinds of glass.

(*d*) The disadvantages attached to the use of a liquid which wets the glass tube, particularly if the liquid in question is very viscous.

(*e*) The sluggishness of thermometers provided with large bulbs, in which connection he advocates the use of a small bulb combined with a fine capillary to secure a reasonably open scale.

Martine appears to have had a very clear idea as to the desirability of introducing standard scales for thermometers, and strongly advocates the employment of the freezing- and boiling-points of water as the two fundamental fixed points. In one of his essays he studies very fully the various scales that had been suggested from the earliest dates, and derives a diagram by means of which these scales can be compared.

§ (3) STANDARD THERMOMETERS.—The determination of temperature by the mercury-in-glass thermometer is based upon the change in volume of mercury in a glass container, and the magnitude of a degree is indicated by a definite proportion of the apparent expansion between two fixed points. The fixed points chosen are the freezing- and boiling-points of water under special conditions, and on the centigrade scale, to which attention may be confined at the moment, the degree is represented by the hundredth part of the expansion between these limits. Thus in the thermometer indicated in *Fig.* 1 the temperature θ is defined as the ratio of the volume of the capillary between the points 0 and θ to that between 0 and 100, so that if V_0, V_θ, and V_{100} be the volumes of the bulb and capillary measured to 0, θ, and 100 respectively, the temperature is given by

$$\theta=\frac{V_\theta-V_0}{V_{100}-V_0}\times 100.$$

The dimensions of a standard thermometer must therefore be such that the mercury is visible in the capillary tube both when the temperature is lowered to the freezing-point of water and when it is raised to the boiling-point. The expansion of the mercury is manifested by the position of the column in the capillary tube forming the stem of the thermometer, and if the volume of this tube between the two fixed points be divided into one hundred equal parts each of these divisions will represent a degree on this particular scale. In a perfect instrument the internal diameter of the capillary tube would be constant throughout its length, and in consequence the division of this volume into one hundred equal parts would be attained by the division of the length between the two fixed points into this number of equal intervals of length. In actual practice the perfect instrument is never attained, and in consequence it becomes necessary to investigate the departure from the ideal conditions and to determine the necessary corrections which may be applied to the readings of the instrument to allow for these accidental variations. The points to which attention must be drawn are as follows:

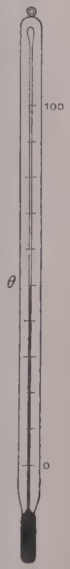

FIG. 1.

(*a*) Calibration Corrections. The capillary will not be such that it is possible to assume that equal intervals of volume will be given by dividing the length of the tube into equal parts; the volume between any two divisions will depend upon the mean cross-sectional area between the two divisions. It would appear, therefore, that the divisions on a thermometer stem should be placed at points corresponding to equal intervals of volume rather than to divide the tube linearly. Such a process would be exceedingly tedious in order to provide a sufficient number of subdivisions, and therefore the method adopted is to divide the tube by means of an accurate dividing machine and then obtain the necessary corrections to convert the readings to volume increments. The process by which these corrections are determined is termed the calibration of the tube, and is performed by measuring the length of a mercury thread in terms of the divisions of the tube when in different positions in the capillary. If the temperature at which the calibration is done is maintained constant the volume of the mercury contained in the thread will also be constant, and the length of thread in different parts of the tube will consequently give a measure of the variation in cross-section corresponding to these positions.

(*b*) and (*c*) Pressure Corrections. In an ideal thermometer the glass envelope should be incompressible, so that changes of pressure either inside the thermometer or exterior to it will have no effect on the position of the mercury column. In actual practice it is necessary to apply corrections to allow for this. Internal pressure arises from differences in the length of the mercury column at different temperatures, thereby producing a hydrostatic pressure within the bulb tending to increase the size of the bulb as the temperature rises, provided the thermometer is used in a position other than the horizontal; the correction is determined by noting the change of reading when the instrument is used in the horizontal and vertical positions at constant temperature. The external pressure upon the instrument depends upon changes in the barometric pressure, together with any additional pressure produced by the immersion of the thermometer below the surface of the liquid whose temperature is being measured. The total external pressure to which the instrument is subjected is the barometric pressure plus the hydrostatic head measured from the level of the liquid in the bath to the centre of the bulb of the thermometer.

(*d*) Zero Correction. The position of the zero of the thermometer may not be accurately determined in the first instance, and in any case the zero is liable to change in course of time, and also according to the temperature to which the instrument has previously been exposed.[1] It therefore becomes necessary to apply a correction to allow for the departure of the freezing-point from the zero of the scale engraved upon the tube.

(*e*) Fundamental Interval. One hundred divisions of the scale will not in general correspond to the difference between the position of the mercury column corresponding to the boiling- and freezing-points of water. This is corrected for by application of the fundamental interval correction.

(i.) *Calibration of Thermometers.*—As previously mentioned, a standard thermometer is constructed by dividing the difference between the fixed points into a definite number of intervals. Owing to the difficulty of making the divisions correspond to equal intervals of volume it is customary to divide the tube into equal intervals of length; it then becomes necessary to evaluate these divisions in terms of volumes. The process is known as calibration of the tube and is performed by the introduction of a thread of mercury which is moved into a series of positions along the tube. In the construction of thermometers of precision it is necessary to make a preliminary calibration of the tube to ascertain whether the bore is uniform, and only those tubes which are satisfactory in this respect should be employed for high-class instruments. The correction at any point should not exceed 0·1°, or at the most 0·2° C., for a thermometer of 50° C. range. It is particularly necessary that there should be no abrupt changes in diameter in passing along the tube, as at these points the value of the correction to be applied to the observed reading will change rapidly, and consequently the accuracy with which it can be determined will be reduced unless a very large series of observations is made. The complete calibration of the tube is in general carried out after the thermometer has been constructed, although it is possible for this to be done prior to making the thermometer. The work of calibrating the highest class of standard thermometer is very considerable, so much so that it is worth putting a new bulb to such a thermometer should the original bulb be broken. A highly skilled glass-blower can affix a new bulb and adjust its size so that the original scale is made use of. It is of course necessary to redetermine the fundamental interval, the pressure corrections and the zero of the thermometer, but the work of recalibrating the tube need not be performed. In the ordinary calibration of the tube by the mercury thread process it suffices to use a thread which will occupy about $\frac{1}{20}$th of the length of the tube it is desired to investigate; but for the highest class of work it is necessary to make use of a series of threads of various lengths.

Reference may first be made to the separation of a thread of mercury from the main column. This may be carried out by heating the tube with a small flame at a point below the end of the mercury column equal to the length of the thread required. Such a method is, however, dangerous, and in consequence should not be used for thermometers of any value. Recourse should be had to a method which, though somewhat tedious, does not involve risk of damage to the thermometer.

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[1] See § (7).

The thermometer is held in a vertical position with the bulb upwards; by lightly tapping the tube the mercury can be caused to run down from the bulb. The tube is then rapidly reversed and is sharply struck with the finger, causing the mercury to break away from the main bulk at the neck of the bulb. The thermometer is then held in a horizontal position and the separated thread of mercury allowed to move along the tube until its end is in a suitable position for observation. The column is then allowed to rejoin by warming the bulb. At the point of junction a very small bubble of residual gas will have accumulated, and this serves to break the column at this point when desired. Hence to break off a definite length of mercury the bulb is allowed to cool slowly until the mercury contained between this point and the end of the mercury column is equal to the desired length. Immediately this is the case a sharp tap will again break the thread, leaving a suitable column for the purposes of calibration.

For convenience in moving the column from point to point along the thermometer tube, in making a calibration, it is desirable to support the thermometer in a horizontal position on a table which may be tilted about a horizontal axis at right angles to the length of the thermometer. By suitably placing stops to limit the amount of tilt, it is possible to move the mercury thread slowly backwards and forwards in the capillary. The table also serves to support reading-microscopes, by means of which the positions of the ends of the thread of mercury are accurately determined in terms of the scale of the thermometer. Successive readings are taken along the tube, and the operation is repeated in the reverse direction, partly to reduce risk of errors of observation, but also to compensate for the small changes in volume of the mercury thread due to slight variations in the temperature.

As an example of the simplest method of calibration by the mercury-thread method the case of a thermometer ranging from 0° to 100° C. will be considered. A thread about 10° in length is first separated off and its lower end (*i.e.* the end nearer to the bulb) is brought to the zero of the scale. The position of the other end is then read in scale divisions by means of a low-power microscope. The thread is then moved along the tube so that its lower end now coincides with the division 10, and the reading of the other end is again observed. Proceeding in this way a series of ten readings will be obtained before the upper end of the thread is in the neighbourhood of the 100° division. The operation is then repeated in the reverse order, the upper end of the thread being brought to the divisions 100, 90, 80, etc., and the corresponding positions of the lower end noted. The mean of each pair of readings is taken, giving a series of values of the length of the thread for each 10° interval along the tube. The differences of these lengths from the mean length of the thread are then calculated. Let these differences be d_1, d_2, d_3, etc., then d_1 is the correction to be applied at the end of the first interval, *i.e.* at the 10° division; d_2 is the correction to be applied for the second interval, *i.e.* it would be the correction to be applied at the 20° division if the 10° division were not in error; the correction here is, however, d_1, so that the actual correction at the 20° division is the sum of these corrections, *i.e.* d_1+d_2. In the same way the correction at the 30° division is $d_1+d_2+d_3$, and so on. Due attention must of course be paid to the sign of d_1, d_2, d_3, etc., and it will be noted that ${}_1\Sigma_{10}\, d$ will be zero, *i.e.* the correction at 100 is zero.

The above method has the disadvantage that errors of observation are cumulative, so that no real increase in accuracy is attained by decreasing the length of the thread and increasing the number of intervals over which the thread is measured. To overcome this difficulty a number of more complicated methods have been devised in which the total length of tube calibrated is divided into a number of sections at points termed "principal points," and each of these sections are in turn investigated. These methods are dealt with by Guillaume[1] and Balfour-Stewart, Rucker and Thorpe,[2] and reference should be made to either of these accounts for details of a full calibration.

(ii.) *Internal Pressure Correction.*—The internal pressure correction to a thermometer is generally determined from observations[3] at the boiling-point of water, when the thermometer is read in the vertical and horizontal positions respectively. The internal pressure correction at the boiling-point of water is given by the difference between the readings in these two positions, while the correction at intermediate positions is taken as proportional to the distance of the end of the mercury column from the centre of the bulb. A table is then calculated so that the magnitude of the correction at each degree of the scale (or other suitable interval) may be readily ascertained.

In the case of thermometers in which the space above the mercury is filled with gas, allowance must be made for its pressure; this has to be calculated from the knowledge of the initial pressure and volume of the gas, together with the change in volume according to the position of the mercury column and of the change of temperature to which the thermometer is submitted. In general this latter correction will not be required, as primary standards cannot be gas-filled if a calibration of the tube is to be carried out by the ordinary method.

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[1] *Traité de la thermométrie*, 1889, pp. 46-99.

[2] "Methods employed in Calibration of Mercury Thermometers," *Brit. Assoc. Reports*, 1882.

[3] See "Determination of Upper Fixed Point," § (3) (v.)

(iii.) *External Pressure Correction.*—To determine the external pressure correction for a thermometer the change in the position of the mercury column at definite temperatures is measured when the thermometer is submitted to different external pressures. It is most convenient to choose atmospheric pressure as one of these values and a pressure of a few millimetres of mercury for the other. The apparatus employed is a glass tube into which the thermometer may be inserted, and which may be connected by means of a two-way stopcock to the atmosphere or to an exhausted reservoir. A mercury gauge serves to measure the pressure to which the thermometer is exposed. The glass tube enclosing the thermometer should contain some mercury at its lower end, in which the bulb of the thermometer is immersed, to enable it readily to pick up the surrounding temperature, and above this glycerine should be introduced to facilitate the reading of the thermometer through the glass tube. The whole of the tube should be enclosed in a bath of water, the temperature of which can be controlled. A bath such as is described later for the comparison of thermometers serves admirably for the purpose. Readings of the thermometer are taken with the glass tube alternately open to the atmosphere and connected to the exhausted vessel.

If t_1 and t_3 are successive readings of the thermometer when exposed to the atmospheric pressure, p_1 and t_2 is an intermediate reading of the thermometer under reduced pressure p_2, then the external pressure coefficient is given by the relation

$$\beta(p_1 - p_2) = \frac{t_1 + t_3}{2} - t_2.$$

The value of β can then be calculated.

This formula holds if the temperature of the bath be slowly changing during the observations, provided the rate of change is uniform and that the successive readings are taken at regular intervals; in fact the most accurate values of β are derived from observations made with the temperature rising at such a rate that the value of t_3 is slightly in excess of t_1, say 0·01° C., as otherwise the reading corresponding to the lower pressure will be taken on a falling meniscus and may therefore be affected by capillarity. A number of observations should be made and the most probable value of the coefficient calculated by the method of least squares. From a knowledge of the coefficient of external pressure as above determined and of the coefficient of elasticity of the glass, it is possible to calculate the mean thickness of the walls of the bulb on the assumption that this bulb is a uniform cylinder. Considerations of the sensitivity of thermometers lead to a reduction of this thickness; the advantage, however, is counterbalanced by the resulting increase in the external pressure correction, which becomes more important the thinner the walls of the bulb. For standard thermometers the thickness may conveniently be from 0·5 mm. to 0·7 mm., and in such a case the external pressure correction is of the order of 0·0001° C. per mm. change in pressure.

For convenience in applying the pressure correction to thermometers it is desirable to calculate a table giving values of the correction for each millimetre change in the external pressure to which the thermometer is likely to be subjected. The external pressure in any observation will be the barometric pressure plus the hydrostatic head corresponding to the difference in level of the centre of the thermometer bulb and the surface of the liquid in which it is immersed.

(iv.) *Determination of the Zero of a Thermometer.*—The zero of a thermometer on the centigrade scale is defined as the temperature of pure ice melting under a pressure of 760 mm. of mercury; in actual practice it is not necessary to make any correction for the changes in barometric pressure, since the lowering of the temperature of fusion is only of the order of 0·008° C. per atmosphere increase of pressure.[1] The determination is most easily carried out in the simple apparatus indicated in *Fig.* 2; a glass bell-jar, of diameter 5-6 in. and depth from 12-15 in., is supported in a suitable case in an inverted position and surrounded by non-conducting material. The bell-jar is provided with an exit tube fitted with a tap. A vacuum vessel of large size, particularly of the type provided with an opening at the lower end, can with advantage be used in place of the lagged bell-jar. The vessel is packed with ice in the form of fine shavings, and is moistened with distilled water; excess of water may be run off through the exit tube, but the ice must not be drained so that it appears white; it must remain thoroughly saturated with water during

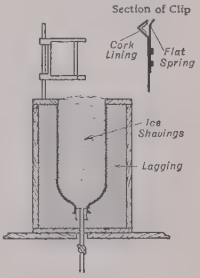

FIG. 2.

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[1] Tammann, 1900.

observations. For the most accurate determinations it is necessary to make the ice from distilled water, very special precautions being taken to prevent contamination with any saline matter. For most practical work, however, natural ice may be used, as the purity of this is very high. The use of ordinary commercial ice is to be deprecated, as this frequently contains salt from the brine circulation employed in its manufacture. The absence of soluble chlorides should be tested for by means of silver-nitrate solution; sulphates should also be looked for by means of barium-chloride solution. Prior to preparing the ice shavings the block of ice should be carefully washed in distilled water. To insert the thermometer under test a hole is made in the tightly packed ice by means of a glass rod of diameter about that of the thermometer under investigation; this is pressed into the ice vertically to a suitable distance; the thermometer is then inserted into the hole so formed and supported by means of the clip shown and its height adjusted so that the mercury column is just visible above the ice. Care must of course be taken that the thermometer does not reach the bottom of the vessel. For thermometers in which the zero point is some distance from the bulb a longer ice-bath may be required. In the other case, in which the ice point is very close to the bulb, it is desirable to pile the ice round the stem above the level of the zero point; a small amount of ice is then removed from in front of the thermometer to permit of observations being made. Readings are made with the aid of a telescope with a micrometer eyepiece; a magnifying power of about 10 is sufficient for most purposes. It may be pointed out here that, in taking a zero reading after exposure to a higher temperature, recovery of the depression begins almost immediately; readings should be taken corresponding to the lowest reached or, alternatively, after a definite interval of time has elapsed since the thermometer was removed from the higher temperature. In making observations of the zero it is often desirable, especially in the case of thermometers of fine bore, to subject the instrument to vibration by sharply striking the table on which the apparatus is standing, before taking a reading. This prevents the holding up of the mercury column by capillarity. It is obviously not possible to overcome this difficulty by taking the ice-point on a rising temperature as in most other thermometric comparisons.

The zero reading, if not coincident with the 0° division on the thermometer, requires correction for the calibration error of the tube, and the internal pressure, while a correction for the external pressure may also be necessary.

(v.) *Determination of the Upper Fixed Point of a Thermometer.*—The upper fixed point of a thermometer, namely 100° C. (or its equivalent 212° F.), is defined as the temperature of the steam issuing from boiling water under a pressure of 760 mm. of mercury corrected to zero temperature, to a latitude of 45°, and to the sea-level. The apparatus in which the boiling-point is usually determined is essentially that used by Regnault, and consists of a cylindrical tube in communication with the vessel in which the water is boiled. The tube is surrounded by a second tube of somewhat larger diameter in such a way that the steam from the boiler passes up the inner tube and then down the outer tube, thence it is conveyed to a condenser open to the atmosphere or is merely allowed to escape; the inner tube is thus provided with a steam-jacket which prevents partial condensation and local variations of temperature. The thermometer is supported so that the bulb and most of the mercury column are within the inner tube; the mercury being just visible above the top of the apparatus. Care must be taken to ensure that the bulb of the thermometer does not reach the level of the water in the boiler and that drops of water cannot fall on it if the ebullition is somewhat vigorous. Two modifications of this apparatus are employed at the National Physical Laboratory and may be referred to here—the first is due to Chappuis, and is such as is used at the International Bureau at Sèvres, while the second is an electrically heated bath for dealing with six thermometers at a time.

(*a*) The apparatus designed by Chappuis[1] is indicated in *Fig.* 3. In this apparatus the jacketed tube in which the thermometer is supported can be turned about a horizontal axis while still in communication with the boiler and the condenser so that the thermometer may be read at the boiling-point of water in the vertical and horizontal positions. This enables the internal pressure correction to be determined (see section on "Internal Pressure Correction").

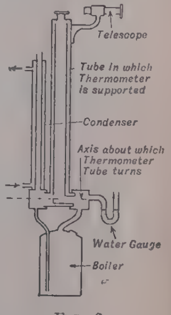
Telescope
Tube in which Thermometer is supported
Condenser
Axis about which Thermometer Tube turns
Water Gauge
Boiler

FIG. 3.

(*b*) *Fig.* 4 shows the construction of the boiling-point apparatus designed at the National Physical Laboratory. Points to which attention may be drawn are the following. A sheet of gauze is supported within the inner tube of the apparatus to prevent any water being splashed on to the bulb of the thermometers. The condensed steam is returned to the boiler, which is further provided with a constant level device. The heating of the water is carried out electrically. Two heater units are provided. These are controlled by a two-way switch with a central "off" position, the connections being made so that the two units are arranged in parallel or in series. In the first position the temperature of the water is rapidly raised to the boiling-point, while in the second or series position the amount of energy supplied is sufficient to keep the water just boiling so that the bath is maintained full of steam for any length of time.

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[1] See Chappuis, *Trav. et Mém.*, 1888, vi.

Both types of baths are provided with water gauges to give indication of the excess pressure of the steam above that of the atmosphere.

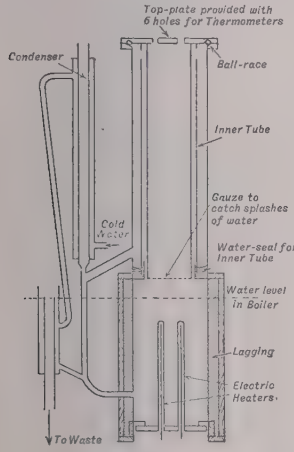
Top-plate provided with 6 holes for Thermometers
Condenser
Ball-race
Inner Tube
Gauze to catch splashes of water
Cold Water
Water-seal for Inner Tube
Water level in Boiler
Lagging
Electric Heaters
To Waste

FIG. 4.

The thermometer is exposed to the steam until the reading remains constant. This will take several minutes in most cases, or even as long as an hour, owing to the change of zero resulting from the heating of the thermometer. Prior to the reading being taken it is desirable that the thermometer be immersed to above the end of the mercury column, the instrument being slightly raised for purposes of reading so that the mercury column is just visible above the end of the tube.

At the same time that the thermometer is read an observation of the barometric height is made; the barometer reading is corrected for temperature, latitude, and height above sea-level as dealt with in the article on "Barometers."[1] The temperature of the boiling-point of water at a particular barometric pressure is derived from the tables calculated by Broch[2] from Regnault's results given in Table I.

(vi.) *Fundamental Interval Correction.*—To obtain the fundamental interval correction to a thermometer the upper fixed point is determined as described in the preceding section and is followed by an immediate determination of the zero point. The difference of these two readings for a perfect thermometer would be 100 divisions, but in practice a value differing slightly from this will be obtained. Let this be represented by $(100+\delta)$ scale divisions. Then each scale division of the thermometer will correspond to $(100+\delta)/100$ degrees, and the number of degrees corresponding to θ divisions will be $(100+\delta)\theta/100$, *i.e.* $\theta+\delta\,.\,\theta/100$; hence the correction for fundamental interval to be applied to a reading θ is $\delta\,.\,\theta/100$ (δ may of course be negative). The correction is applied after the reading has been corrected for errors of calibration and pressure.

§ (4) PRIMARY STANDARDS.—To secure uniformity in the practical measurement of temperature in different countries the International Bureau of Weights and Measures at Sèvres carries out the determination of the corrections to mercury thermometers intended to serve as fundamental temperature standards over the range 0° to 100° C. These instruments are constructed by Baudin of Paris (formerly Tonnelot), and are made of "verre dur." The scale 0° to 100° C. is generally covered by two instruments ranging from 0° to 50° C. and 50° to 100° C. The tubes are carefully chosen for uniformity of cross-section, and are divided to 0·1°; the division lines are very fine and can only be read with the aid of a telescope. The tubes are not provided with enamel backs, so that readings may be taken from behind as well as in front to avoid errors of parallax.

With such instruments readings may be taken to 0·005° or 0·002° by estimation, and after the appropriate corrections are applied to the mean of a series of readings on very slowly rising temperatures agreement between several thermometers is obtained to an accuracy of 0·002°, or in some cases to 0·001°.

To give a clear idea of the method of applying the corrections the following example of an actual series of readings with a pair of Baudin thermometers is given. The zero readings are made immediately after the observations, and the various corrections are obtained from the tables sent out by the International Bureau for the thermometers in question.

Two thermometers, No. 16377 and No. 16378, of range 0° to 50° C. and divided to 0·1° C., were immersed in a bath of water (see § (11)), the temperature of which was very slowly rising, and the readings noted in

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[1] See Vol. III.

[2] Broch, *Trav. et Mém.*, 1881, i.

TABLE I

Mm.	Tenths of a Millimetre.									
	0.	1.	2.	3.	4.	5.	6.	7.	8.	9.
735	99·070	·073	·077	·081	·085	·088	·092	·096	·100	·104
736	·107	·111	·115	·119	·122	·126	·130	·134	·137	·141
737	·145	·149	·153	·156	·160	·164	·168	·171	·175	·179
738	·183	·186	·190	·194	·198	·201	·205	·209	·213	·216
739	·220	·224	·228	·232	·235	·239	·243	·247	·250	·254
740	·258	·262	·265	·269	·273	·277	·280	·284	·288	·292
741	·295	·299	·303	·307	·310	·314	·318	·322	·325	·329
742	·333	·337	·340	·344	·348	·351	·355	·359	·363	·366
743	·370	·374	·378	·381	·385	·389	·393	·396	·400	·404
744	·408	·411	·415	·419	·423	·426	·430	·434	·437	·441
745	·445	·449	·452	·456	·460	·464	·467	·471	·475	·479
746	·482	·486	·490	·493	·497	·501	·505	·508	·512	·516
747	·519	·523	·527	·531	·534	·538	·542	·546	·549	·553
748	·557	·560	·564	·568	·572	·575	·579	·583	·586	·590
749	·594	·598	·601	·605	·609	·612	·616	·620	·624	·627
750	·631	·635	·638	·642	·646	·650	·653	·657	·661	·664
751	·668	·672	·676	·679	·683	·687	·690	·694	·698	·701
752	·705	·709	·713	·716	·720	·724	·727	·731	·735	·738
753	·742	·746	·750	·753	·757	·761	·764	·768	·772	·775
754	·779	·783	·787	·790	·794	·798	·801	·805	·809	·812
755	·816	·820	·823	·827	·831	·834	·838	·842	·846	·849
756	·853	·857	·860	·864	·868	·871	·875	·879	·882	·886
757	·890	·893	·897	·901	·904	·908	·912	·916	·919	·923
758	·927	·930	·934	·938	·941	·945	·949	·952	·956	·960
759	·963	·967	·971	·974	·978	·982	·985	·989	·993	·996
760	100·000	·004	·007	·011	·015	·018	·022	·026	·029	·033
761	·037	·040	·044	·048	·051	·055	·059	·062	·066	·070
762	·073	·077	·081	·084	·088	·092	·095	·099	·103	·106
763	·110	·114	·117	·121	·125	·128	·132	·136	·139	·143
764	·147	·150	·154	·157	·161	·165	·168	·172	·176	·179
765	·183	·187	·190	·194	·198	·201	·205	·209	·212	·216
766	·219	·223	·227	·230	·234	·238	·241	·245	·249	·252
767	·256	·260	·263	·267	·270	·274	·278	·281	·285	·289
768	·292	·296	·300	·303	·307	·311	·314	·318	·321	·325
769	·329	·332	·336	·340	·343	·347	·350	·354	·358	·361
770	·365	·369	·372	·376	·379	·383	·387	·390	·394	·398
771	·401	·405	·409	·412	·416	·419	·423	·427	·430	·434
772	·437	·441	·445	·448	·452	·456	·459	·463	·466	·470
773	·474	·477	·481	·485	·488	·492	·495	·499	·503	·506
774	·510	·513	·517	·521	·524	·528	·532	·535	·539	·542
775	·546	·550	·553	·557	·560	·564	·568	·571	·575	·578
776	·582	·586	·589	·593	·596	·600	·604	·607	·611	·614
777	·618	·622	·625	·629	·632	·636	·640	·643	·647	·650
778	·654	·658	·661	·665	·668	·672	·676	·679	·683	·686
779	·690	·694	·697	·701	·704	·708	·712	·715	·719	·722

Table II. were obtained to the nearest 0·005° by estimation. The instruments were immersed so that the columns were just visible above the top of the bath; the depths of immersion to the centres of the bulbs being 27·9 and 27·8 cm. respectively. The order in which the readings were taken is indicated by the letters (*a*), (*b*), etc., after the readings. This is in accordance with the procedure described in § (11).

In accordance with the standard procedure of the International Bureau the zero corrections given in the above table were obtained by making observations of the freezing-point

TABLE II

Thermometer No.	16377	16378
Readings	30·350 (*a*) ·350 (*d*) ·350 (*e*) ·355 (*h*)	30·360 (*b*) ·360 (*c*) ·365 (*f*) ·370 (*g*)
Means	30·351	30·364
Calibration correction	−0·024	−0·038
Internal pressure	+0·044	+0·041
External pressure *	−0·002	−0·002
Fundamental interval	+0·002	+0·002
Zero correction (see below)	−0·099	−0·096
Corrected reading	30·272	30·271
Mean reading	30·272	
Correction to hydrogen scale	−0·102	
Corrected temperature	30·170	

* External pressure = barometric pressure (752 mm.) + 27·8 cm. of water.

of water immediately after the readings at 30° were obtained. Table III. gives the recorded observations and their reduction. The readings were obtained using a telescope with micrometer eyepiece and two settings were made in each case.

TABLE III

Thermometer No.	16377		16378	
Setting	(*a*)	(*b*)	(*a*)	(*b*)
Micrometer reading on the 0·0 division	30·0	29·5	1·9	2·1
Micrometer reading on column	70·9	70·9	41·6	42·0
Micrometer reading on 0·1 division	76·0	75·9	48·1	47·9
	$\frac{40\cdot9}{46\cdot0}$	$\frac{41\cdot4}{46\cdot4}$	$\frac{39\cdot7}{46\cdot1}$	$\frac{39\cdot9}{45\cdot8}$
Corresponding reading of column	0·089	0·089	0·086	0·088
Means	+0·089		+0·087	
Calibration correction	0·000		0·000	
Internal pressure correction	+0·010		+0·009	
External pressure correction	0·000		0·000	
Fundamental interval correction	0·000		0·000	
Mean zero reading	+0·099		+0·096	

§ (5) CORRECTION TO THE GAS SCALE OF TEMPERATURE.—The subject of gas thermometry and the relationships of the hydrogen and absolute scales of temperature are dealt with elsewhere,[1] so that in this section it is only necessary to deal with the connection between the indication of a mercury thermometer and the gas scale.

At the Fifth Conference of Weights and Measures, held at Paris in 1913, it was agreed that the centigrade thermodynamic scale should be adopted as the fundamental scale of temperature. For the practical realisation of this scale it was further agreed that over the range 0° to 100° C. the scale of the constant-volume hydrogen thermometer coincides with the thermodynamic scale to the accuracy attained by precision mercury thermometers, and that outside this range between the freezing-point of mercury and the boiling-point of sulphur the thermodynamic scale is realised by means of the platinum thermometer under specified conditions[2] to a sufficiently high order of accuracy for practical purposes.

In order to investigate the relationship between the readings of a mercury thermometer and the corresponding temperature on the gas scale, consider the case of a thermometer the volume of whose bulb and that portion of the stem up to the zero division is V_0 measured at a temperature of 0° C.; assume also that the volume of the capillary between successive degree divisions is v_0 also measured at 0° C. A knowledge of the absolute coefficients of expansion of mercury and of the glass of which the thermometer is constructed, both referred to the gas scale of temperature, is also required. Suppose these are represented by $F(T)$ and $f(T)$ respectively, where T is the temperature measured on the gas scale, then at temperature T the volume of mercury originally filling the bulb at 0° C. will be given by $V_0F(T)$, while the volume of the bulb will have changed from V_0 to $V_0f(T)$. Now $V_0F(T)$ will be greater than $V_0f(T)$ as the expansion of mercury is greater than that of glass, so that some mercury will overflow from the bulb into the stem of the thermometer and this volume will be

$$V_0F(T) - V_0f(T).$$

Assume the mercury overflowing will fill θ degrees of the scale, and, since each division originally occupied a volume v_0, the volume of mercury overflowing is also given by the expression

$$\theta \, . \, v_0f(T).$$

Equating these two expressions gives

$$\theta = \frac{V_0}{v_0} \times \frac{F(T) - f(T)}{f(T)}.$$

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[1] See "Temperature, Realisation of Absolute Scale of."

[2] See "Resistance Thermometers" and "Temperature, Practical Scale of."

The functions F(T) and f(T) may be expressed in the form $(1+c_1T+c_2T^2+c_3T^3\ .\ .\ .)$ so that if m_1, m_2, m_3 . . . and g_1, g_2, g_3 . . . are taken for the constants for mercury and glass respectively the above expression reduces to

$$\theta=\frac{V_0}{v_0}\times\frac{\{(m_1-g_1)T+(m_2-g_2)T^2+(m_3-g_3)T^3+\ .\ .\ .\}}{\{1+g_1T+g_2T^2+g_3T^3+\ .\ .\ .\ \}}.$$

V_0/v_0 is dependent solely upon the dimensions of the thermometer under consideration, and its value is determined by putting θ and T each equal to 100° C. as at this point both scales are coincident by definition. Hence

$$100=\frac{V_0}{v_0}\frac{\{(m_1-g_1)100+(m_2-g_2)100^2+\ .\ .\ .\ \}}{\{1+g_1\,.\,100+g_2100^2+\ .\ .\ .\ \}},$$

i.e.

$$\theta=\frac{100(1+g_1100+g_2100^2+\ .\ .\ .\ )\{(m_1-g_1)T+(m_2-g_2)T^2+\ .\ .\ .\ \}}{\{(m_1-g_1)100+(m_2-g_2)100^2+\ .\ .\ .\ \}(1+g_1T+g_2T^2+\ .\ .\ .\ )}.$$

Thus the relationship between the mercury and gas scales is given in terms of the coefficients of expansion of mercury and glass.

A considerable simplification of the above formula would result if the expansion of glass and mercury were both linear, for in this case the value of m_2, m_3, m_4 . . ., g_2, g_3, g_4 . . ., etc., would be zero, and the expression given above reduces to

$$\theta=T\frac{1+g_1100}{1+g_1T}$$

or

$$\theta-T=g_1\frac{T(100-T)}{1+g_1T}.$$

The values of θ and T are therefore not coincident, as might at first be supposed, but are dependent upon the coefficient of expansion of glass. Coincidence between the mercury and gas scales of temperature would only be attained if the glass used in the construction of the thermometer did not expand or contract with change of temperature. In this connection it is of interest to note that owing to the much smaller coefficient of expansion of fused silica than that of the usual thermometric glasses a thermometer constructed with this material should give a scale more closely approximating to the gas scale than in the case of a glass thermometer. This has not received experimental verification, as sufficiently accurate thermometers have not yet been constructed of fused silica.

The formula

$$\theta-T=g_1\frac{T(100-T)}{1+g_1T}$$

does not represent practical results, as it was obtained on the assumption that the expansion of mercury is linear; this is, of course, not the case. Various formulae have been proposed to represent experimental determinations, and from a review of the published data Sears[1] gives the formula

$$V_T=V_0\{1+\alpha T+\beta T^2+\gamma T^3+\delta T^4\},$$

where
$\alpha=181{\cdot}456\times10^{-6}$,
$\beta=0{\cdot}009205\times10^{-6}$,
$\gamma=0{\cdot}000006608\times10^{-6}$,
$\delta=0{\cdot}000000067320\times10^{-6}$.

The expansion of glass is usually represented by a formula of the type $V_T=V_0(1+g_1T+g_2T^2)$, but the numerical values of the coefficients are not known to a high enough order of accuracy to give values of θ in terms of T which agree well with those obtained by experimental methods. The general form of the relationship is, however, in agreement, for the full expression given above may be written in the form

$$\theta-T=T(100-T)(A+BT+CT^2),$$

where A, B, and C are constants depending only on the coefficients in the expansion formulae for glass and mercury. It will be noted that $(\theta-T)$ vanishes if T = 0 or T = 100, since at these points the mercury and gas scales are coincident by definition of the respective scales.

The differences between the mercury and the gas scale have been the subject of extended researches in the case of several of the special glasses used in the construction of mercury thermometers. Reference may be made to the work of Chappuis,[2] who investigated the case of thermometers constructed of "verre dur," the French hard glass used in the construction of the primary standards distributed by the International Bureau at Sèvres. The tests were made by comparison of several mercury thermometers in a horizontal water bath with a constant-volume hydrogen gas thermometer. The results of these tests showed that the difference between temperatures on a "verre dur" mercury thermometer and the hydrogen gas thermometer could be represented by the formula

$$\theta-T=T(100-T)(62{\cdot}296-0{\cdot}48946T+0{\cdot}0012805T^2)\times10^{-6}$$

or

$$\theta-T=\theta(100-\theta)(61{\cdot}859-0{\cdot}4735\theta+0{\cdot}001158\theta^2)\times10^{-6},$$

over the range −35° to 100° C. The second form is more convenient for application of the correction in practical work. Table IV. gives the value of these corrections over the range 0° to 100° C.

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[1] Sears, *Proc. Phys. Soc.*, 1913–14, xxvi. 96.

[2] Chappuis, *Trav. et Mém.*, 1888, vi.

TABLE IV

θ.	Values of Temperature on Mercury Scale – Temperature on Gas Scale (θ – T).*									
	0.	1.	2.	3.	4.	5.	6.	7.	8.	9.
0	0·000	0·006	0·012	0·018	0·023	0·028	0·033	0·038	0·043	0·047
10	0·052	0·056	0·060	0·063	0·067	0·070	0·073	0·076	0·079	0·082
20	0·085	0·087	0·089	0·091	0·093	0·095	0·097	0·098	0·100	0·101
30	0·102	0·103	0·104	0·105	0·106	0·106	0·107	0·107	0·107	0·107
40	0·107	0·107	0·107	0·107	0·107	0·106	0·106	0·105	0·104	0·104
50	0·103	0·102	0·101	0·100	0·099	0·097	0·096	0·095	0·093	0·092
60	0·090	0·089	0·087	0·085	0·084	0·082	0·080	0·078	0·076	0·074
70	0·072	0·070	0·068	0·066	0·064	0·062	0·059	0·057	0·055	0·053
80	0·050	0·048	0·045	0·043	0·041	0·038	0·036	0·033	0·031	0·028
90	0·026	0·023	0·021	0·017	0·016	0·013	0·010	0·008	0·005	0·003
100	0·000									

Thermometers constructed of Jena 16^{III} and Jena 59^{III} glasses have been studied at the Reichsanstalt by Theisen, Scheel, and Sell,[1] who compared thermometers of these glasses with "verre dur" thermometers in both the horizontal and vertical positions; their results were reduced to the hydrogen scale by Sheel,[2] by application of the formula derived from Chappuis above referred to, and lead to the expressions

Jena 16^{III}:

$$\theta - \mathrm{T} = \theta(100-\theta)(67{\cdot}039 - 0{\cdot}473451\theta + 0{\cdot}0011577\theta^2)\times 10^{-6},$$

Jena 59^{III}:

$$\theta - \mathrm{T} = \theta(100-\theta)(31{\cdot}089 - 0{\cdot}473451\theta + 0{\cdot}0011577\theta^2)\times 10^{-6}.$$

Table V. shows for purposes of comparison the magnitude of the corrections computed from the above formulae to "verre dur," Jena 16^{III}, and Jena 59^{III} thermometers over the range −30° to 100° C. at intervals of 10°.

TABLE V

Temperature.	Temp. on Mercury Scale—Temp. on Gas Scale.		
	"Verre dur."	Jena 16^{III}.	Jena 59^{III}.
−30	−0·30	−0·32	−0·18
−20	−0·17	−0·19	−0·10
−10	−0·07	−0·08	−0·04
0	0·00	0·00	0·00
10	0·052	0·056	0·024
20	0·085	0·093	0·035
30	0·102	0·113	0·038
40	0·107	0·120	0·034
50	0·103	0·116	0·026
60	0·090	0·103	0·016
70	0·072	0·083	0·007
80	0·050	0·058	0·001
90	0·026	0·030	−0·002
100	0·000	0·000	0·000

§ (6) THERMOMETRIC GLASSES.—If the zero point of a thermometer, as ordinarily determined by immersion in pure melting ice, is taken periodically it will be noted that variations occur. This is equivalent to stating that the volume of the bulb is not always the same when the temperature of the instrument is brought to a definite value. Although referred to as the zero change, the variations in the indications of the thermometer are manifest at any point in the scale, but readings are usually taken at the zero owing to the greater ease with which observations may be made at this point. In the case of high-range thermometers not divided as low as 0° C. the change may be investigated by observations carried out at the boiling-point of water. This is somewhat less satisfactory owing to the correction which has to be applied for changes in atmospheric pressure.

The change in zero is dependent upon the nature of the glass employed in the construction of the thermometer, and many attempts have been made to introduce special glass to reduce these changes to a minimum. Before dealing with the zero changes in detail, reference may be made to some of the thermometric glasses which have been in use in recent years.[3]

(*a*) English crystal glass and also "Kew" glass, which were in use in this country for many years, were lead-potash glasses which contained a small proportion of soda. These glasses have since been superseded for accurate work by later types, but lead glasses are still employed for the cheaper class of thermometers at the present time, mainly owing to their softer nature and good working properties.

(*b*) "Verre dur" is essentially a soda-lime glass, and is of especial importance, as it has been the subject of much investigation by Guillaume and others in connection with its employment for the primary mercury standards of the International Bureau to which reference has already been made.

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[1] Theisen, Scheel, and Sell, *Zeit. Instrumentenk.*, 1895, xv. 433.

[2] Scheel, *Wied. Ann.*, 1896, lviii. 168-170.

[3] See article "Glass," Vol. IV.

(*c*) Jena 16^{III}.—In 1883 Schott began experiments at Jena on the production of glasses for thermometric work, and soon produced the glass numbered 16^{III}, which was designated "normal thermometer glass"; it is a soda-lime glass containing zinc oxide and alumina. This glass has had very widespread use in the construction of thermometers of ordinary and medium range. Tubes made of this glass may be recognised by a thin purple line throughout their length.

(*d*) Jena 59^{III}.—For work at higher temperatures Schott further developed a harder glass known as Jena 59^{III}, or borosilicate glass, the principal ingredients of which are soda, boric oxide, and alumina.

(*e*) More recently developed glasses to which reference may be made include Powell's normal glass, which is distinguished by a broad deep blue line running through the tubes, Powell's borosilicate glass, and the Corning normal glass which is extensively employed in America. The compositions of the latter glasses are not yet available for publication.

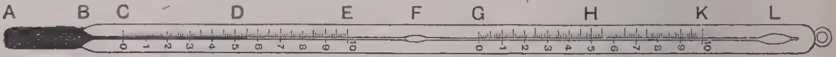

Fig. 5.

Correlation of the analyses of the glasses given in Table VI. with the thermal properties dealt with later shows that the joint presence of soda and potash has a deleterious effect upon the value of the glass from the thermometric point of view, and this was confirmed by Schott [1] in his early work on the subject. The borosilicate glasses give the best results at high temperatures, while the presence of alumina conduces to ease of working and the absence of devitrification.

Table VI

Approximate Percentage Composition of various Glasses

Constituent.	English Crystal.	Kew Glass.	"Verre dur."	Jena 16^{III}.	Jena 59^{III}.
SiO_2 . .	50	53	71	67	72
K_2O . .	12	11·5	0·5	..	..
Na_2O . .	1·5	0·5	12	14	10
PbO . .	34	34	..	..	..
B_2O_3 . .	..	..	..	2	11
CaO . .	1·5	..	14	7	0·5
Al_2O_3 . .	0·5	0·5	2	3	6
ZnO . .	..	..	..	7	..
Fe_2O_3, Mn_2O_3, MgO	0·5	0·5	0·5	..	0·5

The figures in the above table must only be regarded as approximate, as individual samples of glass vary considerably; traces of other oxides are found on analysis.

Although of secondary importance compared with the afterworking effects, some attention must be paid to the suitability of the glass to give good division lines by etching with hydrofluoric acid. The lead glasses appear to offer advantages in this respect, so that English crystal glass is frequently used for the stem of a thermometer in conjunction with the normal glass for the bulb.[2]

§ (7) Zero Changes of Thermometers.—For the experimental investigation of the changes which take place in the zero of a thermometer any thermometer may be employed, but it is preferable to construct a special thermometer of the particular glass under test, to secure a sufficiently open scale in the neighbourhood of the zero to permit of the changes being accurately determined. Such a thermometer is indicated in *Fig.* 5. A range of 5° C. on either side of the fixed point suffices, and a chamber should be blown between these two parts of the scale to avoid having an unnecessarily long tube.

If it is desired to carry out the tests over the complete range of possible use of a mercury-glass thermometer, the instrument must be designed to withstand exposure to a temperature of from 500° to 550° C. for a prolonged period. A chamber must consequently be provided at the upper end of the tube to allow sufficient room for the safe expansion of the mercury up to these temperatures.

To avoid distillation of mercury and the splitting up of the column at the higher temperatures, the tubes must be nitrogen-filled to a pressure of about 16 to 20 atmospheres. For thermometers which are definitely to be used only over a lower range of temperature, say up to 100° C., filling with nitrogen is less necessary.

The bulb AB should be cylindrical and of approximately the same diameter as the stem, with the zero point D at a distance of from 80 to 100 mm. above the upper end B of the bulb. This length is of some importance, as if the zero is close to the bulb it is not possible to ensure that the latter is sufficiently immersed in the ice when taking readings; on the other hand, if this distance is much exceeded the instrument is found to be inconvenient for use in the ordinary type of apparatus employed.

The scale must be chosen so that 10 mm.

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[1] Winkelmann and Schott, *Ann. d. Phys. und Chem.*, 1894, li. 697.

[2] Investigations of the thermometric properties of the Jena glasses 16^{III} and 59^{III} are fully summarised in "Jena Glass" by Hovestadt, of which an English translation by Everett was published in 1902.

corresponds to approximately 1° C., and the graduations should extend about 50 mm. (*i.e.* about 5° C.) on either side of the fixed points (0° and 100° C.). Thus CE and GK in the diagram should each be 100 mm. The tube should be graduated in millimetres. Considerable care should be exercised in the production of the divisions, as their suitability is the controlling factor in obtaining high accuracy of reading. The graduations should be fine, so as to be suitable for reading with a fairly high-power telescope, and each set of divisions should be numbered from 0 to 10 at each 10 mm. as indicated. The freezing- and boiling-points will thus be near the division 5 on each scale respectively. It is unnecessary to obtain exact coincidence, as under the treatment the tube is intended to undergo the position of these points will continually change.

Analysis of the observations will show that three types of variation of zero may occur, and these will be referred to as "Secular Change," "Depression," and "Anneal Effects."

(i.) *Secular Change.* — If the zero of a thermometer be determined periodically it will be found that the readings are successively higher, and the effect is noted even after a period of many years. Joule kept two thermometers under observation for a period of over forty years, and found that even at the end of this time their zeros continued to rise. He showed that the results could be represented by an expression of the form

$$\phi = a - b \,.\, e^{-\frac{t}{c}},$$

where ϕ is the reading of the thermometer, t is the time, and a, b, and c are constants, which will of course depend on the nature of the glass employed. This change of zero does not result from variations in the temperature to which the thermometer may be exposed between observations of the zero. The effect will be found even when the instrument is kept continually in melting ice.

As in the case of the thermometer investigated by Joule it is found that a thermometer shows a comparatively rapid rise of zero when first made, and that this rise decreases in amount with course of time. In a general way thermometers possessing a small zero depression (see next paragraph) have a small secular change. For Jena 16$^{\text{III}}$ glass Allihn[1] has shown that the secular change after the initial period is about 0·01° C. per year. A similar value has recently been found for thermometers constructed of Powell's normal glass as the result of experiments carried out over a period of nearly two years. The cause of this rise of zero is the gradual shrinking of the glass in course of time, thereby reducing the volume of the bulb. Although due to another cause, it is interesting in this connection to record observations in which the zero of certain thermometers falls with use. This on investigation was found to be due to the slow dissolving away of the glass of the bulb, owing to the thermometers being employed continuously in a well-stirred water bath. As the walls gradually become thinner the pressure due to the mercury in the instrument enlarges the bulb, thereby causing the zero to fall. Similar instruments constructed at the same time, and of the same glass, but not in continuous use, showed the usual rise of zero. The magnitude of this fall of zero was not sufficiently constant for its rate to be accurately determined, but it was of the order of three or four times the usual secular change.

(ii.) *Depression of Zero after Heating.* — If the zero of a thermometer be determined immediately after the instrument has been heated it will be found to be lower than its value recorded prior to heating. This reduction is usually termed the depression of zero, and its value depends on the temperature to which the thermometer has been exposed. For comparison it is usual to determine the depression after the exposure of the thermometer to a temperature of 100° C. for a definite period, *e.g.* thirty minutes. The zero must be taken immediately after exposure to the higher temperature to secure accurate results, for recovery begins to take place almost immediately, and the original value of the zero is again obtained after the lapse of several days. To secure uniformity in the observations the thermometer, after removal from the steam bath, is allowed to cool in air until the temperature has fallen to about 50° C.; the instrument is then immersed in ice and the lowest zero value reached is observed. Table VII. shows the value of the depression of zero after exposure to 100° C., and various other temperatures for several of the glasses.

Table VII

Glass.	Zero Depressions.		
	At 25° C.	At 50° C.	At 100° C.
Kew glass	0·04	0·11	0·22
"Verre dur"	0·02	0·05	0·11
Jena 16$^{\text{III}}$	0·01$_5$	0·03	0·07
Jena 59$^{\text{III}}$	0·01	0·02	0·03$_5$
Powell's normal (blue stripe)	0·01	0·02	0·05

The value of the depression is not constant until after the thermometer has been constructed for some time. When the thermometer is new the value of the depression is smaller than that which will be subsequently obtained. The increase in the depression becomes smaller and smaller in course of time until the permanent value is reached.

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[1] *Zeit. f. Anal. Chem.*, 1890, xxviii. 435.

The depression of a thermometer is due to what may be termed a hysteresis effect in the expansion and contraction of the glass on alternate heating and cooling; that is to say, on cooling the glass after previous heating the contraction is less than the expansion, the volume of the glass being temporarily greater than its original value.

It may be remarked here that the depression for fused silica is negligible.

With regard to the recovery from depression it is found that as a general rule low-depression values are associated with quick recovery. Thus Jena 16$^{\text{III}}$ glass recovers in two or three days, while English crystal glass requires from ten to fourteen days.

(iii.) *Anneal Effects.*—The first time a thermometer is heated after construction it will be found that a considerable rise of zero takes place. This is due to the release of the strain which is set up in the glass when it has been allowed to cool fairly rapidly after being in a plastic state. In a well-constructed instrument the change is greatly reduced by carefully annealing the instrument before it is divided. The process of annealing consists in raising the thermometer to a higher temperature than that at which it is intended to be used. The instrument is maintained at the temperature for some hours (up to two days) and is then allowed to cool slowly and uniformly over a long period, preferably some days. The anneal is much more effective the higher the temperature to which the glass can be raised, and the best effects can only be attained when the glass is brought to just below the softening point. Maintaining the glass at such a temperature for an hour is much more effective in removing after effects than if it is only taken up to a moderate temperature for much longer periods. In a high-range thermometer which is not satisfactorily annealed it is not unusual to obtain a rise of 15° to 20° C. on the first occasion the instrument is heated.

§ (8) TYPES OF LIQUID-IN-GLASS THERMOMETERS.—A brief description of each of the various types of liquid-in-glass thermometers will now be given, together with the special points to which attention must be paid in their use.

(i.) *Standard Thermometers.* — The highest class of standard thermometer has already been dealt with under the section dealing with primary standards, but in addition to these the term "standard" is applied to slightly less accurate instruments which are employed in work of high precision and in the pointing and testing of other instruments. Standard thermometers resemble primary standards in that the scale must include one at least of the fixed points for use in determining the change of zero which takes place in the course of time. It is usual for the zero point to be chosen, but for higher-range instruments the boiling-point is sometimes employed instead. The range of these thermometers depends on the use to which the instrument is to be put. Ordinary standards will comprise a range of 0° to 50° C., 50° to 100° C., etc., as in the case of primary standards; but for special purposes —for example, the pointing and testing of clinical thermometers—a much more restricted scale suffices. Standard thermometers are generally divided to 0·1° C., or in some cases to 0·2° C., and this to a certain extent determines the length of the instrument, since it serves no useful purpose to place divisions closer than 0·5 mm. It is preferable that the smallest interval should be 0·8 to 1 mm. in order that accurate subdivision of the readings may be readily effected. A very close division of the scale gives rise to what is generally known as the palisading effect, in consequence of which confusion is caused by the closeness of the lines, and the accuracy of the reading is thereby reduced. Again, attention must be given to the fineness of the divisions, since it becomes impossible to estimate a fraction of a division to any degree of accuracy if the thickness of the division line is too large a fraction of the distance between two consecutive lines. The thickness of the division line should not exceed one-tenth of the distance between lines. The method of use of a thermometer must, however, be considered, for if the thermometer is to be read with the naked eye a coarser division is necessary than if a telescope is employed. The reading of the mercury column is always estimated from centre to centre of the division lines, and to assist in doing this it is useful to turn the instrument so that the mercury column appears against the ends of the division lines. These ends should therefore fall upon a line parallel to the axis of the thermometer. In the other direction every fifth line should be allowed to project to mark either half-degree or degree intervals, while the figuring of the degree lines should be sufficiently frequent to avoid unnecessary labour in taking a reading. Thus open-scale thermometers divided to 0·1 of a degree should be figured at each degree, particularly if employed with a telescope in which the field of vision is necessarily not very large. Less open scale instruments should be figured at every five degrees. The length of the division line also contributes to the ease with which a thermometer may be read. The length of the smallest division line should be one to one and a half times the distance between the lines. *Fig.* 6 shows a scale which satisfies these conditions. The bulb of a standard thermometer is almost invariably cylindrical, and should be of diameter not greater than the stem. Internally the junction of the bulb to the stem should be gradual in order that no shoulder may offer lodgment for bubbles of residual gas; externally there should be no abrupt change of diameter. As already pointed out, the bulb of the standard thermometer must be of glass chosen for its thermometric properties. The stem of the thermometer need not be made of the same glass. Less perfect thermometric properties will have a negligible effect on the indications of the instrument as the quantity of mercury contained in the stem is small compared with that in the bulb. The glass used in the stem should be of such a variety that fine and clear divisions may be etched upon it. English crystal glass was at one time frequently specified for the stems of thermometers as this glass was susceptible of high polish and clean sharp etching. This was especially marked in comparison with Jena 16^{III}. The more modern English thermometric glasses are better in this respect than Jena 16^{III}, so that the use of English crystal glass for stems is of relatively less importance.

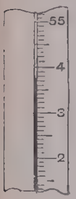

Fig. 6.

The glass employed for the stem of a standard thermometer is generally backed with white enamel. This cannot be used, however, for primary standards, as the latter are read from the back as well as from the front of the instrument. These instruments are therefore made of clear glass, and in consequence require illumination from behind.

As above pointed out, it is necessary that all standard thermometers should be provided with divisions in the neighbourhood of the zero for the determination of zero changes. A scale extending to 2° C. on each side of this point suffices to determine the scale value of the thermometer in this region. If it is not desired that the scale should extend continuously from the zero upwards a chamber is blown in the bore to accommodate the mercury between the zero and the lowest part of the desired scale. All standard thermometers should be provided with a chamber at the upper end of the stem to permit of the temperature being raised above the highest point of the scale. This is necessary in the operation of annealing thermometers, and is also a useful safeguard against breakage of the thermometer should the instrument be heated accidentally to a temperature higher than that to which it is divided. For high-range instruments the chamber should be of such a size that the thermometer may be heated to 550° C. without risk of fracture.

The corrections to standard thermometers are determined by comparison with primary standards. The corrections so determined will change with subsequent changes of zero of the thermometer, and it is consequently useful to tabulate the corrections at various points of the scale on the assumption that there is no error at the zero point of the thermometer; that is to say, the corrections are reduced by an amount equal to the correction at the zero point before the values are tabulated. The true correction for any point at a subsequent time is then given by adding (algebraically) the new correction at the zero point to the tabulated value.

In the construction of standard thermometers the capillary tubes used should be of uniform cross-section, care being especially taken that there are no abrupt changes. It is preferable to employ a tube in which the bore tapers regularly rather than one in which irregular changes of smaller extent occur. The mercury employed must be clean and dry, and particular care must be taken when blowing the bulb that no moisture is introduced. Traces of moisture lead to contamination of the mercury surface, with the result that a sharp meniscus will not be obtained, or in bad cases a trail of mercury will be left in the bore as the column falls.

(ii.) *High-range Thermometers.*—For high-range thermometers of the highest accuracy the precautions and remarks under the heading of "Standard Thermometers" apply, and in addition several other considerations have to be taken into account. The boiling-point of mercury is about 356° C. at atmospheric pressure, and is lower than this at lower pressures; consequently, if a mercury thermometer is constructed in the ordinary way, with the space above the mercury free from gas, it will not be possible to use the instrument at temperatures above 250° C., owing to the splitting up of the column due to the mercury boiling. In fact, at temperatures above 150° C., trouble is caused by distillation of mercury from the top of the column and its subsequent deposition in the cooler parts of the tube. This is specially noticed when the thermometer is used with the whole of the column immersed in the medium whose temperature is being measured. The difficulty is overcome by filling the space above the mercury with a gas at such a pressure that the boiling-point of mercury is raised to a temperature higher than that which it is desired to measure. The gas generally employed for this purpose is nitrogen. The use of air is undesirable, owing to the slow oxidation of the mercury which takes place. Carbon dioxide may also be employed. The presence of moisture must be guarded against. The pressure of the nitrogen required in a thermometer intended to read up to 450° C. is 15 atmospheres, and two methods are available for constructing such instruments. In the first, the thermometer is sealed off with the space above the mercury filled with nitrogen at atmospheric pressure, the dimensions of the chamber at the upper end of the bore having previously been adjusted so that the desired pressure of the gas is obtained by compression as the mercury rises in the bore of the thermometer. To attain a pressure of 15 atmospheres with an initial pressure of 1 atmosphere it will thus be seen that a comparatively small chamber is required. The second method involves the use of a much larger chamber, which is initially filled with gas at the maximum pressure required. A difficulty at once arises in sealing off such thermometers. This is generally done by attaching a short length of capillary tube to the instrument above the chamber and inserting in this tube a small particle of shellac or fusible metal. The instrument is connected to a supply of nitrogen under pressure through the capillary tube. When the instrument has been filled with gas the portion of the tube holding the shellac or fusible metal is warmed, so that the material melts. On cooling it again solidifies, forming a temporary seal, enabling the thermometer to be disconnected from the gas-supply and to be properly sealed off at a few centimetres above the temporary seal. The presence of the shellac or fusible metal has no subsequent effect on the action of the instrument. It is usual, however, to insert a partition across the chamber, as shown in the accompanying diagram (*Fig.* 7), to prevent the sealing material coming in contact with the mercury or the bore of the tube. Another method which is employed is to seal off the thermometer electrically inside a space filled with gas at the desired pressure.

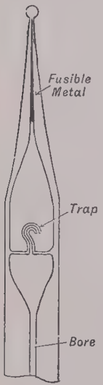

FIG. 7.

The first of these two types of construction is less desirable for three reasons: firstly, although the volume of the chamber may be designed so that the required gas pressure may be obtained for the highest temperatures to which the thermometer is subjected, the amount of compression may not be sufficient at intermediate points; secondly, the internal pressure on the bulb varies from atmospheric to the maximum value in the first method of construction, thereby changing the scale values in different parts of the tube; thirdly, if the thermometer be heated slightly above the maximum temperature for which it is designed a dangerous increase of pressure may result. Attention must also be paid to the shape and thickness of the bulb in the construction of high-range thermometers to enable them to stand the high pressures involved. Should mercury be separated from the main column by distillation it can be joined up by warming the instrument to a temperature above the region in which the mercury has been deposited. On slowly cooling the instrument the column will be found to be rejoined.

In using a gas-filled thermometer it occasionally happens that the mercury column becomes broken by a bubble of gas. If the bubble cannot be removed by carefully tapping the instrument it becomes necessary to cool down the thermometer so that all the mercury is contained in the bulb. This may be done by means of solid or liquid carbon dioxide, care being taken to cool the instrument slowly at first. The instrument is then slowly warmed up again, when the gas will be driven in front of the rising column. It may be noted that a mercury thermometer may be cooled, without risk of fracture to the bulb, in solid carbon dioxide to a temperature below the freezing-point of mercury, owing to the fact that mercury contracts on solidification.

For the construction of high-range thermometers a suitable glass must be chosen. The glass must show no signs of softening at temperatures up to 500° C., and, further, must be of satisfactory thermometric properties. The type of glass employed is a hard borosilicate glass of which Jena 59^{III} affords a satisfactory example. For work up to 450° C. it is usual to construct the bulb and stem of the same material. If Jena 59^{III} be used an enamel backing cannot be employed, and in its place it is customary to grind the back of the thermometer in order to make the divisions more easily visible. With glass of English manufacture (Powell's borosilicate glass) it has been found possible to employ an enamel backing for use up to this temperature, and the resulting instruments are consequently easier to read. The black pigment rubbed into the divisions should be burnt in to secure a permanent effect. All high-range thermometers must be thoroughly annealed before being pointed, as the change in zero may easily amount to 20° or 30° with some instruments.

(iii.) *Chemical Thermometers.*—For convenience in chemical work thermometers are frequently made in sets of seven instruments to cover the range from 0° to 450° C. Such instruments are often termed "Anschutz" thermometers. They are generally provided with small bulbs, and have the zero point (or sometimes a short range in the neighbourhood of 15° C.) on each instrument. These instruments are particularly useful in the determination of the melting- and boiling-points of organic substances. Among the numerous instruments of special ranges may be mentioned the benzol and toluol thermometers, which comprise a short scale in the neighbourhood of the boiling-points of these substances, and which are used in controlling the fractional distillation of these liquids. These instruments are generally employed partially immersed, and the tables of corrections supplied for use with these thermometers should in consequence be determined for this special condition.[1]

In industrial operations the use of very long thermometers is occasionally met with. These thermometers may have the scale starting at a point some 100 to 150 cm. from the bulb. In the case of such instruments care must be taken that definite conditions of immersion are adhered to; otherwise considerable errors may result, owing to the relatively large amount of mercury contained in the capillary stem. To reduce errors arising from this source it is customary for some thermometer-makers to employ a finer-bore capillary for the undivided portion of the stem than is used for the graduated part. This type of thermometer is also frequently enclosed in a metal sheath, and the use of such a sheath must be taken into account when comparing the unmounted thermometer with standards. The enclosure of a thermometer in a metal sheath is in many instances equivalent to using the instrument under conditions of full immersion, owing to the high conductivity of the metal. On the other hand, the employment of a metal sheath tends to make a thermometer move sluggishly in responding to changes of temperature, owing to the increased heat capacity of the instrument.

Other types of chemical thermometer do not call for special consideration. The remarks given above in connection with standard thermometers apply, having regard to the lower accuracy with which ordinary chemical thermometers need to be read.

(iv.) *Meteorological Thermometers.*—In meteorological work a number of special types of thermometer are employed, and the main features and construction of these are dealt with in the following paragraph. The accuracy with which temperatures are required does not in general exceed 0·1°, and the range of the thermometers is limited to those temperatures which occur in natural phenomena. The temperature of the air at any time may be determined by observation of the reading of an ordinary type of chemical thermometer ranging from about $-30°$ F. to 150° F. Such a thermometer, divided to whole or half degrees, can be read by estimation to 0·1° F. For meteorological purposes, however, the value of the air temperatures at any definite time is not so important as the knowledge of the fluctuations which take place over a period, and hence various types of maximum and minimum thermometer have been developed for recording the highest and lowest temperatures attained during a given period.

(*a*) *Maximum thermometers* are of the mercury type, and are provided with a device which allows the mercury to rise in the stem when the instrument is subjected to an increasing temperature, but which prevents the mercury falling again when the temperature is lowered. Three devices to accomplish this end may be mentioned. In the first the stem of the thermometer is bent at a short distance above the bulb, and a small piece of glass is fixed in the capillary at this bend. As the mercury in the bulb expands the pressure enables it to pass the obstruction, which does not completely fill the bore of the tube; but on again contracting there is not sufficient pressure behind the mercury column to force it past the obstacle, consequently the column is broken at this point and a thread of mercury is left in the bore of the tube, of such a length that its farther end indicates the highest temperature to which the instrument has been exposed. Such a thermometer must be perfectly free from air, otherwise the pressure of this gas will drive the mercury back past the obstruction. Furthermore, the instrument is used in the horizontal position to prevent the weight of the mercury column producing the same effect. To reset the instrument for use the thermometer is turned into the vertical position and gently swung with the bulb downwards; the mercury then flows past the obstruction and joins on to that contained in the bulb and lower part of the tube.

In another type of instrument, known as the "air-speck" maximum, the mercury column is interrupted by a small bubble of gas; the short length of mercury above this bubble acts as an index. On rising temperatures the index is pushed forward by the air driven in front of the expanding column of mercury; but on cooling, the index remains at the highest position reached, the space between the index and the main column being occupied by the small amount of air originally introduced. This instrument is also used in the horizontal position to avoid the weight of the mercury in the index forcing it back towards the bulb. The instrument is reset in the way above described.

A third type of maximum thermometer is similar in its action to the first type above mentioned; but in this case the column is broken on reduction of the temperature by a constriction of the bore of the thermometer.

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[1] See "Emergent Stem Correction," § (9).

The method of use is similar to that employed with the instruments already dealt with. "Clinical Thermometers" (*q.v.*) form an example of this type of maximum thermometer.

(*b*) *Minimum Thermometers.*—For the determination of the lowest temperature spirit thermometers are generally used. These are provided with an index of coloured glass, consisting of a length of about 2-3 cm. of very thin glass rod, fused at each end to form a small knob. This type of thermometer is used in a horizontal position, and is set prior to taking an observation by tilting the instrument with the bulb upwards. This allows the index to move down the tube until the end remote from the bulb of the thermometer is in contact with the meniscus of the spirit contained in the tube. When the temperature falls the alcohol contracts and the light glass index is drawn back with the contracting liquid owing to the tension of its free surface. When, however, the liquid expands on rise of temperature it flows past the index, leaving it in the lowest position occupied.

Another type of minimum thermometer is one containing mercury. It is of somewhat more complicated design and works in the following way. A small side tube is attached to the capillary about 1 in. from the bulb. This side tube, which is sealed at its farther end, contains a small platinum plug which does not entirely fill the bore. The construction of the instrument is indicated in the accompanying diagram (*Fig.* 8), in which C represents the platinum plug contained in the side tube B attached to the capillary A. The instrument is first held with the bulb downwards so that the tube B is filled with mercury; then, by raising the bulb end of the tube, the mercury is allowed to flow from B past the plug C until the surface of the mercury reaches C. If the operation is carried out slowly, on further tilting the thermometer the mercury will remain in contact with the plug C, and the instrument is then ready for use when suspended in a horizontal position. On cooling the instrument the mercury will contract and fall in the tube A, but on subsequent increase of the temperature mercury will expand into the tube B past the plug C, leaving the end of the column of mercury in tube A, indicating the lowest temperature to which the instrument has been submitted.

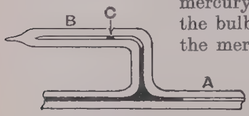

FIG. 8.

(*c*) *Combined Maximum and Minimum Thermometer.*—Another type of thermometer is frequently employed which combines both purposes. This is known as Six's self-registering thermometer. The instrument is not as accurate as is generally required for many meteorological purposes, but suffices in other cases. The accompanying diagram (*Fig.* 9) illustrates the construction of the instrument. A capillary tube bent into a U-tube terminates in a bulb at each end. The capillary contains mercury, while one of the bulbs is completely filled with alcohol and the other partly filled. A glass index is contained in each arm of the tube above the mercury and surrounded by alcohol. Attached to these glass indices are fine iron wires, which serve two purposes: firstly, they act as light springs which press against the walls of the tube and prevent the indices falling under their own weight; secondly, they serve in resetting the instrument, which is done by means of a magnet brought up to the outside of the tube. The magnet is lowered when opposite the indices, and draws them down so that their lower ends rest on the surface of the mercury in the two arms of the U-tube. On exposure of the instrument to an increasing temperature the spirit in the bulb A expands and drives the mercury in the U-tube before it; this raises index B, while when the temperature falls the spirit contracts and pulls the mercury back, which in turn drives the index C up the other arm of the U-tube; scales are attached to each of these tubes and the position of the lower ends of the indices gives the maximum and minimum temperatures respectively to which the instrument has been exposed. It is, of course, obvious that the scale attached to the tube containing the maximum index will be figured upwards, while that of the other arm will be figured downwards. In a variety of this thermometer, known as the Dimenuon thermometer, the U-tube is kept in the horizontal position for reading; the iron springs attached to the indices then become unnecessary and the instrument is reset by tilting it into a position approaching the vertical.

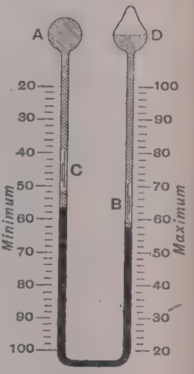

FIG. 9.

(v.) *Earth Thermometers.*—A special type of thermometer is sometimes employed in determinations of the temperature of the soil; the instrument is attached to a chain and is lowered to the required depth within a tube of diameter slightly larger than that of the thermometer, the tube after insertion of the thermometer being closed by a cover from which the chain is supported. The thermometer employed usually differs from an ordinary instrument in that it is made very sluggish in action; this modification enables the temperature to be read before the instrument has time to respond to any change of temperature to which it may be exposed on being withdrawn from the soil, while the sluggishness of the thermometer offers no disadvantage in picking up the temperature of the soil, as the latter changes at a very slow rate. The sluggishness may be obtained in two ways: (*a*) the walls of the bulb are made of very thick glass (3-4 mm.) associated with a large capacity; (*b*) a more ordinary type of thermometer is enclosed in a sealed glass tube, the space around the bulb being filled with paraffin wax.

(vi.) *The Deep-sea Thermometer.*—In oceanographical surveys the determination of the temperature of the sea at varying depths plays an important part in investigations of the direction of currents. An early method of carrying out such an investigation was to send down to the required depth a sampling bottle, and the temperature of the water thereby secured was taken by an ordinary thermometer. The obvious objection to this method is that the temperature of the sample may change while the latter is being drawn up, especially if the depth from which it is taken is considerable. To overcome this difficulty a special type of thermometer known as the "deep-sea reversing thermometer" has been evolved. This instrument consists of a thermometer of special design enclosed in a stout-walled glass sheath capable of withstanding a pressure of three or four tons per square inch. In order to obtain a reading of the instrument it is mounted in a special carrier or frame,[1] by means of which it may be turned upside down when at a depth at which an observation of the temperature is required. On being reversed the mercury column is broken at a modification in the bore and is allowed to flow to the other end of the capillary tube, where it fills an enlargement of the bore and a portion of the stem. The scale of the instrument is arranged in the opposite direction to that of an ordinary thermometer, so that the volume of mercury overflowing is read while the instrument is still in the inverted position. *Fig.* 10 shows the latest form of the instrument: A is the special thermometer employed enclosed in the sheath; C, the portion of the sheath in which the bulb of the thermometer is situated, is partially filled with mercury (E), access to the other portion of the tube being stopped by means of a seal at D; the object of this mercury is to increase the rapidity with which the thermometer picks up the surrounding temperature. The details of the modification are shown in the small diagram, *Fig.* 11. When the instrument is in its normal position mercury flows past the curved portion into or out of the enlargement G and the capillary; on inverting the thermometer the column is broken at the point F, and flows out of the enlargement G into the chamber H at the upper end of the stem, filling this together with a portion of the capillary. Now if, after reversal, the temperature to which the instrument is exposed is increased, mercury will expand from the bulb and will flow past the bend (F), but will be prevented from joining the portion separated off until the enlargement G has been filled. The dimensions of this enlargement are therefore adjusted so that in practice the quantity of mercury resulting from further heating of the instrument will be insufficient to fill this space. The diagram shows a second thermometer B enclosed within the sheath, the bulb of which is in close proximity to the chamber H; the object of this is to enable a correction to be made for any change in volume of the mercury which may result owing to the temperature at which the instrument is read being different from that at which the thermometer was inverted. This is necessary as the mercury contained in the chamber H will expand if its temperature rises, and therefore the reading on the scale will be higher than that at which it was separated off. On each thermometer will be found the volume of this chamber in scale divisions, while the nature of the glass of which the thermometer is constructed is also noted. Suppose the auxiliary thermometer shows a reading higher than that of the deep-sea thermometer by *t*, then the reading of the deep-sea thermometer is higher than its true reading by an amount equal to *t* × volume of chamber H × apparent coefficient of expansion of mercury in the glass of which the thermometer is constructed.

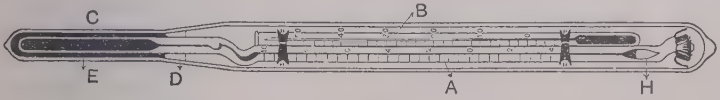

Fig. 10.

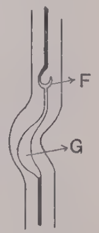

Fig. 11.

(vii.) *The Pentane Thermometer.*—As previously mentioned, the use of pentane offers considerable advantages in the construction of thermometers for work at low temperatures. Its chief value lies in the fact that pentane remains fluid at temperatures as low as that of liquid air, although in this neighbourhood it is somewhat viscous. Care must be exercised in cooling down the thermometer to avoid the liquid sticking to the walls of the capillary, especially at very low temperatures. Commercial pentane is used in the filling of these thermometers.

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[1] See article on "Oceanography," Vol. III.

In the manufacture of ordinary thermometers it is usual to point the instruments at a series of temperatures determined by working standards. To carry this out in the case of a pentane thermometer it would be necessary for a supply of liquid air to be available. This is not always possible, but an accurate instrument may, however, be obtained by making use of previously determined values of the coefficient of expansion of pentane. Rothe[1] has determined these values for technical pentane, and he points out that a low-range thermometer may be calibrated by obtaining the point corresponding to −78·2° C. by the immersion of the thermometer in a mixture of carbonic acid snow and absolute alcohol, the former being in excess. The point corresponding to −100° C. is then obtained by measuring off a length equal to one quarter of the distance between 0° C. and −78·2° C., and marking this below the latter point; while the position of the −200° C. point is given by marking off below the zero point a length 2·23 times the length from 0° C. to −78·2° C. The position of the −200° point may also be obtained by a weighing method, since the volume of the bulb and capillary up to the 0° C. division is 4·47 times the volume contained between the 0° C. and −200° C. divisions. The above method, of course, depends upon the tubing chosen being of uniform cross-section, and this must be verified prior to the construction of the instrument.

(viii.) *Calorimetric and Beckmann Thermometers.*—For calorimetric and similar purposes a thermometer of short range is required which may be capable of being read to a high degree of accuracy; these instruments are consequently divided to hundredths of a degree centigrade, the length of one degree occupying about 50 mm. Using a telescope of low power it is thus possible to read the instrument to 0·002° or even 0·001° C. Many of these instruments have a scale of about 10° or 12°. In order to overcome uncertainties due to capillary effects such instruments are generally provided with a relatively large bulb so as to avoid the use of an unnecessarily fine bore in which the capillary effect would be so large as to reduce the accuracy with which the readings are taken. Trouble due to the sticking of the mercury is greatly reduced by continuously tapping the thermometer or supporting it in such a way that it is subjected continuously to a small amount of vibration; this, of course, must not be so great as to interfere with the accurate reading of the instrument. Thermometers of this type have a large lag, and it is necessary to make allowance for this in some cases. In calorimetric work it is also necessary to allow for the heat capacity of the immersed portion of the instrument.

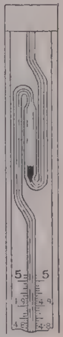

FIG. 12.

In order to avoid instruments of unwieldy length it is necessary that the range should be short to provide the required openness of scale, and it frequently becomes desirable to provide a series of such instruments of different ranges. The Beckmann thermometer obviates this, as it employs a device by means of which the range may be adjusted as required. The scale is generally of a length equivalent to 5° C., and is figured from 0 to 5. The device used is shown in the accompanying diagram, *Fig.* 12, in which it will be seen that a chamber is provided at the upper end of the scale; the object of this is to accommodate the portion of the mercury not required; and by expelling the excess mercury into the chamber the zero of the scale may be adjusted to correspond to any value desired over a fairly wide range. The value of each scale division changes according to the quantity of mercury filling the bulb and stem up to the zero mark, and allowance has to be made for this. A typical set of curves showing the corrections to any part of the scale for definite values of the zero is given in the accompanying diagram, *Fig.* 13. In most work for which this type of thermometer is used a knowledge of any individual temperature to a high order of accuracy is not required; what is required to be known, however, is the difference between two temperatures. Thus, a very exact deter-

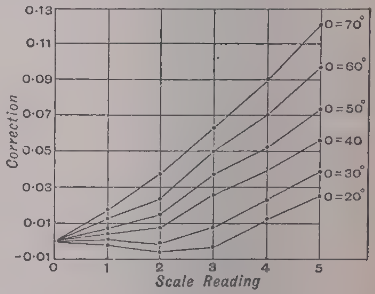

FIG. 13.

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[1] *Zeits. Instrumentenk.*, 1904, p. 47.

mination of the temperature corresponding to any particular zero setting is unnecessary.

The setting of the Beckmann thermometer to any desired range is accomplished by trial. The instrument is warmed by immersion in a bath until the mercury begins to flow into the upper chamber; the instrument is then sharply tapped or jerked, with the result that some mercury breaks off from the main column at the junction of the capillary and its enlarged portion; the temperature is then lowered and a reading taken with an auxiliary thermometer when the mercury stands at zero on the scale of the Beckmann instrument. If this temperature is higher than desired, it is necessary to run the mercury up the tube and break off still more mercury from the main column. This process is repeated until the desired zero is attained. If at any point in the procedure more mercury than necessary has been removed, the column is rejoined by running the mercury to the top of the tube and inverting the instrument. The mercury in the expansion chamber will then join on to the main column and be drawn back into the capillary as the temperature falls; when sufficient mercury has been carried over, the column is again broken by returning the thermometer to its normal position and tapping the instrument. In this way it is possible, after a little experience, to adjust the thermometer to any required zero value after two or three trials. In some instruments the enlarged portion of the capillary which serves as an expansion chamber is graduated with a scale of degrees in order that an estimate may be made of the amount of mercury to be separated off; this scale is, however, so contracted that it is not of great value in obtaining an accurate setting of the zero.

Referring to *Fig.* 13, it will be seen that all the correction curves are brought to a common origin, but that at the upper end of the scale the curves separate by approximately equal amounts for equal differences in zero setting. Correction curves for any other setting of the zero may be drawn by taking proportional amounts of the differences between two curves.

To obtain by calculation the value of a division of a Beckmann thermometer scale corresponding to any given setting of the zero a knowledge of the expansion of mercury and of the glass of which the thermometer is constructed is necessary. Assume that F(T) and f(T) represent the volumes at T° of unit volumes of mercury and glass respectively measured at 0° C. Let V_0 be the volume at 0° C. of the bulb of a Beckmann thermometer together with that portion of the capillary up to the zero division of the scale, and v_0 the volume, also measured at 0° C., of a length of capillary corresponding to one scale division; it is assumed that the capillary is of uniform bore and that the divisions of the scale are equidistant.

Let T be the temperature at which the mercury column coincides with the zero of the scale for a particular setting of the instrument; the volume of mercury contained in the bulb will therefore be $V_0 f(\mathrm{T})$, and this will correspond to an initial volume of mercury measured at 0° C. of $V_0 f(\mathrm{T})/F(\mathrm{T})$.

Now if the temperature be raised to θ so that the scale reading of the thermometer is x_θ divisions, the volume of mercury becomes

$$V_0 \frac{f(\mathrm{T})}{F(\mathrm{T})} \cdot F(\theta).$$

The volume of the bulb has changed to $V_0 f(\theta)$ and the volume of x_θ scale divisions is $x_\theta v_0 f(\theta)$. Hence, equating the volume of mercury and that of the containing vessel,

$$\frac{V_0 f(\mathrm{T}) \cdot F(\theta)}{F(\mathrm{T})} = V_0 f(\theta) + x_\theta v_0 f(\theta),$$

and by transposing,

$$x_\theta = \frac{V_0}{v_0} \left\{ \frac{F(\theta) f(\mathrm{T})}{F(\mathrm{T}) f(\theta)} - 1 \right\}.$$

This expression gives the scale reading corresponding to any temperature θ for any setting T of the zero, in terms of the ratio V_0/v_0.

To evaluate V_0/v_0 the reading of the instrument must be known corresponding to any value of θ other than $\theta = \mathrm{T}$. (The value $\theta = \mathrm{T}$ corresponds to the zero setting of the scale, when $x_\theta = 0$.)

In the construction of a Beckmann thermometer it is usual to point the scale at the upper end under the condition of the zero of the scale corresponding to 0° C., *i.e.* T=0, so that for the usual type of instrument the division 5 will correspond to 5° C. Hence V_0/v_0 for this particular case will be given by

$$\frac{V_0}{v_0} = \frac{5 f(\theta)}{F(\theta) - f(\theta)},$$

where $\theta = 5°$ C.

(ix.) *Clinical Thermometers.*—The ordinary clinical thermometer used for the determination of the temperature of the human body or of animals is a mercury thermometer of short range of the maximum type; that is to say, the thermometer registers the highest temperature to which it has been exposed since re-setting. Clinical thermometers employed in this country, the Colonies, and America are almost invariably divided on the Fahrenheit scale; on the Continent the centigrade scale is generally used together with the Réaumur scale to a much smaller extent. The range of temperature necessary to such a thermometer is 95° to 110° F. or 35° to 45° C. A special type of thermometer, known as the "surface clinical thermometer," employed for the determination of skin temperatures, is divided as low as 85° F. The earliest type of clinical thermometer was known as the "hospital thermometer," and was not self-registering; it was of the normal clinical thermometer range, but was read while in the patient's mouth. An improvement on this was the "Phillip's index thermometer," in which a small thread of mercury was separated from the main column by an air speck; on a rising temperature the short length of mercury advanced in front of the main column, but on again lowering the temperature the main column fell while the index column remained stationary. This thermometer was liable to get out of order when the instrument was re-set by shaking, as the index column frequently joined on to the main column. A later development was the introduction of a slightly narrowed bore which to a certain extent prevents this joining up. This led to the introduction of the modern constricted-bore maximum thermometer, in which the index thread separated from the main column by an air speck is dispensed with. The constriction is made by blowing a small enlargement in the bore of the capillary a short distance above the bulb; heating the tube on one side opposite the enlargement with a very small flame causes this to collapse and so produce a constriction in the tube. The amount of constriction must be such that while the mercury passes freely on expanding when the instrument is raised in temperature, it must be sufficient to prevent the mercury running back when the temperature falls. At the same time the bore must not be so constricted that it is unreasonably difficult to cause the mercury to join up when the instrument is shaken or swung from the end opposite the bulb. A skilled glass-blower can frequently adjust the tightness of the constriction either in the direction of closing it by reheating and causing further collapse of the glass, or opening it by forcing mercury past while the glass is slightly softened in the blowpipe flame. The shape of the constriction has a considerable effect upon the smooth working of a clinical thermometer; if it is not sufficiently sharp the mercury column tends to draw back into the bulb instead of parting cleanly immediately the temperature begins to fall. Further, the mercury column may tend to "jump" as the temperature rises; in this case the column does not rise in the capillary steadily with gradual increase of temperature but proceeds in a series of jerks. In a bad case this jump may exceed a degree Fahrenheit although its ordinary magnitude is of the order of a few tenths of a degree. Another device by means of which mercury thermometers may be rendered self-registering is known as the "choke bore"; this method is employed for thermometers of the enclosed scale type, principally made on the Continent. In these instruments the mercury column is prevented from falling by the introduction of a splinter of glass into the end of the bore of the thermometer at its junction with the bulb; the thread of glass is kept in position by being sealed into the opposite end of the bulb. These instruments are apt to get out of order owing to the fine point of glass forming the choke being broken off in shaking down the mercury column.

As previously mentioned, the operation of resetting a maximum thermometer is shaking or swinging the instrument so that the mercury is forced towards the bulb. Other methods have been introduced to accomplish this with less trouble than is sometimes experienced in shaking down a thermometer. In one device two constrictions are used at a slight distance apart; these constrictions are not so tight as in the ordinary case, thereby allowing the mercury column to be rejoined with much less effort; the double constriction, however, prevents the mercury being drawn back into the bulb when the mercury contracts on fall of temperature. Another method, known as the "Davidson patent," is made by providing a small metal plunger at the upper end of the bore; this plunger works through a metal cap on to a disc of rubber covering an enlargement in the bore of the tube; the enlargement is filled with mercury, which is separated from the main column in the tube by a small quantity of air; pushing in the rubber disc by means of the plunger forces some mercury down the bore, driving the air before it; this in turn causes the mercury column to flow back past the constriction. The instrument is liable to get out of order owing to the perishing of the rubber disc. This difficulty is overcome in another type of instrument known as the "Repello thermometer," in which the movement of the mercury piston is obtained by slight pressure upon a flattened bulb of toughened glass sealed to the upper end of the bore; as in the previous case, the mercury contained in this bulb is separated from the indicating column by means of a small quantity of air.

In taking a temperature by inserting a clinical thermometer in the mouth it is soon realised that the final reading of the instrument is not reached as quickly as it would be if the thermometer were immersed in a bath of water. The reason for this is that the introduction of the thermometer causes local cooling, heat being abstracted in order to warm up the mercury in the thermometer; and an interval of time is necessary before the circulation brings the surrounding temperature back to the actual temperature of the blood. It is obvious that this disturbance will be less the smaller the quantity of mercury contained in the bulb, and hence it is customary to limit the size of the bulb to produce a reasonably rapid instrument. As a consequence of this, it is seen that the bore of the capillary used must also be very small, especially when it is taken into account that an open scale is desirable—the clinical range of 15° generally occupies a length of 2 to 2½ in. With so fine a bore the thermometer becomes difficult to read, and a device was introduced by means of which the mercury column could be magnified in width. This is accomplished in the "lens front" clinical thermometer by employing a capillary tube of section shown in *Fig.* 14. If the instrument be held so that the column is viewed through the sharp edge of the glass a magnified image of the thread is seen. The use of this device is not restricted to clinical thermometers, but is much more frequently employed here on account of the fine bores used. The other device used for rendering a mercury column more easily visible is by means of a flattened bore; this, however, cannot be used with clinical thermometers as the capillarity effects would be too great for the instrument to be reliable.

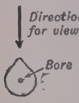

FIG. 14.

It is customary to designate clinical thermometers by terms ½ min., 1 min., and 2 min., etc., to distinguish between the slower and the more rapid instruments. The terms are, however, misleading, as the time taken by a clinical to pick up a temperature depends so much on personal factors as well as on the characteristics of the instrument itself. The ½-min. instrument, for example, will have attained its maximum reading in this period of time when used by some individuals, but in other cases the final temperature will not be reached until after an interval five times as long has elapsed. It, however, may be taken that the classification is dependent on the size of the bulb, and that for any given individual a ½-min. thermometer will be quicker than a 1-min. instrument, which in turn will be quicker than a 2-min. As a result of a discussion between the clinical thermometer trade and the National Physical Laboratory and other authorities, it has been suggested that the bulb of a ½-min. instrument shall not exceed 2·4 mm. in diameter and 20 mm. in length, those designated 1 min., 2·6 mm. in diameter and 20 mm. in length, while instruments outside the latter sizes shall be designated 2 min. In general a 1-min. instrument is easier to read than a ½-min. instrument, and is usually more reliable in construction, the latter being rendered easier by the larger bore of the instrument.

Veterinary thermometers are usually of a much more robust type, and are consequently provided with a larger bulb; they are hence slower in action. The usually accepted "normal points" for veterinary use are shown in *Fig.* 15.

The widespread use of clinical thermometers has led to the introduction of many special patterns; the main types of instruments have been dealt with in the preceding paragraphs, but amongst others may be mentioned the several forms of aseptic thermometers in which special precautions are taken to avoid infection being conveyed by bacteria carried in the divisions on the surface of the tube. The "insulated" thermometer or "Continental" pattern affords an example of this type, and this is probably its main recommendation; the scale is completely enclosed in an outer sheath, and the thermometer may be immersed in an antiseptic solution without damage to the marking. A variation of this type is afforded by a thermometer in which the "solid" stem is traversed by a slot behind and parallel to the bore; the scale is engraved on a thin strip of material such as mica, and is inserted in the slot; the end of the tube is then sealed. Another type of instrument resembles an ordinary clinical thermometer but carries no divisions; for purposes of reading, this undivided thermometer is placed inside an auxiliary glass tube on the surface of which the divisions are engraved. Exact registration is afforded by a ground-glass joint between the two.

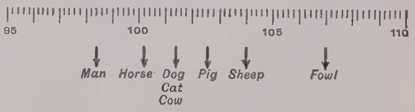

FIG. 15.

In other varieties of clinical thermometers modifications of the scale are introduced to render the use of these instruments easier to the general public. For example, the Sumner's patent has the scale figured with the normal point 98·4° F. as zero, each degree Fahrenheit above or below this being marked +1, +2, . . . or −1, −2, . . ., etc.

Surface clinicals have been mentioned; these are frequently made with a bulb designed to give a larger area of contact than would be obtained with ordinary types of cylindrical bulbs. In one pattern it is arranged in a circular form.

It should be mentioned that in several countries clinical thermometers are required to be tested and approved before they are sold for use.

(x.) *Sodium-potassium Thermometers.*—As pointed out previously in connection with high-range thermometers the boiling-point of mercury is about 356° C., but by the introduction of gas under pressure above the column it is possible to extend the useful range of a mercury thermometer up to about 500° C. This raises the question as to whether alternative liquids are available as a thermometric fluid. It has been found that a mixture of the metals sodium and potassium, in the proportion 4 : 1, is fluid at a temperature as low as −10° C., while the boiling-point of this mixture is sufficiently high for thermometers to be used up to the limit set by the glass envelope without introduction of gas. The use of tin for high ranges has also been attempted, but satisfactory results have not been obtained.

(xi.) *Fused Silica Thermometers.*—Thermometers have been made replacing glass by the transparent variety of fused quartz with a view to increasing slightly the scale of the instrument, and also to avoid the troublesome changes of zero which are experienced with glass thermometers. The depression of zero with a mercury-in-fused-silica thermometer is negligible. The difficulty, however, of making uniform capillary tubes of this material has not yet been overcome successfully for such thermometers to be generally available.

(xii.) *Miscellaneous Thermometers.* — The electrical types of thermometers dealt with elsewhere offer advantages over mercury thermometers in that they can be made to give a continuous record of temperature without great difficulty. Under certain conditions, however, it is possible to obtain a continuous record with a mercury thermometer by photographic means. This has been used in connection with humidity records at certain meteorological stations. A large type of thermometer is employed with a flattened bore, or one where a comparatively wide column of mercury is available. The instrument is supported vertically in front of a slit behind which sensitised paper is carried on a drum with its axis vertical; a beam of light is allowed to fall on the instrument; the mercury column stops part of the light, but the latter is able to pass through the thermometer above the column and affect the sensitised paper; a trace is thus obtained showing the height of the mercury column.

Mercury thermometers may also be employed to give warning of temperature fluctuations outside certain limits. This is achieved by introducing into the capillary platinum wires at points corresponding to the special temperatures; the rise and fall of the mercury makes or breaks electric contact with these wires and gives the desired signals. Such instruments suffer, however, from the disadvantage that the temperatures at which a signal is given cannot be varied at will. The current employed with such instruments must be very small to avoid contamination of the mercury surface, and hence they should only be used in conjunction with relays.

(xiii.) *Transmitting Thermometers.*—In industrial use the ordinary mercury thermometer, while having the great advantage of being a very simple instrument to read, suffers from the disadvantage that glass instruments are very liable to fracture, and further must be read *in situ.* These considerations have led to the introduction of mercury thermometers employing a metal for the envelope instead of glass, while such instruments are made "distant reading" by the use of a metal capillary of any desired length between the bulb and the indicator. The indicator in such an instrument is a Bourdon pressure gauge the scale of which is calibrated to give direct readings of the temperature to which the bulb is exposed. In a recent form of such an instrument a number of the difficulties previously met with are overcome in the following way. The bulb is of steel hermetically connected to a steel capillary terminating in a flattened steel tube wound into a spiral which constitutes the gauge. The capillary may be of any desired length up to 30 or more feet, thereby permitting the indicator to be removed from the point at which the temperature is being determined by a considerable distance. The use of a long capillary at once introduces a potential source of error, as the quantity of mercury contained in the capillary itself may be a large fraction of that contained in the bulb, thereby rendering the instrument largely dependent upon the temperature of the capillary. In the particular form of transmitting thermometer referred to, this objection is overcome by introducing into the capillary a number of lengths of "invar," the ratio of the diameter of which to the internal diameter of the capillary is adjusted so that the effective change in the volume of the capillary is just equal to the corresponding change in the volume of the mercury filling it, whatever be the temperature to which the capillary is exposed. Compensation is also made for changes in temperature of the indicator itself by connecting the pointer to the free end of the Bourdon tube through the intermediary of a bimetallic strip.

§ (9) EMERGENT STEM CORRECTION.—For accurate determinations a thermometer must always be used so that the whole of the mercury contained in the bulb and in the stem is exposed to the temperature it is desired to measure. In many cases, however, this is not convenient, and it then becomes necessary to investigate the effect produced by emergence of a portion of the stem. It will be obvious that the length of the column not immersed in the medium will be longer or shorter according to whether the temperature to which the stem itself is exposed is higher or lower than that of the medium in which the bulb is immersed. The latter case is more usual, but the following reasoning applies equally to either condition, due regard being paid to the sign of the resulting correction. Consider the case in which a thermometer is used so that the mercury column extends n degrees above the level to which the instrument is immersed, while t is the mean temperature of the mercury occupying these n degrees and θ is that of the bath; if the whole of the thermometer had been raised to the temperature θ, the column would have expanded by an amount equal to $n(\theta - t) \times$ apparent coefficient of expansion of mercury in the glass of which the thermometer is constructed; this amount is therefore the correction which has to be added to the reading of the thermometer to give the true temperature of the bath, that is ϕ (the correction required) $= \mathrm{A} \times n \times (\theta - t)$. It will be noted that in the above expression the value θ is the true temperature of the bath which is unknown, but as a first approximation it may be taken to be the reading of the thermometer. To obtain a more accurate value for the correction it is then necessary to substitute for θ the observed reading plus the approximate correction obtained in the first calculation. The resulting value will be sufficiently accurate for all practical purposes owing to the fact that the value of t is somewhat indefinite as discussed later.

The value of the constant A varies with the nature of the glass employed from 0·00015 to 0·00016 if the temperature is measured on the centigrade scale, the corresponding figures for Fahrenheit thermometers are 0·000083 to 0·00009.

The main difficulty in applying this correction to the readings of thermometers is the uncertainty of the value of the mean temperature of the emergent column; it is obvious that this is greater than the temperature of the air owing to the conduction of heat along the mercury column itself; the value is generally obtained by placing an auxiliary thermometer so that its bulb is in contact with the instrument under consideration at a point half-way up the emergent column. Another way of obtaining the temperature is to make use of a special thermometer known as the "Faden" or Thread Thermometer, in which the usual type of bulb is replaced by a capillary tube of length from 10 to 15 cm.: this is placed so that the upper end of its bulb coincides with the upper end of the emergent column whose temperature is desired. For a full description of the methods in which the Faden thermometer can be used, reference should be made to the original paper of Mahlke[1] or a paper by Buckingham.[2]

Chree[3] suggests a slightly different method: instead of taking the mean temperature of the mercury column, the value of the temperature of the air of the room is substituted for t in the expression quoted above. The value of the constant A will therefore be different, and its value has to be determined experimentally, the corrections being obtained by measuring the reduction in the reading when the instrument is immersed to various depths in a steam bath.

Chree points out that the value of this constant varies with different thermometers and with the conditions under which the test is carried out; it is therefore necessary to arrange that these conditions shall approximate as closely as possible to the conditions under which the thermometer is intended to be used.

For instruments intended for use in industrial operations and in certain classes of chemical work, the thermometers are pointed for a specified depth of immersion and the corrections to the thermometers obtained by comparison with standards are tabulated for corresponding conditions. In such cases it is important that the thermometers shall be used in a manner corresponding to the way in which the tests were carried out, as otherwise the corrections will not apply even to a low order of accuracy.

It may be mentioned that for high-range thermometers the correction for emergent column may amount to as much as 20° to 30° C. at 400° C., so that it will be seen this source of error cannot be neglected, even for work in which high accuracy is not essential.

§ (10) Thermometric Lag.—It is a matter of common knowledge that all thermometers exhibit a time lag, that is to say, a thermometer when plunged into a medium at a different temperature does not immediately register the value of that temperature; a certain interval of time must elapse before a final reading is obtained. Generally this is not a matter of great importance, as it is possible to wait long enough for the steady value to be reached. There are, however, some cases where this does not hold, and a correction has to be applied to allow for the lag of the instrument employed. The time taken by a thermometer to acquire the temperature of the medium in which it is immersed is dependent on several factors which include the nature and condition of the medium as well as the type and dimensions of the thermometer.

If T be the temperature of the medium, and θ be the temperature indicated by the thermometer immersed in it at any time t, then by the application of Newton's Law of Cooling

$$\frac{\partial\theta}{\partial t} = \frac{1}{\lambda}(\mathrm{T} - \theta),$$

where λ is a constant with respect to T, θ and t, but which depends upon the type of the thermometer and the nature and conditions of the medium.

Two cases may be considered; the first in which the medium is maintained at a constant temperature, *i.e.* T = constant; and secondly when the temperature of the medium is changing uniformly, *i.e.* T is a linear function of the time *t*.

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[1] *Zeit. Instrumentenk.*, 1893, xiii. 58.

[2] *Bulletin Bureau of Standards*, 1912, viii. 239.

[3] *Phil. Mag.*, 1898, xlv. 314.

Integration of the above equation for the first case gives:

$$\theta - \mathrm{T} = (\theta_0 - \mathrm{T})e^{-\frac{1}{\lambda}t},$$

which is equivalent to stating that the difference between the temperature indicated by the thermometer and that of the medium decreases logarithmically with time, and further that the difference is $1/e$ (*i.e.* 1/2·718) times the original difference in λ seconds.

In the second case, where the temperature of the bath changes regularly with time, integration of the equation gives approximately, after steady conditions have been attained,

$$\theta - \mathrm{T} = -n\lambda,$$

where *n* is the rate at which the temperature of the medium is changing, derived from the expression

$$\mathrm{T} = \mathrm{T}_0 + nt.$$

In this case it is seen that when conditions become steady the temperature indicated by the thermometer lags behind the temperature of the medium by an amount equal to λn. It may be noted that λ has the dimensions of time.

For the ordinary type of chemical thermometer the value of λ is of the order of 5 seconds for immersion in a well-stirred bath of water. Considering the case of a thermometer which initially indicates a temperature 10° C. below that of the bath, application of the above formula shows the difference will be reduced to 0·01° C. in about 35 seconds, while in the case of a thermometer being immersed in a bath of well-stirred water whose temperature instead of being constant is rising at the rate of 0·1° C. per minute, the thermometer will lag behind the bath by nearly 0·01° C. when steady conditions have been attained.

In air a similar result holds, but the values for λ are much larger, that is to say, a thermometer picks up temperatures at a slower rate.

The lag coefficient of a thermometer is determined experimentally in the following way. The thermometer is immersed in a bath maintained at a constant temperature, the liquid being stirred or at rest according to the conditions under which the lag is required. It is essential that the bath be maintained at a constant temperature throughout the observations, and for this purpose it is better that the observations should be carried out at about room temperature, particularly where the conditions do not allow of stirring. Prior to immersion in the bath the thermometer is cooled to a temperature some 15° C. below the bath. The times at which the mercury column crosses various graduations are recorded until the temperature indicated by the thermometer becomes stationary. For slow thermometers a watch may be employed, but for rapid instruments a chronograph is essential. A curve may be plotted giving the relation between the readings and time; this curve will be found to be asymptotic to the ordinate corresponding to the final temperature of the bath. It will be noted that the equation above deduced for the condition of constant temperature is logarithmic, and it may be written in the form

$$\lambda = \frac{t}{\log_e(\theta_0 - \mathrm{T})/(\theta - \mathrm{T})},$$

the value of λ may be obtained by plotting a second curve connecting the time *t* with $\log_e(\theta_0 - \mathrm{T})/(\theta - \mathrm{T})$. Any value for θ on the above curve may be chosen as the starting-point θ_0, provided that the time *t* be reckoned from the instant at which this value was attained. The logarithmic curve will be a straight line, and its slope gives the value of λ, the required lag coefficient.

§ (11) TESTS OF THERMOMETERS BY COMPARISON WITH STANDARDS.—The standardisation of a thermometer from first principles has been dealt with in a previous paragraph. The method is, however, somewhat long and tedious, and furthermore is not applicable to the many types of thermometers whose range does not include the fundamental fixed points, namely 0° and 100° C. For most practical purposes it suffices to compare the instrument under consideration with a standard thermometer the values of whose readings are known in relation to the International Temperature Scale. Methods of carrying out these comparisons have been worked out in detail at the various national testing institutions, and a description of the equipment and methods employed at the National Physical Laboratory of this country will serve to show how the operation is performed.

The essential feature in the comparison of a thermometer with a standard is that the two instruments shall be immersed in a medium the temperature of which may be readily adjusted to any required value and maintained at that value for a reasonable interval of time. Furthermore the medium in which the instruments are immersed must be of uniform temperature throughout its bulk.

To achieve this end the fundamental principle in the construction of comparison baths employed at the National Physical Laboratory is the provision of two vertical tubes cross-connected at their upper and lower ends, the medium in which the comparison is being carried out being circulated round the vessel so formed. The thermometer or thermometers under examination, together with the standard instruments with which they are being compared, are supported in one of the tubes, while in the other devices are arranged for the heating and circulation of the liquid in which the thermometers are immersed. For comparisons between 0° and 100° C. water is used. Circulation of the water is effected by a propeller, supported in the left-hand tube at its upper end. Below the propeller electric heater units are provided. The heaters are protected from contact with the water by being enclosed in pockets constructed of very thin copper sheet. These are of such a size that the heaters fit very closely; the object being to ensure a very rapid transference of heat from the heater itself to the water of the bath, for to a large extent the accuracy and speed with which a comparison can be carried out is dependent on this point. Rapid transference of heat from the heating units ensures that the temperature of the bath will follow without serious lag the changes made in the heating current. It thus becomes an easy matter to adjust the temperature to any desired value and to maintain it there for any required time. This is a matter of some considerable difficulty if there is appreciable lag between the alteration of the heating current and the resultant change in the temperature of the water. The temperature of the baths is controlled directly by the operator, since the employment of any thermostatic device of sufficient sensitivity would slow down the operation to an unnecessary extent, and furthermore a simple device would lack the flexibility required to steady the temperature to any desired point over the range over which such a bath is normally used. Another consideration influencing this method of working is that in making comparisons it is desirable for the temperature of the bath to rise very slowly during the observations, rather than for the temperature to be maintained at a strictly constant value. In carrying out the comparisons at least two standards should always be used; agreement between the readings of these after the application of the necessary corrections will largely tend to check the introduction of accidental errors in reading. At the Laboratory it generally happens that more than one thermometer is under examination at one time, and the usual procedure is to place one of the standards at the beginning of the set of instruments and the other at the end; if more than two standards are employed the others are distributed uniformly among the thermometers being tested. Readings of the instruments in turn are taken at a uniform rate from the first standard to the last, then, without interruption, the readings are continued in the reverse order, the last standard being read again, followed by the instruments under test and finally the first standard. In making high precision comparisons the double set of readings will be repeated. The mean values of the readings are then calculated, the necessary corrections to the standards applied, and the mean value of the standard readings is obtained. As previously pointed out, the corrected means for the standards should be in good accord, and if this is not the case the set should be discarded, after the reason for the discrepancy has been investigated, and a new set of observations made.

In order to avoid difficulty owing to capillarity effects in the fine bore of the majority of mercury thermometers, all comparisons are carried out with the temperature of the bath very slowly rising; consequently if a set of thermometers be read only once, the corrected indication of the last would be slightly in excess of that of the first, while the others will give intermediate values according to the time at which they were read; the object of making a reverse set of readings is now apparent, for, if the bath is rising in temperature at a uniform rate, and if the instruments are read at regular intervals, the means of the two sets of readings taken forward and then backward will give strictly comparable values. In practice the rate of rise of temperature is of course arranged to be very small, of the order of a few hundredths of a degree in the time necessary for making the observations, so that any small irregularities in the rate in which the thermometers are read will be of no importance.

The reading of thermometers of precision is invariably carried out by the aid of a reading-telescope, which is mounted on a substantial support at a suitable distance in front of the thermometers to be observed. The axis of the telescope must be kept at right angles to that of the thermometer; the latter is usually vertical so that the axis of the telescope is made horizontal. With this arrangement the introduction of errors due to parallax is practically eliminated. The telescope need not be of high power for use with ordinary instruments; a magnification of 2 to 3 will be found sufficient in most instances, but for the highest class of work with very finely divided instruments it may be increased to 8 or 10. The use of a micrometer eyepiece is not generally desirable except for reading fixed points; with ordinary skill it is possible to estimate to one-twentieth of a division, while some operators are able to estimate to one-fiftieth. The majority of thermometers will not, however, bear subdivision to this order on account of the width and irregularity of the division lines. In reading a thermometer it is desirable to place it in such a position that the mercury column just fails to cut the ends of the division lines, as in this way the position of the end of the column is not obscured by the division itself.

The illumination of the thermometer is another point to which some attention should be paid. Thermometers of the highest class are not provided with enamel backs and must therefore be illuminated from behind. Glare from the source of illumination is prevented by the interposition of a sheet of ground-glass. Thermometers provided with enamelled backs are best illuminated by a lamp placed in front of the instruments in a direction of about 45° to the line of sight. In this case also it is desirable that the light should be diffused, and a convenient source of illumination is provided by metallic-filament electric lamps with frosted bulbs. The lamps should be at a sufficient distance to prevent heating of the thermometer; and for this reason metallic-filament lamps are preferable to those with carbon filaments. A 20-30 watt lamp at a distance of 12-15″ provides ample illumination.

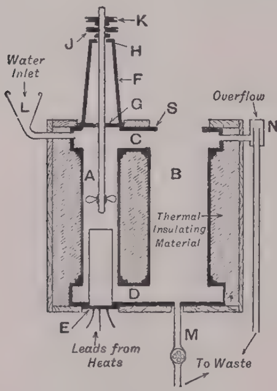

FIG. 16.

(i.) *Construction of Thermometer Comparison Baths.*—The details of construction of the thermometer comparison baths for water are shown in *Fig.* 16. A and B are the two vertical brass tubes joined at the upper and lower ends by means of the gun-metal castings C and D respectively. Tube A contains the heater and stirrer, and is of uniform size, namely, 5 in. diameter, in the various baths employed. The diameter of tube B ranges from 5-12 in. according to the nature of the instruments under test and the number of such instruments being dealt with at any one time. The length of these tubes is also dependent on these factors. The baths at the Laboratory range from 18 in. to about 4 ft. in length. The junction between the tubes and castings is made by flanges soldered to the tubes; the flanges are bolted to the castings with a packing-ring of brown paper covered with a thin layer of red lead and gold size, which produces an efficient joint to withstand the changes of temperature. In some baths, particularly those used for testing clinical thermometers, in which the temperature range is smaller, the tubes are soldered direct to the castings.

In some of the baths the tube B is provided with a rectangular window, through which the readings of the thermometers may be taken with the instruments fully immersed. Such baths are employed for the test of the less accurate types of thermometers, the readings of which are generally obtained with the naked eye or by means of a reading-glass. The window of plate glass is generally carried in a recessed rectangular brass frame bolted or soldered to tube B, and the joint between glass and brass frame is made with red lead and gold size. Such a window will remain water-tight for several years, notwithstanding the considerable fluctuations of temperature to which it is subjected. The castings C and D are each provided with four lugs, by means of which a wooden top and base may be bolted to the metal part of the bath. These wooden frames serve to support the baths and also afford fixing for the outer wooden casing in which the baths are enclosed. The space between the bath itself and the casing is filled with granulated cork, which serves as an efficient thermal insulator for this class of work. A thickness of 2 in. of cork suffices for most purposes. A convenient casing is provided by strips of wood, such as are employed for covering steam cylinders. The cork lagging should not of course be allowed to become wet, as in this condition its insulating properties are impaired.

For convenience in repair, the heater pockets are carried by a plate bolted to the casting D at the lower end of the tube A. In the smaller baths three or four pockets are provided. These are soldered into slots in the brass plate E. The pockets are formed by bending No. 30 gauge copper sheet over a template very slightly larger than the heaters themselves; the edges of the copper sheet are lapped and soldered. In the event of a heater section failing, it sometimes happens that one or more of the pockets are destroyed. This method of fixing makes replacement of a pocket a relatively simple matter.

The stirrer consists of a three-bladed propeller, of diameter about 4½ in., carried on the end of a shaft passing through the support F bolted to the upper casting C; bearings are provided at G and H; rotation of the propeller is obtained by means of a cord passing over the pulley J and driven by a motor. A free pulley K on the end of the shaft serves to carry the cord when the bath is out of use. At the Laboratory it is the practice to drive several baths from one motor by an endless cord passing round the driving pulleys of each. The tension of the cord is adjusted by a weight carried on a free pulley. The baths are supported on a framework of 2- or 2½-in. piping, the weight of the bath being taken by tie-rods fixed to the under side of the lower wooden frame and provided with a right- and left-handed screw connector for purposes of adjustment. The framework referred to serves also to support the water-supply pipes and the necessary switch-gear for regulating the heating current. Cold water is supplied to the bath through the funnel at L, while a waste pipe at M, provided with a stopcock, allows the bath to be emptied when desired. An overflow fitted at N serves to maintain the water at a constant level. In some of the baths this overflow is adjustable.

In baths in which the instruments are fully immersed and are read through the window the exact level of the water is immaterial, but in those where the thermometers are allowed to project above the top it is necessary that the bath should be kept quite full, so that the water laps the under surface of the plate from which the thermometers are carried.

The method of support of the thermometers depends on the type of instrument under examination. Instruments read through the window are carried in spring-clips (of phosphor bronze) on a cage, details of which are shown in *Fig.* 17. The cage consists of an upper plate to which three vertical rods are attached; sliding platforms holding the spring-clips move on these rods. The position of these platforms is adjustable according to the length of the thermometers under test. A spider, provided with curved guides, is employed to direct the cage when it is lowered into the bath and to protect the instruments from contact with the sides of the bath during this process. Simple hydraulic lifts carried on the supporting framework are employed at the Laboratory to raise and lower the cages, since a cage carrying seventy-two thermometers is of considerable weight. Baths with which this type of cage is used are fitted with a ball-race (with phosphor-bronze balls) at S, while gearing, operated by a suitably placed handle, serves to rotate the cage to bring each thermometer in turn opposite the window for purposes of observation.

A different system is employed for precision thermometers. In reading these instruments distortion is avoided by making the observations with the instruments emerging from the bath, instead of viewing them through a window. In most instances the thermometers are compared with the whole of the mercury at the temperature of the bulb, and the instruments are consequently supported in the bath so that the mercury column is only just visible above the top of the bath. For each reading it is thus necessary to readjust the thermometers according to the height of the mercury column, and a simple means of doing this is necessary. This is effected in the following way. A rubber ring is slipped over the stem of the thermometer, of such a size that it grips the stem securely, but yet not so tightly that the thermometer cannot be pushed through the ring to any desired position. Suitable rings can be made by cutting rubber pressure tubing of various sizes into sections about 4 to 5 mm. in length. A brass plate fitting the opening of the bath is drilled with a number of holes and serves to support the thermometer by the aid of the rubber ring. To permit of thermometers of various diameters being satisfactorily supported by a given plate, a series of graded sleeves is employed. The plates are drilled with holes of 10-mm. diameter; the sleeves have an external diameter of slightly under 10 mm., while the internal diameter varies from 4 mm. to 9 mm. The size of sleeve chosen for any thermometer is such that the instrument will just pass through it freely. In addition to the plate carried on the top of the bath, two other plates, drilled to correspond, are carried above and below the main plate by vertical rods; the plates are clamped to the rods to permit of ready adjustment to any desired position. The perforations in these plates register with the holes in the main plate, and their object is to hold the thermometers steady in a vertical position.

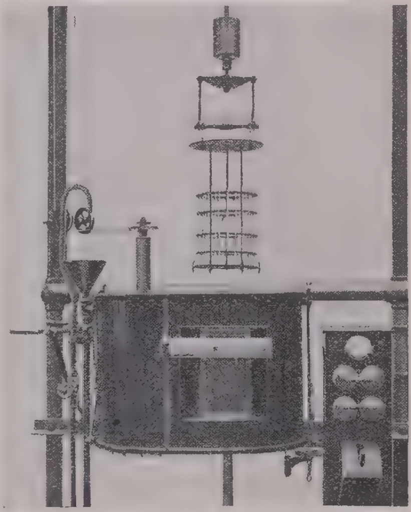

FIG. 17.—Thermometer Comparison Bath, showing "Spring Clip" Type of Cage.

The rapid circulation of the water would otherwise cause the instruments to swing if supported by one point, thereby rendering accurate observation impracticable. *Fig.* 18 will make the arrangement clear. A second rubber ring at the upper end of each thermometer avoids risk of accident should the instrument slip through the first ring. For specially large thermometers auxiliary cages are provided with larger holes. The holes in the plates are arranged either in a circle or in two parallel rows across the centre of the plate. In the former instance the telescope used for reading the thermometers is maintained in a fixed position, and the plate carrying the thermometers is rotated by gearing actuated by means of an endless cord from the observing position. In the second case the plate remains in a definite position, and observations of the different instruments in turn are made by moving the telescope.

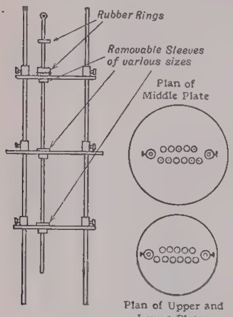
Rubber Rings
Removable Sleeves of various sizes
Plan of Middle Plate
Plan of Upper and Lower Plates

FIG. 18.

As previously mentioned, the circulation of the water is effected by means of a propeller carried in tube A (see *Fig.* 16). The direction of rotation is such that the water is lifted in the tube; it then passes through a top cross-connection and down the main tube, back through the lower casting, and then up past the heaters. The speed of the propeller is roughly 300 to 350 revolutions per minute, which suffices to produce a vigorous circulation of the water. This is an essential feature of this type of comparison bath in order that the water through the main tube may be maintained at uniform temperature. Tests carried out on these baths show that the difference in temperature between the water at the upper and lower ends of the tube on no occasion exceeded 0·01° C. In practice, for the highest precision work the bulbs of the thermometers under comparison are kept as close as possible, thereby reducing any uncertainty in the uniformity of temperature to less than 0·002° C. The direction of circulation of the water is important, for if this is reversed much less uniformity is obtained.

The heaters employed are of uniform size for all the baths. In the smaller baths three or four double heaters are used, while in one larger bath of approximately 80 litres capacity six double heaters are employed. The heaters are in two parts, each of which dissipates 400 watts when used on the standard voltage of 100. Thus the energy can be supplied to the smaller baths at the rate of about 3 kilowatts, and to the larger one at nearly 5 kilowatts. The heaters are constructed of "nichrome" strip wound on a mica frame and protected on each side by mica sheets. *Fig.* 19 shows the construction of the heaters. The size of the strip used is approximately 1·5 mm. by 0·2 mm. in section, and when in position in the bath these units carry a current of 4 amperes without overheating. The actual temperature of the wire is about 400° C., and the energy, of course, can only be dissipated when the pockets in which the heaters are inserted are in contact with water. Switching on the current in an empty bath immediately results in the destruction of the heater and of the pocket. The dissipation of energy is materially assisted by making the pockets of copper so thin that the pressure of the water in the bath collapses the pocket on to the heaters, and the life of the heaters is prolonged by attention to this detail. In ordinary use a carefully made heater will last many months. In some cases heaters have been in continual use for over twelve months. Lack of attention to the points above mentioned, or carelessness in construction, greatly reduces the life of a heater, since these are being run so close to the safe limit. Breaking down of the heater, in general, results from the slow oxidation of the winding, causing hot patches to develop. Another cause of failure is due to the condensation of moisture on the heaters, with consequent short-circuiting of some of the windings if the pockets are not perfectly water-tight, or from the air if the heater does not fit the pocket tightly. This is more especially noticeable if a bath is repeatedly cooled to below the room temperature by the addition of ice. Details of the size of the heaters employed are shown in the accompanying figure, and each half of the heater should have a resistance not less than 25 ohms for use on a 100-volt circuit.

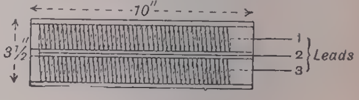

FIG. 19.

Two methods of control of the current through these heaters are employed according to the class of work for which the baths are used. For ordinary cases it suffices to be able to switch on a full load in order to raise the temperature of the bath rapidly from one point to the next. On approaching the desired point the rate of heating is reduced by cutting out six of the eight heaters. On reaching the temperature another heater is cut out of circuit, leaving only one. This last heater has a resistance in series with it so that the current may be adjusted to such an amount that the heat lost by radiation, etc., is just counterbalanced by that supplied electrically. The value of this current varies of course with the temperature at which the bath is being maintained, and with the surrounding temperature, and a small ammeter is provided to assist the operator in adjusting the current to the requisite amount. The value of the current varies with the capacity of each bath, but it will soon become possible to estimate the current required to hold the temperature steady at any point. *Fig.* 20 shows the electrical connections for this case.

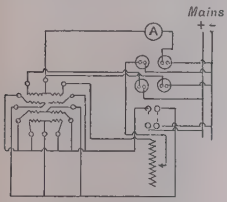

FIG. 20.

Each bath has a slightly different "lag"; that is to say, the rise of temperature which takes place when the second heater is cut out of circuit is different for each bath. It amounts to 0·02° to 0·04° C., and consequently the second heater has to be switched off at a temperature lower by this amount than that at which it is desired to adjust the bath.

For the testing of precision thermometers a rather finer adjustment of heating current is necessary, and a method is employed by means of which the heater units may be connected to the supply circuit in series or in parallel or in a combination of these two ways. In this case also the external regulating resistance is employed in conjunction with one of the heaters. Use is made of a particular type of tumbler-switch having three positions. The central position is "off," while in the other two positions connection may be made to the positive or negative side of the supply mains as desired. *Fig.* 21 shows the connections required. One or two typical examples will be given to show how the heaters may be connected.

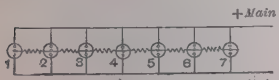

FIG. 21.

(*a*) If the switches are connected alternately to the positive and negative mains, all the heaters will be in parallel on the 100-volt circuit.

(*b*) If the two end switches are connected to the positive and negative mains respectively, the intermediate switches being left in the "off" position, all the heaters will be connected in series across the mains.

(*c*) If switch No. 1 be connected to one pole, and switch No. 4 to the other, the remaining switches being "off," three heaters only will be connected in series across the mains.

(*d*) As an example of the combination of the two, if switches No. 1 and No. 7 are connected to one pole, while switch No. 4 is connected to the other pole, the remainder again being "off," there will be two sets of three heaters in series connected in parallel across the mains.

Numerous other combinations at once suggest themselves, and serve in conjunction with the regulating resistance to obtain a fine adjustment of current to enable the bath to be kept steady at any point in the range from air temperature to 100° C.

With regard to the regulating resistances employed, these are of about 100 ohms resistance and are wound in steps with wire of graded sizes; the smallest wire is large enough to carry a maximum load of 1 ampere, since this is the maximum current obtainable when most of the resistance is included in the circuit. When, however, the resistance is mainly cut out, a current of nearly 4 amperes is being passed, and the coarser wire at this end of the rheostat must, therefore, be capable of carrying 4 amperes.

With regard to temperatures below the temperature of the room the bath is cooled by the addition of ice, and for most practical purposes the rate of rise of temperature is so slow that no special means are required to cool the bath continuously, though this could be done by means of a cold brine circulation.

Other tests below air temperature are carried out in vacuum vessels such as will be dealt with later.

(ii.) *Comparison of Thermometers above* 100° *C.*—For comparison of thermometers above 100° C. water can no longer be used, and recourse has to be had to other liquids. For the range 100° to about 200° C. cotton-seed oil affords a suitable medium; this oil when new is rather viscous at air temperatures, but becomes fluid at temperatures above 100° C. Continued use of the oil causes it to thicken very considerably, but the increase of viscosity above 100° C. is not sufficiently marked to interfere with the efficient circulation of the liquid until it has been in use for a long period. Mineral oils of high flash-point may also be used up to 300° C. Cotton-seed oil cannot be employed as high as this, as continual heating above 200° C. causes decomposition and charring of the oil.

The type of bath used is similar to those used for water in that two vertical tubes are joined top and bottom by cross-connections. Copper tubes are employed, and the connecting tubes are brazed. At the Laboratory gas-heating is employed, although oil baths could be heated electrically if desired. Stirring, as before, is obtained by circulating the oil by means of a propeller carried in the smaller tube The bath itself is enclosed in a sheet-iron double box lagged on the outside with magnesia lagging. Both the vertical tubes are contained in the iron box through which the hot gases from the burners circulate, as otherwise a temperature gradient will exist in the tube in which the thermometers are supported. This tube is preferably lightly insulated with a layer of asbestos cord to prevent local variations of temperature. The speed with which observations can be carried out with gas-heated baths is lower than with those heated electrically, and the accuracy of adjustment of the temperature to any desired value is not so great owing to the larger amount of lagging on these baths. With a bath of this type it is of course impossible to provide a window through which the instruments are viewed, so that all observations are taken with the thermometers so placed that the mercury column is just visible over the cover of the bath. This condition of full immersion is only attained if the oil level in the bath is maintained so that the oil touches the under surface of the plate supporting the thermometers; expansion of the oil is allowed for by the introduction of an overflow pipe just below this level; the overflow pipe is carried to the bottom of the bath through the heating chamber; surplus oil is discharged into a suitably placed receptacle. If the overflow pipe is carried outside the bath, difficulties arise owing to the cooling of the oil and partial stoppage of the flow. A stopcock at the bottom of the bath serves to empty it periodically for cleansing, etc. Comparisons of thermometers should always be carried out at successively higher temperatures, so that it suffices to fill the bath at the beginning of a set of comparisons. The overflow device then ensures that the oil level remains at the desired position.

The thermometers are immersed directly in the oil and supported in the manner previously described for precision instruments.

For temperatures of the range 200° to 450° C. a salt bath is employed, the most satisfactory medium being a mixture of equal parts of sodium and potassium nitrates. At a temperature of 200° C. the salts provide a sufficiently fluid medium for efficient circulation. The bath holding this mixture is again of the type above described, but is cast in one piece in iron in order to withstand the higher temperatures to which it is submitted. Heating is carried out by gas as in the case of the oil bath, and the bath itself is contained in a similar sheet-iron box by means of which it is kept surrounded by hot gas. It is very important that the heating of the salts should be started at the top of the bath, since the mixture expands on heating. If heating, and consequently fusion, is begun at the bottom of the bath, fracture of the vessel is inevitable; but by heating at the top in the first instance fusion commences at the exposed surface and extends downwards. Stirring may be started in the neighbourhood of 180° C. As in the case of the oil bath, an overflow is provided to allow for the expansion of the liquid, together with a tube by means of which the bath may be filled at the beginning of a set of comparisons. Details are shown in the accompanying diagram, *Fig.* 22.

The thermometers themselves are not immersed direct in the fused salts owing to the slow attack of the glass which would result if the instruments were in contact with this medium. It is therefore necessary to provide tubes dipping into the liquid and in which the thermometers may be placed. The tubes are thin-solid-drawn steel, in the lower ends of which plugs are welded. The tubes are carried by the top plate of the bath, and are of different sizes to accommodate instruments of varied diameters. The thermometers are immersed in the smallest tube into which they will conveniently enter, and a small disc of asbestos card slipped on to the stem of the thermometer prevents undue circulation of air into the tube with consequent local cooling. Tubes not in use are closed with a plug of asbestos wool for the same purpose.

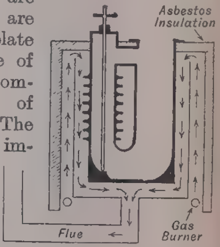

FIG. 22.

In addition to the above baths for the comparison of thermometers over the range above 100° C. it is frequently convenient to determine the correction at a limited number of points without carrying out the above routine. Recourse is then had to a very simple type of vapour bath, in which any of the following substances are used: aniline, naphthalene, benzophenone, or sulphur; each substance requires a separate piece of apparatus, the construction of which is shown in *Fig.* 23. The bath itself consists of a length of iron tubing, 1½ in. to 2 in. in diameter, closed at the lower end by a cap which may be welded on or merely screwed, using asbestos fibre packing. At the upper end this tube is provided with a T-piece and right-angled bend as shown; the bend carries a second length of iron tube about 1 in. in diameter or a tube of hard glass to serve as a condenser, while the thermometers under observation are supported in the vapour through the straight part of the T-piece by suitable means. The thermometer itself may be immersed directly in the vapour, or may be protected from it by provision of a thin steel tube as in the case of a nitrate bath. The main tube is lagged throughout its length with steam-pipe lagging, with the exception of about 6 in. at its lower end. The whole is supported on a convenient stand above a gas burner. The condenser tube is not provided with any lagging, as it is cooled by exposure to the air, thereby serving to prevent loss of vapour, which condenses in the tube and runs back into the main part of the apparatus. These vapour baths are ordinarily about 3 ft. in length, but owing to their simple construction may easily be built of any size to take instruments of abnormal length. One set of tubes at the Laboratory is over 6 ft. in length, and is used for the examination of the very long mercury thermometers sometimes employed in industrial operations. In using these baths, sufficient of the material is introduced into the tube to fill it with vapour when heated; a glass condenser tube is convenient, as the height to which the vapour rises may be readily seen, and in consequence it is known whether the main tube is completely filled or not. Furthermore, care must be taken that sufficient material is available, since if all the substance be vaporised, superheating will ensue. This, however, will not happen so long as there is some unvaporised material present.

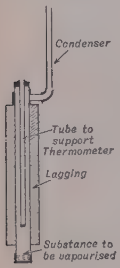

FIG. 23.

The thermometer under observation must be supported in the tube, so that its bulb does not reach the unvaporised material.

The temperature to which the thermometer is exposed is given by the boiling-point of the substance used, but in this type of apparatus it is preferable to rely on the readings of a standard thermometer interchanged with the test thermometer, since with this simple type of apparatus it is not always possible to ensure that impurities may not be introduced. Accurate observations of the boiling-point of such a substance require more elaborate apparatus.[1]

Table VIII. gives the accepted boiling-points of the above substances, with the variation in boiling-point with pressure.

TABLE VIII

Substance.	Boiling-point on Thermodynamic Scale. (p = Pressure in mm. of Mercury.)
Aniline . . .	$183{\cdot}9 + 0{\cdot}051(p - 760)$
Naphthalene .	$217{\cdot}9 + 0{\cdot}058(p - 760)$
Benzophenone	$305{\cdot}9 + 0{\cdot}063(p - 760)$
Sulphur . .	$444{\cdot}5 + 0{\cdot}0908(p - 760) - 0{\cdot}000047(p - 760)^2$

For very small thermometers such as are sometimes employed in chemical observations another type of vapour bath is of service. This consists of a glass tube surrounded by an outer tube through which vapour may be circulated. The vapour is produced by heating a liquid such as aniline by means of a small electric heater at the base of the jacket. The upper end of the vapour jacket is connected through a condenser to an air reservoir, the pressure of which may be varied above or below that of the atmosphere. In this way the corresponding temperature of the vapour may be varied over a fairly wide range. A manometer attached to the air reservoir enables the pressure to be read, and gives an approximate indication of the temperature of the vapour. The temperature to which the thermometer is exposed is given by the reading of a standard thermometer side by side with the thermometer under test. The thermometers are supported in the inner tube by means of a rod passing up this tube, and the bulbs of the thermometers are preferably inserted in a block of copper the high conductivity of which ensures that both instruments will be at the same temperature. Condensation of the vapour in the jacket is reduced by surrounding the jacket with thick rings of felt; these may be moved up and down the outer tube to allow the readings of the enclosed thermometers to be seen.

(iii.) *Low-range Thermometers.*—Thermometers may be compared below 0° C. by immersion in a mixture of ice, salt, and water, which may be contained in a lagged vessel to prevent too rapid a rise in temperature of the mixture, and this must be stirred continuously during the observations. For observations of high precision it is desirable to use a bath the temperature of which may be more accurately controlled; this may be obtained by the use of acetone or ether contained in a vacuum vessel (not silvered) and cooled by the introduction of carbonic acid snow; by this means temperatures as low as $-80°$ C. are obtainable. The liquid must be free from moisture, as otherwise at these low temperatures the moisture would separate out as ice crystals and render it difficult or impossible to take readings of the thermometers immersed in the bath. The liquid must be stirred continuously during the observations; this may be done by bubbling air through the liquid, but a more satisfactory method is to circulate the liquid by a propeller enclosed in a thin brass tube. This tube has openings at the bottom and near the upper level of the liquid in which it is immersed. Rotation of the propeller lifts the liquid in the brass tube and discharges it at the upper orifices; fresh liquid is drawn in from below. Complete circulation is thereby effected, with consequent uniformity of temperature. The brass tube serves as the support for the standard thermometer and the thermometers under observation, while a small electric motor carried at the upper end of the brass tube rotates the propeller.

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[1] Refer to Sulphur Baths in article on "Resistance Thermometers," § 16.

The rate of change of temperature in a well-exhausted vacuum vessel is quite small, so that readings may be taken to a high degree of accuracy with such a device. Cooling of the liquid is effected by adding carbonic acid snow, while raising its temperature may most easily be carried out by removing a portion of the cold liquid and replacing it by liquid which has been exposed to the temperature of the air; for this purpose the type of vacuum vessel provided with an outlet at the lower end is useful. A small electric heater immersed in the liquid would also serve to raise the temperature.

W. F. H.

THERMOSTATS

THERMOSTATS may be divided into three classes: those for use at (1) low temperatures (up to about 100° C.); (2) medium temperatures (100° C. to 350° C.); and (3) high temperatures (above 350° C.). These divisions are, of course, arbitrary, and in many cases the same instrument can be used for any temperature above that of the atmosphere, though, in general, it will be specially suitable for one particular range. In general, also, it is true that instruments suitable for the higher temperatures are less accurate than those which will only work at the lower temperatures.

The number of forms of thermostat is great: it will not, therefore, be possible in this article to do more than refer to one or two pieces of apparatus typical of each of the classes mentioned above.

§ (1) LOW TEMPERATURE THERMOSTATS.—The most important of these instruments is the toluene thermostat. This consists, essentially, of a glass bulb containing toluene, which is immersed in a bath of water or oil. Vessels containing the substance whose temperature is to be kept constant can be suspended in the bath, which is heated either electrically or by means of a gas flame. The expansion of the toluene with rise of temperature is made use of in various ways, some of which are described below, to reduce the gas supply, or the current of electricity. This causes the bath to cool slightly, which produces a contraction of the toluene, with a corresponding increase in the supply of heat.

The vessel containing the toluene has as large a surface as is conveniently possible, and usually takes the form of a long spiral, or of a series of bulbs joined together. *Fig.* 1 represents a common form of regulator for controlling a gas-heated thermostat. Gas enters by the tube A and passes out by the tube B leading to the burner. The expansion of the toluene in the bulbs CC, which takes place when the temperature rises, forces the mercury up the tube D, until the exit of A is closed. A by-pass or pilot jet prevents the gas from being completely extinguished, but is too small to maintain the temperature of the bath, which therefore cools, causing the mercury to descend in the tube and admit gas to the burner again. A side tube E, furnished with a tap, enables the height of the mercury in D to be altered, thus altering the temperature at which the bath is run. A more sensitive form of adjustment can be made by dispensing with the tap and closing the top of the tube E with an air-tight rubber bung, through which a glass plunger passes and dips into the mercury. The diameter of this plunger is not much less than the internal diameter of the tube E. By raising or lowering the plunger, the height of the mercury in both tubes can be regulated. The space between the mercury and the bung in the tube is filled with a mixture of water and glycerine, which serves to exclude all air and to lubricate the plunger.[1]

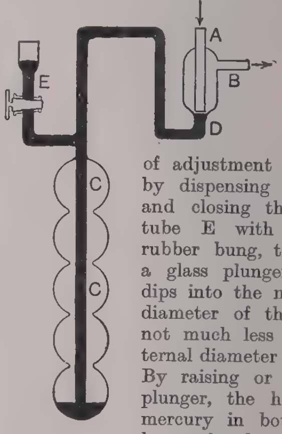

FIG. 1.

For use with an electrically heated bath the tubes A and B are removed, a platinum wire is sealed into D, in any convenient position, and a second platinum wire, carried on the end of a screwed rod, replaces the tube A. The wires are in series with a source of electric current and a relay, the circuit being completed when the mercury makes contact with the second wire, whose position is accurately adjusted by means of the screw. The relay switches on and off the heating current for the bath. In some cases, where only a small current is being used for keeping up the temperature of the bath, it is possible to dispense with the relay and break the heating current directly.

A modification of this method of switching off the current is as follows.[2] On top of the mercury in D floats a plunger, carrying a vertical threaded rod. This rod passes through a flat spring, and carries a nut which depresses the spring when the mercury sinks in the tube. This closes the circuit through the relay, and, on the mercury rising again and lifting the float, the spring reopens the circuit. This apparatus has the great advantage that no current is broken on the surface of the mercury, which, therefore, keeps clean for an indefinite time. Fouling of the mercury by the spark is a serious problem in electrically controlled apparatus, though the use of a condenser, in parallel with the contacts, reduces the spark very considerably.

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[1] Slator, *Soc. Chem. Industry J.*, 1911, xxx. 61.
[2] Cummings, *Faraday Soc. Trans.*, 1911, vii. 253.

There are many other types of regulator. The reader interested will find other examples described in Thorpe's *Dictionary of Applied Chemistry*, 1919 edition, v. 462, and various articles in the *Transactions of the Faraday Society*, the *Journal of Physical Chemistry*, and other periodicals.

When the bath is electrically heated, this is frequently done by means of lamps immersed in the liquid, a convenient form of heater being a lamp with a very long stem which is brought above the level of the liquid, thus enabling the connections to be kept dry.[1] Bare wire has also been used in the bath, while in some cases the wire is wound round the containing vessel.

It is of the utmost importance that the bath should be kept well stirred; the most suitable arrangement is one which produces a circulation from the heater towards the bulb of the regulator.[2]

Various liquids have been used in place of toluene; amongst others may be mentioned benzene, alcohol, and paraffin.

With a toluene thermostat it is a matter of ease to keep the temperature of the bath constant to 0·01° C., while it has been claimed that certain forms can be arranged to keep a temperature steady to less than 0·001° C. for several days.

§ (2) MEDIUM TEMPERATURE THERMOSTATS.—The toluene type of thermostat can be adapted for medium temperatures (say up to 350° C.) by using, in the regulator, a liquid having a sufficiently high boiling-point. Mercury at once suggests itself for this purpose, but it has several disadvantages. Its high specific gravity makes it necessary to use strong-walled containing vessels, and this causes the temperature of the mercury to lag behind the temperature of the bath. In addition the dilatation of mercury is much less than that of toluene, the figures expressing the increase in volume of one litre for one degree rise of temperature being 0·18 c.c. for mercury and 1·1 c.c. for toluene. In spite of these drawbacks, mercury thermostats have been of considerable use, a very simple form, to be employed with electric heating, consisting of a mercury thermometer with a pair of wires sealed into it at suitable points. Such a regulator can, of course, only be used at a single temperature, and is not very sensitive.

More satisfactory results are obtained by the use of the dilatation of solids. It is possible to construct a very sensitive differential expansion regulator, by riveting together a strip of invar and a strip of another metal, such as brass, which has a high temperature coefficient of expansion. Such a strip will bend with change of temperature, and, if clamped at one end, the movement of the other end (suitably magnified, if necessary) can be made to operate a relay for regulating the supply of gas or electricity to a furnace. A description has been published of such a furnace, in which an air space of about a cubic meter capacity, heated electrically, was kept at a temperature of the order of 200° C. with an accuracy of within 0·1° C.[3]

A similar type of thermostat was devised by Gumlich[4] in which the bimetallic strip was curled into a large spiral and actuated an electromagnet which controlled the gas supply. In this case the strip was not housed in the space that was required to remain at a constant temperature, but in a small vessel connected to the same gas supply. The electromagnetic valve controlled the gas supply both to the furnace and the vessel containing the strip. An accuracy of 0·5° C. at 100° C. is claimed for this apparatus.

Another type of thermostat, which is best considered among those working at moderate temperatures, is that depending on boiling-points. There are examples of such apparatus which can work at low temperatures, and others which can be used at high temperatures, but the greater number work between 100° C. and 400° C. They depend upon the constancy of the boiling-point of various liquids. This constancy is made use of in two different ways: (1) A bulb filled with the liquid is placed in the bath or furnace whose temperature is to be regulated, and is connected with a tube containing mercury, or with a cylinder in which a piston moves freely. When the temperature reaches the boiling-point of the liquid, the rapid rise of pressure, due to the evolution of vapour, moves the piston and cuts off the gas supply or the current. With fall of temperature, and consequent condensation of the vapour, the gas or electricity is turned on once more. (2) The specimen, whose temperature is to be controlled, is suspended in the vapour arising from the boiling liquid.

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[1] *Faraday Soc. Trans.*, 1911, vii. 249.

[2] *Ibid.* 263.

[3] Rayner, *Faraday Soc. Trans.*, 1911, vii. 263.

[4] Gumlich, *Zeits. für Instrumentenkunde*, 1898, xviii. 317.

This vapour is led to a condenser, and the liquid is returned to the boiling vessel.

Both these types of thermostat suffer from the disadvantage that they are slightly affected by changes in barometric pressure, and by the more serious disadvantage that, working at atmospheric pressure, the number of temperatures attainable is very limited. By an alteration of pressure it is possible to increase the range of temperatures obtainable, but this introduces serious complications. An ingenious apparatus, which avoids these difficulties, makes use of the fact that, in boiling a mixture of liquids, the concentration of the vapour is different from that of the liquid, and, consequently, if the vapour is allowed to escape, the boiling-point will change continuously. The apparatus is shown diagrammatically in *Fig.* 2, where A is the vessel in which the mixed liquids are boiled, and B is a pipe leading the vapour through the condenser C. As long as the tap D is open, the condensed liquid flows away to the receiver E, and therefore the boiling-point of the liquid in A continues to rise. When the required temperature has been reached, the tap D is closed, with the result that the condensate is returned to A, thus keeping the concentration, and therefore the boiling-point, constant. The specimens to be heat-treated are suspended in the vapour in A.

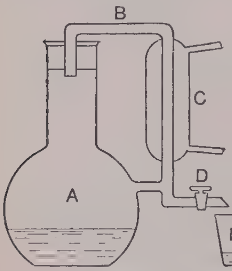

FIG. 2.

§ (3) HIGH TEMPERATURE THERMOSTATS.—The problem of obtaining a steady high temperature is much more difficult than the one of keeping a constant low or medium temperature. In a few isolated cases it can be done by the use of boiling-point methods, but the number of temperatures which can be got in this way is very limited. The expansion of molten tin has also been used in an apparatus similar to that which employs the expansion of mercury as the thermostatic medium, but there are manifest inconveniences in using a substance which is not liquid at ordinary temperatures. An apparatus has also been devised in which the current from a thermocouple is arranged to control the temperature. The current is led to a milliammeter, on the scale of which are two metallic blocks separated by a sheet of mica. These blocks can be moved over the scale so as to place the mica strip below the position occupied by the pointer at any desired temperature. At regular intervals the pointer is depressed by clockwork, and, according as the temperature is above or below that for which the instrument is set, makes contact with one or other of the metallic blocks; these and the pointer are electrically connected with a relay in such a way that resistance is cut into or out of the furnace circuit according to which of the contacts is made. The temperature is thus controlled at regular time intervals, and the amount it varies will therefore depend on the rate of heating and cooling of the furnace. In place of the thermocouple, a platinum resistance and Wheatstone bridge can, of course, be used. This apparatus has only recently been described, and, as far as the writer is aware, has not yet been used to any great extent, but it appears to be promising as a method of controlling large commercial furnaces which only change slowly in temperature, and where variation of a few degrees is not of great importance.

The use of the expansion of a gas in a regulator of the toluene type was early tried. D'Arsonval constructed a thermostat in which the expansion of air in a bulb in the furnace was communicated to a vessel similar to the capsule of an aneroid barometer. The motion of this capsule was used to work a relay for controlling the gas supply to the furnace. A similar arrangement is described by Mellor [1] for use with an electric furnace, the expansion of the air operating on a column of mercury which is arranged to work a relay. This controls the current to the furnace. With Mellor's apparatus, an accuracy is claimed of 10° C. at 400° C. and 30° C. at 700° C.

Both these forms of instrument suffer from two disadvantages. In the first place, the bulb, being inside the furnace, responds more slowly to changes of temperature than does the furnace itself. This permits of wider fluctuations in temperature of the furnace than are desirable. Secondly, the apparatus is seriously affected by changes in barometric pressure. A form of gas thermostat which avoids these disadvantages, and which has certain other advantages peculiar to itself, is illustrated in *Fig.* 3.

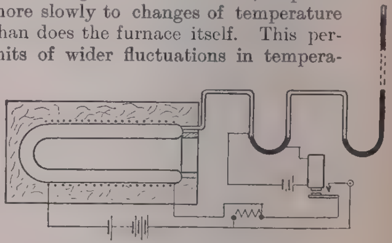

FIG. 3.

The furnace itself [2] consists of a double-walled silica vessel, whose inner space is to be maintained at a constant temperature. The volume between the two walls is filled with air, and round the outer wall is a winding of "nichrome" or other resistance wire. The whole is suitably lagged. The air-space between the walls of the furnace communicates with a U-tube containing mercury, and the expansion and contraction of the air with change of temperature causes a rise and fall of the mercury. This makes and breaks the circuit through a relay, which cuts resistance into and out of the furnace circuit. Since the controlling medium (air) is between the heating wire and the constant temperature space, the fluctuations of temperature of this air will be greater than those of the furnace; this enables the latter to run at a very steady temperature. To protect the apparatus from changes in barometric pressure, the limb of the U-tube not attached to the furnace bulb is connected with the short side of a syphon barometer. The expansion or contraction of the air in the bulb is transmitted through the mercury in the control tube to the mercury in the barometer, which thus rises or falls with alteration of temperature in the furnace. With such a furnace, wound with nichrome wire, it is possible to attain a temperature of 1000° C. and to maintain it constant within $\pm 1°$ C.

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[1] *Clay and Pottery Industries*, p. 133.

[2] *Inst. of Metals J.*, 1915, xiv. 145, and 1917, xviii. 173.

Such a thermostat functions by keeping the mean voltage on the furnace terminals at a definite value. This value is such that the heat input, due to the current produced in the furnace by that voltage, is just sufficient to compensate for radiation and other heat losses. It follows therefore that, given a second furnace which will be affected by changes in room temperature to the same extent as the thermostat furnace, and connecting it in parallel with the terminals of the thermostat furnace, but in series with its relay, resistances, etc., the temperature of the second furnace will be regulated at the same time as that of the first. Nor is there any theoretical limit to the number of furnaces which can be run in this way.

In certain investigations, particularly those dealing with metallic equilibria, it is advantageous to be able to heat or cool specimens at a steady—often very slow—rate. With the apparatus just described this is a matter of considerable ease. All that is necessary is to provide means for gradually lowering the pressure in the apparatus on the furnace side of the U-tube for slow heating and on the barometer side for slow cooling. This is best done by means of a bulb immersed in hot water or oil, contained in a thermos flask or other vessel. By altering the size of the bulb or the rate of cooling of the liquid, it is possible to obtain rates of heating or cooling of the furnace from 1° C. per day upwards. For convenience two bulbs are sometimes fitted, one of which, for slow heating or cooling, is immersed in water in a thermos flask, while the other, for quicker changes of temperature, is in oil in a vessel which carries a winding, by means of which the oil can be heated electrically. *Fig.* 4 illustrates such an apparatus diagrammatically. The relay used is the one described by Barr.[1] J. L. H.

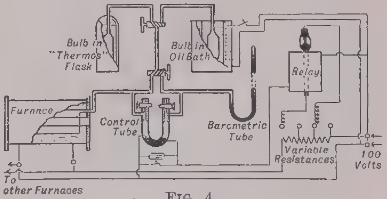

FIG. 4.

THOMSON EFFECT, DEFINITION AND THERMODYNAMIC THEORY OF. See "Thermodynamics," § (65).

THOMSON, JAMES, IDEAL ISOTHERMAL OF. See "Thermodynamics," § (59).

THORNYCROFT BELT-DYNAMOMETER. See "Dynamometers," § (4).

THROTTLING, NATURE AND EFFECTS OF. See "Thermodynamics," §§ (32), (43).

TIDAL POWER. See "Hydraulics," § (30).

TIDE GAUGE. See "Meters, Liquid Level Indicators," § (15) (ii.), Vol. III.

TIMBER TESTS. See "Elastic Constants, Determination of."
- Ball and Cone Hardness Method. § (131).
- Bending. § (128).
- Cleavability. § (130).
- Compression. § (126).
- Crushing Tests on Timber along the Grain. § (126), Table 45.
- Density, Moisture, and General Conditions. § (122).
- Determination of Toughness. § (129).
- Influence of Conditions of Tests upon Results. § (134).
- Resistance to Abrasion and Wear. § (132).
- Size of Test Pieces. § (124).
- Shear. § (127).
- Tension. § (125).

TIME OF EXPLOSION IN INTERNAL COMBUSTION ENGINES. See "Engines, Thermodynamics of Internal Combustion," § (66).

TOOTHED WHEELS. See "Kinematics of Machinery," § (9).

TÖPLER PUMP. See "Air-pumps," § (16).

TORRICELLI'S PUMP. See "Air-pumps," § (15).

TORSION. See "Structures, Strength of," § (29).

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[1] *Phys. Soc. Trans.*, 1921, xxxiii. 53.

Torsional Strain Indicators: description of various forms of apparatus. See "Elastic Constants, Determination of," § (55).

Torsionmeters. See "Dynamometers," § (3) (i.) and (ii.).
  Ayrton and Perry's. See *ibid.* § (3) (ii.).
  Hopkinson-Thring. See *ibid.* § (3) (i.).

Total Heat. See "Thermodynamics."
  Constant in Throttling Process. § (32).
  Definition. § (31).
  Value for Ideal Gas. § (57).
  Value for Mixture. § (62).
  Wet Steam. See "Steam Engine, Theory of," § (14).

Towers' Experiments on Journal Friction. See "Friction," § (26).

Traction Dynamometer. See "Dynamometers," § (5).

Tractor Dynamometer (National Physical Laboratory). See "Dynamometers," § (5) (iii.).

Transformers, Hydraulic. See "Hydraulics," § (62).

Transmission Dynamometers. See "Dynamometers," § (3).

Transmission of Power by Fluid Motion. See "Hydraulics," § (63).

Transmission of Power by Friction: Theory of Belt Friction. See "Friction," § (36).

Triple Point. The temperature at which the three phases of a substance, solid, liquid, and gas, can co-exist in contact with one another and in equilibrium. See "Thermodynamics," § (41); "Phase Rule," § (4).

Trouton, formulation of the law connecting the latent heat, L, of a substance with M, its molecular weight, and T, its absolute temperature, which states that

$$\frac{ML}{T}=\text{constant}.$$

This law is true for members of the same chemical group, such as the hydrocarbons, but is not true for widely different substances. See "Latent Heat," § (11) (i.).

Trouton's Constant, Values of, for different substances, tabulated. See "Latent Heat," § (11) (i.), Table VII.

Tubes, Protecting, for use with thermo-elements. See "Thermocouples," §§ (4), (5).

TURBINE, DEVELOPMENT OF THE STEAM

The following is a list of symbols used, some of which are defined in the text:

A = area of cross-section of a nozzle or of a ring of nozzles (§ (9)). (Square inches § (13).)
a = coefficient (§ (13)).
α = "velocity ratio" of a stage (§ (1)).
$\bar{\alpha}$ = mean over-all "velocity ratio" of a turbine (§ (11)).
b = constant (equation (26), § (11)).
$c=\sqrt{gd\text{P}/d\rho}$; § (9), also coefficient (equation (35), § (15)).
d = mean diameter of blade-ring in inches (equation (2), § (1)).
δ = coefficient (equation (36), § (16)).
E = "curve" blading efficiency (§ (16)).
e = efficiency ratio of a turbine (§ (15)).
η_s = stage efficiency (§ (6)).
η_d = diagram efficiency (§ (4)).
f = principal stress (§ (15)) and periodicity (§ (17)).
g = gravity.
γ = index in isentropic law of expansion leading to the result $\text{PV}^\gamma=\text{constant}$.
H = "Homogeneous head" of steam = 144 PV feet (equation (24), § (10)).
h = blade height or blade length in inches (§ (13)).
I = total heat of steam (§ (14)).
I_A = total available heat (equation (26), § (11)).
I_a = available heat per stage of a turbine (equation (4), § (3)).
J = Joule's mechanical equivalent of heat = 778 ft.-lbs. per B.Th.U.
K = the Parsons coefficient (§ (11)).
K_p = specific heat at constant pressure (§ (6)).
K_v = specific heat at constant volume (§ (6)).
k = coefficient of opening in nozzles or blading $=\dfrac{\text{throat width}}{\text{circumferential pitch}}$ (§ (13)).
L = latent heat (equation (10), § (6)), also output of turbine at coupling (§ (16)).
l = losses (§ (16)).
λ = index in law of expansion $\text{PV}^\lambda=\text{constant}$ (equation (8), § (6)).
  = safety factor (§ (15)), also mechanical losses (§ (16)).
N = number of pressure stages in a turbine (§ (11)).
n = number of velocity stages in a turbine (§ (3)).
P = absolute steam pressure.
p = number of pairs of poles (§ (17)).
Q = total steam consumption per unit time (§ (9)) (lbs. per hour, § (13)).
q = quality of steam or dryness fraction (§ (6)).
R = revolutions per minute of turbine (equation 2, § (1)).
ρ = density of fluid (§ (9)) = 1/V.
S = steam consumption per kilowatt hour (§ (14)).
σ = coefficient (§ (15)).
T = absolute steam temperature (§ (6)).
u = tangential velocity of a blade ring at mean diameter (§ (1)).
V = specific volume of steam (§ (9)) cubic feet per pound (§ (13)).
v = steam jet velocity at any instant (§ (1)).
$\bar{v}$ = change in velocity of steam jet, in any direction (§ (4)).
W = Mg = gravitational units of mass (§ (4)).
$\text{Z}=\text{P}/\text{P}_1$ = reciprocal of pressure ratio (§ (9)).
ω = angular velocity (§ (15)).

I. Introductory

§ (1) General Theory.[1]—The steam turbine is an engine or "prime mover" working on the same principle as the familiar country-side windmill. Instead of a current of air being utilised to rotate a shaft by means of "sails," a current or blast of steam issuing from a number of fixed nozzles is employed to rotate a shaft by means of "vanes," "buckets," or "blades."

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[1] For an account of the theory of jets see "Steam Engine, Theory of," §§ (11), (12).

In the case of windmills the relatively small power obtainable from air, with a reasonable sail area, and the discontinuity of the breeze, render this type of prime mover of relatively little economic importance. With the steam turbine, however, the case is very different. Not only is there the convenience of uniform rotary motion produced by a steady torque applied to a shaft direct (in common with the windmill), but also the unfailing steam blast generated with coal or oil fired boilers, and the enormous output obtainable from a turbine of very moderate dimensions.

The economic value, therefore, of the steam-driven rotary engine, or steam turbine, has been the incentive to its development to the utmost, and its evolution has been successfully carried out during the last thirty years in the face of all obstacles, mainly owing to the efforts of Sir Charles Parsons and his associates, until to-day it is by far the largest and most economical prime mover yet devised. Its efficiency is unrivalled by any other form of steam engine. Units of 30,000 to 40,000 horse-power are becoming commonplace in large land power stations for the generation of electricity, whilst marine installations reach 150,000 shaft horse-power in one vessel.

The difficulties encountered in the inception of the steam turbine were mainly mechanical, brought about by the physical properties of steam. Water turbines had already been worked out by several well-known nineteenth-century engineers, and brought to a considerable state of perfection prior to 1880; it was on the high efficiency of the water turbine that the hope of a successful counterpart in the steam turbine was based. The elastic property and relatively low density of steam, however, in contrast to the incompressibility and relatively high density of water, profoundly complicated the problem. The density of dry saturated steam at 185 lbs. per sq. in. absolute pressure, is 1/2·29 lbs. per cub. ft., or about 1/140 of that of water. At an absolute pressure of ½ lb. per sq. in. it is 1/650 lbs. per cub. ft., or about 1/40,000 of that of water. Further, not only has steam turbine design to deal with large variation in volume of steam, but also with the fact that low density implies high velocity of efflux from a nozzle. The velocity theoretically attainable by a current or blast produced by allowing steam to escape through an orifice or nozzle to a place at lower pressure, is prodigious, as will be seen by a reference to Table I.

TABLE I

THEORETICAL VELOCITY OF EFFLUX OF A STEAM JET

Steam Pressure. Lbs./in.^{2} abs. (dry saturated).	Velocity into Atmosphere. Ft./sec.	Velocity into Vacuum of 28″ Mercury (baro. 30″) Ft./sec.
10	..	2677
15	..	2900
30	1602	3263
50	2108	3510
75	2425	3671
100	2645	3804
125	2777	3905
150	2900	3983
175	3000	4050
200	3075	4100
250	3207	4190

Further, simple mathematical theory shows, and experiment confirms, that the proportion of the available energy in the steam, which can be converted into useful work on the turbine shaft, depends mainly on the relation which is made to exist between the linear velocity of the buckets or "blades," and that of the steam jets which impinge upon them. In other words, the efficiency of a turbine depends mainly upon the ratio

$$\frac{\text{Tangential blade velocity}}{\text{Steam jet velocity}} = \frac{u}{v}, \quad . \quad (1)$$

called the "Velocity Ratio" and usually denoted by the symbol a.

Fig. 1 shows a curve of the inherent efficiency of the usual Parsons blading, plotted on a base of "velocity ratio," leakage and mechanical losses having been eliminated.

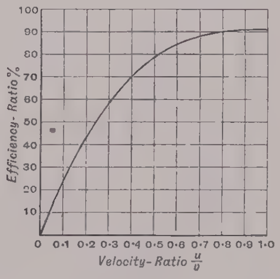

FIG. 1.

From this diagram it will be seen that up to a certain limit, the higher the velocity ratio, or in other words the higher the blade velocity for a given velocity of steam jet, the higher is the efficiency of conversion of heat energy into useful work.

In view of the data furnished in Table I., showing that in the expansion of steam through an orifice, a velocity of several thousand feet per second is to be expected, it becomes clear that to utilise such a jet efficiently in a simple turbine, like a windmill, very high blade velocity will be necessary if a suitable velocity ratio is to be secured.

The mean blade speed of a ring of blades of mean diameter "d" inches, rotating at R revolutions per minute is given by the relation

$$u=\frac{\mathrm{R}d}{229} \text{ feet per second, } \quad . \quad . \quad (2)$$

For the time of one revolution $=60/\mathrm{R}$ seconds, and the distance traversed in this time by a point on the circumference $=\pi d/12$ feet. Hence $u=\pi d\mathrm{R}/12\times 60=\mathrm{R}d/229$ feet per second.

As in general the soundness and reliability of a steel forging diminishes as the size is increased, it follows that a disc or wheel carrying the blades must be of moderate dimensions and therefore rotate at a large number of revolutions per minute.

This, then, was the initial problem encountered in the development of the steam turbine: to construct a shaft and bearings which would render a speed of 15,000 to 30,000 r.p.m. possible and safe. This problem may be subdivided into two parts:

(1) The attainment of continuous and perfect lubrication of the bearings, including the damping out of vibration due to any slight want of running balance in the shaft.

(2) The investigation of the peculiar phenomenon of "critical speeds" of shafts rotating at high revolutions; since at certain speeds a shaft becomes unstable and is liable to dangerous deflection and even fracture.

Although continuous research has been necessary in these matters ever since the commencement of steam turbine engineering, yet at the very outset sufficient experience had to be gained, in order to make this form of prime mover even feasible. It will be realised, therefore, that the physical properties of steam profoundly influenced the mechanical side of the problem from the start, quite apart from any thermodynamic considerations.

Once the practicability of running slender shafts at high rotational speeds, however, had been demonstrated by Parsons in his first turbine of 1884, the path was clear for an attack on the problems of increased efficiency and size of unit. Contemporarily with Parsons, de Laval directed his energies to the perfection of the simple "steam windmill," consisting of one set of nozzles and one wheel carrying blades. In de Laval's turbine, therefore, the steam expanded in one stage from the initial pressure right down to the exhaust pressure, and consequently the inventor was faced with the necessity for extremely high blade speed in order to attain reasonable efficiency. This problem he adequately solved by using a wheel of very small diameter—in order to reduce "windage" losses to a minimum, and make as perfect a forging as possible. Thus a 5-brake-horse-power turbine having a wheel about 4 in. mean diameter would be run at as many as 30,000 r.p.m.

This solution was naturally incomplete in itself, since the speed of rotation was far too high for the direct driving of a dynamo or other apparatus. The inventor surmounted this difficulty by introducing a form of helical reduction gearing which safely and efficiently reduced the speed of the driven shaft to about one-tenth of that of the turbine. This method of dealing with the steam-turbine problem has, however, its limits, inasmuch as no material has yet been found to enable a sufficiently high blade speed to be attained which will give the maximum efficiency, that is a high enough velocity ratio, and furthermore outputs of in excess of 600 or 700 b.h.p. cannot be realised, since the passage of the necessary amount of steam would entail a diameter of wheel such that the "windage" losses would prohibit high efficiency. A peripheral speed of over 1300 feet per second, however, is said to be successfully attained in some modern designs.

With these limitations, the de Laval turbine forms an excellent prime mover for small electric generators, air compressors, etc., and has been made by the perseverance of its inventor a thoroughly practical solution of the problem.

To enable the steam turbine to enter the field monopolised (at the time) by reciprocating steam engines, however—that is to say, to make it suitable for driving machinery at moderate rotational speeds and to enable large b.h.p. to be developed, a radical departure in design was necessary, and this fundamental innovation was introduced by Parsons, who approached the problem in a different way.

The high efficiency already attained by water turbines[1] operating under very moderate "heads" of water, made it seem to him possible that if a number of "simple" one-wheel steam turbines were placed in series on the same shaft, one turbine exhausting into another, and so on, it would be practicable to divide up the energy of expansion of the steam over a number of such elementary turbines, so that the effective "head" of steam (in other words, the amount of expansion) in each would be small. Thus, since the velocity of efflux of the steam jets in each unit turbine would be correspondingly reduced, not only would the latter operate under conditions more analogous to the water turbine, but moderate and practicable blade speeds would be secured also, in conjunction with high "velocity ratio."

This fundamental concept was subjected to practical test by Parsons in 1884, and has since proved to be the only solution. In fact, this principle of "pressure compounding" is the one essential feature of all modern steam turbines of large output and high economy, because it permits reasonable blade velocities to be adopted without sacrifice of efficiency.

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[1] See "Hydraulics," § (45).

This gratifying result, nevertheless, was only finally established after years of patient endeavour, on account of the leakages of steam which take place when it is attempted to divide up the pressure drop into stages. The necessary surface speeds in successful steam turbines are (as has been shown) so high, that leakage over the tips of revolving blades or round the periphery of a shaft where it passes through diaphragms separating one pressure stage or "cell" from another, cannot be stanched by any form of packing which involves actual rubbing contact, on account of the great heat that would be generated. It follows, therefore, that such leakage, if it cannot be altogether suppressed, must be reduced to a minimum. Thus all compound turbines, of whatever design, have fine clearances in certain parts, such fine clearances being essential to steam economy.

In his early work Parsons was faced here with a most formidable difficulty, inasmuch as at the commencement it was only possible to build small machines. It will be evident that small machines cannot be made scale models of larger ones in the matter of clearances. That is to say, small machines must have working clearances relatively very much greater than are required in large ones, and the leakage in the former becomes excessive. The inventor realised that as the size of the turbine was increased, this trouble would simultaneously subside, but even so, the greatest ingenuity was required, and is required today to reduce leakage to the smallest possible amount.

Parsons originated the two principles, now universally admitted, that where the relative motion between two surfaces is very great, and it is desired to limit fluid or gaseous leakage between them—

(1) One of the surfaces must be provided with thin edges or so-called "contacts."

(2) Both surfaces must be serrated or given an interrupted contour in the direction of the pressure gradient.

The first is necessary in order to limit leakage by reducing the leakage area, that is to say, to enable the two surfaces to be run close up to one another at suitable points, without danger of irreparable damage in the event of accidental contact.

The second is necessary in order to limit further the leakage by compelling the fluid to flow in a tortuous path inducing eddies and baffling effect.

If one surface only is serrated, and the other smooth, then the leakage is almost the same as between two smooth surfaces of the same dimensions and clearance.

Fig. 2 shows these principles applied to a Parsons turbine, where it is desired to make the leakage of steam between the stationary cylinder (A) and the revolving shaft (B) as small as possible.

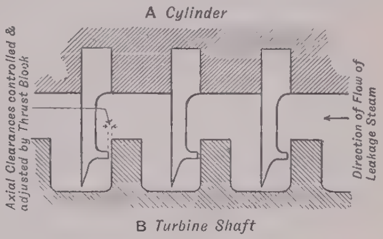
A Cylinder
Axial Clearances controlled & adjusted by Thrust Block
Direction of Flow of Leakage Steam
B Turbine Shaft

FIG. 2.

By such means this great obstacle of leakage loss was gradually overcome, and the principle of pressure compounding established as an essential feature of economical steam turbine design.

§ (2) EVOLUTION INTO DISTINCT TYPES. *Development along Diverging Paths.*—From this stage of development onwards, steam turbine design falls into two distinct categories, viz.:

(i.) *Pressure Compounded "Reaction" Turbine Design*, originated by Parsons in 1884.

(ii.) "*Impulse*" *Turbine Design*—

(*a*) Pure "pressure compounding," adapted by Curtis (1896) to the de Laval type, but developed mainly by Rateau (1898).

(*b*) "Pressure compounding" *and* "velocity compounding," introduced by Curtis (1896) and developed by him.

This divergence has ultimately evolved differences in design so marked that a single glance is all that is necessary in order to distinguish between the two main types, which with few exceptions are built as axial flow turbines with horizontal shafts.

The names "Reaction" turbine and "Impulse" turbine, mentioned above, are more popular than scientific, and it is important to possess a clear idea as to the fundamental difference in design which places a turbine in one or other category.

This fundamental difference lies in the shape of the steam passages between the blades which are mounted on the turbine wheels.

The "nozzles" used in steam turbines to produce the propelling steam jets are fixed and are always the same in principle, although they may differ considerably in proportions. That is to say, there is always a convergent passage connecting a point of higher steam pressure with a point of lower steam pressure, the change good its own internal losses as well as to supply external power, and it is found that the actual efficiency curves, *Fig.* 7, differ from the theoretical curves from two causes :

(1) The greater the number of moving rows, the more serious is the loss in efficiency.

(2) The best efficiency is usually obtained with a velocity ratio somewhat less than the theoretical value given by equation (3). The following range of values, however, fairly represents modern practice in large turbines :

Table II

	$a = u/v$.
Axial flow reaction blading (Parsons)	0·75-0·95
Single-row velocity wheels . . .	0·45-0·52
Two-row velocity wheels . . .	0·22-0·28
Three-row velocity wheels . . .	0·13-0·15

By means of equation (3) a table can be drawn up for velocity wheels with varying number of moving rows, showing their equivalence as regards the utilisation of the available heat derived from the expansion of steam, and with reference to similar points on their respective efficiency curves.

Table III

Equivalent Wheels

	Velocity Stages, n.	Theoretical Best Velocity, Ratio u/v.	Velocity of Jet.	Available Heat.	
Single-row	1	$\frac{1}{2}$	v	I_a	..
Two-row .	2	$\frac{1}{4}$	$2v$	$4I_a$	Equivalent to 4 single wheels
Three-row .	3	$\frac{1}{6}$	$3v$	$9I_a$	Equivalent to 9 single wheels
Four-row .	4	$\frac{1}{8}$	$4v$	$16I_a$	Equivalent to 16 single wheels

If "I_a" is the heat drop or "available heat" in a given case, then JI_a is the "velocity" head in feet ; so that [1]

$$\frac{v^2}{2g} = JI_a, \qquad (4)$$

hence it follows, as will be seen from the above table, that equivalent wheels of a given mean diameter and speed of revolution are proportional to the square of the velocity ratios which give maximum efficiency.

Thus—

$$\frac{\text{One 3-row wheel}}{\text{X single-row wheels}} = \frac{(\frac{1}{6})^2}{(\frac{1}{2})^2} = \frac{1}{9},$$

so that one three-row velocity wheel is the equivalent of 9 single-row wheels, and so on.

In other words, if 9 single-row wheels (with the total heat drop equally divided between them) are operating at a given point on the efficiency curve OA (*Fig.* 7), then if they are replaced by one three-row wheel, utilising the same total heat drop, the latter wheel will operate at a similar point on the efficiency curve OC.

It should be noted, however, that unfortunately this ingenious solution of the steam turbine problem (by means of "velocity compounding") cannot be adopted without sacrifice of efficiency.

The two-row and single-row wheels give reasonable maximum efficiency, but the necessary velocity ratios are relatively so high that all the available energy in high-pressure steam cannot be efficiently used in one wheel, so that the Parsons principle of "pressure compounding" has also to be employed in cases where efficiency is of importance.

Thus a turbine may be both "velocity compounded" and "pressure compounded."

The reason for the inferior efficiency attainable from "velocity compounded" wheels is that it has not been found possible to deviate a high velocity steam jet (in the manner essential to "velocity compounding") without very serious losses due to shock, friction, and eddies.

The Parsons principle of "pure pressure compounding" therefore remains the correct solution ; and it is now universally adopted.

For several years Curtis persevered with "velocity compounding" on account of the advantage it would have in dealing with large pressure drops and so rendering a turbine extremely short, but even when used in conjunction with pressure compounding (*i.e.* a series of velocity wheels separated by nozzles) the efficiency has proved no match for the Reaction type.

Rateau (in 1898), on the other hand, took up (and subsequently developed) Curtis's original adaptation of the Parsons principle, namely pure pressure compounding of the de Laval type. His efforts were attended by great success, such that the type of turbine now known universally as the "Rateau" is the only serious competitor of the pure "Reaction" type.

§ (4) The Velocity Diagram.—The fundamental difference in treatment of the steam in the two main types of pure pressure compounded turbines, namely the "Impulse" and the "Reaction" types, is clearly seen from the respective velocity diagrams (*Figs.* 10 and 11).

In the single-row Impulse wheel, the best velocity ratio is about 0·5, so that v, the jet velocity from the nozzles, is made about twice u, the mean blade velocity. Thus $OA = v$ and $AB = u$, so that

$$\text{The velocity ratio} = \frac{u}{v} = \frac{AB}{OA}.$$

OB is the relative velocity with which the steam enters the blades, and is theoretically constant as the steam passes through the curved blade passages. In practice it is reduced by frictional losses. CD, the relative velocity at exit from the blade passages, is thus somewhat less than OB. CE represents the final absolute velocity of exit of the steam from the wheel. It will be evident that the efficiency will be highest when the value of u/v and the blade exit angle are such that CE is a minimum, which means that the triangle CED will be approximately right-angled.

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[1] See "Steam Engine, Theory of," § (12). In this section heat is measured in Dynamical Units, hence J does not appear.

In reaction blading the best velocity ratio is about 0·9, so that v, the jet velocity from the guide blades, is made nearly equal to u, the mean blade velocity. Thus, as before, the velocity ratio is AB/OA, and it will be seen that the blade inlet angle should be about 90°. The relative velocity OB, however, instead of being nearly constant whilst the steam passes through the blades, increases, due to expansion of the steam in the blade passages, and as the fixed guide blades and the revolving blades are made alike, CD, the relative velocity at exit, will be equal to OA. CE is then the absolute velocity of exit, and, as before, the triangle CED will be approximately right-angled.

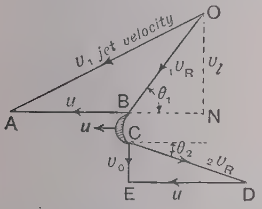

FIG. 10.—Impulse Blading.
Single-row velocity wheel (one pressure stage).

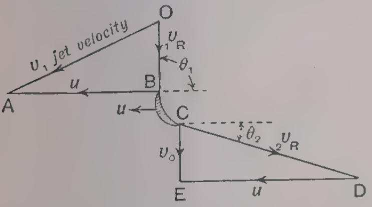

FIG. 11.—Reaction Blading.
One pair of rows (two pressure stages).

The minimum value of the velocity of exit (CE) is determined by the equation of continuity of flow, *i.e.* $QV = Av$. It will be evident that this equation determines the longitudinal velocity that must be maintained at any cross section of the turbine, in order to effect the flow of steam from inlet to exhaust.

Owing to the small drop in pressure over a row of reaction blading, a simple turbine consisting of one row of guide blades and one row of moving blades is never used in practice; but a series of such "pairs" is grouped together. The carry-over of kinetic energy from one row to another throughout the group (represented by CE in *Fig.* 11) is thus utilised in well-designed blading in every row until the final exhaust is reached, when it becomes the "leaving loss" and is a dead loss on the efficiency. It should be noted here that in badly designed blading the kinetic energy carried over from stage to stage may be partially or completely lost, or rather reconverted into heat energy, with consequent thermodynamic loss.

The same is true of impulse blading when pressure compounding is used; and what evidence there is available goes to show that the "carry in" of kinetic energy to any row from the preceding row is about balanced by the frictional losses. That is to say, the jet velocities are very nearly in agreement with the theoretical velocities to be expected from the actual pressure drops.[1]

In the reaction turbine, it will be evident that the current or blast of steam must fill up and traverse every portion of the interior, so that the best blade profile is one which offers the least resistance (as regards shock, friction, and eddying) to the passage of the steam.

The attainment of the best profile is therefore a matter for experiment, and a geometrical construction based on the velocity diagram will be of little assistance.

In the impulse turbine, on the other hand, where there is no pressure drop over the moving blades, the latter must be partially or only just full of moving steam, otherwise either spilling will take place, with consequent loss in driving torque, or if the moving streams of steam do not sufficiently fill up the blade passages in front of the nozzles there may be losses due to eddying. In this case, therefore, a geometrical construction for the blade profile and blade passages is a necessity, and suitable blades are chosen by reference to the velocity diagram, the blade lengths being proportional to the longitudinal components ON, CM, etc. (*Fig.* 12).

This velocity diagram is of use in impulse turbine design for the further reason that from it the "diagram" efficiency can be readily obtained. In the general case of a single-row velocity wheel, not working at any particular velocity ratio, the diagram will be as indicated in *Fig.* 12.

This diagram may be more conveniently drawn (especially when it is extended to multiple-row velocity wheels) as in *Fig.* 13, in which the lettering corresponds to that in *Fig.* 12.

The fundamental relation, viz. change of momentum per second (in direction of motion) is equal to the impressed force, may be written (from the diagram)

---

[1] See § (10).

$$\frac{W}{g}\bar{v}=\frac{W}{g}(\mathrm{AN}+\mathrm{ME})=\frac{W}{g}\,.\,\mathrm{MN}.$$

The "diagram efficiency" is therefore

$$\eta_d=\frac{u\,.\,(W/g)\bar{v}}{\frac{1}{2}(W/g)v_1^{\,2}}=\frac{2u\bar{v}}{v_1^{\,2}}. \qquad (5)$$

In the case of multiple-row velocity wheels, the diagram is extended by drawing the two triangles OAB and CED for each moving row of blades, and if $\bar{v}_1$, $\bar{v}_2$, $\bar{v}_3$, . . . etc. represent the change in momentum per second (in the direction of motion) per lb. of steam for each such row, the diagram efficiency is

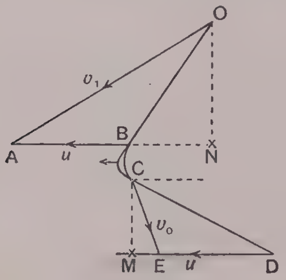

FIG. 12.

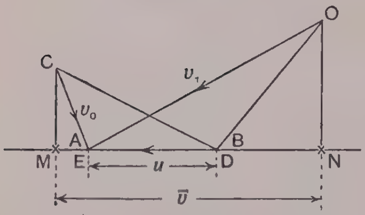

FIG. 13.

$$\eta_d=\frac{2u\Sigma\bar{v}}{v_1^{\,2}}, \qquad (6)$$

which can be compared with the experimental efficiency curves (*Fig.* 7).

In reaction blading, however, experiment has shown that the best blade profile is best at all velocity ratios up to unity, so that the same profile is always used (increased in scale as required by considerations of strength) except in the last few rows of a turbine, where an increased discharge angle is generally necessary to accommodate the low-density steam.

The velocity diagram is therefore not used, but the efficiency of a group is obtained from the experimental curve shown in *Fig.* 1.

§ (5) GENERAL REMARKS ON THE "IMPULSE" AND "REACTION" TYPES.—From the engineer's point of view, steam is not an ideal fluid for use in a turbine, owing to the fact that at high pressures the specific volume becomes very small, whilst at low pressures it becomes very large.

Thus at the high-pressure end of a turbine the difficulty is usually to make the nozzle passages small enough. This difficulty may be got over in the case of the impulse turbine by the use of "partial admission," because as there is no drop in pressure over the moving blades, it becomes possible to admit the steam through nozzles which only occupy a fraction of the periphery of the wheel, but in the reaction turbine, owing to the pressure drop over both fixed and moving blades, this method cannot be applied. The alternative device of diminishing the diameter of the blade rings at the high-pressure end is therefore adopted. The leakage over the tips of the moving blades is best limited by the use of Parsons' "End-tightened" Reaction Blading incorporating fine (adjustable) axial clearances, but very large radial clearances, which ensure perfectly safe operation (*Fig.* 14).

The impulse turbine has fewer "stages" or "simple turbines" in series than the reaction turbine because—

(*a*) For a given available heat per lb. of steam, in order to obtain the best efficiency a higher value of the "velocity ratio" must be maintained in the latter than in the former (*Fig.* 7), thus necessitating the splitting up of a given total available heat into a greater number of parts;

(*b*) The reduction in mean blade ring diameter in the early stages of a reaction turbine involves reduced mean blade speed in these stages and therefore slower rate of conversion of heat energy into mechanical work, so that more stages are required to expand the steam efficiently.

On the other hand, partial admission in the impulse turbine involves serious windage and blade "ventilation" losses, owing to the fact that the blades are only active in driving the wheel when they are passing the nozzles, so that the remainder and the disc itself have to be driven round in an atmosphere of idle steam. Such losses as these are directly proportional to the density of the steam in which the wheel revolves.

In addition, high-velocity steam jets tend to erode the surfaces upon which they impinge, so that the moderate velocities employed in reaction turbines are an advantage in this respect.

The characteristic of the compound impulse turbine (*Fig.* 15) is the splitting up of the "cylinder" or "casing" into a number of compartments, separated from each other by means of diaphragms, through the centre of which the shaft passes (with fine clearances to minimise leakage). Each diaphragm contains nozzles for the impulse wheel immediately after it.

The characteristics of the "Reaction" turbine (*Fig.* 16) are the stepped shaft, the total absence of diaphragms, and the provision of "dummy" or balancing pistons at the high-pressure end. The latter are provided in order to counterbalance the axial thrust (in the direction of the exhaust) due to the difference in the steam pressure over the faces of each moving blade ring, and to the steam pressure on the shoulders of the shaft where it is stepped. A balancing or "equalising" pipe connects each dummy piston to the corresponding shoulder on the shaft, so that the steam pressures are automatically balanced at all loads on the turbine.

More recently (1912) the Swedish engineer, Birger Ljungström, has developed a compound turbine on the outward radial flow principle, and with double rotation in which both nozzles and blades revolve, but in opposite directions. This arrangement involves two driving shafts and two generators, but the efficiency attainable is remarkably high. The turbine is of the pure reaction type.

Owing to the great complexity of the questions involved in successful steam turbine manufacture, many builders compromise by combining the leading principles described above. Thus reaction blading preceded at the high-pressure end by a velocity wheel finds considerable favour on the Continent, whilst the velocity wheel has also been extensively used to precede impulse blading of the "Rateau" type (*Fig.* 15). The present-day tendency, however, appears to be towards a strict adherence to either the pure reaction type as chiefly represented by the Parsons and the Ljungström, or to the pure impulse type as developed mainly by Rateau.

Having thus shortly reviewed the physical considerations which have led to the evolution of the steam turbine into the main types described above, a brief indication of the mathematical treatment of the physical problems involved will next be given.

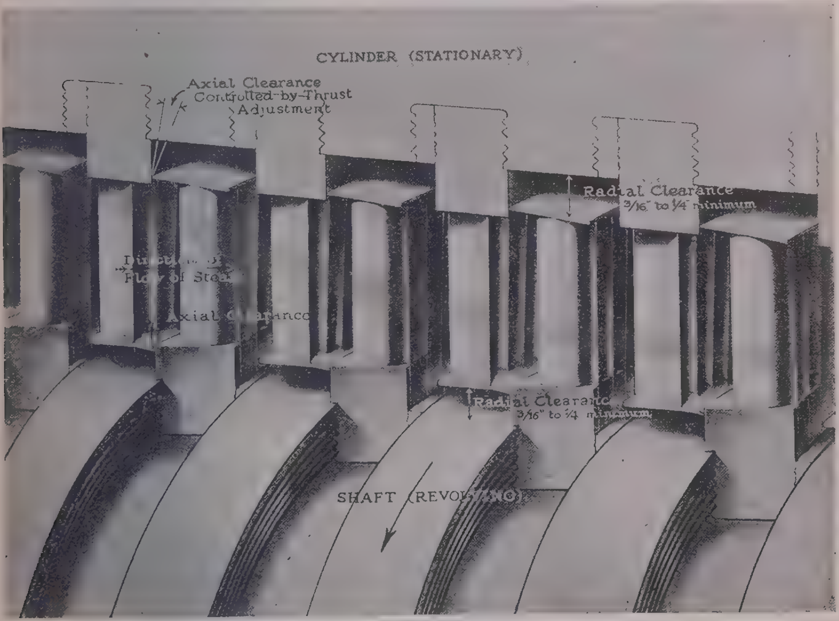
CYLINDER (STATIONARY)
Axial Clearance Controlled by Thrust Adjustment
Radial Clearance 3/16" to 1/4" minimum
Direction of Flow of Steam
Axial Clearance
Radial Clearance 3/16" to 1/4" minimum
SHAFT (REVOLVING)

FIG. 14.—Perspective view of end-tightened reaction blading in cylinder and shaft, showing large radial and fine axial clearances.

§ (6) THERMODYNAMICS OF THE TURBINE.—A discussion of the thermodynamics of the turbine, with an account of the action of the nozzles, will be found in the articles on "Thermodynamics" and "Steam Engine, Theory of."[1]

It is shown there that if η_s be the stage efficiency in any stage of a compound turbine, dI_a the energy available for external work, P the pressure, and V the volume of the unit of mass of steam, then

$$dI_a = \eta_s V dP. \quad . \quad . \quad . \quad (7)$$

[1] See "Steam Engine, Theory of," § (12).

1. Cast steel steam inlet belt.
2. Steam supply pipe to carbon segment gland.
3. Drain from carbon segment gland.
4. Oil supply pipe to No. 1 bearing and thrust block.
5. Steam end pedestal.
6. Emergency release to oil supply to oil relay governor.
7. Shaft runaway governor operating 6.
8. Bearing.
9. Worm driving speed-governor, tachometer, and oil pumps.
10. Thrust bearing (multi-collar type).
11. No. 1 main bearing.
12. Oil baffles.
13. Carbon segment portion of H.P. gland, sealing shaft against atmosphere.
14. Leak off to condenser from "labyrinth" portion of H.P. gland 15.
15. "Labyrinth" portion of H.P. gland.
16. Gland vapour pipe.
17. Primary steam nozzles to velocity wheel.
18. Two-row velocity wheel.
19. First diaphragm.
20. First single-row impulse wheel.
21. Last diaphragm.
22. Last single-row impulse wheel.
23. Carbon segment L.P. gland, sealing shaft against atmosphere.
24. Gland vapour pipe.
25. Oil baffles.
26. No. 2 main bearing.
27. Rigid coupling.
28. No. 3 main bearing.
29. Oil supply pipe to bearings 26 and 28.
30. Drains from carbon segment L.P. gland.
31. Turbine exhaust.
32. Steam supply pipe to carbon segment L.P. gland.
33. Cylinder cast iron casing.
34. Cylinder cleading.
35. Diaphragm "labyrinth" gland.
36. Turbine shaft.

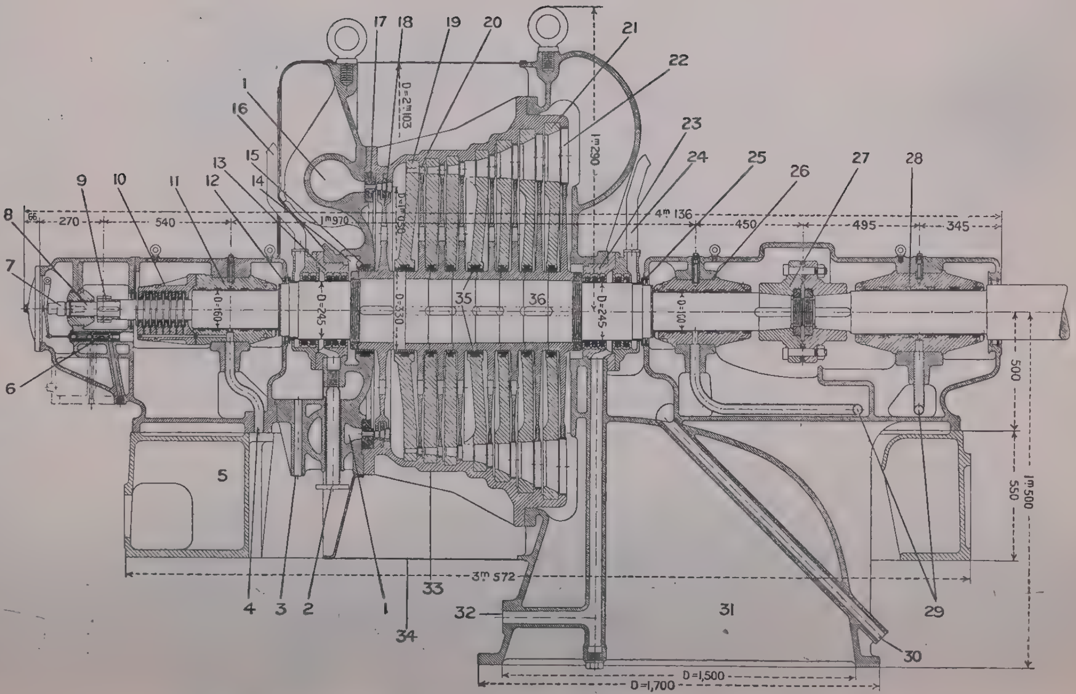

FIG. 15.—6000 kw. 3000 r.p.m. Compound Impulse Steam Turbine (Société Rateau).

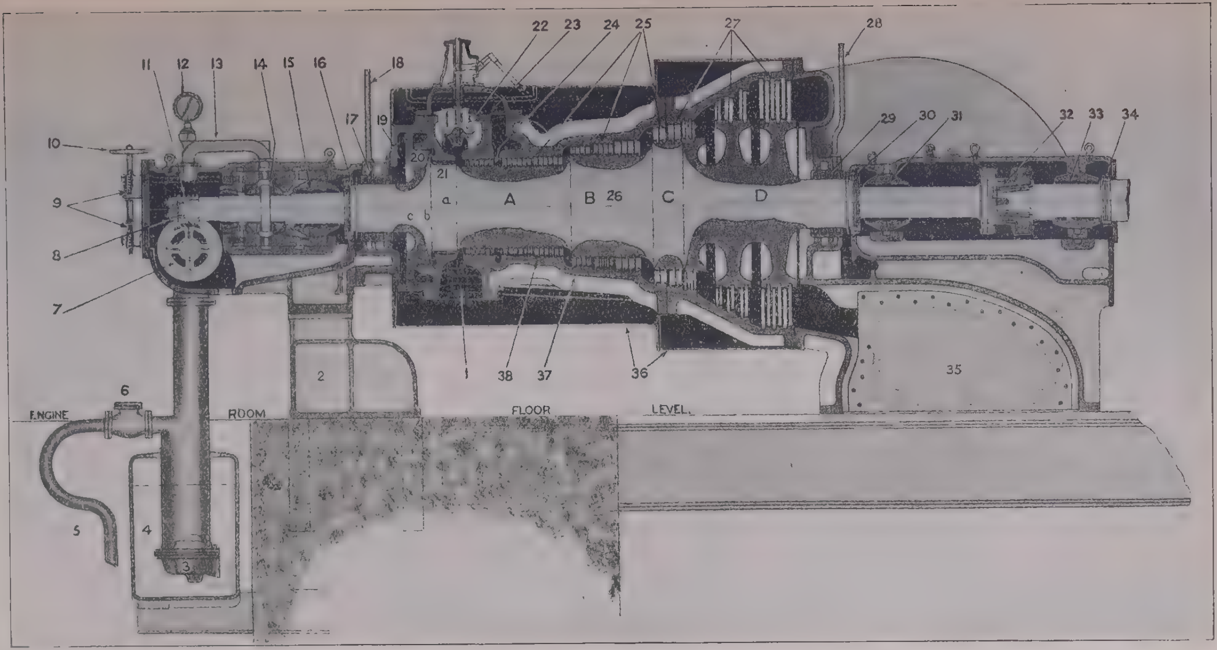

FIG. 16.—Parsons' Compound Reaction Steam Turbine.—6000 kw., 3000 r.p.m.

1. Main steam inlet.
2. Steam end pedestal.
3. Oil pump.
4. Oil tank.
5. Oil delivery pipe.
6. Non-return valve.
7. Worm wheel.
8. Shaft runaway governor.
9. Thrust block adjusting gear
10. Hand-wheel for No. 9.
11. Worm.
12. Tachometer.
13. Oil outlet from thrust bearing.
14. Thrust bearing collar.
15. No. 1 main bearing.
16. Oil baffles.
17. Carbon segment gland.
18. Gland vapour pipe.
19. No. 3 dummy piston.
20. No. 2 dummy piston.
21. No. 1 dummy piston.
22. Overload by-pass double beat valve.
23. Equalising pipe (dotted).
24. Overload by-pass inlet belt.
25. Reaction blading (end tightened).
26. Turbine shaft.
27. Reaction blading (ordinary radial clearance).
28. Gland vapour pipe.
29. Carbon segment gland.
30. Oil baffles.
31. No. 2 main bearing.
32. Flexible coupling.
33. No. 3 main bearing.
34. Oil baffles.
35. Turbine exhaust.
36. Cylinder cleading.
37. Equalising pipe (dotted).
38. Cylinder cast steel centre.

(i.) *Superheated Steam.*—If the steam be superheated and we treat it as a gas of specific heat K_p at constant pressure and K_v at constant volume, then for adiabatic expansion

$$dI_a = K_p dT = \frac{K_p}{K_p - K_v} d(PV).$$

Hence from equation (7) with stage efficiency η_s,

$$K_p P dV + \{K_p - \eta_s(K_p - K_v)\} V dP = 0,$$

and therefore $PV^\lambda = \text{constant}$, ... (8)

where $$\lambda = \frac{K_p}{K_p - \eta_s(K_p - K_v)}. \quad . \quad . \quad (9)$$

If η_s is unity then this expression reduces to K_p/K_v or γ, and we arrive at the ordinary isentropic law.

From the above an expression can easily be found for the energy available for adiabatic expansion.[1]

(ii.) *Saturated Steam.*—In the case of saturated steam, assuming the specific heat of water as unity, then

$$dI_a = dT + d(qL), \quad . \quad . \quad (10)$$

where L is the latent heat at temperature T and q the dryness factor, and for adiabatic expansion with stage efficiency η_s

$$dI_a = \eta_s V dP = \eta_s q L \frac{dT}{T}, \quad . \quad . \quad (11)$$

$$\therefore dT + d(qL) = \eta_s q L \,.\, \frac{dT}{T},$$

or, $$d\left(\frac{qL}{T^{\eta_s}}\right) + \frac{dT}{T^{\eta_s}} = 0,$$

$$\therefore \frac{qL}{T^{\eta_s}} + \frac{T^{1-\eta_s}}{1-\eta_s} = \text{constant}. \quad . \quad (12)$$

From these equations can be found expressions for the energy available for work, which is equal to the heat taken in minus the heat rejected.

§ (7) THE REHEAT FACTOR.—From these expressions we can calculate the ratio of the energy actually available to that which would be available were the isentropic law followed.

This ratio has been defined as the reheat factor.[2] Calculated values have been tabulated by H. M. Martin in his article "A New Theory of the Steam Turbine."[3]

It appears from Tables IV. and V. that the ratio is greater than unity, and also that the values for superheated steam are greater than those for saturated steam.

The discussion in the article "Steam Engine, Theory of," just referred to brings out the reasons for this, and should be consulted for further information.

TABLE IV

REHEAT FACTOR FOR STEAM SUPERHEATED THROUGHOUT THE WHOLE RANGE OF ITS EXPANSION

Values of $X = \frac{P_1}{P}$.	Hydraulic Efficiency $\eta_s =$			
	0·5.	0·6.	0·7.	0·8.
1	1·0000	1·0000	1·0000	1·0000
2	1·0393	1·0312	1·0235	1·0160
4	1·0753	1·0629	1·0468	1·0310
6	1·1033	1·0809	1·0602	1·0397
8	1·1195	1·0934	1·0695	1·0454
10	1·1310	1·1024	1·0762	1·0494
15	1·1554	1·1209	1·0891	1·0585
20	1·1691	1·1313	1·0964	1·0617
25	1·1841	1·1445	1·1057	1·0692
50	1·2219	1·1718	1·1253	1·0810
100	1·2586	1·1998	1·1450	1·0932
200	1·2962	1·2279	1·1643	1·1053

TABLE V

REHEAT FACTORS FOR STEAM INITIALLY IN THE DRY BUT SATURATED CONDITION, AND EXPANDED FROM DIFFERENT INITIAL PRESSURES DOWN TO 1 LB. ABSOLUTE, THERMAL EQUILIBRIUM BEING MAINTAINED THROUGHOUT THE EXPANSION

Abs. Initial Pressure, lb. per sq. in.	Abs. Final Pressure, lb. per sq. in.	Hydraulic Efficiency η_s.				
		0·5.	0·6.	0·7.	0·8.	0·9.
2	1	1·0085	1·0078	1·0065	1·0046	1·0022
4	1	1·0191	1·0163	1·0129	1·0089	1·0043
6	1	1·0284	1·0217	1·0169	1·0114	1·0056
8	1	1·0316	1·0256	1·0195	1·0130	1·0062
10	1	1·0355	1·0290	1·0221	1·0148	1·0071
15	1	1·0435	1·0348	1·0264	1·0181	1·0078
20	1	1·0496	1·0394	1·0294	1·0191	1·0088
25	1	1·0537	1·0427	1·0318	1·0210	1·0104
50	1	1·0669	1·0531	1·0394	1·0261	1·0129
100	1	1·0809	1·0640	1·0475	1·0313	1·0155
200	1	1·0956	1·0755	1·0559	1·0368	1·0162

§ (8) STEAM TURBINE DESIGN.—In any engine design it is necessary so to proportion the scheme that it shall lead to the desired output at reasonable efficiency. The output depends primarily on the weight of fluid which will pass in a given time; the efficiency on the arrangements under which the heat is transmitted through the machine. Thus in a turbine the blading must be (*a*) of such dimensions as to transmit the necessary weight of steam, and (*b*) of such form and arrangement as to expand the steam and do this efficiently.

We need, then, to consider the dimensions and form of the nozzles or passages through which the steam has to pass.

§ (9) STEAM NOZZLE CAPACITY.—The theory of the nozzle discharge has been dealt with in the article[4] on "The Steam Engine," and in considering the capacity of the blading the results of that article should be made use of. It is there shown that if an expanding fluid start from rest at a pressure P_1 and attain a velocity v and a pressure P—assuming frictionless flow and no loss of heat, then writing Z for the ratio P/P_1, and assuming that under the conditions of flow PV^λ is constant, V being the volume of unit mass at pressure P,

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[1] See "Thermodynamics," § (5).

[2] "Steam Engine, Theory of," § (11).

[3] See *Engineering*, July 1918 *et seq.*, cvi.

[4] See "Steam Engine, Theory of," § (12).

$$v=\sqrt{\frac{2g\lambda}{\lambda-1}(1-Z^{(\lambda-1)/\lambda})P_1V_1}, \quad . \quad (13)$$

and $$V=\frac{V_1}{Z^{1/\lambda}}. \quad . \quad . \quad . \quad (14)$$

As the steam traverses the nozzle, Z decreases and v and V both increase.

Again, representing by A the area of the cross-section of the nozzle and Q the quantity of fluid crossing that section per second—this at any time is the same for all the sections—we have

$$QV=\text{volume crossing}=vA.$$

Hence $$A=\frac{QV}{v}. \quad . \quad . \quad . \quad (15)$$

It can be shown that as Z decreases A at first decreases and then increases. The proper form of nozzle depends, then, on the final value of the pressure ratio Z. If this be small the nozzle will have a throat whose position is determined by finding the value of Z which makes Q/A a minimum. The value of Q/A is given by the equation

$$\frac{Q}{A}=\frac{Z^{1/\lambda}}{V_1}\sqrt{\frac{2g\lambda}{\lambda-1}(1-Z^{(\lambda-1)/\lambda})P_1V_1} \quad (16)$$

and from this we find for the pressure ratio at the throat

$$Z_t=\left(\frac{2}{\lambda+1}\right)^{\lambda/(\lambda-1)} \quad . \quad . \quad . \quad (17)$$

Hence we obtain

$$P_t=Z_tP_1=\left(\frac{2}{\lambda+1}\right)^{\lambda/(\lambda-1)}P_1, \quad . \quad (18)$$

$$V_t=\frac{V_1}{Z_t^{1/\lambda}}=V_1\left(\frac{\lambda+1}{2}\right)^{1/(\lambda-1)}, \quad . \quad (19)$$

$$v_t=\sqrt{\frac{2g\lambda P_1V_1}{\lambda+1}}, \quad . \quad . \quad (20)$$

$$\frac{Q}{A_t}=\frac{v_t}{V_t}=\left(\frac{2}{\lambda+1}\right)^{1/(\lambda-1)}\times\sqrt{\frac{2g\lambda P_1}{(\lambda+1)V_1}}, \quad (21)$$

while the area A at any place where the pressure ratio is Z is given by

$$\frac{A^2}{A_t^2}=\left(\frac{Z_t}{Z}\right)^{2/\lambda}\left(\frac{1-Z_t^{(\lambda-1)/\lambda}}{1-Z^{(\lambda-1)/\lambda}}\right). \quad . \quad (22)$$

The reason for the contraction of the nozzle followed by its expansion can be seen from the following. For the reasons given in § (4), the velocity of entry to each ring of nozzles may be neglected for the purpose of calculation of the subsequent exit velocity. Thus we have[1]

$$\frac{v^2}{2g}=\int V dP=\int\frac{dP}{\rho},$$

where ρ is the density and is equal to 1/V. Thus

$$vdv=g\frac{dP}{\rho}=g\frac{dP}{d\rho}\frac{d\rho}{\rho}$$

$$=-c^2\frac{dV}{V},$$

if $c^2=g(dP/d\rho)$ = velocity of sound in the fluid. Thus

$$\frac{v^2}{c^2}=-\frac{dV}{V}\Big/\frac{dv}{v}$$

$$=\frac{\text{Rate of increase of volume}}{\text{Rate of increase of velocity}}. \quad . \quad (23)$$

Hence if v is less than c the velocity increases faster than the volume increases, and the nozzle must contract in order to maintain the continuity of the flow, while if v becomes greater than c the reverse takes place.

One very important result from these formulae is that the discharge through a given orifice under an initial pressure P_1 depends only on the cross-section at the throat and is independent of the back pressure, provided it is not greater than Z_tP_1; by continuing the orifice beyond the throat and lowering the back pressure, the velocity of the issuing fluid is increased, but not the quantity which issues.

§ (10) STEAM CAPACITY OF "REACTION" BLADING.—If we remember that the velocity-energy at entry to the blading is practically equivalent to the energy losses in the latter, so that the jet velocity from one ring of blades depends almost solely on the pressure drop over it, an application of the above formulae, or the direct use of the fundamental equations

$$\frac{v^2}{2g}=VdP,$$

for a *small* drop in pressure and

$$Av=QV,$$

enables the steam capacity in "Reaction" turbines, where the pressure drop per stage is small, to be calculated in terms of the dimensions and initial pressures, regard being had, of course, to the units in which the quantities are measured, for from the above we find

$$PdP=\frac{1}{2g}\frac{Q^2}{A^2}\cdot PV,$$

and Q is given by this equation, which reduces to

$$\frac{Q}{A}=\sqrt{\left\{2g\frac{PdP}{H}\right\}}. \quad . \quad . \quad (24)$$

H being the homogeneous head, for a small but finite drop in pressure.

In practice a number of assumptions are usually made which simplify the calculations, *e.g.* PV is taken as constant over any one stage and equal to the homogeneous head, while PdP is written

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[1] See "Steam Engine, Theory of," § (12).

$$\left(\frac{P_1+P_2}{2}\right)(P_1-P_2) \text{ or } \tfrac{1}{2}(P_1{}^2-P_2{}^2),$$

P_1 and P_2 being pressures at entrance and exit, the difference being small in "Reaction" blading.

§ (11) Equivalent Turbines and Efficiency—Parsons' Coefficient.—The provision of steam capacity in a turbine having been briefly discussed, the second problem—that of efficiency—must now be considered.

The earliest Parsons compound turbines which worked non-condensing were made with almost parallel drums of a single diameter, so that the mean diameter was nearly constant throughout. For the velocity ratio to be constant, therefore, a constant steam-jet velocity was required at every row of blades. The fundamental equation $v^2/2g=VdP$ may be written

$$\frac{v^2}{2g}=H\cdot\frac{dP}{P}, \qquad (25)$$

and shows that for v to be constant at every stage the blade discharge area of each stage must be made $(P+dP)/P$ times that of the preceding stage, assuming, as an approximation, that H is constant. Since the blade discharge area is directly proportional to the blade-length, it follows that the curve of blade-lengths from stage to stage will be approximately logarithmic, so that when low-pressure steam is to be used the blade-lengths on the assumed shaft diameter will soon become impossible of realisation in practice. In other words, it is not, in reaction turbine design, possible to choose a single diameter of shaft which will allow of reasonably large blade-lengths at the high-pressure end and at the same time reasonably small blade-lengths at the low-pressure end.

Parsons therefore introduced the device of stepping the shaft, and placing the blading on successively larger diameters (ABCD, *Fig.* 16), so as to accommodate the ever-increasing volume of the steam. This artifice very greatly reduces the necessary blade-lengths, because—

(1) Doubling the mean diameter doubles the discharge area of a row of blades of given length;

(2) Doubling the mean diameter doubles the mean blade speed, so that double the steam-jet velocity is required in order to maintain the same velocity ratio.

Thus the blade-lengths are inversely as the squares of the mean diameters.

This departure from constant or nearly constant mean diameter, however, soon made it evident that to facilitate calculation some rapid method of comparison of the value of groups of simple turbines on different diameters must be devised. Parsons showed that in a pure pressure compounded turbine, if a given amount of available heat I_A is divided up equally over N pressure stages, then from first principles the velocity ratio

$$=\frac{u}{v}=\frac{\pi dR/12\times 60}{\sqrt{2gJ}\;\sqrt{I_A/N}}$$

$$=b\sqrt{\frac{d^2R^2N}{I_A}}, \qquad (26)$$

a relation which he afterwards extended to the case of a pressure compounded turbine which is also velocity compounded. It will be recalled (§ (3)) that in the case of velocity wheels, their equivalence as regards capacity for utilising heat when operating at similar points on their respective efficiency curves varies as the square of the number of velocity stages (n).

The velocity ratio is therefore given by the general expression

$$\frac{u}{v}=\frac{0{\cdot}62}{n}\sqrt{\frac{d^2R^2n^2N}{10^9I_A}} \qquad (27)$$

(using inches and revolutions per minute), which is the general relation connecting mean diameter, number of pressure stages, number of velocity stages, and the revolutions of a turbine, with the velocity ratio attainable with a given total heat drop.

The expression $(d^2R^2n^2N)10^{-9}$, or more simply, in the case of pure pressure compounded turbines, $(d^2R^2N)10^{-9}$, has always been denoted by "K," and is universally known as "the Parsons coefficient." It enables instant comparison to be made between different blading arrangements. For example, suppose that two 3-row Curtis velocity wheels, each of 36″ mean diameter, are operated at the theoretically correct velocity ratio for maximum efficiency, *i.e.* 1/6 when the revolutions per minute are 3000. How many single-row velocity wheels of 47″ mean diameter must be substituted, in order that the latter may operate (with the same total heat drop as before) at their theoretical velocity ratio for maximum efficiency, *i.e.* 1/2 when the revolutions per minute are 2400?

If x be the number of single-row velocity wheels required, then

$$2(36\times 3000\times 3)^2=x(47\times 2400\times 1)^2,$$

so that $x=16\tfrac{1}{2}$; that is, about 16 wheels.

Again, suppose it is required to ascertain how many "pairs" of reaction blading on 24″ mean diameter rotating at 3600 r.p.m. are equivalent to the above.

It must here be noted that in equation (26) N refers to the number of pressure stages in a turbine, which in the case of impulse turbines is identical with the number of rings of nozzles.

In reaction turbines, however, since there is a drop in pressure over the moving rows of blades as well as over the rows of fixed guide blades (the division of the heat drop per stage being made about equal), it follows that if there are N rows of fixed guides (or "nozzles") there are 2N pressure stages in the turbine. Relation (26) therefore becomes

$$\frac{u}{v}=0{\cdot}62\sqrt{2}\sqrt{\frac{d^2\mathrm{R}^2\mathrm{N}}{10^9\mathrm{I}_\mathrm{A}}}=0{\cdot}87\sqrt{\frac{\mathrm{K}}{\mathrm{I}_\mathrm{A}}}. \quad (28)$$

In this last case, therefore,

$$(47\times2400\times1)^2\times16\tfrac{1}{2}=\{(24\times3600\times1)^2y\}\times\tfrac{1}{2},$$

or $y=56$ reaction "pairs" or 112 rows of blades.

In the general case, therefore, of a turbine consisting of two or more shafts, rotating at different revolutions, then with any arrangement of blading consisting of a series of wheels of different diameters, velocity compounded or not, if the expression

$$\left(\frac{d^2\mathrm{R}^2n^2\mathrm{N}}{10^9}\right)=\mathrm{K}\ . \quad . \quad . \quad (29)$$

has a certain value, then any other blading arrangement giving the same value of K is equivalent to it.

(In the case of reaction blading, the above special modification must, of course, be observed.)

For example, suppose in a pure pressure compounded turbine the total K is 180, and the steam conditions are such that 380 B.Th.Units/lb. are theoretically available for conversion into work, the mean over-all velocity ratio is

$$\bar{a}=0{\cdot}62\sqrt{\frac{180}{380}}=0{\cdot}45,$$

which is about the peak of the curve of efficiency for single-row velocity wheels.

Further, if a velocity compounded turbine (also pressure compounded) with 3-row wheels has a total K of 180 and operates with the same heat drop,

$$\bar{a}=\frac{0{\cdot}62}{3}\sqrt{\frac{180}{380}}=0{\cdot}15,$$

which is about the peak of the curve of efficiency for 3-row velocity wheels.

Lastly, in a reaction turbine, if the total K is $(2\times180)=360$, then with the same available heat

$$\bar{a}=0{\cdot}87\sqrt{\frac{360}{380}}=0{\cdot}85,$$

which is about the peak of the efficiency curve for reaction blading.

The general case, however, is not of the same importance in modern practice as the special case of pure pressure compounding only, for two reasons:

(1) Although a group of 3-row velocity wheels, for instance, may be operating at a point on the corresponding efficiency curve which is similarly situated to the point at which a group of single-row velocity wheels are operating on their efficiency curve, the actual efficiency will be very different (*Fig.* 7), so that equivalent "K" does not mean equal efficiency.

(2) If equivalent "K" were the condition of equal efficiency, then there would be no field for any turbine but the multiple-row single velocity-wheel type (*i.e.* without pressure compounding at all), because three or four rows only would suffice to deal with the entire heat drop, so that for simplicity and shortness this design would be unrivalled.

§ (12) COMPARISON OF TURBINE PERFORMANCES.—This method of comparison of the value of different types of blading, or of different arrangements of blading of any one type, by means of the coefficient "K" enables the data obtained from actual tests of turbines *designed for* different conditions to be marshalled in such a way that they can be compared on a rational basis, and further that the performance of a projected design can be predicted with considerable accuracy without a detailed knowledge of the losses.

From actual test data, coefficients have been arrived at giving the difference in steam consumption obtained with different turbines *designed for*, respectively, different steam pressure, different initial superheat, and different exhaust vacuum.

When the actual steam consumptions of different turbines are corrected to an arbitrarily assumed standard set of steam conditions in this manner, the over-all efficiency ratios calculated therefrom and plotted on a base of total "K" will lie with reasonable accuracy on a curve which is really of similar character to those in *Figs.* 1 and 7.

The curve, having once been established for a given set of steam conditions, can then be used, in conjunction with the correction curves for pressure, superheat, and vacuum, to compare actual performances of past turbines, and to predict the performances of new turbines when *designed for* any other given set of steam conditions and tested under these conditions.

Such a method of comparison, whilst convenient for rapid estimation, does not, of course, render detailed analysis of the losses entirely unnecessary when designing a turbine to meet a rigorous specification, on account of the fact that the steam conditions and other requirements to be met in turbine construction are so widely different that special allowances have to be made in many cases.

For example, it is not possible to compare accurately the performance of a very small turbine with that of a large one by the coefficient method alone, owing to the fact that the losses in a small machine must necessarily be proportionately very much greater than in a large one. Again, in very large turbines, where the maximum output attainable at a given speed and with a given vacuum is required, the "leaving loss," or loss by kinetic energy of the steam leaving the turbine and entering the exhaust, may be very abnormal, a circumstance which will considerably diminish the over-all efficiency of what might otherwise have been a very efficient machine.

§ (13) CALCULATION OF LEAVING LOSS.—Suppose, for instance, that a wheel of mean diameter d has a row of blades of effective length h mounted round its periphery. Let

$$h=ad \text{ or } \frac{h}{d}=a=\text{a constant.}$$

Using the equation

$$QV=Av,$$

where $A=kh\pi d$, the equation

$$Q=\frac{\pi kad^2v}{V} \quad . \quad . \quad . \quad (30)$$

is obtained. A common value of a for the last row of blades in a turbine is $\frac{1}{5}$, so that

$$v=\frac{5QV}{\pi d^2k}\times\frac{144}{3600}=\frac{QV}{5\pi d^2k}\,\frac{\text{feet}}{\text{sec.}} \quad . \quad (31)$$

=the relative velocity of the steam through the last row of moving blades.

From a diagram of velocities (as in *Fig.* 17) the absolute velocity of the steam v_0 as it enters the exhaust passages can be determined. This velocity energy is a dead loss on the available energy of expansion for the whole turbine, and is known as the "leaving loss." If I_a is the heat drop measured in heat units required to produce this velocity, then

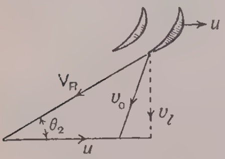

FIG. 17.

$$JI_a=\frac{v^2}{2g}\,\frac{\text{B.Th.U.}}{\text{lb.}},$$

so that

$$\frac{I_a}{I_A}\times 100=\text{the percentage loss in efficiency for the whole turbine.}$$

It is not usual, in high-pressure condensing turbines, to permit I_a to exceed 5 per cent of I_A, and this figure is only tolerated in extreme cases when the exhaust vacuum is very high, and the maximum possible output from a given frame is required.

§ (14) THE ADVANTAGES OBTAINABLE FROM INCREASED VACUUM.—If I_{t0} is the total heat above vacuum temperature (or, more strictly, above condensate temperature t_0), and S_0 is the steam consumption in lbs. per kilowatt-hour, the thermodynamic efficiency of the turbine is

$$\frac{3412}{S_0\times I_{t0}}\times 100 \text{ per cent.} \quad . \quad . \quad (32)$$

If the vacuum be increased, or, in other words, if the condensate temperature be lowered to t_1, the improvement due to the decrease in steam consumption will be to a certain extent discounted by the increased total heat I_{t0}, which has now to be supplied by the boiler plant. In this case the thermodynamic efficiency of the turbine is

$$\frac{3412}{S_1\times I_{t1}}\times 100 \text{ per cent.} \quad . \quad . \quad (33)$$

If, however (as in modern methods of feed-water heating now usually adopted), steam at pressures considerably below atmospheric is led from the turbine and used for heating the feed-water, it is possible to keep t constant, in which case the improvement in heat consumption is identical with the improvement in steam consumption due to higher vacuum.

Assuming that there are about 1000 B.Th. Units remaining in each lb. of exhaust steam, it will be clear that if 1 per cent of the total steam entering a turbine is tapped off at an advanced stage of expansion (so as to minimise the work lost by tapping off), this will be sufficient to raise the temperature of the main condensate by about 10° F. The temperature of the exhaust steam can therefore be lowered by 10° F.—that is, a higher vacuum can be utilised—without altering H_t, which has to be supplied by the boiler plant.

If the turbine has N stages of equal output, and the steam is tapped off in front of the (N − 1)th stage, then the loss of output will be only (2/N) × 1 per cent, whilst the gain due to the raising of the vacuum will be seen from Table VI.,[1] which has been drawn up on the assumption of the following initial steam conditions:

Gauge pressure . . .	300 lbs./sq. in.
Total temperature . .	688° F.
Initial superheat . .	265·8° F.

TABLE VI

EFFECT OF EXHAUST VACUUM AND ECONOMY

Absolute Exhaust Pressure. Inches Hg.	Improvement in Steam Consumption in per cent of that at 2·0″ Hg. Exhaust Vacuum.	Temperature exhaust, ° F.
2·0	0	101·11
1·5	3·32	91·67
1·0	7·90	78·96
0·9	9·15	75·77
0·75	11·15	70·33
0·6	13·45	63·86
0·5	15·35	58·70

This table shows, for example, that a drop from 101·1° F. to 91·67° F., which corresponds to an increase in vacuum from 28″ mercury to 28½″ mercury (barometer 30″), improves the steam consumption of the turbine by 3·3 per cent.

The table also shows the great importance of high vacuum—in other words, reduced exhaust temperature—in effecting steam economy, and especially if the feed-water be heated by means of low-pressure steam tapped off from the turbine.

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[1] *Journal I.E.E.* cccii. 586.

The extremely low density of steam at low pressures, however, is a circumstance inconsistent with the requirement of very large output from a given frame, on account of the leaving loss. Compromise has therefore to be resorted to in such cases, which are becoming numerically greater and of increasing importance in modern land Power Station work.

§ (15) MAXIMUM OUTPUT FROM A GIVEN FRAME.—The stresses f in a given disc, due to its own rotation, are proportional to the square of the product of the angular velocity ω and peripheral diameter, and if the blade height bears a constant ratio to the mean diameter, then $f \propto \omega^2 d^2$.

The additional stresses caused by a peripheral load such as a row of blades of given height-ratio are proportional to the same product $\omega^2 d^2$. Thus, if λ is the factor of safety allowed for the material used, and F is the ultimate strength of that material, then

$$F = \lambda\sigma\omega^2 d^2, \quad . \quad . \quad . \quad (34)$$

σ being a constant. Substituting for d^2 in equation (30), it is seen that

$$Q = \left(\frac{\pi k a v}{V\lambda\sigma}\right)\frac{F}{\omega^2} = c\frac{F}{\omega^2} \text{ for a given case.} \quad (35)$$

The maximum output obtainable under any given steam conditions, therefore, depends on the ultimate strength of the material used, and on the angular velocity or number of revolutions per second.

It will be seen from equation (35) that, other things being equal, the dominating factor affecting output is the exhaust vacuum, because the "leaving loss" depends upon the total volume of steam (QV) forced through the last row of blades per unit time. Since the total weight of steam passed per unit time is inversely as the density, it follows that for a given leaving loss high vacuum reduces total output. On the other hand, high vacuum means greater available heat and therefore —up to a certain point—reduced steam consumption. For a given turbine frame and given vacuum, in other words, the output should not be made to exceed the limit where the benefit of that vacuum is wiped out by the leaving loss, in which case the steam consumption will be practically no better than that which would be obtained with a lower vacuum.

When the exhaust vacuum of a turbine is progressively raised — that is, when the absolute exhaust pressure is progressively lowered—the steam consumption will fall as long as the leaving loss is not excessive, but a point is reached where the gain due to further reduction in exhaust pressure is inappreciable, and the steam consumption curve becomes asymptotic with regard to the x axis.

Assuming a given leaving loss, however, it is a simple matter in any given case to work backwards and find out approximately what value of Q—in other words, what output—gives this percentage loss on the efficiency ratio. An approximation to the probable efficiency ratio e of the turbine will have been determined by the coefficient method just described, so that the probable steam consumption in lbs. per kilowatt-hour will be known, and this divided into Q gives the total output in kilowatts.

Thus,

1 kilowatt-hour

$$= 3412 \text{ B.Th. Units,}$$

$$\frac{3412}{I_A} = \text{the theoretical steam consumption of the turbine in lbs. per kilowatt-hour,}$$

$$\frac{3412}{e\,.\,I_A} = \text{the probable actual steam consumption,}$$

whilst

$$\frac{QeI_A}{3412} = \text{the total output in kilowatts.}$$

§ (16) TURBINE LOSSES.—In calculating the probable efficiency of a turbine at the coupling, the losses which reduce the inherent blading efficiency or "curve" efficiency (*Fig.* 7) in a given case, may be divided into three sections :

(*a*) (1) Losses in the steam chest due to throttling and wire-drawing ; (2) leaving loss from the final stage of the blading ; (3) losses in the exhaust pipe between the turbine and the condenser.

(*b*) (1) Leakage losses through diaphragms (in impulse turbines) and over the blade and dummy piston clearances (in reaction turbines); (2) disc friction and blade ventilation losses (in impulse turbines) ; (3) losses due to the fact that some of the stages are operated by wet steam.

(*c*) (1) Bearing losses ; (2) gland losses ; (3) power required to drive the governor and oil pump.

The losses (*a*) vary as the square of the respective steam velocities, and may therefore be expressed as a fraction of the total isentropic heat-drop I_A.

The losses (*b*) depend upon the steam conditions for a given case, and render what may be termed the "indicated" blading efficiency lower than the "curve" value (*Fig.* 7) for the particular over-all velocity ratio at which the turbine is working. They may be expressed as fractions of the "curve" efficiency E.

Losses (*c*) are independent of the steam conditions or the output of the turbine and are constant for a given frame. They may be termed the "mechanical" losses, and expressed as a fraction of the output at the coupling (L).

Thus if E is the "curve" efficiency and δ is a fraction representing the total correction for losses (*b*) and the gain due to reheating,[1] then the "indicated" efficiency may be expressed as

$$E_1 = E(1-\delta). \quad . \quad . \quad . \quad (36)$$

If there were no further losses, the turbine output in kilowatts would be

$$L_0 = \frac{QE_1I_A}{3412}. \quad . \quad . \quad . \quad (37)$$

With losses (*a*) (designated by *l*, the fraction of the isentropic heat drop I_A) and with mechanical losses (*c*) (designated by λ = the fraction of the effective output L) the effective output (L) at the coupling becomes

$$L = L_0(1-l) - \lambda L$$

$$= \frac{L_0(1-l)}{1+\lambda} = \frac{QE_1I_A}{3412}\left(\frac{1-l}{1+\lambda}\right)$$

$$= \left\{\frac{(1-\delta)(1-l)}{1+\lambda} \cdot \frac{I_A}{3412}\right\} . Q . E \quad (38)$$

= (a coefficient × total steam used per unit time × "curve" efficiency).

Conversely, taking actual test results of different turbines, the calculation may be reversed to determine E, the inherent blading efficiency. *Fig.* 18 shows the result of some calculations of this kind based on actual test data for reaction turbines.

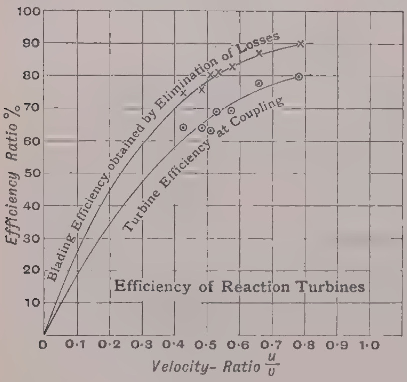

FIG. 18.

§ (17) THE APPLICATION OF THE STEAM TURBINE FOR LAND PURPOSES AND FOR MARINE PROPULSION.—The steam turbine is widely used for driving blowers, compressors, exhausters, and fans in mines, collieries, iron-works, gas-works, etc. The similarity of physical properties of the steam driving the turbine and the gas being compressed enables the turbine and generator to be coupled direct, whilst any speed suitable for the generator is suitable for the turbine.

These two conditions, however, namely, (1) free choice of speed, (2) compatibility of turbine speed, are not simultaneously fulfilled in other very important applications of the turbine, as will be realised from the following considerations:

The principal use of the steam turbine for "stationary" work is in the field of electric power generation, where it is now almost universally adopted for driving alternating current generators or continuous current dynamos.

In the former case the plant is direct coupled (except in small sizes), since turbines and alternators vary in size and speed together. The periodicity or frequency of alternation required for the electric supply, however, dictates certain speeds and certain speeds only at which the alternator can run. In this country the periodicity varies in different systems from 25 cycles to 100 cycles per second; the general tendency being to make 50 the standard. With any given periodicity the possible alternator revolutions are given by the relation

$$f = \frac{\text{periodicity}}{\text{sec.}} = \frac{\text{No. of revolutions} \times \text{No. of pairs of poles}}{60}$$

$$= \frac{R}{60}.$$

Thus if the periodicity is 50, the possible revolutions for direct coupled sets are as in the table below:

Periodicity, *f*.	Pairs of Poles, *p*.	Revolutions per Min., R.
50	1	3000
50	2	1500
50	3	1000
50	4	750
	and so on.	

It will be seen, therefore, that 3000 r.p.m. is the upper limit of revolutions at which the alternator can run, to give a periodicity of 50.

If the over-all dimensions of a turbine be doubled, and the number of revolutions be halved, then the steam capacity will clearly be quadrupled, whilst the rotational stresses will remain unaltered. If, therefore, a turbine has been designed which will give the maximum possible output (under given conditions) at 3000 r.p.m., another turbine can be built to develop four times the output at 1500 r.p.m.

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[1] See § (7).

That is, the maximum output varies inversely as the square of the speed. Since alternators go up in size under the same law, it is always possible to construct a direct coupled turbine driven alternator.

In the case of dynamos, on the other hand, the output of the dynamo does not go up inversely as the square of the speed, and consequently there comes a point where the dynamo speed is far too low for the turbine driving it. For instance, a direct coupled turbo-dynamo for 1000 kw. at 3000 r.p.m. can be built, but at 1500 r.p.m.—at which speed a turbine with the same rotational stresses can develop 4000 kw.—a single dynamo can only be built for an output about twice the previous value. If the turbine is cut down in size so as to develop only 2000 kw. at 1500 r.p.m., then the value of the "K" coefficient will be far too low to permit of good economy.

This incompatibility of speed of turbines and direct current generators has been solved (about 1912) by Parsons, who gradually introduced an accurate form of double helical gearing which safely and efficiently transmits large powers without undue noise, thus permitting both turbine and generator to be run at respectively suitable speeds.

The development of mechanical speed-reduction gearing has also opened up a new field in land work, enabling the high-speed turbine to be coupled either through gearing alone or by means of a rope drive to the driving shafts of textile mills, paper mills, rolling mills, etc.

A further development in the case of mills and factories requiring large and varying quantities of steam for heating and process work has been the introduction of the "Pass-out" turbine, from which steam is continuously tapped off as required from a suitable point on the turbine cylinder, whilst the load on the driven generator is automatically maintained by the governor.

§ (18) THE STEAM TURBINE FOR MARINE PROPULSION.—The steam turbine for marine propulsion has the same characteristics and the same main types as for stationary work. The wide difference, however, between a steam turbine and a water turbine alluded to above, arising from the different density of the medium, made itself immediately apparent in the reconciliation that was necessary between the conditions needed for efficiency of the turbine on the one hand and the screw propeller on the other. The propeller imposes serious limitations on the revolutions of the turbines, especially in the case of vessels of low speed, whilst with vessels of high speed the phenomenon of cavitation is encountered, inimical to high efficiency, and under certain circumstances the cause of erosion of the propeller blades. Provision has also to be made for reversing.

For these reasons, in marine turbines the full expansion of the steam is usually spread over two or more units in separate cylinders, frequently driving separate propeller shafts. This multiplication of expansion stages in separate turbines was further resorted to in order to meet the demands of widely varying output in the case of warship machinery. For cruising purposes one or more additional turbines has been provided through which, at cruising speeds, the steam is partly expanded before admission into the main turbine. Alternatively, additional stages can be provided in the high-pressure turbine, which are by-passed at full power. When such stages consist of impulse wheels, the number of nozzles admitting steam can be varied by means of control valves to maintain the initial pressure.

Without such cruising turbines or stages, at low powers the steam would be wire-drawn at admission, and a considerable part of the available energy would be wasted.

On account of low propeller revolutions, propulsion by means of steam turbines directly coupled to screw propellers was found only practicable for vessels of high speed, such as channel steamers, fast liners, and warships. For vessels of low speed and small power the high-pressure portion of the turbine would be subject to considerable leakage loss, and inferior in efficiency to the high-pressure unit of a reciprocating engine. The low-pressure portion of the turbine would, however, retain its advantage as regards efficiency and economical utilisation of the vacuum, and a combination, therefore, of a high-pressure reciprocating engine and low-pressure turbine was adopted in a few instances, the low-pressure turbine driving a separate propeller. The necessity, however, for such compromises has entirely disappeared with the adoption of mechanical gearing between propeller and turbine, which has practically left the turbine free to be designed at its own best speed.

R. D.

TURBINES: See "Steam Engine, Theory of," § (11).

Types of. See "Turbine, Development of the Steam," § (2); "Steam Turbine, Physics of," §§ (9)-(13).

TURBULENCE: EFFECT OF, ON VISCOSITY OF FLUIDS. See "Friction," § (4).

TURBULENCE IN INTERNAL COMBUSTION ENGINE CYLINDERS. See "Engines, Thermodynamics of Internal Combustion," § (67).

TWEDDELL'S DIFFERENTIAL ACCUMULATOR FOR HYDRAULIC STORAGE. See "Hydraulics," § (56) (ii.).

Two-stroke Internal Combustion Engines. See "Engines, Some Typical Internal Combustion."
  Clerk Engine. § (9).
  Day's Engine (Valveless). § (12).
  Diesel Engine. § (16).
  Koerting Engine. § (10).
  Oechelhauser Engine. § (11).

— U —

Una-flow or Uni-flow Steam Engine. See "Steam Engine, Reciprocating," § (6).
Undershot Wheels. See "Hydraulics," § (47) (iii.).
Units:
  Dynamical. See "Dynamical Similarity, The Principles of," § (4).
  Electrical and Magnetic, Relation between See *ibid.* § (36).
  Kinematic. See *ibid.* § (3).
  Thermal. Variation of Specific Heat of Water with Temperature. See "Calorimetry, Method of Mixtures," § (12).

— V —

Vacuum Pumps, High. See "Air-pumps," § (38).
Valves:
  For Pumps. See "Hydraulics," § (35) (v.).
  Steam Engine. See "Steam Engine, Reciprocating," §§ (2) (ii.), (5).
Van der Waals' Equation: a second approximation to the behaviour of fluids under changes of pressure, volume, and temperature, leading to the equation

$$\left(p+\frac{a}{v^2}\right)(v-b)=\mathrm{RT}.$$

See "Thermal Expansion," § (19) (ii.): "Thermodynamics," §§ (59), (66).
Vapour. See "Thermodynamics."
  Formation under Constant Pressure. § (29).
  Specific Heat of Saturated. § (53).
  Wetness of. § (34).
Vapour Compression Refrigerating Machines. See "Refrigeration," §§ (2), (3).
Variables, Non-dimensional. See "Dynamical Similarity, The Principles of," § (8).
Vauxhall Automobile Engine. See "Petrol Engine, The Water-cooled," § (6) (ii.).
Velocity Images. See "Kinematics of Machinery," § (3) (iii.).
Velocity in Fluids, Critical. The speed at which stream-line motion in fluids breaks down and eddying motion is set up. See "Friction," § (11) (iii.).
Velocity of Flow of Streams. See "Hydraulics," § (10).
Velocity of Mean Square, Definition of. See "Thermodynamics," § (66).
Venturi Meter, used for measuring the flow of liquids in pipes. See "Hydraulics," § (24) (iii.); "Meters, Liquid," § (4), Vol. III.
Vertical Gas-engine, Typical. See "Engines, Some Typical Internal Combustion," § (6).
Vibrations of a Gravitating Liquid Globe, Application of Dynamical Similarity to. See "Dynamical Similarity, The Principles of," § (12).
Violle, 1882, compared gas-thermometers with secondary standards of temperature in the range 500° to 1600°. See "Temperature, Realisation of Absolute Scale of," § (39) (vii.).
Virtual Velocities, Principle of. If the points of application of a system of forces in equilibrium be slightly displaced in any manner consistent with the geometry of the system, then the sum of the products of each force multiplied by the displacement of its point of application resolved in the direction of the force is zero.
Viscometer. See "Viscometry."
  Barbet. § (8).
  Engler. § (6).
  Flow Type of: Poiseuille's theoretical investigation of. § (2).
  Michell. § (10).
  Redwood (Type No. 1): a type of viscometer used chiefly to determine the viscosity of lubricating oils over the normal ranges of temperature for which these oils are used. § (4).
  Redwood (Type No. 2): a type of viscometer used to determine the viscosity of fuel oils at low temperatures. § (5).
  Saybolt. § (7).
  Torsion. § (9).

VISCOMETRY

§ (1) Introduction.—The determination of the viscosity of liquids is an operation of frequent occurrence in applied science, and several types of viscometer of simple design have been evolved for use in this connection. The more theoretical discussions of viscosity and the applications to the flow of fluids through pipes are dealt with elsewhere,[1] and in this section attention will be devoted to those instruments, principally of commercial application, which are employed for oils, etc.

The instruments having the most extended use are the Redwood, the Engler, and the Saybolt viscometers, each of which is of the flow type in which the viscosity of the liquid is determined by noting the time of outflow of a definite volume of the liquid through a short jet under specified conditions. The Redwood instrument, of which there are two designs, is used principally in Great Britain, the Engler viscometer is used on the Continent and to a certain extent in the United States of America, while the Saybolt instrument is used almost exclusively in the U.S.A. The Barbet viscometer, used to a limited extent in France, is also of the flow type.

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[1] "Friction," § (3) (ii.), "Flow of Fluid through a Circular Pipe."

§ (2) THEORY OF FLOW TYPE VISCOMETERS.—The flow of a viscous liquid through a uniform capillary tube of circular cross-section was investigated by Poiseuille, and in the simplest case may be represented by the formula

$$q=\frac{\pi PR^4}{8\eta L},$$

where $q=$volume of liquid flowing in unit time, $P=$fall in pressure of the liquid over a length L of the capillary, $R=$internal cross-section of the capillary, and $\eta=$the viscosity of the liquid. This formula only holds if the flow of the liquid is of the streamline type; at higher velocities the flow becomes turbulent and other relationships apply. The investigations of Osborne Reynolds in 1883 show that the critical velocity at which the flow ceases to be streamline in nature depends upon the viscosity of the liquid and the radius of the tube.[1] His expression is

$$V=\frac{114\cdot2}{Rf},$$

where V is the critical velocity, R the radius of the tube, and f is the relative fluidity of the liquid. (The fluidity is the reciprocal of the viscosity, and in this case the relative value compared with water at 0° C. is employed.) The constant in the above expression refers to values of V and R in C.G.S. units.

For viscous liquids such as oils the critical velocity is not reached in the ordinary types of flow viscometer, but if these instruments are used for liquids as fluid as water the constants given later will not hold.

Poiseuille's formula was derived on the assumption that the capillary tube was indefinitely long in comparison with its diameter, and further, that the liquid does not emerge with any kinetic energy due to the pressure with which it is forced through the tube. Both these conditions are violated in the case of the short jet viscometers under consideration, and modifications have to be made in the formula to represent the actual conditions of working.

A more complete formula which takes account of these conditions is

$$\eta_t=\frac{\pi R^4 g\delta_t hT}{8V(L+nR)}-\frac{m\delta_t V}{8\pi(L+nR)T}.$$

The value q in Poiseuille's formula has been replaced by V/T, where V is the volume outflowing in time T, while for the pressure P, $g\delta_t h$ has been written to correspond with the condition obtaining in flow viscometers in which the pressure under which the liquid flows through the capillary is due to a hydrostatic head h of the liquid under consideration. (g is the gravitational constant and δ_t is the density of the liquid at the temperature t at which the observation is being made.)

Poiseuille's expression holds for a tube of indefinite length, so that the pressure P may be measured over a length L in which the flow is quite uniform, *i.e.* the streamlines are parallel to the axis of the tube. With the case of short jets this condition is not immediately established, and the end effect has to be allowed for. Calculation shows that the length of the tube has in effect to be increased by an amount proportional to the radius of the tube, and hence the length L in Poiseuille's expression becomes $(L+nR)$; the constant n has a value 1·64 if the ends of the tube are plane and at right angles to the axis. In practice this is not the case, and experimental determinations of this and of the other constants are made as described later.

The second part of the expression is the correction for the kinetic energy of the outflowing liquid, and numerous investigations of the value of the constant m, both theoretical and experimental, have been made. If the second term be kept small in comparison with the first it generally suffices to assume that $m=1$ in accordance with the result of Couette's solution of the problem, but in the case of flow viscometers the second term becomes prominent, especially for liquids of lower viscosity, so that an experimental evaluation of the term is necessary.

The general expression above may be written in a simplified form for use in the case of industrial flow viscometers, since the time of outflow is always determined for a definite volume (*e.g.* 50 c.c. or 100 c.c.) for any type of instrument. The expression becomes

$$\frac{\eta_t}{\delta_t}=AT-\frac{B}{T},$$

where η_t and δ_t are the viscosity and density at the temperature t, T is the time of outflow of the specified volume of liquid, and A and B are constants depending only on the dimensions of the apparatus and numerical factors. In obtaining this simplified expression it is, of course, assumed that the initial head is adjusted to a specified value, and in consequence the mean value of h during outflow is constant for the type of instrument under consideration.

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[1] "Friction," § (14) (i.).

For liquids of high viscosity the value of T is fairly large for the usual types of industrial viscometer, so that the term B/T becomes small in comparison with the term AT. Thus for these cases the ratio of the viscosity to the density becomes proportional to the time of outflow, so that it has been the practice generally to assume that the viscosity number for a liquid is merely its time of outflow compared with the time of outflow for a standard liquid. Reference to this will be made when considering the individual types of flow viscometer.

The more satisfactory procedure is, however, to deal with the absolute values of the viscosity as given by the formula

$$\frac{\eta}{\delta}=\mathrm{AT}-\frac{\mathrm{B}}{\mathrm{T}},$$

and it consequently becomes necessary to evaluate the constants A and B. This is done experimentally by determination of the time of outflow for a series of liquids whose viscosities cover a large range, the viscosities also being determined by means of a viscometer of such a type that the absolute values may be calculated.

§ (3) TEMPERATURE COEFFICIENT OF VISCOSITY.—The viscosity of a liquid is dependent to a very marked extent upon its temperature, particularly for the first 40°-50° C. above the setting point of the oil. A typical case is shown in *Fig.* 1, which shows the variation of the viscosity of a Texas fuel oil over a range of about 60° C. The viscosity will be seen to have dropped from 0·007 units to about 0·0003 over this range of temperature. The rate of change in the steeper part of the curve corresponds to about 10 per cent per degree centigrade, so that in order to secure an accuracy of 1 per cent at this point of the curve it is necessary to know the temperature of the oil to the nearest 0·1° C., and further, the temperature of the oil must be maintained constant to this degree of accuracy during the course of an observation. It will thus be seen that attention must be paid to temperature control in the design of viscometers. At temperatures more distant from the setting point of the oil the effect is less marked, and eventually the viscosity tends to become much less dependent on the temperature.

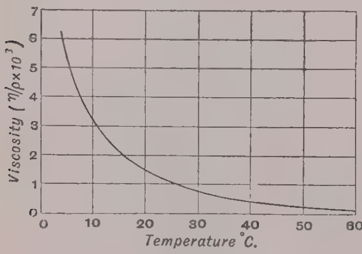

FIG. 1.

The relationship between viscosity and temperature has been investigated on many occasions, but no satisfactory simple formula has been derived to express this relationship to any reasonable degree of accuracy. Slotte put forward the formula

$$\eta_t=\frac{\eta_0}{1+k_1t+k_2t^2},$$

where k_1 and k_2 are constants depending on the nature of the liquid, and η_t and η_0 are the viscosities at the temperatures $t°$ and 0° respectively. A modification of this formula which has been found to suit pure liquids is

$$\eta_t=\frac{\mathrm{A}}{(1+\mathrm{B}t)^c},$$

where A, B, and c are constants. The formula is, however, cumbersome to work with, and even such a formula does not represent satisfactorily the experimental results obtained for oils, mainly owing to the fact that the oils met with in commerce are not definite chemical compounds, but consist of mixtures of a large number of components not readily separated by ordinary chemical means. To represent adequately the relationship between the viscosity and temperature would involve the use of many more constants.

Further reference to the question of temperature will be made when discussing the individual types of viscometer.

§ (4) REDWOOD VISCOMETER[1] (Type No. 1).—The Redwood viscometer, which was designed and introduced by the late Sir Boverton Redwood in 1885, consists of a cylindrical oil cup provided with a thermometer and a filling mark, surrounded by a water-bath by means of which the oil vessel may be maintained at any desired temperature. In the bottom of the oil cup an agate jet is fixed. The agate jet is concave on the upper side to allow a metal ball carried on a stiff wire to serve as a valve for starting or stopping the flow of oil from the oil vessel. The oil cup is silver-plated internally to avoid chemical reaction between the metal parts of the instrument and the acids present in many of the oils which may be tested in it. The essential dimensions in accordance with the official specification are shown in *Figs.* 2 and 3.

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[1] For a full description of the instrument reference should be made to (*a*) *Petroleum and its Products*, Sir Boverton Redwood, 3rd edition; or (*b*) "On Viscometry," Sir Boverton Redwood, *Chem. Ind. Soc. J.*, 1886, v. 121-133.

In the construction of the instrument difficulty is experienced in drilling the hole through the agate jet to exact dimensions, so that it is the practice for the filling point of the instrument to be adjusted, after the jet has been fixed, such that the time of outflow of 50 c.c. of rape oil at 60° F. shall be equal to 535 seconds. If in order to do this it is necessary to place the filling mark at a distance of more than 6·3 mm. from the normal position shown on the diagram, the jet is rejected and a more accurately drilled jet is chosen. The use of steel or other metal for the jet would obviate this difficulty, but such jets would be more liable to damage (mechanically or by corrosion), and hence the use of agate is considered advantageous. In working with this instrument it is of particular importance that the jet should only be cleaned with soft material to avoid chipping at the sharp edges or scratching of the internal surface.

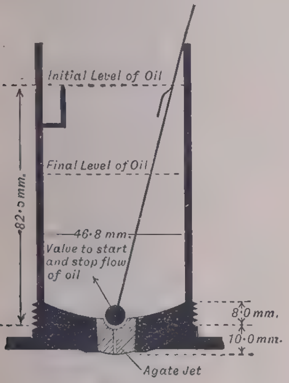

FIG. 2.

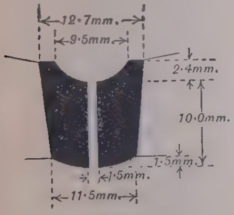

FIG. 3.

An investigation of the Redwood viscometer (No. 1 Type) was made at the National Physical Laboratory[1] in 1912 and 1913. Preliminary work showed that in the use of the instrument it was important that the viscometer should be carefully levelled before adjusting the oil surface to the fixed mark in the oil cup; and further that the oil surface should be carefully adjusted to the level of the mark before making observations. An error of 1 mm. in setting resulted in a change in the time of outflow, and consequently of the viscosity, of about 1·3 per cent.

A number of oils were examined in instruments of normal dimensions, and the times of flow were compared with the viscosities of these oils determined in absolute units by a method similar to that employed by Thorpe and Roger.[2] These tests were made over a wide range of temperature.

To evaluate the constants of the formula

$$\frac{\eta}{\delta} = \mathrm{AT} - \frac{\mathrm{B}}{\mathrm{T}},$$

curves were plotted between T (time of outflow on the Redwood instrument) and η/δ in absolute units. Such curves are found to be sensibly straight for values of T greater than about 200 secs., indicating that the value of B/T is small compared with AT for such values of T; the slope of this straight line consequently gives the value of the constant A.

To obtain the value of B by a simple graphical method, it may be noted that the general formula may be written

$$\frac{\eta}{\mathrm{T}\delta} = \mathrm{A} - \frac{\mathrm{B}}{\mathrm{T}^2}.$$

If now the values of $\eta/\mathrm{T}\delta$ and $1/\mathrm{T}^2$ are plotted a straight line should result. This was found to be the case, and the slope of this straight line gives the value of the constant B. This leads to the following values, $\mathrm{A}=0{\cdot}00260$ and $\mathrm{B}=1{\cdot}715$ in C.G.S. units. The corresponding values for foot-pound-second units are

$$\mathrm{A}=2{\cdot}80\times10^{-6} \text{ and } \mathrm{B}=1{\cdot}85\times10^{-3}.$$

The importance of maintaining the oil at a constant and uniform temperature during observations has already been pointed out. The No. 1 type Redwood viscometer offers difficulty in use on this account if the temperature at which the viscosity is desired is not in the neighbourhood of the air temperature, largely owing to the upper surface of the oil being unprotected and open to the air, while the under side of the jet is also exposed.

For accurate determinations it is necessary for the complete instrument to be enclosed in a constant-temperature chamber. This offers difficulties at the higher temperatures, and reasonably accurate results may be obtained without a constant-temperature enclosure by careful attention to procedure, in view of the fact that the time of outflow for many of the oils of industrial interest is short above 50°-60° C. In such cases the temperature of the water in the bath surrounding the oil cup is taken to a value a few degrees higher than that at which the observation is to be made, and the oil is allowed to pick up the temperature of the water-bath, being stirred thoroughly during this period. The temperature of the water-bath is then allowed to drop very slowly to the required value, the stirring of the oil being continued until its temperature reaches that of the water. Stirring is then stopped and the oil is allowed to flow out into the graduated flask beneath the instrument and the time of outflow is noted. During the time of outflow the temperature of the oil as indicated by the oil-cup thermometer should be fairly steady, a drop of temperature of a fraction of a degree being generally obtained. The mean value of the temperature during the time of outflow is recorded. It is also important that the flask into which the oil flows should be warmed to the temperature at which the observation is being conducted, and that the flask should be protected by some suitable material such as cotton wool during the course of the experiment, in order that the temperature of the oil in the flask may be approximately the same as that in the oil cup, otherwise the volume of oil outflowing will vary according to the temperature instead of being the standard quantity, *i.e.* 50 c.c.

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[1] "Methods and Apparatus used in Petroleum Testing, Part II., Viscometry," W. F. Higgins, *Collected Researches, N.P.L.* xi. 1, or *Chem. Ind. Soc. J.* (Abstract), 1913, xxxii.

[2] *Roy. Soc. Phil. Trans.*, 1894, clxxxv. 430.

§ (5) REDWOOD VISCOMETER (NO. 2 TYPE).—For the determination of the viscosity of lubricating oils over the normal ranges of temperature for which these oils are used, the No. 1 type of Redwood viscometer described above is convenient, but for fuel oils for which a knowledge of the viscosity is required at comparatively low temperatures the determinations with the No. 1 type of instrument occupy an unreasonably long time. In connection with the pumping of fuel oils through pipe lines or from storage tanks in the open the value of these viscosities at 0° C. is frequently desirable. Accordingly Sir Boverton Redwood designed a second type of viscometer known as the Admiralty Fuel Oil Viscometer or the No. 2 type, to enable this determination to be carried out more readily (see *Fig.* 4). The size of the oil cup remains unchanged, but the dimensions of the jet are increased so that the time of outflow is approximately one-tenth of that for the No. 1 type viscometer for the same oils. In addition the bath surrounding the oil cup has been considerably increased in size, and is lagged so that it may be filled with ice for making measurements at 0° C. The oil cup itself is raised in the bath and the jet is placed within a tube so that the under side of the oil cup is also exposed to the temperature of the bath. This is an improvement on the design of the No. 1 type. The upper surface of the oil is, however, exposed to the air, which makes it difficult to ensure that the whole of the oil is at a uniform temperature. In the use of the instrument attention may also be called to the possibility of the condensation of water on the surface of the oil and on the under side of the jet when using the instrument at low temperatures, especially on a damp day. This condensed water may run into the measuring flask if the necessary precautions are not observed. The quantity of oil allowed to flow into the measuring flask is the same, viz. 50 c.c., as in the case of the standard instrument.

The normal method of indicating results for a viscosity determination is to quote the time of outflow in seconds of 50 c.c. of oil. For purposes of subsequent calculation, however, it is more useful to know the absolute value of the viscosity. An investigation was carried out at the National Physical Laboratory on the lines of that referred to in connection with the No. 1 type instrument to determine the new values of the constants A and B in the general expression

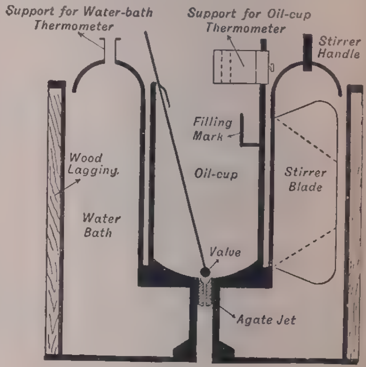

FIG. 4.

$$\frac{\eta}{\delta} = AT - \frac{B}{T}.$$

For the No. 2 type the values are as follows:

$$A = 0{\cdot}0270,$$
$$B = 11{\cdot}2,$$

when the results are expressed in C.G.S. units. The corresponding values for foot-pound second units are

$$A = 2{\cdot}91 \times 10^{-5},$$
$$B = 1{\cdot}20 \times 10^{-2}.$$

§ (6) THE ENGLER VISCOMETER.[1] — The Engler viscometer was introduced about the same time as the No. 1 type of Redwood instrument and, as in the case of the latter, consists of an oil cup provided with a jet at the lower end, but the whole is enclosed in a water-bath the general design of which more closely resembles that of the No. 2 type of Redwood viscometer. The dimensions of the Engler instrument are shown in *Fig.* 5; they differ from those of the Redwood instrument in that the oil cup is wider and shallower, while the jet is longer and of larger diameter. The levelling of the Engler viscometer is carried out by bringing the oil surface into coincidence simultaneously with each of three filling points. The volume of oil dealt with is larger, a flask of 200 c.c. capacity being employed instead of the 50 c.c. for the Redwood instrument. In view of the shallow nature of the oil cup greater care has to be paid to the levelling of this apparatus. An error of 1 mm. in setting the initial level gives an error of 2·5 per cent in the results.

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[1] "Prüfungsbestimmungen für Zähigkeitsmesser nach Engler," *Zeits. Chem. Ind.*, 1907, xxx. No. 9; "Ein Apparat zur Bestimmung der sogenannte Viscosität der Schmieröle," C. Engler, *Zeits. Chem.*, 1885, ix. 189-190; *Schmiermittel und ihre Untersuchung*, A. Kunkler, 119-124.

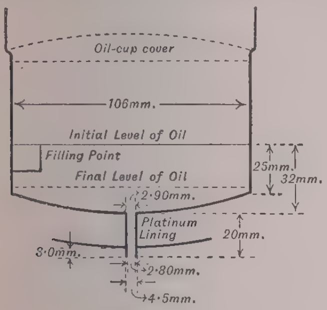

FIG. 5.

In the official specification for the Engler apparatus it is stated that the time of outflow for 200 c.c. of distilled water at 20° C. must lie between 50 and 52 seconds. This variation has to be allowed, as it is impracticable to secure an accuracy of more than 1 per cent in the internal diameter of the jet; this variation takes the place of the variation in height of the filling point which is permitted in the case of the Redwood instrument to compensate for the slightly varying diameters of the jets. It may be pointed out here that the jet of the Engler apparatus tapers slightly, probably for convenience in manufacture; the taper being approximately 1 in 30. This convergence involves a slight modification in the ordinary Poiseuille formula for the flow of a liquid through a tube; the effect, however, due to this is of the order of 0·2 per cent, so need not be taken into account. The flow of a viscous fluid through a circular tube with uniformly converging boundaries has been theoretically investigated by Gibson.[1]

Viscosities determined on the Engler viscometer are expressed in terms of the time of outflow of water, and Ubbelohde has constructed tables to convert times of outflow to absolute viscosities.[2] His formula is

$$Z=\frac{4{\cdot}072T}{T_W}-\frac{3{\cdot}513T_W}{T},$$

where T is the time of outflow from the Engler viscometer of 200 c.c. of the liquid under consideration and T_W is the corresponding time of outflow for water at 20° C. for the instrument. The product $Z\delta$ is known as the "specific viscosity," and is connected with the absolute viscosity by the expression

$$\eta=Z\delta\times0{\cdot}01797.$$

Z is in effect the relative viscosity of the liquid compared with that of water, allowance being made for the density (δ) of the liquid.

For a viscometer of normal dimensions T_W should be 51 secs., so that the above expressions may be combined giving

$$\frac{\eta}{\delta}=0{\cdot}001435T-\frac{3{\cdot}22}{T}.$$

It will be noted that this expression is very similar to that derived for the Redwood viscometer, the values of the constants necessarily being different on account of the different dimensions. The ratio of B to A is greater for the Engler apparatus than for the Redwood, indicating that the kinetic energy effect is of greater importance in the Engler viscometer; this is, of course, due to the larger diameter of the jet.

From the two formulae previously quoted, for the Redwood and Engler viscometers respectively, a comparison of the times of outflow can at once be made. The ratio of these times becomes sensibly constant for values of 150 secs. and upwards for the Redwood viscometer; the constant value of the ratio for instruments of standard dimensions is 1·81, the time of outflow from the Engler viscometer being greater than the corresponding time from the Redwood instrument.

One point to which attention may be drawn in connection with the Engler instrument is the provision of a double cover to the oil cup, which is a marked improvement on the Redwood instrument. Of course, in the case of the Engler viscometer provision of this nature is quite essential in view of the large surface of oil otherwise exposed.

For oils of high viscosity there is no modified form of the Engler viscometer corresponding to the No. 2 type of Redwood instrument, but L. Edeleanu and Mlle. Dulugea have suggested that the time of outflow for thick oils should be determined for quantities of 25, 50, or 100 c.c. instead of for the normal quantity of 200 c.c.; in this way observations can be obtained in comparatively short times. The time of outflow is not directly proportional to the quantity, as the head under which the oil flows through the jet is greater at the commencement of an observation than at the end; consequently the above investigators have determined a series of factors by which the times for the small quantities must be multiplied to give corresponding times for the standard quantity of outflow, namely 200 c.c.

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[1] *Phil. Mag.*, 1909, xviii. 35.

[2] L. Ubbelohde, *Tabellen zum Englerschen Viskosimeter*, 1907.

§ (7) THE SAYBOLT VISCOMETER.—The Saybolt viscometer is used largely in America and was designed by the Standard Oil Company of America for commercial use in that country. Several forms of this instrument have been made, but there appears to be no official specification published dealing with the instrument. This viscometer is similar in construction to the instruments already described in that a short jet is attached to the lower end of an oil cup surrounded by a water-bath. The initial setting of the oil level is accomplished by completely filling the oil cup so that the oil just overflows into a channel surrounding its upper edge. The relationship between the time of outflow and the absolute viscosity of the oil has been investigated recently at the Bureau of Standards by W. H. Herschel.[1]

For a Saybolt Universal viscometer of normal dimensions the constants A and B in C.G.S. units in the formula

$$\frac{\eta}{\delta}=AT-\frac{B}{T}$$

are

$$A=0{\cdot}00226,$$

$$B=1{\cdot}80.$$

§ (8) THE BARBET VISCOMETER.—Barbet's viscometer is of very restricted use, and a brief mention only will be made here. The instrument is of the flow type, but is distinguished from those already dealt with in that the jet is annular instead of cylindrical; the jet consists of an iron rod 4 mm. in diameter supported centrally in a cylindrical hole 5 mm. in diameter in a brass block. This method offers obvious disadvantages in that the accurate centring of the rod within the hole is difficult of attainment.

§ (9) TORSION VISCOMETERS.—The instruments dealt with up to the present have all been of the flow type. A number of other instruments, however, have been devised from time to time, but these are of very limited application and have not met with general use in industrial work. The torsion type of viscometer depends upon the drag exerted by a viscous medium upon the motion of a cylinder rotating within a cylindrical vessel. Thus in the Doolittle viscometer the cylinder is supported by a torsion wire from a torsion head which may be rotated through a definite angle; attached to the cylinder at the lower end of the wire is a pointer moving over a circular scale. The upper end of the wire is rotated through 360 degrees, but owing to the viscosity of the liquid in which the cylinder is immersed the lower end of the wire will not move to the same extent. The cylinder rotates for part of a revolution and then swings back in the opposite direction to a still smaller extent. The retardation between the first and second swings may be taken as an approximate measure of the viscosity. Reference should be made to the article on "Friction"[2] for a complete account of the method of determination of the absolute viscosity by the logarithmic decrement of an oscillating disc.

In the viscometer introduced by Searle[3] for use with very viscous liquids, the torque required to rotate one cylinder within another cylinder where the intervening space is filled with the liquid under test is measured for definite velocities of rotation. The method is especially applicable for use with very viscous substances such as tar, thick syrup, and the heavier crude oils. The ordinary design of this apparatus does not provide for control of temperature, which is of the greatest importance in these determinations.

§ (10) MICHELL VISCOMETER.—A very simple type of viscometer for commercial work has recently been put upon the market, which consists of a steel ball fitting into a steel cup. Complete contact is prevented by three small projections on the inner surface of the cup, which maintain the ball at a definite distance from this surface. In operation a few drops of oil are placed in the cup and the ball is put into position, air being carefully excluded from between the two. The cup is then inverted and the ball is pressed upon a hard surface to ensure close contact with the projections. The cup with ball is then lifted vertically and a stop-watch simultaneously started; after a period has elapsed the ball will detach itself from the cup, and the moment it becomes free the watch is stopped. The period required for the release of the ball is a measure of the viscosity of the oil filling the space, and the action of the instrument depends on the slow creeping of the oil through the semi-spherical shell between cup and ball. The temperature of the oil is indicated by a thermometer inserted in a hole in the handle attached to the cup and the relatively large amount of metal employed in the construction of the instrument compared with the small quantity of oil under examination ensures that the temperature of the oil remains sensibly constant at the value indicated by the thermometer during the test. For more accurate determinations the operation is carried out with the cup and ball completely immersed in oil throughout the observation. In this way surface tension effects at the edge of the oil film in the ordinary method of procedure are eliminated, and the time taken for the ball to detach itself from the cup is solely dependent upon the viscosity and density of the medium.

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[1] *Bureau of Standards*, Techn. paper No. 100, 1917, and No. 112, 1918.

[2] "Friction," §§ (5), (9).

[3] *Cambridge Phil. Soc. Proc.*, 1912, xvi. 600.

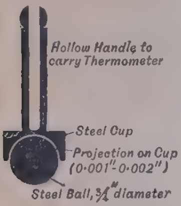
Hollow Handle to carry Thermometer
Steel Cup
Projection on Cup (0·001"-0·002")
Steel Ball, 3/4" diameter

FIG. 6.

Fig. 6 gives dimensions of the standard instrument.

W. F. H.

VISCOSITY :

Defined by Maxwell. See "Friction," § (1).

Of Fluids, Mechanical: the shearing stress which may exist in fluids which are in turbulent motion. See "Friction," § (11).

Of Gases, Experimental Determination. See *ibid.* § (5).

Kinematical, Effect on Heat Convection. See "Heat, Convection of," §§ (2) (iii.) and (4) (v.).

Of Liquids, Effect of Pressure on. See "Friction," § (8).

Of Liquids, Poiseuille's Experiments. See *ibid.* § (5).

Mechanical Method of Determination of Coefficient. See *ibid.* § (12).

Of Solids. See *ibid.* § (10).

Temperature Coefficient of. See "Viscometry," § (3).

Theory. See "Friction," § (2).

Of Thick Oils, Experiments at the National Physical Laboratory. See *ibid.* § (7).

VISCOSITY TACHEOMETER : Air type. See "Meters," § (10) (ii.), Vol. III.

Mercury type. See *ibid.* § (10) (i.), Vol. III.

VISCOUS FLUID :

Dynamical Similarity in the Motion of : the principle that, if a series of bodies, all of the same geometrical shape, be moving in a system of fluids, viscosities $v_1, v_2 \ldots$, with velocities $V_1, V_2 \ldots$, then, provided Vl/ν is maintained constant, photographs of the flow pattern taken on cinematograph films of the same size will all be identical as far as the consecutive geometrical configuration of the stream lines and eddying systems are concerned ; the rates at which the processes unfold themselves will, however, be different. See "Dynamical Similarity, The Principles of," § (17).

Motion of a : Relation between Experiments on Model and on Full Scale, considered by the principle of Dynamical Similarity. See *ibid.* § (18).

Motion of a Body in a, considered by the principle of Homogeneity of Dimensions. See *ibid.* § (14).

VOLUME-COEFFICIENT OF EXPANSION OF A GAS, EXPERIMENTAL DETERMINATION OF, at constant pressure. See "Thermal Expansion," § (16).

VOLUME-COEFFICIENTS OF VARIOUS THERMOMETRIC GASES, tabulated. See "Temperature, Realisation of Absolute Scale of," § (18), Table 3.

VOLUMES, RELATIVE, OF VARIOUS GASES AT HIGH PRESSURES ; Amagat's values, tabulated. See "Thermal Expansion," § (18) (iii.).

VOLUMETRIC EFFICIENCY. See "Petrol Engine, The Water-cooled," § (3).

VOLUMETRIC HEATS : Tables of, for Nitrogen, Carbon Dioxide, and Water Vapour. See "Specific Heat of Gases at High Temperatures," §§ (3), (6).

— W —

WAKE AND HULL EFFICIENCIES. See "Ship Resistance and Propulsion," § (45).

WATER :

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Expansion of, at various temperatures. See "Thermal Expansion," § (11).

Influence of Pressure on Thermal Expansion of, tabulated results of P. W. Bridgman. See *ibid.* § (12).

Latent Heat of Evaporation of, experimental values for, compared with those given by theoretical formulae and tabulated. See "Latent Heat," § (6), Table IV.

Latent Heat of Evaporation of, Griffith's values for temperatures 30 and 40° C., tabulated. See *ibid.* § (2), Table II.

Latent Heat of Evaporation of, A. W. Smith's values for, tabulated. See *ibid.* § (4), Table III.

Latent Heat of Evaporation of, Regnault's values for, tabulated. See *ibid.* § (1) (i.), Table I.

Specific Heat of, at various temperatures. See "Heat, Mechanical Equivalent of," § (7).

Specific Heat of, Regnault's value and application of. See *ibid.* § (7).

WATER-POWER :

Industrial Applications of. See "Hydraulics," III., §§ (46), (54).

Supply available for, and list of largest installations. See *ibid.* § (29).

Water Waves, Speed of. In shallow water, in which the depth h is small compared with λ, the wave-length,

$$V = \sqrt{gh}.$$

In deep water, where h is large compared with λ,

$$V = \sqrt{\left(\frac{g\lambda}{2\pi}\right)}.$$

Water-wheels. See "Hydraulics," § (47).

Watt's Engine. See "Steam Engine, Reciprocating," § (11).

Wave-length, "Effective," determination of, for various temperature ranges. See "Pyrometry, Optical," § (12).

Wave-motion Power Transmission. See "Hydraulics," § (66).

Waves :
- Theory of. See "Ship Resistance and Propulsion," §§ (22) and (23).
- Velocity of, Application of Dynamical Similarity to. See "Dynamical Similarity, The Principles of," § (13).

Whirling Arm Dynamometer for testing Air-screws. See "Dynamometers," § (8).

White, determination of melting-points of silicates, using a thermocouple. See "Thermocouples," § (22) (iii.).

Wien's Displacement Law : the law governing the distribution of energy of radiation in various parts of the spectrum. It states that

$$E_\lambda = \lambda^{-5} f(\lambda\tau),$$

where τ is the absolute temperature,
λ is the wave-length of radiation considered,
E_λ is the density of isotropic energy per unit wave-length,
and f an unknown function, determined by other conditions than those of thermodynamics. See "Radiation Theory," § (5) (ii.), Vol. IV.

Wien's Formula : a formula giving an approximation to the curve of distribution of radiant energy along the spectrum, on the side of the short-wave-lengths. It has the form

$$E_\lambda = \frac{8\pi hc}{\lambda^5} e^{-hc/k\tau\lambda},$$

and was the earliest radiation formula to be suggested. See "Radiation Theory," § (6), Vol. IV.

Wien's Law and Stefan-Boltzmann Law, Comparison of, to 2800° C. See "Pyrometry, Optical," § (2) (iii.).

Willard Gibbs' Thermodynamic Potentials. See "Thermodynamics," § (51).

Witkowski : investigations on the expansion of air and hydrogen at high pressures, comprising temperatures below 0° C. See "Thermal Expansion," § (18) (iv.).
- Values of pv for air at high pressures and various temperatures, tabulated. See *ibid.* § (18) (iv.).
- Values of pv for hydrogen at high pressures and various temperatures, tabulated. See *ibid.* § (18) (iv.).

Wood, Strength Tests on. See "Elastic Constants, Determination of," §§ (122)-(134).

Wood, Thermal Conductivity of. See "Heat, Conduction of," § (4) and Table II.

Worm Gear. See "Mechanical Powers," § (4).

— Y —

Yield Point definition of the yield point and distinction between it and the elastic limit. See "Elastic Constants, Determination of," §§ (21), (62).

Young's Modulus ; a term used, in the theory of elasticity, to denote the constant E in the strain system

$$e_{xx} = \frac{T_1}{E},\ e_{yy} = e_{zz} = -\sigma e_{xx} = -\sigma\frac{T_1}{E},$$

E and σ being constants of the material, e_{xx}, e_{yy}, e_{zz} the stretches in the directions of the axes Ox, Oy, Oz respectively, the material being under a simple tensile stress of amount T_1. See "Elasticity, Theory of," § (4).

— Z —

Zero, Absolute. See "Thermodynamics," §§ (4), (21).

Zero of a Thermometer, Determination of. See "Thermometry," § (3) (iv.).

Zinc :
- Atomic Heat of, at low temperatures, Nernst's values for, tabulated. See "Calorimetry, Electrical Methods of," § (11), Table VI.
- Specific Heat of, at various temperatures, tabulated, with the Atomic Heat. See *ibid.* § (10), Table V.

Zirconia, Fused, a very refractory substance suitable for use as the outer protecting sheath of a thermoelement. See "Thermocouples," § (5) (vi.).

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